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Inferring Long-Run Supply Elasticities From A Short-Run Variable-Revenue Function*

G.E. BOYLE
Maynooth College
H. GUYOMARD
Institut National de la Recherche Agronomique, Rennes

I INTRODUCTION

Recent papers by Kulatilaka (1987, 1985) Squires (1987) and Hertel (1987), using the seminal exposition of Brown and Christensen (1981), which in turn is heavily derivative of the work of Lau (1976, 1978), have emphasised that either the short-run total cost function in the case of Kulatika or the short-run total profit function in the case of Hertel and Squires is a more general specification than their long-run counterparts. In other words, a specification of the cost or profit function in which one or more arguments are assumed fixed in the short run is more general than a specification which assumes that all factors or outputs adjust to their optimal cost minimisation or profit maximisation levels in the period of analysis which is predominantly one year in most studies. This powerful conclusion stems from the fact that long-run responses can be deduced solely from the estimated parameters of the short-run function.

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To date studies have focused on the quasi-fixity of labour and capital in the short run. The motivation for this specification, especially that of family labour, is evident for non-corporate enterprises like family farms. We contend, however, that equally important fixities exist among production outputs. The example that springs to mind immediately is that of cattle output in the agricultural sector. Because of obvious physiological constraints it is simply not possible for cattle output to respond optimally to a price change within one year — the typical periodicity of most supply response studies. In this paper, therefore, we focus on modelling the supply responsiveness of cattle considered as a quasi-fixed output.

In specifying technologies a plethora of simplifying possibilities exist. We have noted the recent work relating to the cost and profit functions. In this paper we consider the revenue function specification. Our motivation for this specification is twofold. First, in the context of model building because of the sheer number of outputs and inputs, it will often be convenient and sometimes mandatory to make simplifying separability assumptions. In this situation the revenue and cost functions become useful constructs relative to the more general profit function. A second justification is pragmatic. The empirical application reported below derives from the findings of a model-building exercise which imposed weak input-output separability (Boyle and O'Neill, 1988).

The plan of the paper is as follows. We first develop for the revenue function the theoretical analogues to the derivation of the long-run cost and profit functions from their respective short-run specifications. In our analysis we consider a single output (for example, cattle production in the agricultural sector) as quasi-fixed. We then obtain pertinent elasticity formulae in the case of the well-known translog specification. We next apply our theoretical results to data for the Irish agricultural sector. We feel the latter is a very good illustration of the methodology as agricultural activity in Ireland is relatively more important than most other Member States of the Community. Moreover the correct specification of the cattle sector is critical to assessing supply response in Ireland because cattle production constitutes about 37 per cent of aggregate farm production. We complete the paper with some conclusions and recommendations for future work.

II THE REVENUE FUNCTION: SOME THEORETICAL PRELIMINARIES

Suppose, y is a vector of m outputs with prices $(p_1, \ldots, p_m) \geqslant 0$, x is a vector of n inputs with prices $(w_1, \ldots, w_n) \geqslant 0$, and $y \geqslant 0; x \geqslant 0$.

The long-run revenue function is defined as

$$R(p,x) \tag{1}$$

This function gives the maximum attainable revenue given the vector of output prices and the vector of production inputs. The properties assumed for this function are (see Diewert (1974)): positively linearly homogeneous, convex and non-decreasing in p; continuous and twice differentiable in p and by Shephard's Lemma: $\delta R(p,x)/\delta p_i = y_i\ (p,x)$, and non-decreasing and concave in x.

In the short run some outputs may be quasi-fixed so we partition our output vector into a sub-vector y^0 of variable outputs and a sub-vector y^1 of quasi-fixed outputs, that is, $y = (y^0,y^1)$, where y^0 is a vector of m–k outputs $(y_1, \ldots ,y_{m-k})$ with typical price element given by (p_i) and y^1 is a vector of k outputs, $(y_{m-k}, \ldots ,y_m)$ with typical price element given by (p_z). The short-run total revenue function is given as

$$R^s(p^0,x,y^1) = RV^s(p^0,x,y^1) + \Sigma_z p_z y_z \tag{2}$$

where RV^s is the short-run variable-revenue function.

In addition to the properties given above for the long-run revenue function, we note that: Shephard's Lemma gives $\delta R^s(p^0,x,y^1)/\delta y^1_z = -p^{*1}_z\ (p^0,x,y^1)$ where, p^{*1}_z is the dual price of quasi-fixed factor z and R^s should also be non-increasing and concave in y^1.

This function gives the maximum attainable revenue given the vector of output prices, the vector of inputs and the vector of quasi-fixed products. Since short-run total revenue cannot exceed long-run revenue we have by the fundamental duality inequality (see Chambers (1988), p. 146)) that

$$R(p,x) \geqslant R^s(p^0,x,y^1) \tag{3}$$

However, if the level of the quasi-fixed output happens to be consistent with maximising short-run variable revenue, then R(p,x) and $R^s(p^0,x,y^1)$ must coincide at some unique price (p^{*1}_z), thus

$$R(p,x) = R^s(p^0,x,y^{*1}) \tag{4}$$

where y^{*1} is the revenue maximising level of output y^1.

From the foregoing discussion then it is apparent that $R^s(p^0,x,y^1)$ is a more general representation than R(p,x) since the former contains the latter as a special case. This opens up rich possibilities for empirical applications. For instance it is possible to test statistically whether R(p,x) equals $R^s(p^0,x,y^1)$ or in other words whether y^1 equates with the level which maximises short-run total revenue. This strategy is preferable to what usually is an arbitrary allocation of outputs and inputs to the variable or quasi-fixed categories. Moreover, even if $y^1 \neq y^{*1}$, it is possible to evaluate the long-run elasticities only from knowledge of the short-run variable-revenue function. In this case, it will be necessary as a first step to calculate the optimal levels of the quasi-fixed outputs which correspond to the long-run equilibrium levels not necessarily achieved by the observed technology.

III AN APPLICATION TO THE TRANSLOG FUNCTION

For expositional convenience we will assume a single quasi-fixed output and input.[1] The translog short-run variable-revenue function is given as

$$\begin{aligned} \log(RV^s(p^0,x,y^1)) &= a_0+\Sigma_i a_i \log(p_i)+a_1 \log(y^1)+a_x \log(x) \\ &+ \tfrac{1}{2}\Sigma_i \Sigma_j a_{ij} \log(p_i) \log(p_j) \\ &+ \tfrac{1}{2}a_{xx} (\log(x))^2 + \tfrac{1}{2}a_{11} (\log(y^1))^2 \\ &+ \Sigma_i a_{i1} \log(p_i)\log(y^1) \\ &+ \Sigma_i a_{ix} \log(p_i)\log(x) + a_{x1}\log(x)\log(y^1) \end{aligned} \tag{5}$$

where the a's are the parameters to be estimated.

Shephard's Lemma gives us

$$\begin{aligned} \delta \log(RV^s)/\delta \log(p_i) &= p_i y_i/RV^s \\ &= M_i = a_i+\Sigma_j a_{ij}\log(p_j)+a_{i1} \log(y^1)+a_{ix} \log(x) \end{aligned} \tag{6}$$

Several elasticity concepts may be derived from (6) but we will concern ourselves here with the price elasticities. For the derivations given below we draw heavily on the results of Guyomard (1988) for the case of the short-run total cost function.

Short-run Price Elasticities

The own-price elasticity is defined as

$$\epsilon^s_{ii} = \delta \log(y_i)/\delta \log(p_i)|y^1;x$$

From (6) we have

$$y_i = (RV^s/p_i)(a_i+\Sigma_j a_{ij}\log(p_j)+a_{i1} \log(y^1)+a_{ix} \log(x)) \tag{7}$$

1. Separability between inputs and outputs allows us to aggregate inputs using an index number. This case is the mirror of the multi-input cost function with only one aggregated output.

Hence,

$$\delta \log(y_i)/\delta \log(p_i)|y^1;x = (a_{ii}+M^2_i-M_i)/M_i \tag{8a}$$

In a similar manner we can obtain the cross-price elasticities as

$$\epsilon^s_{ij} = \delta \log(y_i)/\delta \log(p_j)|y^1;x$$
$$= (M_iM_j+a_{ij})/M_i \tag{8b}$$

Long-run Price Elasticities

In the long run the short-run quasi-fixed input output can adjust to its optimal level in response to price changes. Thus the long-run own-price elasticity is given by

$$\epsilon^\ell_{ii} = (\delta \log(y_i)/\delta \log(p_i))|y^1$$
$$+ (\delta \log(y_i)/\delta \log(y^1))(\delta \log(y^1)/\delta \log(p^{*1}))(\delta \log(p^{*1})/\delta \log(p_i))$$
$$\epsilon^\ell_{ii} = \epsilon^s_{ii} - (a_{i1}+M^{*1}M_i)^2/(a_{11}+(M^{*1})^2-M^{*1})M_i \tag{9a}$$

where, $M^{*1} = -p^1y^{*1}/RV^{*s}$, y^{*1} is the derived optimal level of cattle output and RV^{*s} is the calculated level of short-run variable revenue where cattle is set at its estimated optimal level;[2] p^{*1} is the optimal or "virtual" cattle output price. We note that RV^s must be concave in y^1. Thus $\delta^2 RV^s/(\delta y^1)^2$ must be negative semi-definite so $(a_{11}+(M^{*1})^2-M^{*1})$ must be <0. In (9a), therefore, the Le Chatelier-Samuelson principle holds if the latter condition is satisfied.

In a similar manner we can obtain the various long-run cross-price elasticities. Thus we have

$$\epsilon^\ell_{ij} = \epsilon^s_{ij} - (a_{i1}+M^{*1}M_i)(a_{j1}+M^{*1}M_j)/(a_{11}+(M^{*1})^2-M^{*1})M_i \tag{9b}$$

$$\epsilon^\ell_{i1} = (a_{i1}+M^{*1}M_i)M^{*1}/(a_{11}+(M^{*1})^2-M^{*1})M_i \tag{9c}$$

$$\epsilon^\ell_{1i} = -(a_{i1}+M^{*1}M_i)/(a_{11}+(M^{*1})^2-M^{*1}) \tag{9d}$$

$$\epsilon^\ell_{11} = M^{*1}/(a_{11}+(M^{*1})^2-M^{*1}) \tag{9e}$$

2. This expression could also be evaluated using the derived dual prices and observed output levels. If the dual and observed prices of the quasi-fixed output are equal, then the short-run equilibrium and the long-run Marshallian equilibrium coincide.

If the short-run variable-revenue function is concave with respect to the quasi-fixed output, the denominator of (9e) will be negative and since $M^{*1} = -p^1 y^{*1}/RV^s$ it follows that $\epsilon^{\ell}_{11} > 0$.

IV AN EMPIRICAL APPLICATION TO IRISH FARM DATA

One of the main implications of the preceding sections is that estimation of the short-run variable-revenue function is sufficient to obtain all relevant elasticities. Moreover, estimation of the long-run revenue function may be inappropriate if one or more outputs cannot be adjusted optimally in the short-run. The interesting empirical question raised then is what are the consequences for elasticity estimates of estimating a long-run revenue function when such a strategy is in reality inappropriate?

We examine this question using data for the Irish agricultural sector. Previous work on this data set by Boyle and O'Neill (1988) had estimated a long-run revenue function for the sector which produced a small but negative supply elasticity for cattle output. The revenue function employed was slightly more restrictive than that considered in earlier sections in that the components of aggregate output were assumed to be homogeneously weakly separable from variables outside of this aggregate. Essentially this assumption allows for a multiple stage decision rule. In terms of the specification of the revenue function it implies that the arguments are output prices and the level of aggregate output not the level of aggregate input.

The empirical analysis in this paper considers whether the long-run revenue function is an incorrect specification and if so, we establish the consequences for the derivation of elasticity estimates. Our estimation strategy is as follows. First, we estimate and derive elasticity estimates for the long-run revenue function specification. Second, we estimate a short-run variable-revenue function in which cattle output is specified as quasi-fixed. We then test whether the latter model can be rejected and if not we obtain the implied long-run elasticities and contrast these values with those supplied from the first specification.

The data set spans the period 1960-82.[3] We consider five output categories: milk, cattle, sheep, crops and a residual grouping. The short-run variable-revenue function is estimated jointly with the revenue share equations. We impose linear homogeneity in prices and in aggregate output and symmetry is also imposed in the estimation. All regressors are normalised to 1970=1 and the estimator employed is maximum likelihood.

The regression results for the long-run and short-run variable-revenue functions are given in Table 1. The reported findings for the long-run revenue

3. Details of the data may be found in Boyle and O'Neill (1988) and Boyle (1987).

Table 1: *Regression Results for Alternative Specifications of the Translog Revenue Function*[a]

All Outputs Variable		Cattle Output Quasi-fixed	
a_{00}	5.76 (.002)	$\hat{a}_{00}$	5.33 (.01)
a_{01}	.26 (.006)	$\hat{a}_{01}$	.39 (.01)
a_{02}	.38 (.005)	$\hat{a}_{02}$	-.54 (.15)
a_{03}	.04 (.001)	$\hat{a}_{03}$	.07 (.002)
a_{04}	.15 (.003)	$\hat{a}_{04}$	.24 (.004)
a_{11}	.40 (.08)	$\hat{a}_{11}$	.79 (.11)
a_{22}	.17 (.03)	$\hat{a}_{22}$	-1.90 (1.91)
a_{33}[b]		$\hat{a}_{33}$[c]	
a_{44}[b]		$\hat{a}_{44}$[c]	
a_{12}	.02 (.03)	$\hat{a}_{12}$	-.10 (.15)
a_{13}	-.05 (.02)	$\hat{a}_{13}$	-.14 (.02)
a_{14}	-.12 (.03)	$\hat{a}_{14}$	-.19 (.04)
a_{23}	-.04 (.008)	$\hat{a}_{23}$	.08 (.03)
a_{24}	-.06 (.02)	$\hat{a}_{24}$	-.14 (.05)
a_{34}	.004 (.01)	$\hat{a}_{34}$	-.05 (.17)
Log L	366.8		240.3

Standard errors in parentheses.

Notes: [a]1=milk; 2=cattle; 3=sheep; 4=crops. The numeraire price was residual outputs.

[b]these coefficients are obtained by setting $a_{33}=a_{03}(1-a_{03})$ and $a_{44}=a_{04}(1-a_{04})$ which imply that the long-run own-price elasticities of crops and sheep are zero at the point of approximation.

[c]these coefficients are obtained by setting $\hat{a}_{33}=\hat{a}_{03}(1-\hat{a}_{03})$ and $\hat{a}_{44}=\hat{a}_{04}(1-\hat{a}_{04})$ which imply that the short-run own-price elasticities of crops and sheep are zero at the point of approximation.

function incorporate the restriction that, at the point of approximation, that is 1970, the own-price elasticities of sheep and crops are jointly equal to zero. This restriction could not be rejected with the likelihood ratio test. The coefficient value for a_{22} implies that at the point of approximation the own price elasticity of cattle output is negative. The null hypothesis that the own price elasticity of cattle output was equal to zero was rejected.

The coefficient estimates for the short-run variable-revenue specification also incorporate the unrejected restrictions that the short-run own-price elasticities of sheep and crops are not significantly different from zero at the point of approximation. It is comforting to note that as the sign of the cattle intercept is negative, concavity of the function with respect to the level of cattle output is satisfied at the point of approximation.

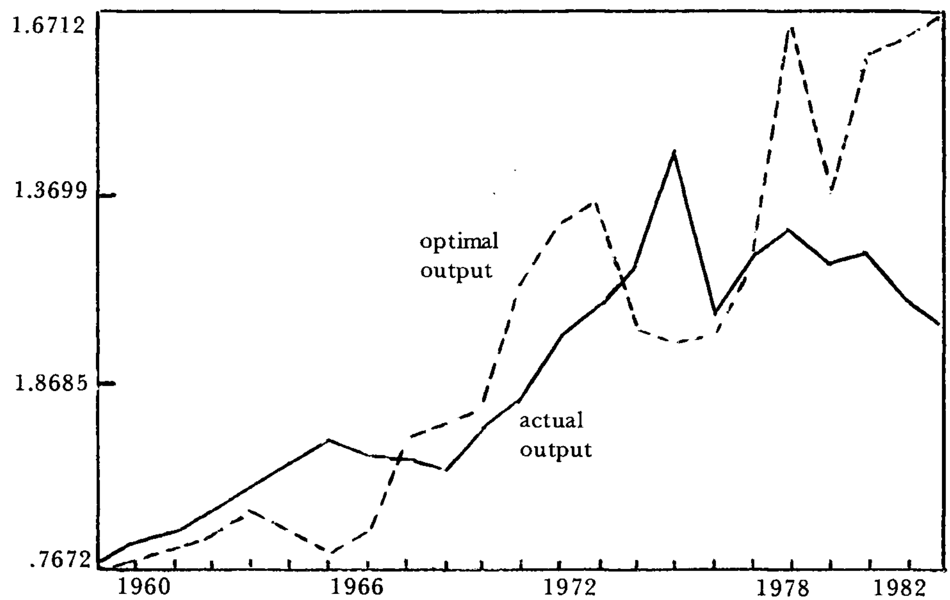
Before we discuss the elasticities associated with these estimates it is necessary to establish whether the variable or quasi-fixed output specification is statistically appropriate. This can be most readily achieved by employing the Conrad and Unger (1987) test. The log likelihood value under the null hypothesis that the observed level of cattle output is at its optimal level is 270.9. Under the alternative hypothesis the log likelihood is 286.4 which is significantly different from the null. Thus the specification of cattle output as short-run variable is rejected for our data.

The implication of this test is that the observed level of cattle output deviates from its implied optimal level. In other words there is evidence of short-run disequilibrium. The same point can be made in a more illuminating way by evaluating the optimal level of cattle output at each data point and contrasting these values with the observed series. The optimal level of cattle output implied by our estimated short-run variable-revenue function is derived by solving Equation (6) for y^{*1}.

The resulting optimal or implied long-run equilibrium level of cattle output is plotted against the observed level in Figure 1. This chart tells us principally that the long-run revenue function specification would not accord with observed behaviour in that the derived revenue-maximising levels of cattle output are different from the observed annual values. The discrepancy is particularly notable for the last few years of the sample.[4]

4. Figure 1 prompts some interesting speculation. Taking one year with another we would expect the observed and optimal series to track each other fairly well in the face of "normal" price developments. Periodically, however, the Irish cattle industry has been beset by "crises" which have caused cattle prices to collapse. Recovery from such "crises" can take a considerable length of time as excessive slaughtering in the period of the "crisis" obviously implies a lead time for stock build up. During this recovery phase we would expect the optimal and observed series to be out of kilter. For our sample period two major "crises" have afflicted the industry, namely, 1965/66 and 1974/75. Recovery from the first slump appears to have been relatively fast and while the optimal and observed series deviate initially they come together relatively quickly. This recovery might have been assisted by the operation

Figure 1: *Observed Versus Optimal Level of Cattle Output (1970=1)*



The set of long-run price elasticities derived from the specification of the revenue function where all outputs are assumed to be short-run variable are exhibited in Table 2. The interesting features of these results are the relatively high own-price elasticity for milk output, the negative, but relatively small, elasticity for cattle and the complementary relationship between cattle and milk production.[5]

Table 2: *Translog Long-run Revenue Function, Long-run Price Elasticity Estimates, 1970*

			Price		
Quantity	*Milk*	*Cattle*	*Sheep*	*Crops*	*Other*
Milk	.80	.46	-.15	-.31	-.80
Cattle	.31	-.17	-.07	-.01	-.07
Sheep	-.99	-.62	.00	.25	1.42
Crops	-.54	-.02	.07	.00	.50
Other	-1.21	-.15	.33	.44	.58

of a government scheme (the "Calved Heifer Scheme" (CHS)) designed to maintain the breeding stock in the wake of the "crisis". The story for the1974/75 "crisis" is quite different. One explanation for the sluggish output performance might have been the absence of any comparable government initiative such as the CHS. A second possible explanation might have been the impact of relative price expectations in the post-EC accession period. The unprecedented increase in relative milk prices superimposed on the 1974/75 price shock might have conspired to substantially retard adjustment.

5. The standard errors of elasticities are not calculated in this study. Indeed as pointed out by Green *et al.* (1987) the delta method used to obtain a first order Taylor's series approximation for the variance of the elasticities is of questionable accuracy in small samples.

The long-run price elasticities derived from the revenue function which specifies cattle as quasi-fixed in the short run are presented in Table 3. These estimates are derived from the formulae (9a)-(9e) and are estimated using the computed optimal level of cattle output. The most interesting aspect of these findings is the contrast with those derived from the long-run revenue function specification. The elasticity for cattle output is now positive. Equally surprising is that we get a vastly greater own-price elasticity for milk output. The long-run own-price crops elasticity is also found to be much larger than the zero estimate yielded by the long-run revenue specification. The sheep own-price value is, however, not different from zero. Substitution rather than complementarity is found between cattle and milk over the long run.

Table 3: *Translog Short-run Variable-revenue Function, Long-run Price Elasticity Estimates, 1970*

			Price		
Quantity	*Milk*	*Cattle*	*Sheep*	*Crops*	*Other*
Milk	1.73	-.33	.03	-.84	-.57
Cattle	-1.70	.02	-.56	1.99	.29
Sheep	.05	-.17	.38	.57	-.80
Crops	-1.09	.50	.46	.17	-.03
Other	-.37	.04	-.32	-.01	.66

V CONCLUSIONS

This paper, utilising the concept of a short-run variable-revenue function, has explored the implications of specifying the cattle product for the Irish agricultural sector as quasi-fixed in the short run. The principal merit of this approach is that all relevant long-run elasticities can be derived from the short-run specification and thus the first methodology is the more general procedure. This more general representation of the production problem is contrasted with the typical approach in supply response studies which is to specify cattle output as variable in the short run.

The main drawback with our approach is that while we present long-run results we do not identify the nature or duration of the adjustment process. We do not feel that this is a serious deficiency since the techniques which are available to model the dynamic adjustment process are typically *ad hoc* and very difficult to empirically implement, especially when the adjustment scheme is multivariate.

To date, in most studies of this genre attention has been focused exclusively on potential factor fixities, e.g., labour and capital. However, it is at least likely that there exist important fixities in production outputs. Cattle output in the agricultural sector is an obvious candidate for the quasi-fixed status given the physiological obstacles which inhibit rapid adjustment to price shocks. In more recent times the existence of the dairy quota in the EC has rendered milk production appropriate for a similar specification.

Our results establish that very different elasticity values are obtained when we specify cattle output as quasi-fixed. The main lesson from our analysis is that if product quasi-fixities are ignored, the resulting elasticities may be substantially unreliable.

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