

The glueball spectrum from novel improved actions.

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Results for the inter-quark potential and low-lying $SU(3)$ glueball spectrum from simulations using a new improved action are presented. The action, suitable for highly anisotropic lattices, contains a two-plaquette term coupling with a negative coefficient as well as incorporating Symanzik improvement.

1. INTRODUCTION

The QCD glueball spectrum has been investigated in low-cost simulations using anisotropic lattices [1]. To reduce the computational overhead, the spatial lattice was kept rather coarse (0.2-0.4 fm) while the temporal spacing was made much finer. The fine temporal grid allows adequate resolution of the Euclidean-time decay of appropriate correlation functions which, for gluonic states are rather noisy and fall too rapidly on coarse lattices.

In these simulations, the scalar glueball suffered from large finite cut-off effects. The mass in units of r_0 fell sharply until the spatial lattice spacing, a_s was about 0.25 fm when the mass rose again; the “scalar dip”.

At the conference last year, we presented results from simulations with an anisotropic Wilson “two-plaquette” action which included a term constructed from the product of two parallel plaquettes on adjacent time-slices [2]. This was found to reduce the scalar dip significantly. Here, we report on the status of simulations in progress using a Symanzik-improved action including a similar two-plaquette term.

In this study, we tune the anisotropy parameter in the lattice action to recover Euclidean invariance in the “sideways” potential. With these parameters fixed, we investigate the inter-quark potential for this action as an initial test that the benefits of the Symanzik program are preserved by the addition of the extra term. We are cur-

rently computing the glueball spectrum for this action.

2. THE ACTION

Following Ref. [2], we begin with the plaquette operator,

$$P_{\mu\nu}(x) = \frac{1}{N} \text{ReTr} U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x).$$

The Wilson (unimproved) discretisation of the magnetic field strength is then constructed from the spatial plaquette.

$$\begin{aligned} \Omega_s &= \sum_{x,i>j} \{1 - P_{ij}(x)\} \\ &= \frac{\xi_0}{\beta} \int d^4x \text{Tr} B^2 + \mathcal{O}(a_s^2), \end{aligned} \quad (1)$$

where i, j are spatial indices and ξ_0 is the anisotropy, a_s/a_t at tree-level in perturbation theory. We introduce a term which correlates pairs of spatial plaquettes separated by one site temporally

$$\Omega_s^{(2t)} = \frac{1}{2} \sum_{x,i>j} \{1 - P_{ij}(x) P_{ij}(x+\hat{t})\}. \quad (2)$$

The separation of the two plaquettes allows the standard Cabibbo-Marinari and over-relaxation gauge-field update methods to be applied. Including two-plaquette terms adds a computational overhead of only 10% to our improved action workstation codes.

It can be shown that for all ω , the operator combination,

$$\tilde{\Omega}_s = (1 + \omega) \Omega_s - \omega \Omega_s^{(2t)} \quad (3)$$

*Poster presented by M.P.

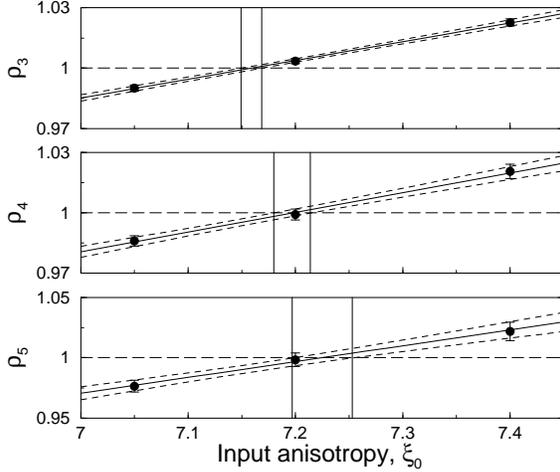


Figure 1. The ratio of Eqn. 5 for $n = 3, 4, 5$

has an identical expansion in powers of $a_{s,t}$ (at tree-level) to Ω_s up to $\mathcal{O}(a_s^4)$. Thus, starting from the improved action S_{II} used in Refs. [1,3], it is straightforward to construct a Symanzik improved, two-plaquette action by simply replacing the spatial plaquette term in S_{II} with the linear combination $\tilde{\Omega}_s$ of Eqn. 3. In full, this action is

$$S_\omega = \frac{\beta}{\xi_0} \left\{ \frac{5(1+\omega)}{3u_s^4} \Omega_s - \frac{5\omega}{3u_s^8} \Omega_s^{(2t)} - \frac{1}{12u_s^6} \Omega_s^{(R)} \right\} + \beta \xi_0 \left\{ \frac{4}{3u_s^2 u_t^2} \Omega_t - \frac{1}{12u_s^4 u_t^2} \Omega_t^{(R)} \right\}, \quad (4)$$

with Ω_t the temporal plaquette and $\Omega_s^{(R)}, \Omega_t^{(R)}$ the 2×1 rectangle in the (i, j) and (i, t) planes respectively. This action has leading $\mathcal{O}(a_s^4, a_t^2, \alpha_s a_s^2)$ discretisation errors and only connects sites on adjacent time-slices, ensuring the free gluon propagator has only one real mode.

The free parameter ω is chosen such that the approach to the QCD continuum is made on a trajectory far away from the critical point in the plane of fundamental-adjoint couplings. Close to the QCD fixed point, physical quantities should be weakly dependent on ω . This provides us with a consistency check, however the data presented here are for one value only, $\omega = 3$.

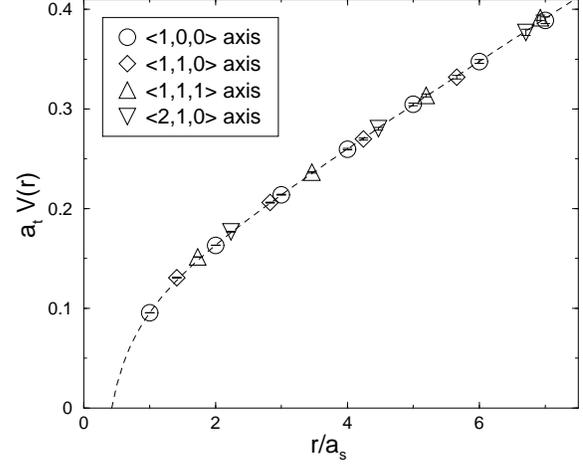


Figure 2. The inter-quark potential of S_{II} with $a_s \approx 0.22$ fm for sources separated along different lattice axis.

3. TUNING THE ANISOTROPY

At finite coupling, the anisotropy measured using a physical probe differs from the parameter in the action at $\mathcal{O}(\alpha_s)$ [4]. In previous calculations, we relied upon the smallness of these renormalisations for the (plaquette mean-link improved) action S_{II} . For the action of Eqn. 4, these renormalisations are larger and thus we chose to tune the input parameter in the action to ensure that the potentials measured along anisotropic axes matched. We follow a similar procedure to Ref. [5]. The potentials between two static sources propagating along the z -axis for separations on both fine and coarse axes, V_t and V_s respectively, are measured using smeared Wilson loops. Since the UV divergences due to the static sources are the same, tuning ξ_0 such that the ratio

$$\rho_n = \frac{a_s V_s(na_s)}{a_s V_t(mna_t)} \equiv 1, \quad (5)$$

implies the anisotropy $\xi_V = m$ ($m \in \mathbb{Z}$). A consistency check is provided by studying different coarse source separations, na_s . Fig. 1 shows this tuning for $n = 3, 4, 5$, where the desired anisotropy is 6. Consistency is observed for $n = 4$ and 5 and the appropriate ξ_0 is found to better than 1%.

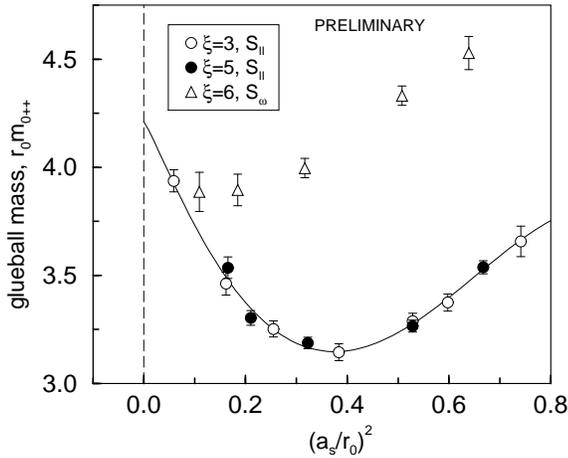


Figure 3. The scalar glueball of both Symanzik improved actions, S_{II} and S_{ω}

4. SIMULATION RESULTS

4.1. The inter-quark potential

The replacement of the spatial plaquette in S_{II} with $\tilde{\Omega}_s$ of Eqn. 3 should lead only to changes in the irrelevant operators responsible for $\mathcal{O}(a_s^4, \alpha_s a_s^2)$ errors. To test this replacement still generates an improved action with the good rotational invariance of S_{II} , the inter-quark potential was computed for a variety of different inter-quark lattice orientations. The potential is shown in 2, and shows excellent rotational invariance. We conclude that the benefits of the Symanzik improvement programme are preserved by including the two-plaquette term for a typical value of ω useful for glueball simulation.

4.2. The glueball spectrum

At present, we are computing the glueball spectrum on the $\xi_V = 6$ tuned lattices. Preliminary data are presented in Figs. 3 and 4. In Fig. 3, the finite-lattice-spacing artefacts in the scalar glueball mass for the new action are compared to those of S_{II} . The lattice cut-off dependence is seen to be significantly reduced and for the range of lattice spacings studied here, the mass rises monotonically with lattice spacing rather than falling first to a minimum. Fig. 4 shows the lattice spacing dependence on the tensor and pseudo-

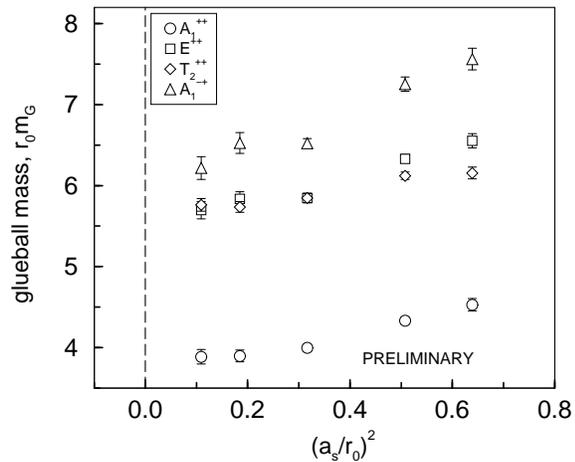


Figure 4. The scalar, tensor and pseudoscalar glueballs of S_{ω}

doscalar glueballs. Their lattice spacing dependence is similar to the form for S_{II} and consistent with leading $\mathcal{O}(a_s^4)$ behaviour.

5. CONCLUSIONS

Preliminary data from our simulations of the Symanzik improved action of Eqn. 4 suggest the scalar dip is removed by inclusion of a two-plaquette term with negative coefficient, consistent with the argument that the poor scaling of the scalar glueball, even after Symanzik improvement, is caused by the presence of a nearby critical point.

The inter-quark potential on this new action exhibits equally good rotational symmetry to the improved actions of Refs. [1,3].

REFERENCES

1. C. Morningstar and M. Peardon, Phys. Rev. D56 (1997) 4043-4061, Phys. Rev. D60 (1999) 034509.
2. C. Morningstar and M. Peardon, Nucl. Phys. B (Proc Suppl.) 73 (1999) 927-929.
3. H. Trotter and N. Shakespeare, Phys. Rev. D59 (1999) 014502.
4. M. Alford *et. al*, in preparation.
5. T. Klassen, Nucl. Phys. B533 (1998) 557-575.