Modelling of Engineering Thermal Problems - An Implementation using CBR with Derivational Analogy

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Abstract An interactive cased based reasoning tool for assisting engineers with the mathematical modelling tasks associated with the analysis of thermal problems is described. By representing fundamental thermal modelling scenarios as cases, complex physical systems are modelled in a piecewise fashion by successive application of matching cases. Retrieval is based on the use of qualitative indices, derivational analogy allows for generative adaption of retrieved cases, thereby providing a basis for validating cases in the context of the problem under consideration. This work represents an alternative perspective to model based reasoning approaches that have been applied to model generation to date.

1 Introduction

This paper describes work in progress which aims to develop an interactive case based reasoning system that assists engineers with the mathematical modelling tasks associated with convection heat transfer analysis. This domain is described mathematically by the thermal partial differential equations (PDEs) and is nowadays usually analysed using numerical simulation techniques such as the finite element method. Mathematical modelling precedes numerical analysis and involves abstracting a mathematical model from a real world problem. This is achieved by applying physical and mathematical idealisations, so as to create a model that is computationally realistic to solve, but, at the same time, still retains the important features of the physical system [1,2]. It is for this modelling task that we propose a case-based reasoning solution.

The case base is made up of episodes that represent valid model simplifications. Each case consists of; a model that is close to the real world problem, a simplified but valid model of this physical system and a set of assumptions and transformations involved in producing this simplified model. These assumptions and transformations are a key component of the case representation and entail the use of generative adaptation in using retrieved cases. This is the derivational analogy approach to CBR as advocated by Carbonell [3].

The paper is organised as follows; firstly we describe the domain of convection heat transfer by examining the various issues associated with mathematical modelling. Next we discuss from a modelling perspective, the conceptual approach that we have taken so that case based reasoning techniques could be applied effectively. We then discuss implementation work carried out to date and demonstrate an early prototype system called CoBRA (Case-Based Reasoning Assistant) that focuses on spatial modelling. Finally we discuss the use of derivational analogy techniques and describe the structure and contents of a typical reasoning trace.

2 Modelling in Heat Transfer Convection

Convection heat transfer problems can be defined as physical systems where heat transfer occurs between a solid body and a surrounding fluid medium, each at a different temperature. Numerical analysis of convection problems is usually carried out in number of stages which have been identified as follows [1]:

- **Behavioural Analysis** This is normally the first task in any analysis episode and it involves reasoning about the physical system with the objective of obtaining a behavioural understanding of the underlying phenomena.
- **Physical and Mathematical Modelling** This phase involves applying idealisations and simplifications to various spatial and phenomenological aspects of the physical system with objective of abstracting an analysis model. This task is the focus of the current work.
- **Numerical Simulation** This phase involves simulating the mathematical model by applying numerical techniques such as the finite element method.
- **Visualisation** This stage involves post processing and visualising of the numerical data produced by the simulation process

In this paper, we focus on task of creating an analysis model (physical and mathematical modelling) which is representative of the physical system. We assume that the engineer has already obtained a behavioural understanding of the physical system[1] and consequently, this task is not addressed in this work. The main objective in analysis modelling, is to abstract a mathematical model acting on a domain, that is computationally

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[1] Much work to date in qualitative physics has focused on predicting the qualitative behaviour of domains described by ordinarly differential equations (ODEs). However, little work has been carried out on problems defined by partial differential equations.

realistic to solve whilst at the same time preserves the essential integrity of the physical system. We consider construction of an analysis model to have two aspects; a physical perspective and a mathematical perspective [4]. Physical modelling focuses on spatial or geometric aspects of the problem domain and involve applying modelling strategies such as; taking a two dimensional idealisation of a three dimensional physical system, applying geometric symmetries or carrying out feature modelling. Strategies used in feature modelling, illustrated in Figure 1, can involve either replacing an existing complex feature with a simpler feature, removing the feature and substituting it with an equivalent boundary condition or removing the feature completely without any compensatory measures.

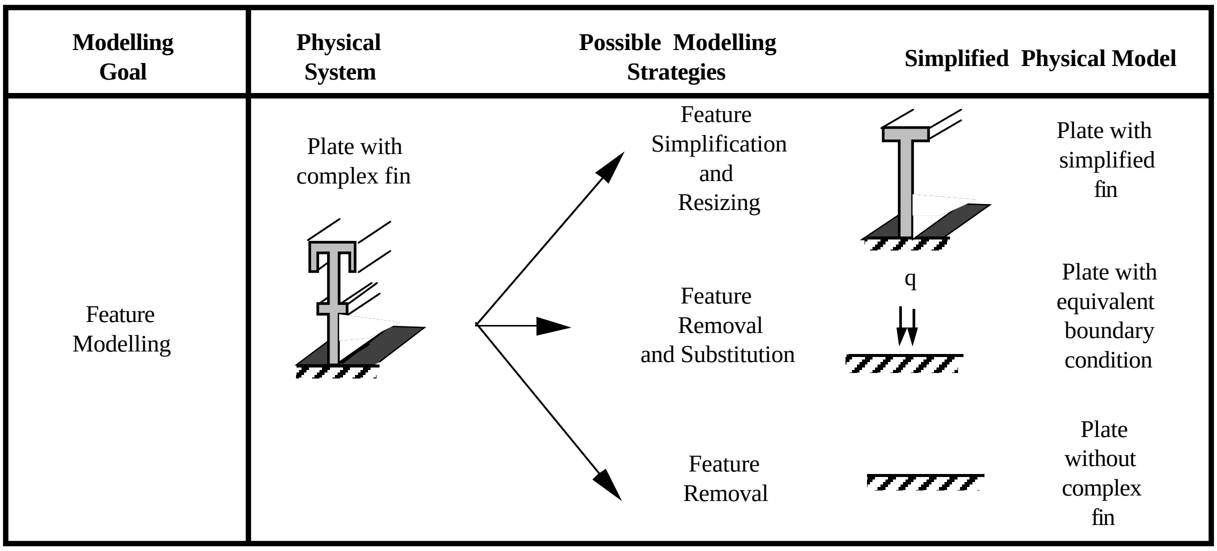


Fig. 1 Feature modelling strategies

Mathematical modelling deals with the construction of a PDE model that describes the thermal heat transfer process. Considering the full thermal PDE, it consists of three sub-equations based on the physical laws of conservation of mass, momentum and energy. Each sub-equation is in turn composed of terms, where each term describes a particular sub-phenomenon. For example, in the energy equation, the diffusion term describes the heat transfer at a molecular level, the advection term describes heat transfer due to bulk motion of the fluid, whereas the viscous dissipation term describes the conversion of mechanical energy to thermal energy due to internal friction effects. In many heat transfer problems it is not necessary to model all these sub-phenomena and therefore terms can be either simplified or even be ignored completely. Another mathematical modelling task (illustrated in Figure 2) is the specification of an analysis volume that defines the extent of the fluid medium to be examined. Although this modelling task has spatial connotations, its specification is essentially governed by type phenomenological analysis that is required by the user.

3 Related Work, Conceptual Matters and Design Issues

3.1 Related work from heat transfer modelling

To our knowledge, no other work with a similar focus and approach has been undertaken to date. However, three related projects that exploit alternative knowledge based techniques in comparable domains are relevant and are briefly discussed here. Ling and Steinberg [5] describe a system that is currently under development which is aimed at modelling conduction heat transfer problems. Model based reasoning is the basis for the approach taken in this work. The system is implemented as part of a greater design system and emphaises is placed on achieving automated modelling decisions without the intervention of the user. Three modelling issues are dealt with and these include; the choice of control region, the determination of the relevant physical processes and the abstraction of appropriate mathematical equations. However, from a geometric perspective, the coverage of this system is confined simple parallelpipid domains. In addition its confinement to conduction based problems makes this domain considerably simpler than convection heat transfer problems. Wentorf and Shephard [2] describe a rule based expert system, that deals with idealisation issues associated with modelling in stress analysis of aircraft. The emphasis in this system is the use of knowledge based techniques to integrate and control interdisciplinary tools in an analysis system such as CAD interfaces, error optimisers and numerical error predictors. Finally, Yip describes a system for simplifying the Navier Stokes (fluid flow) equations using order of magnitude reasoning within a qualitative analysis framework [6]. This system produces idealised PDE models which are mathematically complete, but in many cases have no physical meaning and may sometimes be computationally insolvable. Nevertheless, this work is important as examines how PDE type problems can be tackled using qualitative physics.

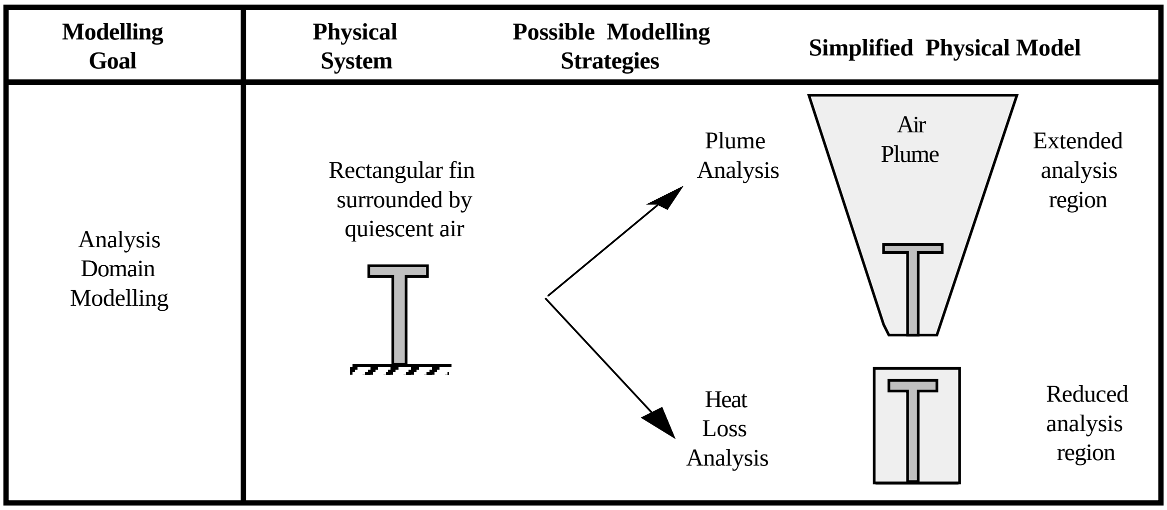


Fig. 2 A subset of mathematical modelling issues

3.2 Conceptual Issues

In this work, particular attention was given to observing how engineers model convection heat transfer problems. These observations have strongly influenced the approach adopted and are reported elsewhere in detail [7]. The key findings are summarised here;

- Engineers usually model complex convection problems in distinct stages. These stages correspond to the physical and mathematical modelling issues outlined in Section 2 and are as follows; spatial modelling, phenomenological modelling, dimensional reduction, temporal modelling and control volume modelling.
- Engineers exploit a number of techniques when modelling convection problems, these include; the use of first principle domain knowledge to reason about modelling strategies, exploitation of previously modelled problems and relying on the guidance from more experienced colleagues. In most modelling episodes, a combination of these techniques are used.
- When investigating a particular modelling stage, e.g., spatial modelling, engineers usually decompose a complex physical system into easily understood sub-problems. These sub-problems are sufficiently low-level to be related to what we call classical engineering modelling scenarios. A scenario typically consists of simple modelling episodes and allow engineering approximations and heuristics to be applied, thereby permitting the modelling issue under consideration to be evaluated easily.

These conclusions influence our approach in two ways; firstly, for an interactive system it is imperative that we aim to accommodate the end-user and therefore the system should attempt to integrate with the modellling patterns used by engineers. Secondly, by capturing engineering first principles, engineering approximations and heuristics within fundamental classical modelling scenarios, it is possible to build a case based reasoning system that is based on episodic based templates that provide guidance for modelling tasks.

3.3 Design Approach adopted in this work

We summarise here our conceptual approach to modelling which forms the basis for the implemented CBR system.

- The system is organised so as to allow modelling to be carried out in distinct stages. In this paper, we consider the stage of spatial feature modelling.
- Within any modelling stage, modelling decisions are taken in a piece wise fashion by examining each modelling issue in turn.
- Case based reasoning with derivational analogy techniques form the core approach. Cases are based on fundamental modelling scenarios and are derived from episodic modelling events.
- Solutions within cases describe a model strategy that can be applied to similar target cases. The strategy is usually in form of some action which is in response to a particular modelling goal.
- Derivational traces describe the full engineering reasoning basis by which a particular modelling solution was reached. They also act as an explanation facility and validator of the case solution. More importantly however, they allow solutions of base cases that are close to the target case to be adapted and applied to the target.

4. Implementation Details

4.1 A Convection Heat Transfer Problem

Figure 3 illustrates a typical convection heat transfer problem that can be tackled by the modelling system. The physical system consists of a finned heat exchanger tube that dissipates heat to the surrounding ambient air. Two complex appendages are attached to the cylindrical base, each appendage has additional minor associated features. The modelling goal in this task is to assess the importance of both the minor features and the appendages themselves, this task corresponds to the spatial modelling phase described in Section 2.

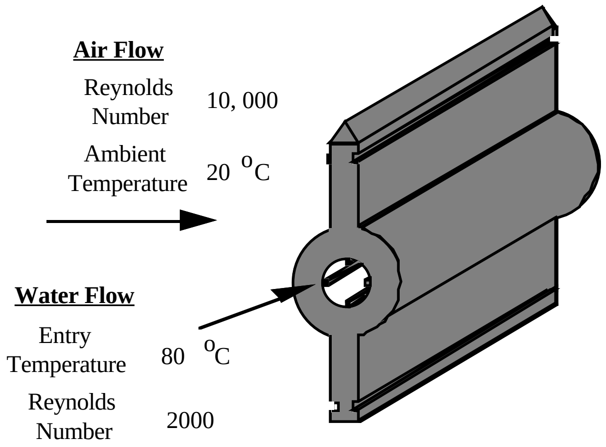


Fig. 3 A finned heat exchanger tube

4.2 Target Case Description

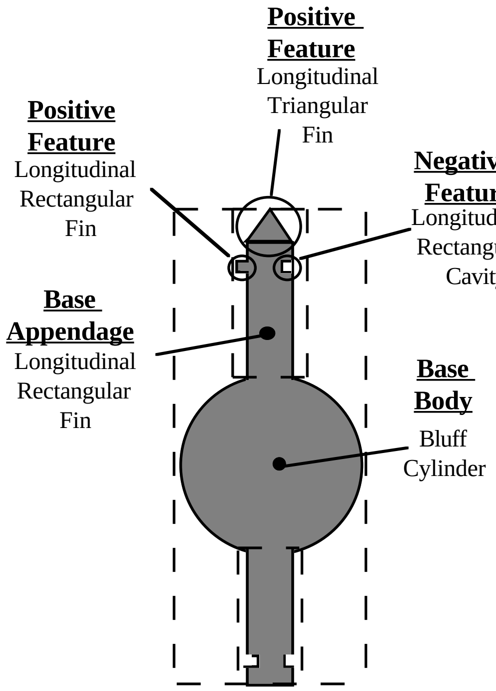
A target case consists of a frame based representation of the physical system. Within a target frame, representation is organised according to the different modelling perspectives; spatial modelling, phenomenological modelling and control volume analysis modelling. Figure 4 illustrates from a spatial perspective how the finned heat exchanger is classified and shows some of the indices used to describe the problem. In this case, a partonomic type relationship at three levels describes the essential components of the physical system, namely; the base cylinder, the complex appendages and their associated minor features. In this problem, the base is classified as a cylindrical bluff body in crossflow, the complex appendage is a rectangular longitudinal fin with features located on its windward, upper parallel and leeward sides. These features are a longitudinal rectangular cavity, a longitudinal triangular fin and a longitudinal rectangular fin. Problem parameters such as geometric data are also included in the target case but are not used as indices, however this information is used in the derivational traces.

4.3 Modelling Approach and Base Case Description

In Section 2, we argued that engineers normally model convection problems by decomposing the problem into well understood scenarios and considering each of these in a sequential manner. By classifying the heat exchanger problem as shown in Figure 4, this decomposition has been effectively achieved. Modelling progresses by firstly examining the role of the minor features with respect to the complex appendage and secondly the role of the complex appendage with respect to the cylindrical base. Each of these modelling episodes are sufficiently fundamental, so that they are comparable in terms of complexity and detail to the classical modelling scenarios discussed in Section 3. Consequently, all base cases are represented at this modelling abstraction level. Figure 5 illustrates one base case, that of modelling a longitudinal positive rectangular feature on the windward side of a rectangular fin. This base case is a classical heat transfer situation, is well understood and can be adapted and applied to a range of similar problems. In this base case, qualitative indices describe the minor feature and the associated base appendage. The modelling action or solution associated with this base case is that the feature can be removed completely without the need for any compensatory action. However, this action is not applied directly but is instead implemented by a process of regenerative transformation by applying the associated derivational trace.

4.4 Matching and Mapping

Case retrieval is implemented in a two stage process, matching (or base filtering) and mapping**.** In our initial prototype matching is implemented using an activation net which is made up of activation units which correspond to the indices of base cases [Ref]. A feature vector is created for each target case which contains the relevant indices of the problem. The feature vector is the basis by which the activation units are initialised and on completion, each case in the case base is contains a value of how many indices it shares with the target case. The mapping stage is concerned on establishing the correspondences between the base cases and the target cases. In our initial prototype, mapping based on establishing the full set of matching features between the target and base cases is the criteria for retrieving useful cases.



Bluff Cylinder with Complex Appendages

Spatial Model

:base
type *bluff_body*
profile *cylindrical*
:c_appendage
type *longitudinal*
profile *rectangle*
boundary_region *windward*
:positive_feature
type *longitudiunal*
profile *rectangular*
boundary_region *upper_parallel*
:positive_feature
type *lomgitudinal*
profile *triangular*
boundary_region *leeward*
:negative_feature
type *longitudiunal*
profile *rectangular*
boundary_region *lower_parallel*

Fig. 4 A spatial classification of the problem with a partial description of the associated target case.

4.5 Derivational Traces

Derivational traces are exploited in this domain, because, although the target and base cases may map qualitatively, small differences between physical parameters such as spatial or medium data can lead to significantly different solutions. Such differences cannot be expected to be captured in the initial qualitative classification of the problem, furthermore, to index all episodes based on both descriptive and parametric indices would result in an intractably large case base. A derivational trace describes the basis of the modelling solution, in this example, the removal of a windward longitudinal feature on a rectangular appendage, the reasoning behind these decisions and the engineering approximations and heuristics used in the evaluation process. In this example, the solution in the base case was derived in two ordered stages; firstly, the influence of the feature on the medium flow field was determined and found to be negligible and secondly the contribution of the feature to total appendage heat transfer was assessed and found to be of minor importance. Figure 6 shows a simplified version of the derivational trace. The fist stage examines the influence of the feature on the flow field and consists of Goals 1a and 1b. This involves determining whether the feature is actually fully contained within a turbulent boundary layer, and if so, the influence of the feature on the flow field is deemed negligible. Goal 2examines the contribution of the feature to overall heat transfer. In the base case, the heat transfer contribution of the feature was of the order of 4% of total heat transfer well within the 5% constraint, so therefore the fin was be removed, in the target case, this contribution was of the order of 3.5% thereby permitting the feature to be removed.

Rectangular Appendage with Fin

Description
direction: *windward*
type *flat plate*
:positive_feature
type *longitudinal*
profile *rectangular*
location *aft*
flow_regime *turbulent*

Solution
Action *Remove fin*
Compensation *None*

Derivational Trace

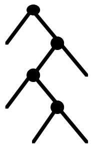
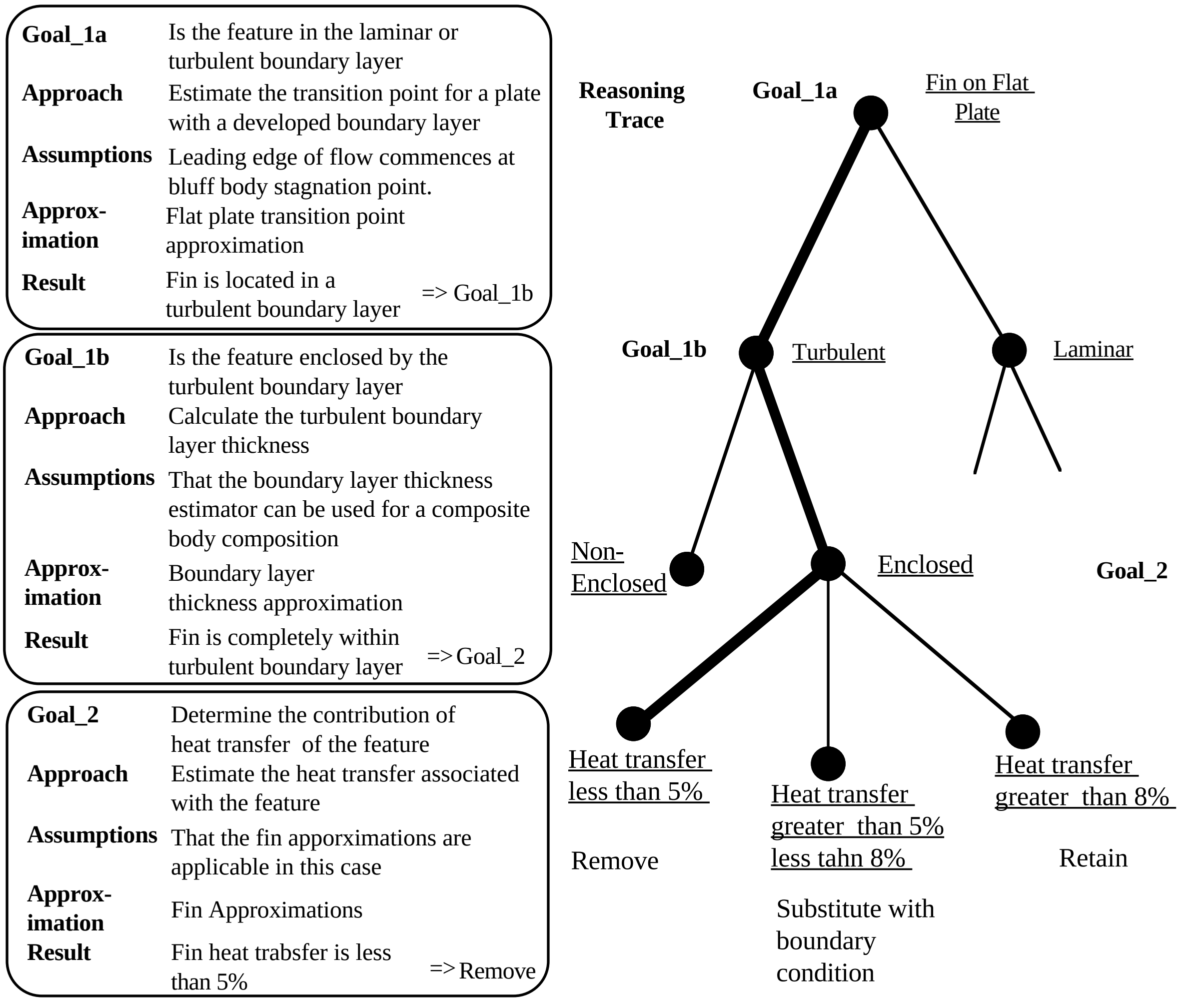


Fig 5 A sample base case for modelling a rectangular appendage with a positive feature

5. Conclusions

We have described a preliminary prototype of an interactive case based reasoning tool for mathematical modelling of thermal engineering problems. Derivational analogy techniques are exploited to provide for generative adaption and validation of base cases. We have found that because of the complexity of the domain, derivational analogy techniques are necessary to provide for case adaption and validation. Nevertheless we believe that this work represents an interesting if not significant alternative perspective to model based reasoning approaches that have been applied to model generation to date.



Goal_1a	Is the feature in the laminar or turbulent boundary layer
Approach	Estimate the transition point for a plate with a developed boundary layer
Assumptions	Leading edge of flow commences at bluff body stagnation point.
Approximation	Flat plate transition point approximation
Result	Fin is located in a turbulent boundary layer => Goal_1b

Goal_1b	Is the feature enclosed by the turbulent boundary layer
Approach	Calculate the turbulent boundary layer thickness
Assumptions	That the boundary layer thickness estimator can be used for a composite body composition
Approximation	Boundary layer thickness approximation
Result	Fin is completely within turbulent boundary layer => Goal_2

Goal_2	Determine the contribution of heat transfer of the feature
Approach	Estimate the heat transfer associated with the feature
Assumptions	That the fin apporximations are applicable in this case
Approximation	Fin Approximations
Result	Fin heat trabsfer is less than 5% => Remove

Fig 6 A derivational trace for a windward finned appendage

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