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Seasonality in Irish Economic Statistics

by

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By C. E. V. LESER*

1. INTRODUCTION

The presence of seasonality is a serious obstacle to an analysis of short-term economic movements, as it may tend to exaggerate or conceal true changes in economic conditions. For any study of current economic conditions in this country it is therefore of some importance, firstly, to ascertain which of the main series are affected by seasonal factors and which are not, and secondly, to investigate whether seasonality can be reliably measured and allowed for by appropriate corrections.

One limitation may be stated at the outset. The quarter is held to be a sufficiently short period for a short-term economic analysis, and no attempt is made to analyse month-to-month fluctuations. In as far as the data analysed are published in the form of monthly series, they have been transformed into

quarterly data by simple totalling or averaging, as the case may be.

Thirty-six quarterly economic series are presently being assembled by the Economic Research Institute in "Statistics of economic level and trend", and it is with these series that the present study is concerned. It will be shown here how the series have been tested for presence or absence of seasonality by an analysis of variance, and how the series have been classified accordingly. Next, an account will be given of the application of various seasonal estimation procedures to four selected series, and of the alternative results obtained. Lastly, it will be shown how the finally adopted method is being used to obtain seasonal indices; and the seasonal indices which are being used to correct the data will be given.

2. TESTING FOR SEASONALITY

Of the 36 economic indicators selected by the Economic Research Institute, seven represent the sum, difference or ratio of two other series presented; these are: production per worker in transportable goods industries, net cash income in agriculture, terms of trade, real earnings in transportable goods industries, volume of imports, volume of exports, and imports excess value. These derived series have not been directly tested for seasonality, but only indirectly through their components.

One series—net passenger movement outward by sea and air—is presented as a twelve-month aggregate, and the question of seasonality does not arise; the quarterly data would be sometimes

positive, sometimes negative, and their time series analysis would demand special treatment. Finally, agricultural minimum wages remain constant for some time and change at irregular intervals; this "institutional" series has also been excluded from the analysis which follows.

The remaining twenty-seven series, which have been investigated for possible seasonality, are as follows:

- Volume of production index (transportable goods)
- Total number of new houses built (State-aided schemes)
- Electricity output (E.S.B.)
- Total sales in agriculture
- Farm costs (excl. wages)
- Sales of insurance stamps
- Employment (transportable goods)

Total live register
 Percentage of insured persons on live register
 Wholesale price index
 Consumer price index
 Agricultural price index
 Import price (unit value) index
 Domestic export price (unit value) index
 Price index of ordinary stocks and shares
 Index of weekly earnings (transportable goods)
 Index of retail sales
 New private motor cars registered
 Total revenue receipts
 Total exchequer expenditure
 Value of imports
 Value of total exports
 Money supply
 Bank debits (ordinary accounts)
 Commercial banking assets: bills, loans and advances
 Commercial banking assets: investments
 External assets of banking system and departmental funds

Table 1: Results of variance analysis

Series	F-ratio		Residual coefficient of variation %
	Between years	Between quarters	
Volume of production	139.4	28.5	2.06
New houses built ..	1.3	0.8	13.48
Electricity output ..	95.4	228.0	2.71
Sales in agriculture ..	12.3	161.2	0.15
Farm costs	18.0	70.9	0.12
Sales of insurance stamps.	7.6	220.9	1.58
Employment ..	769.3	68.8	0.41
Live register ..	37.4	92.8	4.95
% insured on live register.	56.3	55.5	5.11
Wholesale price index	34.9	1.9	0.65
Consumer price index	52.0	1.9	0.89
Agricultural price index.	4.2	6.1	1.59
Import price index ..	4.1	0.9	1.00
Export price index ..	3.8	1.1	1.82
Share price index ..	497.7	19.7	2.75
Weekly earnings index	303.5	31.9	1.18
Retail sales index ..	126.0	138.4	1.17
New cars registered ..	29.9	34.1	0.22
Revenue receipts ..	15.7	133.1	0.14
Exchequer expenditure	2.5	7.0	0.48
Value of imports ..	69.4	10.3	3.36
Value of exports ..	27.6	7.8	5.82
Money supply ..	118.2	25.0	1.60
Bank debits	65.1	8.6	3.60
Banks' bills, loans and advances.	156.6	10.5	2.02
Banks' investments ..	80.6	2.0	1.63
External assets ..	65.2	7.6	1.39

Most of the data are published in the economic series or elsewhere in the *Irish Statistical Bulletin*, or in the *Quarterly Bulletin of the Central Bank of Ireland*; the money supply is obtained as the sum of note and coin circulation and of current accounts, following Oslizlok (1962-63). The agricultural series have been supplied by E. A. Attwood and M. Ross of the Agricultural Research Institute.

No adjustment for working days, which would raise some further problems, has been made to the series representing quarterly totals here or in the later analysis. In so far as variations in number of working days between quarters are regular, their effect is included in the seasonality indicators.

In the majority of cases, 20 observations representing quarterly figures for the years 1958-1962 were utilised. Regarding series for which data were not available as far back, a different or shorter period was chosen, viz. 1959-1963 for both agricultural series, 1961-1963 for retail sales and 1960-1963 for external assets.

The observations were formally subjected to an analysis of variance between years and between quarters, thus testing the hypothesis that no trend and no seasonal variation is present. Except for the two series based on shorter periods, the sum of squares between years thus had 4 degrees, that between quarters 3 degrees, and the residual sum of squares 12 degrees of freedom.

The *F*-ratios were calculated in the usual way and are shown in Table 1. In addition, the magnitude of the irregular movements is indicated by the residual coefficient of variation; this is simply the square root of the residual mean square, expressed as a percentage of the grand mean for the series.

At the 5% significance level, the critical *F*-ratios are 3.26 for the years and 3.49 for the quarters (somewhat higher for retail sales and external assets); at the 1% level, the corresponding ratios are 5.41 and 5.95 respectively. The variation between years thus appears to be significant for all series except new houses built, exchequer expenditure and possibly the agricultural and foreign trade price indices. At first sight, the variation between quarters also appears to be significant for all but 6 of the series analysed; but in interpreting this result, care must be taken as the following reflection will show.

Assume that the time series follows a linear trend. If the trend is rising, this fact will also be reflected in rising quarterly totals from the first to the fourth quarter. If the series is exactly linear, with no true seasonal or irregular component, then it is easy to prove that with *m* years or *4m* observations, the ratio of mean squares between quarters and between years, as customarily defined in the analysis of variance, will be equal to $5/16(m+1)$. With *m*=5, this ratio amounts to 0.0521.

It follows that in cases in which the F -ratio between quarters is about 5% of F between years, a trend effect upon variation between quarters may be suspected and verified by inspection of the annual and quarterly totals. This effect appears to explain the high value of F between quarters entirely in the case of the share price index, and to a substantial part in a few other cases.

With the help of Table 1 and those considerations, it is possible to classify the series into three groups. The series in the first category do not show any marked seasonal variation, and no seasonal corrections are required for them. For those in the second category, the seasonal variation is very pronounced and clearly overshadows such irregular movements as there are. There is a third category of series, apparently influenced by seasonal factors; but the seasonal element is rather moderate here and combined with fairly large irregular fluctuations. Clearly the series in the third category are those which may well present the greatest difficulties when it comes to estimating the seasonal variation. The classification is as follows:

(1) No marked seasonal variation:

- New houses built.
- Wholesale price index.
- Consumer price index.
- Import price index.
- Export price index.
- Share price index.
- Banks' bills, loans and advances.
- Banks' investments.

(2) Clearly marked seasonal variation:

- Volume of production.
- Electricity output.
- Sales in agriculture.
- Farm costs.

- Sales of insurance stamps.
- Employment.
- Live register.
- % insured on live register.
- Weekly earnings index.
- Retail sales index.
- New cars registered.
- Revenue receipts.
- Money supply.

(3) Seasonal variation and irregular movement:

- Agricultural price index.
- Exchequer expenditure.
- Value of imports.
- Value of exports.
- Bank debits.
- External assets.

The result is on the whole in agreement with common sense considerations, though in the case of some series it would hardly have been possible to arrive at the correct classification by *a priori* reasoning. The absence of a clear seasonal pattern in the number of houses built may seem surprising; but the very marked quarter-to-quarter fluctuations which this series exhibits are irregularly spaced instead of recurring at regular intervals. Other series which appear to be free from seasonal influences are the price indices with the exception of agricultural prices and some of the banking statistics. For the remaining series, seasonal correction appears desirable.

It should be clearly understood that the method followed so far does not yield a satisfactory measure of seasonal variation. A mere comparison of quarterly means which did not eliminate the trend effect would lead to biased results. Various valid methods of measuring seasonality are, however, available, and the discussion which follows will be devoted to such methods.

3. METHODS OF OBTAINING SEASONAL INDICES

It has been shown that there are 19 series which apparently require direct seasonal correction by some method or other. In 13 cases, the seasonal variation stands out very clearly, and one may expect to find that the choice of method does not substantially influence the result. In the case of 6 other series, the irregular components of which are not negligible in comparison with the seasonal element, different estimation methods could well lead to widely differing results for the seasonality pattern.

From either group, two series were selected for further analysis. Electricity output and sales of

insurance stamps are examples of series with a large seasonal and a fairly small irregular component. Import value and export value, on the other hand, are series with only moderate seasonality and a substantial irregular component.

It will be assumed that the seasonal variation can best be represented by an index, the simple mean of the four quarters being 100; this seems more appropriate than a constant positive or negative amount for each quarter. Furthermore it is assumed that the seasonal indices are subject to slow changes over time, and that therefore no more than the five previous calendar years should be utilised to obtain

seasonal indices which are applicable to the current period. In the present analysis, the observations over the period 1958-1962 were used.

There is, of course, a wide variety of procedures which may be followed to obtain seasonal indices and which are discussed, for example, in a report of the O.E.C.D. (1961). No exhaustive study is made here but merely an examination of three basic methods, derived from the alternative assumptions that the trend can be represented as a centred unweighted four-quarter moving average, a polynomial, or a quasi-linear trend. The first two of these are standard statistical methods and only their application to the present case need be referred to; the third method will be briefly described.

The moving average may be computed either for the original data or for their logarithms. In the former case, the seasonal index for a quarter is obtained in the case of 20 observations as the unweighted mean of 4 ratios of observation to moving average, multiplied by 100; in the latter case, the logarithm of the seasonal index divided by 100 is the mean of 4 differences between the logarithms of observation and trend. In either case, an additive adjustment is made to ensure that the indices add up to 400 or their logarithms to 0 respectively.

Further refinements may be introduced into the moving average procedure. Geary advocated an adjustment to give a truer representation of the trend at turning points, which may be applied to all moving averages obtained except the first and last ones; effectively, it means that a weighted moving average is now used as trend. A description of the correction and its derivation is given in the Appendix.

The construction of polynomial trends with seasonality in the data by means of orthogonal polynomials is a modification of ordinary least squares. To obtain seasonal indices, the method should be applied to the logarithms of the data. The logarithms of the seasonal indices (divided by 100) are then obtained, with 20 data, as means of 5 differences and add up to 0.

The quasi-linear trend method was developed for the additive case in which the data are thought to be the sum of trend, seasonal variation and residual, and the four seasonal components add up to 0. The method is designed to yield a trend and seasonal pattern which minimise the sum of squares of changes in trend direction plus the sum of squares of residuals. With n observations, where n is a multiple of 4, write Y_1, Y_2, \dots, Y_n for the data; T_1, T_2, \dots, T_n for the estimated trend values; and S_1, S_2, S_3, S_4 for the estimated seasonal components. Also write

$$M_1 = (T_1 - 2T_2 + T_3)^2 + \dots + (T_{n-2} - 2T_{n-1} + T_n)^2$$

$$M_2 = (Y_1 - T_1 - S_1)^2 + \dots + (Y_n - T_n - S_4)^2$$

Then the trend and seasonal components are derived from the condition

$$M_1 + M_2 = \text{Minimum, subject to } S_1 + S_2 + S_3 + S_4 = 0$$

Together with the last-named equation, partial differentiation of the expression to be minimised and elimination of a redundant equation leads to $n+4$ linear equations with $n+4$ unknowns. Exact solutions have been obtained in explicit form for $n=8$ and $n=12$, and approximate solutions for $n \geq 16$.

Regarding the seasonal components, it is only necessary to give the formulae for S_1 and S_2 . The formula for S_3 is obtained writing the numerical coefficients in the formula for S_2 in reverse order; similarly the formula for S_4 reverses the order of the coefficients in S_1 . Thus for:

$$n=8:$$

$$S_1 = (245 Y_1 - 199 Y_2 - 191 Y_3 - 323 Y_4 + 691 Y_5 - 113 Y_6 - 121 Y_7 + 11 Y_8) / 1,248$$

$$S_2 = (-193 Y_1 + 459 Y_2 - 173 Y_3 - 249 Y_4 - 119 Y_5 + 477 Y_6 - 139 Y_7 - 63 Y_8) / 1,248$$

$$n=12:$$

$$S_1 = (5,236 Y_1 - 4,731 Y_2 - 3,723 Y_3 - 5,167 Y_4 + 14,557 Y_5 - 3,048 Y_6 - 4,752 Y_7 - 6,601 Y_8 + 13,435 Y_9 - 3,297 Y_{10} - 2,601 Y_{11} + 692 Y_{12}) / 44,304$$

$$S_2 = (-4,668 Y_1 + 10,695 Y_2 - 3,945 Y_3 - 4,773 Y_4 - 2,913 Y_5 + 11,568 Y_6 - 3,768 Y_7 - 5,043 Y_8 - 3,495 Y_9 + 10,965 Y_{10} - 3,363 Y_{11} - 1,260 Y_{12}) / 44,304$$

Furthermore, for $n=4m$, where $m \geq 4$, integer, the formulae may be written as follows.

$$S_1 = [(31m-36)Y_1 - (33m-35)Y_2 - (19m-9)Y_3 - (19m+8)Y_4 + (77m-36)Y_5 - (25m-35)Y_6 - (25m-9)Y_7 - (25m+8)Y_8 + (75m-36)Y_9 - (25m-35)Y_{10} + \dots - (25m-9)Y_{n-6} - (27m+8)Y_{n-4} + (72m-36)Y_{n-3} - (25m-35)Y_{n-2} - (13m-9)Y_{n-1} + (6m-8)Y_n] / 4m(25m-27)$$

$$S_2 = [-(31m-37)Y_1 + (70m-72)Y_2 - (27m-28)Y_3 - (24m-7)Y_4 - (25m-37)Y_5 + (75m-72)Y_6 - (25m-28)Y_7 - (25m-7)Y_8 - (25m-37)Y_9 + \dots - (25m-7)Y_{n-4} - (29m-37)Y_{n-3} + (71m-72)Y_{n-2} - (24m-28)Y_{n-1} - (6m-7)Y_n] / 4m(25m-27)$$

The derivation of these results is shown in a paper by Leser (1963), in which the last set of formulae is given in a slightly different form. The paper also gives the formulae for the trend values.

To obtain seasonal indices, logarithms should be used. Denoting the logarithms of the data by Y_1, Y_2, \dots, Y_n , the formulae then yield expressions for $\log S'_i / 100$ ($i=1, 2, 3, 4$), where S'_1, S'_2, S'_3 and S'_4 are seasonal indices. In particular we have for $n=20, m=5$:

$$\log(S'_1 / 100) = \cdot 0607 Y_1 - \cdot 0664 Y_2 - \cdot 0438 Y_3 - \cdot 0525 Y_4 + \cdot 1780 Y_5 - \cdot 0459 Y_6 - \cdot 0592 Y_7 - \cdot 0679 Y_8 + \cdot 1730 Y_9 - \cdot 0459 Y_{10} - \cdot 0592 Y_{11} - \cdot 0679 Y_{12} + \cdot 1730 Y_{13} - \cdot 0459 Y_{14} - \cdot 0592 Y_{15} - \cdot 0729 Y_{16} + \cdot 1653 Y_{17} - \cdot 0459 Y_{18} - \cdot 0286 Y_{19} + \cdot 0112 Y_{20}$$

$$\log(S_2'/100) = -0.0602 Y_1 + 0.1418 Y_2 - 0.0546 Y_3 - 0.0577 Y_4$$

$$- 0.0449 Y_5 + 0.1546 Y_6 - 0.0495 Y_7 - 0.0602 Y_8$$

$$- 0.0449 Y_9 + 0.1546 Y_{10} - 0.0495 Y_{11} - 0.0602 Y_{12}$$

$$- 0.0449 Y_{13} + 0.1546 Y_{14} - 0.0495 Y_{15} - 0.0602 Y_{16}$$

$$- 0.0551 Y_{17} + 0.1444 Y_{18} - 0.0469 Y_{19} - 0.0117 Y_{20}$$

As explained previously, the formula for S_3' is obtained by writing the coefficients in the second expression in reverse order; similarly, the formula for S_4' is derived from the first expression. Thus

$$\log(S_3'/100) = -0.0117 Y_1 - 0.0469 Y_2 + \dots - 0.0602 Y_{20}$$

$$\log(S_4'/100) = 0.0112 Y_1 - 0.0286 Y_2 + \dots + 0.0607 Y_{20}$$

One further point may be mentioned. All methods using logarithms, and the last-named method in particular, yield seasonal indices S_1' , S_2' , S_3' and S_4' such that their geometric mean is 100. It is preferable to have seasonal indices S_1 , S_2 , S_3 and S_4 such that the arithmetic mean is 100. The conversion from one to the other set of indices is best achieved by means of a correction factor, thus

$$S_i = 400 S_i' / (S_1' + S_2' + S_3' + S_4') \quad (i=1, 2, 3, 4)$$

If the trend values are required, the anti-logarithms of the derived logarithmic trend values should be inflated in the same proportion as the seasonal indices are deflated. For computation of trend and seasonal variation combined in a single set of figures, it does not matter then whether unadjusted or adjusted figures are used for trend and seasonal variation.

For the chosen series, alternative estimates of S_1 , S_2 , S_3 and S_4 have been derived with the help of seven methods or variants of methods; these are:

- (1a) Ratio to moving average.
- (1b) as (1a) with Geary's correction.
- (1c) Moving average of logarithms.
- (1d) as (1c) with Geary's correction.
- (2a) Linear trend for logarithms.
- (2b) Quadratic trend for logarithms.
- (3) Quasi-linear trend for logarithms.

The results are given in Table 2, together with an indicator R^2 of goodness of fit, which is not to be interpreted as an ordinary coefficient of determination. It is based on the 16 central observations, since the moving average method does not yield trend values for the first 2 or the last 2 observations unless special devices are used. It represents that portion of the sum of squares for the percentage deviations of the 16 observations from their mean which is explained by trend and seasonal variation.

As expected, the various estimates are fairly close to each other with regard to electricity output and insurance stamps sold; for value of imports and exports, wider variations are observed. In no instance are there any substantial differences

between the results of method (1a) to (1d) and between the results of (2a) and (2b). That is to say, if a moving average is adopted, it does not matter much here whether original data or logarithms are used, nor whether Geary's correction is made or not; if a regression method is chosen, the introduction of a quadratic term has no appreciable effect on any of the series analysed.

Some guidance as to choice of method is given by the goodness of fit indicator. The linear and quadratic trend method do not give a good fit in the case of imports and a poor fit in the case of exports; the long-term movements appear to be too complex to be adequately described by a simple quadratic expression. A third-degree polynomial might give a better fit, but the method would then become fairly cumbersome.

A linear trend, if appropriate, would offer certain advantages. Computations involved are simple; and with random residuals, significance tests may be applied and confidence intervals for regression coefficients may be constructed. However, if the actual underlying trend shows clear non-linearities, mechanical computation of a linear trend would lead to excessively large and serially correlated residuals, which would not provide a good basis for statistical analysis.

TABLE 2: ESTIMATES OF SEASONAL INDICES (1958-62) AND GOODNESS OF FIT

Series and method	S_1	S_2	S_3	S_4	R^2
Electricity output:					
(1a)	118.3	86.8	81.4	113.5	.9962
(1b)	118.5	86.9	81.3	113.3	.9948
(1c)	118.1	86.6	81.5	113.8	.9963
(1d)	118.1	86.6	81.5	113.8	.9955
(2a)	118.2	87.0	81.4	113.4	.9943
(2b)	118.2	87.0	81.4	113.4	.9947
(3)	118.1	86.9	81.5	113.5	.9974
Sales of insurance stamps:					
(1a)	114.2	90.1	99.7	96.0	.9903
(1b)	114.2	90.1	99.7	96.0	.9914
(1c)	114.3	90.0	99.7	96.0	.9903
(1d)	114.2	90.1	99.7	96.0	.9913
(2a)	114.9	89.7	99.7	95.7	.9810
(2b)	114.9	89.7	99.7	95.7	.9838
(3)	114.6	89.6	99.8	96.0	.9923
Value of imports:					
(1a)	104.5	101.8	92.3	101.4	.9666
(1b)	104.6	101.6	92.4	101.4	.9691
(1c)	104.5	101.7	92.3	101.5	.9664
(1d)	104.7	101.4	92.3	101.6	.9693
(2a)	103.9	102.2	92.2	101.7	.9144
(2b)	103.8	102.2	92.3	101.7	.9136
(3)	104.3	102.4	92.2	101.1	.9771
Value of exports:					
(1a)	99.1	91.5	105.4	104.0	.9682
(1b)	99.6	91.3	104.9	104.2	.9660
(1c)	99.1	91.4	105.3	104.2	.9684
(1d)	99.5	91.4	105.0	104.1	.9675
(2a)	98.9	93.6	104.6	102.9	.8016
(2b)	98.9	93.6	104.6	102.9	.8029
(3)	97.7	93.2	105.1	104.0	.9789

In the case of the economic series under consideration here, the linear trend method does not appear to be suitable for universal application, nor the quadratic trend method either. As it was felt desirable to use one and the same method for all series, the methods based on simple polynomial trends had to be ruled out.

The biggest differences between results obtained by the moving average methods on one hand and the quasi-linear trend method on the other are found in respect of S_1 and S_2 for the export series. The explanation for the discrepancy lies in a low export value for the first quarter and a high value for the second quarter 1958, combined with the fact that these quarters, being at the beginning of the observation period, carry a very low weight in the estimation of seasonal variation by a moving average method. If the last two quarters of 1957 and the first two quarters of 1963 were added, method (1a) would yield the following estimates:

$$S_1=98.1, S_2=93.4, S_3=104.9, S_4=103.6$$

which are close to the results of method (3). In such circumstances, great time stability cannot be expected from the seasonal indices estimated by any method.

Both the moving average and the quasi-linear trend are sufficiently flexible to give, together with the seasonal indices, a good fit for any of the series. The fit yielded by the quasi-linear trend method is the better one; this is a general feature of the method which is achieved at the expense of a less smooth trend. For some purposes, the relatively large changes in direction for the quasi-linear trend might be a drawback. There is no theoretical difficulty in modifying the method so as to obtain a smoother trend combined with larger residuals, though no explicit formula has as yet been worked out.

In a sense, the quasi-linear trend may be considered as a moving average, though as a weighted one, in contrast to the centred unweighted moving average which offers an alternative. On theoretical grounds, the quasi-linear trend seems to be preferable as it is not arbitrary but derived from a definite theoretical assumption, to the effect that two sets of disturbances in the data, a permanent and a temporary one, have equal variances. Whilst this hypothesis is difficult to prove or disprove, it is at any rate not an unreasonable one on *a priori* grounds.

A further advantage of the quasi-linear trend method over the moving average is that it permits the computation of trend values for the two latest quarters for which data are available. One practical implication is that a basis for forecasting is immediately obtained, however inadequate such a forecast may be in the absence of other information. The

simplest way is to use linear extrapolation of the last two trend values, and by multiplying the projected trend with the most recent seasonal indices appropriate to the quarter under consideration, forecast values are obtained.

As an illustration this method has been applied to the selected series with regard to each 1963 quarter. The following trend values have been obtained by the quasi-linear trend method for the last two quarters of 1962.

Electricity output	..	658.6 ; 668.1
Sales of insurance stamps		6,333 ; 6,272
Value of imports	..	69.60 ; 73.02
Value of exports	..	43.13 ; 43.98

By geometric extrapolation and application of the last set of seasonal indices for each series in Table 2, the 1963 predictions are obtained. These are shown in Table 3, together with the actual figures.

TABLE 3: FORECAST AND ACTUAL FIGURES 1963

Series	Quarter			
	I	II	III	IV
Electricity output (mill. kwh).				
Predicted	800.4	597.4	568.2	802.8
Actual	852.4	617.1	578.2	813.0
Sales of insurance stamps ('000)				
Predicted	7,119	5,512	6,080	5,793
Actual	7,815	5,774	6,285	6,290
Value of imports (£ mill.)				
Predicted	79.89	82.29	77.72	89.40
Actual	69.31	79.87	69.95	87.17
Value of exports (£ mill.)				
Predicted	43.82	42.63	49.03	49.47
Actual	43.09	47.67	52.47	51.98

Of course, this is not an accurate prediction, nor does it claim to be. Regarding electricity output, the forecast could not foresee the abnormally cold winter and high demand in the first quarter 1963; otherwise it is reasonably satisfactory. The trend in insurance stamps sales showed a downturn in the second half of 1962, hence the low forecast values which were belied by an upturn. Imports showed a steep rise in 1962, and the trend somewhat flattened out at the beginning of 1963, which accounts for the discrepancy. The export forecast is remarkably good for the first quarter 1963; after that, a new upswing took place and the rise was somewhat greater than that predicted by the trend formula.

In practice it is unlikely that one would use this forecasting method for more than a quarter or two ahead. Once information and data for the new year

come in, it is possible to modify the forecast by introducing changes in trend direction while maintaining the values of the seasonal indices.

A task which is computationally easier and presents fewer problems than forecasting is the

seasonal correction of quarterly data as they become available, deflating them by the latest estimate of the appropriate seasonal index. For this purpose, it is not necessary to work out trend values; seasonal indices only are required.

4. THE SEASONAL INDICES AND THEIR USE

The preceding analysis suggests that reasonably reliable estimates may be obtained for the seasonal component of almost any Irish economic series which exhibits a seasonal pattern at all, and that the quasi-linear trend method is a suitable one for application in this context. Whilst the use of this method represents an innovation, it seems legitimate to suppose that it gives, with comparatively simple means, as good results as far more complex procedures devised for seasonal correction. The quasi-linear trend method was therefore adopted for seasonal correction of the quarterly economic series assembled by the Economic Research Institute.

It was also decided to compute seasonal indices based on the calendar years 1958-62 for application to each 1963 quarter, seasonal indices based on 1959-63 for application to 1964, and so forth. It was felt that this procedure allows for a possible gradual change in seasonality and ensures that the seasonal indices do not get out of date. Theoretically, recomputation would be possible after each quarter, but the computational effort would hardly be justified by results. If retrospective seasonal adjustment of the series were one of the chief objectives, this procedure would not be the best one and would require modification.

In the case of series not available as far back as 1958, the first set of seasonal indices was computed on the basis of two, three or four calendar years ending in 1962, and the coverage was subsequently extended to include 1963 and 1964, dropping earlier years when the length of the period would exceed five years.

It was found that no satisfactory seasonal indices could be computed for exchequer expenditure, owing to frequent changes in accounting practice which tend to throw the seasonal pattern out of gear. For the remaining 18 out of the 19 series subject to seasonality, three sets of seasonal indices for overlapping five-year (or shorter) periods were worked out, and these are given in Table 4.

In all cases, with the possible exception of new car registrations, the three sets of seasonal indices are reasonably consistent with each other, and the direction of the seasonal variation is clearly shown;

TABLE 4: SEASONAL INDICES FOR 18 ECONOMIC SERIES

Series	Period for which computed	Seasonal index			
		First quarter	Second quarter	Third quarter	Fourth quarter
Volume of production.	1958-62	96.2	105.1	97.7	101.0
	1959-63	96.1	104.6	98.0	101.3
	1960-64	96.5	104.1	97.9	101.5
Electricity output	1958-62	118.1	86.9	81.5	113.5
	1959-63	119.2	86.6	81.4	112.8
	1960-64	119.3	86.8	81.2	112.7
Sales in agriculture	1959-62	75.6	81.3	128.2	114.9
	1959-63	74.9	82.5	127.7	114.9
	1960-64	76.3	83.5	126.6	113.6
Farm costs ..	1959-62	122.4	94.2	93.1	90.3
	1959-63	122.9	95.2	92.7	89.2
	1960-64	122.8	96.0	92.4	88.8
Sales of insurance stamps.	1958-62	114.6	89.6	99.8	96.0
	1959-63	115.9	89.8	98.8	95.5
	1960-64	116.7	90.4	97.8	95.1
Employment ..	1958-62	99.3	100.4	100.1	100.2
	1959-63	99.3	100.2	100.1	100.4
	1960-64	99.3	100.1	100.2	100.4
Live register ..	1958-62	124.2	97.8	76.7	101.3
	1959-63	125.3	97.8	76.2	100.7
	1960-64	125.1	98.0	76.5	100.4
% insured on live register.	1958-62	119.0	100.9	84.9	95.2
	1959-63	120.1	100.8	84.6	94.5
	1960-64	119.7	101.1	84.6	94.6
Agricultural price index	1958-62	101.7	101.1	97.5	99.7
	1959-63	101.6	101.1	97.5	99.8
	1960-64	101.6	101.0	98.0	99.4
Weekly earnings index	1958-62	98.9	100.1	99.8	101.2
	1959-63	98.7	100.1	99.8	101.4
	1960-64	98.9	100.7	99.8	100.6
Retail sales index	1961-62	92.7	100.4	100.9	106.0
	1961-63	92.2	100.4	100.9	106.5
	1961-64	92.0	100.4	101.4	106.2
New cars registered	1958-62	110.2	121.6	95.1	73.1
	1959-63	110.2	122.3	94.4	73.1
	1960-64	107.3	123.0	95.2	74.5
Revenue receipts	1958-62	135.1	83.1	95.4	86.4
	1959-63	136.0	83.4	93.9	86.7
	1960-64	137.2	85.1	92.9	84.8
Value of imports	1958-62	104.3	102.4	92.2	101.1
	1959-63	102.5	102.7	92.9	101.9
	1960-64	102.0	102.8	92.5	102.7

TABLE 4: SEASONAL INDICES FOR 18 ECONOMIC SERIES (continued)

Series	Period for which computed	Seasonal index			
		First quarter	Second quarter	Third quarter	Fourth quarter
Value of exports	1958-62	97.7	93.2	105.1	104.0
	1959-63	97.8	92.5	105.2	104.5
	1960-64	98.9	93.0	105.4	102.7
Money supply ..	1958-62	99.3	99.4	98.3	103.0
	1959-63	99.2	99.4	98.6	102.8
	1960-64	99.8	99.0	98.8	102.4
Bank debits ..	1958-62	101.1	100.8	94.6	103.5
	1959-63	100.9	100.4	95.4	103.3
	1960-64	102.7	99.6	95.2	102.5
External assets ..	1960-62	101.7	99.8	97.3	101.2
	1960-63	101.5	99.4	97.8	101.3
	1960-64	101.5	99.3	98.1	101.1

it is only with bank debits in the second quarter that the direction is uncertain. The hypothesis that the seasonal pattern in some series undergoes a gradual change appears to be provisionally confirmed, though without computations based on a larger number of years no definite conclusion can be drawn. Some more mechanical adjustments of the data such as that for working days, might ensure still greater uniformity.

The series may be conveniently divided up into those with large, medium-size and small seasonal variation, according to whether the difference between the highest and the lowest seasonal index is greater than 20, between 5 and 20, or smaller than 5 index points.

The eight series in the first category comprise electricity output, the two agricultural series, insurance stamps sales, the two alternative unemployment series, car registrations and revenue receipts. In the case of these series, the seasonal pattern is for the most part readily explained by obvious factors such as varying consumption of electricity for heating, crop harvesting and seasonal employment in the summer months; in the case of new car registrations and revenue receipts, conventional and institutional factors are in operation.

Seasonal variations of moderate amplitude are found for industrial production, retail sales, imports, exports and bank debits. Industrial production shows a major peak in the second and a minor peak in the fourth quarter, associated partly with general variations in activity and partly with seasonal food processing. Imports are lower in the third quarter than at other times of the year, whilst exports are greater in the later than in the earlier part of the year, assuming of course other things to be equal. Thus a cursory inspection of trade figures based on the first half of the year would exaggerate the

balance of trade deficit, whilst a study of the third quarter alone would underestimate it. Finally, bank debits are somewhat below their normal level in the third but somewhat above normal in the fourth quarter.

Slight but still clearly recognisable regular variations are discerned for employment and earnings in transportable goods industries, agricultural prices, money supply and external assets. The employment level is somewhat lower in the first quarter than in the others; and earnings show the same drop at the beginning of the year but in addition a slight peak towards the end. Agricultural prices tend to be highest in the first and second quarter and lowest in the third quarter. The supply of money shows the same peak towards the end of the year as bank debits but not their steep drop during the third quarter. Finally, other things being equal, external assets have a tendency to fall first and to rise subsequently during the course of the calendar year.

With the help of the seasonal indices, seasonally corrected data can be derived from the original data; all that is necessary is to divide by the appropriate index. The indices based on 1958-62 are used to correct the 1963 data, the 1959-63 indices for the 1964 data and so on. For example, with the volume of production index standing at 159.5, 174.9, 170.5 and 176.6 respectively in the four 1964 quarters, the seasonally corrected figures become:

$$\begin{aligned} 159.5 \times 100 / 96.1 &= 166.0 \\ 174.9 \times 100 / 104.6 &= 167.2 \\ 170.5 \times 100 / 98.0 &= 174.0 \\ 176.6 \times 100 / 101.3 &= 174.3 \end{aligned}$$

The resulting figures show the trend combined with any irregular movements there may be. In this instance, the rising trend together with a levelling off due to the strikes in the fourth quarter 1964, are clearly demonstrated.

In this example, the average of the four quarters is the same whether calculated from the unadjusted or from the adjusted data. This property does not generally hold with multiplicative seasonal adjustments. Indeed, it could not be expected to hold, since with a rapidly rising or rapidly falling trend an equal proportionate addition and deduction has different quantitative effects at different times of the year.

In appropriate cases, the resulting seasonally corrected figures are multiplied by four to show equivalent annual totals, that is to say the annual rate at which the series tends to run according to the latest indicators. Where the original figure applies to a point in time as in the employment

series, the same strictly holds for the seasonally corrected figure.

In addition to the 18 series for which seasonality corrections have been directly estimated, it is also easy to arrive at seasonally corrected figures for 6 derived series. Thus the seasonally corrected figures for output per head are the ratios of the seasonally corrected figures for production volume and employment in transportable goods industries. The corrected figures for money earnings deflated by the consumer price index which is free from seasonality yields the corrected figure for real earnings. The differences between the seasonally corrected agricultural sales and farm costs, and between corrected imports and exports, yields corrected figures for net cash income in agriculture and for imports excess respectively. Since import and export prices are not subject to clearly recognisable seasonal fluctuations, the seasonal indices for value of imports and exports may be applied to obtain corrected figures for volume of imports and exports.

Most of the remaining series listed in "Statistics of economic level and trend" do not require seasonal correction. This conclusion has been derived from *a priori* considerations for 2 series, viz., net passenger movement outward for the last twelve months, and agricultural minimum wages. For a further 8 series, this has been shown by means of the analysis of variance described in section 2; the series include new houses built, 5 price indices and 2 series of banking statistics. This also applies to 1 further series, viz., terms of trade, which is derived from the indices of import and export prices.

Altogether, then, out of thirty-six series eleven are free from seasonal variation, and twenty-four which are subject to seasonal fluctuations permit correction. Exchequer expenditure is the only series which exhibits a marked degree of seasonality but for which satisfactory seasonal corrections cannot at present be made; seasonally corrected data for new car registrations are also of somewhat uncertain

value. Even without having the seasonally corrected data computed by the Economic Research Institute at hand, it will be a simple matter to derive the deseasonalized 1965 figures from the official data with the help of the latest set of seasonal indices given in Table 4. In the absence of further information, these indices may also be used for later years, though revised indices utilizing 1965 and later data should become available in due course.

The practical value of seasonally corrected data is obvious. With their help, valid comparisons between successive quarters may be effected for short-term economic analysis in order to discern the most recent tendencies.

Seasonally corrected data may also be utilised in the estimation of quarterly econometric models. Although on theoretical grounds, the use of unadjusted data together with dummy variables appears to be preferable, nevertheless the use of adjusted data offers practical advantages from the computational point of view. For large models at any rate, such as the Social Science Research Council model presented by Klein (1964), the use of seasonally corrected data seems to have become standard practice. It may also prove worthwhile to make use of seasonally corrected data for the purpose of econometric studies referring to Ireland.

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APPENDIX

CORRECTION OF MOVING AVERAGES FOR TREND, ESPECIALLY AT HIGH OR LOW TURNING POINTS

By R. C. GEARY

It is known that at monotonic sections of the time curve (ideally when the true trend is linear to time), the unweighted moving average (m.a.) gives a good representation of the true trend, though it may eliminate cycles of length less than the moving average period. At high and low turning points of the trend the moving average (graphed at its mid-time point) must modify the true trend, i.e. at a low turning point the m.a. will run above the trend line and at high turning point below it. This is clear from the fact that at e.g. a high turning point the m.a. is the centre of gravity of a curve section whereas the top point of the trend is on its periphery. The object of this note is to correct the m.a. at these turning points so as to yield the true trend value.

Let the time unit be the calendar quarter and let a turning point, e.g. a minimum, be near $t=0$. Then the individual time points of reference for the original data near $t=0$ will be marked $-\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}, +\frac{5}{2}$, and the centred four-quarter by m.a. points of reference will be $-2, -1, 0, +1, +2$. It is assumed that the true time trend (with seasonality eliminated) will yield the same moving average as does the original series (without seasonality correction). Suppose that near the origin the true trend can be represented by

$$(1) \quad Y_t = a_0 + a_1 t + a_2 t^2$$

and the corresponding m.a. point y_t . Our object in the first place is to compute the coefficients a_0, a_1 and a_2 from the m.a. We have

$$y_{-1} = \frac{1}{4}(Y_{-\frac{5}{2}} + Y_{-\frac{3}{2}} + Y_{-\frac{1}{2}} + Y_{+\frac{1}{2}})$$

and analogous expressions for y_0 and y_{+1} . Then, from (1), we have the following three equations to determine the coefficients from the known m.a. values y_{-1}, y_0, y_{+1}

$$(2) \quad y_{-1} = a_0 - a_1 + \frac{9}{4}a_2 \quad [\text{Note: } \frac{9}{4} = \frac{1}{4}[(-\frac{5}{2})^2 + (-\frac{3}{2})^2 + (-\frac{1}{2})^2 + (+\frac{1}{2})^2] \text{ etc.}]$$

$$y_0 = a_0 + \frac{5}{4}a_2$$

$$y_{+1} = a_0 + a_1 + \frac{9}{4}a_2$$

From (2) we find the following values of a_0, a_1, a_2 :—

$$a_0 = \frac{1}{8}(-5y_{-1} + 18y_0 - 5y_{+1})$$

$$(3) \quad a_1 = \frac{1}{2}(-y_{-1} + y_{+1})$$

$$a_2 = \frac{1}{2}(y_{-1} - 2y_0 + y_{+1}).$$

Now let the m.a. graph near the low point be

$$(4) \quad y_t = a'_0 + a'_1 t + a'_2 t^2$$

The coefficients are found from the values of y_t at $t = -1, 0, +1$:—

$$y_{-1} = a'_0 - a'_1 + a'_2$$

$$(5) \quad y_0 = a'_0$$

$$y_{+1} = a'_0 + a'_1 + a'_2,$$

whence

$$a'_0 = y_0$$

$$(6) \quad a'_1 = \frac{1}{2}(-y_{-1} + y_{+1})$$

$$a'_2 = \frac{1}{2}(y_{-1} - 2y_0 + y_{+1}).$$

Comparing (3) and (6) we see that

$$(7) \quad a'_1 = a_1$$

$$a'_2 = a_2$$

Hence

$$(8) \quad y_0 - Y_0 = a'_0 - a_0 = y_0 - \frac{1}{8}(-5y_{-1} + 18y_0 - 5y_{+1})$$

(from (6) and (3))

$$= \frac{5}{8}(y_{-1} - 2y_0 + y_{+1})$$

or the true Y_0 is found from the (known) m.a. y_0 by deducting

$$(9) \quad \frac{5}{8}(y_{-1} - 2y_0 + y_{+1}).$$

The latter expression will clearly be positive at a low point on the m.a. curve. Obviously at a high point an identical expression with sign changed is added to the m.a. maximum to give the true high point. These turning points will be systematically computed, marked on the chart, and the moving average graph adjusted free-hand near the turning points to give the true trend. Other guide points can be computed as desired, even though these are not maxima or minima. The adjustment (9) can be applied at any point on the m.a. graph, with y_0 as the m.a. ordinate of reference and y_{-1} and y_{+1} the ordinates to left and right. Clearly this adjustment is strictly necessary only where the m.a. graph is markedly curved: the adjustment is zero where the graph is linear. It would not really be onerous to make the correction at every point when one's period is, say, 5 years so that there are only 20 observations to be dealt with.

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