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Estimated Errors in $|V_{cd}|/|V_{cs}|$ from Semileptonic D Decays

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We estimate statistical and systematic errors in the extraction of the CKM ratio $|V_{cd}|/|V_{cs}|$ from exclusive D-meson semileptonic decays using lattice QCD and anticipated new experimental results.

1. INTRODUCTION

High statistics experimental studies of D mesons, such as E831 (FOCUS) and E791, are expected to yield $\mathcal{O}(10^6)$ fully reconstructed D decays with better momentum resolution than preceding experiments[1,2]. Anticipating high precision experimental results, we consider how well the CKM ratio $|V_{cd}|/|V_{cs}|$ can be determined from a ratio of $D \to \pi e\nu$ to $D \to Ke\nu$ decay rates and lattice QCD.

For the Cabibbo allowed $D \to Ke\nu$ decay, lower statistics and poorer resolutions in earlier experiments meant hadronic matrix elements could be described by functions of only two parameters: the CKM matrix element times a form factor at zero momentum transfer and the pole mass in a pole dominance *Ansatz*. The PDG quotes a 3% error for $|V_{cs}|f_+^K(0)$ based upon such techniques[3]. The uncertainty due to this *Ansatz* is difficult to quantify. They quote a 14% uncertainty in the theoretical determination of $f_+^K(0)$ resulting in a 15% uncertainty for $|V_{cs}|$.

$|V_{cd}|$ is determined with an error of 7% from ν-$\overline{\nu}$ production of charm off of valence d quarks[3]. Combining this uncertainty and that for $|V_{cs}|$ we deduce an error of about 17% in the current best value of $|V_{cd}|/|V_{cs}|$.

We examine uncertainties in determining $|V_{cd}|/|V_{cs}|$ directly from a ratio of semileptonic decay rates. We expect that both experimental and theoretical systematic errors common to $D \to Ke\nu$ and $D \to \pi e\nu$ will be reduced in ratios.

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2. PROCEDURE

We suggest the ratio of CKM matrix elements can be determined by a double ratio

$$\frac{|V_{cd}|}{|V_{cs}|} = \frac{E_{\pi/K}(|\mathbf{p}|^{cut})}{T_{\pi/K}(|\mathbf{p}|^{cut})} \,. \tag{1}$$

The maximum recoil momentum cut, $|\mathbf{p}|^{cut}$, will be adjusted to minimize combined experimental and theoretical errors[4]. The ratio of partial widths

$$E^2_{\pi/K}(|\mathbf{p}|^{cut}) \equiv \frac{\int_0^{|\mathbf{p}|^{cut}} d\Gamma(D \to \pi e\nu)}{\int_0^{|\mathbf{p}|^{cut}} d\Gamma(D \to Ke\nu)} \tag{2}$$

is to be determined experimentally. $T_{\pi/K}(|\mathbf{p}|^{cut})$ is provided by theory calculation. In the D-meson rest frame,

$$T^2_{\pi/K}(|\mathbf{p}|^{cut}) \equiv \frac{\int_0^{|\mathbf{p}|^{cut}} d|\mathbf{p}|_\pi \mathbf{p}_\pi^4 |f_+^\pi(E_\pi)|^2/E_\pi}{\int_0^{|\mathbf{p}|^{cut}} d|\mathbf{p}|_K \mathbf{p}_K^4 |f_+^K(E_K)|^2/E_K} \tag{3}$$

where hadronic matrix elements are expressed conveniently by form factors f_+^X.

Matrix elements

$$\mathcal{M}_\mu^X(p_X, p_D) = \frac{\langle X(p_X)|V_\mu|D(p_D)\rangle}{\sqrt{2E_D 2E_X}} \tag{4}$$

are determined in the continuum limit of our lattice results. $\mathcal{M}_\mu^X$ do not depend upon the heavy quark mass in the infinite mass limit. The *continuum* relation

$$\mathcal{M}_\mu^X(p_X, p_D) = \left[(p_D + p_X)_\mu f_+^X(q^2) + (p_D - p_X)_\mu f_-^X(q^2)\right] / \sqrt{2E_D 2E_X} \tag{5}$$

is used to extract the form factors.

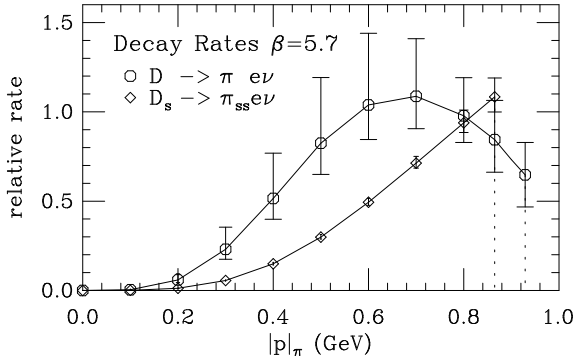

Figure 1. Decay rates (with statistical errors) versus daughter recoil momentum.

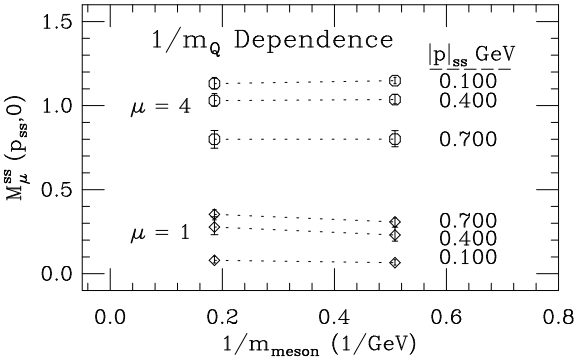

Figure 2. Comparison of $\mathcal{M}_\mu^{s\overline{s}}(\mathbf{p}_{s\overline{s}}, \mathbf{0})$ for the decay of B_s and D_s mesons at $\beta = 5.7$.

3. MATRIX ELEMENTS

We use the tadpole-improved Sheikholeslami-Wohlert quark action with the plaquette-determined mean field coefficient u_0. We obtain $\mathcal{O}(a)$-improved matrix elements using the Fermilab interpretation for heavy quarks and tree-level $\mathcal{O}(a)$-improved currents[5]. Note that the vector current renormalization factor cancels in ratio $T_{\pi/K}$.

We have computed three-point correlators at $\beta = 5.7$, 5.9 and 6.1. We determine the lattice spacing, a, using the charmonium $1P$-$1S$ splitting and the bare charm mass using the spin-averaged $1S$ charmonium kinetic mass. The same gauge-field ensembles and procedures were used to study B and D meson decay constants[6]. Reference [7] describes our $B \to \pi e\nu$ results on these lattices and provides more details on our three-point function calculations.

For illustration, and to reduce some known systematic errors, we take $|\mathbf{p}|^{cut} = 0.7\,\mathrm{GeV}$. For this preliminary study we take our best estimate for matrix elements from $\beta = 5.7$. We approximate the $D \to Ke\nu$ decay by $D_s \to \pi_{s\overline{s}} e\nu$ where a pseudoscalar "$\pi_{s\overline{s}}$" meson consists of s and $\overline{s}$ valence quarks. We denote the ratio in Eq. 3 for the D_s decay by $T_{\pi/s\overline{s}}$. The analysis for $D \to Ke\nu$ is underway.

In Fig. 1 we show differential decay rates for $\beta = 5.7$. The ratio of areas for $|\mathbf{p}|^{cut} = 0.7\,\mathrm{GeV}$ gives $T_{\pi/s\overline{s}} = 1.53 \pm 0.27$ where the error is statistical. Systematic errors are the subject of the rest of the paper.

We use all three lattices to study the cutoff dependence of $D_s \to \pi_{s\overline{s}} e\nu$ matrix elements. We compare partial widths at $\beta = 5.7$ to partial widths computed from $\mathcal{M}_\mu^{s\overline{s}}$ extrapolated to the continuum linearly with the lattice spacing. Matrix elements $\mathcal{M}_\mu^{s\overline{s}}(\mathbf{p}, \mathbf{0})$ for a given $\mathbf{p}$ on each lattice were found by interpolating among $\mathcal{M}_\mu^{s\overline{s}}$ values calculated for discrete values of $\mathbf{p}$. Discretization errors in the width increase with $\mathbf{p}$, as expected, reaching 21% for $|\mathbf{p}| = 0.7\,\mathrm{GeV}$. Momentum dependent errors of this magnitude were anticipated by errors estimates for free-field tree-level quark matrix elements[4]. Errors for the pion width are expected to be similar to those for the $s\overline{s}$ width. These errors, however, will tend to cancel in ratios. Hence we estimate residual cutoff errors are below 10% for $T_{\pi/s\overline{s}}(0.7)$.

Our bare charm masses were determined in quarkonia. Charm masses determined using heavy-light mesons differ due to discretization errors and the quenched approximation. We find at most a 10% difference in bare m_c comparing the two methods. Figure 2, which compares $\mathcal{M}_\mu^{s\overline{s}}(\mathbf{p}, \mathbf{0})$ calculated for D_s and B_s mesons, shows that $1/m_Q$ corrections are small in this case[4]. Numerical results and pole dominance arguments indicate that $\mathcal{M}_\mu^\pi$ has somewhat more

$1/m_Q$ dependence than $\mathcal{M}_\mu^{s\overline{s}}$. We estimate the uncertainty in adjusting m_c leads to a 1% error in $T_{\pi/s\overline{s}}$.

The quenched approximation leads to systematic differences in lattice spacing determinations. We take the charmonium $1P$-$1S$ lattice spacing as our best estimate of a. Repeating our analysis for another reasonable choice for the lattice spacing, $a(f_K)$, determined by the kaon decay constant, changes $T_{\pi/s\overline{s}}$ by about 1%. This almost certainly underestimates the error due to quenching.

Correlators are computed for light quark masses between about 0.4 to 1.2 times m_s. For lighter masses, correlators begin to show increasing fluctuations caused by nearby unphysical Dirac operator zero-modes found on a small subset of our gauge configurations. Hence we must obtain matrix elements for up and down quarks by "chiral" extrapolations of matrix elements computed for masses close to the strange quark. Near zero recoil momentum these extrapolations are more difficult because matrix elements are sensitive to the light quark mass. This can be understood as the effect of nearby poles in a pole dominance picture. The effect upon $T_{\pi/s\overline{s}}$ is small, however, since the decay rate is kinematically suppressed by p^4 (see Eq. 3). Matrix elements show less dependence upon quark mass at larger recoil momenta. Statistical noise increases with recoil momenta, however, making extrapolations there noisier. Hence we find a "window" at moderate recoil momentum where chiral extrapolations are most dependable. By varying our extrapolation procedures we estimate chiral extrapolations contribute a 10% error to $T_{\pi/s\overline{s}}(0.7)$. If correlators could be reliably computed for masses significantly below m_s our chiral extrapolation uncertainties would decrease. The Modified Quenched Approximation shows promise in this respect[8].

Errors due to varying fitting procedures and residual excited state contamination are estimated to be about 2%.

4. CONCLUSIONS

Table 1 summarizes sources of errors in $|V_{cd}|/|V_{cs}|$ for $|\mathbf{p}|^{cut} = 0.7\,\mathrm{GeV}$. These preliminary results are based upon our $\beta = 5.7$ analysis. We are completing the analysis of K and π decays on our finer $\beta = 5.9$ lattice and continuing to study cutoff effects using our 6.1 lattice. Smaller a will result in smaller cutoff effects than our $\beta = 5.7$ results but with similar statistical errors. The error due to the quenched approximation has yet to be fully investigated. A quenching error of 10-20% would not be unexpected, however, in the deviation of $T_{\pi/K}$ from unity.

Table 1
Summary of errors in $|V_{cd}|/|V_{cs}|$ at $\beta = 5.7$

source	%
M.C. statistics	18
fits, excited state contamination	2
residual cutoff dependence	< 10
a determination	1
bare m_s det., extrapolation to m_l	10
bare m_c determination	1

Theoretical uncertainties in $|V_{cd}|/|V_{cs}|$ have been estimated for our procedure with $|\mathbf{p}|^{cut} = 0.7\,\mathrm{GeV}$. We selected this cut to eliminate the largest momentum dependent errors and chiral extrapolation errors. Before quoting a value for $|V_{cd}|/|V_{cs}|$, $|\mathbf{p}|^{cut}$ should be chosen to minimize the total theoretical and experimental errors.

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