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Reappraisal of soil extrusion for geomechanical characterisation

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Reverse extrusion has been promoted over the past four decades for remoulded undrained shear strength (s_u) measurement and consistency-limit determinations. The technique employs a uniaxial-compression test machine to set the die travelling against the confined soil specimen (billet) at a constant displacement rate (v), with the soil extruded through the die orifice under a steady-state extrusion pressure (p_e). This paper presents the first comprehensive literature review of this topic, including a reassessment of extensive data sets presented for many hundreds of fine-grained soils covering an extremely wide plasticity range. Specifically, the paper critically examines (*a*) the soil billet's assumed undrained condition and hence the constancy of the p_e/s_u ratio value, which is central to s_u determinations using this approach; (*b*) the dependence of the steady-state p_e value on both the billet's area reduction ratio (R) and v; (*c*) the role of soil remoulding toughness; and (*d*) the pitfalls of various data analysis and interpolation techniques employed. The author concludes that, depending on mineralogy and gradation, localised billet consolidation may occur for the slow displacement rates employed and high p_e values required, particularly for stiffer soils, such that the present extrusion approach is generally not recommended for s_u measurement or Atterberg-limit determinations.

Notation

A_f	die orifice area
A_o	bore cross-sectional area of the extrusion chamber
a, a'	regression coefficients for semi- and bi-logarithmic correlations, respectively
b, b'	regression exponents for semi- and bi-logarithmic correlations, respectively
c	extrusion coefficient
D_f	die orifice diameter
D_o	bore diameter of the extrusion chamber
d, d'	extrusion coefficients
F	applied force
F_e	force causing extrusion
I_L	liquidity index
I_P	plasticity index
I_{RE}	new consistency index parameter
K_{RE}	reverse-extrusion factor
k	shear yield stress
L	initial billet length
n	number of experimental measurements (data points)
PL_{25}	water content for s_u value 25 times greater than that mobilised at the fall-cone liquid limit
p_e	extrusion pressure
$p_{e(LL)}$	steady-state extrusion pressure for the liquid limit
$p_{e(PL)}$	steady-state extrusion pressure for the plastic limit water content
$p_{e(SL)}$	steady-state extrusion pressure for the shrinkage limit water content
R	extrusion ratio
R^2	coefficient of determination
RE_{25}	water content corresponding to the steady-state extrusion pressure value of $25 \times p_{e(LL)}$
r_e	extrusion rate
s_u	saturated remoulded undrained shear strength
$s_{u(LL)}$	saturated remoulded undrained shear strength at the liquid limit
$s_{u(PL)}$	saturated remoulded undrained shear strength at the plastic limit
T	remoulding toughness
T_{max}	maximum remoulding toughness
t	elapsed time
v	die displacement rate
w	water content
w_L	water content at the liquid limit
$w_{L(cup)}$	water content at the Casagrande liquid limit
$w_{L(FC)}$	water content at the fall-cone liquid limit
w_n	natural water content of undisturbed test specimens
w_P	water content at the standard plastic limit
w_S	water content at the shrinkage limit
w_{ST}	water content at the stiffness transition
w_T	water content at the toughness limit
σ	standard deviation

Soil extrusion fundamentals and experimental approaches

Extrusion as a mechanical process involves the reduction of a billet's cross-sectional area by forcing it to flow through a die orifice under pressure. The concept of extrusion has been applied to soil for brick manufacturing since the mid-nineteenth century (Whyte, 1982) and more recently for the rheological characterisation of various materials, such as food and ceramics (Cheyne *et al*., 2005; Göhlert and Uebel, 2009). The extrusion technique was first introduced for soil characterisation by Timár (1974) and subsequently promoted for determinations of undrained shear strength (Kayabali, 2011a; Kayabali and Ozdemir, 2013; Kayabali and Tufenkci, 2010a; Kayabali *et al*., 2015a, 2015b; Whyte, 1982) and Atterberg limits (Kayabali, 2011a, 2011b, 2012, 2013; Kayabali and Tufenkci, 2007, 2010a, 2010b; Kayabali *et al*., 2015c, 2016; Medhat and Whyte, 1986; Verástegui-Flores and Di Emidio, 2014, 2015). Extrusion has also been employed for three-dimensional clay printing, which allows the construction of physical heterogeneous models (Pua *et al*., 2018), as well as its promotion for possible effective-stress shear strength parameter determinations (Kayabali *et al*., 2015d).

Timár (1974) employed the direct-extrusion (DE) approach, whereby the soil test specimen (billet) is forced along a cylindrical chamber and extruded through a contoured orifice (Figure 1(a)). The material flowing through the orifice has a round bar shape, with a cross-section identical to that of the orifice. However, the method of data interpretation is complicated for DE due to the influence of the frictional resistance generated as the billet is forced along the chamber length to the die, with the value of the pressure causing extrusion (p_e: Equation 1) progressively reducing for the shortening billet (Figure 1(b)).

$$p_e = \frac{F_e}{A_o} \quad (1)$$

where F_e is the applied compressive force causing extrusion to occur and A_o is the extrusion chamber's bore cross-sectional area.

Hence, investigations of soil extrusion after that of Timár (1974) generally adopted the indirect- or reverse-extrusion (RE) approach (Figure 2(a)). For this set-up, the billet is rigidly contained while the die advances towards it at a certain velocity, such that compared with the DE approach, the required value of p_e is significantly lower and attains a steady-state condition for the RE

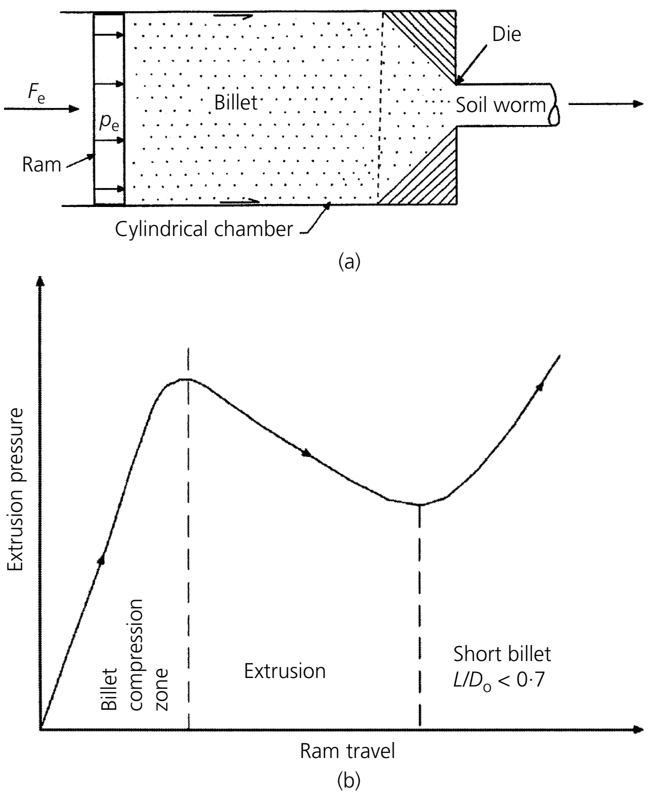

Figure 1. DE: (a) schematic diagram of the experimental set-up; (b) characteristic extrusion pressure plotted against ram displacement trace (adopted from Kayabali and Tufenkci (2010a))

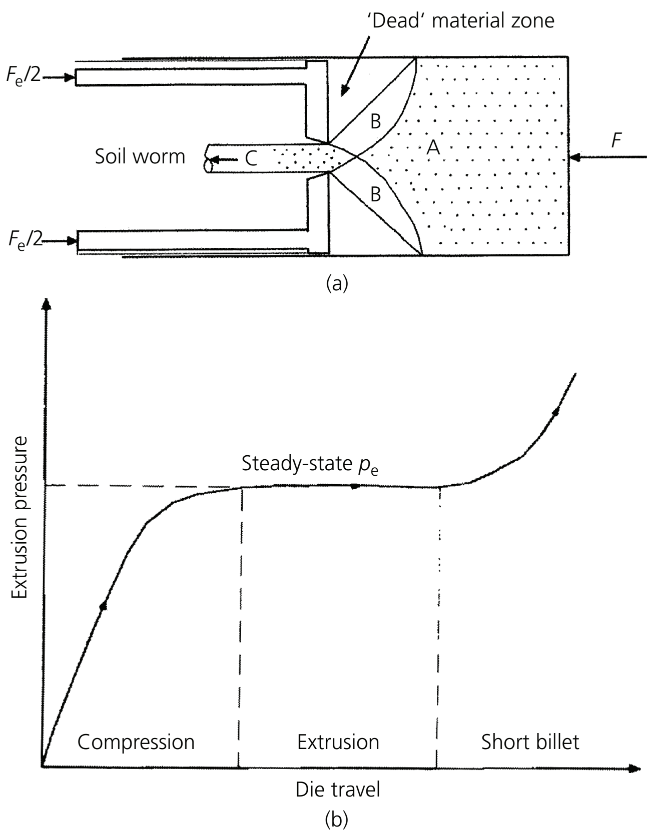

Figure 2. RE: (a) schematic diagram of the experimental set-up; (b) characteristic extrusion pressure plotted against die displacement trace (adopted from Kayabali and Tufenkci (2010a))

approach – that is, since no relative motion occurs between the billet and chamber wall for RE (resulting in much lower mobilisation of interface frictional resistance overall), the value of p_e remains approximately steady with increasing die displacement after the initial billet compression phase (Figure 2(b)). To maintain this steady-state condition, the soil billet must have a sufficient length, with an initial billet length of $1D_o$ to $1{\cdot}5D_o$ typically employed, where D_o is the bore diameter of the extrusion chamber. The soil material yields when it enters the shear zone (fan), located in front of the 'dead' material zone localised between the chamber wall and die face. The initial approximately linear increasing portion of the extrusion pressure against die displacement plot signifies the work required in compressing the placed soil billet to reduce its air void content to practically zero. A small clearance (gap) between the die rim and chamber wall prevents metal friction between them, and the die orifice is also usually chamfered (e.g. see the die orifice shown in Figure 2(a)), so as to reduce the contact friction produced between the die and extruding soil worm to negligible levels.

Some principal physical dimensions of existing soil-extrusion apparatus and associated displacement rate (v) values employed, as reported in the literature, are summarised in Table 1. A 6·0 mm dia. orifice was used in the investigations by Whyte (1982) and subsequently in the many investigations by Kayabali and co-workers, since a standard thread-rolling plastic-limit (PL) test can be quickly performed on the extruded 6 mm dia. soil worm to confirm the soil's consistency, if necessary.

Figure 3(a) presents the RE set-up employed in the studies by Verástegui-Flores and Di Emidio (2014, 2015), which is typical of other soil RE apparatus configurations. Using a uniaxial-compression test machine to provide the desired loading condition (Figure 3(b)), the die is set to travel against the confined soil specimen at a constant displacement rate v (i.e. displacement rate controlled), generally in the range 1–5 mm/min and typical of unconfined compression testing, while the die displacement and corresponding extrusion force are both recorded.

When the soil reaches the shear (plastic) zone within the extrusion chamber (zone labelled B in Figure 2(a)), it distorts and extrudes by way of the die orifice as a soil worm, with the value of the applied compressive force (and the value of p_e computed using Equation 1) assumed to relate to the undrained condition. As such, the soil's resistance capacity is conventionally understood as controlled by its saturated remoulded undrained shear strength, s_u, the value of which is principally dependent for a given test soil on its water content (w) and to a lesser extent on the strain rate. The nature of the soil flow and redundant deformation defining the extent of the dead material zone within the soil billet are dependent on the apparatus extrusion ratio (R: Equation 2) and the die configuration (i.e. shape and geometrical layout of the die orifice(s)). Greater values of p_e are mobilised for higher R values since the associated energy for soil extrusion increases.

2. $$R = A_o/A_f$$

where A_o and A_f are the bore cross-sectional area of the extrusion cylinder and the die orifice area, respectively.

The dependence of the extrusion pressure on extrusion ratio and die displacement rate was investigated experimentally by Verástegui-Flores and Di Emidio (2014). They performed RE testing of a kaolin material at its fall-cone (FC) liquid limit (LL) water content ($w_{L(FC)}$) value, measuring the steady-state p_e value mobilised for R = 4, 16 and 64, with set values of v = 2, 4, 6 and 8 mm/min (Figure 4). As expected, they found that greater steady-state values of p_e were mobilised for higher R and v values. This dependency will be examined in greater detail later in the paper.

The extrusion rate (r_e: Equation 3), which is a function of the extrusion ratio, typically ranges 40–200 mm/min based on the reported RE soil testing set-ups summarised in Table 1.

3. $$r_e = vR$$

Table 1. Various apparatus configurations employed for soil-extrusion testing

Apparatus	D_o: mm	A_o: mm^2	D_f: mm	A_f: mm^2	R	v: mm/min	Reference
DE	23·1	420·0	11·3	100·0	4·2	7·5 (1–4 also)	Timar (1974)
RE	38·0	1134·1	6·0	28·3	40·1	Not reported	Whyte (1982), Medhat and Whyte (1986)
RE	38·0	1134·1	6·0	28·3	40·1	3	Kayabali (2011a)
RE	38·0	1134·1	6·0	28·3	40·1	5 (1–10 also)	Kayabali (2012), Kayabali and Tufenkci (2007, 2010b)
RE	38·0	1134·1	6·0	28·3	40·1	Not reported	Kayabali (2011b, 2013), Kayabali and Tufenkci (2010a)
MPM	30·0	706·9	2·5 (×28)	137·4	5·1	Not controlled	Kayabali *et al.* (2015a)
RE	38·0	1134·1	6·0	28·3	40·1	1	Kayabali and Ozdemir (2013), Kayabali *et al.* (2015b, 2015c)
RE	40·0	1256·6	10·0	78·5	16·0	4	Verástegui-Flores and Di Emidio (2014, 2015)
RE	40·0	1256·6	5·0	19·6	64·0	4	Pua *et al.* (2018)

A_f, die orifice area; A_o, bore cross-sectional area of the extrusion chamber; DE, direct extrusion; D_f, die orifice diameter; D_o, bore diameter of the extrusion chamber; MPM, mud-press machine; R, extrusion ratio; RE, reverse extrusion

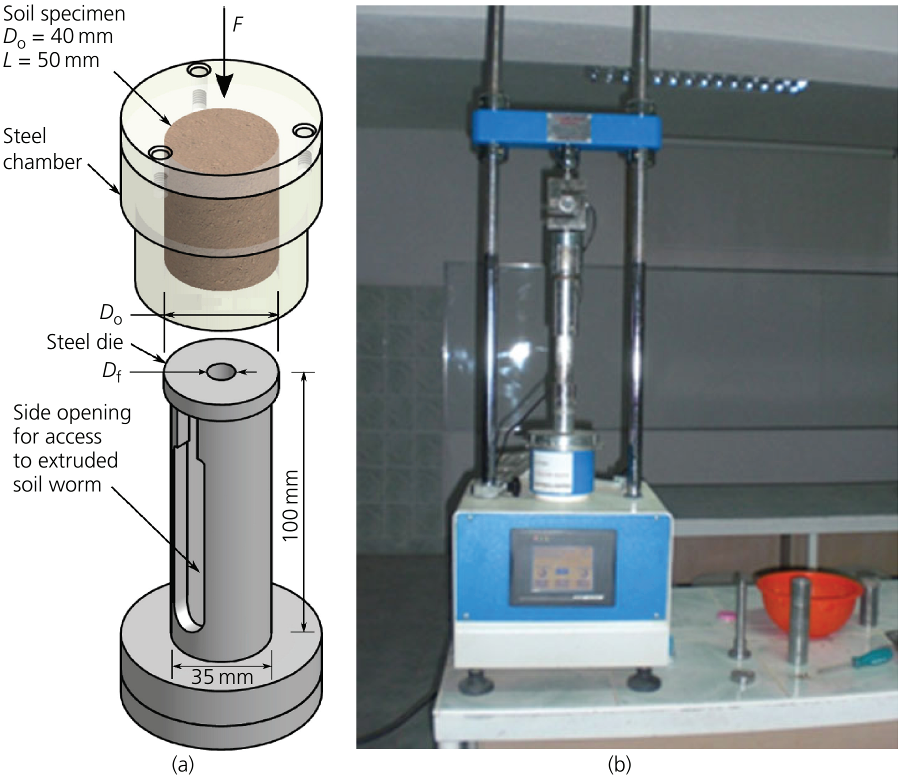

Figure 3. RE testing: (a) schematic diagram of typical extrusion apparatus (adapted from Verástegui-Flores and Di Emidio (2014)); (b) extrusion apparatus fitted in a uniaxial-compression test machine (adopted from Kayabali and Ozdemir (2013))

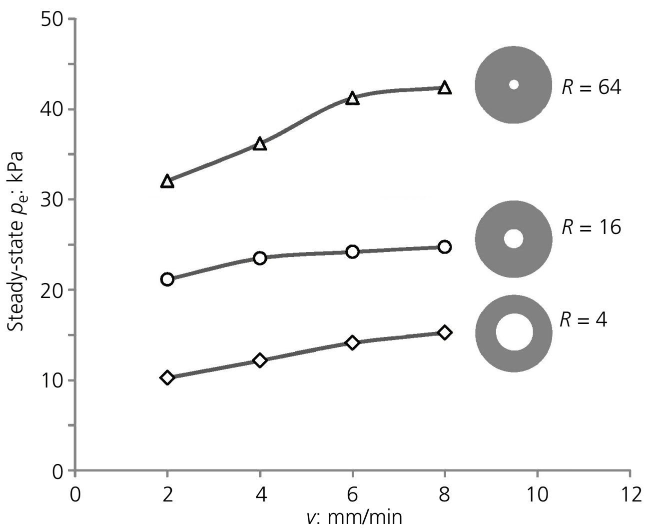

Figure 4. Dependence of the steady-state extrusion pressure on the extrusion ratio and die displacement rate for pure kaolin clay material prepared at its $w_{L(FC)}$ value (adopted from Verástegui-Flores and Di Emidio (2014))

The geometrical layout of the die can be reconfigured to reduce the size of the dead material zone but still maintaining the same R value, by using a die with multiple equally spaced smaller orifices rather than the customary single central orifice. This modification was adopted for the mud-press machine (MPM) extrusion device developed by Kayabali *et al*. (2015a, 2016) and which is described later in this section. Based on the presented p_e against die displacement traces and reported v values in the papers by Kayabali and Tufenkci (2007) and Kayabali and Ozdemir (2013), the steady-state p_e condition (i.e. soil worm extrudes at a steady rate) is achieved within ~1–4 min from the start of the RE test.

An alternative soil-extrusion approach using an MPM device (see Figure 5(a)) was proposed in the papers by Kayabali *et al*. (2015a, 2016) – that is, a miniature DE machine which extrudes the soil billet contained in a 30 mm dia. chamber by way of 28 equally spaced 2·5 mm dia. orifices (i.e. R = 5·1). In this instance, the applied loading is rendered by way of a manually operated mechanical press device, with the value of F_e increasing steadily and quickly, becoming constant as soil worms extrude from the chamber orifices, as evident from the flat portions of the applied force against time traces in Figure 5(b). In other words, the extrusion velocity is not controlled for this experimental set-up, being dependent on the consistency of the test soil, the MPM's R value and the applied force causing extrusion. Based on the extrusion force against time traces presented by Kayabali *et al*. (2015a) and reproduced as Figure 5(b) in this paper, the steady-state extrusion condition is achieved very quickly, typically within ~3 s – that is, compared with the RE testing

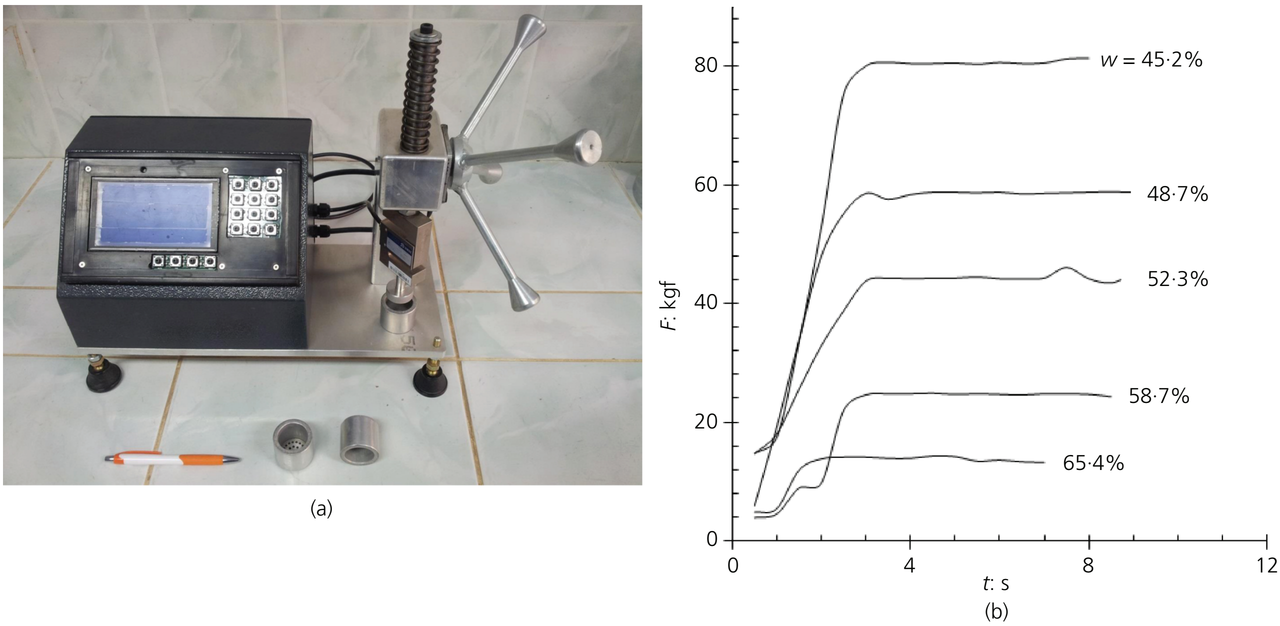

Figure 5. MPM for DE soil testing: (a) photograph of the device, with two 30 mm dia. extrusion chambers included in the foreground; (b) manually applied ram force plotted against time traces for fine-grained soil prepared at various water contents. kgf, kilogram-force (adopted from Kayabali *et al*. (2015a))

described earlier, the MPM device is portable and the test itself is significantly faster (completed within 1 min).

Undrained shear strength measurement

The RE approach has been used for undrained shear strength determinations on remoulded, compacted and undisturbed fine-grained soils (Kayabali, 2011a; Kayabali and Ozdemir, 2013; Kayabali and Tufenkci, 2010a; Kayabali *et al*., 2015a, 2015b; Whyte, 1982). Implicit in this approach are that, for given apparatus R and v values, the soil billet remains in an undrained condition during the course of the RE testing and, furthermore, the steady-state p_e/s_u ratio value is constant among different fine-grained soils – that is, if partial drainage (consolidation) of the soil billet were to occur, its shear strength value would increase, mobilising a higher steady-state p_e value, such that the steady-state p_e/s_u ratio value would increase accordingly. Both of these assumptions are examined in greater detail later in the paper.

In preparing remoulded RE test specimens (e.g. for consistency-limit determinations), excessive compactive effort should not be applied in forming the soil billet inside the extrusion chamber, with the static compaction method preferred. Otherwise, the billet's compaction-induced overconsolidated state (O'Kelly, 2017; O'Kelly *et al*., 2019) would require higher applied extrusion pressures. Compared with specimens in the remoulded state, undisturbed test specimens invariably mobilise higher undrained shear strength in axial compression on account of their inherent fabrics (O'Kelly, 2006) and also possibly overconsolidated state, such that they too generally require higher extrusion pressures.

As described earlier, $p_e = f_n(R, v, s_u)$, such that the steady-state extrusion pressure can be expressed in terms of the soil undrained shear strength for RE testing with given R and v values, as follows

4. $$p_e = K_{RE} s_u$$

where K_{RE} is the RE factor that accounts for the apparatus extrusion ratio, die displacement rate and also apparatus friction and inhomogeneous billet deformation.

Alternatively, knowing the value of the steady-state p_e/s_u ratio for specific experimental R and v values, the undrained shear strength value can be simply determined from the measured steady-state extrusion pressure value. As described earlier, implicit in these calculations is that the soil billet remains in a truly undrained condition throughout the RE testing.

Calibration of the RE device is performed experimentally, with its pertinent value of K_{RE} for specific R and v values producing a one-to-one correspondence between the correlations for measured pairs of steady-state extrusion pressure and undrained shear strength values determined for the reference test material prepared at various water contents. In principle, this approach is the same as that presented in the papers by O'Kelly (2014a, 2018) for the calibration of an FC device using unconsolidated-undrained (UU) triaxial-compression data obtained for test specimens prepared at various water contents. The described approach is particularly versatile since the RE device calibration can be performed for pertinent confinement pressure and shearing rate values that are controlled in performing the triaxial-compression tests. The MPM device can be calibrated in the same manner, but, because the extrusion velocity is not constant (depending on the MPM R value, soil undrained shear strength and maximum manually applied extrusion force), the MPM-derived strength values should be regarded as approximations only.

Steady-state p_e/s_u ratio

Provided that the soil billet remains in a truly undrained condition, following directly from Equation 4, one would expect for a given RE die configuration and set R and v values that the experimental steady-state p_e/s_u ratio would have the same value for various fine-grained soils investigated. In this section, the author investigates this central assumption of the RE approach for s_u measurement, making use of deduced steady-state p_e and measured s_u values presented in the paper by Kayabali and Ozdemir (2013), which is the only published research that reports s_u and associated steady-state p_e values at the same water contents for various fine-grained soils.

Specifically, Kayabali and Ozdemir (2013) reported pairs of s_u and steady-state p_e values corresponding to the PL water contents (i.e. $s_{u(PL)}$ and $p_{e(PL)}$, respectively) for 60 remoulded fine-grained soils (i.e. $n = 60$, where n is the number of samples). They also reported pairs of undrained shear strength and steady-state p_e values for 75 undisturbed natural clay soils with various natural water content (w_n) values. The steady-state p_e values were determined from RE testing for R and v values of 40·1 and 1 mm/min, respectively. Undrained shear strength was determined from unconfined compression testing of 38 mm dia. test specimens, with the remoulded specimens prepared by static compaction at the steady-state p_e values corresponding to their PL water content (w_P) values. Their reported tabulated data are plotted as the $p_{e(PL)}/s_{u(PL)}$ ratio against both w_P and plasticity index (I_P) and also as the steady-state p_e/s_u ratio against w_n in Figures 6(a)–6(c). Consistency-limit values were reported by Kayabali and Ozdemir (2013) for the 60 remoulded soils, such that these could be categorised in the present investigation as either clay or silt materials according to the standard plasticity chart.

As evident from Figure 6, the value of the steady-state p_e/s_u ratio for the set R and v values varied widely for both the remoulded and undisturbed fine-grained soil specimens, with mean values of $p_{e(PL)}/s_{u(PL)} = 13·9$ (standard deviation, $\sigma = 3·2$ for $n = 60$) and steady-state $p_e/s_u = 14·9$ ($\sigma = 4·7$ for $n = 75$), respectively. The marginally higher p_e/s_u ratio value for the undisturbed test specimens is most likely explained by the effects of their inherent fabrics and strength anisotropy, considering the different shearing mechanisms occurring in extrusion compared with uniaxial compression. Further, the regression lines fitted to the $p_{e(PL)}/s_{u(PL)}$ ratio values for the 60 remoulded specimens indicated an overall reducing trend with increasing w_P (Figure 6(a)) and also with increasing plasticity (Figure 6(b)), more so for the clay soils than the silt soils investigated. Assuming that the s_u measurements are sound, these observations point to a fundamental issue with extrusion testing that is soil dependent. As the constancy of the steady-state p_e/s_u ratio value is central to the RE s_u measurement approach, this aspect merits further in-depth investigation and is fully addressed later in the paper.

p_e/s_u relationships with R and v

For extrusion of metals, the relationship between the steady-state p_e and R has been shown to be well fitted by Equation 5, which takes account of apparatus friction and inhomogeneous billet deformation.

$$\frac{p_e}{k} = c + d \ln R \quad (5)$$

where c and d are extrusion coefficients and k is the yield stress in shear.

Among the first soil-extrusion investigations, based on RE testing of two fine-grained soils, Whyte (1982) and Medhat and Whyte (1986) tentatively proposed two correlations of the form given by Equation 5 to relate the steady-state p_e with the soil resistance capacity – that is, with the s_u parameter replacing the k parameter in Equation 5. This relationship was further investigated by Kayabali (2011a), considering deduced $p_{e(PL)}$ values and an assumed $s_{u(PL)} = 160$ kPa for 60 remoulded fine-grained soils. The deduced values of the coefficients c and d reported in these investigations are listed in Table 2. However, since the value of $s_{u(PL)}$ is not unique, potentially varying over a very wide range between different soils (Haigh *et al.*, 2013; Nagaraj *et al.*, 2012; O'Kelly, 2013a), the c and d coefficient values reported for the Kayabali (2011a) data set can at best be regarded as mean approximations. Further, none of these investigations considered the dependence of the steady-state p_e/s_u ratio value on the die displacement rate in the case of soil-extrusion testing.

In order to investigate the relationship between R, v and the steady-state p_e/s_u ratio the author has reinterpreted the data presented in Figure 4 for RE testing of kaolin specimens prepared at their FC LL water content ($w_{L(FC)}$) values, as determined using the British Standard (BS) 30°–80 g FC in accordance with BS 1377-2:1990 (BSI, 1990). Knowing that the corresponding saturated remoulded undrained shear strength (i.e. $s_{u(LL)}$) value is ~1·7 kPa (O'Kelly *et al.*, 2018), steady-state p_e/s_u ratio values have been computed from the steady-state p_e data values in Figure 4 and are replotted in Figure 7. Also, included in Figure 7 are

- reported p_e/s_u–R correlations in the papers by Whyte (1982) and Medhat and Whyte (1986)
- representative correlation deduced by the author as part of the present investigation for the 60 remoulded fine-grained soils investigated in the paper by Kayabali (2011a)
- the mean $p_{e(PL)}/s_{u(PL)}$ ratio value of 13·9 deduced for the 60 remoulded fine-grained soils investigated in the paper by Kayabali and Ozdemir (2013)

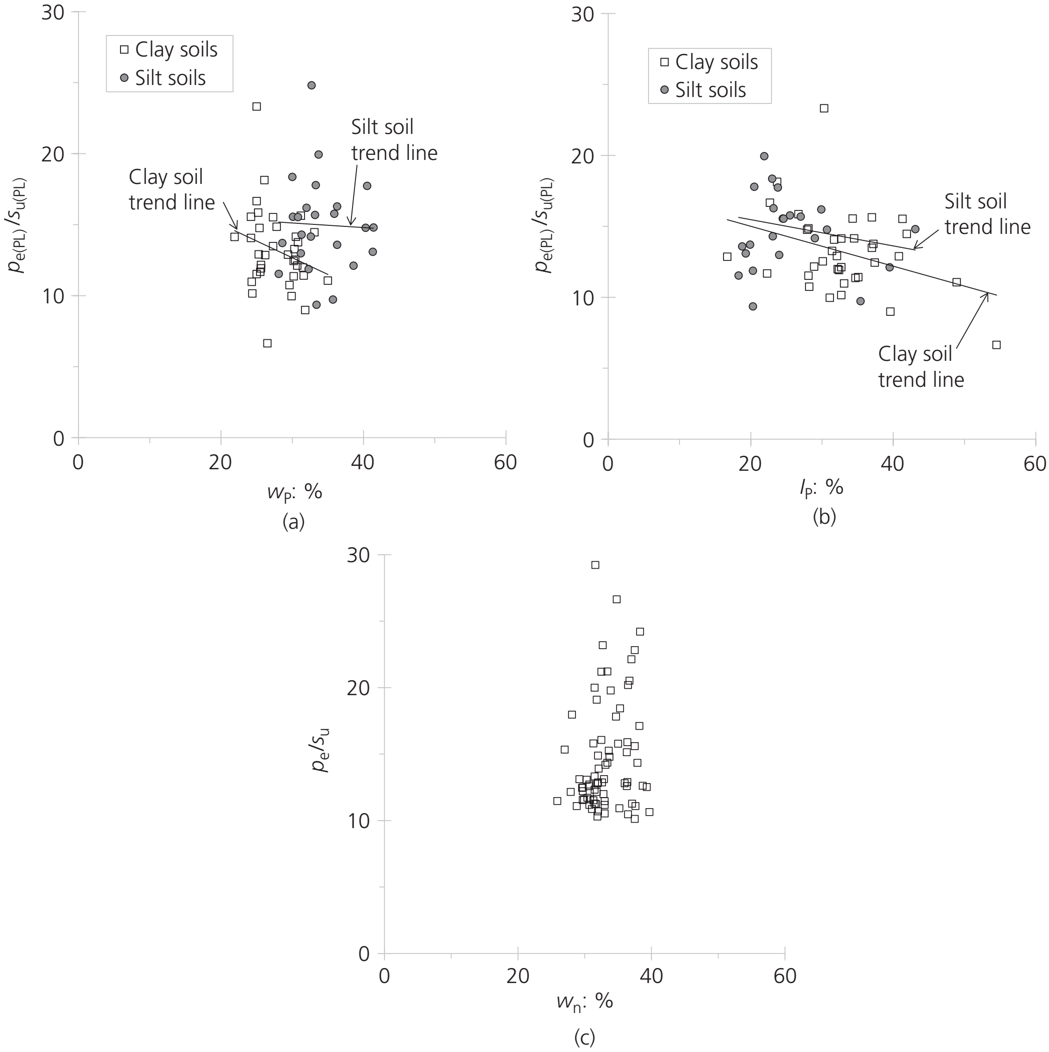

Figure 6. Steady-state extrusion pressure to undrained shear strength ratio: (a) and (b) show variation in $p_{e(PL)}/s_{u(PL)}$ with PL water content and plasticity index, respectively, for various remoulded soils; (c) variation in p_e/s_u with natural water content for various undisturbed soils (produced from tabulated data presented by Kayabali and Ozdemir (2013))

Table 2. Deduced values of coefficients *c* and *d* in Equation 5 for RE testing of remoulded fine-grained soils

Coefficient		v: mm/min	Reference	Comment
c	*d*			
1·6	4·3	Not reported	Whyte (1982)	One low-plasticity soil investigated ($w_{L(cup)}$ = 32·5%, w_P = 16·5%)
0·5	5·8	Not reported	Medhat and Whyte (1986)	Flixton Clay material
2·87 (σ = 0·94)	*3·93 (σ = 1·16)*	3	Kayabali (2011a)	Sixty different fine-grained soils and assuming $s_{u(PL)}$ = 160 kPa
0·098	*4·508*	2	Verástegui-Flores and Di Emidio (2014)	Kaolin clay at its BS $w_{L(FC)}$ value
−0·098	*5·118*	4		
−0·441	*5·781*	6		
0·147	*5·781*	8		

Data entries in italics were computed as part of the present investigation from data reported in these studies
$w_{L(cup)}$ and $w_{L(FC)}$, water contents at Casagrande-cup and FC LLs, respectively

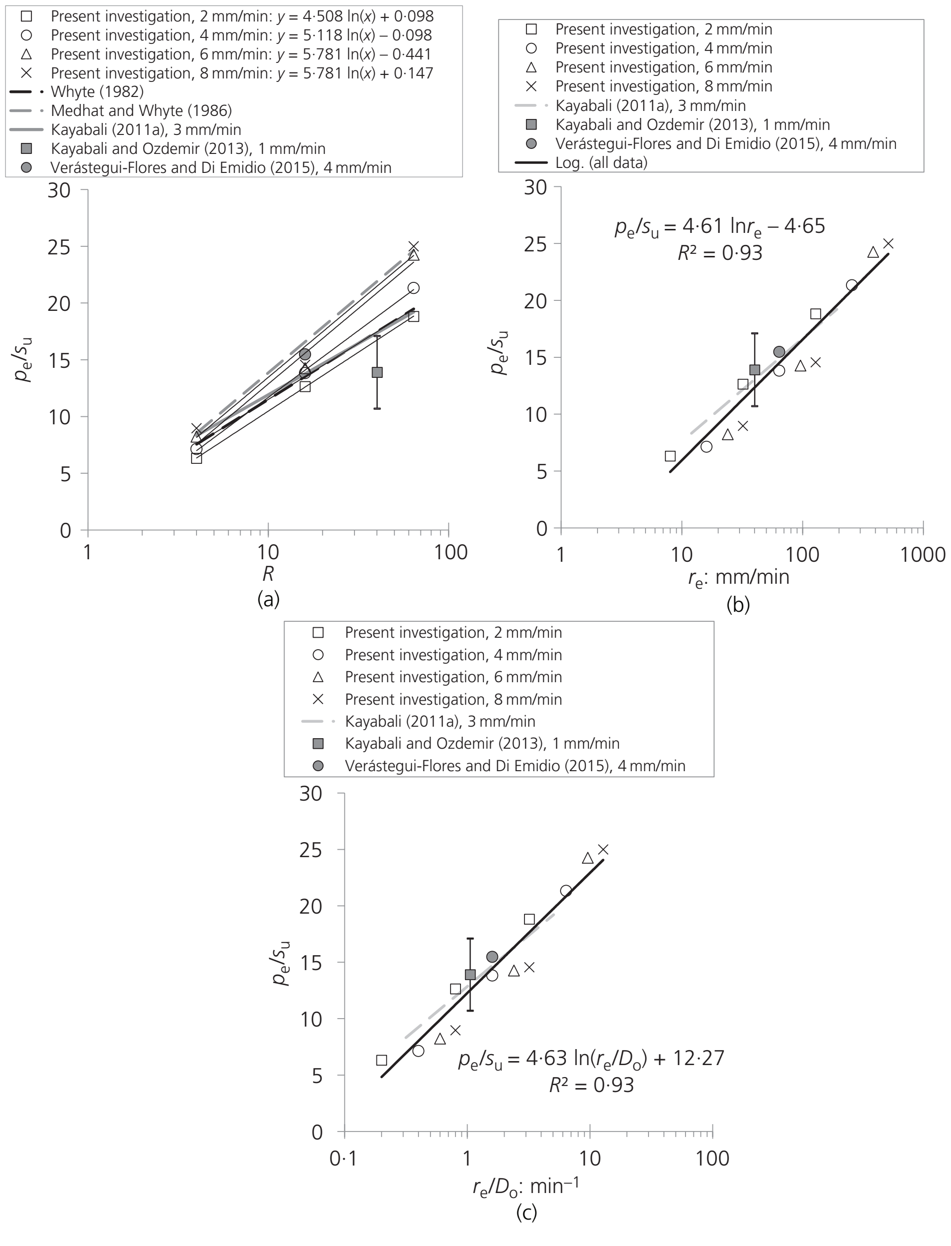

Figure 7. Steady-state p_e/s_u ratios for various remoulded fine-grained soils as functions of: (a) extrusion ratio, (b) extrusion rate and (c) strain rate

- steady-state extrusion pressure for the LL water content ($p_{e(LL)}$ = 23·6 kPa (Verástegui-Flores and Di Emidio, 2015)) expressed as a ratio of FC $s_{u(LL)}$ (1·7 kPa) for set R and v values of 16 and 4 mm/min, respectively – that is, $p_{e(LL)}/s_{u(LL)}$ = 15·5.

As evident from Figure 7(a), even for the relatively narrow v data range 1–8 mm/min available, the value of the steady-state p_e/s_u ratio is strongly dependent on the values of both v and logarithm R, with its dependency on the former appearing to increase with increasing R.

Note that the deduced (reported) coefficient c values listed in Table 2 are generally very small. In other words, for RE testing and assuming no apparatus friction effect, one would expect a steady-state p_e value of zero for the quasi-static extrusion of a zero-shear-resistance material, so that Equation 5 would take the form of $p_e/s_u = d' \ln R$, which is analogous to stress equals a modulus coefficient times strain, with the true strain of the soil billet given by $\ln R$.

To account for die displacement rate, the steady-state p_e/s_u data are plotted against extrusion rate r_e (= vR (mm/min)) in Figure 7(b) and also against r_e/D_o in Figure 7(c), with their best-fit regression lines given by Equations 6a–6c. Equation 6b is the result of the steady-state p_e/s_u–$\ln r_e$ regression analysis for the particular case with the value of coefficient c set as zero.

6a. $$\frac{p_e}{s_u} = -4{\cdot}65 + 4{\cdot}61 \ \ln r_e \quad (R^2 = 0{\cdot}93)$$

6b. $$\frac{p_e}{s_u} = 3{\cdot}57 \ \ln r_e \quad (R^2 = 0{\cdot}88)$$

6c. $$\frac{p_e}{s_u} = 12{\cdot}27 + 4{\cdot}63 \ \ln \frac{r_e}{D_o} \quad (R^2 = 0{\cdot}93)$$

The extrusion rate (r_e) is the velocity of the soil worm. When normalised by a characteristic dimension, here taken as the bore diameter of the extrusion chamber, the r_e/D_o ratio (min^{-1}) gives the strain rate. Since the compiled data set considers only two D_o values (i.e. 38 and 40 mm), Equation 6c is only tentatively proposed, although if steady-state p_e/s_u data were available for a wide D_o range, it is suggested that this form of relationship would provide superior fitting for the enlarged data set. Further, this form of relationship would allow valid comparisons of s_u measurements obtained using different RE apparatus employing various R and v value combinations.

The strain-rate range 0·2–12·8 min^{-1} for soil extrusion (see Figure 7(c)) is comparable with that usually employed for UU triaxial-compression testing but substantially lower than the shear strain rate of the order of 170 min^{-1} typical of the range of penetration depths measured in practice for the BS FC LL test (Koumoto and Houlsby, 2001; Sivakumar *et al*., 2015). The very high strain rates for the FC test specimen are such that it undergoes undrained shearing for all practical purposes and triaxial specimens are fully enclosed by a sealing membrane to ensure a fully undrained condition. In contrast, the combination of relatively high confinement pressure acting on the soil billet and slow strain rate can potentially allow some consolidation to occur with drainage by way of the die orifice. This potential major shortcoming of the soil-extrusion approach, one that has received almost no attention in the published literature, is explored in detail later in the paper.

Atterberg-limit determinations

Since first introduced in soil mechanics approximately 45 years ago, the extrusion approach has been promoted for LL and PL determinations by a handful of research groups on the premise that, for given apparatus R and v values, the w_L and w_P values are associated with unique values of $p_{e(LL)}$ and $p_{e(PL)}$, respectively. Furthermore, Kayabali (2012, 2013) proposed that the RE approach could also be used for shrinkage-limit (SL) determinations, claiming that the SL water content (w_S) value was also associated with a unique steady-state extrusion pressure (i.e. $p_{e(SL)}$) value. Implicit in these approaches are that the steady-state p_e/s_u ratio value is constant among different fine-grained soils and the soil billet remains in a truly undrained condition throughout the RE test. Further, and, importantly, the determination of the consistency limits using the RE approach relies on them having strength-based definitions, and, while this holds true for $w_{L(FC)}$ (O'Kelly *et al*., 2018), it is definitively not the case for the standard w_P and w_S (Barnes and O'Kelly, 2011; Haigh *et al*., 2013; Nagaraj *et al*., 2012; O'Kelly, 2013a; O'Kelly *et al*., 2018; Sivakumar *et al*., 2016). In other words, as elaborated in the following sections, it should not be unexpected if the experimental values of both $p_{e(PL)}$ and $p_{e(SL)}$ for various fine-grained soils were to vary over wide ranges.

Experimental approaches employed for the determination of RE pressures corresponding to consistency limits

Two approaches have been employed in previous RE investigations for the determination of the values of $p_{e(LL)}$ and $p_{e(PL)}$ associated with a given experimental set-up. In their investigations, Kayabali and co-workers employed regression and extrapolation of the steady-state p_e–w correlation obtained for a minimum of five specimens of the test soil prepared at various water contents within its liquidity index (I_L) range 0–0·5. As reported by Kayabali *et al*. (2015b), the values of water content that they investigated were limited to this I_L range because they found that RE testing became very difficult to conduct for I_L > 0·5, presumably because their experimental set-up did not have the required sensitivity for the p_e values mobilised by a lower-shear-strength material. Typical die force–displacement traces for high-plasticity soil are shown in Figure 8(a), from which data pairs of steady-state p_e and corresponding water content values were obtained and plotted in a semi-logarithmic chart (Figure 8(b)). The values of the y-intercept, a (i.e. for w = 0), and gradient $-b$ were then determined for the best-fit correlation line (Equation 7) from regression analysis. Knowing the soil-dependent a and b values, the $p_{e(LL)}$ and $p_{e(PL)}$ values were computed by data extrapolation for the particular soil investigated on inputting, in turn, its conventionally measured w_L and w_P values in Equation 7. As discussed later in the paper, the same approach was used in the papers by Kayabali (2012, 2013) for the determination of the steady-state p_e values corresponding to the w_S values measured for various fine-grained soils.

7. $$p_e = ae^{-bw}$$

A different experimental approach was adopted in the papers by Verástegui-Flores and Di Emidio (2014, 2015). The value of $p_{e(LL)}$ was obtained from repeat RE tests performed on the reference kaolin clay material prepared at its $w_{L(FC)}$ value. The $p_{e(PL)}$ value was determined for the reference bentonite clay material from extrapolation of its best-fit steady-state p_e–w correlation to its measured thread-rolling w_P value, when analysed in a bi-logarithmic chart (Equation 8).

8. $$p_e = a'w^{-b'}$$

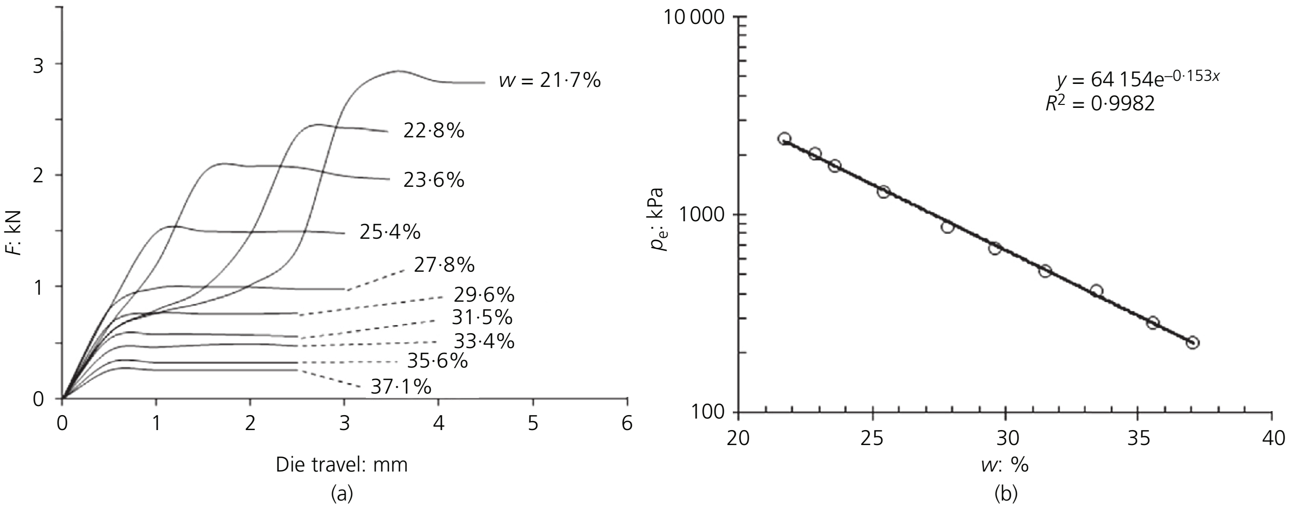

Figure 8. RE of high-plasticity soil ($w_{L(FC)}$ = 61%; w_P = 24%) for R = 40·1 and v = 1 mm/min: (a) die force–displacement traces for various water contents; (b) regression analysis of deduced steady-state extrusion pressure plotted against water content data (adopted from Kayabali *et al*. (2015b))

where a' and b' are the y-intercept and gradient, respectively, of the best-fit correlation line in a bi-logarithmic chart.

Recommended RE approaches for consistency-limit determinations

LL determination

From the author's perspective, the value of $p_{e(LL)}$ would be best determined from RE testing of reference fine-grained soil prepared at its measured $w_{L(FC)}$ value – that is, since the $w_{L(FC)}$ value is strength based, with the BS FC LL (BS 1377-2:1990 (BSI, 1990)) condition corresponding to $s_{u(LL)} \approx 1{\cdot}7$ kPa, the RE approach is valid for $w_{L(FC)}$ determinations provided that the steady-state p_e/s_u ratio remains constant, having the same value for different fine-grained soils. As explained earlier, this assumption is inherently dependent on the soil billet remaining in a truly undrained condition throughout the RE test.

The aforementioned approach was employed in the investigation by Verástegui-Flores and Di Emidio (2014), who reported a steady-state $p_{e(LL)}$ value of 23·6 kPa for RE of a pure kaolin clay material prepared at its $w_{L(FC)}$ value, as determined from three repeat RE tests with R = 16 and using v = 4 mm/min. As expected, good agreement was reported between the derived RE LL water content and the BS-measured $w_{L(FC)}$ value for various artificial fine-grained soil mixtures composed of different percentages of kaolin and bentonite materials, thereby covering an extremely wide range of plasticity.

Should RE testing prove problematic at the LL water content owing to the soil's very low undrained shear strength, the RE tests can instead be performed on the reference soil material for various lower water contents that produce values of steady-state p_e within the approximate range 10–50 kPa. The value of $p_{e(LL)}$ is then determined from the steady-state p_e–w regression correlation as the extrapolated extrusion pressure value corresponding to the measured $w_{L(FC)}$ value. Once the pertinent value of $p_{e(LL)}$ has been established for the particular experimental RE set-up, the latter (extrapolation) approach can be employed for subsequent LL determinations on other fine-grained soils.

With a suitably large R value, it is probable that the MPM device used in combination with a rapid water content sensor device (e.g. see the papers by Caldwell *et al*. (2018) and Dettmann and Bechtold (2018)) could be used to obtain quick estimates of the steady-state p_e–w correlation for a given test soil. Hence, having previously established the MPM $p_{e(LL)}$ value, the value of $w_{L(FC)}$ could be estimated from extrapolation of the obtained correlation. This aspect may merit further research.

PL determination

A significant shortcoming of the extrusion approach (and other strength-based methods, including FC (e.g. see the paper by O'Kelly *et al*. (2018)) is that it cannot demonstrate the significant change in behaviour, from ductile to brittle, obtained for rolling out of soil threads at water contents each side of the PL. The approach employed in previous RE investigations of soil consistency relied on correlations with the thread-rolling PL method to configure the RE apparatus set-up accordingly. As described earlier in the paper, when considering various fine-grained soils, however, any agreement between measured thread-rolling PL and RE-derived PL values is purely coincidental. In other words, the RE approach cannot consistently provide reliable w_P values for different soils investigated, and, as such, the author recommends that the RE approach not be used for PL determinations.

A new parameter, RE_{25}, termed the extrusion pressure parameter (analogous to the FC PL_{25} parameter first proposed in the paper by O'Kelly *et al*. (2018)), is introduced at this stage and is defined as the water content value corresponding to a steady-state p_e value 25 times greater than the $p_{e(LL)}$ value – that is, corresponding to $s_u \approx 42 \cdot 5$ kPa. Like PL_{25}, the water content corresponding to this new extrusion pressure parameter bears no relationship with the standard PL water content, any equivalence in their values for a given soil being purely coincidental. The extrusion pressure range considered is limited to a 25-fold increase (i.e. encompassing soft to medium-stiffness clays), since fine-grained soils are invariably in a plastic state over the associated water content range (O'Kelly *et al.*, 2018) – that is, widening the range of steady-state p_e values considered in this analysis any further would incur the increasing probability of soil materials occurring in a brittle state near the associated lower-bound water content value.

Using the newly introduced RE_{25} parameter allows significantly better correlations to be achieved between the steady-state p_e (and hence undrained shear strength) value and a new consistency index (I_{RE}: Equation 9) parameter than can be obtained with the conventional liquidity index parameter.

9. $$I_{RE} = \frac{\log w_{L(FC)} - \log w}{\log w_{L(FC)} - \log RE_{25}}$$

with I_{RE} being defined in logarithmic form since compared with the semi-logarithmic form, the bi-logarithmic steady-state p_e (s_u) against w correlation for a given soil material generally provides a regression coefficient value closer to unity.

Rearranging Equation 9 and assuming $s_{u(LL)} = 1 \cdot 7$ kPa for $I_{RE} = 0$, the value of saturated remoulded undrained shear strength for water content values in the range $w_{L(FC)} < w < RE_{25}$ can be approximated in the proposed framework as (O'Kelly *et al.*, 2018)

10. $$s_u = 10^{1 \cdot 4 I_{RE} + 0 \cdot 23}$$

RE pressure values reported for consistency limits

With $R = 40 \cdot 1$ and $v = 3–5$ mm/min, respective mean values for $p_{e(LL)}$ and $p_{e(PL)}$ of approximately 20 and 2000 kPa for $n = 120$ (Kayabali, 2011a) and 100 (Kayabali, 2012), 30 and 2250 kPa for $n = 20$ (Kayabali and Tufenkci, 2007), 35 and 3000 kPa for $n = 31$ (Kayabali and Tufenkci, 2010b) and 40 and 3100 kPa for $n = 30$ (Kayabali and Tufenkci, 2010a) have been determined experimentally for different groups of fine-grained soils. Using the same RE apparatus ($R = 40 \cdot 1$) but for a slower die displacement rate of 1 mm/min, Kayabali *et al.* (2015c) reported respective mean values for $p_{e(LL)}$ and $p_{e(PL)}$ of approximately 15 and 2300 kPa for $n = 70$, with the lower $p_{e(LL)}$ value deduced for this experimental set-up consistent with the earlier presented theory that requires overall lower values of steady-state p_e for a slower die displacement rate.

Compared with $p_{e(PL)}$, the mean $p_{e(LL)}$ range 20–40 kPa deduced in the various studies by Kayabali and co-workers for $R = 40 \cdot 1$ and $v = 3–5$ mm/min was much narrower. This is largely explained by the compatibility of the RE, FC LL and to a less extent the Casagrande LL measurement approaches, in that they are all essentially strength based, whereas the standard PL is not a strength-based phenomenon but a demarcation between ductile (or plastic) and brittle behaviour (Barnes and O'Kelly, 2011; Haigh *et al.*, 2013; O'Kelly, 2013a; O'Kelly *et al.*, 2018; Sivakumar *et al.*, 2016).

Using an RE set-up with $R = 16 \cdot 0$ and $v = 4$ mm/min, Verástegui-Flores and Di Emidio (2014, 2015) reported repeatable $p_{e(LL)} = 23 \cdot 6$ kPa and $p_{e(PL)} = 558 \cdot 3$ kPa (linked to the water contents at the BS FC LL (BS 1377-2:1990 (BSI, 1990)) and standard thread-rolling PL) for various soil samples prepared by mixing kaolin and bentonite clays in different proportions. These $p_{e(LL)}$ and $p_{e(PL)}$ values are significantly lower than the mean values of $p_{e(LL)}$ and $p_{e(PL)}$ ranging approximately 20–40 and 2000–3100 kPa, respectively, reported in the earlier-mentioned studies by Kayabali and co-workers, who investigated $R = 40 \cdot 1$ and $v = 3–5$ mm/min. Again, this is consistent with the presented theory that requires overall lower values of steady-state p_e for a lower R value.

As expected, good agreement between the derived RE LL and BS-measured $w_{L(FC)}$ values was reported by Verástegui-Flores and Di Emidio (2014) but not for the Casagrande-cup measured values (i.e. $w_{L(cup)}$), with strong divergence occurring between $w_{L(cup)}$ and BS $w_{L(FC)}$ for increasing bentonite content in the tested soil materials (Figure 9).

The systematic bias producing $w_{L(cup)} > w_{L(FC)}$ for high-LL materials is well documented (e.g. see the papers by Haigh (2012), O'Kelly (2019) and Vardanega *et al.* (2018)). Specifically, BS $w_{L(FC)}$ corresponds to an $s_{u(LL)}$ value of ~1·7 kPa (O'Kelly *et al.*, 2018), whereas $w_{L(cup)}$ corresponds to a specific strength (i.e. ratio of s_u to soil density) value of ~1 m^2/s^2 (Haigh, 2012). The latter equates to a relatively narrow Casagrande $s_{u(LL)}$ range of ~1–3 kPa (O'Kelly, 2019), reducing approximately linearly in value with increasing water content at the LL, when plotted in a semi-logarithmic chart (Youssef *et al.*, 1965). As such, strong divergence between $w_{L(FC)}$ and $w_{L(cup)}$ values occurs for increasing LL beyond 120% water content (O'Kelly, 2013a; O'Kelly *et al.*, 2018; Škopek and Ter-Stepanian, 1975; Wasti, 1987). Hence, along with differences in die displacement rates employed ($v = 1–5$ mm/min), the disparity in w_L values obtained for a given soil tested using the Casagrande-cup and FC LL methods may partly explain inconsistencies in the mean $p_{e(LL)}$ values associated with BS $w_{L(FC)}$ and $w_{L(cup)}$ for a given extrusion ratio value.

Kayabali (2012) and Kayabali and Tufenkci (2010a) argued that, because the consistency limits are threshold water contents, they should also correspond to threshold undrained shear strengths and hence fixed steady-state p_e values for a given RE set-up and die

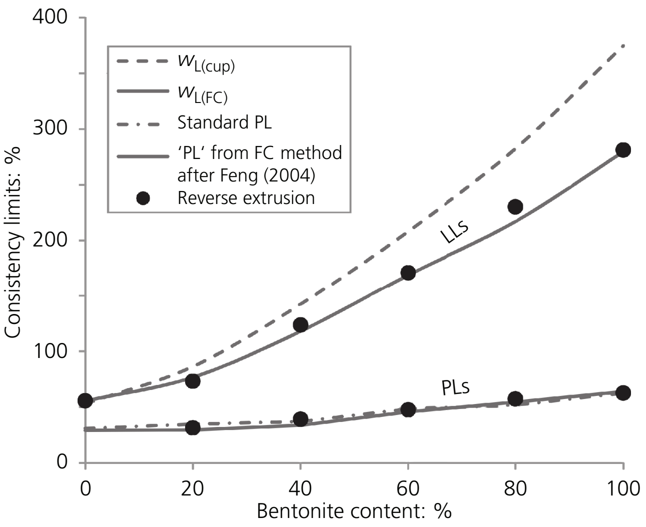

Figure 9. Good agreement between BS FC LL, standard PL and corresponding values deduced using RE apparatus with R = 16·0 and using v = 4 mm/min, $p_{e(LL)}$ = 23·6 kPa and $p_{e(PL)}$ = 558·3 kPa for artificial fine-grained soil samples containing various bentonite contents (adopted from Verástegui-Flores and Di Emidio (2014))

displacement rate. As explained earlier, while this statement holds true for $w_{L(FC)}$, it has been definitively established that the value of $s_{u(PL)}$ is not unique but can vary over a wide range in considering different fine-grained soils (Haigh *et al*., 2013; Nagaraj *et al*., 2012; O'Kelly, 2013a). A major factor explaining the very wide mean $p_{e(PL)}$ range 2000–3100 kPa reported by Kayabali and co-workers for their RE set-up with R = 40·1 and using v = 3–5 mm/min is, therefore, the expected variation in $s_{u(PL)}$ values for different fine-grained soils. Other contributing factors include differences in die displacement rates employed and also the fact that different methods were adopted for PL determinations, with Kayabali and Tufenkci (2010a, 2010b) employing the standard thread-rolling method, whereas Kayabali (2012) and Kayabali *et al*. (2015b, 2015c) employed the PL rolling device described in ASTM D 4318-17e1 (ASTM, 2017). Specifically, PLs obtained using this rolling device generally underestimate the standard thread-rolling PLs (Bobrowski and Griekspoor, 1992), most likely because the paper covering the two rolling plates tends to cause inhomogeneity of the soil thread, the outside becoming drier than its centre, during the rolling-out procedure (O'Kelly *et al*., 2018). As such, compared with standard thread-rolling, values of $s_{u(PL)}$ and hence $p_{e(PL)}$ for a given fine-grained soil would be expected to be higher overall for PLs obtained using the rolling device.

As mentioned earlier, the RE approach has also been proposed for the determination of the values of w_S for fine-grained soils. For instance, with R = 40·1, $p_{e(SL)}$ values of 12 000 kPa for v = 5 mm/min (n = 100 (Kayabali, 2012)) and 15 000 kPa (n = 120 (Kayabali, 2013)) were reported as providing reasonably good estimates of the values of w_S, as determined using the mercury method in accordance with ASTM D 427-98 (ASTM, 1998). As such, the RE approach may appear as an attractive proposition for SL determination. However, since the SL is the transition water content below which no further reduction in the soil volume occurs (i.e. not a strength-based definition), the RE approach cannot consistently provide reliable w_S values. Consequently, the author does not recommend the RE approach for SL determination.

Relationships between measured consistency limits and deduced RE pressure values

In this section, the extensive data sets presented in the papers by Kayabali (2011a, 2012, 2013), Kayabali and Ozdemir (2013), Kayabali and Tufenkci (2007, 2010b) and Kayabali *et al*. (2015b, 2015c), which together report original pairs of $p_{e(LL)}$ and $p_{e(PL)}$ values deduced for the consistency limits of 457 different fine-grained soils, are combined and reassessed as part of the present investigation. These data sets relate to an RE apparatus with an R value of 40·1 and employed die displacement rates of 1 (Kayabali and Ozdemir, 2013; Kayabali *et al*., 2015b, 2015c), 3 (Kayabali, 2011a) and 5 mm/min (Kayabali, 2012; Kayabali and Tufenkci, 2007, 2010b) to investigate a very wide range of soil plasticity; w_L = 26·4–105·0%, w_P = 15·4–46·9% and I_P = 8·3–75·9% (see Table 3 and Figure 10). Using this large data set, potential relationships between reported consistency limits and their steady-state p_e values are investigated, including the effects of changes in soil plasticity categories (levels) and die displacement rate. Further, the $p_{e(LL)}$ and $p_{e(PL)}$ values deduced for the consistency

Table 3. Atterberg-limit ranges for various RE soil investigations performed by Kayabali and co-workers

Reference	n	w_L: %	w_P: %	I_P: %
Kayabali and Tufenkci (2007)	20	29·5–82·6	16·7–35·0	10·8–47·6
Kayabali and Tufenkci (2010b)	31	26·4–83·6	16·7–46·9	8·3–46·5
Kayabali (2011a)	120	42·9–90·3	21·9–43·8	16·7–55·1
Kayabali (2012)	100	38·2–100·2	22·6–39·0	9·7–65·8
Kayabali (2013)	120	42·9–90·3	21·9–43·8	16·7–55·1
Kayabali and Ozdemir (2013)	60	42·9–84·5	21·9–41·4	16·7–54·5
Kayabali *et al*. (2015a)	60	29·3–117·0	15·4–29·1	12·4–90·0
Kayabali *et al*. (2015b)	120	46·0–91·0	22·0–44·0	20·0–57·0
Kayabali *et al*. (2015c)	70	29·3–105·0	15·4–29·1	12·4–75·9
Kayabali *et al*. (2016)	275	29·3–166·0	15·4–43·8	12·4–134·9

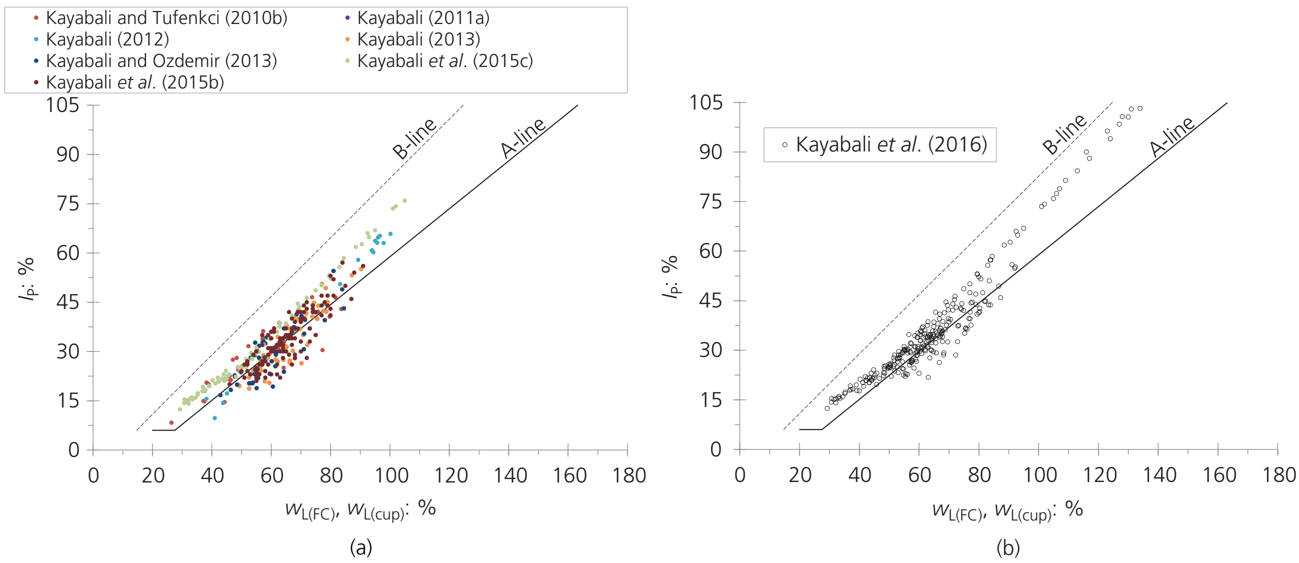

Figure 10. Plasticity characteristics of fine-grained soils investigated using (a) RE and (b) MPM testing approaches

limits of another 275 different fine-grained soils using the MPM device approach and reported in the paper by Kayabali *et al.* (2016) are separately reassessed as part of the present investigation for the same purposes.

As already mentioned, different methods were adopted in these studies for the determination of the consistency limits – that is, the Casagrande-cup or BS FC methods for LL determinations and standard thread-rolling or rolling-device methods for PL determinations. Multiple repeat tests were performed in these studies towards ensuring good reliability of reported LL and PL values. For instance, the $w_{L(FC)}$ and w_P values reported for each soil sample investigated in the paper by Kayabali and Tufenkci (2010b) are the computed mean of 15 or 22 pairs of LL and PL test results obtained for each soil sample by various experienced operators in eight different soil mechanics laboratories. However, the nuances of the experimental methods themselves and their derived results must be considered when interpreting the various data trends for the combined data sets.

RE pressure corresponding to LL

Considering the entire combined RE data sets for v = 1–5 mm/min, $p_{e(LL)}$ had a mean value of 22·3 kPa (σ = 16·0 kPa) and demonstrated general overall strong trends of reducing exponentially from approximately 90 to 2 kPa for increasing w_L from 35 to 95% (Figure 11(a)) and also for increasing I_P (Figure 11(b)). The effects of adopting different LL determination methods were unexpected and counterintuitive. For a given RE set-up, since $w_{L(FC)}$ corresponds to a particular reference s_u value, the associated $p_{e(LL)}$ values should, in theory, remain approximately constant for different LL water content values. However, this was not the case; instead, they exhibited a strong exponentially reducing trend with increasing $w_{L(FC)}$, as evident for the Kayabali *et al.* (2015b, 2015c) data sets presented in Figure 11(a). Further, the Kayabali *et al.* (2015c) data set was obtained for the lowest die displacement rate investigated (1 mm/min) and generally plotted as a lower bound for the entire combined data set, indicating a significant extrusion (strain) rate effect. Also unexpected was the values of $p_{e(LL)}$ corresponding to the $w_{L(cup)}$ values for the Kayabali (2012) and Kayabali and Tufenkci (2010b) data sets generally plotting above the Kayabali *et al.* (2015c) data set – that is, $w_{L(cup)} > w_{L(FC)}$ for higher LL water contents implies lower values of $p_{e(LL)}$ expected for $w_{L(cup)}$. The relationship between $p_{e(LL)}$ and w_P was also investigated, but no consistent overall trend was apparent.

RE pressure corresponding to PL

Considering the entire combined data sets for v = 1–5 mm/min, $p_{e(PL)}$ had a mean value of 2240 kPa (σ = 839 kPa), with 900 and 5300 kPa appearing as approximate tentative lower- and upper-bound values (Figure 12). Apart from the Kayabali and Tufenkci (2007, 2010b) data sets, the overall trend suggests that the values of $p_{e(PL)}$ appear to reduce exponentially from approximately 4000 to 1100 kPa when w_P increases from 17 to 37% (Figure 12(a)). The apparent opposing trend of increasing $p_{e(PL)}$ values with increasing w_P observed for the Kayabali and Tufenkci (2007, 2010b) data sets may be related to the fact that they were determined for the highest die displacement rate of v = 5 mm/min investigated – that is, overall higher steady-state p_e values could be anticipated on account of the greater strain-rate effect compared with, for instance, the Kayabali *et al.* (2015c) data set obtained for v = 1 mm/min, which clearly exhibited a decreasing trend with increasing value of w_P.

In relation to the plasticity index, the overall trend considering the entire combined data sets is that the value of $p_{e(PL)}$ marginally increases with increasing I_P (Figure 12(b)) – that is, apart from the Kayabali *et al.* (2015c) data set, which again may be explained by

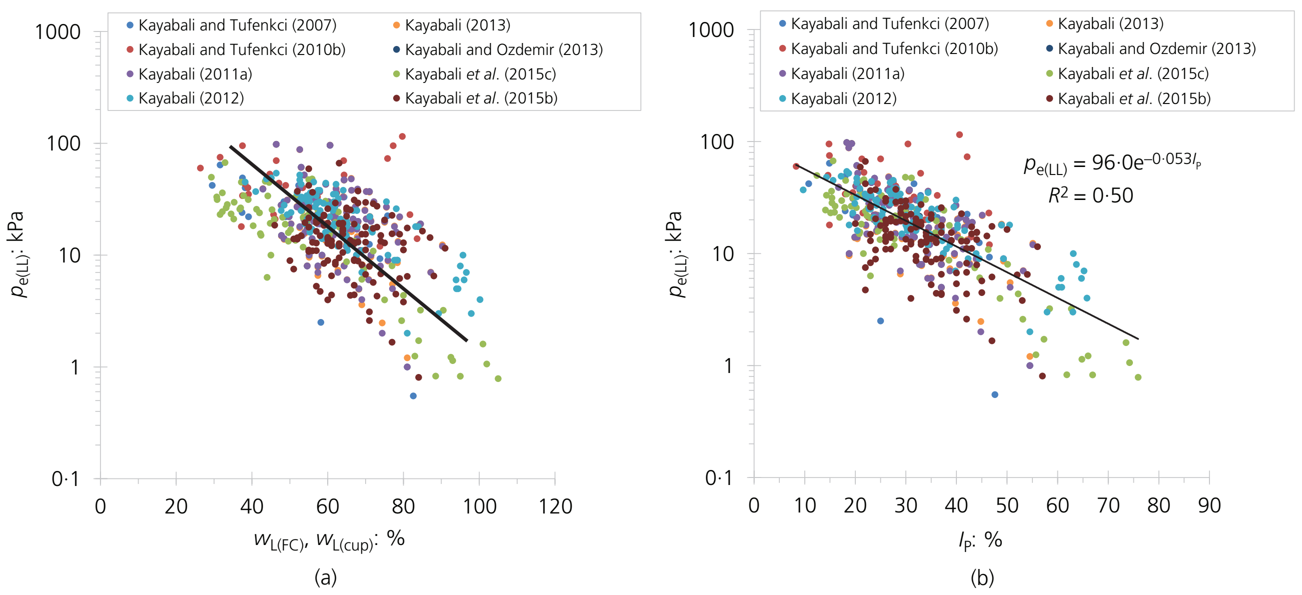

Figure 11. Steady-state extrusion pressure at LL deduced for $R = 40{\cdot}1$ and $v = 1$–5 mm/min plotted against (a) water content at LL and (b) plasticity index

the lower die displacement rate employed for that investigation. When the entire combined data sets were plotted in a bi-logarithmic chart, the value of $p_{e(PL)}$ was found to increase approximately with increasing $p_{e(LL)}$ according to a power-law relationship (see Figure 12(c)). The relationship between $p_{e(PL)}$ and w_L was also investigated, although no consistent overall trend was apparent.

Ratio of RE pressures at consistency limits

The ratio of steady-state extrusion pressures corresponding to the consistency limits (i.e. $p_{e(PL)}/p_{e(LL)}$) had a mean value of 204 but with an extremely wide range, showing a general strong trend of increasing exponentially from approximately 30 to 2000 when w_L increases from 35 to 100% (Figure 13(a)) and also when I_P increases from 10 to 75% (Figure 13(b)). In relation to PL, regression of the entire combined data sets indicated that the $p_{e(PL)}/p_{e(LL)}$ ratio reduces approximately exponentially in value with increasing w_P, although the Kayabali and Tufenkci (2007, 2010b) and Kayabali *et al.* (2015c) data sets tended to suggest that the opposite occurs.

DE testing using the MPM device

As evident from Figure 14, the same general trends in terms of the steady-state DE pressures for the consistency limits are found from reanalysis of the Kayabali *et al.* (2016) data set obtained using the MPM device with $R = 5{\cdot}1$ – that is, a significant approximately exponential decrease in the value of $p_{e(LL)}$ occurs for increasing $w_{L(FC)}$, whereas the value of $p_{e(PL)}$ does not exhibit this trend with increasing I_P (appearing independent of one another for $I_P > {\sim}50\%$), such that the value of the $p_{e(PL)}/p_{e(LL)}$ ratio exhibits a general strong exponentially increasing trend for increasing I_P. Referring to Figure 14, the ranges of the MPM $p_{e(LL)}$ and $p_{e(PL)}$ values are different from those presented for RE testing in Figures 11–13 since the die configurations and values of the R and v parameters were significantly different between these two experimental approaches.

Discussion

The regression correlations included in Figures 11 and 12 would suggest that the values of the steady-state RE pressures corresponding to the consistency limits are not unique but instead are strongly dependent on the soil plasticity, with the values of $p_{e(PL)}$ and, in particular, $p_{e(LL)}$ both reducing exponentially for increasing values of w_P and w_L, respectively (Figures 11(a) and 12(a)). However, there are a number of major anomalies between these suggested behaviours and the actual soil behaviour.

In relation to LL, within the limits of experimental error and following directly from Equation 4, the value of $p_{e(LL)}$ should remain constant for increasing value of $w_{L(FC)}$, since the associated $s_{u(LL)}$ value remains approximately constant – for example, at ~1·7 kPa in the case of the BS FC LL. However, the $p_{e(LL)}$-against-$w_{L(FC)}$ data set from Kayabali *et al.* (2015c) presented in Figure 11(a) clearly shows that their deduced values of $p_{e(LL)}$ substantially decrease for increasing value of $w_{L(FC)}$, with values of $p_{e(LL)}$ as low as 0·8–2 kPa for $w_{L(FC)} > 80\%$. In other words, the trend in the Kayabali *et al.* (2015c) data set would imply that, for given R and v values, the value of $p_{e(LL)}$ corresponding to the $w_{L(FC)}$ is not uniquely related to the soil $s_{u(LL)}$ value, but it is also significantly dependent on its plasticity characteristics. If this statement were true, the non-uniqueness in the value of $p_{e(LL)}$ associated with $w_{L(FC)}$ would invalidate the extrusion approach for LL determination and undrained shear strength measurement. Further, for increasing values of $w_{L(cup)}$, it is well documented that the associated values of $s_{u(LL)}$ decrease exponentially over a

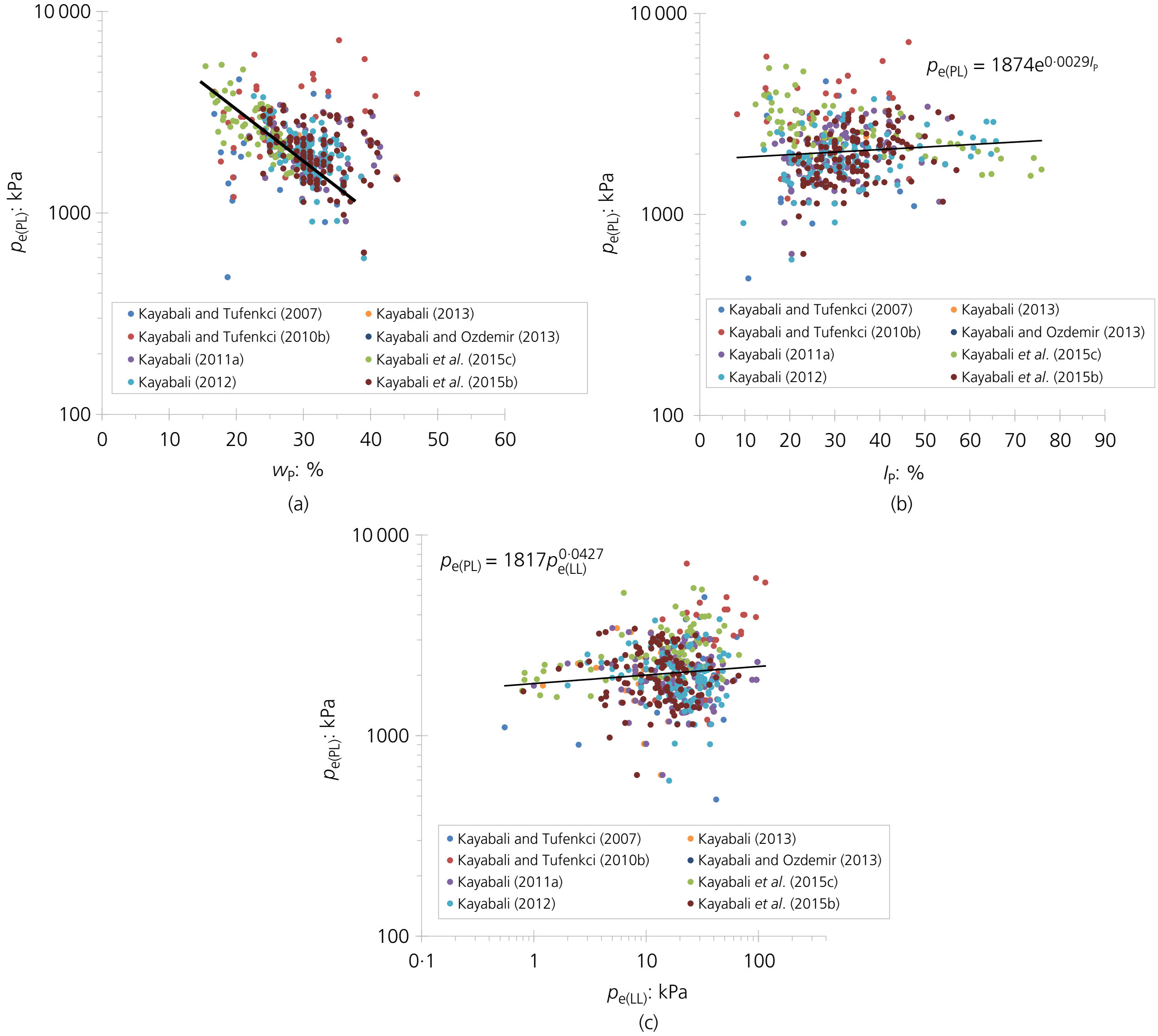

Figure 12. Steady-state extrusion pressure at PL deduced for R = 40·1 and v = 1–5 mm/min plotted against (a) water content at PL, (b) plasticity index and (c) extrusion pressure at LL water content

relatively narrow range of ~1–3 kPa (O'Kelly, 2019; Youssef *et al.*, 1965). Hence, with $p_{e(LL)}$ solely dependent on the soil billet's s_u value for given R and v values according to Equation 4, the value of $p_{e(LL)}$ would also be expected to exhibit a proportionate exponential decrease with increasing value of $w_{L(cup)}$. However, as evident in Figure 11(a), an approximate two-orders-of-magnitude decrease in the value of $p_{e(LL)}$ (reducing from approximately 90 to 2 kPa) occurred for increasing $w_{L(cup)}$ from 35 to 95%. Clearly, such an enormous reduction in the value of $p_{e(LL)}$ cannot be reconciled with the more modest reduction occurring in the value of Casagrande $s_{u(LL)}$ for increasing $w_{L(cup)}$. Taken together, these observations indicate a major issue with the values of $p_{e(LL)}$ reported by Kayabali and co-workers, particularly for w_L > ~60% and I_P > ~35%, as tentatively deduced from the general pattern of the data spread evident in Figures 11 and 13.

In relation to PL, consistent with the general overall trend expected for $s_{u(PL)}$ with increasing soil plasticity, one would anticipate that the values of $p_{e(PL)}$ and the $p_{e(PL)}/p_{e(LL)}$ ratio would decrease overall with increasing value of I_P. However, the data trends in Figures 12(b) and 13(b) seem to suggest that this was not the case, with the values of $p_{e(PL)}$ and particularly the $p_{e(PL)}/p_{e(LL)}$ ratio increasing exponentially with increasing I_P for the various fine-grained soils investigated. For instance, the value of the $p_{e(PL)}/p_{e(LL)}$ ratio increased from approximately 30 to 2000 for increasing I_P from 10 to 75%. Further, with Equation 4 implying that the steady-state p_e is analogous to s_u, this approximately 67-fold increase in the value of the $p_{e(PL)}/p_{e(LL)}$ ratio (and hence the $s_{u(PL)}/s_{u(LL)}$ ratio) for increasing I_P of fine-grained soil is simply not credible – see the discussion on typical strength gain over plastic range presented in the paper by O'Kelly (2013a). Based on all of the aforementioned

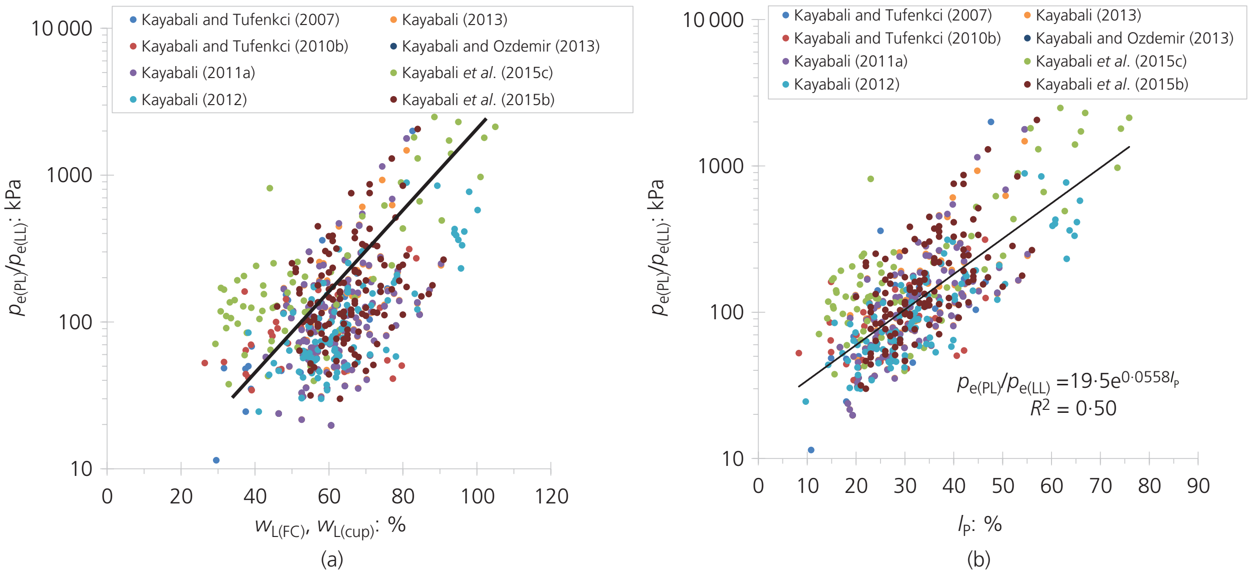

Figure 13. Ratio of steady-state extrusion pressures at the consistency limits deduced for $R = 40{\cdot}1$ and v = 1–5 mm/min plotted against (a) water content at LL and (b) plasticity index

observations, it is evident that other influencing factors are at play, with three significant ones identified by the author investigated in the following sections – that is, it is the author's contention that inconsistencies in the reported $p_{e(LL)}$ and $p_{e(PL)}$ values and their inferred correlations with the consistency limits largely occur for the following reasons.

- Some consolidation of the soil billet may occur under the compressive force applied by the slowly advancing die, particularly for stiffer soils that require very high extrusion pressures and particularly those with higher hydraulic conductivity values. In these instances, the value of shear strength mobilised in the billet's yielding zone would correspond to a partially drained condition (i.e. greater than the assumed undrained shear strength value).
- For the water content range of $w_P \leq w < w_T$ (where w_T is the transition water content below which the soil has measurable toughness), the property governing extrusion resistance capacity is the soil remoulding toughness (T), rather than simply its undrained shear strength.
- When considering a wide range of water contents, the general piecewise-linear nature of the steady-state p_e–w relationship impacts on the reliability of extrusion pressure values deduced from regression analyses and data extrapolation.

Partially drained condition of the soil billet?

Relative to the consistency of the soil billet, the extrusion pressures required for plastic soils are very high, with their values significantly increasing for soil with greater shear strength and for higher experimental R and v values. For instance, $p_{e(LL)}$ and $p_{e(PL)}$ values ranging approximately 20–40 and 2000–3100 kPa, respectively, were reported for the RE apparatus with an R value of 40·1 employed in the investigations of various fine-grained soils by Kayabali (2011a, 2012) and Kayabali and Tufenkci (2007, 2010a, 2010b). For the slow die displacement rates of typically 1–5 mm/min employed, it is entirely plausible that some localised consolidation of the soil billet may occur, caused by drainage of the highly pressurised pore water from the die end of the billet, by way of the extruding soil worm which is under no confinement pressure. More pronounced localised consolidation would be expected for stiffer soils that require higher extrusion pressures, those with greater hydraulic conductivity and also for lower die displacement rates, such that the drainage time increases. In this event, the shear strength mobilised in the billet's yielding zone would correspond to a partially drained condition, representing a significant departure from the extrusion theory that was adopted from metallurgy. If substantiated, this anomaly could adversely impact or possibly even invalidate the RE approach for the determination of the true undrained shear strength for fine-grained soil and by association its consistency-limit values. A full review by the author of the pertinent literature produced only two experimental evidences, which are elaborated below, concerning this particular aspect of RE soil testing.

The most compelling evidence supporting the billet consolidation hypothesis occurs in connection with RE testing of pure kaolin clay (w_L = 53·2%; w_P = 31·0%) prepared at water contents slightly higher than its PL, as reported in the papers by Verástegui-Flores and Di Emidio (2014, 2015). Rather than reaching a steady-state p_e value, the mobilised extrusion pressure continuously increased with increasing die displacement, from which Verástegui-Flores and Di Emidio (2014, 2015) concluded that not only did the kaolin clay billets undergo extrusion but they were also undergoing consolidation within the RE chamber. This

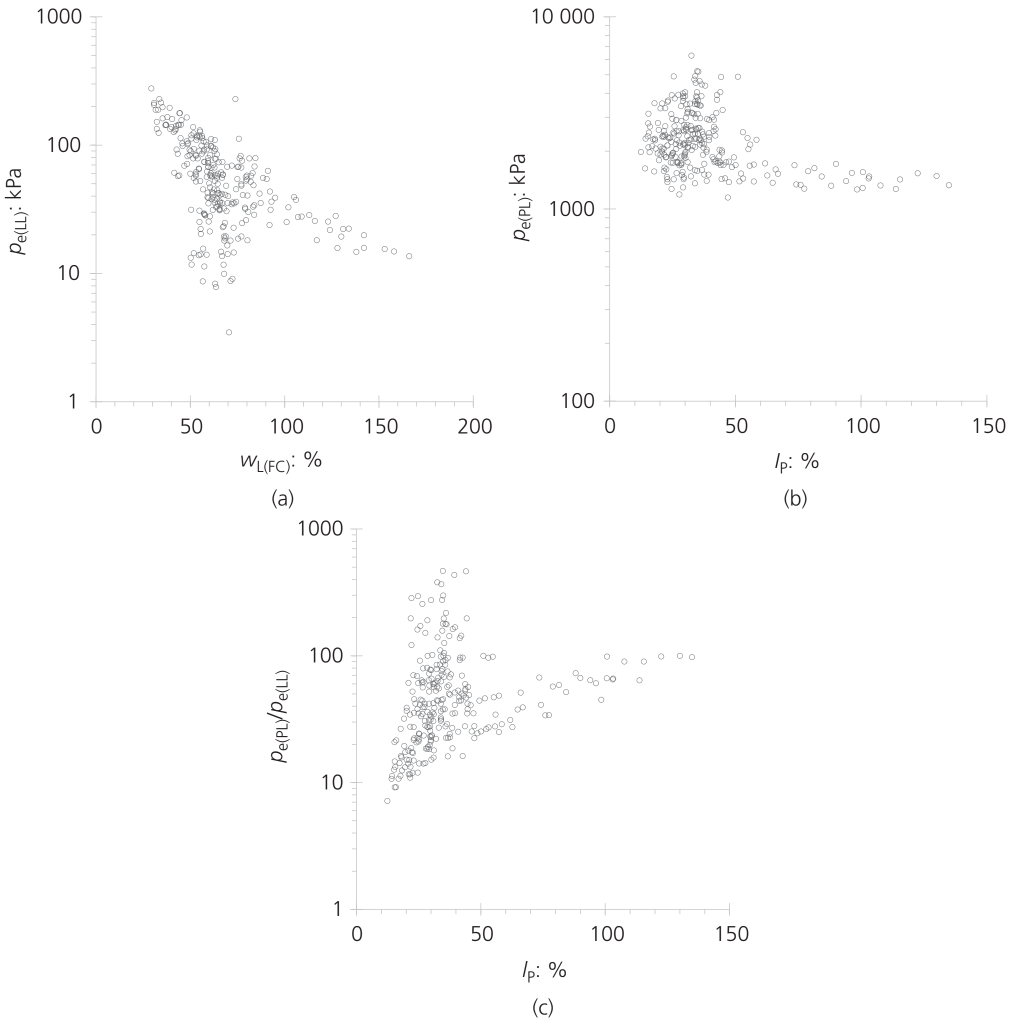

Figure 14. MPM DE testing of 275 fine-grained soils for R = 5·1: values of steady-state extrusion pressure corresponding to (a) $w_{L(FC)}$ and (b) plasticity index; (c) ratio of extrusion pressures at the consistency limits (reproduced from tabulated data presented by Kayabali *et al.* (2016))

assessment was confirmed by the extruded soil worm having an increased value of water content compared with the remaining compressed billet. In other words, consolidation of the soil billet (with drainage by way of the die orifice) under the very high applied extrusion pressures progressively increased the shear strength value of the kaolin clay billet within the extrusion chamber, such that a steady-state p_e value could not be achieved for large die displacements. These researchers reported no similar issues for RE testing of the same kaolin clay at its w_L value, or for natural Boom Clay containing illite (w_L = 51·0%; w_P = 32·4%) and pure bentonite clay (w_L = 374·7%; w_P = 62·9%) prepared at water contents slightly higher than their w_P values. As such, it would appear that fine-grained soil with higher shear strength can be susceptible to billet consolidation and that the other major determining factors are clay mineralogy and gradation. The dependence of shear resistance and compressibility on clay mineralogy is well documented (O'Kelly, 2014b) – that is, among kaolinite, illite and smectite clay minerals, kaolinite has a smaller specific surface area and shows higher hydraulic conductivity, so it is more prone to releasing water when subjected to confinement stress.

On the other hand, based on <1% difference between the measured water contents of the soil billet and worm parts for 12 specimens of a single high-plasticity clay soil (w_L = 57·2%; w_P = 23·0%) reverse-extruded at various water contents, Kayabali and Tufenkci (2010b) concluded that the soil material in the billet's yielding zone remained in a fully undrained condition. However, a water volume balance requires that pore water draining out of the extrusion chamber by way of the die orifice brings about recovery for the extruded soil worm of any reduction in its water content value that occurred earlier within the yielding zone to (approximately) the billet's initial water content value. As such, this experimental evidence presented by Kayabali and Tufenkci

(2010b) does not, in itself, conclusively support their assertion of a truly undrained billet condition.

Compared with the high-plasticity clay soils investigated by Kayabali and Tufenkci (2010b) and Verástegui-Flores and Di Emidio (2014, 2015), any billet consolidation effect would be significantly more pronounced for fine-grained soils with substantially higher hydraulic conductivity (e.g. silty fine sand material), which is dependent on the soil gradation. Further, in terms of the experimental set-up, a higher degree of localised consolidation and hence greater gain in shear resistance capacity of these soils could be expected for higher applied extrusion pressures and slower die displacement rates. In other words, under these circumstances, the soil material within the billet's yielding zone could be partially drained and would therefore mobilise a higher shear resistance capacity compared with its initial s_u value. Further, different degrees of localised billet consolidation for various fine-grained soils with the same starting s_u value would produce different levels of strength increase, thereby mobilising different values of the steady-state p_e for these soils. Consequently, the values of the steady-state p_e/s_u ratio for these soils would be different.

It is the author's opinion that billet consolidation, combined with data analysis and extrapolation issues discussed in the following sections, largely explains the very wide $p_{e(LL)}$ and $p_{e(PL)}$ value ranges reported for various fine-grained soils investigated using the RE apparatus with set R and v values – that is, consistent with the data trends evident in Figures 11, 12(a) and 14(a), fine-grained soils with lower plasticity, which typically have higher hydraulic conductivity, generally mobilise significantly greater values of $p_{e(LL)}$ and $p_{e(PL)}$. As such, the extrusion approach may not be a reliable method for undrained shear strength determinations on low-plasticity soils, producing unconservative (high) shear strength values if some localised billet consolidation occurs during the testing. Counteraction measures include reducing the apparatus R value, thereby reducing the pressure values required to cause extrusion, and (or) increasing the die displacement rate, thereby reducing the time period over which billet consolidation could occur. Other potential shortcomings of the RE experimental set-up include required maintenance of the extrusion chamber and the risk of the die jamming, particularly considering the high extrusion pressures required as the PL water content is approached.

Soil remoulding toughness, *T*

Compared with the predefined shear surfaces for direct shear and shear vane strength-measurement methods and the shearing plane or general plastic deformation of the test specimen for triaxial compression (e.g. see the papers by O'Kelly (2013b, 2013c, 2015)), the extrusion process is distinctly different in that it involves plastic flow of the soil element through the die orifice. At the PL, clayey silt and clayey silty sand materials have slight toughness, whereas low- to medium-plasticity soils have medium toughness and high-plasticity clay soils have high toughness, toughness here referring to the effort required in (re)moulding. As such, soil with greater toughness would be expected to provide higher resistance to extrusion – that is, greater energy would be required to cause extrusion, noting that the extrusion pressure against die displacement plot can be regarded as the work or power diagram of extrusion.

Typical bi-linear T–w relationships determined for clay soils using a PL thread-rolling device developed by Barnes (2013a, 2013b) are presented in Figure 15. In this figure, the plastic range is subdivided based on the deduced toughness relationship into an adhesive (sticky) plastic region and workable soft- and stiff-plastic regions, as delineated by the reducing water content values of w_L, w_T, the stiffness transition (w_{ST}) and w_P. For water contents within the adhesive-plastic region ($w_T \leq w < w_L$), fine-grained soil exhibits zero toughness. In this instance, for given apparatus R and v values and a truly undrained condition, the steady-state extrusion pressure value is entirely controlled by the soil's undrained shear strength. Based on experimental data for rolling out of soil threads presented by Barnes (2013b), measurable toughness for fine-grained soil is typically first evident at $I_L \approx 0{\cdot}4$.

For the water content range of $w_P \leq w < w_T$, however, it is the author's contention that the steady-state extrusion pressure value relates to the soil toughness. Compared with low-plasticity fine-grained soil, greater toughness for higher-plasticity soil would be expected to provide greater resistance capacity to extrusion for water contents in the workable soft-plastic and, in particular, stiff-plastic regions. This interpretation is consistent with the earlier observation in connection with Figure 12(b) of overall increasing value of $p_{e(PL)}$ with increasing I_P. Further, as discussed in the next section, whether plotted in normal, semi- or double-logarithmic charts, the s_u–w (and hence steady-state p_e–w) correlation can generally be expected to be piecewise linear. This can have major implications for the reliability of regression analyses and extrapolation of steady-state p_e–w data.

An approach commonly employed for quantifying relative remoulding toughness is to compute the area enclosed by the experimental shear stress–shear strain curve. For the RE test, under the action of the steady-state extrusion pressure, the die displacement rate and hence extrusion velocity of the soil worm remain constant, meaning that the associated work rate ($=p_eA_ov$) is also constant. As such, for the workable soft- and stiff-plastic regions, the steady-state p_e value mobilised for a given apparatus die configuration and R and v values provides an assessment (measure) of the test soil's remoulding toughness. The maximum remoulding toughness, T_{max}, occurs for the PL water content. In assessing the relative value of T_{max} using the RE approach, the extrusion testing should be limited to soil water contents within the workable stiff-plastic region, given the characteristic approximately bi-linear nature of the T (steady-state p_e)–w correlation (see Figure 15(a)). Having determined the value of w_P from standard thread-rolling of the extruded 6 mm dia. soil worm (RE apparatuses with 6 mm dia. orifices are the norm), the associated value of $p_{e(PL)}$ can then be deduced from regression of the obtained experimental steady-state p_e–w correlation, thereby providing a relative measure of the test soil's T_{max} value.

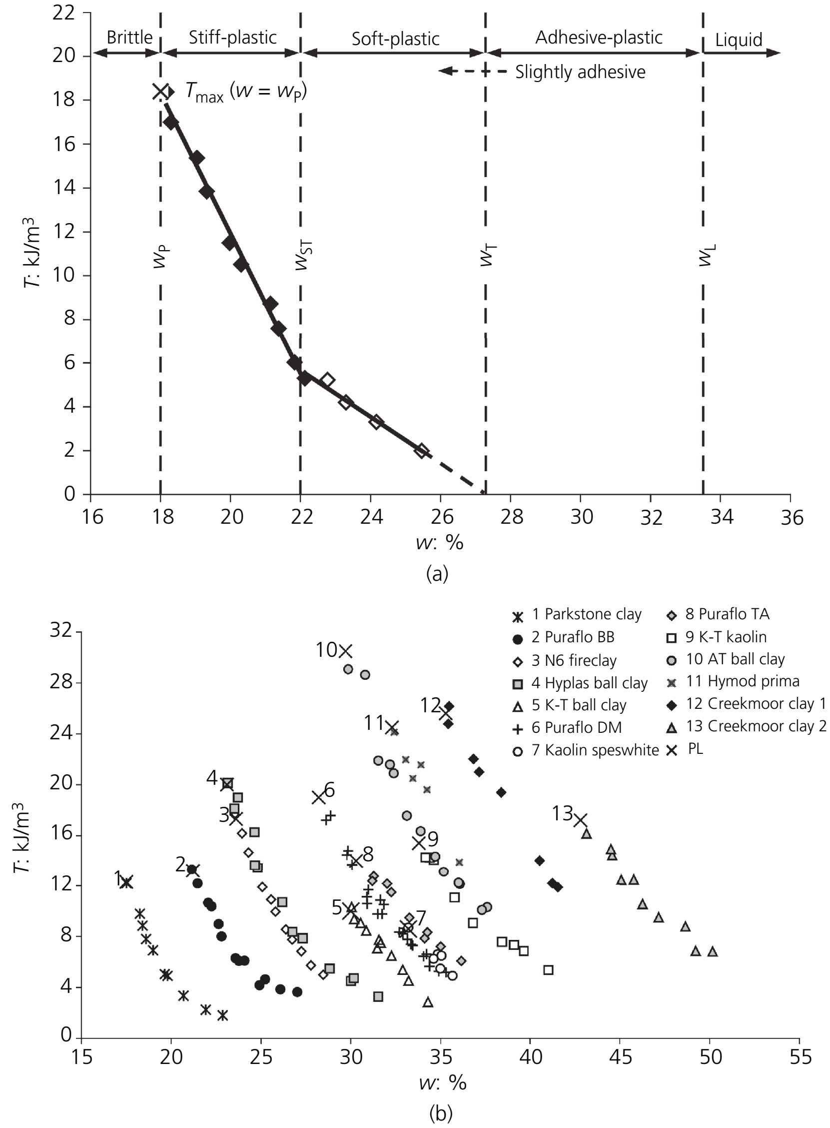

Figure 15. Relationships between toughness and water content deduced using a mechanical thread-rolling device for (a) Weald Clay (brick clay) and (b) various ceramic clays (adopted from Barnes (2013a)). T_{max}, maximum remoulding toughness; w_{ST}, stiffness transition; w_T, toughness limit

General piecewise-linear nature of the steady-state p_e–w correlation and implications for regression outputs and data extrapolation

The piecewise-linear or curved nature of the steady-state p_e–w correlation covering the plastic range for a given fine-grained soil impacts to varying degrees on the results obtained from regression analyses and data extrapolation, such that deduced values of $p_{e(PL)}$ and, in particular, $p_{e(LL)}$ may not be reliable. For example, the tabulated values of $p_{e(LL)}$ and $p_{e(PL)}$ reported in the studies by Kayabali and co-workers were determined from data extrapolation using Equation 7, inputting the values of the a and b fitting parameters obtained from regression analysis of typically five p_e and w data pairs confined to the I_L range 0–0·5 – that is, in this instance, significant extrapolation of the experimental data was required to determine the value of $p_{e(LL)}$ corresponding to I_L = 1·0.

The principal reasons for the piecewise-linear or curved nature of the steady-state p_e–w correlation are the following.

- As described earlier, depending on the consistency of the soil billet, different mechanical properties can control its resistance capacity – that is, remoulding toughness for $w_P \leq w < w_T$ and shear strength for $w \geq w_T$.
- When presented in semi- or bi-logarithmic charts, compared with intermediate- and high-plasticity fine-grained soils which exhibit approximately linear relationships for $w_P \leq w > w_L$ (Figure 16(a)), some extremely-high-plasticity soils were found to exhibit a piecewise-linear behaviour at near their w_L values. The latter behaviour is most evident for the FC s_u–w power correlations presented for two pure bentonite clays in Figure 16(b)). Compared with intermediate- and high-plasticity soils,

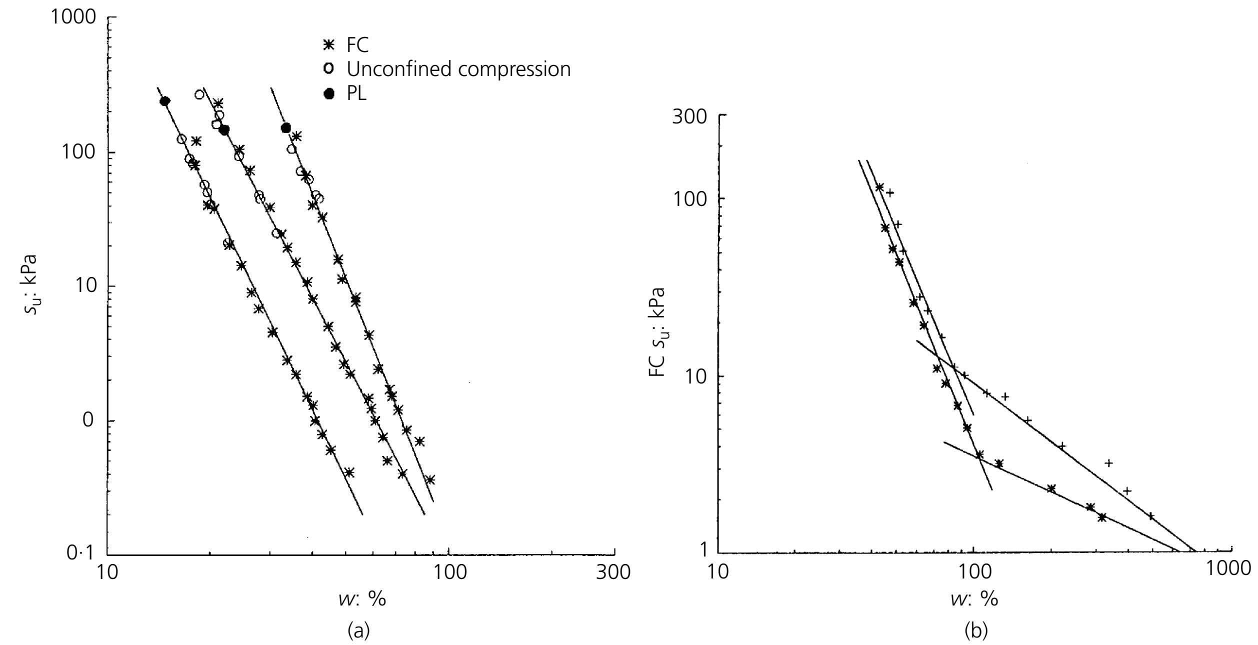

Figure 16. Undrained shear strength–water content correlations reported for (a) three remoulded fine-grained soils of intermediate and high plasticity and (b) two pure bentonite clays (adopted from Sharma and Bora (2003) with permission from the American Society of Civil Engineers)

this distinctly different behaviour is rationalised as the LL of kaolinitic soils mainly due to their fabric, as governed by interparticle attractive forces (Sridharan *et al.*, 1988), whereas that of montmorillonitic soils is largely due to the diffuse double-layer-held water (Sridharan *et al.*, 1986).

- When investigating a wide range of water contents, experimental steady-state p_e (s_u)–w data often exhibit some curvature when plotted in a semi-logarithmic chart. For this reason, bi-logarithmic presentations of the data are usually preferred (Butterfield, 1979; Koumoto and Houlsby, 2001; O'Kelly, 2014a, 2016; Sharma and Bora, 2003). Experimental supporting evidence in terms of RE testing is provided in the papers by Verástegui-Flores and Di Emidio (2014, 2015), with the bi-logarithmic steady-state p_e–w plot of their data covering the I_L range 0·05–0·44 reported as producing a very well-defined linear correlation, whereas the semi-logarithmic presentation of the same data was non-linear.

In the studies by Kayabali and co-workers, their experimental steady-state p_e–w correlations presented in a semi-logarithmic chart were obtained for $0 < I_L < 0{\cdot}5$ – that is, the various fine-grained soils tested had measurable toughness. In view of the issues highlighted earlier, it is evident that the significant extrapolation of the steady-state p_e–w correlation from $I_L < 0{\cdot}5$ to an I_L value of unity in order to establish the value of $p_{e(LL)}$ will likely result in an underestimation of its true value. As explained earlier using the soil behaviours shown in Figure 16(a) as examples, the underestimation may be substantial for extremely-high-plasticity soils investigated ($w_L >$ 90%) and possibly also for some high-plasticity soils (w_L = 70–90%). This interpretation is consistent with the extremely low values of $p_{e(LL)}$ reported for these soil plasticity categories – that is, with R = 40·1 and using v = 1–5 mm/min, Kayabali and co-workers deduced $p_{e(LL)}$ values ranging 0·8–10 and 0·8–30 kPa for extremely high- and high-plasticity soils, respectively (refer to Figure 11(a)) based on the described data extrapolation approach. Although these issues also affect the deduced values of $p_{e(PL)}$, the resulting errors are generally likely to be smaller since the required data extrapolation is not as great. However, consideration of the impact of the reported bi-linear T–w relationship on the reliability of the described regression and extrapolation approaches merits further investigation.

Computed values of $p_{e(LL)}$ and $p_{e(PL)}$ can also be particularly sensitive to reduced precision (rounding) of the deduced b or b' fitting parameter values in Equations 7 and 8. For instance, Figure 17 compares the values of $p_{e(LL)}$ and $p_{e(PL)}$ reported for 31 fine-grained soils in the paper by Kayabali and Tufenkci (2010b) with those computed by the author as part of the present investigation from tabulated p_e and w data pairs reported in their paper. The author performed the same computation as used in the original study – that is, a and b values were deduced for each soil from regression analysis of its reported steady-state p_e–w data set, from which the values of $p_{e(LL)}$ and $p_{e(PL)}$ were computed using Equation 7 for the reported values of w_L and w_P, respectively. The only difference between these analyses was that the author's deduced b exponent values had an enhanced precision of four decimal places. Not surprisingly, as evident from Figure 17, the deduced values of $p_{e(LL)}$ and $p_{e(PL)}$ can be significantly affected by rounding errors. As evident from the regression lines presented in Figure 17 for the reported and author-determined data values, this can have knock-on effects for inferred steady-state p_e–w

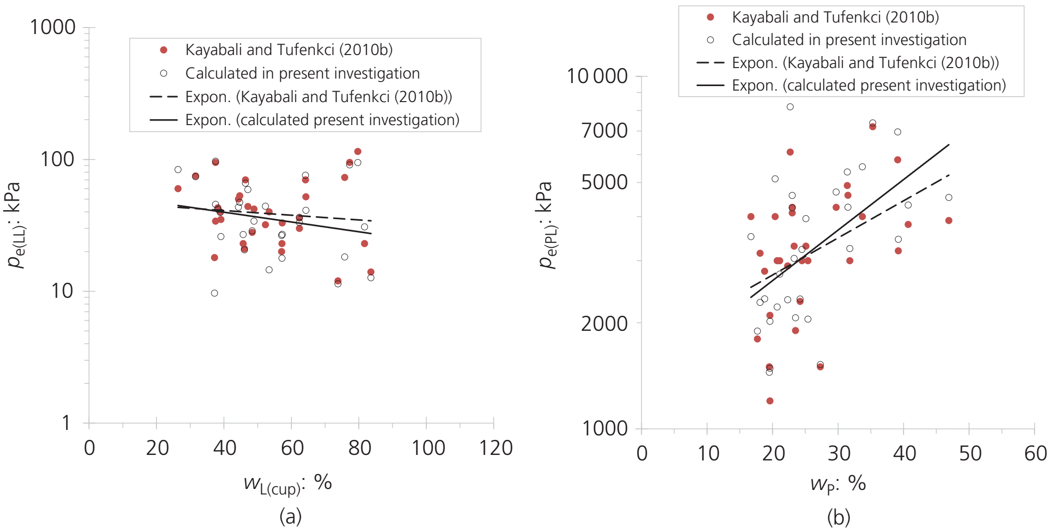

Figure 17. Comparisons of steady-state RE pressures reported for R = 40·1 and v = 5 mm/min in the paper by Kayabali and Tufenkci (2010b) with those deduced in the present investigation from reanalysis of their tabulated p_e–w data sets: (a) against Casagrande LL; (b) against PL

relationships. For instance, the author-derived $p_{e(LL)}$–$w_{L(cup)}$ correlation presented in Figure 17(a) is consistent with the trend of reducing value of Casagrande $s_{u(LL)}$ for increasing LL water content (O'Kelly, 2019; Youssef *et al.*, 1965), whereas the Kayabali and Tufenkci (2010b) regression correlation is ambiguous regarding this accepted trend.

Finally, as mentioned earlier, the RE approach was proposed by Kayabali (2012, 2013) for the determination of the SL water content, with the value of w_S supposedly associated with a particular steady-state p_e value (i.e. $p_{e(SL)}$). As evident from Figure 18 and with s_u–I_L as a proxy for the steady-state p_e–w, the pattern of relationships

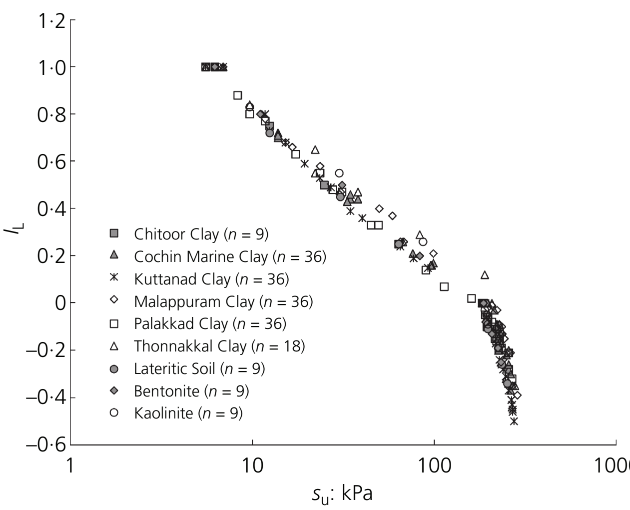

Figure 18. Vane undrained shear strength–liquidity index relationship for various remoulded clay soils of high and extremely high plasticity (adopted from Vinod *et al.* (2013))

undergoes a noticeable transition at the PL (I_L = 0). As such, the proposed back-extrapolation of the logarithmic steady-state p_e–w correlation determined for the plastic range to water contents in the semi-solid region does not seem a valid approach for the determination of the w_S value.

Summary and conclusions

The steady-state p_e value for RE of fine-grained soil is strongly dependent on its water content and the logarithm of extrusion rate, r_e (= Rv). The following are concluded from reassessment of the combined RE data sets presented in the literature for many hundreds of remoulded fine-grained soils covering an extremely wide plasticity range.

- The value of the steady-state p_e/s_u ratio and hence K_{RE} can vary widely between soils, appearing to decrease overall with increasing plasticity. For instance, a standard deviation of 3·2 occurred for the mean $p_{e(PL)}/s_{u(PL)}$ = 13·9 deduced for 60 remoulded fine-grained soils investigated in the paper by Kayabali and Ozdemir (2013).
- The values of both $p_{e(LL)}$ and $p_{e(PL)}$ exhibited substantial exponentially reducing trends with increasing values of w_L and w_P, respectively. This behaviour was also strongly apparent for the value of the steady-state $p_{e(LL)}$ corresponding to the strength-based BS $w_{L(FC)}$ deduced for $s_{u(LL)} \approx$ 1·7 kPa.
- The values of $p_{e(PL)}$ and the $p_{e(PL)}/p_{e(LL)}$ ratio increased overall with increasing plasticity, although one would have anticipated the contrary, given the general expected trend of $s_{u(PL)}$ reducing in value with increasing plasticity.

Based on these points, it is concluded that, for given apparatus R and v values, the steady-state p_e/s_u ratio value does not remain constant among different remoulded fine-grained soils. Depending on mineralogy and gradation, some localised billet consolidation may occur for stiff low-plasticity soils under the very high extrusion pressures required and low strain-rate range of 0·2–12·8 min^{-1} typically employed, particularly for those soils with higher hydraulic conductivity values. Evidence of this effect was presented in the papers by Verástegui-Flores and Di Emidio (2014, 2015) for pure kaolin clay near its PL water content. With drainage to atmosphere allowed by way of the die orifice, some consolidation of soil material within the billet's highly stressed shear zone would have the overall effect of increasing the value of the steady-state p_e/s_u ratio.

Further, compared with standard strength-measurement approaches (e.g. vane shear, direct shear and triaxial compression), extrusion is distinctly different in that it involves plastic flow of the soil, such that remoulding toughness (rather than simply undrained shear strength) is the mechanical property governing resistance capacity for water contents in the range of $w_P \leq w < w_T$. This interpretation is consistent with the overall observed trend of the experimental $p_{e(PL)}$ value increasing with increasing soil plasticity, given that higher-plasticity soils have greater toughness at the PL.

Caution is urged in interpolating values of extrusion pressure and (/or) water content from regression analysis of steady-state p_e–w correlations and particularly for their extrapolation over a wider water content range on account of the probable piecewise-linear nature of p_e–w relationships when considering the full plastic range and beyond.

Since the standard thread-rolling PL relates to the ductile–brittle transition, as essentially a strength-based approach, the extrusion technique cannot be reliably employed for its determination. Further, the standard SL relates to the water content below which no further reduction in soil volume occurs, which again cannot be reliably determined using the strength-based extrusion approach. The standard FC and Casagrande LL are strength based, but, as mentioned already, the various data sets presented by Kayabali and co-workers indicate that the value of $p_{e(LL)}$ generally reduces significantly with increasing soil plasticity. Therefore, in conclusion, the substantial experimental evidence is such that the present extrusion approach is not recommended for undrained shear strength measurement or for Atterberg-limit determinations.

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