*Built-in Flexibility of Taxation and Stability when Tax Liabilities Respond with a Time Lag**

Part I: *A Comment* JAMES F. BRADLEY

In the October 1975 issue of *The Economic and Social Review*, D. J. Smyth examined the built-in stability of a system when tax liabilities are a lagged function of income. Taking an unlagged and a distributed lag consumption function he showed that the existence of a lagged tax function could be destabilising, causing damped, regular or explosive oscillations. The purpose of this comment is:

(*a*) to argue that the lagged tax function will have a stabilising effect on Smyth's second model, which uses a distributed lag consumption function,

(*b*) to show how a lagged tax function will affect the stability of Smyth's first model when consumption is related to income with a one-period lag.

(*a*) Smyth reduces his second model to a first order linear difference equation with root

$$\frac{\lambda-(1-\lambda)ct}{1-c(1-\lambda)} \tag{1}$$

where λ is a weight <1 but >0, c is the marginal propensity to consume out of permanent income and t is the marginal tax rate.

The root will take a negative value, causing two period oscillations, when

$$\text{(i) } \lambda-(1-\lambda)ct<0 \qquad \text{or} \qquad \text{(ii) } t>\frac{\lambda}{c(1-\lambda)} \tag{A}$$

The oscillations will be explosive when

$$t>\frac{1-c}{c}+\frac{2\lambda}{c(1-\lambda)} \tag{B}$$

Smyth suggests that oscillations could occur when plausible values for t are taken. This holds only if very low values are given to the weight λ, see Figures 2 and 3 in the original article. However, estimates of λ are much closer to unity than

*The author has benefited from discussion with J. M. Matatko.

zero (Evans, 1969). This would imply a marginal tax rate of over 100 per cent if the tax is to cause oscillations within the system and a marginal tax rate of over 200 per cent for explosive oscillations to occur. In estimating the consumption function labelled (12) by Smyth, Friedman (1957) found that the constant term c_o was very close to zero, c took a value of 0·88 and $c(1-\lambda)$ equalled 0·33. According to Friedman's estimate, λ will equal 0·625. Consequently a marginal tax rate of 189·4 per cent would be required for (Aii) to hold and a tax rate of 392·4 per cent would be required for (B) to hold. On empirical grounds it is therefore highly unlikely that the introduction of a lagged tax function into this system will cause the root to become negative and generate oscillations.

Indeed the introduction of the tax function into the second model will have a stabilising effect. Without built-in flexibility, i.e., $t=0$, the root is

$$\frac{\lambda}{1-c(1-\lambda)}.$$

With built-in flexibility the root is smaller and therefore more stable since the stability of a system with a positive root is directly related to the size of that root, i.e., the smaller the root, the faster the system approaches equilibrium if the root is <1; the smaller the root the slower the rate at which the system will explode in an uncyclical fashion if the root is >1.

(*b*) Standard dynamic built-in flexibility models generally use a one-period lag consumption function. Taking the first model used by Smyth and making

$$C_t = c_o + c(Y_{t-1} - T_{t-1})$$

gives

$$Y_t = c_o + a_o + ct_o + cY_{t-1} - ctY_{t-2} \tag{2}$$

The system without built-in flexibility will reduce to a first order difference equation with root c. With built-in flexibility the roots of the auxiliary equation derived from (2) are given by

$$r = 0{\cdot}5(c \pm \sqrt{c^2 - 4ct})$$

When $c^2 = 4ct$ or $c = 4t$ the real and equal roots are smaller than c. When $c > 4t$ there are two distinct real roots which are both smaller than c. Consequently the system is completely stable when $c \geqslant 4t$.

When $c < 4t$ the roots are complex and the system generates oscillations. However, the cyclical behaviour of the system will depend on the constant term, ct, of the auxiliary equation (see Black and Bradley, 1973). Since $ct < 1$ the oscillations will be damped. Explosive or regular oscillations could not occur within this model unless implausible values are assigned to c or t.

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