The Financial Economics of White Precious Metals - A Survey

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Abstract

Stuff

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1. Introduction

The excellent paper on the financial aspects of gold produced by O'Connor et al. (2015) provided fellow researchers with a great overview on the state of past research on the matter and allows for an easier formulation of future research and academic contribution. In the light of the great amount of work undertook on gold, the work that has been done about white precious metals is far greater than one might initially believe. Though silver is clearly overshadowed by gold as an investment vehicle, it’s importance in the financial world seems recognised when one considers the amount of academic literature published on its economic role over the past few years (Figure 1). Interestingly, the increase in academic output on white precious metals is in line with their increased role as investment vehicles, as it can be seen for silver in Figure 2\(^1\). Indeed, the introduction of a silver ETF in April 2006 followed by platinum and palladium ETFs in 2010 heightened the attractiveness of white precious metals as investment assets. In contrary to gold, New York is the more important market for silver, where the turnover is nearly about twice the size than the London market (Figure 3\(^2\)), regarding platinum and palladium, Zrich remains the biggest market while the importance of London as a market place was growing in recent years (Gold Field Mineral Services Ltd. (2015)). This survey is providing the reader with an overview of all of the existing literature about the financial economics of silver, platinum and palladium until March 2017. We try to produce an overview as thorough as possible about previous research on the economic and financial aspects and implications of white precious metals as an asset, but have deliberately left out papers dealing with historical aspects of silver in both monetary policy aspects and in the economy in general. Here, the cutoff line was made depending on the era studied. Recent events and

\(^1\)Figures obtained from the annual World Silver Surveys.

\(^2\)Figures obtained from The Silver Institute (2016).
implications of silver for currencies that are believed to still have an effect on today’s economy will be presented further down, while other historical aspects are, though fascinating, deliberately excluded from our research.

2. A *Very Brief History of Gold, Silver and Money*

   Herodotus believes that the first precious metal coins were minted in the Kingdom of Lydia about 700 BC (Blanco (2013)) and were oval and much thicker than modern coins (Weatherford (1997)). The advantages of using precious metals as a currency lied in their rarity and their quality of being unaffected by corrosion, while other Nations, such as Byzantium, used iron coins which proved to be too heavy and too easily rusting to be used as an effective currency (Averbury (1903)).

   During the reign of Gyges of Lydia, from 716 BC to 678 BC, one of the most important decrees issued by the King was to prohibit the private issue of coins (Bernstein (2000)). Henceforth, only the State was allowed to mint coins from a gold and silver alloy found in the bed of a river nearby; the different quantity of gold in every coin led to an uneven quality amongst
Figure 2: Global Demand for Silver by Type in Mio. Oz

Figure 3: Annual Turnover of the COMEX and the LBMA in Mio. Oz
the coins called *electrum* (Averbury (1903)). His son Ardys printed values on the coins to guarantee weighting and value, while Alyattes, grandson of Ardys, minted the first coins made of pure gold.

Finally, Croesus son of Alyattes, recalled all *electrum* coins produced and separated both metals in order to produce distinct gold and silver coins. The gold coins were used for foreign commerce while the silver coins satisfied the domestic market: the bimetallic system was born (Vaupel and Kaul (2016)).

About the same time, the city of Athens used thin silver bars as a currency. Six of these bars were called *drachma* (a handful) and would later become the name of the official Greek currency (Averbury (1903)). About 600 BC, Athens started minting coins with a content of 4.34 grams of silver, a value that would only decrease by about 3% until the end of the Athenian *drachma* six hundred years later (Vaupel and Kaul (2016)).

Taking over the concept of the Athenian *drachma*, the Roman Republic started minting the *denarius* about 268 BC with the exact same weight and value as the Greek model. The Roman victory over the Greeks at the battle of Pydna 168 BC sealed the supremacy of Rome in modern day Greece. A growing empire called for an organised and efficient coinage system that was placed in the temple of Juno on the Capitoline Hill in Rome for more than four hundred years (Aicher (2004)). The Goddess Juno was also known by her byname *Moneta* (from *monere*: to warn) after saving Rome from a surprise attack of the Gauls around 390 BC. The English word *money* is derived from that byname and the Juno temple housing the mint of the Roman Empire (Averbury (1903)).

Between 54 and 68, Emperor Nero reduced the amount of silver in the *denarius* to 90%, and between 98 and 117, Emperor Trajan reduced it to 85%, while Marcus Aurelius reduced the amount of silver to 75% between 161 and 180 and when Emperor Septimius Severus died in 211, the amount of silver in the *denarius* was at 50% (Vaupel and Kaul (2016)) while a growing amount of countries stopped accepting the Roman currency - leading to
severe problems for Rome to finance its expenses. Finally when Romulus Augustulus was deposed as the last Emperor of the Western Roman Empire in 476, the amount of silver in the denarius was at a mere 0.02% (Bonner and Wiggin (2009)). The downfall of the Roman Empire goes hand in hand with the devaluation of the denarius. On the other hand, the solidus, a gold coin of 4.55 grams that replaced the aureus in 312 survived for nearly seven centuries due to its constant weight and amount of gold it contained (Bernstein (2000)).

During the Middle Ages, the Commercial Revolution (de Roover (1942)), international trading flows (North (1994)) and regional differences in the access to natural resources led to a monetary schism of more than five centuries in which the Christian world used a silver currency and the Islamic world a gold currency. This division would only be broken up in 1252, when the cities of Genoa and Florence would introduce gold coins (North (2009)).

The discovery of America allowed Spain to have access to enormous new precious metal reserves. It is estimated that between 1500 and 1540, approximately 1,500 kilograms of gold and 300 tons of silver were shipped to the new every year (Davies (2002)) - doubling the amount of silver in Europe in only a few decades (Vaupel and Kaul (2016)). This immense access to new gold and silver led to growing imports in Spain and to a severe economic crisis when the precious metal flow started falling towards the end of the 16th century. Spain decided to borrow money from abroad in order to continue the importation of goods which led to a chain of insolvencies in the 16th and 17th centuries (Vaupel and Kaul (2016)) and to a price inflation of 400% in the 16th century alone (Bonner and Wiggin (2009)).

While it took the Roman Empire nearly four hundred years to reduce the amount of silver in the denarius from 99% to 0.02%, it took the Tudor Monarchy nine years, between 1542 and 1551, to cut the amount of silver in the pound by more than 80% (Reinhart and Rogoff (2009)), flooding the English domestic market with low quality coinage. However, part of the
English peace treaty with Spain ending the Anglo-Spanish War of 1625 was the Establishment of the *Cottington Treaty* in 1630, allowing the English mint to have direct access to Spanish silver arriving from the New World (Davies (2002)). With a considerable part of the silver obtained used for the minting of new coins (Craig (1953)), the quantity of coins minted under the reign of Charles I amounted to £9,000,000 (Davies (2002)); almost twice the amount coined during the entire reign of Queen Elizabeth I (Feavearyear (1963)). Since the new coins entering the English domestic market contained more silver, *Gresham’s Law* led to a hoarding of the new coins and to an increased usage of the old coins as the regular method of payment (Britannica Concise Encyclopedia (2006)). Even though an equivalent to banks existed in continental Europe since the 12th century (North (2009)), the development of modern banks in England appeared only during the 17th century.

King Charles I of England, notorious for his execution in 1649, had severe problems to finance his expenses and decided in 1640 to mint three hundred thousand pounds with a silver content of only 25% (Davies (2002)). Anticipated inflation was immediately noticeable with prices increasing by in the very short term (Andréadès (1966)). King Charles I therefore decided against the original plan and simply confiscated an amount between £100,000 and £130,000 deposited in the Tower of London by goldsmiths and merchants of London (Davies (2002)). The King was only willing to return the money if granted a credit over two hundred thousand pounds (Andréadès (1966)). The loss of trust from English citizens in the Tower of London as a safe haven for their wealth was sealed; henceforth, the Londoners decided to deposit their precious metal with goldsmiths in possession of a vault. The citizens who deposited their money with goldsmiths received a receipt over the amount of pounds entrusted in their vault. This piece of paper, that guaranteed the bearer to obtain a fixed amount of pounds from the goldsmith, evolved in becoming a smooth and uncomplicated median for financial transactions: paper money gain acceptance in England. On the
other hand, the goldsmiths realised that more money was deposited in their vaults than the amount withdrawn, allowing them to lend money to clients (Skousen (1996)) and hence producing more receipts than the amount of precious metals actually held in their vault; this was the birth hour of modern banking (Withers (1920)). Financing his State affairs by borrowing excessive amounts from goldsmiths, Charles II of England ordered the Great Stop of the Exchequer in January 1672 in order to freeze interest and redemption payment from the Crown (Ferguson (2008)). The result was a substantial wave of exchange of goldsmith receipts back into precious metals; and early days bank run. Since the gold and silver was mainly lent to the King, the goldsmiths were unable to honour their debts and became insolvent (Goodman (2009)). While there were forty-four active goldsmiths in London in 1677, only twelve or fourteen were still operating in 1695 (Vaupel and Kaul (2016)). In 1691, Scottish trader William Paterson suggested a bank that would lend money to the Government at a reasonable interest rate and issue bank notes backed by Government debt (Goodman (2009)). Three years later, the Ways and Means Act would turn this idea into reality and give birth to the Bank of England, which would over the centuries obtain the monopoly to print money.

The French Revolutionary Wars and the associated expenses were disastrous for the gold reserves of the Bank of England. By the 31st of August 1796, the Bank of England was holding gold worth £2,100,000 against liabilities of about £16,000,000 (Cannan (1919)). A French invasion of approximately 1,400 men in February 1797 on the shores of Wales immediately spread panic across Britain and led to a bank run (Vaupel and Kaul (2016)). Facing a situation of inability to exchange all liabilities into gold, an Order in Council was issued on the 27th of February 1797 forbidding the exchange of paper money into gold (University of Northern Iowa (1867)). Different reasons assured a certain stability of the fiat pound in comparison to the previous examples in America and France. On the day of the announce-
ment, 3,000 London traders declared to accept all payments in paper money at the nominal value, furthermore, on the medium and long term, the early days of industrialisation and the implementation of the income tax led to an increase of the national wealth (Vaupel and Kaul (2016)). With the increase in money supply leading to an increase of the gold price (Ricardo (1810)) and the general public getting accustomed to the use of paper money (Vaupel and Kaul (2016)), the Government passed a law obliging the Bank of England to exchange paper money against gold coins on demand from the 1st of May 1823 onward. After 25 years of fiat money, Great Britain returned to the gold standard.

The Coinage Act of 1873 technically ended bimetallism in the United States of America and laid the foundation of the American gold standard by abolishing the right of bullion holders to struck silver into legal tender US Dollar coins (United States of America (1904)). While the silver Dollar didn’t play an role under the new regulations set out in 1873, old silver coins were used predominantly as shrapnel (Friedman (1990)). However, in the light that their metal value was far higher than their nominal value, most of the coins were melted to extract their silver content (Vaupel and Kaul (2016)). While the United States of America turned to gold in order to erase the price fluctuations of gold in trading with the United Kingdom, other countries introduced the gold standard as well, in order to have a currency linked to that of the British hegemon. Figure 4 illustrates which country introduced the gold standard in which year.

A direct result of the gold standard was a drop of the silver price due to the large quantities of the white precious metal being thrown on the market in the light of the nonnecessity of silver as a currency (Friedman (1990)). An event that clearly facilitated the establishment of the global gold standard was the gold rush of the 1840’s, with great mine discoveries

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3Graphic obtained and modify form Vaupel and Kaul (2016).
in Australia, Russia, and notoriously: California. However, with more and more economies in need of gold to feed their monetary systems, the amount of gold available declined around the 1880’s and therefore the money supply; leading to a deflation in the USA of about 1.7% per year and of 0.8% per year in the United Kingdom between 1875 and 1896 (Friedman (1990)). In the United States of America, some politicians suggested a partial return to a silver linked currency, which was very cheaply and easily available due to the price plunge in the previous years. Indeed, while this very idea was a fighting point of Democratic Presidential Election candidate William Jennings Bryan, his defeat against the Republican William McKinley in 1897 destroyed all hopes of the silver lobbyists to see the white precious metal exercise a more important role in monetary policy (Friedman (1990)). In 1887, the invention of the MacArthur-Dingus cyanidation process by Scottish chemists John Stewart MacArthur and Dr. William Dingus led to a boom of the gold production in South Africa and allowed the United States and the United Kingdom to considerably increase their money supply (Friedman and Schwartz (1963) and Friedman (1994)).

However, the First World War led many countries to abandon the gold standard, such as Germany as early as 1914. Reparation costs faced by the Mittelmächte, alongside precarious central bank liquidity positions on
the side of the *Triple Entente* led to an end of the European *gold standard* during the interwar period (Officer (2005)).

In July 1944, delegates of all the 44 Allied Nations met in Bretton Woods (NH) and agreed to follow a new set of rules and guidelines for international financial and commercial relations amongst each other. The *Bretton Woods System* was very similar to the *international gold standard* in that it fixed the exchange rates of individual national currencies to the US Dollar which could then be exchanged to gold at a fixed ratio (Gale Encyclopedia of U.S. Economic History (1999)). While the system was arguably successful, it virtually assigned the US Dollar the same power as physical gold, leading many countries, most of all France, to exchange their Dollar holdings into gold around the late 1960’s (Garber (1993)). The unwillingness of the USA to significantly drive down the amount of gold it held in order to meet the demands of other countries, coupled with a persistent balance of payment deficit due to the Vietnam War, led President Richard Nixon to end the convertibility of the US Dollar into gold on the 15\textsuperscript{th} of August 1971; leading to an international *fiat money* system.

3. About the Role of Silver in a Bimetallic System

Some empirical work has been done on the implications and effects of bimetallism, where the paper by Oppers (1996) is an early example. Looking into both the causes and the consequences of the switch from bimetallism to the gold standard in the early 1870s, the author argues that the German demonetisation of silver after the unification caused the fall of the price of silver in 1871. Looking at a bigger and more global picture via a composite-good model consisting of money supply, price level, the interest rate and the exchange rate between the gold and silver currencies of the different areas under study, the author challenges the belief that the end of bimetallism in the early 1870s was inevitable since it was accompanied by neither an excessively abundant supply of silver nor by a shortage of gold.
A few years later, the same author published a further paper focused on issues revolving around bimetallism - namely at the apparent stability of the ratio between gold and silver in the absence of arbitrage possibilities for bimetallic countries in the 19th century. Oppers (2000) believes that Gresham’s law should be reviewed in the light of *arbitrage anticipation*. Stating that ”good money drives out bad money” (Greenfield and Rockoff (1995)), Gresham’s law affirms that in a bimetallic system, coins trade at a fixed official relative price whereas the price of the precious metal is linked to fluctuations in the commodity markets. If the market price of silver would deviate from the mint ratio, a profit could be realised by transferring the more valuable coin (*good money*) into the commodity market. Considering data from the major bimetallic economies of the past centuries, Oppers (2000) argues that anticipation of arbitrage opportunities occurring from Gresham’s law act as a stabilising power for both the gold and silver price.

Velde et al. (1999) work with a more complex economic model in order to offer support to the findings of Oppers (1996), namely that the sudden end of bimetallism in the 1870s was by no means inevitable. The authors take into account a large selection of countries within a steady-state analysis but fail to find a rational explanation for the simultaneous abandonment of bimetallism by so many countries in 1873; even more puzzling, evidence show that the shift of Germany in 1871 from a silver linked currency to a gold linked currency relaxed the constraints exercised on bimetallism.

A recent paper that has a far more pessimistic view on the possible future of bimetallism is that of Meissner (2015) who predict a collapse of bimetallism would have occurred by latest 1875. The author bases his conclusion by working within a conditional model developed by Flandreau (1996), and finds an explanation for the sudden global exit of bimetallism in 1873 in the apparent lack of monetary and economic knowledge of most actors at the time and their poor willingness to cooperate.
4. Structural Issues around White Precious Metals

4.1. The Pricing Structure of Silver, Platinum and Palladium

In an early paper on the rates of return of gold and silver, Frank and Stengos (1989) examine possible predictability of the rate of returns of the two precious metals. Results from daily, weekly and biweekly data between the mid 1970’s and the mid 1980’s for silver point to the possibility of an underlying martingale process, indicating the likelihood that a nonlinear process generates observed gold and silver returns and that a certain degree of randomness is present in generating the price of silver.

Lashgari (1992) takes into account daily, weekly and monthly gold and silver prices between January 1970 and December 1989. Optimal silver price forecasts are obtained based on prior observed values by minimising the amount of information inaccuracy resulting from the divergence between actual and forecasted price changes; essentially relying on an exponential smoothing time series model. Results indicate that the information of past prices for gold and silver is stronger when short-term previous prices are considered, while this effect vanishes away when looking at weekly and monthly prices. While the dependence on past prices is found to be stronger for gold than for silver, no trading profits can be realised for both gold or silver.

Fassas (2012) look into a better understanding of the structure of precious metal prices by considering the period between the 1st of May 2007 and the 28th of February 2011. The author argues that the price increase in precious metals was partly due to the flow of precious metals into Exchange-Traded Products (ETPs). Looking at 28 precious metal ETPs and the weekly spot returns of silver, evidence suggests a statistical significant positive correlation between silver returns and the flows into silver Exchange-Traded Products exists, though this relationship is stronger for gold. The evidence for correlation between the two variables should, however, be considered in the light that a Granger causality test (Granger (1969)) rejects the hypothesis that precious metal ETPs flows caused the return of silver.
On the other hand, results for platinum and palladium show that returns of these metals *granger cause* flows into their respective ETPs.

A few years later, Demiralay and Ulusoy (2014) published an applied risk-management approach to predict the value-at-risk (VaR) implications of gold, silver, platinum and palladium. Working with daily data of both long and short trading positions between the 4th of January 1993 and the 29th of November 2013, Demiralay and Ulusoy (2014) fit the data into three different non-linear long memory volatility models (FIGARCH, FIAPARCH, HYGARCH) and find that for all three white precious metals, a FIAPARCH model with Student $t$ distribution is able to capture long memory and asymmetry as well as fat tails and outperforms the other long-memory volatility models in predicting one-day-ahead VaR positions for trading. The fractionally integrated asymmetric power ARCH (FIAPARCH) model was proposed by Tse (1998) and is a more complex case of the fractionally integrated GARCH (FIGARCH) model proposed by Baillie et al. (1996) that can distinguish between short and long memory in the conditional variance process.

In a recent article looking at the pricing structure of gold, silver, rhodium, platinum and palladium, Gil-Alana et al. (2015b) rely on a fractional integration modelling framework in order to identify structural breaks in the monthly data of the series between January 1972 and December 2012. Results indicate evidence of long memory processes for platinum, in contrary to silver and palladium where strong evidence for mean reversion is obtained. However, taking into account the respective structural breaks identified for the three white precious metals, all series seem to be non-stationary, so that exogenous shocks will affect the long memory behaviour of the series - hence advising policy makers to adopt measures in case white precious metals drift away from their original trend.

In another paper looking at long memory behaviour of the price of silver, Gil-Alana et al. (2015a) take into account annual silver prices between 1792 and 2013 and find that real silver prices are mean reverting; indicating that
no long-run memory behaviour exists in the silver inflation rate. This result indicates that exogenous shocks will affect silver prices less intensely than gold prices.

Urquhart (2016) analyses return predictability for gold, silver and platinum by fitting daily data between the 5th of January 1987 and the 30th of September 2014 into a battery of tests. Results indicate time-variation in the predictability potential of silver and platinum returns, where a joint-rank test indicates that silver returns could only have been predicted with confidence until April 1999, while the time frame is longer for platinum prices, which could have been predicted until March 2001. However, results are inconclusive when considering different testing procedures, leading Urquhart (2016) to work with a rolling window approach highlighting the best prediction potential for platinum and the worst prediction potential for silver.

Figuerola-Ferretti and McCrorie (2016) base their research on the explosive/multiple bubble technology developed by Phillips et al. (2015) to analyse the effect of the Global Financial Crisis on the price behaviour of gold, silver, platinum and palladium by looking at weekly data between 2000 and 2013. Evidence points towards short periods of mildly explosive behaviour in the prices of all precious metals, furthermore, while the Global Financial Crisis led the gold price to deviate from fundamentals, it seems that silver and palladium were rather affected by the launch of ETFs rather than the financial crisis as such.

Working with 12,187 daily observations of the price of silver between January 1968 and March 2016, alongside 6,561 daily observations of the prices of platinum and palladium between April 1990 and March 2016, Almudhaf and AlKulaib (2016) underline the importance of outliers for white precious metal prices by identifying that a traditional buy-and-hold strategy outperforms an attempted market timing strategy. Indeed, Almudhaf and AlKulaib (2016) show that large price increases of precious metals tend to
be pooled around single days and that missing these individuals days would come at high cost for an investor, who should rather follow a more long-term classical investment strategy.

Recently, Zhang and Zhang (2016) examined the Value-at-Risk (VaR) and statistical properties in the daily price returns of gold, silver, platinum and palladium between the 11th of January 2000 and the 9th of September 2016. A complex two stage methodology relying on different GARCH models reveal that gold has the highest and most steady VaR values of the group while palladium has the most volatile and lowest VaR values of all four precious metals. Further results indicate that the VaR values of silver are more volatile than those of platinum, while the residuals of both metals are characterised by heavy-tail distributions.

4.2. Econometric Issues around the Pricing Patterns of White Precious Metals

Roberts (2009) is looking into the behaviour of metal prices by working with monthly data from January 1947 to December 2007. Considering highs and lows in the price of certain precious metals, amongst which platinum and silver, the author finds that the duration of these phases can be explained through cyclicality and not through mere randomness. The mentioned cyclicality is supported by evidence from a Bry and Boschan (1971b) procedure that identifies turning points in time series by adjusting them for outliers and replacing these outliers to create a smoothed time series. Looking more into how to model the duration of both phases and cycles of high and low silver and platinum prices, results for silver issued from a batteries of tests indicate that a random walk is not the optimal formalisation to apply to the London Price of Silver.

Set in a framework of autoregressive conditional heteroscedasticity (ARCH) models, Nadarajah et al. (2015) build upon the work of Zhu and Zinde-Walsh (2009) and Zhu and Galbraith (2010) in order to test which GARCH specification performs better when modeling the returns of five different com-
modities amongst which gold and silver. A GARCH model is composed of two components: the volatility component and the innovation component. The latter is assumed to come from either the Gaussian distribution or the Student $t$ distribution; or from a skewed extension of these distributions. Considering daily price returns from the 12th of March 1993 to the 13th of March 2013, Nadarajah et al. (2015) find that the best fitting model for silver price returns is the Skewed Exponential Power (SEP) distribution, results in line with Cheng and Hung (2011). The SEP distribution is a particular case of the Asymmetric Power Distribution (APD) (Zhu and Zinde-Walsh (2009)) - the most general form of the Normal Distribution.

Though the findings of Nadarajah et al. (2015) suggest to model the return of silver with a Gaussian Distribution, Cochran et al. (2016) model the return of aluminium, copper, gold and silver, using an asymmetric GARCH (AGARCH) model with a conditional skewed generalised $t$ (SGT) distribution. Similar to Demiralay and Ulusoy (2014) before them, the authors study the performance of value-at-risk measures obtained from an AGARCH model with a SGT distribution to results obtained from an AGARCH model with a normal and student $t$ distribution. For the case of silver, an AGARCH model with the SGT distribution offers the best fit. An interesting difference to both Demiralay and Ulusoy (2014) and Nadarajah et al. (2015), who take into account a similar time frame, is the much shorter period of time used by Cochran et al. (2016): daily spot returns of silver between the 2nd of January 1999 and the 29th of January 2010. Further findings point towards time-variation in the skewness for silver, as well as in the peakedness and tail thickness parameters of silver returns. Results from a Wald test (Engle (1984)) implies that higher order moments of silver returns, like skewness and kurtosis, are time-varying.

In a recent paper attempting to forecast the returns of gold and silver prices, Pierdzioch et al. (2016a) discuss the statistical and economic performance of different forecasting models in regard to the choice of the
Information Criteria selected to determine the boosting algorithm. Using monthly data from January 1987 to September 2014 and using a large set of predictors to forecast the excess return of silver to the 1-month LIBOR, the authors develop a trading algorithm in which an investor should buy silver if the forecasted excess return is above the historical real-time mean of excess returns. In such a scenario, the trading rule performs better under the Akaike Information Criterion (AIC) then under the Minimum Descriptive Length (MDL) proposed by Bühlmann and Hothorn (2007). Even though the forecasting model for silver performs well, Pierdzioch et al. (2016a) warn the readers that the outlined model might not survive an economic performance evaluation.

Degiannakis and Potamia (2016) base their research on the recommendations of the Basel Committee on Banking Supervision and examine whether inter-day or intra-day model provide accurate predictions for reliable Value-at-Risk (VaR) and Expected Shortfall (ES) forecasts. Daily data for silver between the 3rd of January 2000 and the 5th of August 2015, indicates that a GARCH-skT model, relying on inter-day data, provides better results than a HAR-RV-skT model, as it satisfies most of the conditions implied in VaR and ES forecasting, but that it overall fails to provide accurate forecasts of the risk measures implied.

5. Behavioural Aspects of White Precious Metals

5.1. Market Efficiency

In a rather early paper, Solt and Swanson (1981) analyse if the efficient-markets theory (Fama (1970)) can be applied to the gold and silver market and the results of their endeavour lead to several conclusions. First of all, the author finds evidence for strong heteroscedasticity in the variances of the price changes for silver, except for the logarithmic price series, where the mean of these price changes are nonzero and not stationary, as well as not merely drifting. Also, even though there seems to exist a positive dependence
in each of the price change series, this dependence cannot be used to generate
abnormal returns. Solt and Swanson (1981) conclude by saying that the
silver market does not conform with traditional market efficiency models
and that trading silver could be considered more a speculative endeavour
rather than an investment activity.

Only a few years later, Goss (1983) builds upon the findings of Solt
and Swanson (1981) and focuses on two different questions. First, if actual
futures prices of silver are unbiased anticipations of consequent cash prices,
and second, whether or not the silver market is efficient in a weak form
sense. A market is efficient in the weak form of the term if silver futures
prices reflect all information contained in past silver futures prices (Jensen
(1978)). The first question is addressed through a General Instrumental
Variable Estimation (GIVE) which corrects for autocorrelation and is not
limited to first order processes (Hansen and Singleton (1982)). Evidence
shows that futures prices of silver overstate the maturity date cash prices
at levels below $3.82 to $4.42 depending on the lag length, and understate
maturity date cash prices above these levels. Regarding the Efficient Market
Hypothesis (EMH) for silver, the author worked with silver futures daily
closing prices between 1973 and 1979 and addresses the question by testing
the random walk hypothesis; assessing that subsequent price changes are
independent and identically distributed - they can never be described as
either too high or too low (Shiller and Perron (1985)). Results point towards
a rejection of the Efficient Market Hypothesis, a finding in line with Solt and

A rather new paper to look at the efficiency of the silver and platinum
markets is that of Charles et al. (2015). The authors take into account log
returns of daily closing spot prices between the 3rd of January 1977 and the
23rd of October 2013 and apply an automatic Portmanteau test and a vari-
ance ratio test to check for weak-form efficiency of the silver market. The
automatic Portmanteau test builds upon a robustified Box-Pierce Q statistic
(Lobato et al. (2001)) in which the optimal value of \( p \) is determined by the data fed into the system (Escanciano and Lobato (2009)). The automatic Portmanteau testing procedure has multiple advantages: the order of the autocorrelation tested doesn’t need to be specified, there is no need to use a bootstrap procedure to estimate critical values and the test is robust to the presence of conditional heteroscedasticity of unknown form. The automatic variance ratio test used builds upon the findings of (Kim (2009)) who developed a way to improve the performance of the Choi (1999) testing procedure. It should be pointed out however, that Kim (2009) developed their testing procedure for \emph{small samples}, Charles et al. (2015) work with 9,603 observations, which arguably qualifies as a \emph{small sample}. Charles et al. (2015) conclude their findings by saying that both markets fit the criterion of the \emph{adaptive market hypothesis} (Lo (2004)) that takes into account behavioural economics, and that the silver market has gradually shifted towards the efficient market requirements over the time period considered.

A very recent paper on the subject of market efficiency is that of Batten et al. (2016) looking into possible gold and silver price manipulation. 5 minutes tick data was collected between the 1\textsuperscript{st} of January 2010 and the 30\textsuperscript{th} of April 2015, while a cluster analysis procedure is used to detect price manipulation. Regarding silver, results point towards a large concentration of returns around the derivative expiry date, suggesting possible manipulation. Furthermore, a three component mixture model indicates abnormal market behaviour, which is also supported by a further method clustering the silver returns. However, Batten et al. (2016) warn the author in jumping to conclusions about possible manipulation of the silver market as the evidence provided is merely indicative and not a legal prove for \emph{foul play}.

5.2. Seasonality and Trading Behaviour of Investors

Yang and Brorsen (1993) focus on futures prices and argue that past models have failed to successfully explain non-normality and dependence in speculative price changes. The authors apply a GARCH model and deter-
ministic chaos processes to daily closing futures prices of silver, platinum and palladium between January 1979 and December 1988 in order to detect market anomalies. Building upon a GARCH model that generates data with fatter tails than the Gaussian distribution (Bollerslev (1986)) and reviewing previous research on the application of stochastic processes in finance, the methodology proposed by Yang and Brorsen (1993) captures day-of-the-week effects (Junkus (1986)), seasonality in variance (Anderson (1985) and Kenyon et al. (1987)) and maturity effects (Milonas (1986)) of silver futures prices. The GARCH model is augmented with a Residual Test in order to limit forecasting errors (Brock et al. (1996)). Test results indicate a strong calendar-day effect for silver since the variance of silver futures prices is larger on certain days of the week and after holidays. Concerning platinum and palladium, a calendar-day effect on the variance is observed on Tuesday and after holidays; Monday is only found to be significant for palladium. Further results point towards seasonality in the variance.

Focusing as well on seasonality effects is a paper by Lucey and Tully (2006a) that studies daily COMEX silver cash and futures contracts between January 1982 and November 2002. More specifically, the authors test both the unconditional and conditional means and variances of silver cash and futures prices. The unconditional means and variances are modelled with a dummy variable regression and tested via the $t$ and the $F$ Statistic; where robustness is ensured through the iterative re-sampling model proposed by Rousseeuw and Leroy (2005) that captures outliers better. The approach of Lucey and Tully (2006a) also controls for heteroscedasticity and autocorrelation (White (1980)) and tests for seasonality in the unconditional variance (Levene (1960)). The conditional means and variances are modelled starting with a GARCH-in-mean (GARCH-M) framework adding a heteroscedasticity term into the mean equation (Elyasiani and Mansur (1998)) and accounting for potential asymmetric responses in the conditional variance by adding a leverage term to the variance itself (Glosten
et al. (1993)), hence resulting in a Leveraged GARCH (LGARCH) model. Results point towards a Monday effect that increases the variance of silver cash prices, findings in line with Yang and Brorsen (1993), and a Wednesday effect decreasing the variance of silver futures prices. In a next step, Lucey and Tully (2006a) augment their GARCH framework by including day-of-the-week dummy variables in both the mean and the variance (a procedure similar to Clare et al. (1998) and Lucey (2000)) and find that the seasonality in silver price returns is not due to daily variation in risk - an important conclusion for silver investors as this means the risk-return relationship does not hold.

A recent paper looking at calendar-effects in precious metal returns is that of Auer (2015) finding no Friday the 13th effect for silver returns. The author builds upon the findings of Kolb and Rodriguez (1987) and works with a dummy-augmented GARCH model to understand the impact that certain days exercise on the conditional means of silver returns. The methodology used is slightly different from Lucey and Tully (2006a) and relies upon the Generalised Autoregressive Conditional Heteroscedasticity model with time-varying Skewness and Kurtosis (GARCHSK) proposed by León et al. (2005) and estimated under the assumption of a Gram-Charlier expansion of the normal density function for the error term (Jondeau and Rockinger (2001)) - a series easier to estimate than the non-central t distribution proposed by Harvey and Siddique (1999). Auer (2015) extends the GARCHSK model by adding lagged returns to the mean equation in order to capture potential serial correlation (a method similar to Bhattacharya et al. (2003)) and by adding two dummy variables in the mean equation according to calendar-days of interest. Even though the main finding of the author is that no Friday the 13th effect can be observed in silver, platinum and palladium returns, another important conclusion is that there is significant evidence of time-variation for both the skewness and the kurtosis of white precious metal returns.
In the same year, Caporin et al. (2015) focus on the behaviour of silver, platinum and palladium spot prices return, volatility and liquidity. A first major contribution to the field is the data used in the paper: trading quotes issued by the Electronic Brokerage Services (EBS) and provided by ICAP plc. The time frame observed ranges from the 27th of December 2008 to the 30th of November 2010, where 100,962,954 quotes were observed for silver, amounting to 27,638 trades and a volume of 1,173,425,000 oz of silver traded. Caporin et al. (2015) develop a battery of well fitted models depending on the feature of silver under study. When looking at return and volatility, the authors specify an Exponential Generalised Autoregressive Conditional Heteroskedasticity (EGARCH) model as proposed by Nelson (1991) and superior to traditional GARCH models when exogenous variables are included. Even though the authors are aware of the issue of presence of long memory in high-frequency returns volatility (Andersen and Bollerslev (1997a,b, 1998), Bordignon et al. (2007, 2009)), Caporin et al. (2015) argue that introducing long memory in the conditional variance equation would further increase the model’s complexity - hence resorting to a solution proposed by Corsi (2009) specifying the dynamic of the variance following a Heterogeneous Autoregressive (HAR) structure. The approach of Corsi (2009) allows the researcher to approximate the long-memory behaviour by reproducing the volatility persistence term of the HAR model. Results from modeling the return and volatility time series indicated the presence of a stochastic periodic behaviour and the possible presence of long-range dependence in the volume time series. In response to these features, Caporin et al. (2015) work with a multi-factor Generalised Autoregressive Moving Average (GARMA) model as proposed by Woodward et al. (1998) which allows for long-memory behaviour associated with specific periodic frequencies. The silver volume GARMA modeling approach of Caporin et al. (2015) can be used to forecast both volume levels and volume density if upgraded with a GARCH or EGARCH equation. The authors conclude their work by saying that white
precious metals have features comparable to those of more traditional assets and that market liquidity of silver, platinum and palladium is characterized by intra-day seasonalities and very strong commonality. More specifically, platinum is found to be the least liquid and least volatile metal of the precious metals considered.

Another paper that looks at trading data is that of Mutafoglu et al. (2012) focusing on the question if trader positions can predict the direction of gold, platinum and silver spot price movements. The authors take into account the weekly Commitment of Traders (COT) reports for silver and platinum prices obtained from the Commodity Futures Trading Commission (CFTC) between January 1993 and December 2009. Mutafoglu et al. (2012) build their methodology upon a basic VAR model with two variables: precious metal spot price returns and trader positions. Unclear about exogeneity issues of system parameters, they treat each variable symmetrically (Enders (2014)); furthermore, as argued by Harvey (1997) and Clements and Hendry (2001), failures of economic forecasts can be due to structural breaks and hence lead to unreliable estimation results of model parameters. Mutafoglu et al. (2012) address this by using a generalised structural break test with unknown break points (Andrews (1993)) to test the stability of model parameters over the entire sample period. If structural breaks are detected in the manner of Bai (1994, 1997), the VAR analysis is repeated for each sub-sample period. Results from the entire sample show that white precious metal market returns explain trader’s positions and that a major structural break happened in the early 2000’s, after which the tendency of trader positions to follow returns became much stronger.

Recently, Lucey and O’Connor (2016) identified whether or not investors in the gold and silver markets are affected by psychological barriers in regard to prices ending in 0 and in 00. Intraday silver prices between the 2nd of March 1975 and the 30th of April 2015 are taken into account and tested for the uniformity of their distribution. While evidence is found for the
existence of psychological barriers in the price of gold, no such evidence is observed for silver, who’s price seems unaffected by the format.

One paper that stands out, though technically concerned with calendar-effects as well, is the work from Lucey (2010) finding evidence for the existence of a lunar cycle on precious metal returns. Considering daily PM fixing prices for silver traded in London between January 1998 and September 2007 against lunar phases from the Munich Astronomical Archive, Lucey (2010) applies a battery of classical descriptive and analytic tests and finds that returns around the full moon tend to be negative in contrast to positive returns around the new moon. The existence of lunar seasonality for silver is in line with previous findings from Dichev and Janes (2003) and Yuan et al. (2006) who did similar work on stock market returns. On the other hand, the evidence for a lunar cycle on the return of platinum prices is very weak.

5.3. Silver Price Forecasts

Two papers emerge from the field of behavioural aspects of white precious metals by looking at the forecasting abilities of survey forecasts of the price of silver.

Fritsche et al. (2013) consider silver price forecasts obtained from Consensus Economics Inc. for different forecast horizons between June 1995 and August 2012 and rely on the market-timing approach proposed by Pesaran and Timmermann (1992, 1994) in order to test for the accuracy of silver price forecasts. Results for silver are different than those obtained for gold; indeed, it is proven that silver price survey forecasts contain information on the subsequent price changes, while forecasts on the price of gold are not accurate in predicting future price movements.

Working with the exact same data set provided by Consensus Economics Inc. between June 1995 and August 2012, Pierdzioch et al. (2013) model the behaviour of the authors of gold and silver price forecasts and find evidence for irrational behaviour on both markets. More specifically, results based
on the asymmetric loss function proposed by Elliott et al. (2005) to test for the rationality of forecasts, indicate a herding behaviour of some forecasters, while others tend to issue more *extreme* forecasts in order to differentiate themselves from others. Indeed, this change in behaviour tends to occur depending on the customers of silver price forecasts, resulting in biased forecasts in order to assure a loyalty of the main group of customers.

6. The Investment Benefits of White Precious Metals

6.1. Adding White Precious Metals into a Portfolio

A very early classic on the matter is the paper from Jaffe (1989), which looks at implications of adding gold to hypothetical portfolios of varying risk. In his work, Jaffe (1989) also considers the monthly silver price from September 1971 to June 1987, a time frame during which the average monthly return of silver was about 1.50% and the beta (Markowitz (1952), Sharpe (1964)) to the S&P 500 was of 0.34 (considerably higher than that of gold at 0.09) while the standard deviation was of 11.58% (against 7.88% for gold). Looking at the relationship between precious metals and other assets in a next step, silver shares a rather high correlation coefficient with gold (0.744) and with Toronto Stock Exchange (TSE) gold stocks (0.589); the lowest correlation coefficient is that between silver and the German Mark/US Dollar exchange rate (-0.252).

McCown and Zimmerman (2007) investigate the inflation hedging abilities of silver but also look into the investment performance of the metal by using multiple asset pricing models. The data used is the monthly spot price of silver and the US CPI between January 1970 and December 2006. When looking at investment performance, McCown and Zimmerman (2007) start by estimating the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966) for silver. The risk-free rate considered is the yield on 30 days US Treasury Bills and the market portfolio is given by three different proxies: the Morgan Stanley Capital International (MSCI)
World Index, the MSCI World Index denominated in US Dollars (showing the returns a US investor would get if he does not hedge foreign exchange risks), and finally, the MSCI World Index denominated in local currencies (showing the return an investor would get if he completely hedged foreign exchange risks). The Beta results for silver are quite similar independent of the index used as market portfolio proxy; small Betas (around 0.33) are recorded for a period of up to six months and they become negative after one year. The findings suggest that silver is a less volatile investment to the market on the short-run, and that it moves in opposite direction than the market on the long-run - arguments in support of silver’s ability to be used as a hedging tool against stock markets. In a next step, McCown and Zimmerman (2007) model the return on silver using the multiple-factor Arbitrage Pricing Theory (APT) proposed by Ross (1973, 1976). The APT models the return of an asset through a linear combination of macroeconomic variables; a discussion about which variables to use can be found in Chen et al. (1986), Chen (1991) and Shanken and Weinstein (2006) amongst others. The results of the APT point towards the growing importance that inflation has on silver when a larger time set is considered. In a final step, the authors look more into the relationship between silver and expected inflation. Based on the Fischer (1930) equation, they derive expected inflation as the spread between US Treasury securities and Treasury Inflation Protected Securities (TIPS). Results point towards a high correlation between the price of silver and expected inflation, making it a good indicator of the latter, though gold proved to perform better at this role than silver.

A few years later, Conover et al. (2009) continue the discussion about the investment benefits of precious metals by taking into account daily returns of silver and platinum between the 17th of January 1973 and December 2006, but also by looking at equity performance of precious metal companies. The tactical allocation of assets in the portfolio is made based on changes in the direction of the Fed discount rate. This technique suggested
by Jensen et al. (1996) propose that a direction change of the Fed discount rate marks a turning point in the Fed’s monetary policy (the close alignment between discount rate turning points and federal fund rate turning points is discussed further in Thornton (1998) and Jensen and Mercer (2006)). Multiple findings emerge from the results of Conover et al. (2009). First of all, an investment into the equity of precious metal firms proves to be much more efficient than a direct commodity investment; adding a proportion of 25% of the mentioned stocks to a portfolio of US equities leads to an increase in return by 1.65% and a decrease in the portfolio’s standard deviation by 1.86%. A further finding is the superiority of an investment in gold against an investment in white precious metals, results in line with Hillier et al. (2006), also, the investment benefits held by precious metals varied over time and grew during the later years of the sample (a formal derivation of the time-varying nature of silver’s investment benefits is provided by Bampinas and Panagiotidis (2015)). The last finding is about the relation between the Fed policy and precious metals. Adding silver to a portfolio is much greater when the Fed policy is restrictive rather than expansive, findings somewhat in line with Conover et al. (2008) who advise investors to use monetary conditions as a guideline for portfolio allocation; however, Conover et al. (2009) point towards the impossibility of knowing that return patterns are caused by monetary policies.

Bruno and Chincarini (2010) focus on calculating the ideal amount of commodities in a portfolio by generating optimal mean-variance portfolios. Even though the authors look at more than 15 different countries between 1930 and 2009, the only national portfolios explicitly advised to have holdings in silver where the cases of Germany, Mexico and the United States of America between 1970 and 2009. For both Germany and the U.S.A., an optimal allocation of 0.01% to silver is advised (along with 4.36% and 3.07% in gold for Germany and the US respectively), whereas Mexico stands out with a much higher proportion of 1.98% of silver (along with 5.95% of gold).
Also focused on a portfolio framework are Belousova and Dorfleitner (2012) who look at traditional assets held by a European investor: a mixture of global stocks, European sovereign debt and the money market. Looking at monthly data between January 1995 and December 2010, the authors build upon the framework of Markowitz (1952) and apply a mean-variance spanning method introduced by Huberman and Kandel (1987) and formally developed by Kan and Zhou (2012) to check the impact that the introduction of an asset has on the portfolio performance. Results are divided into bull and bear markets and show that adding silver to a portfolio during bull markets reduces volatility and enhances return. During bear markets on the other hand, silver is shown to reduce portfolio risk, results somewhat in line with Hillier et al. (2006) who look at a different time frame and conclude that even though adding silver to a portfolio is beneficial, gold and platinum are more beneficial investments. Regarding platinum, adding it to a portfolio in a bullish market environment lowers portfolio volatility, while platinum is found to lose its diversification properties during bear markets.

Hammoudeh et al. (2013) look at the daily downside risk associated with gold, silver, platinum, palladium, oil and the S&P 500 Index between the 2nd of January 1995 and the 5th of July 2011 within a Value-at-Risk (VaR) framework. The VaR metrics are computed at the 99% confidence level (in accordance with the Basel Accord rules for computing capital requirements (Basel Committee on Banking Supervision (1988, 1995, 1996, 2006)) and the RiskMetrics model from J.P. Morgan (1996) which proved to be superior to eight other proposed models. Two Asymmetric Power Autoregressive Conditional Heteroscedasticity (APARCH) models proposed by Ding et al. (1993) are used to forecast results. The difference in the two APARCH models lies in the distribution, where the first one follows a normal student-$t$ distribution and the second a skewed-$t$ distribution (Tu et al. (2008) look at Asian markets and provide evidence for the superior performance of the skewed-$t$ distribution - results in line with Mittnik and Paolella (2000) and Giot and
Laurent (2003)). With VaR results on hand, Hammoudeh et al. (2013) follow the portfolio construction approach proposed by Campbell et al. (2001) using a performance metric based on the mean-variance approach. Three optimal portfolios are constructed: the first consisting only of the four precious metals, the second consisting of all six assets under study, and the third consisting of gold, oil and the S&P 500. Portfolio one has the highest annual return, around 9%, but also a higher standard deviation than the second, containing the greatest amount of different assets, which scored an average yearly return of 8.625%. Looking at portfolio efficiency, the authors conclude that an optimal portfolio should hold a higher proportion of gold than any other asset (even though silver was the best performing asset over the time frame observed), and that overall, the pure precious metal portfolio proved to be the least efficient one.

6.2. Debt and Equity Hedging Abilities of White Precious Metals

Quite some attention was paid to the performance of white precious metals as a hedging tool against stock market movements over the past years. Hillier et al. (2006) look at the return of the Zurich silver price in US Dollar per kilogram and the London Free Market platinum price in US Dollar per Troy ounce against the S&P 500, the MSCI Europe, the MSCI Australia and the MSCI Far East Index between the 1st of January 1976 and the 1st of April 2004 and apply a GARCH(1,1) model proposed by Bollerslev (1986). During the period observed, silver has proven to be a very volatile asset, with a daily standard deviation of about 2.15% - about twice the standard deviation of the S&P 500, while platinum provided the highest mean daily return of approximately 9% per year. Subsample analyses indicate a decreasing correlation of silver and platinum with the price of gold but a constant very small correlation with the S&P 500 of an order between -0.05 and 0.05. Looking into the diversifying properties of an investment in white precious metals, the persistent negative elasticity of silver and platinum at the 1% confidence level presumes that it was a valuable diversifying asset against
an S&P 500 portfolio between 1976 and 2004 but not so in regard to the MSCI indices. Focusing in a next step more on periods of high volatility and poor returns of stock markets, silver’s hedging abilities prove to be stronger than those of platinum. However, when looking at what metals to optimally hold in a portfolio, silver did not perform as well as both gold and platinum which scored higher returns over the period.

Taking again a VaR approach in the quest of optimising portfolios, Sarafrazi et al. (2014) focus on daily downside risk of euro-zone national equity and sovereign bond markets by classifying the countries in two distinct groups: the PIIGS (Portugal, Italy, Ireland, Greece and Spain) and the Core (Germany, France, Austria, The Netherlands and Finland). The period observed ranges from the 31st of March 1999 to the 20th of November 2012 and the methodology used is the same as Hammoudeh et al. (2013). The results point to multiple interesting findings. By means of the Sharpe (1966, 1975, 1994) ratio, the optimal portfolio over the full period is composed of 4% Austrian bonds, 2% Finnish bonds, 3% French bonds, 34% German bonds, 5% Dutch bonds, 13% of gold, 11% of copper, 11% of oil, 9% of silver and 7% of platinum (sic). When looking at the very troublesome subperiod between July 2007 and November 2012, results show that from all the commodities observed (gold, silver, platinum, palladium, copper and oil), only gold and silver prove to contribute to diversification benefits to stock and bond portfolios - adding silver to both portfolios increases the Sharpe ratio.

Another paper of the same year to look at the hedging capacities of silver, platinum and palladium against sovereign bonds is that of Agyei-Ampomah et al. (2014) questioning the commonly held belief that gold is a good protection against bonds (see Baur and Lucey (2010) and Baur and McDermott (2010)) by looking at whether or not other metals might not have proven to be more effective in the time period from July 1993 to June 2012. The methodology used is that of Baur and McDermott (2010) capturing extreme
movements of the bond prices by assigning a dummy variable corresponding to a return in the lowest 1st, 5th or 10th percentile. On average, the returns of silver are negatively correlated with bond returns; looking at the 95% confidence level through a Wald (1943, 1949) test, the hedging ability is particularly strong for Austria, Belgium, Germany, Italy, Portugal and the UK. Regarding platinum, the hedging ability is even stronger while it is much weaker for palladium, where the white precious metal only served as a hedge against debt in Austria, Germany, Greece, the Economic and Monetary Union and the UK. Looking into three different sub-periods, July 1993 to December 2000, January 2001 to December 2006, and finally, January 2007 to June 2012, the hedging ability of silver against sovereign bonds was only statistically significant between July 1993 and December 2000 and during no other period, while statistical significance is observed for platinum between July 1993 and December 2000 as well as between January 2007 and June 2012. For palladium finally, strong hedging potential is uncovered for the later time period of the sample, between January 2001 and June 2012. A very interesting finding is the great hedging potential from industrial metals over the time period.

Auer (2014) looks at the hedging ability of diamonds against stock market and currency risk and contrasts the performance with both gold and silver; hence also deriving implications for the two precious metals. Looking at weekly data from January 2002 to July 2012, the author builds upon a Multivariate GARCH model of constant conditional correlations (CCC-GARCH) proposed by Bollerslev (1990) that he augments with a time-variation element proposed by Engle (2002) to capture volatility and correlation amongst the series. The results from different popular performance measurements indicate that even though some individual diamonds (in regard of their quality grade) are more volatile than precious metals, a well diversified diamond portfolio bears less risks than investments in precious metals. An investment in silver however offered the better investment performance and should be
preferred against diamonds as a portfolio component.

Another paper to use the methodology from Baur and McDermott (2010) is the paper of Lucey and Li (2015) who look at the potential of the four main precious metals to act as a safe haven against the S&P 500 and US 10 year bonds taking into account daily data between January 1989 and July 2013. The methodology is upgraded with the approach of Ciner et al. (2013) to identify time-variation in the safe-haven property of silver, platinum and palladium and graphically depict it. Results for silver have interesting implications for investors; concerning equity, silver was only a safe haven during the last quarter of 2009 and the first quarter of 2010, the performance of gold, platinum and palladium was much better and it had safe haven properties during more quarters - concerning bonds though, silver was a safe haven at times during which gold failed to be, but also during far more quarters than both platinum and palladium. Empirically however, gold should be considered the better safe haven investment for it acts as one more often than white precious metals.

In a recent paper, Mensi et al. (2015) look at the linkages between silver and other commodities and develop implications for Saudi-Arabian investors derived from daily data between the 1st of June 2005 and the 13th of August 2013. The FIAPARCH model is a symbiosis between the FIGARCH model proposed by Baillie et al. (1996) and the APARCH model proposed by Tse (1998). The FIAPARCH model allows: asymmetric volatility responses to both positive and negative shocks, an endogenic determination of the power of returns by the data itself, and finally, long memory in the volatility dependence (Conrad et al. (2011) applied and tested the efficiency of FIAPARCH models on different national stock markets and find the forecasting ability to be superior). In the paper of Mensi et al. (2015), a traditional FIAPARCH model is then upgraded with a DCC model by Engle (2002) to identify time-varying correlations amongst the variables. Initial results point towards a negative linear correlation between silver and Saudi Arabian
stock returns - a finding upgraded by the DCC-FIAPARCH model showing evidence for time-varying conditional correlations between both series, disproving the capacity of silver to be used as a hedge or a safe haven against the Tadawul. In order to extract structural breaks in the correlation relationship, Mensi et al. (2015) work with a modified Inclan and Tiao (1994) Iterated Cumulative Sums of Squares (ICSS) test proposed by Sansó et al. (2004) to detect structural breaks in the volatility (applications of the ICSS test can be found in Rapach and Strauss (2008), Vivian and Wohar (2012) and Ewing and Malik (2016) amongst others). Two structural break dates are observed between silver and the Tadawul: the first in November 2005 and the second in July 2012.

Bredin et al. (2017) continue the investigation into the equity hedging abilities of silver and platinum by considering daily, weekly and monthly price returns between 1980 and 2014. The hedging potential is identified via a Value-at-Risk (VaR) procedure detecting the level of tail- and downside risk associated with silver and platinum investments, and measures the respective cost or benefit of such investments by considering risk-adjusted returns against an S&P 500 portfolio. Results indicate the superiority of gold to act as a hedge, while the equity risk reduction potential of silver and platinum only seems to be strong on a short time horizon but not on long time horizons.

6.3. White Precious Metals and Inflation

Taylor (1998) is the first paper to look at the inflation hedging ability of white precious metals. The dataset used by the author is rather extensive and covers the US Consumer Price Index and monthly silver, platinum and palladium returns between January 1914 and April 1996, broken down into subperiods. A Jarque-Bera test (Jarque and Bera (1980, 1987)) indicates non-normality as well as high leptokurtosis across the time series, therefore, an Ordinary Least Squares (OLS) method would minimise the sum of the squared residuals and yield inefficient coefficient estimates. Especially
when working with precious metal prices, Weston (2012) points towards the importance of outliers caused by intense speculative activity. Dielman and Pfaffenberger (1990) propose a Least Absolute Deviations (LAD) procedure proven to be more powerful in presence of outliers for it minimises the sum of absolute deviations instead of minimising the sum of squared residuals. A second procedure used by Taylor (1998) is the Huber M-Estimation (Huber (1964, 2011)), minimising the sum of a function of the residuals, a mixture between OLS and LAD estimations. Both procedures are augmented with a Granger and Newbold (1974) test for non-stationarity to ensure that the relationship is not spurious in nature. The results for silver indicate that it was a long-run hedge over the period observed, but that it also served as a short-run hedge against the US CPI over many subperiods of the sample. A noteworthy finding is that silver was a hedge during the second Organisation of Oil Exporting Countries (OPEC) crisis of 1979, but not during the first OPEC crisis of 1973. For platinum and palladium, Johansen cointegration results indicate that the two white precious metals served as a long-run inflation hedge, while evidence also points towards the short-run hedging abilities of platinum.

A few years later, Adrangi et al. (2003) continue the investigation into the relationship between silver and US inflation. The authors work with monthly averages of London Fix silver prices between April 1967 and November 1999 as well as the American Industrial Production Index (IP) and the Consumer Price Index - both issued by the International Financial Statistics of the International Monetary Fund (IMF). Adrangi et al. (2003) build their methodology upon two theoretical frameworks. The first one, based on the works of Fama (1981), Fama and Gibbons (1982) and Geske and Roll (1983) predicts that a rising inflation rate leads to a reduction of both the economic activity and the demand for money (Motley (1998) provides evidence that persistent inflationary pressures could lower US Gross Domestic Product (GDP) growth) - hence to a reduction of corporate profits and their
stock price; an impact known as the proxy effect (Fama (1981)). Adrangi et al. (2003) argue that this would on the one hand lead to a reduction of industrial demand for silver, but might lead to an increase of the investment demand side due to silver’s alleged ability to act as a hedge during inflationary times. The second framework is build upon the portfolio equilibrium model of Feldstein (1980) who developed it looking at gold only, though Adrangi et al. (2003) apply it to silver as well. The framework is based on the assumption that the demand for gold and bonds in a portfolio is a function of expected real after-tax returns of the two assets. However, Feldstein et al. (1977) showed that the net after-tax return on gold is higher than the after-tax return of bonds as long as the capital gains tax is lower than the ordinary income tax rate; therefore, during inflationary periods, the relative price of gold rises, making it a good inflation hedge. Checking for cointegration (Johansen (1991)) and causality (Granger (1969)), results for silver are multiple. First of all, evidence proves that silver was a good hedge against inflation over the time period observed. Second, the Fischer (1930) hypothesis holds, in other words: real silver returns are not adversely affected by inflation. However, econometric results do no offer support in favour of the Fama (1981) proxy hypothesis. A final finding is the positive relationship between silver and the CPI in the long-run equilibrium and the short-run dynamics derived from it.

A recent paper that added to the knowledge of silver as a tool against inflation is by Bampinas and Panagiotidis (2015) taking into account annual data of the price of gold and silver and consumer prices in the United Kingdom and the United States of America between 1791 and 2010. Reinhart and Rogoff (2011) work with historical inflation data going back this long and their work is used as a data source for the paper of Bampinas and Panagiotidis (2015). Both the linear Hodrick and Prescott (1997) filter and an asymmetric Christiano and Fitzgerald (2003) band-pass filter were used to derive expected CPI series. Bampinas and Panagiotidis (2015) are one of
the few papers to look at the relationship between silver and inflation in a time-varying manner; the methodology is based upon the Johansen (1991, 1995) cointegration framework and upgraded with a Bierens and Martins (2010) test for time-varying cointegration. For both countries, the UK and the US, silver shares no long-run relationship with inflation; however, in a time-varying framework, a strong long-run relationship does exist between silver and UK inflation. Since a time-varying relationship still fails to exist for the US, the authors conclude that silver was not an effective inflation hedging instrument against American inflation in the past 200 years.

6.4. Exchange-Rate Hedging

Pierdzioch et al. (2016b) consider the ability of daily gold, silver, platinum and palladium prices between January 1999 to December 2015 to act as a hedge against exchange rate movements from the US Dollar versus the Yen, the Canadian Dollar, the Euro, the Pound Sterling and the Australian Dollar. Results from a Bayesian additive regression tree indicate that silver is the overall better exchange rate hedge than platinum and palladium. Silver is found to be an asymmetric hedge against exchange rates: it is a weak hedge in times of US Dollar appreciation, but a strong hedge, even a safe haven, during times of US Dollar depreciation. Platinum and palladium are not very effective currency hedgers except for their pronounced ability to hedge depreciation of the US Dollar against the Canadian and the Australian Dollar.

7. Volatility Issues around White Precious Metals

An early paper to look at silver price volatility is that of Barnhill and Powell (1981) building upon the silver market crash of March 1980. More specifically, the authors derive which determinants caused the great volatility of the silver price between July 1979 and April 1980 by looking into the demand and supply for silver, as well as into the relationship between the
price of silver and macroeconomic indicators - amongst which the yield on 3 months Treasury Bills and the CPI. Barnhill and Powell (1981) point towards multiple explanations for the sharp increase of the silver price in late 1979. A first reason was a constant and considerable shortfall of silver production relative to the commercial demand for silver, leading to the belief that silver was undervalued. A second reason is found to be the important acquisition of silver by certain parties, the most prominent being the Hunt family (implications of the role of the Hunt family on the silver market can be found in Fay (1982), in Krehbiel and Adkins (1993) and in Williams (1995)). Other driving forces identified were a rising investment demand for silver due to unattractive real returns offered by conventional investments and political actions undertaken by the government of India and the United States of America restricting both access and reallocation of a large portion of above-ground silver stocks. In a next step, Barnhill and Powell (1981) identify the reasons leading to the collapse of the silver price in early 1980: on the demand side, a fall in industrial U.S. commercial demand by 40% accompanied by a slow-down of cash silver acquisition by the Hunt family is observed. On the supply side, an increase of 200% in scrap supply as well as an increase of 80% in recycled silver overshadowed the demand. Finally, a growing attractiveness of alternative investments mixed with a negative investor sentiment against silver led to a very heavy drop of the price in late March 1980.

Focusing more on volatility implications for the industry, Labys et al. (1998) derive evidence for the existence of business cycles in the price of silver in order to obtain greater forecasting information to improve investor’s behaviour. The data observed is the monthly Handy and Harman silver price quoted in US Dollars between January 1960 and December 1995 and the methodology used in the paper is threefold. First, a standard business cycle identification procedure developed by the National Bureau of Economic Research (NBER) looking at turning points and time series peaks is
used (more details can be found in Bry and Boschan (1971a) and in Moore (1983)). This procedure is upgraded with a Weibull test of cyclical duration proposed by Davutyan and Roberts (1994) and assuming a linear relationship between the duration and the hazard function (Zuehlke (2003)), finally, cyclical components of the silver price series are modelled through the Structural Time Series (STS) model proposed by Harvey (1985, 1990, 1994) that divides a time series into the trend component, the cyclical component, and the irregular component. The statistical properties necessary to specify and interpret the model are outlined in Harvey (1989, 1994) and the data was divided into sample periods to assure that the irregular component meets the required conditions of non-autocorrelation and homoscedasticity (Harvey and Koopman (1992)). Results for silver point towards a decrease in volatility over the period under study and a total of 17 individual cycles observed though they are largely time-invariant. On average, contraction phases of the silver price tend to be longer then the more seldom occurring expansion phases.

Plourde and Watkins (1998) study the volatility of the monthly oil price between 1985 and 1994 and compare it to other commodities, hence also drawing conclusions for the price of silver. The notion of volatility is detected via three different aspects: first, the dispersion of the monthly price changes, second, the size of the absolute value of the price changes, and third, the frequency distribution of the absolute value of the price changes. Results for the dispersion of monthly price changes are obtained through a modified Levene (1960) proposed by Brown and Forsythe (1974) and a Fligner and Killeen (1976) test, as they allow for variation amongst the data of the different variables observed and perform better in terms of both robustness and power (Conover et al. (1981)). The location of absolute values of monthly price changes are derived through a Mann-Whitney-Wilcoxon (MWW) test (proposed by Wilcoxon (1945) and Mann and Whitney (1947)) verifying if the measures of central tendency of the time-series have the same value by
generating a normally distributed test statistic for each individual sample. Finally, the distribution of absolute values of monthly price changes are obtained via a specific $\chi^2$ test proposed by Mosteller and Rourke (1973) and focused on the overall shape of the distributions, assigning less weight to individual outliers. Results point towards a consistency of the volatility between oil and industrial metals, but large volatility differences between oil and silver. Not surprisingly, oil is found to be far more volatile than silver in both the short- and the long-term.

Nowman and Wang (2001) look at the monthly price of silver between February 1970 and May 1997 using a battery of nine continuous time models. The authors follow the previous work of Nowman (1998) on interest rate models and build upon the general stochastic differential equation proposed by Chan et al. (1992) that consists of a drift component, a mean reversion parameter and a variable measuring the dependence of the conditional volatility on the level of the price modelled via a Wiener process. By assigning different values to the individual component of the equation from Chan et al. (1992), nine testing models can be designed to analyse the volatility of the price of silver. Results of the different models for silver show that the volatility of the price depends strongly on the price level, furthermore, a rather interesting finding from an econometric point of view is that both the drift and mean reversion parameters are insignificant at the 5% level across all models.

A few years later, Hammoudeh and Yuan (2008) focused on the volatility of the daily price of silver in the presence of crude oil and interest rate shocks between the 2nd of January 1990 and the 1st of May 2006. The authors work with three different models when looking at different aspects of silver price volatility: a standard GARCH model to study the impact and the persistence of oil and interest rate shocks on the volatility of silver future prices, an EGARCH model to understand the effect of both good and bad news on silver price volatility, and finally, a Component Autoregressive
Conditional Heteroscedasticity (CGARCH) model to study the extent and persistence of transitory and permanent volatility in the short- and long-run. It should be mentioned, what specifications the CGARCH contains; a Component GARCH allows mean reversion to be time-varying (Grier and Perry (1998)) and to distinguish between convergence and persistence of volatility on the short- and long-run (Guo and Neely (2008), Bauwens and Storti (2009), Engle and Sokalska (2012)). Results point towards a certain cyclicality and volatility persistence for silver (findings in line with Roberts (2009)), alleged to it’s feature as being both a precious and an industrial metal. Furthermore, the EGARCH results point towards silver’s ability to be a good investment in anticipation of bad news and to a cooling effect that past positive oil shocks have on the volatility of silver.

Sari et al. (2010) look at information transmission amongst the daily spot prices of gold, silver, platinum, palladium, oil and the US Dollar/Euro exchange rate between the 4th of January 1999 and the 19th of October 2007. Methodologically, the authors use a generalised impulse response functions proposed by Koop et al. (1996) and the generalised impulse response function from Pesaran and Shin (1998) in order to extract and analyse the impact and responses to shocks. The main advantage of the methods used is that the ordering of the variables in the VAR is not relevant any longer; the numerical results extracted show how much of the variance of a variable can be explained by shocks to another variable. The long run impact of gold on silver is quite important, explaining about 16% of the variation in silver; findings in line with Lucey and Tully (2006b). Interestingly, this relationship is also observed the other way around: silver explains 23% of the variance in the gold price (results later rejected by Balcilar et al. (2015)). On the short-run, unexpected shocks of gold, platinum and palladium have a positive and significant impact on the price of silver and vice versa - further evidence point towards a positive short-run impact of the exchange rate on the silver price. The effect amongst white precious metals is also quite important,
where silver explains about 10% of the variations of both platinum and palladium prices, while platinum and palladium explain about 22% of their respective price fluctuations.

As a reaction to metal prices reaching new record highs and facing a very uncertain future, Chen (2010) work with a very large data set of more than a hundred years to derive volatility insights for silver and platinum amongst other metals. Chen (2010) uses annual prices between 1900 and 2007 obtained from the US Geological Survey (USGS) and groups different precious metals together in order to gain insights of the price volatility of precious vs. industrial metals. The results indicate much higher volatility figures for industrial metals, probably pointing towards the ability of precious metals to act as hedges and safe havens (see Baur and Lucey (2010) and Baur and McDermott (2010)). In a second step, Chen (2010) uses a sub-section analysis proposed by Chan and Clements (2007) to understand the importance of within- and between-group volatility and shows that the volatility is higher within the metal groups than between the metal groups; in other words: the volatility is higher amongst precious metals than between precious and industrial metals. In a final step, Chen (2010) applies a single factor asset pricing model and finds that the importance of global macroeconomic factors in explaining silver and platinum price volatility has increased over the time period observed. Between 1900 and 1971 (the year the USA departed from the Bretton Woods system) over 90% of the price volatility of silver can be attributed to commodity-specific risk against under 10% to be attributed to global macroeconomic risk factors; between 1972 and 2007, these numbers shifted and the global macroeconomic risk share is above 36% against nearly 62% for commodity-specific risk factors. A similar picture is observed for platinum, where the share of commodity-specific risk between 1900 and 1971 is of more than 93% against around 87% between 1972 and 2007.

In the same year, Choi and Hammoudeh (2010) look at the importance
of financial and geopolitical crises on the volatility of commodity prices and extend previous studies by looking at correlation amongst commodity prices. Looking at weekly closing spot prices between the 2nd of January 1990 and the 1st of May 2006, Choi and Hammoudeh (2010) use distinctive methodologies to assess the issues of volatility and correlation. In order to measure the switch in return volatility between variance regimes and quantify their duration, the authors work with a univariate Markov-switching heteroscedasticity model (Krolzig (2013)) allowing to distinguish between high- and low-variance regimes (Haas et al. (2004)) where the probability of each regime is given by Kim (1994). Results of the Markov-switching model point to a duration of the low volatility state for silver of about 50 weeks against a duration of the high volatility state of about 25 weeks. Also, even though silver seems to be highly volatile around the same time periods as gold, it is especially in the early 2000’s that silver was much less volatile than gold. In a second step, Choi and Hammoudeh (2010) work with a DCC multivariate GARCH (Engle (2002)) to understand the development of the correlation between gold and silver. Though the correlation between them is always fairly high and remains constant through time, a peak in correlation is observed in the second half of 1992 while the lowest correlation is observed around late 1996.

Focusing on interactions between precious metals, Morales and Andreoss-O’Callaghan (2011) investigate the nature of volatility spillovers amongst daily precious metal returns between the 1st of January 1995 and the 4th of November 2010 by means of a GARCH (Bollerslev (1986)) and an EGARCH (Nelson (1991)) model. Results of their work are multiple. First of all, descriptive statistics indicate that only palladium has a higher standard deviation than silver; furthermore, the standard deviation of daily silver returns is more than two times bigger than the standard deviation of gold returns. A second finding is that over the entire period, there is evidence of volatility running from gold to silver but not the other way around. Results from the
GARCH model point towards a significantly positive relationship between precious metal returns, where the prices of gold, silver, platinum and palladium tend to appreciate simultaneously and also depreciate at the same time. On the other hand, results from the EGARCH model show that bad news have a greater impact on the silver market than good news.

A few years later, the same authors upgraded their work by adding oil and equity returns to their choice of variables under observation. Morales and Andreosso-O’Callaghan (2014) examine volatility persistence on precious metal returns and derive the relationship amongst the commodities up until the global financial crisis of 2008 by looking at daily silver and platinum returns between the 1st of January 1995 and the 25th of May 2008. The methodology is similar to that of Morales and Andreosso-O’Callaghan (2011) but upgraded with an Iterative Cumulative Sums of Squares (ICSS) algorithm to identify structural breaks and sudden changes in the variance of returns. The ICSS algorithm was designed by Inclan and Tiao (1994) and assumes that a time series has a stationary unconditional variance over a given time period until a break takes place. After such a break is detected, the unconditional variance is again assumed to be stationary until the next change occurs and so on - a method leading to a possible overstatement of the actual number of breaks in variance (Fernández (2004)). Also, ICSS results are not reliable under the presence of conditional heteroscedasticity (Bacmann and Dubois (2002)), Morales and Andreosso-O’Callaghan (2014) solve this problem by filtering the series with a GARCH(1,1) model. Results point towards 25 breaking point for silver series, less then for gold and platinum. Concerning the relationship between silver and equities, the GARCH and EGARCH results point toward an insignificant relationship with the Dow Jones index, but to a significant positive relationship with both the FTSE100 and the Nikkei225. A significant positive relationship between silver returns and Brent returns is also indicated at the 5% significance level. Concerning platinum, a significant positive relationship is observed with
both Dow Jones and Nikkei225 returns. As for silver, a significant positive relationship between platinum and Brent returns is observed.

A paper that uses a very similar approach as Morales and Andreosso-O’Callaghan (2014) two years beforehand is that of Vivian and Wohar (2012). The authors look at daily spot prices of different commodities between the 2nd of January 1985 and the 30th of July 2010 and work with a GARCH(1,1) model and an ICSS algorithm to capture volatility and structural breaks of the series. Even though the time period under observation is longer than that of Morales and Andreosso-O’Callaghan (2014), Vivian and Wohar (2012) find evidence for fewer breaking points in the precious metal series while the GARCH model results indicate high volatility persistence of the silver, platinum and palladium prices.

Arouri et al. (2012) analyse the return and volatility of gold, silver, platinum and palladium in order to investigate both long memory properties and the potential of structural changes of the price series. The authors take daily spot and futures prices between the 4th of January 1999 and the 31st of March 2011 into account and use squared returns to test for long memory (Lobato and Savin (1998) and Choi and Hammoudeh (2009) amongst others use squared returns as well as a proxy for conditional volatility). Testing for long memory components in return and volatility is done via the Geweke and Porter-Hudak (1983), the Robinson and Henry (1999) and the Sowell (1992) test statistics, as they are all extensively used in relevant literature. Structural breaks are identified through an ICSS algorithm (Inclan and Tiao (1994)), similar to Vivian and Wohar (2012) and Morales and Andreosso-O’Callaghan (2014). The distinction between long memory and structural breaks is assured through the tests proposed by Shimotsu (2006) comparing full- and sub-sample parameters and testing them for stationarity (in a manner proposed by Phillips and Perron (1988) and Kwiatkowski et al. (1992)). Finally, fractionally integrated models are build into an Autoregressive Moving Average (ARMA) model (Whittle (1951), Box et al. (2008))
and a GARCH model (Bollerslev (1986)). The ARFIMA-FIGARCH model allows to test the accuracy of long memory because it should be considered fallacious if due to the presence of structural breaks. Results from Arouri et al. (2012) indicate long memory properties only for spot silver and not for silver futures, results rejected by the ARFIMA-FIGARCH model that finds no long memory evidence for spot silver. The ICSS test results indicate only three breaks for each the silver spot and future price; the breaks found are not in line with Vivian and Wohar (2012), but the spot price breaks on the 21st of June 2001 and the 2nd of January 2004 are also observed by Morales and Andreosso-O’Callaghan (2014). Platinum futures returns are found to exhibit the highest long memory in the variance equation, suggesting that platinum might not be able to function as a good hedging instrument; long memory patterns are also observed for palladium futures prices.

In the same year, Cochran et al. (2012) also work with a FIGARCH model to examine the return and long memory properties of daily return volatility for copper, gold, platinum and silver between the 4th of January 1999 and the 10th of March 2009. Testing for the existence of long memory in the return of silver and platinum is done through a modified Range over Standard Deviation (R/S) test proposed by Lo (1991). The silver return series is not modelled as an autoregressive process, but as a multi-index CAPM proposed by Chang et al. (1990) in which the MSCI World Index serves as the market proxy, the VIX is the level of implied equity market volatility, and macroeconomic variable proxy the economic environment. Finally, volatility processes are captured via a FIGARCH approach (Baillie et al. (1996)), in a method similar to Arouri et al. (2012). Results point towards a negative effect of interest rates movement on silver but not on platinum (results in line with Jaffe (1989) and Hammoudeh and Yuan (2008)) and a negative relationship between exchange rates and both silver and platinum returns, proving that the law of one price holds for silver. FIGARCH results show evidence of long memory characteristics for the two white precious metals,
while the metals return volatility share a positive relation with changes in the VIX. Finally, seasonal dummies indicate an increase in volatility of the prices of the three precious metals since the Global Financial Crisis.

One year later, Sensoy (2013) contrasts the findings of Cochran et al. (2012) and finds that the Global Financial Crisis of 2008 has no effect on volatility levels of silver, but indeed caused an upwards shift in the volatility levels of platinum and palladium. The data taken into account are the daily gold, silver, platinum and palladium spot prices between the 2nd of January 1999 and the 15th of April 2013 filtered through an ARMA(p,q) process. The methodology from Lavielle (2005) is used to automatically detect mean shifts in dynamic correlation levels and volatility shifts in precious metal returns. In a next step, the consistent Dynamic Conditional Correlation (cDCC) from Aielli (2013) is used to detect relationships between precious metals price fluctuations; the cDCC build a parameter combining a GARCH like model and an innovation criterion into the traditional DCC model of Engle (2002). The results show that while the turbulent year 2008 has no effect on volatility levels of silver, silver is found to have a volatility shift contagion effect on platinum and palladium. On the other hand, it is found that platinum and palladium have no volatility shift contagion effect on one another; Sensoy (2013) explain this by arguing that platinum and palladium were historically not considered a store of value, creating an insensitivity in the correlation dynamics between the two mentioned white precious metals.

Papadamou and Markopoulos (2014) look at volatility transmission between currency exchange rates and gold and silver prices using hourly data between the 1st of January 2010 and the 27th of March 2012. In a first step, a Quandt-Andrews test for parameter instability (Quandt (1960), Andrews (1993, 2003)) indicates no structural breaks for the time series; an unrestricted BEKK-GARCH model proposed by Engle and Kroner (1995) is used to obtain information about the conditional covariance and explain volatility transmissions amongst the series. The BEKK-GARCH model al-
allows the researcher to define both the conditional variance and covariance of the time series in order to capture possible asymmetric effects. Evidence points towards volatility transmission from gold, the EUR/USD exchange rate and the GBP/USD exchange rate to silver.

Another paper to look at volatility transmission between exchange rates and precious metals is that of Antonakakis and Kizys (2015) taking weekly data between the 6th of January 1987 and the 22nd of July 2014 into account. The methodology applied is that of Diebold and Yilmaz (2009, 2012), using a generalised VAR framework in which Forecast Error Variance Decomposition (FEVD) is invariant to the order of the variables. Silver is a net transmitter to the FEVD of other variables. On average, silver returns contribute to the FEVD of other variables by 52.78% and receives an average of 50.90% from other variables. On the other hand, platinum contributes 52.39% to the FEVD of other variables and receives an average of 49.61%. In contrast to the two, palladium is found to be a net receiver of return spillovers. A similar picture is observed for volatility, where silver and platinum are net transmitters, while palladium is a net receiver of volatility spillovers. Even though the Global Financial crisis of 2008 weakened the role of silver returns as a net transmitter of shocks, it strengthened the role of platinum as a net transmitter of return shocks. However, the Global Financial Crisis had no significant effect on the role of white precious metals as transmitters of volatility shocks.

Another paper to use the methodology from Diebold and Yilmaz (2009, 2012) is that of Batten et al. (2015) taking into account monthly futures closing prices for gold, silver, platinum and palladium traded on the New York Mercantile Exchange between 1984 and 2012. The results contrast the findings of Antonakakis and Kizys (2015): on the one hand, silver is found to be a net transmitter of return spillovers because it contributes 52% to the return of gold, platinum and palladium and only receives 49%; but in contrary to the findings of Antonakakis and Kizys (2015), silver proves to
be a net receiver of volatility spillovers, it contributes 29% to the volatility of the other three metals and receives 31%. A time-varying approach shows that the Global Financial Crisis of 2008 increased the importance of silver as a net recipient of spillovers - a finding somewhat in line with Antonakakis and Kizys (2015) who observe a weakening transmission of shocks for silver during that time. Conflicting results to those of Antonakakis and Kizys (2015) are also observed for platinum, which is neither a net transmitter nor a net receiver of return spillovers, but is a net receiver of volatility spillovers. Palladium however, is indeed a net receiver of both return and volatility spillovers.

Also looking at commodity prices on the COMEX, Bunnag (2015) examine volatility co-movements and spillovers for gold, silver, platinum and palladium using daily data between the 28th of October 2009 and the 21st of August 2014. The authors works with three Multivariate GARCH models (Kroner and Ng (1998)) amongst which the BEKK-GARCH model (Engle and Kroner (1995)) previously used by Papadamou and Markopoulos (2014). The other two models are the diagonal VECH model and the CCC model. The VECH approach was proposed by Bollerslev et al. (1988) and models every conditional variance and covariance as a function of lagged conditional variances and covariances in addition to lagged squared returns and cross-products of returns. The diagonal VECH model tends to be quite restrictive as it doesn’t account for the interaction between the different conditional variances and covariances. Another drawback of the system is the necessity to impose nonlinear inequality restrictions because the model might otherwise not yield a final positive covariance matrix (Kraft and Engle (1983)). The Constant Conditional Correlations (CCC) model was proposed by Bollerslev (1990) and models the covariance matrix by estimating a conditional correlation matrix - therefore, the conditional correlation is assumed to be constant while the conditional variances are varying and follow a univariate GARCH process. Results point towards an important short
run persistence of shocks on the dynamic conditional correlation for gold with silver, and towards an important long run persistence of shocks on the dynamic conditional correlation for palladium with silver.

Balcilar et al. (2015) study information transmission between oil, precious metals and the US Dollar/Euro exchange rate of daily spot prices between January 1987 and February 2012 using a methodology different to that of Sari et al. (2010) - the methodology from Balcilar et al. (2015) builds upon a Bayesian Markov-Switching Vector Error Correction (MS-VEC) model. Hamilton (1990) introduced a Markov switching approach that can deal with structural changes in time series models which was later applied to VEC models by Krolzig (1996, 2013). Markov switching series are generated by nonlinear dynamic properties (Fan and Yao (2003)) and are very efficient to apply on data that includes very influential events; multiple studies used MS models to analyse macroeconomic time series (Diebold et al. (1994), Kim and Yoo (1995), Filardo and Gordon (1998)) and stock returns (Pagan and Schwert (1990), Kim and Nelson (1998), Kim et al. (1998)). The VEC model used relies on time-varying parameters to reflect regime switching and allows for regime dependent conclusions for the impulse response analysis; MS-VEC models have been used by Krolzig et al. (2002), Francis and Owyang (2003) and Psaradakis et al. (2004) amongst others. Results show that during high volatility regimes, the impact of change of the gold price on silver is about 1.25%; against an impact of about 0.07% from silver on gold - results contrary to Sari et al. (2010) who found that the effects of changing gold and silver price returns mirror each other. The impact of change of the gold price on both platinum and palladium is of about 0.8%, while the impact of change of silver prices is practically inexistent on both platinum and palladium prices. Another noteworthy finding is the relatively important impact of an exchange rate shock on silver in both low and high volatility periods, of about 0.4% and 0.8% respectively, as well as the heavy impact of changes in palladium prices on both oil prices and exchange rates.
Bosch and Pradkhan (2015) take a different approach to volatility and examines if the position of speculators can be used to predict returns and return volatility of precious metal futures. The authors work with futures contracts traded on the COMEX between the 13th of June 2006 and the 31st of December 2013 and use a Brunetti and Buyuksahin (2009) rolling procedure to create a continuous series. The data of trading positions is obtained from Disaggregated Commitments of Traders (DCOT) and COT reports, similar to Mutafoglu et al. (2012) who also work with Commitments of Traders reports. The relationship between the variables is detected through a Johansen (1991, 1995) and an Engle and Granger (1987) test followed by a Vector Error Correction Model (VECM) and a VAR Model to detect short-term impacts. Results point towards a herding behaviour of traders on the silver market between October 2007 and December 2013 and on the platinum and palladium market between the entire estimation period from June 2006 to December 2013. Furthermore, evidence point towards a trend-following behaviour of non-commercial traders, a finding in line with Mutafoglu et al. (2012).

A recent paper by Luo and Ye (2015) looks at predictability potential of the Shanghai silver futures market using the CBOE Silver ETF Volatility Index (VXSLV). The authors take into account 139,500 observations between the 10th of May 2012 and the 31st of December 2014 and differentiate between realised volatility and implied volatility. Realised volatility is defined as the root of the sum of squared returns (Andersen and Bollerslev (1998)) using a sampling frequency of 25 minutes (Zhang et al. (2005)). Previous literature showed that option-implied volatility contains information on future volatility (see Blair et al. (2001), Busch et al. (2011) and Benavides and Capistrán (2012) for examples), Luo and Ye (2015) believe this information can be found in options on the iShares silver trust fund. Following Gallant et al. (1992), Andersen (1996), Girma and Mougoué (2002) and Doran and Ronn (2005) amongst others, trading volume, open interest and a
momentum variable are added in the empirical model. Results show that the VXSLV has significant power in predicting daily and weekly volatility forecasts. Furthermore, adding trading volume, open interest and momentum leads to a significant improvement in forecasting the volatility of the Shanghai silver futures market.

Sarwar (2016) takes a different approach and identifies the interaction between stock market volatility, quantified by the VIX, and the volatility on Treasury note, gold and silver markets. Implied futures market volatility is considered in the dataset and the time period between the 7th of August 2007 and the 15th of March 2009 is chosen to reflect the equity market crisis period (Bekaert et al. (2014)). Results indicate a Granger relationship between increases in stock market volatility and increases in the silver market, implying that the impulse for investors to rebalance their portfolio towards gold and silver begins with increases in the VIX.

Lyócsa and Molnár (2016) propose identifying one-day forward volatilities of gold and silver with a battery of tests relying on two specific approaches: the Heterogeneous Autoregressive model (HAR) proposed by Corsi (2009) enabling to capture the persistence of variance, and the Generalised Heterogeneous Autoregressive (GHAR) model proposed by Barunik and Čech (2016) that enables to exploit cross-sectional dependences between error-terms. High frequency data from January 2008 to December 2014 reveals that gold is more volatile than silver and that forecasts are less accurate in times of high market volatility. More interestingly, the GHAR type models provided above average forecasts, pointing towards the inferiority of univariate models in predicting the volatility of silver prices.

Kang et al. (2017) examine volatility spillover effects on six commodity futures markets, amongst which gold and silver, by relying on the DECO-GARCH model (Engle and Kelly (2012)) eliminating computational and presentational difficulties of high-dimension systems. Results obtained from weekly silver futures prices between the 4th of January 2002 and the 28th
of July 2016 indicate that equicorrelation between commodity futures increased during the recent Global Financial Crisis, and remains high during periods of economic and financial turmoil. Across all commodities considered, evidence points towards bidirectional return and volatility spillovers that again increased during the crisis; from a more silver-specific point of view, it is found that silver is a net information transmitter to other commodity futures markets while further evidence points towards a flight-to-quality phenomenon for both gold and silver during the financial crisis.

8. Research About Exchange Traded Products on White Precious Metals

The iShares silver trust fund mentioned earlier is part of the study of different other papers looking at the performance of silver Exchange Traded Funds (ETFs). One such example is Ivanov (2013) who study the relationship between silver ETFs, future prices and spot prices using 1 minute intradaily data between the 1st of March 2009 and the 31st of August 2009. The relationship between the variables is given via the Hasbrouck (1995, 2003) methodology representing a VECM as a vector moving average in order to extract the information share as a function of the impact of the change in standard deviation from one variable on another and the variance of each series itself. Silver ETFs largely dominate the information share in contrast to spot and futures prices with a value above 89%; interesting findings knowing that Ivanov (2013) claims that price discovery traditionally originates in futures data.

Taking an investment perspective on gold and silver ETFs, Naylor et al. (2011) look at daily returns of three silver ETFs traded on the NYSE between the 5th of May 2006 and the 31st of December 2009 to understand whether or not abnormal returns could have been realised via an investment in these securities. The methodology is based upon a CAPM and a Classic Linear Regression Model (CLRM). Building the CAPM, the market
under consideration is the S&P 500 and the risk-free rate is given as the US 90 day Treasury bill rate. Results show that the behaviour of silver ETFs is very similar to that of physical silver returns, particularly, the price movements do not follow a random walk. An important conclusion for investors is that using a filter trading rule (Fama and Blume (1966), Solt and Swanson (1981)), abnormal returns can be generated through ETFs, hence outperforming a passive investor.

A few years later, Naylor et al. (2014) investigate the microstructure of silver investment funds and look more closely at tracking ability, tracking deviation, and the impact of market panics on ETF dynamics. Daily share prices, trading volumes and assets under management of the Silver Trust and the Physical Silver Shares fund starting from the 21st of April 2006 and the 24th of July 2009 respectively are taken into account, where the timeframe spans to December 2011. Following Frino and Gallagher (2001), results point towards an average daily tracking error of 112 basis points for silver ETFs which is maximised in times of high market volatility. Furthermore, following Baur (2013) a monthly February effect is found, in which returns of silver and silver ETFs are positively significant.

Similar to Luo and Ye (2015) who look at the iShares Silver Trust, but looking more specifically at market contagion during the 2010 Flash Crash, MacKenzie and Lucey (2013) use 4,695 one minute intraday observations between the 26th of April 2010 and the 12th of May 2010. Results point towards the vulnerability of the iShares Silver Trust to contagion, hence questioning the portfolio theory of Markowitz (1952) since historically uncorrelated assets can become susceptible to contagion in financial turmoils.

Lau et al. (2017) opens the ETF investigation to platinum and palladium and considers daily ETF prices for gold, silver, platinum, palladium, oil and global equity between the 19th of June 2006 and the 18th of June 2016. An E-GARCH procedure is used in order to ensure that the conditional variance is strictly positive and augmented with a frequency dynamics of connectedness
procedure and a hidden semi-Markov model to measure the dynamics and intensity of return spillovers as well as to analyse the return characteristics of white precious metals. Results identify a strong relationship between gold and silver ETFs, but a relatively unimportant relationship between oil and white precious metals, where oil price movements spill over on silver and platinum but not on palladium. Regarding the relationship of white precious metal ETFs with the global equity ETF, results do point towards a cointegration relationship between equity and precious metals, but the effect of equity ETFs on white precious metals ETFs are relatively unimportant.

9. The Macroeconomic Determinants of White Precious Metal Prices

Very early on, Radetzki (1989) looks at the factors influencing the price of gold, silver and platinum in the medium and long term. The analysis is done by differentiating between driving forces of supply and driving forces of demand between 1972 and 1987. In contrary to gold, mine production is found to be a more important factor of supply for silver and is also more evenly spread across countries. On the silver demand side, the importance from industry is dominating; namely from the photographic, the electrical and the jewellery industry. These findings lead Radetzki (1989) to conclude that two factors drive the price of silver: demand from industry and private inventories. On the other hand, oil prices and official inventories are not believed to be amongst the major driving forces of the silver price, even though they are important in determining the price of gold. On the supply side of platinum, an interesting result is the relative unimportance of scrap supply in comparison to gold and silver. The nature of platinum demand is similar to that of silver, where industry is a key player. More specifically, Radetzki (1989) finds the following industries to have the largest demand for platinum: chemical, electrical, glass and petroleum, while the author also notices a very large increase of demand over the years, namely that
stemming from the production of automobile catalysts.

Christie-David et al. (2000) use 15 Minutes intraday data between the 3rd of January 1992 and the 29th of December 1995 to determine the effect of macroeconomic news announcements on the price of gold and silver futures. The authors work with the Brown and Forsythe (1974) modified Levene (1960) statistic, a methodology used by fellow researchers for similar works (see Lockwood and Linn (1990) and Ederington and Lee (1993)). Results show that silver future prices respond strongly to the announcement of capacity utilisation and the unemployment rate. Weak responses are found for the announcement of the CPI (a finding in line with Frankel and Hardouvelis (1985) who use daily data between the 7th of July 1980 and the 5th of November 1982), hourly wages, business inventories, and construction spending. Announcements for durable goods orders, nonfarm payroll, the GDP, housing starts, merchandise trade deficit, leading indicators, the PPI, retail sales, industrial production, factory inventories, NAPM survey, new single home sales, personal income, personal spending, federal deficit, installment credit and consumer credit are not found to have an effect on the price of silver futures.

A few years later, Thorbecke and Zhang (2009) build upon two hypotheses in order to understand the effect of monetary policy surprises on commodity prices, amongst which silver. The first theory, by Romer and Romer (2000) states that federal funds rate increases might lead to an increase in inflation by revealing the Fed’s private information about inflation. The second theory, by Gürkaynak et al. (2005) presents evidence that an increase of the federal funds rate leads to a decrease in long-term expected inflation. Thorbecke and Zhang (2009) consider the time period between 1974 and 1979 as well as between 1989 and 2006; the period between 1980 and 1989 is omitted because the Fed abandoned fund rate targeting in 1979, and only since the appointment of Alan Greenspan as Chairman in the late 1980s did the fund rate become the leading indicator of Fed policy (Jones
For the first period, the changes in the Fed’s target for the federal funds rate are obtained from Cook and Hahn (1989) and augmented with an unrestricted ordinary least squares prediction equation to measure the anticipated change in monetary policy. The data for the second time window is obtained from Kuttner (2001). Regression results differ substantially for both time periods considered. Between 1974 and 1979, an increase in the federal funds rate led to an increase in the price of silver as a reaction to an increased demand in answer to anticipated inflation. Between 1989 and 2006 however, an increase in the federal funds rate led to a decrease in the price of silver in expectation of increasing short-term real term interest rates; a finding in line with Frankel (2008).

Focusing more on explaining the price volatility of gold, silver, platinum and palladium through macroeconomic determinants, Batten et al. (2010) work with monthly data between January 1986 and May 2006 and express the expected return of silver as a function of the information available at a previous time interval while estimating the conditional standard deviations with the methodology of Davidian and Carroll (1987). The following macroeconomic variables are considered in the analysis: the S&P 500 and its dividend yield, the World excluding US stock index and its dividend yield, the difference in interest rate yields between a US 10 years bond and a US 3 months Treasury bill, US M2 money supply, US industrial production, US inflation, the US Dollar index, and finally, US consumer confidence. The authors argue that these variables contribute to the effects of the business cycle, monetary environment and financial market sentiment on asset returns. Results show that neither monetary nor financial market variables are significant for silver price volatility. Instead, the volatility from the other precious metals markets has an effect on silver price volatility. These findings are in line with Sensoy (2013) concluding that the same macroeconomic factors do not jointly influence the price series of the four main precious metals. Results for platinum indicate a significant effect of stock market volatility.
during a part of the sample period under observation, while the effect of macroeconomic variables seems much stronger on palladium. Here, it seems that the volatility of the S&P 500 and its dividend yield significantly effect the volatility of palladium prices.

In the same year, Roache and Rossi (2010) examine the effect of macroeconomic news announcements on the price of silver futures relying on daily data between the 1st of January 1997 and the 31st of December 2009 using a GARCH model that accounts for the US Dollar effect in order to assure that the sensitivity observed reflects the relationship between the commodity and the macroeconomic announcement and not between the commodity and other financial assets. Similar to Christie-David et al. (2000), the set of macroeconomic variables under observation is quite extensive but only the German IFO survey and the US Dollar index, both lagged by one day, have an effect on the price of silver over the entire time period considered. More variables are found to be significant for platinum. Here, results point towards the FOMC interest rate decision, alongside the lagged value of changes in non-farm payrolls, existing home sales, the German IFO survey and the US Dollar index. Finally, palladium is found to be influenced by the following variables: the GPD, industrial production, the employment cost index, existing home sales, the German IFO survey and the US Dollar index.

Similar to the work from Christie-David et al. (2000), Elder et al. (2012) work with intraday data between January 2002 and December 2008 to analyse the impact of US macroeconomic news announcements on the return, volatility and trading volume of gold, silver and copper futures. After cleaning the data by removing week-end announcements, the authors build a control sample of trading intervals free of announcements (a method proposed by Ederington and Lee (1993)) to assure the robustness of the results. Advance retail sales, changes in nonfarm payrolls, durable goods orders, business inventories, construction spending, and new home sales announcements have a statistically significant negative influence on silver futures prices; only
trade balance announcements are positively associated with silver futures prices. These results are in contrast to the findings of Christie-David et al. (2000) who observe an importance for the announcements of capacity utilisation, the unemployment rate and the CPI, all of which are not found to be statistically significant in influencing the price of silver futures in the work of Elder et al. (2012); indicating time-variation in the importance of macroeconomic announcements on the price of silver.

Apergis et al. (2014) look into the nature of spillovers between the prices of gold and silver, stock markets, and different macroeconomic variables of the G7 countries using monthly data between January 1981 and December 2010. The authors use the Zurich silver price in US Dollar per kilogram and consider a vast amount of variables reflecting industrial production, inflation, unemployment, exchange rates, commodity prices, interest rates, government debt, money supply, equity prices, market capitalisation, price/earnings ratios, and price to book ratios. Apergis et al. (2014) work with a Factor-Augmented Vector Autoregressive (FAVAR) approach proposed by Bernanke et al. (2005) and better suited then a standard VAR model when considering large amounts of variables. Results indicate that the price of silver responds negatively to positive shocks of industrial production and interest rates, furthermore, a higher inflation and unemployment rate have a negative impact on the price of silver. The authors explain this negative relationship with inflation by saying that a higher inflation rate deteriorates the macroeconomic environment while adding to macroeconomic uncertainty, hence turning investors away from precious metal markets. On the other hand, FAVAR test results indicate that both positive money supply shocks and positive stock market shocks lead to higher silver prices.

Finally, Adrangi et al. (2015) consider intraday data of gold, silver and copper futures prices traded on the COMEX between January 1999 and December 2008, as well as 18 macroeconomic variables believed to influence the behaviour of financial markets in the United States. The results are then di-
vided into different categories: thirty-minute return responses, thirty-minute return responses taking into account the surprise element, standardised inventory holdings and return responses, and finally, local inventory clustering and return responses. Across all different categories, capacity utilisation and industrial production have a positive significant relationship with the price of silver, results somewhat in line with Christie-David et al. (2000), but contrary to Elder et al. (2012) who find no significant relationship between either capacity utilisation nor industrial production with silver futures prices.

Fernandez (2017) focuses her research on gold, silver and Platinum-group elements, amongst which platinum and palladium, and derives the differences in macroeconomic determinants for annual prices between 1930 and 2014 and for monthly and weekly prices between July 1992 and July 2016. On a yearly basis, a relationship between white precious metals prices and both global production and US consumption is identified. On a monthly basis, a strong relationship is identified between white precious metals and US industrial production as well as US monetary supply; the effects of South African mine production are also revealed. Finally, a very strong relationship is identified between the prices of gold and silver on a weekly basis during bullish environments, while platinum and palladium have a strong relationship with silver during bearish periods. A very interesting finding is the rise in importance of the price of white precious metals and consumer confidence and exchange rates in the United States, in line with the rise in importance of white precious metals as an investments asset.

Ciner (2017) takes a very specific approach and predicts the price of silver, platinum and palladium by looking at the level of the South African Rand in relying on daily data between the 7th of October 1996 and the 29th of July 2016. Results indicate a unilateral effect from exchange rates to the price of white precious metals but not the other way around, while the effect is strongest on palladium, than platinum and finally silver, in that order. Indeed, the ability of exchange rates movements of the Rand to forecast
palladium prices remains significant under different model specifications, which it doesn’t for both platinum and silver.

10. The Relationship between White Precious Metals and Other Commodities

10.1. About the Relationship between Gold and Silver

Early on, Koutsoyiannis (1983) considers daily commodity prices between the 29th of December 1979 and the 31st of March 1981 to erect a short-run pricing model for gold. Amongst different macroeconomic variables, the relationship between the price of gold and the price of silver is assessed through an OLS regression. Although Koutsoyiannis (1983) argues that silver, alongside stocks, offers an alternative speculative investment opportunity to gold, the low value of elasticity of the price of gold to changes in the price of silver ($\eta = 0.08$) suggests that gold and silver are not very close substitutes.

Looking at the alleged characteristic of silver to be considered a substitute of gold, Ma (1985) tests if a trading strategy can be derived based on changes in the equilibrium parity ratio between gold and silver. Lang (1983) states that an investor should buy gold and sell silver if actual parity drops below the equilibrium level, and sell gold to buy silver when the parity is above the equilibrium level. Ma (1985) looks at daily gold and silver prices between January 1974 and October 1984 and tests whether a technical trading rule would generate profits based on gold/silver ratio spreads. A CAPM framework indicates that substantial excess profits can be generated before transaction costs if the investor executes trades on a frequent basis; underlining the very fast adjustment of the market to a new equilibrium parity.

A few years later, Ma and Soenen (1988) extend the findings of Ma (1985) and find that the parity between gold and silver is also observed for futures prices. Daily closing prices from January 1976 to October 1986
fitted in an autoregressive model are used to investigate the parity between both securities. The results suggest that a profitable trading strategy can be derived; where a leveraged position linked to lower transactions costs in the futures market proves to be more profitable than executing the given strategy on the spot market.

In an attempt to understand the relationship between gold and silver prices, Chan and Mountain (1988) consider the price of gold and silver traded in Toronto and the Canadian bank rate using weekly basis between March 1980 and February 1983 and test for causality amongst these variables using the approach proposed by Granger (1969). The optimal lag structure for each variable in the equation is determined by the Schwarz Bayesian Information Criterion (SBIC) (Schwarz (1978)), and by Akaike’s minimum Final Prediction Error (FPE) criterion (Akaike (1969, 1970)). In the later method, a major advantage is in the choice of the significance level based on minimising the mean squared prediction error (Hsiao (1979, 1981)). Results point towards a feedback causal relationship between the price of gold and the price of silver. Furthermore, interest rates exert a causal influence on the price of silver while this relationship is not reciprocal: the price of silver does not determine changes in interest rates.

Wahab et al. (1994) build upon the Efficient Market Hypothesis of Fama (1970) and argue that given risk neutral investors and perfect markets, the EMH implies that futures prices, or a linear combination of them, should evolve as a simple martingale process over time. Arguing that market participants could employ two sets of information: the first one contained in the price history of each time series, and the second one contained in the price history of other relevant time series, Wahab et al. (1994) examine the relationship between gold and silver prices with a focus on intertemporal behaviour and the predictability of the gold-silver spread. The authors work with daily cash and futures prices between January 1982 and July 1992 and use a variety of classical econometric approaches such as the Dickey...
and Fuller (1979) procedure to test for unit roots, the Engle and Granger (1987) test of cointegration, and an Autoregressive Integrated Moving Average (ARIMA) model to predict the behaviour of the time series. The main contribution of the paper is the construction of a moving average model generating trading signals based on the unconstrained spreads between gold and silver prices. Though adopting the proposed trading strategy generates profits, these profits come at high risk. Further results point towards a very high correlation between gold-silver cash and futures prices.

A few years later, Escribano and Granger (1998) study the long-run relationship between gold and silver prices by focusing on three aspects in particular. First, the authors look at the effects of the 1979 silver price bubble on the cointegration relationship, in a second step, Escribano and Granger (1998) test whether or not including error-correction terms in a non-linear way performs better than an out-of-sample random walk model, and finally, the authors focus on the existence of a simultaneous relationship between the rates of return of gold and silver. Escribano and Granger (1998) use monthly data from 1971 to 1990 and address each individual question using a different econometric approach. Evidence is found for a long-run cointegration relationship between the two precious metals which is especially strong during the silver price bubble from September 1979 to March 1980. Furthermore, a battery of testing procedures indicates that non-linear forecasting models for silver perform better than random walk processes; however, this is only the case for in-sample analyses, as the predictive power vanishes for the out-of-sample period. Finally, a linear regression indicates a strong simultaneous relationship between gold and silver returns across the entire time period considered.

findings consistent with Escribano and Granger (1998) who show that the stable relationship between gold and silver broke in the 1990s. Ciner (2001) also concludes that a trading strategy based on the gold-silver parity would not be profitable, a finding in direct contradiction with Ma (1985), Ma and Soenen (1988), and Wahab et al. (1994). An interesting final comment of Ciner (2001) based on his findings of no cointegration between gold and silver, is that of advising market participants not to consider gold and silver as substitutes when hedging against similar risks.

Liu and Chou (2003) employ a general method of fractional cointegration analysis to study the spreads between gold and silver prices; in both cash and futures markets. Results indicate that gold-silver and silver-gold parities are slow-adjustment long-memory processes and that the futures and cash spreads between gold and silver are cointegrated. Further results point towards the ability of future spreads to reflect information before cash spreads.

Lucey and Tully (2006b) review the findings of Ciner (2001) and extend the data period considered. Lucey and Tully (2006b) take into account weekly COMEX prices between January 1978 and November 2002. In contrary to Ciner (2001), Lucey and Tully (2006b) examine cointegration between the time series through a dynamic cointegration analysis proposed by Hansen and Johansen (1992), involving estimations of the Johansen (1988) and the Johansen and Juselius (1990) cointegration approaches over various time windows. A visual application of the Hansen and Johansen (1992) method (used before by Rangvid (2001) for example) shows that the stable relationship between gold and silver prevails over time. Considering that results indicate no cointegration between gold and silver from 1992 to 1998, Lucey and Tully (2006b) concludes that the findings of Ciner (2001) might only be driven by the choice of the time period considered and should not be regarded as empirical.

Gerolimetto et al. (2006) extent the time window considered by studying
monthly gold and silver prices between 1971 and 2004. In order to detect cointegration properties, Gerolimetto et al. (2006) apply a Relevant Vector Machine (RVM) proposed by Tipping (2001), which relies on a Bayesian framework in order to derive rigorous prediction models. A major advantage of the RVM is that density functions over the model weights depend on parameters which are endogenously estimated from the data; this methodology enables Gerolimetto et al. (2006) to contrast previous findings and claim that a dynamic long run relationship between gold and silver exists.

Batten et al. (2013) consider daily gold and silver futures prices between January 1999 and December 2005 to investigate the dynamics of the bivariate relationship between both metals. Looking at spread returns, Batten et al. (2013) detect long-term dependence by using statistical techniques proposed by Hurst (1951, 1956). The Rescaled Adjusted Range (RAR) technique of Hurst is flexible in setting the sample period considered and provides a comprehensive understanding of the direction of the equilibrium reverting process. Results indicate a dominant positive dependence between the spread returns, even though negative dependence is also observed during the sample. An important result for investors is the proposed trading strategy derived from Hurst coefficients which indicate when to buy or sell and out-perform both a simple buy-and-hold strategy and a moving-average strategy.

In response to the study of Batten et al. (2013), Auer (2016) considers daily spot prices of gold and silver between January 1979 and March 2015 and augments the Hurst procedure considered in order to derive a simple trading rule considering transaction costs. Auer (2016) produces a trading strategy that is found to outperform a traditional buy-and-hold strategy even though it is time-varying - advising the reader to be careful in the attempt to derive empirical observations.

Building upon the results of Escribano and Granger (1998), Baur and Tran (2014) enlarge the sample considered and look at monthly gold and
silver prices between January 1970 and July 2011 in order to analyse their potential long-run relationship. Closely following the method proposed by Escribano and Granger (1998), results show that the price of gold drives the price of silver and therefore the long-run relationship. Time-variation is observed in the relationship, namely the disconnection of gold and silver prices in the 1990s.

In light of the growing amount of papers studying time-variation in the cointegration relationship between gold and silver, Pierdzioch et al. (2015) propose to use a different methodology to gain further insights into the gold-silver relationship. Considering monthly data between January 1970 and May 2015, Pierdzioch et al. (2015) work with a Residual Augmented Least Squares (RALS) test for noncointegration. The RALS is a correction of the standard test for noncointegration proposed by Dickey and Fuller (1979) and revises the equation with new regressors accounting for skewness and excess kurtosis (Im and Schmidt (2008)). Pierdzioch et al. (2015) stay in line with previous studies by computing the RALS test following the specifications of Taylor (1998). Even though gold and silver are cointegrated during major parts of the whole sample, evidence for noncointegration is found in the mid 1990s and the early 2000s.

Zhu et al. (2016) take a quantile regression approach to the relationship between gold and silver by considering weekly data from the 3rd of April 1968 to the 27th of April 2016. A Quantile Autoregressive Distributed Lag model based on the Johansen procedure detects a positive long-run relationship between gold and silver that is mainly driven by the tail quantiles outside the interquantile range. Further results indicate that prices of silver are more susceptible to contemporaneous price changes of gold, though this adjustment is stronger when silver prices are in their extremes.

10.2. The Relationship Amongst Precious Metals

Kearney and Lombra (2009) is one of the few to focus on the relationship between gold and platinum prices between 1985 and 2006 focusing on
the short-term shifts from positive to negative correlation. Results obtained from the hedge books of 93 gold mining companies between 1996 and 2006 uncover that forward sales are negatively related to gold prices and equilibrium errors and therefore altered the return on gold and explain the shift towards a negative relationship with platinum prices.

Hammoudeh et al. (2010) look at conditional volatility, correlation dependency and interdependency for all four major precious metals and the US Dollar/Euro exchange rate using daily spot prices between the 4th of January 1999 and the 5th of November 2007. A VARMA-GARCH model (Ling and McAleer (2003)) and a Multivariate Dynamic Conditional Correlation GARCH (DCC-MGARCH) model are used to detect coherences between the metals. Results point towards high conditional correlation between gold and silver; furthermore, silver exhibits strong sensitivity to exchange rate volatility during monetary policy regimes, underpinning gold’s safe haven characteristic. Results for platinum and palladium indicate a high correlation amongst the return of the two white precious metals, but to a very low correlation between gold and palladium. Interestingly, the authors point towards superior portfolio diversification benefits of platinum in comparison to silver, hence advising investors to hold more platinum than silver.

Looking again at volatility and correlation dynamics of gold, silver, platinum and palladium, Hammoudeh et al. (2011) base their methodology on the VaR approach using daily spot prices between the 4th of January 1995 and the 12th of November 2009 - hence including the financial crisis. The VaR specifications are similar to those of Hammoudeh et al. (2013) and fitted for daily data following Christoffersen (2009). Relevant results for portfolio managers show that a VaR GARCH-t specification delivers the best prediction results even though they are linked to lower profitability. Regarding silver specifically, the high VaR values observed in 2006 are linked to the launch of the first silver ETF on the American Stock Exchange. Relying on an efficient portfolio framework under different specifications, results fro
platinum and palladium show that the individual returns are far off the capital market line.

Chng and Foster (2012) argue that gold is demanded by investors during good economic times as a safe haven (see Baur and Lucey (2010) and Baur and McDermott (2010)), but that during good economic times, firms stockpile silver, platinum and palladium for industrial consumption. Using daily data between January 1996 and July 2010, Chng and Foster (2012) test whether or not the implied convenience yield of precious metals affect the return, volatility and volume dynamics of other precious metals. The authors derive time series for the implied convenience yield of a precious metal taking into account both the spot and futures prices of the metal, as well as the sum of the risk-free rate (Bank of England official bank rate) and the percentage storage costs of the metal (0.43% annual fee charged by ETF Securities Ltd. on their Physical Precious Metal Basket ETF). A VAR model augmented with a sub-sample analysis point towards significant cross-metal interactions amongst convenience yields of precious metals, though gold and silver seem to be more influential depending on the state of the economy. Silver is more convenient to hold during positive economic times since it carries the heaviest industrial usage. Regarding the relationship between gold and silver, it seems that a long-run equilibrium relation is only observed during normal economic times. Further findings point towards the greater influence of the convenience yields of gold and silver in comparison to that of platinum and palladium.

Kucher and McCoskey (2016) consider weekly data for gold, silver and platinum prices between the 3rd of January 1975 and the 13th of February 2015 in order to understand the long-run relationship between the three precious metals. Results indicate a cointegration relationship between gold and silver prices as well as gold and platinum prices, though these relationships are time-varying: they tend to decline around business cycle peaks and increase during recessions. Finally, while Kucher and McCoskey (2016)
finds that the prices of precious metals are influenced by the actual economic condition, the long-run relationship amongst them seems to be unaffected by short term macroeconomic shocks.

10.3. The Relationship Between White Precious Metals and Oil

Soytas et al. (2009) base their research on the Turkish economy. The authors examine both short-run and long-run information transmission between the Brent oil price, the Turkish interest rate, the Turkish Lira/US Dollar exchange rate, and finally, the domestic spot price of gold and silver. Soytas et al. (2009) consider daily data between the 2nd of March 2003 and the 1st of March 2007 and examine the long-run relationship between the series through a Toda-Yamamoto procedure (Toda and Yamamoto (1995)) allowing to run a VAR model in levels, therefore avoiding information loss from differencing. Also, the Toda-Yamamoto procedure does not require testing the congregation properties of the system, hence avoiding biases associated with unit roots and cointegration tests (Zapata and Rambaldi (1997) and Clarke and Mirza (2006)). Short-run dynamics are identified using generalised impulse responses in a VAR model, as proposed by Koop et al. (1996) and Pesaran and Shin (1998) who overcome the orthogonality problem of traditional VAR models, therefore making the ordering of the variables irrelevant. Results show that Turkish interest rates Granger cause silver prices and that there is evidence for unidirectional causality from gold to silver in the long-run. Concerning oil, evidence shows that price shocks have a negative impact on the price of silver, pointing to silvers’ industrial importance in Turkey. On the other hand, the silver spot price quoted on the Istanbul Gold Exchange (IGE) is found to have a significant positive impact on the Brent oil price in the short-run.

A few years later, Bhar and Hammoudeh (2011) take a more global approach and look at the relationship between WTI oil, copper, gold, silver, short-run US interest rates, a trade-weighted average index of the value of the US dollar, and finally, the MSCI world equity index. Bhar and Ham-
moudeh (2011) fit weekly data between the 2nd of January 1990 and the 1st of May 2006 into a regime-dependent VAR model based on the Markov Switching (MS) configurations outlined by Hamilton (1989, 1994) and Kim and Nelson (1999) (a similar approach can be found in Alexander and Kaeck (2008) on the Credit Default Swap (CDS) market). Results relying on different model specifications show no evidence for a significant relationship between oil prices and silver. However, a positive relationship between silver and the US Dollar exchange rate is observed, indicating that silver can’t be considered a hedge against a depreciating dollar.

Jain and Ghosh (2013) conduct an analysis similar to that of Soytas et al. (2009) and focus their endeavours on India by looking at cointegration relationships and Granger causality between daily prices of Brent oil, gold, silver, platinum, and the Indian Rupee/US Dollar exchange rate between the 2nd of January 2009 and the 30th of December 2011. Testing for cointegration is done via an Auto Regressive Distributed Lags (ARDL) approach relying on appropriate critical $F$ values for large samples (Pesaran and Shin (1998), Pesaran et al. (2001)), while a Toda and Yamamoto (1995) version of Granger causality identifies causation amongst the variables. Finally, a Generalised Forecast Error Variance Decomposition (GFEVD) analysis measures how a change in one variable affects the value of another variable. Evidence points towards a strong relationship between gold and silver - also in explaining each others error variances. Granger causality is observed between oil and three variables (exchange rates, gold and silver), implying that the silver price has predictive powers for international oil prices. Results also uncover a bi-directional causality between gold and platinum, indicating that platinum starts to behave like an alternative investment to gold; as well as a somewhat important explanation factor of platinum on oil, where 9.55% of the variance of oil is explained by the variance of platinum.

Charlot and Marimoutou (2014) work with daily time series between the 3rd of January 2005 and the 19th of October 2012 in order to examine
the volatility and correlation amongst WTI oil, gold, silver, platinum, the Euro/US Dollar exchange rate and the S&P 500. The methodology used is an extension of the Hidden Markov Model (HMM) (see Baum and Petrie (1966), Baum and Eagon (1966), Baum et al. (1970) and Rabiner (1989)) and provides a fully probabilistic decision tree. Charlot and Marimoutou (2014) work with the Hidden Markov Decision Tree (HMDT) proposed by Jordan et al. (1997), that reconciles the Factorial Hidden Markov Model (FHMM) of Ghanhramani and Jordan (1997) and the Coupled Hidden Markov Model (CHMM) of Brand (1997) in order to maximise the amount of possible interactions of the variables. Decision tree results indicate that the volatility of silver responds strongly to economic shocks and is linked to specific events such as the Financial Crisis of 2008, while the response of platinum is very slow. Further results indicate a low correlation between oil and silver (0.41) that increased since 2009 and a high correlation between silver and platinum/gold (0.74 and 0.83 respectively). An increase in correlation since the Global Financial Crisis is also observed between oil and platinum (0.44), while a strong correlation also exists between platinum and gold (0.72).

Focusing more on the effect that Brent oil price shocks have on the volatility of precious metal prices, Behmiri and Manera (2015) consider daily spot prices of aluminum, copper, lead, nickel, tin, zinc, gold, silver, platinum and palladium between July 1993 and January 2014. Volatility persistence is revealed through a GARCH model allowing to detect regime dependency of price changes and rate of returns (Bollerslev (1987)) and augmented with the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model (Glosten et al. (1993)) to analyse asymmetry and leverage effects in the GARCH process. Outliers in the GARCH model are detected via the Doornik and Ooms (2005, 2008) procedure inspired by Chen and Liu (1993) allowing to distinguish between outliers affecting the levels of the series and outliers affecting future conditional variances (a discussion on the different types of outliers and their effects on GARCH models can be found in Sakata
and White (1998) and in Hotta and Tsay (2012)). This approach provides Behmiri and Manera (2015) with two sets of data: the original time series and a data set corrected for outliers. Finally, the Mork (1989) method developed for oil prices is used to differentiate between positive and negative shocks. Empirical results indicate that negative oil price shocks do not affect the volatility of silver, while positive oil price shocks decrease the volatility of silver prices. Regarding platinum, it is found that negative oil price shocks increase the volatility of platinum while positive oil price shocks decrease the volatility of platinum prices. The volatility of palladium prices is always increased, disregarding the type of oil price shock. However, considering the data set corrected for outliers, only the positive oil price shocks remain significant in affecting silver price volatility. Behmiri and Manera (2015) give two possible reasons for the fact that negative oil price shocks become insignificant when the series are corrected for outliers: first, detected outliers in the silver market are due to shocks in the oil market, and second, the volatility of both the silver and the oil price are likely to be affected by the same events.

Similar to Jain and Ghosh (2013), the paper by Bildirici and Türkmens (2015a) analyses cointegration and causality relationship between oil, gold, silver and copper using monthly data between January 1973 and November 2012. The oil price considered is the equally weighted average of the spot price of Brent, Dubai and West Texas Intermediate (WTI) oil. A Brock et al. (1996) test indicates nonlinearity in the series, hence leading Bildirici and Türkmens (2015a) to perform a nonlinear ARDL test of cointegration: an asymmetric extension of the linear ARDL approach proposed by Pesaran et al. (2001) (examples of nonlinear ARDL tests and their applications can be found in Katrakilidis and Trachanas (2012) and Bildirici and Türkmens (2015b)). Regarding causality amongst the variables, Bildirici and Türkmens (2015a) argue that a standard linear Granger approach might not be appropriate due to the nonlinear nature of the series. The authors
therefore propose to work with a Hiemstra and Jones (1994) modified Baek and Brock (1992) test to reveal information about the positive and negative nature of shocks, and a Kyrtou and Labys (2006) test to detect response asymmetry from one variable to another. Results indicate a positive long-run relationship between oil and silver, where a 1% increase in the price of oil results in a 1.33% increase in the price of silver. However, the different causality tests indicate conflicting results about the relationship between oil and silver; Bildirici and Türkmen (2015a) therefore advice the reader to be cautious when interpreting these results.

Reboredo and Ugolini (2016) examine the impact of strong bidirectional oil price movements on ten metal prices, amongst which gold, silver, platinum and palladium, using weekly spot prices between the 7th of January 2000 and the 23rd of October 2015. Results point towards an asymmetric effect of oil price spillovers, where the effects of upward price movements were larger than those of downwards price movements. Reboredo and Ugolini (2016) also take a specific look at the Global Financial Crisis and find that it did not affect the spillover effects running from oil to the three white precious metals, evidence that the relationship between oil and silver, platinum and palladium is not time-varying.

Considering volatility spillovers from oil to gold and silver amongst other commodities, Awartani et al. (2016) work with daily data between the 27th of July 2012 and the 3rd of June 2015 and the Diebold and Yilmaz (2012) procedure, results indicate moderate risk spillovers from oil to silver of about 11%.

11. Research on Silver Futures

An early paper by French (1983) focuses on the relationship between silver futures and forward prices. Arguing that both contracts are similar in nature, French (1983) examines the relationship between both types of contracts from 1968 to 1980 relying upon different pricing models assuming
neither taxes nor transaction costs. In the light that the daily return from a futures contract is transferred between the traders on a daily basis, whereas the return from holding a forward contracts accumulates until maturity, the first two models used (proposed by French (1982) and Cox et al. (1981)) presume that the price for a futures contract is equal to the present value of the product of the underlying’s maturity spot price and the return from rolling over one-day bonds, while the price of a forward contract is related to the interest rate of a long-term bond with comparable maturity date. The last model used, proposed by Richard and Sundaresan (1981), emphasises that the marginal utility of a sum of dollars today must be equal to the expected marginal utility of that same sum of dollars in the future as an investor could otherwise increase his life-time expected utility with a portfolio of bonds and forward contracts to transfer money across time. Even though the models used are not able to explain but only to detect intra-sample variations in futures-forward prices differences, the results emphasise that the two type of contracts are to be considered different assets for the case of silver.

In the same year, Garbade and Silber (1983) study the extent at which hedgers use futures contracts to shift the price risk of silver. Building upon the assumption that the futures price for a commodity should be equal to the cash price plus a premium reflecting the deferred payment on the futures contract (see McCallum (1977), McCormick (1979) and Phaup (1981) for a similar assumption on different futures markets), Garbade and Silber (1983) develop a pricing model that is able to assess whether or not futures contracts are good substitutes for a cash market position and detect in which market price changes first appear. Results for silver indicate that the elasticity of supply is quite high, the authors argue that this is due to the low storage costs for silver and the ability to sell short silver very easily. Another interesting finding indicates that in contrary to gold, where the spot price depends largely on the futures price, price discovery for silver is more evenly distributed amongst the spot and the futures price.
Aggarwal and Sundararaghavan (1987) work with silver futures contracts between 1980 and 1984 and test for the efficiency of the silver futures market in regard to the information contained in the time series. Markov chain models are used to study price changes in the series (a method previously used by Fielitz and Bhargava (1973) and Fielitz (1975)) and derive upward and downward cycles that could possibly be exploited by traders. Results point towards a change in behaviour of silver prices since 1981 due to the Hunt brothers investments. Considering the cycles pointed out by the methodology applied, traders could have earned excess returns, pointing to weak-form inefficiency in the silver futures market between 1981 and 1983.

Chang et al. (1990) build upon the theory of normal backwardation (Keynes (1930)) stating that due to the volatility of futures prices, long traders must be rewarded with a risk premium for carrying the risk of investing in futures contracts. The authors test this hypothesis by applying the CAPM model of Sharpe (1964) and Lintner (1965) and estimate if investing in silver futures between January 1964 and December 1983 was linked to carrying systematic risk and whether or not carrying such risk was rewarded. The Dow Jones Cash Commodity Index is used as a proxy for the market portfolio and the risk-free rate used is the end-of-period return on one-month Treasury-bills. Results show that investing in silver futures yielded higher returns than investing in futures contracts of other metals, but was also linked to higher volatility. Furthermore, silver outperformed the stock market over the period observed and has a $\beta$ close to 1 (a finding later supported by Cochran et al. (2012)), on the other hand, the $\beta$ of platinum was of 0.848 for the same period. Chang et al. (1990) conclude by finding that based on standard deviations of returns, silver and platinum futures are riskier than common stocks but earn less return per risk unit than equity - therefore strongly challenging the attractiveness of white precious metals futures as an investment asset.

Looking again at risk premium in the silver futures market, Kocagil
and Topyan (1997) follow Bessembinder (1992) on currency and agricultural futures and analyse the possible relationship between risk premium, futures trading and the S&P 500 by Kalman filtering the time series (Kalman (1960)). A theoretical model is used to illustrate the equilibrium relationship between cash and futures prices and fitted with daily data between 1990 and 1994. Evidence uncovers a positive relationship between risk premium and daily futures trading and to a negative relationship with the S&P 500, pointing towards silver’s role as an inflation hedge (Kocagil and Topyan (1997)).

Ntungo and Boyd (1998) consider weekly silver futures and try to understand whether or not neural network models (Kaastra and Boyd (1995)) outperform traditional ARIMA models in predicting silver prices. Neural network procedures were designed based on the structure of the brain and consists of a collection of input units and processing units receiving the data. In contrary to other forecasting procedures, neural networks compare the results of their analysis with the desired output, adding a *machine learning* element to the procedure (Hecht-Nielsen (1990)). While the predictive models generated positive returns, neural network results for silver indicate that the more complex models did not perform better than the traditional ARIMA model considered; an explanation provided by the authors is that the data considered might not have been highly nonlinear.

In a more technical paper, Longin (1999) derives optimal margin levels required in silver futures contracts in the light that it should be high enough to protect brokers against insolvent customers, but also low enough to minimise the additional costs for investors and hence keep the market attractive. Longin (1999) works with observations derived from the COMEX between the 3rd of January 1975 and the 30th of June 1994 and develops a new method for setting optimal margin levels. Considering extreme price movements, the author takes an atypical approach and examines price changes observed over a given period rather than general price changes as such.
Results for long investors show that optimal margins are smaller than the historically observed margin requirements for a probability of margin violation above 10%, a similar observation is found for short investors where the optimal margin requirement is smaller than the historically observed margin requirements for a probability of margin violation above 25%. Results to handle with care, since correlation between different futures markets might require higher margin levels (Edwards and Neftci (1988)).

Varela (1999) examines the relationship between 15-, 30-, 45-, and 60-day silver futures and their realised cash and delivery settle prices. The period considered ranges from July 1971 to September 1995 and the results rely on a Phillips and Perron (1988) unit root test and an augmented Dickey and Fuller (1981) test. Silver series are found to be stationary in levels, making it impossible to predict cash prices with futures using cointegration and Error Correction Models (ECM). However, a simple regression model indicates that closest to delivery silver futures are a good predictor of the future cash price; similar results are observed with gold futures, pointing to the parity between gold and silver cash (Ma (1985)) and futures (Ma and Soenen (1988)) prices.

Looking at a smaller time window between the 27th of December 1993 and the 30th of December 1995, Adrangi et al. (2000) collect 15 minutes intraday data of silver futures amounting to 11,979 observations. A bivariate GARCH model is used to examine the relationship between gold and silver futures and takes into account time varying volatility, volatility spillovers, and asymmetrical effects of spreads variation. Results point towards a bi-causal relationship between gold and silver returns where the silver contracts carry the burden for spread convergence - Adrangi et al. (2000) believe that this is due to a faster reaction of the gold market to macroeconomic factors. Also noteworthy are the strong volatility spillovers from the gold to the silver market.

Another paper that looks at margin levels of silver futures is that of
Chatrath et al. (2001a) using daily data between January 1986 and March 1995. Market volatility is measured with two different models: the first method follows Hoaglin et al. (1986) and Hoaglin and Iglewicz (1987) and is based on the dispersion of prices, while the second method is based on a GARCH model. The costs imposed by futures margins are estimated with a Two Stage Least Square (2SLS) model and differentiates between different types of traders. Chatrath et al. (2001a) open up the results section to a debate: if margins are considering opportunity costs, the trading activity will be affected by margin changes far from the contracts maturity date; if, however, margins impose only transaction costs, the trading activity will be affected by margin changes close to maturity. Empirical results for silver futures reveal that open interest and trading volume are relatively insensitive to margin changes far away from the maturity date - evidence not in harmony with the opportunity-cost hypothesis. Finally, regarding differences amongst traders, results show that small traders and speculators are more sensitive to margin changes than hedgers and spreaders.

In another paper from the same year, Chatrath et al. (2001b) consider daily prices of gold and silver futures between January 1975 and June 1995 and apply a battery of tests in order to understand the structure of both futures markets. In a first step, Chatrath et al. (2001b) test for correlation dimension following the specifications of Grassberger and Procaccia (1983), while estimating the test statistic following the proposition of Brock and Sayers (1988) and Frank and Stengos (1989) for limited data samples. In a second step, the authors build upon the correlation integral and use a Brock et al. (1996) test to detect nonlinearity and deterministic chaos in the time-series. Finally, a Kolmogorov entropy measures the degree at which the time series movements are predictable. Results reveal that the silver series has nonlinear dependencies, but that these are not consistent with chaos, therefore allowing for a certain degree of predictability.

Looking at the silver futures market in the United States and Japan in
particular, Xu and Fung (2005) analyse daily gold, silver and platinum futures data between November 1994 and March 2001 to detect across-market information flow amongst the contracts through a bivariate asymmetric ARMA-GARCH model. The methodology used is effective in detecting interactions between time series (Karolyi (1995) and Kearney (2000)) and augmented with a factor allowing for a long-run equilibrium error in the conditional mean equation (Engle and Granger (1987)) and a specification differentiating between positive and negative shocks of asymmetric impacts on volatility (Glosten et al. (1993)). Intraday information transmission is measured with a Seemingly Unrelated Regression (SUR) framework consisting of a multitude of regressions (Zellner (1962)). Results show that pricing transmission is strong between the markets and originates in the USA, pointing towards the leading role of the American market. Furthermore, SUR results point towards the speed at which new information is incorporated in the market - usually not longer than a day.

Erb and Harvey (2006) take a portfolio approach and analyse the benefits of silver futures for an investor. Data between December 1982 and May 2004 shows that silver has very low correlation with other major commodities except gold (0.66) and has had constant negative excess, spot, and roll returns. An interesting finding is the statistically significant negative relationship between silver and inflation, pointing to the inability of silver to serve as a hedge against rising prices. However, $R^2$ results seem to indicate that inflation is only able to explain a moderate portion of silver’s return variation. Regarding silver’s ability to serve as a hedging instrument against sovereign debt, Erb and Harvey (2006) apply the liability hedging technique proposed by Sharpe and Tint (1990) based on the risk tolerance-adjusted covariance of an asset’s return and the return of a given liability. Credit for silver to be used as a hedge is given for Treasury Inflation Protected Securities (TIPS) and 10 Year Treasury Bonds between 1997 and 2004 but not for long term US debt between 1982 and 2004.
Believing that WTI oil futures contracts can help estimate the price of silver futures, Cortazar and Eterovic (2010) work with different daily in-and out-of-sample panels between 2004 and 2007 fitted into two different methodological approaches. The first one, a two-factor model proposed by Schwartz and Smith (2000), considers the long-term equilibrium spot price as well as short term deviations. However, Cortazar et al. (2008) argues that individual-commodity models make poor extrapolation for long maturities and propose multicommodity models as a solution; Cortazar and Eterovic (2010) therefore consider the price of copper futures and Brent futures in the model as well. Results show that due to silver’s low correlation with WTI oil, a multicommodity model is recommended when using WTI futures prices to accurately predict the long-term price of silver futures.

Similar to Xu and Fung (2005) before them, Aruga and Managi (2011) focus on the relationship between the US and the Japanese silver futures market. Johansen test results indicate that the daily prices considered between January 2001 and June 2010 are cointegrated while a causality test indicates that American prices are dominant in driving the cointegration relationship. Furthermore, results indicate that the Law of One Price (LOP) did not hold over the entire period, hence allowing investors to realise profits through arbitrage.

Paschke and Prokopczuk (2012) consider daily data of crude oil, copper, gold and silver between the 1st of January 2006 and the 30th of June 2008 in order to understand whether or not continuous time pricing models can be used to reveal mispriced commodity futures prices. A set of different models reveals that excess returns can be realised based on the pricing errors present in the silver futures market - the evidence for gold is much weaker, which the authors blame on the much bigger size of the yellow precious metal market.

Considering the futures market of precious metals, Narayan et al. (2013) look at whether or not daily futures prices of gold, silver, platinum and oil predict the spot prices of their respective market. The window considered
is the longest for gold and silver, ranging from 1980 to 2011, while it is somewhat shorter for platinum, starting in 1983. Results of linear and non-linear models show that futures returns only predict spot returns in the case of silver but not in the case of platinum, leading to a potential of profit realisation in the case of silver, but not in the case of platinum. However, time-variation results indicate that the profit potential was lowest during the Global Financial Crisis.

Cifarelli and Paladino (2015) analyse the behaviour of futures returns for five commodities amongst which silver, by relying on daily data between the 3rd of January 1990 and the 26th of January 2010. The motivation on the paper relies in differentiating between the two main agents on the futures market, hedgers and speculators, in order to understand how they affect the functioning of the individual commodity market. More specifically, Cifarelli and Paladino (2015) rely on a non-linear CCC-GARCH to model the reaction of hedgers and speculators to volatility shifts in the silver futures markets; furthermore, a two-state Markov-switching procedure highlights the impact of changes in the behaviour of futures markets. Finally, individual periods of high futures return volatility are associated to specific intensified trading activities from hedgers or speculators respectively. Results for silver indicate that the effect of hedgers on the futures market is far more important than the effect of speculation activities, while the behaviour of speculators is very much depending on the level of volatility in the silver futures market. It is also noteworthy to mention that the optimal spot-futures hedge ratio for silver increases during high volatility periods.

More recently, Reboredo and Uddin (2016) work with a quantile regression approach and analyse the impact of financial stress and policy uncertainty on weekly gold, silver platinum and palladium futures prices between the 4th of January 1994 and the 10th of February 2015. Results show that there is no Granger causality between the prices of commodity futures and financial uncertainty, but that financial stress has a positive effect on gold and
silver prices, in contrary to platinum and palladium. Interestingly, Reboredo and Uddin (2016) finds limited evidence that stock market uncertainty influences commodity futures prices.
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