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ASSESSMENT OF THE PARAMETRIC ROLL BEHAVIOUR OF A POINT ABSORBER WAVE ENERGY CONVERSION DEVICE IN IRREGULAR WAVES.

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Abstract. The response of the Wavebob wave energy conversion device has investigated numerically in long crested irregular waves generated using the JONSWAP wave spectrum. Only long crested waves are considered as the roll mode will not be directly excited by the incident wave since the device is axisymmetric. As a result all roll response is due to mode coupling associated with parametric roll. The simulations were conducted at 17th scale to allow direct comparison with the regular wave case. The simulations were conducted for a significant wave height of 0.2m and a range of wave peak periods. The response of the device is simulated at each condition for 2000 seconds of simulated time. In all conditions, it was found that there was a significant roll response at the roll natural frequency for all wave excitation frequencies, including excitation frequencies outside the main zones of parametric resonance, even though the roll mode is not directly excited in long crested waves. It is concluded that there will always be some level of parametric excitation in irregular waves due to broadband excitation. At the tuning factor of 2, there are sudden bursts of roll motion due to parametric roll, however the large roll amplitudes are not sustained as they were in regular waves with parametric resonance.

1 INTRODUCTION

Ocean Waves as a renewable energy source offer a relatively high energy density and limited impact on competing land uses (e.g. agriculture, tourism). The estimated resource is significant. For example, it has been estimated that the North-Eastern Atlantic could be as much as 290GW [1]. However, the technology to capture and convert the energy is still immature, as evidenced by the numerous approaches proposed and under development. Falcão estimated there were over 1000 patents on converting wave energy. Detailed reviews of these technologies have been presented by, for example Drew et al. [2] and Falcão [3].

The particular wave energy converter (WEC) under consideration is called the Wavebob. This WEC is an axisymmetric, self-reacting point absorber with power take-off only in the heave mode. It consists of two concentric floating buoys: a torus and a float-neck-tank (FNT).

The FNT is a long, heavy, largely cylindrical component. The FNT pierces the sea surface in the centre of the torus. The two floats are mechanically constrained so that the only possible relative motion between them is in the heave direction. As the masses and hydrodynamic properties of the two floats are quite different, an incident wave train excites a relative heave motion between the two floats. The Torus is effectively a wave follower, while the FNT is the reaction mass. In scale models, this arrangement has been found experimentally to experience parametric roll, at least in regular waves. It is well known that floating bodies, such as self-reacting WECs, may exhibit parametric resonance. This condition occurs when the incident wave frequency is approximately twice the pitch or roll frequency. The coupled fluid structure system exhibits coupling of at least two degrees of freedom, and the response amplitude may be large even in degrees of freedom which are not excited directly. While survivability may be an issue ultimately, for the Wavebob, the onset of parametric roll causes a dramatic reduction in the relative heave, and hence power take-off, as energy is transferred to the roll mode. Parametric roll is not unique to Wavebob. Spar like structures in general will be susceptible and there is evidence that other WEC devices may exhibit this type of behaviour [4]. The large amplitude motions associated with parametric roll necessitate a non-linear model of the system. A numerical scheme for modelling this situation has previously been described [5], and has been demonstrated for the Wavebob in regular waves. The results were validated against 17$^{\text{th}}$ scale wave tank test results, and found to agree well.

An irregular wave analysis is desirable important to ascertain whether or not a point absorber is susceptible to parametric motion in more realistic seas, and if so, if the behaviour of the phenomenon is similar to that which occurs in regular waves. Parametric roll has been proven to be a dangerous phenomenon for several categories of ships occurring in realistic sea states, as outlined by France et al. [6]. In particular, it is to be noted that the occurrence of parametric roll of ships in irregular seas is much more dangerous and subtle than the same phenomenon in regular seas. This is due to the fact that when the parametric excitation is random, the build-up of roll can occur suddenly and abruptly after very long periods of quiescence, with a fast increase in the roll amplitude after only a few roll cycles [7].

2 NUMERICAL SCHEME

A time-domain nonlinear numerical model is used to investigate the dynamic response of the Wavebob. The pressure of the incident wave is integrated over the instantaneous wetted surface to obtain the nonlinear Froude-Krylov excitation force and the nonlinear hydrostatic restoring forces, while first order diffraction-radiation forces are computed by a linear potential flow formulation. These four components have been calculated directly.

In addition to these fluid forces, a non-linear viscous drag force is included using a form similar to the Morison equation. The drag coefficient matrix has been estimated experimentally with free decay tests in quiescent water. The mooring is treated as a linear spring.The power take off (PTO) provides a linear damping proportional to the relative heave motion only. Details of the modeling approach and numerical scheme used can be found in [5].

3 IRREGULAR WAVE MODEL

Assessment of the motion response of the device in irregular waves requires a suitable model of the real ocean environment. Irregular waves can be classified as either long crested or short crested based upon the direction of wave propagation. If the irregularities of the observed waves are only in the dominant wind direction, so that there are mainly mono-directional wave crests with varying separation but remaining parallel to each other, the sea is referred to as a long crested irregular sea. Throughout this paper the sea state will be considered long crested. This has the advantage that only heave, pitch and surge are directly excited and so response in roll can be attributed to non-linear interaction.

A linear superposition of N regular wave components, each having different amplitude, frequency, and phase angle is used to generate long crested waves. The wave elevation time series can therefore be represented as

$$\zeta_w(x,t) = \sum_{n=1}^{N} \zeta_{no} \cos(k_n x - \omega_n t + \theta_n) \tag{1}$$

where θ_n is the phase of component n, with amplitude ζ_{no} and wave number k_n. The equally spaced frequencies are given by $\omega_n = 2\pi n/T_H$, where T_H is the length of the time history.

The energy per square meter of the sea surface of the nth wave component is

$$E = \frac{\rho g \zeta_{no}^2}{2} \tag{2}$$

The wave spectrum are represented by the symbol $S_\zeta(\omega_n)$ and is given by

$$S_\zeta(\omega_n) = \frac{\zeta_{no}^2}{2\delta\omega} \tag{3}$$

where $\delta\omega$ is the fixed interval between frequencies. Therefore, from Eqn. 2, the total energy per square meter of the wave system is equal to the total area enclosed by the wave spectrum multiplied by the factor ρg.

The inverse transformation can also be made in order to generate a time history according to Eqn. 1 from a given wave spectrum. The amplitude of the nth component sinusoidal wave is given by

$$\zeta_{no} = \sqrt{2 S_\zeta(\omega_n)\delta\omega} \tag{4}$$

Therefore, the time history of wave elevation is obtained by substituting Eqn. 4 into Eqn. 1 such that

$$\zeta_w(x,t) = \sum_{n=1}^{N} \sqrt{2 S_\zeta(\omega_n)\delta\omega} \cdot \cos(k_n x - \omega_n t + \theta_n) \tag{5}$$

The wave spectrum used in the current work is the JONSWAP (Joint North Sea Wave Project) spectrum described by Hasselmann et al. [8]. A fully developed sea exists in conditions where there is sufficient fetch available, and the wind blows at a constant velocity for long enough such that the rate at which energy is absorbed by the waves will eventually be exactly balanced by the rate of energy dissipation. The waves in the JONSWAP spectrum thus continue to evolve with distance (or time). The spectral ordinate of the JONSWAP spectrum is defined as

$$S_{J\zeta}(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-1.25\frac{\omega_p^4}{\omega^4}\right]\gamma^a \tag{6}$$

where

$$a = \exp\left[-\frac{(\omega-\omega_p)^2}{2\omega_p^2\sigma^2}\right]$$

and

$$\sigma = \begin{cases} 0.07 & \text{if } \omega \le \omega_p \\ 0.09 & \text{if } \omega > \omega_p \end{cases}$$

The parameter γ in Eqn. 6 is called the peak-enhancement factor, the effect of which is to increase the peak of the spectrum. The other parameters to define the sea state are the peak frequency, $\omega_p = 2\pi/T_p$, which is the frequency with the maximum value in the spectrum; T_p, which is the peak period of the sea state; and H_s, the significant wave height which is defined as the average height of the highest one-third wave peaks in a wave spectrum. The parameter α in Eqn. 6 is included to take into account the growth of the waves with distance.

The motion response spectrum for the translational motion (surge, sway and heave) is calculated by filtering the wave energy spectrum with the motion transfer function. The translational motion transfer function is defined as the motion amplitude divided by the wave amplitude in the particular degree of freedom for each wave frequency. The filtering procedure is achieved by multiplying the wave spectrum $S_\zeta(\omega)$ by the square of the translational motion transfer function. Note that in order to distinguish the wave spectrum from the motion response spectrum, the ordinates of the motion response spectrum are denoted by S_{ξ_j}, corresponding to the particular degree of freedom ξ_j. The motion response energy spectrum ordinate at each wave frequency for the translational degrees of freedom ($\xi_j,\ j = 1,2,3$) of a single body system is given by

$$S_{\xi_j}(\omega) = S_\zeta(\omega)\left(\frac{\xi_j}{\zeta_o}(\omega)\right)^2 \tag{7}$$

A similar procedure is followed to calculate the motion response spectra for the angular motion responses (roll, pitch and yaw), except that the transfer functions are normalised by dividing by the wave slope amplitude $k\zeta_o$. Also, the wave slope spectrum $S_\alpha(\omega)$ is used instead of the wave energy spectrum $S_\zeta(\omega)$ for calculating the motion response spectrum in the case of the rotational modes. The wave slope spectrum is given by

$$S_\alpha(\omega) = \frac{\omega^4}{g^2} S_\zeta(\omega) \tag{8}$$

The motion response energy spectrum for the roll, pitch and yaw degrees of freedom ($\xi_j,\ j = 4,5,6$) of a single body system is then given by

$$S_{\xi_j}(\omega) = S_\alpha(\omega)\left(\frac{\xi_j}{k\zeta_o}(\omega)\right)^2 \tag{9}$$

Sea State	H_s [m]	T_p [s]	ω_p [rad/s]	ω_p/ω_4	B_{PTO} [Ns/m]
1	0.2	1.25	5.027	4.7059	5000
2	0.2	2.94	2.137	2	5000
3	0.2	3.33	1.887	1.7665	5000

Table 1: Simulated conditions in long crested waves

where $k = \omega^2/g$ is the wavenumber. The variance, m_{o_j}, of the response motion in degree of freedom j is obtained by integrating the motion energy spectrum according to

$$m_{o_j} = \int_0^\infty S_{\xi_j}(\omega)d\omega \tag{10}$$

The root-mean-square (RMS) of the motion in degree of freedom j is then given as

$$\xi_{j_{rms}} = \sqrt{m_{o_j}} \tag{11}$$

4 RESULTS

The response of the Wavebob to three sea states are presented each with a significant wave height of $H_s = 0.2$m. Table 1 outlines the parameters associated with each condition. The tuning factor is also shown, which is defined for irregular waves as the ratio of the peak wave frequency to the roll natural frequency (ω_p/ω_4). This set of conditions were chosen as they correspond to a low period wave (Sea State 1), a wave at the parametric resonance tuning factor of 2 (Sea State 2) and a high period wave (Sea State 3).

The time traces for the relative heave, roll and pitch motions for the three sea states are presented in Figs. **??** respectively. The corresponding wave energy spectra, transfer functions and motion response energy spectra for the same conditions are shown in Figs. 4 - 6. Since only long crested waves are considered in this analysis, there is no direct wave excitation in the roll mode. Since there is no roll excitation, and hence no roll transfer function, only the roll motion response energy spectrum is presented for each test, which is calculated as the spectral density of the roll response.

4.1 Sea State 1

In Fig. 4(a) (Sea State 1), the frequency range of greatest wave energy content ($\omega_p = 5.027$ rad/s) is far from the heave natural frequency of 2.199 rad/s. For this reason, a small amount of the wave energy is transferred into first order heave motion, as observed from the relative heave transfer function and relative heave energy spectrum. Furthermore, as can be seen in Fig. 4(b) the frequency range of greatest wave energy content is far from the pitch natural frequency of 1.068 rad/s. Therefore, the pitch transfer function and pitch energy spectrum show almost zero motion, with most of the energy occurring at the pitch natural frequency. From Fig. 4(c), it is seen that the roll motion for this test shows a significant response, even though the roll is not being directly excited. The tuning factor of 4.7059 is far from the primary region of parametric resonance, however, as the system is being excited by broadband excitation, there will always be some level of parametric excitation at twice the roll natural frequency, which accounts for the roll response.

4.2 Sea State 2

For Sea State 2, at the parametric resonance tuning factor of $\omega_p/\omega_4 = 2$, the RMS of the relative heave is over six times greater than that of Sea State 1, since the peak of the wave energy spectrum for this test condition ($\omega_p = 2.137$ rad/s) is located around the heave natural frequency of 2.199 rad/s. Therefore, a large amount of the wave energy is transferred to the heave motion, as shown in Fig. 5(a).

The corresponding time trace in Fig. 2 reveals evidence of parametric roll occurring, as seen from the sudden bursts of roll motion taking place, in particular at around 1400 seconds, where the roll angle reaches a maximum amplitude of .07 radians. The roll motion energy spectrum in Fig. 5(c), shows that the RMS of the roll is over twice the value it was in Sea State 1 at the lower period of 1.25s, whereas the maximum value of the spectral ordinate in the roll motion energy spectrum is over five times greater. In Fig. 5(b), it can be seen that the pitch motion RMS is almost the same in this test as Fig.4(b). However, most of the energy for the pitch motion is again, at the pitch natural frequency.

A time-frequency analysis of Sea State 2 is shown in Figs. 7 to illustrate how the response spectrum of the different modes evolve over time. The dominant tuning factor for this condition is 2, and it can be seen that the roll motion responds mainly at it's own natural frequency for the duration of the test. For the first 200s, the pitch motion is responding with a low amplitude, at both the wave excitation frequency and the pitch natural frequency. After 200s, the largest pitch response amplitude is at the pitch natural frequency, which is typical of parametric motion. The heave motion in this test is responding at the wave peak frequency ($\omega_p = 2.137$ rad/s) and at the heave natural frequency of 2.199 rad/s.

4.3 Sea State 3

In the final condition there is a slight reduction in the relative heave compared to Sea State 1, however, the heave mode is still absorbing a large amount of the wave energy as shown in Fig. 6(a) as the wave energy peak frequency ($\omega_p = 1.887$ rad/s) is still close to the heave natural frequency. Fig. 6(c) shows the roll RMS is approximately half the value it was in Sea State 1, as there is less frequency content at twice the roll natural frequency, and hence less parametric excitation occurring in this condition.

5 CONCLUSIONS

For the long crested sea states shown here, there is always a roll response at the roll natural frequency, with the greatest amplitudes occurring at the wave tuning factor of 2 (Sea State 2). In irregular waves, there is always going to be a significant level of roll response occurring, even though there is no direct wave excitation in the roll mode since there will always be frequency content occurring at twice the roll natural frequency due to the broadband nature of the excitation. This is different from the case in regular waves [5], where significant roll response was observed only in waves which had frequencies in the region of the tuning factors of 1 or 2. For waves outside these unstable zones, the roll response was almost zero. In the case of the pitch mode, it was seen that the dominant response in regular waves was close to the excitation frequency for all wave excitation frequencies, except at a frequency ratio of 2 when parametric motion occured. In the case of long crested waves, most of the pitch motion energy is concentrated at the pitch natural frequency, not just at the tuning factor of 2 but also at tuning factors of 4.7059 and 1.7665, as is seen in the roll mode.

It seems likely that since any realistic sea state with irregular waves will always have some level of excitation at twice the pitch/roll natural frequency due to broadband excitation, there will always be some level of parametric motion taking place. The non-linear coupling will therefore give rise to significant roll motion even in the absence of direct excitation.

6 ACKNOWLEDGEMENTS

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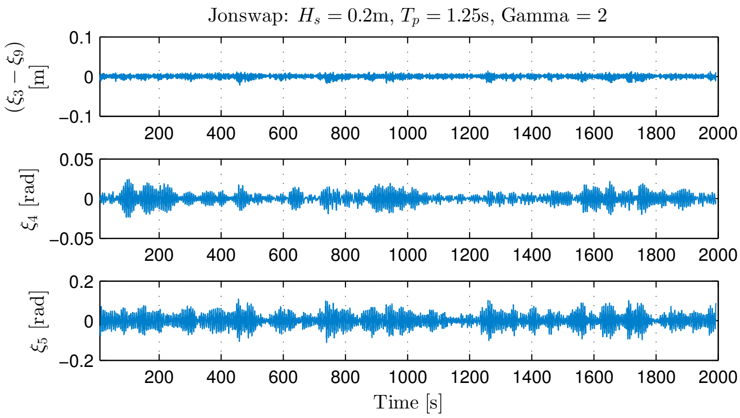


Figure 1: Time series of relative heave, roll and pitch motion for Sea State 1 ($T_p = 1.25$s)

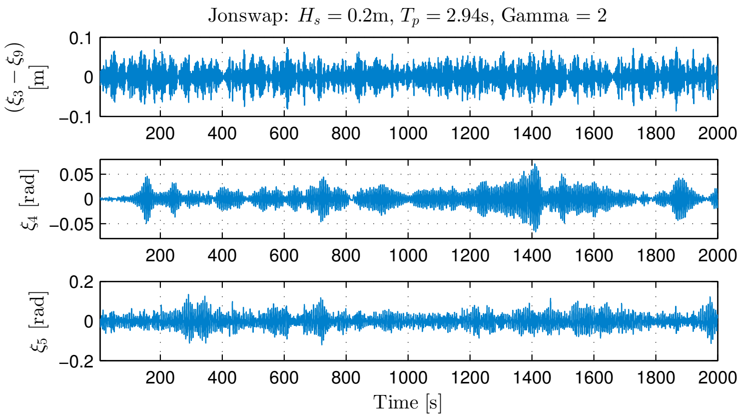


Figure 2: Time series of relative heave, roll and pitch motion for Sea State 2 ($T_p = 2.94$s)

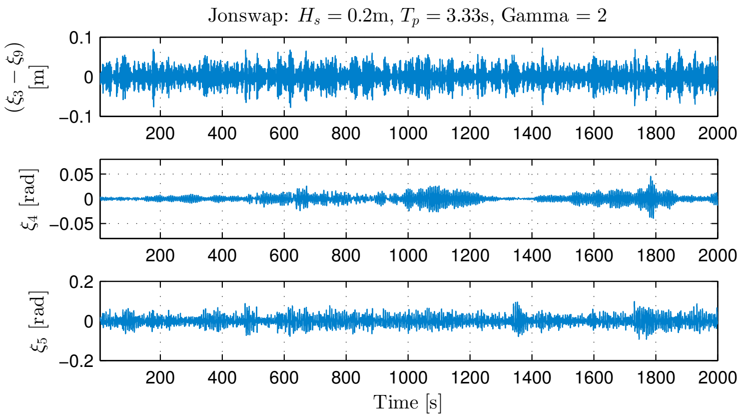
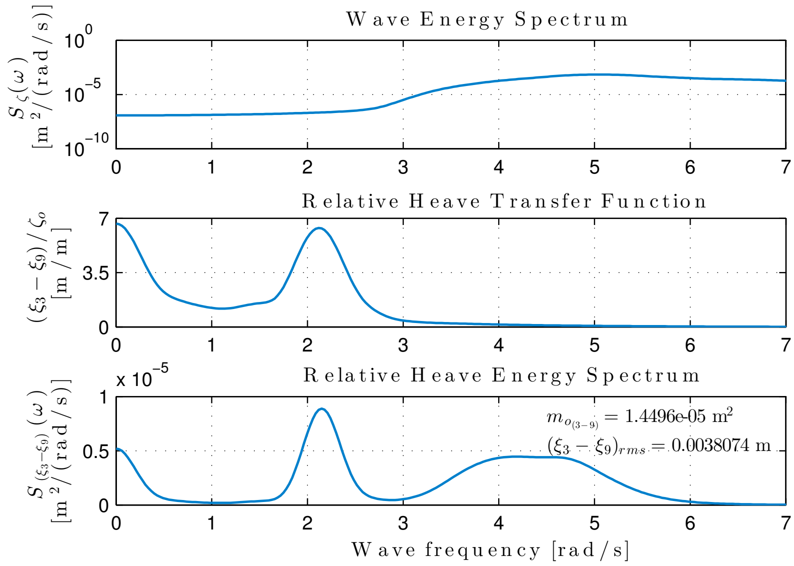
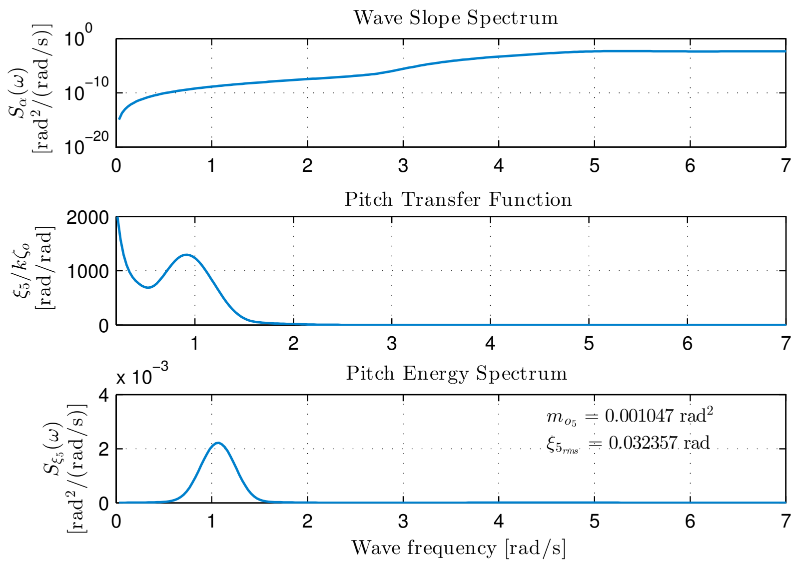


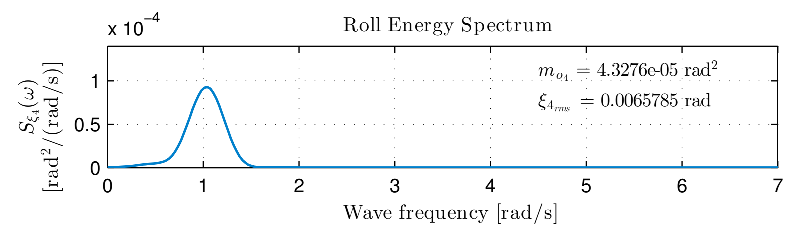
Figure 3: Time series of relative heave, roll and pitch motion for Sea State 3 ($T_p = 3.33$s)



(a) Relative heave spectra for test 1

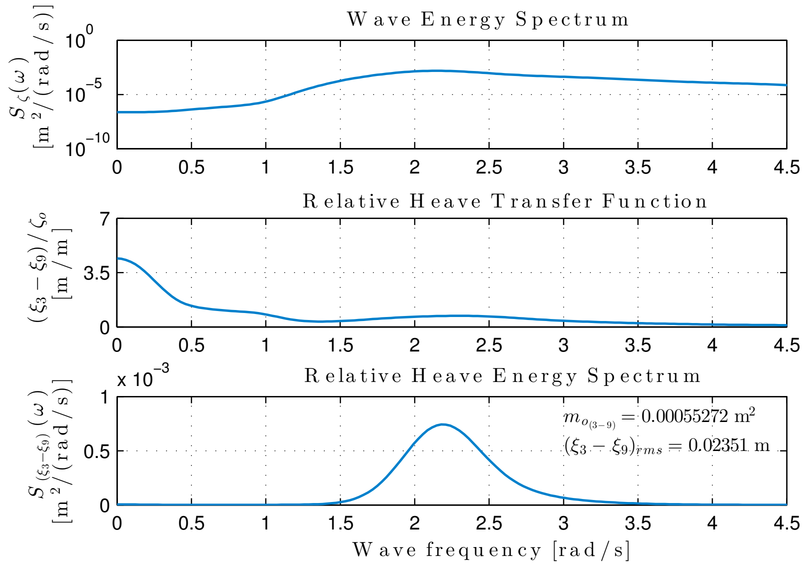


(b) Pitch spectra for test 1

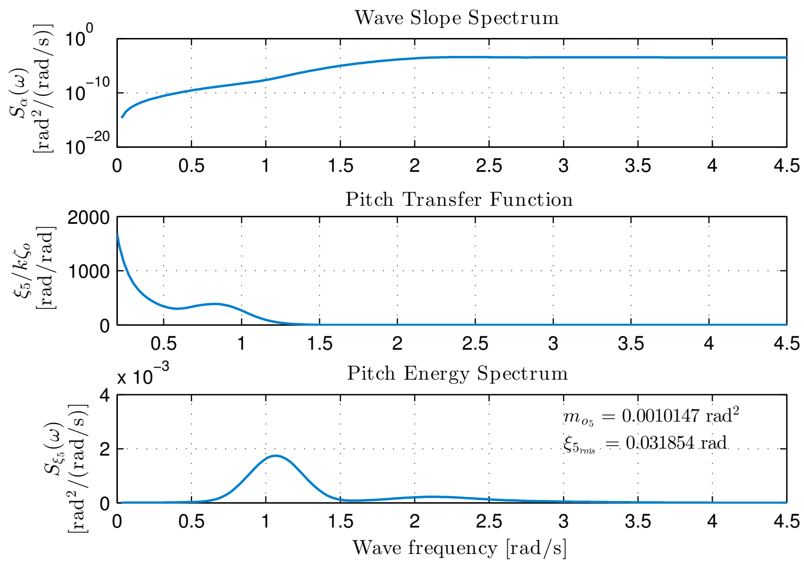


(c) Roll motion spectrum for test 1

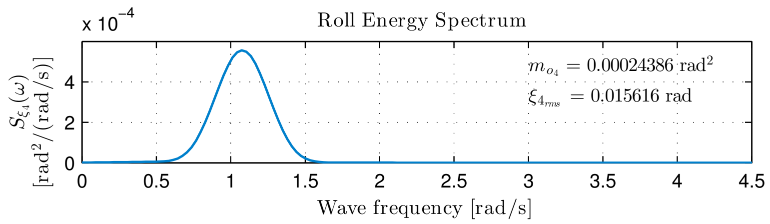
Figure 4: Calculation of motion energy spectra for (a) relative heave (b) pitch and (c) roll for Sea State 1 $H_s = 0.2$m, $T_p = 1.25$s and $B_{PTO} = 5000$ Ns/m



(a) Relative heave spectra for Sea State 2

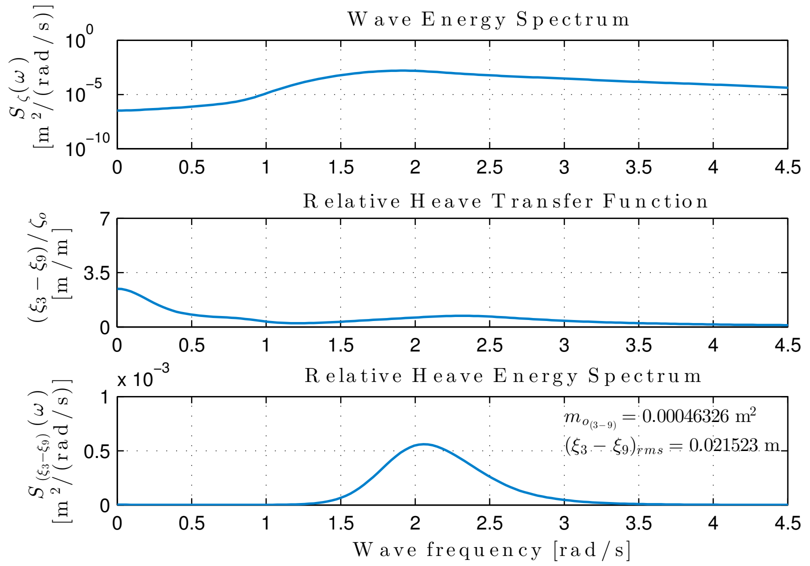


(b) Pitch spectra for Sea State 2

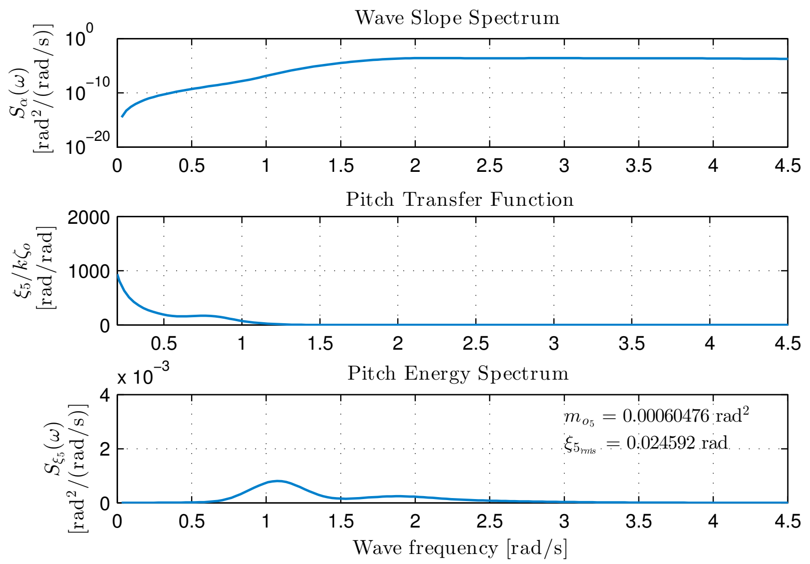


(c) Roll motion spectrum for Sea State 2

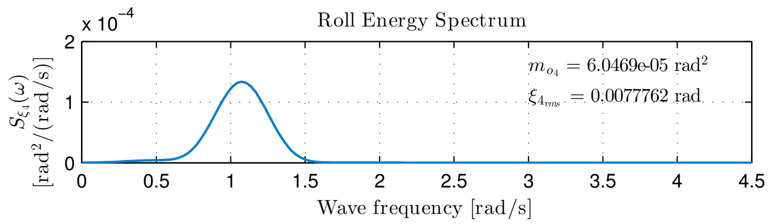
Figure 5: Calculation of motion energy spectra for (a) relative heave (b) pitch and (c) roll for Sea State 2 $H_s = 0.2$m, $T_p = 2.94$s and $B_{PTO} = 5000$ Ns/m



(a) Relative heave spectra for Sea State 3

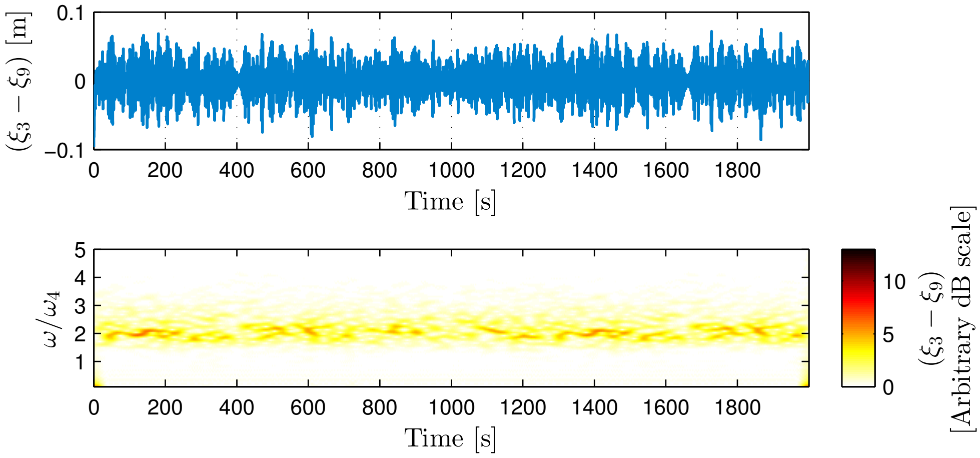


(b) Pitch spectra for Sea State 3

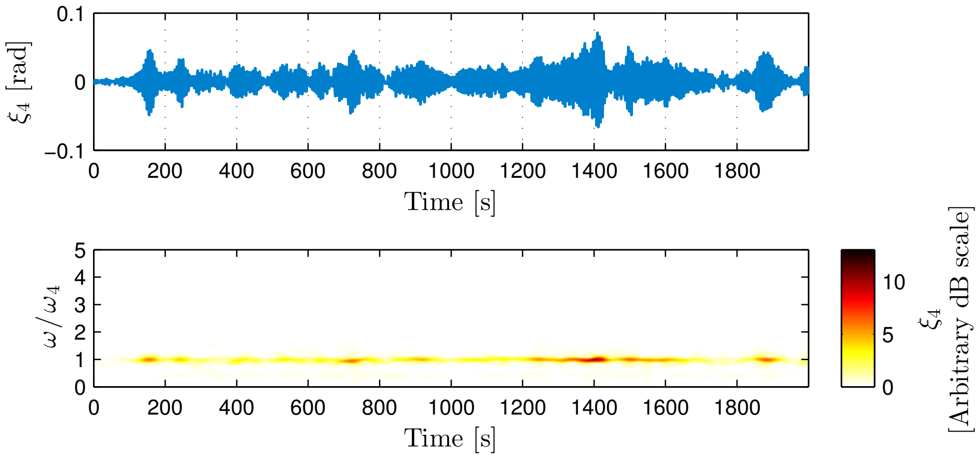


(c) Roll motion spectrum for Sea State 3

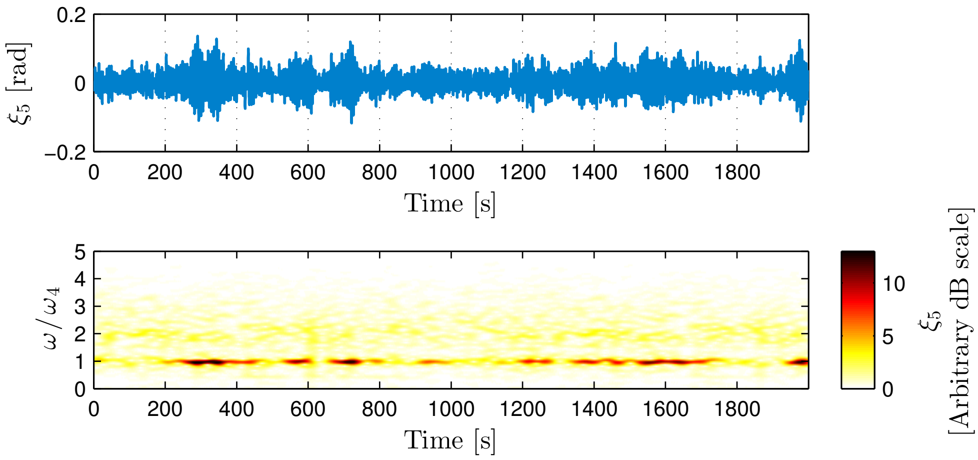
Figure 6: Calculation of motion energy spectra for (a) relative heave (b) pitch and (c) roll for Sea State 3 $H_s = 0.2$m, $T_p = 3.33$s and $B_{PTO} = 5000$ Ns/m



(a) Time-frequency analysis of relative heave motion



(b) Time-frequency analysis of roll motion



(c) Time-frequency analysis of pitch motion

Figure 7: Time frequency analysis of response to Sea State 2 for (a) relative heave (b) roll and (c) pitch for $H_s = 0.2$m, $T_p = 2.94$s and $B_{PTO} = 5000$ Ns/m