

## **Dynamic Hedging with the IFOX Long Gilt Futures: New Evidence\***

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*Abstract:* This paper assesses the hedging effectiveness of the IFOX long gilt futures contract. The paper builds on earlier work by Hogan which was hindered by small sample size, thin trading and incomplete specification of the data generation process. We establish that spot and futures long gilt prices are cointegrated and hence have an error correction representation. We test a number of nested sub-models and find some support for the contention that equilibrium spot and futures are unit elastic. The hedging effectiveness of the futures contract is high, and dynamic hedging strategies which enhance effectiveness are established.

### I INTRODUCTION

**H**ogan (1990) conducted a preliminary study of the hedging effectiveness and cost of the IFOX long gilt futures contract. He concluded that the long gilt futures provides an effective hedging vehicle against weekly but not daily cash market volatility and estimated the cost of a full weekly hedge to be 5 per cent over the 15 week period studied.<sup>1</sup>

1. Hogan shows that unhedged positions in 9 per cent Capital 2006 and 8.5 per cent Capital 2010 yielded 7.2 per cent and 7.6 per cent, respectively, in the 15 week period under study; the fully hedged returns over the same period were 2.2 per cent and 2.7 per cent, respectively. Hence hedging costs in terms of foregone yield were of the order of 5 per cent. (See Hogan op. cit., p. 382.)

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We reassess the evidence. We update the sample period in order to mitigate the effects of thin trading and increase its size over fourfold to improve the quality of parameter determination. We further show that the bivariate regression model employed by Hogan is dynamically misspecified.

Spot and futures long gilt prices are cointegrated and hence have an error correction representation. We employ Hendry's methodology in identifying an appropriate dynamic specification. We use this model to identify empirically the long-run equilibrium relationship between spot and futures prices. We also draw on the model estimates in deriving the dynamic risk-minimising hedging strategy.

## II PREVIOUS FINDINGS

Hogan's sample period dates from the exchange's initiation on May 29, 1989 to September 18, 1989, two days before the delivery date for the September long gilt futures. Although four futures contracts (September 1989, December 1989, March 1990 and June 1990) were available from the exchange's inception active trading occurred only in the nearby September futures contract. Moreover, although four gilts (9 per cent Capital 2006, 8.5 per cent Capital 2010, 8.25 per cent Capital 2008 and 8.75 per cent Capital 2012) were deliverable against the long gilt futures only the former two gilts were actively traded over the nearby futures' term to maturity. On 10 per cent of the days studied no trades took place in these more active stocks. Daily trading volumes were also low; £1.25m for the September futures and £27m per day for medium and long gilts.

Hogan estimated the following bivariate regression,

$$\Delta S_t = a + h\Delta F_t + U_t \quad (1)$$

where  $S$  is the gilt spot or cash price,  $F$  is the long gilt futures price,  $U$  is an error term,  $\Delta$  is the first difference operator and  $a$  and  $h$  are fixed coefficients. It is well known, Ederington (1979), that, given Equation (1), the slope coefficient,  $h$ , is the risk minimising hedge ratio and the regression's coefficient of determination,  $R^2$ , is the associated degree of risk reduction on portfolio returns when the hedge ratio,  $h$ , is used to hedge a cash position with futures contracts.

Hogan's daily results were poor; the results for weekly data are reproduced for convenience.

$$\begin{aligned} 9\% \text{ Capital 2006: } \quad \Delta S &= -0.03231 + 1.09250\Delta F \\ &\quad (0.098891) \end{aligned}$$

$$\text{Nobs} = 15, \text{SER} = 0.45, R^2 = .90, \text{DW} = 2.85$$

$$8.5\% \text{ Capital 2010: } \Delta S = -0.00723 + 1.044354 \Delta F \\ (0.126874)$$

$$\text{Nobs} = 15, \text{SER} = 0.58, R^2 = .84, \text{DW} = 1.73$$

The sharp deterioration on daily data is accounted for by thin trading, i.e., on days when either the spot or futures contracts but not both were traded coincident movements in the prices of both instruments is scarcely possible. While the hedging effectiveness over weekly periods improves dramatically (to 90 per cent and 84 per cent, respectively, for the two cash stocks) the low degrees of freedom, 13, compounded by an inconclusive DW statistic for the shorter dated gilt is a source of concern.

We test the robustness of the above results by revising and extending the sample period from 3 January 1990 through 5 July 1991. The data set consists of Wednesday closing prices for the nearby long gilts futures contract and for each of the four gilts deliverable against it.

### III COINTEGRATION RESULTS

We begin by testing if the spot and futures prices are cointegrated. If they are they have an error correction representation, Engle and Granger (1987). Spot and futures prices are cointegrated if (a) each is  $I(1)$ , integrated of order one; and (b) there exists a linear combination of the two that is  $I(0)$ , integrated of order zero, i.e., a stable long run equilibrium relationship exists between the relevant variables.

The Dickey and Fuller regression,

$$\Delta S_t = \alpha_0 + \alpha_1 T + \alpha_2 S_{t-1} + \sum_{i=1}^m \beta_i \Delta S_{t-1} + \varepsilon_t \quad (2)$$

was estimated for weekly spot gilt prices; an identically specified regression (F replacing S) was run for futures prices where  $m$  is of sufficient order to ensure white noise errors,  $\varepsilon_t$ .  $T$  denotes a time trend. If  $S_t$  ( $F_t$ ) is  $I(1)$  without a drift we cannot reject the hypothesis  $\alpha_0 = \alpha_1 = \alpha_2 = 0$ ; if  $S_t$  ( $F_t$ ) is  $I(1)$  with a drift we cannot reject the hypothesis  $\alpha_1 = \alpha_2 = 0$ . Wednesday closing prices for spot and futures prices were used over the full sample period which yielded 78 observations. Plots for each of the raw series are provided in Figure 1. The test results are reported in Table 1.

The ADF statistics (Part a, Table 2) cannot reject the null hypotheses that all series are  $I(1)$  at the 1 per cent level of significance. The results (Parts b and c, Table 2) are inconclusive in that neither of the null hypotheses that the series follow a random walk both with and without drift can be rejected at the

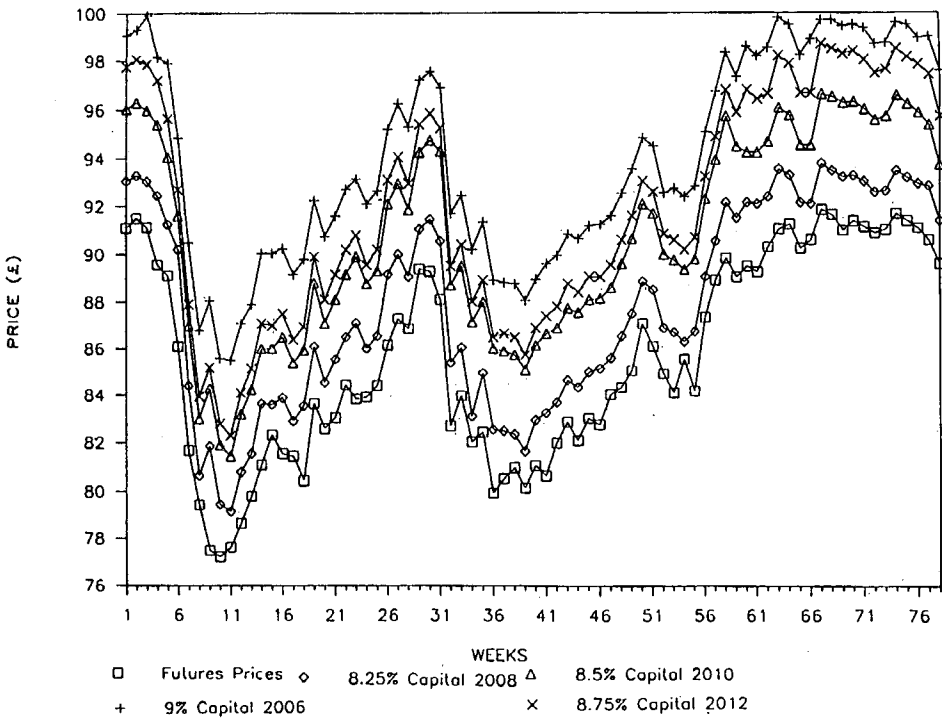


Figure 1: Price Series (£)

Table 1: Unit Root Test Results

Series	$T_t$	$m$	$n$	Critical Values		
				$n =$		
IFOX Long Gilt Nearby Futures	-4.093	2	76	1%	-4.15	-4.04
9% Capital 2006 Stock	-3.4706	2	76	5%	-3.5	-3.45
8.25% Capital 2008 Stock	-3.6459	2	76	10%	-3.18	-3.15
8.5% Capital 2010 Stock	-3.5978	2	76			
8.75% Capital 2012 Stock	-3.9288	2	76			

*All series are weekly*

same significance level. The test for cointegration between  $S_t$  and  $F_t$  was conducted by estimating,

$$\Delta U_t = \rho U_{t-1} + \sum \delta \Delta U_{t-1} + v_t \tag{3}$$

where  $U_t$  are the estimated residuals from a bivariate linear regression of  $S_t$  on  $F_t$ . The t-statistics for  $\rho$  are given in Table 2, along with the critical values at conventional significance levels.

Table 2(a): *Random Walk with No Drift Test*

Series	$T_t$	$m$	$n$	Critical Values		
				$n =$	50	100
IFOX Long Gilt Nearby Futures	5.8833	2	76	1%	7.02	6.50
Capital 2006 Stock	4.3127	2	76	5%	5.13	4.88
8.25% Capital 2008 Stock	4.7388	2	76	10%	4.31	4.16
8.5% Capital 2010 Stock	4.5846	2	76			
8.75% Capital 2012 Stock	5.3099	2	76			
<i>All series are weekly</i>						

Table 2(b): *Random Walk with Drift Test*

Series	$T_t$	$m$	$n$	Critical Values		
				$n =$	50	100
IFOX Long Gilt Nearby Futures	8.8164	2	76	1%	9.31	8.73
9% Capital 2006 Stock	6.4588	2	76	5%	6.73	6.49
8.25% Capital 2008 Stock	7.0994	2	76	10%	5.61	5.47
8.5% Capital 2010 Stock	6.8569	2	76			
8.75% Capital 2012 Stock	7.9526	2	76			
<i>All series are weekly</i>						

Table 2(c): *Test of Non-Cointegration*

Series	$T_t$	$m$	$n$	Critical Values		
				$n =$	50	100
9% Capital 2006/ Long Gilt Futures	-4.4057	2	76	1%	-4.15	-4.04
8.25% Capital 2008/ Long Gilt Futures	-4.2388	2	76	5%	-3.5	-3.45
8.5% Capital 2010/ Long Gilt Futures	-3.6178	2	76	10%	-3.18	-3.18
8.75% Capital 2012/ Long Gilt Futures	-5.2295	2	76			
<i>All series are weekly</i>						

The evidence rejects the null hypothesis of non-cointegration at conventional significance levels for each of the deliverable gilts, as one would expect.

Given that the ADF test for unit roots has notoriously low power especially when the disturbance term in the unit root regression is serially correlated as well as heteroscedastic<sup>2</sup> we additionally applied the  $Z_{\alpha}$ -Phillips-Ouliaris test for cointegration to the data. Our choice of this test is motivated by the following considerations; Phillips and Ouliaris (1990) compare the asymptotic properties of several residual based tests for cointegration including the ADF test. They conclude that the  $Z_{\alpha}$  test should have superior power properties in

2. We are grateful to an anonymous referee for drawing this point to our attention.

small samples and is the preferred test for cointegration. Monte Carlo evidence tends to support this conclusion. Hansen (1990) finds that systems based tests have less power than residuals based tests. Gregory (1991) concludes that the ADF and  $Z_\alpha$  tests perform better than either the remaining tests considered by Phillips and Ouliaris or systems based multivariate tests (Johansen, 1988, 1991; Johansen and Juselius, 1990 and Stock and Watson, 1988, are examples of the latter). Haug (1991) finds that the ADF test has the least size distortions amongst the residual based tests considered but has less power than the  $Z_\alpha$  test.<sup>3</sup>

The  $Z_\alpha$  test is based on the residuals,  $\phi_t$ , of the cointegration regression,

$$y_t = \sum_{i=1}^n \beta_i x_{it} + \phi_t \quad (4)$$

Under the null hypothesis of no cointegration,  $\phi_t$  is integrated of order 1; under the alternative hypothesis of cointegration  $\phi_t$  is integrated of order 0, i.e., it is stationary.

The residuals from Equation (4) are used in the following regression,

$$\phi_t = \alpha \phi_{t-1} + k_t \quad (5)$$

The  $Z_\alpha$  statistic is,

$$Z_\alpha = T(\alpha - 1) - (1/2) \left( S_{Tj}^2 - S_k^2 \right) \left( T^{-2} \sum_{t=2}^T \phi_{t-1}^2 \right)^{-1} \quad (6)$$

where,  $S_k^2 = T^{-1} \sum_1^T k_t^2$

$$S_{Tj}^2 = T^{-1} \sum_1^T k_t^2 + 2T^{-1} \sum_{s=1}^j w_{sj} \sum_{t=s+1}^T k_t k_{t-s}$$

and some lag window e.g.,  $w_{sj} = 1 - s/(1+j)$ .

The critical values for  $Z_\alpha$  are provided by Phillips and Ouliaris for a sample size of 500 only. Fortunately, the critical values for smaller sample sizes have recently been tabulated by Haug (1992). The  $Z_\alpha$  statistic for each of the four gilt contracts was calculated using Equation (8) beneath and setting  $j=2$ . The  $Z_\alpha$  values for the four gilt contracts lie in the range  $-68$  to  $-75$  which is considerably less than the critical values for  $Z_\alpha$  (demeaned) at all conventional

3. These findings are discussed in Haug (1992).

significance levels at sample sizes of 50 and 100.

We therefore reject the null hypothesis of no cointegration and conclude that spot and futures are cointegrated. This result is scarcely surprising as no-arbitrage markets imply that the spot price converges on the futures price as contract maturity approaches and the cost of carry falls to zero. Both prices should enjoy a long run equilibrium relationship.

#### IV MODEL IDENTIFICATION AND SELECTION

Since spot and futures prices are cointegrated they have a cointegrating factor "d" and there exists an Error Correction Model (ECM) of the form,

$$S_t - S_{t-1} = a[S_{t-1} - dF_{t-1}] + b[F_t - F_{t-1}] \quad (7)$$

We now proceed to identify the parameters of the above model using Hendry's methodology. It is convenient to rearrange Equation (7) as a conventional auto-distributed AD (1,1) model,

$$S_t = \alpha_1 S_{t-1} + \beta_0 F_t + \beta_1 F_{t-1} + v_t \quad (8)$$

The formal equivalence between Equations (7) and (8) may be seen by reparameterising Equation (8) as,

$$\Delta S_t = (\alpha_1 - 1)S_{t-1} + \beta_0(F_t - F_{t-1}) + (\beta_0 + \beta_1)F_{t-1} + v_t \quad (8')$$

where  $\Delta$  is the first difference operator. Simplifying Equation 8' gives,

$$\Delta S_t = \beta_0 \Delta F_t + (\alpha_1 - 1) [S_{t-1} - (\beta_0 + \beta_1)/(1 - \alpha_1)F_{t-1}] + v_t \quad (8'')$$

Equation (8'') is formally identical (with suppression of the error term  $v_t$ ) to Equation (7);  $\beta_0 = b$ ,  $(\alpha_1 - 1) = a$  and  $(\beta_0 + \beta_1)/(1 - \alpha_1) = d$ .

The structural parameters have the following interpretation:  $\beta_0$  represents the *impact* effect of F on S,  $(\beta_0 + \beta_1)/(1 - \alpha_1)$  represents the *equilibrium* effect of F on S and  $(\alpha_1 - 1)$  represents the *adjustment or feedback* effect, i.e., the term in square brackets in Equation (8'') represents the previous period's deviation of S from its equilibrium value, and  $(\alpha_1 - 1)$  is the amount by which that *error* is adjusted or *corrected* by the current period movement of S. In short, all AD (1,1) models are ECMs. We will, however, hereafter use the term "the general model" for Equation (8'') for ease of presentation.

A special form of the ECM arises where  $(\beta_0 + \beta_1)/(1 - \alpha_1) = 1$ . In this case F has a unitary *equilibrium* effect on S and long-run exact proportionality or

unit elasticity exists between the two variables. This hypothesis may be tested using Equation (8) by testing if the implied restriction  $\alpha_1 + \beta_0 + \beta_1 = 1$  is supported by the data. We use the term "Error Correction Model" hereafter in the restricted sense of applying only to this model, for ease of presentation. A second sub-model of special interest to us is the growth-rate model used by Hogan, Equation (1) above, and commonly used in similar research on futures markets. Equation (1) is also a restricted version of Equation (8). The restrictions are:  $\alpha_1 = 1$  and  $\beta_1 = -\beta_0$ . These restrictions are readily amenable to testing. If they are not supported by the data then the growth-rate model is dynamically misspecified and estimates based on that model are biased. It is also well established (see Hendry and Mizon, 1978) that such dynamic misspecification will frequently result in serially correlated errors.

In fact, Equation (8) nests 9 commonly used dynamic linear models. It is for this reason that we have described Equation (8) as the General Model. The nested sub-models together with the coefficient restrictions they imply are presented in Table 3.

Table 3: *Models Nested in the General Model*

<i>Type of Model</i>	<i>Equation</i>	<i>Restrictions on General Model</i>
Static	$S_t = \beta_0 F_t + \varepsilon_t$	$\beta_1 = \alpha_1 = 0$ (no dynamics)
Univariate AR	$S_t = \alpha_1 S_{t-1} + \varepsilon_t$	$\beta_0 = \beta_1 = 0$ (no covariates)
Growth Rate	$\Delta S_t = \beta_0 \Delta F_t + \varepsilon_t$	$\alpha_1 = 1$ $\beta_1 = -\beta_0$ (no levels)
Leading Indicator	$S_t = \beta_1 F_{t-1} + \varepsilon_t$	$\beta_0 = \alpha_1 = 0$ (no contemporaneity)
Distributed Lag	$S_t = \beta_0 F_t + \beta_1 F_{t-1} + \varepsilon_t$	$\alpha_1 = 0$ (finite lags)
Partial Adjustment	$S_t = \alpha_1 S_{t-1} + \beta_0 F_t + \varepsilon_t$	$\beta_1 = 0$ (no lagged F)
Common Factor	$S_t = \beta_0 F_t + \mu_t$	$\beta_1 = -\alpha_1 \beta_0$ (one common factor)
Error Correction	$\Delta S_t = \beta_0 \Delta F_t + (\alpha_1 - 1) S_{t-1} + (\beta_0 + \beta_1) F_{t-1} + \varepsilon_t$	$\alpha_1 + \beta_0 + \beta_1 = 1$ (long run unit elasticity)
Dead Start	$S_t = \alpha_1 S_{t-1} + \beta_1 F_{t-1} + \varepsilon_t$	$\beta_0 = 0$ (lagged information only)

#### *Model Estimation and Test Procedures*

We proceed in two stages. First, we test if the *general model* is a well defined statistical model, i.e., is correctly specified. To this end we employ a number of diagnostic tests for misspecification. These are, Lagrange multiplier tests of residual serial correlation and auto-regressive conditional heteroscedasticity (ARCH), LM and F tests of heteroscedasticity, a Ramsey RESET test for functional form, the Bera-Jarque Skewness-Kurtosis test for normality (of errors), Cusum and Cusumsq tests of the structural stability of



the models. The models are estimated using the first 69 sample observation points by OLS, with the exception of the Common Factor model which is estimated by IV. The final 9 of the 78 sample observation points are withheld and used to calculate a Chow test of predictive failure (Chow's second test), Chow's test for the stability of the regression coefficients (Chow's first test) and to calculate Theil's inequality coefficient,  $U$ , as a further useful measure of the predictive power of the models.

Second, provided the assumptions of classical linear regression model are upheld by the preceding tests of the *general model*, we impose the coefficient restrictions associated with each of the sub-models. The validity of the imposed restrictions for each sub-model is tested separately using the adjusted F test:

$$F_{d,n-k} = [(ESS_r - ESS_u) / d] / [ESS_u / (n-k)]$$

Where  $ESS_r$  ( $ESS_u$ ) is the error sum of squares from the restricted (unrestricted) regression,  $d$  is the number of restrictions (i.e., on the sub-model under test) and  $n-k$  is number of degrees of freedom in the unrestricted regression (i.e., the general model).

Where the restrictions are supported the relevant sub-models are further subjected to the same specification tests as the general model. When more than one sub-model passes both the coefficient restriction and specification tests conventional criteria including the residual sum of squares, standard error of the regression and coefficient of determination are used in selecting "the" appropriate model. (A comprehensive treatment of the methodology is provided by Spanos (1986)).

We augment the models with an intercept term and following a suggestion by Gilbert (1986) reparameterise the general model as,

$$S_t = \alpha_0 + (\alpha_1 + \beta_1)S_t + \beta_0 F_t + \beta_1(F_{t-1} - S_{t-1}) + v_t \quad (9)$$

This reparameterisation is intended to mitigate the effects of high correlation between  $F_t$  and its lagged value,  $F_{t-1}$ . We refer to this model as the corrected model in the results presented beneath. Finally, we omit the Univariate AR model in our reported results since it has no direct bearing on the hedging efficacy of the futures contracts under study.

The models were estimated using Datafit (see Pesaran and Pesaran (1987)) which also contains an account of each of the diagnostic tests used.<sup>4</sup> The estimation results for each of the deliverable long gilts are presented in

4. Pindyck and Rubinfeld (1981, pp. 364-365) provide an account of Theil's inequality coefficient,  $U$ .

Tables 4, 5, 6 and 7. Diagnostic test details are suppressed in the interest of parsimony of presentation in the Tables which simply signify the tests which were failed for each of the models.

### *The Results*

The cheapest to deliver gilts over the estimation period were 9 per cent Capital 2006 and 8.75 per cent Capital 2012. Our discussion of results, accordingly, centres on these two gilts (Tables 4 and 7) since the appropriate hedging strategy and the efficacy of the long gilt futures contract is best judged in relation to them. Estimation details for the remaining two gilts, 8.25 per cent Capital 2008 and 8.5 per cent Capital 2010, are provided in Tables 5 and 6 for inspection.

The *general model* passes all diagnostic tests in relation to the 9 per cent Capital 2006; it marginally fails the heteroscedasticity test for the 8.75 per cent Capital 2012 at the 5 per cent level but passes it comfortably at the 1 per cent level.

It fails the Normality and Heteroscedasticity tests for 8.25 per cent Capital 2008 but passes both at the 1 per cent level; it fails the functional form test for the 8.5 per cent Capital 2010 at the 5 per cent level but passes it at the 1 per cent level.

The salient features of the remaining results are:

- (1) With regard to 9 per cent Capital 2006, the Error Correction Model passes the adjusted F test on the coefficient restrictions comfortably,  $F = 0.1456$ . It fails the normality test at the 5 per cent level but passes it at the 1 per cent level. It has a lower residual sum of squares and standard error of regression than any of the remaining sub-models tested. The growth rate model displays a notably poor performance and is clearly (dynamically) mis-specified.
- (2) With regard to the 8.75 per cent Capital 2012, the Error Correction Model passes all diagnostic tests but fails the coefficient restriction adjusted F-test (at the 5 per cent confidence level) as do all of the remaining sub-models. The growth rate model, once again, performs poorly.
- (3) With regard to the 8.5 per cent Capital 2010, the coefficient restrictions are upheld for the Distributed Lag, Partial Adjustment and Error Correction Models. Again, the growth rate model performs poorly.
- (4) With regard to the 8.25 per cent Capital 2008, the Static, Distributed Lag, Common Factor, Partial Adjustment and Error Correction models all pass the coefficient restrictions test. The Error Correction

Table 4: Regression Results for 9 Per Cent Capital 2006

Model Name	Model Specification and Parameter Estimates	Passes all Diagnostic Tests	Failed Diagnostic Tests	RSS	Standard Error of Regression	Adjusted F-statistic	Theil's U	R <sup>2</sup>
General Model	$S_t = 10.26 - 0.1625S_{t-1} + 0.8693F_t + 0.2869F_{t-1}$ (5.84) (-1.41) (20.17) (2.3929)	Yes		17.58	0.524		0.0022	0.98
Static Model	$S_t = 9.454 + 0.9873F_t$ (6.623) (58.79)	No	Normality Heteroscedasticity	20.60	0.558	5.64	0.0018	0.98
Leading Indicator	$S_t = 13.8366 + 0.9356F_{t-1}$ (3.8695) (22.24)	No	Normality, Cusum Heteroscedasticity	129.4	1.400	203.7	0.0023	0.88
Growth Rate	$\Delta S_t = 0.00713 + 0.8461\Delta F_t$ (0.0707) (12.614)	No	Autocorrelation ARCH, Cusumsq	45.59	0.831	51.0	0.325	0.71
Distributed Lag	$S_t = 8.6982 + 0.8668F_t + 0.1294F_{t-1}$ (6.337) (19.98) (2.9837)	No	Normality, Cusumsq Heteroscedasticity	18.12	0.528	2.00	0.0017	0.98
Error Correction	$\Delta S_t = 9.6920 + 0.8723\Delta F_t - 1.1578(S_{t-1} - F_{t-1})$ (10.14) (20.72) (-10.16)	No	Normality	17.61	0.5026	0.1456	0.3855	0.88
Common Factor	$S_t = 9.1972 + 0.9903F_t$ (7.631) (69.82)	No	Normality	19.86	0.553	5.81	0.0017	0.98
Dead Start	$S_t = 14.485 - 0.0669S_{t-1} + 1.0015F_{t-1}$ (3.089) (-0.2158) (3.249)	No	Normality, Cusum Cusumsq	129.3	1.411	407.1	0.0025	0.88
Partial Adjustment	$S_t = 8.00 + 0.0952S_{t-1} + 0.8998F_t$ (5.21) (2.2211) (21.1013)	No	Normality, Cusumsq Heteroscedasticity Autocorrelation	19.15	0.543	5.76	0.0022	0.98
General (Corrected)	$S_t = 10.26 + 0.1245S_{t-1} + 0.869F_t + 0.2869(F_{t-1} - S_{t-1})$ (5.84) (2.88) (20.17) (2.393)	No	Normality	17.57	0.524		0.0017	0.98
Dead Start (Corrected)	$S_t = 14.485 + 0.9346S_{t-1} + 1.0015(F_{t-1} - S_{t-1})$ (3.089) (21.915) (3.249)	No	Normality	129.3	1.411	407.1	0.0025	0.88

- Notes: 1. The Common factor model was estimated using instrumental variables.  
2. Estimation was on weekly data from January 3,1990 to April 24,1991. (69 observations). Nine observations, from May 1,1991 to June 26,1991 were withheld for prediction and for calculating Theil's Inequality Coefficient, U.  
3. Splice points, which arise when the second nearby futures becomes the nearby futures, were included as a continuous series in levels and increments is required in estimating some of the sub-models.  
4. The hedge factors calculated on the above estimations are untailed; tailing procedures are of operational and not methodological significance.

Table 5: Regression Results for 8.25 Per Cent Capital 2008

Model Name	Model Specification and Parameter Estimates	Passes all Diagnostic Tests	Failed Diagnostic Tests	RSS	Standard Error of Regression	Adjusted F-statistic	Theil's U	R <sup>2</sup>
General Model	$S_t = 1.95 - 0.126S_{t-1} + 0.852F_t + 0.280F_{t-1}$ (1.281) (-1.1) (18.09) (2.249)	No	Normality Heteroscedasticity	21.00	0.573		0.00248	0.98
Static Model	$S_t = 2.505 + 0.9962F_t$ (1.597) (53.97)	No	Normality, Cusumsq	24.85	0.6142	5.94	0.0019	0.97
Leading Indicator	$S_t = 6.666 + 0.9472F_{t-1}$ (1.866) (22.533)	No	Normality, Cusum, Cusumsq	129.2	1.399	164.9	0.262	0.89
Growth Rate	$\Delta S_t = 0.0042 + 0.8578\Delta F_t$ (0.039) (11.89)	No	Autocorrelation ARCH, Cusumsq	52.76	0.8941	48.40	0.2639	0.68
Distributed Lag	$S_t = 1.608 + 0.8532F_t + 0.1536F_{t-1}$ (1.078) (18.09) (3.259)	No	Normality, Cusumsq Heteroscedasticity	21.39	0.5737	1.209	0.0017	0.98
Error Correction	$\Delta S_t = 2.4622 + 0.8488\Delta F_{t-1} - 1.1276(S_{t-1} - F_{t-1})$ (9.5545) (18.48) (-9.899)	No	Normality	21.04	0.5689	0.122	0.3664	0.87
Common Factor	$S_t = 2.3294 + 0.9983F_t$ (1.648) (60.01)	No	Normality, Cusumsq	24.52	0.6142	5.87	0.0019	0.97
Dead Start	$S_t = 7.148 + 1.1285F_{t-1} - 0.1823S_{t-1}$ (1.95) (3.986) (-0.6476)	No	Normality,	128.4	1.4055	327.4	0.00265	0.89
Partial Adjustment	$S_t = 1.535 + 0.1131S_{t-1} + 0.8917F_t$ (0.986) (2.53) (19.83)	No	Normality, Cusumsq Heteroscedasticity Autocorrelation	22.66	0.5905	5.06	0.0018	0.98
General (Corrected)	$S_t = 1.949 + 0.154S_{t-1} + 0.851F_t + 0.2802(F_{t-1} - S_{t-1})$ (1.281) (3.278) (18.09) (2.249)	No	Normality	21.00	0.5728		0.0248	0.98
Dead Start (Corrected)	$S_t = 7.15 + 0.9462S_{t-1} + 1.1285(F_{t-1} - S_{t-1})$ (1.95) (22.39) (3.986)	No	Normality	128.4	1.4055	327.4	0.0026	0.88

See Notes to Table 4.

Table 6: Regression Results for 8.5 Per Cent Capital 2010

Model Name	Model Specification and Parameter Estimates	Passes all Diagnostic Tests	Failed Diagnostic Tests	RSS	Standard Error of Regression	Adjusted F-statistic	Theil's U	R <sup>2</sup>
General Model	$S_t = 4.377 + 0.005S_{t-1} + 0.874F_t + 0.1283F_{t-1}$ (2.551) (0.0467) (17.38) (0.9901)	No	Functional Form	23.9	0.6121		0.0025	0.98
Static Model	$S_t = 5.189 + 0.9987F_t$ (3.197) (52.30)	No	Heteroscedasticity	26.6	0.6353	7.07	0.0021	0.97
Leading Indicator	$S_t = 9.589 + 0.9468F_{t-1}$ (2.605) (21.86)	No	Normality, Cusum Cusumsq	137	1.441	151.02	0.00296	0.88
Growth Rate	$\Delta S_t = 0.00497 + 0.8774\Delta F_t$ (0.0469) (12.44)	No	Autocorrelation ARCH, Cusumsq	50.4	0.874	38.41	0.2747	0.70
Distributed Lag	$S_t = 4.41 + 0.874F_t + 0.1338F_{t-1}$ (2.79) (17.51) (2.683)	Yes		23.9	0.6074	0.002	0.0025	0.97
Error Correction	$\Delta S_t = 5.0535 + 0.8701\Delta F_t - 0.9957(S_{t-1} - F_{t-1})$ (3.3869) (17.72) (-8.44)	Yes		23.9	0.6121	0.1805	0.3818	0.85
Common Factor	$S_t = 5.2029 + 0.9985F_t$ (3.155) (51.46)	Yes		26.6	0.6401	7.23	0.002	0.97
Dead Start	$S_t = 9.81 - 0.0399S_{t-1} + 0.987F_{t-1}$ (2.45) (-0.142) (3.473)	No	Normality, ARCH, Cusum	137	1.4524	301.93	0.0030	0.88
Partial Adjustment	$S_t = 3.905 + 0.1139S_{t-1} + 0.8931F_t$ (2.37) (2.472) (19.21)	No	Heteroscedasticity	24.3	0.612	0.9805	0.0026	0.98
General (Corrected)	$S_t = 4.38 + 0.134S_{t-1} + 0.874F_t + 0.128(F_{t-1} - S_{t-1})$ (2.55) (2.66) (17.38) (0.99)	No	Functional Form	23.9	0.6121		0.0025	0.98
Dead Start (Corrected)	$S_t = 9.802 + 0.9467S_{t-1} + 0.987(F_{t-1} - S_{t-1})$ (2.449) (21.69) (3.473)	No	Normality ARCH	137	1.4524	301.93	0.0030	0.88

See Notes to Table 4.

Table 7: Regression Results for 8.75 Per Cent Capital 2012

Model Name	Model Specification and Parameter Estimates	Passes all Diagnostic Tests	Failed Diagnostic Tests	RSS	Standard Error of Regression	Adjusted F-statistic	Theil's U	R <sup>2</sup>
General Model	$S_t = -2.2 - 0.1697S_{t-1} + 0.946F_t + 0.335F_{t-1}$ (-1.53) (-1.49) (20.95) (2.572)	No	Heteroscedasticity	19.32	0.5495		0.0014	0.99
Static Model	$S_t = -1.148 + 1.087F_t$ (-0.753) (60.62)	No	Normality Heteroscedasticity	23.48	0.5965	6.90	.00103	0.98
Leading Indicator	$S_t = 3.552 + 1.032F_{t-1}$ (0.9165) (22.63)	No	Normality, Cusum	152.01	1.5176	219.76	0.00187	0.89
Growth Rate	$\Delta S_t = 0.0095 + 0.09236\Delta F_t$ (0.088) (12.99)	No	Autocorrelation ARCH, Cusumsq	51.21	0.8808	52.81	0.2420	0.71
Distributed Lag	$S_t = -2.044 + 0.944F_t + 0.1534F_{t-1}$ (-1.42) (20.72) (3.369)	No	Normality, Cusumsq Heteroscedasticity	19.99	0.5546	22.34	0.00136	0.98
Error Correction	$\Delta S_t = 5.2508 + 0.8994\Delta F_t - 0.8423(S_{t-1} - F_{t-1})$ (6.9189) (16.541) (-6.947)	Yes		29.388	0.6724	33.36	0.6341	0.84
Common Factor	$S_t = -1.391 + 1.09F_t$ (-1.078) (71.79)	No	Normality	22.66	0.5904	6.80	0.0009	0.98
Dead Start	$S_t = 3.479 + 1.125F_{t-1} - 0.0864S_{t-1}$ (0.889) (3.244) (-0.272)	No	Normality, Cusum	151.84	1.5284	438.95	0.00196	0.88
Partial Adjustment	$S_t = -1.711 + 0.1058S_{t-1} + 0.98F_t$ (-1.54) (2.57) (21.74)	No	Normality, Cusumsq Heteroscedasticity Autocorrelation	21.32	0.5728	6.64	0.00176	0.98
General (Corrected)	$S_t = -2.20 + 0.165S_{t-1} + 0.946F_t + 0.335(F_{t-1} - S_{t-1})$ (-1.54) (3.609) (20.95) (2.57)	No	Normality	19.32	0.5495		0.00139	0.99
Dead Start (Corrected)	$S_t = 3.479 + 1.0387S_{t-1} + 1.125(F_{t-1} - S_{t-1})$ (0.889) (19.61) (3.244)	No	Normality Heteroscedasticity	151.84	1.5284	438.95	0.00196	0.88

See Notes to Table 4.

Model has the lowest residual sum of squares and standard error of regression of these sub-models. However, it fails the normality test at the 5 per cent level; it passes it at the 1 per cent level of significance.

- (5) A pairwise comparison of the Distributed Lag Model with the Error Correction Model shows that the coefficient restrictions imposed are supported for both models in the case of three of the four gilts (the exception in both cases being the 8.75 per cent Capital 2012). Where both sets of coefficient restrictions are supported by the data the Error Correction Model has both a lower residual sum of squares and standard error of regression for two of the three contracts and fails fewer of the diagnostic tests. (The distributed lag model has a marginally lower residual sum of squares and standard error of regression for the 2010 contract but note that the dependent variable for the error correction model is the change in as against the level of the spot price). The results for the general model suggest that there is little to divide the models once both are related to the same dependent variable.
- (6) As noted above, a number of the models fail the Bera-Jarque test for normality of residuals. This test is, however, very sensitive to outliers and for this reason is frequently used as an informal diagnostic tool. Accordingly, the residuals of the general model for the two cheapest-to-deliver gilts were inspected (the plots for each are provided in Figures 2 and 3) and reveal sizeable outliers for the 8th, 9th and 54th observations for both contracts; there is an additional outlier for the 15th observation which is singular to the 8.75 per cent Capital 2012 gilt. These outliers underly the failure of the Bera Jarque test. When the outliers are modelled by inclusion of dummy variables the normality tests are passed. However, as we can advance no plausible account for these outliers we have resisted the temptation to purge the reported results of their effects.<sup>5</sup>
- (7) None of the models displays heteroscedasticity at the 1 per cent level of significance although a few of the sub-models do so at the 5 per cent level. If Equation (8) is solved iteratively forward it is easy to show that the error term  $v_t$  is heteroskedastic.

5. Inclusion of the dummy variables also reduces occasional breaches of the heteroscedasticity tests at the 5 per cent level. None of the results display heteroscedasticity at the 1 per cent level, with or without the inclusion of dummies.

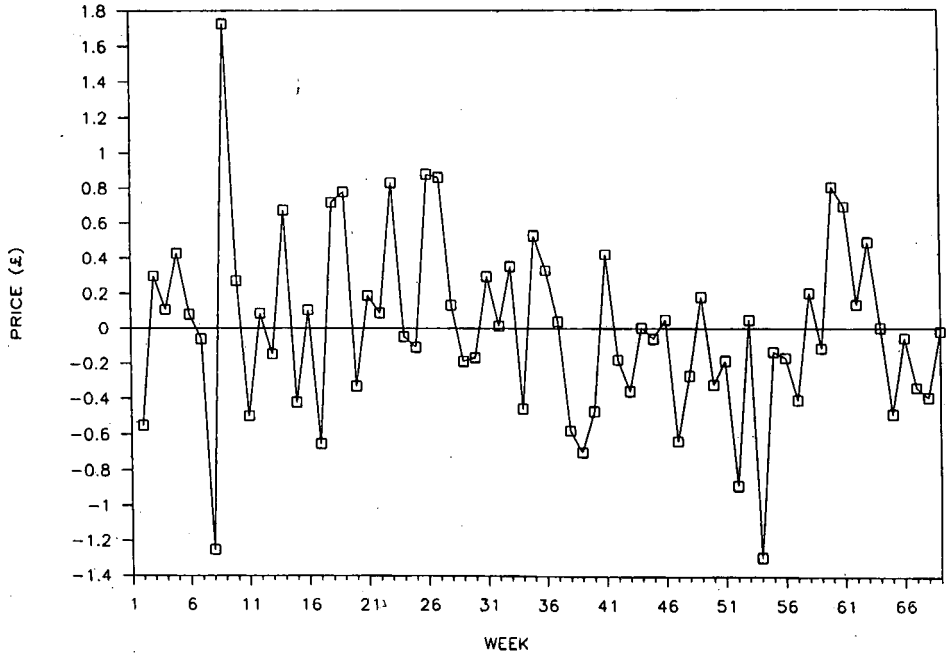


Figure 2: 9 Per Cent Capital 2006 Residuals for General Model

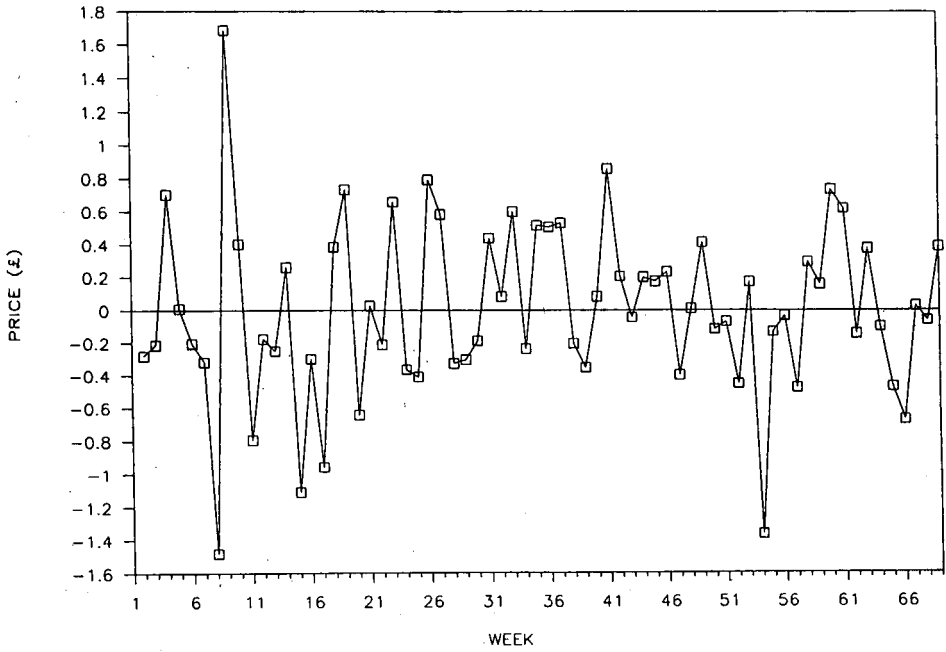


Figure 3: 8.75 Per Cent Capital 2012 Residuals for General Model



In our reported estimations the scale of the imparted heteroscedasticity is too small to lead to a single breach of the ARCH tests. Some minor gain in the efficiency of parameter determination is possible from explicitly modelling the pattern of heteroscedasticity over the individual lives of each futures contract. However, since the estimators are unbiased and consistent and the general quality of parameter determination is high we have left the OLS estimates stand.

- (8) All of the models pass Chow's (first) test for structural stability . As noted in the Tables a number of the models breach the 5 per cent bounds for the Cusum and Cusumsq tests (the power of these latter tests is low).
- (9) All of the models also pass the Chow's (second) test for adequacy of predictions (both LM and F versions). In addition to the adequacy of predictions it is also useful to gauge their out-of-sample accuracy. Theil's inequality U coefficient performs this task. Its numerical value is bounded in the zero-one interval with a numerical value of zero signifying a perfect fit between the 9 predicted and actual values. The reported U values indicate a high accuracy of forecast for the competing models, particularly so for the general corrected, static and common factor models. In view of the extremely low U values we considered it unfruitful to decompose U into its bias, variance and covariance proportions. Finally, note that the U values for the growth rate and error correction models are not directly comparable to the remaining models since they relate to forecasts of the change in rather than the level of spot prices.
- (10) Subject to the exception of 8.75 per cent Capital 2012, the results imply that equilibrium spot and futures prices are equal. Thus over extended time intervals a naive (100 per cent) hedge is efficient. Over short time horizons the price relationship follows an error correction process and efficiency gains may result from designing a dynamic hedging strategy which incorporates the process.

In summary we conclude that the Error Correction Model is the best sub-model of the data generation process and that the growth rate model receives little support from the data.

#### *Dynamic Hedging Implications*

The *total* return,  $R_t$ , to a hedged portfolio consisting of spot and futures over a *single* discrete time interval from  $t-1$  to  $t$  is given by,

$$R_t = \Delta S_t - h_t \Delta F_t \quad (10)$$

where  $h_t$  is the hedge ratio of future to spot contracts. Substituting the general model, Equation (9), for  $\Delta S_t$  gives,

$$R_t = \alpha_0 + \beta_0 \Delta F_t + (\alpha_1 - 1) [S_{t-1} - dF_{t-1}] + v_t - h_t \Delta F_t \quad (11)$$

where  $d = (\beta_0 + \beta_1)/(1 - \alpha_1)$ .

Letting  $\text{Var}(R_t)$  represent return risk, inspection of Equation (11) shows that risk is minimised by setting  $h_t$  equal to,

$$h_t^* = \beta_0 + (\alpha_1 - 1) [S_{t-1} - dKF_{t-1}] / \Delta F_t \quad (12)$$

Letting  $h_t = h_t^*$  in Equation (8) yields,

$$R_t = \alpha_0 + v_t. \quad (13)$$

Consequently,  $\text{Var}(R_t) = \text{Var}(v_t)$ . In short, risk is reduced to an irreducible white noise error term.

In contrast, a static hedge strategy entails setting  $h_t = \beta_0$ . This is a less efficient procedure. Further, when the coefficient estimate for  $\beta_0$  is based on the incorrectly specified growth rate model it will be biased (omitted variable bias). A static hedge does, however, carry the advantage of being operationally easy to implement; a dynamic hedging strategy requires forecasts of  $\Delta F_t$  and may entail volatile changes in the dynamic hedge ratio and higher transactions costs which will mitigate the net returns to the strategy. (Implementing a *multi-period* dynamic hedging strategy within an expected utility maximising framework is treated more fully in Kenneally and Cronin (forthcoming)). The general model nests the remaining sub-models and, hence, dynamically efficient hedging strategies, associated with each sub-model, are obtained by imposing the appropriate coefficient restrictions, as set out in Table 3.

The hedging techniques considered in this paper have all been regression based. The advantages of these models is that they are parsimonious in the data required to estimate the model parameters and, as illustrated, yield efficient hedging strategies which are easy to derive and implement. The cost of this simplicity is that some factors which affect the spot-futures price relationship have been set aside. In particular, it is well known that spot and futures prices are linked via the cost-of-carry and that the evolution of the term structure of interest rates has an important bearing on the latter. Also, the fact that any one of four gilts may be delivered against the long gilt futures contract implies that the contracts contains imbedded options. We

have not attempted to model either of these features.

Duration based techniques in contrast, represent a popular alternative procedure for hedging or immunising the return on a gilt portfolio. They also have explicit regard to the convexity of the gilt's price-yield relationship and the stochastic process driving the evolving term structure. Duration measures summarise the time structure and distribution of gilt cash flows. Risk reduction is achieved by exploiting the opposing directional effects of changing market yield conditions on the interest-on-interest and capital gain components of portfolio return. Broadly, this is achieved by equating the portfolio's duration and the investor's holding period. Some important historical contributions in the development of duration techniques are provided in Hawawini (1982). Bierwag, Kaufman and Toevs (1983) and Gay and Kolb (1983) outline its use in bond portfolio risk management. A comparison of alternative hedging methodologies is provided by Toevs and Jacob (1984). Gilibert and Girard (1988) provide a survey of recent uses. A detailed modelling of stochastic process risk which would facilitate a detailed comparison of regression and duration based hedging strategies is beyond the scope of this paper.<sup>6</sup>

## V CONCLUSION

The short-run hedging effectiveness of the IFOX long gilt futures contract is enhanced by adopting a dynamic hedging strategy. The appropriate dynamic hedging strategy requires that the underlying relationship between spot and futures prices be correctly specified. Our findings show that the data generation process is well represented by an AD(1,1) model. We used this model to derive a formula for effective dynamic hedging.

If a static hedging strategy is preferred for operational ease or to curtail transactions costs then a naive hedge seems in order given the strong support for equality between spot and futures equilibrium prices.

The growth rate model is commonly used in identifying effective static hedge ratios. It is, however, dynamically misspecified and yields biased estimates of the required hedging strategy parameters.

Finally, given an efficient hedging strategy, risk reduction entails reduction in expected return. An empirical delineation of the efficiency frontier and the expected utility maximising choice of position on the frontier are the subject of further research.

6. Hogan provides some evidence for the relative efficacy of regression, duration and conversion based hedging strategies in relation to the IFOX long gilt futures.

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