Collaborative, Contextual, and Technology-Mediated Mathematics Learning Activities: Design Heuristics and Effects on Student Engagement

A Thesis Submitted in Fulfilment of the Requirements for the Award of Doctor of Philosophy

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Declaration

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SUMMARY

This dissertation describes an approach to mathematics activity design that aligns the affordances of off-the-shelf technologies with relevant mathematics pedagogy. The aim is to create transformative learning experiences with the potential to overcome some of the well-documented impediments to mathematics teaching and learning.

A review of existing literature identifies a number of areas in mathematics education as problematic, with the lack of student engagement with the subject seen as an area of particular concern. Although technology has been heralded as having the potential to address such issues, there is evidence of a need for explicit design heuristics to guide the development and implementation of technology interventions in mathematics education. This is seen to be particularly relevant within the context of 21st Century learning, in which the key skills of mathematical creativity, critical thinking, problem-solving, communication and collaboration are emphasised. In order to gauge the current trends in technology-mediated mathematical research, and to identify whether the issues highlighted in the general literature review are being addressed, a systematic analysis of recent empirical studies of technology interventions in mathematics education has been undertaken. This has informed the development of a system of classification of the types of technology, the pedagogical foundations, the level of integration of the technology, and the goals of the interventions in which those technologies are used.

This research attempts to align appropriate educational theories of mathematics with the affordances of readily available technologies, in order to create learning experiences that have the potential to overcome some of the issues with engagement and confidence evident in the literature. A set of guidelines for practitioners and design heuristics, for the development of such learning experiences, are devised, along with a suite of sample activities created in accordance with them. Data relating to changes in student engagement and confidence through participation with the learning experiences are collected, and emergent issues relating to effective classroom orchestration are considered. The research questions thus relate to the development of a model for the creation and implementation of contextualised, collaborative and technology-mediated mathematics activities (RQ1); the impact this model of teaching and learning has on the student cohort; and the reasons underpinning how and why such an impact is being effected (RQ2).

Case-study within a design-based research paradigm has been chosen as the research methodology for the research, and a mixed methods approach is adopted with the collection of both qualitative and quantitative data. In order to gauge the suitability of the learning activities, and to refine the research questions, an initial, exploratory case study of pilot interventions in a laboratory-school setting is described. For the purposes of analysing the underlying causes of change in student engagement, and identifying the factors pertinent to the design heuristics, an explanatory case study is then presented, with multiple embedded units representing interventions in authentic school settings. A further exploratory case study, which details teachers' experiences with the heuristics, is also outlined.

The findings of the two primary case studies confirm that activities designed in line with the approach developed in this research have the potential to increase student engagement with and confidence in mathematics, thus addressing some of the issues identified in the analysis of the literature. Analysis of the final case study indicates that the design heuristics and guidelines give a requisite level of support to teachers, resulting in benefits to their own practice as well as to their students' experiences.

The main contributions of this research are:

i) A system of classification for technology interventions in mathematics education.
ii) A set of design heuristics and guidelines for practitioners for the development of collaborative, contextual and technology-mediated mathematics activities.
iii) A suite of activities developed in accordance with the design heuristics and integrated into the curriculum.
iv) An evaluation of the efficacy of the proposed approach to teaching and learning in addressing some of the problems in mathematics education that have been identified in the literature.

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I would like to dedicate this dissertation to my grandfather, Paddy Reilly, who would have been so proud.

Related Publications

Journal Articles

Bray, A., & Tangney, B. (2015). Enhancing Student Engagement through the Affordances of Mobile Technology: A 21st Century Learning Perspective on Realistic Mathematics Education. *Mathematics Education Research Journal*, 1 - 25.

Book Chapters

Tangney, B., Bray, A., & Oldham, E. (2015). Realistic Mathematics Education, Mobile Technology & The Bridge21 Model for 21st Century Learning - A Perfect Storm. In H. Crompton & J. Traxler (Eds.), *Mobile Learning and Mathematics: Foundations, Design, and Case Studies* (pp. 96 - 106). Oxon, UK: Routledge.

Peer Reviewed Conference Papers

Bray, A., Oldham, E., & Tangney, B. (in press). *Technology-Mediated Realistic Mathematics Education and the Bridge21 Model: A Teaching Experiment*. Ninth Congress of European Research in Mathematics Education (CERME9), Czech Republic, February 2015.

Bray, A., & Tangney, B. (2014). *Barbie Bungee Jumping, Technology and Contextual Learning of Mathematics*. 6th International Conference on Computer Supported Education (CSEDU 2014), 3, 206 - 213.

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Tangney, B., & Bray, A. (2013). *Mobile Technology, Maths Education and 21C Learning*. 12th World Conference on Mobile and Contextual Learning (Mlearn2013), Qatar, Oct 2013, pp 20 - 27.

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Bray, A. (2014). Barbie Bungee Jumping, Technology and the Contextualised Learning of Mathematics. Presented at Mathsfest 2014, Dublin, 2014.

Bray, A. (2015). Technology-Mediated Realistic Mathematics Education and the Bridge21 Model. Presented as part of the symposium: The Junior Cycle in Transition – the Bridge21 Model and Delivering the Curriculum. Presented at the ESAI conference 2015, Maynooth, 2015.

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1. Introduction

There is ongoing international debate about the quality of mathematics education at post-primary level. Research suggests that, while the capacity to use mathematics constructively will be fundamental to the economies of the future, many graduates of the secondary-school system have a fragmented and de-contextualised view of the subject, leading to issues with engagement and motivation (Ayinde, 2014; Boaler, 1993; Maaß & Artigue, 2013; Schoenfeld, 2004; Star et al., 2014b). This study examines how the affordances of readily available digital technology, combined with a 21st Century approach to teaching and learning, can be exploited to create mathematical activities that address common issues in mathematics education.

1.1 Research Problem

A retrospective view of curriculum developments since the 1970s indicates a gradual move away from a perception of mathematics as a purely formal body of facts and procedures as exemplified by the New Math movement (Schoenfeld, 2004; Treacy, 2012), to a conception of the subject as a dynamic and evolving discipline with an emphasis on exploration, conjecture and context (Schoenfeld, 2004; Van den Heuvel-Panhuizen, 2000). However, despite these trends in curriculum development the "implemented curriculum" – that is, what is actually taught in schools (Voogt & Pelgrum, 2005) – remains influenced by the New Math movement, frequently incorporating an explanation-exposition-practice approach to teaching, with procedure emphasised over concepts (Conway & Sloane, 2005; Oldham, 2001; Treacy, 2012). While many teachers strive to incorporate innovative practice into their daily teaching, the strictures of curriculum and assessment can curtail their efforts (Conneely, Lawlor, & Tangney, 2013; Conway & Sloane, 2005; Dede, 2010a; Hoyles & Noss, 2009). Traditional, behaviourist approaches to teaching and learning tend to persist, with an emphasis on formal, abstract mathematics (Albert & Kim, 2013; Conway & Sloane, 2005; Dede, 2010a; Maaß & Artigue, 2013). As a consequence, students' experience of the subject can be fragmented and lacking in context, compounding problems related to motivation and engagement (Boaler, 1993; Star et al., 2014b).

When combined with appropriate pedagogy, it has been suggested that digital tools may have the potential to address some of these issues, having the capacity to facilitate realistic, problem-solving and collaborative approaches to teaching and learning, which provide coherency and context for the mathematics. However, a review of existing literature reveals that although use of technology in the classroom is increasing, the outcomes of its utilisation do not live up to their perceived potential to enhance the learning experience (Geiger, Faragher, & Goos, 2010; Reed, Drijvers, & Kirschner, 2010;

Selwyn, 2011; Voogt & Pelgrum, 2005). Although the majority of students and teachers engage in the creative use of digital technologies every day, they do so less frequently in an educational context; here, it is more often used in a traditional manner, with didactic teaching methods, and an emphasis on de-contextualised procedure as an end in itself (Ainley et al., 2011; Hyde & Jones, 2013; Oldknow, 2009).

Tools such as Dynamic Geometry Systems (for example, GeoGebra, Cabri and Geometer's Sketchpad), Computer Algebra Systems (for example, Maple and Mathematica), tablet/smartphone apps, and educational websites all provide mathematics teachers with readily accessible, and often free, tools to help their students overcome the challenges in becoming mathematically creative and proficient. However, teachers can be overwhelmed when faced with an array of technologies and pedagogical theories, and could benefit from a framework to guide the development of activities that meaningfully integrate technology into their teaching so that it is not used in such a way as to merely re-instantiate aspects of traditional mathematics teaching (Dede, 2010a; Laborde, 2002; Olive et al., 2010; Sinclair et al., 2010; Trouche & Drijvers, 2010).

1.2 Problem Statement

Two related problem areas have been identified through the literature review. The primary issue is that students tend to have low levels of engagement and confidence with mathematics when it is taught in the traditional manner in schools. The second, related problem is that teachers can find it difficult to know how to integrate technology and 21st Century pedagogies that emphasise collaboration and inquiry, in mathematics education in such a way as to transform the topic into a vibrant and meaningful subject.

- **Problem Statement 1 (PS1)**

The negative aspects of a traditional approach to mathematics education are well documented in the literature: traditionally, a behaviourist approach to mathematics education has been adopted, manifesting in didactic teaching methods with an emphasis on procedure over understanding, and content over literacy (Conway & Sloane, 2005; Luhan, Novotna, & Kriz, 2013; Ozdamli, Karabey, & Nizamoglu, 2013; Star et al., 2014b). In this environment, the mathematics is frequently presented without context and lacks inter- or intra-disciplinary connections that could lend it a level of coherency (Boaler, 1993; Schoenfeld, 2004). This has been shown to impact negatively on students' engagement with, and confidence in, the subject (Boaler, 1993; Star et al., 2014b).

- **Problem Statement 2 (PS2)**

While digital technologies may have the capacity to facilitate realistic, problem-solving and collaborative approaches to teaching and learning, providing coherency and context for mathematics, they remain under-exploited in many secondary school education systems (Bredeweg, McLaren, & Biswas, 2013; Laborde, 2002; Trouche & Drijvers, 2010). This can be linked to a number of factors such as inadequate resources and continuous professional development (CPD), systemic issues at school and policy level, and fundamental issues typically related to teachers' core beliefs (Dede, 2010a; Donnelly, McGarr, & O'Reilly, 2011; Euler & Maaß, 2011; McGarr, 2009).

1.3 Research Aims

This research aims to develop a technology-mediated, inquiry-based and collaborative approach to activity design. The focus of the research will be on the changes in students' perceptions and experiences (engagement and confidence) of the subject as well as on the reification of a set of design heuristics for teachers' use.

Following from the problem statements, the research aims can be listed as a number of inter-connected goals:

- To develop an overview of current research trends in technology-enhanced mathematics education.
- To illustrate an approach to the design and implementation of activities and interventions that align relevant mathematics pedagogy with the affordances of readily available technology. Interventions that conform to these design heuristics should encourage the development of skills such as mathematical problem-solving and creativity in a technology-mediated, team-based environment.
- To collect evidence to show that student participation in such activities has the potential to increase levels of engagement with, and confidence in, mathematics.
- To determine the primary aspects of the interventions that lead to any changes in engagement and confidence.
- To provide guidelines for teachers to facilitate the design and implementation of such activities.

1.4 Research Questions

In pursuit of these research aims, two primary research questions have been identified, each of which consists of two parts. The first question (***RQ1***) relates to the development of a set of design heuristics and guidelines to assist teachers in the creation and implementation of collaborative, contextual and technology-mediated mathematics activities that encourage the desired skill set.

RQ1 (a) What are the desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence?

RQ1 (b) What are the key elements of a practitioner's guide for the creation and implementation of such interventions within the traditional school environment?

RQ1 is explored through an iterative approach to the design and implementation of a number of pilot student interventions and teacher workshops. This was conducted using an exploratory case study approach and is described in detail in chapter 6.

Throughout this research, a second research question (***RQ2***) is also investigated, which relates to the impact of the approach on students. The first part of RQ2 relates to the effect the interventions have on students' engagement and confidence, and the second part explores the reasons behind any changes:

RQ2 (a) What effects on student engagement and confidence does participation in activities designed in accordance with the heuristics and implemented using the practitioner's guide have?

RQ2 (b) What are the primary factors that cause such a change in engagement?

The explanation of the reasons that motivate changes in engagement and confidence feeds into the iterative development of the design heuristics and strengthens the argument for the use of the model within mathematics education.

1.5 Overview of Research Methods

The primary objective of this research – the development of a set of design heuristics and associated activities that have the potential to increase student engagement with the subject of mathematics – is a complex proposal that poses many difficulties in relation to the identification and testing of hypotheses and causality. In an attempt to address these difficulties, two case studies are proposed, to be conducted within a design-based research paradigm.

1.5.1 Research Methods

Anderson and Shattuck describe design-based research (DBR) as a methodology "designed by and for educators that seeks to increase the impact, transfer, and translation of education research into improved practice. In addition, it stresses the need for theory building and the development of design principles that guide, inform, and improve both practice and research in educational contexts." (2012, p. 16). Although this study does not aim to build a formal theory, the iterative approach to the development of design principles, in collaborative, authentic settings, with a practical application fit very well with the aims of the research. Within this framework, two case studies are conducted. The first is an exploratory case study relating to the first research questions (***RQ1***), which is used to pilot aspects of the study and to refine the research questions. The second is an explanatory case study, investigating the motivations for any recorded increase in engagement and confidence (***RQ2***).

Exploratory case studies are useful for the identification and refinement of research questions, hypotheses, or procedures that will be used in further research (Yin, 2014). Within this study, with the aim of addressing some of the challenges inherent in measuring attributes such as student engagement and motivation as well as piloting initial activities and generating hypotheses, an Exploratory Case Study is used (Chapter 6). This case study is made up of a number of pilot interventions conducted in a laboratory school environment (Bridge21[1], (Lawlor, Marshall, & Tangney, 2015)) within Trinity College Dublin. The aims of this study were to:

- Explore the activities with the students
- Explore effective classroom orchestration for interventions of this kind
- Investigate and refine data collection instruments
- Generate and refine research questions

Seven pilot interventions were conducted, with a total of 74 students taking part in the activities. Each intervention is viewed as an embedded unit (Yin, 2014), with the unit of analysis being student engagement and confidence.

While the use of an exploratory case study provides an opportunity to achieve some of the research aims, such as refining the design of the activities and the instruments, and honing the research questions, it is through the implementation of a second, explanatory case study, that a greater depth of understanding can emerge (Yin, 2014). The purpose of this second case study (Chapter 7) is to explain how and why student engagement with, and confidence in, mathematics changes through participation with activities designed and implemented in accordance with the design heuristics and practitioner's guide.

1.5.2 Research Design

The two case studies have multiple embedded units (Figure 1.1), and employ a mixed methods approach to data collection and analysis. The embedded units in the exploratory study are made up

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[1] Bridge21 is a learning and research centre in Oriel House, Trinity College Dublin, which was established to innovate, evaluate and refine 21st century learning methodologies (www.bridge21.ie).

of the seven Bridge21 pilot studies. Each of the interventions (Int 1 – Int 4) in schools constitutes an embedded unit in the explanatory study. In each case, the context is post-primary education in Ireland, differing only in the setting (Bridge21 or Authentic School Setting), and the overarching unit of analysis is student engagement. There is potential for a further case study to be added to this model, the context of which would be 'Post-Primary Education – Authentic Setting, Direct Observer role', in which the researcher would not take an active role in the classroom, but would observe the activity.

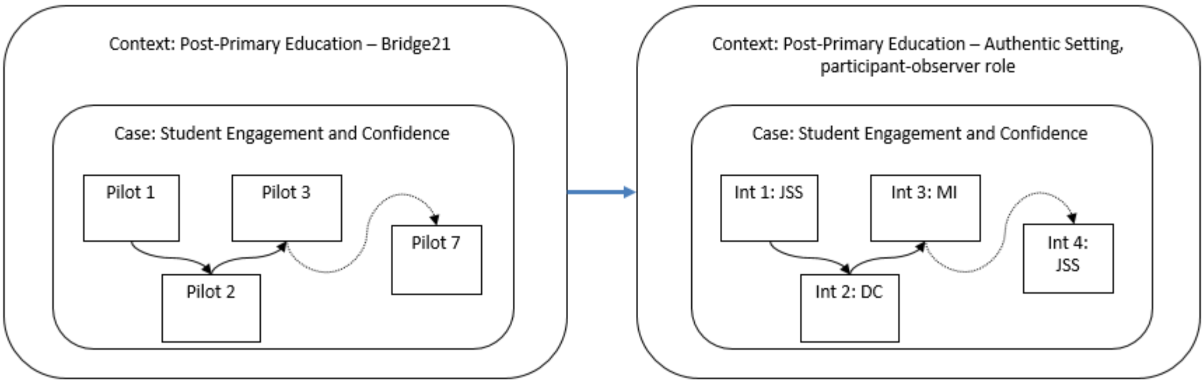


Figure 1.1: The two Case Studies, with sequence of embedded units

Each study is a single case study with multiple embedded units, in which all of the schools and students are part of a larger main unit of analysis (Yin, 2014). The choice of this single-case design, rather than a multiple-case, replication design, was made as the focus of the case studies is engagement and confidence, and not the individual schools or pilot studies. For this reason, the data from the embedded units (interventions) are pooled across schools/pilot interventions for the purposes of analysis.

1.5.3 Data Collection and Analysis - Mixed Methods

Mixed methods research refers to studies in which the researcher synthesises ideas, techniques, approaches, methods, and concepts from quantitative and qualitative research, within a single study (Johnson & Onwuegbuzie, 2004; Johnson, Onwuegbuzie, & Turner, 2007).

In this research, each embedded unit relates to a single intervention (or pilot study), in which both quantitative and qualitative data are collected. Both types of data are collected concurrently with emphasis given to quantitative data in the exploratory study, and qualitative data (Figure 1.2) in the explanatory one. Pre- and post-questionnaires are used to quantify the impact of the intervention on student engagement, while the analysis of interviews, observation, journal entries and comments explore the transformation in greater depth (Creswell, 2003), providing a rich and detailed description of *how* and *why* it has emerged.

Quantitative data is gathered using the Mathematics and Technology Attitudes Scale (MTAS) (Pierce, Stacey, & Barkatsas, 2007), which is administered to the students before and after each intervention. MTAS is a 20-item Likert-type scale, with five subcategories that relate to affective and behavioural engagement, mathematical and technical confidence and attitude to using technology for learning mathematics.

The exploratory case study uses directed content analysis (Elo & Kyngäs, 2008; Krippendorff, 2004) in order to examine the student data, and the explanatory case study employs a combination of an initial directed content analysis, followed by a second analysis using constant comparative techniques (Strauss & Corbin, 2008).

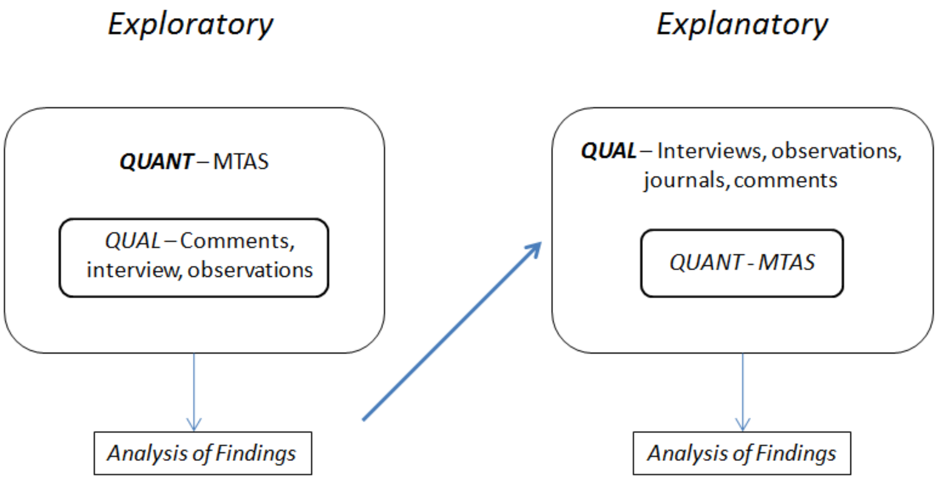


Figure 1.2: Mixed Methods Approach

The research design and process that informs the case studies described above is illustrated in detail in figure 1.3.

In addition to the examination of the student data, a total of 22 teachers used the design heuristics and practitioner's guide developed in this research to create and implement activities in their own schools as part of a continuous professional development course associated with the School of Education in Trinity College Dublin. Their written reflections on this process are also analysed with the purpose of identifying the benefits and barriers that the teachers may experience, with a view to further refining the heuristics. These reflections are analysed using constant comparative techniques (Strauss & Corbin, 2008).

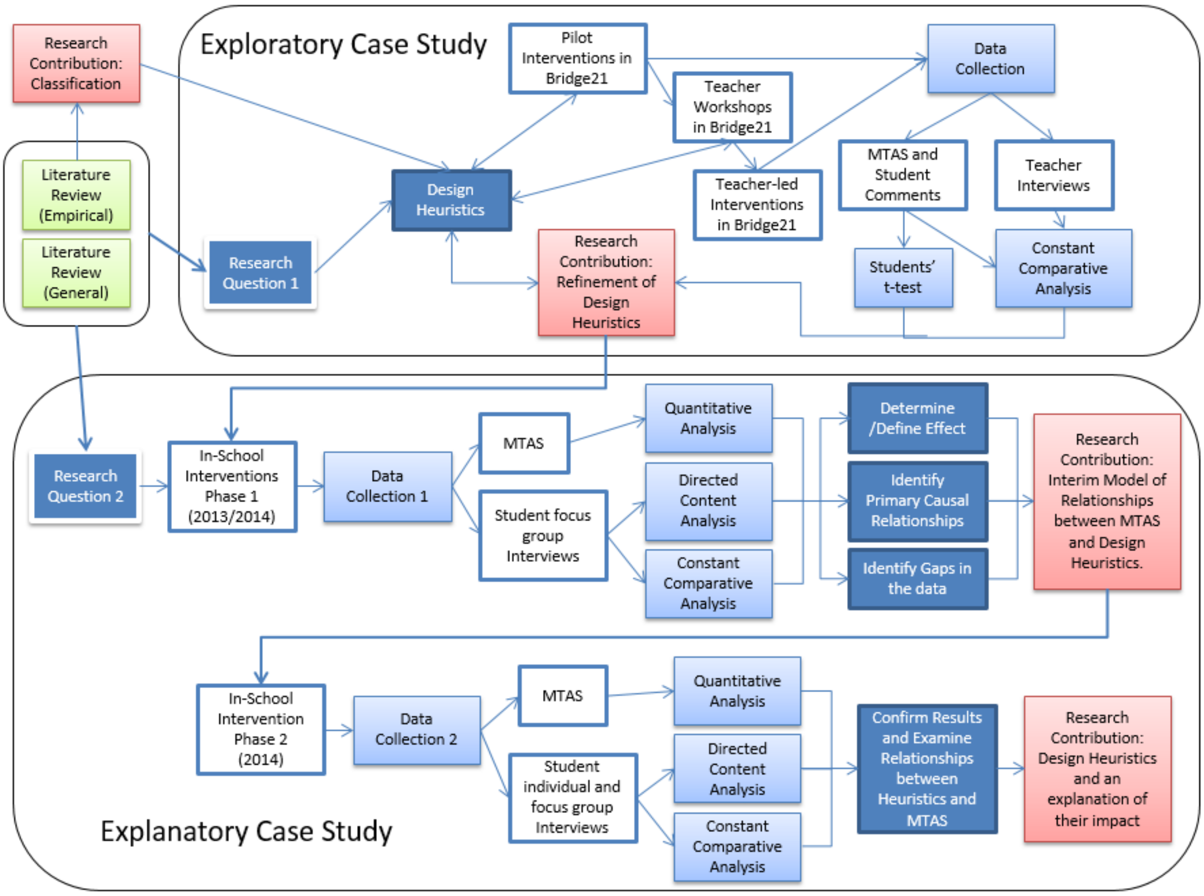


Figure 1.3: Research Design and Process

1.6 Contributions

To date, this research has been presented at three national conferences (IMS2012, Mathsfest 2014 and ESAI 2015), and has been published in peer-reviewed proceedings of five international conferences (CERME8, CSEDU5, ICTMT11, CSEDU6, and CERME9). Aspects of the work have also been incorporated into an international, peer-reviewed conference paper on mobile learning (MLEARN2013), and a chapter in the book *Mobile Learning and Mathematics: Foundations, Design, and Case Studies* (Crompton H., & Traxler J., (Eds), Routledge). An article for a special edition of the Mathematics Education Research Journal relating to the use of mobile technologies in mathematics education is currently in press and due for publication in March 2016. In addition to these publications, the contributions described in sections 1.6.1, 1.6.2, and 1.6.3 have also been made.

1.6.1 Development of a System of Classification

In order to generate an informed idea of current trends in research and to identify any gaps in the field, a review of recent interventions was carried out. A systematic analysis of this research was conducted through the lens of a classification system developed specifically for this purpose. The development of the classification system and results of the analysis are detailed in chapter 3.

1.6.2 Development of Design Heuristics

The results of the analysis of papers that detail empirical interventions (chapter 3), in conjunction with a more general literature review (chapter 2), has led to the evolution of a set of design heuristics, for the development of collaborative, contextual and technology-mediated learning experiences in mathematics, and a practitioner's guide to their implementation. These heuristics have been refined through a process of iterative development, informed by the literature review and classification, the analysis of implementations of related learning experiences in laboratory and natural settings, and reflections of teachers who have put the approach to use in their own classrooms. The development of the heuristics are discussed in detail in chapter 4.

1.6.3 Sample Activities

A number of sample activities have been developed consonant with the emerging design heuristics. Five activities have been piloted in the initial exploratory case study, and results indicate that they are pragmatic to implement and have the potential to help increase learner engagement with, and confidence in, mathematics. Three of the activities have also been integrated into larger scale interventions in authentic school settings. The activities are aligned with the curriculum areas of Statistics and Probability, Geometry and Trigonometry, Number, and Functions, while lending themselves to cross-curricular learning. They are discussed in Chapter 4 (4.3).

1.7 Structure of the Thesis

Following the introduction to the research presented in this chapter, the work is discussed in more detail over the next eight chapters.

Chapter 2 provides a general review of literature relating to ICT in Education and 21st Century Learning in order to give a broad background to the research. To further situate this study, a more focused perspective on Issues in Mathematics Education and Technology-Enhanced Mathematics Education is also presented.

Chapter 3 describes the development of a system of classification for empirical technology interventions in mathematics education. The development of the system is discussed, and the results of analysis of the classified interventions are provided.

Chapter 4 introduces the Design Heuristics, detailing the process of their development, and the rationale underpinning each of the points. In addition, five examples of learning activities designed in accordance with the heuristics are presented.

Chapter 5 considers a range of methodological issues and approaches, identifying a combination of design-based research and the case-study method as particularly appropriate for this research. Data collection and ethical considerations are also discussed in this chapter.

Chapter 6 presents an exploratory case study, which examines the impact that the pilot activities described in chapter 4, have on student engagement and confidence. This study focuses primarily on the quantitative data provided by the MTAS instrument, with the qualitative data focusing on the relationship between five factors identified by the quantitative instrument, MTAS, and the design heuristics. In addition to the pilot interventions, a number of teacher workshops are discussed along with an emerging practitioner's guide.

Chapter 7 discusses four interventions in that took place in authentic school settings in an explanatory case-study. The research in this section focuses on answering the 'how' and 'why' questions around the change in engagement and confidence of the students. Once again, the qualitative analysis focuses on the relationships between the subcategories of MTAS, and the design heuristics, with an in-depth analysis of the motivating factors and the types of learning experienced by the students.

Chapter 8 provides an initial analysis of the teachers' reflections on the use of the design heuristics for the creation and implementation of contextual mathematics activities in their own classrooms. The purpose of this chapter is to provide anecdotal triangulation of the impact of the approach on the students, as well as to present insights into its effects on the teachers' beliefs regarding their role in the classroom and the nature of teaching and learning.

Chapter 9 concludes the thesis by drawing together the findings of the case studies in order to answer the research questions. Additional outcomes of the research along with its limitations and directions for future research are also outlined in this section.

2. Literature Review

2.1 Introduction

This literature review aims to describe, synthesise and evaluate the broad areas of ICT in education, 21st Century Learning, mathematics education, and technology-enhanced mathematics education. The purpose is to outline the field of research, inform the development of the research questions, and contextualise the current research within the broader field.

Sections 2.2 and 2.3 provide a broad context to the field of research into which this dissertation is grounded – ICT in Education and 21st Century Learning.

- The discussion of **ICT in Education** begins with a brief historical background. This is followed by a review of the barriers to successful integration of ICT in education, as well as of the factors that particularly facilitate its usage. The impact of ICT on students and teachers will also be investigated.
- **21st Century Learning** (21CL) (2.3) is discussed in light of the previous section around the integration of ICT in classrooms, and incorporates some other pedagogically relevant aspect that make up the definition of 21CL used in this research. Barriers to its integration are also discussed along with ways to address these issues.

The subsequent sections review topics that comprise the main focus of this work. Section 2.4 gives a brief background to the field of Mathematics Education, and section 2.5 focuses more specifically on field of Technology Enhanced Mathematics Education.

- The review of **Mathematics Education** begins by providing a brief historical background to curriculum developments in the field from the 1960s. This is followed by a discussion of any mismatch between intended and implemented curricula, and where the barriers may lie.
- The **Technology Enhanced Mathematics Education** section takes a historical view of the area, but also looks at the types of tasks and tools that are best suited to using technology for teaching and learning mathematics. Once again, challenges to the integration of technology in mathematics education are discussed. A brief discussion of recent technology interventions in the subject is provided, which will be expanded upon in chapter 3.

Thus, from the literature review, the research problem emerges and is used to define the primary research aims of this thesis:

a. To identify the desirable attributes of technology-mediated mathematics learning activities that have to potential to increase student engagement and confidence.

b. To develop an understanding of the reasons for such changes, through the identification of relationships between the activity attributes and specific aspects of the students' experiences of mathematics.
c. To develop a set of design heuristics and a model for the creation and implementation of such activities by teachers in their school environment.

2.2 ICT in Education

2.2.1 Historical background of ICT in Education

In broad historical terms, the integration of technology into schools has tended to follow three main phases (Conole, 2008; Martin & Grudziecki, 2006; McGarr, 2009). The first was one of exploration and mastery of the technology, the main proponents of which were teachers of mathematics. Emphasis was placed on acquiring the specialist knowledge to understand how the computer worked and how to program it.

Following this early technical adoption in some schools, came the development of specific informatics subjects, with an emphasis on learning about how to use the technology, rather than learning with it (McGarr, 2009). However, the use of applications and software changed the focus of the definition of computer literacy away from specialist knowledge and towards practical competences. Simple graphical user interfaces allowed technology to be seen as a tool to be used in work and home environments, widening its appeal to non-experts (Martin & Grudziecki, 2006).

The third (current) phase is one of attempting to integrate technology into the curriculum (McGarr, 2009). However, how we interact and communicate has changed dramatically in recent times and the rising ubiquity of computer mediated communication along with the interactivity afforded by emerging digital technologies forms the basis of this shift (Conole, 2008; Ertmer & Ottenbreit-Leftwich, 2010; Fullan & Langworthy, 2014). Students increasingly expect to be able to work, access information, and interact wherever and whenever they want. In the workplace, learning and innovation skills are increasingly recognised as those that distinguish students who are ready for the complex, connected environment of modern society (OECD, 2012). The traditional idea of instructional technologies as a means of merely transmitting information to students (Jonassen, Carr, & Yueh, 1998; Li & Ma, 2010) is no longer seen as adequate. This may be partly fuelled by the realisation that some of the student-centred, constructivist and collaborative pedagogies proposed by innovative educators since the 1960s can be facilitated through appropriate use of technology in education (Fullan & Langworthy, 2014; Martin & Grudziecki, 2006). Olive et al. (2010) draw attention to the fact that technology has the potential to encourage new forms of practice, learning, and knowledge and should not be simply assimilated into traditional curricula: used innovatively, ICT could have the capacity to help redefine classrooms.

2.2.2 Barriers to the Meaningful Integration of ICT in Education

Many authors contend that, although use of technology in the classroom is increasing, its take up and perceived potential to enhance the learning experience lags behind its implementation in the classroom (Conneely, Lawlor, et al., 2013; Dede, 2010b; Donnelly et al., 2011; Ertmer & Ottenbreit-Leftwich, 2010; Hoyles & Lagrange, 2010; McGarr, 2009; Means, 2010; Office of Standards in Education, 2008; Psycharis, Chalatzoglidis, & Kalogiannakis, 2013; Voogt & Pelgrum, 2005). While many students engage in the creative use of digital technologies on a daily basis, they do so less frequently in an educational context (Conole, 2008; Oldknow, 2009; Pimm & Johnston-Wilder, 2005). Thus, the abilities of our students out of school, where they are rapidly acquiring new skills and sharing them with their peers, are not yet being reflected in many classrooms. While the conception of computers in classrooms has been described as being akin to the infrastructural role once reserved for pen and paper (Noss et al., 2009), technology has the potential to do much more than simply replace the traditional tools with a screen, without altering the tasks. Be that as it may, in schools technology is still frequently used as it was in the 1990s, to simply convey information to students (Conole, 2008; Ertmer & Ottenbreit-Leftwich, 2010; Jonassen et al., 1998) and to "transfer the traditional curriculum from print to computer screen"(Kaput, 1992, p. 516).

The simple addition of technology to a classroom is not enough to instigate educational change; alteration of the pedagogical approach and the learning experience of the students is also required, and this is fundamentally dependent on the actions and beliefs of teachers (Donnelly et al., 2011; Ertmer & Ottenbreit-Leftwich, 2010; McGarr, 2009). Although Dede (2010a) contends that the barriers to adjusting pedagogic practices are no longer conceptual, technical or economic, but are rather psychological, political and cultural, Donnelly et al. (2011) provide a less radical viewpoint highlighting two levels of barriers. The first considers resources such as equipment and support, which can be relatively easily overcome with financial input. Second level barriers relate more to Dede's definition, in that they are rooted in teachers' beliefs around their role in the classroom, teaching methods, and modes and purposes of assessment. It is easy to see how a third level, also consistent with Dede's view, could be added to this, relating to more systemic barriers such as curriculum and assessment. Thus we can view the barriers as being at micro, meso and macro levels. However, this delineation is not clear cut. While some of the micro-level barriers relating to infrastructure and training can be overcome with adequate financial support, others are not so trivial. Lack of support and effective communities of practice may be more related to the overall school culture rather than to any economic obstacles.

Regarding meso-level barriers, issues around teachers' beliefs can be very deep-rooted and difficult to change (Donnelly et al., 2011; Ertmer & Ottenbreit-Leftwich, 2010). In the traditional conception of a classroom, the teacher commands a dominant position, is regarded as "knower", and their role is one of transmission of information (Conneely, Murchan, Tangney, & Johnston, 2013). Often the pedagogic approaches that are complemented by technology do not fit into this teaching culture (Ertmer & Ottenbreit-Leftwich, 2010; Fullan & Langworthy, 2014; Voogt & Pelgrum, 2005). Attempts to alter the teacher's role from initiator and controller to facilitator and guide, through the integration of technology and associated "21st Century" pedagogies into the classroom, can be seen as undermining (Euler & Maaß, 2011). As a result teachers may accommodate the technology to conform to their current, "lecture-based" practice rather than alter their approach to make best use of the technology (Ertmer & Ottenbreit-Leftwich, 2010; McGarr, 2009; Voogt & Pelgrum, 2005). Furthermore, even when teachers are keen to integrate innovative practices, more systemic issues such as large class sizes and short class periods also tend to hamper the meaningful use of technology (Dede, 2010b; Voogt & Pelgrum, 2005).

At the macro-level, Means (2010) points out that many teachers will only expend the effort required to integrate technology into their teaching practice when they can see that there are significant benefits in terms of learning outcomes. However, current forms of standardised, high-stakes testing and assessment prevalent in many countries (including Ireland, despite efforts to the contrary), tend to focus on routine skills, and not on the kinds of problem-solving, creativity and decision-making skills that can be facilitated by the interactive, communicative and accessible nature of technology (Conole, 2008; Dede, 2010a; Fullan & Langworthy, 2014; Star et al., 2014b). Until evidence is provided that the use of technology will be of benefit, and that the skills that can be developed through its use are valued in assessment, it will remain difficult to convince teachers to change their practice (Donnelly et al., 2011).

2.2.3 Facilitating Factors

From the teacher's perspective, learning to teach with technology is not a trivial task (Conole, 2008; Ertmer & Ottenbreit-Leftwich, 2010; Trouche & Drijvers, 2010). Trouche and Drijvers (2010) use the metaphor of "orchestration" to highlight the importance of designing good tasks, but also take into account the technical aspects of the environment. Classroom orchestration acknowledges the importance of good classroom management (Means, 2010), as well as other pedagogic factors. A number of authors have conducted meta-analyses of the integration of technologies in school environments, with a particular focus on identifying what does and does not work (Li & Ma, 2010; Means, 2010; Voogt & Pelgrum, 2005). Drawing on their work, it appears that the positive effects of technology on learning were strongest when combined with a constructivist, team-based, project-based pedagogic approach, and non-standardised assessment methods (Ertmer & Ottenbreit-Leftwich, 2010; Li & Ma, 2010; Voogt & Pelgrum, 2005). In addition, larger positive effects on learning are identified when the students did not have a one-to-one relationship with the technology (Means, 2010), possibly owing to the increased emphasis on collaboration. Voogt and Pelgrum (2005) identify that in successful interventions the teachers act as facilitators to the students, providing structure and advice and keeping track of their progress. Students present the results of their work at the end of the project, which is then assessed by peers.

In addition to the appropriate methods of teaching and learning, Donnelly et al. (2011) and Fullan and Langworthy (2014) suggest that for change to be successfully accomplished, teachers require resources, practical examples and support from colleagues and management. The empirical results from research by Means (2010) concurs with this, concluding that support from the principal and colleagues is a factor that is present in all of the schools studied that exhibited learning gains with technology.

2.2.4 Impact of ICT in Education

The impact that successful technology-mediated interventions have on education can be categorised into three main areas: students, teachers and assessment.

2.2.4.1 Impact on Students

In their meta-analyses of technological interventions, both Li and Ma (2010) and Voogt and Pelgrum (2005) identify the positive impact that innovative, collaborative, technology-based approaches had on students' attitudes, problem-solving ability, content knowledge, and technological expertise. With a particular focus on the use of graphics calculators in mathematics lessons, it is noted that students' conceptual understanding of mathematics increases and that no differences are found in their procedural skills (Li & Ma, 2010). The acquisition of metacognitive skills is also identified as a positive impact of some of the interventions: "The biggest difference that the innovative practice brought about was that students changed from receivers who simply swallow presented materials to constructors who create their personal knowledge" (Voogt & Pelgrum, 2005, p. 171). These positive results are found irrespective of whether the technology is used to enhance collaboration, as an exploratory environment or as an information resource (Li & Ma, 2010; Voogt & Pelgrum, 2005).

2.2.4.2 Impact on Teachers

Perhaps it is not surprising that some of the positive outcomes that are identified at the student level also manifest among the teachers. In schools where the technology is successfully integrated, regardless of the nature of the intervention, an increase in the levels of collaboration and collegial support is identified (Fullan & Langworthy, 2014; Means, 2010; Voogt & Pelgrum, 2005). Many authors recognise the concerns of teachers to keep up with technological development (Ertmer & Ottenbreit-Leftwich, 2010; Means, 2010; Sinclair et al., 2010; Voogt & Pelgrum, 2005); the increase of collaborative skills is seen as being explicitly related to the formal and informal methods of continuous professional development, or CPD, that support the teachers' implementation of the interventions. Not surprisingly, the collaboration and support also leads to an increase in technological expertise.

From another perspective, the collaborative skills of teachers also has an impact on their role within the classroom as their once dominant position as the holder and transmitter of information and knowledge changes to one of co-learner and facilitator (Conneely, Murchan, et al., 2013; Fullan & Langworthy, 2014).

2.2.4.3 Impact on Assessment

Voogt and Pelgrum (2005) note that the methods of assessment in some schools was altered in line with the technological developments. Generally this manifested as an increase of emphasis on formative assessment. Regarding summative assessment, evaluation of products and presentations, rather than merely the results of traditional testing, is viewed as important for the assessment of technology-mediated activities (Li & Ma, 2010; Voogt & Pelgrum, 2005). The reason for this is likely to be related to the fact that the use of technology as a calculation device can trivialise many of the lower level, procedural questions common on traditional tests (Oates, 2011).

2.3 21st Century Learning

There is no single, universally recognised definition of 21st Century skills or of the types of teaching and learning required to achieve them. However, in their comparative analysis of international frameworks for 21st Century competences, Voogt and Roblin (2012) identify a common recognition of the development of skills relating to communication and collaboration, problem-solving and creativity, as well as technological fluency as being fundamentally important. Many of these skills can be defined as higher-order thinking and learning skills, or "life-skills", and they are seen as being transversal (not subject-specific) and multi-dimensional, impacting on attitudes and knowledge (Dede, 2010a; Voogt & Roblin, 2012). Although there is recognition that the benefit of these skills is not new, an approach that emphasises the importance of acquiring them in an integrated manner throughout curricular activities, combined with the potential of technology to assist in their realisation, can be viewed as innovative (Conole, 2008; Dede, 2010a, 2010b; Voogt & Roblin, 2012). The change in focus will require a shift in teaching and learning approaches, de-emphasising the more traditional procedural skills still common in educational practice, and increasing emphasis on the more complex skills that require an understanding of 'why' as well as 'how' these skills should be used (Conneely, Lawlor, et al., 2013; Dede, 2010b; Fullan & Langworthy, 2014; Voogt & Roblin, 2012). The role of technology is seen as important, not just in the delivery of the life-skills described above, but also in that it requires specific competences regarding the effective use, management and evaluation of information across many different platforms (Martin & Grudziecki, 2006; Voogt & Roblin, 2012).

It is generally recognised that 21CL can be best supported through specific pedagogic approaches such as Inquiry-Based Learning (IBL), Problem-Based Learning (PBL), and collaboration, as well as a more formative approach to assessment (Conneely, Murchan, et al., 2013; Conole, 2008; Fullan & Langworthy, 2014; Voogt & Roblin, 2012). Some of these approaches as well as barriers to their implementation are discussed in detail in the following sections.

2.3.1 Facilitating 21st Century Learning

Bearing in mind that technology is fundamental to the development of 21st Century skills, the facilitating factors for the integration of ICT in education identified in section 2.2.3, should form the basis of any attempt to implement 21CL in schools. Thus the foundation of a 21CL learning environment should be based on a constructivist, collaborative approach to teaching and learning, in which the teacher acts as a facilitator and guide of the learning (Conneely, Lawlor, et al., 2013; Fullan & Langworthy, 2014; Maaß & Artigue, 2013).

However, the ability to collaborate effectively is not necessarily an innate skill (Blatchford, Kutnick, Baines, & Galton, 2003). In order to increase the effectiveness of collaboration and group work, Baines, Blatchford, and Kutnick (2008) found that participants benefit from some related training. In their attempt to address the discrepancy between the potential of group work to influence motivation and learning, and its limited use in classrooms, the SPRinG project found that their training methods lead to increased student learning and motivation, and have a positive effect on their attitude and behaviour within the classroom. Thus in attempting to introduce 21CL into classrooms, it is beneficial to provide students and teachers with appropriate time and methods to develop their collaborative skills (Conneely, Lawlor, et al., 2013; Fullan & Langworthy, 2014).

An inquiry-based approach (Inquiry-Based Learning, or IBL) to the development of 21st Century skills is identified, within a European context, as the method of choice to increase young people's interest and achievement in certain educational domains (Euler & Maaß, 2011). Many terms and concepts have been used interchangeably with IBL, including discovery learning, problem-based learning and constructivist learning (Euler & Maaß, 2011; Maaß & Artigue, 2013). IBL promotes student engagement with processes such as diagnosing problems, critiquing approaches, distinguishing alternatives, planning investigations, researching and justifying conjectures, searching for information, and presenting coherent arguments (Maaß & Artigue, 2013).

As with the integration of ICT, the classroom atmosphere is important for the successful implementation of a 21CL approach to teaching and learning. This includes the teacher facilitating students' construction of their own knowledge through inquiry, and encouraging a shared ownership of what emerges in the classroom. The views and opinions of all participants in the classroom should be seen as valid, leading to an increased sense of ownership for the students (Euler & Maaß, 2011; Maaß & Artigue, 2013).

However, while IBL is recognised as being an "important ingredient for good education" (Euler & Maaß, 2011, p. 8), the authors also highlight that a balance between the exploration and the presentation of information is required. An effective implementation of 21CL should integrate a focus on content and core subjects as well as the higher-order learning and thinking skills (Dede, 2010b).

In order for the successful implementation of 21CL to be achieved in classrooms, appropriate assessment procedures need to be in place to determine whether the desired learning outcomes have been achieved (Dede, 2010b; Fullan & Langworthy, 2014; Voogt & Roblin, 2012). While standardised assessment of discrete knowledge may measure some of the skills and understanding that students require, it is not adequate for the measurement of the more complex competences associated with 21CL (Fullan & Langworthy, 2014; Voogt & Roblin, 2012). Therefore, a balance of summative and formative assessments that integrate traditional approaches with complex tasks that necessitate students to apply their understandings in collaborative, authentic scenarios is required (Dede, 2010b; Voogt & Pelgrum, 2005; Voogt & Roblin, 2012). Dede (2010b) contends that the assessment of core subjects and 21st Century skills should be combined in order to meaningfully integrate the knowledge and skills, and also that technology should be utilised in the assessment process. The use of e-portfolios and virtual learning environments have been suggested as having potential in this regard (Voogt & Pelgrum, 2005; Voogt & Roblin, 2012).

2.3.2 Barriers to the implementation of 21st Century Learning

Euler and Maaß (2011) identify three groups of problems associated with the implementation of a 21CL approach to teaching and learning: the overarching school system, a lack of resources including CPD, and teachers' beliefs. Similar to the issues surrounding the integration of ICT into the classroom, there are problems at the macro-level relating to policies and curriculum. In particular the confines of short class periods and existing assessment practices have been noted as restrictive; although assessment is generally "a primary driver of students' activity" (Hoyles & Lagrange, 2010, p. 84), traditional high-stakes exams do not generally test the kinds of skills prioritised by 21CL (Fullan &

Langworthy, 2014). Despite the fact that many countries identify the cultivation of 21st Century skills as a national objective (Voogt & Roblin, 2012), given the overcrowded nature of the majority of syllabi, a significant issue at the policy level relates to articulating what to emphasise in the curriculum in order to maintain a balance between the existing core subjects and a focus on the acquisition of such skills (Dede, 2010b).

Problems relating to classroom management and the difficulties that teachers may have in redefining their role are also identified, and may contribute to the gap between the intended curricula, which tend to recognise the importance of 21st Century skills, and that which is actually implemented (Conneely, Murchan, et al., 2013; Euler & Maaß, 2011; Voogt & Roblin, 2012). Not only are teachers expected to facilitate the acquisition of 21st Century skills amongst their students, but they are also expected to possess the skills themselves (Voogt & Roblin, 2012). Discussion alone is not sufficient to address these issues, rather a shift in the beliefs and practices of policy-makers and practitioners is required (Dede, 2010b). Educators need to be provided with adequate support and continuous professional development in order to master the necessary skills and teaching strategies, but also to ‘unlearn’ the beliefs and assumptions that underpin the traditional industrial-model of classroom practice (Conneely, Lawlor, et al., 2013; Dede, 2010b; Voogt & Roblin, 2012).

2.4 Review of Mathematics Education

The coincidence of technology, 21CL and mathematics education form the context for the work presented in this dissertation. While the former are relatively new disciplines, the history of mathematics education stretches back for millennia. This review provides an overview of the more recent developments in the field.

2.4.1 A Recent History of Curriculum Developments

Historically, the goals of mathematics curricula have varied depending on how mathematics is conceptualised: if is primarily understood as a body of facts and procedures relating to quantities and forms and the relationships between them, then the goal of mathematics education is mastery of these routines. On the other end of the spectrum, if mathematics is conceived of as a dynamic and evolving discipline, then the goals of mathematics education are to encourage exploration, the formulation of conjectures and the seeking of empirical evidence (Schoenfeld, 1992).

As a reaction to the Soviet success in their launching of the Sputnik satellite in 1957, the Americans developed a new, ‘modern’ mathematics curriculum that incorporated formal, abstract concepts such as set theory and symbolic logic. The ‘New Math’ curriculum radically altered the focus of the subject to prioritise structure, proof, generalisation and abstraction (Schoenfeld, 2004; Treacy, 2012).

The concepts behind New Math had an impact on curricula and pedagogy at an international level, which persisted in many countries for decades (Treacy, 2012).

In 1970s America however, there was a dramatic reaction against the ‘New Math’ movement. Parents and teachers had felt disenfranchised by the radically different and abstract focus of the 1960s curriculum. As a result, a ‘back-to-basics’ curriculum marked a return to the earlier curricular content, which largely focuses on skills and procedures (Schoenfeld, 2004).

By 1980, analysis of the previous decade’s concentration on basic skills indicated that students showed no improvements in the mastery of core mathematical procedures, and, not surprisingly, limited problem-solving ability (Schoenfeld, 1992, 2004). As a result, the focus of the curriculum turned to problem-solving (Schoenfeld, 2004; Van den Heuvel-Panhuizen, 2000). However, problem-solving within the mathematics curriculum at this time was frequently interpreted as merely replacing straightforward calculation with simple word problems (Schoenfeld, 2004).

The development of the National Council of Teachers of Mathematics (NCTM) Standards for School Mathematics (National Council of Teachers of Mathematics, 1989) led to a period of debate and change within mathematics education in the United States that came to be known as ‘Math Wars’ (Schoenfeld, 2004; Treacy, 2012). The NCTM Standards called for significant reform of instruction methods to include project work, group work and a more discursive relationship between the students and teacher. However, some cohorts of teachers and parents, whose sole experience of mathematics instruction had followed the traditional approach, found the reform methods inaccessible and alien (Schoenfeld, 2004). By the end of the 20th Century, the Math Wars had reached national scale in America.

From a European perspective, the New Math movement had a significant influence, which persists in many countries (Conway & Sloane, 2005; Treacy, 2012). However, a view of mathematics that focuses on real-world problem solving has become increasingly prominent (Conway & Sloane, 2005). This new curricular culture has been strongly influenced by Piagetian constructivism, situated cognition, and realistic mathematics education (RME) (Freudenthal, 1991). Constructivism and situated cognition both have their roots in cognitive educational and developmental psychology and view knowledge as something constructed by learners in social and material contexts. The RME approach stems from the work of Hans Freudenthal (Freudenthal, 1991; Van den Heuvel-Panhuizen, 2000). An emphasis on realistic context is the key idea that unites these ideologies and is in stark contrast to the New Math view of abstraction as the most important value in mathematics education.

2.4.2 Problems in Mathematics Education

What emerges through the above synopsis of recent historical curriculum directions is that a view of mathematical competence as solely related to procedures and concepts, and as accumulating with practice, is naïve and incomplete. There are equally important aspects of mathematical proficiency that relate to metacognitive skills such as creativity and problem-solving. However, there remains an unfortunately prevalent belief that mathematics is a collection of unrelated facts, rules, and 'tricks', and that mathematics education is about memorisation and execution of procedures that should lead to unique and unquestioned right answers (Ernest, 1997; Maaß & Artigue, 2013; Schoenfeld, 1992, 2004); that mathematics is "hard, right or wrong, routinised and boring" (Noss & Hoyles, 1996, p. 223). This has contributed to a behaviourist approach to teaching and learning, with an emphasis on formal, abstract mathematics remaining dominant in many countries (Albert & Kim, 2013; Conway & Sloane, 2005; Dede, 2010a; Maaß & Artigue, 2013; Ozdamli et al., 2013; Treacy, 2012). In this context, the teacher is frequently viewed as the absolute authority on the subject, their primary purpose being the transmission of information to the students. In conjunction with a strong focus on assessment, this has led to an environment in which mathematics is presented as a disjoint set of rules and procedures rather than a complex and interrelated conceptual discipline (Garofalo, 1989; Schoenfeld, 1992). Didactical teaching methods prevail, with an emphasis on procedure rather than understanding. Content is often favoured over mathematical literacy and learners are not encouraged to explore alternative answers or to seek out their own solutions (Conway & Sloane, 2005; Dede, 2010a; Maaß & Artigue, 2013; Schoenfeld, 1992). The resultant fragmented and de-contextualised view of the subject frequently leads to issues with motivation and engagement (Boaler, 1993; Star et al., 2014b).

Efforts to address some of these issues have been undertaken, but results have had limited success. The importance of embedding mathematics within meaningful context has been recognised, however this often resulted in pseudo-real-world problems – traditional computational problems with a thin veneer of 'real-world' through translation into simple word problems (Boaler, 1993; Foster, 2013; Olive et al., 2010; Schoenfeld, 1992). Not only are the problems frequently uninteresting from the point of view of the students, but they are also presented in such a way that they are not actually authentic problems, but are routine, practice problems in disguise. Many of the activities remain too well-defined in an attempt to reduce their complexity. Often all of the information to solve the problem, generally without surplus, is provided in the question, and the learner is reduced to following a procedure of putting data into appropriate formulae to get the 'right' answer (Buteau & Muller, 2006; Dede, 2010b; Maaß & Artigue, 2013). As a result of this narrow view of context, lack of emphasis on problem-solving, and overt focus on the mastery of routines and algorithms, students tend to lack the ability to apply their mathematical knowledge in anything but the most familiar contexts (Maaß & Artigue, 2013; Treacy, 2012). Their limited experience of tackling challenging problems can lead to numerous unproductive beliefs about the nature of mathematics (Schoenfeld, 2004). Students are more likely to believe that mathematical problems have only one right answer, that there is only one correct approach to achieve that answer, and that the answer should be achievable within a short period of time (Schoenfeld, 1992). When faced with problems in the real world, it is unlikely that their school-based strategies will be applicable and students may feel a sense of failure and perceive mathematics as a particularly difficult subject (Boaler, 1993; Schoenfeld, 1992; Treacy, 2012).

The difficulties that students may encounter in mathematics education can thus be summarised by the following points:

- A formal, abstract and fragmented approach to the subject.
- A perception of the teacher as an authority, who transmits information.
- Over-emphasis on a didactical approach to teaching.
- An emphasis on content over literacy and procedure over understanding.
- A lack of context.
- A lack of ownership of the subject for the students.

Mathematical competence is related to numerous factors. Having a strong understanding of the basic material is fundamental, but so is the ability to make use of this knowledge, which relates to meta-cognitive skills such as problem-solving strategies and creativity, and a "productive belief about oneself and the mathematical enterprise" (Schoenfeld, 2004, p. 263; Star et al., 2014b).

Education is however, a complex system involving schools, teachers, students and knowledge, and is subject to constraints of policy, curriculum, assessment, timetabling, and the beliefs of all stakeholders (Laborde, 2002). Teachers' epistemological and pedagogical beliefs tend to determine the nature of the classroom (Ertmer & Ottenbreit-Leftwich, 2010; Schoenfeld, 1992). Despite widespread acknowledgment that mathematics education should be embedded in authentic contexts, and that mathematical knowledge is culturally and socially based, classroom practice remains largely transmission-based and individualistic in nature (Boaler, 1993; Conneely, Lawlor, et al., 2013; Maaß & Artigue, 2013). Ruthven, Hennessy, and Deaney (2008) suggest that the mismatch between the intended curriculum, which highlights key skills such as problem-solving, creativity and collaboration, and the content-heavy curriculum that is actually implemented in classrooms stems from the fact that teachers tend to select and adapt the curricular materials so that it is assimilated into existing, traditional classroom practice. In order to address the issues at the student level, it will not be sufficient to simply change the instructional materials. Changes at the systemic level are required, which, in order to be successful, will require a shift in teachers' beliefs as well as changes within schools and policy.

2.5 Technology-Enhanced Mathematics Education

The use of digital technologies in mathematics education has the capacity to address many of the issues identified in the previous section, opening up diverse pathways for students to construct and engage with mathematical knowledge, embedding the subject in authentic contexts and returning the agency to create meaning to the students (Drijvers, Mariotti, Olive, & Sacristán, 2010; Olive et al., 2010). In addition to its computational power, modern technologies can help increase collaboration and bring about more of an emphasis on practical applications of mathematics, through modelling, visualisation, manipulation and the introduction of more complex scenarios (Geiger et al., 2010; Noss & Hoyles, 1996; Olive et al., 2010). For these reasons, the use of technology in mathematics education is becoming increasingly prioritised in international curricula (Geiger et al., 2010).

2.5.1 A Historical Overview of Technology-Enhanced Mathematics Education

Sinclair and Jackiw (2005) refer to the history of the relationship between technology and mathematics education as consisting of three waves. The focus of the first wave differs somewhat in emphasis from the history of the more general integration of technology in education described in section 2.2, but the development of the second and third waves broadly align.

2.5.1.1 First Wave

The first wave grew out of diametrically opposed approaches to learning and gave rise to widely differing technologies: *Logo* (Papert, 1980) on the one hand, was widely expressive in terms of mathematics and encouraged a constructionist approach to learning, supporting the link between students' actions and symbolic representations (Olive et al., 2010). On the other hand, the multiple-choice tests of *computer-assisted instruction technology* (CAI) had a very narrow level of expressivity, and embodied a behaviourist approach to learning. Unlike what has been classed as the first wave of the integration of technology in schools, the main proponents of which were the teachers (Section 2.2), in both of these cases, there is an exclusive focus on the student's engagement with the mathematical content, with little regard for the role of the teacher, classroom, or social environment. Possibly as a result of the lack of focus on classroom practice, teacher beliefs and the curriculum, neither types of technology have been particularly successfully incorporated in practice (Sinclair & Jackiw, 2005).

2.5.1.2 Second Wave:

The second wave expands from a focus on the individual's relationship with mathematics to incorporate the teacher and the curriculum. Figure 2.1 is adapted from Sinclair and Jackiw (2005) and Wright (2010), and represents some of the categories of technology described as making up the second wave, such as Computer Algebra Systems (CAS), and Dynamic Geometry Environments. The data analysis application *Fathom* (fathom.concord.org), is included in Figure 2.1 as an example of software that was designed with a focus on a particular aspect of the curriculum.

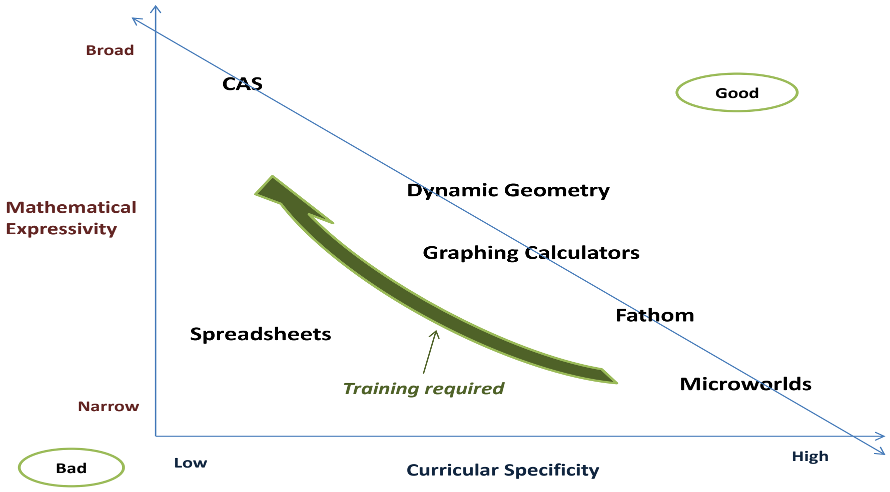


Figure 2.1: Some Second Wave Technologies

A high level of curricular specificity combined with the potential for broad mathematical expressivity is described as the most desirable blend of attributes. However, few technologies meet these criteria; the dominant linearity of the plot reflects the trade-off between curricular specificity and potential for mathematical expressiveness (Sinclair & Jackiw, 2005; Wright, 2010). The linear relationship between expressivity and specificity can also be seen as representing the amount of training required to use the particular technology for learning mathematics (Wright, 2010).

Some of the technologies that are considered as belonging to the second wave are: *Dynamic Geometry Environments or Systems* (DGE/DGS) such as GeoGebra, Cabri Geometry and Geometer's Sketchpad. These are highly expressive technologies, but are not specifically linked to curriculum and their "comprehensive unifying scope runs counter to the particularized, balkanizing manner in which most school curricula chop up mathematics" (Sinclair & Jackiw, 2005, p. 241). The original four function calculators of the 1970s, were quickly developed into scientific calculators and *Graphics calculators* (GC), which are algebraic and data analytic tools that cover a range of mathematical topics (Pimm & Johnston-Wilder, 2005). With these handheld devices, there has traditionally been a trade-off between power, speed, memory and size, and portability and price. However, the development of increasingly powerful handheld devices such as tablets and smartphones is lessening the need to compromise (Trouche & Drijvers, 2010). Some more specific technologies such as the software *Fathom*, have a narrower more focused level of expressivity, which leads to greater alignment with the curriculum, and a closer resemblance to professional software designed with a particular application in mind. *Microworlds* such as MathSticks (Noss, Healy, & Hoyles, 1997) are described as being constructivist learning environments that instantiate a specific, well-defined sub-domain of the subject. Thus, by definition, they are specific to a particular area of mathematics, usually closely related to the curriculum. Wright (2010) suggests that the very specificity of Microworlds may lead to students becoming bored or disengaged and points to the need for activities to be carefully designed in such a way as to encourage the use of technologies as problem-solving tools.

2.5.1.3 Third Wave:

Sinclair and Jackiw (2005) look to future developments, which have been largely realised in the intervening decade, as defining the third wave of technology usage in mathematics education. These developments have further expanded the pedagogic focus of the integration of technology to include "relationships among individual learners, groups of learners, the teacher, the classroom, classroom practices and the world outside the classroom", (Sinclair & Jackiw, 2005, p. 244). Networked calculators and other handheld technologies, along with the increasingly ubiquitous interactive white boards, are recognised as having the potential to create social and collaborative learning environments (Pimm & Johnston-Wilder, 2005; Trouche & Drijvers, 2010). This perceived capacity is further enhanced by wireless capabilities along with the widespread use of tablets, laptops, and smartphones, which allow for mobility and access to the internet as well as to increasingly sophisticated mathematics packages (Trouche & Drijvers, 2010). Virtual Learning Environments (VLEs) such as MOODLE are becoming widely utilised in schools, offering students an opportunity to learn anytime and anywhere, while also facilitating teachers' management of learning and assessment (Psycharis et al., 2013).

2.5.2 Facilitating the use of technology for teaching and learning mathematics.

The evolution of the use of technology in mathematics education has increased the perception that problem-solving and inquiry, and not just the memorisation of a catalogue of facts and procedures, should be at the heart of mathematics education (Geiger et al., 2010; Hoyles & Lagrange, 2010). The availability of technology in a classroom environment will not however, ensure the development of a collaborative and explorative classroom (Geiger et al., 2010; Olive et al., 2010). As with the integration of ICT discussed above (Section 2.2), the role of the teacher, appropriate task design and consideration of the learning environment, are fundamental for the facilitation of a discursive, inquiry-focused atmosphere in the mathematics classroom (Conneely, Lawlor, et al., 2013; Geiger et al., 2010; Laborde, Kynigos, Hollebrands, & Strässer, 2006; Olive et al., 2010; Swan, 2007).

It is necessary to carefully consider the kinds of gains to mathematics learning that can be made through the introduction of technology (Sinclair et al., 2010), and to design tasks accordingly. Artigue (2002) distinguishes between the pragmatic and epistemic value that technology can bring to tasks: digital technology can act as efficiency tools, to increase the speed and accuracy of computations (pragmatic), or they can contribute to students understanding of the mathematics (epistemic), thus becoming a conceptual toolkit and a "source of questions about mathematical knowledge" (Artigue, 2002, p. 248; Oates, 2011; Olive et al., 2010; Ruthven et al., 2008).

Laborde (2001, 2002) recognises four levels of technology integration in task design: at the lowest level are tasks that use the tools to directly substitute for traditional practice, such as measuring and drawing; at the second level the technology is used to facilitate exploration and analysis, such as dragging in a dynamic graphical environment; the third level is characterised by significant redesign of the tasks through the use of the technology; and the highest level on the scale constitutes tasks that could not have been conceived of without the use of technology. This classification of technology integration is mirrored in the more general framework provided by Puentedura (2006), which is discussed in more detail in section 3.1. Several researchers argue that it is preferable to utilise technology in tasks that are transformed by its application – i.e. that fit into the two higher levels distinguished by Laborde (ibid.) – rather than in tasks that could have been completed without its use (Laborde, 2001, 2002; Noss et al., 2009; Oates, 2011; Olive et al., 2010).

As a starting point for task design, several authors highlight the importance of genuine and engaging contexts for the activities in order to create compelling goals that the students require mathematics to solve, and in which the technology has an important role (Confrey et al., 2010; Geiger et al., 2010; Olive et al., 2010). Oldknow (2009) suggests possible criteria for such activities can be summed up by the acronym Al Fresco: Accessible, Lively, Fun, Reliable, Easily set up, Safe, Cheap, Open-ended. In such an environment, students are given an opportunity to use technical tools as experimental instruments to make practical use of mathematics for genuine and productive purposes, rather than for the application of rote-learned formulae and procedures to contrived scenarios (Olive et al., 2010).

The inquiry-based approach that was identified in section 2.3 as being particularly appropriate for 21CL has also been recognised as relevant for technology-enhanced mathematical tasks (Confrey & Maloney, 2007; Geiger et al., 2010; Psycharis et al., 2013). The affordances of technology for modelling, experimentation and testing of ideas, as well as the visualisation of abstract mathematical concepts, can change the nature of the mathematics classroom from a transmission-based, teacher-led environment, to a student-centred, investigative and constructivist one (Olive et al., 2010). A fundamental concept of task design in an inquiry-based learning (IBL) environment relates to the open-ended nature of the activities (Geiger et al., 2010). The use of digital technologies in mathematics education can allow for diverse routes for learners to solve problems and reach their goals (Hoyles & Lagrange, 2010), giving students control over their progress through the material (Buteau & Muller, 2006; Olive et al., 2010; Wright, 2010). Supporting students' autonomy over their learning in this manner has the potential to strengthen their mathematical confidence and increase their enjoyment of the subject (Boaler, 1993; Noss et al., 2009).

In order to be in a position to design tasks that take advantage of the affordances of available technologies, teachers tend to want to have some level of proficiency with the tools themselves, as well as to be familiar with the appropriate pedagogies for their usage (Trouche & Drijvers, 2010; Voogt & Pelgrum, 2005). Sinclair et al. (2010) discuss the trade-off between learning a few tools well enough to use them fluently, and the fact that this may curb the adoption of constantly emerging new software that potentially better addresses emerging needs. Indeed, Wright (2010) hypothesises that the adoption of new technologies by teachers and students is most successful if it does not incur a large investment of time and effort, and where the gains offered by the technology are easily identified.

2.5.3 Challenges to the use of technology for teaching and learning mathematics

The challenges of the integration of technology in mathematics education reflect many of the barriers discussed earlier in this chapter. There are issues centred on the confines of curriculum and assessment, the strictures imposed by the infrastructure at school level, and the intransigence of some teachers' beliefs (Buteau & Muller, 2006; Dede, 2010b; Oates, 2011; Olive et al., 2010; Trouche & Drijvers, 2010).

Some authors have identified a shortfall in theory relating to the integration of the IBL approach and traditional instruction (Li & Ma, 2010; Maaß & Artigue, 2013; Noss et al., 2009). Similarly, difficulties are highlighted in altering the role of the teacher from instructor to facilitator, indicating that such a role can be demanding and difficult to implement in a traditional classroom setting, and pointing to a need for a structured approach based on sound research (Noss et al., 2009).

2.5.3 Recent Interventions

In order to assess the current situation with regard to technology usage in mathematics education, a systematic review of recent literature was undertaken early in this research project, with the intention of devising a system of classification. An initial classification of 25 papers was conducted prior to the development of the first activities. Throughout the research, further papers were added and classified leading to an overall collection of 114 classified articles that describe empirical studies of the use of technology in mathematics education. The process and results of the classification are described in detail in Chapter 3, but are synopsised in this section.

It quickly became clear that there was little evidence of 'first-wave' technologies in recent literature. The first wave is characterised by an exclusive focus on the student's engagement with the mathematical content, with little focus on the role of the teacher, classroom, or social environment (Sinclair & Jackiw, 2005). An exception to this was a study on the use of video podcasts (digital files made available on the Internet for download) for the instruction of specific procedural mathematical problems (Kay & Kletskin, 2012).

The majority of the papers reviewed showed a marked focus on curriculum alignment (second-wave) and used diverse technologies, with particular prominence given to Dynamic Graphing Environments (DGE) and Computer Algebra Systems (CAS) (e.g. Santos-Trigo and Cristóbal-Escalante (2008), Maracci, Cazes, Vandebrouck, and Mariotti (2009), and Geiger et al. (2010)).

A significant number of papers also concentrated on the effects of the technology on the interactions between the students, teachers and the environment, and the effects of the technology-mediated community of learners on the mathematical learning (third-wave) (e.g. Arzarello, Ferrara, and Robutti (2012), Hitt (2011), and Kynigos and Moustaki (2013)).

2.6 Discussion

It is evident from the general literature review and the classification, that a wide range of technologies are being researched in different environments, with different agendas and from varying theoretical standpoints. What they have in common seems to be a desire to create engaging environments in which the technology is used to increase the students' interest, motivation and performance. The pervasive perception of mathematics education emerging from the literature is one that focuses on understanding of relations, processes and purposes, as opposed to the requirement to learn a fixed body of knowledge; as such, the role of the technology is more as a "conceptual construction kit", as opposed to an efficiency tool (Olive et al., 2010). There is a move towards connection, coherency and context as important aspects of mathematics education that can be facilitated by technology.

Some of the technological interventions classified in chapter 3 address issues around the absolute authority of the teacher, creating a shift in empowerment from the external authority of the teacher, to the students as “generators of mathematical knowledge and practices” (Drijvers et al., 2010). The technology is used to “motivate students to take on, more and more, the responsibility of mediator in their own mathematics learning” (Buteau & Muller, 2006, p. 77). Tools are also being used to give students new ways to visualise concepts and approach problems in a dynamic way; authentic contexts and realistic data can be used without becoming overbearingly complex.

While digital technology has the potential to open up new routes for students to construct and comprehend mathematical knowledge and new approaches to problem-solving, this requires a change in the pedagogical approach in the classroom in terms of student engagement with learning (Drijvers et al., 2010). The development and deployment of innovative interventions may give food for thought but, in many cases, this is also the limit to academic involvement. Pimm and Johnston-Wilder (2005) liken the mismatch between theoretical and technological developments and their impact on pedagogy to “attempting to walk on a shale hillside” (p. 6). Exemplars are developed, often relying on the assistance of the research team to deliver them in practice, but when the researchers move on teachers are left to their own devices. Hence many interesting educational technology innovations remain at the periphery of practice and do not make their way into the mainstream (Boaler, 2008; Maaß & Artigue, 2013; Tangney & Bray, 2013). Ruthven et al. (2008) suggest that teachers in everyday classrooms frequently do not use the available technology in its intended, exploratory way, possibly even restricting exploration in order to avoid outcomes that do not match up with their intended learning trajectory. Numerous reasons have been cited as impacting on this divide between theory and practice. Geiger et al. (2010) point out that research on digital tool use in classroom environments is complicated and requires methodologies that are capable of accommodating “educational phenomena that are situated, temporal and complex” (p. 56). In order to avoid this complexity, research in the area can be too narrowly focused, and restricted in scope (Boaler, 2008; Maaß & Artigue, 2013). Within the wider research community, there can be a lack of appreciation of the types of research needed to facilitate effective and sustainable impact on practice – because of the complexity of the school environment, it is often impossible to follow a rigorously scientific approach involving control groups (Maaß & Artigue, 2013).

Olive et al. (2010) make the point that “it is not the technology itself that facilitates new knowledge and practice, but technology’s affordances for development of tasks and processes that forge new pathways” (p154). It is essential to conduct research into the design and development of tasks and activities that provide engaging environments, in which the mathematics is seen as relevant by the students, with goals that they find compelling (Confrey et al., 2010; Laborde, 2001; Oldknow, 2009). More specifically, Laborde (ibid.) argues for the development of tasks that are transformed through the use of technology, and that new mathematical practices, such as modelling of real-life events, and observations based on deductions should take precedence over tasks that could be just as easily completed without technology.

It is also important to consider the enormous potential in the technology that is readily accessible to students and educators. Oldknow (2009) suggests that the transformative potential of ICT is not restricted to new, or purpose built technology, but also lies in the innovative uses of everyday equipment such as cameras and mobile phones. In order to harness this potential, the affordances of off-the-shelf technology to alter the teaching and learning should be considered, providing for the investigation of challenging and interesting problems, and the development of flexible and creative solving strategies, in easily replicable situations. Students and teachers need to be encouraged to be flexible when it comes to the adoption of new technologies that may fulfil emerging needs (Sinclair et al., 2010).

Oates (2011) and Geiger et al. (2010) provide evidence that the outsourcing of computation through the use of technologies such as Computer Algebra Systems (CAS) has the potential to do more than just improve speed and accuracy of calculations. It can also provide increased opportunity for the development of investigative skills and problem-solving strategies by alleviating the need to learn off a catalogue of procedural techniques. The emphasis in teaching can thus be placed much more on the *why* than on the *how*. Oates (2011) also highlights the fact that the use of technology can change the relative value of some topics such as routine algebraic skills, often reducing their usefulness and even questioning their place in the development of mathematical knowledge.

In order to integrate technology into teaching and learning, issues around assessment frequently arise; it is viewed as an essential part of the learning cycle and is frequently a strong motivator for students (Drijvers et al., 2010). Some of the technologies analysed in the process of classification in chapter 3, have the capacity to provide summative assessment either for the student during the course of the task in hand, or for the teacher. Assessment can be administered through computer based testing, intelligent tutoring systems, use of collaborative documents or knowledge fora (Lazakidou & Retalis, 2010), or student devices networked to the teacher console (Noss et al., 2012). Data collected in this manner can be used for individual student reports, or for summary-level information for the class as a whole. It is also noted (Means, 2010) that teachers who actively facilitate and scaffold their students interactions with the technology are in a position to use their insights to refine the activities and inform instruction. In essence, the students’ interactions with the technology can contribute to their formative assessment.

Prior to the development of any technology-mediated intervention, it is necessary to look at the circumstances under which learning can be enhanced by technology. Kieran and Drijvers (2006) contend that mathematical tasks that make use of technology should not be studied without also paying careful attention to the classroom environment and the role of the teacher. Innovation and preparation, with regard to the working environment and class routine, are necessary in order to ensure that the full potential of technology is exploited (Means, 2010). In addition, the creation of an atmosphere in which students play a participative and collaborative role, with shared ownership of the learning and a dialogic relationship with the teacher, are more likely to lead to success (Euler & Maaß, 2011; Li & Ma, 2010). Means (2010) points out that, contrary to popular belief, higher learning gains are evident when there is not a one-to-one relationship between the student and the technology, thereby encouraging collaboration and team-work. Higher learning gains are also associated with classrooms in which an established routine is in place for moving between technology-mediated and traditional activities (ibid).

The requirement for sustained, integrated support and professional development of teachers, both in-service and pre-service, emerges as essential for the integration of technology and associated 21st Century pedagogies in educational settings (Ponte, 2008). In order to bring about change, such professional development should have relevance for day-to-day teaching include resources and practical support for change, and are most likely to succeed if they include collegial support (Donnelly et al., 2011; Maaß & Artigue, 2013; Means, 2010). Thompson and Wiliam (2008) advocate for a “tight but loose” approach to CPD, with strict adherence to design principles, but flexibility regarding the constraints and needs of specific contexts.

2.7 A Theoretical Framework: RME and Bridge21

It has been suggested that within an appropriate pedagogical framework, the use of technology can make mathematics more meaningful, practical, and engaging (Ainley et al., 2011; Drijvers et al., 2010; Hoyles & Lagrange, 2010; Olive et al., 2010). Similarly, the use of context and the process of mathematization in Realistic Mathematics Education (RME) have the potential to address some of the limitations associated with the more formal and abstract traditional mathematics education (Gravemeijer, 1994; Noss et al., 2009; van den Heuvel-Panhuizen, 2002). Social constructivist educational theories are advocated as aligning particularly well with the affordances of technology (Bray & Tangney, 2013; Li & Ma, 2010; Patten, Arnedillo Sánchez, & Tangney, 2006), and are also highly compatible with RME. However, activities combining mobile technology, social constructivism and RME tend not to fit well with the didactic teaching and short class periods common in the conventional classroom (Wijers, Jonker, & Kerstens, 2008); pedagogical models more in line with 21st Century learning may be more appropriate (Bray, Oldham, & Tangney, in press; Voogt & Roblin, 2012). In this section, key features of the elements of the framework that underpin the Design Heuristics – RME and the Bridge21 model of 21CL – are outlined.

2.7.1 Realistic Mathematics Education (RME)

RME is an approach to mathematics education that involves students in the development of their understanding through the exploration and solution of problems set in contexts that engage their interest. Rather than transmitting information in a lecture-based manner, teachers scaffold the "reinvention" of the mathematics that the students encounter (Freudenthal, 1991). Five essential characteristics of RME have been identified:

1. "The importance of problems set in contexts that are real to the students,
2. The attention paid to the development of models,
3. The contributions of the students by means of their own productions and constructions,
4. The interactive character of the learning process, and
5. The intertwinement of learning strands." (Tangney, Bray, & Oldham, 2015)

It should be noted that the contexts do not have to be drawn from the real world; the important aspect is that the students find them meaningful (van den Heuvel-Panhuizen, 2002).

These five characteristics guide a process called 'progressive mathematization' (Gravemeijer, 1994). This involves:

1. Presentation of a problem set in a context;
2. Identification of the relevant mathematical concepts involved;
3. Gradual refinement of the problem so that it becomes a mathematical one that represents the original situation;
4. Solution of that problem; and
5. Interpretation of the solution in terms of the original situation.

Mathematization has two aspects, known as 'horizontal mathematization' and 'vertical mathematization'. These are described by Dickinson, Hough, Searle, and Barmby in terms of modelling: "The process of using a model to solve a particular problem is known as 'horizontal mathematisation', while that of using the model to make generalisations, formalisations etc. is known as 'vertical mathematisation'" (2011, p. 48). As the students engage in the process of progressive mathematization, they encounter the mathematical concepts first informally, then 'pre-formally', and only eventually at a formal level. Geiger et al. (2010) highlight that while the conceptualisation of the model is primarily a human activity, the abstraction of the model and solution of the problem can be enhanced through the incorporation of technology.

Realistic Mathematics Education has been influential at an international level over the last decades, as evinced by its coverage in successive ICMEs in the 1990s; attempts to adopt or adapt it to local settings in various countries such as USA, England, South Africa, Germany, Denmark, etc. (De Lange, 1996, 1998); and its consonance with constructivism and problem-solving approaches (Karp, 2013). At a national level RME, along with constructivism, problem-solving and other approaches, has had a noteworthy impact on the development of the new Irish mathematics syllabus, Project Maths (Treacy, 2012). It is hoped that an approach to task design underpinned by RME, with a particular emphasis on mathematization, will achieve many of the desirable activity attributes described in this chapter.

2.7.2 The Bridge21 Model of 21st Century Learning

Bridge21 is a pedagogic model of 21CL that integrates the use of technology in a seamless, meaningful and transformative manner, encouraging participants to discover, analyse, synthesise, visualise and create (Lawlor, Conneely, & Tangney, 2010; Lawlor et al., 2015). It was originally conceived of as an out-of-school outreach programme, and in recent years has been adapted for use in Irish post-primary schools. Currently it is being trialled in a number of schools as part of a systemic reform process in Irish education (Johnston, Conneely, Murchan, & Tangney, 2014). Bridge21 embodies a collaborative, inquiry-based approach to the development of basic (Maths, History, English, etc.) and transversal (Digital and Information Literacy, etc.) skills.

The Bridge21 pedagogic approach was developed with the concept of teamwork at its core. In combination, the elements of the model (Figure 2.2) facilitate collaboration, and exploit the positive outcomes of learning in a team-based environment (Lawlor et al., 2015). Collaboration and teamwork are not necessarily innate skills however, and thus the Bridge21 programmes incorporate initial team-skills development activities, in accordance with the recommendations of Blatchford et al. (2003).

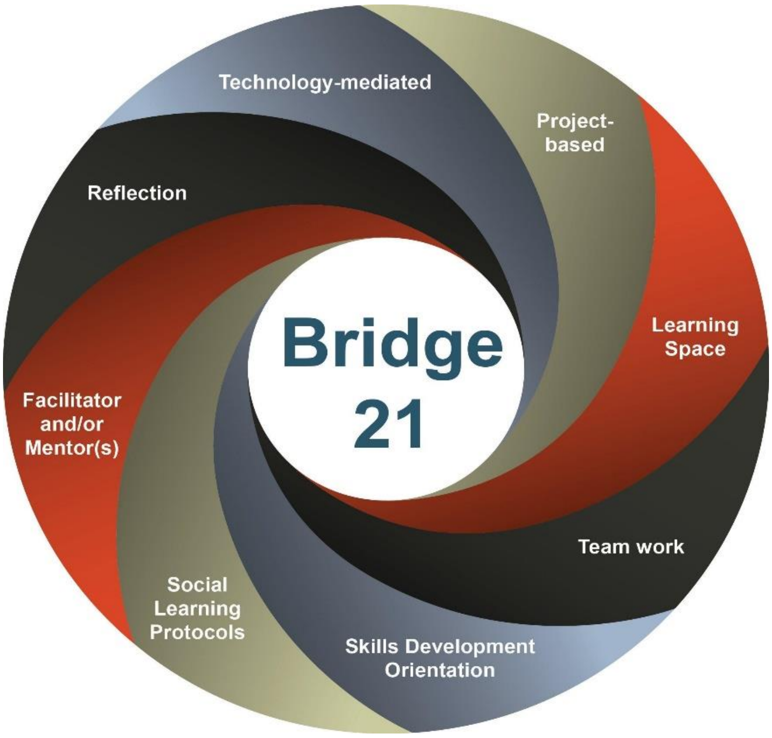


Figure 2.2: Bridge21 Pedagogic Model

Bridge21 activities are mediated by technology – the tools are integral to the activities, yet are not the primary focus of the learning. Students are expected to assimilate the skills required for new applications through exploration and peer-learning, with the guidance and support of a facilitator/mentor (Mitra, 2010). Sharing of resources is a distinguishing feature of the Bridge21 approach, with each team of 4/5 members equipped with no more than two devices in order to encourage collaboration among the participants.

Bridge21 activities feature an *inquiry*, and *project-based* approach to learning, involving complex and challenging problems that are authentically situated, require collaboration as well as autonomous effort, have strict time limits, and feature reflection as well as the production of an artefact or presentation (Lawlor et al., 2015). The focus of these projects is the development of skills, with an emphasis on encouraging participants to reach their own potential as opposed to normative assessment.

A Bridge21 learning environment is designed to be supportive of teamwork, with workspaces assigned to each team, as well as breakout and presentation areas. Such a physical environment is supportive of socially constructivist pedagogy (Blatchford et al., 2003) and encourages an open, friendly and relaxed atmosphere. The social learning protocols in a Bridge21 environment are based on trust, support and responsibility, rather than on control. The intention is to encourage independent and autonomous learners.

The Bridge21 *Activity Model* outlines a structure for the implementation of 21CL activities in educational environments: its innovative approach to classroom orchestration provides a set of steps to facilitate a successful intervention, thus addressing some of the barriers to the implementation of 21CL identified in the literature (Euler & Maaß, 2011; Voogt & Roblin, 2012). The steps typically include: team formation; a divergent-thinking, ‘warm-up’ activity; investigation of the problem/challenge; planning; an iterative phase of task execution/problem solving/artefact creation; presentation; and reflection. Strict deadlines are enforced to encourage planning and ensure the teams stay on-task. However, the “tight-but-loose” approach advocated by Thompson and Wiliam (2008) is adopted in recognition of the constraints of individual contexts. The physical learning space is configured to support a collaborative, project-based, cross-curricular and technology-mediated approach, with an emphasis on individual and group reflection. The activity model is examined in more detail in Chapter 6.

2.7.3 Bridge21 and Continuous Professional Development (CPD)

Bridge21 also supports an innovative approach to CPD, strongly influenced by the Japanese model of Lesson Study that uses an iterative cycle of goal setting, planning, teaching and observation, review, and revision (Lewis, Perry, & Hurd, 2009). In this way, groups of teachers form communities of practice to engage in a process of systematic examination of their practice, with the goal of becoming more effective teachers and optimising their lessons (Maaß & Artigue, 2013; Takahashi & Yoshida, 2004). The initial experience for teachers engaging with the Bridge21 CPD model involves active participation in immersive and authentic activities, which enables them to understand the power of the approach at a personal level. Throughout the process, participants are provided with the resources, practical designs and collegial support that Donnelly et al. (2011) highlight as necessary conditions to motivate change amongst teachers. In addition, the teachers are encouraged to become co-researchers, facilitating dissemination of the research into practice and addressing the practice-research gap (Boaler, 2008; Maaß & Artigue, 2013; Pimm & Johnston-Wilder, 2005).

2.8 Conclusion

Having identified certain aspects of task design and classroom orchestration (e.g., open-ended tasks, meaningful contexts, inquiry-based learning, collaboration) that can have a positive impact on students learning and attitudes to mathematics with technology (Boaler, 1993; Li & Ma, 2010; Maaß & Artigue, 2013; Oldknow, 2009), this research aims to extrapolate a set of design heuristics that support an approach to the design and implementation of such activities. In order to achieve this aim in a systematic manner, a theoretical framework to underpin the design heuristics is required.

The transformative use of technology (Laborde, 2002; Puentedura, 2006) within the Bridge21 pedagogical framework underpins the amalgamation of 21st Century Learning and RME, providing an appropriate theoretical frame to scaffold the design of collaborative, technology-mediated and contextual learning activities (Tangney et al., 2015). When used in this context, the technology becomes a ‘mindtool’, encouraging students to “learn with, not from technology” (Jonassen et al., 1998) and the full potential of RME can be exploited for the creation of engaging learning scenarios. In these conditions, mathematics can be recognised by the students as a human activity and not just a formal discipline, and the teacher can guide students to ‘reinvent’ the mathematics that they require, hence overcoming many of the issues with traditional approaches to mathematics education identified earlier in the chapter.

2.8.1 Direction of this Research

This chapter has provided a background to the areas of ICT in Education, 21st Century Education, Mathematics Education and Technology Enhanced Mathematics Education. Through the literature review, some of the widespread problems associated with each of these areas of interest were identified, along with some suggested approaches to their resolution. From this, a theoretical framework was identified, to underpin the development of an approach to the design of technology-mediated, contextual and collaborative mathematics learning activities. In the following chapters, the evolution of a set of design heuristics that fits within this framework, will be described.

It is also important to clearly identify, through rigorous research, how the integration of Bridge21/RME activities in the classroom can be of benefit, and to use this information to back up arguments for change. This research will attempt to achieve this through analysis of the relationships between the aspects of the design heuristics and the types of learning and attitudinal changes observed in the students. In this way it will be possible to identify potential links between the kinds of knowledge and attitudinal changes that were motivated through participation in the intervention, and specific aspects of the activities.

2.8.2 Research Questions

This analysis of the literature has drawn attention to two related research questions that this study attempts to address; each main question is comprised of two parts as follows:

RQ1 (a) What are the desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence?

RQ1 (b) What are the key elements of a practitioner's guide for the creation and implementation of such interventions within the traditional school environment?

RQ2 (a) What effects on engagement and confidence does participation in activities created in accordance with the design principles have on students?

RQ2 (b) What are the primary factors that cause these changes?

These questions are answered through the implementation of activities developed in accordance with the emerging design heuristics, initially in a purpose designed learning space in Trinity College Dublin, and subsequently in school settings. In addition a number of teacher workshops conducted throughout the study are used to harness the expertise of practitioners as well as to facilitate the dissemination of the method. These aspects of the study are discussed in chapters 6, 7 and 8.

3. A System of Classification for Technology Interventions in Mathematics Education

At the outset of the research process, it became clear that some kind of system of classification would be beneficial, in order to put a framework onto the current trends in the literature. The intention in doing so was not to provide a definitive description, but rather to facilitate the emergence of conjectures about the current situation, based in empirical research, with a view to informing a set of design heuristics for the development of interventions in the field. Subsequent to the initial phase of analysis, relevant papers were collected on an ongoing basis, in order to extend the reach of this aspect of the research. A timeline for the development of the classification system and analysis of the results is provided in Figure 3.1.

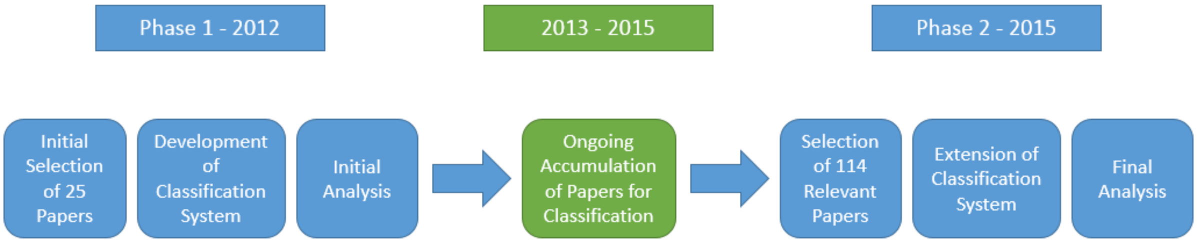


Figure 3.1: Timeline of the Development and Analysis of the Classification

The development of the classification system is described in sections 3.1 – 3.3, with results of the analysis of the second, more comprehensive, phase of the classification provided in section 3.4.

A classification should be dynamic and be able to keep pace with changes to the status quo; it should also permit generalisation, and provide a basis for the explanation of an emerging argument. In this research, an ongoing, systematic review of recent literature in which technology interventions in mathematics education are described provides the data for the classification. The electronic databases searched for the initial review of recent literature were chosen for their relevance to education, information technology and mathematics: ERIC (Education Resources Information Center), Science Direct, and Academic Search Complete, were the sources of the initial papers, and subsequently Google Scholar was also used. The general search terms used were:

math* AND (technolog* OR tool*) AND education

These were used in an initial pass over the databases, and the results were then refined by limiters such as 'secondary education' in order to increase relevance to the particular field of interest, without skewing the results in favour of any particular theoretical, philosophical or technological approach. The results were further restricted to recent articles, issued since 2009, and full text availability was required. A preliminary set of thirty four papers were selected for analysis, and of these, twenty five made up the initial data set. The remaining nine papers were not included as they do not discuss specific interventions. However, a number of them compare interventions in general and have been useful in informing the set of design heuristics that aim to describe a method of successful integration of technology in mathematics education.

This first pass over the relevant literature has been expanded, and the classification, like the design heuristics under development in this research (Chapter 4), is subject to an iterative process of review and refinement. In addition to the articles that emerged through the search facility, papers from the technology working groups at the Congress of European Research in Mathematics in 2013 and 2015 (CERME8 and CERME9) are included in the classification. This biennial conference facilitates a working group specifically for researchers with an interest in using technology for teaching and learning mathematics, thus providing a particularly relevant pool of work from which to draw. In addition, five papers published between 1997 and 2008 are also included. These older papers are referenced in other classified papers and are incorporated owing to their particular relevance to the field.

In order to inform the development of the classification presented in this report, some existing systems of classification were identified and considered. Although none of these provided a sufficiently comprehensive structure from the point of view of this research, three areas emerged as being of particular interest: technology, levels of adoption, and learning theories.

3.1 Background to the Classification

3.1.1 Existing Classifications of Technology

The classification systems of Clarebout and Elen (2006) and Passey (2012) were considered, but were unsuitable due to issues around relevance to mathematics and levels of complexity. Two classifications of technology for mathematics education by Hoyles and Noss however, are influential in this research. They are specific to mathematics education and, while being concise, provide an appropriate level of detail.

The first (Hoyles & Noss, 2003), distinguishes between programming tools and expressive tools. *Programming tools*, such as microworlds, are defined as lending themselves to individual expression and collaboration. *Expressive tools* on the other hand, provide easy access to the results of algorithms and procedures, without the user being required to understand the intricacies of their calculation. The category of expressive tools is further broken down into *pedagogic tools*, designed specifically for the exploration of a mathematical domain, and *calculational instruments*, which are frequently adapted to, rather than designed for, pedagogic purposes. Dynamic Geometry

Environments (DGE) such as GeoGebra, are examples of pedagogic tools, and spreadsheet programs would fall into the category of calculational instruments.

In their later research, Hoyles and Noss (2009) classify tools according to how their usage shapes mathematical meanings. They refine and extend their previous framework differentiating between: DGEs such as Cabri and Geometers Sketchpad; tools that outsource the processing power, of which computer algebra systems (CAS) are an example; new semiotic tools, which may have the potential to influence how mathematics is represented; and tools that increase connectivity, such as knowledge fora.

3.1.2 Classifications of Technology Adoption

There are a number of available theories that describe general technology adoption, such as Gartner's Hype Cycle (Lowendahl, 2010) and Roger's Innovation Adoption Lifecycle (Rogers, 1962). Two perspectives were identified that categorise technology adoption within specific interventions: the FUIRE model (Hooper & Rieber, 1995) and the SAMR hierarchy (Puentedura, 2006). While the FUIRE model provides information on an individual's use of the technology and their level of adoption of it in the classroom, the SAMR model is better fitted to describing the level of adoption present in a given intervention and as such, is the model selected for this classification of the papers. The significant overlap between the SAMR model and the four-level model specific to Dynamic Geometry Environments presented by Laborde (2001), make it particularly suited to this work.

The SAMR hierarchy (Figure 3.2) can be divided into the two broad categories of Enhancement and Transformation, each of which has two further subsections. The lowest level of Enhancement is classed as Substitution. This describes situations in which the technology is used as a direct substitute for the traditional method, without functional change as exemplified by the reading of classic texts online. The second level is that of Augmentation, in which the technology is used as a substitute for an existing tool, but with some functional improvement regarding exploration and analysis, e.g. if the text being read contains links to online study guides.

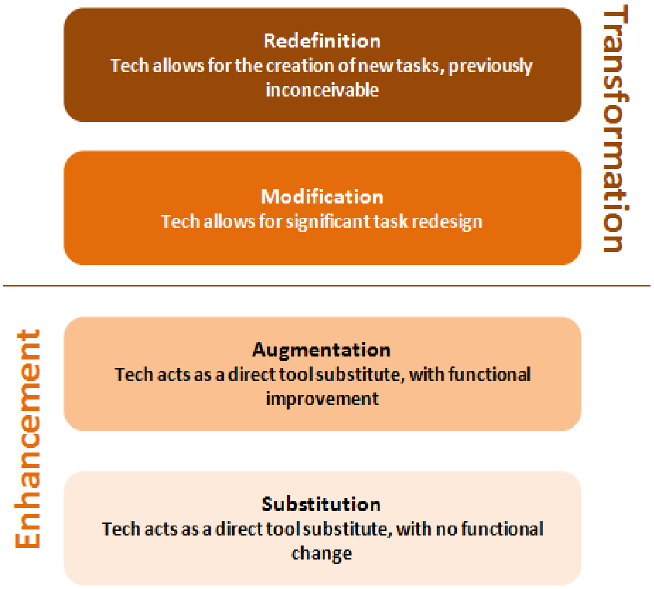


Figure 3.2: The SAMR Hierarchy

The Transformation space on the SAMR hierarchy describes interventions that offer tasks that are significantly changed through the use of the technology (modification), or that use the affordances of the technology to design new tasks that would previously have been inconceivable (redefinition).

The SAMR hierarchy relates to levels of integration of non-specific technology, in education in general. At a more specific level, Laborde (2002) similarly distinguishes between four levels of tasks in her paper on the integration of technology into mathematical tasks using the dynamic geometry environment Cabri-Geometry. These four levels are described as follows:

1. Tasks which technology facilitates, but does not change, such as measuring and drawing in a graphics program. (~SAMR level - Substitution)
2. Tasks for which technology facilitates increased exploration and conjecture, such as the dragging of objects. (~SAMR level - Augmentation)
3. Tasks in which the technology facilitates completely new approaches, such as the construction of a square with a given side. In this case, the technological task requires a higher level of mathematical knowledge relating to the properties of the square and the circle, than the equivalent with pencil and paper, which relies mainly on perception. (~SAMR level - Modification)
4. Tasks that could not be posed without the use of the technology. These can be tasks in which the technology permits the use of strategies that would not be possible using pen and paper, or tasks that could only be carried out in the specific environment. For example, students could be presented with a diagram on-screen and asked a related question. The students would then manipulate the diagram in order to solve a problem, in which the invariants of the figure are the “tools of solution” (Laborde, 2002, p. 311). (~SAMR level - Redefinition)

3.1.3 Classification of Learning Theory

The learning theories initially considered in this classification fall into the two main camps of Behaviourism (Skinner, 1938) and Cognitivism (Bruner, 1977). Some cognitive learning activities can be further classified as Constructivist (Kolb, 1984; Piaget, 1955), and within

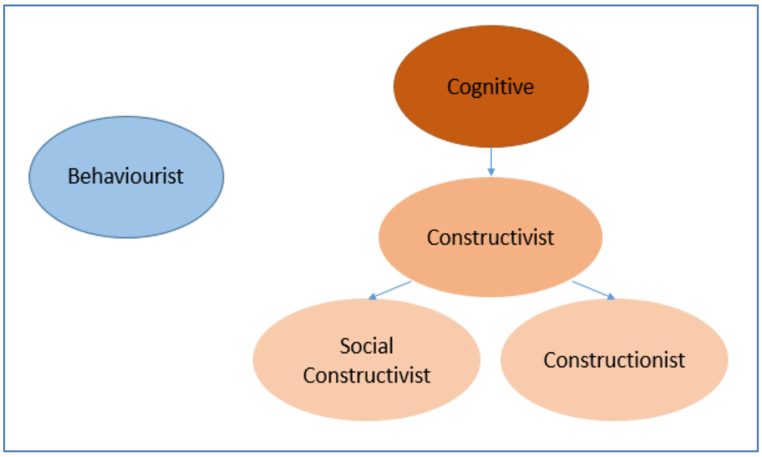


Figure 3.3: Learning Theories

this, as Constructionist (Papert, 1980), and Social Constructivist (Vygotsky, 1978) (Figure 3.3).

Behaviourist theory holds that (a) learning is manifested by a change in behaviour, (b) the environment shapes behaviour, (c) events must occur in quick succession, and be reinforced in order for a bond to be formed. Thus, learning is the acquisition of new behaviour through (classical or operant) conditioning.

In cognitive learning theories, learning is viewed as a combination of internal mental processes consisting of insight, information processing, memory and perception. From a cognitive perspective therefore, education should focus on building intelligence and on cognitive and metacognitive development in such a way that the learner will develop capacity and skills to improve learning.

Constructivism falls within the cognitive domain, and is founded in the belief that knowledge is constructed rather than transmitted (DiSessa, 1983; Piaget, 1955). In constructivist learning environments “the problem drives the learning, rather than acting as an example of concepts and principles previously taught” (Jonassen, 1999, p. 218). Social constructivism adds another layer to this, and has its foundations in social learning theory (Bandura, 1977; Vygotsky, 1978), which stems from the perspective that people learn within a given context and that the effects of culture and interactions with people play a significant role in how we learn. In particular, Vygotsky believed that the potential to learn is greatly enhanced through interaction with a ‘more able other’, where learners are challenged close to, but slightly above, their current level of ability.

Papert is the main proponent of constructionism. His thesis is that learning can happen most effectively when people are actively engaged in the creation of tangible objects. Constructionism involves experiential, problem-based learning and builds on the theory of constructivism. Learning is viewed as a construction, as opposed to a transmission, of knowledge, and is most effective when the activity involves the construction of a meaningful product - "learning by making".

3.2 Process of Classification

The process of classifying the papers was facilitated by the qualitative analysis software NVivo10. Initial coding was directed by the elements of the classification as described in the previous section (3.1). Throughout the second phase of analysis however, it emerged that the initial classification was insufficient, and a number of changes and extensions were required. The methodology underpinning the classification process thus initially followed a directed coding technique (Hsieh & Shannon, 2005; Krippendorff, 2004; Namey, Guest, Thairu, & Johnson, 2007), with a subsequent emic approach, not based on a-priori theoretical distinctions (Yin, 2014) used to identify emerging themes.

"Systems of classification are not hatracks, objectively presented to us by nature" (Gould, 1987). The process of classification is not always clear-cut and it is important to bear in mind a number of points when considering the analysis that follows. Firstly, the classification is based on the perspective of the researcher. In certain instances, classification of a given intervention was not straightforward and a level of personal judgement was required. In order to be rigorous, the analysis would benefit from a coding comparison from the perspective of a second researcher – that is however, outside the scope of this research owing to the volume of papers. In addition, a number of the classified papers considered more than one intervention, had multiple goals, or used various technologies. Therefore, although the total number of papers analysed to date in this classification is 114, the number of interventions in the analysis add up to more than that (circa 130).

3.2.1 Emerging Classification of Technology

In the second phase of this study, the classifications by Hoyles and Noss (Hoyles & Noss, 2003, 2009) are further refined and amalgamated to provide the foundation for the technological component of an emerging classification. There is no evidence thus far in the papers reviewed, of semiotic tools that change the representational infrastructure of mathematics, and it is thus not represented in the emerging system of classification. Through the ongoing review of the papers a number of extensions to the Hoyles and Noss classification have been required. The category of *toolkit* has been added as a distinct class. Integral to the definition of the toolkit category is the design of technologies in accordance with a specific pedagogical approach, along with the provision of support for the student and the teacher through tasks and lesson plans, and feedback for assessment, all founded in the relevant didactic theory. The category of Multiple Linked Representations (MLR) describes tools that integrate diverse representations of single mathematical entities. MLR would be used to describe, for example, a tool that integrates the capacity of a Dynamic Geometry Environment (DGE) and Computer Algebra System (CAS) in a single, dynamically linked system. A required division was identified in the original category of Outsourcing of Processing Power. A number of the interventions originally classified as belonging to this category relate to the outsourcing of content to the technology. Therefore, the Outsourcing category was split into 'Outsourcing – Computational' and 'Outsourcing – Content'. The resulting technological aspect of the classification is thus as follows:

- Collaborative by Design
- Dynamic Geometry Environments (DGE)
- Multiple Linked Representations (MLR)
- Outsourcing – Computational
- Outsourcing – Content

- Programming Tools
- Toolkit

3.2.2 Emerging Classification of Learning Theory

Very few of the papers discussed interventions in which the technology had a drill and practice facility, and those that did, couch it within a cognitive approach to learning. Thus, none of the interventions classified to date come under the category of behaviourist. The refined, phase 2 classification of learning theories is influenced by Li and Ma's (2010) distinction between traditional and constructivist teaching. The traditional approach is described as being generally teacher-centred and whole-class, which can be seen as aligning with cognitive learning theory. The constructivist approach to teaching is viewed as being student-centred and incorporating discovery and problem-based learning. This approach aligns well with the constructivist family of learning theories. Thus, the refined classification of learning theories includes the following elements:

- Cognitive
- Constructivist
- Social Constructivist
- Constructionist

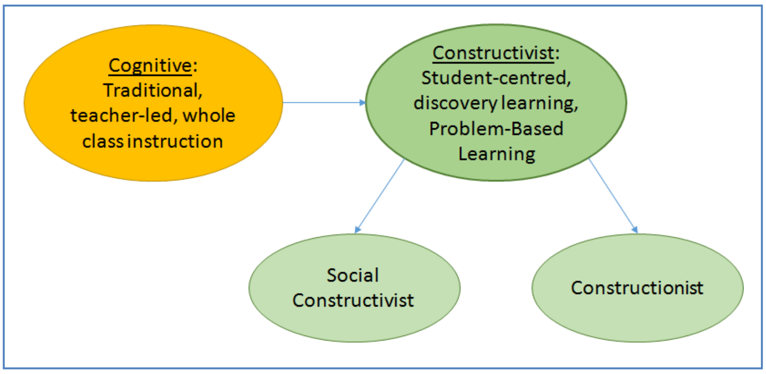


Figure 3.4: Refined Classification of Learning Theories

3.2.3 Emerging Classifications of Technology Adoption

The papers reviewed for this classification did not discuss the usage of technology at the level of Substitution on the SAMR hierarchy. There are a variety of possible reasons for this, the most likely being that although technology is still being used in a substitutive manner, this kind of usage is not being researched or reported in the literature. Therefore, only three levels of the SAMR hierarchy appear in the analysis of the results of the classification:

- Augmentation
- Modification
- Redefinition

3.2.4 Classification of Purpose

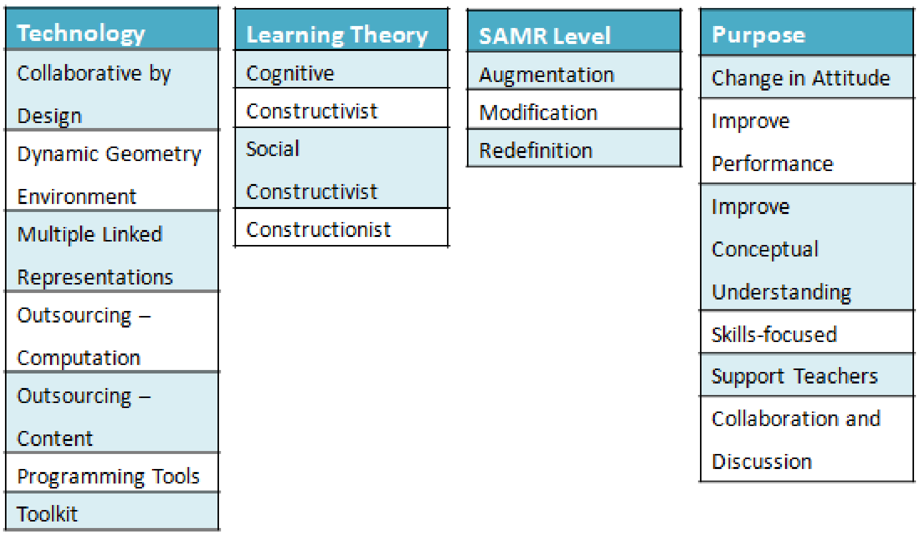
An additional layer to the classification is also identified in the phase 2 analysis, which categorises the primary purpose, or aim, of the interventions. The method of identification of the elements of this category was emergent, and arose throughout the process of classification. The aims identified in the 114 final papers are as follows:

- Change in attitude
- Improved Performance
- Development of Conceptual Understanding
- Skills-focused
- Support Teachers
- Collaboration and Discussion

The requirement to occasionally code interventions as having more than one aim may indicate that some of these goals are inextricably linked. Although it was not always explicit, it is likely that many of the interventions had more than one underlying purpose. The majority of the categories in this section of the classification are self-explanatory, however the ‘Change in attitude’ encompasses issues around motivation, self-efficacy and engagement, and ‘Skills-focused’ relates to the generation of collaborative, problem-solving, and creative skills amongst others.

3.2.5 Final Classification Components

The components that make up the system of classification used for the phase 2 analysis of papers are outlined in Figure 3.5.



Technology	Learning Theory	SAMR Level	Purpose
Collaborative by Design	Cognitive	Augmentation	Change in Attitude
Dynamic Geometry Environment	Constructivist	Modification	Improve Performance
Multiple Linked Representations	Social Constructivist	Redefinition	Improve Conceptual Understanding
Outsourcing – Computation	Constructionist		Skills-focused
Outsourcing – Content			Support Teachers
Programming Tools			Collaboration and Discussion
Toolkit			

Figure 3.5: Components of the Classification

Each intervention in the 114 reviewed papers was categorised according to the technology used, the learning theory underpinning the intervention, the level of integration of technology and the overarching purpose of the tasks. As discussed in section 3.2, a number of the papers were not confined to a single intervention, learning theory, or purpose, and have thus been classified at more than one of the elements of a single class.

3.3 Examples of Classified Interventions

The first phase of the classification of papers is published in Bray and Tangney (2013). This has been significantly extended and to date interventions 114 papers have been classified according to the lenses of technology, learning theory, level of technology adoption, and purpose. The examples presented in this section are from the second phase of the classification.

In order to illustrate the process of coding the papers for the classification, three of the interventions that have been examined and classified are presented in this section. Each sample intervention is representative of one of the three upper levels on the SAMR hierarchy: *Augmentation* (section 3.3.1), *Modification* (section 3.3.2), and *Redefinition* (section 3.3.3). A rationale for the classification of each of the examples, according to each of the categories of technology, learning theory, level of adoption and aim, is provided in Tables 3.1, 3.2 and 3.3.

3.3.1 Augmentation

The paper chosen as representative of the category of *augmentation*, by Hampton (2014), investigates why some students choose to view online instructional videos, and investigates differences in the levels of motivation and self-efficacy between those who do and do not view such material. Table 3.1 provides a rationale for the designation of this paper at each section of the classification.

Table 3.1: Augmentation

	Classification	**Rationale**
Learning Theory	Cognitive	In general, the use of online tutorial material reflects a view of learning as an internal mental process including insight, information processing, memory and perception.
Technology	Outsourcing – Content	In this paper, the role traditionally associated with the teacher to deliver content has been outsourced to the technology.
SAMR Level	Augmentation	The technology acts as a substitute for the teacher, with the added potential for ‘anytime, anywhere’ learning, and the ability to pause and rewind.
Purpose	Change in Attitude	The primary purpose of this research is to investigate the levels of motivation and self-efficacy associated with the use of the technology in question.

3.3.2 Modification

Granberg and Olsson's (2015) reflection on the impact that the use of GeoGebra may have on students' collaboration and creative reasoning is selected as representative of the category of *modification*. In this paper, the authors examine how pairs of 16 and 17 old students attempt to solve linear functions in a dynamic geometry environment.

Table 3.2: Modification

	Classification	Rationale
Learning Theory	Social Constructivist	In this paper, the students work in pairs in order to solve "non-routine" tasks. In this way, their learning is constructed in a social environment, through collaboration with their peers.
Technology	DGE	The dynamic geometry environment GeoGebra is utilised for this study.
SAMR Level	Modification	In this study, the technology facilitates new approaches to the solution of the problem, through the dynamic aspects of the software. In addition, the distribution of the process of problem-solving amongst the participants (each student can manipulate and interact with the technology), is beneficial for collaboration.
Purpose	Skills-focused	The main aim of the tasks described in this paper is to increase collaboration and mathematical creativity.

3.3.3 Redefinition

Only 17 interventions of the 130 classified are categorised as using the technology to facilitate activities that would not have been conceivable without the digital tools – i.e., *redefinition*. One example (Table 3.3) is provided in the research of Kynigos and Moustaki (2013), who discuss how students' meaning making processes are shaped by online and face-to-face collaboration, as they try to make sense of mathematical problems in a what the authors term a 'half-baked' microworld. This is an environment that is, by design, incomplete. The students must deconstruct the mathematical problems in order to make sense of the behaviour of the environment. Particular emphasis is placed on how the students' mathematical activity is shaped by their need to explicitly articulate their own ideas in order to share them online, and by the ideas that others bring to the discussion.

Table 3.3: Redefinition

	Classification	Rationale
Learning Theory	Social Constructivist and Constructionist	In this intervention, the students use a computer supported collaborative learning environment to communicate. They collaboratively construct artefacts using the "3d Math" Authoring Tool, a constructionist environment (http://etl.ppp.uoa.gr/malt).

Technology	Toolkit	A variety of technologies are used together, in accordance with a specific pedagogic approach, along with the provision of support for the student and the teacher. In this case, Exploratory Learning Environments are combined with Computer Supported Collaborative Tools in one web platform.
SAMR Level	Redefinition	Both the online communication and the exploratory, 3-dimensional mathematical tasks presented in this paper would not have been possible without the use of the technology.
Purpose	Increased Conceptual Understanding	This research focuses particularly on the impact that the collaborative technology, in conjunction with the specific tasks, have on the students' meaning making processes.

3.4 Analysis of the Interventions

A full list of the 114 classified papers is provided in Appendix 3.A. An overview of the sources and years of publication is given in Figure 3.6 below.

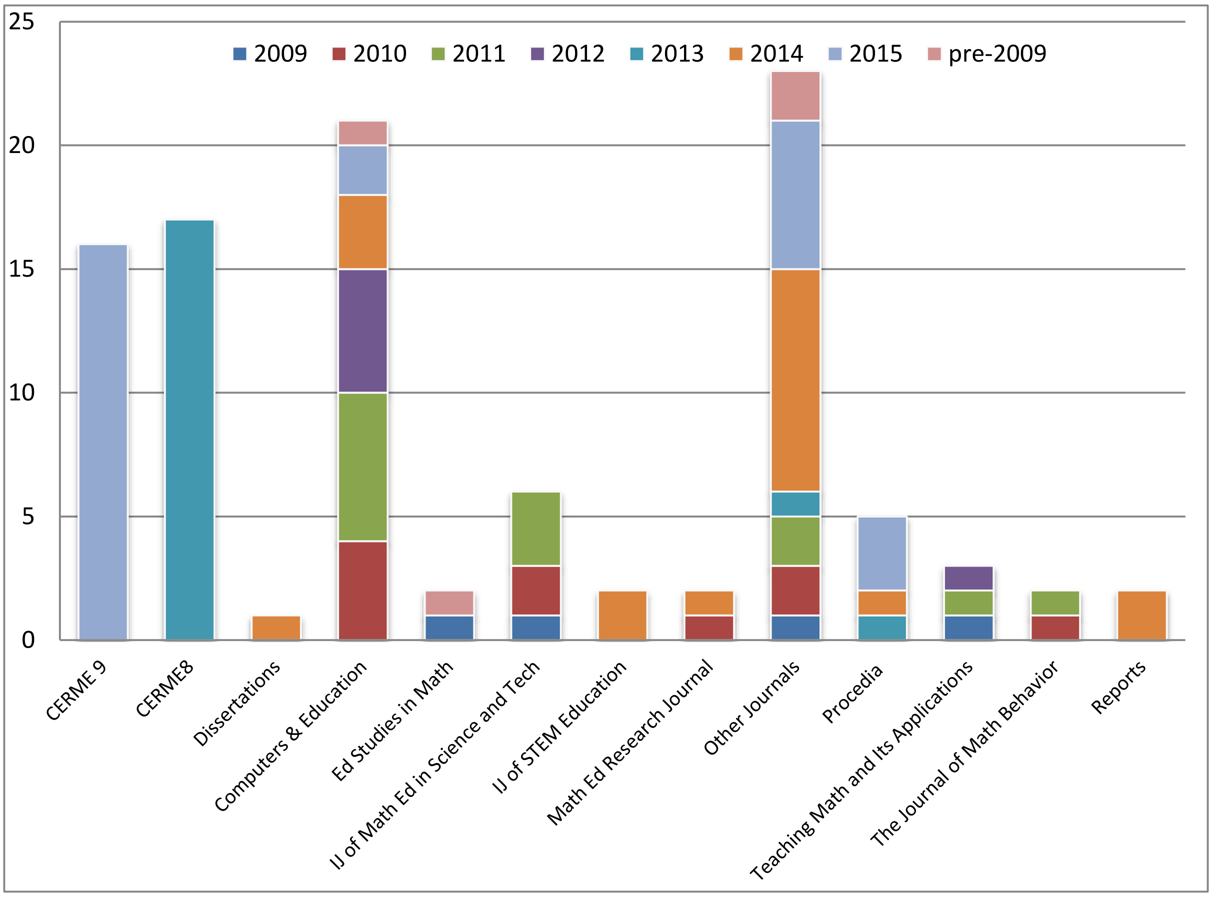


Figure 3.6: Overview of the sources and years of publication

In this chart, the row entitled "Other Journals" refers to 26 journals that were only referenced once, which include ZDM, BJET and Technology, Pedagogy and Education.

The process of classification of the 114 papers according to the categories of technology, learning theory, SAMR level and purpose, was expedited by the use of the qualitative analysis software NVivo10. This tool facilitated further analysis and visualisation of the data through a process of matrix coding.

Through this process, a number of interesting patterns have emerged. Figure 3.7 illustrates the clear constructivist (46%) and social constructivist (35%) trend in the literature, possibly supporting the perception that technology has the potential to realise some of the student-centred, constructivist and collaborative pedagogies proposed by innovative educators since the 1960s (Martin & Grudziecki, 2006; Voogt & Pelgrum, 2005).

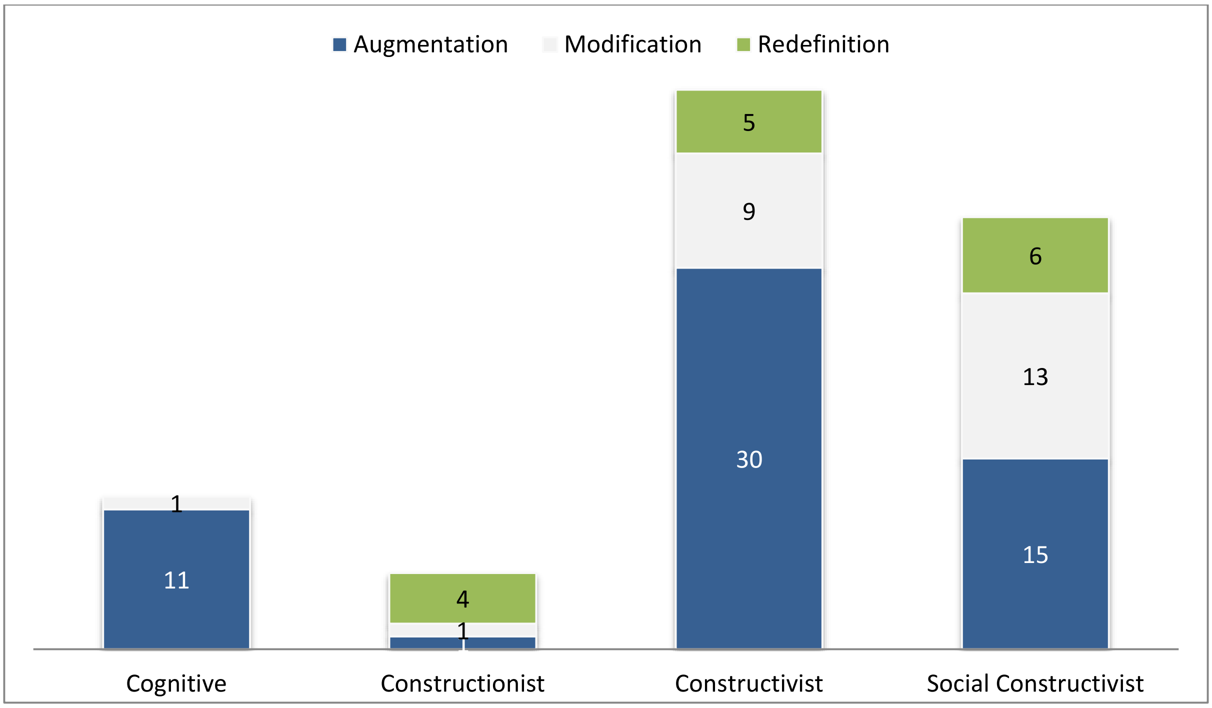


Figure 3.7: Learning Theory v SAMR

Figure 3.7 also illustrates the spread of the levels of technology adoption. The majority of interventions (59%) were classified as Augmentation; this means that the technology was used as a substitute for traditional approaches, but with some functional improvement; for example, an increased ability to explore and analyse. Although several researchers have argued that it is preferable to utilise technology in tasks that are transformed by its application – that is, that fit into the two higher levels on the SAMR hierarchy (Noss et al., 2009; Oates, 2011; Olive et al., 2010) – only 41% of the interventions have been classified in this way, with only 15% classified at Redefinition. If these transformative uses of technology are indeed preferable, this analysis serves to bolster claims that although use of technology in the classroom is increasing, its implementation in the mathematics classroom still lags behind its perceived potential to enhance the learning experience

(Conneely, Lawlor, et al., 2013; Dede, 2010a; Hoyles & Lagrange, 2010; Psycharis et al., 2013). Despite the small numbers, the high proportion of constructionist tasks classified at Redefinition may indicate a possible synergy, potentially indicating that if technology is being used in a constructionist environment, it is likely to be facilitating tasks that would not be possible without its use.

The predominant classification of interventions in the cognitive domain as being at the level of Augmentation (figure 3.7) reflects the increasing number of interventions that are using technology to outsource content. This claim is supported by the data in figure 3.8 below, which compares the technology and the learning theories.

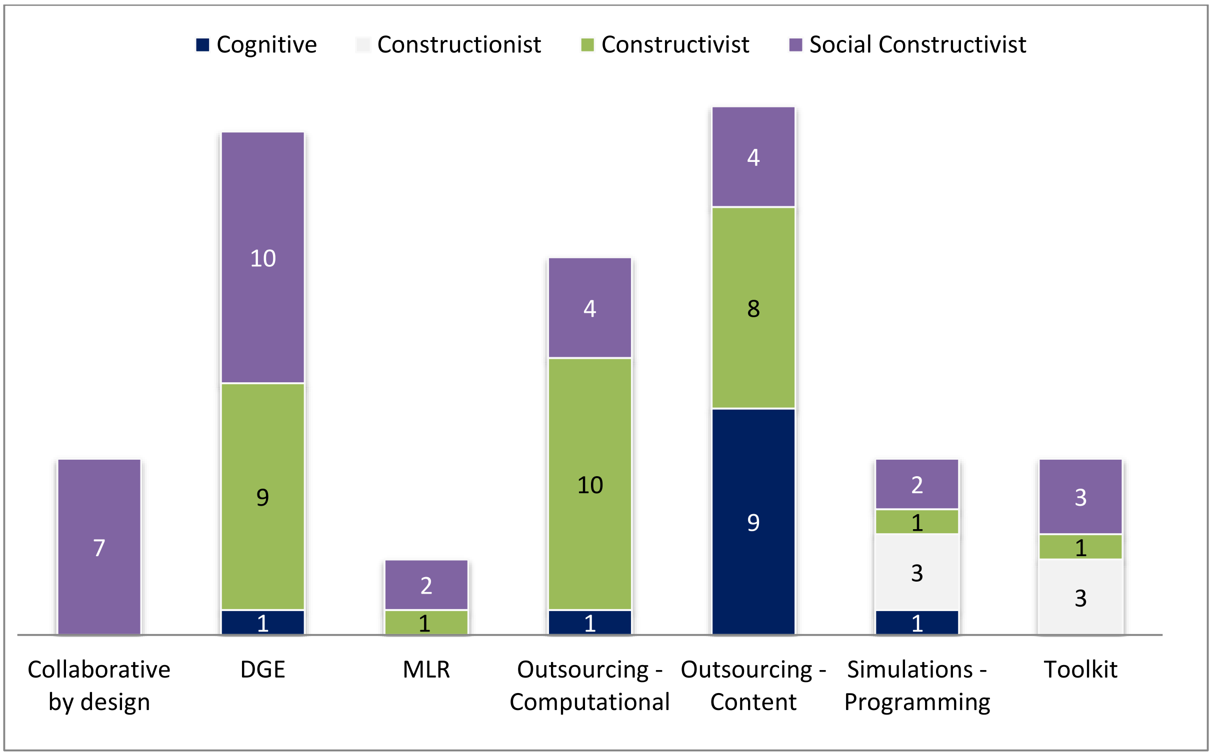


Figure 3.8: Technology v Learning Theory

In this illustration of the data, it is evident that using technology to outsource the delivery of content is of interest to the research community, making up 26% of the total number of classified interventions. 25% of the total interventions made use of Dynamic Graphical Environments and 19% used technology that outsourced the computation. A constructionist environment can be seen to align well with the Simulations – Programming category. It could be regarded as surprising that an intervention classified as Cognitive, should also fall under the label of Simulations – Programming (Figure 3.8). On further analysis, the particular paper classified in this way, by Star et al. (2014a), refers to the use of an immersive virtual environment in which the player is introduced to mathematical concepts and is required to solve puzzles. This particular intervention is a good example of one in which the researcher struggled with the classification. That is, without more information about the puzzles than was provided in the paper, it is difficult to determine whether the learning theory should be categorised as Cognitive or as Constructivist.

The crossover between the technologies and the SAMR hierarchy is illustrated in Figure 3.9. In this graph, the high correlation between the uses of technology to outsource the delivery of content and the SAMR level of Augmentation is particularly notable. This could be interpreted as suggesting that technology used in this way has not, to date, had a major influence on task design. However, none of the interventions classified in this way took into account the potential for diversifying activities in the classroom owing to the fact that the bulk of the required content had already been covered. This aspect of "flipping" the classroom to facilitate a more inquiry-based, exploratory school environment is something that may benefit from further research.

A majority of papers classified as Outsourcing – Computational, were also categorised at the level of Augmentation. This is possibly owing to the fact that the primary functions of computer algebra systems and graphics calculators are to increase speed and accuracy and to facilitate exploration – that is, to augment traditional practice. Moving focus to the DGE and Toolkit categories, it is possible to identify a shift in the way the technology is being used into the more transformative arena.

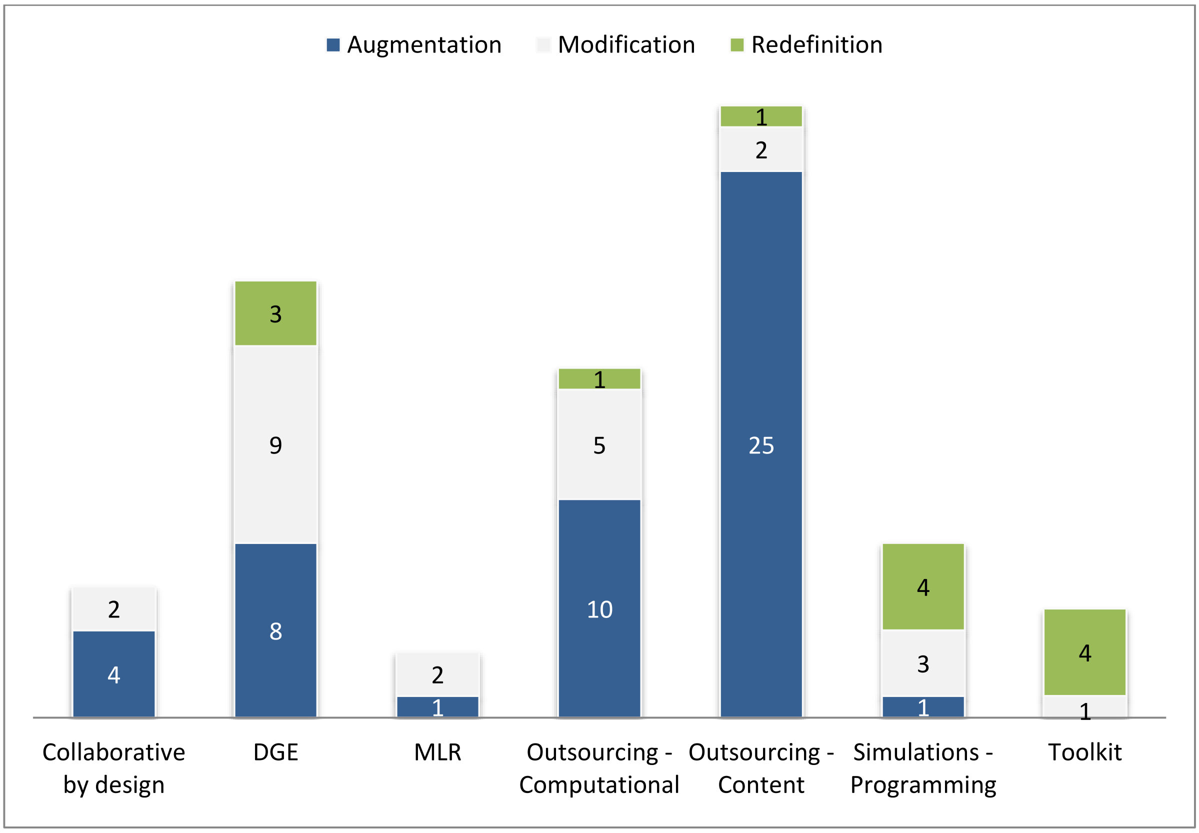


Figure 3.9: Technology v SAMR

The aspect of the classification that was added in the second phase of the development and analysis of the classification (Figure 3.1), relates to the purpose of the various interventions. Three graphs have been generated to illustrate the crossover between Purpose and Technology (Figure 3.10), Purpose and Learning Theory (Figure 3.11), and Purpose and SAMR hierarchy (Figure 3.12).

The main result of analysis of the first of these comparisons (Figure 3.10) relating Purpose and Technology is that a diverse assortment of technologies are being employed in an attempt to achieve various aims. Owing to the extensive amount of data to be represented, a different style of graph has been used for this illustration. The most common goal of the interventions, at 34%, was to improve students' Conceptual Understanding, with Improved Performance constituting the primary aim of 26% of the interventions, and a Change in Attitude, 24%. It is important to recognise however, that a number of these goals can be seen as being linked, and a number of the interventions reported having more than one goal.

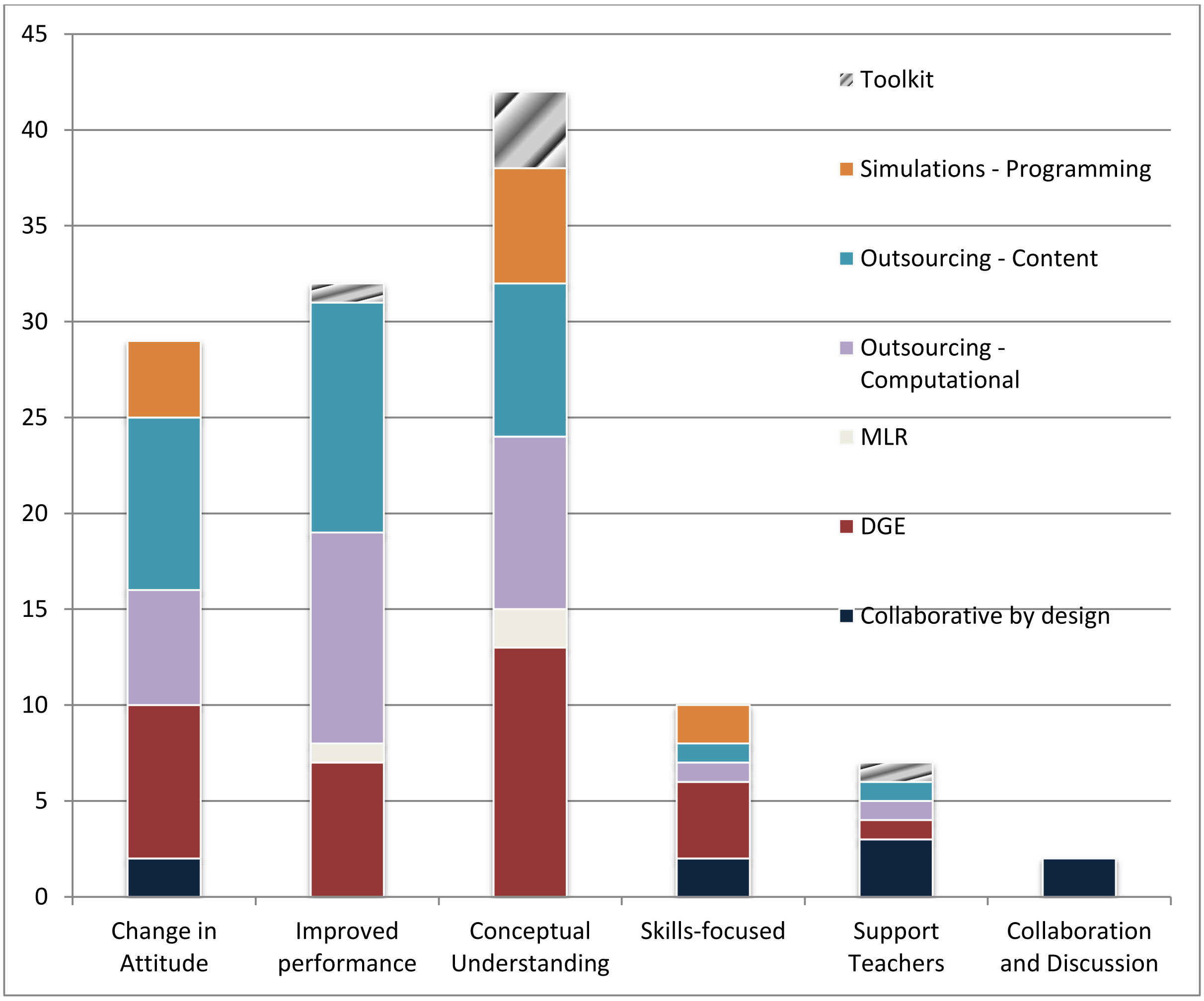


Figure 3.10: Purpose v Technology

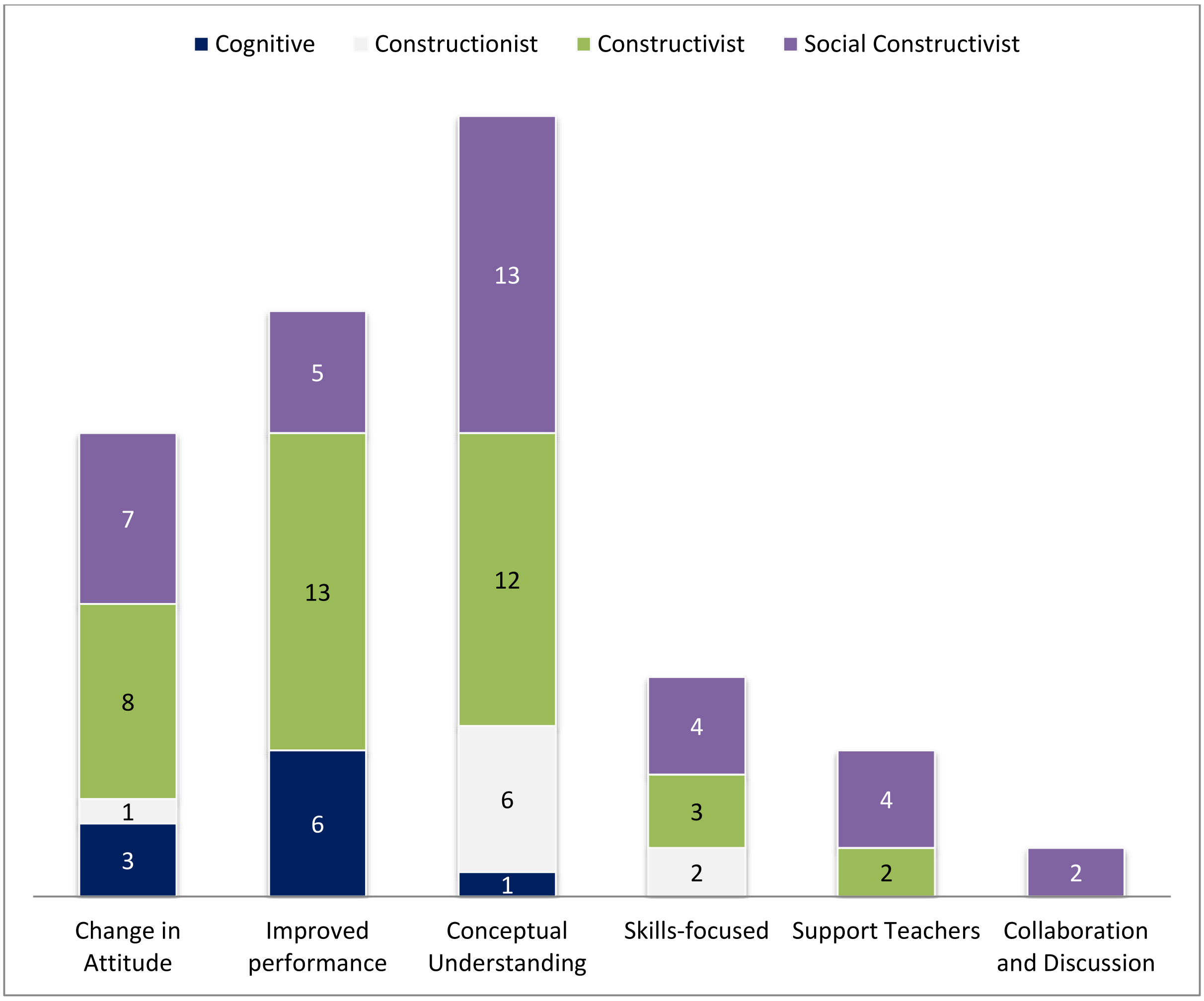


Figure 3.11: Purpose v Learning Theory

The comparison of Purpose and Learning Theory in Figure 3.11 indicates that, with a couple of exceptions, there is a relatively even spread of constructivism and social constructivism across the aims. The clustering of interventions that employed a cognitive learning theory among the more common goals could be representative of the fact that a skills-focused intervention, one that supports collaboration, or one that is supportive of teachers is unlikely to fall within the cognitive learning domain. A constructionist learning environment appears to be mostly associated with the goal of increased conceptual understanding, although proportionally, constructionism is more dominant in skills-focused interventions.

An illustration of Purpose compared with the SAMR hierarchy is provided in Figure 3.12. Using technology at the level of augmentation dominates in most of the interventions, but makes up a particularly high proportion of those that aim to improve performance. Interestingly, interventions that aim to improve conceptual understanding or those that are skills-focused show a slightly higher proportion of technology being utilised in a more transformative manner.

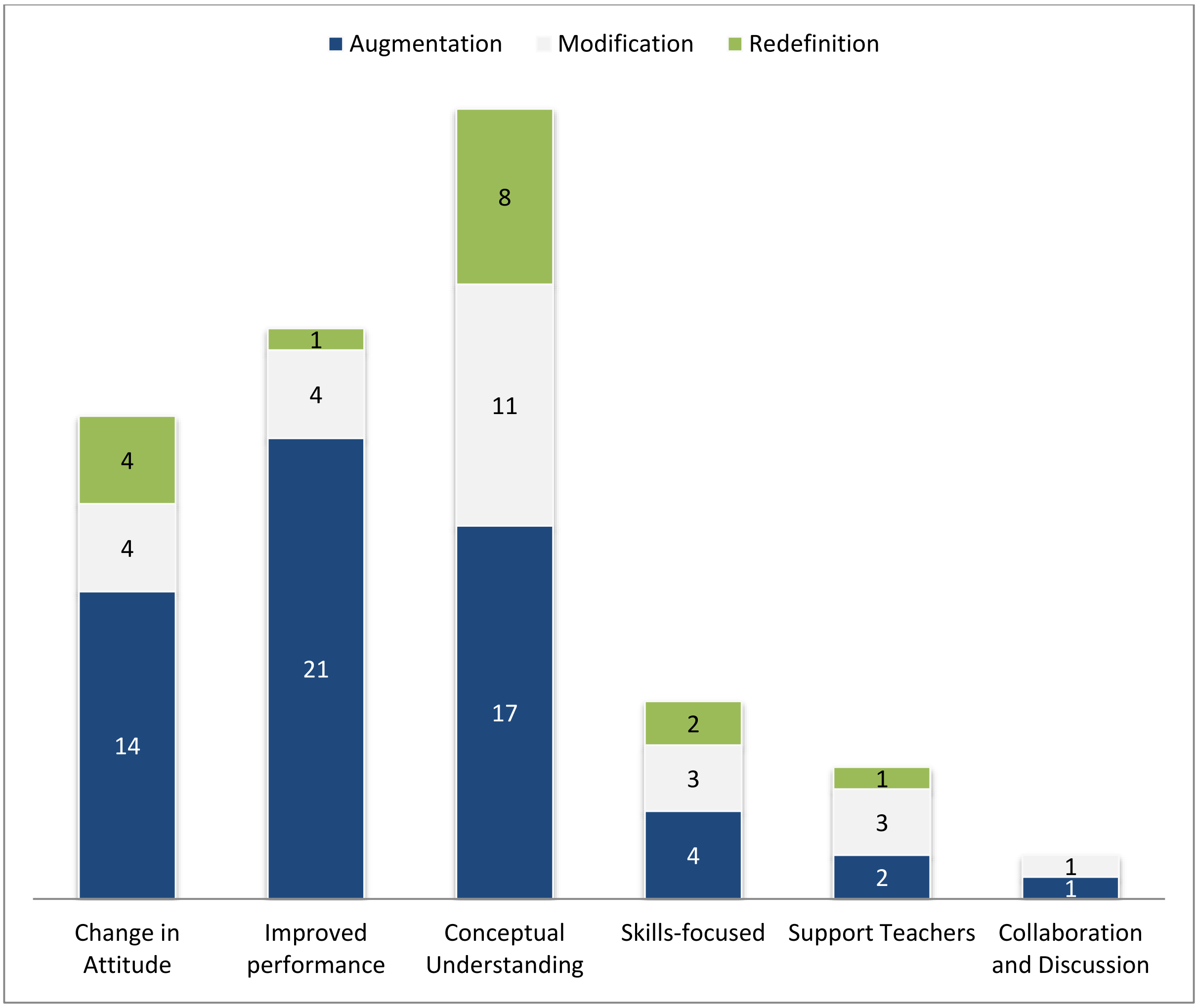


Figure 3.12: Purpose v SAMR

3.5 Discussion

The initial intention in carrying out a classification of the literature was to develop an empirical understanding of the status quo. Although this classification does not purport to give a definitive picture of what is going on generally in classrooms (which is likely to be quite different to what is going on in the focused research interventions), it does provide quite a clear indication of current research trends. The predominance of Constructivist and Social Constructivist tasks in the classified interventions may be indicative of a realisation of the potential for technology to support some of the student-centred and collaborative approaches that are associated with 21CL and key skills (Martin & Grudziecki, 2006; Voogt & Pelgrum, 2005).

Although none of the classified research discusses uses of technology at the SAMR level of substitution, there is evidence that digital tools are frequently utilised in this manner in classrooms (Ottenbreit-Leftwich et al., 2012; Thinyane, 2010), with basic technologies, such as the internet and PowerPoint, reported as being the technologies of choice among Irish teachers (Egan, FitzGibbon, &

Oldham, 2013). At the research level, it appears that the majority – almost 60% – of technological interventions in mathematics are using digital tools to augment traditional practice. This majority usage of technology at the lower levels of the SAMR hierarchy can be considered as corroboration of claims that the perceived potential of technology to improve the learning experience is not being achieved in classrooms (Hoyles & Lagrange, 2010; Pimm & Johnston-Wilder, 2005; Psycharis et al., 2013). In fact, a further conjecture could be made, that even within the relevant field of research, the perceived potential of the technology to “transform” the learning experience of students, is not being harnessed. Predominantly, technology is not being used in tasks that are transformed by its application – that is, tasks that fit into the two higher levels of the SAMR hierarchy, and are identified as preferable by several researchers (Noss et al., 2009; Oates, 2011; Olive et al., 2010) – but rather in tasks that could have been completed without its use.

Technology that was used for “Outsourcing - Content” was one of the more commonly researched areas. Once again, the tasks that were most co-referenced with this class of technology fell within the level of Augmentation. Used in conjunction with innovative approaches to task design, it is easy to see how the outsourcing of content could be a very profitable use of technology. However, in these interventions, the adoption of the technology did not generally lead to significant transformation of the tasks.

A number of the barriers to the integration of ICT and 21CL methodologies in classrooms, and the problems with mathematics education that were identified in Chapter 2, are likely to have an impact on the level of technology usage evident from the classification. In particular, a very significant proportion (80%) of interventions classified as having “improved performance” as their primary aim, were also classified at the SAMR level of augmentation. It stands to reason that if the purpose of a task is to increase student attainment in an existing form of assessment, then the purpose of the technology is to achieve an improved, and not necessarily different, version of what went before. In order to radically change the predominant pattern of technology usage in classrooms, it is likely that a change in focus away from the predominant high-stakes assessment and associated curriculum pressures will be required.

Predominantly, the papers classified in this research report positive outcomes, although a few comparative papers did not achieve a significant difference between control and experimental groups and a number of drawbacks to the use of technology were recorded (Borba, Azevedo, & Barreto, 2015; Kebritchi, Hirumi, & Bai, 2010; Triantagyllou & Timcenko, 2015). However, very few of the papers reported on longitudinal studies (one exception to this is the Migen project reported on by Noss et al. (2009), Noss et al. (2012), and Geraniou and Mavrikis (2015)) or incorporated dissemination plans into the research. Thus, it is possible to conclude that although they were successful in the short-term, they may fall into the practice-research gap identified by Boaler (2008), Maaß and Artigue (2013), and Pimm and Johnston-Wilder (2005). Teachers are most likely to be influenced by other teachers and they look for pragmatic examples of activities and tasks that are possible with their own resources. For this reason, many of the innovative uses of technology that are described in research projects remain at the periphery of general use and do not transition into mainstream classrooms (Tangney & Bray, 2013).

3.6 Conclusion

This empirical classification of current literature has served to substantiate the findings of the general literature review in chapter 2, leading to a picture of the research terrain as one in which technology is most frequently being used to augment traditional practice.

In his plenary keynote at the ICMI17 conference, Papert challenged participants not to be too bound by current constraints, but instead to spend a portion of their time considering the new kinds of mathematical knowledge and practices that might emerge through the use of technology (Stacey, 2011). There is the potential for the transformative use of technology integrated in a structured way, and with sustainable support for teachers, to have a significant, positive impact on the domain. However, according to the analysis in this chapter, this is not yet being achieved on a large scale.

The chapters that follow describe the development of a technology-mediated, inquiry-based and collaborative approach to teaching and learning that aims to engender increased levels of student engagement and mathematical confidence. The work aims to address Papert's challenge and attempts to engage students in mathematical activities that change their relationship with the subject. The analysis focuses primarily on the changes in students' perceptions and experiences of mathematics through participation in a set of activities developed in accordance with the described approach. A supplementary work that describes teachers' experiences of the creation and implementation of such tasks, within the confines of a sustainable CPD program is also provided in Chapter 8.

4. Design Heuristics and Examples of Transformative Mathematics Activities

Having reviewed the general literature on uses of technology in mathematics education (Chapter 2), this study has taken a structured approach, facilitated by a classification system, to analyse specific interventions (Chapter 3). The results of the analysis, in conjunction with the general literature review, provide the theoretical foundations for a set of design heuristics for the development of innovative, technology-mediated, mathematical activities. Using a first iteration of the design heuristics, a number of activities have been devised and trialled in the exploratory Bridge21 environment (Chapter 6). The results of these pilot studies, combined with the ongoing analysis of the literature, have been used to iteratively revise and refine the heuristics. A timeline of these activities is provided in figure 4.1.

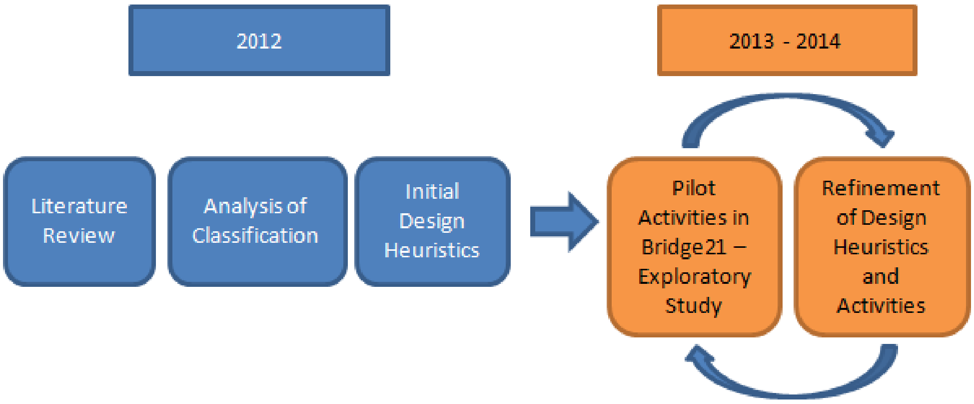


Figure 4.1: Timeline for the development of the Design Heuristics

The primary desired outcome of student participation in activities designed in accordance with the heuristics is an increase in engagement with, and confidence in, mathematics.

Through the development of the design heuristics, this research aims to support teachers' development of transformative, technology-mediated mathematics activities and to provide an understanding of how to support student learning and engagement within such scenarios. The heuristics aim to emphasise general characteristics of the development process that are deemed important to foster engagement and confidence amongst students, focusing on activity attributes and environmental aspects.

Therefore, the goal of the development of design heuristics and related activities is to address the first research question with a particular focus on the first of its associated parts:

RQ1 (a) What are the desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence?

4.1 Theoretical Foundations of the Design Heuristics

In order to address the research question relating to the desirable attributes of transformative mathematics learning activities (RQ1(a)), the literature review and subsequent classification of papers are used to provide a theoretical foundation for the development of a set of design heuristics.

4.1.1 Social Constructivism

Li and Ma (2010) and Voogt and Pelgrum (2005) acknowledge the positive impact on learning that can be brought about by a collaborative environment in which students act as constructors rather than recipients of knowledge. Confidence in the benefits of a constructivist and socially constructivist approach to learning is also evident from the results of the classification. Combined, the interventions categorised as incorporating constructivist or socially constructivist learning theories made up 81% of the total (46% and 35% respectively). Therefore, the first point of the design heuristics indicates that:

- *Activities should be team-based and encourage collaboration, in accordance with a socially constructivist approach to learning*.

4.1.2 Importance of Meaningful Context

The analysis of the classification of recent literature clearly illustrates the diversity of technologies that are being researched from differing theoretical points of view and with various goals. A common thread that runs through the literature however, is a desire to create engaging environments in which the tools under investigation are used to increase student understanding, engagement and performance in mathematics. The view of mathematics education emerging from the papers is one that focuses on understanding, connections, and context, and not only on a requirement to learn a fixed body of knowledge (e.g., Ayinde (2014) and Contreras (2014)). The importance of embedding mathematics within a context that is meaningful to the students has been long been recognised by many authors (Boaler, 1993; Freudenthal, 1991; Geiger et al., 2010; Hoyles & Lagrange, 2010; Schoenfeld, 1992). Thus, in line with the framework provided by RME, the design heuristics state that:

- *Activities should be situated in contexts that are interesting and meaningful for the students*.

4.1.3 Technology Integration

Despite the fact that the more general literature review highlights a need for the development of tasks that are transformed through the use of technology, providing activities that are relevant and of interest to the students, and which have compelling goals (Confrey et al., 2010; Laborde, 2002; Oldknow, 2009), only 40% of the classified interventions describe using the technology in a transformative manner. Oldknow (2009) proposes that technology can provide students with opportunities to experiment and make practical use of mathematics in meaningful and contextual scenarios that are not contrived, that is, that fall into the two upper levels on the SAMR hierarchy (Puentedura, 2006).

However, technologies that outsource the burden of calculation have also proven to be an interesting area of research, explicitly making up 19% of classified interventions (this figure does not take into account the use of other technologies, which may have been utilised to 'outsource' other aspects of the mathematics such as graphing and measuring). Using technology in this manner has the potential to not only improve the speed and accuracy of students engaged in procedural tasks, but also to allow an increased emphasis to be placed on meaning as opposed to routine operation (Geiger et al., 2010; Oates, 2011; Oldknow, 2009). For this reason, the design heuristics reflect a belief that:

- *Activities should exploit the transformative as well as the computational capabilities of the technology*.

The use of a variety of accessible, free technologies has also emerged as an important point, not only due to issues of equity, but also to engender flexibility amongst students and teachers (Oldknow, 2009; Sinclair et al., 2010), therefore:

- *Activities should make use of a variety of technologies (digital and traditional) suited to the task; in particular, non-specialist technology such as mobile phones and digital cameras that students have to hand*.

4.1.4 Key Skills and Task Design

Although only 10% of classified interventions are identified as having a primary goal focused on the development of key 21st Century skills such as problem-solving, communication, creativity, and technical fluency, the constructivist nature identified in the majority of papers would lend itself to the development of such skills (Li & Ma, 2010). An inquiry-based approach to the development of key skills, has been identified as being particularly suited to the domains of mathematics and science (Euler & Maaß, 2011; Maaß & Artigue, 2013). In addition, the meaningful integration of core content in tasks that require higher order thinking and learning is seen as being an efficient way of encouraging the development of 21st Century skills (Dede, 2010a). The Inquiry-Based Learning (IBL) approach can be viewed as having particular resonance with the RME view of mathematization. For this reason, the design heuristics state that:

- *Tasks should involve problem-solving, investigation and sense-making, moving from concrete to abstract concepts. The tasks should be open-ended and require the integration of content,* *knowledge and skills from other domains. In addition, the tasks should have a 'low-floor', encouraging less mathematically proficient students to engage, and a 'high-ceiling', in order to promote and maintain participation of more able students*.

4.1.5 Classroom Environment

Issues surrounding the classroom environment are not considered in the classification of the literature. However, authors such as Hoyles and Lagrange (2010) suggest that the design of mathematical tasks that make use of technology should take into account the classroom environment created by the teacher. The term 'classroom environment' in this instance relates to the atmosphere created by the teacher, which should be one of facilitation rather than transmission, encouraging and supporting student exploration. The physical environment is however, also an important topic for consideration. If teamwork and inquiry is to be promoted, then a traditional approach to classroom layout and timetabling is not appropriate (Dede, 2010a; Euler & Maaß, 2011). Therefore, the heuristics suggest that:

- *The learning experience should be interesting and immersive/real wherever possible, which should include adapting the environment and class routine as appropriate*.

4.1.6 Structured Approach

Bearing in mind the issues raised in the literature review regarding the challenges associated with the successful integration of both IBL and technology-mediated tasks, a requirement for a structured approach to the implementation of teamwork and technology has been identified (Baines et al., 2008; Euler & Maaß, 2011; Means, 2010; Noss et al., 2009). In order to address this, the design heuristics developed in this research propose the use of the Bridge21 model as a viable solution. Therefore:

- *Activities should be structured in accordance with the Bridge21 model (or a suitable alternative) of 21st Century Learning and activity design*.

4.1.7 Ongoing development of the Design Heuristics

The theoretical foundations of the design heuristics thus describe an approach to the design of learning experiences that aim to combine the educational potential of off-the-shelf technology with appropriate pedagogy. They resonate with a view of mathematics as a problem-solving activity and of mathematics education as involving students in constructing their knowledge via the social formulation and solution of problems. Moreover, they seek to counteract a conception of mathematics as a collection of unrelated facts, rules, and 'tricks', and of mathematics education as consisting of memorisation and execution of procedures that should lead to unique and unquestioned right answers (Albert & Kim, 2013; Ernest, 1997; Maaß & Artigue, 2013; Schoenfeld, 2004).

The development process of the design heuristics however, is an iterative one. What is described above is an initial step in this process, providing a sound theoretical basis for the heuristics, grounded in relevant literature. Chapters 6 and 7 describe the continuing development and refinement of the heuristics, based on empirical interventions with students and teachers. Throughout these chapters, a practitioner's guide to creation and implementation of activities that conform to the heuristics is developed, along with a deep understanding of the impact that the different aspects of the activities have on student engagement with and confidence in mathematics.

4.2 Learning Activities

In order to provide a practical illustration of the design heuristics, this section describes five activities that have been developed in accordance with them. Each of the activities has been piloted in an exploratory learning environment in our institution (described in Chapter 6), and a number of them have been implemented in authentic school settings (described in Chapter 7). The pilot interventions have provided data relating to the practicality and efficacy of the tasks and a starting point from which to begin the iterative process of development. The descriptions that follow are the final, refined versions of the activities. A discussion of their alignment with the design heuristics is provided in section 4.3.

4.2.1 Scale Activity

Participants in this activity work collaboratively, in teams of 3 or 4, to develop a dynamic presentation about scale, orders of magnitude and scientific notation. The learning objectives include the development of an understanding of how to recognise appropriate technological and mathematical techniques for measuring and estimation. The students, working actively and collaboratively, are required to select objects to measure and to make sense of their information, figuring out how to measure objects of diverse size, and to present their results using scientific notation. The activity is suitable for years 8 – 10 (ages 13 - 16), and is particularly aligned with the Irish mathematics syllabus area "Number". Crossover with other areas of the syllabus, such as "Geometry and Trigonometry" through the process of measurement, is also included in this activity.

Smartphones are used to gather information, take measurements and perform some trigonometric calculations. Instruments utilised include tools for measuring distance and angles of elevation from the MobiMaths app (Tangney et al., 2010). Participants are required to determine into which 'Power of 10' each measurement fits and have the option of further populating their collection using Google

Earth, Google Maps, and other internet resources. The target for each group is to have two or three objects within each band of measurement and to cover at least five consecutive orders of magnitude. Prezi[2] is suggested (although not required) as an appropriate tool for creating the presentations, as it is straightforward to use and facilitates a zooming effect to simulate a perception of increasing and decreasing size.

Although a more traditional pencil-and-paper approach to this project would be possible, the adoption of technology to mediate this activity has permitted significant task redesign. The students use smartphones for scientific calculation, capturing images, and a variety of measurements. Online mapping tools permit the measurement of greater distances than would be practical to calculate by traditional means. The activity is thus classified at the level of *modification* on the SAMR hierarchy.

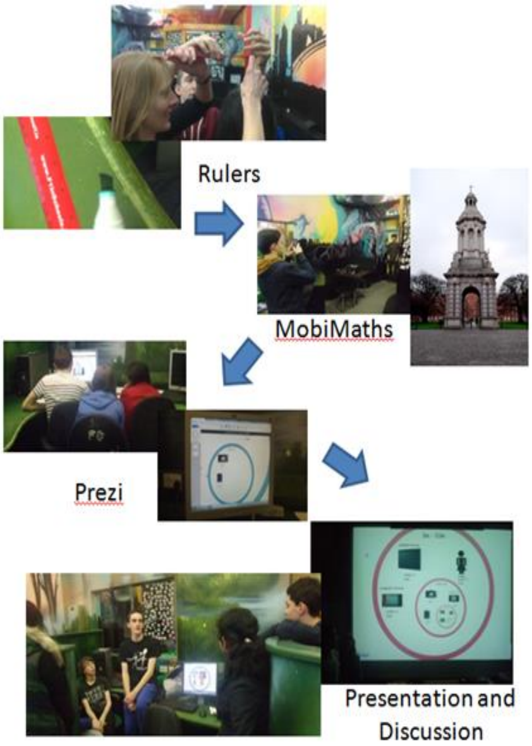


Figure 4.2: Scale Activity

This activity is designed to help students develop a sense of when and why different mathematical approaches and notations are required, and to acquire a realistic idea of scale and estimation, based on concrete examples. Final presentations and discussion allow for formative assessment of these learning goals, and for the scaffolding of deeper engagement with the topic.

4.2.2 The Barbie Bungee

Although the Barbie Bungee activity is not a novel concept,[3] it has been significantly redesigned through the application of the design heuristics. Student groups are provided with a doll and some rubber bands, and are confronted with the problem of determining how many bands they will require to give Barbie a safe, but exhilarating jump from an, as yet unknown, height. Each team is given access to smartphones, and laptops with free spreadsheet[4] and video-analysis software

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[2] www.prezi.com

[3] illuminations.nctm.org/Lesson.aspx?id=2157

[4] www.openoffice.org/product/calc.html

(Kinovea).[5] While estimation is encouraged, students are dissuaded from using guesswork or trial-and-error approaches. As the students do not initially know the distance their doll will need to fall, their problem-solving skills are put to the test while they attempt to develop a mathematical model of the relationship between distance and rubber bands. This activity is suitable for years 9 – 11 (ages 14 - 17), and incorporates aspects of the Irish mathematics syllabus areas of “Statistics and Probability”, “Number”, “Algebra”, “Functions”, as well as potential for elements of “Geometry and Trigonometry”.

The use of smartphones and video analysis facilitates accurate estimates of the distances that the Barbie falls with differing numbers of bands. The students then use spreadsheets to create tables, scatter plots, line-of-best-fit, and linear functions representing the relationship between the distances and numbers of bands. This mathematical model represents the relationship between their Barbie and the number of bands required to drop her from any height. Throughout the activity, they are introduced to the concepts of correlation, causality, line-of-best-fit and extrapolation, along with data collection and analysis.

Figure 4.3: Barbie Bungee

The activity concludes with a competition between the teams, as the Barbies “jump” from a designated height (Figure 4.3). A possible extension to the activity incorporates the use of a variety of methods to calculate and estimate the height from which the Barbies are to jump. Clinometers, both in digital and physical form, estimation using a known measure, and the affordances of the video analysis software Kinovea, can all be used to gauge the distance of the drop and encourage discussion.

The loosely scaffolded, team-based and technology-mediated approach to the design of the Barbie Bungee gives rise to an activity that is contextualized and meaningful, and is rich in promoting both generic 21st Century skills (such as collaboration and problem-solving) and mathematical content (such as collection, representation and analysis of data, line of best fit, correlation and causality, and extrapolation). The activity makes use of a variety of personal devices, including smartphones and

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[5] www.kinovea.org

laptops, in different contexts. The learners interact with the content in a thought-provoking fashion, allowing the mathematical concepts to emerge from the activity.

A pencil and paper approach to the Barbie Bungee activity forms the basis of the described task, however, the use of technology in the activity described above has permitted significant task redesign, and it is thus classified at the level of *modification* on the SAMR hierarchy.

4.2.3 Catapult Activity

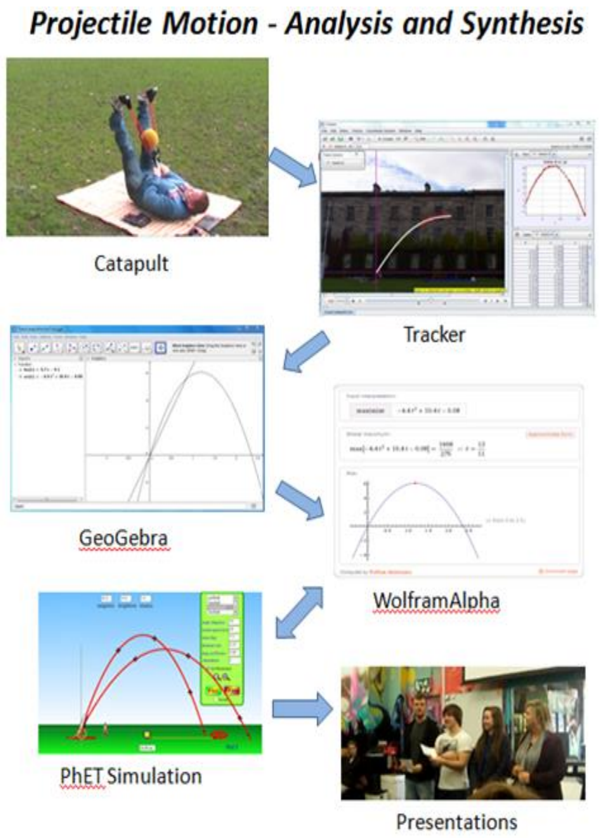

Figure 4.4: Catapult Activity

In the Catapult Activity (Figure 4.4), students work in teams of 3 or 4, to investigate the properties of projectile motion. They are provided with a scenario in which they have to catapult a parcel with essential supplies to their partner who is stranded on a rock 15 metres out to sea, in shark-infested waters. Particular emphasis is placed on functions relating height, horizontal distance and time; angles; rates of change; and velocity. Students use an oversized slingshot along with readily available, free software to conduct their investigations, moving from a concrete exploration of trajectory, to mathematical modelling of the activity, with verification of the results using a projectile motion simulation. This activity is suitable for years 10 – 12 (ages 15 - 18), and aligns with the Irish mathematics syllabus areas of “Functions”, while incorporating aspects of “Algebra” and “Geometry and Trigonometry”.

Initially the students record videos of their team using the catapult to fire a foam ball. The trajectory of the ball is analysed using the free software Tracker[6] to trace the flight path, and also to generate functions relating height to time, horizontal distance to time, and height to horizontal distance. GeoGebra[7] is used for further analysis of the functions, enabling the students to estimate the angle of projection and initial velocity of the projectile. The investigations are guided and scaffolded by an

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[6] http://www.cabrillo.edu/~dbrown/tracker/

[7] www.geogebra.org

instruction sheet, with suggested explorations provided. The computational website www.wolframalpha.com can be used for routine calculations and for checking answers. Once the students have calculated the data required, a projectile motion simulation[8] is used to stage a competition and to check the validity of their results. Group presentations and whole class discussion conclude the activity, providing scope for formative assessment as well as an opportunity for the students to consolidate and demonstrate their learning.

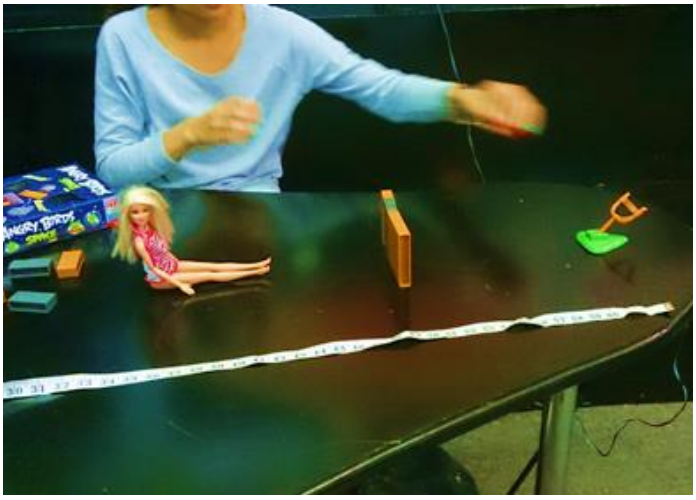
Figure 4.5: Angry Birds Catapult

The catapult activity described above requires a sufficient amount of space (ideally outdoors) to fire the catapults and record their trajectory. There are a number of reasons why such an activity may not be feasible, including inclement weather conditions, insufficient staff for supervision of the students, or other confines of school policy. For this reason, a variation of the task was devised that can be completed in a more standard classroom. The Angry Birds variation uses an inexpensive toy (Figure 4.5) to fire a catapult on a small scale. The students then follow the same procedure as described above.

The tasks involved in this exercise would not have been possible without the use of technology, leading to its classification as *redefinition* in terms of the SAMR hierarchy. The students are required to make extensive use of the computational facility afforded by the technology in a task that is designed to be engaging and immersive.

4.2.4 Probability and Plinko

Plinko is a game of chance based on a Galton board: a board with evenly spaced pegs arranged in staggered order, to form a triangle (Figure 4.6). Balls should be funnelled onto the board from directly above the top peg. If the pegs are symmetrically placed, the marbles have an equal probability of bouncing left or right. A number of evenly placed slots form the base of the board, into which the marbles will fall.

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[8] phet.colorado.edu

Figure 4.6: Plinko and Probability

In this activity, participants work in teams to develop a game for a casino. They are required to devise a set of rules and a scoring system in such a way that the game will be appealing to players, but that the casino owners will win overall. They are provided with a Galton/Plinko board template, a cork-board and some pins and marbles, smartphones, laptops with open-source spreadsheet software and the free video-analysis software Kinovea installed. They are also given a sheet of exploratory questions relating to the possible paths on a Galton/Plinko board. This activity is suitable for years 8 – 10 (ages 13 - 16), and aligns particularly with the Irish mathematics syllabus areas of “Statistics and Probability” and “Number”, while encouraging crossover with other curricular areas such as Art and Design, and History of Mathematics, through the creation of an attractive and appropriate game board.

The aim of the activity is to encourage the students to make sense of what appears to be random behaviour. In particular, they are encouraged to identify that, starting from the top, the number of routes to the pegs in the grid form Pascal’s Triangle, and also to understand the probability of a marble landing in a particular bin if the board were perfect. In addition, they analyse their own boards, using the spreadsheet to tabulate and visualise 100 rolls. They are thus able to see how well their game conformed to a digitally generated one,[9] introducing the notions of bias and fairness. They use video tracking to see if any of the marbles they roll follow the same path to any one bin, developing a practical understanding of the concept of probability.

The Probability and Plinko task would conceivably be possible without the use of digital tools, however, the technological mediation of the activity permits a richer, more inquiry-based approach, placing the activity at the level of *modification* on the SAMR hierarchy.

4.2.5 Pond-filling Activity

The Pond-filling activity involves problem-solving, estimation, area and volume. Participants, in teams of between 3 and 5 members, are given the challenge of determining the length of time it would take to fill a local pond (or any nearby, irregularly shaped space) with water, using only buckets

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[9] http://phet.colorado.edu/en/simulation/plinko-probability

filled from a tap in their school. Reasonable assumptions can be made about the depth of the pond and students can ignore the effect of evaporation and rainfall.

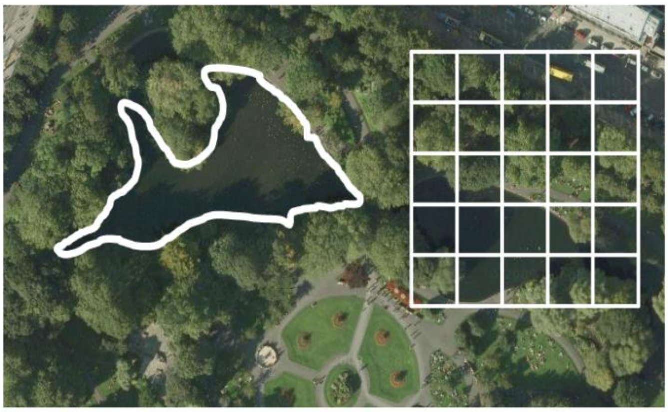

Figure 4.7: Pond-filling Activity

While it may be possible to attempt the 'pond filling' exercise as a desk-based activity, in order to truly harness the contextual aspects of the activity, participants are required to physically fill buckets and carry them to the pond, using the affordances of the mobile phone technology to track the distance travelled and the time taken. The Pond-filling problem challenges the learners' powers of estimation and approximation, their understanding of area and volume, and their approach to problem-solving, thus aligning with the Irish mathematics syllabus area of "Number". It also lends itself to cross-curricular learning and can be used as an entry point into discussions on rainfall, evaporation or human rights: The question of what percentage of the world's population lives more than 1km from a clean water supply, and the implications of this, may become more meaningful after carrying a full bucket of water for 500m. The activity is suitable for years 8 – 10 (ages 13 - 16).

A variety of technology-based tools is available to the students to help tackle this problem. Many, such as GPS tools to measure distance walked, are freely available as smartphone apps.[10] Also of use, is access to a web browser and Google Earth. The use of these tools in context adds to the realistic nature of the activity and its authenticity. At the core of the solution to the pond-filling problem, is approximation of the area of the pond as a number of small squares. This can be achieved by overlaying a grid on a Google Earth image of the pond, or through the use of smartphone apps such as MobiMaths (Tangney et al., 2010), which includes functionality to overlay a resizable grid on an image.

The requirement for the groups to present their results at the end of the activity in line with the Bridge21 activity model provides an opportunity for the students to justify their approach and for the facilitator to assess their understanding of the domain.

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[10] E.g., http://runkeeper.com/

It is difficult to conceive of how this task would be possible without the use of technology, and it is therefore classified as being at the level of *redefinition* on the SAMR hierarchy.

4.3 Discussion

The design heuristics developed through the course of the literature analysis and classification suggest that the desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence are as follows:

1. *Activities should be team-based and encourage collaboration, in accordance with a socially constructivist approach to learning.*
2. *Activities should exploit the transformative as well as the computational capabilities of the technology.*
3. *Activities should make use of a variety of technologies (digital and traditional) suited to the task; in particular, non-specialist technology such as mobile phones and digital cameras that students have to hand.*
4. *Tasks should involve problem-solving, investigation and sense-making, moving from concrete to abstract concepts. The tasks should be open-ended and require the integration of content, knowledge and skills from multiple domains. In addition, the tasks should have a 'low-floor', encouraging less mathematically proficient students to engage, and a 'high-ceiling', in order to promote and maintain the participation of more able students.*
5. *Activities should be structured in accordance with the Bridge21 model (or a suitable structured alternative) of 21st Century Learning and activity design.*

Using these theoretically-grounded foundations of activity design, five activities were developed for pilot testing, with the aim of further refining and developing both the design heuristics and the activities themselves. The exploratory and explanatory case studies described in Chapters 6 and 7 proved a detailed discussion of their implementation in both pilot and authentic settings. A discussion the correspondence between these activities and the design heuristics is provided below.

4.3.1 Influence of Design Heuristics on the Activities

It is evident from the descriptions provided in section 4.2, that all of the activities adhere to a socially constructivist approach to learning. Teams are formed in the initial stages of each activity and are required to collaborate in order to achieve the goals that have been set.

Each of the activities makes use of the computational affordances of the technology, outsourcing calculations and computations, but they also fall within transformation space on the SAMR hierarchy - the technology is used in a way that is integral to the task design, and meaningful in the context of the activities. In order to accomplish this, as well as to engender flexibility and adaptability in the users, a variety of free, off-the-shelf technologies are utilised in each of the activities. Participants are scaffolded in their recognition of what constitutes a suitable tool for each task as well as in their use of the technology.

Each of the activities begins with a problem set in a context that is meaningful, or real in the RME sense, to the student, promoting interest and engagement. Participants are not provided with a strict set of steps or procedures to follow, but are instead scaffolded and guided in their discovery of the mathematics required to solve the problem. They are however, required to justify the choices that they make. The nature of the tasks involves using knowledge and skills from a variety of different subject domains, but also draws on 21st Century skills (communication, creativity, problem-solving) and associated literacies (information literacy, technical fluency). In order to encourage students of all abilities to engage in the activities, they were designed to have a low-floor – an easily accessible entry point. However, all of the activities have the flexibility and potential to support in-depth levels of exploration.

The Bridge21 activity structure provides a set of steps to scaffold the implementation of 21st Century learning activities. The steps typically include: team formation; a divergent-thinking, 'warm-up' activity; investigation of the problem/challenge; planning; an iterative phase of task execution/problem-solving/artefact creation; presentation; and reflection. While all of the activities described in section 4.2 have been designed to be implemented within this framework, a tight-but-loose approach is advocated (Thompson & Wiliam, 2008), and not all of the steps are required for every intervention. The purpose of using a structure such as that provided by Bridge21 is to mitigate some of the barriers that teachers have reported as hindering their integration of this approach to teaching and learning, it should not become a further restraint or confine that needs to be adhered to. Used flexibly, the steps provided by Bridge21 encourage and scaffold collaboration and teamwork in an inquiry-based, technology-mediated environment. The presentation and reflection steps encourage peer-learning and require students to take responsibility for their work.

4.4 Conclusion

Involving students in activities of this kind, which provide meaning, context and coherency to the mathematics, may have the potential to increase conceptual understanding, confidence and engagement in the mathematics classroom. The relationship between these attitudinal constructs and the design heuristics will be investigated in Chapters 6 and 7. Prior to examining such relationships however, Chapter 5 provides an in-depth discussion of the methodology that has been selected for the gathering and analysis of data presented in the subsequent chapters.

5. Methodology

A central objective of educational research is to contribute to the development of knowledge and wisdom, specifically through informing educational judgements and decisions and improving educational action (Bassey, 1999). Assumptions about the nature of knowledge – how it can be found, recognised and used – are fundamental considerations when conducting research in this field. Decisions about research frameworks are informed by researchers' beliefs regarding the nature of reality (ontology) and of knowledge (epistemology), which in turn have implications for methodological considerations, instruments and data collection (L. Cohen, Manion, & Morrison, 2007).

When considering research frameworks, two related concepts need to be addressed. The first, *Research Methodology*, relates to how the data is interpreted – the rationale, beliefs and ideas that underpin the research. The second, *Research Methods*, is the collection of tools and techniques used to collect and interpret the data. Methodological considerations determine the framework within which the research questions are formulated, and influences the types of methods that will be considered appropriate for answering them.

5.1 Methodological Rationale

There is a range of methodological approaches from which the methodological rationale can be drawn (Morrison, 2007):

- Positivistic/reductionist approaches emphasise cause-and-effect relationships, with controllable variables, and maintain that facts exist independently of the knower. Pre-existing theory informs initial hypotheses.
- Naturalistic/interpretive techniques emphasise individual, personal meaning. Thus research within this paradigm is grounded in people's experience. Researchers recognise that their presence and actions will impact on the participants' experience, and that their own interpretation of events will colour the research findings.
- Critical theories accept that values are central to all research. The researcher does not take a neutral stance in the research, but rather has a transformative, emancipatory agenda.
- Pragmatic knowledge claims relate to actions, situations and consequences as opposed to antecedent conditions. The primary concern is with what works – actions and solutions to problems.

Reflexivity is a process, by which the researcher comes to understand how they are positioned in relation to the research that they are producing (Morrison, 2007). The context in which this research is founded is undoubtedly influenced by social, political, cultural and historical factors. In particular, it is situated in a post-primary education system which is undergoing systemic reform. Furthermore, it is not possible to separate who we are from the analysis process and the personal philosophies and history of the researcher will have a significant influence on perspectives and interpretations, although the nature of this impact is impossible to quantify. For this reason, it is important for the researcher to be self-reflective about how the research process is approached (Strauss & Corbin, 2008).

Identification of a research paradigm that reflects the researcher's ontological and epistemological beliefs, and embraces related research tools, is seen as an 'acknowledgement of the researcher's belief systems and of the impact a researcher can have on the object of research' (Grogan & Simmons, 2007, p. 37). It will impact on the use of qualitative versus quantitative methods. In this study, the importance of pre-existing theory is recognised, and the possibility of the existence of facts and truths independent of the knower is acknowledged. However, the importance of individual perception and the impact of social and cultural influence is also taken into account.

The overall purpose of the current research inquiry is to draw together elements that combine to create learning opportunities in which students can increase their engagement with mathematics, and to develop structures to facilitate the design and implementation of such activities; it thus falls within the pragmatic paradigm and is primarily concerned with "what works" (Johnson & Onwuegbuzie, 2004; Johnson et al., 2007; Morrison, 2007).

5.2 Research Methods

Quantitative methods are in general favoured by those with a positivist outlook, emphasising measurable, causal, statistically generalisable relationships. Qualitative research methods are generally employed by those with a more interpretivist philosophy, who believe that there is no measurable, objective reality that exists outside of the meanings that human beings bring to it (Johnson et al., 2007; Morrison, 2007). In qualitative research, a detailed description of the setting is provided in order to understand the data within the broader social and historical context.

More recently, an approach that combines qualitative and quantitative methods has come to the fore. *Mixed methods* research refers to studies in which the researcher mixes quantitative and qualitative research techniques, approaches, methods, concepts etc., within a single study, for the broad purposes of breadth of understanding and corroboration (Johnson & Onwuegbuzie, 2004; Johnson et al., 2007). Practically, mixed methods can be used to approach different aspects of the research question, within a careful and coherent research design (Creswell, 2003). Also, a "routine combination of methods creates researchers with an increased ability to make appropriate criticisms of all kinds of research" (Gorard & Taylor, 2004, p. 7), and thus the research itself should be more robust to the criticisms of either of the other two methods in isolation.

From an epistemological point of view, there are questions about how true can the combination of approaches be when stemming from different epistemological perspectives. However, Hammersley (1992) argues that both qualitative and quantitative researchers generally accept that their accounts are constructed, and that they themselves do not create reality. Thus, a pragmatic approach, which is consequence-oriented, problem-centred and pluralistic in nature, is favoured in mixed methods research (Johnson et al., 2007).

This research draws on data that is quantitative in nature in order to ascertain whether a change in student engagement has emerged through participation in the activities. An examination of the nature of the change however, requires a more interpretive perspective. Thus, the assessment of the impact of the approach on engagement and confidence has necessitated a combination of qualitative and quantitative methods – a *mixed methods* approach.

The research itself was conducted in both laboratory and real-life, or natural, settings, and the researcher was cognisant of the fact that the phenomenon under examination would be affected by the context in which it occurred. Bearing this in mind, three alternative methodologies that compliment a mixed methods approach have been considered.

5.2.1 Action Research

Action research aims to improve future practice through a process of iterative change. It makes use of a continuous system of feedback to solve particular problems in a specific setting, and to produce guidelines for best practice in this context (Denscombe, 2010; K. Green, 1999). The defining characteristics of action research are (Denscombe, 2010):

- Practical – action research aims to come up with solutions to specific problems within their context, in the real-world.
- Change is regarded as integral to this research process, both in terms of solving the specific problem, and through developing a deeper understanding of the phenomenon.
- Cyclical process – action research is characterised by its iterative nature. Initial findings generate possibilities for change, which are implemented and evaluated in a feedback loop.
- Practitioner participation – practitioners in the field are heavily involved in the research within the action research paradigm.

Although many of the attributes of action research are suited to the research problem identified in this work, there are two factors that make it inappropriate. Firstly, the rationale that underpins the identification of a problem and the implementation of a solution tend to be unique to a specific, local context and are not typically generalisable (K. Green, 1999). Therefore, this study – which aims to develop design heuristics that are applicable in any post-primary school context – is not suited to action research. Secondly, the initiator of an action research project tends to be the practitioner seeking to improve their own practice; it is not researcher-led.

5.2.2 Design-Based Research

Design-Based Research (DBR) is a methodology "designed by and for educators that seeks to increase the impact, transfer, and translation of education research into improved practice. In addition, it stresses the need for theory building and the development of design principles that guide, inform, and improve both practice and research in educational contexts" (Anderson & Shattuck, 2012, p. 16). From a literature review of articles relating to DBR published between 2002 and 2011 Anderson and Shattuck (2012) identify seven principles of what they consider constitutes good quality design-based research:

- *Authentic Context*: Being situated in a realistic educational context lends validity to the research, ensuring that the results can be used to assess, inform and hopefully improve practice in at least the current environment, with the potential for scalability.
- *A focus on the Design and Testing of Interventions*: This is viewed as a collaborative effort between practitioners and researchers. The creation of the intervention is founded in the local context and is informed by relevant literature, theories and other relevant contexts. The intervention should be designed to either improve local practice, or to overcome a particular problem.
- *Mixed Methods*: Typically DBR uses mixed methods and a variety of research tools and techniques, in accordance with dynamic requirements.
- *Iterative*: Design, in whatever field, generally involves the generation of prototypes followed by an iterative process of testing and refining. In DBR, the design and theory evolve continuously, in an authentic setting.
- *Collaborative*: Design-based research involves collaboration between practitioners and researchers. The nature of this collaboration is one area that distinguishes it from action research. In DBR, it is the researcher who takes initiative in the process with respect to both the research and the design. Action research on the other hand, is usually initiated by the practitioner, with researchers facilitating the process.
- *Evolving Design Principles*: The design of successful interventions is grounded in, and leads to the development of practical principles, patterns and theories. These should not be de-

contextualised principles or grand theories that function with the same weight in all scenarios, but should reflect the context in which they are utilised.

- *Practical Impact*: One of the fundamental tenets of DBR is that it is at least partly conducted in the natural environment and that it generates practical guidelines, principles and theory. As such it “should be able to migrate from our experimental classroom to average classrooms operated by and for average students and teachers, supported by realistic technological and personal support” (Brown, 1992, p. 143).

The cyclic nature of the DBR approach is well suited to the development of the design heuristics, and can be seen as an appropriate overarching methodological approach to address the research questions in this dissertation. It is particularly suited to the development of the heuristics and practitioner’s guide. However, within the design-based approach, a more structured research method would be beneficial, in order to scaffold the analysis of the relationship between the heuristics and the changes in engagement and confidence experienced by the students.

5.2.3 Case Study

A case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when boundaries between phenomenon and context cannot be drawn clearly or unambiguously. Case studies are most suited to study ‘how’ or ‘why’ questions about events over which the investigator has little or no control. They are situated within real-life scenarios, where an understanding of the phenomenon is linked to its context (Yin, 2014).

Propositions for case studies are developed from existing literature, prior to data collection. The theory emerging from the literature will have an impact on the type of case study chosen, the cases to be studied, analysis of the data and interpretation of the findings (Yin, 2003, 2014). Observations, interviews and documents provide the most common forms of data sources for case studies. Approaches for analysis vary depending on the intended outcome of the study, the final result of which should be a rich description of the case.

Features that identify a project as a case study include (Bassey, 1999; Creswell, 1998; Yin, 2014):

- The “case” for the study is clearly identified.
- It is an empirical study, conducted within specific time and spatial boundaries.
- The phenomenon is explored within its natural context.
- Extensive, multiple sources of information are used in data collection leading to a rich description of the case, and allowing for plausible interpretation and verification of trustworthiness.

- A rich description of the context or setting of the case is provided.
- Results of the study will be compared to propositions emerging from the literature, for the purpose of analytic, or “fuzzy” (Bassey, 1999) generalisation to the broad context of the study.

There are various types of case study; the two considered for this research were exploratory and explanatory, which will be discussed in the following section.

5.2.4 Research Method of Choice: DBR and Case Study

Table 5.1 below provides an overview of the crossover between the research objectives of this study and the different research methods that were considered. The shaded areas relate to instances in which the research objective could be addressed by the research method.

Table 5.1: Research Objectives and Methods

Research Methodologies v Research Objectives	Action Research	Design-Based Research	Exploratory Case Study	Explanatory Case Study
Develop theoretical foundations of the Design Heuristics	▨	▨	▨	▨
Create Sample Activities	▨	▨	▨	
Pilot Sample Activities	▨	▨	▨	
Gauge Student Reactions through questionnaires and comments	▨	▨	▨	
Gauge teacher reactions through workshops	▨	▨	▨	
Adjust Heuristics and Activities	▨	▨	▨	▨
Trial Activities in Authentic Settings	▨	▨		▨
Gauge Student Reactions through questionnaires and Interviews		▨		▨
Develop a deep understanding of the relationship between the Heuristics and changes in attitude		▨		▨
Gauge teacher reactions through written reflections		▨		▨
Develop a final version of the Heuristics and a practitioner's guide		▨		▨

Although Action Research has many strengths, it is clear from Figure 5.1 that it does not adequately match the objectives for this project.

Design-Based Research aligns particularly well with RQ1, and with the research approach in general. However, RQ2 requires a level of analysis and explanation more suited to a case study. For this reason, a combination of the two has been selected and the research method to be employed in this dissertation can be described as a design-based case study, with the *case* identified as “student engagement and confidence”, and the *context* of the case being Irish post-primary mathematics education. In effect, the study incorporates an exploratory and an explanatory case study within an overarching DBR paradigm.

Exploratory case studies are used to identify and refine research questions or procedures that will be used in further research (Yin, 2014). In this study, exploratory case study methods are used to generate hypotheses and to pilot the activities and research instruments.

While the use of an *exploratory case study* provides an opportunity to achieve some of the research aims, such as refining the design of the activities and the instruments, and honing the research questions, it is through the implementation of a second, *explanatory case study*, that a greater depth of understanding emerges. The purpose of this second case study is to explain how, and why some condition is achieved (Yin, 2014). In this case, we examine changes in student engagement and confidence through participation with activities designed and implemented in accordance with the design heuristics arising from the exploratory study.

In order to prevent ‘bleed’ from the exploratory study, the subsequent explanatory study, while based on the improved understanding that originated from the original study does not incorporate the original exploratory data.

The two case studies are made up of multiple embedded units – 7 Pilot Studies in the exploratory case study and 4 interventions (Int), in the Explanatory case study (Figure 5.1) – and employ a mixed methods approach to data collection and analysis. The context is post-primary education in Ireland, differing only in the setting, and the overarching unit of analysis is student engagement.

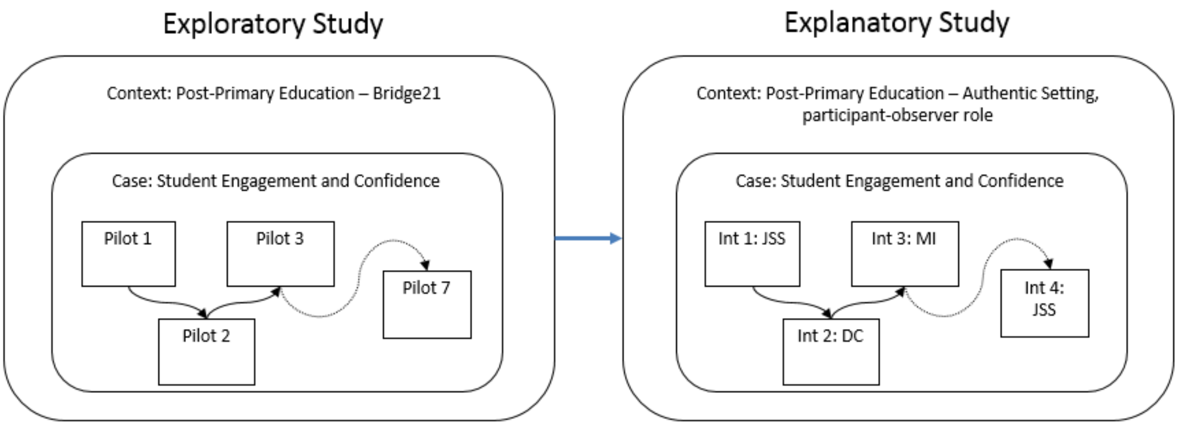


Figure 5.1: The Case Studies

As the data from the embedded units (Bridge21 pilots and in-school interventions) are pooled across schools, a multiple-case, replication design is not used. Both the exploratory and explanatory case studies are embedded, single case studies, in which all of the schools and students are part of a larger main unit of analysis (Yin, 2014). The methodological rationale for the choice of a single case study with multiple embedded units rather than a multiple case study approach, stems from a view of the overarching context of post-primary education as more relevant to this study than individual school contexts.

In this research, each embedded unit relates to a single intervention (or pilot study), in which both quantitative and qualitative data are collected. Both types of data are collected concurrently with emphasis given to quantitative data in the exploratory study and qualitative data in the explanatory study (Figure 5.2). A pre-experimental design, using pre- and post-questionnaires is used to attempt to quantify the impact of the intervention on student engagement, while analysis of interviews, observation, journal entries and comments explore the transformation in greater depth (Creswell, 2003), providing a rich and detailed description of *how* and *why* it has emerged.

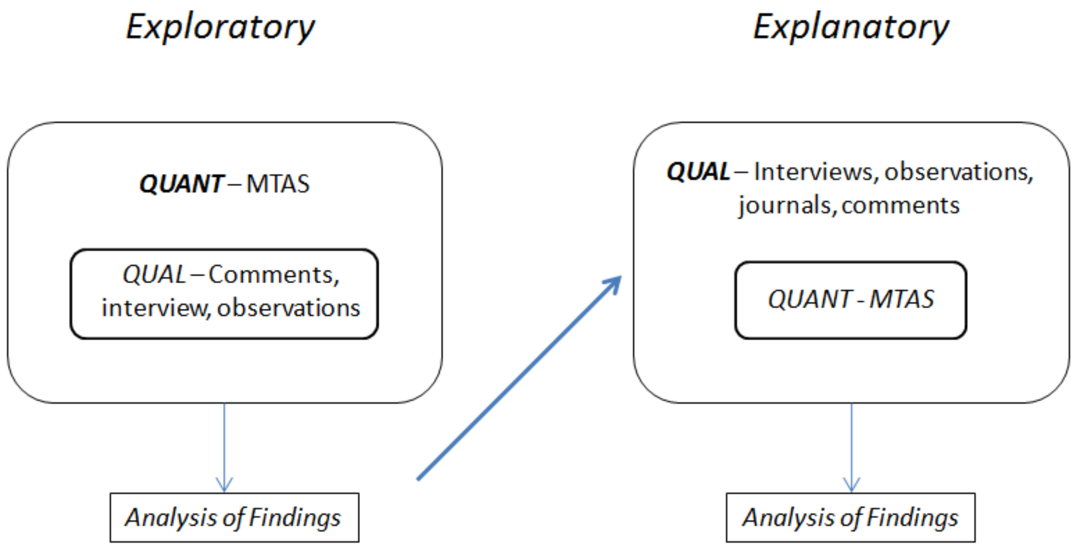


Figure 5.2: The Mixed Methods Approach

There is potential for a further case study to be added to the model described in Figure 5.1, the context of which would be ‘Post-Primary Education – Authentic Setting, non-Participant Observer role’. Initial research at this level is provided through analysis of written reflections from teachers who participated in Contextual Mathematics modules on a Postgraduate Certificate in 21st Century Learning in Trinity College Dublin. These practitioners were required to design activities in accordance with the heuristics and guidelines developed in this research, and to implement them in their classrooms. Preliminary results are discussed in Chapter 8.

5.3 Data Collection and Analysis Methods

Bassey (1999) highlights the need for sufficient data for a thorough exploration of the phenomenon, to test for trustworthiness and validity, and to permit interpretation. However he warns against collecting a surfeit of data such that there is insufficient time for a thorough exploration of the emerging themes. Many authors suggest a qualitative analysis of documents, archival records, interviews, artefacts and observation as appropriate sources of data for case study (Bassey, 1999; L. Cohen et al., 2007; Creswell, 1998; Yin, 2014). Yin (2014) adds to this by suggesting that case studies can also include quantitative evidence when relevant.

5.3.1 Quantitative Data

This research uses a pre-experimental design in a pre-test/post-test study, in order to generate quantitative data. In this type of pre-experimental design, the researcher provides each group of participants with a pre-test, conducts an intervention, and then post-tests the participants in order to measure change. This design does not have a control group to compare with the experimental group (Creswell, 2003), however, for reasons of feasibility and access this approach was deemed most appropriate for this work.

In order to determine a baseline of students' attitudes to, and confidence with mathematics and technology, the Mathematics and Technology Attitudes Scale (MTAS) (Pierce et al., 2007) is used as a pre- and post-test for each cohort. This is a 20 item questionnaire with a likert-type scoring system that measures five affective variables related to technology enhanced mathematics learning (For the full questionnaire, see Appendix 5.A):

- Behavioural Engagement (BE): how students behave when learning mathematics
- Affective Engagement (AE): how students feel about the subject
- Mathematical Confidence (MC): students' conceptions of their ability to do well in the subject and to handle difficulties
- Confidence with Technology (TC): students' confidence in their ability to master technological procedures required of them and resolve difficulties
- Attitude to using Technology for Learning Mathematics (MT): the degree to which students feel that technology provides relevance, aids their learning, and contributes to their achievement in mathematics.

The questionnaire designers use factor analysis and reliability analysis, with the data satisfying the underlying assumptions of the Principal Component Analysis and indicating an acceptable/strong internal consistency in each subscale (Pierce et al., 2007, p. 294). Pierce et al. (ibid.) highlight that "students' vocabulary and behaviour indicating confidence and engagement will be dependent on local culture and context" (p. 289). As MTAS was initially designed and trialled in Australia, the reliability tests described in the original paper were applied to 148 responses from Irish students. Factor analysis confirmed the five-factor structure of the Australian scale, and while the Chronbach's Alpha test highlighted some cultural differences – Irish students appear to be less likely to try to answer questions asked of them by the teacher – the scores remained satisfactory for each of the subscales (MC, .89; MT, .81, TC, .91; BE, .68; and AE, .74).

The questionnaire is given as a pre-test to each cohort of students prior to taking part in the activities. At this point they are asked to reflect on their general mathematics classes in the school. After the intervention, participants are requested to score the questionnaire again, this time relating their answers to the period of the intervention.

Results are tested for normalcy and then either Student's paired sample t-test (if a normal distribution is identified), or Wilcoxon Signed Rank tests (if the data are not normally distributed) are used to check for statistically significant changes in each of the subscales. This choice of statistical analysis is considered appropriate according to the criteria put forward by Jaykaran (2010). Cohen's *d* is used to determine the effect size, or practical significance, of the results.

5.3.2 Qualitative Data Collection

The qualitative student data collected in this study includes interview, observation (direct and participant), journals and written comments. Qualitative teacher data is entirely drawn from written reflections. The approaches to data collection used in this study are discussed below.

5.3.2.1 Interviews

Yin (2014) suggests that interviews are one of the most important sources of case study data, describing them as guided conversations, with a fluid stream of questions within a consistent line of inquiry (p110). Interviews can be differentiated according to their duration.

- Prolonged case study interviews that take place over two, or more, hours.
- Shorter case study interviews tend to be more focused and generally take place in under an hour. While the questions may be open-ended and assume a conversational manner, the major purpose of such interviews is not to ask about topics of a broad, open-ended nature, but might simply be to corroborate certain findings that have already been tentatively established. (Yin, 2014).

They can also be classified according to their level of structure.

- Unstructured, or open interviews rely solely on open-ended questions in which the interviewee is free to form their own response (L. Cohen et al., 2007).

- Semi-structured interviews follow a pre-determined set of questions, but also allow for deep exploration of participant responses through discussion and clarification questions.
- Structured, or Survey interviews, use closed questions with a fixed set of possible responses. These structured questionnaires can also be conducted as surveys, and are generally used to produce standardised, comparable and quantitative data (L. Cohen et al., 2007; Yin, 2014).

Interviews can also be one-to-one, or in focus-groups, and they can be conducted in person, by telephone, using video-conferencing software such as Skype of Google Hangouts, or using synchronous/asynchronous text-based communications such as email or instant messaging.

Semi-structured, focus-group and individual interviews are the primary forms used in this study. A total of nine interviews were conducted with students, comprising roughly 4.5 hours of data. The decision to use both individual and focus-group interviews stems from a pragmatic viewpoint. Initial interviews were group-based (four in total, one per intervention) in order to develop a strong overview of the experiences of the students. Individual interviews (five in total, conducted after the final intervention) complemented this by allowing the researcher to delve into particular issues that had been highlighted through theoretical sampling of the initial qualitative data (Strauss & Corbin, 2008).

Each interview involved the researcher travelling to the school in which the intervention had taken place in order to speak to individual participants, or groups of 5/6 students, one from each team that had participated in the activity. The purpose of these interviews was to develop an understanding of the phenomena under investigation, that being the extent of change in affective behaviours in relation to mathematics learning.

5.3.2.2 Observation

Owing to the level of researcher control implicit in interviews, Stake (1995) displays a preference towards observation as a data collection method. He suggests that observation should provide an accurate description of events and should be directed by pre-identified research concerns. Yin (2014) differentiates between direct observation, in which the researcher is a passive observer, and participant-observation, wherein the researcher can take on a variety of roles and may participate in the action. Participant-observation can be a more feasible option in terms of facilitating access to groups. However with this kind of observation there is a greater potential to introduce bias through engagement with the group under study, and it can be difficult to conduct adequate observation whilst participating in the activity.

While the researcher in the present study engaged in a participant-observer role, this research has endeavoured to also have an independent observer present at the some of the interventions. The direct observer was in a position to take notes on the activities based on a pre-defined observation protocol that structured the approach, encouraging a balanced observation of the groups (Appendix 5.2). It was not always possible however, to enlist another researcher in this way, and therefore observation does not make up a large portion of the data to be analysed.

5.3.2.3 Journals and written comments

Depending on the duration of the intervention, the students were asked either to keep a reflective journal, which addressed four pre-defined headings, or to address the same headings as comments on the post-test sheet:

- Did you enjoy the session?
- Why?
- List two things you learned.
- List two things you found difficult.

Yin (2014) describes such documents as corroboratory data, suggesting that they may provide records of events that were unobservable, but are rarely however without bias, as they are written with a particular audience in mind. The reflection documents in this study however are used as a primary source as they are considered to be an unobtrusive form of data collection, in which the students are likely to be uninhibited in their representations of events.

5.3.2.4 Teacher Reflections

As a part of their assignment for the Contextual Mathematics module on the Postgraduate Certificate in 21st Century Teaching and Learning, which is discussed in detail in Chapter 8, participants were required to create and implement an activity in line with the design heuristics developed in this research. The assignments required the teachers to give a detailed description of the activity along with its rationale and expected learning outcomes. They were also required to provide samples of student work, their marking rubric, and a written reflection on the overall process and experience. These written reflections comprise the teacher data.

5.3.3 Qualitative Data Analysis

There are numerous qualitative analysis techniques appropriate for the analysis of textual data. L. Cohen et al. (2007) differentiate in particular between grounded theory and content analysis. The grounded theory approach uses Constant Comparative analysis techniques in the generation of a theory that emerges from the data. Both directed content analysis (Section 5.3.3.1) and constant comparative analysis (Section 5.3.3.2) are used in this dissertation.

In this research, a systematic analysis of the literature led to the development of initial hypotheses, which were tested in a pilot setting with the generation of further research questions. These questions constitute the purpose and direction of the research. For this reason, content analysis has been selected as an appropriate analytic strategy for the initial stages of this study.

However, constant comparative techniques are used for a secondary analysis of the student interview data, in order to ensure that emerging themes are not overlooked and to develop a deep understanding of the relationship between the design heuristics and the changes in engagement and confidence. This is considered particularly appropriate due to the diversity in the nature of the embedded units.

5.3.3.1 Content Analysis

Content analysis is described by Krippendorff (2004) as “a research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use” (p. 18). Tipaldo (2014) updates this definition, referring to the process as a wide and heterogeneous set of manual or technology-enhanced techniques for the contextualised analysis of documents produced through communication or signification processes, with an ultimate goal of the production of valid and trustworthy inferences.

Zhang and Wildemuth (2009) differentiate between *quantitative content analysis*, a method of counting occurrences of text evident in documents, with *qualitative content analysis*, which goes beyond a simple word count, allowing meanings, themes and patterns to emerge from the data. The techniques used in qualitative content analysis are referred to by Mayring as an “empirical methodological and controlled analysis of text within their context of communication, following content analytic rules and step by step models, without rash quantification” (2000, p. 2). This type of qualitative analysis permits the researcher to understand the data in a subjective, but scientific manner (Zhang & Wildemuth, 2009).

Hsieh and Shannon (2005) further differentiate between three types of qualitative content analysis: summative, conventional and directed. The choice of content analysis thus depends on the particular research question or purpose of the study, with the distinction based on the way the codes and categories are derived from the text, i.e., inductively or deductively (Moretti et al., 2011).

- Summative Content Analysis
  The term *summative content analysis* is used for an initially quantitative approach that goes on to include analysis of latent meanings and themes (Hsieh & Shannon, 2005; Humble, 2009; Zhang & Wildemuth, 2009).

- Conventional Content Analysis
  Conventional techniques are used when the purpose of the study is to describe a phenomenon. No preconceived categories are used in the analysis and it is therefore a useful technique when existing theory or research in the area is limited. Researchers immerse themselves in the text and allow the coding categories and names to emerge inductively from the raw data. The approach has parallels with grounded theory, which utilises a similar approach in its initial data analysis (constant comparison), but goes beyond basic content analysis to generate a theory (L. Cohen et al., 2007; Moretti et al., 2011).

- Directed Content Analysis
  Directed content analysis on the other hand is guided by a more structured approach, and allows for the pre-existence of theory to guide the process of data analysis. This deductive approach begins with a theory or relevant research findings, which the researcher uses to identify key concepts and variables that can be used as coding categories (Moretti et al., 2011). The pre-defined categories are then used in the analysis of the data, with any passages that do not seem to fit with the pre-determined codes assigned a new one (Hsieh & Shannon, 2005).

As the primary thrust of the qualitative aspect of this research is to examine the impact of an approach using the pre-defined categories from the MTAS questionnaire, a directed approach to content analysis is utilised, similar to that described in Lev-Zamir and Leikin’s (2013) study on creativity in mathematics teaching. In the present study, the content analysis is directed by a search for patterns that indicate the pre-defined MTAS categories of confidence and engagement on one hand, and the design heuristics on the other, with an inductive approach to the identification of new, but related categories.

5.3.3.2 Constant Comparative Analysis

Approaches that work the data from ‘the ground up’, such as constant comparison, contrast directly with directed content analysis, in that they are not based on a-priori theoretical propositions (Yin, 2014). Although directed content analysis remains open to emergent coding (Namey et al., 2007), this is generally restricted to portions of the data that have not been already coded according to the theory. Constant comparison considers the entire data set with an open mind, and thus allows for a broader spectrum of emergent themes, and a fuller understanding of the properties of and relationship between such themes. As the embedded units within this case study are diverse in many ways (location, duration, organisation, number of students, socio-economic background and so on), constant comparison has been used in addition to the directed approach, in order to be sensitive to emerging themes and to give a richer understanding of the relationships between the categories.

Constant comparative techniques are also used for the analysis of the teachers' written reflection, as no a-priori theory had been developed to guide the analysis.

5.3.3.3 Use of Software in Data Analysis

Yin (2014) and L. Cohen et al. (2007) identify that software can be useful when processing and analysing large quantities of qualitative data. As highlighted by Yin (2014), computer assisted qualitative data analysis software, such as NVivo, has become more sophisticated in recent years, and now has the capacity to deal with evidence that has not necessarily been converted into textual format, such as video-based and photographic data. However, it is important to remember that these packages are merely tools to assist in the process of data analysis. It is the researcher who must interpret the meaning, remaining central in the data analysis process.

Throughout this research, NVivo10 has proven to be invaluable, and has provided the researcher with a range of tools to support the qualitative data analysis. It was possible to transcribe the interviews directly into NVivo10, which maintained a link between the text and the auditory data. For the initial stages of directed content analysis, the a-priori codes were inputted into the software, and were then used to code the interview transcripts. NVivo matrix coding facilitated easy comparison of the elements of the transcripts that had been coded at the MTAS subcategories, with those that were coded according to the design heuristics, allowing for meaningful conjectures to be drawn.

The software also permitted a coding comparison query to be run on a second coding of the data by a researcher from outside the project. This comparison of codes was useful to ensure reliability of the results.

Constant comparative analysis techniques require significant documentation of the process and again the NVivo10 software was very useful in this regard. Codes and categories were constructed and merged as required and memos could be created and directly linked to specific codes or categories, recording queries and decisions in a dynamic and apposite manner. In this way, a coherent record of the analysis was maintained.

5.4 Trustworthiness of the Study

Validity, reliability and objectivity are the criteria most commonly used to assess trustworthiness and quality within the conventional positivist paradigms (Zhang & Wildemuth, 2009). As interpretivist research methods differ fundamentally from positivist ones, in their processes, assumptions and purpose, making use of reliability and validity as constructs to assess quality, can be problematic in qualitative analysis and case-study research (Bassey, 1999; Zhang & Wildemuth, 2009). However, validity and reliability are important within qualitative research (Morse, Barrett, Mayan, Olson, & Spiers, 2002; Yin, 2014) although perhaps with slightly different terminology and interpretations such as trustworthiness, credibility, transferability, dependability, and confirmability (Creswell & Miller, 2000; Morse et al., 2002). Yin (2014) proposes case study ‘tactics’ (p. 45) for four tests of quality of research design: construct validity, internal validity, external validity and reliability.

- *Construct Validity* refers to the identification of correct operational measures for the concepts under consideration and refers to methods of data collection and composition. In order to test for construct validity in this research, triangulation, i.e., the collection of multiple sources of evidence in order to develop converging lines of inquiry, is used.
- *Internal validity* in explanatory studies, seeks to establish causal relationships between conditions. In qualitative research, this is affected by research design and can be tested for at the point of data analysis. This research uses a process of pattern matching, explanation building and exploration of rival explanations in an attempt to ensure internal validity.
- *External validity* relates to generalisability and theory building. In order that the analysis of the data generates results that are deemed to be applicable in different settings, external validity needs to be addressed. Although this research does not purport to generate a formal theory, the application of the analysis across multiple embedded units provides a certain level of extensibility to the findings.
- *Reliability* relates to the capacity to demonstrate that the procedures undertaken in a study are repeatable, with the same findings and conclusions. This does not mean that the outcomes of a case study can be replicated in another, rather that the processes undertaken throughout the research could be accurately followed. In this research, protocols have been developed to document the procedures to be undertaken in interviews, observation, and for the administration of the MTAS questionnaires, operationalising as many steps as possible. In order to ensure reliability of the content analysis, small samples of text were first analysed, and alterations where made to the coding and categorisation where appropriate (Weber, 1990; Zhang & Wildemuth, 2009). In addition, a coding comparison was conducted by a researcher from outside the project.

Creswell (2003) suggests that validity is in fact a strength of qualitative research and presents eight primary approaches to ensure validity: triangulation, member-checking (or peer-validation), rich description, clarification of researcher bias, presentation of negative or discrepant information, prolonged time in the field, peer debriefing, and the use of an external auditor.

Owing to constraints on time and resources in this study, only triangulation, rich description and clarification of researcher bias are amalgamated with Yin's tactics for ensuring trustworthiness of the study. Triangulation is approached using three of Denzin's (1978) four basic types:

- *Methodological* triangulation is achieved through the analysis of quantitative and qualitative data, as well as through the use of different sources of qualitative data.
- *Data* triangulation is accomplished through the analysis of the multiple embedded units across the various interventions.
- *Investigator* triangulation emerges through the integration of non-participant observation conducted by a secondary researcher and a coding comparison of the directed content analysis.

The fourth basic type, *theory* triangulation – that is the use of more than one theoretical framework in the analysis of the data – is outside the scope of this research.

The use of a *rich description* to increase validity is evident within this research as it is required by the nature of the case study approach.

Finally, although Miles and Huberman (1994) present a number of strategies, including triangulation, to minimise the impact of researcher bias, it is the view of this researcher that such bias cannot be completely negated. Therefore, it is acknowledged that the personal history, beliefs, values and assumptions that a researcher brings to their role as primary research instrument and analyst, has had an impact on the results of the study. Creswell and Miller (2000) recognise that the lens used by the researcher along with the paradigm assumptions to which s/he adheres will have an impact on the chosen validity procedures. In this research, the lens combines that of the researcher with people external to the study, within a pragmatic paradigm. Consequently, triangulation, an awareness of the possibility of rival explanations, careful documentation of procedures and rich description are all used in an attempt to ensure reliability and validity and to counter researcher bias.

5.5 Generalisation of Findings

Opposing views are held by some authors regarding the potential for generalisation of qualitative research in general and case study research in particular. For example, Stenhouse (1985, cited in Bassey, 1999) maintains that a case study should be able to stand on its own, without generalisation, through good reporting. L. Cohen et al. (2007) on the other hand, suggest that the overall purpose of the case study is to "probe deeply and to analyse the multifarious phenomena that constitute the life cycle of the unit [under observation] with a view to establishing generalisations about the wider population to which that unit belongs". Sturman, in Keeves (1994), speaks to the holistic nature of case studies, arguing that a deep investigation into the relationships between component parts is required in order to understand why things happen as they do, and that generalisation requires an "in-depth investigation of the interdependencies of parts and of the patterns that emerge", p61. Yin (2014) argues that if the empirical results of a case study back up a previously developed theory, then replication can be claimed, leading to *analytic generalisation*. *Fuzzy generalisation* is the term coined by Bassey (1999), and relates to the type of generalising statement that makes no concrete claims to knowledge, but identifies what may be possible or even likely in circumstances with characteristics similar to those of the case study from which the generalisation arose. Bassey (1999) suggests that this should be the key objective of an educational case study. In this study, owing to the broad nature of the case and the variety offered by the embedded units, the capacity for fuzzy generalization to support prediction is seen to have considerable potential.

In particular, the integration of the two case studies within a design-based paradigm recognises an explicit intent to develop practical principles, patterns and theories. Such transferability does not anticipate the development of a formal theory that functions with the same weight in all scenarios, but rather a substantive theory, elements of which can be transferred to contexts similar to the one under investigation (Strauss & Corbin, 2008).

5.6 Ethics

Ethical considerations involve more than mere courtesy and etiquette, but are concerned with the appropriate treatment of people in a free society (Best & Kahn, 1989). The growing awareness of ethical concerns in research is evident from the increase in related literature and the introduction of regulatory codes in research institutions and professional bodies (L. Cohen et al., 2007). In this study, a research approach has been drawn up that adheres to the ethics standards required by the School of Computer Science & Statistics, Trinity College Dublin. Approval for the overarching Bridge21 project was already in place, and in order to obtain approval from the Research Ethics Committee for this particular project, the researcher was obliged to submit a School of Computer Science and Statistics Research Ethical Application Form, that gave an outline of the purpose of the project along with the research methods and instruments to be used.

An opportunistic, or convenience sampling strategy was used to select schools and individuals (L. Cohen et al., 2007), and participants were recruited from schools that had given the researcher permission to conduct the research project. Approval was obtained through the Board of Management of each school, and the school principal, as well as from individual participating teachers.

1. Both parental and student consent to take part in the research was required to include each student in the data collection activities.
2. Teachers took part in the project on a voluntary basis and gave their written consent to participate in the research.
3. Consenting participants on the Postgraduate Certificate in 21st Century Learning provided written permission to have their assignments included in the research.
4. Students in participating teachers' classes were involved in the learning activities as part of their normal school curriculum day, regardless of whether consent had been given for their data to be used in the research.

5.7 Discussion

Identification of a research approach highlights two fundamental aspects of the study. Firstly, the approach adopted reflects the ontological and epistemological beliefs of the researcher, situating them within a particular paradigm. Secondly, these beliefs impact on the methods that will be adopted to ensure relevance, validity and reliability of the research.

This research study is pragmatic in nature and is situated within an overarching design-based paradigm, incorporating two distinct case studies; one exploratory and one explanatory. The purpose of the studies is to identify and examine the nature of the impact that a particular approach to mathematics education has on student engagement and confidence with the subject. Analysis of the narrative emerging from various forms of qualitative data is used to support the identification of tentative generalisations about the nature of the impact of the approach in general. The design-based aspect supports the iterative approach to the extrapolation of the design heuristics and practitioner's guide.

A mixed-methods approach to data collection and analysis is implemented. A focus on the more quantitative aspects is used in the exploratory study, generating initial data and hypotheses. The explanatory case study emphasises a more qualitative approach, with the quantitative data used for corroboration and triangulation.

Analysis of quantitative data emerging from the pre- and post-questionnaires, as well as the results from the analyses (directed and comparative) of the various forms of qualitative data provides a rich and detailed picture of the impact of this particular approach to mathematics education. In combination with the results of the analysis of the teacher reflections, this supports the iterative, design-based development of a framework to guide teachers in the creation and implementation of transformative, technology-mediated, and collaborative mathematics learning activities.

6. Exploratory Case Study – Bridge21

6.1 Introduction

The design heuristics introduced in chapter 4 have theoretical foundations in the results of the analysis of the classification of literature relating to technology-mediated mathematics interventions (chapter 3) and a general literature review (chapter 2). A number of activities were designed in accordance with these heuristics, and trialled in the Bridge21 learning space. The results of these pilot interventions have fed back into the theoretical foundations of the research, serving to revise and refine the design heuristics. The motivation for the development of these heuristics and activities is twofold: firstly to increase student engagement with and confidence in mathematics, and secondly to facilitate and support teachers in the creation and implementation of learning activities of this kind, emphasising the general characteristics that are deemed important to foster engagement and confidence. The pilot interventions constitute the exploratory case study described in this chapter.

6.2 Research Aims and Questions

The aims of exploratory case studies are to generate or refine research questions when there is insufficient literature to address the question, and to pilot aspects of the study (Yin, 2014). The specific research aims of this exploratory case study are:

1. To develop a number of activities consistent with the design heuristics.
2. To explore the learning experiences of students who engage with these activities.
3. To explore the experiences of teachers engaging with the design and implementation of such activities.
4. To refine the design heuristics and develop a guide for practitioners.
5. To generate relevant questions for future research.

Prior to piloting the activities, it was not possible to be sure whether student participation in the interventions would have a positive impact on their levels of confidence and engagement, and it was not clear how teachers could be supported in the design and implementation of activities of this kind. For this reason, while aiming to maintain the broad exploratory aims, the following research questions (relating to RQ1) were used to focus discussion of the findings:

a. What are the desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence?
b. What are the key elements of a practitioner's guide for the creation and implementation of such interventions within the traditional school environment?

In order to address these questions, a number of the activities described in section 4.3, which were created in accordance with a first iteration of the design heuristics, were conducted with students in a purpose designed learning space (Bridge21) in Trinity College Dublin. Once the data emerging from the activities were analysed, and the set of design heuristics were refined, three continuous professional development (CPD) workshops were conducted with teachers. Analysis of the student and teacher experiences with the process are presented in this chapter.

The following chapter (chapter 7) provides a more focused analysis of the relationship between the design heuristics and the MTAS constructs relating to students' behavioural and affective engagement, mathematical and technical confidence, and attitude to using technology for learning mathematics (Pierce et al., 2007), within authentic school environments.

6.3 Context: Bridge21

Bridge21 is a model of collaborative, project-based learning that has been developed at the author's institution. It embodies many of the aspects associated with 21st century learning including teamwork, extensive use of technology and the teacher as orchestrator rather than director of learning. Since its inception in 2007, over 8,500 students have participated in out-of-school, team-based, technology-mediated workshops. The workshops typically run for 3.5 consecutive days (22 hours total) and take place during the school day in a purpose-designed learning space on the university campus. The model is currently being trialled in a number of post-primary schools as part of a systemic reform process in Irish education (Conneely, Girvan, & Tangney, 2015; Lawlor et al., 2015). The Bridge21 model encompasses the following elements (Lawlor et al., 2010).

- A structured team-based pedagogy influenced by the Patrol System learning method of the World Organization of the Scout Movement (Bénard, 2002).
- A physical learning space designed and configured to support team-based learning.
- Adult support that seeks to guide and mentor, with teachers orchestrating and scaffolding team activities.
- Engagement with content through student-led projects.
- Technology used as an integral tool in the process.
- Incorporation of team and individual reflection as a regular part of the learning.
- Cross-curricular thematic learning.

It is within the context of the Bridge21 learning space and using the associated pedagogic model that a number of mathematics learning activities were initially trialled with students. This was a cyclic process, in which the students responded to pre- and post-questionnaires and added comments about what they did and did not like, both about the process and the activities. The purpose of this was to identify whether the activities had the potential to effect positive change in the subscales identified by the MTAS questionnaire (Section 5.3.1), and to identify what might be the common attributes of such activities that could be used to refine the set of design heuristics. The activities followed the Bridge21 activity model, as described below.

6.3.1 A Bridge21 Activity Model

A Bridge21 learning experience involves a number of steps (Figure 6.1):

1. **Set-Up:** Ice breaker and team formation.
2. **Warm-Up:** Divergent thinking activity**.**
3. **Investigate:** Explanation of the problem context.
4. **Plan:** Group planning.
5. **Create**:
    a. Exploration with resources.
        i. In the field.
        ii. In the classroom.
    b. Modelling and Calculation:
        i. Analysis and Synthesis.
6. **Present:** Competition and/or Presentations.
7. **Reflect:** Reflection and Discussion.

Figure 6.1: Bridge21 Activity Model

In some instances, not all the steps may be required; for example, team formation is not required if the learners are in functioning teams already. However, it is important that tight deadlines are set for each phase to ensure the team stays on-task.

6.3.2 Initial Pilot Interventions

A total of five activities were designed and refined through the exploratory process. These are described in detail in section 4.3. All of the students who took part in these activities had prior experience working in the Bridge21 environment and were familiar with the activity structure and with working in teams. They were therefore, in a position to concentrate fully on the learning activity and the emerging mathematics.

The first activity to be tested was the Human Catapult activity (4.2.3) in which 16 students (16-18 years old) volunteered to participate in a day-long workshop. In this instance, pre-tests were not recorded, so no statistical analysis could be conducted. During the day, the activity was added to and refined. The initial plan did not include the use of the simulation engine (phet.colorado.edu), but as the activity progressed, it became clear that a move from the abstract modelling phase back to more concrete experimentation would be beneficial.

The second activity conducted in the Bridge21 environment was the Scale activity (4.2.1). This activity was run on two separate occasions with groups of 12 and 10 students (15-16 years old). Refinements were made to the lesson plan after the first instance, putting an increased emphasis on estimation and narrowing the scope of the activity. Pre- and post-tests were conducted in both cases and a small number of comments were collected for qualitative analysis.

The third intervention consisted of 12 students aged 15-16, who were split into 4 groups, two of which engaged with the Plinko and Probability activity (4.2.4), one with the Human Catapult, and one with its indoor equivalent – the Angry Birds Activity (4.2.3). Once again, Pre- and post-tests provide quantitative data, and some qualitative comments were collected.

6.3.3 Adapted Pilot Interventions

In addition to the original activities that were piloted in Bridge21, a further three activities were conducted in collaboration with other researchers. The Human Catapult activity was adapted by two researchers for their own projects, and the Timepiece activity was a collaborative design between a practicing teacher and the author. These three collaborative endeavours served to enhance understanding of the general development process for activities of this kind.

6.3.3.1 Human Catapult e-Book

A fourth year computer science undergraduate student developed an e-book based on the Human Catapult activity and implemented a pilot study in Bridge21 using the activity model. Four students took part in the pilot, in two pairs. The author ran the session, with the assistance of the undergraduate student. Once again, MTAS was used to collect quantitative data, and comments provided qualitative data.

6.3.3.2 Digital Pen Supported Reflection

A teacher enrolled on the Technology and Learning M.Sc. in Trinity College Dublin adapted the Human Catapult activity for use with his students. The activity was conducted in Bridge21, using the activity structure, but the students were required to keep extensive reflective notes throughout, using a digital pen, which digitally captured their written and audio data. Students completed a pre- and post-questionnaire and gave a short interview at the end of the day, providing qualitative data.

6.3.3.3 The Timepiece – a Teacher-designed activity

In addition to the adapted activities, a physics teacher used the design heuristics to develop an activity in collaboration with the author, which was also piloted in Bridge21. The Timepiece activity incorporated both mathematics and physics, and involved using video analysis techniques to estimate acceleration due to gravity. Students were then required to use their estimates in the construction of a pendulum that could accurately measure one minute. MTAS was once again administered before and after the activity, which generated quantitative data.

6.4 Data Collection and Analysis

This exploratory study took a mixed-methods approach, with an emphasis on quantitative data (Figure 6.2).

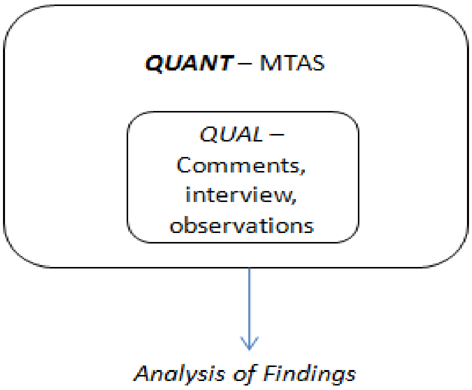


Figure 6.2: Quantitative-led Mixed Methods Approach

A pre-experimental design was used in an attempt to quantify the impact of the approach on student engagement with and confidence in mathematics (Creswell, 2009). Directed content analysis techniques were employed for the qualitative analysis of students' comments and interview data.

6.4.1 Pre-Experiments Data Collection and Analysis

The Mathematics and Technologies Attitudes Scale (MTAS) (Pierce et al., 2007) was utilised to gather quantitative data. MTAS is a 20-item, five-point Likert-type questionnaire with five subscales of *Affective Engagement* (AE), *Behavioural Engagement* (BE), *Mathematical Confidence* (MC), *Confidence with Technology* (TC), and *Attitude to using Technology for Learning Mathematics* (MT) (Section 5.3.1).

While a total of 74 students took part in the interventions, only 55 completed both pre and post-tests. The instrument was administered to these students before and after the interventions, and Wilcoxon Signed-Rank tests were used to analyse the data. Wilcoxon Signed-Rank tests are appropriate in this instance as the Shapiro Wilk test of normality indicated that the data are not normally distributed (Field, 2009). Statistically significant differences are identified in all subscales at the $p < 0.05$ level (Table 6.1).

Cohen’s *d* is used to estimate the effect size, showing the effect as 0.3 (MC and TC), 0.4 (MT and BE) and 0.5 (AE). While effect sizes of this magnitude are generally classified as moderate, they are considered notable in naturalistic educational research of this kind (J. Cohen, 1988; Elliot & Sammons, 2004; Lipsey et al., 2012).

Table 6.1: Wilcoxon Signed-Rank test results

	Mean-pre	SD-pre	Mean-post	SD-Post	*z*(54)	P
AE	14.42	2.69	15.60	2.30	-3.87	<0.001
BE	14.33	2.32	15.35	2.80	-3.54	<0.001
MC	13.87	3.50	14.75	3.36	-3.09	0.002
TC	14.44	2.64	15.15	2.76	-3.08	0.002
MT	15.11	3.10	16.38	3.17	-3.47	0.001

The results of the quantitative analysis strengthened confidence that the sample activities, designed in accordance with the design heuristics, have the potential to positively impact on the engagement and confidence levels described by the MTAS instrument.

6.4.2 Qualitative Data Collection and Analysis

In order to further explore the results of the quantitative data, the students were requested to add written comments about their experiences with the pilot activities, relating to the activities themselves, the technology, and the structure of the day. A total of 22 comments and reviews were collected. In addition to these written comments, a short interview was conducted with one cohort of students, who took part in one of the adapted Human Catapult activities. This interview included 14 students, was conducted immediately following the activity session in Bridge21, and was of approximately five minutes total duration.

6.4.2.1 Directed Content Analysis

Directed content analysis is a deductive technique and is appropriate when the structure of the analysis is operationalised based on prior knowledge or theory (Elo & Kyngäs, 2008). A key feature of content analysis is that the data under analysis is classified into smaller content categories (Weber, 1990). According to Elo and Kyngäs (2008), the first step in this process is to determine a coding or categorization matrix. In this exploratory case study, three coding matrices were developed, one based on the Mathematics and Technologies Attitudes (MTAS) Scale, one on the design heuristics, and one on the traditional school approach.

Figure 6.3 illustrates the coding matrix used for the MTAS-directed coding. Within the Generic Categories, AE refers to Affective Engagement, BE to Behavioural Engagement, MC to Mathematics Confidence, TC to Confidence with Technology, and MT to attitude towards the use of Technology for learning Mathematics. The sub-categories break down each of the generic categories into positive and negative references to the interventions.

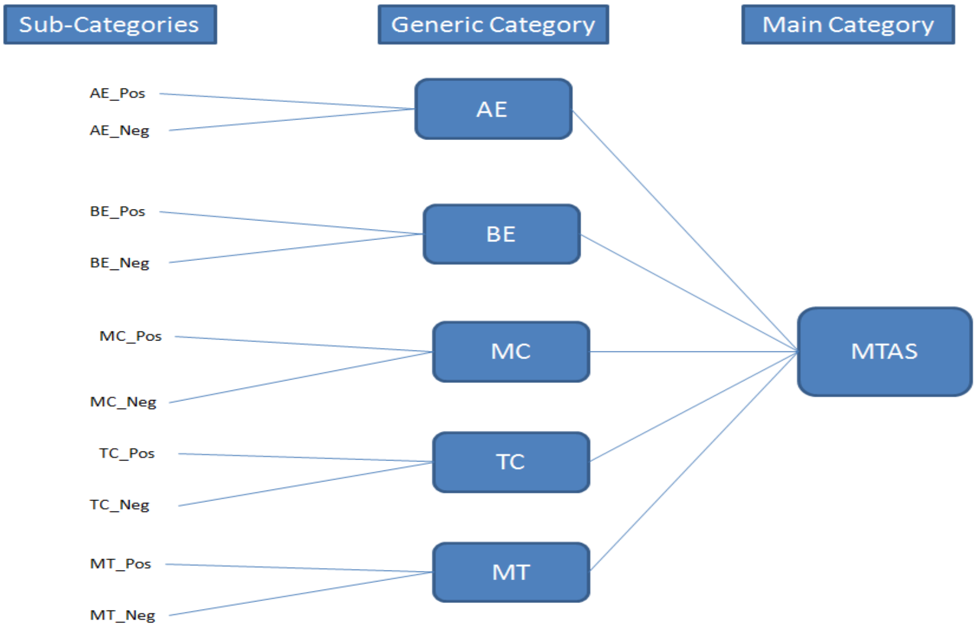


Figure 6.3 MTAS Categorization Matrix

Figure 6.4 displays the coding matrix used for the design heuristics-directed coding. The generic categories list the primary headings of the heuristics, and the sub-categories highlight various aspects of each.

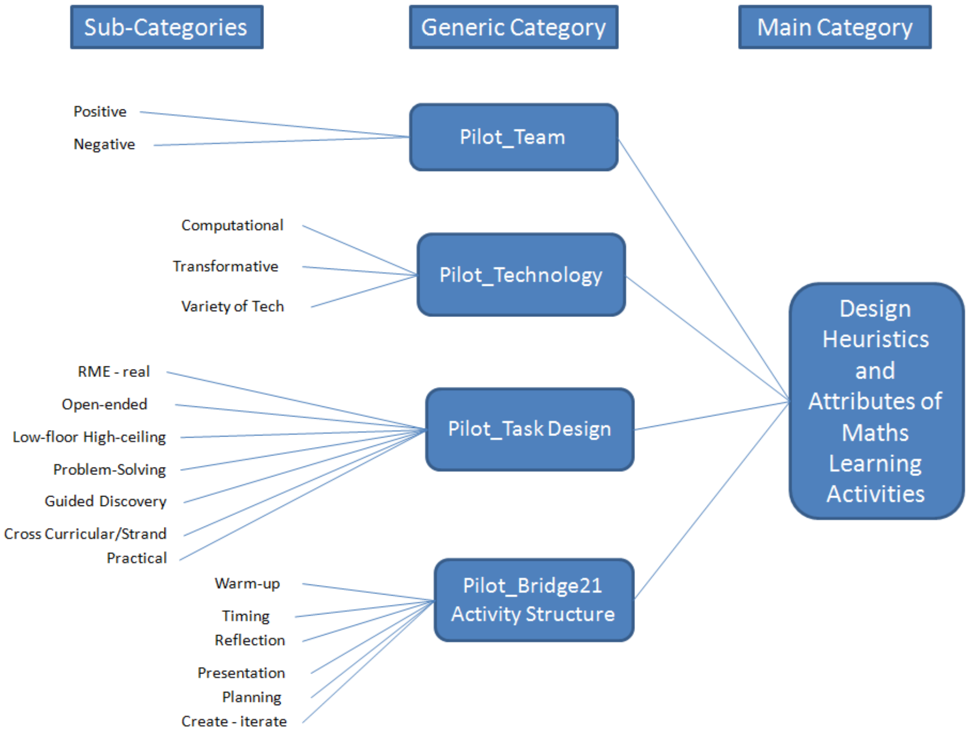


Figure 6.4: Design Heuristics Categorization Matrix

A final coding matrix (Figure 6.5) is used to identify codes related to the traditional, in-school approach as experienced by the students. This matrix is significantly less complex in that an exploration of the school experience was not the main focus of the study. It is however considered relevant for comparative purposes.

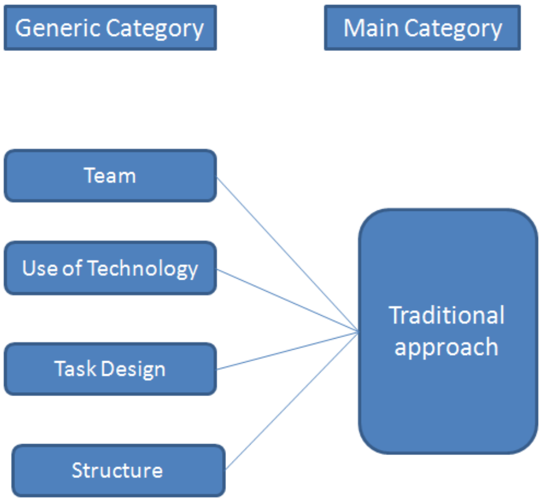


Figure 6.5: Traditional Approach Categorization Matrix

After the categorization matrices were developed, the comments and interviews were coded according to their correspondence with each of the predefined categories, using the qualitative analysis software tool, NVivo10. Coding schema were then developed (De Wever, Schellens, Valcke, & Van Keer, 2006; Weltzer-Ward, 2011), detailing how the categories were operationalised, and providing keywords, and exemplary segments coded at each sub-category. The MTAS coding scheme is provided in Table 6.2, below. The design heuristics and traditional approach coding schemes are available in Appendix 6.A. These coding schema are particularly valuable for ensuring reliability of the study, as they operationalise the coding process in such a way that it can be followed by other analysts (Yin, 2014).

Matrix coding, facilitated by the NVivo10 software, was subsequently used to cross-reference the design-heuristics codes and the traditional approach codes, with the MTAS codes, in order to investigate the relationships between the approaches and the subscales.

Table 6.2: MTAS Coding Scheme

Category	Sub category	Keywords	Examples	Operationalisation
Affective Engagement	**AE_Pos** (positive)	Enjoyable, Reward, Satisfaction, Interesting, Fun, Like.	This was really enjoyable and interesting. I had a great time, thank you	Segments in which the students refer to how they feel about the subject.
	AE_Neg (negative)		We listen to the teacher talk about, like, boring things	
Behavioural Engagement	**BE_Pos** (positive)	Work, Try, Answer, Learn, Do.	I found myself trying out and exploring lots of different sums.	Segments that relate to how students behave in learning the subject
	BE_Neg (negative)		And then you're like: but I still don't understand. And he goes, well, deal with it and write, so we just write.	
Mathematics Confidence	**MC_Pos** (positive)	Confident, I can, Understand, Figure it out.	Yeah, because we learnt much more. Because we learned by what we did. It was me and not just what someone said.	Segments that relate to the student's perception of their ability to achieve good results in mathematics and their confidence in handling difficulties in the subject.
	MC_Neg (negative)		He goes so fast that you can't like follow or understand.	
Confidence With Technology	**TC_Pos** (positive)	I am good at, I can fix, I can master.	Well, we already have some classes using GeoGebra and it's not such a big deal.	Segments that relate to students' confidence in the use of computers and in their ability to master procedures required of them. Also, segments that refer to confidence in the use of a broad range of technology.
	TC_Neg (negative)		I felt that more tutorials would have been better. GeoGebra and Tracker do require some knowledge of various systems to use.	
Attitude towards use of Technology for learning Mathematics	**MT_Pos** (positive)	Like, More interesting, Learn better, Worthwhile, Easier.	I enjoyed the use of technology in maths. It makes maths fun and interesting.	Segments that refer to the how students perceive that the use of technology in mathematics learning activities provides relevance, aids their learning, and contributes to their achievement in the subject.
	MT_Neg (negative)		N/A	

6.5 Findings – Student Data

Quantitative analysis of the pre/post MTAS data strongly indicates that participation in the activities had a positive effect on students' engagement with and confidence in mathematics, with particularly significant effects on affective and behavioural engagement (AE and BE) and attitude to using technology for learning mathematics (MT). In order to understand these changes in more detail, and to attempt to identify any causal relationships that may exist between the changes and the design heuristics, directed content analysis of the comments and interview was undertaken.

Considering the number of cross-referenced codes between the design heuristics and MTAS, the first thing to note is the high quantity of positive instances of the MTAS subscales. In Table 6.3, any rows/columns made up entirely of zeros (that is, no instances were cross-referenced at these codes) are hidden from view. All bar one columns record positive associations between the design heuristics and the MTAS subscales.

Table 6.3: Design heuristics v MTAS (Pilots)

Design Heuristics v MTAS (Pilot)	AE_Pos	BE_Pos	MC_Pos	MT_Pos	TC_Pos	TC_Neg
Pilot_Technology	10	3	5	13	7	2
Pilot_Task_guided discovery	6	3	5	1	1	0
Pilot_Tech_Computational	3	2	4	8	4	0
Pilot_Task_practical	3	1	4	2	1	0
Pilot_Task_cross-strand	3	1	2	1	1	0
Pilot_Task_open-ended	3	4	4	0	0	0
Pilot_Task_problem-solving	3	2	2	0	0	0
Pilot_Bridge21 Activity Structure	3	2	1	0	0	0
Pilot_Variety of Tech	2	1	1	2	1	0
Pilot_Task_RME-real	2	1	3	0	0	0
Pilot_Tech_Transformative	1	1	0	1	1	0
Pilot_Task_low-floor_high-ceiling	1	0	1	0	0	0
Pilot_Bridge21_Presentation	1	1	0	0	0	0

Further analysis of the data indicates that the use of *technology* (computational and transformative) in the activities played a very important part in this positive relationship, as it is the most commonly cross-referenced code with AE_Pos, MC_Pos, MT_Pos and TC_Pos, and comes second in the hierarchy for BE_Pos. The following quotations have been chosen to highlight some of these relationships:

> *"I enjoyed the use of technology in maths. It makes maths fun and interesting."* (AE)
> *"In the future I can use it to check my calculations in my homework/study"* (MC)
> "Using technology was a better way of learning and teaching maths." (MT)
> *"I found myself trying out and exploring lots of different sums. Very fun."* (BE)
> *"The simulations were very fun and easy to use!" (TC)*

The next most commonly cross-referenced design heuristic code is *Task_guided discovery*, which appears to have a particularly positive impact on AE and MC.

> *"Yeah, because we learnt much more. Because we learned by what we did. It was me and not just what someone said." (MC)*

The *open-ended* aspect of task design appears to be positively associated with BE and MC

> *"It's cool because we had to try our best to resolve the problem. So like when we have everything there to do […] it would be quite boring because you don't have to think about it, you are just following instructions. So it's really good." (BE)*

Other elements of task design, such as the practical, cross-strand and problem-solving aspects of the activities, also appear to have a positive impact on student engagement and confidence.

Interestingly, no references were made to the collaborative aspect of the activities, and very few to the Bridge21 activity structure. It is likely that this is due to the fact that the students had all had prior experiences in Bridge21 and were thus familiar with the structure and the way of working; it was an expected environment and was therefore not worth commenting upon.

A total of two segments that relate to the MTAS subscales and the design heuristics are recorded as negative; both of these are associated with technological confidence. Further investigation reveals that the students felt they would have benefitted from some prior knowledge of the Tracker and GeoGebra software that were used in the intervention.

Further negative instances of engagement and confidence are identified in association with the students' experiences of mathematics in their school environment (Table 6.4).

Table 6.4: Traditional Approach v MTAS (Pilots)

Trad v MTAS (Pilot)	AE-Neg	BE_Neg	MC_Neg	TC_Pos
Pilot_Trad_Task Design	3	5	4	0
Pilot_Trad_Structure	3	4	2	0
Pilot_Trad_Use of Tech	0	0	0	1

Quotes to illustrate the students' negative associations with the traditional approach to mathematics education are provided below:

> *"We listen to the teacher talk about, like, boring things and then we just take them down." (AE)*
>
> *"And then you're like: but I still don't understand. And he goes, well, deal with it and write, so we just write." (BE)*
>
> *"He goes so fast that you can't like follow or understand." (MC)*

The only somewhat positive comment that was associated with the traditional approach related to the use of GeoGebra in class:

> *"We already have some classes using GeoGebra and it's not such a big deal" (TC)*

6.6 Contextual Mathematics Teacher Workshops

In addition to the pilot student sessions, a number of pilot continuous professional development (CPD) workshops for teachers were conducted in Bridge21. These were focused on familiarizing practitioners with the activities, the Bridge21 model and the design heuristics for use in classrooms. Three day-long workshops were run, in which participants (post-primary mathematics and physics teachers), engaged in immersive experiences of the mathematics learning activities, providing practical exposure to the Bridge21 methodology and model in action. Participants were required to work in groups and follow all of the steps of a standard Bridge21-contextual mathematics activity.

The workshops were conducted over the period April 2013 to May 2014, introducing a total of 25 teachers to the approach. The final pilot workshop included a co-design element, in which the participants and the researcher followed a detailed, lesson study-style planning session (described in section 2.7.3) around the introduction of a lesson (the Barbie Bungee) to a traditional class. One of the participants then ran the lesson in his school, which was video recorded for the purpose of observation by the others.

Continuing the iterative process of the development of the design heuristics, a follow-up meeting led to some adjustments and additions. There was some indication that the level of scaffolding provided to the students needed to be supplemented. As a result of this, the level of instruction given, particularly around very procedural tasks such as capturing video, has been increased in the activities, and some pre-recorded examples of the software have been created. In addition, there was a suggestion that rather than relying purely on presentation for the assessment of the students' work, perhaps they should be required to write up a structured reflection of the activity. These suggestions have been incorporated into the set of heuristics to be provided to teachers.

6.7 A Practitioner's Guide for the Creation and Implementation of Contextual Mathematics Activities

As a result of the analyses of the Teacher and Student workshops, a practitioner's guide to the creation and implementation of contextual mathematics activities has been formulated. The activity development and implementation process is broken down into three parts as follows:

1. The Beginning:
   a. Begin with an interesting problem – this should reflect the interests of your own students and should be situated in a realistic/meaningful context.
   b. Use the Design Heuristics and the Bridge21 activity model to guide the development of the activity.
   c. If possible, provide a related problem in the divergent thinking session.
2. The Middle:
   a. Have a roadmap for guiding the students - a set of steps that the students must be guided through. However, this should not be too prescriptive. It is an open-ended approach.
   b. Provide all the resources that the students will require, with extras, but let them try to figure out what they will need.
   c. Encourage the students to think. Use Socratic questioning for guidance when possible.
   d. Use team-lead meetings for guidance to encourage peer-learning.
3. The End:
   a. Allow for different trajectories and answers. However, all approaches and results must be justified.
   b. Finally, present and compare results, and discuss approaches.
   c. For assessment purposes, the students can be asked to provide a written reflection.

This practitioner's guide has been presented at a number of conferences (Mathsfest and ESAI) and workshops and is being used in a contextual mathematics module on a postgraduate certificate (PG-Cert) course in 21st Century Teaching and Learning. The contextual mathematics teacher workshops described in section 6.6 have been adapted to make up one module on this course. Further details about the Postgraduate Certificate Workshops will be presented in Chapter 8.

6.8 Discussion

The student and teacher pilot workshops described in this chapter have provided an opportunity to undertake an exploratory case study to investigate the design and implementation of contextual mathematics activities, their impact on student engagement and confidence, and the development and use of such activities by practicing teachers, thus addressing the research aims identified in section 6.2. This section critically examines the findings in order to answer the guiding research questions:

1. What are the desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence?
2. What are the key elements of a practitioner's guide for the creation and implementation of such interventions within the traditional school environment?

The section concludes by discussing areas for future study, with a particular focus on the generation and refinement of the research questions for the explanatory study.

6.8.1 Activity Design – Refined Designed Heuristics

The initial activities that were piloted with the students had been created in accordance with a set of design heuristics with theoretical foundations in related literature. As the different activities were trialled, these heuristics were iteratively revised and refined based on responses from students and observations of what appeared to be successful during the intervention.

One example of this revision process is the flow of the activities from a concrete problem to abstract modelling and back to a concrete solution/implementation. This is evident in Realistic Mathematics Education theory, but had not initially been identified as a specific design heuristic. Throughout the first pilot activity, it became evident that having a concrete goal to the exercise is of benefit, transforming it from a purely modelling activity to one that had a more practical focus. This in turn leads to an element of competition, thus capturing the students' interest.

The repeat of the Scale Activity with two groups of students also led to a refinement of the design heuristics. After the first iteration, it was evident that the scope of the activity had been too wide and that the use of 'Liberating Constraints' (Davis, Sumara, & Luce-Kapler, 2000) would lead to a more achievable task, and would increase the students' creative freedom. The requirements of the activity were reduced, and participants were constrained to the acquisition of data with more explicit criteria. In the second iteration, the students were able to achieve more meaningful results in the time allotted to them.

As a result of these revisions, the task design aspect of the design heuristics has been extended as follows, tasks should:

- involve problem-solving, investigation and sense-making,
- involve guided discovery,
- be situated in a meaningful/real context,

- move from concrete to abstract concepts, with a final concrete goal grounded in the initial context.
- be open-ended but with constraints,
- be cross-curricular/cross-strand,
- be focused on skill development as well as on content,
- have a ‘low-floor’ and a ‘high-ceiling’.

6.8.2 Impact on Students

The bulk of the student data in this exploratory study is quantitative, consisting of pre- and post-questionnaires that students used to assess their engagement and confidence. The pre-questionnaires gather data about students’ experiences in their usual mathematics lessons, and the post-questionnaires assess how they feel about the pilot activities. Results of the quantitative data analysis are very positive, and indicate a substantively significant difference between students’ engagement and confidence in their usual lessons and when participating in the contextual mathematics lessons. This has led to confidence that the design heuristics are a powerful guide for the creation of activities that can lead to this kind of positive effect.

In order to further investigate the impact of the activity design on the subscales of MTAS, qualitative data analysis, facilitated by NVivo10, has permitted the cross-referencing of data associated with elements of the design heuristics and the MTAS subscales. This process allowed the generation of conjectures as to which aspects of the heuristics have the most significantly positive effect.

The results point to the use of technology as a significant factor in the positive change across all of the subscales, and are particular strongly associated with how the students feel about mathematics (AE) and how they perceive the technology as being relevant for their learning and contributing to their achievement (MT).

In terms of task design, all of the elements in the design heuristics are positively associated with the MTAS codes, with the guided-discovery, open-ended and practical aspects appearing most influential.

While useful for providing an indication of participants’ experiences, the written qualitative data was voluntarily provided and not comprehensive in scope, relying on participants to “add comments about what you did/did not like about the activities”. The dearth of references to the Bridge21 activity structure and the use of teamwork and collaboration are most easily explained by the fact that the activities took place in a familiar out-of-school environment in which this was the expected approach. Assessment is also not referenced, possibly for the same reason.

Overall, the results of the exploratory study indicate that the design heuristics are a good basis for the creation of mathematics learning activities with the potential to increase student engagement with and confidence in mathematics. However, a richer, qualitative analysis, provided in Chapter 7, will provide further explanation of the primary factors that cause any changes in engagement and confidence.

6.8.3 A Practitioner's Guide

Using the set of design heuristics as the basis for a practitioner's guide to the creation and implementation of contextual mathematics activities, three workshops were conducted with teachers. The purpose of the workshops was to give teachers a chance to explore the activities and to investigate how they might create and implement similar activities in their own classrooms. Feedback from the teachers was mostly positive in these workshops although there was some concern regarding the practicality of implementing such large-scale activities within the confines of the school timetable. A variety of different activities were proposed, many relating to sporting activities and trajectory, or time/speed/distance.

One of the workshops led to further collaboration with one of the teachers and to the creation of the Timepiece activity and its piloting in Bridge21. The collaborative approach to the design of the activity facilitated the identification of the different steps in the development process highlighted in section 6.7.

The final teacher workshop was particularly focused on the integration of this kind of activity into the school environment and followed a lesson-study style approach of: set goals, plan, teach and observe, review, and revise (Lewis et al., 2009; Takahashi & Yoshida, 2004). The group of teachers collaborated in the detailed planning of an in-school lesson in which a class of students took part in the Barbie Bungee activity. The duration of the class was two hours, taking up a triple period. The class itself was recorded and transcribed (by the author) for the purpose of observation. Each of the workshop participants observed the recorded lesson and reconvened the following week to discuss the activity and its implementation. The points that emerged regarding the structuring, scaffolding and implementation of the activity are incorporated into the practitioner's guide.

6.8.4 Further Research

At the beginning of this research one objective was to identify the desirable attributes of mathematics learning activities with the potential to effect positive change on students' engagement and confidence. This exploratory study provides strong evidence that the set of design heuristics described in chapter 4 fulfil that objective. The quantitative evidence indicates a significant positive effect on each of the subscales identified by the MTAS instrument, and this is backed up by the qualitative findings.

However, some gaps have been identified in the qualitative data, particularly in relation to the structure of the activities, and the use of teamwork and collaboration. It is important to trial the activities in a school environment in order to identify whether these aspects of the heuristics have a positive effect. In addition, the fact that the activities were piloted in an out-of-school setting is likely to have had an impact on the students' experiences. In order provide a more robust test, it is important to test the activities in more authentic settings. In order to address these issues, the explanatory study described in Chapter 7 is undertaken, identifying whether the positive effect can be maintained, and providing a rich and detailed description of *how* and *why* any positive changes emerge.

An adaptation of the teachers' workshops (section 6.6) that integrates the practitioner's guide (section 6.7) makes up one module on this the Postgraduate Certificate (PG-Cert) Course in 21st Century Teaching and Learning run by the TCD School of Education, in association with the Trinity Access Programme (TAP) and Bridge21. This module requires teachers to create and implement a contextual mathematics activity using the design heuristics. Teacher reflections on the process make up a part of the assignment for the course and analysis of these reflections have provided further data about the implementation of such activities in authentic environments. Details about the Postgraduate Certificate Workshops are presented in Chapter 8.

7. Explanatory Case Study – Schools

7.1 Introduction

The exploratory study described in the previous chapter examines the learning experiences of students who took part in activities designed in accordance with the design heuristics, in an experimental setting, with a particular focus on their levels of engagement and confidence. The Mathematics and Technology Attitude Scale, or MTAS (Pierce et al., 2007), is utilised in an attempt to develop a quantitative measure of changes in Behavioural Engagement (BE), Affective Engagement (AE), Mathematical Confidence (MC), Technological Confidence (TC), and Attitude to using Technology for learning Mathematics (MT). Qualitative analysis pays particular attention to the relationship between the MTAS subcategories and the design heuristics, in an attempt to develop tentative conjectures about the effects of particular aspects of the activity design on engagement and confidence.

An additional focus of the exploratory study relates to teachers' experiences with the design and implementation of transformative, technology-mediated, collaborative activities, as described by the heuristics.

The results of the MTAS pre/post tests are positive and show a statistically significant increase in student scores across all of the subscales. The process of piloting the activities has led to an extension of the initial design heuristics proposed in section 4.3, with a more detailed description of the desirable task attributes provided in section 6.8.1. Working with teachers has led to the development of a practitioners' guide (section 6.7) to support teachers in the development and implementation process of activities in traditional school settings.

However, the importance of trialling the activities in a school environment has emerged as fundamental in order to provide a more robust test of the effectiveness of the activities in increasing student engagement and confidence, and to provide a rich and detailed description of *how* and *why* any positive changes may emerge.

7.2 Research questions

The purpose of an explanatory case study is to explain how, and why some conditions have been achieved (Yin, 2014). The study presented in this chapter aims to address the second research question, namely:

RQ2 (a) What effects on student engagement and confidence does participation in activities designed in accordance with the heuristics and implemented using the practitioner's guide have?

RQ2 (b) What are the primary factors that cause such a change in engagement?

7.3 Context: Embedded Units

Activities were run with 51 students in three secondary schools during the 2013/2014 academic year and a further 18 students in one school in the following academic session. Participating schools were drawn from a network of institutions that are working with our research centre to roll out the Bridge21 pedagogic model into mainstream classrooms (Conneely et al., 2015). All participating students had previously engaged in workshops in which they were introduced to the Bridge21 model of learning. The researchers provided laptops, smartphones and any other tools required for the activities.

The students involved were from year 10 (age 15/16), known as ‘Transition Year’ in the Irish system. This is a one-year school programme that focuses on personal, social, vocational and educational development, providing opportunities for students to experience diverse educational inputs in a year that is free from formal examinations (Department of Education and Science, 2004). Timetabling is more flexible than in other school years, facilitating teaching experiments that are not constrained by short class periods.

7.3.1 Intervention 1 – School A (2013)

The first intervention took place in a co-educational, private school, for two hours per day, over the course of a week. The class consisted of 21 mixed-ability students, assigned to 6 groups of 3-4 students each. The working area comprised two interconnecting rooms, with readily moveable tables and chairs. Students participated in the Barbie Bungee (Section 4.2.2) and Human Catapult (Section 4.2.3) activities.

7.3.2 Intervention 2 – School B (2014)

The second intervention was conducted in an all-boys school in a disadvantaged area, and took place over the course of two days, running from 10am to 4pm each day. Twenty mixed-ability students were organised into 5 teams. The work environment was a large, standard classroom, with moveable desks and chairs. During the course of this intervention, students engaged with the Barbie Bungee and the Probability and Plinko (Section 4.2.4) activities.

7.3.3 Intervention 3 – School C (2014)

The third intervention was significantly shorter than the others. It was conducted in a disadvantaged, all-girls school, and took place over the course of two hours in a single afternoon. 10 students participated in the Barbie Bungee activity in a standard classroom with moveable furniture, and in the school gymnasium.

7.3.4 Intervention 4 – School A (2014)

The final intervention took place in October 2014. The choice of school for this intervention (that is, a return to School A, albeit with a different cohort of students) was guided by opportunistic sampling (L. Cohen et al., 2007). This particular school has altered the traditional timetable for their Transition Year classes, in order to facilitate a two hour project block every day. This has had the impact of familiarising the students with this kind of work and thus potentially negating some of the novelty aspect that may be experienced in other settings. In addition, it was straightforward to accommodate a sustained intervention over the course of a week. The class consisted of 18 mixed-ability students (an entirely new cohort), assigned to 5 groups of 3-4 students each. The working area was as described in section 7.3.1. In this session, three activities were completed: the Barbie Bungee, the Angry Birds Catapult (Section 4.2.4), and Probability and Plinko.

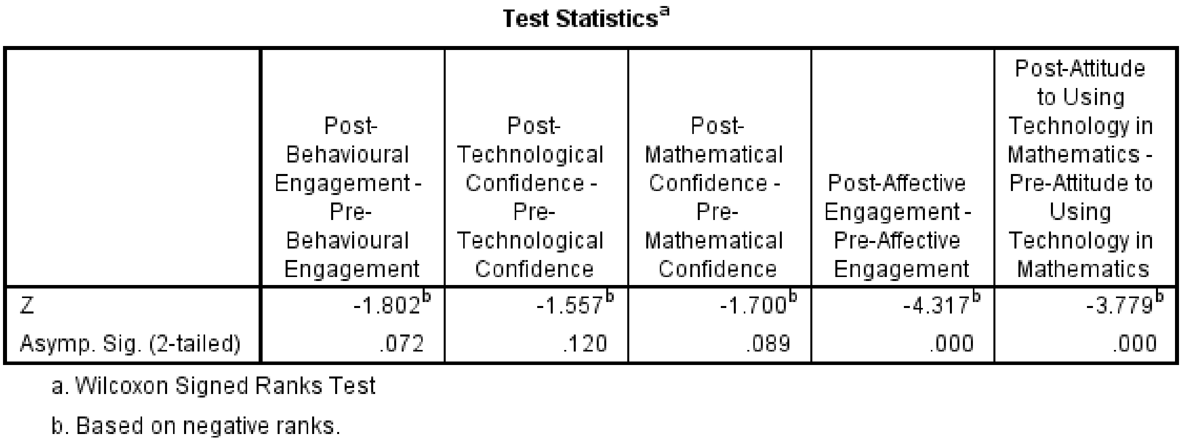
In order to provide a strong overall picture, the data that emerged from each of the interventions was aggregated, and analysed as a single data set. This methodological choice was taken early on in the research process, and had an impact on the research methods chosen (Section 5.2.4). It is important to acknowledge that the different school cultures, socio-economic factors, gender balance, timetabling, and duration of activities, along with numerous other factors will likely have had an impact on the results of each of the individual interventions, however it is outside the scope of the current research to take all of these elements into consideration. One point that has been considered is that the interventions were conducted during a non-standard year in the students' schooling. This has been taken into account throughout the interview process through the use of questions that relate specifically to the standard, more exam-focused years for more authentic comparative purposes (Table 7.2).

7.4 Pre-Experiments Data Collection and Analysis

The MTAS questionnaire was once again used to gather pre- and post-test data. Prior to each of the interventions, students were requested to reflect on their "usual" mathematics classes in order to fill in the pre-test. Subsequently, they were asked to consider the period of the intervention in order to fill in the post-test.

The data that emerged from the pre- and post-tests are not normally distributed, and therefore the Wilcoxon Signed Rank tests is used to check for statistically significant changes in the MTAS subscales. There were gains in all subtest scores, with significant differences identified ($p < 0.05$) in the Affective Engagement (AE) and Attitude to using Technology for learning Mathematics (MT) subscales (Table 7.1).

Table 7.1: Wilcoxon Signed Rank Test MTAS Results



Test Statistics[a]

	Post-Behavioural Engagement - Pre-Behavioural Engagement	Post-Technological Confidence - Pre-Technological Confidence	Post-Mathematical Confidence - Pre-Mathematical Confidence	Post-Affective Engagement - Pre-Affective Engagement	Post-Attitude to Using Technology in Mathematics - Pre-Attitude to Using Technology in Mathematics
Z	-1.802[b]	-1.557[b]	-1.700[b]	-4.317[b]	-3.779[b]
Asymp. Sig. (2-tailed)	.072	.120	.089	.000	.000

a. Wilcoxon Signed Ranks Test

b. Based on negative ranks.

A number of reasons for the less significant changes in the MTAS subscales are possible. Certainly, the change in context from the exploratory Bridge21 environment to the traditional classroom is likely to have had an effect: the students who attended the exploratory sessions had generally volunteered for the sessions, whereas in the explanatory study the intervention was not a choice for the students. In addition, the technology in the schools was not as reliable as it was in Bridge21. However, despite the somewhat less positive quantitative results, the qualitative data is largely supportive of the positive trend in all of the subscales.

7.5 Qualitative Data Collection and Analysis

Qualitative data collection involved individual and focus-group interviews conducted between 2 and 4 weeks after each intervention, non-participant and/or participant observation for the duration of each learning experience, and students' written reflections, collected after the final plenary session. The interviews provide the primary source of data for analysis in this explanatory case study.

A total of four focus-group, and five individual interviews were conducted. The duration of the interviews was between 20 and 40 minutes, and focus-groups were made up of between 4 and 6 participants. Each interview opened with questions about the students' experience in their usual mathematics classes, differentiating between the exam-focused years (years 7 – 9, or 1st to 3rd year in the Irish System) and Transition Year, followed by an exploration of their understanding of what the approach described in this research is trying achieve and how they felt about it. While the interviews were open, they were focused by the research questions, through which the researcher prompted participants to discuss what they liked and did not like about the activities, what their reasons were, what mathematics emerged and how they felt about the mathematics and the technology. An interview protocol (Table 7.2) provided a general guiding structure.

Table 7.2: Interview Protocol

Main Questions	Additional Questions	Clarifying Questions
1. Can you describe your usual maths class? Is there a difference between 1st – 3rd year and TY? How do you feel about it? Why? 2. Can you tell me what you think I was trying to achieve? What was different to your usual class? How did you feel about it? Why?	Has the experience changed how you feel about maths? Are you more curious about where/when/how maths is used? Would more experiences like this have a positive/negative impact on your relationship with maths? Why?	What? Why?
3. What did you learn? 4. What did you like/dislike about the experience? Would you suggest doing anything differently?	Did you learn any new mathematical content that you had not seen in class? Did you get a better understanding of the mathematical concepts that you had previously learned in class? Has this changed your confidence in your ability to understand or use the maths? Do you think you gained any new skills – mathematical or otherwise?	Can you expand on this? Can you tell me anything else? Can you give me some examples?
Conclusion of Interview		
Do you want to add anything? In one word, how would you sum up the experience?		

7.6 Qualitative Data Analysis

There is no single, right way to approach the qualitative analysis of data, and often an assortment of different approaches that build upon each other is preferable (Namey et al., 2007). In this study, the researcher elected to initially use a directed approach to the content analysis (Hsieh & Shannon, 2005; Krippendorff, 2004), followed by a re-examination of the data using constant comparative techniques (Glaser, 1965; Glaser & Strauss, 1967; Strauss & Corbin, 2008).

As discussed in Chapter 5, a directed approach to content analysis provides a framework on which to base initial analysis of the qualitative data in this explanatory case study. *Directed content analysis* (DCA) is a theory driven approach that provides the researcher with an opportunity to focus on areas of particular interest (Krippendorff, 2004; Namey et al., 2007; Yin, 2014), with results emerging from a quantitative analysis of the researcher's interpretation of the textual data. Approaches that work the data from 'the ground up', such as c*onstant comparison*, contrast directly with DCA, in that they are not based on a-priori theoretical propositions (Yin, 2014). While the definition of DCA remains open to emergent coding (Namey et al., 2007), it is generally restricted to portions of the data that have not been already coded according to the theory.

Prior to embarking on the data analysis process, it is important to consider issues relating to sampling (Elo & Kyngäs, 2008; Krippendorff, 2004). Graneheim and Lundman (2004) suggest that where possible, entire interviews or observational protocols are appropriate to use as sampling units. This is considered an appropriate approach for this research and so each interview in its entirety is used as a sample unit. In order to become immersed in and sensitised to the data, the researcher listened to the audio recordings of the interviews, transcribed them, and then listened to them again a number of times, and read and re-read the transcriptions.

Yin (2014) describes the process of data analysis as consisting of "examining, categorizing, tabulating, testing, or otherwise recombining evidence to produce empirically based findings" (p132). Once initial coding has taken place Yin (2014) proposes five techniques that are useful to provide structure in the analysis of the data:

1. Pattern Matching,
2. Explanation Building,
3. Time-Series Analysis,
4. Logic Models,
5. Cross Case Analysis.

The analysis process in this dissertation makes use of the techniques of *Pattern Matching* and *Explanation Building*, as described in the following sections.

Pattern Matching

Pattern Matching is used to compare empirical patterns emerging from matrix coding of the data in NVivo, with predicted patterns that suggest that the MTAS subcategories would be positively affected by student participation in the Contextual Maths learning activities. This hypothesises five outcomes (>AE, >BE, >MC, >TC, >MT), each representing different dependent variables to be assessed using qualitative and quantitative measures. The pattern matching process was expedited by the NVivo10 matrix coding facility and the generation of models, discussed in section 7.6.1.

Explanation Building

This analytic technique is a particular instance of pattern matching that aims to explain a phenomenon. The process is described by Yin (2014) as stipulating “a presumed set of causal links about it [the phenomenon], or about “how”, or “why” something happened.”, (p147). In this study, explanations are backed up by the quantitative analysis of the textual data, which suggests links between certain attributes of the activities inherent in the design principles, and the positive effect on the MTAS subscales. The explanations that have emerged from this research are the result of a series of iterations of the *directed content* and the *constant comparative* analysis processes. The initial explanatory proposition emerged from the exploratory case study (Chapter 6), and suggests that usage of the design principles could lead to the creation of maths learning activities with the potential to increase student confidence and engagement with the subject. The explanation building process is discussed in depth in section 7.7

The remaining three processes were not applicable in this research, for the following reasons:

Time-Series Analysis

The duration of the interventions that constitute the embedded units within the case study are not sufficiently long to warrant time-series analysis. There is no way of examining relevant “how” or “why” questions about the relationships of the events over time. This could be considered a limitation of the study, as the students’ attitudes could change if the contextual maths style of activities were more routine in their schooling. However the students’ own opinion on the matter was that familiarity with this kind of teaching and learning would not breed disinterest:

> *“It would be something to look forward to if you were stuck in the same class. Maybe even like once a week, a class like that.”*
>
> *“Yeah, and then the rest of your maths could be going back to it and taking from it and building on it.”*

Logic Models

Similar to time-series analysis, the duration of the case study is not sufficient to generate a meaningful logic model. A longitudinal approach to the research would be useful in order to analyse any lasting change that might result from consistent engagement with activities such as those described, but it is outside the scope of this study.

Cross-Case Synthesis

Cross-Case Synthesis aggregates findings across a series of individual studies. This technique only applies to the analysis of multiple case studies. As described in chapter 5, a methodological choice was made early in this research to consider each of the interventions as embedded units rather than as separate case studies. For this reason, the data has been aggregated for analysis and a cross-case synthesis is not a relevant approach.

7.6.1 Directed Content Analysis of Interviews

As described in Section 6.4.2.1, directed content analysis is a deductive technique and is appropriate when the analytical approach is operationalised, according to prior knowledge or theory (Elo & Kyngäs, 2008). In this case study, two categorisation matrices have been developed to guide the analysis; one is based on the Mathematics and Technologies Attitudes (MTAS) Scale and the other on the Design Principles.

Figure 7.1 shows the categorization matrix used for the MTAS-directed coding. Within the Generic Categories, AE refers to Affective Engagement, BE to Behavioural Engagement, MC to Mathematics Confidence, TC to Confidence with Technology, and MT to attitude towards the use of Technology for learning Mathematics. The sub-categories break down each of the generic categories into positive and negative references to the traditional approach, and positive and negative references to the Mathematics Learning Activities (MLAs) used in the interventions. For example, BE_MLAsPos relates to positive references to behavioural engagement associated with one of the mathematics learning activities described in Chapter 4.

Figure 7.2 displays the categorization matrix used for the Design Principles-directed coding. The generic categories list the primary headings of the guidelines, and the sub-categories highlight various aspects of each of the principles.

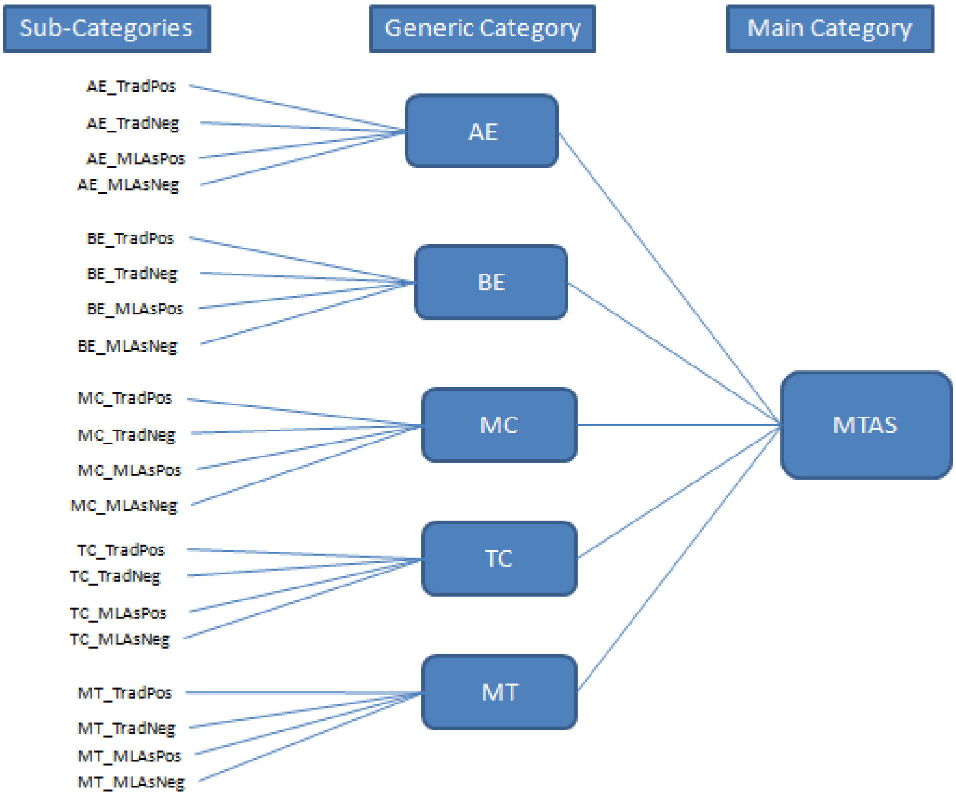


Figure 7.1: MTAS Categorisation Matrix

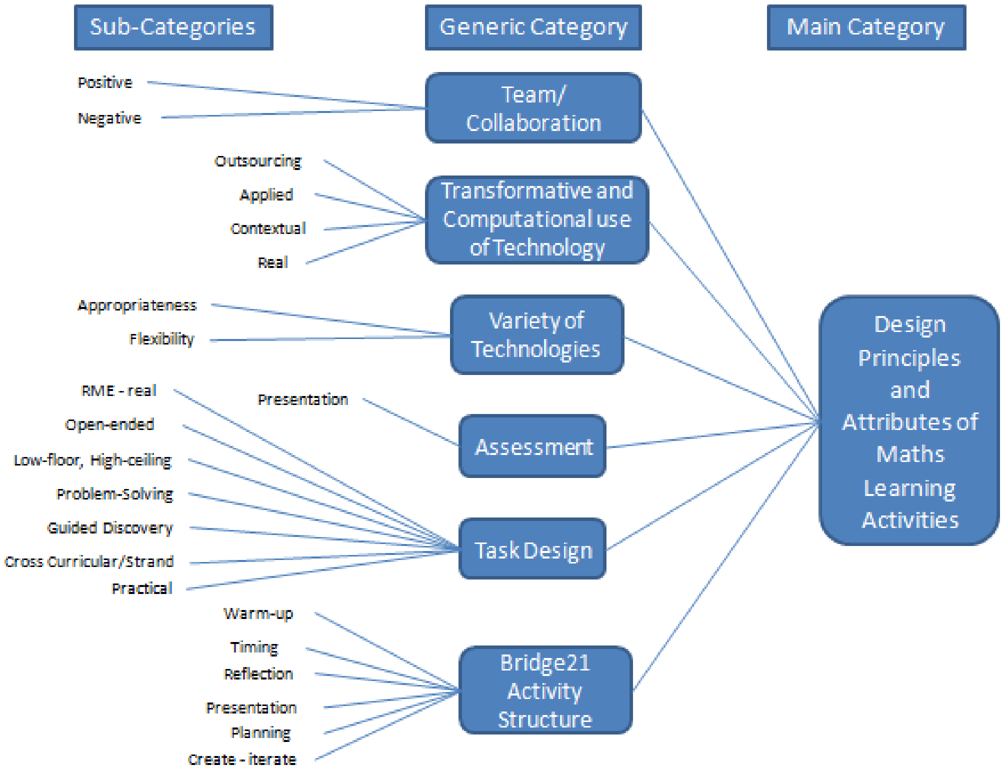


Figure 7.2: Design Principles Categorisation Matrix

7.6.1.1 Coding and Theming

After the categorisation matrices were developed, the data were coded based on their correspondence with each of the predefined categories. A coding scheme was then developed (De Wever et al., 2006; Weltzer-Ward, 2011), which details how the categories were operationalised, provides keywords and exemplary segments coded at each sub-category. The MTAS coding scheme and the Design Principles coding scheme are provided in Appendix 7.A

7.6.1.2 Coding Comparison

In order to ensure reliability of the results of the directed content analysis, a second coding of the data was conducted by a researcher from outside the project. Coding matrices and schema were supplied to the researcher, in order to direct the analysis of the interviews. An NVivo coding comparison query was used to compare the two analyses. The average Cohen's Kappa coefficient across all of the data was 0.8, which Landis and Koch (1977) suggest demonstrates a "substantial" to "almost perfect" agreement between the coders. This result reflects very positively on the level of reliability of the research.

7.6.1.3 Analysis of Relationships

Once all of the text was coded, coding matrices generated by NVivo10 facilitated comparisons between the sub-categories of the design heuristics and the MTAS subscales. This permitted the generation of tentative conjectures as to the primary factors that cause the change in student engagement and confidence evident in the MTAS scores. Tables 7.3 to 7.7 show the coding matrices that relate the codes associated with the heuristics to those related to each of the MTAS subscales. The data has been filtered from largest to smallest on the column that relates to positive instances of engagement or confidence during the intervention. The numbers in each of the cells relate to the number of segments that were co-coded at the associated row and column. The results of this process can be viewed as indicative of associations between the heuristics and the MTAS subscales. The tentative associations between the approach to activity design and implementation, and their impact on engagement and confidence are discussed in the following sections:

Affective Engagement (AE)

The data comparison from the coding matrix (Table 7.3) points to the task design – in particular, the realistic, practical, guided discovery, open-ended, cross-strand and problem-solving aspects – as having a positive impact on the students' AE. In addition, the Bridge21 activity structure and the transformative use of technology, which facilitated the realistic nature of the tasks, are all positively associated with AE.

Table 7.3: Coding Matrix for Design Heuristics v Affective Engagement

	AE_MLAsNeg	AE_MLAsPos	AE_TradNeg	AE_TradPos
Task_RME-real	0	43	2	1
Task_practical	0	30	1	0
Task_guided discovery	0	18	0	0
Task_open-ended	0	17	0	0
Task_cross-strand	0	15	0	0
Task_problem-solving	0	15	0	1
Team_Positive	0	10	0	0
Task_high-ceiling	0	9	0	0
Tech_Transformative	0	8	1	0
Bridge21_Reflection	0	7	0	0
Bridge21_Timing	0	6	0	0
Bridge21_Planning	0	5	0	0
Bridge21_Presentation	0	4	0	0
Bridge21_Create/iterate	0	3	0	0
Bridge21_Warm up	0	3	0	0
Task_low-floor	0	3	1	1
Tech_Computational	0	2	0	0
Variety of Tech	0	2	0	0
Team_Negative	1	0	0	0

Very few instances that relate to the learning interventions were coded as having a negative impact on AE, and those that did refer to the students' discomfort at working in groups. Other negative instances of AE are recorded when the students refer to the "normal", or traditional, class environment, in particular owing to the lack of realistic or practical tasks.

Behavioural Engagement (BE)

Table 7.4: Coding Matrix for Design Heuristics v Behavioural Engagement

	BE_MLAsNeg	BE_MLAsPos	BE_TradNeg	BE_TradPos
Team_Positive	0	33	0	0
Task_RME-real	0	28	1	0
Task_practical	0	25	1	0
Task_guided discovery	0	20	0	0
Task_open-ended	0	20	0	0
Task_problem-solving	0	20	0	0
Task_high-ceiling	0	13	0	0
Task_cross-strand	0	9	0	0
Task_low-floor	0	8	0	0
Tech_Transformative	0	8	1	0
Bridge21_Planning	0	7	0	0
Bridge21_Reflection	0	6	0	0
Bridge21_Create/iterate	0	4	0	0
Bridge21_Timing	0	4	0	0
Bridge21_Warm up	0	3	0	0
Tech_Computational	1	3	0	0
Bridge21_Presentation	0	2	0	0
Variety of Tech	0	2	0	0
Team_Negative	4	1	0	0

In terms of BE, the most significantly positive factor appears to be related to teamwork (Table 7.4). The realistic, practical, guided, open-ended, problem-solving, cross-strand and low-floor aspects of the task design also seem to have a positive effect on the students' BE throughout the interventions. The impact the structure of the activities (Bridge21) and the transformative and computational use of technology also appears to have a positive impact.

Once again, there are very few negative associations between the heuristics and BE. However, teamwork is not always perceived as positive, with four instances of negative coding relating behavioural engagement and the collaborative aspects the interventions. Two instances of negative behavioural engagement that relate to the students' traditional mathematics class were also noted.

Mathematical Confidence (MC)

The most notable positive effect on MC appears to be connected to the task design, in particular, to the realistic and practical nature of the tasks and to the students' perception of guided discovery (Table 7.5). The use of technology also seems to have a positive influence, as does the collaborative aspect of the activities. The influence of the Bridge21 activity structure is also positively associated with MC.

Table 7.5: Coding Matrix for Design Heuristics v Mathematical Confidence

	MC_MLAsNeg	MC_MLAsPos	MC_TradNeg	MC_TradPos
Task_practical	0	28	3	0
Task_RME-real	0	24	4	0
Task_guided discovery	0	22	0	0
Task_problem-solving	0	13	0	0
Team_Positive	0	13	0	0
Task_open-ended	1	10	0	0
Tech_Transformative	0	9	2	0
Bridge21_Reflection	0	7	0	0
Tech_Computational	2	6	0	0
Task_cross-strand	0	5	0	0
Task_high-ceiling	0	5	0	0
Task_low-floor	0	5	0	0
Bridge21_Create/iterate	0	3	0	0
Bridge21_Presentation	0	3	0	0
Bridge21_Planning	0	2	0	0
Variety of Tech	0	2	0	0
Bridge21_Warm up	0	1	0	0
Bridge21_Timing	0	0	0	0
Team_Negative	0	0	0	0

Some negativity is recorded around the use of technology however, with one student feeling that it made the mathematics too abstract. Within the confines of what the students describe as their "normal/traditional" classes, MC appears to have been negatively affected by the lack of context and practicality of the subject.

Technological Confidence (TC)

It is perhaps not surprising that the use of technology appears to have the most significant impact on the students' TC (Table 7.6). The variety of technology is noted as leading to flexibility, and the transformative and computational affordances of the technology seem to facilitate greater relevance of the tasks, leading to increased conceptual understanding.

Table 7.6: Coding Matrix for Design Heuristics v Technological Confidence

	TC_MLAsNeg	TC_MLAsPos	TC_TradNeg	TC_TradPos
Variety of Tech	3	25	1	0
Tech_Transformative	0	20	0	0
Tech_Computational	1	14	1	0
Task_practical	1	3	0	0
Task_cross-strand	0	2	0	0
Task_guided discovery	0	2	0	0
Bridge21_Planning	0	1	0	0
Task_high-ceiling	0	1	0	0
Task_low-floor	0	1	0	0
Task_RME-real	0	1	0	0
Team_Positive	0	1	0	0
Bridge21_Create/iterate	0	0	0	0
Bridge21_Presentation	0	0	0	0
Bridge21_Reflection	0	0	0	0
Bridge21_Timing	0	0	0	0
Bridge21_Warm up	0	0	0	0
Task_open-ended	0	0	0	0
Task_problem-solving	0	0	0	0
Team_Negative	0	0	0	0

There are a few instances in which the technology is highlighted in a negative light, where the students' experienced higher levels of confusion and did not find the technology easy to manage. A small amount of negativity is also noted in relation to the lack of technology use in the more traditional approach.

Attitude towards using Technology for Learning Mathematics (MT)

The variety of technology, and its use in a transformative and computational manner, in conjunction with the approach to task design highlighted by the heuristics, appears to have a positive influence on students' MT (Table 7.7). The guided discovery approach along with the realistic and cross-curricular/cross-strand aspects of the tasks are particularly notable as having a positive impact.

Once again, the small number of instances of negatively coded instances relate to the level of complexity of the programs. It would not be unwarranted however, due to the paucity of such codes, to assume that the majority of students did not find the software overly complex.

Table 7.7: Coding Matrix for Design Heuristics v Attitude to using Technology for Learning Mathematics

	MT_MLAsNeg	MT_MLAsPos	MT_TradNeg	MT_TradPos
Tech_Transformative	0	28	0	0
Tech_Computational	2	16	0	0
Variety of Tech	1	14	1	0
Task_guided discovery	0	4	0	0
Task_RME-real	0	4	0	0
Task_cross-strand	0	3	0	0
Task_high-ceiling	0	3	0	0
Bridge21_Planning	0	2	0	0
Task_practical	0	2	0	0
Task_problem-solving	0	2	0	0
Bridge21_Create/iterate	0	1	0	0
Bridge21_Reflection	0	1	0	0
Task_low-floor	0	1	0	0
Task_open-ended	0	1	0	0
Team_Positive	0	1	0	0
Bridge21_Presentation	0	0	0	0
Bridge21_Timing	0	0	0	0
Bridge21_Warm up	0	0	0	0
Team_Negative	0	0	0	0

Each of these findings will be discussed in more depth in section 7.7.

7.6.2 Constant Comparative Analysis of Interviews

After completion of the process of directed coding, and owing to the diversity of the embedded units of analysis in the case study, a second analysis of the data using *constant comparative* techniques was conducted. The purpose of this was to attempt to fully grasp any emerging themes in the hope of providing a full description of the case. Constant comparative analysis is a method of reducing qualitative data to emic codes (i.e., emerging from within the data) that retain much of the richness of the original data. In this way, the results of the analysis are used to create a rich picture of the students' experience, and potentially, an understanding of the theoretical propositions of the alignment between participation in the activities and changes in engagement and confidence.

The constant comparative process used in this research follows the analytic strategies and techniques laid out by B. G. Glaser (1965), Merriam (1998) and Strauss and Corbin (2008). The first step involves detailed analysis of the text with codes being assigned to words, phrases or sections therein, and the comparison of segments assigned to the same code. It is important to recognise that it is not the words themselves, but the underlying contextualised meanings that are of primary importance (Miles & Huberman, 1994). Memos are used as a reflection of the mental dialogue that occurs between the data and the researcher. Each segment that is coded in a particular way is compared to all other instances of the code. This comparison starts to generate theoretical properties of the related category. After repeatedly attributing a code to different segments of text, the researcher may begin to question whether the code is an accurate enough representation of the concept and thus conflict can arise during the process. Memos are created throughout this process to record the emerging ideas, questions and comparisons.

The second step involves the integration or reduction of the categories. The units that are being compared are the properties of the categories that resulted from the initial coding comparison of the text. As the categories and their properties become integrated, the researcher must make some theoretical sense of the comparisons. At this point, summary memos and models of codes can be useful. These can be used to help identify the list of concepts/codes and emerging themes as well as to identify possible relationships between them.

Step three relates to determining a potential theory (particularly relevant to the grounded theory approach, which is not adopted in this research), theoretical saturation of the categories, reduction of the terminology and the resultant increase in scope of applicability of the theory. In grounded theory, the final phase in the process involves writing the theory. The categories form the themes of the theory, with the memos providing the rationale for their development and the coded data the resources to turn to for the purposes of clarification. This research does not aim to develop a formal theory. However, the relationships between the categories have been used to generate a substantive theory, i.e. “a theoretical model that provides a “working theory” of action for a specific context” (Strauss & Corbin, 2008, p. 57).

It should also be noted however, that although this is laid out as a step-by-step process, it is not the case that one phase finishes before the other begins. This is rather, an iterative process in which previous stages remain operational until the entire process terminates.

7.6.2.1 Phase 1: Emergent Codes

The first phase of the constant comparative data analysis aims to look at the data as a ‘new user’, in order to identify emergent themes without the preconceptions of the directed content analysis. A number of months had passed between these phases of analysis, leading to a reasonably ‘fresh’ perspective on the data. This section provides a detailed description of the process of analysis, detailing how the main codes and themes emerged through the use of memos and diagrams. However, constant comparative data analysis is a non-linear, often messy process. An attempt at illustrating this is provided in Figure 7.3 below and the subsections that follow describe the process in detail.

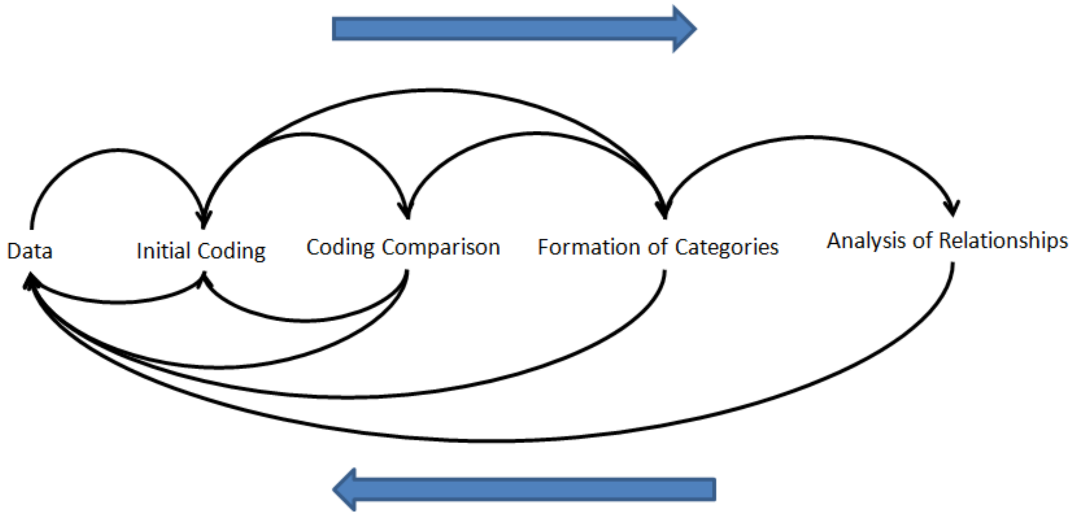
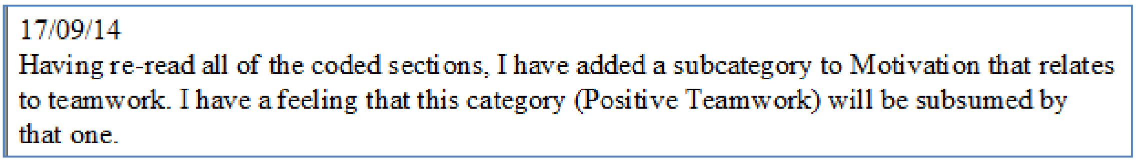


Figure 7.3: The constant comparative analysis process

The end product of the analysis process is a set of emergent categories, sub-categories and associated codes. These have been useful for the development of a rich description of the case as well as providing an opportunity to develop further hypotheses relating to the relationship between the design heuristics and the perceived increases in engagement and confidence.

7.6.2.2 *Generation of initial codes and categories*

The first interview to be analysed was the School A (2013) focus group interview. Following an initial sensitising read of the data, an open coding process was used to assign emergent codes to sections of the text. Initial codes, concepts and categories emerging from the data at this point were considered provisional, and were frequently returned to and re-classified in light of subsequent analysis. The following memo highlights one instance of this process:



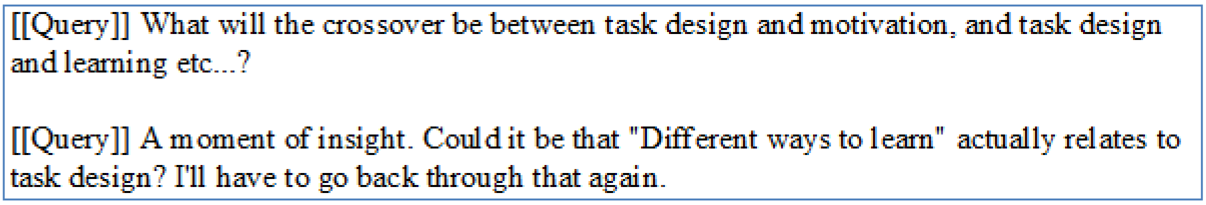
17/09/14
Having re-read all of the coded sections, I have added a subcategory to Motivation that relates to teamwork. I have a feeling that this category (Positive Teamwork) will be subsumed by that one.

Figure 7.4: Sample School A (2013) memo

Analysis of the first Interview generated an overall total of 34 codes, and even at this early stage, some of these codes were assigned to tentative categories. For example, the category “Learning” already had nine associated codes (Technology, Real, Practical, Peer, Problem-solving, Estimation, Content, Connections and Concepts).

All of the memos written during the process were re-read, analysed and condensed into a summary memo of the first interview. This highlighted the need for two more codes relating to Task Design and the Traditional Approach to teaching. Re-coding the interview for these, led to the identification of 8 new codes, with an overall total of 42 codes.

Before looking in detail at further connections highlighted through the process of coding and memoing, the second focus group interview – School B (2014) – was analysed. The analysis process was slightly different for the second interview, in that there were fewer memos written associated with individual codes. Instead, a primary memo was used to record the generation of new codes and queries regarding possible relationships between the codes. Figure 7.5 shows an example of a portion of this memo.



[[Query]] What will the crossover be between task design and motivation, and task design and learning etc...?

[[Query]] A moment of insight. Could it be that "Different ways to learn" actually relates to task design? I'll have to go back through that again.

Figure 7.5: Sample School B (2014) memo

Following the first pass over the interview, 23 new codes were generated, making an overall total of 65 codes. A process of comparison of all of the codes led to the generation of a further four codes, bringing the total to 69, made up of 14 categories and 55 sub-codes.

Only one new item was added through the first pass over the School C (2014) interview. This related to the Bridge21 element of the task design. A memo relating to this was also added as it seemed curious that this element of task design was only explicitly emerging at this point. Instinct indicated that there may be other instances of this code in the other interviews.

Having coded and formed initial categories from the first 3 interviews, all of the coded segments of text were compared once again, in order to ascertain whether they were accurately classified, in light of any new codes, and to examine the internal validity of the coding. Throughout this process, an attempt was made to integrate and reduce the number of categories. This process is described in the following section (7.6.2.3). Following the coding comparison and reduction of codes a final set of codes for this phase of the analysis was created (Appendix 7.B).

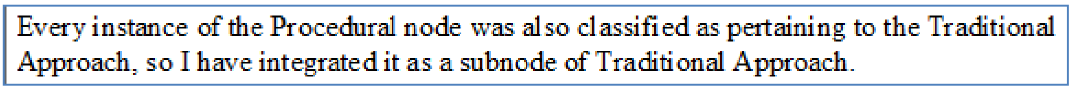
A six month period had elapsed between the School C (2014) interview and the return to School A (2014). The initial analysis of the first three interviews had been conducted during this period, which permitted a level of theoretical sampling to inform the subsequent interviews (Strauss & Corbin, 2008). Theoretical sampling is a sequential approach to data collection and analysis, which influences the gathering of data based on emerging concepts. In particular, during the interviews, a more explicit focus was placed on each of the MTAS subcategories and on the elements of the design heuristics in order to further probe the relationships between the two.

Prior to beginning analysis of the second collection of interviews, the first set of codes were re-read once again. Through the analysis of the first of the School A (2014) interviews, two new codes were created (Motivation_Creativity and Learning_Teamwork). Analysis of the second interview led to a new code (Negative_Learning) and a new category (Participation). Throughout the remaining three interviews, no further codes were added, permitting the researcher to conclude that a reasonable level of saturation had been achieved.

One of the themes that emerges as particularly relevant in the School A (2014) interviews relates to student participation. A re-coding of the three initial interviews however, reveals few segments that could be coded at this new category; these codes emerged through the second set of interviews, which, through the process of theoretical sampling, were more focused on themes relating to the MTAS subcategories, such as behavioural engagement. For this reason, it makes sense that participation is more prevalent in the later interviews.

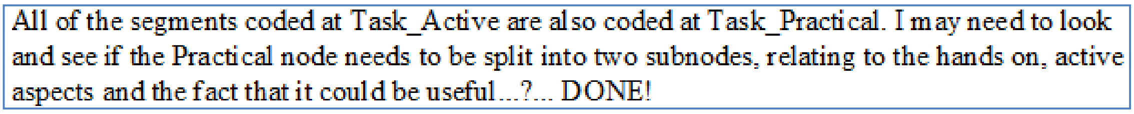
7.6.2.3 Reduction of Codes

The process of integration and reduction of codes was ongoing throughout the analysis, but is particularly evident in the final coding comparison. This is highlighted through some of the later memos such as:



Every instance of the Procedural node was also classified as pertaining to the Traditional Approach, so I have integrated it as a subnode of Traditional Approach.

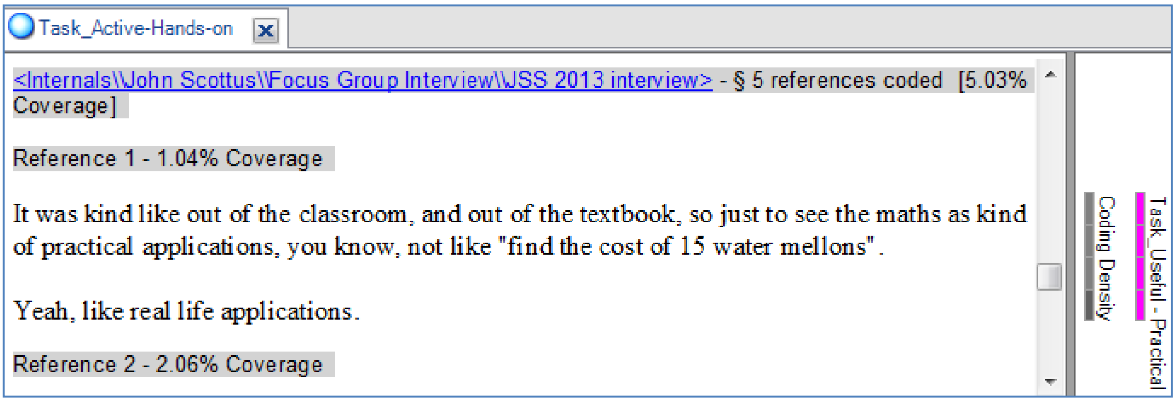
Figure 7.6: Sample from Summary Memo



All of the segments coded at Task_Active are also coded at Task_Practical. I may need to look and see if the Practical node needs to be split into two subnodes, relating to the hands on, active aspects and the fact that it could be useful...?... DONE!

Figure 7.7: Sample from Summary Memo

Following the split referred to in Figure 7.7, all of the references coded at each code were compared, confirming that the new codes Task_Useful-Practical and Task_Active-Hands-on were distinct, with only one reference that overlapped the two codes. The NVivo10 software was particularly useful for facilitating this kind of analysis, through the use of the “viewing stripes” functionality, which provides a clear view of cross coding (Figure 7.8).



<Internals\\John Scottus\\Focus Group Interview\\JSS 2013 interview> - § 5 references coded [5.03% Coverage]

Reference 1 - 1.04% Coverage

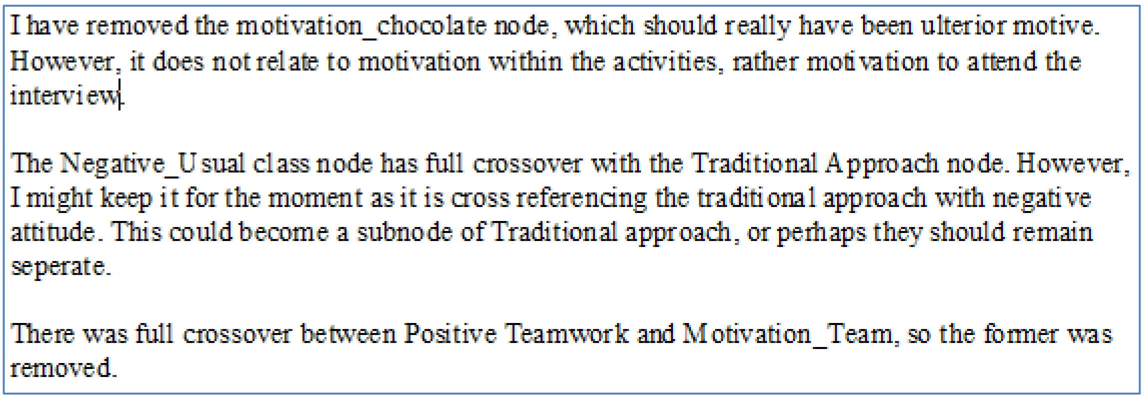
It was kind like out of the classroom, and out of the textbook, so just to see the maths as kind of practical applications, you know, not like "find the cost of 15 water mellons".

Yeah, like real life applications.

Reference 2 - 2.06% Coverage

Figure 7.8: Example of coding stripes to highlight cross coding

A number of potential redundancies and overlaps between codes were also identified at this point, as illustrated in the following memo (Figure 7.9). Once again, the coding stripes functionality of NVivo10 facilitated this phase of the analysis


I have removed the motivation_chocolate node, which should really have been ulterior motive. However, it does not relate to motivation within the activities, rather motivation to attend the interview.

The Negative_Usual class node has full crossover with the Traditional Approach node. However, I might keep it for the moment as it is cross referencing the traditional approach with negative attitude. This could become a subnode of Traditional approach, or perhaps they should remain seperate.

There was full crossover between Positive Teamwork and Motivation_Team, so the former was removed.

Figure 7.9: Sample Memo from Summary

7.6.2.4 Process of Analysis of Relationships

A number of memos suggest possible relationships between codes and categories. These "query" memos (Appendix 7.C) were easy to break down into different areas in order to look at them in greater detail. The queried relationships that emerge most frequently, and which were most relevant to the Research Questions, are between the different elements of the task design and their impact on Motivation and on Learning. Interestingly, some of the queries also relate to the relationships between self-belief, confidence, negative attitude, and motivation; these areas could provide interesting avenues for future work. The matrix coding functionality of the NVivo10 software supported the analysis of the emerging relationships.

The following sections describe the process of teasing out the potential relationships between the different elements of Task-Design, Student Motivation and Learning.

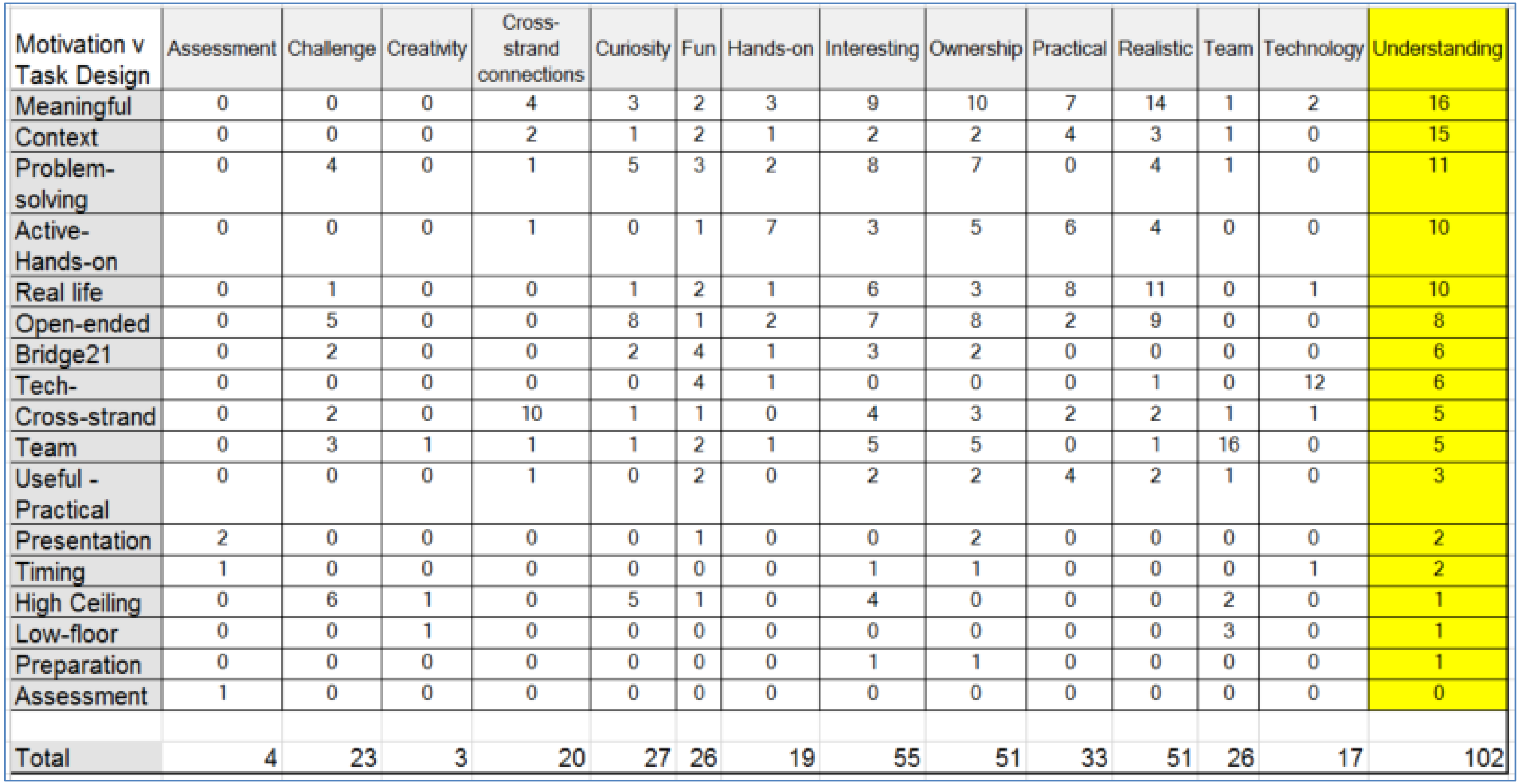
Task Design and Motivation

Segments coded in the Motivation category relate to aspects that motivated the students to engage with and enjoy the activities, for example:

> *"You were trying to show us where you get the numbers from, how they relate to each other and, you know, why. You know, answers to the questions that we never get answers to, like how, why, those kinds of questions."*

Matrix coding has been used to facilitate analysis of the potential relationships between Task Design and Motivation (Table 7.8) (note – any zero rows and columns are hidden for reasons of size).

Table 7.8: Motivation v Task Design



Motivation v Task Design	Assessment	Challenge	Creativity	Cross-strand connections	Curiosity	Fun	Hands-on	Interesting	Ownership	Practical	Realistic	Team	Technology	Understanding
Meaningful	0	0	0	4	3	2	3	9	10	7	14	1	2	16
Context	0	0	0	2	1	2	1	2	2	4	3	1	0	15
Problem-solving	0	4	0	1	5	3	2	8	7	0	4	1	0	11
Active-Hands-on	0	0	0	1	0	1	7	3	5	6	4	0	0	10
Real life	0	1	0	0	1	2	1	6	3	8	11	0	1	10
Open-ended	0	5	0	0	8	1	2	7	8	2	9	0	0	8
Bridge21	0	2	0	0	2	4	1	3	2	0	0	0	0	6
Tech-	0	0	0	0	0	4	1	0	0	0	1	0	12	6
Cross-strand	0	2	0	10	1	1	0	4	3	2	2	1	1	5
Team	0	3	1	1	1	2	1	5	5	0	1	16	0	5
Useful - Practical	0	0	0	1	0	2	0	2	2	4	2	1	0	3
Presentation	2	0	0	0	0	1	0	0	2	0	0	0	0	2
Timing	1	0	0	0	0	0	0	1	1	0	0	0	1	2
High Ceiling	0	6	1	0	5	1	0	4	0	0	0	2	0	1
Low-floor	0	0	1	0	0	0	0	0	0	0	0	3	0	1
Preparation	0	0	0	0	0	0	0	1	1	0	0	0	0	1
Assessment	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	4	23	3	20	27	26	19	55	51	33	51	26	17	102

These results motivated an in-depth examination of the segments coded at the highly cross-referenced nodes. For example, looking at the number of instances coded at each of the sub-nodes of Motivation, it is clear that *Understanding* is the most referenced code. The breakdown of the aspects of Task Design that impact on Understanding as a motivating factor for the students is given in the following table.

Table 7.9: Relationship between Task Design and Motivation_Understanding

Motivation v Task Design	Motivation_ Understanding
Meaningful	16
Context	15
Problem-solving	11
Active- Hands-on	10
Real life	10
Open-ended	8
Bridge21	6
Tech-mediated	6
Cross-strand	5
Team	5
Useful - Practical	3
Presentation	2
Timing	2
High Ceiling	1
Low-floor	1
Preparation	1
Assessment	0

In order to fully understand the potential relationships highlighted through this kind of quantitative data analysis, a number of these references were examined within a wider context, through a return to the raw data. Using the coding stripes facility of NVivo10, all of the codes that were referenced on the segments that linked Task_Meaningful and Motivation_Understanding were identified. The crossovers, with the number of times they were cross-referenced, are highlighted in the following diagram.

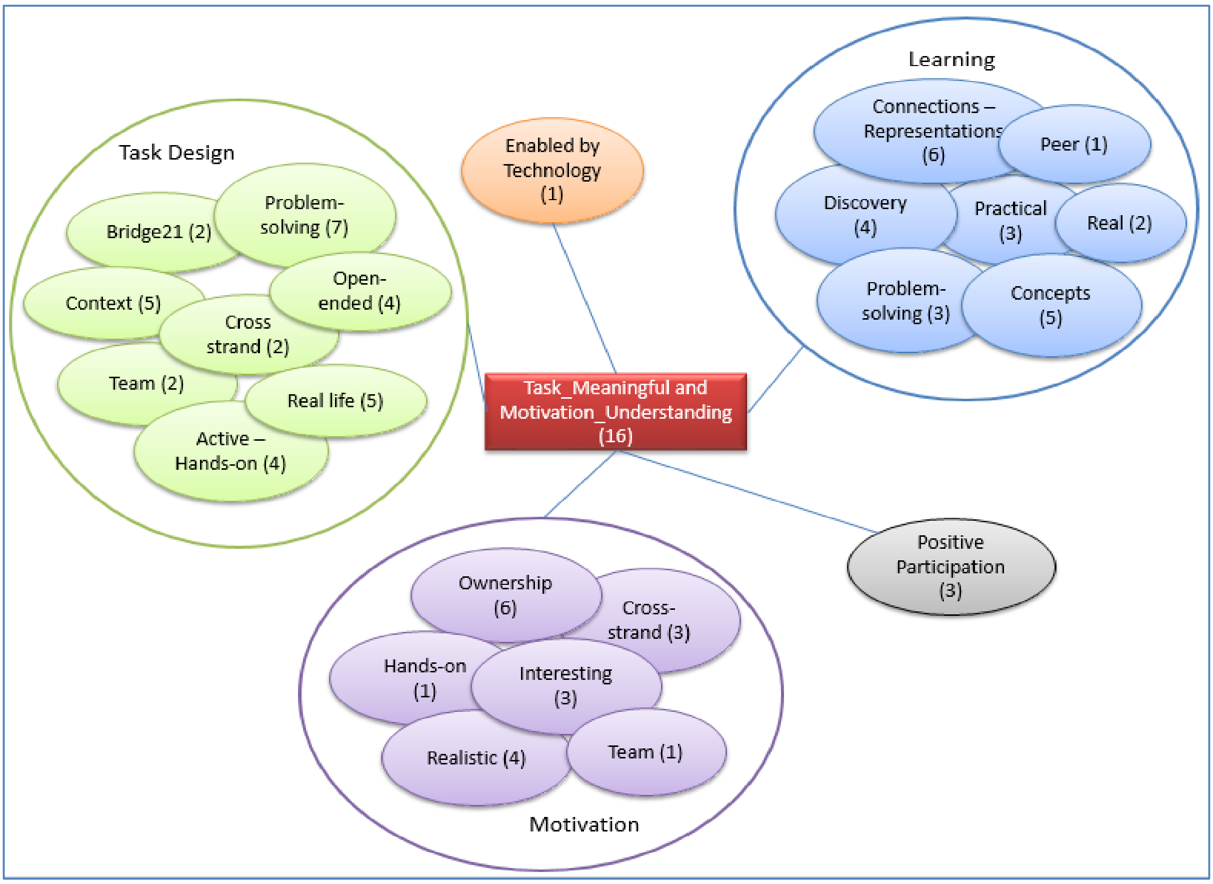
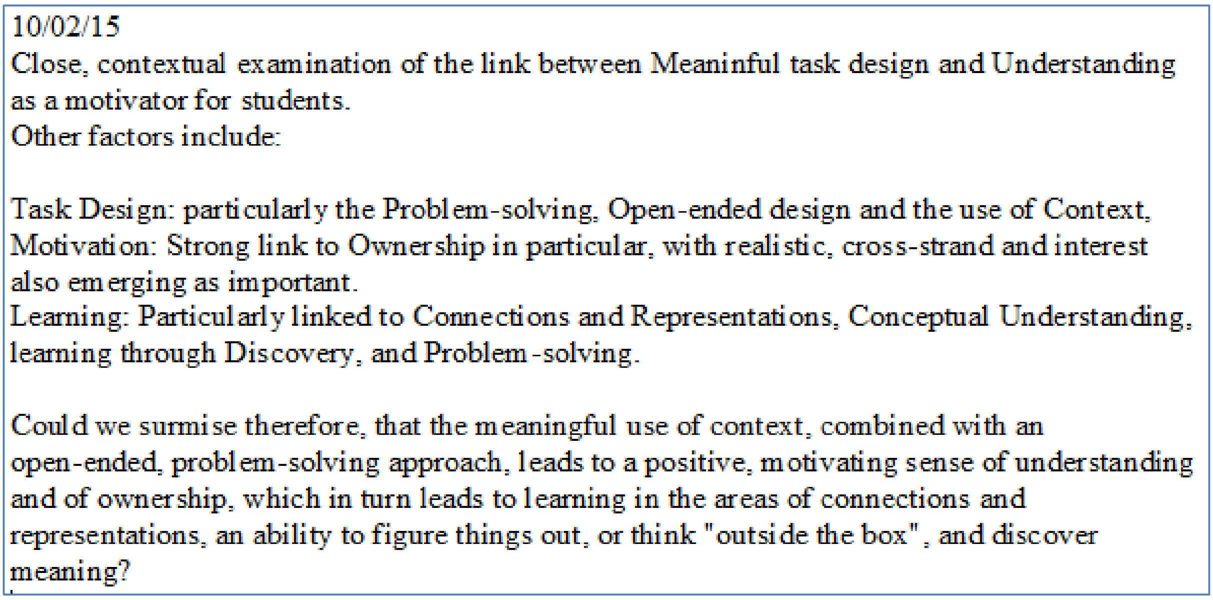


Figure 7.10: Breakdown of cross coding

Following the grouping of codes that were found to cross over with these 16 co-referenced sections of interview, tentative directional associations were made. These were noted in the following memo:



10/02/15
Close, contextual examination of the link between Meaninful task design and Understanding as a motivator for students.
Other factors include:

Task Design: particularly the Problem-solving, Open-ended design and the use of Context,
Motivation: Strong link to Ownership in particular, with realistic, cross-strand and interest also emerging as important.
Learning: Particularly linked to Connections and Representations, Conceptual Understanding, learning through Discovery, and Problem-solving.

Could we surmise therefore, that the meaningful use of context, combined with an open-ended, problem-solving approach, leads to a positive, motivating sense of understanding and of ownership, which in turn leads to learning in the areas of connections and representations, an ability to figure things out, or think "outside the box", and discover meaning?

Figure 7.11: Sample memo highlighting possible relationships

Similarly, close examination of the interplay between Task_Context and Motivation_Understanding led to the development of the following relational diagram:

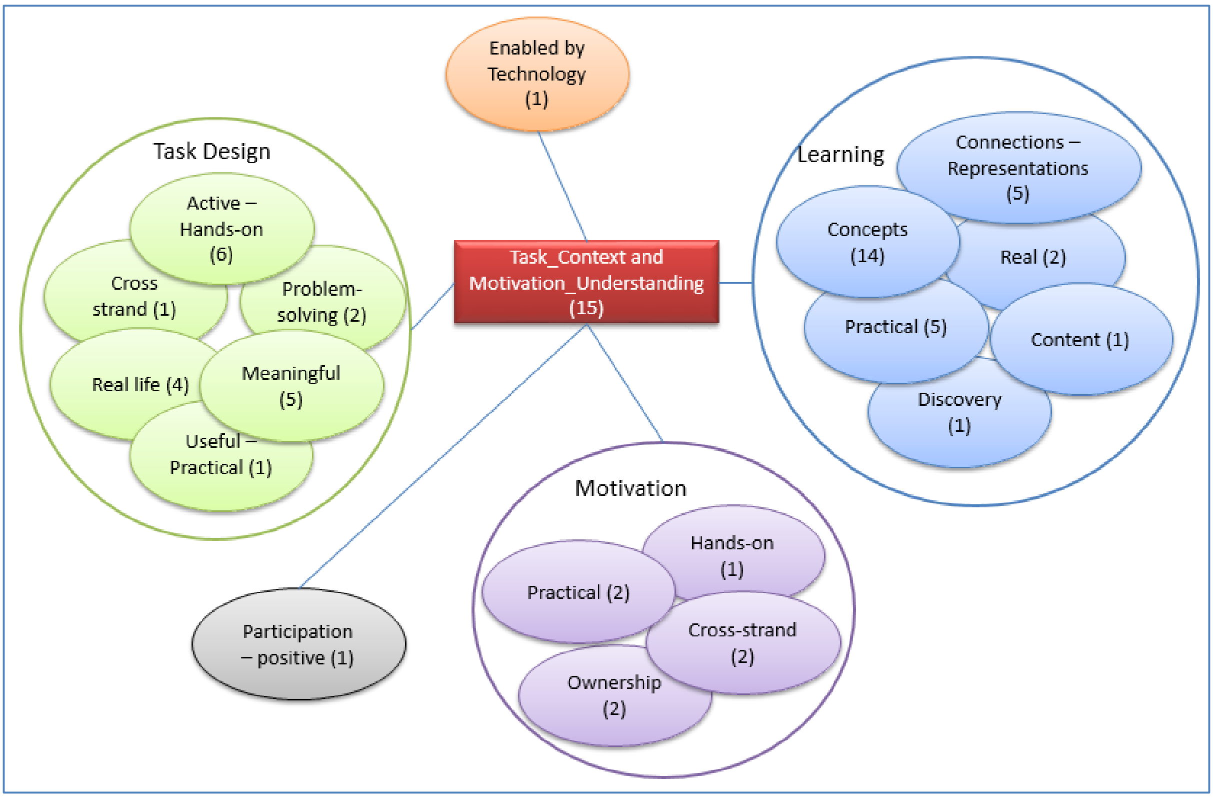


Figure 7.12: Model of cross coding – Task_Context and Motivation_Understanding

This model could be perceived as indicative of a link between the contextual, meaningful, hands-on design of the activities, a desire for understanding among the students, and the development of conceptual understanding of mathematics.

Although some of the relationships seem reasonably self-evident, through this kind of analysis, such relationships can be probed in more depth. An example of this is the relationship between the technology-mediated aspect of the task design, and the students' motivation through the use of technology (Figure 7.13). The modelling process is able to expand upon some of the possible reasons why the technology motivated the students. In particular, many of the students felt that the software helped them to be capable of accomplishing the tasks, even having a positive impact on their attitude towards mathematics and on their level of participation:

> *"Because it was on a computer, it was much easier for me to deal with."*

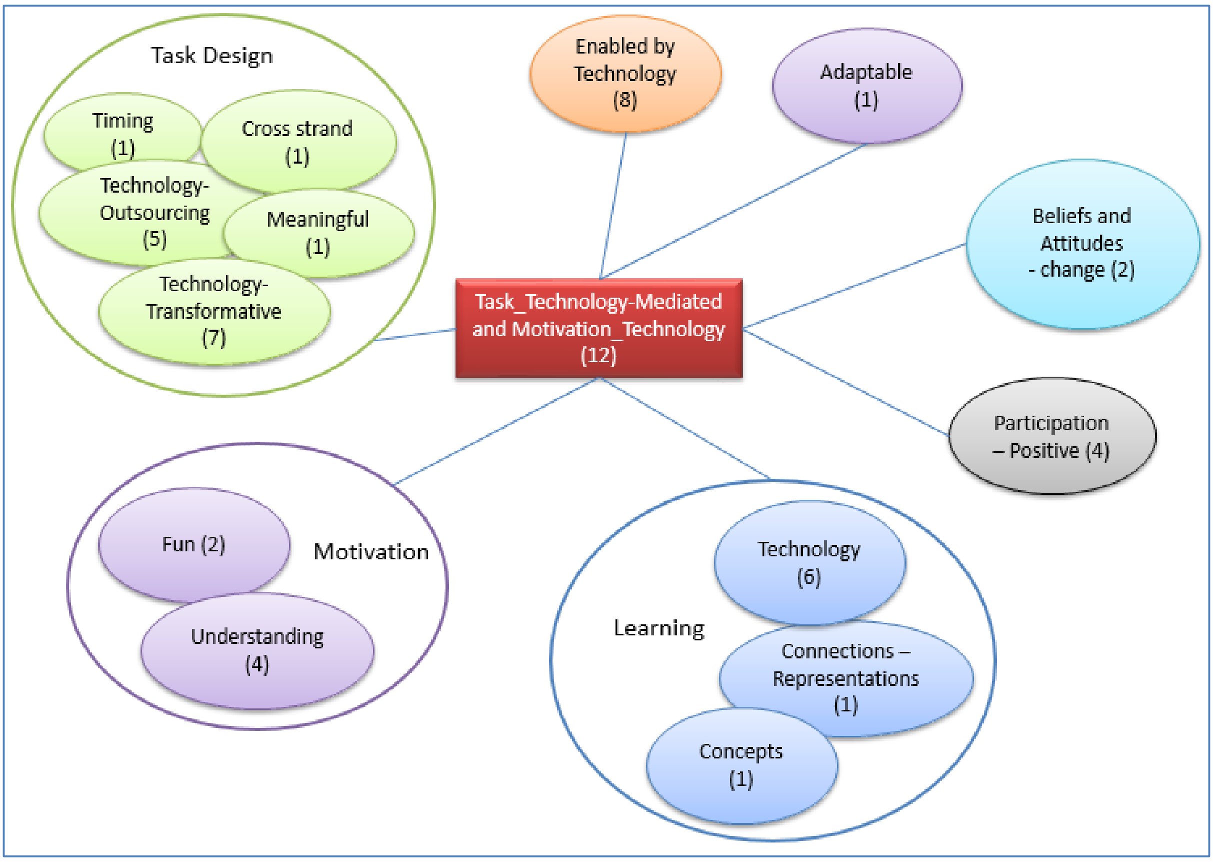


Figure 7.13: Model of cross-coding – Task_Technology-mediated and Motivation_Technology

However, not all of the potential overlaps yield interesting relationships. For example, the relationship between Task_Team and Motivation_Team is very straightforward and, although there are 16 overlaps of references at this point, there are no other significant contributing or resultant factors.

This process of in-depth exploration was repeated for some of the most significant apparent relationships between the Task-Design, Motivation, and Learning categories, leading to the generation of further tables (Tables 7.11 and 7.12) and relational diagrams similar to the above. An overall total of 457 instances of cross-coding were evident in the Task-Design/Motivation coding matrix alone, and for this reason, only those with in excess of 10 co-coded segments were examined in-depth using the process described above. This decision was made partly due to time constraints, but also because as the number of cross-references diminished, the complexity of the relationships also decreased and no new conjectures could be drawn.

Learning and Motivation

The comparison of segments that are co-coded at Learning and Motivation has strengthened the conjectures that had been drawn in the previous section relating to the importance of student understanding as a motivating factor. Table 7.10 provides a clear indication that a desire for understanding has had an impact on conceptual understanding and on the connections between different areas of mathematics. In fact, understanding is the most significantly cross-referenced code in eight of the eleven codes associated with Learning, with a total of 81 segments co-coded at Motivation_Understanding and Learning.

Table 7.10: Learning and Motivation coding Matrix

Learning v Motivation	Concepts	Connections	Content	Discovery	Further	Prob-solve	Peer	Practical	Real	Team work	Tech
Understanding	24	17	4	6	0	4	5	9	6	0	6
Ownership	7	5	0	6	0	2	3	3	1	1	1
Realistic	4	4	0	3	0	1	0	4	5	0	0
Technology	3	3	0	0	1	0	0	0	0	0	8
Fun	2	1	0	0	0	0	0	1	0	0	2
Practical	2	2	0	3	0	1	0	3	5	0	1
Hands-on	2	2	0	2	0	0	0	4	1	0	0
Cross-strand	2	11	0	0	0	0	1	0	0	0	0
Challenge	1	1	1	2	1	2	2	0	0	0	2
Interesting	1	1	3	4	1	3	1	2	2	0	0
Team	0	1	0	0	0	0	6	0	0	1	0
Curiosity	0	0	0	3	0	4	1	0	1	0	0
Assessment	0	0	0	0	0	0	0	0	0	0	0
Creativity	0	0	0	0	0	0	0	0	0	0	0

The two most highly co-coded nodes associated with the Learning category are Concepts (relating to conceptual understanding), and Connections-Representations (relating to an understanding of the connections within and outside mathematics), each of which has a total of 48 cross-referenced segments.

Learning and Task Design

Conceptual understanding again appears to be the most significant form of learning, with 65 cross-referenced codes related to Task Design. It seems to be the contextual, realistic, active and meaningful aspects of the task design that have the greatest impact on conceptual understanding (Table 7.11).

Interestingly, the Meaningful element of the task design category is the most highly co-coded with Learning, although it does not have the highest number of co-coded segments in any one node. This may potentially reflect that the meaningful nature of the task design is a fundamentally important attribute of the tasks’ relationship with all areas of student learning.

Table 7.11: Learning and Task Design coding Matrix

Learning v Task Design	Concepts	Connec tions	Content	Discovery	Further	Prob-solve	Peer	Practical	Real	Team work	Tech
Context	17	5	1	2	0	1	0	5	3	0	0
Real life	11	3	1	4	0	2	0	7	6	0	2
Active-Hands-on	10	4	3	5	1	1	0	12	4	0	0
Meaningful	8	8	1	7	0	5	1	6	7	0	0
Problem-solving	6	5	2	6	0	9	2	4	2	0	0
Open-ended	4	2	1	8	1	8	1	2	1	0	0
Tech Transformative	2	4	0	1	0	0	0	0	1	0	5
Useful - Practical	2	1	0	1	0	1	0	2	1	0	0
Bridge21	2	4	1	4	0	1	3	0	0	0	0
Presentation	2	2	1	0	0	0	2	0	0	0	0
Cross-strand	1	11	1	1	0	2	1	0	1	0	1
Tech Outsourcing	0	0	0	0	1	0	0	0	1	0	3
Team	0	2	1	2	0	1	7	0	0	2	1
High Ceiling	0	0	0	1	3	2	1	0	0	0	0
Low-floor	0	0	0	0	0	0	1	0	0	0	0
Preparation	0	1	0	1	0	0	0	0	0	0	0

7.6.3 Validity - Triangulation

As discussed in chapter 5, issues around validity and reliability need to be addressed. Measures to establish the reliability of the analysis have been taken by way of coding comparison of portions of the analysis (section 7.6.1.2), and the rich description and explanation building inherent in the case study approach. A process of triangulation is also used to establish validity in this research. This has involved the collection of multiple sources of evidence through interviews, observation, journals and comments, as well as quantitative data, in order to develop converging lines of inquiry.

7.6.3.1 Observational Data

Qualitative analysis of the observation records was undertaken using the directed coding matrices relating to MTAS and the Design Heuristics described in section 7.6.1. This form of data primarily relates to observable levels of engagement, and less so to confidence. For this reason, the MTAS subscales that were most commonly referenced are behavioural and affective engagement (Table 7.12). Zero rows and columns have been removed.

Behavioural engagement appears to be particularly positively associated with the problem-solving aspect of the task design and the Bridge21 style of activities. A return to the raw data reveals that the nature of the tasks involved the teams in a process of hypothesising and testing, using their prior knowledge to discuss the tasks and to come up with conjectures. The use of team-lead meetings and mentor scaffolding to guide the students’ progress through the activities has had a positive impact on keeping the teams on task. This accords with the analysis of the interview data that highlights the positive impact of teamwork, the Bridge21 activity structure and the problem-solving, guided nature of the task design as particularly impactful on BE.

Table 7.12: Observational data - MTAS v Design Heuristics

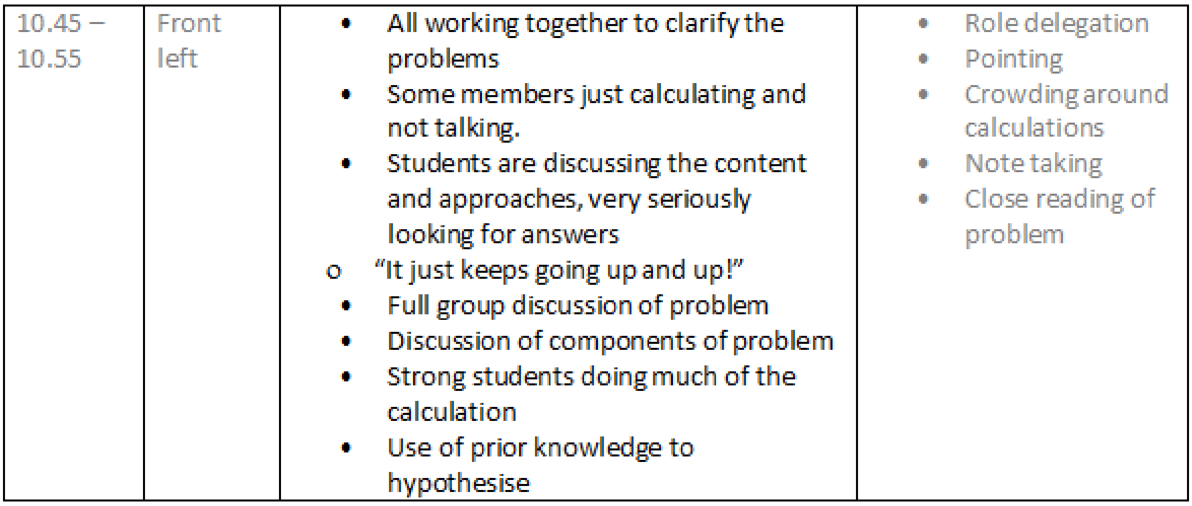
MTAS v Design Heuristics	AE_pos	BE_Neg	BE_Pos	MC_Pos	MT_Pos
Problem Solving	5	0	13	5	2
Bridge21 Structure	3	0	13	1	1
Team_Pos	5	0	9	2	1
Guided discovery	3	0	8	2	1
Open ended	1	0	6	3	2
RME real	1	0	4	2	0
Technology	0	0	3	1	4
Practical	0	0	3	2	2
Cross strand	0	0	3	1	0
Low floor	0	0	2	1	1
High ceiling	1	0	2	1	0
Team_Neg	0	1	0	0	0

Affective engagement also appears to be positively impacted by the problem-solving, guided discovery and open-ended aspects of the task design and by the Bridge21 structure, although AE is less evident through observation. AE is generally identified through observations such as “smiling”, “laughing”, and “gesticulating”.

Identification of mathematical confidence is made through observer recordings of discussions involving phrases such as “ah, I understand now”, and observation of students’ active participation in mathematical discussions in their teams and in plenary sessions. As with the interview data, the observation indicates that the problem-solving, open-ended, and guided discovery aspects of the task design supports this kind of discussion.

Students’ positive attitudes to using technology for learning mathematics are observed through their experimentation with different approaches in an attempt to find the most accurate answers.

Figure 7.14 shows the notes that were made through the observation of one group over a 10 minute period, which illustrates a number of these points.



10.45 – 10.55	Front left	• All working together to clarify the problems • Some members just calculating and not talking. • Students are discussing the content and approaches, very seriously looking for answers o "It just keeps going up and up!" • Full group discussion of problem • Discussion of components of problem • Strong students doing much of the calculation • Use of prior knowledge to hypothesise	• Role delegation • Pointing • Crowding around calculations • Note taking • Close reading of problem

Figure 7.14: Observed team problem-solving

Some negative behavioural engagement is observed in relation to the teamwork. This particular area becomes more explicit through the students' journals and written comments, which are discussed in the following section.

7.6.3.2 Journals and Written Comments

Regarding the written data, students were explicitly requested to detail two things they liked, two things they did not like, what they found difficult and what they learned during the intervention. The resulting data is particularly useful therefore, for detailing the aspects of the interventions that the students experienced difficulties with, and for confirming some of the issues that had been identified, but are less evident in the interview and observational data.

Table 7.13 highlights a lack of technical confidence experienced by some of the students throughout the course of the activities (the highest co-coded cells in each column are highlighted). Inspection of the raw data identifies that the students are not always comfortable using technology that they are not familiar with. Unsurprisingly, the use of technology in the activities is also most associated with a negative attitude towards using technology for learning mathematics. However, it is also most significantly related to negative affective engagement – some of the students did not like being out of their comfort zone with regards to the technology.

The most significant factor that appears to have had a negative impact on Behavioural engagement relates to unfavourable experiences of group work. This is primarily associated with the perception of an unfair distribution of the workload and of some participants not pulling their weight.

Table 7.13: Written Data – Negative MTAS v Design Heuristics

Negative MTAS v Design Heuristics	AE_MLAsNeg	BE_MLAsNeg	MC_MLAsNeg	MT_MLAsNeg	TC_MLAsNeg
Technology	6	2	0	3	18
Task_high-ceiling	2	0	2	0	1
Team_Negative	4	12	0	0	1
Task_RME-real	0	0	0	0	0
Task_problem-solving	0	0	0	0	0
Task_practical	2	0	0	0	0
Task_guided discovery	0	0	0	0	0
Task_cross-strand	0	0	0	0	0
Variety of Tech	0	0	0	0	0
Task_open-ended	2	1	4	0	0
Task_low-floor	2	0	0	0	0
B21 Activity Structure	0	0	0	0	0

Negative mathematical confidence seems to be mainly associated with the open-ended task design. The lack of clear, procedural instructions led some of the students to feel out of their depth:

> *"I found the whole thing hard; I wasn't sure what I was doing with the angles."*

Returning to the more positive impacts on students' engagement and confidence, table 7.14 relates the Positive MTAS subcategories to the design heuristics.

Table 7.14: Written data - Positive MTAS v Design Heuristics

Positive MTAS v Design Heuristics	AE_MLAsPos	BE_MLAsPos	MC_MLAsPos	MT_MLAsPos	TC_MLAsPos
Task_RME-real	44	14	20	18	11
Task_practical	28	17	16	11	5
Technology	20	11	18	34	34
Task_problem-solving	12	13	14	9	6
Team_Positive	10	14	3	1	1
Task_high-ceiling	9	6	2	2	5
Variety of Tech	8	3	2	9	1
Task_cross-strand	7	1	6	7	2
Task_guided discovery	5	4	6	4	5
B21 Activity Structure	5	3	0	0	0
Task_open-ended	4	8	2	0	0
Task_low-floor	3	0	1	0	0

It is apparent that the realistic aspect of task design is most associated with positive affective engagement. Realistic in this context refers to the RME sense of the word, which can also be understood as 'meaningful to the students' (van den Heuvel-Panhuizen, 2002). Examination of the segments of text that have been coded as RME-real reveals that making the mathematics meaningful for the students has a positive impact on their enjoyment of the subject.

> *"I learned that maths is not just numbers and can be made fun just by being given a few elastic bands and a Barbie doll."*

The practical element of the tasks, including the fact that the students were required to go outside in order to physically gather data, also appears to have a positive effect on AE, as does the challenge provided by the high-ceiling and problem-solving elements of the tasks, and the use of technology.

The aspects of the design heuristics that positively impact on AE appear to have a similar effect on mathematical confidence, leading to segments such as:

> *"I also learned how functions can help find how many bands are needed to drop a Barbie out of a window."*
>
> *"I learned how to use Tracker and this included a lot. I found this useful and it helped me to understand functions more".*

Behavioural engagement appears to be most affected by the practical aspects of the task design, with positive experiences of teamwork also having a beneficial effect:

> *"Yes, I enjoyed the session – I think my team worked well together and I enjoyed doing functions in a practical way."*

The two MTAS subcategories associated with the use of technology have significantly more positive than negative associations, with students generally finding the use of the tools enjoyable and helpful.

> *"I learned that using technology can make maths enjoyable and easier."*
>
> *"I really enjoyed using the equipment provided for us (laptops, phones etc.)"*
>
> *"I learned how to use technology to help solve maths problems."*

Once again, results of the analysis of the data from the journals and written comments confirms what has emerged through analysis of the interviews.

The fact that the positive effect identified through the qualitative analysis relating to the MTAS subcategories, occurs across all of the student cohorts (embedded units) and from each of the data sources indicates theoretical replication and demonstrates robust validity, permitting strong conclusions to be drawn (Yin, 2014).

7.7 Findings – Student Data

There were two primary motivators for analysing the qualitative data using the techniques described in sections 7.6.1 and 7.6.2: one was to ensure that nothing relevant had been missed during the initial directed content analysis of the interviews, and the other was to further tap into the rich interview data in order to deepen understanding of the relationship between the MTAS results and the design heuristics. At first glance, the only theme common to both methods of analysis is Task Design. In reality however the categories of Motivation, Learning, and Beliefs are easily mapped to different aspects of affective or behavioural engagement, mathematical or technical confidence, or attitude to using technology for learning mathematics. The process of mapping has served to deepen the understanding of the different motivations for, and learning outcomes of the MTAS subcategories.

In order to probe these relationships, the codes and categories developed through constant comparison have been compared and mapped to the codes from the initial directed content analysis. NVivo10 matrix coding was used to identify crossover of the different areas of the analyses. Table 7.15 below presents the crossover of references coded using the design heuristics categorisation matrix and the relevant codes from the constant comparative approach. Table 7.16 identifies relationships between the analysis directed by MTAS and the relevant aspects of the constant comparative approach.

7.7.1 Crossover of Directed Content and Constant Comparative Coding Relating to Design Heuristics

The first of these tables (Table 7.15) identifies codes related to the design heuristics that are consistently coded across both the directed content and the constant comparative analysis. It has become apparent, through analysis of the relationships between these codes, that some of the Task Design aspects of the Design Principles could be more subtly differentiated (Figure 7.15), allowing for clearer descriptions of the attributes of the tasks, and thus a more comprehensive guide for teachers. Figure 7.15 illustrates the relationship between the directed coding of Task_Practical, Task_RME - real, Task_guided discovery, and their most highly associated counterparts. These particular codes are highlighted as their expansion through the process of constant comparison was particularly meaningful. The solid line is indicative of the strongest level of overlap.

Table 7.15: Crossover between Directed Content and Constant Comparative Coding Relating to Design Heuristics

Directed Content v Constant Comparison	Bridge21 Structure	cross-strand	guided discovery	high-ceiling	low-floor	open-ended	practical	problem-solving	RME-real	Tech_ Computational	Tech_ Transformative
Real life	0	1	2	0	0	1	19	1	16	0	3
Meaningful	2	4	9	4	0	5	16	8	17	0	4
Active-Hands-on	2	0	6	1	0	0	15	3	10	0	2
Context	0	5	3	0	0	0	12	0	14	0	3
Useful - Practical	0	2	1	0	0	0	10	1	5	0	2
Problem-solving	3	4	13	6	0	11	6	16	6	0	0
Open-ended	7	2	10	7	0	22	3	13	6	0	1
Cross-strand	0	13	1	2	0	2	2	2	3	0	2
Bridge21	15	0	4	0	0	1	1	7	1	0	0
Team	2	3	1	3	3	0	1	3	2	0	0
High Ceiling	2	1	0	10	3	4	1	2	1	0	0
Timing	8	1	0	0	0	0	0	0	0	1	1
Presentation	5	0	0	0	0	0	0	0	0	0	0
Preparation	2	0	1	0	0	0	0	0	0	0	0
Low-floor	2	0	0	4	7	0	0	0	0	0	0
Tech Transformative	1	0	0	0	0	0	0	1	1	6	13
Tech Outsourcing	0	0	0	0	0	0	0	0	0	7	5

Legend		
	🟥	Highest score in both rows and columns
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	🟩	Highest score in columns (DC)

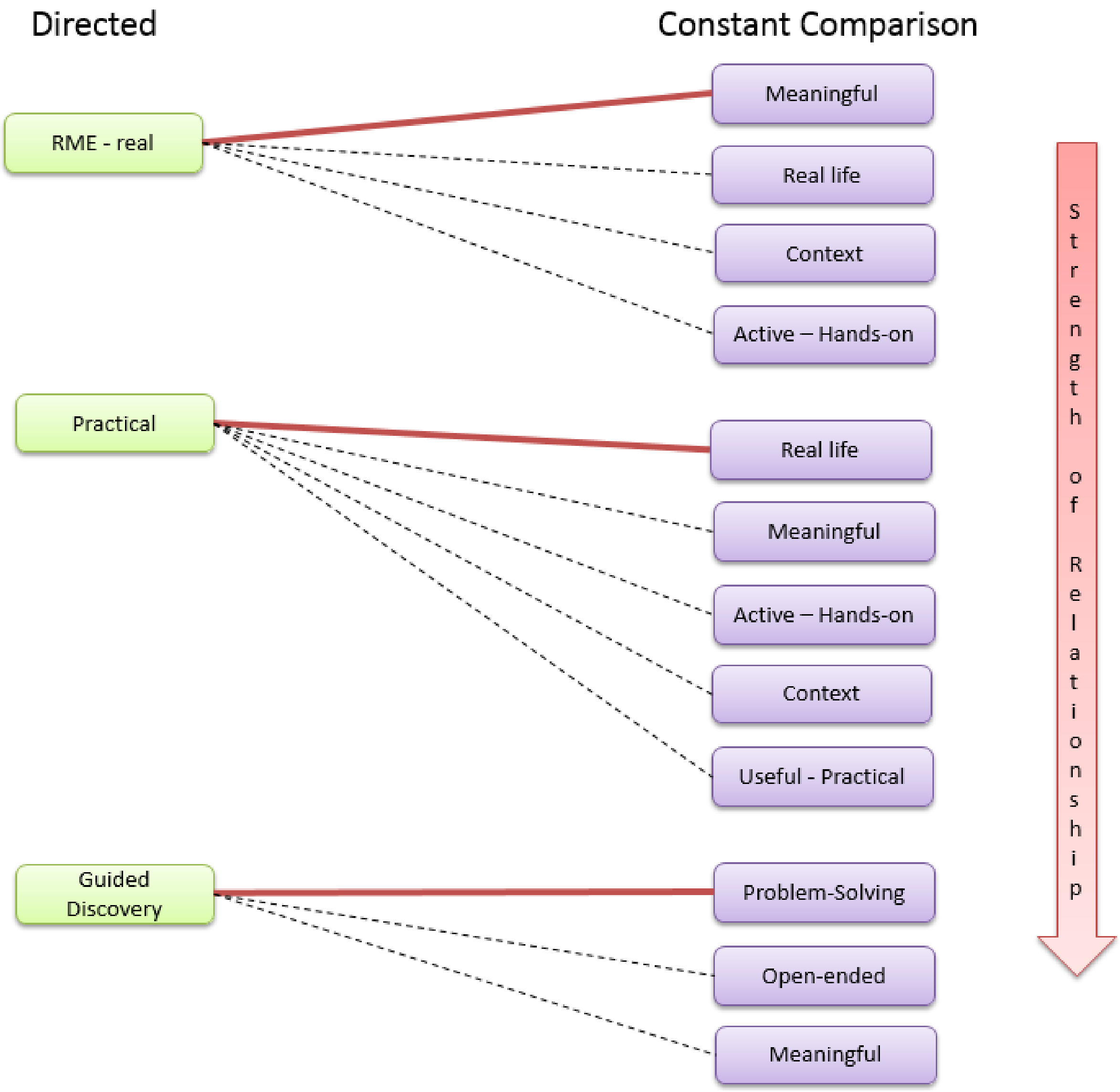


Figure 7.15: Diagram of the relationship between some Constant Comparative and Directed codes related to task design

The comparative diagram above (Figure 7.15) confirms the relationship between the RME concept of ‘realistic’ and how it has been understood in the analysis as based in contexts that are meaningful to the students. The differentiation of the category of Practical into Active and Useful had already been identified through the constant comparative process. It is interesting to note that Guided Discovery did not emerge as a category in its own right through constant comparison. It appears to have been most prominently associated with problem-solving and open-ended task design

Table 7.16: Crossover between Directed Content MTAS and Constant Comparative Coding

DC MTAS v Constant Comparison	AE_MLAs Pos	AE_Trad Neg	BE_MLAs Neg	BE_MLAs Pos	BE_Trad Neg	MC_MLAs Pos	MC_Trad Neg	MT_MLAs Neg	MT_MLAs Pos	TC_MLAs Neg	TC_MLAs Pos	TC_Trad Neg
Beliefs_Change	9	0	0	6	0	6	1	0	4	0	3	0
Beliefs_Negative	0	6	0	0	9	2	4	0	1	0	1	0
Beliefs_Positive	4	0	0	5	0	2	0	0	1	0	1	0
Learning_Concepts	15	0	0	19	1	46	5	0	7	0	3	0
Learning_Connections	17	0	0	13	1	21	2	0	6	0	4	0
Learning_Content	4	0	0	1	0	15	2	0	1	0	0	0
Learning_Discovery	12	1	0	12	1	10	1	0	3	0	0	0
Learning_Estimation	1	0	0	1	0	1	0	0	0	0	0	0
Learning_Further	1	0	0	1	0	2	0	0	2	0	1	0
Learning_ Prob-solving	6	1	0	6	0	3	0	0	1	0	0	0
Learning_Peer	4	0	0	14	0	11	0	0	1	0	1	0
Learning_Practical	13	0	0	12	1	15	1	0	3	1	1	0
Learning_Real	6	1	0	7	0	7	1	0	3	1	1	0
Learning_Teamwork	1	0	0	2	0	0	0	0	0	0	0	0
Learning_Technology	4	0	0	5	0	6	1	0	19	2	31	1
Motivation_Assessment	1	0	0	1	0	2	0	0	0	0	0	0
Motivation_Challenge	11	0	0	11	0	6	0	0	2	0	2	0
Motivation_Creativity	3	0	0	3	0	2	0	0	0	0	0	0
Motivation_Cross-strand	7	0	0	6	0	6	0	0	4	0	1	0
Motivation_Curiosity	14	1	0	10	0	6	0	0	1	0	0	0
Motivation_Fun	21	0	0	15	0	5	0	0	4	0	4	0
Motivation_Hands-on	5	0	0	6	0	7	0	1	2	1	0	0
Motivation_Interesting	37	2	0	18	1	8	0	0	1	0	0	0
Motivation_Ownership	14	0	0	23	0	16	0	0	5	0	2	0
Motivation_Practical	14	2	0	6	2	5	1	0	2	0	1	0
Motivation_Realistic	21	4	0	13	2	12	0	0	0	0	0	0
Motivation_Team	11	0	0	26	0	10	0	0	0	0	0	0
Motivation_Technology	3	0	0	7	0	9	0	0	18	0	15	0
Motivation_Understanding	25	0	0	30	1	41	5	0	15	0	9	0
Participation_Negative	0	2	2	0	7	0	1	0	0	0	0	0
Participation_Positive	21	0	1	44	0	18	0	0	6	0	6	0

7.7.2 Crossover of Directed Content Codes Relating to MTAS and relevant Constant Comparative Codes

Table 7.16 provides an overview of the areas that have emerged as relevant to the MTAS subcategories through the constant comparative approach. The primary categories that are identified as being relevant are Beliefs, Learning, Motivation and Participation. Each of these will be discussed in the following sections.

MTAS and Beliefs

The Beliefs category has three associated codes, relating to changing beliefs and perceptions of mathematics, positive, and negative beliefs about the nature of mathematics. Analysis of the relationships between these nodes and the MTAS subcategories indicates that positive aspects of each of the subcategories associated with the interventions may effect a change in students' beliefs and perceptions about mathematics.

> *"I remain unconvinced, if admittedly shaken in my absolute use of the term 'hate' and more on the side of 'mildly dislike'. Well done!! :D"*

Negative beliefs appear to be particularly associated with negative behavioural and affective engagement in the traditional classroom. Positive beliefs appear to be most significantly related to positive affective and behavioural engagement in the interventions.

MTAS and Learning

The areas of Learning that are most significantly associated with positive affective engagement through the interventions are practical, discovery and problem-solving. This may imply that these approaches help the students to learn and understand the mathematics, and leads to an increase in their enjoyment of the subject.

In terms of an impact on behavioural engagement, the students' increasing conceptual understanding, which emerges through the discovery of connections and peer support, and a practical, realistic and problem-solving approach, all appear to have had a positive effect.

The focus on conceptual understanding emerges as particularly important for mathematical confidence, and is again supported by the discovery of connections in a practical manner, supported by technology. The technology-mediated approach to learning also appears to have a positive impact on students' attitudes to using technology for learning mathematics, and on their technological confidence.

MTAS and Motivation

Students' affective engagement appears to be particularly positively impacted by various themes related to motivation. In particular, their interest in the activities provided to them, leading to a desire for understanding, motivates the students and increases their levels of enjoyment and satisfaction.

A desire for understanding also motivates an increase in positive behavioural engagement throughout the process. Behavioural engagement is also positively associated with the motivation stemming from being a part of a team, and the sense of ownership of their learning that emerges through the discovery and open-ended approach in the activities. The interesting and fun aspects of the activities also positively affects the students' behaviour during the interventions.

Positive effects on mathematical confidence (MC) appear to be strongly related to a desire for understanding and interest in the topics presented. The realistic nature of the tasks, and the support of the teams also has a positive impact on MC. Once again, a positive relationship with the use of technology in the activities appears to positively impact on students' attitude to using technology for learning mathematics, and on their technological confidence.

MTAS and Participation

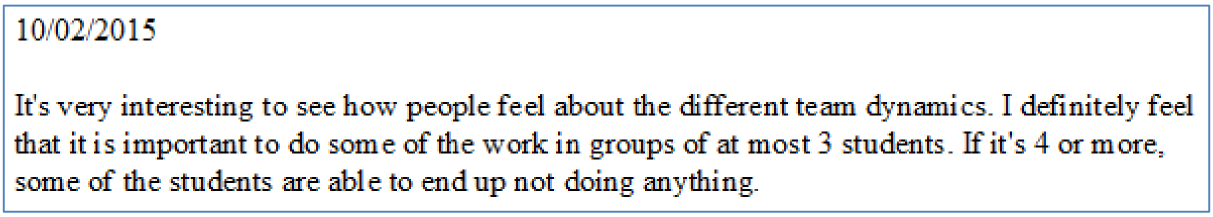
Participation emerged as a category throughout the School A (2014) set of interviews, and at the time, a conjecture was drawn about the possible relationship between participation and engagement. The analysis evident in Table 7.16 goes some way to validate this conjecture. It is clear from the data that positive participation is highly associated with positive behavioural engagement in the interventions, and also has a significantly positive association with affective engagement and mathematical confidence. Negative participation appears to be most strongly related to negative behavioural engagement in the traditional mathematics class.

7.7.3 Understanding of negative categories

A number of themes have emerged through the process of constant comparison that were not explicit in the directed content analysis. Participation is one of the themes that has emerged through the second analytical technique, as is students' beliefs about the nature of mathematics and education. Their sense of ownership and autonomy over their learning has also emerged as important. These areas have been discussed in sections 7.6.3.1 and 7.6.3.2. However, the students' negative experiences were also explored through the constant comparative analysis. Although negative associations with the MTAS subcategories were identified in the directed analysis, the comparison of the two techniques has led to a richer understanding of these less favourable attitudes. Table 7.17 illustrates the relationships between the negative constant comparative codes and the negative MTAS subcategories.

The traditional approach is clearly negatively associated with affective and behavioural engagement and mathematical confidence. Delving deeper into this relationship reveals that students frequently find the traditional approach to be boring and monotonous. Negative beliefs and associations regarding the nature of mathematics and of education appears to be strongly associated with low levels of participation and behavioural engagement. Maths anxiety, the pressure of assessment, and the impact that has on teaching and learning are also negatively associated with BE in the traditional classroom. A lack of context and the scope of the curriculum are negatively associated with mathematical confidence. As technology does not appear to have played a major part in the traditional classroom in any of the schools, the MTAS subcategories of TC_TradNeg and MT_TradNeg do not emerge as particularly relevant.

Exploring the negative MTAS subcategories associated with the interventions, it is interesting to note that three segments relating to the teamwork aspect are negatively correlated with behavioural engagement. Interestingly, one of the memos from the final stages of coding refers to the importance of good team dynamics, particularly in relation to balancing the workload. It is conjectured that teams of three may be particularly appropriate in order to ensure that engagement is maintained amongst all members of the group.



10/02/2015

It's very interesting to see how people feel about the different team dynamics. I definitely feel that it is important to do some of the work in groups of at most 3 students. If it's 4 or more, some of the students are able to end up not doing anything.

Figure 7.16: Memo relating to the importance of good team dynamics

There are four cross-referenced sections that negatively relate the use of technology with attitudes to using technology for learning mathematics and with technological confidence. Most of these refer to frustration with technology when it does not work as it is supposed to.

Table 7.17: : Crossover between DC Negative MTAS and CC Negative codes

DC Negative MTAS v CC Negative	AE_MLAs Neg	AE_Trad Neg	BE_MLAs Neg	BE_Trad Neg	MC_MLAs Neg	MC_Trad Neg	MT_MLAs Neg	MT_Trad Neg	TC_MLAs Neg	TC_Trad Neg
Beliefs_Negative	0	6	0	9	0	4	0	0	0	0
Confusion	0	1	0	0	0	0	0	0	0	0
Negative_Assessment	0	1	0	3	0	2	0	0	0	0
Negative_Boring	1	8	1	5	0	0	0	0	0	0
Negative_Curriculum	0	0	0	1	0	2	0	0	0	0
Lack of context	0	0	0	0	0	2	0	0	0	0
Negative_Learning	0	1	0	1	1	1	0	0	0	0
Maths anxiety	0	1	0	3	0	1	0	0	0	0
Monotonous	0	4	0	3	0	0	0	0	0	0
Negative_Teacher	0	1	0	1	0	1	0	0	0	0
Negative_Teams	1	0	3	0	1	0	0	0	0	0
Negative_Technology	1	0	1	0	1	0	4	0	4	2
Negative_Usual class	0	10	0	2	0	1	0	1	0	1
Participation_Negative	0	2	2	7	0	1	0	0	0	0
Traditional Approach	0	29	0	19	0	11	0	1	0	2

Legend
🟥 Highest score in both rows and columns
🟪 Highest score in rows (CC)
🟩 Highest score in columns (DC)

7.8 Discussion

The purpose of the study presented in this chapter is to address the second research questions, which relate to the effects on students of participation in mathematics learning activities that are consistent with the design heuristics described earlier in the study. In particular this explanatory study aims to identify the effects on engagement and confidence that participation in such activities can have, and to determine the primary factors that cause these changes.

7.8.1 Summary of the findings emerging from student data

The findings discussed in section 7.7 provide a compelling story of the positive effects that activities designed in accordance with the design heuristics can have. The most highly referenced nodes in the MTAS subcategories are AE_MLAsPos (183 references), MC_MLAsPos (160 references), and BE_MLAsPos (159 references), followed by TC_MLAsPos and MT_MLAsPos (63 and 61 references). Negative references in each of the categories associated with the mathematics learning activities all number under ten.

Exploration of the relationship between the positive aspects of the MTAS subcategories and the design heuristics has led to identification of the activity attributes that appear to have the most significantly constructive impact, and has also underlined some of the possible rationales for the associations. In particular, the (RME) realistic activities provide students with tasks that are situated in contexts that they perceive as meaningful. They are interested in solving the problems and challenges and want to understand the mathematics in order to be able to achieve this. This attribute of the tasks is particularly positively associated with affective engagement (AE), behavioural engagement (BE), and mathematical confidence (MC).

The practical nature of the task design also emerges as strongly correlated with AE, BE and MC. However, through the constant comparative analysis, it became clear that the practical code could be further differentiated between Useful and Hands-on, and is also associated with tasks that deal meaningfully with realistic mathematics.

The guided discovery approach, which requires problem-solving of open-ended tasks, also appears to be positively associated with AE, BE and MC. This seems to be related to a sense ownership and autonomy in the students over their own learning processes, and appears to lead to an increase in conceptual understanding of the mathematics involved in the activities.

The impact of teamwork on AE, BE and MC is also predominantly positive. Most students seem to like working in teams, which leads to increased enjoyment of the subject and improved participation in the class. In addition, the mixed-ability groups facilitate peer learning in a supportive and exploratory environment. Also associated with this is the ‘low-floor’ and ‘high-ceiling’ aspect of the tasks, which permit all of the students to meaningfully engage with the activities.

The MTAS subcategories of MT and TC were unsurprisingly most associated with the technological aspects of the design heuristics. In the main, the students find the technology helpful and reasonably straightforward to use, with some evidence of it supporting their development and understanding of mathematical concepts, connections and representations.

The small number of negative associations with the interventions are mainly related to unsuccessful teamwork, and technological failures, although a few of the students displayed a preference for working without technology. This emphasises the importance of careful selection of the team members in order to ensure that they can work well together. It also emerged through the analysis that teams should perhaps be made up of no more than three members, in order to encourage active participation of all participants. Technological problems can be difficult to anticipate, but careful planning and practice have proven to be somewhat effective in alleviating problems.

It thus appears likely that the design heuristics described in sections 4.3 and 6.8.1 adequately describe an approach to the design and implementation of RME/Bridge21-style mathematics learning activities that have the potential to increase student engagement and confidence with the subject. Students have been motivated by problems set in meaningful contexts that appeal to their interests. They have developed a sense of ownership and autonomy over their learning all, of which has improved their attitudes, behaviour and confidence.

7.8.2 Methodological factors, Limitations and Future Research

A number of methodological decisions have been taken in this research that have an impact on the data collection and analysis, and on the results. It is important to be clear about some of the factors that may influence the findings and to identify any limitations of the study.

As was discussed in chapter 5, this research has chosen to amalgamate the data from each of the interventions into one case study, viewing the individual schools as embedded units. This decision was taken as the focus of the research is on the impact of the design heuristics and not on developing a rich description of each of the school contexts. However this means that the contextual differences in each of the interventions are not taken into account in the analysis of the data. Each intervention was run with a different cohort of students and all but two of them took place in very different school settings. It could be argued that the decision to run the interventions in a private co-educational school, an all-boys underprivileged school and an all-girls underprivileged school gives a sufficient spread of contexts; however, any potential differences between these contexts are not explored.

The decision to use Transition Year students – that is, students who are not in a standard, exam-focused class – for the interventions was primarily due to reasons of access. It is important to recognise however, that this may have had an impact on the results. An attempt to address this limitation was made in the interviews, in which the students were explicitly asked to discuss the exam-focused classes as well as their experiences of transition year classes and of the interventions.

In addition to the school environment, differences between the interventions themselves are also not examined. It would be interesting to explore whether the various activities are received differently, or whether the duration and set up of the activities has an impact. These are all areas that can be addressed in future research.

Decisions with respect to the process of the data analysis should also be highlighted. In particular, the choice to conduct an initial directed content analysis is driven by the fact that the research explores the relationship between two existing frameworks (MTAS and the design heuristics). The subsequent constant comparative analysis is conducted in order to fully explore themes emerging from the data that may have been missed by the directed analysis. The combination of the two methods is an attempt to ensure that all of the themes in the data are identified, and to develop a rich and detailed description of the interventions, in order to go some way to addressing the limitations raised in the previous paragraph. Despite the researcher’s efforts to counter the influence of the first set of analysis on the second, it is clear from the development of the codes and categories that this has not been completely avoided. While it is important to identify this fact, it does not negate the depth or richness that was brought to the findings through the use and particularly the comparison, of the two methods of analysis. This combination of methods, in conjunction with the triangulation of data sources, has permitted a greater number of conjectures and conclusions to be drawn than would have been possible with any one method in isolation.

8. Exploratory Case Study – Teacher Experiences

8.1 Introduction

One of the findings that emerges throughout the analysis of the literature in Chapter 2, is a need for ongoing support and continuous professional development (CPD) for teachers in order to facilitate the development of 21st Century pedagogies and the integration of technology, as well as to scaffold their changing role in the classroom (Conneely, Murchan, et al., 2013; Dede, 2010a; Euler & Maaß, 2011; Voogt & Roblin, 2012). The teacher workshops described in Chapter 6 were useful for the development of a structured CPD module that has been incorporated into a larger Postgraduate Certificate (PG Cert) course in 21st Century Teaching and Learning, coordinated by the School of Education in TCD (Bridge21, 2014). This course began in September 2014 and the first cohort of teachers has recently completed the certificate programme.

The Contextual Mathematics module on the PG Cert requires each of the attending teachers to create and implement an activity using the design heuristics developed in this research. This chapter provides an analysis of their work with particular emphasis on their reflections on the process, and their experiences with it. The purpose of its inclusion in this research is to provide some evidence of the effectiveness of the design heuristics and practitioner guidelines. However, owing to the small scale of this aspect of the study to date, it is regarded as "anecdotal triangulation", and will require further development.

8.2 Research Aims and Questions

The work is framed as an exploratory case study, as it aims to investigate teachers' experiences, with a view to developing hypotheses and research questions for future research. The research design of the case study is similar to those described in chapters 6 and 7 in that it is a single case study, with multiple embedded units, each consisting of one of fifteen teachers' implementation and reflection. It differs however in two respects: firstly, the context is Post-Primary Education – Authentic Setting (the researcher is not an observer); and secondly, the case relates to Teacher Experience and their perceptions of their students' engagement (Figure 8.1).

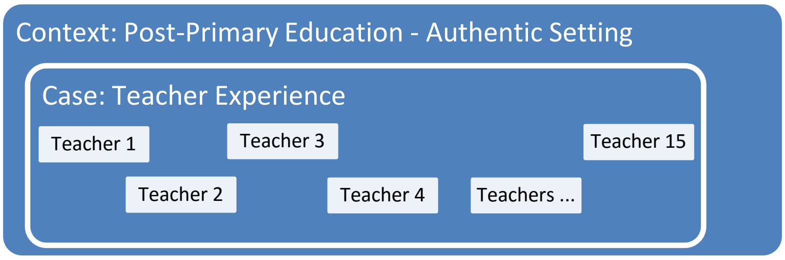


Figure 8.1: The Case Study Model

The specific aims of this exploratory case study are:

1. To explore the CPD module as a means of scaffolding teachers' use of the design heuristics for the creation of their own activities.
2. To explore the experiences of teachers in the creation and implementation of such activities, with particular emphasis on their perceived barriers to, and benefits of, the approach.
3. To explore the teachers' perceptions of their students' experiences with the activities.
4. To generate research questions for future work.

8.3 Context and Teacher Profiles

The Postgraduate Certificate in 21st Century Teaching and Learning is a new CPD course on offer to in-service teachers with a minimum of one year's teaching experience in schools. It is coordinated by the School of Education in Trinity College Dublin, in collaboration with the School of Computer Science and Statistics, the Centre for Research in IT in Education (CRITE) and the Trinity Access Program (TAP). The rationale underpinning the course modules is as follows:

"The course modules reflect a number of intersecting concerns on the current landscape of Irish education, particularly reform of curriculum and pedagogy in the Junior Cycle of education; development of enhanced leadership capacity within schools and across the system generally; development of STEM/CS capacity within schools; and enhanced support for students from disadvantaged backgrounds." (Bridge21, 2014)

8.3.1 Postgraduate Certificate

The PG Cert is a part time course in which students attend four core modules and two (out of eight) optional modules. The Contextual Mathematics course is one of the optional modules. Two opportunities to attend the module were offered to teachers on the PG Cert, one in December 2014 and one in March 2015. Each of these consisted of day-long Saturday workshops with a three hour assignment-support session on the subsequent Friday. Teachers who were not enrolled on the PG Cert were permitted to attend the workshops if the numbers of certificate attendees allowed. There were 5 PG Cert attendees at the December workshop, out of a total of 14 workshop participants. In the March session, 17 attendees (out of a total of 26 participants) were registered on the PG Cert. Thus a total of 22 teachers enrolled on the PG Cert opted to take the Contextual Mathematics module in the 2014/2015 academic session.

Attendees at the Saturday workshops engaged in a full cycle of the Bridge21 activity model, from ice-breaker and warm-up, to presentation and reflection. They were divided into teams and were required to solve at least one of the mathematics activities described in Chapter 4. This was followed by a period of reflection on the activities, on what had been successful, what had been challenging, and what the barriers might be for implementation in the classroom. The teams then brainstormed possible activities that followed the design heuristics, and that they would be able to implement with their own students.

The assignment support session involved the development of these activities, with particular attention paid to overcoming any potential barriers and pitfalls identified by the teachers. The assignment itself (Appendix 8.A) involved the creation and implementation of a mathematics learning activity developed in accordance with the design heuristics. The teachers were required to write a report on the experience, detailing the rationale of the design, the subject content to be covered and skills to be developed, and including evidence of student learning. A multimedia presentation of aspects of the learning experience and a written reflection were also required.

8.3.2 Teachers

A total of 22 teachers attended the contextual mathematics module as a part of the certified course. These teachers came from a wide range of different schools and had levels of teaching experience ranging from 3 to 19 years. Of the 22 participants, 15 were female and 7 were male. In a number of cases, two or three of the teachers came from the same school, which promoted greater levels of collaboration in the design and implementation of their activities, but also strengthened the communities of practice within the schools.

In terms of assessment, 9 of the teachers achieved over 70% (distinction) in their assignment, 9 attained between 50% and 69% (pass) and 2 were awarded less than 50% (fail). Two assignments were not received.

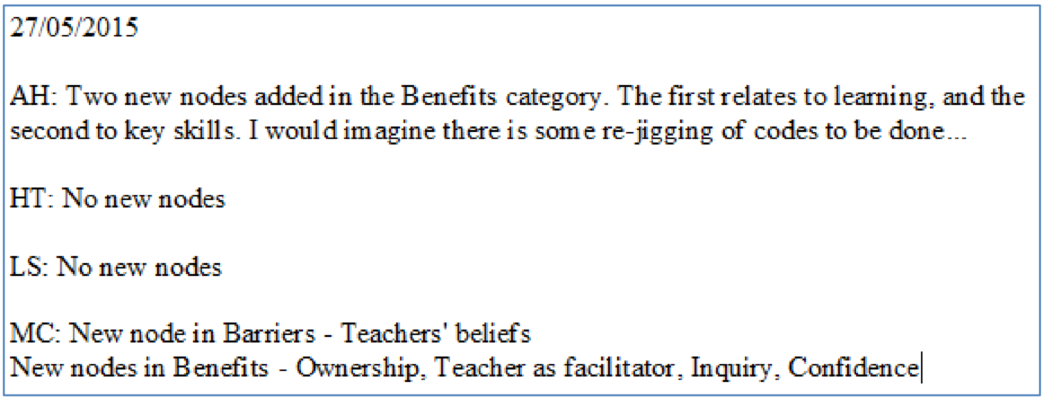
8.4 Data Collection and Analysis

The data that has been collected for this exploratory study is purely qualitative and comes from the written reports of the teachers. Not all of the teachers provided authorisation for their work to be included in this research (Section 5.6) and thus the total number of assignments that have been analysed comes to fifteen. A number of the teachers worked collaboratively on the design of the activities, and joint implementation was permitted. Appendix 8.B provides an overview of the 12 activities created by the 15 teachers. All but one of the activities (Children’s birthday party) were deemed to be creative, contextual and transformative in their use of technology. Of particular relevance to this research are the reflective pieces, which provide insight into the teachers’ experiences with the implementation of contextual mathematics learning activities and into the barriers and benefits of the approach.

As this aspect of the research study is purely exploratory, and is not based on detailed preliminary research, a *constant comparative* approach to the analysis of the data is most suitable. As described in Chapter 7, constant comparison is a method of reducing qualitative data to codes emerging from within the original source, while retaining much of the richness of the original data. Thus, the results of the analysis can be used to create a rich picture of the teachers' experiences, potentially identifying any common themes or categories. The steps in this process are fully described in section 7.6.2., and follow the procedure outlined by Glaser (1965) and Strauss and Corbin (2008).

8.4.1 Generation of Initial Codes and Categories

NVivo10 was used to facilitate the process of coding and theming. After the first five assignments were analysed, a total of 23 codes had been identified. These fell into the two main categories of Barriers, with five associated codes, and Benefits, with 18 associated codes. All segments of text at each of these codes were re-examined and compared before moving on to the next set of assignments. The next four assignments led to the addition of five new codes, four under the category of Benefits, and one under Barriers. At this point, the process of memoing was very useful for highlighting areas that could potentially benefit from re-organisation (Figure 8.2). In particular, the codes associated with the category of Benefits seemed to be developing into a number of subcategories, some relating to teachers and some to students, some to the development of key skills, and so on.



27/05/2015

AH: Two new nodes added in the Benefits category. The first relates to learning, and the second to key skills. I would imagine there is some re-jigging of codes to be done...

HT: No new nodes

LS: No new nodes

MC: New node in Barriers - Teachers' beliefs
New nodes in Benefits - Ownership, Teacher as facilitator, Inquiry, Confidence

Figure 8.2: Sample Memo

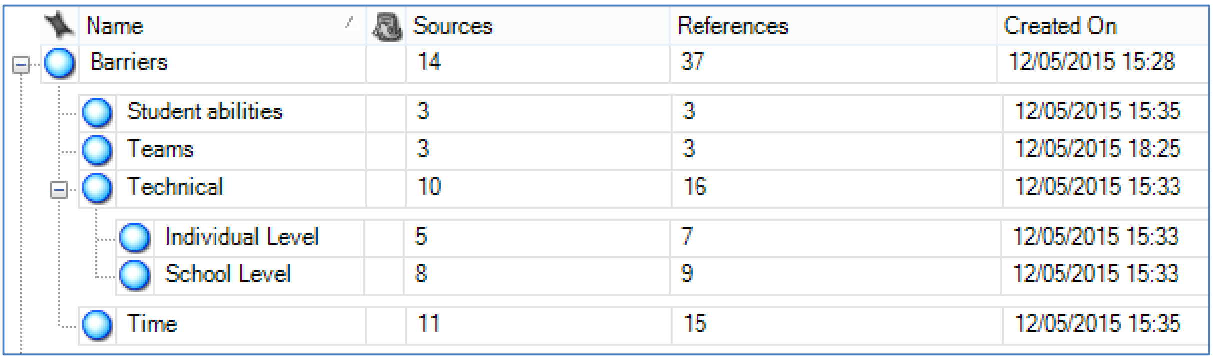
The remaining six assignments only led to the generation of two more codes, leading to the tentative conclusion that a reasonable level of saturation may have been reached.

All of the text was re-examined after each session of analysis, and particularly after the addition of new codes, in order to compare the coded text within their assigned nodes and also to identify whether they could be associated with any other codes. This process of constant evaluation and comparison has led to a rigorous association of codes and text.

8.4.2 Reduction of Codes

Once the initial development of codes and categories had been completed, the process of reducing and merging the codes, and developing sub-categories began. This involved an examination of the codes and the coded segments in order to determine whether there was any crossover of themes.

The Barriers category had significantly fewer references than Benefits, and included student abilities, teams, technical difficulties (at individual and school level), and time constraints (Figure 8.3).



Name	Sources	References	Created On
Barriers	14	37	12/05/2015 15:28
Student abilities	3	3	12/05/2015 15:35
Teams	3	3	12/05/2015 18:25
Technical	10	16	12/05/2015 15:33
Individual Level	5	7	12/05/2015 15:33
School Level	8	9	12/05/2015 15:33
Time	11	15	12/05/2015 15:35

Figure 8.3: Barrier Category

The category of Benefits had a total of 295 references, in comparison to only 37 in the Barriers category. At this point in the analysis, a number of subcategories were confirmed in the Benefits category. These related to benefits to the students (key skills, other outcomes, associated task attributes) and benefits to the teachers (change in beliefs, teacher as facilitator, and teacher as learner). All of these subcategories are expanded in Appendix 8.C.

8.4.3 Process of Analysis of Relationships

In line with the earlier research, the process of analysis of relationships used the coding matrix facility of NVivo10. Analysis focused on the relationships between the teachers' perceptions of the task design elements that had an impact on themselves and on their students, and their perceived *benefits*. No associations between task attributes and *barriers* were identified. Table 8.1, below, provides a numerical analysis of the number of times that segments of text were co-coded with a particular aspect of task design and a perceived benefit. The most significant elements of the *task design* columns and *perceived benefits* columns have been highlighted.

Using the ***sum*** functionality at the end of each row and column, it is clear that the fact that the tasks were student-led has had the most significant impact on perceived benefits, particularly on the sense of student ownership or autonomy, on their conceptual understanding, and on engagement. The student-led approach also seems to be significant in affecting a change in the role of the teacher in the classroom.

Table 8.1: Matrix Coding of Task Design and Perceived Benefits

Task Design v Perceived Benefits	Contextual	Cross-curricular	Hands-on	High Ceiling	Inquiry	Meaningful	Multiple Learning Styles	Open ended	Peer Learning	Student -led	*sum*
Ownership	1	0	0	0	0	0	1	1	2	7	*12*
Conceptual Understanding	5	1	0	1	1	3	0	2	2	6	*21*
Engagement	5	1	0	0	1	5	2	2	3	5	*24*
Teacher as Learner	1	1	1	0	1	1	0	0	2	4	*11*
Confidence	0	0	0	0	0	0	0	0	1	4	*5*
Communication	2	0	0	0	0	0	1	1	5	3	*12*
Teacher as facilitator	0	0	0	0	0	0	0	0	1	3	*4*
Collaboration	3	0	0	0	1	2	0	2	6	2	*16*
Technological competence	1	0	0	0	0	0	0	0	2	2	*5*
Problem-Solving	0	0	0	0	1	0	0	0	1	2	*4*
Flexibility	3	1	1	0	0	1	0	0	1	1	*8*
Creativity	0	0	0	0	0	0	0	0	1	1	*2*
Teacher Beliefs	0	0	0	0	1	0	0	0	0	1	*2*
Enjoyment	2	1	0	0	1	2	1	1	1	0	*9*
Organisation	1	0	0	0	0	0	1	0	0	0	*2*
Presentation	0	0	0	0	0	0	0	0	1	0	*1*
Sum	***24***	***5***	***2***	***1***	***7***	***14***	***6***	***9***	***28***	***41***	

Peer learning and the contextual nature of the task design also appear to have had beneficial effects on the students and teachers, particularly in the areas of collaboration, communication and engagement.

In terms of perceived benefits, it appears that the task design has had most impact on student engagement, with the tasks set in contexts that were meaningful to the students and the student-centred nature of the activities appearing to have the greatest effect.

Conceptual understanding is highlighted as the second highest co-coded perceived benefit, and this seems to be related to tasks that are set in contexts that are meaningful to the students, as well as the student-led nature of the learning.

8.5 Findings

The findings that have emerged through analysis of the relationships between task design and the perceived benefits of the approach, go some way to confirm the findings of the earlier research. In particular, the apparent link between the student-led, contextual and meaningful approach to activity design, and a perception of increased engagement and conceptual understanding (easily linked to confidence), can be seen as an endorsement of the results of the analyses in chapters 6 and 7. However, in addition to these relationships, a number of other findings have emerged relating to the teachers' perceptions of the barriers to the implementation of activities of this kind, and also of the benefits that engagement with these tasks can engender.

8.5.1 Barriers

Although the CPD module addressed some of the barriers to the integration of technology and the implementation of new teaching and learning strategies that had been highlighted in the literature review, such as a need for a structured and supportive approach (Conneely, Lawlor, et al., 2013; Dede, 2010a; Euler & Maaß, 2011; Means, 2010; Voogt & Roblin, 2012), many of the more systemic barriers remain and have been identified by the teachers. The most significant of these relates to time constraints and the difficulty that implementing a project-based, inquiry activity in a series of 40 minute classes, which was identified as a problem in 10 of the 15 assignments:

> *"Having a longer block of time would have been more productive, having to stop after 40 minutes and then pick up again a day or two later was inconsistent, especially when we were running into problems" (AH)*

Technical barriers were an issue for nine of the teachers, with five identifying personal difficulties with the technology, which would be easily rectifiable on a re-run of the project:

> *"The camera we were using ran out of battery power during the penalty shoot outs... More cameras would need to be made available, especially if more teachers were to start working with this approach. If this were the case, it would be advisable to get the camera the day before to charge it up etc. Still I had a backup plan of using my iPhone. This quickly ran out of space and required some quick deletions of some other video. A tripod would have also been desirable as one of the student's hands were too shaky and the video captured by them was not usable later as measuring distances would have become pointless." (WMI)*

Eight of the teachers identified technical barriers at the school level, which primarily related to inadequate access to the technology:

> *"Resourcing fully functioning laptops could be a challenge - I need to ensure that the limited number of laptops are available for at least three class periods." (IS)*

Other barriers that were identified by the teachers referred to lower than expected levels of students' technical expertise, and difficulties relating to the development of well-functioning teams.

8.5.2 Benefits

The perceived benefits associated with the approach far outweigh the barriers, and can be broken down into benefits for teachers and benefits for students.

8.5.2.1 Benefits to Teachers

The teachers perceived a number of changes to their beliefs and to their role in the classroom. Two of the teachers in particular discussed the impact that teaching in this way has had on their beliefs about mathematics teaching:

> *"Overall for me, this module has changed the way I think about teaching maths, with such pressures to cover all the curriculum and each topic, I never felt there was space to make a topic interesting, fun and engaging. After trying this, my eyes have been opened to the possibilities of covering the curriculum, but by changing the setting of the learning, you can teach a lot more effectively to an audience who are stimulated and engaged." (JPF)*
>
> *"This is wonderful in theory but has taken its time for my own thought process and in turn teaching style to change and develop. I will be honest that I found it more difficult to change my teaching style when it came to Maths. I was teaching the way I was taught, which was with very little understanding." (MC)*

It appears that the role of the teacher in the classroom is significantly affected through the implementation of these activities. The change in role from transmitter of information to facilitator of learning was not a comfortable one for some of the teachers; however, in all cases, it was hailed as a positive development, empowering the students to take ownership of their own progress.

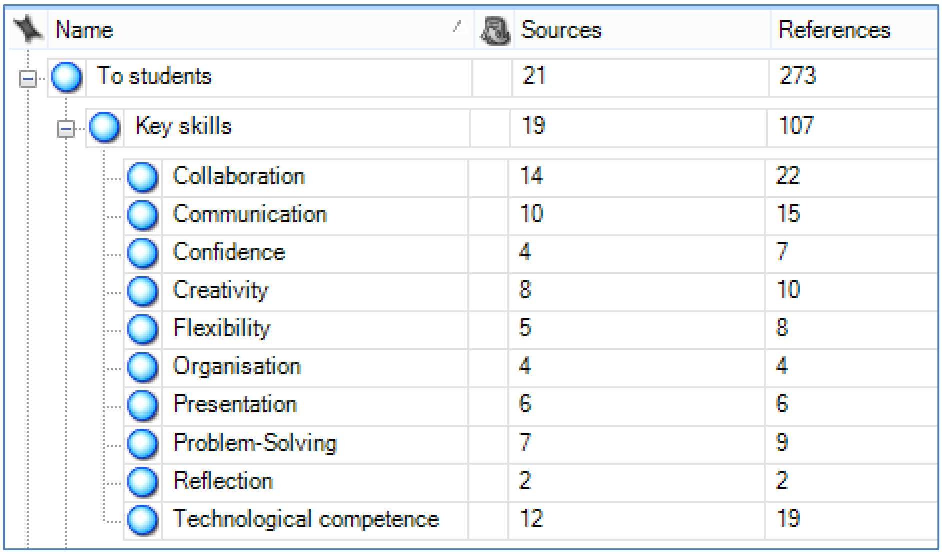
> *"I decided to tell the students of how this was as much of a learning curve to me as it was to them. This was because I really did feel that they would lose confidence in me if they felt that I was trying to teach them rather than facilitate them. This seemed to empower them as they felt that even though I wasn't part of their team, I was learning and teaching as opposed to teaching and learning with them." (MC)*

In addition to the change in role from teacher to facilitator, six of the teachers also identified themselves as co-learners in the classroom, both in terms of learning about the technology with and from the students, and learning about how to make activities of this kind more successful in the future.

8.5.2.2 Benefits to Students

The benefits to the students have been deconstructed into the subcategories of 'key skills', 'other outcomes' and 'associated task attributes'. The relationships between the task attributes and the perceived benefits of the approach have already been discussed in section 8.4.3. This section will therefore focus on the perceived benefits of the approach to students, without dwelling on their associations with the task design.

The key skills subcategory is made up of the following codes (Figure 8.4)



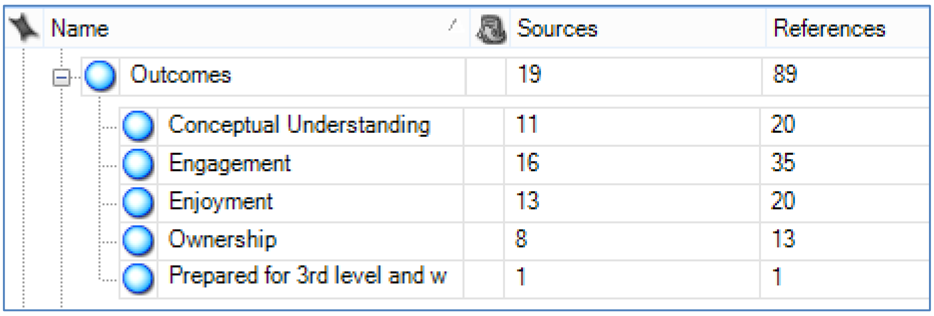
Name	Sources	References
To students	21	273
Key skills	19	107
Collaboration	14	22
Communication	10	15
Confidence	4	7
Creativity	8	10
Flexibility	5	8
Organisation	4	4
Presentation	6	6
Problem-Solving	7	9
Reflection	2	2
Technological competence	12	19

Figure 8.4: Key Skills

It is clear from this figure that the most common skills that were developed relate to collaboration and communication, technological confidence and creativity and problem-solving. The students generally seemed to enjoy working in teams and learning with and from their peers. Many of the teachers recognised the potential that technology has to facilitate a deeper understanding of the mathematics involved in the activities, as well as increasing the students' technological skills.

> *"The resounding theme of the [student] reflection was that they could really engage with one another and more importantly that they could engage more with the abstract topics of maths because of their ability to use technology in everyday maths." (DR)*

In addition to the development of key skills, a number of other beneficial outcomes emerged through students' participation in the activities designed by the teachers. These outcomes are listed in figure 8.5.



Name	Sources	References
Outcomes	19	89
Conceptual Understanding	11	20
Engagement	16	35
Enjoyment	13	20
Ownership	8	13
Prepared for 3rd level and w	1	1

Figure 8.5: Other Beneficial Outcomes

An increase in student engagement relating both to how they felt about the subject (affective engagement) and how they behaved in the classroom (behavioural engagement), was evident through the teachers' reflections. Comments such as those provided below, clearly illustrate the sense of engagement and motivation experienced by students and teachers alike.

> *"All the team members were fully engaged in the activity; their pride in and ownership of their learning was clearly expressed… It's really heartening to encounter such a level of motivation and commitment." (DD)*
>
> *"Please let's do more of this stuff! It's brought Maths to life! I really get it now! ☺" (Student)*
>
> *"This project was a thoroughly enriching experience for both the students and teachers assisting them." (DOC)*
>
> *"After this contextual Maths workshop, they asked for a Maths club. To me that is success!" (MC)*

There is a high level of cross-coding of segments of text coded at engagement and at enjoyment. However, a deeper analysis of the text coded at enjoyment indicates that this code is particularly closely related to affective engagement. Any segments that are coded at enjoyment and not at engagement relate specifically to the idea of having fun in the class, both from the point of view of the students, and the teachers:

> *"This project has highlighted one of the most enjoyable pieces of technology that I have used in my teaching career" (IB)*

> *"I feel that the students enjoyed this realistic contextualized activity and by taking part they have taken a step forward in developing their technological skills, becoming better problem solvers and gaining attributes in working as part of a team." (AH)*

> *"The creative building and testing stages are always enjoyed by the students." (HT)*

> *"The students also had fun, which they said that they thought they would never be able to say about Maths. " (MC)*

An increase in students' conceptual understanding and confidence was identified in nine of the analysed reports. This appears to be particularly closely associated with the contextual and meaningful nature of the tasks, a relationship that is clearly captured in the following quote:

> *"I am sure that none of these students will ever forget how they deepened their understanding of quadratic functions: the next time they video a friend kicking a football or teeing off in golf they will visualise that ball moving across the Cartesian plane, describing a smooth parabola." (DD)*

In addition, the open-ended task design and the student-led approach within the classrooms appears to have led to a deepening of the students' understanding:

> *"The open-ended nature of the activity produced a new energy in the teams: they were not working to find one answer (already known to me) but were engaged in a meaningful exploration of the topic." (DD)*

Seven of the reports refer to the increased sense of student ownership of their work, leading in turn to pride, engagement and motivation.

> *"Students came into their own when given the opportunity to work as a group and they seemed to grow as individuals even in the short space of time while working in groups with their peers" (DR)*

> *"Moreover, I feel that if I had taken over this aspect of the project… I would be impacting on their self-efficacy." (DF)*

By handing the responsibility for the learning to the students, they were seen to develop as individuals and as members of a group, with the apparent increase in levels of motivation and pride in their learning leading to higher levels of conceptual understanding.

> *"all the participants felt that they had created their own quadratic function and understood that it could be mathematically analysed.." (DD)*

In summary, these findings provide a compellingly positive picture of the approach to the development and implementation of mathematics learning activities that correspond to the design heuristics described in this research.

8.6 Discussion

The analysis of teachers' reflections described in this chapter has provided an opportunity to explore various aspects of the participants' experiences of the Contextual Mathematics module on the Postgraduate Certificate, thereby addressing the research aims identified in section 8.2. In particular, analysis of the data permitted:

- An investigation of the Bridge21 CPD model as a means of scaffolding teachers' use of the design heuristics for the creation of mathematics learning activities.
- Examination of the experiences of teachers in the creation and implementation of such activities, paying particular attention to the barriers to, and benefits of, the approach.
- Exploration of the teachers' perceptions of their students' experiences with the activities.

These topics have been explored throughout this chapter. This discussion will explore aspects of the reflections that mirror concerns that emerged in the literature review, and will also set out the primary limitations of the exploratory study.

8.6.1 Addressing the Issues

Throughout the analysis of the teachers' reports, it was interesting to see that many of the problems associated with mathematics education that had been identified through the literature review, were also highlighted by the teachers taking part in this module. The predominantly formulaic approach to text-book questions (Boaler, 1993) was identified by one teacher as an area that the approach advocated in the CPD module, had the potential to address.

> *"These problems involved being given the function, algebraically or graphically, and all the information required to answer some fairly predictable questions. There was never any redundant information either: just enough and not too much to apply the usual procedures. While it is of course important to be familiar with the procedures, the syllabus does urge teachers to "use real-life problems as motivation for the study and application of functions. I considered that setting the students the task of creating their own quadratic curve would give them a real sense of ownership and a greater insight into the nature of quadratic functions." (DD)*

The teachers' reflections indicate a belief that this approach may go some way to address the fragmented, and de-contextualised nature that frequently pervades school mathematics (Albert & Kim, 2013; Dede, 2010a).

> *"It was useful for students to see different aspects of Maths used in one place rather than the disjointed treatment that they usually receive in a text book." (WMI)*

In addition, the use of personal devices, such a mobile phones, to generate mathematical models, contextualised the mathematics for the students, providing a relevance and meaning to the topic (Oldknow, 2009).

> *"For students, to discover that they can take their ubiquitous phone out of their pocket and create a mathematical model of an everyday event grounds Maths in the real world." (DD)*

The issues surrounding teachers' beliefs and their changing role in the classroom can also be seen to be addressed through the structured, immersive and supportive nature of the CPD program. The provision of a specific structure (Bridge21) and set of design heuristics provide the teachers with an approach that has been tested and shown to work. The teachers all seemed to have been empowered by this, and were confident to approach their classes in a different way. The results appear to have been beneficial for both teachers and students.

> *"I have worked with this particular class group on two other 21st Century Teaching and Learning Assignments previously. Their development throughout the course of the year has been astounding. The flair with which they now competently and confidently use technology to gather and analyse information, and present their findings is very impressive. This project was a thoroughly enriching experience for both the students and teachers assisting them." (DOC)*

8.6.2 Limitations

It is clear from analysis in this chapter that this approach to the creation and implementation of mathematics learning activities that has been developed in this research has the potential to address many of the issues that were highlighted in the literature review (Chapter 2). However, it is also important to identify the limitations of this portion of the research.

Firstly, the sample that is used in this exploratory study consists of teachers who have opted to be a part of the research, and who are participants on a CPD course that they have chosen to attend. It is therefore a self-selecting sample of a self-selecting sample and cannot be seen as representative.

Another point that needs to be highlighted is that the reflective pieces provided by the teachers were all submitted for assessment purposes. There is a possibility that the participants therefore emphasised the positive aspects of their experiences more than the negative. This is a limitation of the study to date, which could be overcome through interviews with participants and their students and non-participant observation of the classes. Due to time constraints however, this will be considered as future work and will not be included in this thesis.

Another drawback of this exploratory study is its small size. The analysis of fifteen teachers' reports is unlikely to permit the generation of any substantive theory. However, the consistency of the results do allow the generation of hypotheses and research questions to follow up on the initial, very promising, findings.

This is a very recent portion of the overall research presented in this thesis, and will require further expansion in order to fully examine the emerging themes. It is a very encouraging however, seeing such positive results emerging from the work with teachers. In particular, the following quote from one of the attendees on the Contextual Mathematics module highlights the teachers' understanding of the intention behind this research.

> *"The importance of 21st Century teaching and learning and indeed the B21 model can be seen by Green and Hannon who state, "In an economy driven by knowledge rather than manufacturing, employers are already valuing very different skills, such as creativity, communication, presentation skills and team building. Schools are at the frontline of change and need to think about how they can prepare young people for the future workplace" (2007, p. 15). As such a huge emphasis is being placed on STEM subjects/activities in schools, RME in conjunction with the B21 model helps to contextualise maths for our students, increasing their engagement and allowing them to use technology in a meaningful way." (MC)*

9. Discussion and Conclusion

9.1 Introduction

This dissertation describes an approach to mathematics activity design that aligns the affordances of off-the-shelf technologies with relevant mathematics pedagogy and 21st Century approaches to teaching and learning, in order to create transformative learning experiences. The potential of such activities to overcome some of the well-documented impediments relating to student engagement with, and confidence in, mathematics has been demonstrated throughout the study.

There is a wealth of literature that highlights different problem areas in mathematics education, and in the integration of technology in education, which were summarised as Problem Statements 1 and 2 (Chapter 1). Problem Statement 1 (PS1) relates to the lack of context, meaning, and connections in mathematics education when it is taught in a traditionally didactic fashion, which can lead to students becoming disengaged with the subject (Boaler, 1993; Maaß & Artigue, 2013; Noss & Hoyles, 1996; Schoenfeld, 1992, 2004; Star et al., 2014b). Problem Statement 2 (PS2) refers to the general under-exploitation of technology in secondary school mathematics, despite its potential to go some way to addressing PS1, when integrated in an appropriate manner (Conneely, Lawlor, et al., 2013; Dede, 2010b; Donnelly et al., 2011; Hoyles & Lagrange, 2010). Different levels of obstacles are proposed as contributing to these problems, and hampering efforts to address them, ranging from top-level issues relating to curriculum, assessment, and school cultures, through more technical difficulties associated with equipment and resources including CPD, down to the more individual level of teacher beliefs (Donnelly et al., 2011; Euler & Maaß, 2011; McGarr, 2009).

Stemming from these problem statements and the understanding of the barriers to their solution that emerged through the literature review in chapter 2, this research aimed to realise a number of inter-connected goals, including:

- A review of current research trends in technology-enhanced mathematics education; in particular, the creation of a system of classification that will be beneficial on an ongoing basis to keep abreast of developments in the area over time.
- The formalisation of a set of design heuristics that describe an approach to the development and implementation of activities and interventions that align relevant mathematics pedagogy with the affordances of readily available technology. Such interventions should encourage student engagement with, and confidence in mathematics, in a technology-mediated, team-based environment;

- The collection of evidence to demonstrate that student participation in activities developed in accordance with these design heuristics has the potential to increase levels of engagement with, and confidence in, mathematics, and to determine the primary aspects of the interventions that can affect such positive changes.
- The development of a set of guidelines for practitioners to facilitate the design and implementation of such activities and a reflection on their experiences.

9.2 Addressing the Aims of the research

In order to address these aims, the following sections present the development of the research reported in this thesis. Particular aspects are discussed in detail, beginning with the analysis of the results of the system of classification, and the development of a theoretical framework, which forms the basis of the development process of the design heuristics. This is followed by an explanation of the design heuristics themselves, which will address the first pair of research questions:

> ***RQ1 (a) – Design Heuristics:*** What are the desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence?
>
> ***RQ1 (b) – Guidelines:*** What are the key elements of a practitioner's guide for the creation and implementation of such interventions within the traditional school environment?

The impact that engagement with activities developed in accordance with the design heuristics has on students, will then be discussed, in answer the second set of research questions:

> ***RQ2 (a) – Effects:*** What effects on student engagement and confidence does participation in activities designed in accordance with the heuristics and implemented using the practitioner's guide have?
>
> ***RQ2 (b) – Reasons:*** What are the primary factors that cause such a change in engagement?

The experiences of teachers engaged in the creation and implementation of activities consonant with the heuristics will also be addressed, followed by a section that highlights the limitations of the study. Possibilities for future, related strands of research will be identified prior to the conclusion of the thesis.

9.2.1 A Classification of the Literature

As illustrated through the literature review in chapter 2, the common problems associated with mathematics education, the meaningful integration of technology in classrooms, and the adoption of 21st Century teaching and learning methodologies, are well documented. However, the systematic review of literature provided in chapter 3, which focuses on classifying and analysing 114 empirical studies of technology-enhanced mathematics interventions, provides evidence that current trends in the field fall short of addressing the identified problems. Analysis of the results of the classification provides evidence that a wide range of technologies are being researched, with various agendas and from differing theoretical standpoints. Although constructivist and social constructivist learning environments are favoured across the classified interventions, the use of technology for the delivery of traditional content make up a significant proportion of the studies. Analysis of the goals of the research projects reveal that most are focused on improving student performance and conceptual understanding, with a change in student attitudes also emerging as an important area of discussion.

Although the majority of interventions appear to have a common desire to create engaging environments in which the technology is used to increase the students' interest, motivation and performance, the technology is used primarily to augment traditional practice. In addition, in most of the interventions there is limited evidence to suggest continued support of the projects after the period of the research. Furthermore, there is little in-depth exploration of the reasons or motivations for any positive effects identified by the research.

Having recognised through this analysis that a number of the issues identified in the general literature review are not being addressed by the current practices of integrating technology in mathematics classrooms, this research has taken a holistic approach, focusing on combining suitable pedagogies with the affordances of readily available technology, to develop a system of design heuristics that have the potential address some of the aforementioned obstacles.

9.2.2 Combining RME and Bridge21 – A Theoretical Framework

The difficulties that students can experience with mathematics education are frequently related to didactical approaches to teaching and learning that emphasise a formal, abstract, fragmented and de-contextualised perspective of the subject, with content prized over literacy and procedure over understanding (Albert & Kim, 2013; Dede, 2010a). The teacher is generally perceived as the authority in the classroom, and mathematics is viewed as a subject made up of absolute answers (Albert & Kim, 2013; Ayinde, 2014; Boaler, 1993; Buteau & Muller, 2006; Ernest, 1997). In this kind of environment, students are prone to experiencing a lack of ownership of the mathematics and of their learning, leading to issues with motivation and engagement (Boaler, 1993; Star et al., 2014b).

In this dissertation, Realistic Mathematics Education (RME) has been selected as a mathematical pedagogy with the potential to address some of the limitations associated with the more formal and abstract approach to traditional mathematics education (Gravemeijer, 1994; Noss et al., 2009; van den Heuvel-Panhuizen, 2002). In particular, the use of context and the process of mathematization in

RME are directly relevant to introducing meaning and purpose to the subject. RME encourages student-driven discovery of the mathematics, with particular attention paid to the development of mathematical models that are necessary to solve problems that are meaningful to the students. The teacher guides and scaffolds their inquiry and the students are encouraged to collaborate with each other in their endeavour.

It is clear that an approach to task design underpinned by RME has the potential to address many of the issues associated with traditional mathematics education described above. Furthermore, Geiger et al. (2010) highlight that while the conceptualisation of a mathematical model is primarily a human activity, the use of technology can greatly enhance the abstraction of the model and the solution of the problem. The approach to the usage of technology described in this research aims to combine two of its perceived affordances. The first is the capacity for technology to be used to increase the speed and accuracy of calculations – that is, to *enhance* traditional practices. The second is the potential for technology to be used in such a way as to facilitate the development of realistic, meaningful mathematical activities that would not be easy to achieve without its integration – that is, to *transform* traditional practices (Laborde, 2002; Puentedura, 2006).

A technology-mediated approach to RME aligns easily with a commonly advocated approach to education that encourages the development of “21st Century skills”. Although there is no universally recognised definition of these skills or of the types of teaching and learning required to achieve them, they are generally related to communication and collaboration, problem-solving and creativity, and technological fluency (Dede, 2010a; Voogt & Roblin, 2012). However, many barriers have been identified as hampering the introduction of 21st Century learning in classrooms. Addressing some of these problems, such as the overarching school system, and the constraints of curriculum and assessment, are outside the scope of this research. Similarly, issues around the acquisition of appropriate technology and resources are also not dealt with in this research, other than the fact that only free software packages are used, and many of the required technologies could be personal devices provided by the students themselves.

It has also been identified that educators require adequate support and continuous professional development in order to master the necessary 21st Century skills and teaching strategies, but also to ‘unlearn’ the beliefs and assumptions that underpin the traditional industrial-model of classroom practice (Conneely, Lawlor, et al., 2013; Dede, 2010b; Voogt & Roblin, 2012). To this end, this research has made use of the structure provided by the Bridge21 pedagogic approach, which incorporates a focus on the development of the desired 21st Century skills through the use of a particular activity model. Use of the activity model provides teachers with a structured approach to the design of technology-mediated, project-based activities that engender collaboration, inquiry, problem-solving and creativity amongst students. The activity model is described in section 6.3.1.

Accordingly, in order to generate a set of design heuristics for the development of mathematics activities that have the potential to provide students with context and meaning, while at the same time providing teachers with an adequately structured approach, firmly embedded in appropriate pedagogy, a combination of RME and Bridge21 have provided a framework for the design aspect of this work. Within the RME/Bridge21 framework, this research has developed a set of design heuristics and guidelines for practitioners that provide a structured approach to the development and implementation of technology-mediated, collaborative, inquiry-based mathematical activities with the potential to engage students in the creation of meaningful and contextual mathematics. The heuristics and guidelines, along with associated rationale, are described in detail in chapters 4 and 6. Therefore, in answer to ***RQ1(a) – Design Heuristics***, the following points make up the final set of heuristics providing the desirable attributes of technology-mediated mathematics learning activities that have the potential to increase student engagement and confidence:

1. *Activities should be team-based and encourage collaboration, in accordance with a socially constructivist approach to learning.*
2. *Activities should exploit the transformative as well as the computational capabilities of the technology.*
3. *Activities should make use of a variety of technologies (digital and traditional) suited to the task, in particular, non-specialist technology such as mobile phones and digital cameras that students have to hand.*
4. *Tasks should:*
    - *involve problem-solving, investigation and sense-making,*
    - *involve guided discovery,*
    - *be situated in a meaningful/real context,*
    - *move from concrete to abstract concepts,*
    - *be open-ended but with constraints,*
    - *be cross-curricular/cross-strand,*
    - *be focused on skill development as well as on content,*
    - *have a 'low-floor' and a 'high-ceiling'.*
5. *Activities should be structured in accordance with the Bridge21 model (or a suitable structured alternative) of 21st Century Learning and activity design.*

Use of these heuristics, in conjunction with the practitioner guidelines described in section 6.8.3, address ***RQ1(b) – Guidelines***, providing the key elements of a practitioner's guide for the creation and implementation of such interventions within the traditional school environment.

In an attempt to answer the second set of research questions, this research has involved the design and testing of a number of activities that adhered to the design heuristics in order to determine their effects on student engagement with mathematics. This involved two case studies, in laboratory and natural settings, with the collection of both qualitative and quantitative data. The MTAS instrument (Pierce et al., 2007) was used to gather quantitative data relating to students' engagement and confidence, and its subscales were also used to direct the analysis of the qualitative data. The relationship between the design heuristics and their impact on students' behavioural and affective engagement, mathematical and technological confidence, and their attitude to using technology to learn mathematics as described by MTAS, were examined in great detail through the comparison of two methods of data analysis. This will be discussed in more detail in the following section.

In addition to an examination of the student data, teachers' requirements have also been explored. In order to examine teachers' experiences of working with the activity model and design heuristics, the reflections from a number of teachers who took part in a contextual mathematics module as part of a continuous professional development certificate, have been analysed. The course was an immersive experience for the teachers, in which they participated in at least one of the pre-designed activities, fully engaging with the Bridge21 activity model in a problem-solving, guided-discovery approach to the open-ended problems. For their assignment, the teachers were required to develop and implement an activity that made use of the design heuristics and the Bridge21 activity model, and then to report on their experience. Their reflections were very insightful, highlighting many of the barriers that had been discussed in the literature, but also identifying the manifold benefits of the approach. These reflections serve to corroborate the findings of the exploratory and explanatory studies, providing some evidence that the design heuristics are practical and do indeed incur the desired impact on students' experience of mathematics.

9.2.3 Impact on Students

RQ2(a) – Effects queries the effects on student engagement and confidence that participation in activities designed in accordance with the heuristics and implemented using the practitioner's guide might have. In order to address this question, an exploratory study has been undertaken, which is described in detail in Chapter 6. ***RQ2(b) – Reasons*** questions what the primary factors that cause any changes in engagement and confidence might be. This part of the second set of research question is addressed by way of an explanatory case study in Chapter 7.

9.2.3.1 Exploratory study

The first implementation of activities designed in accordance with the heuristics in an exploratory case study, provided an opportunity to broadly examine the impact of the activities on students' engagement and confidence. This first study focused on gathering quantitative data through the use of the MTAS instrument in order to assess changes in students' affective and behavioural engagement (AE and BE), mathematical and technological confidence (MC and TC), and attitude to using technology for learning mathematics (MT). The data showed statistically significant increases across all of the MTAS subscales at the $p < 0.05$ level, and a moderate effect size, which is a noteworthy result for this kind of educational research (J. Cohen, 1988; Elliot & Sammons, 2004; Lipsey et al., 2012).

Qualitative data was accumulated from students' written comments and one short interview. These data were analysed using directed content analysis, as described in chapter 6. The results that emerged portrayed a very positive picture of students' responses to the interventions. Positive references to use of technology were particularly notable across all of the subscales, reinforcing the views of many authors that the use of digital technologies in mathematics education has the capacity to open up varied routes for students to construct and engage with the subject, providing authentic contexts and meaningful problems (Drijvers et al., 2010; Geiger et al., 2010; Noss & Hoyles, 1996; Olive et al., 2010). All of the task design elements of the design heuristics were also positively associated with the MTAS codes. However, the results of the exploratory study were also used to refine the design heuristics, placing an increased emphasis on the RME view of modelling and mathematization, as a process of moving from a problem set in context, to the development of a mathematical model of a solution, and back to the original context to test the solution (Dickinson et al., 2011; Gravemeijer, 1994).

Overall, the results of the exploratory study provide an answer to ***RQ1(a) – Design Heuristics***, indicating that the activities described in chapter 4, and implemented in the Bridge21 setting, have the potential to effect significantly positive changes in student engagement and confidence. However, in order to form a practical model, implementations in authentic school settings were required, as well as an in-depth examination of the primary factors that cause any changes in engagement and confidence. The explanatory case study described in Chapter 7 addresses these issues.

9.2.3.2 Explanatory study

The purpose of the explanatory study was twofold. This research is primarily interested in the development of design heuristics that can be used by teachers for the creation of mathematics activities that can increase student engagement and confidence (that is, ***RQ1(a) – Design Heuristics***, and ***RQ2(a) – Effects***). In order to achieve these aims, the activities required evaluation in authentic settings, to identify any potential barriers that were not evident in the laboratory environment of Bridge21. In addition, an in-depth analysis of the reasons behind any changes in students' engagement and confidence is important in order to pinpoint how and why, as well as if, the design heuristics were having an impact (***RQ2(b) - Reasons***).

Results of the quantitative data were less significant in the second case study: although all of the subscales showed positive change, only affective engagement (AE) and attitude to using technology for learning mathematics (MT) achieved statistically significant differences between pre- and post-tests. A number of reasons are possible for the less positive statistical results in the explanatory study. In particular, the change in context from an out-of-school setting, to a traditional school setting is likely to have had a significant effect on students' attitudes and participation in the interventions and could be responsible for the less conclusive quantitative data. However, the research design for the explanatory study was more focused on the qualitative data, which largely confirmed the positive aspects of the quantitative data and endorsed a very optimistic picture of students' experiences with the activities and the effects that their participation in the interventions had on their engagement with mathematics (AE and BE) and their confidence in the subject (MC).

Analysis of the data in the explanatory study permitted exploration of the relationships between the positive aspects of the MTAS subcategories and the design heuristics, which led to the identification of the activity attributes that appear to have the most significantly constructive impact on engagement and confidence. The comparison of the results from the directed content analysis and the constant comparative analysis enabled conjectures to be made relating some of the possible rationales for the associations between the heuristics and the positive effects, thereby addressing ***RQ2(b) - Reasons***.

Table 9.1 broadly summarises what has emerged from the analysis of the data in the explanatory study. The realistic (in the RME sense) aspect of the task design has had a powerfully positive impact on the students' mathematical experiences throughout the interventions. Because the activities were based in contexts that were of interest to the participants, they were motivated to engage – they wanted to understand the mathematics in order to be able to solve the problems or win the challenges presented to them. They were interested in the mathematics, as it had a meaning and a purpose for them. They were guided to discover connections within the discipline and with other fields, which led to increased conceptual understanding of the topics. The use of technology empowered them to explore, model and visualise the mathematics, discovering multiple representations and permitting engagement with complex, real mathematics. Collaboration and teamwork encouraged peer learning, ensuring that the students learnt from each other as well as from the teacher.

Table 9.1: Effects of Design Heuristics

Design Heuristics Qualitative Codes	Impact	Positive effects
Meaningful/Realistic Context, Practical	Curiosity, Fun and Interest => Desire for understanding	AE, BE, MC
Meaningful tasks, Intertwined strands	Interest => Desire for understanding	AE, BE
Guided Discovery, problem-solving, open-ended	Interest, Curiosity, Ownership, Connections => Desire for understanding,	AE, BE, MC
Use of technology	Meaningful tasks, Connections, Practical => Desire for understanding	MC, TC, MT, AE, BE
Teams	Collaboration, peer learning, interest, Ownership => Desire for understanding	BE, MC

It is clear from these results that the approach to activity design described in this study has a largely positive effect on participating students (***RQ2(a) - Effects***), and has the potential to address many of the problems identified in the literature review, both in relation to mathematics education in general, and to the integration of technology within the subject. In particular, it emphasises the use of meaningful context, open-ended problems and guided discovery, addressing problems associated with the fragmented, de-contextualised approach frequently associated with traditional mathematics education, along with its tendency towards a formulaic approach to problems (Boaler, 1993; Dede, 2010a; Ernest, 1997; Garofalo, 1989; Noss & Hoyles, 1996; Olive et al., 2010; Schoenfeld, 2004).

The explanations of the positive experiences of the students (***RQ2(b) - Reasons***) identified in this research are also be backed up by evidence from the literature review. Students develop confidence in their mathematical abilities and increased levels of conceptual understanding through the discovery and modelling of mathematics that is grounded in contexts that are engaging and of interest to them (Dickinson et al., 2011; Euler & Maaß, 2011; Gravemeijer, 1994; Maaß & Artigue, 2013; Star et al., 2014b). The use of technology and the collaborative approach, combined with the elements of task design described above, have all facilitated the generation of truly meaningful contexts, and have returned the agency to create and understand mathematics to the students (Drijvers et al., 2010; Geiger et al., 2010; Noss & Hoyles, 1996; Olive et al., 2010).

9.2.4 Impact on Teachers

Some of the primary difficulties with the introduction of 21st Century pedagogies and with the integration of technology in schools that were identified in the literature review, relate to teachers' need for effective CPD, a structured approach to classroom management, and continuous, ongoing support within the school environment (Conneely, Lawlor, et al., 2013; Dede, 2010a; Euler & Maaß, 2011; Means, 2010; Voogt & Roblin, 2012). Teachers' beliefs about the nature of mathematics, and regarding their role in the classroom, are also recognised as potential barriers to change (Buteau & Muller, 2006; Dede, 2010a; Euler & Maaß, 2011; Laborde, 2002; Schoenfeld, 1992). In addition, the theory-research gap is highlighted as an issue regarding the propagation of practices that are developed as one-off research-led interventions (Boaler, 2008; Geiger et al., 2010; Maaß & Artigue, 2013).

Through the development of the design heuristics, which incorporate the activity structure provided by the Bridge21 model, many of these issues have been addressed. In addition, a set of guidelines for practicing teachers has been developed in collaboration with a number of teachers who participated in workshops conducted in Bridge21 (chapter 6). These workshops provided teachers with an opportunity to engage with the activities themselves, experiencing them from the perspective of the students. They then worked together in order to address some of the potential barriers to the implementation of such activities in traditional classrooms, and to develop guidelines for their development. This was an iterative process, in which the teachers collaboratively planned, implemented, observed, reflected on, and refined a lesson that adhered to the heuristics, thus consolidating ***RQ1(a) – Design Heuristics*** and addressing ***RQ1(b) - Guidelines***.

The process of refinement of the heuristics and guidelines with the students and teachers culminated in the development of a Contextual Mathematics module on a Postgraduate Certificate in 21st Century Teaching and Learning administered by the School of Education in Trinity College Dublin. To date the module has been run twice, with a total of 22 teachers participating. For their assignment, participants were required to design and implement a mathematics activity that adhered to the heuristics and the Bridge21 activity model. The results of their experiences are discussed in detail in chapter 8, and are summarised below.

The teachers' reflections confirm a number of the systemic barriers that had been identified in the literature review. These are mainly focused on the difficulties associated with the short class periods (Dede, 2010a; Voogt & Pelgrum, 2005; Wijers et al., 2008), and technical issues, both at the general level of availability of resources, and at the more personal level relating to their unfamiliarity with the tools themselves (Dede, 2010b; Donnelly et al., 2011; Euler & Maaß, 2011).

The benefits of the experience identified by the teachers far outweigh any of the difficulties. Analysis of the benefits identify two primary categories, relating to the teachers themselves, and to the students. The subcategories associated with the teachers relate to changes in their fundamental beliefs about mathematics education, the redefinition of their role from instructor to facilitator, and their experiences as co-learners in the classroom. Each of these aspects are viewed as positive transformations, with beneficial impacts for the teachers and their pupils, and served to corroborate the findings that address ***RQ2(a) – Effects***, and ***RQ2(b) - Reasons***.

The advantages for the students can be further deconstructed into three subcategories relating positive effects of specific aspects of the task design, the development of skills, and affective outcomes. The team-based aspect of the task design can be related to the development of collaborative skills, and peer-learning. The meaningful/contextual nature of the tasks are recognised as leading to an increase in student engagement and conceptual understanding. The student-led, open-ended design is acknowledged as being related to an increase in confidence, a sense of ownership and responsibility, and increased engagement and confidence. In addition to collaborative and communication skills, creativity, flexibility, and problem-solving skills all appear to be motivated through participation in the activities. Students' technological competence is also seen to increase, as are their organisation, reflection and presentation skills. An increase in students' engagement and conceptual understanding is identified in all of the analysed reflections.

The teachers' experiences with the design heuristics and the CPD provided through the contextual mathematics module appears to have been largely successful. Analysis of the reflections have corroborated the results of analysis of the student data that has been presented in this thesis, strengthening the claims that have been made relating to each of the research questions. In addition, the model of CPD – involving an immersive experience of the activities, followed by the use of the heuristics for the creation and implementation of activities in their own classrooms and culminating in a reflection – appears to have the capacity to address at least some of the difficulties with changing pedagogic practices that are identified in the literature review. Teachers' own experiences in the CPD module gave them confidence in the benefits of the approach, the heuristics and the Bridge21 activity model appears to have provided the requisite level of structure, and the use of free software and off-the-shelf technology allowed for ease of experimentation. The resulting shift in teachers' beliefs about their role in the classroom and the nature of mathematics education that is identified in the analysis provides evidence that this approach has the potential to alleviate some of the most problematic barriers to change.

9.2.5 Limitations

Case study research in educational settings is a messy process involving diverse, often unidentified factors that can influence any outcomes. Although rigour and validity are striven for throughout the research described in this thesis, there are certain limitations and confounding factors that need to be taken into account. Probably the most significant limitation to consider relates to the opportunistic sampling methods that have been used in each of the case studies.

In the exploratory case study described in chapter 6, the students who attended the interventions were volunteers with previous experience of activities in Bridge21. While this allowed the research to focus on their experience with the mathematical activities, it is likely that they were already positively disposed to the methodology, which may have impacted on the results.

The choice of schools for the explanatory study was driven by availability and accessibility, and the number of class groups that took part in this study is modest. In addition, the student cohorts, the duration of the interventions, and the specific activities involved, were not consistent across the groups. However the positive results of triangulation of the different forms of data gathered in the interventions go some way to address concerns in this regard. Similarly, the sole use of transition year students in the case studies was linked to accessibility – as it is not an exam-focused year, students, teachers, and parents tend to be more flexible - and, as discussed in chapter 7, this may have impacted on the results. However, the teacher designed activities (Appendix 8.B) were conducted with students across four different year groups (ranging in age from 12 – 16), providing evidence that activities designed in this way are also suitable for implementation with standard student cohorts.

In terms of the CPD, the teachers involved in the course made up a primarily self-selecting sample, and, as such, are likely to have been positively disposed to engaging in innovative and transformative practices. However, this does not account for the level of personal development evident in a number of the reflections related to changes in fundamental beliefs, nor does it account for the improvements that the teachers recognised in their students levels of engagement, understanding and skill development.

The limitations relating to sampling that have been identified in this section are common in case study research. They do not however, suggest that the findings are not suitable for generalisation, or that they do not form a valuable contribution to the field. In particular, the successful expansion of the context from Bridge21 laboratory school, to researcher-led school settings, to teacher-led school settings, with similar findings being propagated throughout, provides evidence of the generalisability of the approach.

9.3 Additional Research

In addition to addressing the initial aims and research questions of this dissertation proposed in Chapter 1, this thesis has demonstrated the value of a particular model of CPD. The Bridge21 model of CPD is influenced by the cyclical approach advocated by Japanese Lesson Study. This involves iterating through a cycle of goal setting, planning, teaching and observation, reflection, and revision. Throughout this process, the teachers are encouraged to develop communities of practice, both within and outside their schools. This supports a critical examination of practice, with the ultimate goal of improving student experiences, optimising lessons and becoming more efficient teachers (Maaß & Artigue, 2013; Takahashi & Yoshida, 2004).

Teachers enrolled in the contextual mathematics CPD module actively engaged in immersive and authentic activities, which enabled them to appreciate the power of the approach through personal experience. Throughout the process, the participants had access to resources, practical activity designs and a community of support that Donnelly et al. (2011) highlight as necessary conditions to motivate change amongst teachers. Furthermore, the teachers were required to become co-researchers and to report and reflect on their practice, further facilitating the dissemination of the research and addressing the practice-research gap (Boaler, 2008; Maaß & Artigue, 2013; Pimm & Johnston-Wilder, 2005).

9.4 Future Research

The overarching aim of the research presented in this dissertation is to formalise a set of design heuristics that describe an approach to the development and implementation of activities and interventions that align relevant mathematics pedagogy with the affordances of readily available technology. The goal of activities developed in this manner is to encourage student engagement with and confidence in mathematics, in a technology-mediated, team-based environment.

The research has progressed from exploratory to explanatory case studies that analyse students' experiences with activities designed in this way. Although some limitations have been identified, the results of the research to date have been very positive. In order to progress the research further, teachers' experiences with the design heuristics have been investigated. Once again, while limitations have been recognised, the results remain overwhelmingly positive.

In order to further extend this research, a number of potential routes could be taken. The most obvious would be to run more in-school interventions in order to determine whether the results are sustainable. In particular, it would be of interest to identify whether the findings can be replicated – both for repeated use with similar students, and for greater numbers of classes following syllabi leading to state examinations. In the limitations section (9.2.5), it was acknowledged that the differences between the interventions, both in terms of the student cohorts, and duration, were not considered in the analysis of the data. It would be very interesting to delve further into the data in order to draw out differences between gender and socio-economic groupings, as well as the impact of the different activities and implementations.

The contextual aspect of the task design emerged as a very important motivator for students. The design of more contextual activities, incorporating other curricular areas, would be an interesting extension to the work presented in this thesis.

Regarding the work with teachers, a number of research avenues have been identified.

- The use of technology and the flipped classroom for the delivery of content, is an area of research that emerged through the process of classification in chapter 3. Combining this methodology with the approach described in this thesis may have the potential to address some of the barriers to its implementation associated with the constraints of curriculum.
- The power of collaboration that is evident amongst the students engaged in this research was also an important factor from at the teacher level, for the success of the CPD. Further development and maintenance of communities of practice will be fundamental for the success and propagation of this method.
- One area that the design heuristics and the Bridge21 model do not adequately address is assessment. A number of assessment rubrics have been developed by the teachers engaged in the CPD process, and these will be evaluated and developed as the research progresses.

These three areas (flipped classroom, communities of practice, and assessment) have been proposed as particular areas of focus in a transnational funding proposal that has recently been accepted by the European Commission as a Key Action 2 project,[11] which will be managed by the author.

9.5 Conclusion

This thesis has formalised and tested design heuristics that align the pedagogies of Realistic Mathematics Education and the Bridge21 model of 21st Century learning with the affordances of readily available technology, for the creation of mathematics learning activities that have the potential to increase students' engagement with and confidence in the subject. In order to achieve this, a number activities were designed in accordance with the heuristics, which were then tested in experimental and authentic settings. Participation in the activities has been shown to have significant

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[11] https://eacea.ec.europa.eu/erasmus-plus/actions/key-action-2-cooperation-for-innovation-and-exchange-good-practices_en

positive effects on students' engagement, motivation and confidence in mathematics as well as their development of skills such as creativity, communication, reflection and problem-solving, technological competence and conceptual understanding.

The testing of the design heuristics and practitioner's guide by teachers in their own classrooms, has further demonstrated the potential of the approach to provide a meaningful environment for students to explore and create their own mathematics. Activities designed in this way have been shown to create an environment in which the students are motivated to engage with the subject through a desire to understand and solve problems that having meaning to them.

Further to the effect of the interventions on the students, the approach to teachers' CPD that has emerged throughout this research has proven successful in addressing many of the barriers to the integration of technology and the implementation of 21st Century pedagogies that are identified in the literature review.

In addition to the design heuristics, the development of a system of classification of literature relating to empirical, technology-meditated mathematics research provides a tool to identify whether the issues that are identified in more general research are being addressed by ongoing studies. This classification tool has the capacity to be of benefit on an ongoing basis, highlighting fields of research that would benefit from further investigation.

While the design heuristics, the model of CPD and the system of classification are all still developing, this thesis provides a strong foundation for continued research in the area.

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Appendix 3.A List of Classified Papers

Paper Classification	Authors	Year	Source
Conference Proceedings	Amado, N.;Carreira, S	2015	CERME9
Conference Proceedings	Arnau, D.;González-Calero, A.;Arevalillo-Herráez, M.	2015	CERME9
Conference Proceedings	Avraamidou, A.	2015	CERME9
Conference Proceedings	Bairral, M.;Arzarello, F.	2015	CERME9
Conference Proceedings	Biton, Y.;Hershkovitz, S.;Hoch, M.	2015	CERME9
Conference Proceedings	Borba, R.;Azevedo, J.;Barreto, F.	2015	CERME9
Conference Proceedings	Geraniou, Eirini;Mavrikis, M.	2015	CERME9
Conference Proceedings	Haddif, G. N.;Yerushalmy, M.	2015	CERME9
Conference Proceedings	Jacinto, H.;Carreira, S.	2015	CERME9
Conference Proceedings	Misfeldt, Morten;Ejsing-Dunn, Stine	2015	CERME9
Conference Proceedings	Pinkernell, G.	2015	CERME9
Conference Proceedings	Promodrou, T.;Lavicza, Z.;Koren, B.	2015	CERME9
Conference Proceedings	Schumacher, S.;Roth, J.	2015	CERME9
Conference Proceedings	Soldano, C.;Arzarello, F.;Robutti, O.	2015	CERME9
Conference Proceedings	Swidan, O.	2015	CERME9
Conference Proceedings	Triantagyllou, E.;Timcenko, O.	2015	CERME9
Journal Article	Arbain, Nazihatulhasanah;Shukor, Nurbiha A	2015	Procedia - Social and Behavioral Sciences
Journal Article	Bhagat, Kaushal Kumar;Chang, Chun-Yen	2015	Eurasia Journal of Mathematics, Science & Technology Education
Journal Article	Chang, Mido;Evans, Michael A;Kim, Sunha;Norton, Anderson;Samur, Yavuz	2015	Educational Media International
Journal Article	Coelho, Artur;Cabrita, Isabel	2015	Journal of the European Teacher Education Network
Journal Article	Granberg, Carina;Olsson, Jan	2015	The Journal of Mathematical Behavior
Journal Article	Gunbas, N	2015	The Journal of Computer Assisted Learning
Journal Article	Ipek, Jale;Orhan, Sevil;Akbasoglu, Ruya;Kaplan, Serkan	2015	Global Journal of Information Technology
Journal Article	MacLeod, Cheri	2015	Learning and Teaching in Higher Education- Gulf Perspectives

Journal Article	Moeller, Korbinian;Fischer, Ursula;Nuerk, Hans-Christoph;Cress, Ulrike	2015	Computers in Human Behavior
Journal Article	Ochkov, Valery F;Bogomolova, Elena P	2015	Journal of Humanistic Mathematics
Journal Article	Regec, Vojtech	2015	Procedia - Social and Behavioral Sciences
Journal Article	Rodríguez, Georgina;Spiegel, Alejandro;Salviolo, Melina;Peña, Alicia	2015	Procedia - Social and Behavioral Sciences
Journal Article	Takači, Djurdjica;Stankov, Gordana;Milanovic, Ivana	2015	Computers & Education
Journal Article	Taleb, Zahra;Ahmadi, Amineh;Musavi, Maryam	2015	Procedia - Social and Behavioral Sciences
Journal Article	Tan, Choo-Kim;Tan, Choo-Peng	2015	Computers & Education
Journal Article	Voskoglou, Michael Gr	2015	American Journal of Educational Research
Conference Proceedings	Roorda, Gerrit;Vos, Pauline;Drijvers, Paul;Goedhart, Martin	2014	PME 2014
Conference Proceedings	De Vita, Mauro;Verschaffel, Lieven;Elen, Jan	2014	ICEMST 2014
Conference Proceedings	Muir, T;Chick, H	2014	MERGA 2014
Electronic Article	Star, Jon Robert;Chen, Jason A;Taylor, Megan W;Durkin, Kelley;Dede, Christopher J;Chao, Theodore	2014	International Journal of STEM Education
Journal Article	Ayinde, Olatoye Mukaila	2014	Middle Eastern & African Journal of Educational Research
Journal Article	Contreras, José	2014	Journal of Mathematics Education at teachers college
Journal Article	Ersoy, Mehmet;Akbulut, Yavuz	2014	Computers & Education
Journal Article	Fakomogbon, Michael Ayodele;Omiola, Matthew Adetayo;Awoyemi, Samson Oyebode;Mohammed, Ridwan Enuwa	2014	European Scientific Journal
Journal Article	Hampton, Charles Edgar	2014	Instructional Technology Education Specialist Research Papers
Journal Article	Heo, Hae Ja;Choi, Min Ryeol	2014	Advanced Science and Technology Letters
Journal Article	Katmada, Aikaterini;Mavridis, Apostolos;Tsiatsos, Thrasyvoulos	2014	Electronic Journal of e-Learning
Journal Article	Ke, Fengfeng	2014	Computers & Education
Journal Article	Ku, Oskar;Chen, Sherry Y;Wu, Denise H;Lao, Andrew CC;Chan, Tak-Wai	2014	Educational Technology & Society
Journal Article	Kuiper, Els;de Pater-Sneep, Martie	2014	Mathematics Education Research Journal
Journal Article	Loc, N. P.	2014	Journal Of International Academic Research For Multidisciplinary
Journal Article	Sommerauer, Peter;Müller, Oliver	2014	Computers & Education

Journal Article	Star, Jon R;Chen, Jason A;Taylor, Megan W;Durkin, Kelley;Dede, Chris;Chao, Theodore	2014	International Journal of STEM Education
Journal Article	Tajudin, Nor'ain Mohd;Idris, Noraini	2014	Asia Pacific Journal of Multidisciplinary Research
Journal Article	Turan, Burcu	2014	Procedia - Social and Behavioral Sciences
Report	Böhmer, Bianca;Burns, Justine;Crowley, Luke	2014	Report
Report	Lentin, Jamie;Jonsdottir, Anna H;Stern, David;Mokua, Victoria;Stefansson, Gunnar	2014	Report
Conference Proceedings	Sollervall, Håkan	2013	CERME8
Conference Paper	Persson, PerEskil	2013	CERME8
Conference Proceedings	Balgalmis, Esra;Shafer, Kathryn G;Cakiroglu, Erdinc	2013	CERME8
Conference Proceedings	Fredriksen, Helge	2013	CERME8
Conference Proceedings	Grønbæk, Niels;	2013	CERME8
Conference Proceedings	Joubert, Marie	2013	CERME8
Conference Proceedings	Kaya, G.;Akcakin, Veysel;Bulut, Mehmet; (2013) - 374	2013	CERME8
Conference Proceedings	Kilic, Hulya	2013	CERME8
Conference Proceedings	Kynigos, C.;Moustaki, F	2013	CERME8
Conference Proceedings	Lagrange, J.B.;Psycharis, G.	2013	CERME8
Conference Proceedings	Mackrell, K;Maschietto, M;Soury-Lavergne, S	2013	CERME8
Conference Proceedings	Misfeldt, Morten	2013	CERME8
Conference Proceedings	Pilet, Julia;Chenevotot, Françoise;Grugeon, Brigitte;El, Naïma	2013	CERME8
Conference Proceedings	Rieß, Michael;Greefrath, Gilbert	2013	CERME8
Conference Proceedings	Robová, Jarmila;Vondrová, Naďa	2013	CERME8
Conference Proceedings	Weigand, Hans-Georg	2013	CERME8
Journal Article	Applebaum, Mark;Freiman, Viktor	2013	European Journal of Science and Mathematics Education
Journal Article	Awang, Tuan Salwani;Zakaria, Effandi	2013	Procedia - Social and Behavioral Sciences
Journal Article	Ozdamli, Fezile;Karabey, Dervis;Nizamoglu, Besime	2013	Procedia - Social and Behavioral Sciences
Journal Article	Zakaria, Effandi;Daud, Md Yusoff	2013	Asian Journal Of Management Sciences & Education
Conference Proceedings	Clark-Wilson, A.	2013	CERME 8

Journal Article	Arzarello, F.;Ferrara, F.;Robutti, O.	2012	Teaching Mathematics and Its Applications
Journal Article	Bokhove, C.;Drijvers, P.	2012	Computers & Education
Journal Article	Kay, Robin;Kletskin, Ilona	2012	Computers & Education
Journal Article	Lai, Kevin;White, Tobin	2012	Computers & Education
Journal Article	Noss, Richard;Poulovassilis, Alexandra;Geraniou, Eirini;Gutierrez-Santos, Sergio;Hoyles, Celia;Kahn, Ken;Magoulas, George D.;Mavrikis, Manolis	2012	Computers & Education
Journal Article	Tan, C.K.	2012	Computers & Education
Journal Article	Aydin, H.;Monaghan, J.	2011	Teaching Mathematics and Its Applications
Journal Article	Chan, C.K.K.;Chan, Y.Y.	2011	Computers & Education
Journal Article	Cheng, C.K.;Paré, D.E.;Collimore, L.M.;Joordens, S.	2011	Computers & Education
Journal Article	Dogan, M.;Içel, R.	2011	International Journal of Human Sciences
Journal Article	Erbas, Ayhan Kursat;Yenmez, Arzu Aydogan	2011	Computers & Education
Journal Article	Hitt, F.; (2011) - 107	2011	International Journal of Mathematical Education in Science and Technology
Journal Article	Leng, N. W.	2011	International Journal of Mathematical Education in Science and Technology
Journal Article	McCulloch, A	2011	The Journal of Mathematical Behavior
Journal Article	Naftaliev, Elena;Yerushalmy, Michal	2011	Computers & Education
Journal Article	Shirley, M.L.;Irving, K.E.;Sanalan, V.A.;Pape, S.J.;Owens, D.T.	2011	International Journal of Science and Mathematics Education
Journal Article	Sokolowski, A.;Yalvac, B.;Loving, C.	2011	International Journal of Mathematical Education in Science and Technology
Journal Article	Tan, Choo-Kim;Harji, Madhubala Bava;Lau, Siong-Hoe	2011	Computers & Education
Journal Article	Topcu, A.	2011	International Journal of Mathematical Education in Science and Technology
Journal Article	Vos, N.;van der Meijden, H.;Denessen, E.	2011	Computers & Education
Reference	Euler, Manfred	2011	
Journal Article	Berger, M.	2010	Computers & Education
Journal Article	Fu, Meiyu;Li, Zunbai;Fu, Yuanming	2010	British Journal of Educational Technology
Journal Article	Geiger, V.;Faragher, R.;Goos, M.	2010	Mathematics Education Research Journal
Journal Article	Kebritchi, Mansureh;Hirumi, Atsusi;Bai, Haiyan	2010	Computers & Education

Journal Article	Lagrange, J.B.	2010	International Journal of Mathematical Education in Science and Technology
Journal Article	Lavy, Ilana;Shriki, Atara	2010	The Journal of Mathematical Behavior
Journal Article	Lazakidou, G.;Retalis, S.	2010	Computers & Education
Journal Article	Lee, J. A.;McDougall, D. E.	2010	International Journal of Mathematical Education in Science and Technology
Journal Article	Tangney, B.;Weber, S.;O'Hanlon, P.;Knowles, D.;Munnelly, J.;Salkham, A.;Watson, R.;Jennings, K.	2010	MLearn2010
Journal Article	Wang, Q.	2010	Computers & Education
Journal Article	Wright, David	2010	Technology, Pedagogy and Education
Journal Article	Lassak, Marshall	2009	International Journal of Mathematical Education in Science and Technology
Journal Article	Noss, R.;Hoyles, C.;Mavrikis, M.;Geraniou, E.;Gutierrez-Santos, S.;Pearce, D.	2009	ZDM Mathematics Education
Journal Article	Oldknow, A.	2009	Teaching Mathematics and Its Applications
Journal Article	Ruthven, K.;Deaney, R.;Hennessy, S.	2009	Educational Studies in Mathematics
Journal Article	Ruthven, Kenneth;Hennessy, Sara;Deaney, Rosemary	2008	Computers & Education
Journal Article	Santos-Trigo, Manuel;Cristóbal-Escalante, César	2008	Journal of Computers in Mathematics and Science Teaching
Conference Proceedings	Kieran, C.;Drijvers, P.	2006	ICMI 17
Journal Article	Laborde, Colette	2002	International Journal of Computers for Mathematical Learning
Journal Article	Noss, Richard;Healy, Lulu;Hoyles, Celia	1997	Educational Studies in Mathematics

Appendix 5.A MTAS Questionnaire

No.	Statement	Strongly Disagree	Disagree	Not Sure	Agree	Strongly Agree
1	I concentrate hard in mathematics					
2	I try to answer questions the teacher asks					
3	If I make mistakes, I work until I have corrected them.					
4	If I can't do a problem, I keep trying different ideas.					
5	I am good at using computers					
6	I am good at using devices like games consoles, iPods, smartphones etc.					
7	I can fix a lot of computer problems					
8	I can master any computer programs or apps needed for school					
9	I have a mathematical mind					
10	I can get good results in mathematics					
11	I know I can handle difficulties in mathematics					
12	I am confident with mathematics					
13	I am interested to learn new things in mathematics					
14	In mathematics you get rewards for your effort					
15	Learning mathematics is enjoyable					
16	I get a sense of satisfaction when I solve mathematics problems					
17	I like using Technology for learning mathematics					
18	Using Technology tools in mathematics is worth the extra effort					
19	Mathematics is more interesting when using Technology tools.					
20	Technology tools help me learn mathematics better					

Appendix 5.B Observational Protocol Sheet - Sample

Date: 23/01/2014 Time: 14:00 – 15:45

Lesson Topic: Plinko and Probability

Time	Group	Activity	Behaviour
14.07 – 14.17	Back right	• One very strong personality, but all contributing • Whole group very engaged in discussing the odds – all members understand the process; "how about this..."all involved; lots of calculating • Students are hypothesising, interacting around the content, engaging in group discussion around the concepts of probability • When asked for explanation, lots of hand gesturing in an explanatory way – excellent! • All students explaining their idea for the game – deciding between low and high odds ○ "The stats are here!" ○ "We get more money if we do it my way!" ○ "Just think about it, we make more money!" • Voting to decide which idea to use	• Nodding • Calculating • Voting • Gesturing • Laughing • Raising voices • Calculating on computer • Whole group discussion • Interactions with each other, teacher and mentor • Lots of pointing to the board
14.19 – 14.29	Centre Group	• One team member on PC, doing nothing. • Looking at phone – do they understand? • Very little cohesion of team • Whole team have gone • Aibhín talking to team, explaining procedure. Aibhín doing full explanation. • Not even looking for an answer • Move from looking at the (Plinko) board to painting the board.	• Tapping pens on table; • playing on keyboard; • looking at other groups; • heads down on desk; • low levels of interaction; • hands propping up face; • getting up and walking around; • low levels of smiling; • Using phones.
14.30 – 14.40	Back Left	• Majority of team members on task • Need Aibhín to guide process • Aibhín checking in – fun interaction ○ "Ah lads come on, enough messing now" ○ "I told him that, but he wouldn't listen" ○ "This is a sinking ship" ○ "We can salvage this really quickly" • Get into roles and make a pitch • Paint board and Create table of odds	• Laughing; • pointing in group; • animated; • concentration on task; • pride in work; • having fun; • mix of interaction

Appendix 6.A Coding Schema for Exploratory Study

Table 6.A.1: Design Heuristics Categorization Matrix

Category	Sub category	Keywords	Examples	Operationalisation
Pilot_Team	Team_Positive	Groups, Ask, Asking, Team, Teamwork, Pairs, Pull together.	N/A	Segments that refer to working in teams or collaboration with others.
	Team_Negative		N/A	
Pilot_ Technology	Tech_ Transformative	Different, Exciting, Easier, Saves time, Make concrete, Involving, Understand.	What an interesting day! Playing with catapults was enjoyable and using technology was a better way of learning and teaching maths.	Segments that refer to the students' use of the technologies in the class and how the use of technology affected the participants' experiences with the task.
	Tech_ Computational		The simulations were very fun and easy to use. I found myself trying out and exploring lots of different sums. Very fun.	
	Tech_Variety		Very cool. I'd say the simulation website and wolfram alpha can be very useful, more than Google as it gives options and different solutions to quite possibly everything!	

Category	Sub category	Keywords	Examples	Operationalisation
Pilot_Task Design	Task_RME-real	Realistic, Real-life, How, Why, Practical, Fun, Outside the box, Independent, Progressing, Sense making, Hints, Other areas, Problem solving, What it's for	Because it's a real situation.	Segments that refer to aspects of the task design that impacted on participants' experience of the activities.
	Task_problem-solving		It's cool because we had to try our best to resolve the problem	
	Task_practical		Yeah, because we learnt much more. Because we learned by what we did.	
	Task_open-ended		Then you think about it and you find the beginning and then you can continue.	
	Task_low-floor/high-ceiling		This programme is open to everybody – good at maths, bad at maths	
	Task_guided discovery		Doing it in this way, creating it yourself, by analysing your own creations... it definitely gave a better understanding of it.	
	Task_cross-strand		You were not forced to have a whole fact file and understanding of maths	
Pilot_ Bridge21 Activity Structure	Bridge21	Warm-up Plan, Create, Reflect, Present, Mentor, Timing.	No, but it's better, because what we did today, we had, but it's difficult to do what we did today every day because we had more teachers, eh like persons, who helped us	Segments that refer to aspects of the Bridge21 Activity Structure that impacted on participants' experience of the activities.
	Bridge21_ presentation		I liked the overall experiment, the way we presented it and worked at it.	

Table 6.A.2: Traditional Approach Categorization Matrix

Category	Examples	Operationalisation
Use of Technology	Well, we already have some classes using GeoGebra and it's not such a big deal.	Segments that refer to the use of technology in the students' usual mathematics class.
Task Design	We listen to the teacher talk about, like, boring things and then we just take them down.	Segments that refer to aspects of the task design that impacted on participants' experience of the activities.
Structure	He goes so fast that you can't like follow or understand.	Segments that refer to aspects of the structure of their usual mathematics class that impact on students' experiences.

Appendix 7.A Coding Schema for Explanatory Study

Table 7.A.1: MTAS Coding Scheme

Category	Sub category	Keywords	Examples	Operationalisation
AE (Affective Engagement)	**AE_TradPos** (Traditional Approach - positive)	Enjoyable, Reward, Satisfaction, Interesting, Fun, Like.	The questions with words are better. I like the questions with words better than maths, I mean better than ones with just like numbers because it's much easier	Segments in which the students refer to how they feel about the subject.
	AE_TradNeg (Traditional Approach - negative)		because like you're really just doing maths all week, taking notes off the board, the teacher sets you some problems to do, and you're doing that like last class on a Friday, and it's just, you don't have any interest. You don't want to be doing it.	
	AE_MLAsPos (Contextual Maths Approach - positive)		But having to go I have to use... I mean the software was the push for me, using the computers was really handy, because it meant that I could understand it and have fun with it	
	AE_MLAsNeg (Contextual Maths Approach - negative)		People just like got pushed aside and lost motivation to do anything and were just a bit bored.	
BE (Behavioural Engagement)	**BE_TradPos** (Traditional Approach - positive)	Work, Try, Answer, Learn, Do.	It's good if you have people in the class who also work as well.	Segments that relate to how students behave in learning the subject
	BE_TradNeg (Traditional Approach - negative)		Because you know when we're doing maths, we don't really understand it in school. You know, we just learn a procedure, or a formula, and get an answer	

Category	Sub category	Keywords	Examples	Operationalisation
	BE_MLAsPos (Contextual Maths Approach - positive)		And you know that kind of breakthrough moment when we actually got all the way there? That's because we were watching back and looking at the techniques that we had used, we were looking at two different techniques and what was wrong and what was right and then we tried to use that to manipulate the way we were doing it before to get where we want to go, and we had to do it by ourselves	
	BE_MLAsNeg (Contextual Maths Approach - negative)		But my whole team was like "no, I'm not doing anything" and I was stuck going - okay, I guess I'll do it myself then.	
MC (Mathematics Confidence)	**MC_TradPos** (Traditional Approach - positive)	Confident, I can, Understand, Figure it out.	If you actually just fly through the book and get the work done, it's not that bad.	Segments that relate to the student's perception of their ability to achieve good results in mathematics and their confidence in handling difficulties in the subject.
	MC_TradNeg (Traditional Approach - negative)		you're building up for a test at the end of the week or something, and then you could not understand it, and then you do bad, and then it comes to the Junior Cert or Leaving Cert, and then you do bad in that.	
	MC_MLAsPos (Contextual Maths Approach - positive)		It's like you're adaptive to it. It's something you've never seen before and you get someone just to show you how to do it and then... you might not be able to do it yourself, but you'll be able to figure out a way to do it and you'll eventually get there	
	MC_MLAsNeg (Contextual Maths Approach - negative)		Ah no, I don't understand how to do it, I just trust the technology	

Category	Sub category	Keywords	Examples	Operationalisation
TC (Confidence With Technology)	**TC_TradPos** (Traditional Approach - positive)	I am good at, I can fix, I can master.	N/A	Segments that relate to: Students' confidence in the use of computers and in their ability to master procedures required of them; Increased student confidence in their answers when they are supported by computers; Confidence in the use of a broad range of technology.
	TC_TradNeg (Traditional Approach - negative)		...all we used were calculators and online calculators like	
	TC_MLAsPos (Contextual Maths Approach - positive)		Well, it kind of made it funner, like using computers, like at the start using the technology was a bit complicated, but once you learned how to use it, and you could understand it, you learned more about it, and you understand the maths more as well	
	TC_MLAsNeg (Contextual Maths Approach - negative)		Our main problem was that we didn't know what we were doing. Like, nothing worked. You gave us the thing and we then got off graphs and it just kinda confused us all	

Category	Sub category	Keywords	Examples	Operationalisation
MT (Attitude towards use of Technology for learning Mathematics)	**MT_TradPos** (Traditional Approach - positive)	Like, More interesting, Learn better, Worthwhile, Easier.	N/A	Segments that refer to the how students perceive that the use of technology in mathematics learning activities provides relevance, aids their learning, and contributes to their achievement in the subject.
	MT_TradNeg (Traditional Approach - negative)		And to see that I have never ever used computers in maths, like, is nuts	
	MT_MLAsPos (Contextual Maths Approach - positive)		Yeah, if you get computers involved, it gets a bit easier, if you use software. It's just instead of like, having to measure it and like hold it up against a giant ruler, and then drop it down, you can just use that software that we were using. It's a lot easier. Saves a lot of time	
	MT_MLAsNeg (Contextual Maths Approach - negative)		…but on a computer it seems very abstract. It's like over there, it's not here	

Table 7.A.2: Design Principles Coding Scheme

Category	Sub category	Keywords	Examples	Operationalisation
Team - Collaboration	Team_Positive	Groups, Ask, Asking, Team, Teamwork, Pairs, Pull together.	Yeah, they pulled the team really well together, because they had fun doing it instead of just being told what to do.	Segments that refer to working in teams or collaboration with others.
	Team_Negative		Yeah, people were just wandering off. And that was really annoying. Some teams had a better advantage and others were disadvantaged.	
Use of Technology	Tech_Transformative	Different, Exciting, Easier, Saves time, Make concrete, Involving, Understand.	It was kind of like maths through computers and things and ways, different ways to learn maths basically, more exciting and involving ways for the people.	Segments that refer to how the use of technology affected the participants' experiences with the task.
	Tech_Computational		I mean the software was the push for me, using the computers was really handy, because it meant that I could understand it and have fun with it, without having to stress about getting it wrong. Because, as long as I typed in the right numbers, it was going to be okay	
Variety of Technologies	Var_Tech_tool for task	Accessible, Simple, Learned.	I knew they existed, but I didn't know they were so easily accessible, and you could actually use them to do stuff.	Segments that refer to the students' use of the technologies in the class.
	Var_Tech_flexibility		It only took us about 5 minutes to get used to it. Yeah, it was pretty simple to use anyway.	
Task Design	Task_RME-real	Realistic, Real-life, How, Why, Practical, Fun, Outside the box, Independent, Progressing, Sense-	If it relates to everyday life, you know, not that Barbies do, but	Segments that refer to aspects of the task design that impacted on participants' experience of the activities.

Category	Sub category	Keywords	Examples	Operationalisation
	Task_problem-solving	making, Hints, Other areas, Problem-solving, What it’s for.	it really got you thinking, first of all to try and know how to cut the thing, and the way you did it was like, you're looking at a piece of paper and thinking this is impossible, like, how is this possible?? I was just like, I can't do this, and like when you did it I was just like ... when you gave us kind of hints and stuff for that, like there are different ways, I started thinking of shapes and everything you used to do in maths, so I started thinking of shapes you know and different ways you can do,	
	Task_practical		Well like we were out there doing stuff, we could see how the actual maths related to real life.	
	Task_open-ended		It's kind of, like it's not just simple problem solving, like when you get this big long-winded question, and you've to find... okay, you know it's simultaneous equations, or you know it's going to be graphs. This is like, it doesn't tell you what it is, you just have to figure it out yourself.	
	Task_low-floor		No, I wouldn't say I like it, I'm like Rory, I prefer just asking the questions and gathering data and all that. I wouldn't be a fan of numbers.	
	Task_high-ceiling		you kind of asked us how many seconds were in our like, and well, me and Dualtagh took that a bit literally and like really counted leap years, but like I thought that was really interesting because you had to think about all these things, like exactly what time of the day you were born, and what time of the day it is now. And kind of things like that. So it kind of made you think about it.	

Category	Sub category	Keywords	Examples	Operationalisation
	Task_guided discovery		It's cool because we had to try our best to resolve the problem, so like when we like have everything there to do, after maybe an hour, it would be quite boring because you don't have to think about it, you are just following instructions, so it's really good.	
	Task_cross-strand		Well it was like I realised that everything could, or at least most things in maths are about graphs, which I really didn't realise. Because they give these functions, like find all the different properties, and find out what x is, and it's all related to graphs, which I really hadn't realised until we had to do it. And even the two sets of data, the weight and the number of elastic bands, I mean distance and elastic bands, was actually, like... I mean I knew in my head what it was, but it's different when you see it like, put into a graph. You kind of know it and it makes sense.	
Bridge21 Activity Structure	Bridge21_Warm up	Warm-up, Plan, Create, Reflect, Present, Timing.	I really liked how we did the warm ups, I actually think they were like really good. I mean like it really got you thinking	Segments that refer to aspects of the Bridge21 Activity Structure that impacted on participants' experience of the activities.
	Bridge21_timing		I know it's probably not a maths class that you could do every day because there is like, so much in it and so much to do, but like it was good. Just even like, to do once. It really works to change your outlook on maths	
	Bridge21_Reflection		I even think it might've been better if we'd had another day, just to look over everything.	
	Bridge21_Presentation		You know the way that you have to present it at the end of the day? If you were presenting that to people from different schools, it would be better fun.	

Category	Sub category	Keywords	Examples	Operationalisation
	Bridge21_Planning		Yeah, it opens your eyes a bit more, to take in everything before you start like	
	Bridge21_Create-iterate		You know that kind of breakthrough moment when we actually got all the way there? That's because we were watching back and looking at the techniques that we had used, we were looking at two different techniques and what was wrong and what was right and then we tried to use that to manipulate the way we were doing it before to get where we want to go	
Assessment	Assessment	Presentations	Some groups didn't really have a proper presentation. I think more time should have been put towards that.	Segments that relate to aspects of the activities that could be assessed.

Appendix 7.B Nodes and Categories from Constant Comparison

Name	Sources	References
Adaptable	1	4
Beliefs and attitudes relating to Maths	2	26
Beliefs_Change	2	9
Beliefs_Negative	2	14
Beliefs_Positive	2	13
Confusion	1	1
Enabled by technology	3	13
Learning	4	114
Learning_Concepts	4	24
Learning_Connections - Representations	3	11
Learning_Content	4	14
Learning_Discovery	3	8
Learning_Estimation	1	2
Learning_Further	1	2
Learning_Outside the box - Prob-solving	4	9
Learning_Peer	3	9
Learning_Practical	3	9
Learning_Real	2	7
Learning_Technology	4	14
Motivation	4	153
Motivation_Assessment	1	1
Motivation_Challenge	3	6
Motivation_Cross-strand-connections	2	6
Motivation_Curiosity	3	14
Motivation_Fun	3	9
Motivation_Hands-on	3	6
Motivation_Interesting	4	16
Motivation_Ownership	4	16
Motivation_Practical	4	8
Motivation_Realistic	4	11
Motivation_Team	3	16
Motivation_Technology	3	11
Motivation_Understanding	4	30
Negative attitude	3	61
Negative_Assessment	2	3
Negative_Boring	1	4
Negative_Curriculum	1	2
Negative_Lack of context	1	1
Negative_Maths anxiety	1	3
Negative_Monotonous	3	6
Negative_Self belief	2	5
Negative_Teacher	3	5
Negative_Teams	3	14
Negative_Technology	1	6

Negative_Usual class	3	7
Precision	1	4
Suggestion	1	4
Sum it Up	3	16
Task Design	4	171
Task_Active-Hands-on	4	16
Task_Assessment	1	1
Task_Bridge21	2	6
Task_Context	2	14
Task_Cross-strand	2	6
Task_High Ceiling	4	9
Task_Meaningful	4	32
Task_Open-ended	4	14
Task_Preparation	2	3
Task_Presentation	1	3
Task_Problem-solving	3	11
Task_Real life	4	7
Task_Team	3	13
Task_Team_Mixed ability	2	8
Task_Technology-mediated	3	15
Task_Technology-mediated_Outsourcing	2	7
Task_Technology-mediated_Transformative	3	10
Task_Timing	1	10
Task_Useful - Practical	2	10
Traditional Approach	4	55
Traditional_Procedural	3	9
Traditional_TY	1	7

Appendix 7.C Relational Query Memos

[[Query]] What is the link between the realistic aspect, the conceptual understanding and the technology? In fact, what is the link between all of the different levels of learning???

[[Query]] Could it be that the realistic aspect of the activity has led to the motivation?

[[Query]] Is there a link between self-belief and “Negative_Maths anxiety”? - It will be interesting to note the link that might emerge between maths anxiety and negative attitude. My guess is that there will be a directional link from anxiety to negative attitude, but will it go the other way as well?

[[Query]] My next question is what is the link between maths anxiety and self-belief? Presumably this is related to the concept of Confidence. I guess this is where this work should come into its own. How can confidence be affected by this kind of activity?

[[Query]] If the steps are clearly laid out, the student with a negative attitude can do the work. What is the link then between understanding, or lack thereof, and anxiety???

[[Query]] Is a change from negative attitude – linked to “Enabled by Technology” - some students seem to feel that the technology has increased their confidence. This could relate to a belief that by using the technology they will get it right, or it could be more to do with the fact that they are freer to play around with the mathematics, and it doesn't really matter if they gets it right or wrong. One student claims that she doesn't know how to do it, she just trusts the technology. A lot of people would probably consider that "knowing how to do it". Does this girl require a deeper conceptual understanding in order to be able to feel comfortable with the procedure? (I know how she feels, if this is correct, it is very like how I would have been). Perhaps it is important to encourage some kids that "outsourcing" can be ok at times. We don't have to know how a car works to be able to drive. This could be an interesting potential barrier.

[[Query]] Self Belief or Confidence? - This is an interesting question! Are self-belief and confidence the same thing? I have a feeling that they are linked but are not actually the same. I think confidence is something that can be more easily changed than self-belief. This could be an interesting question for our questionnaire as well!

[[Query]] What is Realistic? - I have just added a section to this node, relating to how working in a team is good because it is something that they will have to do in "real life", although it was not easy. How does this kind of 'Realistic' relate to the 'Realistic' aspect of the task design? I would imagine that these will need to be different subcategories.

[[Query]] TEAMWORK - What is the impact of the type of school on teamwork?

[[Query]] Link to Negative attitude – Linked to “Traditional Approach” - It is interesting to note that 7 of the 8 sections of text that were coded at Traditional Approach were also coded as Negative attitude.

[[Query]] if we have a section relating to the traditional approach, perhaps we should have a section that relates to the Maths learning activities (MLAs) or could that still be a part of the "sum it up" node? I'll leave it as the latter for the moment, but it's something to bear in mind. There is the "sum it up" aspect, and then the motivation aspect, so "what was it like", and "why was it good"?

[[Query]] What will the crossover be between task design and motivation, and task design and learning etc...?

[[Query]] Learning_Further and Taks_High Ceiling - there is a link between these two new codes!

[[Query]] I really think there is a link between the motiviation afforded by a sense of ownership, and learning.

[[Query]] Open ended and Further Learning – Linked to “Task_High-ceiling”: There is a link to the open ended task design and the fact that the boundaries of the subject are being pushed by the more able students.

[[Query]] Change – Are beliefs static? – Linked to “Beliefs and Attitudes relating to Maths”: 17:08.1 - 17:23.7: Are beliefs static or hard to change? This student claims that participation in the activities worked to change his outlook on maths. However, most of the students have very fixed ideas about whether or not they are "good" at maths/with numbers. This is a very interesting area to pursue...

[[Query]] What about changes in BELIEFS and ATTITUDE? What affects that? I see a link to OWNERSHIP and also to use of TECH...

[[Query]] Link between Ownership and Change in Belief/Attitude – Linked to “Motivation_Ownership” - I definitely feel that there is a link between the sense of ownership and a feeling of being more confident with the mathematics.

[[Query]] Link to Understanding – Linked to “Task_Context” - I think I need to look through task design in general, to see where it links to understanding and meaning. It seems quite evident in this node on context.

[[Query]] Understanding and Concepts – Linked to “Learning_Concepts” - So, what is the link between "concepts" and "understanding"? Should this be renamed? Also, what about content and concepts? Are they both just subsections of understanding?

[[Query]] Interesting and Thinking – Linked to “Motivation_Interesting” - A number of the segments coded at Motivation_Interesting actually relate to being stimulated to think. Is this actually the same as being motivated by interest? Should there be another, separate node?

[[Query]] Hands-on and Practical – Linked to “Motivation_Hands-on” - What's the difference between hands-on and practical? Could they be merged into one node or is there a place for the two of them? Perhaps hands-on relates to physical manipulation, whereas practical can just be something that is of use in everyday life...

Appendix 8.A PG Cert Contextual Mathematics Assignment

Postgraduate Certificate in 21st Century Teaching and Learning

Contextual Mathematics MODULE ASSESSMENT

PURPOSE

The purpose of the assignment is to help deepen teacher knowledge and practical experience in the creation, delivery and reflection on an innovative, technology-mediated, team and project-based learning experience. The content to be covered can reflect any area of the mathematics curriculum. The target learners should be able to demonstrate deep conceptual understanding of the content and as well as 21st Century learning skills.

STUDENT TASK

Design, implement and report on a contextual mathematics, 21st Century learning experience. *Collaboration* at the design and reflection stages is encouraged, but not compulsory. All reports must be *individual* work. The report should include the following elements:

1. A completed lesson planning template for the learning experience, detailing:
    a. The rationale underpinning the design of the learning experience.
    b. A description of the learning activity and desired learning intentions.
    c. The content that is covered.
    d. Three central key skills for development.
    e. Schedule and resources required.
    f. Evidence to demonstrate student learning.
2. A multi-media presentation showing aspects of the learning experience being delivered to the learners (no more than 2 min/20 slides in length).
3. Three examples of assessed student work: below average, average and above average. Details and rationale are required.
4. A 600 word written reflection on the experience with the exercise.

Notes: Students are free to use whatever digital media they wish for submission of their report, but the written component should not exceed 2,500 words. Also, if you chose to include video content from the classroom then informed consent must be provided for all students that are filmed.

ASSESSMENT FOCUS

On successful completion of this module, the student will be able to do the following:

1. Demonstrate an understanding of how to create, deliver and assess a 21C maths learning experience.
2. Demonstrate technical competence in a number of digital formats.
3. Provide a deep and rich reflection on the experience.

MARKING

The assignment is marked using the marking scheme and grade descriptions associated with the PG Cert (see PG Cert Handbook 2014/15).

Appendices contained in the PG Cert Handbook 2014/15 contain general advice on planning and writing an assignment, including expected conventions for referencing.

Appendix 8.B Contextual Mathematics Activities

Activity	**Class**	**Description**
Heights with Helium	Transition Year (age 15/16)	A helium balloon and technology was used to find the measure of certain heights around our school. Similar to the Barbie Bungee activity, this meant dealing with only two variables, Height and Time and being able to use the free video analysis software Kinovea (*www.kinovea.org*) to obtain these variables and a Spreadsheet to graph the data.
Functions in context: analysing the trajectory of a ball.	2nd Year (age 13/14)	o Each team of students will take video clips of attempts to throw a ball into a basket. o They will then use appropriate software to analyse the trajectory of a successful shot. o Using a suitable graph, they will compare successful and unsuccessful shots.
Distance, Speed and Time	3rd Year (age 14/15)	Students will be asking themselves “how fast am I running?”. Based on their introduction to Kinovea and their knowledge of Microsoft Excel, they will be asked to answer this question and illustrate their answers in the form of graphs and tables.
Statistics/Measuring Heights/Distance, Speed and Time	1st Year (age 12/13)	Working in groups of five members, students are tasked with comparing the speed of the shortest and tallest members of their group over a specified distance. The data collected, and analysis of their findings, will be done using Kinovea.
Egg Drop Challenge	Transition Year (age 15/16)	Teams of four students work to design a method of safety dropping an egg from a first floor window. They use smart phones, digital camera and iPads to visually record the activity (photos and video). They generate data from the activity and use a video App and mathematical analysis software to provide mathematical evidence for their teams approach.
Quadratic equations, functions and algebra.	Transition Year (age 15/16)	The students will be asked to plot the quadratic function for the flight of their shot in a football crossbar challenge. The students will be dived into groups of 3-4 students. Each group will work with the tracker software to analyse the best shot that is closest to hitting the crossbar. The students will use tracker software called Kineova and excel to find and plot the flight of

		their shot. Finally the students will be asked to analyse the graph produced.
Children's Birthday Party	2nd Year (age 13/14)	Given an advertisement for a party hire company, the students are encouraged to use GeoGebra to explore different combinations of tables etc. to get the best value for money.
Speed, Distance, Time revision and application	Transition year (age 15/16)	The students will be presented with the problem 'Who is the fastest in the class?'. In their groups they must produce a method of experiment and a Microsoft excel presentation of their results.
Shoot a basket!	2nd Year (age 13/14)	In groups, the students will develop a different way to analyse and make 'real' quadratic functions through group work and peer teaching and learning. Students will learn to select, create, and use many new forms of technology, such as GeoGebra and Tracker. The groups will be briefly introduced to the programmes but need to decide if it will help answer the question, "What makes a successful shot successful?" As the students gain experience working with the programmes, they become more aware of the technology available around them.
Speed\Distance\Time and Statistics	1st Year (age 12/13)	The students will undertake a study to determine if the speed of the ball affects the chances of scoring goal. On the SAMR Hierarchy (Puentedura, 2006, as cited by Bray and Tangney,2013), this activity falls into the Transformation space, arguably into the Redefinition category as without the use of technology, measuring the time taken for a ball to travel such a short distance would be inconceivable.
Speed Camera	2nd Year (age 13/14)	In groups students were required to use technology to analyse the speed of cars passing by the school. They needed to represent the data using an appropriate chart and to come up with a hypothesis as to whether different coloured cars were more prone to breaking the speed limit based on their data.

Appendix 8.C List of Codes from Analysis of Teacher Reflections

Name	**Memo Link**	**Sources**	**References**	**Created On**	**Modified On**
Barriers		14	37	12/05/2015 15:28	13/05/2015 13:38
Student abilities		3	3	12/05/2015 15:35	27/05/2015 09:58
Teams		3	3	12/05/2015 18:25	09/06/2015 14:40
Technical		10	16	12/05/2015 15:33	27/05/2015 09:19
Individual Level		5	7	12/05/2015 15:33	09/06/2015 14:23
School Level		8	9	12/05/2015 15:33	09/06/2015 14:50
Time		11	15	12/05/2015 15:35	09/06/2015 15:00
Benefits		21	295	12/05/2015 15:28	13/05/2015 13:38
To students		21	273	03/06/2015 13:23	03/06/2015 13:39
Key skills		19	107	27/05/2015 10:08	03/06/2015 12:14
Collaboration		14	22	12/05/2015 15:30	09/06/2015 15:14
Communication		10	15	27/05/2015 11:07	09/06/2015 14:42
Confidence		4	7	27/05/2015 11:01	09/06/2015 14:49
Creativity		8	10	27/05/2015 11:08	09/06/2015 15:17
Flexibility		5	8	13/05/2015 13:33	09/06/2015 15:15
Organisation		4	4	13/05/2015 13:19	09/06/2015 14:41
Presentation		6	6	13/05/2015 13:18	09/06/2015 15:15
Problem-Solving		7	9	12/05/2015 15:30	09/06/2015 15:17
Reflection		2	2	03/06/2015 11:36	09/06/2015 15:15
Technological competence		12	19	13/05/2015 11:42	09/06/2015 15:14
Outcomes		19	89	03/06/2015 13:28	03/06/2015 13:39
Conceptual Understanding		11	20	12/05/2015 15:30	09/06/2015 15:16
Engagement		16	35	12/05/2015 15:30	09/06/2015 15:17
Enjoyment		13	20	12/05/2015 15:28	09/06/2015 15:17
Ownership		8	13	27/05/2015 10:54	09/06/2015 15:13

Prepared for 3rd level and workplace		1	1	09/06/2015 14:23	09/06/2015 14:23
Task attributes		17	77	03/06/2015 13:26	03/06/2015 13:39
Contextual		9	12	12/05/2015 15:29	09/06/2015 15:12
Cross-curricular		5	5	13/05/2015 13:34	09/06/2015 15:12
Hands-on		3	3	12/05/2015 15:29	03/06/2015 12:24
High Ceiling		3	3	12/05/2015 15:37	03/06/2015 12:14
Inquiry		2	3	27/05/2015 11:00	03/06/2015 12:14
Meaningful		7	10	12/05/2015 15:32	09/06/2015 15:16
Multiple Learning Styles		3	3	12/05/2015 15:31	03/06/2015 12:31
Open ended		4	4	09/06/2015 14:46	09/06/2015 15:33
Peer Learning		9	16	12/05/2015 18:24	09/06/2015 15:14
Student-led		8	18	27/05/2015 10:03	09/06/2015 15:13
To teachers		8	22	03/06/2015 13:23	03/06/2015 13:39
Beliefs		2	4	27/05/2015 10:51	09/06/2015 14:26
Teacher as facilitator		3	5	27/05/2015 10:55	09/06/2015 15:13
Teacher as Learner		6	13	27/05/2015 10:05	09/06/2015 15:12