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A Simple Macro-economic Growth Model.   Part I.

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At constant prices in year t (with t = 0 in the base year) let

		1960 £m
Y_t = net national product at market prices		626
C_t = national consumption at market prices		567
V_t' = net fixed capital formation		51
V_t'' = change in stock		9
X_t = current exports		255
(1) M_t' = current imports (positive or negative) in respect of profit, interest etc in consequence of investment from abroad N_t		0
M_t'' = other current imports		256
N_t = net investment from abroad		1
S_t = national saving		59

National consumption includes general government as well as household consumption. The values for Ireland in 1960 are indicated.

Identities and equations

We then have four national accounting identities as follows:-

(i) Product account   : $Y_t = C_t + V_t' + V_t'' + X_t - M_t' - M_t''$

(ii) External account   : $X_t - M_t' - M_t'' + N_t = 0$

(2)

(iii) Capital-Saving account : $V_t' + V_t'' = S_t + N_t$

(iv) Consumption account   : $C_t + S_t = Y_t$

These four accounts are articulated (or double-entry), for it will be noted that each of the nine entities specified at (1) occurs twice, once on the left side and once on the right side of the identities (2).

In consequence, only three of the four identities are independent: any one of the four can be derived from the remaining three. In sum, there are three relations between the nine entities; six other relations are required to obtain a model from which each of the entities may be determined in any year t, given the values in base year $t = 0$. It is assumed that during the growth period import and export price indexes (base year o) are the same so that no entry for the trading gain is required in relations (2). It will be noted that N_t is positive when current imports exceed current exports and negative in the contrary case.

These six behaviouristic equations (deterministic, as distinct from stochastic, in character) are found as follows. First the growth equation

$$(3) \qquad Y_t = (1 + r)^t Y_o,$$

where r is the annual growth rate of the economy. A consumption equation is

$$(4) \qquad C_t = (1 - s)Y_t,$$

so that, from (2)(iv), saving is given by

$$(5) \qquad S_t = sY_t.$$

The fixed capital relation is derived from the incremental capital-output ratio k whereby

$$(6) \qquad V'_t = k(Y_{t+1} - Y_t).$$

But, by definition of r, the growth ratio,

$$(7) \qquad Y_{t+1} - Y_t = rY_t.$$

Hence, from (6) and (7) we have the fixed capital equation

$$(8) \qquad V'_t = krY_t.$$

As regards changes in stocks, let stock P_t at beginning
of year t be

(9) $\qquad P_t = pY_t.$

Then since $V_t'' = p(Y_{t+1} - Y_t)$ we have, from (7), the
stock equation

(10) $\qquad V_t'' = prY_t.$

It will be convenient to take investment from abroad
N_y next. The required external investment equation,
derived from (2)(iii), using (5), (8) and (10), is

(11) $\qquad N_t = (kr + pr - s)Y_t.$

If n is the rate of interest, total interest M_t'
payable or receivable in year t in respect of foreign
investment during the period of review is

(12) $\qquad M_t' = n \sum_{t'=0}^{t-1} N_{t'}.$

Hence, from (3) and (11),

$$M_t' = n(kr + pr - s)Y_0 \sum_{t'=0}^{t-1} (1 + r)^{t'}$$

$$= n(kr + pr - s)Y_0 \{(1 + r)^t - 1\}/r$$

(13) $\qquad = n(kr + pr - s)(Y_t - Y_0)/r,$

the external investment interest equation.

The import equation is

(14) $\qquad M_t'' = mY_t.$

Finally the export equation is derived from (2)(i)
by substitution from (4), (8), (10) and (14). It
is unnecessary to write the complicated formula down
since in practice (i.e. with actual figures) the
value can be readily found.

The model accordingly consists of a system

of nine equations (of which three are the accounting
identities (2) to determine the nine entities
specified at (1).   The six behaviouristic equations
involve six parameters r, s, k, p, n and m -- actually
they would all appear in the export equation.   Each
of the six equations expresses the relevant entity
linearly in terms of Y_t and Y_o or, if Y_t be regarded
as given by (3) in terms of Y_o alone.   Hence, given
Y_o, the initial value of Y_t, and the parameters the
values of all the entities specified at (1) are
determinable in such a way that they must satisfy
the accounting identities (2).

Application in Ireland.

Though this is designed mainly as a
theoretical exercise some reality will be lent to
it by some consideration of the values of some of the
parameters in Ireland.

The capital-output ratio, k.   In the appended Table
1 some relevant macro-data are displayed covering
the period 1947-1961.   From Chart 1 showing gross
national product and net national product at
constant prices -- the difference is depreciation --
it will be apparent that 1947-1955 was a period of
small but regular advance in the economy.
Accordingly it seems appropriate to estimate the
average net capital-output ratio for this period.
This estimate is made by using the model

$$(15) \quad (Y_{t+1} - Y_t) - k'V_t' = u_t, \quad t = 1946, \ldots, 1954,$$
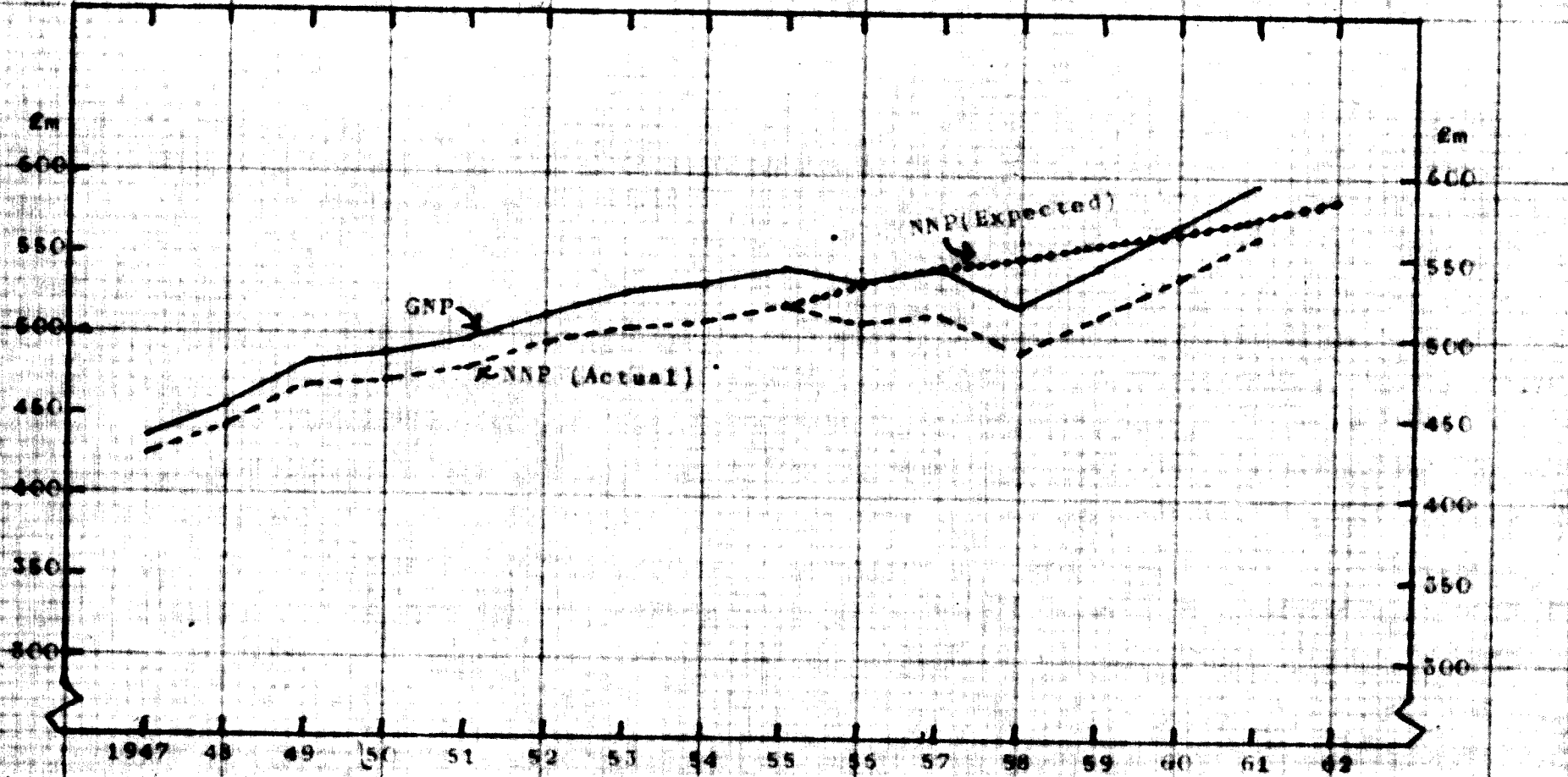
where u_t is a random variable and

$$(16) \qquad k' = 1/k,$$

the inverse of the capital-output ratio.   By least
squares procedure applied to (8) the estimate of
k', namely $\hat{k}'$, is found from

$$(17) \qquad \hat{k}' = \Sigma (Y_{t+1} - Y_t)V_t'/V_t'.$$

Chart 1   Gross National Product and Net National Product at Constant (1953)
Prices, Ireland 1947-1961.

£m

600

550

500

450

400

350

300

GNP

NNP (Actual)

NNP(Expected)

£m

600

550

500

450

400

350

300

1947   48   49   50   51   52   53   54   55   56   57   58   59   60   61   62

Applying this formula to the data in columns (6) and
(7) in Table 1 we find $\hat{k}' = 0.2088$ which, for
convenience, may be taken as 0.2, yielding a
capital-output ratio for the period 1947-1955 of
5. By international standards this value is large.
There are three possible contributory causes for
this:-

(a) under-utilisation of capacity;

(b) the sizable proportion borne by dwellings --
see column (4) of Table 1 -- with a high
capital-output ratio in total gross fixed
capital formation;

(c) the low output increment for agriculture
during the period.

Extrapolating the constant price value of
output using the formula

$$(18) \qquad \Delta y_t = y_{t+1} - y_t = 0.2V_t$$

and starting with the actual value of net national
product for 1955, namely £518 million, the
following "expected" values are found (in £ million)
with the actual Y_t for comparison:-

Year	Δy_t	y_t	Y_t
1955	13	518	518
1956	11	531	508
1957	8	542	513
1958	8	550	490
1959	8	558	513
1960	9	566	537
1961	11	575	562
1962		586	

The y_t are graphed as the dotted line (from 1955)
on Chart 1. Comparison of the actual and
"expected" development (i.e. of Y_t with y_t) of
NNP from 1955 clearly indicates the period 1956-
1958 as one of a marked recession and the period
1959-1961 as merely one of recovery. During the

whole period 1956-1961 fixed capital was under-
utilised by reference to the probably not too exalted
standard of 1947-1955.   If the favourable trend of
1959-1961 continues into 1962 the economy will have
just reached the point it would have reached if the
1947-1955 trend had continued.   In fact the
"expected" graph may understate what the actuality
might have been, since net fixed capital formation
(on which the estimated annual increments depend
pursuant to formula (18)) would presumably have been
larger had there been no recession.*   Of course,
there is the unprecedently healthy feature about the
recovery phase 1959-1961 that it was based largely
on exports and the impetus induced in this sector
once achieved may not slacken.   Furthermore, as
will presently appear, the capital-output ratio of
5 is far too big for an economy aspiring to a
large and sustained rate of growth, given the Irish
propensity to save which is considered later.   The
chart makes manifest the fact that no safe
inference can be made as to the magnitude of the
incremental capital-output ratio on the experience
of 1959-1961.   The writer refrains from quoting the
rate since he regards it as being too good to be
true.   The critical year will really be 1963:   will
the actual NNP at constant prices be the extra-
polation of the actual (broken) graph on the chart,
which will establish a new and vigorous growth
impulsion;   or will the figure be merely an
extrapolation of the (dotted) expected graph which
will indicate merely a reversion to the 1947-1955
trend with the unhealthy large net capital-output
ratio of 5?

Admittedly a capital-output ratio
extending to the whole economy is not particularly
significant for comparative purposes in time or
between countries, in particular because sectors
of the economy differ so much in capital (as
distinct from labour) intensity.   This is

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* In The Economic Research Institute Paper No. 6
"Prospects of the Irish Economy in 1962" (in press),
A. Kuehn anticipates only a small increase (if any)
in exports in 1962.

particularly the case in Ireland where agriculture
is an important sector (accounting for about a
quarter of gross domestic product) and in agriculture
one surmises that the role of fixed capital in
promoting development is less important than in
industry in the sense that current expenditure in
the form of fertilisers, insecticides, medicines etc
is likely to be more conducive to growth than is
capital expenditure.   Apart from this point, in
comparing capital-output ratios in Ireland and the
United Kingdom it seems desirable to eliminate
agriculture from the Irish computation; this is
scarcely necessary in the case of UK where
agriculture accounts for only one-twentyfifth of
gross domestic product.

 Omitting agriculture from Irish capital
formation and output, the gross capital-output ratio
for the period 1953-1959, calculated by formula (18)
is 6.8, practically identical with the UK figure
similarly calculated for the same period of 6.7.
It should be pointed out, however, that, for the
purpose of this calculation, GDP for the years
1956-1959 was based on "expected" net national
product shown on Chart 1, not on actual since the
figure obviously related to years of recession.
The gross ratio has been used (i.e. in formula
(10)); V_t has been taken as gross fixed capital
formation and Y_t as gross domestic product, both
at constant market prices, not only because
separate figures for depreciation in the agricultural
sector have not been published for Ireland but also
because some expert opinion favours the use of the
gross concept in preference to the net on theoretical
grounds for which there is much to be said.   It is
satisfactory to observe that on this admittedly
over-generalised test, the Irish non-agricultural
economy does not make a bad showing.   It may be
worth observing that, in advanced economies the
gross ratio is about double the net ratio.   On
the net basis, therefore, an incremental ratio of
3 for Ireland (including the agricultural sector)
is not an unreasonable aspiration for the future.

<u>Stock ratio, p.</u>   The stock ratio p, given by (9),
at constant prices in each of the last five years,
was as follows (source CSO.):-

$$p$$

Year	p
1957	61.9
1958	63.1
1959	58.8
1960	59.7
1961	58.3
Average	60.4

The relative trendlessness in the figure will be
noted, also its magnitude, due mainly to the con-
tribution of livestock in which so much working
capital is perennially locked up in Ireland.   A
reduction in this stock ratio would be desirable for
rapid economic progress.   It will have been noted
that in the six behaviouristic equations the term
kr is always accompanied by pr indicating that
<u>pro tanto</u> a reduction in p has precisely the same
effect as a reduction in the marginal net capital-
output ratio k.   Clearly a prime objective in economic
policy should be earlier maturity of dry cattle.
For this paper p will be taken as 0.6.

<u>Saving ratio,s.</u>   As saving fluctuates considerably
from year to year (see columns (9) and (10) of
Table 1) it will be convenient to divide the 15
years 1947-1961 into three quinquennia and to use
annual averages to the following effect,

Period	Annual average (£ million at current market prices)				As percentage of national income(2) - (3) (4)	
	Net national income	Saving	Net investment from abroad	Net capital formation at home		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1947-1951	367.7	15.2	30.2	45.4	4.1	8.2
1952-1956	506.2	46.1	14.3	60.4	9.1	2.8
1957-1961	602.4	54.8	0.1	54.9	9.1	0.0

Of course, column (3) plus (4) equals column (5).
In 1947-1951, a period of restocking, especially of
consumer durables, the saving ratio was low (column
(6)) and foreign disinvestment high.   The proportion
borne by investment from abroad in capital formation
at home rapidly declined to practically zero (on
average) in the latest quinquennium.   It should be
emphasized that the figures in column (4) are net,
i.e. they represent the balance of gross extern
investment (direct and portfolio, including drawing
down of Irish-owned assets abroad, reserves of
extern-owned companies, subsidiaries and branches in
Ireland invested in Ireland as well as direct
investment of externs) over Irish investments abroad.
These two gross totals are not known.   Since foreign
investment in the State during the period 1957-1961
was known to be large, so also must have been Irish
investments abroad.   It would be useful and
revealing to have the gross figures separately
analysed into their main constituents.

From the present point of view, mainly of
note is the fact that in each of the periods
1952-1956 and 1957-1961 the saving ratio equalled
9.1.   In any speculations as to the magnitude of the
ratio during the next decade or so it would appear
unrealistic to assume a large departure from 9 or
10 per cent.

The import ratio, m.   For any Irish model, m is
possibly the most important parameter because of the
relative magnitude of external trade in the national
economy.   The simple average values of the ratio in
the last three quinquennia were as follows:-

| Period	Imports as percentage GNP (constant prices)
1947-51	41.7
1952-56	38.0
1957-61	40.2

The absence of trend will be observed, more apparent

perhaps from the single year percentages in column (8)
of the appended Table 1. One might be inclined to infer
from these very stable figures that it would be proper
to assume a 40% ratio in any Irish growth model. This
would not be correct, for, as the following analysis
shows, the stability of the ratio in Ireland has been
very probably a reflection of the comparatively slow rate
of growth of Irish GNP at constant prices.

From the UN National Account Yearbook 1960
linear time trends were fitted to log (constant price
GDP) and to log (ratio of constant price imports to
GDP) during the seven year period 1953-1959 for all
the countries for which these data were available, for
the purpose of determining exponential growth rates
of both entities. Ultimately three countries were
omitted, namely Columbia, Ecuador and Iceland, where
it was evident that there had been some interference
with the "natural" trend of imports; the same indeed
might be said of Ireland itself because of the Special
Levies of 1956 but it was decided to retain Ireland
since the near zero position of the country according
to the two variables would obviously not be much
affected even if the levies had not been imposed.
The results for the twenty-one countries are displayed
on Chart 2, which makes the relationship between the
two variables fairly evident to the eye. This is
confirmed by the fact that the coefficient of
correlation between the two is 0.67, highly significant
with 21 pairs of observations. The tendency for a
hypothetical straight line of relationship to pass
through the origin also appears to be a tenable
hypothesis, i. e. that zero ratio of growth of GNP
will be associated with zero rate of growth of the
import ratio: Ireland itself is an illustration.
In fact the averages for the 21 countries are 3.6%
for GNP and 4.1% for the import ratio. Accordingly
the simple hypothesis that a 1% rate of growth of
GNP will be accompanied by a 1% growth in the ratio
seems reasonable, if a little conservative as
regards the growth in the ratio.

It might be thought that as Ireland has
already a comparatively high ratio of 40% its rate

Chart 2. **Annual Average Percentage Rate of Growth**
1953-59 in (a) Gross National Product
and (b) Ratio of Imports to Gross
National Product, in Twenty-one Countries
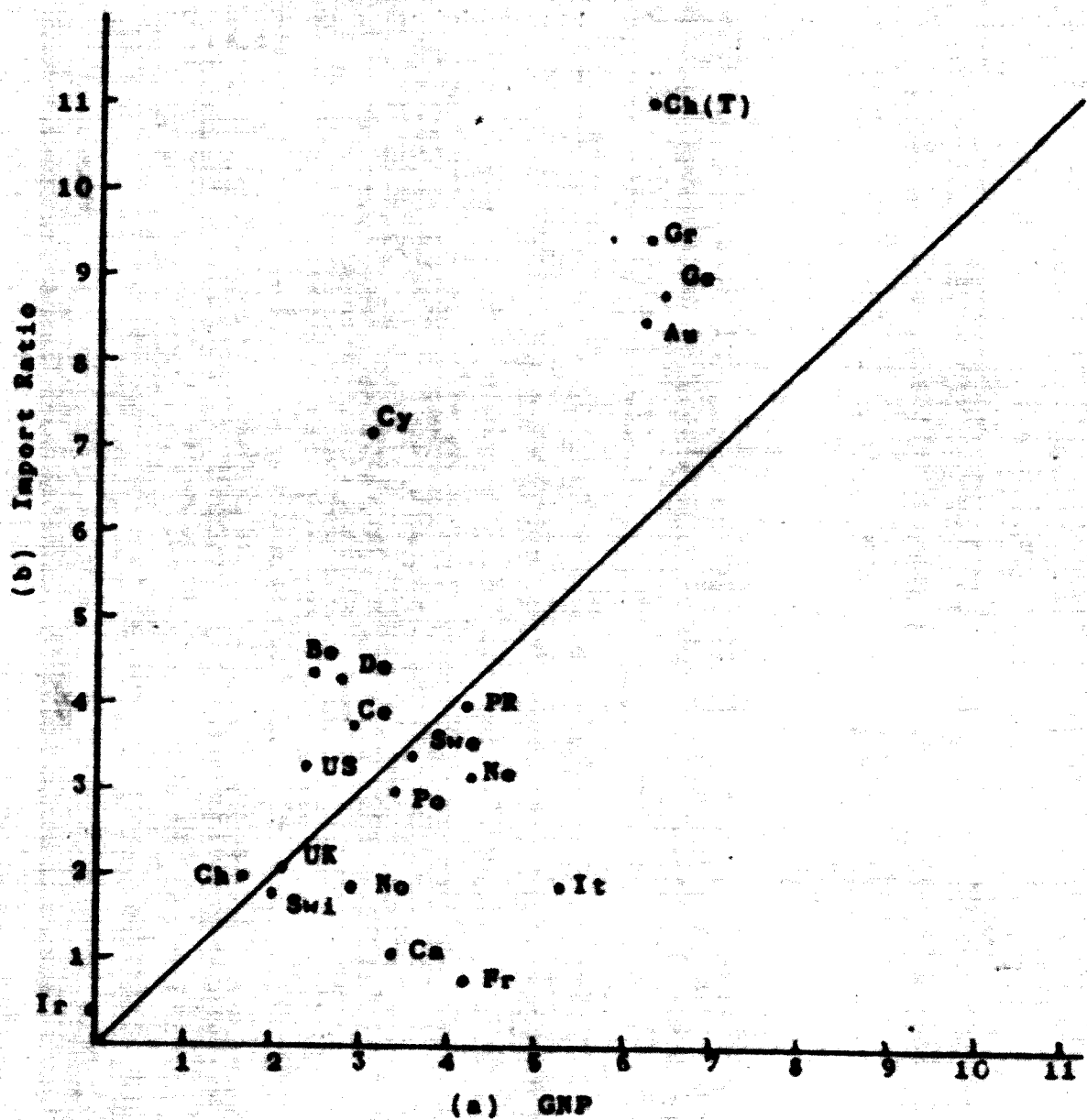
Source: Table 2

of growth for each 1% in GNP might be less than that
of countries with a lower percentage, i.e. that there
would be a tendency in all countries of "tailing-off"
of the ratio at some fairly high percentage (60, 70,
80?). Such does not appear to be the case in any very
marked degree. In fact, the correlation between the
mean level of the ratio and its rate of growth is
only - 0.10 which is certainly not statistically
significantly different from zero though the minus
sign will be noted. The <u>possibility</u> of a slight
tendency towards tailing-off is allowed for in
depressing the relationship to 1:1. It should be
added that no account is taken of Ireland's prospective
EEC membership in assessing this relationship. It is
beyond doubt that the ratio m would increase more
steeply on entry into the Common Market.

All parameters will accordingly be given
fixed values in the experimental forecasts to be
undertaken, except m which will increase percentage-
wise with Y_t. For example if the series starts with
m = 0.40 and a rise of 3% is postulated for the first
and subsequent years in Y_t m will assume the
successive values 0.40, 0.412, 0.424, ..., each 3% in
excess of the preceding value.

The break-even case.

If it be necessary to assume that the nation
is not to develop a systematic import excess the
solution of the problem of economic growth is an
extremely simple one, in fact it may be derived from
relation (11) above. Since the assumption implies
that N is positive or zero, we have

(19) $s \geq (k + p)r$

as a necessary condition. Given that p = 0, the
values of s, the net saving ratio, can now be
calculated for different sets of values of k, the net
fixed capital-output ratio, and r, the rate of increase
of net national product. Four values of k, namely
2, 3, 4, 5, are postulated and three values of r, namely

0.03, 0.05 and 0.07. It will be recalled that the value of k in Ireland for the period of continual growth 1947-1955 was very close to 5. The following are the values of s for all twelve combinations of values under the assumption that

$$(20) \qquad s = (k + 0.6)r,$$

i.e. that imports always equal exports so that foreign net investment is zero:-

k	r	s	k	r	s
2	0.03	0.078 *	4	0.03	0.138 *
2	0.05	0.130 *	4	0.05	0.230
2	0.07	0.182	4	0.07	0.322
3	0.03	0.108 *	5	0.03	0.168
3	0.05	0.180	5	0.05	0.280
3	0.07	0.252	5	0.07	0.392

It will be recalled that during the last decade s averaged 0.091 in Ireland so that, realistically, only the asterisked combinations can be contemplated if one has to postulate an external break-even with a saving ratio which the Irish people would tolerate. During the growth period 1947-1955 the actual rate of growth of NNP at constant prices in Ireland was 2.4% and the value of k was about 5. It will be recalled, however, that during this period external trade was markedly in deficit. It was this deficit which made the modest rate of growth possible, given the Irish pattern of economic behaviour.

The following conclusions emerge from this analysis, under the assumption that in future foreign trade is in balance.

(1)   To maintain a 3% rate of growth with the net fixed capital-output ratio of 5 which obtained in the growth period 1947-1955 it would be necessary to increase the net saving ratio from its recent level of 9% to 17½%. It is very unlikely that this level would be attained

through voluntary saving and forced saving
(e.g. by taxation) would have a disincentive
effect.

(2) To attain and maintain a rate of growth of 3%
or more a drastic reduction of the capital-
output ratio will be necessary. Analysis has
shown that the ratio is very high in the
agricultural sector in which the rate of growth
in the future as in the past is likely to be
slow, whatever the rate in the rest of the
economy. Hence fixed capital investment should
be deflected from agriculture where it seems
likely that current inputs of fertiliser, seeds
etc efficiently applied, as well as improved
standards of farm management, should be conducive
to considerable economy in application of fixed
capital, in arterial drainage etc.

(3) Through skilled management the closest attention
should be given to increasing output from fixed
capital stock at any level. This implies an
incremental capital-output ratio of zero.
Clearly the achievement of a zero ratio over
wide areas of the economy will result in a
substantial reduction in the existing level of
the ratio which, in the non-agricultural sector
was about 3-4 in the post-war period, not very
different, as has been shown, from the UK level
but not good enough for a regular substantial
growth rate in the economy.

(4) Attention should be given to economy in working
capital. The present high level of 0.6 for the
national ratio of stock to NNP could be
drastically reduced by earlier maturity of
cattle.

(5) Entrepreneurs should be encouraged to apply the
net capital-output test (output of course being
forecast) in all considerations of increasing
fixed capital. One surmises that at present
"hit-or-miss" obtains in considerable degree

where the necessary calculations could be made
with little trouble.

(6)   The propensity of the ratio imports to net
national product to increase with net national
product has been stressed.   The 1% - 1%
relationship should be regarded as a conserv-
ative one.   EEC membership would entail a sharp
increase in the ratio.   On any reasonable
assumption (and not only the present restrictive
one of equality) of the relationship between
current imports and exports in the aggregate,
exports must be envisaged as increasing
percentagewise in volume more than net national
product.   Accordingly the attainment of a great
and continuing increase in volume of exports is
a necessary condition for a substantial rate
of growth in the Irish economy.   To this end a
substantial reduction in prices, internal as well
as external, in relation to European prices, will
be necessary.   The present gap is a large one:
a recent sample inquiry conducted by CTT has
shown that, on average, Irish industrial products
are exported at a price about 15% on average
below the home price.   Under EEC conditions it
is obvious that a large part (though perhaps
not all because of imperfect competition) of
this margin must be swept away.

(7)   Having regard to possible modifications in the
right direction in the coefficients of the economy
it does not appear possible to attain even a 5%
rate of growth without tolerating a fairly
substantial external deficit in trade for many
years to come.   This aspect will be examined in
subsequent Parts.

Table 1. Some Macro-economic Data, Ireland, 1947-1961

Year	At constant (1953) prices						Imports as % of GNP	At current prices		(10) as percentage of (9)
	Gross national product	Gross fixed capital formation	Dwell-ings	Deprec-iation	Net national product	Net fixed capital formation		Net national product	Saving	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	£ million						%	£ million		%
1947	436.2	37.7	5.1	12.7	423.5	25.0	41.7	325.5	14.8	4.5
1948	453.7	45.0	8.1	12.5	441.2	32.5	41.1	349.8	20.9	6.0
1949	480.5	59.7	14.5	13.7	466.8	46.0	38.7	373.0	26.4	7.1
1950	486.8	70.6	17.4	16.8	470.0	53.8	42.8	384.1	15.4	1.0
1951	496.6	80.1	19.5	17.5	479.1	62.6	44.1	406.3	-1.5	-0.4
1952	512.7	78.2	18.6	17.9	494.8	60.3	35.6	461.0	42.9	9.3
1953	525.6	79.5	16.3	21.2	504.4	58.3	39.2	504.4	58.0	11.5
1954	532.4	85.5	15.2	23.4	509.0	62.1	37.9	505.9	50.5	10.0
1955	541.9	89.1	15.8	24.2	517.7	64.9	41.1	526.9	40.0	7.6
1956	533.7	83.4	15.9	26.0	507.7	57.4	36.4	533.0	39.3	7.4
1957	540.6	69.6	10.5	27.7	512.9	41.9	34.4	552.2	49.6	9.0
1958	518.9	68.6	8.6	28.6	490.3	40.0	39.5	561.1	36.8	6.5
1959	543.1	71.8	9.3	30.1	513.0	41.7	41.3	600.8	60.7	10.1
1960	569	76	11	32	537	44	41.7	626	59	9.4
1961	595	86	n.a.	33	562	53	44.2	667	68	10.2

Sources: Economic Statistics issued prior to the Budget 1962;   Statistical Abstracts;   CSO.

Notes:
General: When revision in published figures was noted figures previously published were revised by
            linkage at year of change.   GNP and NNP are expressed at market prices.

Col.(3):   Current price data deflated by capital price index (CSO).
Col.(4):   Included in col.(3).
Col.(5):   Current price data deflated by capital price index (CSO).
Col.(6):   Col.(2) less col.(5).
Col.(7):   Col.(3) less col.(5).

Table 2.   Percentage Annual Average Rates of
           Increase in Gross National Product
           and in the Import Ratio at Constant
           Prices in Twenty-one Countries,
           1953-1959.

Country	Rate of Increase		
	Gross national product	Import ratio	Mean import ratio
(1)	(2)	(3)	(4)
	%	%	%
Germany, FR	6.5	8.8	18.9
China (Taiwan)	6.3	11.0	23.8
Greece	6.3	9.4	22.7
Austria	6.2	8.5	26.0
Italy	5.3	1.9	14.2
Netherlands	4.3	3.2	51.5
France	4.2	0.8	13.1
Porto Rico	4.2	4.0	69.3
Sweden	3.6	3.4	28.7
Canada	3.4	1.1	24.0
Portugal	3.4	3.0	23.3
Cyprus	3.1	7.2	62.8
Ceylon	2.9	3.8	40.0
Norway	2.9	1.9	44.7
Denmark	2.8	4.3	34.6
Belgium	2.5	4.4	32.0
U.S.A.	2.4	3.3	4.8
United Kingdom	2.1	2.1	21.7
Switzerland	2.0	1.8	28.5
Chile	1.7	2.0	11.5
Ireland	-0.1	0.4	36.8

Source: Based on data from UN Yearbook of National
          Account Statistics 1960.

Note: Countries arranged in descending order of
          rates in column (2).   Rates in columns
          (2) and (3) are exponential.   Means in
          column (4) are geometric, in consequence
          lower than the arithmetic averages for
          Ireland used in the text: for 1953-59
          simple arithmetic average is 38.5%
          compared with the geometric 36.8% shown
          in the table.