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LINEAR AND NON-LINEAR SPECTRAL ANALYSIS

OF

OFFSHORE LATTICE STRUCTURES

BY

MICHAEL HARTNETT

DISSERTATION SUBMITTED TO THE UNIVERSITY OF DUBLIN IN
PARTIAL FULFILMENT OF REQUIREMENTS FOR THE DEGREE OF Ph.D.

Department of Civil, Structural and Environmental Engineering

May 2000
DECLARATION

I declare that this dissertation, in whole or in part, has not been submitted to any University as an exercise for a degree. I further declare that, except where reference is given in the text, the work is entirely my own.

Michael Hartnett

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Michael Hartnett
24th July 2000
DEDICATION

This work is dedicated to my family: to my wife Cathleen, who has been being a 'Ph.D. widow' for too long, and to my children Mary and Conor.
ABSTRACT

Most of the offshore structures that are used in oil and gas exploration and production are lattice jacket structures. Until recently these structures have been sited in relatively shallow water, however, the trend now is to exploit the oil and gas resources in deep water. This dissertation addresses the spectral response of lattice structures in shallow and deep waters by developing linear and non-linear approaches respectively.

An efficient linear spectral response model is developed using a novel application of the unit wave technique. The approach adopted relies on linearity between surface waves and structural response and employs mode superposition. Using this method the extensive computations required to analyse large platforms are reduced by half. Complete details of the mathematical development are presented, followed by a validation of various components of the model. The eigensolver is validated against different test cases and the spectral response model is validated against the results of a structural monitoring programme of a large gas production platform, Kinsale Platform Alpha. Phasing effects due to leg spacing can play an important part in the response of jacket platforms and are incorporated in the above development. These effects had not previously been incorporated into non-linear analyses. A non-linear model is developed which extends the work of previous researchers by including leg spacing effects. Full details of the model development are presented. The linear and non-linear models are applied to some simple structures for various storm conditions; response spectra are computed and compared. Interesting differences between the results are obtained, indicating that interactions between higher order harmonics forces and leg spacing give rise to larger structural responses spectra. This phenomenon had not previously been observed and could have important consequences for deep-water structures, such as fatigue assessment.

The models developed in this research are generic in nature and could also be applied to structures other than offshore lattice platforms, such as pipelines and risers. Also, results obtained suggest that other systems subjected to nonlinear random forces may experience significant interactions between higher order forces and system spatial extent.
ACKNOWLEDGEMENTS

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1.1 PREAMBLE

Surface water waves incident on an offshore structure induce complex forces distributed along the structure. These waves are generally random function of time, thus, the resulting forces are also random in nature. Offshore structures subjected to these random forces must, therefore, be analysed to determine the dynamic response of the structure and the interactions between the structure and the surrounding water. In particular, the greater the depth of water in which a jacket platform is deployed, the more important it is to consider the dynamic behaviour of the structure. In the 1970's jacket platforms were regularly located in water depths of about 100m, such as the Kinsale Head Gas Production Platform off the South Irish Coast. However, recent finds of oil and gas deposits in deeper waters have necessitated the deployment of platforms, such as the Bullwinkle jacket, in nearly 500m of water in the Gulf of Mexico. The possibility of using jacket type structures in water depths of approximately 1,000m is currently being investigated. Because of the tendency to deploy structures in deeper water, it is imperative to perform more accurate dynamic response analyses of these structures.

Different types of dynamic analyses are performed on offshore structures depending on the type of structure in question. For large structures such as gravity platforms and buoyancy modules of tension leg platforms, where it is considered that the structure interferes with the flow field, a diffraction analysis is necessary to determine the distribution of pressure around the structure. For slender structures, such as jacket platforms it is assumed that, because the structural members are of small diameter relative to wavelengths, that they do not interfere with the flow field and thus
characteristics of the undisturbed flow field are used to compute the incident forces. In this thesis only slender structures are considered.

The dynamic analysis of a jacket platform can be carried out either in the time domain or in the frequency domain. However, because the wave loadings on a platform are random in nature, it is often useful to present the results of the analysis in a statistical framework. This can be achieved most succinctly in the frequency domain using spectral analysis techniques. This is the approach adopted in this thesis.

1.2 SCOPe OF WORK

In 1950 Morison, O'Brien, Johnson and Schaaf assumed that the wave force on a structure was comprised of an inertia force and a drag force. Their work led to the development of the Morison equation for computing wave forces on fixed slender offshore structures. The Morison equation assumes that the inertia force at a point is a function of the water particle acceleration at that point and that the drag force is a function of the square of the water particle velocity at the point. When structures are placed in water depths of greater than about 30m, effects of wave-structures interaction must generally be taken into account. These wave-structure effects are significant because the velocities and accelerations of the structure will modify the inertia and drag forces induced by the waves. Modified versions of the Morison equation have been developed in which the inertia terms use the relative acceleration between the water particles and the structure and the drag terms use the relative velocity between the water particles and the structure. In order to calculate the wave forces it is necessary to employ a wave theory to compute the water particle velocities and accelerations. Many different wave theories exist such as Linear Airy wave theory, Stokes Fifth-Order wave theory
and Cnoidal wave theory; in this work Linear Airy wave theory is used to compute relevant water particle kinematics.

The wave-structure system contains several different non-linearities that may be significant during the design and analysis of offshore structures. Some of the most important non-linearities of such a system are outlined below:

(i) The non-linear relationship between wave height and force as postulated in the drag term of Morison's equation.

(ii) The non-linear relationships between wave kinematics and wave heights for most wave theories.

(iii) The non-linear stiffness coefficients associated with large displacements of structural elements such as risers.

(iv) The non-linear interactions between foundation piles and the surrounding soil.

(v) The plastic behaviour of structural elements during extreme loading conditions.

All of the above non-linear effects can be significant for particular structural systems and their relative importance must be appreciated before proceeding with detailed designs. However, in this study the only non-linearity being considered is that due to drag term of Morison's equation. Linear Airy wave theory is widely used for analysing jacket platforms and has been found to produce good designs. A useful feature of this wave theory is that it postulates a linear relationship between wave height and the velocity and acceleration of a water particle underneath the wave. In this thesis, Linear Airy wave theory is used in the modified version of the Morison equation to compute wave forces along slender members of jacket platforms. Large displacements are not generally associated with the motions of jacket platforms and thus linear stiffness coefficients are assumed. The structures analysed are considered fixed at their bases and so non-linear
soil-structure interactions are ignored. Finally, the structures analysed are assumed to behave linear-elasticly.

In this thesis, there are two primary objectives:

(i) The development and validation of an accurate and efficient general purpose finite element based model to perform linear spectral analysis of offshore jacket platforms.

(ii) The development of a spectral analysis model incorporating non-linear drag terms to investigate the effects of platform leg spacings on response spectra.

The effects of incorporating non-linear drag force terms on stick models of jacket platforms has been the subject of previous investigations, however, the effects of these terms on spatially extended structures has not previously been examined. The approach developed below is generic in nature and may be applied, with some further developments, to such systems as pipelines and risers.

1.3 ORGANISATION OF THESIS

Chapter 2 presents a review of research on determining forces on slender offshore structures. Relevant aspects of Morison's equation are examined and factors affecting its evaluation are discussed. Different approaches to performing equivalent linearisation of Morison's equation are presented and techniques for performing linear spectral analysis using the linearised equation are considered. An outline of the linear model developed in this thesis is presented. Research leading to the development of non-linear spectral response models is reviewed and an outline of the non-linear model developed herein is presented.

In Chapter 3, the development of the linear spectral analysis model is presented. A finite element model is developed using 3-dimensional beam elements to represent the...
structure elements of a jacket platform. The equations of motion for the wave-structure system are derived by developing a novel equivalently linearised version of Morison's equation. Mode superposition is used to evaluate the transfer functions after equivalent diagonalisation of the hydrodynamic damping matrix is performed. Unit wave theory is employed in a novel manner to efficiently compute transfer functions relating output spectra of displacements to input spectra of sea surface elevations.

The validation of the model developed in Chapter 3 is presented in Chapter 4 by applying the model to a number of different structures. The model is first applied to a simple structure to compute the first few natural frequencies and mode shapes of the structures. The natural frequencies and mode shapes are also computed by a commercial package and the results compared. The model is then used to perform a full spectral analysis of an offshore structure idealised as a cantilever and the results compared with a semi-analytical solution. Finally, the model is applied to a large operational jacket platform, namely, Kinsale Head Gas Production Platform Alpha. Following a structural monitoring programme, the first five natural frequencies and mode shapes are known for this platform and a spectrum of displacement is available for the platform deck associated with a known spectrum of sea surface elevations. The model computes the first five natural frequencies and mode shapes of the structure and computes a spectrum of displacement for the known spectrum of sea surface elevation; these results are then compared with the results from the structural monitoring programme. The validated program is then used to consider some of the phasing effects of leg spacing on response spectra.

Details of the development of the novel numerical model for non-linear spectral analysis of jacket structures are presented in Chapter 5. Again, the finite element method is used to model the structural system as 3-dimensional beam elements. The drag term in
Morison's equation is developed such that a system of equations with time-varying damping coefficients is developed. A solution to this system is obtained by employing a perturbation procedure that transforms the original system of equations into a set of recursive equations. The non-linear drag force terms are expressed as a series of Hermite polynomials such that the spectra of wave force on the structure include terms involving convolutions of spectra of water surface elevations. Transfer functions relating wave force spectra to spectra of displacements are obtained by applying series of harmonic forces of unit amplitudes at each degree of freedom in turn over a range of frequencies of interest. Finally, the spectra of displacements are expressed in terms of the transfer functions and the spectra of force obtained by using a relevant closure technique.

Applications of the model developed in Chapter 5 are presented in Chapter 6. The non-linear model is firstly applied to a simple cantilever structure and then to a number of portal frame structures and the results are compared to the results obtained by applying the linear model to the same structures. A discussion of the results obtained using the two models is then presented and comparisons are made with results obtained by other researchers in this subject area.

Finally, a summary of the work undertaken and conclusions arrived at are presented in Chapter 7. The main findings of the research are discussed and areas of relevant further research are outlined.

This work contributes, in particular, two original elements to research in the field of offshore structural dynamics. Firstly, the unit wave technique has been used in an efficient way to reduce force computations by half. Secondly, the nonlinear model that has been developed illustrates the need to consider interactions between higher order frequencies and spatial effects when calculating structural responses during low to moderate sea states.
CHAPTER 2

REVIEW OF SPECTRAL RESPONSE ANALYSIS OF OFFSHORE PLATFORMS

2.1 INTRODUCTION

A thorough understanding of the interactions between waves and offshore jacket platforms is essential for the safe and economical design of such structures. The time-varying forces induced by waves on these structures are generally computed using a version of Morison's equation. Structural responses to the waves can be determined either in the time-domain, probability domain or the frequency-domain. In the time-domain, a time integration technique is used to solve the equations of motion as the wave propagates forward in time and space. Non-linearities, such as non-linear drag force terms, can be accommodated relatively easily in the time-domain approach using appropriate iterative procedures. Other positive aspects of this approach are that water-structure interactions and the presence of ambient currents can be incorporated into the model with little difficulty. This methodology has been used widely and generally gives good results. However, a major problem associated with time-domain analysis is that it is computationally relatively expensive when applied to complex offshore structures with thousands of degrees-of-freedom.

Probabilistic analyses are based on the probabilistic properties of the structural responses; these analyses usually take the form of determining the long-term distribution of stress reversals for fatigue analysis and on the long-term distribution of peak values of responses during the service life of the structure. Najafian and Burrows (1994) state that assuming that water particle kinematics are jointly Gaussian distributed, the probability distribution is of Pierson-Holmes type and is fully expressed by its first four statistical
moments. The calculation of these moments is extremely time consuming, however, recently Najafian (1999) has developed a methodology that has considerably reduced the computationally effort.

Spectral techniques have been used extensively in the past few decades to perform platform response analysis in the frequency domain. Spectra of structural displacements are calculated from known spectra of sea states and platform receptance functions. Most of the spectral response models that have been developed linearise the drag force terms and apply linear spectral theory. For most jacket platforms in relatively shallow water this approach gives good results and is more efficient than time-domain analysis. However, non-linearities, which may be significant in deeper waters, cannot be easily incorporated into spectral response analysis. A number of methods have been developed over the last 15 years or so to devise non-linear spectral response models for research purposes. Such models often include higher order effects due to non-linear drag force terms.

Regardless of whether response analysis is performed in the time-domain or the frequency-domain Morison’s equation must be invoked. Various aspects of Morison’s equation are considered in the next section of this chapter, including the modified Morison’s equation. The following two sections present reviews of linear spectral response models and non-linear spectral response models respectively. The final section of this chapter summaries the above reviews and outlines the novel developments that are presented in subsequent chapters.
2.2 FORCES ON SLENDER OFFSHORE STRUCTURES

2.2.1 Morison's Equation

For fixed slender offshore structures, such as jack-ups, guyed towers and jacket platforms, wave forces are usually computed using the Morison equation. In their landmark paper Morison, O'Brien, Johnson and Schaaf (1950) assumed that the in-line wave force at a point on a structure is given as the sum of an inertia force and a drag force. For the case of a vertical cylindrical section of diameter \( D \) the force per unit length is

\[
F = C_m \rho A \dot{V} + C_d \rho D V |V| \tag{2.1}
\]

where \( \rho \) is the fluid density

\( V \) and \( \dot{V} \) are the undisturbed velocity and acceleration of the flow respectively.

\( C_d \) and \( C_m \) are the drag and inertia coefficients respectively.

The inertia force is analogous to that on a body in a uniformly accelerating flow of an ideal fluid; while the drag force is analogous to the force on a body in a steady flow of a real fluid with a fully developed wake formation behind the body. Morison's equation does not take into account transverse or lift forces due vortex shedding. These forces can be incorporated into the expression to find the total force on a structural member, but it is usual to ignore them during initial appraisals of a structure. Various models of transverse forces have been developed, see Bearman et al (1984) and Verley (1992).

Morison's original equation is empirical and thus was proposed as an approximate solution to a very complicated problem. It is recognised that the above drag and inertia coefficients are functions of Reynolds number and Keulegan-Carpenter number, amongst other variables. The original linear-quadratic sum equation works very well in practice.
outside a small range of Keulegan-Carpenter numbers and these small discrepancies can
be considered unimportant relative to other vagaries within the design process,
Chakrabarti (1994). Morison's equation appears to provide acceptable answers when the
Keulegan-Carpenter number is either smaller than about 8 or larger than about 25.
Numerous attempts have been made to improve upon Morison's equation or develop
new force equations with varying degrees of success, Barnouin (1979) and Sarpkaya and
Isaacson (1981). More recently, Otsuka and Ikeda (1996) have successfully developed
simple formulae to estimate inertia forces on horizontal cylinders in regular and irregular
waves at low Keulegan-Carpenter numbers; however, Morison's equation applies to
general flow situations for combined drag and inertia forces and thus has more
widespread application than these formulae. Sarpkaya (1999) has developed a modified
Morison equation that includes three terms, the third term is proportional to the
acceleration of the flow. The advantage of this development over other improvements of
Morison's equation is that it does not introduce new coefficients. Initial results show
that the modified equation developed by Sarpkaya provides good predictions of forces
throughout the complete flow range.
Morison's equation is a function of drag and inertia coefficients and hence their
determination is very important to the correct implementation of the equation. Much
research has been undertaken to determine their values for various flow regimes; a brief
review of some of this research is presented in the next subsection. Originally, the
Morison equation was developed for calculating in-line hydrodynamic forces on a rigid
vertical pile subjected to regular waves. For engineering purposes, the equation has been
extended to account for the effects of relative velocity of a flexible structure, ocean
currents, arbitrary orientation and cross-section of the member as well as random sea
states.
The force equation used in this study is the original Morison equation modified to include relative motion effects and its application to inclined members. This approach, recommended by the American Petroleum Institute (1993), has been used extensively by offshore design engineers over the past number of decades and has not led to wave induced failures in North Sea platforms. Both of these modifications are considered in subsequent subsections.

2.2.2 Hydrodynamic Coefficients

Much research has been undertaken to determine how the drag and inertia coefficients specified in Morison's equation are related to different flow regimes during harmonically oscillating flows. Keulegan and Carpenter (1958) carried out major studies on cylinders to investigate the relationship between the coefficients and flow regime. Resulting from this work it was thought that the coefficients were dependent only on the Keulegan-Carpenter number, \( K \), where

\[
K = \frac{V_m T}{D}
\]

\( V_m \) is the amplitude of velocity

\( T \) is the period of oscillation

\( D \) is the cylinder diameter

However, Sarpkaya (1976a, 1976b, 1977) demonstrated the dependency of the coefficients on both the Keulegan-Carpenter number and Reynolds number, \( \text{Re} \), through a frequency parameter, \( \beta \). This frequency parameter is defined as

\[
\beta = \frac{\text{Re}}{K} = \frac{D^2}{\mu T}
\]

where

\( \mu \) is the dynamic viscosity of water
Some of the results obtained by Sarpkaya are presented in Figures 2.1-2.4. Figures 2.1 and 2.2 show $C_d$ versus $K$ and $C_m$ versus $K$ respectively for five different values of the frequency parameter and hence the dependency on Reynolds number. These figures show that the coefficients do not vary significantly with $Re$ for values of $Re$ less than 20,000 which explains the conclusions reached by Keulegan-Carpenter. Figures 2.3-2.4 show how the coefficients are related to $Re$ for constant values of $K$. From these figures it is seen that $C_d$ decreases with increasing $Re$ to a value of about 0.5 and then rise to a constant value for large $Re$. $C_m$ increases for increasing $Re$, peaks and then decreases to a constant value of about 1.85 for large $Re$.

When offshore platforms are initially placed in-situ the outside surfaces of the members are generally smooth. However, over time various types of marine growth attaches itself to the members changing its roughness characteristics. Sarpkaya (1976b, 1977) carried out experiments on cylinders having different degrees of roughness to study the relationship between roughness and the hydrodynamic coefficients during sinusoidally oscillating flows. Surface roughness of a cylinder is generally expressed in terms of $k/D$, where $k$ is the typical height of the surface irregularities and $D$ is the cylinder diameter. Some of his findings are presented in Figures 2.5-2.6, which illustrate how $C_d$ and $C_m$, respectively, vary with $Re$ for different values of roughness when $K$ is kept constant.

At low $Re$ the drag coefficient for a rough cylinder does not vary substantially from that of a smooth cylinder. As $Re$ increases $C_d$ for a rough cylinder quickly decreases until a minimum is reached and then increases so that for post-supercritical flows the values are almost constant. Thus the effects of roughness are more pronounced for higher $Re$ than for lower $Re$. In general, when $Re$ is low the values of $C_d$ for roughened cylinders are less than the values for smooth cylinders, whereas for higher $Re$ the values for roughened cylinders are much greater than the values for smooth cylinders. When the degree of
relative roughness of the cylinder is quite small, \( k/D = 1/800 \), it is seen from Figure 2.5 that the asymptotic value of \( C_d \) is about 1.35, whereas for a smooth cylinder the value of \( C_d \) is about 0.59. Hence it was shown that small increases in roughness can have significant effects on \( C_d \).

The variations of the inertia coefficient and Re for various degrees of relative roughness are presented in Figure 2.6. For low values of Re the values of \( C_m \) for roughened cylinders are substantially less than the values for smooth cylinders, whereas for higher values of Re the values of \( C_m \) for roughened cylinders are substantially less than the values for smooth cylinders. Again at relatively high Re values \( C_m \) values remain almost constant for a given relative roughness. For both smooth and rough cylinders it is seen from the various experimental results that a rise in the drag coefficient is accompanied by a decrease in the inertia coefficient.

Figure 2.7 presents results obtained by Sarpkaya (1976a) showing the relationship between \( C_m \) and \( C_d \) for roughened cylinders and the roughness Reynold’s number, \( Re_k \), where \( Re_k = (V_m \, k) / \mu \), for \( K=100 \). This figure shows that for \( Re_k \) larger than about 300 \( C_m \) and \( C_d \) are virtually independent of \( k/D \). Thus for large values of \( Re_k \) the drag and inertia coefficients are primarily functions of the roughness height rather than of the cylinder diameter. Obviously the effective diameter of the cylinder is important as the drag and inertia forces are both functions of it. As the effective diameter depends on marine growth around the cylinder, it is necessary to obtain as much information regarding marine-fouling on offshore structures as possible.

The experiments carried out above by Sarpkaya to consider the forms of \( C_d \) and \( C_m \) were for sinusoidally varying flows. Burrows et al (1997) undertook flume tests to derive the wave force coefficients during random seas. After extensive analysis of the data they
concluded that values of $C_d$ and $C_m$ obtained from wave-by-wave analyses may be used also when performing frequency domain analysis.

### 2.2.3 Flexible Cylinders

The Morison equation in the form given above has been extensively used in the design of rigid offshore structures. These structures have been built in relatively shallow water, that is water depths less than approximately 30m, where the effects of wave-structure interaction are insignificant or are easily taken care of with a linearization procedure. In shallow water an offshore structure is quite stiff with natural frequencies usually much greater than the wave frequencies. In deeper water, however, offshore structures become more flexible with natural frequencies approaching the wave frequencies and consequently consideration must be given to the possibility of dynamic effects increasing the response. The phrase wave-structure interaction is used to describe the coupling that occurs when the relatively large response of the structure influences the force on the structure that in turn determines the structural response. In order to include the effects of wave-structure interaction, a modified form of the Morison equation which includes structural velocities and structural accelerations is often assumed to apply. One popular method is to assume that the drag force is proportional to the square of the relative velocity between the fluid and the structure and the inertia force is proportional to the relative acceleration between the fluid and the structure. This formulation, called the relative velocity formulation, has been used by Malhotra and Penzien (1970) and Greeco and Hudspeth (1983) amongst others. For a single degree of freedom system Morison's equation as applied to a flexible cylinder is generally written as

$$F = C_m \rho A (\dot{V} - \dot{v}) + C_d \rho D (V - v)\sqrt{V - v}$$

(2.2)
where

\[ \mathbf{v} \] is the velocity of the structure

\[ \dot{\mathbf{v}} \] is the structural acceleration

Equation (2.2) has been used extensively to carry out deterministic and stochastic dynamic response of offshore platforms. From the work carried out by Burrows et al (1997) the hydrodynamic coefficients employed to compute wave forces on rigid cylinders can also be used to compute the wave forces on flexible cylinder when using the relative velocity form of Morison’s equation.

The second method that can be used to modify the Morison equation to include the effects of wave-structure interaction is the independent flow fields formulation described by Laya et al.(1984). This method considers two independent flow situations, the first being a far field flow in which the fluid force is given by the original form of the Morison equation and arises from the wave force acting on a stationary structure. The second is a near field flow caused by the structure oscillating in an otherwise quiescent fluid. In this case the fluid force is again given by the Morison equation written in terms of the structure’s velocity and acceleration. Laya et al.(1984) discuss the relative merits of each of these two formulations for wave-structure interaction and qualitatively suggest their respective range of applicability.

In this dissertation the relative velocity form of Morison’s equation is used.

2.2.4 Inclined Cylinders

Many of the structural members of offshore platforms are not vertical but are at arbitrary orientations and thus Morison’s equation as defined in equation (2.1) cannot be used. The effect of pile inclination in the plane of the flow on wave induced forces has been
the subject of much research in steady flows. Studies by Hoerner (1965) and Smith et al (1972), amongst others, have found that the steady flow problem is a very complex one to solve. Thus it is generally recognised that the problem of determining the forces due to wave motion on oblique cylinders is even more difficult.

Wade and Dwyer (1976) examined four methods, each of which was generally considered appropriate, for calculating wave forces on arbitrarily oriented cylinders. Each of the four methods are summarised below.

The first approach is known as the projected area method and assumes that the resultant drag pressure acts on an area projected on a plane normal to the total normal water-particle velocity and that the resultant inertia pressure acts on an area projected normal to the total water-particle acceleration.

Secondly, the resultant drag and inertia pressures were postulated as being resolved into components normal and tangential to the member axis and the tangential component ignored.

The third method assumes that the resultant water-particle velocities and accelerations are resolved into components normal and tangential to the axis of the member and the tangential components ignored.

Finally, the last method assumes that the resultant water-particle velocities and accelerations act normal to the axis of the member, which is generally not considered to be a very valid assumption.

Wade and Dwyer (1976) carried out tests on two test structures and applied the above methodologies using constant values of $C_m$ and $C_d$ throughout each test. During the tests the structure base shear values were calculated using each of the four approaches. The results of the tests showed that each of the methods agreed to within about 12% of each other.
Probably the most consistent approach is the third method above; this approach is widely used in industry, Chakrabarti (1994), and is adopted in this dissertation. Chakrabarti et al (1977) conducted tests in a wave tank and measured normal and transverse forces on inclined cylinders. From these tests it was concluded that values of $C_m$ are virtually unaffected by angle of inclination, whereas values of $C_d$ appear to increase slightly with angle.

2.3 SPECTRAL RESPONSE MODELS

2.3.1 Introduction

During a particular storm, ocean wave heights can generally be characterised as being random in nature, thus the resulting forces exerted by the waves on offshore structures are also random in nature. Also, the random wave heights for a particular storm are considered to be stationary and ergodic, Kinsman (1965) and usually conform closely to a Gaussian distribution. Time-varying spectral analysis associated with nonstationary random processes, as discussed by researchers such as Liu (1972), were not considered during the course of this study. If the relationship between incident wave heights and structural responses are considered to be linear then the distribution of structural responses can also be considered to be Gaussian, Robson (1963) and Bendat and Piersol (1971). In order to compute the probability density of a Gaussian response it is necessary to know only the variance and the mean value of the response. Linear spectral techniques have been used extensively to compute response spectra of displacements and stresses of offshore structures; Malhotra and Penzien (1970) developed a spectral response model for jacket platforms and they applied it to several offshore towers demonstrating its usefulness. From these response spectra the response distributions and probability density functions can be easily obtained. Nath and Harleman (1970) showed
good correlation between measurements and analysis for a one degree of freedom linear system.

The relationship between wave heights and structural responses, however, are not linear for a number of reasons as outlined in Chapter 1. In particular, Morison's equation is a quadratic function of water particle velocities and may significantly effect response distributions. Until fairly recently, many offshore structures involved in oil and gas developments were of the jacket platform type and were deployed in relatively shallow water depths, 30-100m. Consider for example Kinsale Head Production Platform Alpha operated by Marathon, this structure is located in approximately 100m of water. From a structural monitoring study of the platform, Brennan (1983), it was ascertained that the fundamental natural frequency of the platform is about 1.7Hz. This frequency is much higher than likely frequencies of incident waves and thus resonance is unlikely; for example, the peak frequency during a storm recorded during the monitoring study was about 0.2Hz. For stiff platforms such as Kinsale Alpha, it has been common to perform spectral analyses by linearising Morison's equation in some manner and using a linear wave theory to compute response spectra, Malhotra and Penzien (1970), Barltrop and Adams (1991), WS Atkins (1996) and Chakrabarti (1994). This method has been found to give results for stiff, jacket platforms and has been recommended by the American Petroleum Institute (1993) for spectral analyses of such platforms.

Over the past 15 years or so much attention has been focussed on the analysis and design of structures for deployment in deeper waters such as West of Shetlands and West of Ireland where water depths can reach 1000m. Various types of structures have been considered for use in locations including compliant towers and guyed towers. Typically deep water structures such as these have natural frequencies much lower than those in shallow water and dynamic effects become more significant when determining
responses. Wilson and Orgill (1984) illustrated how natural frequencies of offshore structures are functions of water depth and type of structure; Figure 2.7 compares typical frequencies of structures and sea-state. From this figure, it is clear that for jacket platforms in 300m of water the fundamental natural frequency is just above sea state peak frequencies, whereas guyed towers in this water depth have fundamental natural frequencies that are just below sea state peak frequencies. In both of the above situations it is expected that dynamic responses would contribute significantly to structural responses. When fundamental natural frequencies are close to peak sea-state frequencies effects due to non-linear drag forces may become important. Deterministic models including non-linear effects are well established, Selna and Cho (1972). However, if non-linear drag forces must be included in frequency-domain analysis then response spectra cannot be computed using linear spectral theory. Several approaches have been developed to consider including these effects as detailed below.

When non-linearities have significant effects on structural responses then the distribution of the responses may not be adequately described by a Gaussian distribution. Pierson and Holmes (1965) developed a distribution, which bears their names, to describe wave force distributions based on including the effects of the non-linear drag force terms. The work done by Pierson and Holmes showed that the force distributions are not accurately modelled by Gaussian distributions, particularly for storms of higher significant wave heights. They also showed that responses actually follow the Pierson-Holmes distribution rather than the often used Gaussian distribution. Lipsett (1985) used Edgeworth series to develop probability distributions associated with structural response when non-linear effects are significant. Difficulties associated with determining either the Pierson-Holmes distribution or the distribution developed by Lipsett is that the third and fourth moments, skewness and kurtosis respectively, of the distributions are required
to define them. The computation of these moments are often very time consuming and thus make them impracticable for many realistic engineering problems.

2.3.2 Linear Spectral Response Models

Many engineering problems pertaining to random vibration processes include non-linearities and the solutions to these problems have generated considerable interest amongst the research community, Vanmarke (1976) and Davenport and Novak (1976). Non-linearities generally render problems of a stochastic nature difficult or impossible to solve directly and thus the problems are often transformed in some manner to a more desirable form. One of the most commonly applied methods of dealing with non-linearities is statistical linearisation, Roberts and Spanos (1990). Statistical linearisation involves replacing the original set of non-linear differential equations by an equivalent set of linear equations and the difference between the two sets is minimised in some appropriate manner. The technique has been extensively used to study deterministic non-linear problems for many years and was first applied to stochastic problems by Booton et al (1953). Caughey (1963) developed an approach for using statistical linearisation to solve non-linear stochastic problems in structural dynamics and since then it has found widespread use in this field. The method is now commonly referred to as statistical linearisation, stochastic linearisation or equivalent linearisation.

Statistical linearisation is a very versatile technique in that it can easily cope with multi-degree of freedom systems, non-white noise excitations and can be generalised to deal with non-stationary excitations and responses. The method has found widespread use in engineering, for example it has been applied to sloshing of liquids in a tank due to earthquake motions, Sakata et al (1984) and to non-linear soil-structure interactions during dynamic response of buildings.
The non-linearity of concern in this study is that due to the drag force term of Morison’s equation. This non-linear effect makes it difficult to determine the probability distribution of structural responses. Borgman (1965a, 1965b, 1969), in his seminal reports and papers, developed a statistical theory for hydrodynamic forces on submerged piles and applied statistical linearisation to the non-linear Morison equation. From his research he developed an equivalently linearised version of Morison’s equation. Borgman assumed that the water-particle velocities in a random sea, in the absence of an ambient current, could be described by a zero mean Gaussian distribution; linear wave theory was used to derive water-particle kinematics from wave heights. Ignoring relative velocity, the non-linear drag force term considered by Borgman is given by

\[ F(t) = c \, V|V| + k \, \ddot{V} \]  \hspace{1cm} (2.3)

Borgman (1965a) derived the theoretical covariance function for \( F(t) \) using ensemble averaging and a Gaussian random wave model to be

\[ R_{FF}(\tau) = c^2 \, \sigma_v^4 \, G( \, R_{vv}(\tau) / \sigma_v^2 \, ) + k^2 \, R_{aa}(\tau) \]  \hspace{1cm} (2.4)

where

\[ G(r) = \frac{\left[ (2+4r^2) \arcsin (r) + 6r \, (1-r^2)^{1/2} \right]}{\pi} \]

- \( R_{FF}(\tau) \) is the covariance of \( F(t) \)
- \( R_{vv}(\tau) \) is the covariance the water particle velocities
- \( R_{aa}(\tau) \) is the covariance of the water particle accelerations
- \( \sigma_v \) is the standard deviation of water particle velocity
- \( c \) and \( k \) are Morison coefficients
Borgman then used the following series approximation to the function \( G(r) \)

\[
G(r) = \frac{1}{\pi} \left[ 8r + 4r^{3/3} + r^{5/15} + r^{7/70} + 5r^{9/1008} + \ldots \right]
\]

By retaining the first term in the above approximation Borgman then developed the following linearised version of Morison’s equation

\[
F^*(t) = c \sigma \left( \frac{8}{\pi} \right)^{1/2} V(t) + k A(t)
\] (2.5)

The above expression has found widespread use in the analysis and design of offshore structures, however, it does not include relative motion effects between water particles and structure. Grecco and Hudspeth (1983) developed a linearised frequency domain analysis model for two-dimensional structures incorporating relative motions. In their approach the modified Morison equation was linearised in a time-average, mean square sense, a technique which is extensively used in statistical linearisation for various engineering applications, Roberts and Spanos (1990). In the research carried out by Grecco and Hudspeth, developed for vertical members, the linearised model is very similar to that developed by Borgman and again is widely used for platform analysis. When relative motions are included, added mass and damping matrices are generated from the inertia and drag terms respectively of Morison’s equation. Grecco and Hudspeth used modal superposition to decouple the general equations of motion and developed an approach to decouple the generalised added damping matrix.

A number of approaches have been developed to compute response spectra of displacements for a given storm using a linearised version of Morison’s equation. One approach is to relate response spectra to spectra and cross-spectra of forces using a complex frequency response function as described by Grecco and Hudspeth (1983). However, a more efficient approach is to relate response spectra to sea state spectra
directly using the unit wave approach as outlined by Barltrop and Adams (1991). The unit wave methodology consists of subjecting the platform to a series of unit amplitude complex waves over different frequencies and calculating the receptances of the structure due to these waves. Response spectra are then easily obtained from the receptances and the spectrum of sea-state using linear spectral theory, Robson (1963).

Varying degrees of sophistication have been applied to the development of structural response models. Some of the most commonly types of models that have been used are single degree of freedom models, lumped node models, Angelides (1978), and consistent techniques using the finite element method, Barltrop and Adams (1991). Finite element models are probably the most commonly used type of response model in recent years and have been found to produce good results; no North Sea jacket platform has yet failed due to single excursion wave loading. ASAS as developed by WS Atkins (1996) is one such model that has been extensively used for analysis of North Sea jackets. This model employs the unit wave approach outlined above to compute responses and can also be used to perform fatigue analysis.

The drawback to any linearization method is that certain features of the nonlinear solution are not reproduced by the linear solution. Specifically, the probability distribution of the response of a nonlinear system will in general be non-Gaussian even though the forcing is Gaussian. The departure from a Gaussian distribution is most evident in the tails of the distribution, making the accurate prediction of extreme events of the nonlinear process very difficult. Also, the response spectral density of the nonlinear system may have multiple peaks, corresponding to super-harmonic and sub-harmonic effects, Li (1998), whereas the spectral density of the corresponding linear system may only have a single peak. In spite of these shortcomings the method of equivalent linearization has proved to be a very powerful method to solve randomly
forced nonlinear differential equations. Specifically, the mean square value predicted by the method of equivalent linearization has been found to be very accurate, even for cases of large nonlinearity.

2.3.3 Non-Linear Response Models

The response spectrum obtained when employing linear spectral theory only spans the same frequency range as the excitation spectrum. The existence of sub-harmonics and super-harmonics, however, in response spectra due to Morison loading has been confirmed both analytically by Liaw (1987) and experimentally by Borwick and Herbert (1988). Furthermore, when equivalent linearisation is employed then, if the sea state has a Gaussian distribution, the distribution of response is also considered to be Gaussian, but it is known that the response is in fact not Gaussian. Thus, equivalent linearisation cannot be used to predict the non-Gaussian distribution of response and hence non-linear response models must be used for this.

The subject of randomly forced nonlinear differential equations has had a very active research period beginning in the early 1960's. Reviews of the subject have periodically appeared in the literature, for example: Crandall (1963), Caughey (1971), Spanos (1980), Roberts (1981), Crandall and Zhu (1983), and Roberts (1984a, 1984b). In addition, the books by Soong (1973) and Nigam (1983) also treat the problem of nonlinear differential equations. More recently Li (1998) discusses non-linear stochastic responses of offshore platforms and the statistical distributions of the responses. Some of the more commonly used approaches are briefly reviewed below.

The moment closure method is used so that appropriate moments and cumulants of a distribution can be computed to allow the distribution to be defined. In non-linear systems moments are usually defined by an infinite hierarchy of coupled equations. In
order to solve this problem it is necessary to introduce a 'closure approximation', which involves retaining cumulants to a particular order. One of the difficulties of this system is that the moment equations become quite complex when the order of the closure increases. Results have shown that significant improvements in distribution accuracy can be obtained by progressing beyond simple Gaussian closure, Lin and Wu (1984). The disadvantage of the moment closure method is that the complexity of the moment equations dramatically increases as the order of closure increases, Li (1998).

Caughey (1986) developed an approach whereby the original set of non-linear differential equations are replaced by an equivalent set of non-linear differential equations which can be solved exactly. The main problem with this approach is that there are very few sets of non-linear equations that can be solved and hence the method is quite restricted in its usefulness.

Li (1998) discusses methods whereby the original non-linearity is replaced by an equivalent polynomial expansion and the resulting equivalent non-linear system is solved in the frequency domain using Volterra series as described by Schetzen (1980). Li states that equivalent statistical 'quadratisation' as developed by Donley and Spanos (1990) reduces to equivalent linearisation in the absence of a current and thus is of limited use. Li then proposed and developed a higher-order method termed statistical 'cubicisation' as an improvement over the equivalent statistical 'quadratisation' approach.

The perturbation technique is often used when system non-linearities are relatively small. The approach has been used extensively in deterministic analysis, Stoker (1950), and was generalised to the case of stochastic analysis by Crandall (1963). Basically this method is the extension to the randomly forced case of the well-known regular perturbation method. As with most perturbation methods, the nonlinear equation is transformed into a set of recursive linear equations that are then solved by a standard
method such as spectral analysis. Practically, the computations become very involved for all but the first or second terms in the perturbation series. Also, usually only the moments of the response can be found directly using this method, with only the lower order moments being easily found.

Monte Carlo simulation is used to generate a set of pseudo-random numbers belonging to a population with a specified probability density function. When applied to the analysis of an offshore structure the process starts by simulating a possible realisation of a random sea surface. The criterion that is used to generate this realisation is the wave height spectra often used in design. Spanos (1983) reviewed the techniques that can be used for this simulation process. The fluid velocity and acceleration are then calculated using linear wave theory and the equations of motion are numerically integrated using the simulated velocity and acceleration records in the Morison equation to obtain the time history of the response. The main difficulty with simulation techniques is that long computer runs are necessary so that the statistical characteristics associated with the time histories of responses are close to those of the actual responses.

A totally different approach to the problem can be taken when the random forcing can be assumed to be equivalent to white noise. Then it can be shown that the response is a Markov vector process for which the response probability can be completely described by a transition probability density and an initial condition. An equation for the transition probability density called the Fokker-Planck-Kolmogorov equation, FPKE, can be found from the original equation, Caughey (1963). Once a solution of the FPKE has been obtained the response statistics of interest can be found in a straightforward fashion. The FPKE is a second order linear partial differential equation for the type of equations of motion considered here. Note that the problem has changed from trying to solve a nonlinear ordinary differential equation to trying to solve a linear partial differential
equation, both being quite difficult. Unfortunately there are only a few known solutions to the FPKE in a few special cases. Even the stationary FPKE, whose solution yields the stationary probability density, has only a few known solutions and approximate methods are often required in order to find a solution. Nigam (1983) reviews some of the methods that have been proposed to find solutions to the FPKE.

As mentioned above the FPKE approach can only be applied when the forcing can be considered to be white noise. White noise is a physically unrealistic model for natural processes because it implies infinite energy for the process. However, in the case of a wide banded process applied to a lightly damped system, the assumption of white noise as a model for the forcing results in very realistic solutions. The usefulness of white noise as a model for real processes in this case may be considered similar to the usefulness of the concept of ideal point loads in the theory of elasticity. If a natural process is not wide banded, as is the case of a random sea state, then the FPKE approach can still be used if an enlarged system of equations is used. The white noise process is first passed through an auxiliary system which shapes the input to the real system. The disadvantage here is that the resulting FPKE is even more difficult to solve. The class of non-linear random vibration problems for which the FPK equation can be solved is quite limited, Li (1998), and restricted usually to a one degree of freedom system, Roberts and Spanos (1990). Thus the application of the Markov method is quite limited.

Since the 1970’s offshore engineers have applied some of the above techniques to non-linear spectral response analysis of offshore structures. A summary of some of the relevant applications is presented below.

Moe (1977) was one of the first to investigate the effects of incorporating the non-linear drag terms on response spectra of jacket platforms. Moe did not carry out detailed modelling of a structure but instead qualitatively considered the magnitudes of the
contributions of linear and non-linear terms in the force spectra at resonance. Moe considered, firstly, a single cantilever and deduced that for a single, vertical and uniform pile of 1m diameter that non-linear effects are not very significant because the fundamental natural frequency of the pile is sufficiently removed from the sea state peak spectral frequency. However, when Moe analysed a more realistic structure having spatial extent, such as a portal frame, he concluded that the non-linear term may become the largest when the spatial correlation effects are included. This is because for certain leg spacings linear forces on different legs will cancel each other out, but the forces due to the non-linear force may be almost fully correlated. Thus the non-linear forces may be the dominant forces for a given frequency of surface wave and will have an associated frequency of three times the wave frequency.

Eatock-Taylor and Rajagopalan (1982) and Rajagopalan and Eatock-Taylor (1982) describe approaches they developed to perform non-linear spectral analysis of an offshore tower using a perturbation technique and using stochastic averaging. In the approach outlined in Eatock-Taylor and Rajagopalan (1982) they included the non-linear drag effects using spectral convolutions as suggested by Moe (1977). Eatock-Taylor and Rajagopalan’s model was based on the modified Morison equation on a dynamically responsive structure. In this model the drag term was approximated to give a time-varying hydrodynamic damping term based on the assumption that the structural velocities are much less than the water-particle velocities; this is a reasonable assumption for jacket platforms. The structure Eatock-Taylor and Rajagopalan analysed was a plane frame jacket reduced to a lumped mass model, as presented in Figure 2.9. The main findings from this study are: (a) the drag force term can lead to significant excitation at three times the wave frequency, particularly at resonance (b) the relative velocity term leads to a significant source of hydrodynamic damping, and (c) the influence of the non-
linear drag force term is smaller at lower sea states. Eatock-Taylor and Rajagopalan conclude by stating that the superharmonic resonance effects in drag dominated compliant structures is unknown and warrants investigation.

Sigbjørnsson and Morch (1982) developed a method for non-linear stochastic response analysis of offshore towers applying the hierarchy closure technique considering second-order statistics of wave forces and structural responses. One of the important aspects of this work is that they showed that the autospectral density of horizontal wave force acting on a single vertical pile is comprised of the following contributions: approximately 32% inertia force, 50% linear drag force and 9% non-linear drag force. Thus they postulated that the stochastic linearisation technique fails to give a good representation of the power spectral densities of wave forces due to possible secondary peaks induced by superharmonics present in the drag forces.

Sigbjørnsson and Morch (1982) applied their model to a cantilever and used a version of Morison's relative velocity equation by assuming that the water particle velocities are generally greater than structural velocities, similar to the approach adopted by Eatock-Taylor and Rajagopalan. The closure scheme used was the correlation discard approximation, Soong (1973), which approximates expectations of products of random variables by the products of the individual expectations. A comparison of computed linear and non-linear response spectra for the top of the cantilever suggested that non-linear effects contribute only to increased displacements at the fundamental natural frequency of the cantilever; this finding agrees with one of the primary conclusions of the work by Eatock-Taylor and Rajagopalan.

Lipsett (1985, 1986a, 1986b) considered the dynamics of a flexible offshore structure in both regular and random seas using linearisation and perturbation techniques. Lipsett used a multiple-scale perturbation technique which allowed him to model the relatively
velocity form of the drag force term without approximating it, as was done by Eatock-Taylor and Rajagopalan. Using these approaches he developed response models of single degree of freedom systems. Comparing results obtained for random seas obtained by a numerical simulation method with the results of a perturbation method and the equivalent linearisation method Lipsett showed that the perturbation method gives a better estimate of the mean square value of the response than does the method of equivalent linearisation. Lipsett’s results also show that the non-linear effects mainly occur at resonant frequencies, as found by the other researchers above. One of the specific recommendations for further research by Lipsett (1985) was that the perturbation method should be extended so that it could be applied to a more realistic multi-degree of freedom structure.

Isaacson’s (1985) review of non-linear wave effects on slender offshore structures considers the perturbation techniques of Eatock-Taylor and Rajagopalan and Lipsett and suggests that perturbation solutions to Morison’s modified equation may be viable alternatives to time-consuming simulation approaches. In this review, no other applications of perturbation techniques to solve Morison’s equation are referred to. Jain and Datta (1987) present a technique for non-linear frequency domain analysis of offshore towers and compare the results with linear analysis. However, the technique uses the Newton-Raphson method to solve for displacement amplitudes for different frequencies and does not show how response spectra are related to sea state spectra. Results of their analysis suggest that non-linear response spectra of displacements have lower peaks at resonance than linear response spectra.

Borgman (1982) suggested that Volterra kernels may have good potential for solving non-linear frequency domain problems associated with Morison’s equation. Olagnon et al (1988) developed a model to analyse a single vertical cylinder subjected to
unidirectional wave and current loadings using Volterra integrals up to order 3. Good results were obtained against measurements of spectra of structural acceleration and strain. One of the main conclusions drawn by Olagnon et al was, however, that the complexity of the computations may restrict their use to simple structures.

Armand et al (1991) carried out a detailed review of stochastic modelling with particular emphasis on modelling non-linear systems between sea state and displacement. They found that most non-linear models were developed and applied to single degree of freedom systems and were aware that responses become much more difficult to compute and interpret when multi-degree of freedom systems are developed. They pointed out that as a result of this designers are confronted with the dilemma of choosing between a sophisticated linear model and a crude non-linear model, realising that the crude non-linear model may predict phenomena that could not be predicted by the linear model.

Li (1998) developed an equivalent non-linear model of the drag force using statistical cubicisation and solved the problem in the frequency domain using Volterra series. He developed numerical models and applied them to single degree of freedom systems of stiff and flexible platforms; he then uses the results of these models to carry out fatigue analyses of the structures and assess the non-linear effects on fatigue calculations. Li concludes that statistical cubicisation is an improvement over statistical linearisation and quadratization when compared with results from simulations and that the main non-linear effects occur at resonant frequencies. He also states that for low wave states non-linear effects are not significant and they, therefore, do not affect fatigue calculations. One of the main recommendations for further work suggested by Li is that the method he developed be extended to multi-degree of freedom systems, acknowledging that multi-degree of freedom systems should be considered.
2.4 CONCLUSIONS

The above sections detail some of the research that has been undertaken to develop linear and non-linear spectral response models for offshore platforms. There is obviously considerable scope for further research in this area, such as considering the non-linearities of various wave theories. The author has extended the above research by developing a linear spectral response model and a non-linear spectral model as summarised below.

Linearised versions of Morison's equation have been widely used in frequency domain models for analysing offshore jacket platforms that are located in relatively shallow waters. The application of such models to real structures with thousands of degrees of freedom, however, can be quite time consuming; thus in this thesis an accurate and computationally efficient linearised model has been developed. The unit wave method was applied in the development of this model in a novel manner that reduces the number of computations required to calculate response spectra by about half. Also, during the development of this model a novel development for the linearised drag coefficient for inclined members is presented which further reduces computational times. The model is formulated within the finite element framework and is validated against results from an industry standard computer code and measured data from a large gas production platform.

As offshore oil production moves into deeper water, there are increasingly more fixed and compliant platforms with fundamental frequencies close to two or three times the peak frequency of wave excitation, for these platforms analysis using equivalent linearisation may significantly underestimate spectral responses, Li (1998). The nonlinear models discussed in Section 2.3 above have been developed for either single degree of freedom systems or for cantilever models of offshore structures and many
commentators have suggested that more realistic structural response models should be developed. The author has extended the technique proposed by Eatock-Taylor and Rajagopalan (1981) so that it can be applied to multi-degree of freedom structures that are spatially extended, in this case portal frames; however, the method can be applied to structures that are spatially extended in three-dimensions. It is well-known that leg spacings have phase effects on the responses of offshore jacket structures, these effects have thus far not been considered in any of the above non-linear model developments. One of the main objectives of this development is to ascertain if phasing effects due to leg spacing are important processes when determining non-linear response spectra. Both Eatock-Taylor (1997) and Lipsett (1997) suggest that non-linear spectral response models should be developed to consider this.
Figure 2.1 - Cd versus K for various values of the frequency parameter (Sarpkaya 1976a)
Figure 2.2 - $C_m$ versus $K$ for various values of the frequency parameter
(Sarpkaya 1976a)
Figure 2.3 - Cd versus Reynolds number for various values of K
(Sarpkaya 1976a)
Figure 2.4 - $C_m$ versus Reynolds number for various values of $K$
(Sarpkaya 1976a)
Figure 2.5 - Cd versus Reynolds number for various rough cylinders, K=20 (Sarpkaya 1976a)
Figure 2.6 - Cm versus Reynolds number for various rough cylinders, K=20
(Sarpkaya 1976a)
Figure 2.7 - Drag and inertia coefficients as a function of the roughness Reynolds number
(Sarpkaya 1976b)
Figure 2.8 - Relationship between Structural and Sea State frequencies
(Wilson and Orgill 1984)
Figure 2.9 - Reduction of Space Frame to Lumped Mass Model (Eatock-Taylor and Rajagopalan 1981)
CHAPTER 3

DEVELOPMENT OF A LINEAR SPECTRAL MODEL

3.1 Introduction

The development of a model to predict spectra of structural response of offshore lattice platforms due to know spectra of sea states incident on platforms using linear spectral theory is presented in this chapter. The structure-water system is modelled using the finite element method based on the virtual work formulation and is developed in a general-purpose manner so that most structural configurations can be modelled. The structural members are modelled using three-dimensional beam elements having six degrees of freedom at each node. Using this approach the coefficient matrices of the system, namely mass, damping and stiffness, are represented in a consistent rather than in a lumped manner. Modal damping is employed to represent structural damping.

The wave forces on submerged structural members are computed using the modified Morison’s equation which considers both the relative motion between water particles and the structure and arbitrary spatial orientation of a structural member. Linear Airy wave theory is employed to calculate water particle velocities and accelerations beneath surface gravity waves. The nonlinear drag force term in Morison’s equation is linearised in an equivalent manner and a new expression for the drag coefficient is developed. Relevant added mass and added damping terms form Morison’s equation are incorporated in the element mass and damping matrices. Element coefficient matrices are assembled into global coefficient matrices to develop the governing equations of motion for the system. The undamped free vibration problem is solved to compute the eigenvalues and
eigenvectors of the system using subspace iteration. The orthogonality properties of the eigenvectors are used to decouple the global governing equations of motion in which the hydrodynamic damping is decoupled in an equivalent manner. The decoupled equations are then solved for the receptances of the system due to the application of a series of unit amplitude sinusoidal surface gravity waves. The response spectra of displacements are obtained from the input spectrum of sea state and receptance functions.

3.2 Finite Element Formulations

The dominant time-varying forces due to waves acting on submerged elements of jacket structures act normal to the longitudinal axes of the elements, thus the structural members are modelled as three-dimensional beam elements. Because such structures are designed as lattice space frames shear deformations are ignored with bending deformations most significant. Typical structural members are hollow cylinders and a schematic diagram of such a member is shown below in Figure 3.1; this figure also defines the local coordinate system of a member.

The local coordinate system of the beam is defined by the rectangular axis system X, Y, and Z, translational displacements in these directions are given respectively by $u$, $v$, and $w$ and rotational displacement about the X axis is given by $\gamma_x$. The point j above lies on the neutral axis of the beam and it's displacements are represented by $(u, v, w, \gamma_x)$; the objective of this development is to determine displacement spectra of points such as j. Rotations about the Y and Z axes respectively are $\gamma_y$ and $\gamma_z$. 
In Figure 3.1, Section A–A shows a steel hollow section which is filled with water and on which marine growth is attached, both of these affect structural responses and are included in the following development.

The finite element method is used to determine the governing equations of motion of such members as shown in Figure 3.1 subjected to wave loading. The displacement-based method is used in the finite element formulation by employing the following expression for the principle of virtual work, Bathe (1996).

\[
\int_{Vol}^{\bar{T}} \varepsilon \tau dVol = \int \bar{U}^T \bar{f}^B dx
\]  

(3.1)

where

\(\bar{\varepsilon}\) is the vector of virtual strains
\(\tau\) is the vector of actual stresses
Vol is the volume of the beam
\(\bar{U}\) is the vector of virtual displacements
\(\bar{f}^B\) is the vector of actual applied forces
\(l\) is the length of the beam

The strains considered in this analysis are: axial strain in the X-direction, \(\varepsilon_{xx}\), due to axial deformation and bending about the Y and Z axes and the shear strains \(\varepsilon_{yx}\) and \(\varepsilon_{zz}\) due to torsion of the beam about the longitudinal axis. Using elasticity theory these strains at a point such as \(j\) can be expressed as follows, Dym and Shames (1973):

\[
\varepsilon_{xx} = \frac{du(x)}{dx} - z \frac{d^2w(x)}{dx^2} - y \frac{d^2v(x)}{dx^2}
\]

(3.2)
Thus, the vector of strains required to describe this problem is:

\[
\epsilon = \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yx} \\ \epsilon_{yy} \end{bmatrix}
\] (3.5)

Considering only linear elastic isotropic material, the stress vector is related to the strain vector as follows, Timoshenko and Goodier (1970):

\[
\tau = [C] \epsilon
\] (3.6)

where

\[
[C] = \begin{bmatrix} E & 0 & 0 \\ 0 & 4G & 0 \\ 0 & 0 & 4G \end{bmatrix}
\] (3.7)

\[
E = \text{Modulus of Elasticity}
\]

\[
G = \frac{E}{2(1 + \mu)}
\]

\[
\mu = \text{Poisson’s ratio}
\]

The finite element procedure as detailed by Bathe (1996) is employed to develop the governing equations of motion using the following displacement functions for the neutral axis of a beam element as prescribed by Brebbia and Walker (1979):

\[
u(x) = \alpha_1 + \alpha_2 x
\] (3.8)

\[
v(x) = \alpha_3 + \alpha_4 x + \alpha_5 x^2 + \alpha_6 x^3
\] (3.9)

\[
w(x) = \alpha_7 + \alpha_8 x + \alpha_9 x^2 + \alpha_{10} x^3
\] (3.10)

\[
\gamma_x (x) = \alpha_{11} + \alpha_{12} x
\] (3.11)
where

the $\alpha$'s are generalised coordinates

$x$ represents location along the $X$ axis of the beam

The rotations of the neutral axis at a point $j$ about the $Y$ and $Z$ axes respectively are obtained from the above displacements as follows:

$$\gamma_y(x) = -\frac{dw(x)}{dx} \quad (3.12)$$

$$\gamma_z(x) = \frac{dv(x)}{dx} \quad (3.13)$$

Equations (3.8) - (3.13) may be expressed in matrix form as

$$U = [Q]\alpha \quad (3.14)$$

where

$U$ is the vector of displacement

$[Q]$ is the matrix of coordinate variables

$\alpha$ is the vector of generalised coordinates

Thus the nodal displacements for a beam element may be expressed as

$$\begin{bmatrix} U^1 \\ U^2 \end{bmatrix} = [A]\alpha \quad (3.15)$$

where

$U^1$ is the vector of displacements at node 1

$U^2$ is the vector of displacements at node 2

$[A]$ is the matrix of nodal coordinate variables
\( \alpha \) is the vector of generalised coordinates

From equation (3.15) we can express the vector of generalised coordinates in terms of the vector of nodal displacements as:

\[
\alpha = [A]^{-1} \begin{bmatrix} U^1 \\ U^2 \end{bmatrix} \tag{3.16}
\]

Thus, from equations (3.14) and (3.16) the displacements at any point \( j \) along the element may be written in terms of nodal displacements as:

\[
U = [Q][A]^{-1} \begin{bmatrix} U^1 \\ U^2 \end{bmatrix} = [G] \begin{bmatrix} U^1 \\ U^2 \end{bmatrix} \tag{3.17}
\]

where 

\([G]\) is the matrix of shape functions, which are detailed in Appendix I.

Using the relationship between strains and displacements, equations (3.2) - (3.4), and equation (3.17), an expression relation strains to nodal displacements may be written as follows:

\[
\varepsilon = [B] \begin{bmatrix} U^1 \\ U^2 \end{bmatrix} \tag{3.18}
\]

where the entries in matrix \([B]\) are relevant spatial gradients of shape functions.
Because the external forces on the beam elements induce structural accelerations, the virtual work equation (3.1) must be extended to incorporate d'Alembert's principle as follows:

\[
\int_{\text{Vol}} \varepsilon^T \tau d\text{Vol} = \int \bar{U}^T \bar{f}^B dx - \int_{\text{Vol}} \bar{U}^T \rho_s \begin{bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{w} \\ \ddot{\gamma}_x \end{bmatrix} d\text{Vol}^S
\]

(3.19)

where

\( \rho_s \) is the density of the steel of the beam

\( r_s \) is the radius of gyration of the cross-section of the beam

\( \ddot{u} \) is the translational acceleration in the X direction of point j

\( \ddot{v} \) is the translational acceleration in the Y direction of point j

\( \ddot{w} \) is the translational acceleration in the Z direction of point j

\( \ddot{\gamma}_x \) is the rotational acceleration of point j about the X axis

\( \text{Vol}^S \) is the volume of steel

Re-writing equation (3.19) in terms of nodal displacements and accelerations using equations (3.6), (3.17) and (3.18), and then re-arranging, we can write the following expression:

\[
\begin{bmatrix} \bar{U}_1^T \\ \bar{U}_2^T \end{bmatrix} \int_{\text{Vol}} \bar{f}^B [B]^T [C][B] d\text{Vol}^S \begin{bmatrix} \bar{U}_1^T \\ \bar{U}_2^T \end{bmatrix} + \begin{bmatrix} \bar{U}_1^T \\ \bar{U}_2^T \end{bmatrix} \int_{\text{Vol}} \rho_s [G]^T [G] d\text{Vol}^S \begin{bmatrix} \bar{U}_1^T \\ \bar{U}_2^T \end{bmatrix} = \begin{bmatrix} \bar{U}_1^T \\ \bar{U}_2^T \end{bmatrix} \int^T [G]^T f^B d\text{Vol}^S
\]

(3.20)
The virtual displacements \( \begin{bmatrix} \mathbf{U}^1 \\ \mathbf{U}^2 \end{bmatrix} \) are arbitrary, thus equation (3.20) may be re-written as:

\[
[M]^n \ddot{\mathbf{U}}^n + [K] \mathbf{U}^n = \int [G]^T \mathbf{f}^B dx
\]

Equation (3.21)

where

\( \mathbf{U}^n \) is the vector of beam nodal displacements

\( \ddot{\mathbf{U}}^n \) is the vector of beam nodal accelerations

\([M]^n\) is the consistent mass matrix of the steel beam

\([K]\) is the consistent stiffness matrix of the beam

When beam elements such as those shown in Figure 3.1 accelerate the additional inertial forces due to the entrained water and marine growth should be included in the analysis. These effects are included in the model developed herein by including relevant terms in the virtual work expression. The virtual work equation including these inertia effects is now given as

\[
[M] \ddot{\mathbf{U}}^n + [K] \mathbf{U}^n = \int [G]^T \mathbf{f}^B dx
\]

where

\([M]\) is the consistent mass matrix of the beam-water-growth element.

Assuming that structural damping is viscous in nature the general finite element formulation for the beam can be written as:

\[
[M] \ddot{\mathbf{U}}^n + [C] \dot{\mathbf{U}}^n + [K] \mathbf{U}^n = \int [G]^T \mathbf{f}^B dx
\]

Equation (3.22)

where

\([C]\) is the damping matrix of the beam.
\( \mathbf{U}^n \) is the vector of beam nodal velocities

The element mass matrices obtained by carrying out the integrations in equation (3.20) are also presented in Appendix II.

### 3.3 Wave Forces

In equation (3.22) the vector \( \mathbf{f}^b \) represents the externally applied forces distributed over the surface of a beam element due to long-crested gravity waves travelling in an arbitrary direction. As discussed in Chapter 2, a modified version of Morison's equation is used to evaluate the wave-induced forces on the arbitrarily oriented submerged members of jacket platforms incorporating relative motions between water and structure.

Consider a structure subjected to an incident surface gravity wave, then from Morison’s equation the force term on the right hand side of equation (3.22) is given by:

\[
\int \mathbf{[G]}^T \mathbf{f}^b \, dx = \int \left( \mathbf{f}^i + \mathbf{f}^d \right) \, dx
\]

where

- \( \mathbf{f}^i \) is the vector of inertia force at \( j \)
- \( \mathbf{f}^d \) is the vector of drag force at \( j \)

The drag force and inertia force terms in equation (3.23) are defined respectively as follows

\[
\mathbf{f}^i = \int \mathbf{[G]}^T \begin{bmatrix} \dot{v}_2 \\ \dot{v}_3 \\ 0 \end{bmatrix} \, dx + \int \mathbf{[G]}^T \begin{bmatrix} 0 \\ \dot{v}_2 - \ddot{v} \\ \dot{v}_3 - \ddot{w} \\ 0 \end{bmatrix} \, dx
\]
Equation (3.23) contains a non-linear relationship between wave forces and water particle velocities in the drag force term. A linear spectral model, which is developed in this chapter, derives a relationship between the input spectrum of sea state and the response spectrum of displacement; this model requires a linear relationship between sea state and structural displacements. In order to develop a linear spectral model equation (3.23) is linearised in an equivalent manner as follows.

Lin (1967) developed a general method for performing equivalent linearisation of non-linear equations, this approach is used to linearise the above drag force terms. Lin’s method is based upon the postulation of a linear equivalent of a non-linear system and
then minimising the square of the error term in some manner. Other researchers such as Malhotra and Penzien (1970) and Grecco and Hudspeth (1983) have applied this approach to the non-linear drag term of Morison's equation for lumped mass systems. The development outlined below and is applicable to any three-dimensional jacket structure using the consistent approach of the finite element method.

Consider the following components of the drag force vector:

\[ F_D = C_D \begin{bmatrix} (v_2 - \dot{v}) r \\ (v_3 - \dot{w}) r \end{bmatrix} \]  

(3.24)

where \( C_D \) is a drag coefficient.

The linearised drag force terms are postulated as:

\[ F_D = C_D^*(x) \begin{bmatrix} (v_2 - \dot{v}) \\ (v_3 - \dot{w}) \end{bmatrix} \]  

(3.25)

where \( C_D^*(x) \) is a revised drag coefficient due to linearisation and is a function of location along the longitudinal axis of a member. Borgman (1965b) developed an expression for \( C_D^*(x) \) for a simple vertical pile and ignored relative velocity. Grecco and Hudspeth (1983) developed an expression for \( C_D^*(x) \) a vertical pile incorporating relative velocity. Below an expression is developed for \( C_D^*(x) \) for a randomly oriented pile having two normal components of velocity incorporating relative velocity.
The error vector between the non-linear and linear drag force vectors at point j on a submerged member is obtained by subtracting equation (3.25) from equation (3.24):

\[ E = C_D \left[ (v_2 - \dot{v})|r| - C_D^*(x) \left( (v_2 - \dot{v}) \right) \right] - C_D^*(x) \left( (v_3 - \dot{w}) \right) \]  \hspace{1cm} (3.26)

Now, the square of the above error vector is given by

\[ (E)^2 = [C_D(v_2 - \dot{v})|r| - C_D^*(x)(v_2 - \dot{v})]^2 + [C_D(v_3 - \dot{w})|r| - C_D^*(x)(v_3 - \dot{w})]^2 \]  \hspace{1cm} (3.27)

In accordance with Lin's methodology, the square of the error term is now minimised in a mean square sense with respect to the revised drag coefficient. Thus, the square of the error vector is differentiated with respect to the revised drag coefficient to obtain the following

\[ \frac{d(E)^2}{dC_D^*} = -2[C_D(v_2 - \dot{v})|r| - C_D^*(x)(v_2 - \dot{v})](v_2 - \dot{v}) - 2[C_D(v_3 - \dot{w})|r| - C_D^*(x)(v_3 - \dot{w})](v_3 - \dot{w}) \] \hspace{1cm} (3.28)

It is easily shown that the second derivative of \((E)^2\) with respect to \(C_D^*(x)\) is positive ensuring that equation (3.28) does actually represent the minimum situation.

The revised drag coefficient is obtained by setting the time average of equation (3.28) to zero

\[ \left< \frac{d(E)^2}{dC_D^*(x)} \right> = 0 \] \hspace{1cm} (3.29)

where \(< >\) represents the expected value of a random function.
Equation (3.29) can be solved for $C^*_D(x)$ to give

$$C^*_D(x) = \frac{C_D \left[ \langle |r|^2 \rangle + \langle |r'|^2 \rangle \right]}{\langle (r_2)^2 \rangle + \langle (r_3)^2 \rangle}$$  \hspace{1cm} (3.30)$$

where

- $r_2$ is the relative velocity in the Y direction
- $r_3$ is the relative velocity in the Z direction

A similar expression to equation (3.30) was developed and applied by Grecco and Hudspeth (1983) for a one-dimensional problem. To apply this expression at a point j, the denominator terms are variances of the components of the relative velocities at j and are easily obtained by integrating the spectra of relative velocity components. The expected values in the numerator of equation (3.30) are not as easily computed as those in the denominator. To compute the numerator directly from the above expression the individual terms of the numerators have been computed from a two-dimensional Gaussian distribution as shown.

Let

$$x = r_2 \quad \text{and} \quad y = r_3$$

then if x and y are normal random variables both with zero means, then the joint probability distribution function of the two variables is given by, Papoulis (1965),
The expectation of a function of the two random variables with a probability distribution function \( p(x, y) \) is given by

\[
\langle f(x, y) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) p(x, y) \, dx \, dy \tag{3.32}
\]

where

\[
f(x, y) = \left( \sqrt{x^2 + y^2} \right)^2 \left( x^2 + y^2 \right)
\]

Equations (3.31) and (3.32) are used to compute the terms of the numerator of equation (3.30) using numerical integration and hence evaluate \( C^*_D(x) \). Langley (1984) using a similar approach obtained an expression analogous to equation (3.32) when linearising the drag force term when current was included and also used numerical integration of a double integral to compute the revised coefficient.

Although equation (3.32) is used to compute \( C^*_D(x) \) it is instructive to expand equation (3.30) as outlined below.

The numerator terms in equation (3.30) are determined as follows

\[
\langle \hat{r} \rangle \langle \hat{r}_2^2 \rangle + \langle \hat{r} \rangle \langle \hat{r}_2 \rangle^2 = \tag{3.33}
\]

\[
\langle \hat{r} \rangle \langle \hat{r}_2^2 \rangle + \langle \hat{r}_2 \rangle^2 = \langle \sqrt{\hat{r}_2^2 + \hat{r}_3^2} \rangle \langle \hat{r}_2 \rangle^2 + \langle \hat{r}_3^2 \rangle
\]

Now let the modulus of the relative velocity be written as
When employing Airy wave theory, it can be assumed that there is a linear relationship between wave height and relative velocity and since wave height can be described by a Gaussian distribution relative velocity can also be described by a Gaussian distribution. Let the probability density function of relative velocity at \( x \) be given by \( p_r(x) \), then from Papoulis (1965), the probability density function of the modulus of relative velocity at \( x \) is given by

\[
p_h(x) = 2 p_r(x) U(x)
\]

where \( U(x) \) is the unit step function.

Now equation (3.33) can be re-written as

\[
\langle H^3 \rangle = \int_{-\infty}^{\infty} H^3 p_h(H) dH
\]

\[
= \int H^3 2p_r(H) dH
\]

where

\[
p_r(H) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{H^2}{2\sigma^2}}
\]

where

\( \sigma \) is the standard deviation of the relative velocity.

Thus the expectation of equation (3.36) may be written as
Now let

\[ b = \frac{-1}{2\sigma^2} \]

Consider now the integral \( \int_0^\infty 2H^3 e^{bH^2} \, dH \)

let \( y = H^2 \)
then \( dy = 2H \, dH \)

Substituting for \( H^2 \) in the above integral and integrating from 0 - \( a^2 \), gives the following expression for the integral

\[ I = \int_0^{a^2} y e^{by} \, dy \]

The integral \( I \) is evaluated by integrating by parts as follows

let
\[ f = y \quad \text{and} \quad dg = e^{by} \, dy \]
then
\[ df = dy \quad \text{and} \quad g = \frac{1}{b} e^{by} \]

The integral can now be rewritten as

\[ \langle H^3 \rangle = \int_0^\infty \frac{2H^3}{\sigma\sqrt{2\pi}} e^{-\frac{H^2}{2\sigma^2}} \, dH \]

\[ = \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty 2H^3 e^{-\frac{H^2}{2\sigma^2}} \, dH \]

(3.38)
\[
\int_{-a}^{a} ye^{-by} dy = \frac{1}{b} \left( ye^{-by} \right)_{0}^{a} - \frac{1}{b} \int_{0}^{a} e^{-by} dy \\
= \frac{y}{b} e^{-by} \bigg|_{0}^{a} - \frac{e^{-by}}{b^2} \bigg|_{0}^{a} \\
= \frac{a^2}{b} e^{ba^2} - \frac{e^{ba^2}}{b^2} + \frac{1}{b^2} \\
= \frac{e^{ba^2}}{b} \left( a^2 - \frac{1}{b} \right) + \frac{1}{b^2} \\
= -2\sigma^2 \exp \left( -\frac{a^2}{2\sigma^2} \right) \left( a^2 + 2\sigma^2 \right) + 4\sigma^4
\]

Thus, the numerator of equation (3.30) can be written as

\[
\langle |r|(r_2)^2 \rangle + \langle |r|(r_3)^2 \rangle = \\
\frac{1}{\sigma(\sqrt{2\pi})} \left[ -2\sigma^2 \exp \left( -\frac{a^2}{2\sigma^2} \right) \left( a^2 + 2\sigma^2 \right) + 4\sigma^4 \right] = \\
\left( \sigma^3 \sqrt{\frac{8}{\pi}} - \sigma \sqrt{\frac{2}{\pi}} \exp \left( -\frac{a^2}{2\sigma^2} \right) \left( a^2 + 2\sigma^2 \right) \right)
\]

Considering the second term of equation (3.40), when the limits of integration \(a\) is infinity, then this term becomes zero and hence the numerator becomes

\[
N = \sigma^3 \sqrt{\frac{8}{\pi}}
\]

The denominator of equation (3.30) is developed as follows

\[
D = \langle (r_2)^2 \rangle + \langle (r_3)^2 \rangle = \\
\langle (r_2)^2 + (r_3)^2 \rangle = \langle H^2 \rangle
\]
Now from Papoulis (1965), the expected value of the square of the modulus of a random variable, such as \( H \), is equal to the variance of the random variable itself. Thus we can write the denominator of equation (3.30) as

\[
D = \sigma^2
\]

(3.43)

And finally the revised drag coefficient of equation (3.30) can be written as

\[
C_D^*(x) = C_D \sigma \sqrt{\frac{8}{\pi}}
\]

(3.44)

By inspection of equation (3.44) it is easily seen that this coefficient is analogous to that postulated by Borgman for the simpler case showing that they are functionally identical.

The computation of the above drag coefficient requires the standard deviations of the components of relative velocities, \( r_2 \) and \( r_3 \). At the beginning of the calculations the structure is motionless and thus these relative velocities are initialised to the water particle velocities and an iterative procedure is employed to compute relative velocities. Details of this are outlined in Section 3.6 below.

Incorporating the linearised drag force in equation (3.23) and substituting this into equation (3.22) the general equations motion for the system may be written as
\[
[M]\ddot{U}^n + [C]\dot{U}^n + [K]U^n = \\
\int_v [G]^T f_1 \left[ \begin{array}{c} \dot{v}_2 \\ \dot{v}_3 \\ 0 \end{array} \right] dx + \int_v [G]^T f_2 \left[ \begin{array}{c} \ddot{v}_2 - \dot{v} \\ \ddot{v}_3 - \dot{w} \\ 0 \end{array} \right] dx + \int_v [G]^T f_4 \left[ \begin{array}{c} 0 \\ \ddot{v}_2 - \dot{v} \\ \ddot{v}_3 - \dot{w} \end{array} \right] dx
\]

(3.45)

where

\( f_4 \) is a drag force coefficient based on the linearised drag force

The components of the second and third terms on the right hand side of equation (3.45) that are functions of structural velocities and accelerations are brought over to the left-hand side as added mass and hydrodynamic damping terms respectively. Thus, the governing equations of motion for any element may now be written as:

\[
[M]\ddot{U}^n + [C]\dot{U}^n + [K]U^n = \\
\int_v [G]^T f_1 \left[ \begin{array}{c} \dot{v}_2 \\ \dot{v}_3 \\ 0 \end{array} \right] dx + \int_v [G]^T f_2 \left[ \begin{array}{c} \ddot{v}_2 - \dot{v} \\ \ddot{v}_3 - \dot{w} \\ 0 \end{array} \right] dx + \int_v [G]^T f_4 \left[ \begin{array}{c} V_2 \\ V_3 \\ 0 \end{array} \right] dx
\]

(3.46)

where

\([M]\) is the element mass matrix including added mass

\([C]\) is the element damping matrix including added damping

It is necessary to develop the governing equations of motion for the entire system, in order to achieve this two additional coordinate systems are required, namely, structure
global and wave coordinate systems as shown in Figure 3.2. The following notation is used to define these systems:

\( X_g \) is the global structure x coordinate direction
\( Y_g \) is the global structure y coordinate direction
\( Z_g \) is the global structure z coordinate direction
\( X_w \) is the wave x coordinate direction
\( Y_w \) is the wave y coordinate direction
\( Z_w \) is the wave z coordinate direction

The \( X_w \) direction represents the direction of wave propagation. Both coordinate systems are located at the mean water line and the model has been developed to consider the structural responses due to waves travelling in any particular direction by specifying an angle between \( X_w \) and \( X_g \).

The wave forces acting on a member are obtained in equation (3.46) by calculating water particle velocities and accelerations normal to the member as suggested by Chakrabarti. These quantities are computed using relevant transformations between the wave and local member coordinate systems.

The member local coordinate system is defined such that the local X axis is defined as being in the direction from node 1 to node 2 and the Y and Z local axes are chosen to construct an orthogonal right handed system. Coordinate transformations are applied relating terms in the local member coordinate system to terms in the structure global coordinate system to construct the structure global coefficient matrices.
The governing equations of motion for the overall structure are assembled from the element equations given by (3.46) as detailed by Bathe and Wilson (1982). The assembled governing equations of motion can then be written as:

\[
[M][\ddot{U}]_g + [C][\dot{U}]_g + [K][U]_g = F_g
\]  

(3.47)

where the subscripts \( g \) imply that the vectors and matrices are in terms of structure global coordinates. Equation (3.47) represents a set of simultaneous equations relating structural response to applied force in a linear manner.
3.4 Solution of the General Equations of Motion

When analysing a large offshore structure the problem will consist of many hundreds of degrees of freedom, thus to reduce the order of the matrices the general equations of motions are solved using the method of mode superposition. Consider an n-degree of freedom system whose \( \omega_i \) \(^{th}\) natural frequency is \( \omega_i \) and whose first m mode shapes are the columns of the matrix \([\Phi]\) of order \(n \times m\). Let

\[
U_g = [\Phi]Z \tag{3.48}
\]

where

\(Z\) is the vector of generalised coordinates.

If equation (3.48) is substituted into equation (3.47) and the equation is multiplied across by \([\Phi]^T\), the general equations of motion can be expressed in terms of the generalised coordinates as:

\[
\ddot{Z} + [C_h]Z + [D]Z + [\Omega]Z = R \tag{3.49}
\]

In equation (3.49) the generalised hydrodynamic coefficient matrix, \([C_h]\) is not diagonalised, since it contains the hydrodynamic damping terms, the matrix \([D]\) contains the diagonalised structural damping terms. The matrix \([C_h]\) can be diagonalised in an equivalent manner in a mean square sense as described by Grecco and Hudspeth (1983). \([\Omega]\) is a diagonal matrix in which the entries on the diagonals are the squares of the natural frequencies of the system.

Now assume \([C_h]^*\) is the equivalent diagonalised matrix, then an error vector is defined as follows:

\[
E_h = [C_h]Z - [C_h]^T \dot{Z} \tag{3.50}
\]

Consider the \(i^{th}\) term of the error vector.
where

\[ \hat{Z}^i \] is the \( i^{th} \) generalised velocity

The error term is now minimised in a mean square sense with respect to \( C_H^*(i,i) \) as follows:

\[
\left( \frac{d(E_i^H)^2}{dC_H^*(i,i)} \right) = 0
\]

then it can easily be shown that

\[
C_H(i,i) + \sum_{j=1}^{m} C_H(i,j) \langle \hat{Z}^i \hat{Z}^j \rangle
\]

\[
C_H^*(i,i) = \frac{\sum_{j=1}^{m} C_H(i,j) \langle \hat{Z}^i \hat{Z}^j \rangle}{\langle \hat{Z}^i \hat{Z}^j \rangle}
\]

The \( C_H^*(i,i) \) are the entries on the diagonals of the equivalent diagonalised hydrodynamic damping matrix \([C_H]^*\). Now substituting \([C_H]^*\) for \([C_H]\) in equation (3.49) and the decoupled system of equations can now be written as:

\[ \hat{Z} + [C_D] \hat{Z} + [\Omega]Z = R \] (3.53)

The \( i^{th} \) equation of (3.53) can now be written as:

\[ \hat{Z}^i + (C_H^*(i,j) + 2\xi_i \omega_i) \hat{Z}^i + \omega_i^2 Z^i = R^i \] (3.54)

where

\[ \xi_i \] is the percentage of damping employed in the \( i^{th} \) mode of vibration

To solve for any generalised coordinate \( Z^i \) of equation (3.53) the generalised force \( R^i \) must be obtained from Morison's equation and wave kinematics, these are considered below.
3.5 Wave Kinematics

The forcing functions on the right hand side of equation (3.45) are linear functions of water particle velocities and acceleration. Linear Airy wave theory is used to calculate these water particle velocities and accelerations. Airy wave theory may be considered as a first approximation of a complete theoretical description of ocean surface wave behaviour. However, it has been used extensively for force calculations on offshore structures with good results, Borgman (1965b). It is used in this development because it postulates linear relationships between water particle velocities and accelerations and wave heights, this allows linear spectral theory to be used to relate response spectra of displacements to input spectra of sea surface elevations.

Consider a long crested wave travelling in the positive X-direction of the wave coordinate system as defined in Figure 3.2. Airy wave theory describes the free surface of the water at location $X^w$ at time $t$ as:

$$\eta = a \cos(\omega t - kX^w)$$  \hspace{1cm} (3.55)

where

- $k = 2\pi/L = $ wave number
- $L = $ wavelength of wave
- $\omega = 2\pi/T = $ angular frequency
- $T = $ period of wave
- $a = $ wave amplitude
Using Airy wave theory the following expressions are obtained for water particle velocities and accelerations at a point \((X'^w, Z'^w)\) expressed in the wave co-ordinate system.

\[
V_1^w = \frac{a \cosh[k(Z'^w + d)]}{L \cosh(kd)} \cos(\omega t - kX'^w) \\
V_3^w = \frac{a \sinh[k(Z'^w + d)]}{L \cosh(kd)} \sin(\omega t - kX'^w)
\]  

\[
\dot{V}_1^w = a2\pi g \frac{\cosh[k(Z'^w + d)]}{L \cosh(kd)} \sin(\omega t - kX'^w) \\
\dot{V}_3^w = a2\pi g \frac{\sinh[k(Z'^w + d)]}{L \cosh(kd)} \cos(\omega t - kX'^w)
\]

where

- \(d\) is the overall depth of water
- \(V_1^w\) is the water particle velocity in the \(X'^w\) direction
- \(V_3^w\) is the water particle velocity in the \(Z'^w\) direction
- \(\dot{V}_1^w\) is the water particle acceleration in the \(X'^w\) direction
- \(\dot{V}_3^w\) is the water particle acceleration in the \(Z'^w\) direction

Equation (3.58) and (3.59), as stated above, represent horizontal and vertical components of water particles expressed in the wave coordinate system. Now, the governing equations of motion, equation (3.46), require components of water particle velocities and accelerations in the member coordinate system. These are obtained as shown in the development presented below, which is a variation on the development carried out by Chakrabarti et al (1975).
Consider a structural member in some arbitrary orientation as shown in Figure 3.3 with respect to the wave co-ordinate system. Define a unit vector, $\mathbf{e}_w$, along the longitudinal axis of a beam element as shown above. The complete vector of water particle velocity expressed in the wave coordinate system, $\mathbf{V}_w$, can be resolved into two perpendicular directions, one component, $\mathbf{V}_t^w$, that is tangential to the member axis and a component, $\mathbf{V}_n^w$, that is perpendicular to the member axis. In this development only velocity components normal to member axes are considered to contribute to external forces, the tangential components are ignored.

Now the tangential component is given by

$$\mathbf{V}_t^w = (\mathbf{V}_w \cdot \mathbf{e}_w) \mathbf{e}_w$$  \hspace{1cm} (3.60)

but

$$\mathbf{V}_w = \mathbf{V}_n^w + \mathbf{V}_t^w$$  \hspace{1cm} (3.61)

$$\therefore \hspace{1cm} \mathbf{V}_n^w = \mathbf{V}_w - (\mathbf{V}_w \cdot \mathbf{e}_w) \mathbf{e}_w$$  \hspace{1cm} (3.62)

Equation (3.61) can be expressed in terms of its component parts as

$$V_{n1}^w = V_1^w - e_1^w (e_1^w V_1^w + e_3^w V_3^w )$$  \hspace{1cm} (3.63)

$$V_{n2}^w = -e_2^w (e_1^w V_1^w + e_3^w V_3^w )$$  \hspace{1cm} (3.64)

$$V_{n3}^w = V_3^w - e_3^w (e_1^w V_1^w + e_3^w V_3^w )$$  \hspace{1cm} (3.65)

The components of water particle velocities required in equation (3.46) are related to equations (3.62) - (3.64) by defining the transformation $[T_{wn}]$ which relates quantities
expressed in the wave coordinate system to those quantities expressed in member coordinate system. Thus we can write the following relationship

\[
\begin{bmatrix}
0 \\
V_2 \\
V_3 \\
0
\end{bmatrix}
= \begin{bmatrix}
V_{n1}^w \\
V_{n2}^w \\
V_{n3}^w
\end{bmatrix} = f \begin{bmatrix}
T_{wm}
\end{bmatrix}
\]

(3.66)

A similar expression can be obtained between water particle accelerations in the two coordinate systems. Thus equation (3.46) can now be re-written as:

\[
\begin{bmatrix}
V_{n1}^w \\
V_{n2}^w \\
V_{n3}^w
\end{bmatrix} = \begin{bmatrix}
\dot{V}_{n1}^w \\
\dot{V}_{n2}^w \\
\dot{V}_{n3}^w
\end{bmatrix} + \int \begin{bmatrix}
\dot{f}_4^T \\
\dot{f}_4^T \\
\dot{f}_4^T
\end{bmatrix} \begin{bmatrix}
T_{wm} \\
T_{wm} \\
T_{wm}
\end{bmatrix} \begin{bmatrix}
V_{n1}^w \\
V_{n2}^w \\
V_{n3}^w
\end{bmatrix} dx
\]

(3.67)

### 3.6 Receptances and Response Spectra

#### 3.6.1 Linear Spectral Theory

Bendat and Piersol (1971) illustrate that for a linear system the relationship between the spectrum, \( S_x(\omega) \), of a random input signal, \( x(t) \), and the spectrum, \( S_y(\omega) \), of the output signal, \( y(t) \) is

\[
S_y(\omega) = |H(\omega)|^2 S_x(\omega)
\]

(3.68)

where

\( H(\omega) \) is the receptance of the system
Robson (1963) shows that the spectrum of response of a single degree of freedom linear system subjected to an excitation force, whose spectrum is given by $S_x(\omega)$, may be represented by

$$S_y(\omega) = \alpha(\omega) \alpha(\omega)^* S_x(\omega)$$

where

$\alpha(\omega)$ is the complex response of the system to an excitation of $e^{i\omega t}$

Further details of linear spectral theory are presented in Appendix IV.

The drag force term in Morison’s equation has been linearised and thus equation (3.46), which represents the general equations of motion of an offshore structure subjected to long-crested waves, defines a linear system between sea state and structural response. Thus the approach outlined by Robson (1963) can be applied to the current development. For a given long-crested wave the distributed forces throughout the offshore platform are obtained using Morison’s equation and Airy wave theory. In general, the receptance function for any degree of freedom, $H(\omega)$, is obtained by applying a series of unit waves to the platform and computing the structural response at each frequency. It is shown below that the receptances can be determined more efficiently by applying unit amplitude complex cosine waves rather than the complex wave $e^{i\omega t}$.

### 3.6.2 Unit Wave Method

Consider a linear system subjected to an input signal $e^{i\omega t}$ the response is

$$y(t) = \alpha(\omega) e^{i\omega t}$$

where

$$\alpha(\omega) = \alpha_1(\omega) + i\alpha_2(\omega)$$

and

$$\alpha(\omega) = \alpha(\omega)^*$$

(3.69)
\[ H(\omega) = \alpha(i\omega) \times \alpha(i\omega)^* \]  

(3.70)

and

* represents complex conjugate

Thus the response spectrum at frequency \( \omega \) is

\[ S_y(\omega) = \left| (\alpha_1(\omega))^2 + (\alpha_2(\omega))^2 \right| \text{s}^2 \]  

(3.71)

Now, equation (3.63) can be rewritten as

\[ y(t) = (\alpha_1(\omega) + \alpha_2(\omega))(\cos \omega t + i \sin \omega t) \]

\[ = (\alpha_1(\omega) \cos \omega t - \alpha_2(\omega) \sin \omega t) + i(\alpha_2(\omega) \cos \omega t + \alpha_1(\omega) \sin \omega t) \]  

(3.72)

By inspection of equation (3.66) it can be seen that the response to the real part of the input signal, \( \cos \omega t \), is

\[ y(t) = \alpha_1(\omega) \cos \omega t - \alpha_2(\omega) \sin \omega t \]  

(3.73)

From equation (3.67), if a surface gravity wave of unit amplitude and defined by \( \cos \omega t \) is incident on a structure and the structure response is

\[ y(t) = U_1(\omega) \cos \omega t + U_2(\omega) \sin \omega t \]  

(3.74)

then

\[ \alpha_1(\omega) = U_1(\omega) \]  

(3.75)

and

\[ \alpha_2(\omega) = -U_2(\omega) \]

and thus we get

\[ \alpha(\omega) = U_1(\omega) - iU_2(\omega) \]  

(3.76)

Thus, by applying a surface gravity wave described by \( \cos \omega t \) to the structure both the real and imaginary parts of the response of equation (3.66) can be obtained which can be used
to compute the receptance $H(\omega)$, this is much simpler method of determining the receptances that applying $e^{i\omega t}$. The receptances for all degrees of freedom over the frequency range of interest are obtained by computing structural responses due to a series of unit amplitude cosine waves with varying frequencies.

This novel approach reduces the number of calculations required to compute the receptances by about half when compared with the conventional way that the unit wave method is used. Figure 3.4 from Barltrop and Adams (1991) shows that usually a pair of load cases are required to represent the real and imaginary components of the applied complex load at each frequency. The approach described above requires only one load case and thus is considerably more efficient.

If a unit amplitude cosine wave is applied to the origin of the wave co-ordinate system, then using the water particles velocities and accelerations as defined in equation (3.56) - (3.59) and by some algebraic manipulations equations (3.67) can be expressed as

$$[M]\ddot{U}^n + [C]\dot{U}^n + [K]U^n = F_1 \cos \omega t + F_2 \sin \omega t$$  \hspace{1cm} (3.77)

Equation (3.77) represents the governing equations of motion in the member coordinate system, this is now expressed in terms of structure global co-ordinates by using an appropriate coordinate transformation matrix, $[T_{gm}]$, and then multiplied across by the transpose of this matrix to give

$$[T_{gm}]^T[M][T_{gm}]\ddot{U}^g + [T_{gm}]^T[C][T_{gm}]\dot{U}^g + [T_{gm}]^T[K][T_{gm}]U^g = J_1 \cos \omega t + J_2 \sin \omega t$$

72
Equation (3.78) is rewritten in terms of structure global coordinates as

$$[M]^{s} \ddot{U}^{s} + [C]^{s} \dot{U}^{s} + [K]^{s} U^{s} = J_1 \cos \omega t + J_2 \sin \omega t$$  \hspace{1cm} (3.79)

Using the mode superposition technique as outlined in Section 3.4, equation (3.79) can be decoupled to give $r$ equations, where $r$ is the number of mode shapes used to approximate and also decouple the general equations of motion. The $i^{th}$ equation of this decoupled system is given by

$$\ddot{Z}^i + \left( C_{ii}^i + 2 \xi_i \omega_i \right) \dot{Z}^i + C_{ii}^i Z^i = R_1^i \cos \omega t + R_2^i \sin \omega t$$  \hspace{1cm} (3.80)

The solution to equation (3.80) for the $i^{th}$ generalised coordinate is given by

$$Z^i = Z_{1i}^i \cos \omega t + Z_{2i}^i \sin \omega t$$  \hspace{1cm} (3.81)

From equation (3.81) we get the $i^{th}$ generalised velocity and acceleration respectively as

$$\dot{Z}^i = -\omega Z_{1i}^i \sin \omega t + \omega Z_{2i}^i \cos \omega t$$  \hspace{1cm} (3.82)

$$\ddot{Z}^i = -\omega^2 Z_{1i}^i \cos \omega t - \omega^2 Z_{2i}^i \sin \omega t$$  \hspace{1cm} (3.83)

Substituting for $Z^i$, $\dot{Z}^i$ and $\ddot{Z}^i$ in equation (3.80) we obtain

$$-\omega^2 Z_{1i}^i \cos \omega t - \omega^2 Z_{2i}^i \sin \omega t + \left( C_{ii}^i + 2 \xi_i \omega_i \right) - \omega Z_{1i}^i \sin \omega t + \omega Z_{2i}^i \cos \omega t$$

$$+ \omega^2 Z_{1i}^i \cos \omega t + \omega^2 Z_{2i}^i \sin \omega t = R_1^i \cos \omega t + R_2^i \sin \omega t$$  \hspace{1cm} (3.84)

Collecting the coefficients of $\cos \omega t$ and $\sin \omega t$ on the left hand side of equation (3.84), we can write:
\[
\left[ -\omega^2 Z_1^i + \omega Z_2^i \left( C_H^s(i,i) + 2\xi_i\omega_i \right) + \omega_1^2 Z_1^i \right] \cos\omega t + \\
\left[ -\omega^2 Z_2^i - \omega Z_1^i \left( C_H^s(i,i) + 2\xi_i\omega_i \right) + \omega_1^2 Z_2^i \right] \sin\omega t \\
= R_1^i \cos\omega t + R_2^i \sin\omega t
\]  
(3.85)

By comparing the coefficients of \(\cos\omega t\) and \(\sin\omega t\) on both sides of equation (3.85) we obtain

\[
Z_1^i = \frac{R_1^i - \omega Z_2^i \left( C_H^s(i,i) + 2\xi_i\omega_i \right)}{\omega_1^2 - \omega^2}
\]  
(3.86)

and

\[
Z_2^i = \frac{R_2^i + \omega Z_1^i \left( C_H^s(i,i) + 2\xi_i\omega_i \right)}{\omega_1^2 - \omega^2}
\]  
(3.87)

Re-arranging equations (3.86) and (3.87), we obtain the following expressions for the components of the \(i\)th generalised coordinates.

\[
Z_1^i = \frac{-R_1^i \left( \omega_1^2 - \omega^2 \right) - R_2^i \omega \left( C_H^s(i,i) + 2\xi_i\omega_i \right)}{\omega^2 \left( C_H^s(i,i) + 2\xi_i\omega_i \right)^2 + \left( \omega_1^2 - \omega^2 \right)^2}
\]  
(3.88)

and

\[
Z_2^i = \frac{R_1^i \omega \left( C_H^s(i,i) + 2\xi_i\omega_i \right) - R_2^i \left( \omega_1^2 - \omega^2 \right)}{\omega^2 \left( C_H^s(i,i) + 2\xi_i\omega_i \right)^2 + \left( \omega_1^2 - \omega^2 \right)^2}
\]  
(3.89)

Thus the vector of generalised co-ordinates can be expressed in component form as

\[
Z = Z_1^i \cos\omega t + Z_2^i \sin\omega t
\]  
(3.90)

The vector of nodal global displacement is then obtained from equations (3.73) and (3.90) as

\[
U^g = [\Phi]Z_1^i \cos\omega t + [\Phi]Z_2^i \sin\omega t \\
= U_1^g \cos\omega t + U_2^g \sin\omega t
\]  
(3.91)
Then the vector of receptances is obtained from equations (3.63), (3.68), (3.69) and (3.91) as

$$\alpha^8(\omega) = \alpha^8_f(\omega) + \alpha^8_\Sigma(\omega)$$
$$= U_f^*(\omega) - iU_\Sigma^*(\omega)$$ (3.92)

In order to compute the spectrum of displacement for a particular degree of freedom, $j$, the structure is subjected to a series of cosine surface water waves of unit amplitudes and of varying frequencies over the frequency range of interest and the receptances calculated at each frequency. The spectrum of the displacement for degree of freedom $j$, $S_j(\omega)$, associated with a storm event incident on the structure having a water surface elevation spectrum of $S_\eta(\omega)$ is then given by

$$S_j(\omega) = \alpha_j(\omega) \times \alpha_j(\omega)^* S_\eta(\omega)$$ (3.93)

where

$\alpha_j(\omega)$ is the $j^{th}$ entry of vector $\alpha^8(\omega)$

### 3.6.3 Statistics of Relative Velocity

The drag force term of equation (3.43) requires the calculation, in the local member coordinate system, of the linearised drag force coefficient, $C_D^e(x)$, as it varies along each member. To compute this it is, therefore, first necessary to calculate the standard deviations and cross-standard deviations of the components of relative velocities, $r_2$ and $r_3$, at integration points along each member. The following approach is adopted.
Ignoring components of water particle velocities that are tangential to member axes, the vector of water particle velocities, in local member coordinates, at a point along a member is given by equation (3.66) as

\[
\begin{bmatrix}
0 \\
v_2 \\
v_3 \\
0
\end{bmatrix} = [T_{wn}]
\begin{bmatrix}
V_{n1}^w \\
V_{n2}^w \\
V_{n3}^w \\
0
\end{bmatrix}
\tag{3.94}
\]

From equations (3.63)-(3.65), the components of the right hand side vector are functions of $V_1^w$ and $V_3^w$ and from equations (3.58) and (3.59) it can be shown that for a unit amplitude cosine gravity wave of frequency $\omega$ they are given by

\[
\begin{align*}
V_1^w &= V_1 \cos \omega t + V_2 \sin \omega t \\
V_3^w &= V_3 \cos \omega t + V_4 \sin \omega t
\end{align*}
\tag{3.95}
\]

Thus it can be shown that

\[
\begin{align*}
V_{n1}^w &= E_1 \cos \omega t + E_2 \sin \omega t \\
V_{n2}^w &= E_3 \cos \omega t + E_4 \sin \omega t \\
V_{n3}^w &= E_5 \cos \omega t + E_6 \sin \omega t
\end{align*}
\tag{3.96}
\]

Thus the components of velocity normal to the member in the local member coordinate system is

\[
\begin{align*}
v_2 &= E_7 \cos \omega t + E_8 \sin \omega t \\
v_3 &= E_9 \cos \omega t + E_{10} \sin \omega t
\end{align*}
\tag{3.97}
\]

From equation (3.91) it can be shown that the components of structural velocities normal to the member in the local member coordinate system, due to the unit wave, are given by
\[ \dot{v} = E_{11}\cos\omega t + E_{12}\sin\omega t \]  
(3.98)

\[ \dot{w} = E_{13}\cos\omega t + E_{14}\sin\omega t \]

From equation (3.97)-(3.98) the components of relative velocity at a point, due to a unit wave, can be written as

\[ r_2 = v_2 - \dot{v} = E_{15}\cos\omega t + E_{16}\sin\omega t \]  
(3.99)

\[ r_3 = v_3 - \dot{w} = E_{17}\cos\omega t + E_{18}\sin\omega t \]

Thus the receptances of the relative components are given by

\[ \alpha_{r2} = E_{15} - iE_{16} \]  
(3.100)

\[ \alpha_{r3} = E_{17} - iE_{18} \]

From linear spectral theory the spectra of the components of relative velocity resulting from a storm having an associated spectrum of \( S_\eta(\omega) \) are given by

\[ S_{r2}(\omega) = (\alpha_{r2})(\alpha_{r2})^*S_\eta(\omega) \]

\[ S_{r3}(\omega) = (\alpha_{r3})(\alpha_{r3})^*S_\eta(\omega) \]  
(3.101)

\[ S_{r2r3}(\omega) = (\alpha_{r2})(\alpha_{r3})^*S_\eta(\omega) \]

The variances and cross-variances of the relative velocities are then computed by integrating the above spectra over the frequency range and standard deviations obtained from these.

### 3.7 Linear Spectral Analysis Computer Model

A general-purpose computer based model has been developed to implement the methodology detailed in the above sections. The model has been developed so that any
configuration of a lattice offshore structure can be analysed. The computer program is written in FORTRAN 90 and has been successfully implemented on a VAX mini-system and on a Pentium PC. For any particular structure the program is structured so that spectra of displacements for any degree of freedom of the structure are computed for any given input spectrum of sea-state. Thus, structure responses for a whole range of sea states may be examined. The program is developed to consider only unidirectional seas; however, analyses can be performed for storms incident on a structure from any particular direction.

The computer code is divided into three main computational blocks as summarised in Figure 3.5. Each of these computational areas is described below in detail.

3.7.1 Formulation of Coefficient Matrices

Many different methods have been developed to efficiently formulate the coefficient matrices in structural analysis programs. The program developed herein follows the structure described by Bathe (1996) using the active column storage scheme. The main steps involved in this methodology are presented in the flowchart in Figure 3.6 and the associated FORTRAN computer code as given in Appendix VII

The program first reads in the structure geometry by reading in the total number of nodes used to model the structure and the coordinates of these nodes. Next the information defining the structural elements are read in. This data includes the connectivity, boundary conditions and material properties; each member being allowed different material properties. Six degrees of freedom are allowed at each element node, depending on the
nodal constraints the specified boundary condition determines whether a particular degree of freedom is free or fixed.

Employing equation (3.46) and using the input data, the element stiffness matrix and mass matrix are defined; when entrained water and marine growth are relevant they are also incorporated into the element mass matrix. For each member a transformation matrix is computed which relates a vector in the global co-ordinate system to the vector expressed in local member co-ordinates. Using these matrices, the mass and stiffness matrices are then transformed so that they are expressed in terms of the global co-ordinate system. They are then assembled into the global mass and stiffness vectors. The storage scheme used to store these vectors makes use of the symmetrical and bounded properties of the element matrices and is explained below.

Consider the upper part of the structure stiffness matrix in global co-ordinate, given by \([K]\) as

\[
[K] = \begin{bmatrix}
k_{11} & k_{12} & 0 & 0 & 0 \\
k_{22} & k_{23} & k_{24} & 0 \\
k_{33} & k_{34} & 0 & 0 \\
k_{44} & k_{45} & & & \\
& & & & & k_{m}
\end{bmatrix}
\]

(3.102)

The storage scheme employed in this development is to store only the entries below the skyline of matrix \([K]\) from the upper portion of \([K]\), known as the active columns of \([K]\),
in a one-dimensional array, A. Thus less than half of all entries are retained in vector A. In order to keep track of these entries a specific procedure for addressing the elements of [K] in A is used.

In the [K] matrix of equation (3.102) the row number of the first non-zero element in column i is defined as the variable \( m_i \). Thus, for each column the set of variables \( m_i \) define the skyline of the matrix as shown in equation (3.102). Also define the variable

\[
CH_i = i - m_i
\]  

(3.103)

as the column height associated with column i. In general, the column heights will vary from column to column, thus for efficiency it is important not to include any zeros outside the skyline. Obviously, the zeros within the skyline are stored as they may subsequently be acted upon to become non-zero.

When the column heights are determined, all the elements below the skyline a stiffness matrix [K] are stored in the one-dimension array A. The following expression illustrates the locations in A that these elements are allocated for a hypothetical situation laid out as they would appear in a matrix.

\[
[K] = \begin{bmatrix}
A(1) & A(3) & A(9) \\
A(2) & A(5) & A(8) \\
A(4) & A(7) & A(15) \\
A(6) & A(11) & A(14) \\
A(10) & A(13) & A(17) \\
A(12) & A(16) & A(18)
\end{bmatrix}
\]  

(3.103)
Thus, for this problem the storage vector $A$ has the following entries

$$A = \begin{bmatrix}
A(1) \\
\vdots \\
\vdots \\
A(18)
\end{bmatrix}$$

(3.104)

Thus, for the above case the 64 entries in an 8 x 8 stiffness matrix are stored in the 18 entries of $A$.

In addition to defining $A$ and $CH$, it is also necessary to define a vector, $IA$ which stores the locations where the diagonal elements of $[K]$ are placed in $A$. Thus, the location of the $i^{th}$ diagonal on $[K]$ in $A$ is stored in $IA(i)$. Therefore, using the three vectors, $A$, $CH$ and $IA$, the location of each element of $[K]$ be easily located in $A$. The same procedure as the above is also used to efficiently store the global element mass matrix expressed in global coordinates.

The computer code consists of a main program, SPEC, and associated subroutines. The links between the various routines are shown schematically in Figure 3.7 and the routines are briefly described below.

**Speclin**

This is the main routine that controls the overall structure of the program. This program opens relevant input and output data files, initialises various working vectors and sets up a pointer system which monitors the locations of the variables in the main working vectors. Spec stores input data and element-generated data efficiently in either of two vectors: $A$ stores real variables and $IA$ stores integer variables. Pointers are defined to enable the
storage locations of these data to be tracked as outlined above. *Speclin* calls the relevant subroutines to perform specific functions.

**Input**

This routine reads in the X, Y, and Z co-ordinates of each node of the structure and a flag to indicate the free degrees of freedom at each node. The total number of equations to be solved is also computed here.

**Elcall**

This routine loops over all different element groups.

**Beamel**

This routine allows the user choose the type of finite element being used; in this case only beam elements are used but other elements can easily be coded in.

**Storbeam**

In this routine storage space is set aside for the element stiffness and mass matrices for beam elements

**Beam**

*Beam* is called twice during an analysis; the first time it is called the element material properties and element connectivities are read in. The second time *Beam* is called it performs the following functions:

* computes local member axes
* defines element stiffness matrices
* defines element mass matrices – including marine growth and entrained water where relevant
* includes soil stiffness if member is in foundation
* includes point masses where relevant
* computes member-to-global transformation matrix
* expresses matrices in global coordinates

Diagad

This routine calculates the addresses of the diagonal elements in the banded matrix whose column heights are known.

Heights

Here the column heights associated with each element stiffness and mass matrices are computed to define the skyline of the matrices.

Assembl

In this routine the upper triangular element stiffness and mass matrices are assembled into the compacted global vectors.

3.7.2 Solution of the Eigenproblem

3.7.2.1 Subspace Iteration

In Section 3.4 it is stated that use is made of the system eigenvectors and eigenvalues to solve the general equations of motion (3.47). In this section the methods used to compute the eigenvectors and eigenvalues are presented.

Equation (3.47) with damping ignored and the global forces set to zero reduces to the undamped free vibration problem as given by

\[
[M]_g \ddot{U}_g + [K]_g U_g = 0 \quad (3.105)
\]

The solution to equation (3.105) can be postulated to be of the form

\[
U = \phi \sin(\omega t) \quad (3.106)
\]

where
ϕ is a vector of order n

t is the time variable

ω is the frequency of vibration, rads/s)

By substituting equation (3.106) into equation (3.105) the generalised eigenproblem is obtained as follows

\[ [K] \phi = \omega^2 [M] \phi \]  

(3.107)

where the subscript g is dropped for convenience.

It can be shown that the solution to the generalised eigenproblem yields the n eigensolutions \((\omega_1^2, \phi_1), (\omega_2^2, \phi_2), \ldots, (\omega_n^2, \phi_n)\), where the vector \(\phi_i\) is the \(i^{th}\) mode shape and \(\omega_i\) is the corresponding frequency of vibration.

The matrix \([\Phi]\) is defined such that its columns are the eigenvectors \(\phi_i\), and the diagonal matrix \([\Omega]\) is defined such that the entries on the diagonals are the squares of the eigenvalues \(\omega_i\), that is

\[ [\Phi] = [\phi_1, \phi_n, \ldots, \phi_n] \]  

(3.108)

and

\[ [\Omega] = \begin{bmatrix}
\omega_1^2 & 0 & \cdots & 0 \\
0 & \omega_2^2 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \omega_n^2
\end{bmatrix} \]  

(3.109)
The subspace iteration technique, as detailed by Bathe (1996), that is used to solve the undamped free vibration problem, is very similar to vector inverse iteration, and thus inverse iteration is briefly described.

Consider the general eigenproblem from equation (3.107)

\[
[K] \phi = \omega^2 [M] \phi
\]

re-written as

\[
[K] \phi = \lambda [M] \phi
\] (3.110)

Assume an initial trial vector \( X_1 \) for \( \phi_1 \) and a value of unity for \( \lambda \), then the right hand side of equation (3.110) may be re-written for the first mode shape as

\[
R_1 = [M] X_1
\] (3.111)

Since \( X_1 \) is arbitrary we can re-write equation (3.110) as

\[
[K] X_2 = [M] X_1 = R_1
\] (3.112)

where equation (3.112) is solved for \( X_2 \) to obtain a better approximation to the eigenvector \( \phi_1 \) than \( X_1 \) was.

The inverse iteration procedure then continues by using the last approximation to \( \phi_1 \) to determine a revised right hand side as in equation (3.110) and then to compute a new approximation until convergence is reached. Proof of convergence to the first and subsequent eigenvectors is presented in Hurty and Rubenstein (1964).

The subspace iteration technique is widely used in engineering practice and is particularly suited when it is required to determine the lowest few eigenvalues and eigenvectors of
large finite element systems. The basic steps involved in solving for p eigenvectors and eigenvalues are:

(1) Establish q starting iteration vectors, where q > p.
(2) Use simultaneous inverse iteration on the q vectors and Ritz analysis to compute the "best" eigenvalue and eigenvector approximations for the q iteration vectors.
(3) After convergence the Sturm sequence check is performed to verify that the required eigenvalues and associated eigenvectors have been calculated.

In this approach the iteration is equivalent to iterating with a q-dimensional subspace rather than simultaneous iteration with q individual iteration vectors. The starting vectors span the subspace $E_1$ and iteration continues until, to sufficient accuracy, the subspace defined by the eigenvector, $E_\infty$ is spanned.

To iterate from one subspace to the next, the following algorithm is used in a manner analogous to the vector iteration method.

The following equation is solved for $[Y]_{k+1}$

$$[K][Y]_{k+1} = [M][Y]_k$$

(3.113)

where $[Y]_k$ is the approximation to $[Y]$ after iteration $k$ and $[Y]_{k+1}$ is the first approximation to $[Y]$ in iteration $(k+1)$.

The Rayleigh-Ritz procedure is next invoked to maintain reasonable sizes of the numbers in the calculations and to orthogonalise the vectors. Thus, the $(k + 1)^{st}$ generalised coordinate stiffness and mass matrices are written as
The eigenproblem of the generalised matrices are then solved, using the generalised Jacobi technique, for the generalised co-ordinate mode shapes in the \((k + 1)^{\text{st}}\) iteration, from

\[
[K]_{k+1} = \bar{Y}_{k+1}^T [K] \bar{Y}_{k+1}
\]

(3.114)

and

\[
[M]_{k+1} = \bar{Y}_{k+1}^T [M] \bar{Y}_{k+1}
\]

(3.115)

The improved approximation to the eigenvectors, from the Rayleigh Ritz procedure, is given by

\[
[Y]_{k+1} = [\bar{Y}]_{k+1} [Q]_{k+1}
\]

(3.117)

For sufficient numbers of iterations Bathe (1996) states that, in general,

\[
[Y]_{k+1} \rightarrow [\Phi]
\]

(3.118)

and

\[
[\Lambda]_{k+1} \rightarrow [\Lambda]
\]

(3.119)

Bathe (1996) also states, however, that there is no formal proof that the subspace iteration method always converges to the eigenpairs.

### 3.7.2.2 Starting Iteration Vectors

It has been shown that starting iteration vectors should be constructed to excite those degrees of freedom with which large mass and small stiffness are associated. Based on
this observation the following algorithm has been used for the selection of the starting
iteration vectors.

In the first iteration, the first column in \([M] [Y]_1\) is the diagonal of \([M]\), thus all mass
degrees of freedom are excited. The other columns of \([M] [Y]_1\), are unit vector with the
entry (+1) at the degrees of freedom with the smallest ratios of \((k_{ii} / m_{ii})\)

where

\[ k_{ii} \text{ is the entry on the } i^{th} \text{ diagonal of the stiffness matrix} \]
\[ m_{ii} \text{ is the entry on the } i^{th} \text{ diagonal of the mass matrix} \]

3.7.2.3 Convergence

The starting vectors described above have been found to give good results in practice.

Now although, as observed above, there is no formal proof that this method converges,
the following convergence criteria have been found to work well in practice. To
determine if convergence to the required eigenpairs has been attained assume that in
iterations \((k-1)\) and \((k)\) eigenvalue approximations \(\lambda^k_i\) and \(\lambda^k_i\), \(i=1,\ldots,p\) have been
calculated. Convergence is considered to have been reached when

\[
\frac{|\lambda^{k+1}_1 - \lambda^k_1|}{\lambda^{k+1}_1} \leq \text{tol} \tag{3.120}
\]

where

\[ \text{tol} = 10^{-25} \]

and the eigenvalues will be accurate to 25 digits.
During any iteration, \( k \), equation (3.116) is solved for \([Q]_{k+1}\) using the generalised Jacobi method. After convergence has been reached and the \( p \) eigenvectors and eigenvalues computed, it is necessary to check that these eigenpairs are the \( p \) lower pairs. This is ascertained by applying a Sturm sequence check to the computer eigenvalues. This involves using the Sturm sequence property of the characteristic polynomials of the problem

\[
[K][\Phi] = \lambda[M][\Phi]
\]

at a shift \( \mu \), where \( \mu \) is just larger than the largest eigenvalue. If \([K] - \mu[M]\) is factorised into \([L][D][L]^T\) using Gauss factorisation, then the number of negative elements in \([D]\) is equal to the number of eigenvalues smaller than \( \mu \). Thus, the case in the case in question there should be \( p \) negative elements in \([D]\).

Further details of this technique are presented in Bathe (1996), the associated FORTRAN code, which has been revised to ensure numerical stability, is given in Appendix VIII for completeness.

3.7.3 Evaluation of Receptances and Spectra of Structural Displacements

The main computational features of the subroutines in Block III are outlined below in Figure 3.8 and the FORTRAN code is given in Appendix VII. Because of the non-linear drag force term, the system receptances are functions of the relative velocities. However, initially the structural velocities are unknown and, therefore, an iterative procedure must be used, this is shown clearly in the flowchart of Figure 3.8.

As outlined above in Section 3.6.2 the unit wave method is used to compute the system receptances. The consistent nodal forces on any member, as described in equation (3.46),
are obtained by integrating functions of water particle velocities and acceleration over the length of the member. Because these cannot be analytically evaluated, numerical integration is used to compute the forces. Thus, each finite element is divided into a number of integration points and the nodal forces are calculated using the Simpson’s composite rule, Carnahan et al (1969).

Brief descriptions of each of the main subroutines are presented below.

Spec

This routine computes the spectrum of water surface elevations for either the Jonswop or Pierson-Moskowitz formulations over the frequency range of interest at discrete frequencies. These are the most commonly used synthetic spectra for North Sea sea-states and are defined respectively as

(i) Pierson-Moskowitz Spectrum

\[
S_n(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left[ -\beta \left( \frac{g}{\omega W} \right)^4 \right]
\]

where

\[
\alpha = 8.1 \times 10^{-3}
\]
\[
\beta = 0.74
\]
\[
W = \text{wind speed, m/s}
\]

(ii) JONSWAP
The JONSWAP spectrum constitutes a modification to the Pierson-Moskowitz spectrum to account for the effects of limited fetch and to provide for a more sharply peaked spectrum

\[ S_n(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left[ -\frac{5}{4} \left( \frac{\omega}{\omega_0} \right)^{-4} \right] \gamma^a \]

(3.122)

where

\[ \gamma = 3.30 \quad \text{(mean value)} \]

\[ a = \exp \left( -\frac{(\omega - \omega_0)^2}{2\tau^2\omega_0^2} \right) \]

\[ \tau = 0.07 \quad \omega < \omega_0 \]

\[ \tau = 0.09 \quad \omega > \omega_0 \]

\[ \alpha = 0.076(X^{0.22}) \]

X = fetch

The routine also allows the user to input a recorded spectrum in the form of discrete ordinates at corresponding frequencies. Spectrum ordinates are specified at user defined frequency intervals, usually 0.04rads/s.

**Storage**

Many of the variables used throughout Block III are efficiently stored in a single vector. This routine sets up the required storage space and a series of pointers locating where these data will be stored.
This routine computes the co-ordinate transformations between the structure global co-ordinate system and the wave co-ordinate system, \([T]_{gw}\) and also the co-ordinate transformations between the wave co-ordinate system and the local member co-ordinate system, \([T]_{wm}\). These transformations are required to express the coordinates at integration points in the wave coordinate system.

At each frequency a check is carried out to ascertain if the frequency represents a wave in deep water or shallow water. If the wave is in deep water, the associated wave number required by equations (3.56) – (3.57) is computed by

\[
k = \frac{2\pi}{\omega L}
\]  
where

\[
L = \frac{gT^2}{2\pi}
\]

\[T = \text{wave period}\]

When the wave is not in deep water, the following dispersion relationship is used to compute the wave number

\[
\omega = gk \tanh(kd)
\]

where \(\omega = \text{wave frequency}\)

\(d = \text{water depth}\)

For a given water depth and wave frequency, equation (3.125) cannot be solved explicitly for \(k\), the Newton-Raphson iterative method is thus used to solve for \(k\) as follows:
\[ k_{n+1} = k_n - \frac{f(k_n)}{f'(k_n)} \]  

where \( n \) = iteration number

\( f(k_n) \) in general is a function of \( k_n \)

\( f'(k_n) \) is the derivative of \( f(k_n) \) with respect to \( k_n \)

Repeated iterations of equations are performed until convergence is reached; this procedure is carried out in the function sub-program \textit{Waveno}.

In equation (3.46), the linearised drag force coefficient uses the standard deviation of the relative velocity at an integration point. During the first iteration the variances of relative velocities are set to the respective variances of water particle velocities at the integration points. The variances of water particle velocities are obtained from linear Airy wave theory and the spectrum of sea state by numerical integration.

The cross-variances of generalised velocities are required to compute the equivalent diagonalised hydrodynamic damping matrix in equation (3.53), initially these are set to zero. Structural velocities and accelerations are also initialised to zero.

\textit{Cbarj}\)

The linearised drag coefficient in equation (3.30) is computed in this subroutine at each integration point. In order to compute \( C^d \phi(x) \) the double integral of equation (3.32) must be numerically evaluated, namely:

\[ \langle f(x,y) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)p(x,y)dx\,dy \]

It was found that, though the limits of integration are between \( \infty \) and \(-\infty \), practical limits of between +/- 3 standard deviations of relative velocity components appeared sufficient for convergence and was used for all results presented herein.
Addamp

The element hydrodynamic damping matrix obtained from equation (3.46) is computed using numerical integration over the length of the member. This element matrix is next expressed in the global coordinate system using the appropriate coordinate transformation matrix.

The norm of the difference between the global hydrodynamic damping matrix of the current iteration and the previous is computed, if it is less than a preset value convergence is considered to have been achieved and results are output, otherwise the computation proceeds to next iteration.

Assem

The element hydrodynamic damping matrix expressed in global coordinates is assembled into the global damping matrix.

Modal

The generalised damping matrix of equation (3.53) is constructed.

Force

The element force vectors of equation (3.77) are computed using numerical integration. The nodal forces are obtained by using the composite Simpson’s Rule to integrate the distributed forces over the length of each. The element force vectors are transformed into the global coordinate system and assembled into the correct locations of the global force vectors. The global force vectors are acted upon by the matrix of eigenvectors to obtain the generalised force vectors of equation (3.80).

The generalised displacements, as defined by equations (3.88) and (3.88), are next computed.
Finally, the variances of relative velocities and cross-variances of generalised displacements are computed from the input spectrum and the computed displacements.

Results

The procedure used in Speclin is iterative since the revised drag force coefficient requires knowledge of relative velocities. Thus, as shown in Figure 3.8, after each iteration the drag force coefficients are revised and the hydrodynamic damping matrix updated. Convergence is considered to have been reached when the difference between the norms of the hydrodynamic damping matrix of the current iteration and the previous iteration is less that a set tolerance. When convergence has been reached spectra and standard deviations of displacements are output to file.
Section A - A

Figure 3.1 - Definition Sketch of Beam Element

Figure 3.2 - Global Structure and Wave Coordinate Systems
Figure 3.3 - Unit Vector in Wave Coordinates

Figure 3.4 - Unit Wave Loads on Members (Barltrop & Adams 1991)
Figure 3.5 - Main Computational Blocks of Linear Spectral Model
Read nodal data
- Node numbers
- Nodal coordinates

Read element data
- Connectivity
- Boundary Conditions
- Material Properties

Generate element stiffness and mass matrices

Transform local element matrices to global structure coordinates

Assemble element matrices into global structure vectors

Figure 3.6 - Formulation of Matrices Flowchart
Figure 3.7 - Flowchart Block I
Figure 3.8 - Flowchart Block III
CHAPTER 4

APPLICATION OF THE LINEAR SPECTRAL MODEL

4.1 Introduction

In this chapter details are presented of a number of applications of the linear spectral analysis computer model, SPECLIN, developed in Chapter 3. The various applications have been chosen in order to validate the model as fully as possible and to consider effects of leg spacing on response spectra.

The validation exercise was carried out in two stages: when computing the system receptances the model makes use of the system eigenvalues and eigenvectors in the mode superposition method, thus during the first stage of the procedure the eigensolver of the program was tested. Two test cases are considered to validate the eigensolver. Firstly, a model is constructed of a typical offshore jacket platform and the first five eigenpairs computed using SPECLIN and the commercially available software ASAS. The results from the two analyses are then compared. The eigensolver is also validated by developing a model of an existing gas production platform for which the first five natural frequencies and mode shapes are known. SPECLIN is then used to compute the natural frequencies and mode shapes and the results compared with the known values. Offshore platforms are designed so that some hollow cylindrical members are flooded and others are dry, however, during the course of their lifetime dry members may become flooded due to material failure. The global mass matrix of an offshore structure is a function of the entrained water and hence, so are the
natural frequencies and mode shapes. SPECLIN was used to consider the effect on natural frequencies when various members are flooded or dry.

The second stage of the validation exercise consists of calculating response spectra for a number of different structures and comparing the results with either other solution schemes or measured spectra. The models developed in stage two of the validation exercise range from a simple cantilever to an existing gas production platform with over a thousand degrees of freedom. The different analyses presented in this section illustrate that the model is correctly computing response spectra of displacement.

It is generally known that the spacing of the main legs of an offshore jacket structure can cause phasing effects on incident loads on the legs. This physical effect means that the overall response of an offshore structure is a function of the main leg spacings of the platform. The computer model developed in this study has been used to analyse this phase effect and to consider how optimum leg spacings may reduce overall structural response. A simple portal frame model has been developed by Mitchell (1996) who applied SPECLIN to illustrate this phasing effects by considering receptances and response spectra for portal frames with different leg spacings. Finally, the effect of changes in the angle of incidence of a storm on receptance functions for a space frame due to leg phase effects are assessed.

4.2 Eigensolver Analyses

The eigensolver used in SPECLIN has been used to compute the natural frequencies and mode shapes for various structures, both on land and immersed in water. Two analyses are presented below to validate the accuracy of the solver used and a further analysis is performed to consider effects of entrained water on natural frequencies.
4.2.1 Eigensolver Validation - Analysis I

Angelides (1978) devised a test structure on which to perform various spectral model studies, the mode shapes and natural frequencies of this structure were calculated using SPECLIN and ASAS. Line drawings of two orthogonal side elevations of this doubly symmetric steel jacket structure are shown in Figure 4.1.

The structure was defined with 137 node points connecting 418 members. Each node point was allowed six degrees-of-freedom; the total number of free degrees-of-freedom was 750, giving coefficient matrices of order 750x750. The structural elements are all tubular hollow sections of high strength steel, with external diameters ranging from 914mm to 2286mm and wall thickness ranging from 19mm - 57mm. Details of member sizes, dimensions and connectivities are given in Angelides (1986). The deck superstructure was modelled by specifying lumped masses at the nodes at the top of the structure which contribute only to the relevant translational degrees of freedom. The structure is considered to be fully fixed at the mudline.

The first five natural frequencies and mode shapes were calculated for the above structure using the two software packages by Mitchell (1996) and the results are given below. Table 4.1 shows a comparison between the results from both analyses and the percentage difference between both methods; this comparison shows that there is very close agreement between the natural frequencies as computed by the two different programs.

Both programs also predicted the same mode shapes for associated frequencies; these mode shapes are presented in Figures 4.2 - 4.6 below. The first four of the above mode shapes
contribute to overall structure motions, whereas the fifth mode contains only vibrations of some local members at the top of the structure.

<table>
<thead>
<tr>
<th>Mode</th>
<th>ASAS (rads/sec)</th>
<th>SPEC (rads/sec)</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.305</td>
<td>6.078</td>
<td>5.3</td>
</tr>
<tr>
<td>2</td>
<td>9.523</td>
<td>9.676</td>
<td>1.6</td>
</tr>
<tr>
<td>3</td>
<td>11.148</td>
<td>10.287</td>
<td>8.4</td>
</tr>
<tr>
<td>4</td>
<td>13.240</td>
<td>12.502</td>
<td>5.9</td>
</tr>
<tr>
<td>5</td>
<td>17.569</td>
<td>18.386</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Table 4.1. - Comparison of Natural Frequencies

4.2.2 Eigensolver Validation - Analysis II

4.2.2.1 Kinsale Platform Alpha

The second validation exercise was carried out on an existing platform. Kinsale Gas Head Production Platform Alpha consists of an eight-legged steel jacket structure which supports the accommodation module, production deck and ancillary systems associated with gas production operations. The structure is located off the southern Irish coast, as shown in Figure 4.7, and hence is exposed to a relatively harsh wave environment. It is generally considered that current induced loadings are insignificant at the site relative to wave induced loadings.

A schematic diagram of the platform is presented in Figure 4.8. The horizontal dimensions of the platform at cellar deck level are approximately 53m x 25m. The total height of the platform above the seabed is approximately 122m and it is located in approximately 92m of water at low water spring tide. The foundation material is a very stiff chalk having a Young’s
modulus of approximately $3 \times 10^9$ N/m$^2$; a 3m layer of sand overlies this chalk. The main piles anchoring the platform to the seabed were driven 12m into the chalk and were further anchored by drilled and grouted insert piles which extend to about 48m into the chalk. The eight main jacket legs are 1560mm in diameter, which are braced by horizontal and raking tubular sections and I-beams as shown in Figure 4.8. Eolas, the Irish offshore certification authority, carried out a structural monitoring project to determine certain dynamic characteristics of the platform, see Brennan (1983); from the study the first five natural frequencies and mode shapes were estimated. A brief outline of the monitoring project is presented below, further details of the structure and the structural monitoring project can be found in Brennan (1983).

4.2.2.2 Monitoring Objectives and Methodology

The primary purposes of the monitoring project were: (i) to obtain information that could be used to calibrate and validate numerical models and (ii) to investigate correlations between ocean surface waves and structural responses. To achieve these objectives, a programme was devised to obtain information regarding dynamic response characteristics of the structure in both the time-domain and frequency-domain. The frequency-domain data that was required from the monitoring programme is:

- the estimation of the fundamental normal modes of vibration of the structure.
- the determination of degree of correlations between records of wave heights and record of structural accelerations.
The principal components of this work concentrated on the determination of the prevailing environmental conditions and the deflected shapes of the platform associated with these conditions.

*Environmental Conditions*: Hydrodynamic loads had to be estimated as they could not be measured directly. Continuous time-histories of sea surface elevations were recorded for selected 17-minute intervals which allowed estimates of wave loadings to be performed. Current velocities were estimated from measurements made by the Institute of Oceanographic Science.

*Deflected shape*: Three pairs of accelerometers were installed on the structure above the water level. These recorded continuous time-histories of structural accelerations in two mutually perpendicular horizontal directions for selected 17-minute periods. A computer program subsequently converted these to displacement records, thus providing time traces of the deflection of the structure at three locations.

**4.2.2.3 Accelerometer Deployment**

Accelerometers were deployed on the platform as part of the monitoring project to measure structural responses during storm conditions. Offshore structures subjected to sea waves experience dynamic forces having typical associated frequencies of 0.1 - 0.2 Hz, and frequencies in storms, when accurate measurement is most important, can be expected to be as low as 0.05 Hz. The accelerometer chosen to meet these requirements was the SUNSTRAND Q-flex servo-accelerometer, type QA 1300. The main features of the overall data acquisition system were:
Continuous recording for a minimum of 6 hours

Detection limits of wave heights 0 – 30m

Detection limits of wave frequencies 0.02Hz – 0.5Hz

Detection limits of accelerations $10^{-5}g$ – 0.3g

To achieve the project objectives, it was decided to deploy accelerometers at three fixed locations which automatically recorded data at predetermined times. The relatively small number of locations precluded the possibility of estimating any but the first few fundamental modes of vibration. However, this constraint was not considered too stringent since most structural motion generally occurs in these fundamental modes.

Having decided that accelerometers were to be deployed at three locations, it was relatively easy to choose the locations. The three positions ought not to be in the same plane, they should be separated as far as feasible about the structure, and they should be above water to avoid complications due to diving. One set was thus placed at the centre of the cellar deck on one of the legs, another at the same level on a corner leg. The third was placed at the lower, spider deck, level on the same pile. In this way translation of the structure in both directions could be picked out, as could the first torsion mode. Higher order pile bending modes could also be discerned. The accelerometer deployment locations are shown in Figure 4.9. The accelerometer located at the lower level is biaxial, the other two accelerometers are triaxial, which made possible, if desired, the study of vertical movements of the platform.

4.2.2.4 Sea-State Measurement

A water surface elevation recorder, wavestaff, was deployed to allow correlations be made between hydrodynamic loads and structural response. The instrument was a simple unit
consisting of two parallel wires carrying a current and passing through the sea surface. The current passes down one wire, across the sea surface and up the second wire. As the sea surface level changes, so does the total resistance in the circuit. A voltage is measured which can be calibrated to indicate the sea surface elevation above a defined base.

Wave direction was not recorded, however, the Irish Meteorological Service provided estimates of wave direction based on calculations using wind data. This method proved quite successful and appeared to be adequate, particularly when complemented by the information contained in the accelerometer records. The location of the deployment of the wavestaff is shown in Figure 4.9.

4.2.2.5 Normal Modes Estimation

Many data sets were recorded during the project and these were used to estimate the natural frequencies and mode shapes of the platform. Plots of sections of the wave height records and corresponding records of structure accelerations in the longitudinal direction of the structure at cellar deck level are shown in Figure 4.10 for two storms.

The first data set above was recorded over a 17-minute period during which the prevailing wave direction was from the west and the significant wave height during this storm was 7.0m. The second data set was recorded over a 17-minute period during which the prevailing wave direction was again from the west and the significant height was 2.5m.

An inspection of a typical wave trace, such as those above, shows that wave heights and periods are random functions of time, only capable of being fully described by the use of spectral methods. It is, consequently, difficult to adequately define structural response by deterministic methods. Hence, in order to estimate the natural frequencies of the platform, it
was necessary to convert time traces of the recorded structure response accelerations into spectra and then the perform analysis in the frequency domain.

Determination of the natural frequencies of Kinsale Platform Alpha was achieved by analysing the signals from the accelerometers using cross-correlation techniques. The cross-correlation analysis were carried out on records associated with seas of very different intensities, although the normal mode frequencies were not expected to change with the intensity of the sea state. In particular, the following functions of displacements

cross-spectral density magnitudes
cross-spectral density phases
coherence functions

were obtained from the accelerometer recordings and analysed to estimate the eigenpairs, Brennan (1983). The results obtained from assessing the above functions are present in Table 4.2. The mode shapes presented in Table 4.2 are qualitative estimates of the actual mode shapes, since in order to accurately quantify the mode shapes many more accelerometers would need to be deployed throughout the structure.

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Frequency (rads/s)</th>
<th>Mode Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.756</td>
<td>Main Translation</td>
</tr>
<tr>
<td>2</td>
<td>0.756</td>
<td>Transverse</td>
</tr>
<tr>
<td>3</td>
<td>1.03</td>
<td>Main Torsion</td>
</tr>
<tr>
<td>4</td>
<td>1.73</td>
<td>Higher Order</td>
</tr>
<tr>
<td>5</td>
<td>1.98</td>
<td>Higher Order</td>
</tr>
</tbody>
</table>

Table 4.2 – Estimated Natural Frequencies and Mode Shapes
4.2.2.6 Computation of Normal Modes

A detailed model developed for Platform Alpha using SPECLIN was used to compute the first five natural frequencies of the platform. The structural model developed for this analysis consisted of 239 nodes and 724 beam elements. Because the platform is founded in stiff chalk, the main legs are considered to be encastre at the seabed, this assumption was incorporated into the model by. The model was represented in total by 1188 free degrees-of-freedom; line drawings of the finite element model are shown in Figure 4.11.

The main legs of the platform are comprised of composite sections with an insert pile inside each leg and the annulus between the leg and pile is grouted. Also, the eight main legs of the platform are flooded and were specified accordingly in the model, all other hollow members were assumed to be dry internally. A schematic diagram of a cross-section through a typical main leg this is shown in Figure 4.12. This composite cross-section is incorporated into the model.

The topside modules such as the accommodation units were included as translational lumped masses at the top nodes of the model. The risers transporting the gas from the seabed to the production deck are loosely connected to the structure and are also included as lumped masses at the relevant nodes; it is assumed that the risers do not provide any stiffness to the structural system. The finite element model has been developed only for beam elements, thus the plates at the deck level are not included. The primary effect of the plates with respect to the current model is that they provide large lateral resistance at the top of the platform. This large stiffness is achieved in this model by specifying beams with high moments of inertia, this increases the structural stiffness but does not affect structural mass. To accurately model the
structure, 90 different material property types were specified and the relevant type assigned to each of the 724 beam elements.

SPECLIN was run for the model of Kinsale Platform Alpha detailed above and the first five natural frequencies and mode shapes computed. Table 4.3 presents a comparison of the computed natural frequencies and those estimated from the monitoring project.

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Natural Frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPECLIN</td>
</tr>
<tr>
<td>1</td>
<td>0.703</td>
</tr>
<tr>
<td>2</td>
<td>0.756</td>
</tr>
<tr>
<td>3</td>
<td>0.896</td>
</tr>
<tr>
<td>4</td>
<td>1.945</td>
</tr>
<tr>
<td>5</td>
<td>2.149</td>
</tr>
</tbody>
</table>

Table 4.3 - Comparison of Computed and Estimated Natural Frequencies

By inspection of Table 4.3 it can be seen that there is good agreement between the computed and estimated frequencies. The maximum difference between the two sets of values is 13%; such close agreement between model results and frequencies estimated from a monitoring project for such a large structure is considered good validation.

The first five computed mode shapes are presented in Figures 4.13 – 4.17. The first two modes are translational modes about the two main horizontal axes of the structure, the third mode is a
torsional mode and the fourth and fifth modes are bending modes about the two main horizontal axes of the structure. Table 4.4 presents a comparison between the computed mode shapes and those estimated from the monitoring project.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Mode Shapes Computed by SPECLIN</th>
<th>Mode Shapes Estimated from Monitoring Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Main Translation</td>
<td>Main Translation</td>
</tr>
<tr>
<td>2</td>
<td>Transverse Translation</td>
<td>Transverse Translation</td>
</tr>
<tr>
<td>3</td>
<td>Main Torsion</td>
<td>Main Torsion</td>
</tr>
<tr>
<td>4</td>
<td>Main Bending</td>
<td>Higher Order Mode</td>
</tr>
<tr>
<td>5</td>
<td>Transverse Bending</td>
<td>Higher Order Mode</td>
</tr>
</tbody>
</table>

Table 4.4 – Comparison of Computed and Estimated Mode Shapes

The comparisons presented in Table 4.4 illustrate that the computed mode shapes are in general agreement with the estimated ones.

As part of the structural monitoring project, the computer code ASAS was also used to compute the natural frequencies and mode shapes of the structure, IIRS (1982). A comparison between the SPECLIN, estimated and ASAS computed natural frequencies are presented in Table 4.5. From inspection of Table 4.5 it is seen that both SPECLIN and ASAS computed frequencies differ from the estimated frequencies by about the same degree. The first five mode shapes computed by ASAS are the same as computed by SPECLIN and summarised in Table 4.4.


<table>
<thead>
<tr>
<th>Mode Number.</th>
<th>Estimated</th>
<th>SPECLIN</th>
<th>ASAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.756</td>
<td>0.703</td>
<td>0.7633</td>
</tr>
<tr>
<td>2</td>
<td>0.756</td>
<td>0.756</td>
<td>0.8530</td>
</tr>
<tr>
<td>3</td>
<td>1.03⁰</td>
<td>0.896</td>
<td>0.8820</td>
</tr>
<tr>
<td>4</td>
<td>1.73</td>
<td>1.945</td>
<td>1.512</td>
</tr>
<tr>
<td>5</td>
<td>1.98</td>
<td>2.149</td>
<td>1.726</td>
</tr>
</tbody>
</table>

Table 4.5 – Comparison of Natural Frequencies of Kinsale Platform Alpha

4.2.3 Effects of Entrained Water on Natural Frequencies

During the lifetime of a structure it is possible that some hollow members that are initially dry become flooded due to material failure, this additional entrained water will affect the system mass matrix and hence may affect the natural frequencies of the structure.

In this section the validated model, SPECLIN, was used to consider the effects of variations in hydrodynamic mass on the natural frequencies of Kinsale Platform Alpha. Only the first five lowest natural frequencies and mode shapes were computed, corresponding again to the measured values. The sensitivity of the natural frequencies of the platform to the following scenarios were considered:

(1) main legs flooded

(2) all submerged members flooded.
(3) no member flooded.

(4) structure in air.

For each of the four scenarios above the total system mass varies, Table 4.6 details the mass of the system for each of these situations.

<table>
<thead>
<tr>
<th>Analysis No</th>
<th>Analysis Description</th>
<th>Mass (kg)</th>
<th>$\alpha_i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Main Legs Flooded</td>
<td>$1.029 \times 10^7$</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>All Members Flooded</td>
<td>$1.374 \times 10^7$</td>
<td>0.94</td>
</tr>
<tr>
<td>3</td>
<td>No Members Flooded</td>
<td>$0.915 \times 10^7$</td>
<td>1.06</td>
</tr>
<tr>
<td>4</td>
<td>Structure in Air</td>
<td>$0.818 \times 10^7$</td>
<td>1.21</td>
</tr>
</tbody>
</table>

$\alpha_i = \sqrt{\frac{M}{M_i}}$ where $M$ is the total mass for analysis no. 1 and $M_i$ is the total mass for analysis no. i

Table 4.6 – System Mass as a Function of Hydrodynamic Mass

In each of the four analysis outlined in Table 4.6 the mass due to the steel of the structure was $4.8625 \times 10^6$ kg and the mass of the platform superstructure was specified as being $3.3175 \times 10^6$ kg. The total mass contributed to the system from fouling and marine growth was $0.2053 \times 10^6$ kg. The mass due to marine growth was estimated based on measurements taken during the monitoring project. The first five natural frequencies for each of these analyses are presented in Table 4.7 along with the percentage variation from the respective frequencies of the prototype structure, namely analysis no. 1.
where $\beta_{ij} = F_{ij} / F$, where $F_{ij}$ is the frequency of $i^{th}$ mode associated with the $j^{th}$ analysis and $F$ is the frequency associated with analysis no. 1.

Table 4.7 - Effects of Hydrodynamic Mass on Natural Frequencies

In each case there is very little difference between the first three natural frequencies, although there is a significant variation in the total system masses. However, the natural frequencies associated with mode shapes four and five vary appreciably for different analyses. The first three mode shapes are dominated by large displacements of the platform at deck level which are induced by topside masses such as accommodation modules and operational equipment. The fourth and fifth modes are main bending and transverse bending modes respectively and hence are more strongly influenced by the distribution of masses throughout the structure. Since the hydrodynamic masses vary considerably throughout the structure for the different analyses, it is reasonable that such variations should affect the natural frequencies associated with mode shapes four and five. As expected, it can be seen that when $\alpha_i$ is larger than 1 the natural frequencies associated with the $i^{th}$ analysis are larger than the natural frequencies of the prototype. Conversely, when $\alpha_i$ is less than 1 the natural frequencies associated with the $i^{th}$ analysis are less than the natural frequencies of the prototype.
4.3 Spectral Response Validation

The eigensolver is considered to be well-validated from the above exercises and the next step in the process is to validate the computer code that computes spectral responses. The model is validated against other numerical methods and against an estimated structural response spectrum measured during the structural monitoring of Kinsale Head Gas Production Platform Alpha. Details of this validation are presented below.

4.3.1 Spectral Response Validation - Analysis I

Brebbia and Walker (1979) developed an expression for the response spectrum for translational displacement at the top of a cantilever when subjected to a random storm event. When the cantilever is modelled as a single-degree-of freedom system the displacement spectrum at frequency $\omega$, $S_u(\omega)$, is related the spectrum of force, $S_f(\omega)$, as follows

$$S_u(\omega) = |H(\omega)|^2 S_f(\omega) \quad (4.1)$$

The receptance relating input and output spectra in equation (4.1) is given by

$$|H(\omega)|^2 = \frac{1}{M^2 \omega_r^4 \left\{1 - \left(\frac{\omega}{\omega_r}\right)^2\right\}^2 + \left(2\gamma \frac{\omega}{\omega_r}\right)^2} \quad (4.2)$$

where
M is the system equivalent mass, incorporating hydrodynamic mass

\( \omega_r \) is the first natural frequency

\( \gamma \) is the critical damping ratio, incorporating hydrodynamic damping

The spectrum of force was obtained from the water surface spectrum assuming a linearised Morison’s equation. The sea state spectrum was computed using a Pierson-Moskowitz spectrum as given by

\[
S_n(\omega) = \frac{\alpha g^2}{\omega} \exp \left[ -\beta \left( \frac{g}{\omega W} \right)^4 \right]
\]

where

- \( W \) - wind speed (m/s)
- \( g \) - gravity
- \( \alpha = 0.0081 \)
- \( \beta = 0.74 \)

Brebbia and Walker applied their theory to calculate response spectra for the cantilever offshore structure shown in Figure 4.18; the same structure was also modelled in SPECLIN and the resulting response spectra compared.

The water-structure system shown in Figure 4.17 has the following properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexural rigidity</td>
<td>EI 2250x10^9 Nm^2</td>
</tr>
<tr>
<td>Cross-sectional area of member</td>
<td>A 29m^2</td>
</tr>
<tr>
<td>Cross-sectional area including marine growth</td>
<td>A_{tot} 78m^2</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>Density of material of member</td>
<td>$\rho_c \quad 2.5 \times 10^3 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>Outside diameter of member and marine growth</td>
<td>$D_{\text{tot}} \quad 10 \text{ m}$</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>$C_D \quad 1.0$</td>
</tr>
<tr>
<td>Inertia coefficient</td>
<td>$C_M \quad 1.0$</td>
</tr>
<tr>
<td>Lumped mass</td>
<td>$M_c \quad 2 \times 10^4 \text{ kg}$</td>
</tr>
</tbody>
</table>

The above structure was subjected to a storm event defined by using a wind speed, $W$, of 20 m/s in the Pierson-Moskowitz spectrum; the spectrum of sea surface elevation is shown in Figure 4.19 below.

The response spectra of the displacement of the top of the cantilever were calculated using both methods. The results are presented in Figure 4.20 for the response spectrum calculated using equations (4.1) and (4.2) and in Figure 4.21 for the response spectrum calculated using SPECLIN.

By inspection of Figures 4.20 and 4.21 it can be observed that both spectra are similar in shape, each peaking at both the maximum storm energy frequency and at the first natural frequency of the structure, 1.2 rads/s. The integral of a spectrum over its relevant frequency range results in the variance of the random signal, from which the standard deviation of the signal may be computed. In this case, the standard deviations for the displacement of the top of the cantilever as calculated by Brebbia and Walker and by SPECLIN are 0.314 m and 0.291 m respectively. From the above analyses it is shown that SPECLIN is accurately able to calculate response for this simple offshore structures.
4.3.2 Spectral Response Validation - Analysis II

The second validation of the SPEC model was performed against a spectral analysis carried out using ASAS on the three-dimensional space frame structure detailed in Section 4.2.1 above. The structure is subjected to a random storm event as defined by the ISCC spectrum shown in Figure 4.22 below, see Angelides and Connor (1980).

For the purposes of this analysis it is considered that the storm is unidirectional and that the direction of wave propagation is orthogonal to the broadside of the structure as shown in Figure 4.23. Hydrodynamic coefficients of $C_m = C_d = 1.0$ and a 6% damping ratio was selected by Angelides and used here for both analyses.

Response spectra were computed by Mitchell (1996) using both ASAS and SPEC for translational displacements in the direction of wave propagation at three node points as shown in Figure 4.23. The results of the spectral analyses using both programs for node 1 are presented in Figure 4.24 below, further comparisons are given in Mitchell (1996). By inspection it is clear that the results form both analyses are quite similar. In both cases the spectra peak at about 0.4 rads/sec, which is the frequency at which the water surface elevation spectrum peaks. Because the fundamental natural frequencies of the structure in this case are so large relative to the peak frequency of the sea state spectrum there is no resonant peak. Also, the shapes of the spectra as computed by ASAS and SPEC are very similar and both programs predict similarly decreasing response spectra at decreasing water depth from node 1 to node 3, Mitchell (1996), as expected. However, there are some differences between them which is to be expected from two different numerical procedures.
The standard deviations associated with the displacement spectra of the three nodes are presented in Table 4.8. Again the results show good agreement between the values computed using the two computational procedures with a maximum difference of 4%.

<table>
<thead>
<tr>
<th>Node</th>
<th>ASAS (m)</th>
<th>SPEC (m)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.87E-03</td>
<td>6.601E-03</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2.66E-03</td>
<td>2.66E-03</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.45E-03</td>
<td>1.41E-03</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4.8 - Comparison of Standard Deviations for Test Structure (Mitchell 1996)

4.3.3 Spectral Response Validation - Analysis III

The final validation of the spectral model was carried out on the Kinsale Head Gas Production Platform Alpha. As outlined in Section 4.2.2 above, a structural monitoring study was carried out on Kinsale Platform Alpha to determine some of its dynamic characteristics. In particular, one objective of the study was to relate input spectra of sea state to response spectra of structural displacements. During this study wave heights and structure responses were recorded simultaneously, from these records spectra of sea surface elevations and topside displacements were derived. During 9th Oct 1981 wave heights of a storm incident on the platform were recorded, Figure 4.25 shows a time trace of wave heights that were recorded during part of the storm. The sea state record shown above in Figure 4.25 has a significant wave height of 7.2m and the estimated spectrum associated with this sea state is shown in Figure 4.26. The sea state spectrum shows that most of the energy contained
within the storm is contained around the low frequency of 0.1Hz, this is typical of the energy
distribution in gravity waves during such storms.

A structural model of Kinsale Platform Alpha was developed using SPECMOD as outlined
above and spectra of displacements were computed for all degrees of freedom throughout the
structure due to the sea state spectrum of Figure 4.26. The waves incident on the structure
during this storm were observed as being long crested waves such that the wave rays made an
angle of 21° to the X-axis of the structure. Figure 4.27 shows a schematic drawing of a plan
view of the platform at mean water level and the angle between the structure and the waves.
Because the wave ray are not co-linear with global structure X-axis this is quite a rigorous
test of the model as it also tests that part of the code which computes the transformation
matrices between the wave and global coordinate systems.

During the structural monitoring project an accelerometer was located at the cellar deck level
and recorded structural accelerations in the X direction. Part of the time trace of this record
for the above storm is shown in Figure 4.28. Brennan (1983) derived a spectrum of
displacement associated with the spectrum shown in Figure 4.28, this is presented in Figure
4.29.

By inspection of the response spectrum in Figure 4.29 it is seen that most of the dynamic
response of the structure occurs at the frequency of the peak of the sea state spectrum,
0.1Hz, and that some lesser response occurs at the first natural frequency of the structure. It
is expected that there will be little resonance since the frequency of the peak of the sea state
spectrum and the first natural frequency of the structure are well separated.

Spectral displacements were calculated for the translational degree of freedom of the
structure in the structure X direction at the cellar deck using SPECLIN. Following the
recommendations of Brennan, the coefficients of mass and inertia used in the analysis were
$C_m = 1.0$ and $C_d = 1.0$ and 3.5% modal damping was used. The system receptances were
computed over the frequency range of $0 - 4$Hz at a frequency interval of 0.04Hz, this
frequency interval was found to be sufficiently refined so that good accuracy could be
obtained achieved. The displacement spectrum computed by SPECLIN is presented in
Figure 4.30.

By inspection of both spectra in Figures 4.29 and 4.30 it is seen that they are in excellent
agreement, having similar shapes and peaking at similar values. The above illustrates that
SPECMOD has accurately predicted the response of the top-side of Kinsale Alpha Platform
during the storm that prevailed on the 9th October 1981. This is a very good test for the
model and the author has not seen any model been validated in such a manner against the
results of a monitoring study of an existing large jacket platform.

4.4 Effects of Leg Spacing on Structural Response

Many frequency domain analyses of fixed offshore lattice platforms are carried out as lumped
analyses as opposed to consistent analyses; Brebbia and Walker (1979) illustrate such an
approach. In a lumped analysis the structural properties of the three-dimensional platform
are reduced to equivalent properties of a cantilever having a few degrees of freedom. This
section considers the spectral response of offshore jacket structures due to incident long
crested waves taking into account the effect of structure leg spacing on the phase of wave
loading on different parts of the structure.
Because a structure has a spatial extent in the direction of the propagation of a long-crested surface gravity waves phase differences due to these waves have effects on structural responses; these effects can best be illustrated by considering the response of a structure in the frequency domain. The lumped analysis approach cannot incorporate such phasing effects because of the one-dimensional nature of the structural model and thus cannot be confidently used in the analysis of offshore structures. To illustrate the difference in results between a structure modelled in one-dimension and a structure modelled in three-dimensions, typical receptance functions for the cantilever detailed in Section 4.2.1 and Kinsale Platform Alpha are presented in Figure 4.31 and 4.32 respectively.

It is seen from Figures 4.31 and 4.32 that the receptances are quite different in form from each other. The receptance function for the cantilever is a regular function and it exhibits only one peak, which occurs at the first natural frequency of the cantilever which is typical of a receptance function for such a system.

The receptance function of Kinsale Platform Alpha is more complex in shape and exhibits many peaks and troughs. The peaks and troughs associated with the receptance of Kinsale Platform Alpha illustrate the phase effect of harmonic waves of different frequencies on structure response. These phase effects are due to the fact that different parts of the platform are submerged under different parts of the wave simultaneously and, therefore, experience forces of different magnitudes and directions. For example, one leg may be under the crest of a wave generating a horizontal force in the direction of the wave ray when another leg is under a trough of the wave which is generating a force of similar magnitude but in the opposite direction. The net effect of the above example is that the overall structural motion is very small, this causes the troughs in the receptance functions in the above figures. This
phenomenon can be important in determining the overall structural configuration during the
design process and is considered in more detail below where the analyses of a portal frame and
a space frame are presented.

**Portal Frame Analysis**

A model was developed of the portal frame shown in Figure 4.33, which is located in 75m of
water and extends 25m to the top of the superstructure, Mitchell (1996). The structure was
modelled as an assemblage of 11 finite elements interconnected at 12 node points. At the
two top nodes lumped masses are applied to represent effects of superstructure masses on the
platform. Although the model is conceptually simple, and, therefore, easier to model, it
contains many of the main features affecting the dynamic response of actual jacket platforms.
For the purposes of this analysis it is considered that the structure is fully fixed at the seabed.
The input wave spectrum is described by a Pierson-Moskowitz spectrum, with a wind speed
of 20 m/s. For simplicity, only the first two natural frequencies and the associated mode
shapes of the portal frame were used in the analysis. The first two natural frequencies are
1.2rads/s and 1.402rads/s respectively. The first mode shape is an out-of-plane oscillation of
the frame and the second mode shape is an in-plane oscillation of the frame. In the analyses
only the transfer functions of the top node of the portal frame in the direction of wave
propagation will be considered. In the first analysis the direction of wave propagation is
normal to the plane of the frame; the resulting transfer function is shown in Figure 4.34. The
transfer function consists of a single peak, occurring at the first natural frequency of the
structure in the direction of loading, as expected. If the direction of wave propagation is now
turned through 90°, most of the response occurs from the second mode and thus the second
natural frequency is excited. The transfer function due to the second storm, shown in Figure 4.35, shows this function is quite different in nature to than the one shown in Figure 4.34. No longer does it contain a single peak, but instead there are many peaks.

The transfer function shown in Figure 4.34 is a represented by a smooth curve peaking at the first natural frequency of the structure. Because the wave crests are parallel to the plane of the frame, the transfer function is not influenced by wave phasing effects. However, the irregular transfer function shown in Figure 4.35 is caused by dissimilar forces acting on the legs of the structure from the long crested waves. At certain frequencies, the crest of a wave will be located at the center of one leg and the trough of the wave will be located at the center of the other leg; this condition causes exactly equal and opposite forces to be experienced at the respective legs simultaneously. These frequencies lead to a zero net displacement, no-load frequencies, of the structure and thus the irregular type of transfer function results when waves of varying frequencies are applied to the structure.

The consequences of the no-load frequencies were considered in more detail by considering portal frames with leg spacings of 20, 30, 40, 50 and 60 meters which were analysed using SPECLIN to consider optimum leg spacing. Again, a Pierson-Moskowitz input spectrum with wind speed 20 m/s was specified with the direction of wave propagation in the plane of the portal frames. During these analyses, only the second natural frequencies were excited, and these are listed in Table 4.9 for the various configurations.

The receptance functions obtained were similar in form in all cases to that shown in Figure 4.35. The frequencies for which the net horizontal loads are zero can easily be calculated from linear wave theory by determining the frequency of those waves that have multiples of
Table 4.9 - Second Natural Frequencies for Various Leg Spacings

<table>
<thead>
<tr>
<th>Leg spacing (m)</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd nat. frequency [rad/sec]</td>
<td>1.604</td>
<td>1.529</td>
<td>1.510</td>
<td>1.483</td>
<td>1.402</td>
</tr>
</tbody>
</table>

half their wavelengths equal to the leg spacing of the portal frames. Such frequencies were calculated for the five structures, Mitchell (1996), and the no-load frequencies as predicted by the SPECLIN receptances and linear wave theory are presented in Table 4.10. The results obtained from both methods were found to be in close agreement.

Table 4.10 - No-load Frequencies Computed by SPEC and Wave Theory (Mitchell 1996)

<table>
<thead>
<tr>
<th>Spacing</th>
<th>20 m</th>
<th>30 m</th>
<th>40 m</th>
<th>50 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>Theory</td>
<td>SPEC</td>
<td>Theory</td>
<td>SPEC</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>1.24</td>
<td>1.24</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>2.15</td>
<td>2.16</td>
<td>1.76</td>
<td>1.76</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>2.78</td>
<td>2.80</td>
<td>2.27</td>
<td>2.28</td>
</tr>
<tr>
<td>$\omega_4$</td>
<td>3.28</td>
<td>3.28</td>
<td>2.68</td>
<td>2.68</td>
</tr>
<tr>
<td>$\omega_5$</td>
<td>3.72</td>
<td>3.72</td>
<td>3.04</td>
<td>3.04</td>
</tr>
<tr>
<td>$\omega_6$</td>
<td>4.12</td>
<td>4.12</td>
<td>3.36</td>
<td>3.36</td>
</tr>
<tr>
<td>$\omega_7$</td>
<td>4.48</td>
<td>4.48</td>
<td>3.65</td>
<td>3.64</td>
</tr>
<tr>
<td>$\omega_8$</td>
<td>4.81</td>
<td>4.80</td>
<td>3.93</td>
<td>3.92</td>
</tr>
<tr>
<td>$\omega_9$</td>
<td>7.12</td>
<td>7.12</td>
<td>4.18</td>
<td>4.16</td>
</tr>
<tr>
<td>$\omega_{10}$</td>
<td>7.41</td>
<td>7.40</td>
<td>4.42</td>
<td>4.40</td>
</tr>
</tbody>
</table>

Figure 4.36 presents a 3-dimensional plot of the transfer functions computed for each of the 5 structures. It is worth considering the variations in the shapes of the transfer functions and
comparing the peak ordinates of the various functions. When the legs spacing is relatively small then only high frequency waves will contribute to no-load conditions and thus over the frequency range of interest there will be few frequencies corresponding to this situation. As the leg spacing is increased the number of no-load frequencies also increases within the frequency range considered. When the leg spacings are at 20m, 30m, 50m and 60m it can be observed from Figure 4.36 that the transfer functions display one large peak occurring at the second natural frequency of the structure and some secondary peaks in between the no-load frequencies. However, the transfer function for the structure that has a leg spacing of 40m displays two similar size peaks either side of the second natural frequency of the structure. The reason for this is that the second natural frequency of this latter structure occurs at 1.52 rad/sec which is a no-load frequency for this configuration of structure. Because of the coincidence of this natural frequency with a no-load frequency, there will not be any resonant behaviour at the natural frequency. This is an interesting situation and can be used when designing structures subjected to certain types of random vibrations to avoid resonance.

The spectra of response due to the applied Pierson-Moskowitz spectrum for each of the structures are shown in Figure 4.37. All the displacement response spectra show peaks occurring at the frequency of peak energy in the wave spectrum, that is the first peak in each case. The structures with leg spacings at 20m, 30m, 50m and 60m also show relatively large spectral peaks at their second natural frequencies, whereas the structure with a leg spacing of 40m shows two other peaks, one either side of it’s second natural frequency. It is noted that the structure having a leg spacing of 60m exhibits the largest peak of all analyses at its natural frequency. This maximum peak occurs because the structure’s natural frequency is the
frequency at which a surface wave is exactly in phase with the leg spacing and hence maximum forces are experienced simultaneously on both legs.

The standard deviations of the structures with leg spacing up to 50m are provided in Table 4.11. The standard deviations show that for the four structures considered the least displacement is expected for the structure with 40m leg spacing since the no-load frequency occurs at the natural frequency.

<table>
<thead>
<tr>
<th>Leg spacing (m)</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
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<td>1.483</td>
<td>1.402</td>
</tr>
</tbody>
</table>

Table 4.11 – Standard Deviations for Various Leg Spacings

Space Frame Analysis

The second structure analysed under this section was a modified version of the space frame detailed in Section 4.2.1 above. The superstructure masses were adjusted so that the first five natural frequencies are as shown in Table 4.12.

<table>
<thead>
<tr>
<th>Natural Frequency</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>[rad/sec]</td>
<td>2.5</td>
<td>5.4</td>
<td>6.9</td>
<td>8.67</td>
<td>9.23</td>
</tr>
</tbody>
</table>

Table 4.12 – Space Frame Natural Frequencies
This structure was then subjected to a number of separate storm events and the receptances of the displacements of the top of the structure in two orthogonal horizontal directions computed. Each storm event was defined by the sea-state spectrum presented in Figure 4.38, but the angle of incidence between the wave rays and the structure, $\alpha$, as defined in Figure 4.39, varied.

During the first storm $\alpha$ is $0^\circ$ and the displacement receptances in the $X_g$ and $Y_g$ directions at the top of the structure are given in Figures 4.40 and 4.41. Figure 4.40 shows that the second mode of vibration is excited by this storm, as expected since this oscillation takes place in the plane of the propagation of the wave. It is also seen that at the second natural frequency the receptance is very small, as this frequency is a no-load frequency. Figure 4.41 shows that the magnitudes of this receptance function in the $Y_g$ direction are much less than the values in the $X_g$ direction, as expected, with just a single peak being experienced at the first natural frequency.

During the second storm $\alpha$ is $45^\circ$ and the displacement receptances in the $X_g$ and $Y_g$ directions are given in Figures 4.42 and 4.43. Figure 4.42 shows that the second mode of vibration is excited by this storm, again, this is as expected since there is a wave force component acting in the $X_g$ direction. It is also seen, again, that at the second natural frequency the receptance is very small. Figure 4.43 shows in this case that the magnitudes of this receptance function in the $Y_g$ direction are much greater than the values in the $X_g$ direction. This is expected since now the angle of wave attack is such that there is relatively quite a large force being applied in the plane of oscillation of the first mode shape.
Similar results were obtained for other directions of wave propagation. Because SPECLIN can compute receptances due to waves propagating from different directions the model can easily be extended to compute offshore structural responses due to short-crested waves.

4.4 Summary and Conclusions

The finite element based linear spectral response model developed in Chapter 3 was applied to various structures. The model eigensolver was first validated against a commercial program and against results from a monitoring survey and the spectral response model was also validated against a commercial program and results from the same monitoring survey. In all cases very good agreement was obtained between SPECLIN and the other available data. SPECLIN has been configured to run on a PC and the above analyses have been performed on a 330MHz Pentium PC. The eigenvectors of Kinsale Platform are computed in about one minute and the full spectral analysis is carried in 12 minutes, this demonstrates that the program is both accurate and computationally efficient. The linearised drag coefficient developed in Section 3. provides a new analytical expression for this coefficient and this improvement over previous numerically computed coefficients improves the accuracy and the efficiency of the model.

The program SPECLIN must iterate to the required solution because initially structural velocities are assumed to be zero; from the applications it has been found that the program converges after 4 iterations. It has also been observed that a maximum of 21 integration points are necessary to accurately compute the hydrodynamic forces induced on the structure.
One of the most interesting results from the linear spectral analyses carried out above is the effect of leg spacings on spectral responses through the receptance functions. To date the author is not aware of any analysis that has previously been performed to consider the effects of leg spacings on non-linear spectral responses and thus considers these effects in the next two chapters.

In the above analyses Airy wave theory was used to compute forces on structural members. It is important to realise that this wave theory has certain limitations and its range of application must be appreciated. Airy wave theory is based on the assumption that the wave height is small in comparison to the wavelength and the water depth. Because of this assumption the nonlinear terms which involve products of terms of order of the wave height are negligible in comparison with the remaining terms and the shape of the resulting wave crest is sinusoidal. Among the shortcomings associated with Airy wave theory are crest kinematics; Stokes second order shows that second order components at twice wave frequency are superimposed on the fundamental components predicted by Airy wave theory inducing steeper crests and flatter troughs than are calculated using a sinusoidal profile. Also, Airy wave theory restricts the computation of forces in the vertical to the mean water line; other theories account for the vertical movement of the water line as the wave passes through the structure. However, although Airy wave theory has these and other shortcomings it is used regularly to solve practical engineering problems and, as shown above, is capable of producing accurate results.
Figure 1 - Test Structure (Angelides 1978)
Figure 4.2 - Test Structure First Mode Shape

Figure 4.3 - Test Structure Second Mode Shape

Figure 4.4 - Test Structure Third Mode Shape
Figure 4.5 - Test Structure Fourth Mode Shape

Figure 4.6 - Test Structure Fifth Mode Shape
Figure 4.7 - Location of Kinsale Head Gas Field
Figure 4.8 - Schematic Diagram of Kinsale Platform Alpha

Figure 4.9 - Accelerometer Deployment
Figure 4.10 - Wave Height and Structural Acceleration Records (Brennan 1983)
Figure 4.11 - Finite Element Model of Kinsale Platform Alpha

Figure 4.12 - Composite Cross-Section of Main Leg
Figure 4.13 - Kinsale Platform Alpha First Mode Shape
Figure 4.14 - Kinsale Platform Alpha Second Mode Shape
Figure 4.15 - Kinsale Platform Alpha Mode Shape Three
Figure 4.16 - Kinsale Platform Alpha Mode Shape Four
Figure 4.17 - Kinsale Platform Alpha Mode Shape Five
Figure 4.18 - Cantilever Structure

Figure 4.19 - Sea State Spectrum (Brebbia and Walker 1979)
Figure 4.20 – Brebbia’s Response Spectrum (Brebbia and Walker 1979)

Figure 4.21 – Response Spectrum - SPECLIN
Figure 4.22 – ICC Sea-State Spectrum (Angelides 1982)

Figure 4.23 – Response Nodes
Figure 4.24 – Response Spectrum of Node 1 (Mitchell 1996)
Figure 4.25 - Incident Wave Height Record
9th October 1981 (Brennan 1983)

Figure 4.26 - Sea State Spectrum 9th October 1981
(Brennan 1983)
Figure 4.27 - Relationship Between Wave and Structure Coordinate System

Figure 4.28 - Acceleration Record at Cellar Deck
9th October 1981 (Brennan 1983)
Figure 4.29 - Estimated Displacement Spectrum
9th October 1981 (Brennan 1983)

Figure 4.30 - Computed Displacement Spectrum
9th October 1981
Figure 4.31 - Typical Receptance Function for a Cantilever

Figure 4.32 - Typical Receptance Function for Kinsale Platform Alpha
Figure 4.33 - Portal Frame Model (Mitchell 1996)

Figure 4.34 - Transfer Function for Top Node
Figure 4.35 - Transfer Function for Top Node (60m leg-spacing)

Figure 4.36 - Receptances as a Function of Leg-spacing (Mitchell 1996)
Figure 4.37 - Response Spectra as a Function of Leg-spacing
Figure 4.38 – Sea-State Spectrum Incident on Space Frame

Figure 4.39 – Angle of Incidence of Wave on Structure
Figure 4.42 - Space Frame Receptance – $X_g$: $\alpha = 45^\circ$

Figure 4.43 - Space Frame Receptance – $Y_g$: $\alpha = 45^\circ$
Figure 4.44 - Space Frame Receptance – $X_{g}$: $\alpha = 90^\circ$

Figure 4.45 - Space Frame Receptance – $Y_{g}$: $\alpha = 90^\circ$
CHAPTER 5

DEVELOPMENT OF A NON-LINEAR SPECTRAL MODEL

5.1 Introduction

In Chapter 3 the details of a linear spectral model for analysing offshore platforms was presented. The linear model was developed by equivalently linearising the Morison drag force term, thereby allowing linear spectral theory to be applied to the problem. The relationship between sea state and structural displacements are not actually linearly related because of the non-linear drag force term and other nonlinearities of the wave kinematics.

In this chapter details are presented of a non-linear spectral model using a perturbation technique. In this development, some of the non-linear dependencies of excitation on wave kinematics through the drag force term are maintained; again, linear Airy wave theory is used to describe the wave kinematics. The methodology is based on the modified Morison equation incorporating the effects of relative velocities and accelerations and the hydrodynamic damping terms are time varying. However, the latter terms are assumed small and thus the equations of motion are formulated as a system of successive linear time invariant equations for each successive term of the perturbation expansion.

In particular, this development is an extension of research carried out by Lipsett (1985) and Eatock-Taylor and Rajagopalan (1982) who considered the effects of non-linearities on single degree of freedom systems and lumped mass systems respectively. In Chapter 4
it is shown that phasing effects due to leg spacings affect structural responses. The non-linear model described herein has been developed to model structures in three-dimensions and is used to investigate the effects of leg spacing on spectral response due to the higher order harmonics in the force term. Previous researchers have not investigated these effects on the non-linear response of platforms.

5.2. Finite Element Formulation

The non-linear model is developed for planar jacket structures with vertical members of constant cross-sectional area only; these members are relatively easy to model but yet the effects of structure spatial extent can be fully assessed. A typical structural offshore platform that can be analysed using this model is shown schematically in Figure 5.1. The finite element method was used to compute nonlinear spectra of structural responses due to incident waves. The virtual work approach, as outlined in Chapter 3, is also used here to derive the governing equations of motion.

Consider a long-crested wave propagating in the X direction, the force at a point on a structure is again, using Morison's equation, comprised of a drag force term and an inertia force term. Following the same approach as adopted in Section 3.3, the vector of drag force at a point on a member in the X-Z plane is given by

\[
F_D = C_D \begin{bmatrix} 0 & (v_2 - \dot{\gamma})|r| \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ f \\ 0 \end{bmatrix}
\]

(5.1)
Since only in-plane motions of vertical members are considered, then

$$|r| = |v_2 - \dot{v}|$$  \hspace{1cm} (5.2)

Now, following the methodology of Eatock-Taylor and Rajagopalan (1982), let

$$f(v_2) = v_2|v_2|$$  \hspace{1cm} (5.3)

In order to find \( f(v_2 - \dot{v}) \), the Taylor series is used to expand \( f(v_2) \) about \( \dot{v} \), thus \( f(v_2 - \dot{v}) \) can be written as

$$f(v_2 - \dot{v}) = f(v_2) + (-\dot{v})f'(v_2) + \frac{(-\dot{v})^2}{2!}f''(v_2) + \frac{(-\dot{v})^3}{3!}f'''(v_2) + \ldots$$  \hspace{1cm} (5.4)

It can be shown, Thomas and Finney (1980) that

$$f'(v_2) = 2|v_2|$$  \hspace{1cm} (5.5)

$$f''(v_2) = 2\operatorname{sgn}(v_2)$$  \hspace{1cm} (5.6)

$$f'''(v_2) = 4\delta(v_2)$$  \hspace{1cm} (5.7)

Substitution equations (5.5) to (5.7) into equation (5.4) leads to

$$f(v_2 - \dot{v}) = v_2|v_2| - 2\dot{v}|v_2| + \dot{v}^2\operatorname{sgn}(v_2) - 2\frac{\dot{v}^3}{3}\delta(v_2) + \ldots$$  \hspace{1cm} (5.8)
In equation (5.8) it is considered that the structural velocities are small relative to the water particle velocities, also the second and greater terms lead to hydrodynamic damping that is time varying and non-linear. The linear hydrodynamic damping component is generally considered the most significant term under storm conditions as shown by Rajagopalan and Eatock-Taylor (1982) from a stochastic averaging analysis. Thus equation (5.8) is reduced to

\[(v_2 - \dot{v})|v_2 - \dot{v}| = v_2|v_2| - 2\dot{v}|v_2| \quad (5.9)\]

The drag force vector of equation (5.1) can thus be written as

\[\mathbf{F}_D = C_D \begin{bmatrix} 0 \\ v_2|v_2| - 2\dot{v}|v_2| \\ 0 \end{bmatrix} = C_D \begin{bmatrix} 0 \\ v_2|v_2| \\ 0 \end{bmatrix} + C_D \begin{bmatrix} 0 \\ -2\dot{v}|v_2| \end{bmatrix} \quad (5.10)\]

If equation (5.10) is used to develop the equations of motion of a structural member then the second term on the right hand side would lead to a time varying damping matrix in a system of linear coefficient matrices. The hydrodynamic damping is simplified by utilising statistical properties of the water particle kinematics. Consider the water particle velocities to be zero mean stationery ergodic Gaussian random processes, then the expected value of $|v_2|$, Papoulis (1965), is
Thus, the hydrodynamic damping term of equation (5.10) may be re-written as

\[ 2C_D|v_2|\dot{v} = 2C_D E|v_2|\dot{v} + 2C_D |v_2| - E|v_2|\dot{v} = \]

\[ C_D \frac{8}{\pi \sigma_{v_2}} \dot{v} + 2C_D \left[ |v_2| - \frac{2}{\sqrt{\pi}} \sigma_{v_2} \right] \dot{v} \]  

The first term on the right hand side of equation (5.12) represents constant hydrodynamic damping and is added to the corresponding structural damping terms on the left-hand side of the general equations of the motion analogous to the procedure adopted when developing the linear model. The second term represents the fluctuating component of hydrodynamic damping and remains on the right side of the general equations of motion.

Beam elements are used to represent structural members, as for the linear model. Applying the finite element method to the water-structure system, analogous to the development of Section 3.3, the governing equations of motion may be written as follows for a member of the structure.
\[
[M]\ddot{u} + [C]\dot{u} + [K]u = \int_0^1 [G]^T f_1 \begin{bmatrix} 0 \\ \dot{v}_2 \\ 0 \\ 0 \end{bmatrix} \, dx + \int_0^1 [G]^T f_2 \begin{bmatrix} 0 \\ \dot{v}_2 \\ 0 \\ 0 \end{bmatrix} \, dx + \\
\int_0^1 [G]^T f_3 \begin{bmatrix} 0 \\ v_2 \dot{v}_2 \\ 0 \\ 0 \end{bmatrix} \, dx - 2 \int_0^1 [G]^T f_3 \begin{bmatrix} 0 \\ v_2 \dot{v}_2 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \, dx
\]

(5.12)

The terms on the right hand side of equation (5.12) are determined by substituting the shape functions into \([G]\), and the relevant parameters into \(f_1\), \(f_2\) and \(f_3\). From Appendix 1 we have

\[
[G]^T = \begin{bmatrix}
g_1 & 0 & 0 & 0 \\
0 & g_3 & 0 & 0 \\
0 & 0 & g_3 & 0 \\
0 & 0 & 0 & g_1 \\
0 & 0 & g_7 & 0 \\
0 & g_4 & 0 & 0 \\
g_2 & 0 & 0 & 0 \\
0 & g_5 & 0 & 0 \\
0 & 0 & g_5 & 0 \\
0 & 0 & 0 & g_2 \\
0 & 0 & g_8 & 0 \\
0 & g_6 & 0 & 0
\end{bmatrix}
\]

(5.13)

Therefore
For the purposes of this development, it is assumed that the nodal velocities and accelerations are sufficient to compute the forces in equation (5.12) and thus are considered constant in equation (5.14). Thus, the evaluation of equation (5.14) depends on the integration of the relevant shape functions and the length of the member as shown below.

\[ \int_{0}^{1} g_3(x) \, dx = \int_{0}^{1} \left[ 1 - \frac{3x^2}{1} + 2x^3 \right] \, dx = \frac{1}{2} \]  

\[ \int_{0}^{1} g_4(x) \, dx = \int_{0}^{1} \left[ x - 2x^2 + \frac{x^3}{1} \right] \, dx = \frac{1}{12} \]  

\[ \int_{0}^{1} g_5(x) \, dx = \int_{0}^{1} \left[ 3x^2 \frac{1}{1} - 2x^3 \frac{1}{1} \right] \, dx = \frac{1}{2} \]
\[
\int_0^1 g_6(x)dx = \int_0^1 \left[-\left(\frac{x^2}{1}\right) + \left(\frac{x^3}{l^2}\right)\right]dx = -\frac{l^2}{12}
\]  

(5.18)

In equation (5.12)

\[f_1 = f_2 = C_M^* \rho_w A\]

where

A is the cross-sectional area of the member.

Thus the first two terms of equation (5.12) may be combined and written as:

\[
F_M = C_M^* \begin{bmatrix}
0 \\
\rho_w Al\dot{\nu}_1^2 \\
0 \\
0 \\
0 \\
0 \\
\rho_w Al^2 \dot{\nu}_2^2 \\
0 \\
0 \\
0 \\
0 \\
-\rho_w Al^2 \dot{\nu}_2^2 \\
6
\end{bmatrix}
\]  

(5.19)

where
\( \dot{v}_2 = \) horizontal component of water particle acceleration at node 1 of the element

\( \dot{v}_2 = \) horizontal component of water particle acceleration at node 2 of the element

By following a similar development as the above, the third terms on the right hand side of equation (5.12) can be written as

\[
F_{D1} = C_D \\
\begin{bmatrix}
0 \\
\rho_w Dl v_2' v_2' \\
2 \\
0 \\
0 \\
0 \\
\rho_w Dl v_2' v_2' \\
2 \\
0 \\
0 \\
0 \\
\rho_w Dl v_2' v_2' \\
12
\end{bmatrix}
\]

(5.20)

where

\( v_2' = \) horizontal component of water particle velocity at node 1 of the element

\( v_2' = \) horizontal component of water particle velocity at node 2 of the element
Similarly, the fourth term on the right hand side of equation (5.12) may be written as

\[
F_{D2} = \begin{bmatrix}
0 \\
\rho_w DlF_1^* \hat{v}^1 \\
0 \\
0 \\
0 \\
\rho_w Dl^2 F_1^* \hat{v}^1 \\
0 \\
\rho_w DlF_2^* \hat{v}^2 \\
0 \\
0 \\
\rho_w Dl^2 F_2^* \hat{v}^2 \\
\end{bmatrix}
\]  

(5.21)

where

\[
F_1^* = |v_1^*| - \frac{2}{\sqrt{\pi}} \sigma_{\hat{v}_1}
\]

\[
F_2^* = |v_2^*| - \frac{2}{\sqrt{\pi}} \sigma_{\hat{v}_2}
\]

\( \hat{v}_1 \) = horizontal component of structural velocity at node 1

\( \hat{v}_2 \) = horizontal component of structural velocity at node 2

Equation (5.21) is now rewritten as:
Thus the general equations of motion for a beam element may be written as:

\[
[M] \ddot{U} + [C] \dot{U} + [K] U = F_M + F_{D1} + F_{D2} = F_1 + F_2 \tag{5.23}
\]

where

\[ F_1 = F_M + F_{D1} \]

The equations of motion for each element are then assembled to give the global equations of motion of the structure-water system as

\[
[M]_g \ddot{U}_g + [C]_g \dot{U}_g + [K]_g U_g = F_{ig} + F_{2g} \tag{5.24}
\]

The second term on the right hand side of equation (5.24) is a function of the structural velocities, thus the solution to this equation can be obtained in a number of ways, such as perturbation or an iteration procedure. In this instance a perturbation technique is used, a
summary of the general perturbation method is presented in Appendix VI. It is assumed that the fluctuating damping term is relative small and hence the force terms on the right hand side of equation (5.24) may be written as

$$ P = F_{tg} + \varepsilon F_{2g} $$  \hspace{1cm} (5.25)

where

$\varepsilon$ is the perturbation parameter

From the perturbation technique, the solution to equation (5.23) to the force $P$ is expressed in an asymptotic series as

$$ U_g = U_g^{(0)} + \varepsilon U_g^{(1)} + \varepsilon^2 U_g^{(2)} + \ldots \ldots $$  \hspace{1cm} (5.26)

where

$U_g^{(0)}$ is the zero order vector of global displacements

$U_g^{(1)}$ is the first order vector of global displacements

$U_g^{(2)}$ is the second order vector of global displacements

It is generally considered only necessary to retain terms up to $\varepsilon^2$, Lipsett (1986). Substituting equation (5.26) into equation (5.23) and equating terms of the same order of $\varepsilon$, the following expressions can be written

$$ [M]_g \ddot{U}_g^{(0)} + [C] \dot{U}_g^{(0)} + [K] U_g^{(0)} = P^{(0)} $$  \hspace{1cm} (5.27)

$$ [M]_g \ddot{U}_g^{(1)} + [C] \dot{U}_g^{(1)} + [K] U_g^{(1)} = P^{(1)} $$  \hspace{1cm} (5.28)
\[
[M]_g \ddot{U}^{(2)}_g + [C] \dot{U}^{(2)}_g + [K] U^{(2)}_g = P^{(2)}
\]  

(5.29)

where

\begin{align*}
P^{(0)} &= F_{1g} \\
P^{(1)} &= F_{2g}^{(0)} \\
P^{(2)} &= F_{2g}^{(1)}
\end{align*}

(5.30) \quad (5.31) \quad (5.32)

and \(F_{2g}^{(0)}\) uses the structural velocities \(\dot{U}_{g}^{(0)}\) to compute the force vector and \(F_{2g}^{(1)}\) uses the structural velocities \(\dot{U}_{g}^{(1)}\) to compute the force vector.

5.3 System Receptances

Soong (1973) shows that the response spectral density at any degree of freedom \(x\) due to the application of the perturbation method is given by

\[
S_{xx}(\omega) = S_{x^{(0)}x^{(0)}}(\omega) + \varepsilon [S_{x^{(0)}x^{(0)}}(\omega) + S_{x^{(0)}x^{(0)}}(\omega)] + \varepsilon^2 [S_{x^{(0)}x^{(0)}}(\omega) + S_{x^{(0)}x^{(0)}}(\omega) + S_{x^{(0)}x^{(0)}}(\omega)]
\]

(5.33)

where

\(S_{x^{(0)}x^{(0)}}(\omega)\) is the cross-spectrum of the \(i^{th}\) order response of degree of freedom \(x\)

and the \(j^{th}\) order response of degree of freedom \(x\).
Robson shows the cross-spectrum of response between two degrees of freedom, in general, are related to cross-spectra of force by the following relationship

\[ S_{xy}(\omega) = \sum_{r=1}^{n} \sum_{s=1}^{n} \alpha_{xp,r}(\omega) \alpha_{yp,s}(\omega) S_{p,p_s}(\omega) \]  

(5.34)

where

\[ \alpha_{xp,r}(\omega) \] is the complex receptance of response at frequency \( \omega \) at \( x \) due to a force \( e^{i\omega t} \) applied at degree of freedom \( r \)

\[ S_{p,p_s}(\omega) \] is the cross-spectra of force between degrees of freedom \( r \) and \( s \) at frequency \( \omega \).

\( n \) is the total number of degrees of freedom

Thus in order to solve for \( S_{xx}(\omega) \) in equation (5.33), it is first necessary to compute the system receptances at all degrees of freedom due to unit harmonic loading applied to each degree of freedom separately.

The receptances at all degrees of freedom due to a unit harmonic load at, say, degree of freedom \( k \) having a frequency \( \omega \) are obtained as follows. Consider equation (5.24) with only a unit harmonic load applied at degree of freedom \( k \)

\[
\begin{bmatrix}
0 \\
\vdots \\
0 \\
\vdots \\
0
\end{bmatrix} + [C] \begin{bmatrix} 0 \\
\vdots \\
0 \\
\vdots \\
0
\end{bmatrix} + [K] \begin{bmatrix} 0 \\
\vdots \\
0 \\
\vdots \\
0
\end{bmatrix} = e^{i\omega t} \quad \text{Row} \; k
\]

(5.35)
Assume the system eigenvalues are given by $[\Omega]$ and eigenvectors by $[\Phi]$, which have been defined above in Section 3.4.

Now define $r$ generalised coordinates $Z^k$ such that

$$U^k = [\Phi]Z^k$$  \hspace{1cm} (5.36)

Substitute for $U^k$ and its derivatives in equation (5.35) and then pre-multiply across by $[\Phi]^T$ to get

$$[\Phi]^T[M][\Phi]Z^k + [\Phi]^T[C][\Phi]Z^k + [\Phi]^T[K][\Phi]Z^k = e^{i\omega t}$$  \hspace{1cm} (5.37)

where

$r$ is the number of eigenpairs used

Equation (5.37) can be simplified to give the following uncoupled system

$$[I]Z^k + [\Theta]Z^k + [\Omega^2]Z^k = e^{i\omega t}$$  \hspace{1cm} (5.38)
assuming that the damping matrix $[C]$ from equation (5.37) is diagonalised in an equivalently linearised manner as detailed for the linear spectral model in Chapter 3.

For any degree of freedom, say $l$, of the generalised coordinate system the following equation can be written

$$\ddot{Z}_l^k + C_i \dot{Z}_l^k + \omega_i^2 Z_l^k = \phi_m e^{i\omega t} \quad (5.39)$$

where

$Z_l$ is the $l^{th}$ generalised coordinate

$C_i$ is the entry on the $i^{th}$ diagonal of $[\Theta]$

$\omega_i$ is the $i^{th}$ natural frequency

The general solution to $Z_l^k$ in equation (5.39) is

$$Z_l^k = Z_0^k e^{i\omega t} \quad (5.40)$$

$$\dot{Z}_l^k = i\omega Z_0^k e^{i\omega t} \quad (5.41)$$

$$\dddot{Z}_l^k = -\omega^2 Z_0^k e^{i\omega t} \quad (5.42)$$

Thus by substituting equations (5.40) - (5.42) into equation (5.39) the following expression can be written

$$(- \omega^2 + i\omega C_1 + \omega_i^2) Z_0^k = \phi_m \quad (5.43)$$
\[ Z^k = \frac{\phi_m}{-\omega^2 + i\omega C_1 + \omega_i^2} \]
\[ = \frac{\phi_m (\omega_i^2 - \omega^2 - i\omega^2 C_1)}{(\omega_i^2 - \omega^2)^2 + \omega^2 C_1^2} \]
\[ = Z_{11}^k + iZ_{21}^k \]  

where

\[ Z_{11}^k = \frac{\phi_m (\omega_i^2 - \omega^2)}{(\omega_i^2 - \omega^2)^2 + \omega^2 C_1^2} \]  

\[ Z_{21}^k = \frac{-\phi_m \omega^2 C_1}{(\omega_i^2 - \omega^2)^2 + \omega^2 C_1^2} \]

Thus, in general, for all degrees of freedom

\[ Z_0^k = Z_{11}^k + iZ_{21}^k \]  

and

\[ Z^k = Z_0^k e^{i\omega t} \]

Now substituting for \[ Z^k \] back into equation (5.36) we can write

\[ U_k^e = [\Phi] Z_0^k e^{i\omega t} \]
\[ = \alpha_k e^{i\omega t} \]

where \[ \alpha_k \] represents the complex system receptances of all degrees of freedom in the global coordinate system at frequency \( \omega \) due to a unit harmonic load applied at degree of freedom \( k \).
5.4 Response Spectra

As shown in equation (5.33) the spectra of structural responses are comprised of a linear combination of zero order, first order and second order response spectra. The significance and evaluation of these spectra are now considered. The general approach adopted in evaluating the response spectra follows the approach used by Rajagopalan and Eatock-Taylor for the one-dimensional model and extended here to consider space frames.

5.4.1. Zero Order Response Spectra

The zero order response spectra of equation (5.33) are computed by solving equation (5.27) that is rewritten here

\[ [M] \ddot{U}_g^{(0)} + [C] \dot{U}_g^{(0)} + [K] U_g^{(0)} = P^{(0)} \]  

(5.50)

The force term on the right hand side of equation (5.50) contains drag forces terms that are non-linear with respect to water particle velocities as illustrated by equation (5.20). These non-linear functions are expressed as a series of Hermite Polynomials which possess useful orthogonality properties. It has been shown that Hermite Polynomials converge rapidly to the exact solution and that the approach can easily be extended to the situation when steady currents prevail, see Rajagopalan and Eatock-Taylor (1982).

Consider, in general, a process \( p \) that is a zero mean ergodic random Gaussian process, having unit variance, then the following expression may be written
\[ p|p| = \sum_{k=1,3,...}^{\infty} a_k He_k(p) \]  

(5.51)

where

\[ a_k \] are coefficients to be solved for

\[ He_k(p) \] are the Hermite Polynomials which are discussed in Appendix VI.

By approximating the non-linear function of \( p \) to the Hermitian Polynomial as shown in equation (5.51) an error is introduced into the original non-linear function:

\[ Er(p) = p|p| - \sum_{k=1,3,...}^{\infty} a_k He_k(p) \]  

(5.52)

Then the coefficients \( a_k \) are determined by minimising the above error term in an mean square sense as follows:

\[ \frac{\partial}{\partial a_k} \langle |Er(p)|^2 \rangle = 0 \]  

(5.53)

From equation (5.53) it can be shown that

\[ \sum_k \langle He_k(p)He_j(p) \rangle a_k = \langle p|p|He_j(p) \rangle \]  

(5.53)

Now from Appendix VI the orthogonality relationship between Hermite Polynomials can be written as
Since \( p \) is a Gaussian function, the expected value of a function \( f(p) \) is

\[
\langle f(p) \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(p) e^{-\frac{p^2}{2}} dp
\]  

(5.55)

Thus by incorporating equations (5.54) and (5.55) into equation (5.53) the following expression is obtained for the coefficient \( a_k \)

\[
a_k = \frac{2}{k!\sqrt{2\pi}} \int_0^{\infty} p^2 He_k(p) e^{-\frac{p^2}{2}} dp, \quad k \text{ odd}
\]  

(5.56)

The integral in equation (5.56) is integrated by parts using the following relationships derived by Abplanowitz and Stegun (1964)

\[
\int_0^z He_k(z)e^{-\frac{z^2}{2}} dz = He_{k-1}(0) - e^{-\frac{z^2}{2}} He_{k-1}(z)
\]  

(5.57)

and

\[
He_k(0) = \begin{cases} 
0, & k \text{ odd} \\
\left(\frac{-1}{2}\right)^{\frac{k}{2}} k!, & k \text{ even} \\
\frac{(-1)^{\frac{k}{2}} k!}{2^{\frac{k}{2}} \left(\frac{k}{2}\right)!}, & k \text{ even}
\end{cases}
\]  

(5.58)
Equation (5.56) can then be integrated to give

\[ a_k = \frac{(-1)^{k+1/2}}{(2^{k-6} \pi)^{0.5} k(k-1)(k-2)((k-3)/2)!} \quad \text{k odd} \] (5.59)

and

\[ a_1 = \sqrt{\frac{8}{\pi}} \] (5.60)

In a similar manner it can be shown that if we write \(|p|\) as

\[ |p| = \sum_{k=0,2,4,\ldots}^{\infty} b_k H_k(z) \] (5.61)

then it can be shown that

\[ b_k = \frac{(-1)^{k(2-k)}}{(2^{k-3} \pi)^{1/2} k(k-1)(k/2-1)!} \quad \text{k even} \] (5.62)

and

\[ b_0 = \sqrt{\frac{2}{\pi}} \] (5.63)

In equation (5.50) the drag force terms at node \(i\) can be written as

\[ f_i^D = K_{Di} v^i_2 |v_2| \] (5.64)
where

\( K_{Di} \) is a drag force coefficient associated with node \( i \)

\( v^i \) is the horizontal water particle velocity at node \( i \) due to surface wave action.

By substituting equation (5.51), into equation (5.64) the drag force terms may be written as

\[
f^D_i = K_{Di} \sigma^2 v^i \sum_{k=1,3}^\infty a_k \bar{H}_e_k(\bar{v}^i_2)
\]

(5.65)

where

\[
\bar{H}_e_1(\bar{v}^i_2) = \bar{v}^i_2 \\
\bar{H}_e_3(\bar{v}^i_2) = \bar{v}^i_2^3 - 3\bar{v}^i_2 \\
\bar{H}_e_5(\bar{v}^i_2) = \bar{v}^i_2^5 - 10\bar{v}^i_2^3 + 15\bar{v}^i_2
\]

and \( \bar{v}^i_2 = \frac{v^i_2}{\sigma v^i_1} \)

\( a_1 = 2\sqrt{\frac{2}{\pi}} \)

\( a_3 = \frac{1}{3}\sqrt{\frac{2}{\pi}} \)

\( a_5 = -\frac{1}{60}\sqrt{\frac{2}{\pi}} \)

etc.

The zeroth order response spectrum in equation (5.33) is given by

\[
S_x(\omega) = \sum_{r=1}^{n} \sum_{s=1}^{n} \alpha_{xp} r(\omega) \alpha_{xp} s(\omega) S_{p(p)p(p)}(\omega)
\]

(5.66)

From equation (5.66) it can be seen that in order to compute the response spectrum it is necessary to determine expressions for the zeroth order cross spectra of force. \( S_{p(p)p(p)} \)
These spectra are obtained by first finding approximations for the cross-correlations of force as defined by

$$ R_{P_{1}P_{2}}(\tau) = \langle P_{1}^{(0)}(t)P_{2}^{(0)}(t + \tau) \rangle $$  \hspace{1cm} (5.67) 

Now, from equations (5.20), (5.31) and (5.65) the zeroth order non-zero force terms at any degree of freedom $i$ is given by terms such as

$$ P_{i}^{(0)} = K_{m_i} v_{i}^{1} + K_{D_i} \sigma_{v_{i}^{2}} \sum_{k=1,3,\ldots}^{\infty} a_{k} H_{e_{k}}(v_{2}^{l}) $$ \hspace{1cm} (5.68) 

where

$$ K_{m_i} $$ is an inertia force coefficient for node $i$

Thus the cross-correlations of force of equation (5.67) are obtained as follows. Substitute in for nodal forces as given by expression (5.68), substituting for the polynomials $H_{e_{k}}(v_{2}^{l})$ as defined above and the using the expression derived by Eatock-Taylor and Rajagopalan (1982) for the expected value of the product of 2 functions as outlined in Appendix VI

$$ R_{P_{1}P_{2}}^{(0)} = K_{m_i} K_{M_i} \langle \dot{v}_{1}^{1}(t)\dot{v}_{2}^{2}(t + \tau) \rangle + K_{D_i} K_{M_i} \sqrt{\frac{8}{\pi}} \sigma_{v_{2}^{2}} \langle \dot{v}_{2}^{2}(t)\dot{v}_{2}^{2}(t + \tau) \rangle 
+ K_{D_3} K_{M_3} \sqrt{\frac{8}{\pi}} \sigma_{v_{2}^{2}} \langle \dot{v}_{2}^{2}(t)\dot{v}_{2}^{2}(t + \tau) \rangle + K_{D_2} K_{M_2} \sigma_{v_{2}^{2}} \sigma_{v_{2}^{1}} \sum_{k=1,3,\ldots}^{\infty} a_{k}^{2} k! \rho_{k}^{k} $$ \hspace{1cm} (5.69)

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\[ \rho_n(\tau) = \frac{\langle v'_n(t)v'_n(t+\tau) \rangle}{\sigma_{v'_n} \sigma_{v'_n}} \]  

(5.70)

The thus the zero order \((r-s)\) cross-spectral density of force is obtained by taking the Fourier transform of equation (5.69) and is given by

\[
S_{p_{l'},p_{j'}}(\omega) = K_{m}, K_{M}, S_{v_{i'}v_{j'}}(\omega) + K_{D_{r}}, K_{M_{s}} \left( \frac{8}{\pi} \sigma_{v_{i'}} S_{v_{i'}v_{i'}}(\omega) \right) + K_{D_{i}}, K_{D_{s}} \left( \frac{4}{3} \pi \left( \frac{1}{\sigma_{v_{i'}} \sigma_{v_{i'}}} \right) S_{v_{i'}v_{i'}}^{*3}(\omega) \right) + \ldots
\]

(5.71)

where

\[ S_{v_{i'}v_{i'}}^{*3}(\omega) \text{ is the } 3^{rd} \text{ convolution of } S_{v_{i'}v_{i'}}(\omega) \]

In theory, the zeroth order force spectral density matrix may be computed to any level of accuracy depending on the number of terms retained. In the derivation of the cross-spectrum of force above, convolutions up to the third order only are included in accordance with Borgman's suggestions. Appendix V outlines the relationship between spectra and their convolutions which are used in non-linear spectral analysis.

5.4.2. First Order Response Spectra

Equation (5.33) shows that the general response spectrum of the non-linear system being considered contains first order response spectra such as \( S_{x(\theta),0}(\omega) \), which are related to
first order force spectra as shown by equation (5.34). Consider the first order force cross correlation function between the zero order force at degree of freedom \( r \) and the first order force at degree of freedom \( s \)

\[
R_{\eta_{r}\eta_{s}}(\tau) = \langle F_{1r}(t)F_{2s}^{(0)}(t + \tau) \rangle = \langle F_{1r}^{i}(t)F_{2s}^{i}(t + \tau)\bar{U}^{s}(t + \tau) \rangle
\]  

(5.72)

where

\( F_{1r}^{i}(t) \) is the component of \( F_{1g} \) acting at degree of freedom \( r \)

\( F_{2s}^{(0)}(t) \) is the component of \( F_{2g}^{(0)} \) acting at degree of freedom \( s \)

\( F_{2s}^{i}(t + \tau) \) is the drag force coefficient acting at degree of freedom \( s \)

\( \bar{U}^{s}(t + \tau) \) is the structural velocity of degree of freedom \( s \) due to the zero order force.

Equation (5.72) shows that the first order cross correlation is a function of the structural responses due to the zero order forces.

The right hand side of equation (5.72) cannot be determined exactly, a number of approximate techniques have been developed to evaluate such expressions. In this development, the cumulant discard hypothesis described by Soong (1973) and Lin and Cai (1995) is used to evaluate the expectation in equation (5.72). Using this approach it is assumed that the joint cumulants of order higher than two can be neglected and thus the average can be approximated in terms of a combination of lower order averages. Thus equation (5.73) may be approximated as follows
Now the functions $F_i^r(t)$, $F_D^s(t)$, and $\dot{U}^{s^0}(t)$ are zero mean stochastic processes, therefore, their expectations and hence the right hand side of equation (5.73) becomes zero and thus we obtain

$$R_{p_i^r,p_k^s}(\tau) = 0 \quad (5.74)$$

Thus the first order terms associated with equation (5.33) are set to zero.

### 5.4.3 Second Order Response Spectra

In equation (5.33) the terms of order $\varepsilon^2$ are of two types. The first type involves cross-spectral terms of zero order response and second order response terms such as $S_{x^{(0)}x^{(0)}}(\omega)$. Using a cumulant-neglect closure scheme, it can be shown that these terms can be set to zero in the same manner as the first order terms are set to zero.

The second type of terms are spectra of cross first order terms, such as $S_{x^{(0)}x^{(0)}}(\omega)$. In general, these terms are non-zero and they are also determined using a cumulant-neglect closure scheme, details of this are presented in Appendix VIII. The spectra terms are determined by firstly computing the associated correlation functions as follows:
\[
R_{p^0,p^0}(\tau) = \left( F_{f}^{r}(t)F_{D}^{r}(t+\tau)\dot{U}^{r}(t)\dot{U}^{r}(t+\tau) \right) \tag{5.75}
\]

Using the results from Appendix VIII, equation (5.75) may be expressed as:

\[
R_{p^0,p^0}(\tau) = E\left[ F_{f}^{r}(t)F_{D}^{r}(t+\tau) \right] E\left[ \dot{U}^{r}(t)\dot{U}^{r}(t+\tau) \right] + \\
E\left[ F_{f}^{r}(t)\dot{U}^{r}(t) \right] E\left[ F_{D}^{r}(t+\tau)\dot{U}^{r}(t+\tau) \right] + \\
E\left[ F_{f}^{r}(t)\dot{U}^{r}(t+\tau) \right] E\left[ F_{D}^{r}(t+\tau)\dot{U}^{r}(t) \right] 
\]

\[
(5.76)
\]

In equation (5.76) terms such as \( \dot{U}^{r}(t) \) are linear functions of the force vector \( F_{1g} \). The terms of \( F_{1g} \) are odd functions of water particle velocities and the terms such as \( F_{f}^{r}(t) \) are even order Hermite Polynomials of water particle velocities, thus these two functions are uncorrelated and hence, \( F_{f}^{r}(t) \) and \( F_{D}^{r}(t) \) are uncorrelated with \( \dot{U}^{r}(t) \) and \( \dot{U}^{r}(t) \), Eatock-Taylor and Rajagopalan (1982). Using this, equation (5.76) may be written as

\[
R_{p^0,p^0}(\tau) = R_{r,r}^{f}(\tau) R_{U^{r},U^{r}}(\tau) \tag{5.77}
\]

Now, from equation (5.21) the drag force terms in the above equation are of the following form

\[
F_{D}^{r}(t) = K_{D}^{r} \left[ v_{i}^{2}(t) - \frac{2}{\sqrt{\pi} \sigma_{v}} \right] \tag{5.78}
\]
Eatock-Taylor and Rajagopalan (1982) used Hermite polynomials to compute the expectation of equation (5.77). Below it is computed in a simpler manner by directly expanding the terms of the expectation.

Consider the first expectation on the right hand side of equation (5.77) as follows

\[ R_{D',D'}(\tau) = K_{D',D'} \left[ \left( |v_2(t)| - \frac{2}{\sqrt{\pi}} \sigma_v \right) \ast \left( |v_2(t+\tau)| - \frac{2}{\sqrt{\pi}} \sigma_v \right) \right] \]  

(5.79)

Expanding the expectation on the right hand side of equation (5.79) gives

\[ \left\langle \left( |v_2(t)| - \frac{2}{\sqrt{\pi}} \sigma_v \right) \ast \left( |v_2(t+\tau)| - \frac{2}{\sqrt{\pi}} \sigma_v \right) \right\rangle = \]

\[ \left\langle |v_2(t)| |v_2(t+\tau)| - |v_2(t)|^2 \frac{2}{\sqrt{\pi}} \sigma_v + |v_2(t+\tau)|^2 \frac{2}{\sqrt{\pi}} \sigma_v + \frac{2}{\sqrt{\pi}} \sigma_v \sigma_v \right\rangle = \]  

(5.80)

\[ \left\langle |v_2(t)| |v_2(t+\tau)| - \frac{2}{\sqrt{\pi}} \left[ \left( |v_2(t)| \sigma_v \right) + \left( |v_2(t+\tau)| \sigma_v \right) \right] + \frac{2}{\sqrt{\pi}} \sigma_v \sigma_v \right\rangle \]

Obtaining expressions for the expectations of a modulus of a random variable from Papoulis, equation (5.80) can then be written

\[ \left\langle \left( |v_2(t)| - \frac{2}{\sqrt{\pi}} \sigma_v \right) \ast \left( |v_2(t+\tau)| - \frac{2}{\sqrt{\pi}} \sigma_v \right) \right\rangle = \]

\[ \left\langle |v_2(t)| |v_2(t+\tau)| - \frac{2}{\sqrt{\pi}} \left[ \left( |v_2(t)| \sigma_v \right) + \left( |v_2(t+\tau)| \sigma_v \right) \right] + \frac{2}{\sqrt{\pi}} \sigma_v \sigma_v \right\rangle = \]  

(5.81)

\[ \left\langle |v_2(t)| |v_2(t+\tau)| - \frac{2}{\sqrt{\pi}} \sigma_v \sigma_v \right\rangle \]

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In their development Eatock-Taylor and Rajagopalan (1982) obtained the following similar expression

\[
\langle \left| v_x'(t) - \frac{2}{\sqrt{\pi}} \sigma_{v_x} \right| \cdot \left| v_y'(t + \tau) - \frac{2}{\sqrt{\pi}} \sigma_{v_y} \right| \rangle = \langle v_x'(t) \cdot v_y'(t + \tau) \rangle
\]  

(5.82)

In order to compute equation (5.77), cross-correlations of zero-order nodal structural velocities must first be evaluated. From consideration of equation (5.66), cross-spectra of zero-order structural velocities between node x and node y are given by

\[
S_{x^{(0)}y^{(0)}}(\omega) = \sum_{r=1}^{n} \sum_{s=1}^{n} \alpha^{*}_{x_{p_{r}}^{(0)}}(\omega) \alpha_{y_{q_{s}}^{(0)}}(\omega) S_{r^{(0)}q^{(0)}}(\omega)
\]  

(5.83)

where

\[
\alpha^{*}_{x_{p_{r}}^{(0)}}(\omega)
\]

is the complex receptance of structural velocity at frequency \( \omega \) at degree of freedom x due to a force \( e^{i\omega t} \) applied at degree of freedom r.

From equations (5.41) and (5.49) the vector of structural velocities due to a unit force of \( e^{i\omega t} \) applied at degree of freedom k is given by

\[
\dot{U}_g = i\omega U_g = i\omega K^k e^{i\omega t}
\]  

(5.84)

Thus the cross-spectra of equation (5.83) can be computed. The cross-correlations of zero-order structural velocities are then obtained by taking the inverse Fourier transform.
of the above cross-spectra. Since equation (5.77) can now be fully evaluated, the cross-
spectra of the second-order force terms can be obtained by taking the Fourier transforms
of $R_{p_i p_j}(\tau)$. Finally, the cross-spectra of the second-order response displacements are
obtained from linear spectral theory as outlined above.

From the zero-order and second-order displacement response spectra the complete
displacement response spectrum is obtained from

$$S_{xx}(\omega) = S_{x(0)x(0)}(\omega) + \epsilon^2 S_{x(0)x(0)}(\omega)$$

(5.85)

5.5. Non Linear Spectral Analysis Computer Model

In order to apply the methodology developed in the previous sections of this chapter to a
multi-degree of freedom structure, a computer program was written. Generally speaking,
the program was developed analogously to the linear spectral analysis computer program
described in Section 3.5., namely, as a general purpose finite element program that can be
run on most common hardware platforms. The non-linear spectral analysis computer
program also has three main computational blocks as shown in Figure 5.2 below.

The computational Blocks I and II are identical to the corresponding blocks of the linear
spectral computer program and thus are not discussed further.

Computational Block III of the non-linear spectral analysis consists of three distinct
modules:
Evaluation of receptances

Determination of zero-order spectra

Determination of second-order spectra

Details of these modules are presented below in the following sections.

5.5.1 Evaluation of Receptances

In the linear spectral response model system receptances are obtained by applying a series of unit surface waves to the structure and calculating the resulting structural responses. Because of the non-linear relationship between wave kinematics and forces induced on a structure, the unit wave method cannot be applied to compute the non-linear system receptances. The approach adopted in the non-linear model is to compute receptances at all degrees of freedom due to harmonic forces, $e^{i\omega t}$, being applied at all degrees of freedom separately over the relevant frequency range. This approach allows the displacement receptance terms of equation (5.32) and the velocity receptance terms of equation (5.84) to be computed. Figure 5.3 presents a flowchart outlining how the receptances are computed.

In Section 5.3 it is shown that use is made of the orthogonality characteristics of the eigenpairs of a system to compute the receptances. Thus, the first stages in determining the receptances are the assembly of system mass and damping matrices and then solving for the eigenvalues and eigenvectors of the system as detailed in Section 3.7.2. The other subroutines shown in Figure 5.3 are described below.

The FORTRAN code associated with the model is presented in Appendix VII.
The hydrodynamic damping matrix is a function of variances of water particle velocities, thus these are computed first as for the linear analysis. The element hydrodynamic damping matrix is then computed using equation (5.12). By applying appropriate coordinate transformations the element hydrodynamic damping matrix is expressed in global coordinates and the structure hydrodynamic damping matrix is then assembled in subroutine ASSDAM.

The generalised equations of motion given by equation (5.37) contain an uncoupled generalised damping matrix, \([ C ]\). This uncoupled damping matrix is diagonalised in an equivalent manner as detailed in Section 3.4. The entry on the \(i\)th diagonal of \([ C ]\) is

\[
C(i,i) = 2\xi_i\omega_i + \frac{\sum_{j=1}^{m} C_H(i,j) \langle \dot{Z}_i \dot{Z}_j \rangle}{\langle \dot{Z}_i \dot{Z}_i \rangle} \tag{5.86}
\]

where

\(CH(i,j)\) is the \((i,j)\)th entry of the structure hydrodynamic damping matrix and the \(\langle \dot{Z}_i \dot{Z}_j \rangle\) are obtained from the results of a linear analysis using the model described in Section 3 above.

In equation (5.86) the values for \(\langle \dot{Z}_i \dot{Z}_j \rangle\) are obtained from the results of the linear spectral analysis model.
The receptance of displacement at any degree of freedom $x$ due to a force $e^{i\omega t}$ at any degree of freedom $r$, $\alpha_{xp}(\omega)$, over the frequency range of interest in both generalised coordinates and the global structure coordinates are computed in RGEN. In this routine the force $e^{i\omega t}$ is specified at each free degree of freedom in turn and the components of the generalised receptances obtained from equations (5.45) and (5.46) for each frequency interval $\omega$. From equation (5.49) the receptances in global structure coordinates are obtained using the mode superposition. From this the velocity receptances of equation (5.85) are computed.

5.5.2 Evaluation of Zero-Order Spectra

The zero-order displacement spectrum is the response spectrum due to the applied random force vector $P^{(0)}$ as defined in equation (5.30). The receptances as computed in Section 5.5.1 are combined with cross-spectra of forces to compute the required displacement spectra. This response spectrum is given by equation (5.66). Figure 5.4 presents a flowchart of the computer code developed to compute this spectrum for a space frame. Details of the subroutines listed on Figure 5.4 are given below.

SPECIN

This is the same subroutine that is described in Section 3.5.3 and it computes the relevant spectrum of sea state for given storm conditions.

FORCE
This subroutine computes the coefficients of the drag force term and inertia force term developed in equations (5.19), (5.20) and (5.21) at each node of the structure. These coefficients are then assembled into global vectors that are used to compute the global force terms such as those expressed in equation (5.69).

**SPECTRA**

The cross-spectra of force defined by equation (5.71) are functions of cross-spectra of water particle velocities and accelerations. The cross-spectra of water particle velocities and accelerations between the different nodes of the structure, which are functions of wave kinematics and the spectrum of sea state and which are defined in Appendix IV are calculated in this subroutine. The relevant standard deviations of water particle velocities are then obtained by numerical integration.

**CONSPE**

Equation (5.71) shows that convolutions of certain spectra are required to compute response spectra, these convolutions are computed in this subroutine. Appendix V presents relevant background information pertaining to convolutions and their evaluations. The third order convolution of a spectrum with itself is obtained by firstly finding the inverse Fourier transform of the spectrum and then raising the ordinates of the resulting function to the power of three. The third order convolution is then evaluated by taking the Fourier transform of this latter function. CONSPE reads in the relevant spectra, performs the above Fourier analyses and manipulations and returns the third order convolutions.
The system receptances, as calculated in Section 5.5.1, and the zeroth-order force terms, as expressed in equation (5.71), are combined in this subroutine as shown in equation (5.66) to compute the zeroth-order spectra of displacements.

5.5.3 Evaluation of Second-Order and Total Response Spectra

Equation (5.33) shows that the total displacement response spectrum is a linear combination of the zeroth-order, first-order and second-order response spectra. Details of the evaluation of the zeroth-order spectra are described in Section 5.5.2 and it shown in Section 5.4.2 the first-order spectra can be approximated to zero. Figure 5.5 presents a flowchart that outlines the main computational aspects associated with evaluating the second-order and total response spectra. These computational blocks are briefly described below.

From equation (5.77) cross-correlations of water particle velocities must be computed. Firstly, the relevant cross-spectra of these velocities are evaluated using wave kinematics and the input spectrum of sea-state, as shown in Appendix IV. Then, inverse Fourier transforms of these spectra are obtained. The require cross-correlation functions of equation (5.79) are then computed using the cross-correlation functions of the velocity spectra, the drag coefficients and standard deviations of water particle velocities. These functions are then stored and used when required below.
CROSSX

The second-order force cross-correlation terms as shown in equation (5.77) require the functions computed in Subroutine CROSSV and also the cross-correlation functions of the zeroth-order structural velocities. Equation (5.83) shows how the cross-spectra of the zeroth-order structural velocities are evaluated using the structural velocity receptances and the cross-spectra of the zeroth-order force terms. The structural velocity receptances are read in from RGEN. Then using the zeroth-order cross-spectra of forces as computed in Subroutine SPEC1, the cross-spectra of velocities are computed. Finally, the cross-correlation functions of zero-order structural velocities are obtained by taking the inverse Fourier transform of the above cross-spectral terms.

SPEC2

The second-order force cross-correlation terms are computed by obtaining the products of the relevant ordinates of the second-order cross-correlations of structural velocities and the cross-correlations of drag force terms. The Fourier transforms of these force cross-correlations are obtained to give the second-order cross-spectral force terms. The second-order response spectra are obtained from these force spectra and system receptances.

SPECOUT

The spectra of structural displacements, incorporating up to second-order effects, are computed using the cross-spectra computed in the Subroutines SPEC1 and SPEC2 in accordance with equation (5.82).
Figure 5.1 - Schematic of Planar Structure
Figure 5.2 - Main Computational Blocks of Non-Linear Spectral Model
Blocks I and II
- Formulate Coefficient Matrices
- Solve Eigenproblem

\textit{ALPHAU}
- Compute variances of water velocity
- Compute element hydrodynamic matrix
- Assemble global damping matrix

\textit{CGEN}
- Compute generalised damping matrix from mode shapes and eigenvalues

\textit{RGEN}
- Loop over all frequencies
- Loop over all degrees of freedom
- Applied a load $e^{int}$ at each DOF in turn
- Compute receptances of displacement
- Compute receptances of velocity

\textbf{Figure 5.3 - Flowchart for Evaluation of Receptances}
**SEPCIN**
- Choose sea-state spectrum
- Compute spectrum over frequency range

**FORCE**
- Loop over all submerged elements
- Compute:
  - Inertia force coefficient terms
  - Drag force coefficient terms
- Assemble global force matrix

**SPECTRA**
- Loop over all submerged elements
- Loop over all frequencies
- Compute velocities cross-spectra
- Compute accelerations cross-spectra
- Call **CONPSE**
- Compute standard deviations of water particle velocities.

**CONPSE**
- Compute third convolution of sea-state spectrum

**SPECI**
- Loop over all degrees of freedom
- Loop over all frequencies
- Loop over all degrees of freedom
- Compute cross-spectra of force
- Compute zeroth-order response spectra

Figure 5.4 - Flowchart for Evaluation of Zeroth-Order Response Spectra
**CROSSV**
- Loop over all nodes
- Obtain inverse Fourier transform of cross-spectra of water particle velocities
- Compute cross-correlations of drag force terms

**CROSSX**
- Loop over all nodes
- Compute cross-spectra of velocities
- Obtain inverse Fourier transform of cross-spectra of zeroth-order structural velocities

**SPEC2**
- Loop over all nodes
- Compute second-order force cross-correlation terms
- Compute second-order response spectra from Fourier transforms of second-order force cross-correlation terms and system receptances

**SPECOUT**
- Loop over all nodes
- Spectra of total displacements computed from combination of zeroth-order and second-order displacement spectra.

Figure 5.5 - Flowchart for Evaluation of Second-Order and Total Response Spectra
CHAPTER 6
APPLICATION OF THE NON-LINEAR SPECTRAL MODEL

6.1 Introduction

The non-linear spectral model developed in Chapter 5 was applied to perform analyses on a cantilever and plane frame structures subjected to wave loadings. The results from these analyses were compared with the results from applications of the linear model to the same structures; details of these analyses and comparisons are presented in this chapter. Because the linear spectral model has been well validated in Chapter 4 the comparisons made between the linear and non-linear models in this chapter provide good validation for the non-linear model.

Since the non-linear responses utilise the third-order convolutions of water particle velocity spectra, it is instructive to consider the third-order convolution of a velocity spectrum to see the effect of this non-linearity on the force spectrum. Thus, this is presented first in the next section below; also, in this section spectra computed by NONLIN are compared with similar spectra from literature.

The main objective in developing the non-linear model is to determine whether or not structure spatial extent has a significant effect on the non-linear structure response during a random storm as suggested by Moe (1977). In order to achieve this objective, a non-linear model of a cantilever was developed and results compared with results from a linear analysis; in this analysis there are no spatial effects and it will serve as an initial check on the results of the non-linear model. Both linear and non-linear models are then applied to
a simple offshore portal frame to consider spatial effects; more complex structures such as Kinsale Platform Alpha would take much more time to analyse and not provide any better insight into the processes. The analyses of two different portal frame models and the results obtained are presented in Section 6.4 below.

6.2 Spectra of Velocities and Forces

Lipsett (1986b), in the development of his single degree of freedom model, considers the response of a structure to a storm event represented by a Pierson-Moskowitz spectrum. Lipsett defined the sea state spectrum by specifying a mean wind speed of 21.8m/s, and he computed the third order convolution of a velocity spectrum due to this sea state spectrum. The sea state spectrum is reproduced below in Figure 6.1 and the convoluted velocity spectrum is reproduced in Figure 6.2.

The non-linear model developed in Chapter 5 was also used to compute the convoluted velocity spectrum. Figure 6.3 shows the sea-state spectrum computed by NONLIN for a wind speed of 21.8m/s. The third order convolution of water particle velocity computed by NONLIN is presented in Figure 6.4. By inspection, it is clear that the spectra presented in both Figures 6.1 and 6.3 are very similar and those presented in Figures 6.2 and 6.4 are similar thus validating an important component of the non-linear model.

It is also obvious from considering the convoluted velocity spectra that, as well as containing energy at the frequency, \( \omega \), that is the frequency at which the peak in the sea state spectrum occurs, it contains considerable energy at three times this frequency, \( 3\omega \), and some energy at zero frequency. In general, the first natural frequency of a jacket
platform is reasonably removed from \( \omega \) thus avoiding resonant responses, however, because \( 3\omega \) will be closer to the first natural frequency greater structural responses may be encountered.

Figure 6.5 shows the relationship between spectra of velocity convolutions as a function of water depth, as computed by NONLIN. This illustrates that the superharmonic terms of the convolutions become relatively more significant with increasing water depth as shown by the steeper second peak at increasing depths, this agrees with the findings of Sigbjornsson and Morch (1982) who consider this important when computing the total force on a member over its length.

Li (1998) presents a graph comparing the relative magnitudes of the total wave force spectrum at a point and the force spectrum term due to the convolution of water particle velocity, this is reproduced here as Figure 6.6. The program NONLIN also calculates these spectra and they are presented in Figure 6.7. From inspection, it can be seen that both figures present spectra of similar form, illustrating that the spectra of force computed by NONLIN appear correct.

By inspection, also, it is seen that the ordinates of the convoluted spectrum are much less than those of the spectrum of total force at lower frequencies. However, at higher harmonics the convoluted spectrum contributes relatively more to the spectrum of total force. This higher harmonic signal, due to the convoluted spectrum, will contribute more to the resonance response of a jacket structure than the lower spectral peak and thus it may be important to assess its effect on response spectra.
6.3 Cantilever Model

The non-linear model was first applied to the simple cantilever as described in Chapter 4 and shown below again in Figure 6.6. The cantilever response spectra were calculated by representing the cantilever displacements as a linear sum of the first three mode shapes. The cantilever has a fundamental natural frequency of 1.25rads/s and the fundamental mode shape is translational displacement.

The cantilever structure was subjected to three different storm events to consider structural responses to different forcing frequencies. These storms are characterised by the Pierson-Moskowitz spectra shown in Figure 6.9 for wind speeds of 10m/s, 20m/s and 30m/s. Response spectra of the translational displacements of the structure, in the direction of wave propagation, at the water line were calculated using both the linear and non-linear models and the results compared.

6.3.1 Cantilever Analysis No.1 - Wind Speed 10m/s

The structure was first subjected to a storm characterised by the Pierson-Moskowitz spectrum with a 10m/s wind speed as shown in Figure 6.9. The spectra of response of the translation of the top of the structure in the direction of the wave ray due using the linear and non-linear models are presented in Figure 6.10.

By inspection of Figure 6.10 it is clear that there is very little difference between the linear and non-linear response spectra. Lipsett (1985) studied response characteristics of a single degree of freedom system comparing results, inter alia, from a perturbation analysis and an equivalent linearisation analysis. Figure 6.11 show results obtained by Lipsett when the forcing function frequency is similar to the first natural frequency. Lipsett's results are quite similar to that obtained using NONLIN in that there is little
difference between the linear and non-linear analysis. The non-linear results in both cases are marginally greater than the linear results.

6.3.2 Cantilever Analysis No.II - Wind Speed 20m/s

The cantilever structure was next subjected to a storm characterised by the Pierson-Moskowitz spectrum with a 20m/s wind speed as shown again in Figure 6.9. The spectra of response of the translation of the top of the structure in the direction of the wave ray using the linear and non-linear models are presented in Figure 6.12. Figure 6.13 show the result obtained by Lipsett (1985) for his single degree of freedom system when the relationship between the forcing function frequency and the first natural frequency of the system are similar to the present situation.

Again, the results obtained by the models developed herein compare favourably with the results obtained by Lipsett. For this analysis case the response spectra show two peaks, one at the frequency of the peak energy of the input spectrum and the other at the first natural frequency of the structure. In this case, the two peaks are obviously because the forcing and natural frequencies are sufficiently far apart. Both the current analysis and Lipsett’s analysis show that the ordinates of the non-linear spectrum are marginally larger than those of the linear spectrum.

6.3.3 Cantilever Analysis No.III - Wind Speed 30m/s

The cantilever was finally subjected to a storm characterised by the Pierson-Moskowitz spectrum with a 30m/s wind speed as shown again in Figure 6.9. The spectra of response of the translation of the top of the structure, in the direction of wave propagation, due to
linear and non-linear analyses are presented in Figure 6.14. Figure 6.15 presents the results from a similar analysis performed by Lipsett (1985). In these figures it is clearly seen that there are two distinct spectral response peaks associated with each analysis, due to the separation of the excitation spectrum peak frequency and the structures first natural frequency. The general form of the results obtained by the present analysis is quite similar to that obtained by Lipsett, in that the linear and non-linear response spectra are quite similar with the non-linear being more conservative. Similar observations were made by Eatock-Taylor and Rajagopalan (1982) and Sighjorsson and Morch (1982).

6.4 Portal Frame Models

Two simple models of offshore portal frames were constructed to assess the effects of spatial extent on non-linear spectral responses due to unidirectional waves and to compare the results with linear spectral responses. Figure 6.16 presents a schematic diagram of a typical frame structure with horizontal bracing provided at superstructure level, mean water level and midway between mean sea level and the seabed. The two frames considered were identical except that the spacing between the legs was varied to assess the effects on response spectra. The leg spacings considered are 20m and 40m and each structure is subjected to three storm events defined by Pierson-Moskowitz spectra of wind speeds 10m/s, 20m/s and 30m/s respectively. These spectra are presented in Figure 6.9 above. The incident storms are all specified so that the wave rays are in the plane of the frames. In all cases the structure had lumped masses to represent topside modules and forces were calculated only on vertical legs. Although in reality jacket structures usually have inclined members, the structure shown below will serve to assess
whether or not non-linear spectral response is significantly affected by spatially extended structures.

The finite element models of the frames consist of 10 equally spaced nodes along each main leg and only in-plane motions were considered. The first two mode shapes in the plane of the frames were used in the mode superposition analysis to decouple the equations of motion; these modes represent translational displacement and bending respectively.

6.4.1 Frame I - 20m Leg Spacing

The first frame analysed has a leg spacing of 20m and a fundamental natural frequency of 3.1 rad/s. The response spectra of this structure to the above three storms are presented in the following sections.

6.4.1.1 Frame I Analysis No. I - Wind Speed 10m/s

Frame I was subjected to a random storm defined by a Pierson-Moskowitz spectrum having a 10m/s wind speed. Both the linear and non-linear spectral analysis models were used to predict the resulting response spectrum of the horizontal displacement of the superstructure. The results of both analyses are presented in Figure 6.17.

Figure 6.17 shows a very interesting difference between the linear and non-linear response spectra. The linear spectrum shows a number of distinct peaks that start at the peak frequency of the excitation spectrum, reaching a maximum at the first natural frequency of the frame and then decaying rapidly to zero. In between each peak the spectrum
decreases to zero at the frequencies corresponding to the zero load frequencies as discussed in Chapter 4. However, the non-linear spectrum varies considerably from the linear spectrum in the lower frequency range. By inspecting Figure 6.17 it is clear that, whereas the linear spectrum goes to zero at about 1.2 rad/s, the non-linear does not go to zero at this frequency. The primary reason for the difference between the spectra is due to the contributions of the velocity convolutions that induce forces at higher harmonics than the wave frequency. In the linear model certain wave frequencies give rise to phase effects due to leg spacings that generate zero net forces on the structure. However, at these wave frequencies in the non-linear model higher forcing function frequencies are also included which give rise to relatively large responses as shown.

6.4.1.2 Frame I Analysis No.II - Wind Speed 20m/s

Frame I was next subjected to a random storm defined by a Pierson-Moskowitz spectrum having a 20m/s wind speed. Both the linear and non-linear spectral analysis models were used to predict the resulting response spectrum of the horizontal displacement of the superstructure. The results of both analyses are presented in Figure 6.18. A similar, but less, pronounced difference between both spectra is evident again during this analysis. This is because the difference between the excitation frequency and the structures natural frequency is greater than in the previous analysis, thereby decreasing resonance effects.

6.4.1.3 Frame I Analysis No.III - Wind Speed 30m/s

Frame I was finally subjected to a random storm defined by a Pierson-Moskowitz spectrum having a 30m/s wind speed. Again, both the linear and non-linear spectral analysis models were used to predict the resulting response spectrum of the horizontal
displacement of the superstructure. The results of both analyses are presented in Figure 6.19. This shows a similar situation to that observed from the results of the previous analysis, that is that the for more intense storm events the non-linear effect appears to be less significant as leg spacing phase effects are reduced.

6.4.2 Frame II Analyses

The fundamental natural frequency of the portal frame with a leg spacing of 40m is 2.76 rad/s, this is less than Frame I as expected.

Frame II was subjected in turn to the three random storms defined by a Pierson-Moskowitz spectrum having wind speeds of 10m/s, 20m/s and 30m/s respectively, as in the previous section for Frame I. In all three cases again the linear and non-linear spectral analysis models were used to predict the resulting response spectrum of the horizontal displacement of the superstructure. The results of both analyses are presented in Figures 6.20 – 6.22.

By inspection of Figures 6.20-6.22 it is clear that similar differences between the linear and non-linear spectra as were found in the analysis of Frame I. In particular, Figures 6.20 and 6.21 show an appreciable difference between the response spectra at the lower end of the frequency scale due to leg phase effects. During larger storms, such as that represented by the 30m/s storm, there is less of a difference due to the fact that the forcing function and natural frequencies are further apart.

The results of the analyses performed in chapter will be discussed in Chapter 7.
Figure 6.1 - Sea State Spectrum (Lipsett, 1985)

Figure 6.2 - Velocity Convolution (Lipsett, 1985)
Figure 6.3 - NONLIN Sea State Spectrum

Figure 6.4 - NONLIN Velocity Convolution
Figure 6.5 - Depth Attenuation of Velocity Convolutions
Figure 6.6 - Comparison of Force Components (Lin 1998)

Figure 6.7 - Comparison of Force Components (NONLIN)
Figure 6.8 - Cantilever Model
Figure 6.9 - Sea State Spectra
Figure 6.10 - Comparison of NONLIN V’s Linear Response Spectra
Cantilever 10m/s

Figure 6.11 - Comparison of Perturbation V’s Linear Response Spectra
(Lipsett 1985)
Figure 6.12 - Comparison of NONLIN V's Linear Response Spectrum
Cantilever - Wind Speed 20m/s

Figure 6.13 - Comparison of Perturbation V's Equivalent Linear Spectrum
(Lipsett 1985)
Figure 6.14 - Comparison of NONLIN V's Linear Response Spectra
Cantilever - Wind Speed 30m/s

Figure 6.15 - Comparison of Perturbation V's Linear Spectra
(Lipsett 1985)
Figure 6.16 – Frame structure
Figure 6.17 - Comparison of NONLIN V's Linear Response Spectra
Frame (20m leg spacing) - Wind Speed 10m/s
Figure 6.18 - Comparison of NONLIN V’s Linear Response Spectra
Frame (20m leg spacing) - Wind Speed 20m/s
Figure 6.19 - Comparison of NONLIN V's Linear Response Spectra
Frame (20m leg spacing) - Wind Speed 30m/s
Figure 6.20 - Comparison of NONLIN V’s Linear Response Spectra
Frame (40m leg spacing) - Wind Speed 10m/s
Figure 6.21 - Comparison of NONLIN V’s Linear Response Spectra Frame (40m leg spacing) - Wind Speed 20m/s
Figure 6.22 - Comparison of NONLIN V's Linear Response Spectra
Frame (40m leg spacing) - Wind Speed 30m/s
CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 SUMMARY

This thesis is concerned with developing structural response models of fixed, lattice structures subjected to unidirectional waves. The primary objectives of the thesis are twofold: firstly, to develop an accurate and efficient general-purpose linear spectral response model; secondly, to investigate effects of structure spatial extent on non-linear response spectra. The linear model has particular application to numerous jacket structures deployed in relatively shallow water. Many of these platforms are reaching the end of their service life and decommissioning of them will require further re-analysis. The industry is now beginning to exploit deposits of oil and gas in deep waters, Marshall (1992); a consequence of this is that structure natural frequencies may be lowered and non-linear effects may become important. The non-linear model developed was used to carry out some investigations into a particular problem that may effect such structures. Both of the above models were developed within a finite element framework so that various realistic structural configurations could be considered.

The linear spectral response model was developed using a novel application of the unit wave technique. The approach used linear systems theory relying on an assumed linear relationship between surface waves and structural response. The drag force was linearised in an equivalent manner to obtain an equivalently linearised drag force coefficient. Mode superposition was employed to decouple the general equations of motion and compute system receptances. The eigensolver was validated against different
test cases and the spectral response model is validated against the results of a structural monitoring programme of a major gas production platform, Kinsale Platform Alpha. Linear spectral theory was used to relate response spectra to the input spectrum by means of the receptances. Using this method the extensive computations required to analyse large platforms are reduced by half. The linear model was also applied to a number of different portal frame structures and phasing effects of leg spacing investigated. The conclusions drawn from these analyses are presented and discussed in the next section.

The non-linear model developed extends work of previous researchers by including leg spacing effects, such effects had not previously been incorporated into non-linear analyses of lattice structures. The non-linear drag force term was included in the model using perturbation, keeping terms up to second-order. Use was made of the orthogonality properties of Hermite polynomials to compute the response spectra. System receptances were again required to obtain response spectra, however, because of the non-linear system the unit wave method could not be used. Receptances were obtained by applying unit forces at each degree of freedom in turn and then computing associated responses. A cumulant neglect closure was required to compute relevant expectations used in computing the response spectra.

Components of the non-linear model were compared with published data to check their validity, such as force calculation routines. The model was then used to compute response spectral for a cantilever and for portal frames; the model results were compared with similar analyses carried out using the linear model. Both models computed similar
response spectra for the cantilever and agreed with similar investigations reported in the literature; this served as a good validation of the non-linear model. The two models were then applied to the portal frames for a number of different storm conditions. The primary conclusions drawn from these analyses are presented and discussed below.

7.2 CONCLUSIONS

The work carried out during this research has led to the following conclusions:

(a) During the future exploration and production of subsea deposits of oil and gas activities will take place in both shallow and deep waters. Regarding shallow water platforms, as well as the deployment of new structures the safe decommissioning of existing structures will require efficient and accurate structural analysis models. The search for new deposits is leading the industry into deeper water and hence it is important that processes in these more difficult environments are well understood. Thus the tools developed herein are very relevant to the offshore industry.

(b) The model developed in Chapter 3 and applied in Chapter 4 is an accurate tool for performing modal and linear spectral analysis of offshore jacket platforms. The model development includes a novel solution to the linearised drag force coefficient for the three-dimensional problem. Further, the development also applies the unit wave technique in a unique manner resulting in an efficient model in which computations are halved. The model validation carried out in Chapter 4 is very extensive comparing model predictions with measured structural responses for an existing large platform; the author is not aware of any other model being so extensively validated. From this validation exercise it is concluded that the model is capable of
(i) accurately computing mode shapes and natural frequencies of jacket platforms and

(ii) efficiently and accurately predicting structural response spectral for unidirectional waves.

(c) The linear model was used to assess leg spacing effects on structural responses, the results of this analysis lead to the conclusion that leg spacing effects are important when considering structural response and should be incorporated into all such analyses.

(d) The non-linear spectral response model developed in Chapter 5 and applied in Chapter 6 is an extension of research carried out by others in that it incorporates leg spacing phase effects. The model was firstly applied to a cantilever and results obtained were similar to those obtained by previous researchers; namely, that the linear and non-linear models predicted similar responses. This indicates that the non-linear model is well-behaved.

(e) The non-linear model was then also to portal frames for different storm intensities. The results of this analysis show that there is an appreciable difference between the results of the linear and non-linear models. In particular, it is shown that response spectra computed using the non-linear model may be larger than corresponding spectra computed using the linear model. This difference is due to interactions between the higher order force terms and the phase effects caused by the leg spacings of the portal frame. The author is not aware of this phenomenon being observed by previous researchers and is an important conclusion of this work.
From their research Eatock-Taylor and Rajagopalan (1982) concluded that ‘The influence of the non-linearities has been demonstrated to be smaller at lower sea-states’. For the structures considered, the current research shows that when leg spacing is included in the analysis then the discrepancies between the linear and non-linear response spectra are greatest at lower sea-states. This is an important conclusion because lower sea-states have a higher frequency of occurrence and thus the fatigue life of structural members may be affected, Isaacson (1992).

Eatock-Taylor and Rajagopalan (1982) also concluded that the difference between the linear and non-linear response spectra ‘...is negligible around the frequencies of maximum wave energy, but significant at resonance.’ The analysis carried out in this research on a cantilever leads to similar conclusions. However, this is not the situation when analysing portal frames; in this situation the main difference occurs close to the frequencies of maximum wave energy due to the higher order effects interacting with the leg spacing phase effects. Thus, not only is it important to avoid resonance problems but structural spatial extent may also be an important consideration in the design process because of non-linearities arising from drag force terms.

7.3 RECOMMENDATIONS FOR FURTHER RESEARCH
The non-linear spectral model developed herein considered a particular problem associated with structures used in deep-water oil and gas exploitation. Based on the current work it is considered that research should be carried out in the following areas to further our understanding of the behaviour of deep-water structures:
Carry out fatigue analysis based upon the above non-linear spectral response model to quantify more fully the effects of the interactions between the higher order forcing terms and structure spatial extent.

Using Edgeworth series, compute the statistical distributions of structural responses and compare these with Gaussian responses obtained when carrying out linear analysis.

Application of the developments carried out in this work to compliant structures

Extend current development to include short-crested waves

Include higher order wave theories into non-linear spectral response model
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APPENDIX I

FINITE ELEMENT SHAPE FUNCTIONS
Finite Element Shape Functions

The matrix of shape functions used to develop the mass and stiffness matrices was derived to be

\[
[G]=\begin{bmatrix}
g_1 & 0 & 0 & 0 & 0 & g_2 & 0 & 0 & 0 & 0 \\
0 & g_3 & 0 & 0 & 0 & g_4 & 0 & g_5 & 0 & 0 \\
0 & 0 & g_3 & 0 & g_7 & 0 & 0 & g_5 & 0 & g_8 \\
0 & 0 & 0 & g_1 & 0 & 0 & 0 & 0 & g_2 & 0
\end{bmatrix}
\]

where

\[
g_1 = 1 - \xi \\
g_2 = \frac{x}{l} \\
g_3 = 1 - 3(\xi^2) + 2(\xi)^3 \\
g_4 = x - 2 \frac{x^2}{l^2} + \frac{x^3}{l^2} \\
g_5 = 3(\xi)^2 - 2\left(\frac{x}{l}\right)^3 \\
g_6 = -\frac{x^2}{l^2} + \frac{x^3}{l^2} \\
g_7 = -x + 2 \frac{x^2}{l^2} - \frac{x^3}{l^2} \\
g_8 = \frac{x}{l} - \frac{x^2}{l^2}
\]

x is distance along the member in the local X direction

l is the member length
APPENDIX II

FINITE ELEMENT MATRICES
Finite Element Matrices

The stiffness and mass matrices obtained by using the principle of virtual work as detailed in Chapter 3 are given below.

Stiffness matrix

$$
[K] = E \begin{bmatrix}
\frac{A}{l} & 0 & 0 & 0 & 0 & 0 & \frac{-A}{l} & 0 & 0 & 0 & 0 & 0

0 & \frac{12A}{E l^3} & 0 & 0 & 0 & \frac{6A}{E l^3} & 0 & \frac{-12A}{E l^3} & 0 & 0 & 0 & \frac{6A}{E l^3}

0 & 0 & \frac{12A}{E l^3} & 0 & \frac{6A}{E l^3} & 0 & 0 & \frac{-12A}{E l^3} & 0 & \frac{-6A}{E l^3} & 0 & 0

0 & 0 & 0 & \frac{J}{2(1+v)} & 0 & 0 & 0 & 0 & \frac{J}{2(1+v)} & 0 & 0 & 0

0 & 0 & \frac{-6A}{E l^2} & 0 & \frac{4A}{E l^2} & 0 & 0 & \frac{-6A}{E l^2} & 0 & 0 & 0 & \frac{2A}{E l}

0 & \frac{-12A}{E l^3} & 0 & 0 & \frac{6A}{E l^3} & 0 & \frac{12A}{E l^3} & 0 & 0 & 0 & \frac{-6A}{E l^2}

0 & 0 & \frac{-12A}{E l^3} & 0 & \frac{6A}{E l^3} & 0 & 0 & \frac{-12A}{E l^3} & 0 & \frac{6A}{E l^3} & 0 & 0

0 & 0 & 0 & \frac{-J}{2(1+v)} & 0 & 0 & 0 & 0 & \frac{-J}{2(1+v)} & 0 & 0 & 0

0 & 0 & \frac{-6A}{E l^2} & 0 & \frac{4A}{E l^2} & 0 & 0 & \frac{-6A}{E l^2} & 0 & \frac{J}{2(1+v)} & 0 & 0

0 & \frac{6A}{E l^2} & 0 & 0 & \frac{-J}{2(1+v)} & 0 & 0 & \frac{2A}{E l} & 0 & \frac{-6A}{E l^2} & 0 & 0
\end{bmatrix}
$$

where

- \( E \) is Young's modulus of elasticity
- \( A \) is the member cross-sectional area
- \( l \) is the member length
- \( I_k \) is the second moment of area about axis \( k \)
- \( J \) is the polar moment of area
- \( v \) is Poisson's ratio
Mass matrix

\[
[M] = \frac{\rho A_1}{420}
\]

\[
\begin{bmatrix}
140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\
0 & 156 & 0 & 0 & 0 & 22 \times 1 & 0 & 54 & 0 & 0 & 0 & -13 \times 1 \\
0 & 0 & 156 & 0 & -22 \times 1 & 0 & 0 & 0 & 54 & 0 & 13 \times 1 & 0 \\
0 & 0 & 0 & \frac{140 \times 1}{A} & 0 & 0 & 0 & 0 & 0 & \frac{70 \times 1}{A} & 0 & 0 \\
0 & 0 & -22 \times 1 & 0 & 4 \times 1^2 & 0 & 0 & 0 & -13 \times 1 & 0 & -3 \times 1^2 & 0 \\
0 & 22 \times 1 & 0 & 0 & 0 & 4 \times 1^2 & 0 & 13 \times 1 & 0 & 0 & 0 & -3 \times 1^2 \\
70 & 0 & 0 & 0 & 0 & 0 & 140 & 0 & 0 & 0 & 0 & 0 \\
0 & 54 & 0 & 0 & 0 & 13 \times 1 & 0 & 156 & 0 & 0 & 0 & -22 \times 1 \\
0 & 0 & 54 & 0 & -13 \times 1 & 0 & 0 & 0 & 156 & 0 & 22 \times 1 & 0 \\
0 & 0 & 0 & \frac{70 \times 1}{A} & 0 & 0 & 0 & 0 & 0 & \frac{140 \times 1}{A} & 0 & 0 \\
0 & 0 & 13 \times 1 & 0 & -3 \times 1^2 & 0 & 0 & 0 & 22 \times 1 & 0 & 4 \times 1^2 & 0 \\
0 & -13 \times 1 & 0 & 0 & 0 & -3 \times 1^2 & 0 & -22 \times 1 & 0 & 0 & 0 & 4 \times 1^2 \\
\end{bmatrix}
\]

where

\( \rho \) is material density
APPENDIX III

LINEAR SPECTRAL THEORY
Linear Spectral Theory

Concepts of a Random Process

To say a process is random means that it is governed by a probability law. The word "process" suggests something that evolves in time, the follow description is based on ideas described by Kinsman (1965). Consider the Kinsale platform located in the Celtic Sea as an example. Imagine an unlimited supply of Celtic Seas and that it was possible to set identical macroscopic conditions e.g. mean wind velocity, fetch length etc. in each sea. Further, suppose that in each you select corresponding fixed points (for example a point close to the Kinsale platform) and made records of the waves. You would not expect these records to be identical or even closely similar in detail, since you know that the records taken simultaneously at adjacent points subject to the same weather system show little correlation. However, there is a sense in which these, although dissimilar in detail, are all similar. It is this similarity which we are interested in describing mathematically.

Let \( \eta^1(t) \) be the point wave record taken in the first Celtic Sea, \( \eta^2(t) \) be that taken in the second, \( \eta^3(t) \) in the third and in general \( \eta^k(t) \) is the kth. Such a collection of records, of a process is called an ensemble (also referred to as sample functions). It may be either finite, if \( k \) is finite, or infinite, if \( k \) is infinite. Each record begins at time \( t = -\infty \) and ends at time \( t = \infty \). Obviously an ensemble is a conceptual idea since none of us can wait that long. The statistical quantities about which we will talk always have an exact meaning in terms of the ensemble. The computable statistics with which we must content
ourselves are always calculated from partial approximations to the ensemble, and their physical meanings are often obscure (e.g. cross-spectra).

Stationary

For a number \( N \) and any fixed times \( t_1, t_2, t_3, t_4, \ldots, t_N \) the quantities \( \eta^k(t_1), \eta^k(t_2), \eta^k(t_3), \ldots, \eta^k(t_N) \) represent \( N \) random variables over the index \( k \). In general a particular sample function \( \eta^k(t) \), would not be suitable for representing the entire random process \( \{\eta^k(t)\} \) to which it belongs. Under certain conditions it turns out that for the special class of ergodic random processes, it is possible to derive desired statistical information about the entire random process from analysis of a single arbitrary sample function. Consider the arbitrary random process \( \{\eta^k(t)\} \). The first statistical qualities of interest are the ensemble mean values at arbitrary fixed values of \( t \), where \( \eta^k(t) \) is a random variable over the index \( k \).

The mean is defined as

\[
\mu_\eta(t) = E[\eta^k(t)]
\]

In general this mean value is different at different lines. That is

\[
\mu_\eta(t_1) \neq \mu_\eta(t_2)
\]

The next statistical quantities of interest are the covariance functions at arbitrary fixed values of \( t_1 = t \) and \( t_2 = t + \tau \).
This function is defined by

\[ C_\eta(t, t + \tau) = E[(\eta^k(t) - \mu_\eta(t))(\eta^k(t + \tau) - \mu_\eta(t + \tau))] \]

In general these quantities are different for different combinations of \( t_1 \) and \( t_2 \). If the mean value \( \mu_\eta(t) \) together with the covariance \( C_\eta(t, t + \tau) \) yield the same result for all fixed values of \( t \), the random process is said to be weakly stationary. If all possible probability distributions involving \( \{\eta^k(t)\} \) are independent of time translations, the processes are strongly stationary. For Gaussian random processes, these two stationary concepts coincide.

For stationary random process \( \{\eta^k(t)\} \), the mean value is a constant, independent of time. That is for all \( t \)

\[
\mu_\eta = E[\eta^k(t)] = \int_{-\infty}^{\infty} \eta p(\eta) d\eta
\]

where

\( p(\eta) \) is the probability density function of \( \eta \).

For arbitrary fixed \( t \) and \( \tau \): define the correlation function as

\[ R_\eta(\tau) = E[\eta^k(t)(\eta^k(t + \tau))] \]

where
\[ R_\eta(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 x_2 p(x_1, x_2) dx_1 dx_2 \]

where

\[ x_1 = \eta(t) \]

and

\[ x_2 = \eta(t + \tau) \]

**Ergodic.**

Now consider a Gaussian stationary random process. The stationary random process is ergodic if the mean values and covariance function which are defined already by ensemble averages, may be calculated performing corresponding time averages on an arbitrary sample function. It is assumed that the random process of ocean surface waves incident on a pile is both stationary and ergodic.

**Response to Periodic and Transient Loading**

Robson (1963) present a methodology for relating response spectra to input spectra based on structural responses to unit loads, this approach is followed below.

Consider the response of a single degree of freedom spring-mass system to a sinusoidally varying force

\[ P(t) = P_0 e^{i\omega t} \]

The equation of motion may be written as

\[ M\ddot{x} + C\dot{x} + Kx = P_0 e^{i\omega t} \]
The particular integer for the above equation is

\[ x = \frac{P_0}{K - M\omega^2 + i\omega C} e^{i\omega t} \]

where

\[ \alpha(\omega) = \frac{1}{K - M\omega^2 + i\omega C} \]

The quantity \( \alpha(\omega) \) is the response of the system to a complex force of unit modulus and it is called the receptance of the system.

The response to a transient force can be determined from Duhamel's integral, which expresses the response in terms of the response to unit impulse.

An impulsive loading can be expressed as

\[ P(t) = I \delta(t) \]

where

- \( I \) is the magnitude of the impulse
- \( \delta(t) \) is the Dirac \( \delta \)-function

The response to such an impulse loading may be expressed as

\[ X(t) = W(t) I \]
where the function $W(t)$, which gives the response to a unit impulse, must be determined for the particular system being considered.

Let us look at the one degree of freedom system subjected to an impulse $I$ at time $t = 0$ and starting from rest; then the equation of motion for this system is given by

$$M\ddot{x} + C\dot{x} + Kx = I\delta(t)$$

and its solution may be shown to be

$$x = \frac{I}{M\omega} e^{-\alpha t} \sin \omega t$$

where $\omega$ and $\alpha$ are functions of $K$, $M$ and $C$ only.

Thus the response to a unit impulse, $W(t)$, is thus given by

$$W(t) = \frac{I}{M\omega} e^{-\alpha t} \sin \omega t$$

The above result can be used to obtain the response to a continuous transient loading, $P(t)$. The transient loading is considered as a series of contiguous impulse loads, the impulse due to $P(\tau)$ at time $t$ being given by $P(\tau)\, d\tau$. The response at time $t$ to a single impulse load occurring at time $\tau$ is
\[ x(t) = W(t - \tau) P(\tau) \delta \tau \]

The total response at time \( t \) to all impulse loads up to that time is given by the integral.

\[ X(t) = \int_{-\infty}^{t} W(t - \tau) P(\tau) \, d\tau \]

The above integral is the well-known Duhamel’s integral.

Robson (1963) showed that the following important relationship exists between \( \alpha(\omega) \) and \( W(\tau) \)

\[ \alpha(\omega) = \int_{0}^{\infty} W(\tau) e^{-i\omega \tau} \, d\tau \]

that is, \( \alpha(\omega) \) is the Fourier transform of \( W(\tau) \).

**Response to Random Loading**

Again denote the random force by \( P(t) \) and assume it to have autocorrelation function \( R_p(\tau) \) and spectral density function \( S_p(\omega) \). The resulting displacement is again denoted by \( X(t) \) and its autocorrelation function and spectral density by \( R_x(\tau) \) and \( S_x(\omega) \) respectively.

Robson (1963) showed, using the results from the previous section, that

\[ R_x(\tau) = \int_{0}^{\infty} W(\tau_1) \int_{0}^{\infty} W(\tau_2) R_p(\tau_1 - \tau_2 + \tau) \, d\tau_2 \, d\tau_1 \]
and that

\[ S_x(\omega) = \alpha^*(\omega) \alpha(\omega) S_p(\omega) \]

where

* denotes complex conjugate.

The last expression above relating response spectra to input spectra is used extensively in the linear spectral model described in Chapters 3 and 4 of this dissertation.

Robson (1963) also illustrates the important relationship that exists between the spectrum and auto-correlation function of a random process; namely, that they are a Fourier pair

\[ S(f) = \int_{-\infty}^{\infty} 2R(\tau)e^{-i2\pi ft} d\tau \]

and

\[ R(\tau) = \int_{-\infty}^{\infty} \frac{1}{2} S(f)e^{i2\pi ft} df \]
APPENDIX IV

LINEAR WAVE THEORY
Linear Wave Theory

Ocean surface waves are generated primarily by the drag of the wind on the water surface, and account for about 70% of total loading on an offshore structure. For engineering purposes, it is customary to analyse the effects of surface waves on structures either by use of a single design wave chosen to represent extreme storm conditions in the area of interest or by use of a statistical representation of the waves. Either way, it is necessary to relate surface-wave data to water particle velocities and accelerations beneath the waves; this is achieved by the use of an appropriate wave theory. Many such theories exist, and a comprehensive description of these may be found in Dawson (1983). Most of these theories describe a non-linear relationship between wave elevation and resulting water particle velocities and accelerations. In this dissertation linear Airy wave theory, as described by Dawson (1983), is used.

Airy wave theory describes ocean surface waves as having a sinusoidal form. The theory assumes that the wave height $H$ is small in comparison with the wavelength $\lambda$ and the water depth $h$ as shown in Figure AIV.1. The surface wave elevation $\eta$ at any point $x$ is given by

$$\eta = \frac{H}{2} \cos(kx - \omega t)$$

where

$$k = \text{wavenumber defined in terms of the wave length } \lambda \text{ as}$$

$$k = \frac{2\pi}{\lambda}$$
and

\[ \omega = \text{wave frequency, defined in terms of wave period}, \ T, \text{ as} \]

\[ \omega = \frac{2\pi}{T} \]

Using potential theory, Chakrabarti (1994), the horizontal velocity in the direction of wave propagation, \( u \), and vertical velocity, \( v \), of the water particles at position \( (x,y) \) are given

\[ u = \frac{gTH \cosh k(h-y)}{2L \sinh kh} \frac{\cos(kx - \omega t)}{\cosh kh} \]

\[ v = \frac{gTH \sinh k(h-y)}{2L \sinh kh} \frac{\sin(kx - \omega t)}{\sinh kh} \]

Airy theory relates \( \omega \) and \( k \) through the dispersion equation

\[ \omega^2 = gk \tanh kh \]

where

\( g \) is the acceleration due to gravity

In order to calculate the forces imparted by waves to offshore using, say, Morison's equation, the resulting water particle velocities and accelerations are required. Thus we need to solve equation the dispersion in order to find \( k \).
Rearranging the dispersion equation we get

\[ k = \frac{\omega^2}{g \tanh kh} \]

which is solved using some iteration technique. In this dissertation Newton-Raphson is used to solve the dispersion equation, Carnahan et al (1969).

Finally, for given \( k, \omega, \) and \( H \) the water particle accelerations \( a_x \) and \( a_y \) are given as

\[
\begin{align*}
    a_x &= \frac{\partial u}{\partial t} \\
    a_y &= \frac{\partial v}{\partial t}
\end{align*}
\]

Using the expression for velocities from above the accelerations can be written as

\[
\begin{align*}
    a_x &= \frac{\pi g H \cosh k (h - y)}{L \sinh ky} \sin(kx - \omega t) \\
    a_y &= \frac{-\pi g H \sinh k (h - y)}{L \sinh ky} \cos(kx - \omega t)
\end{align*}
\]

for time \( t \) at location \((x,y)\).

Figure AIV.2 shows how the horizontal water particle velocity decays as a function of depth. Similar trends are observed for vertical water particle velocities and also for accelerations. Thus in the calculation of forces induced by gravity waves it is common
practice to assume a constant force below a certain depth, such as twice the wavelength of the incident wave. In this development wave kinematics are calculated for all depths. Another interesting aspect of ocean waves is the path of particle motion. Sarpkaya and Isaacson (1981) show that the horizontal and vertical water particle displacements, $X$ and $Y$ respectively, are given as

$$X = -\frac{H \cosh ky}{2 \sinh kh} \sin(kx - \omega t)$$

and

$$Y = \frac{H \sinh ky}{2 \sinh kh} \cos(kx - \omega t)$$

Using the trigonometric relationship

$$\cos^2 \theta + \sin^2 \theta = 1$$

it can be shown that

$$\frac{X^2}{\left(\omega H \cosh ky \right)^2 \sinh^2 kh} + \frac{Y^2}{\left(\omega H \sinh h^2 ky \right)^2 \sinh^2 kh} = 1$$

The above equation describes an ellipse with semi-axes

$$\left| \frac{\omega H \cosh ky}{2 \sinh kh} \right|$$

horizontally
The motion described by the particles is shown schematically in Figure IV.3. An important aspect of the particle motions is that the direction of the motion changes when it is beneath different parts of the wave, for example, two particle move in opposite directions when one is under the wave crest and the other is under the wave trough. This has important consequences for structural responses when structure spatial extent is incorporated into analyses.

Spectral Relationships

Using the above expressions for water particle velocities and accelerations Borgman (1967) presents the following relationships for cross-correlation function of water particle velocities and accelerations during a random unidirectional storm event:

$$R_{V_V}(y,y',\tau) = 2 \int_0^\infty S(f)(2\pi f)^3 \frac{\cosh ky \cosh ky'}{\sinh^2 kh} \cos[k(x_2 - x_1) - 2\pi ft]df$$

$$R_{V_A}(y,y',\tau) = 2 \int_0^\infty S(f)(2\pi f)^3 \frac{\cosh ky \cosh ky'}{\sinh^2 kh} \sin[k(x_2 - x_1) - 2\pi ft]df$$

$$R_{A_V}(y,y',\tau) = -2 \int_0^\infty S(f)(2\pi f)^3 \frac{\cosh ky \cosh ky'}{\sinh^2 kh} \sin[k(x_2 - x_1) - 2\pi ft]df$$
\[
R_{A_m A_n}(y, y', \tau) = 2 \int_{0}^{\infty} S(f)(2\pi f)^a \frac{\cosh ky \cosh ky'}{\sinh^2 kh} \cos[k(x_2 - x_1) - 2\pi ft] df
\]

where

\( A_m \) is the horizontal component of water particle acceleration degree of freedom \( m \), which is at elevation \( y \) and horizontal location \( x_1 \), at time \( t \)

\( A_n \) is the horizontal component of water particle acceleration degree of freedom \( n \), which is at elevation \( y' \) and horizontal location \( x_2 \), at time \( t \)

\( V_m \) is the horizontal component of water particle velocity degree of freedom \( m \), which is at elevation \( y \) and horizontal location \( x_1 \), at time \( t \)

\( V_n \) is the horizontal component of water particle velocity degree of freedom \( n \), which is at elevation \( y' \) and horizontal location \( x_2 \), at time \( t \)

\( R_{A_m V_n}(y, y', t) \) is the cross-correlation function between \( A_m \) and \( V_n \)

\( S(f) \) is the spectrum of sea-state at circular frequency \( f \).

Spectra of the above terms are obtained by taking relevant Fourier transforms. The above expressions are used to compute required cross-spectral terms of Chapter 5.
Figure AIV.1 - Wave Definition Diagram (Mitchell 1996)

Figure AIV.2 - Velocity Attenuation with Depth (Mitchell 1996)
Figure AIV.3 - Schematic of Water Particle Motions (Mitchell 1996)
APPENDIX V

FREQUENCY CONVOLUTION
Frequency Convolution

In the development of the non-linear spectral model convolutions of spectra were carried out, some aspects of this important process are presented here based on the work of Papoulis (1962) and Lipsett (1985).

Papoulis (1962) presents time and frequency convolution theorems that form the basis for computing spectral convolutions as required in Chapters 5 and 6. In the time domain he considers two functions \( f_1(t) \) and \( f_2(t) \) and the convolution of these two functions is defined as

\[
f(t) = \int_{-\infty}^{\infty} f_1(y)f_2(t-y)dy
\]

Papoulis proves that the Fourier transform \( F(\omega) \) of the convolution of the two functions \( f_1(t) \) and \( f_2(t) \) equals the product of the Fourier transforms \( F_1(\omega) \) and \( F_2(\omega) \) of these two functions. That is if

\[
f_1(t) \leftrightarrow F_1(\omega) \quad f_2(t) \leftrightarrow F_2(\omega)
\]

then

\[
\int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)d\tau \leftrightarrow F_1(\omega)F_2(\omega)
\]

where

\[
\leftrightarrow \quad \text{represent Fourier pairs}
\]
Using the above result Papoulis showed that the Fourier transform $F(\omega)$ of the product $f_1(t) f_2(t)$ of two functions equals the convolution $F_1(\omega) \ast F_2(\omega)$ of their respective transforms $F_1(\omega)$ and $F_2(\omega)$ divided by $2\pi$, that is

$$f_1(\omega)f_2(\omega) \longleftrightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(y)F_2(\omega - y)dy$$

The third-order water particle velocity convolutions are computed using fast Fourier transforms that are built into the FORTRAN compiler. These efficient algorithms are required because of the complex nature of the calculations. In order to assess the effects of convolution it is useful to consider convolution calculations for a simpler harmonic system as outlined by Lipsett (1985).

This procedure illustrates how convolution spectra of orders 2 and 3 are computed for a simple harmonic wave. For more complex spectral shapes such as the JONSWAP spectrum the same procedure can be used but the Fourier transforms are efficiently computed by the fast Fourier transform technique.

The two sided spectral representation of an harmonic wave with frequency $\omega_0$ and mean square value $a^2$ is given by

$$S(\omega) = \frac{a^2}{2} \delta(\omega - \omega_0) + \frac{a^2}{2} \delta(\omega + \omega_0)$$

where $\delta(\cdot)$ is the Dirac delta function.
The procedure to find the convolution spectrum \([S(\omega_0)]^r\) starts by finding the correlation function \(R(\tau)\) which is the Fourier transform of the spectrum. That is

\[
R(\tau) = \int_{-\infty}^{\infty} S(\omega)e^{i\omega \tau} d\omega \\
= a^2 \cos \omega_0 \tau
\]

Next raise the correlation function to the power \(r\) and take the Fourier transform to obtain the desired convolution spectrum. This procedure will be shown for \(r = 2\) and \(r = 3\) below. Therefore

\[
R^2(\tau) = a^4 \cos^2 \omega_0 \tau \\
= \frac{a^4}{2} + \frac{a^4}{2} \cos 2\omega_0 \tau
\]

and

\[
R^3(\tau) = a^6 \cos^3 \omega_0 \tau \\
= \frac{3a^6}{4} \cos \omega_0 \tau + \frac{a^6}{4} \cos 3\omega_0 \tau
\]

Originally the correlation function contained only one frequency \(\omega_0\) but in raising the correlation to some power the correlation function and hence the energy content is distributed to other frequencies such as \(2\omega_0\) and \(3\omega_0\). In the development of Chapter 5 it can be seen that third-order convolutions of water particle velocities are used resulting in wave loadings being induced at three times the associated wave frequencies.

Taking the Fourier transform then gives the convolution spectrum which in general is
\[ \{S(\omega)\}^r = \frac{1}{2\pi} \int_{-\infty}^{\infty} R^r(\tau)e^{-i\omega\tau}d\tau \]

and for the specific examples being considered here are

\[ \{S(\omega)\}^2 = \frac{a^4}{2}\delta(\omega) + \frac{a^4}{2}\delta(\omega + 2\omega_o) + \frac{a^4}{2}\delta(\omega - 2\omega_o) \]

and

\[ \{S(\omega)\}^3 = \frac{3a^6}{8}\delta(\omega + \omega_o) + \frac{3a^6}{8}\delta(\omega - \omega_o) + \frac{a^6}{8}\delta(\omega + 3\omega_o) + \frac{a^6}{8}\delta(\omega - 3\omega_o) \]

Figure AV.1 shows the relationship between the original spectrum and the *2 and *3 convolution spectra graphically. For the case of a more complex initial spectral shape, such as the Pierson-Moskowitz spectrum, the convolution spectra are obtained in an analogous fashion. However, now the Fourier transforms must be obtained numerically and are efficiently evaluated using the fast Fourier transform algorithm.
Figure AV.1 - Discrete Convolutions
APPENDIX VI

HERMITE POLYNOMIALS
Hermite Polynomials

Hermite polynomials, $H_k(z)$, find frequent use in applied problems because they possess the important property that they are orthogonal with respect to a zero mean unit variance Gaussian probability density weighting function. That is

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} H_j(z) H_k(z) dz = k! \delta_{jk}$$

where

$$\delta_{jk} = 1 \text{ only if } i = k \text{ and is zero otherwise.}$$

Because of this property it is often useful to use Hermite polynomial expansions for Gaussian random variables or for functions of Gaussian random variables. Specifically it is very easy to evaluate expected values of Hermite polynomial expressions of Gaussian variables due to the orthogonality relationship. In what follows some useful properties of Hermite polynomials are described.

Abramowitz and Stegun (1964) define the Hermite polynomials $H_k(z)$ by the Rodrigues formula

$$H_k(z) = (-1)^k e^{z^2/2} \frac{d^k}{dz^k} e^{-z^2/2} \quad k = 1, 2, \ldots$$

The lowest few Hermite polynomials are
The expected value of a function of a zero mean unit variance Gaussian random variable $z$ is given by

$$
\langle f(z) \rangle = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} f(z) dz
$$

which can be easily evaluated using the orthogonality relationship if $f(z)$ is expanded in an Hermite polynomial series. That is, if $f(z)$ is expressed as a linear function of Hermite polynomials

$$
f(z) = \sum_{k=0}^{\infty} c_k H_k(z)
$$

then

$$
\langle f(z) \rangle = \sum_{k=0}^{\infty} c_k \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} H_k(z) H_0(z) dz
\quad = c_0
$$

That is the expected value of $f(z)$ is simply given by the first coefficient $c_0$ of the Hermite polynomial expansion of that function.

In employing Hermite polynomials it thus becomes necessary to determine the series coefficients. The coefficients $c_k$ can be found in a number of equivalent ways. Three different methods are described here.
Orthogonality

Multiplying both sides of the above equation by $e^{-z^2/2} H_j(z)/\sqrt{2\pi}$ and integrating over the range of $z$ gives

$$
\int_{-\infty}^{\infty} f(z) e^{-z^2/2} H_j(z) dz = \sum_{k=0}^{\infty} c_k \int_{-\infty}^{\infty} H_k(z) e^{-z^2/2} H_j(z) dz
$$

Using the orthogonality relationship gives

$$
c_k = \frac{1}{k! \sqrt{2\pi}} \int_{-\infty}^{\infty} f(z) e^{-z^2/2} H_k(z) dz
$$

Least square minimisation

The coefficients $c_k$ can also be found by minimising the expected value of the squared error of the approximation. That is minimising with respect to the coefficients gives

$$
\frac{\partial}{\partial c_k} \left( (f(z) - \sum_{k=0}^{\infty} c_k H_k(z))^2 \right) = 0
$$

which can be shown to yield the same expressions for $c_k$ as given above.

Weighted residuals

The residual $R$ of the expansion process is

$$
R = f(z) - \sum_{k=0}^{\infty} c_k H_k(z)
$$
The coefficients \( c_k \) can be found by setting the expected value of the weighted residual to zero. That is

\[
\langle W_j(R) \rangle = \langle W_j(f(z) - \sum_{k=0}^{\infty} c_k H_k(z)) \rangle = 0
\]

where

\( W_j = W_j(z) \) are the weighting functions.

If the weighting functions are themselves Hermite polynomials, \( W_j(z) = H_j(z) \), then the same expression for the coefficients \( c_k \) will again result.

In Chapter 5 the coefficients are obtained using the least squares minimisation technique as applied by Eatock-Taylor and Rajagopalan (1982).

Eatock-Taylor and Rajagopalan (1982) also showed that the expected values of the product of two functions can be found using Hermite polynomial expansions. For example if

\[
f(z(t_1)) = \sum_j a_j H_j(z(t_1))
\]

and

\[
g(z(t_2)) = \sum_k b_k H_k(z(t_2))
\]

then the correlation function is given by

A34
Lin (1976) showed that when $z$ is Gaussian that the joint probability density function is

$$p(z_1, z_2) = \frac{1}{2\pi} \exp \left[ -\frac{z_1^2 + z_2^2}{2} \right] \sum_r \frac{\rho^r}{r!} \text{He}_r(z_1)\text{He}_r(z_2)$$

where

$$z_1 = z(t_1)$$
$$z_2 = z(t_2)$$
$$\rho = \langle z_1 z_2 \rangle$$

Using the orthogonality relationship for Hermitage polynomials gives the following expression

$$\langle f(z_1)g(z_2) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(z_1, z_2) f(z_1)g(z_2) dz_1 dz_2$$

$$= \sum_a a_r b_r r! \rho^r$$

Again, this approach is followed in Chapter 5.
APPENDIX VII

FORTRAN COMPUTER CODE
INCLUDE 'SPEC.CMN'
DIMENSION VEC(100,100)
REAL ALPHA
DIMENSION HED(20)

DO 23 I=1,1000000
IA(I)=0
23 CONTINUE
ITWO=1
DO 22 I=1,7000000
IA(I)=0
22 CONTINUE
MTOT=7000000

OPEN RELEVANT FILES
OPEN(17,FILE='OUTPUT.DAT',STATUS='OLD')
OPEN(18,FILE='ROOTS.DAT',STATUS='OLD')
OPEN(19,FILE='DAMPING.DAT',STATUS='OLD')
OPEN(50,FILE='STRAP.DAT',STATUS='OLD')
OPEN(51,FILE='TYPE.DAT',STATUS='OLD')
READ(19,556)AA3B,RMI,ETA,IDAMP,CDAMP
556 FORMAT(4F10.0,I10,F10.0)
SQMI=RMI**2.
JELMNT=1
ILOAD=2
IN=5
IOUT=17
ISTRP=50
ITYPE=51

200 NUMEST=0
MAXEST=0

INPUT PHASE
READ CONTROL INFORMATION
OPEN(20,FILE='INPUT.DAT',STATUS=' OLD')
READ(20,1000JEND=888) HED,NUMNP,NMEG,NLCASE,MODEX,
CLOSE(19)
CLOSE(20)
IF (NUMNP.EQ.0) STOP
WRITE (IOUT,200)HED,NUMNP,NMEG,NLCASE,MODEX

READ NODAL POINT DATA
CALL INPUT(IA(N1),A(N1),A(N2),A(N3))
NEQ=NEQ+1
READ,GENERATE AND STORE ELEMENT DATA
IND=1
CALL ELCALL
SOLUTION PHASE
C ASSEMBLE STIFFNESS & MASS MATRICES
CALL DIAGAD(IA(IN5),IA(IN2))
C
MM=NWK/NEQ
C IN DATA CHECK ONLY MODE WE SKIP ALL FURTHER CALCULATIONS
C
IF (MODEX.GT.0) GO TO 100
GO TO 120
C
100 NNL=NWK+NEQ
C
IND=2
C
REWIND 1
DO 1345 NP=1,JMEG
C CALL ASSEM
CALL STORBM
1345 CONTINUE
C
C READ VALUES FOR SOLVING EIGENVALUE PROBLEM
READ(18,1347)NROOT,NITEM,NCT,JEXPE
1347 FORMAT(3I10,F10.0)
RTOL=10.**(-JEXPE)
NC=NCT+NROOT
IF(NC.GT.NEQ)THEN
NC=NEQ
ELSE
CONTINUE
END IF
NNC=NC*(NC+1)/2
IFPR=1
IFSS=1
NSTIP=7
NNM=NEQ+1
C
NN17=NEQ*NC
N17=N16+NN17
N18=N17+NC
C
N19=N18+NEQ
N20=N19+NC
N21=N20+NNC
N22=N21+NNC
N23=N22+NC
N24=N23+NC
N25=N24+NC
N26=N25+NC
PRINT*,TESTr
WRITE(IOUT,*)' REAL TOTAL=',N26
WRITE(IOUT,*)' INTEGER TOTAL=',IN6
C
CALL SSSPACE(A(N14),A(N15),IA(IN5),A(N16),A(N17),
1 RTOL,NC,NNC,NITEM,IFSS,IFPR,NSTIP,
1 NNM,VEC,A(N18),A(N19),A(N20),A(N21),A(N22),A(N23),
1 A(N24),A(N25))
C
C FACTOR EIGENVALUES
DSCAB=SCA-SCB
DSCBA=SCB-SCA
DO730 IPP=1,JROOT
NPP=N17-1+IPP
A(NPP)=A(NPP)*(10.**DSCAB)
WRITE(IOUT,*)'ROOT',IPP,'=',A(NPP)
730 CONTINUE
C
SQSCB=SQRT(10**SCB)
DO IRY=1,NN17
IRYM1=ARY+1
A(ARYM1)=A(ARYM1)/SQSCB
END DO
C
OPEN(19,FILE='INTE.DAT',STATUS='OLD')
READ(19,777)CD,IXO,IXN,IFYM,IFYM
777 Format(F10.0,4I10)
CLOSE(19)

CALL SPEC

C INITIALISE FOR ITERATION
CALL STORAGE(ALPHA)

C NUME=NPAR(2)
ISPT=SPT
ISPNUM=ISPT*NUME

C CALL CBARJ(A(N112),NUME,NC,ALPHA,ISPT,A(N111),
A(N113),NC,A(N114),A(N12),1A(IN4),ISPNUM,
A(N131),A(N132),A(N133))
N1=1

C STORAGE SPACE FOR 'MODAL FOR'
NEQRT=NEQ*NROOT
N116=NI15+NEQRT
N117=NI16+NEQRT
N118=NI17+NEQRT
N119=NI18+NROOT

C 120 CONTINUE
GO TO 200

C 888 PRINT*, 'END OF DATA PROGRAM FINISHED'
STOP

1000 FORMAT(20A4/5I5/6F10.0)
1010 FORMAT(2I5)

C 2000 FORMAT (1H1,20A4///,
1 45H CONTROL INFORMATION //,5X,
2 45H NUMBER OF NODAL POINTS ... (NUMNP) =J5,
3/5X,
4 45H NUMBER OF ELEMENT GROUPS ... (NUMEG) =J5
5/5X,
5 45H NUMBER OF LOAD CASES ... (NL CASE) =J5
5/5X,
6 45H SOLUTION MODE ........... (MODEX) =J5
5/5X,
7 45H EQ.0 DATA CHECK /,5X,
8 45H EQ.1 EXECUTION )

2005 FORMAT(1H1,26H LOAD CASE DATA)
2010 FORMAT(///4X,28H LOAD CASE NUMBER .......... =J5,5X,
1 32H NUMBER OF CONCENTRATED LOADS =,I5)
2015 FORMAT(1H1,9H LOAD CASE,J3)
2020 FORMAT(1X,40H ** ERROR LOAD CASES ARE NOT IN ORDER )
2025 FORMAT(1H1,)

1 17H TOTAL SYSTEM DATA,///,5X,
2 45H NUMBER OF EQUATIONS ........... (NEQ) =J5/5X,
3 45H NUMBER OF MATRIX ELEMENTS ........ (NW K) =J5/5X,
4 45H MAXIMUM HALF BANDWIDTH ........ (MK ) =J5/5X,
5 45H MEAN HALF BANDWIDTH .......... (MM ) =J5

END

C FUNCTION GG1(X1MJ,CL)
GG1=1.-X1MJ/XL
RETURN
END

FUNCTION H2(X1MJ,XL)
H2=X1MJ/XL
RETURN
END

FUNCTION XI3(X1MJ,XL)
XI3=1.-3*(X1MJ/XL)**2.+2.*((X1MJ/XL)**3.
RETURN
END

FUNCTION J4(X1MJ,XL)
J4=X1MJ^2.*((X1MJ**2.)/XL+(X1MJ**3.)/(XL**2.))
RETURN
END

FUNCTION KS(X1MJ,XL)
RETURN
END

FUNCTION L6(X1MJ,XL)
L6=-(X1MJ**2.)/XL+X1MJ**3./(XL**2.)
RETURN
END

FUNCTION M7(X1MJ,XL)
M7=-X1MJ+2.*X1MJ**2./XL-X1MJ**3/(XL**2.)
RETURN
END

FUNCTION NN8(X1MJ,XL)
NN8=0.0+(X1MJ**2.)/XL-(X1MJ**3.)/(XL**2.)
RETURN
END

FUNCTION RKC(X,D,SOILK)
RKC=SOILK*(-X+D)
RETURN
END

A40
When changing problems you should also change two dimension statements in RUSS.FOR which define ranges for:

- XIFL, PT1, PT2, PT3, NTFL & XL

These are all currently set to:

724

**COMMON**

- /INTE/CD,IXO,IXN,JYO,JYM,ITEFLG,ITERFLG,RATIO(40)
- /MS/MASSFL(724),WT(724)
- /SOL/NUMNP,NEQ,NWK,NUMEST,MIDEST,MAXEST,LMK
- /DIME/N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15
- /DM1/N16,N17,N18,N19,N20,N1,N2,N3,N4,N5,N6,N7,N8
- /SPEC1/N100,N101,N102,N103,N104,N105,N106,N107,N108,N109
- /SPECE/N110
- /SPEC2/N111,N112,N113,N114,N115,N116,N117,N118,N119,N120
- /SPEC3/N121,N122,N123,N124,N125,N126,N127,N128,N129,N130
- /EL/IND,NPAR(10),NUMEG,MTOT,NFIRST,NLAST,ITWO
- /VAR/NG,MODEX
- /TAPES/IEMNT,ILOAD,JIN,JOUT,JSTRP,JTYPE
- /NX2/N21,N22,N23,N24,N25,N26
- /IA(2000000),A(10000000),T(12,12,724)
- /RT/NROOT
- /PM/STEP,SPECTR(202),WLST
- /CONV/CON,ITER
- /MUD/XMUD,XFOUND,IFOUND(724),INUME(724),JNUME(724)
- /SP/SPT,SCA,SCB
- /DAMP/AA3B,CM,SQMI,ETA,IDAMP,CDAMP,wht
- /ZS/Z1(10,202),Z2(10,202) IMH INCLUDED 11/5/95

**DATA WHT/1.0/**
SUBROUTINE INPUT(ID,X,Y,Z)

PROGRAM
TO READ, GENERATE, AND PRINT NODAL POINT INPUT DATA
TO CALCULATE EQUATION NUMBERS AND STORE THEM IN ID ARRAY

N = ELEMENT NUMBER
ID = BOUNDARY CONDITION CODES (0 = FREE, 1 = DELETED)
X, Y, Z = COORDINATES
KN = GENERATION CODE
I.E. INCREMENT ON NODAL POINT NUMBER

INCLUDE 'SPEC.CMN'
DIMENSION X(1), Y(1), Z(1), ID(6, NUMNP)

READ AND GENERATE NODAL POINT DATA

WRITE (10,2000)
WRITE (10,2010)
KNOLD = 0
NOLD = 0

N = 0
OPEN (10, FILE = 'ID.DAT', STATUS = 'OLD')
10 N = N + 1
READ (10, 1000) (ID(I, N), I = 1, 6)
   X(N), Y(N), Z(N), IFOUND(N)
WRITE (10, 2031) N, X(N), Y(N), Z(N)

IF (N.NE. NUMNP) GO TO 10

WRITE COMPLETE NODAL DATA

EQUATIONS NUMBERS

NEQ = 0
DO 100 N = 1, NUMNP
   DO 100 I = 1, 6
      IF (ID(I, N)) 110, 120, 110
120   NEQ = NEQ + 1
      ID(I, N) = NEQ
   GO TO 100
110   ID(I, N) = 0
100   CONTINUE
   CLOSE (10)
RETURN

1000 FORMAT (6I5, 3F10.0, 15)
2000 FORMAT (1H1, 33H NODAL POINT DATA //)
2010 FORMAT (18H INPUT NODAL DATA //)
2030 FORMAT (15, 6X, 6I5, 6X, 3F10.4, 3X))
2031 FORMAT (15, 6F10.4)
END
SUBROUTINE ELCALL

C ...................................................
C . PROGRAM . TO LOOP OVER ALL ELEMENT GROUPS FOR READING,
C . GENERATING AND STORING ELEMENT DATA
C . ...................................................
C INCLUDE 'SPEC.CMN'
C LOOP OVER ALL ELEMENTS GROUPS
C DO 100 N=1,NUMEG
IF (N.NE.1) WRITE (IOUT,2010)
C OPEN(30,FILE='NPAR.DAT',STATUS='OLD')
READ(30,1000)NPAR
C CALL BEAMEL
C WRITE(IOUT,7654) MAXEST
IF(MIDEST.GT.MAXEST) MAXEST=MIDEST
WRITE(IOUT,7654) MAXEST
C 7654 FORMAT(1X,14HMAXEST(ELCALL)=J10)
C 100 CONTINUE
C 1000 FORMAT(10I5)
2000 FORMAT((1H1,36HELEMENT GROUP DATA //))
2010 FORMAT((1H1))
C END
SUBROUTINE BEAMEL

C
C
C          PROGRAM
C          TO CALL THE APPROPRIATE ELEMENT SUBROUTINE
C
C
C
INCLUD' SPEC.CMN'

C
C  NOTE: HERE WE DETERMINE WHICH TYPE
C  OF AN ELEMENT WE ARE TO LOOK AT
C  NPAR(1)=1......A BEAM
C  NPAR(1)=2......A SPACE FRAME
C
C
C  GO TO (1,2,3),NPAR(1)
1  CALL STORBM
   RETURN
C
C  AT THIS STAGE WE COULD USE OTHER TYPES
C  OF MEMBERS
C
2  CALL STORBM
   RETURN
3  RETURN
C
C
END
SUBROUTINE STORBM

C PROGRAM TO SET UP STORAGE AND CALL THE ELEMENT SUBROUTINE

C

INCLUDE 'SPEC.CMN'
NPAR1=NPAR(1)
NUME=NPAR(2)
NUMMAT=NPAR(3)

C

N5=N4+NUMMAT
N6=N5+NUMMAT
N7=N6+NUMMAT
N8=N7+NUMMAT
N9=N8+NUMMAT
N10=N9+NUMMAT
N11=N10+NUMMAT
N12=N11+NUMMAT
N13=N12+NUMMAT
N14=N13+6*NUME

C

N15=N14+NWK
N16=N15+NWK

C

PRINT*,N16=N16
IN3=IN2+NEQ
IN4=IN3+12*NUME
IN5=IN4+NUME
IN6=IN5+NEQ

C

CALL BEAM(I(A(N1),A(N1),A(N2),A(N3),LA(IN2),A(N4),A(N5),
A(N6),A(N7),A(N8),A(N9),A(N10),A(N11),A(N12),
A(N13),IA(IN3),IA(IN4))

C

RETURN

C

END
WRITE(IOUT,*)' CHECK ON CONNECTIVITY ETC.'
WRITE(IOUT,*)' »****»*»♦*♦**♦»♦**♦*♦***«*♦ ' 
READ AND GENERATE ELEMENT INFORMATION
n o o o
3 " = g 2 
c/3 K >
S> s>
I — *  o o o
M  >> z  Z  2;
^ d d is 
W
f j
SUBROUTINE beam(id, x, y, z, mht, e, area, area wt, area gw, areator)
AR E T 0 T ^M ;X I3M K >L A 1U )T 0T ^Z , LMMATP)
WRITE(OUT,*)' ELEMENT NODE1 NODE2 TYPE MASSFL ASPT WTMASS ENTRFL,
XIFL PT1 PT2 PT3'

PRINT*, READ IN VALUE FOR NUMBER OF SIMPSONS INTEGRATION POINTS
READ(5,*)SPT
ISPT=SPT

DO INO=1,NUME
  READ(15,1020,END=999)NJNUME(INO),JNUME(INO),MTYP,MASSFL(INO),
  ASPT,WT(INO),NTFL(INO),XIFL(INO),PT1(INO),PT2(INO),PT3(INO)
  I=INUME(INO)
  J=JNUME(INO)
  WRITE(IOUT,1025)N,INUME(INO)JNUME(INO),MTYP,MASSFL(INO),
  SPT,WT(INO),NTFL(INO),XIFL(INO),PT1(INO)JPT2(INO),PT3(INO)
  C SAVE ELEMENT INFORMATION
  XYZ(1,INO)=X(I)
  XYZ(2,INO)=Y(I)
  XYZ(3,INO)=Z(I)
  C
  XYZ(4,INO)=X(J)
  XYZ(5,INO)=Y(J)
  XYZ(6,INO)=Z(J)
  C
  MATP(INO)=MTYP
  C
  DO 390 L=1,12
  LM(L,INO)=0
  390 CONTINUE
  DO 400 LU=1,6
  LM(LU,INO)=ID(LU,I)
  LM(LU+6,INO)=ID(LU,J)
  400 CONTINUE
  UPDATE COLUMN HEIGHTS AND BANDWIDTH
  CALL HEIGHTS (MHTJMT),LM(1,INO))
  C
  END DO
  CLOSE(15)

999 RETURN

C ' STIFFNESS AND MASS COMPILATION'

C

OPEN(16,FILE='POLAR.DAT',STATUS='OLD')
READ(16,623)POI,STLDEN,WATDEN,CLADEN,GRODEN,CHECK,SOILK
WRITE(IOUT,'*)' MATERIAL CONSTANTS
WRITE(IOUT,*)' POISSON STLDEN WATDEN CLADEN GRODEN CHECK SOILK
WRITE(IOUT,621  )POI,STLDEN,WATDEN,CLADEN,GRODEN,CHECK,SOILK
621 FORMAT(1X,6F10.4,JF12.2)
623 FORMAT(6F10.0,F15.0)
P(1,1)=130.0
P(2,1)=50.0
P(3,1)=50.0
P(1,2)=30.0
P(2,2)=120.0
P(3,2)=50.0
P(1,3)=50.0
P(2,3)=50.0
P(3,3)=50.0
P(1,4)=50.0
P(2,4)=50.0
P(3,4)=50.0
P(1,5)=100.0
P(2.5)=-100
P(3.5)=100
P(1.6)=15.3
P(2.6)=12.9
P(3.6)=100
GROWT=0.0
GROWT=0.0
WATOUT=0.0
WATIN=0.0
WATTOT=0.0
WEIGHT=0.0
DO 500 N=1,NUME
C
C INITIALIZE MATRICES
DO 1011=1,12
DO 1011=1,12
CLYSTF(I,J)=0.0
T(UJ4)=0.
TGC(I,J)=0.
TRANS(IJ)=0.
TK(I,J)=0.
1011 CONTINUE
C
IXP=1
IFLGMS=0
MTYPE=MATP(N)
XL2=0.
DO 505 LW=1,3
D(LW)=XYZ(LWJ<)-XYZ(LW+3J^)
505 XL2=XL2+D(LW)*D(LW)
C NOTE; XL IS THE LENGTH OF THE ELEMENT IN QUESTION
C
XL(N)=SORT(XL2)
XX=EMTYPE)*AREA(MTYPE)*XL(N)
XL3=XL(N)**3.
C
C READ IN MATRIX R, THIS MATRIX IS USED TO
C COMPILE THE AXES TRANSFORMATION MATRIX
C
A1=-D(1)/XL(N)
A2=-D(2)/XL(N)
A3=-D(3)/XL(N)
T(1,1,N)=A1
T(1,2,N)=A2
T(1,3,N)=A3
C
C READ IN DIAGONAL ELEMENTS FIRST
C
TK(1,1)=AREA(MTYPE)/XL(N)
TK(2,2)=12.*X12M(MTYPE)/XL3
TK(3,3)=12.*X12M(MTYPE)/XL3
TK(4,4)=POLAR(MTYPE)/(2.*XL(N)*(1.+POI))
TK(5,5)=4.*X12M(MTYPE)/XL(N)
TK(6,6)=4.*X12M(MTYPE)/XL(N)
DO 5011=7,12
TK(y)=TK(I-6J-6)
5011 CONTINUE
TK(1,7)=TK(1,1)
TK(7,1)=TK(1,7)
TK(2,6)=(6.*X12M(MTYPE))/XL2
TK(6,2)=TK(2,6)
TK(2,8)=TK(2,2)
TK(8,2)=TK(2,8)
TK(2,12)=TK(2,6)
TK(12,2)=TK(2,12)
TK(3,5)=-6.*X12M(MTYPE)/XL2
TK(5,3)=TK(3,5)
TK(3,9)=TK(3,3)
TK(9,3)=TK(3,9)
TK(3,11)=TK(3,5)
TK(11,3)=TK(3,11)
TK(4,10)=TK(4,4)
TK(10,4)=TK(4,10)
TK(5,9)=TK(3,5)
TK(9,5)=TK(3,9)
TK(5,11)=2.*X12M(MTYPE)/XL(N)
TK(11,5)=TK(5,11)
TK(6,8)=-TK(2,6)
TK(8,6)=TK(6,8)
TK(6,12)=0.5*TK(6,6)
TK(12,6)=TK(6,12)
TK(9,11)=TK(5,9)
TK(11,9)=TK(9,11)

DO IY=1,12
DO JY=1,12

TK(IY,JY)=EMTYPE*TK(IY,JY)
END DO
END DO

CAL. THE TRANSFORMATION

IF (XIFL(N).EQ. 1) THEN
  P1=(PT1(N)-XYZ(1,N))/XP
  P2=(PT2(N)-XYZ(2,N))/XP
  P3=(PT3(N)-XYZ(3,N))/XP
ELSE
  P1=(P1,IXP)-XYZ(1,N)/XP
  P2=(P2,IXP)-XYZ(2,N)/XP
  P3=(P3,IXP)-XYZ(3,N)/XP
END IF

COTETA=A1*P1+A2*P2+A3*P3
TETA=ACOS(COTETA)
TETA1=0.174532 ! 10 DEGREES

IF (TETA.LE.TETA1) THEN
  IXP=IXP+1 ICOLINEAR
  IF (IXP.EQ.6) THEN
    PRINT* POINTS EXCEEDED'
    STOP
  ELSE
    CONTINUE
  END IF
  GO TO 508
END IF

RC=1/COTETA
(A3-RC*P3)**2.))

B1=TL*A1-TL*RC*P1
B2=TL*A2-TL*RC*P2
B3=TL*A3-TL*RC*P3
T(2,1,N)=B1
T(2,2,N)=B2
T(2,3,N)=B3

AX=A2*B3-A3*B2
BX=A3*B1-A1*B3
CX=A1*B2-A2*B1

RTS=SQRT(AX**2.+BX**2.+CX**2.)
C1=AX/RTS
C2=BX/RTS
C3=CX/RTS
T(3,1,N)=C1
T(3,2,N)=C2
T(3,3,N)=C3
DO JT=1,3
  DO IT=1,3
    T(IT+JT+6,JT+6)=T(IT,JT,N)
    T(IT+JT+9,JT+9)=T(IT,JT,N)
  END DO
END DO
A50

\[ (x,y,z) \mid (x+y) = x+y \]

C

C

END
DO IT=1,12
DO JT=1,12
T(IT,JT,N)=T(IT,JT,NM1)
END DO
END DO
ELSE
CONTINUE
END IF
C
IF(XYZ(3,N).EQ.XMUD)THEN
T(7,7,N)=1.
T(7,9,N)=A1*C1+A2*C2+A3*C3
T(8,8,N)=1.
T(8,9,N)=B1*C1+B2*C2+B3*C3
T(9,7,N)=C1*A1+C2*A2+C3*A3
T(9,8,N)=C1*B1+C2*B2+C3*B3
T(9,9,N)=1.
DO IT=7,9
DO JT=7,9
JT3=IT+3
JT3=JT+3
T(IT3,JT3,N)=T(IT,JT,N)
END DO
END DO
C
ELSE
T(1,1,N)=1.
T(1,2,N)=A1*B1+A2*B2+A3*B3
T(1,3,N)=A1*C1+A2*C2+A3*C3
T(2,2,N)=1.
T(2,3,N)=B1*C1+B2*C2+B3*C3
T(3,1,N)=C1*A1+C2*A2+C3*A3
T(2,3,N)=C1*B1+C2*B2+C3*B3
T(3,3,N)=1.
C
DO IT=1,3
DO JT=1,3
IT3=IT+3
IT6=IT+6
IT9=IT+9
C
JT3=JT+3
JT6=JT+6
JT9=JT+9
T(IT3,JT3,N)=T(IT,JT,N)
T(IT6,JT6,N)=T(IT,JT,N)
T(IT9,JT9,N)=T(IT,JT,N)
END DO
END DO
C
END IF
C
ELSE
CONTINUE
END IF
ELSE
CONTINUE
END IF
C
DO I=1,12
DO J=1,12
TRANS(IJ)=T(J,I,M)
TGC(IJ)=T(J,I,M)
END DO
END DO
C
NOW MULTIPLY TRANS,K AND T TOGETHER
C
DO 111=1,12
DO 111=1,12
A51
Q(UH) = 0
CONTINUE
DO 20 I=1,12
DO 20 J=1,12
Q(UH) = T(K(LM))*TGC(M, J) + Q(UH)
CONTINUE
DO 30 I=1,12
DO 30 J=1,12
ST(I, J) = 0.
CONTINUE
DO 40 I=1,12
DO 40 J=1,12
DO 40 M=1,12
ST(I, J) = TRANS(I, M) * Q(M, J) + ST(I, J)
CONTINUE
C THE ELEMENT STIFFNESS MATRIX IS
C COMPLETELY COMPILED IN THE
C GLOBAL CO-ORDS, FOR A SPACE FRAME
C WE MAY NOW POST THE UPPER HALF TO
C THE STRUCTURE STIFFNESS
C MATRIX
C
KL = 0
DO 600 L=1,12
DO 600 M=L,12
KL = KL + 1
S(KL) = ST(L, M)
CONTINUE
CALL ASSEMBL (A(N14), IA(IN5), S, L, M(1, N), ND)
C
C ONCE THE ELEMENT STIFFNESS HAS BEEN ASSEMBLED
C ASSEMBLE THE ELEMENT MASS
C
IFLGMS = 1
DO 7002 ISM=1,12
DO 7002 JSM=1,12
SM(ISM, JSM) = 0.0
STLM(ISM, JSM) = 0.0
CONTINUE
C READ IN DIAGONALS FIRST
C
WEIGHT = AREA(MTYPE) * XL(N) * STLDEN + WEIGHT
XL(N) = SQRT(XL2)
CONST = STLDEN * AREA(MTYPE) * XL(N) / 420.00
SM(1, 1) = 140.0 * CONST
SM(2, 2) = 156.0 * CONST
SM(3, 3) = 156.0 * CONST
SM(4, 4) = 140.0 * POLAR(MTYPE) * CONST / AREA(MTYPE)
SM(5, 5) = 4.0 * XL2 * CONST
SM(6, 6) = 4.0 * XL2 * CONST
C
DO 7001 IKK=7,12
SM(IKK, IKK) = SM(IKK-6, IKK-6)
CONTINUE
C READ IN THE REMAINDER OF ELEMENTS
SM(1, 7) = 70.0 * CONST
SM(7, 1) = SM(1, 7)
SM(2, 6) = 22.0 * XL(N) * CONST
SM(6, 2) = SM(2, 6)
SM(2, 8) = 54.0 * CONST
SM(8, 2) = SM(2, 8)
SM(2, 12) = -13.0 * CONST * XL(N)
SM(12, 2) = SM(2, 12)
SM(3, 5) = -22.0 * XL(N) * CONST
SM(5, 3) = SM(3, 5)
SM(3, 9) = 54.0 * CONST
SM(9, 3) = SM(3, 9)
SM(3,11)=13.*XL(N)*CONST
SM(11,3)=SM(3,11)
SM(4,10)=70.*POLAR(MTYPE)*CONST/AREA(MTYPE)
SM(10,4)=SM(4,10)
SM(5,9)=-13.*XL(N)*CONST
SM(9,5)=SM(5,9)
SM(5,11)=-3.*XL2*CONST
SM(11,5)=SM(5,11)
SM(6,8)=13.*XL(N)*CONST
SM(8,6)=SM(6,8)
SM(6,12)=-3.*XL2*CONST
SM(12,6)=SM(6,12)
SM(8,12)=-22.*XL(N)*CONST
SM(12,8)=SM(8,12)
SM(9,11)=22.*XL(N)*CONST
SM(11,9)=SM(9,11)

C DO IAP=1,12
DO JAP=1,12
STLM(IAP,JAP)=SM(IAP,JAP)
END DO
END DO

C THE ABOVE IS THE ELEMENT MASS OF THE STEEL
C IN THE LOCAL CO-ORDS OF THE MEMBER

IF(XYZ(3,N).LE.0.0.AND.XYZ(6,N).LE.0.0)THEN
C MEMBER IN EITHER THE WATER OR IN THE FOUNDATION

PI=3.142857
IF(GRODEN.GT.0)THEN
DOUT=DTOT(MTYPE)*0.05
ELSE
DOUT=DTOT(MTYPE)
ENDIF
AREAGW(MTYPE)=(DOUT**2-DTOT(MTYPE)**2)*PI/4
POLGRO(MTYPE)=(DOUT**4-DTOT(MTYPE)**4)*PI/64
AREAOUT(MTYPE)=(DOUT**2)*PI/4
DO IP=1,12
DO JP=1,12
IF(IP.EQ.4.AND.JP.EQ.4)THEN
SM(IP,JP)=SM(IP,JP)+STLM(IP,JP)*POLGRO(MTYPE)*GRODEN/
(POLAR(MTYPE)*STLDEN)
ELSE IF(IP.EQ.10.AND.JP.EQ.10)THEN
SM(IP,JP)=SM(IP,JP)+STLM(IP,JP)*POLGRO(MTYPE)*GRODEN/
(POLAR(MTYPE)*STLDEN)
ELSE IF(IP.EQ.4.AND.JP.EQ.10)THEN
SM(IP,JP)=SM(IP,JP)+STLM(IP,JP)*POLGRO(MTYPE)*GRODEN/
(POLAR(MTYPE)*STLDEN)
ELSE IF(IP.EQ.4.AND.JP.EQ.4)THEN
SM(IP,JP)=SM(IP,JP)+STLM(IP,JP)*POLGRO(MTYPE)*GRODEN/
(POLAR(MTYPE)*STLDEN)
ELSE
SM(IP,JP)=SM(IP,JP)+STLM(IP,JP)*WATDEN*AREAOUT(MTYPE)/
(STLDEN*AREA(MTYPE))
END IF
END DO
END DO
WATOUT=WATOUT+WATDEN*XL(N)*AREAGW(MTYPE)
GROWT=GROWT+GRODEN*XL(N)*AREAGW(MTYPE)
ENDIF
ENDIF

IF(NTFL(N).EQ.1)THEN
DO IAD=1,12
DO JAD=1,12
IF(IAD.EQ.1.OR.IAD.EQ.4.OR.IAD.EQ.7.OR.IAD.EQ.10)THEN
GOTO 108
ELSE
GOTO 108
END IF
END DO
END DO

A53
ELSE
    SM(IAD,JAD)=SM(IAD,JAD)+STLM(IAD,JAD)*((WATDEN*AREAWT(MTYPE))/
    (STLDEN*AREA(MTYPE)))
END IF
END IF

108 CONTINUE
END DO
END DO

WATIN=WATIN+(AREAWT(MTYPE)*XL(N)*WATDEN)

C
C MEMBER IN THE AIR
CONTINUE
END IF
C
C CHECK FOR A POINT MASS ON NODE ONE OF THIS DECK MEMBER
IF(MASSFL(N).EQ.1) THEN
    PTWT=WT(N)+PTWT
    SM(1,1)=SM(1,1)+WT(N)
    SM(2,2)=SM(2,2)+WT(N)
    SM(3,3)=SM(3,3)+WT(N)
ELSE
    CONTINUE
END IF
C
C "GLOBALISE" THE MASS MATRIX
DO 7003 IAD=1,12
DO 7003 JAD=1,12
QM(IAD,JAD)=0.0
GMM(IAD,JAD)=0.0
ADM(IAD,JAD)=0.0
AMOR(IAD,JAD)=0.0
ADMER(IAD,JAD)=0.0
7003 CONTINUE

DO 7004 IQM=1,12
DO 7004 JQM=1,12
DO 7004 KQM=1,12
QM(IQM,JQM)=SM(IQM,KQM)*TGC(KQM,JQM)+QM(IQM,JQM)+QM(IQM,JQM)
7004 CONTINUE

GMM(IQM,JQM)=TRANS(IQM,KQM)*QM(KQM,JQM)+GMM(IQM,JQM)

C
C STORE ELEMENTS OF MASS MATRIX IN A VECTOR A(N15)
KL=0
DO 130=1,12
DO 131=130,12
KL=KL+1
GMK(KL)=GMM(I30,I31)
130 CONTINUE

CALL ASSEMBLY(A(N15),JA(IN5),GMK,LM(1,N),ND)

WATTOT=WATIN+WATTOT+PTWT+GROWT
500 CONTINUE

TOTWT=WEIGHT+WATTOT+PTWT+GROWT
RETURN
14H ELEMENT TYPE .13(2H .),17H( NPAR(1) ) . .  =J5/,
25H EQ.1, TRUSS ELEMENTS/,
29H EQ.2, ELEMENTS CURRENTLY/,
25H EQ.3, NOT AVAILABLE/,
20H NUMBER OF ELEMENTS .10(2H .),17H( NPAR(2) ) . .  =,
15/)
2010 FORMAT (42H MATERIAL DEFINITION ///,
137H NUMBER OF DIFFERENT SETS OF MATERIAL
2/32H AND CROSS-SECTIONAL CONSTANTS,
3/4(2H .),17H( NPAR(3) ) . .  =,)
2020 FORMAT (/1I5,4X,E12.5,3(2X,E14.6))
2040 FORMAT (1H1,40H ELEMENT INFORMATION ///,
18H ELEMENT ,5X,4HNODE,5X,4HNODE,7X,8HMATERIAL/,
29H NUMBER-N,6X,1HI,8X,1HI,7X,10HSET NUMBER/)
2050 FORMAT (1I5,6X,I5,4X,I5,7X,I5)
C END
C
FUNCTION RKC(XASOILK)
RKC=SOILK*(-X+D) !D=XMUD LE ITS NEGATIVE
RETURN
END
SUBROUTINE HEIGHTS (MHT, ND, LM)

C ....................................................
C ....................................................

SUBROUTINE HEIGHTS (MHT, ND, LM)

C ....................................................
C . PROGRAM
C . TO CALCULATE COLUMN HEIGHTS
C . ....................................................

C INCLUDE 'SPEC.CMN'

COMMON/SOUNUMP,NEQ,NWK,NUMEST,MIDEST,MAXEST,MK
DIMENSION LM(1), MHT(1)

C LS=1000000
DO 100 I=1, ND
IF (LM(I)) 110, 100, 110
110 IF (LM(I)-LS) 120, 100, 110
120 LS=LM(I)
100 CONTINUE

DO 200 I=1, ND
II=LM(I)
IF (II.EQ.O) GO TO 200
ME=II-LS
IF (ME.GT.MHT(I)) MHT(I)=ME
200 CONTINUE

C RETURN
END
SUBROUTINE ADDRES(MAXA,MHT)

C PROGRAM TO CALCULATE ADDRESSES OF DIAGONAL ELEMENTS IN BANDED MATRIX WHOSE COLUMN HEIGHTS ARE KNOWN
C MHT=ACTIVE COLUMN HEIGHTS
C MAXA=ADDRESSES OF DIAGONAL ELEMENTS
C
C INCLUDE 'SPEC.CMN'
C COMMON/SOL/NUMNP,NEQ,NWK,NUMEST,MIDEST,MAXEST,MK
C DIMENSION MAXA(1),MHT(1)
C CLEAR ARRAY MAXA
C
NN=NEQ+1
DO 20 I=1,NN
20 MAXA(I)=0
C
MAXA(1)=1
MAXA(2)=2
MK=0
IF (NEQ.EQ.1) GO TO 100
DO 10 I=2,NEQ
IF (MHT(I).GT.MK) MK=MHT(I)
MAXA(I+1)=MAXA(I)+MHT(I)+1
C
10 CONTINUE
100 MK=MK+1
NWK=MAXA(NEQ+1)-MAXA(1)
C
RETURN
END
SUBROUTINE ASSEMBL (AX,MAXA,S,JLM,ND)

C
C ............................................................................................
C PROGRAM
C . ASSEMBLES UPPER TRIANGULAR ELEMENT STIFFNESS & MASS MATRICIES TO COMPACTED
C . GLOBAL STIFFNESS
C . A=GLOBAL STIFFNESS
C . B=GLOBAL MASS
C . S=ELEMENT STIFFNESS,OR ELEMENT MASS
C . ND=NO. OF DEGREES OF FREEDOM IN ELEMENT STIFFNESS
C ............................................................................................
C DIMENSION S(1),AX(1),MAXA(1),LM(1)
C
C
C
NDI=0
DO 200 I=1,ND
II=LM(I)
IF (II) 200,200,100
100 MI=MAXA(II)
KS=I
DO 220 J=1,ND
JJ=LM(J)
IF (JJ) 220,220,110
110 J=II-JJ
IF (J) 220,210,210
210 KK=MI+JJ
KSS=KS
C
C PRINT*,KK,KSS J.O'B 11-5-94
C
C IF (J.GE.I) KSS=J+NDI
AX(KK)=AX(KK)+S(KSS)
220 KS=KS+ND-J
200 NDI=NDI+ND-J
DO 300 IR=1,12
II=LM(IR)
IF (II.EQ.0) GO TO 202
MI=MAXA(II)
202 CONTINUE
300 CONTINUE
RETURN
END
SUBROUTINE SSPACE(A4,B,MAXA,R,EIGV,RTOL,NC,NNC,
   NITEM,IFSS,IFPR,NSTIF,NNM,VEC,TT,D,
   AR,BR,RTOLV,BUP,BLO,BUPC)
C
TO SOLVE THE GENERAL EIGENPROBLEM USING THE
SUBSPACE ITERATION METHOD
C
---- INPUT VARIABLES-------
C. A4(NWK) = STIFFNESS MATRIX IN COMPACTED FORM
C. B(N,N) = MASS MATRIX IN COMPACTED FORM
C. MAXA(NNM) = VECTOR CONTAINING ADDRESSES OF DIAGONAL
   ELEMENTS OF STIFF MATRIX A
C. EIGV(N) = VECTOR STORING EIGENVALUES ON SOLUTION EXIT
C. R(NN,NC) = EIGENVECTORS ON SOLUTION EXIT
C. RTOL = CONVERGENCE TOLERANCE ON EIGENVALUES (USUALLY SET TO C.10**-12)
C. IFPR = FLAG FOR PRINTING DURING ITERATION
   EQ.0 NO PRINTING
   EQ.1 INTERMEDIATE RESULTS ARE PRINTED
C. IFSS = FLAG FOR STURM SEQUENCE CHECK
   EQ.0 NO CHECK
   EQ.1 CHECK
C. IOUT = OUTPUT DEVICE NUMBER
C. NN = ORDER OF MATRICES A AND B
C. NNM = NN+1
C. NWK = NO. OF ELEMENTS BELOW THE SKYLINE OF STIFF MATRIX
C. NWM = NO. OF ELEMENTS BELOW THE SKYLINE OF MASS MATRIX
C. NROOT = NO. OF REQUIRED EIGENVALUES AND EIGENVECTORS
C. NC = NO. OF ITERATION VECTORS USED
C. USUALLY SET TO MIN(2*NROOT,NROOT+8), BUT NC CANNOT BE
   LARGER THAN THE NO. OF MASS D.O.F.
C. NNC = NC*(NC+1)/2 DIMENSION OF STORAGE VECTORS AR, BR
C. NITEM = MAX NO. OF SUBSPACE ITERATIONS PERMITTED
C. USUALLY SET TO 16
C. THE PARAMETERS NC AND/OR NITEM MUST BE
   INCREASED IF A SOLUTION HAS NOT CONVERGED
C. NSTIF = SCRATCH FILE TO STORE STIFFNESS MATRIX
C.
C. ---- OUTPUT ----
C.
C. EIGV(NROOT) = EIGENVALUES
C. R(NN,NROOT) = EIGENVECTORS
C.
C. INCLUDE 'SPEC.CMN'
DIMENSION A4(NWK),B(NWK),R(NEQ,NC),TT(NEQ),W(2000),EIGV(NC),
   1D(NC),Z(20),VEC(NC,NC),AR(NNC),BR(NNC),RTOLV(NC),BUP(NC),
   1BLO(NC),BUPC(NC),MAXA(NNM),RADS(20),AMX(10,10)
C
C NWM=NWK
NN=NEQ
PRINT *,NC="NC"
IFPR=0
PI=3.141592654
C
C...OPEN EIGENVALUE FILES
OPEN(12,FILE='VECT.DAT',STATUS='OLD')
OPEN(16,FILE='VCT.DAT',STATUS='OLD')
OPEN(7,FILE='SCR1.DAT',STATUS='UNKNOWN',FORM='UNFORMATTED')

C
C...INITIALIZATION...
OPEN(30,FILE='CONST.DAT',STATUS='OLD')
READ(30,13,END=999)SCA,SCB
33 FORMAT(2F10.0)
WRITE(OUT,*) 'SCA=',SCA,'SCB=',SCB
999 CLOSE (30)
C
C
C...READ IN FLAG TO EXECUTE OR SKIP SSSPACE
PRINT*, 'READ JSKIP (1 TO SKIP SSSPACE, 0 TO EXECUTE SSSPACE)
READ(*,2026)JSKIP
2026 FORMAT(I1)
C
IF(JSKIP.EQ.1)THEN
READ(12,1006)(EIGV(I),I=1,NROOT)
WRITE(16,1006)(EIGV(I),I=1,NROOT)
DO 1300 IR1=1,NROOT
READ(12,1005)(R(K1,IR1),K1=1,NN)
WRITE(16,1005)(R(K1,IR1),K1=1,NN)
1300 continue
RETURN
C
ELSE
C
C...SET TOL FOR JACOBI ITERATION...
TOL=0.0000000000001
C
DO 1303 IO=1,NWK
A4(IO)=A4(IO)/(10.**SCA)
B(IO)=B(IO)/(10.**SCB)
1303 continue
QUOT=1000.0
ICONV=0
NSCH=0
NSMAX=12
N1=NC+1
NC1=NC-1
REWIND NSTIF
WRITE(NSTIF) A4
D 0 60I=1J4C
D a > = 0 .
60 continue
C
C...ESTB. STARTING ITER VECTOR
ND=NN/NC
IF(NWK.GT.NN) GO TO 4
J=0
DO 2 I=1,NN
II=MAXA(I)
R(I,1)=B(I)
R(I,1)=B(I)/A4(II)
IF(NC.LE.J) GO TO 16
WRITE(IOUT,1007)
STOP
4 CONTINUE
DO 10 J=1,NN
II=MAXA(I)
R(I,1)=B(I)
W(I)=B(I)/A4(II)
10 CONTINUE
16 DO 20 J=2,NC
DO 20 I=1,NN
20 R(I,J)=0.
C
L=NN-ND
DO 30 J=2,NC
RRT=0.
DO 40 I=1,L
IF(W(I).LT.RRT) GO TO 40
RRT=W(I)
30 CONTINUE
40 continue

\[ U = I \]

\begin{verbatim}
40 CONTINUE
   DO 50 I = L, NN
   IF(W(I) .LE. RRT) GO TO 50
   RRT = W(I)
   I = I + 1
50 CONTINUE
   TT(I) = FLOAT(TT(I))
   W(I) = 0.
   L = L - ND
30 R(U, J) = 1.
   WRITE(IOUT, 1008)
   WRITE(IOUT, 1002) (TT(J) = TT(J) + 2, NC)
C C...FACTORIZE MATRIX 'A' INTO (L)*(D)*(L(T))
   ISH = 0
   CALL DECOMP (A4, MAXA, NN, ISH, IOUT)
C C...START OF ITERATION LOOP...
   NITE = 0
   IF(IFPR .EQ. 0) GOTO 90
   WRITE(IOUT, 1010) NITE
   C C...CALCULATE THE PROJECTIONS OF A4 AND B
90 I = 0
   DO 10 J = 1, NC
      K = I; W
   10 TT(K) = R(K, J)
      CALL REDBAK(A4, TT(I)AXA,)
      DO 20 J = 1, NC
         ART = 0.
         DO 30 K = 1, NN
            ART = ART + R(I, K) * TT(K)
   20 U = U + 1
   30 AR(I) = ART
      DO 40 K = 1, NN
   40 R(K, J) = TT(K)
   110 CONTINUE
   I = 0
   DO 120 J = 1, NC
      CALL MULT(TT, B, R(I, J), MAXA, NN, NWK)
   120 BRT = 0.
      DO 130 K = 1, NN
   130 BRT = BRT + R(K, I) * TT(K)
   140 BR(U) = BRT
   150 R(K, J) = TT(K)
110 CONTINUE
   I = 0
   DO 160 J = 1, NC
      CALL MULT(TT, B, R(I, J), MAXA, NN, NWK)
   160 CONTINUE
   C C...SOLVE FOR EIGENSYSTEM OF SUBSPACE OPERATORS...
   IF(IFPR .EQ. 0) GO TO 320
   IND = 1
210 WRITE(IOUT, 1020)
   I = 0
   DO 300 I = 1, NC
      ITEMP = I + NC - I
      WRITE(IOUT, 1005) (AR(J), J = ITEMP)
   300 I = I + 1
   WRITE(IOUT, 1030)
   I = 0
   DO 310 I = 1, NC
      ITEMP = I + NC - I
      WRITE(IOUT, 1005) (BR(J), J = ITEMP)
   310 I = I + 1
      IF(IND .EQ. 2) GO TO 350
320 CALL JACOBI (AR, BR, VEC, EIGV, HZ, W, NC, NNC, TOL, NSMAX, IFPR, IOUT, MAXA, NWK)
C
A61
\end{verbatim}
C IF(IFPR.EQ.0)GO TO 350
WRITE(OUT,1040)
IND=2
GO TO 210

C ...ARRANGE EIGENVALUES IN ASCENDING ORDER...
350 IS=0
II=1
DO 360 I=1,NC1
ITEMP=II+NI-1
IF(EIGV(I+1).GE.EIGV(I)) GO TO 360
IS=IS+1
EIGVT=EIGV(I+1)
EIGV(I+1)=EIGV(I)
BT=BRITEMP
BRITEMP=BR(II)
BR(II)=BT
DO 370 K=1,NC
RRT=VEC(K,I+1)
VEC(K,I+1)=VEC(K,J)
370 VEC(K,J)=RRT
360 II=ITEMP
IF(IS.GT.0)GO TO 350
IF(IFPR.EQ.0) GO TO 375
WRITE(OUT,1035)
WRITE(OUT,1006) (EIGV(I),I=1,NC)

C ...CALCULATE B TIMES APPROX EIGENVECTORS (ICONV.EQ.0)
C OR FINAL EIGENVECTOR APPROX (ICONV.GT.0)
375 DO 420 I=1,NN
DO 422 J=1,NC
TT(J)=R(J)
DO 424 K=1,NC
RRT=0.
DO 430 L=1,NC
RRT=RRT+TT(L)*VEC(L,K)
424 R(I,K)=RRT
420 CONTINUE
IF(ICONV.GT.0) GO TO 500

C ...CHECK FOR CONVERGENCE OF EIGENVALUES ...
DO 380 I=1,NC
DIF=ABS(EIGV(I)-D(I))
380 RTOLV(I)=DIF/EIGV(I)
IF(IFPR.EQ.0)GO TO 385
WRITE(OUT,1050)
WRITE(OUT,1005) (RTOLV(I),I=1,NC)
385 DO 390 I=1,NC
IF(RTOLV(I).GT.RTOL) GO TO 400
390 CONTINUE
WRITE(OUT,1060) RTOL
ICONV=1
GO TO 100
400 IF(NITELT.NITEM) GO TO 410
WRITE(OUT,1070)
ICONV=2
IFSS=0
GO TO 100

C DO 440 I=1,NC
440 D(I)=EIGV(I)
GO TO 100

C ...END OF ITERATION LOOP ...
C
500 DO 445 I=1,20
HZ(I)=0
RADIS(I)=0
445 CONTINUE
DSCAB=SQA-SCB
DO 510 III=1,NROOT
EIGVA=0
EIGV = EIGV(II)*(10**DSCAB)
RADS(II) = SQRT(EIGV)

DO 520 IHI = 1, NROOT
HZ(IHI) = RADS(IHI)*(2*PI)

WRITE(12,1006) (EIGV(I), I = 1, JJROOT) ! added to write out to vect.dat
WRITE(IOUT,1099) (EIGV(I), I = 1, JJROOT)
WRITE(IOUT,1100) (HZ(IH), IH = 1, NROOT)

DO 520 J = 1, NROOT
TST = 0.
DO 540 ITST = 1, N
AB1 = ABS(R(ITST,J))
AB2 = ABS(TST)
IF(AB2.LT.AB1) THEN
TST = R(ITST,J)
ELSE
END IF
END DO
C
WRITE(IOUT,1005) (R(KK,J), KK = 1, J<LO)

C...CALCULATE AND PRINT ERROR NORMS...
REWIND NSTIF
READ(NSTIF) A4
DO 580 L = 1, NROOT
RRT = EIGV(L)
CALL MULT(TT,A4,R(1,L),MAXA,NN,NWK)
VNORM = 0.
DO 590 I = 1, NN
T T(I) = T T(I) - RRT*W(I)
VNORM = VNORM + TT(I)**2
END DO
VNORM = SQRT(VNORM)

DO IH1 = 1, NROOT
AMX(IH1,L) = 0.0
DO IH2 = 1, NN
AMX(IH1,L) = AMX(IH1,L) + W(IH2)*AMX(IH2,L)
END DO
END DO
WRITE(IOUT,1115) AMX(IH3,L), IH3 = 1, NROOT

DO 580 J = 1, NROOT
WRITE(IOUT,1106) R(J), J = 1, NROOT
WRITE(IOUT,1115) R(J), J = 1, NROOT

C...APPLY STURM SEQUENCE CHECK...
IF(IFSS.EQ.0) GO TO 700
CALL SCHECK(EIGV,RTOLV,BUP,BLO,BUPC,NC,NEI,RTOL,SHIFT,IOUT)
C WRITE(IOUT,1120) SHIFT
C
C...SHIFT MATRIX A...
   REWIND NSTIF
   READ(NSTIF),A4
   IF(NWM.GT.NN)GO TO 645
   DO 640 I=1,NW
   A4(D)=A4(n)-B©*SHIFT
   GO TO 660
640 645 D 0 I=1JW K
   A4(I)=A4(I)-B(I)*SHIFT
C
C...FACTORISE SHIFTED MATRIX...
650 660 ISH=1
   CALL DECOMP(A4,MAXA,NN^SH,IOUT)
C
C...COUNT NO.OF NEGATIVE DIAGONAL ELEMENTS...
   NSCH=0
   NSCH=NSCH+1
   IF(NSCH.LT.MIT.AN) NSCH=NSCH+1
   CONTINUE
   GO TO 670
664 IF(NSCH.EQ.NE) GO TO 670
   NMIS=NSCH-NEI
   WRITE(IOUT,1130) NMIS
   GO TO 700
670 WRITE(IOUT,1140)NSCH
700 RETURN
C
C END IF
C
C 1002 FORMAT (1HO.IOFIO.O)
1005 FORMAT (12E10.4)
1006 FORMAT (6E22.14)
1007 FORMAT (///,38HSTOP,NC LAGER THAN NO OF MASS DEGREES ,
   1 11H OF FREEDOM )
1008 FORMAT (///,36H DEGREES OF FREEDOM EXCITED BY UNIT ,
   1 27H STARTING ITERATION VECTORS )
1010 FORMAT (1X,30HIT E R A T I O N  NUMBER  ^4)
1020 FORMAT (IX^SPROJECTION OF A (MATRIX AR))
1030 FORMAT (IX^SPROJECTION OF B (MATRIX BR))
1035 FORMAT (1X,30HEIGENVALIES OF AR-LAMBDA*BR )
1040 FORMAT (1X,43HRELATIVE TOLERANCE REACHED ON EIGENVALUES )
1050 FORMAT ( ///,30HCONVERGENCED REACHED FOR RTOL ,E10.4)
1060 FORMAT ( ///,30HCONVERGENCED IN MAXIMLIM NUMBER OF ,
   1 20HITERATIONS PERMITTED/28HWE ACCEPT CURRENT ITERATION ,
   2 6HVALUES,42HTHE STURM SEQUENCE CHECK IS NOT PERFORMED ,
   3 18HVALUES ARE )
1099 FORMAT (///,31HTHE CALCULATED EIGENVALUES ARE )
1100 FORMAT (///,40HTHE CALCULATED EIGENVALUES IN RADS/S ARE )
1101 FORMAT (///,40HTHE CALCULATED EIGENVALUES IN HERTZ ARE )
1115 FORMAT (///,36HPRINT ERROR NORMS ON THE EIGENVALUES )
1110 FORMAT (///,36H THE CALCULATED EIGENVECTORS ARE //)
1120 FORMAT (///,23H CHECK APPLIED AT SHIFT ,E22.14)
1130 FORMAT (///,10H THERE ARE ,14,21HEIGENVALUES MISSING )
1140 FORMAT (///,20H WE FOUND THE LOWEST ,14,12HEIGENVALUES )
C
C END
SUBROUTINE DECOMP(A4,MAXA,NN^ISH,IOUT)
C
C . PROGRAM
C . TO SOLVE FINITE ELEMENT STATIC EQUILIBRIUM EQUATIONS IN
C . CORE USING COMPACTED STORAGE AND COLUMN REDUCTION SCHEME
C . ---INPUT VARIABLES---
C .
C . A4(NWK) =STIFFNESS MATRIX STORED IN COMPACTED FORM
C . V(NN) =RIGHT-HAND-SIDE LOAD VECTOR
C . MAXA(NNM) =VECTOR CONTAINING ADDRESSES OF DIAGONAL ELEMENTS OF
C . STIFFNESS MATRIX IN A
C . NN =NUMBER OF ELEMENTS
C . NWK = NUMBER OF ELEMENTS BELOW SKYLINE OF MATRIX
C . NNM = NN+1
C . KKK = INPUT FLAG
C . EQ. 1 TRIANGULARIZATION OF STIFFNESS MATRIX
C . EQ. 2 REDUCTION AND BACK-SUBSTITUTION OF LOAD VECTOR
C . IOUT = NUMBER OF OUTPUT DEVICE
C —— OUTPUT ——
C . A4(NWK) = D AND L - FACTORS OF STIFFNESS MATRIX
C . V(NN) = DISPLACEMENT VECTOR
C ....................................................................
DIMENSION A4(1),MAXA(1)
IF(NN.EQ.1) RETURN
C
C • PERFORM L*D*L(T) FACTORIZATION OF STIFFNESS MATRIX •
DO 140 N=1,NN
   KN=MAXA(N)
   KL=KN+1
   KU=MAXA(N+1)-1
   KH=KU-KL
   IF(KH)304,90,210
210 K=N-KH
   IC=0
   KLT=KU
   DO 80 J=1,KH
      IC=IC+1
      KLT=KLT-I
      KI=MAXA(K)
      ND=MAXA(K+1)-KI-1
      IF(ND)80,80,60
60 KK=MIN(IC,ND)
   C=0.
   DO 70 L=1,J
      C=C+A4(KI+L)*A4(KLT+L)
   A4(KLT)=A4(KLT)-C
   70 K=K+I
   90 K=N
B=0.
DO 100 KK=KL,KU
   K=K-1
   KI=MAXA(K)
   C=A4(KK)/A4(KI)
   IF(ABS(C.GT.1.E07)) GO TO 290
   WRITE(IOUT,10)N,C
   STOP
290 B=B+C*A4(KK)
100 A4(KK)=C
   A4(KN)=A4(KN)-B
304 IF (A4(KN).EQ.0.) A4(KN)=-1.E-16
120 IF(ISH.EQ.0.) GO TO 320
   IF(A4(KN),EQ.0.) A4(KN)=-1.E-16
   GO TO 140
320 WRITE(IOUT,2000)N,A4(KN)
STOP
140 CONTINUE
RETURN
2000 FORMAT(//42HSTIFFNESS MATRIX NOT POSITIVE DEFINITE //
1 32H NONPOSITIVE PIVOT FOR EQUATION ,14 //
2 10H PIVOT = ,E20.12 )
2010 FORMAT(//47HSTOP-STURM SEQUENCE CHECK FAILED BECAUSE OF ,
135HMULTIPLIER GROWTH FOR COLUMN NUMBER J4,7HMULTIP=,
2 E20.8)
END
SUBROUTINE REDBAK(A4,V,MAXA,NN)
C
C * REDUCE RIGHT-HAND-SIDE LOAD VECTOR *
C
C
DIMENSION A4(1),V(1),MAXA(1)
C
DO 180 N=1,NN
KL=MAXA(N)+1
KU=MAXA(N+1)-1
IF(KU-KL)180,160,160
160 K=N
C=0.
DO 176 KK=KL,KU
K=K-1
176 C=C+A4(KK)*V(K)
V(N)=V(N)-C
180 CONTINUE
C
C BACK SUBSTITUTE
C
DO 200 N=1,NN
K=MAXA(N)
V(N)=V(N)/A4(K)
200 CONTINUE
IF (NN.EQ.1) RETURN
N=NN
DO 230 L=2,NN
KL=MAXA(N)+1
KU=MAXA(N+1)-1
IF(KU-KL)230,210,210
210 K=N
DO 220 KK=KL,KU
K=K-1
V(K)=V(K)-A4(KK)*V(N)
220 CONTINUE
230 N=N-1
RETURN
END
SUBROUTINE SCHECK(EIGV,RTOLV,BUP,BLO,BUPC,NC,NEIV,1 RTOL,SHIFT,OUT)
C...TO EVALUATE SHIFT FOR STURM SEQUENCE CHECK...
DIMENSION EIGV(NC),RTOLV(NC),BUP(NC),BLO(NC),BUPC(NC),1 NEIV(NC)
FTOL=0.01
DO 100 I=1,NC
BUP(I)=EIGV(I)*FTOL
100 BLO(I)=EIGV(I)*(-FTOL)
NROOT=0
DO 120 I=1,NC
120 IF(RTOLV(I).LT.RTOL)NROOT=NROOT+1
IF(NROOT.GE.1) GO TO 200
WRITE(OUT,1010)
STOP
C...FIND UPPER BOUNDS ON EIGENVALUE CLUSTERS...
200 DO 240 I=NROOT
240 NEIV(I)=1
IF (NROOT.NE.1) GO TO 260
BUPC(I)=BUP(I)
LM=1
I=1
GO TO 295
260 L=1
I=2
270 IF(BUP(I-1).LE.BLO(I))GO TO 280
NEIV(L)=NEIV(L)+1
I=I+1
280 IF(L.LE.NROOT) GO TO 270
BUPC(L)=BUP(I-1)
I=I+1
IF(L.LE.NROOT) GO TO 270
BUPC(L)=BUP(I-1)
290 LM=L
IF(NROOT.EQ.NC) GO TO 300
295 IF(BUP(I-1).LE.BLO(I))GO TO 300
IF(RTOLV(I).GT.RTOL) GO TO 300
BUPC(L)=BUP(I)
NEIV(L)=NEIV(L)+1
NROOT=NROOT+1
IF(NROOT.EQ.NC) GO TO 300
I=I+1
GO TO 295
C...FIND SHIFT...
300 WRITE(OUT,1020)
WRITE(OUT,1005) (BUPC(I),I=1,LM)
WRITE(OUT,1030)
WRITE(OUT,1006) (NEIV(I),I=1,LM)
LL=LM-1
IF(LL.LE.0) GO TO 310
330 DO 320 I=1,LL
320 NEIV(L)=NEIV(L)+NEIV(I)
L=L+1
LL=LL-1
IF(LL.LE.0) GO TO 330
310 WRITE(OUT,1040)
WRITE(OUT,1006) (NEIV(I),I=1,LM)
L=0
DO 340 I=1,LM
L=L+1
340 CONTINUE
350 SHIFT=BUPC(L)
NEI=NEIV(L)
RETURN
1005 FORMAT(1HO,6E22.14)
1006 FORMAT(1HO,6I22)
1010 FORMAT(*ERROR SOLUTION STOP IN *SCHECK*/12X, 21HN0 EIGENVALUES FOUND.,/IX)
1020 FORMAT(///,37HUPPER BOUNDS OF EIGENVALUE CLUSTERS )
1030 FORMAT(34HONO OF EIGENVALUES IN EACH CLUSTER )
1040 FORMAT(42HONO OF EIGENVALUES LESS THAN UPPER BOUNDS )
END
SUBROUTINE JACOBI(A4,B,X,EIGV,HZ,D,N,NWA,RTOL,NSMAX,
IFPR,IOUT,MAXA,NWK)
C.........................................................................................
C
C DIMENSION A4(NWA),B(NWA),X(N,N),EIGV(N),HZ(N),D(N),MAXA(NWK)
C
C INITIALISE EIGENVALUE AND EIGENVECTOR MATRICES
C
N1=N+1
II=1
DO 10 I=1,N
IF (A4(I,N).GT.0. AND. B(I,N).GT.0.) GO TO 4
WRITE (IOUT,20)N
STOP
4 D(I)=A4(I,N)/B(I,N)
EIGV(I)=D(I)
N=N+1
10 CONTINUE
DO 20 J=1,N
20 X(I,J)=0.
J = N - 1
IF (N.EQ.1) RETURN
C
C INITIALISE SWEEP COUNTER AND BEGIN ITERATION
C
NSWEEP=0
NR=N-1
40 NSWEEP=NSWEEP+1
IF(IFPR.EQ.1) WRITE(IOUT,2000) NSWEEP
C
C...CHECK IF PRESENT OFF-DIAGONAL ELEMENT IS LARGE ENOUGH TO REQUIRE
C ZEROING...
SWEEP=NSWEEP
EPS=(0.01**SWEP)**2
DO 210 J=N-1,NR
JP1=J+1
JM1=J-1
LJ=JM1*N-JM1*J/2
JJ=UK+J
DO 210 K=J+1,NR
KM1=K-1
KK=KM1*N-KM1*K/2
UK=JM1*N-JM1*J/2
JJ=UK+J
DO 210 K=J+1,NR
KM1=K-1
KK=KM1*N-KM1*K/2
EPTOLA=(A4(JK)*A4(JK))/(A4(JJ)*A4(KK))
EPTOLB=(B(JK)*B(JK))/(B(JJ)*B(KK))
IF((EPTOLA.LT.EPS).AND.(EPTOLB.LT.EPS)) GO TO 210
C
C...IF ZEROING IS REQUIRED CALCULATE THE ROTATION MATRIX ELEMENTS CA AND CG...
AKK=A4(KK)*B(JK)-B(KK)*A4(JK)
AJJ=A4(JJ)*B(JK)-B(JJ)*A4(JK)
AB=A4(JJ)*B(KK)-A4(KK)*B(JJ)
CHECK=(AB*AB+4.*AKK*AJJ)/4.
IF(CHECK)50,60,60
50 WRITE(IOUT,2020)
STOP
60 SQCH=SQRT(CHECK)
D1=AB/2.+SQCH
D2=AB/2.-SQCH
DEN=D1
IF(ABS(D2).GT.ABS(D1)) DEN=D2
IF(DEN)80,70,80
70 CA=0.
CG=-A4(JK)/A4(KK)
GO TO 90
80 CA=AKK/DEN
CG=AJJ/DEN
10 CONTINUE
C
C...PERFORM THE GENERALIZED ROTATION TO ZERO THE PRESENT OFF-DIAGONAL ELEMENT
CONTINUE

DO 222 IKP=1,N
IF(A4(IIP).GT.0.0.AND.B(IIP).GT.0.0)GO TO 222
WRITE(IOUT,2020)IIP,A4(IIP),B(IIP),ICONT1

222 IIP=I+1
CONTINUE

DO 223 IKP=1,N
IF(A4(IIP).GT.0.0.AND.B(IIP).GT.0.0)GO TO 223
WRITE(IOUT,2020)IIP,A4(IIP),B(IIP),ICONT2

223 IIP=I+1
CONTINUE

C...UPDATE THE EIGENVECTOR MATRIX AFTER EACH ROTATION...
DO 200 I=1,N
XI=X(I,J)
XX=X(I,K)
X(J)=XI+CG*XX

200 XI=XX
CONTINUE

C...UPDATE THE EIGENVALUES AFTER EACH SWEEP...
II=1
PRINT*,NSWEEP=NSWEEP

A69
ICONT3=3
DO 220 I=1,N
IF(A4(I),GT,0.0,.AND.B(I),GT,0.0)GO TO 215
WRITE(IOUT,2020)II,A4(I),B(I),ICONT3
STOP
215 EIGV(I)=A4(I)/B(I)
220 II=II+1-N-1
IF(IFPR.EQ.0) GO TO 230
WRITE(IOUT,2030)
WRITE(IOUT,2010) (EIGV(I),J=1,N)
C
C...CHECK FOR CONVERGENCE...
230 DO 240 I=1,N
TOL=RTOL*D(I)
DIF=ABS(EIGV(I)-D(I))
IF(DIF.GT.TOL) GO TO 280
240 CONTINUE
C
C...CHECK ALL OFF-DIAGONAL ELEMENTS TO SEE IF ANOTHER SWEEP IS REQUIRED...
EPS=RTOL**2
DO 250 J=1,NR
JM1=J-1
JP1=J+1
LJK=JM1*N-JM1*J/2
JJ=LJK+J
DO 250 K=JP1,N
KM1=K-1
JK=LJK+K
KK=KM1*N-KM1*K/2+K
EPSB=(B(JK)*B(JK))/(B(JJ)*B(KK))
IF(EPSA.LT.EPS).AND.(EPSB.LT.EPS)) GO TO 250
GO TO 280
250 CONTINUE
C
C...FILL OUT BOTTOM TRIANGLE OF RESULTANT MATRICES AND SCALE EIGENVECTORS...
255 II=1
DO 275 I=1,N
BB=SQRT(B(I))
DO 270 K=1,N
270 X(I)=X(K,I)/BB
275 II=II+1-N-1
RETURN
C
C...UPDATE D MATRIX AND START NEW SWEEP,IF ALLOWED...
280 DO 290 I=1,N
290 D(I)=EIGV(I)
IF(NSWEEP.LT.NSMAX)GO TO 40
GO TO 255
2000 FORMAT(27HOSWEEP NUMBER IN *JACOBI* =,I4)
2010 FORMAT(1H0,6E20.12)
2020 FORMAT(25H ***ERROR SOLUTION STOP /)
1 31H MATRICES NOT POSITIVE DEFINITE /
2 4H II=I,JHA(I)=E20.12,6HII=I,E20.12,1X,1J)
2023 FORMAT(1H0,DIAGONALS OF STIFF AND MASS MATRICES')
2030 FORMAT(36H0CURRENT EIGENVALUES IN *JACOBI* ARE,/)
END
SUBROUTINE SPEC
INCLUDE 'SPEC.CMN'
DIMENSION W(202)

OPEN(10,FILE='SPECT.DAT',STATUS='OLD')
OPEN(8,FILE='SPECT.OUT',STATUS='OLD')
OPEN(72,FILE='FREQ.DAT',STATUS='OLD')
READ(72,990)WFIRST,WLAST
CLOSE(72)

STEP=0.04
WLST=WLAST/STEP

PI=3.1415927
WO=0.00

READ(10,1000)HS,TO,WIND,G,X,SFLG,H,TZ

*H in feet, Tz in seconds. This is for angelides spectrum

990 FORMAT(2F10.0)
1000 FORMAT(6F10.0)

READ IN SPECTRUM

IF(SFLG.EQ.1)THEN
OPEN(11,FILE='SPEC.DAT',STATUS='OLD')
DO 101 IIP=1,200
READ(11,2000)W(IIP),SPECTR(IIP)

CLOSE(11)
ELSE IF (SFLG.EQ.2)THEN
B=(H**2)*Tz/(8*Pi*Pi)
SPECTR(1)=0.0
DO I=2,200
IF(LT.WLST)THEN
SPECTR(I)=0.0
ELSE
DI=1
W(I)=WO+STEP*(DI-1.0)
C=Tz*W(I)/(2*Pi)
E=B/C**5
F=-1/(Pi*(C**4))
G=EXP(F)
SPECTR(I)=E*G
ENDIF
END DO
ELSE
WM=((4.*0.74/5.)**.25)*9.81/WIND
A1=0.0081
STEP=0.04
DO 100 I=2,200
IF(LT.WLST)THEN
SPECTR(I)=0.0
ELSE
DI=1
W(I)=WO+STEP*(DI-1.0)
R=W(I)
IF(R.LE.WM)THEN
SIG=0.07
ELSE
SIG=0.09
ENDIF
A1=0.01
A2=5.00
IF(R.GE.A1.AND.R.LE.A2)THEN
SPECTR(I)=SPIONS(R,WM,A1,SIG,G)
ELSE
IM1=I
A11=W(IM1)
RTA=SPECTR(IM1)
SPECTR(I)=TYLSPEC(A11,WM,A1,SIG,G,R,RTA)
ENDIF
ENDIF
END IF

END IF

CONTINUE
II=3
DO 200 I=1,2
DI=II
W(I)=WO+STEP*(DI-1.0)
R=W(I)
IF(R.LE.WM)THEN
  SIG=0.07
ELSE
  SIG=0.09
END IF
IP=II+1
'A1=W(IP1)
RTA=SPECTR(IP1)
II=3-I
200 CONTINUE
C

C

C

END IF
WRITE(8,*)This is the input spectrum.'
WRITE(8,*)'  Snm(m2s/i:ad) w(rad/s)
DO IP=1,200
IF(SPECTR(IP).LT.O.OR.IP.GT.WLST)THEN
  SPECTR(IP)=0.
ELSE
  CONTINUE
END IF
WRITE(8,2000)SPECTR(IP),W(IP)
END DO
C
CLOSE(10)
CLOSE(8)
RETURN
C
FUNCTION SPIONS(R,RM,RAL,SG,RG)
A1=RAL*(9.81**2.)
B=(-5./4.)*(RM**4.)
C=(2.*(SG**2.)*(RM**2.))
SPIONS=(A1/(R**5.))*EXP(B/(R**4.))*
1  RG**EXP(-((R-RM)**2.)/C)
RETURN
C
ENTRY TYLSPEC(WI,WM,AL,SIG,G,AW,RTA)
C NOTE: HERE WI IS THE POINT ABOUT WHICH THE
C TAYLOR SERIES IS BEING EXPANDED
C AW IS THE FREQUENCY OF INTEREST
A2=AL*(9.81**2.)
B=(-5./4.)*(WM**4.)
C=2.*(SIG**2.)*(WM**2.)
U1=A22*(WI**(-5.))
U2=EXP(B/(WI**4.))
U3=G**EXP(-(WI-WM)**2.)/C)
U4=-5.*A22/(WI**6.)
U5=-4.*B/(WI**5.)
U6=EXP(-(WI-WM)**2.)/C)
U7=-2.*(WI-WM)/C
F13=(-2.*A22*(WI-WM)/(C*(WI**5.)))
AB8=8.*A22*B*(WI-WM)/(C*(WI**10.))
AW8=-(8.*A22*WI-10.*A22*WM)/(C*(WI**6.))
AW4=(4.*A22*(WI-WM)**2.)/(C*(WI**5.))
A4=(A22*(WI-WM)**20.*WM-12.*WI)/(C*(WI**5.))
C
S1 IS THE FIRST DERIVATIVE
S1=U4*U2*U3+(U1*U2*U5*U3)+U1*U2*U3*U6*U7*LOG(G)
C
S2 IS THE SECOND DERIVATIVE
S2=(30.*A22/(WI**7.))*U2*U3+(U4*U2*U5*U3)+
1  (U4*U2*U3*U6*U7*LOG(G))+

A72
1 (U4*U2*U5*U3)+(U1*U2*U5*U3)+(U1*U2*(20.*B/(WI**6.))*U3+)
1 (U1*U2*U5*U3)*U6*U7*ALOG(G)+
1 F13*U2*U3*U6*U7*ALOG(G)+F13*U2*U3*U6*U7*ALOG(G)+
1 F13*U2*U3*U6*U7*ALOG(G)
C

S3 IS THE THIRD DERIVATIVE
S3=(+210.*A22/(WI**8.))*U2*U3+(30.*A22/(WI**7.))*U2*U5*U3+
1 (30.*A22/(WI**7.))*U2*U3*U6*U7*ALOG(G)+
1 (-220.*A22*B/(WI**12.))*U2*U3+(20.*A22*B/(WI**11.))*U2*U5*U3+
1 (20.*A22*B/(WI**11.))*U2*U3*U6*U7*ALOG(G)+
1 ((-72.*A22*B/(C*(WI**11.)))+(80.*A22*B*WM/(C*(WI**11.))))*U2*U3*U6*U7*ALOG(G)+
1 (-220.*A22*B*(WI**(-12)))*U2*U3+(20.*A22*B*(WI**(-11)))*U2*U5*U3+
1 (20.*A22*B*(WI**(-11)))*U2*U3*U6*U7*ALOG(G)+
1 ((-72.*A22*B*(C*(WI**(-11.)))+(80.*A22*B*WM/(C*(WI**-11.)))))*U2*U3*U6*U7*ALOG(G)
S3=S3+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
1 AB8*U2*U3*U6*U7*ALOG(G)+
SUBROUTINE STORAGE(ALPHA)

C INCLUDE 'SPEC.CMN'
OPEN(10,FILE='INT.DAT',STATUS='OLD')
READ(10,1000)NINT
1000 FORMAT(I10)
CLOSE(10)
NUME=NPAR(2)
N100=N26
C
N111=N100
NRT=NROOT*NROOT
N112=N111+NRT
C
ISPT=SPT
NUP=ISPT*NUME
N113=N112+NUP
N114=N113+NWK
N115=N114+NWK
NEQRT=NEQ*NROOT
N116=N115+NEQRT
N117=N116+NEQRT
N118=N117+NEQRT
N119=N118+NROOT
N120=N119+NROOT*NROOT
C
F1G
N121=N120+NEQ
C
F2G
N122=N121+NEQ
C
TRPHI
N123=N122+NROOT*NEQ
C
R1
N124=N123+NROOT
C
R2
N125=N124+NROOT
C
Z1
N126=N125+NROOT
C
Z2
N127=N126+NROOT
C
PHI1
N128=N127+12*NROOT
C
PHI2
N129=N128+8*NROOT
C
PHI3
N130=N129+8*NROOT
C
CDIFF
N131=N130+NWK
C
SIGR22
N132=N131+ISPT*NUME
C
SIGR33
N133=N132+ISPT*NUME
C
SIGR23
N134=N133+ISPT*NUME
print*,N134 = '134'
CALL INITIAL(NUME,NINT,A(N13),ALPHA,A(N111),
A(N131),A(N132),A(N133),NUP)
1 RETURN
END

SUBROUTINE INITIAL(NUME,NINT,XYZ,ALPHA,SIGMZZ,
SIGR22,SIGR33,SIGR23,ISPNUM)
SUB EVAluates E1W,E2W,E3W,TWM1....TWM4,S1G,S2G,S3G
C
INCLUDE 'SPEC.CMN'
DIMENSION SIGMZZ(NROOT,NROOT),SIGR22(ISPNUM),SIGR33(ISPNUM),
SIGR23(ISPNUM)
DIMENSION TGW(3,3),TWM(3,3),TXW(6),DIV(nume),E1W(1),E2W(2),E3W(3),
IDIV(nume),XYZ(6,1),XW(6,1),KK(2000)
CHARACTER *2 C(10)
CHARACTER RTY*4,DD*2
CHARACTER *1 EE(9)
CHARACTER NAME*5, NAME1*4
DATA C/7',8',9',10',11',12',13',14',15',16'/
DATA EE/1',2',3',4',5',6',7',8',9'/

GO TO 4444
3333 PRINT*, 'ERROR=', JO
CALL EXIT

4444 NINT1=100
ICOUNT=0
JCOUNT=0
PI=3.1415927

C

C OPEN SCRATCH FILES FOR TWMI
DO II=1,9
II=II+20
DD=EE(II)
NAME='TWMI'/DD(:1)
OPEN (II,FILE=NAME,STATUS='NEW')
END DO
PRINT*, 'OPENED'
END

C

C SCRATCH FILES FOR INCLINED PILE CO-ORD SYS
DO JJ=1,3
JJ=JJ+10
DD=EE(JJ)
NAME='EI'/DD(:1)'
OPEN (JJ,FILE=NAME,STATUS='UNKNOWN')
END DO
PRINT*, 'S1 FILES OPENED'

C

C SCRATCH FILES FOR STANDARD DEVIATIONS
DO JJ=1,3
JJ=JJ+30
DD=EE(JJ)
NAME='SI'/DD(:1)'G'
CALL SCRATCH(JJ,NAME)
END DO
PRINT*, 'S1 FILES OPENED'

C

C SCRATCH FILES FOR XW1 AND XW3
IL=41
NAME1='XW1I'
CALL SCRATCH(IL,NAME1)
IL=42
NAME1='XW13'
CALL SCRATCH(IL,NAME1)
PRINT*, 'XW FILES OPENED'

C

C SCRATCH FILE FOR FACTOR XK
NAME1='XKI I'
CALL SCRATCH(43,NAME1)

C

C SCRATCH FILES FOR FORCES
NAME1='BT1I'
CALL SCRATCH(44,NAME1)
NAME1='BTI2'
CALL SCRATCH(45,NAME1)

C

C SCRATCH FILES TO HOLD VALUES DURING ITERATIONS
NAME='SIG1I'
CALL SCRATCH(46,NAME)
NAME='SIG12'
CALL SCRATCH(47,NAME)
NAME='SIG13'
CALL SCRATCH(48,NAME)

C

C

C ALL SCRATCH FILES OPENED
C

A75
PRINT*.'ALL SCRATCH FILES OPEN'
DELTAW=STEP
DELTAW=0.04
DO 10 II=1,3
  DO 10 JJ=1,3
    TGW(IJ)=0.0
10 CONTINUE
C
C NOTE: HERE ALPHA IS THE ANGLE THE WAVE AXIS MAKES
C WITH THE STRUCTURE AXIS
C
TGW(1,1)=COS(ALPHA)
TGW(1,2)=SIN(ALPHA)
TGW(2,1)=-SIN(ALPHA)
TGW(2,2)=COS(ALPHA)
TGW(3,3)=1.
C
C LOOP OVER ALL ELEMENTS
C
ISPT=SPT
D=-XMUD
IEW=0
PRINT*,XMUD=,XMUD
INK=0
DO 100 N=1,NUME
  IF(XYZ(3,N).LE.0.0.AND.XYZ(6,N).LE.0.0)THEN
    IEW=IEW+1
    XL=0.
    XL2=0.
    DO IP=1,3
      XL=XYZ(IP,N)-XYZ(IP+3,N)
    END DO
    XLN=SQRT(XL2)
    SPTM1=SPT-1.
    DEL=XLN/SPTM1 ! IDIV(N)=(NO. OF SIMPS PTS -1)/6
    DELX=0.0
  END IF
20 CONTINUE
TWM(IMW,IMW)=0.0
TWM(1,1)=T(1,1,N)*COS(ALPHA) + T(1,2,N)*SIN(ALPHA)
TWM(1,2)=-T(1,1,N)*SIN(ALPHA) + T(1,2,N)*COS(ALPHA)
TWM(1,3)=T(1,3,N)
TWM(2,1)=T(2,1,N)*COS(ALPHA) + T(2,2,N)*SIN(ALPHA)
TWM(2,2)=-T(2,1,N)*SIN(ALPHA) + T(2,2,N)*COS(ALPHA)
TWM(2,3)=T(2,3,N)
TWM(3,1)=T(3,1,N)*COS(ALPHA) + T(3,2,N)*SIN(ALPHA)
TWM(3,2)=-T(3,1,N)*SIN(ALPHA) + T(3,2,N)*COS(ALPHA)
TWM(3,3)=T(3,3,N)
C
C EXPRESSS THE NODAL CO-ORDS IN TERM OF THE
C WAVE CO-ORD SYSTEM.
XW(1)=TGW(1,1)*XYZ(1,N)+TGW(1,2)*XYZ(2,N)+TGW(1,3)*XYZ(3,N)
XW(2)=TGW(2,1)*XYZ(1,N)+TGW(2,2)*XYZ(2,N)+TGW(2,3)*XYZ(3,N)
XW(3)=TGW(3,1)*XYZ(1,N)+TGW(3,2)*XYZ(2,N)+TGW(3,3)*XYZ(3,N)
XW(4)=TGW(1,1)*XYZ(4,N)+TGW(1,2)*XYZ(5,N)+TGW(1,3)*XYZ(6,N)
XW(5)=TGW(2,1)*XYZ(4,N)+TGW(2,2)*XYZ(5,N)+TGW(2,3)*XYZ(6,N)
XW(6)=TGW(3,1)*XYZ(4,N)+TGW(3,2)*XYZ(5,N)+TGW(3,3)*XYZ(6,N)
C
E1W(N)=-(XW(1)-XW(4))*XLN
E2W(N)=-(XW(2)-XW(5))*XLN
E3W(N)=-(XW(3)-XW(6))*XLN
WRITE(11,REC=N)E1W(N)
WRITE(12,REC=N)E2W(N)
WRITE(13,REC=N)E3W(N)
C
WRITE(21,REC=N)TWM(1,1)
WRITE(22,REC=N)TWM(1,3)
WRITE(23,REC=N)TWM(2,1)
WRITE(24,REC=N)TWM(2,2)
WRITE(25,REC=N)TWM(3,1)
WRITE(26,REC=N)TWM(3,2)
WRITE(27,REC=N)TWM(1,2)
WRITE(28,REC=N)TWM(2,3)
WRITE(29,REC=N)TWM(3,3)

WRITE(41,REC=N)XW(1)
WRITE(42,REC=N)XW(3)

C
DO 310 IP=1,JSPT ! NO. OF SIMPS POINTS
IEW=IEW+1
SIP=IP
WL=7000.
JCOUNT=JCOUNT+1
DELX=(SIP-1)*DEL
XW1=XW(1)+TWM(1,1)*DELX
XW3=XW(3)+TWM(1,3)*DELX
SIGR22(JCOUNT)=0.0
SIGR33(JCOUNT)=0.0
SIGR23(JCOUNT)=0.0
XNT=NINT1
DO 320 XNF=1,WLST
IXN=XNF
DNF=XNF

C
WF=DELTAW*(DNF)
IF(WF.EQ.0.0)THEN
PRINT*,DELT="DELTAW"
STOP
ELSE
CONTINUE
END IF

C
TW=2.*3.1415927/WF !WAVE PERIOD (HERTZ). FREQUENCY (RADIANS)
DWL=DAVL
IF(DWL.GT.0.5)THEN
C DEEP WATER
WL=9.81*(TW**2.)/(2.*PI)
DK=2.*3.1415927/WL
B1=2.*PI*(EXP(DK*XW3))*COS(DK*XW1)/TW
B2=2.*PI*(EXP(DK*XW3))*SIN(DK*XW1)/TW
B3=2.*PI*(EXP(DK*XW3))*SIN(DK*XW1)/TW
B4=-2.*PI*(EXP(DK*XW3))*COS(DK*XW1)/TW
ELSE
C
B1=BB1(TW,DK,XW3,D,XW1,WL)
B2=BB2(TW,DK,XW3,D,XW1,WL)
B3=BB3(TW,DK,XW3,D,XW1,WL)
B4=BB4(TW,DK,XW3,D,XW1,WL)
END IF

END IF

C
H1=B1-E1W(N)*(E1W(N)*B1+E3W(N)*B3)
H2=B2-E1W(N)*(E1W(N)*B2+E3W(N)*B4)
H3=E2W(N)*(E1W(N)*B1+E3W(N)*B3)
H4=E2W(N)*(E1W(N)*B2+E3W(N)*B4)
H5=B3-E3W(N)*(E1W(N)*B1+E3W(N)*B3)
H6=B4-E3W(N)*(E1W(N)*B2+E3W(N)*B4)
H7=TWM(2,1)*H1+TWM(2,2)*H3+TWM(2,3)*H5
H8=TWM(2,1)*H2+TWM(2,2)*H4+TWM(2,3)*H6
H9=TWM(3,1)*H1+TWM(3,2)*H3+TWM(3,3)*H5
H10 = TWM(3,1)*H2 + TWM(3,2)*H4 + TWM(3,3)*H6
IF(XNF.EQ.1.) THEN
  XK1 = H7*H9 + H8*H10
  IF(XK1.EQ.0) THEN
    XK = 0.
    ELSE
    XK = XK1/(H9**2. + H10**2.)
  END IF
  WRITE(43,REC=N) XK
  ELSE
    CONTINUE
  END IF

C NEXT INITIALISE SIGR22(IP,N) TO SIGV22(IP,N) ETC.
IF(XNF.EQ.1. OR XNF.EQ.WLST) THEN
  RULL = 1.
  IIR = 2
  ELSE IF(IIR.EQ.2) THEN
    RULL = 4.
    IIR = 1
    ELSE
    RULL = 2.
    IIR = 2
  END IF

C CALCULATE THE SPECTRUM ORDINATE FOR THE PARTICULAR FREQUENCY
DO I=1,WLST
  BI = I
  STEPI = BI*STEP
  IF(STEPI.GE.WF) GO TO 101
END DO
101 CONTINUE
  DIFF = WF - STEP*(I-1)
  DIFSPC = SPECTR(I) - SPECTR(M)
  SPE = SPECTR(IXN+1)
  IF(I.EQ.1) SPE = 0.0
  IF(I.EQ.1) THEN
  ELSE
    CONTINUE
  END IF
  SlGR22(JCOUNT) = (H7**2 + H8**2)* SPE* RULL* DELTAW/3. + SIGR22(JCOUNT)
  SIGR33(JCOUNT) = (H9**2 + H10**2)* SPE* RULL* DELTAW/3. + SIGR33(JCOUNT)
  SIGR23(JCOUNT) = H7*H9* SPE* RULL* DELTAW/3. + SIGR23(JCOUNT)
  320 CONTINUE

  SG1 = SIGR22(JCOUNT)
  SG2 = SIGR33(JCOUNT)
  SG3 = SIGR23(JCOUNT)
  IF(SG1.LT.0.0 OR SG2.LT.0.0) THEN
    STOP
    ELSE
    CONTINUE
  END IF
  310 CONTINUE

C WRITE(11,REC=N) E1 W(N)
C WRITE(12,REC=N) E2 W(N)
C WRITE(13,REC=N) E3 W(N)
C WRITE(21,REC=N) TWM(1,1)
C WRITE(22,REC=N) TWM(1,3)
C WRITE(23,REC=N) TWM(2,1)
C WRITE(24,REC=N) TWM(2,2)
C WRITE(25,REC=N) TWM(3,1)
C WRITE(26,REC=N) TWM(3,2)
C WRITE(27,REC=N) TWM(1,2)
C WRITE(28,REC=N) TWM(2,3)
C WRITE(29,REC=N) TWM(3,3)
C END IF
100 CONTINUE

C SET SIGMZZ(IJ) TO ZERO
DO 460 IRT=1,NROOT
DO 460 JRT=1,NROOT
IF(IRT.EQ.JRT)THEN
SIGMZZ(IRT,JRT)=1.
ELSE
SIGMZZ(IRT,JRT)=0.0
END IF
460 CONTINUE
PRINT*,'END OF INITIAL.FOR' RETURN
END

C FUNCTION WAVENO(WF,D)
C THIS CALCULATES THE WAVE NUMBER
C AND CAN BE USED TO CALCULATE THE WAVELENGTH FOR A FREQUENCY
C INITIALISE DK
WNO=0.6
TOL=1.E-3
ITMAX=40
DO I=1,ITMAX
FK=-9.81*TANH(WNO*D)-WF**2.
FKD=9.81*TANH(WNO*D)+9.81*WNO*D*((COSH(WNO*D))**(-2))
FACT=FK/FKD
WN1=WNO-FACT
C CHECK FOR CONVERGENCE
CHECK=FACT*WN1
IF(ABS(CHECK).LT.TOL)GO TO 100
WNO=WN1
END DO
100 CONTINUE
WAVENO=WN1 RETURN
END

C FUNCTION BB1(TW,DK,XW3,D,XW1,WL)
BB1=9.81*TW*COSH(DK*(XW3+D))*COS(DK*XW1)/(WL*COSH(DK*D)) RETURN
ENTRY BB2(TW,DK,XW3,D,XW1,WL)
BB2=9.81*TW*COSH(DK*(XW3+D))*SIN(DK*XW1)/(WL*COSH(DK*D)) RETURN
ENTRY BB3(TW,DK,XW3,D,XW1,WL)
BB3=-9.81*TW*SINH(DK*(XW3+D))*SIN(DK*XW1)/(WL*COSH(DK*D)) RETURN
ENTRY BB4(TW,DK,XW3,D,XW1,WL)
BB4=-9.81*TW*SINH(DK*(XW3+D))*COS(DK*XW1)/(WL*COSH(DK*D)) RETURN
END
SUBROUTINE CBARJ(CDBARJ,NUME,NC,ALPHAJSF,SSIGMZ, 
XYZ,NNC,CAP,DTOT,MTP,ISPNUM,SRGR22,SRGR33,SRGR23) 
C SUB.TO CALCULATE THE DRAG COEFF. DUE TO LINEARISATION OF MORRISON'S EQUATION 
INCLUDE 'SPEC.CMN'
DIMENSION CDBARJ(ISPT,NUME),SIGMZ(NROOT,NROOT),XYZ(6,NUME), 
MTP(1),CAP(NWK),DTOT(1),SRGR22(ISPNUM), 
SRGR33(ISPNUM),SRGR23(ISPNUM)
ICNT=0 
PI=3.1415927
ITERFLG=ITERFLG+1
ISPT=ISPT
DO 100 I=1,NUME
MTYPE=MTP(I)
CD1=CD*100072.
IF(XYZ(3,I).LE.0.0.AND.XYZ(6.I).LE.0.0)THEN 
READ(43,REC=I)XK 
DO 110 J=USPT 
ICNT=ICNT+1 
DENOM=SRGR22(ICNT)+SRGR33(ICNT)
A2=(SRGR22(ICNT)**2.)*SRGR33(ICNT)-(SRGR23(ICNT))**2. 
XO=2.*SRGR22(ICNT) 
XN=2.*SRGR33(ICNT) 
YO=-2.*SRGR33(ICNT) 
YM=2.*SRGR33(ICNT) 
GR22=SRGR22(ICNT) 
GR33=SRGR33(ICNT)
C IF(SRGR22(ICNT),EQ.0.0.AND.SRGR33(ICNT),EQ.0.0)THEN 
CDBAR(J,J)=0.0
ELSE 
IF(XK.EQ.O.)THEN 
CDBAR(J,J)=0.
ELSE 
IF(AB.XT.0.000001  )THEN 
SRXK=SQRT((1.+XK**2.)) 
E1=-(XN**2.)/(2.*SRGR22(ICNT)) 
E=EXP(E1) 
C=(SRGR22(ICNT)**2.)*PI)**0.5 
XNUM1=SRXK*(4.*(SRGR22(ICNT)**2.))*(E1*E-E+1.)*C 
OVER=XNUM1+XNUM2 
CDBAR(J,J)=CD*OVER/DENOM
ELSE 
SRGR22=SRGR22(ICNT) 
SRGR33=SRGR33(ICNT) 
SRGR23=SRGR23(ICNT)
FACT1=(P(SRGR22,SRGR33,SRGR23,XO,YO,XO)+ 
P(SRGR22,SRGR33,SRGR23,XN,YO,XN)+ 
P(SRGR22,SRGR33,SRGR23,XO,YM,XO)+ 
P(SRGR22,SRGR33,SRGR23,XN,YM,XN))
FACT1Y=(P(SRGR22,SRGR33,SRGR23,XO,YO,YO)+ 
P(SRGR22,SRGR33,SRGR23,XN,YO,YO)+ 
P(SRGR22,SRGR33,SRGR23,XO,YM,YM)+ 
P(SRGR22,SRGR33,SRGR23,XN,YM,YM))
FACT10=0.0
FACT2Y=0.0 
XOP1=XO+0.005 
XNM1=XN-0.005 
DO 120 XI=XOP1,XNM1,0.005 
XI=XI 
FACT2=FACT2+((P(SRGR22,SRGR33,SRGR23,XI,YO,XI)+ 
P(SRGR22,SRGR33,SRGR23,XI,YM,XI)) 
FACT2Y=FACT2Y+((P(SRGR22,SRGR33,SRGR23,XI,YO,YO)+ 
P(SRGR22,SRGR33,SRGR23,XI,YM,YI)) 
A70
1  \( P(SGR22, SGR33, SGR23, XI, YM, YM) ) \)

120 CONTINUE
YOP1=YO+0.005
YMM1=YM-0.005
DO 130 XI=YO+1,YMM1,0.005
YJ=XJ
FACT2=FACT2+(P(SGR22, SGR33, SGR23, XO, YJ, XO)+
1  P(SGR22, SGR33, SGR23, XN, YJ, XN))
FACT2Y=FACT2Y+(P(SGR22, SGR33, SGR23, XO, YJ, YJ)+
1  P(SGR22, SGR33, SGR23, XN, YJ, YJ))
130 CONTINUE
FACT3=0.
FACT3Y=0.
DO 140 XIM=XO+1,XNM1,0.005
YN=XM
DO 140 XIM=XO+1,XNM1,0.005
XNI=XIM
FACT3=FACT3+(P(SGR22, SGR33, SGR23, XNI, YN, XNI))
FACT3Y=FACT3Y+(P(SGR22, SGR33, SGR23, XNI, YN, YN))
140 CONTINUE
C
C
C
C
CD=0.005*0.005*0.25*(FACT1+FACT1Y+(2.*(FACT2+FACT2Y))
1  +(4.*FACT3+FACT3Y))
CDB1=SQRT(CDB)
CDB2=CDB1*CDB1
PISQRT=SQRT(8/PI)
CDBARJ(J,I)=PISQRT*CDB1 *CD1
CDBAR=CDBAR*PISQRT*CDB2*CD1/DENOM
C
END IF
END IF
END IF
110 CONTINUE
C
ELSE
DO 901 J=USPT
ICNT=ICNT+1
CDBARJ(J,I)=0.0
901 CONTINUE
C
100 CONTINUE
C
CALL ADDAMP(A(N13),A(N113).CAP,IA(IN4),IA(IN5),IA(NM2),NC,SIGMZZ,
1  CDBARJ,NUME,NNC,ALPHA,A(N130)^(N12),ISPNUM,SIGR22,SIGR33,SIGR23,JSPT)
RETURN
END
C
C
FUNCTION P(SGR22, SGR33, SGR23, R2J, R3)
R2SQ=R2**2.
R3SQ=R3**2.
SRT=(R2SQ+R3SQ)
SRT1=SQRT(SGR22*SGR33-SGR23**2.)
CON=1.4145927*SRT1
TOP=SGR33*(R2**2.)*2.*SGR2*R2+SGR22*(R3**2.)
BOT=2.*(SGR22*SGR33-(SGR23**2.))
ETB=EXP(-(TOP/BOT))
PE=CON*ETB
R3=1
P=SRT*PE
RETURN
END
C
C
FUNCTION TO EVALUATE THE PROBABILITY
C
DENSITY P(X,Y)
R2SQ=R2**2.
R3SQ=R3**2.
SRT=(R2SQ+R3SQ)
SRT1=SQRT(SGR22*SGR33-SGR23**2.)
CON=1.4145927*SRT1
TOP=SGR33*(R2**2.)*2.*SGR2*R2+SGR22*(R3**2.)
BOT=2.*(SGR22*SGR33-(SGR23**2.))
ETB=EXP(-(TOP/BOT))
PE=CON*ETB
R3=1
P=SRT*PE
RETURN
END

A81
SUBROUTINE ADDAMP(XYZ,C,CAP,MATP,LM,MAXA,MHT,NC,1
SIGMZZ,CDBARJ,NUME,NNC,ALPHA,1
CDIFF,DTOT,ISPNUM,SIGR22,SIGR33,SIGR23,ISPT)
C
C SUB. TO CALCULATE AND ASSEMBLE THE ADDED DAMPING
C
INCLUDE 'SPEC.CMN'
C
DIMENSION CA(12,12),CDD(100),TR(12,12)
C
DIMENSION GG2(12,12),G2G(I2,12),Q(12,12),XYZ(6,1),CAP(NWK),1
1 MATP(1),LM(12,1),MAXA(1),MHT(1),C(NWK),D(3),1
1 SIGMZZ(NROOTJ4ROOT),CDBARJ(ISPT,NUME)J)TOT(100),1
1 CDIFF(NWK),SIGR22(ISPNUM),SIGR33(ISPNUM),1
1 SIGR23(ISPNUM)
OPEN{38,FILE='SIGMZ.DAT,STATUS='LnSIKNOWN)
ISPT=ISPNUM/NUME
SPT=ISPT
ND=12
DO 301 IU=1,NWK
C(IU)=0.
CDIFF(IU)=0.
301 CONTINUE
CON=1.0
DO 100 N=I,NUME
DO 151 IP=1,12
DO 151 JP=1,12
G2G(IPJP)=0.0
151 CONTINUE
C
IF(XYZ(3JvI).LE.0.0.AND.XYZ(6,N).LE.0.0)THEN
XL2=0.
DO 306 IP=1,3
DIP=XYZ(3,N)-XYZaP,N)-XYZaP+3,N)
XL2=XL2+DIP*DIP
306 CONTINUE
XL=SQRT(XL2)
MTYPE=MATP(N)
SPTM1=SPT-1.
DELXL=XL/SPTM1
DO110IN=1,12
DO110JN=1,12
CA(IN,JN)=0.0
110 CONTINUE
DO120IM=1,12
DO 120 JM=1,12
GG2(IM,JM>=0.00
120 CONTINUE
C
C PERFORM INTEGRATION
DO130IL=1,ISPT
RIL=IL-1
XIMJ=RIL*DELXL
GG2(2,2)=G3(X1MJ,XL)*G3(X1MI,XL)
GG2(2,6)=G3(X1MJ,XL)*G4(X1MJ,XL)
GG2(2,12)=G3(X1MJ,XL)*G6(X1MJ,XL)
GG2(3,3)=G3(X1MJ,XL)*G3(X1MJ,XL)
GG2(3,5)=G3(X1MJ,XL)*G7(X1MJ,XL)
GG2(3,9)=G3(X1MJ,XL)*G5(X1MJ,XL)
GG2(3,11)=G3(X1MJ,XL)*G8(X1MJ,XL)
GG2(5,3)=G7(X1MJ,XL)*G3(X1MJ,XL)
GG2(5,5)=G7(X1MJ,XL)*G7(X1MJ,XL)
GG2(5,9)=G7(X1MJ,XL)*G5(X1MJ,XL)
GG2(5,11)=G7(X1MJ,XL)*G8(X1MJ,XL)
GG2(6,2)=G4(X1MJ,XL)*G3(X1MJ,XL)
GG2(6,6)=G4(X1MJ,XL)*G4(X1MJ,XL)
GG2(6,8)=G4(X1MJ,XL)*G5(X1MJ,XL)
GG2(6,12)=G4(X1MJ,XL)*G6(X1MJ,XL)
GG2(8,2)=G5(X1MJ,XL)*G3(X1MJ,XL)
GG2(8,6)=G5(X1MJ,XL)*G5(X1MJ,XL)
GG2(8,8)=G5(X1MJ,XL)*G5(X1MJ,XL)
GG2(8,12)=G5(X1MJ,XL)*G6(X1MJ,XL)
GG2(9,3)=G5(X1MJ,XL)*G3(X1MJ,XL)
GG2(9,5) = G5(X1MJ,XL)*G7(X1MJ,XL)
GG2(9,9) = G5(X1MJ,XL)*G5(X1MJ,XL)
GG2(9,11) = G5(X1MJ,XL)*G8(X1MJ,XL)
GG2(11,2) = G3(X1MJ,XL)*G6(X1MJ,XL)
GG2(11,6) = G6(X1MJ,XL)*G4(X1MJ,XL)
GG2(11,8) = G6(X1MJ,XL)*G5(X1MJ,XL)

IF(IL.EQ.1.OR.IL.EQ.ISPT) THEN
  RUL = 1.
  IIP = 2.
ELSE IF(IIP.EQ.2) THEN
  RUL = 4.
  IIP = 1.
ELSE
  RUL = 2.
  IIP = 2.
END IF

DO 140 IK = 1, 12
  DO 140 JK = 1, 12
  140 CONTINUE

130 CONTINUE

C
C NEXT WE MUST PRE- AND POST- MULTIPLY
C BY THE TRANSFORMATION MATRICES TO
C CONVERT TO GLOBAL CO-ORDS

NN = 0
DO 150 IP = 1, 12
  DO 150 JP = 1, 12
    Q(IP,JP) = 0.0
  150 CONTINUE

DO 160 IT = 1, 12
  DO 160 JT = 1, 12
    DO 160 IR = 1, 12
      Q(IT,JT) = G2G(IT,IR) * T(IR,JT,N) + Q(IT,JT)
    160 CONTINUE
150 CONTINUE

C CONVERT TO LOCAL CO-ORDS

DO 170 IA = 1, 12
  DO 170 IB = 1, 12
    CA(IA,IB) = TR(IA,IB,0.0)
170 CONTINUE

K = 0
DO 180 LF = 1, 12
  DO 180 LK = LF, 12
    K = K + 1
    CDD(K) = CA(LF,LK)
  180 CONTINUE

ELSE
  K = 0
  DO 185 LF = 1, 12
    DO 185 LK = LF, 12
      K = K + 1
      CDD(K) = 0.0
  185 CONTINUE
180 CONTINUE

C NOTE HERE WE MUST ENSURE TO ADD THE ADDED DAMPING TO THE CORRECT
C ELEMENT

END IF

CALL ASSEMBLE(CA(IJ),CA(IN5),CDD,LM(N,N),ND)

100 CONTINUE

C
C FIND NORM OF DIFFERENCE OF C(I) AND CAPI)
C IF THIS VALUE IS GREATER THAN A GIVEN VALUE THEN PUT
C C (IJ) = CAP(IJ), OTHERWISE OUTPUT THE RESULTS

AXRC = 0.0
AXRCP = 0.0
DO 307 IK = 1, NWK

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CDIFF(IK)=C(IK)-CAP(IK)

CONTINUE

DO 50 IC=1,NEQ
MAXIC=MAXA(IC)
MAXHT=MAXA(IC) + MHT(IC)
SUMIC=0.0
SUMCAP=0.0
DO 190 IX=MAXIC,MATH
SUMIC=ABS(C(IX))+SUMIC
 SUMCAP=ABS(CDIFF(IX))+SUMCAP

190 CONTINUE

JIC=IC+1
JNEQ=NEQ+1
L=0
DO 200 J=HIC,JNEQ
L=L+1
IF(MHT(J).GE.L)THEN
MAXJL=MAXA(J)+L
SUMIC=SUMIC+ABS(CDIFF(MAXJL))
ELSE
CONTINUE
END IF

200 CONTINUE

IF(SUMIC.GT.AXRC) AXRC=SUMIC
IF(SUMCAP.GT.AXRCP) AXRCP=SUMCAP

50 CONTINUE

IF(AXRC.EQ.O)THEN
RATIO(ITERFLG)=0
PRINT*,'AXRC = 0'
ELSE
RATIO(ITERFLG)=AXRCP/AXRC
END IF

IF(ITERFLG.EQ.ITERFLG1)THEN
CALL RESULTS(A(N16))
ELSE
DO 304 INWK=1,NWK
CAP(INWK)=C(INWK)
304 CONTINUE

CALL MODAL(C(1),A(N16),NC,IA(In5),IA(IN2),SIGMAZ, 
1 A(N15),A(N16),A(N17),A(N118),A(N18),A(N21), 
2 NNC,A(N119),ALPHA,CDBARJ,NUME,CAP,ISPNUM, 3 SIGR22,SIGR33,SIGR23,ISP)

C

FUNCTION SUBPROGRAMS
FUNCTION G1(X1MJ,XL)
G1=1-X1MJ/XL
RETURN
ENTRY G2(X1MJ,XL)
G2=X1MJ/XL
RETURN
ENTRY G3(X1MJ,XL)
G3=1-3.*(X1MJ/XL)**2+2.*(X1MJ/XL)**3.
RETURN
ENTRY G4(X1MJ,XL)
G4=X1MJ-2.*(X1MJ**2.)/XL+(X1MJ**3.)/(XL**2.)
RETURN
ENTRY G5(X1MJ,XL)
G5=3.*(X1MJ/XL)**2-2.*(X1MJ/XL)**3.
RETURN
ENTRY G6(X1MJ,XL)
G6=(X1MJ**2.)/XL+X1MJ**3.)/(XL**2.)
RETURN
ENTRY G7(X1MJ,XL)
G7=X1MJ+2.*(X1MJ**2.)/XL-X1MJ**3.)/(XL**2.)
RETURN
ENTRY G8(X1MJ,XL)
G8=(X1MJ**2.)/XL-(X1MJ**3.)/(XL**2.)
RETURN
END
SUBROUTINE MODAL(CA,RJJC,JMAXAJVIHT.SIGMZZ.CAS.
RT,CASTAR,CADIAG,TT,BR,NNC,BBR,
1 ALPHACDBARJ,NUMECAP,ISPNUM,
1 SIGR22,SIGR33,SIGR23,JSPT)
C
INCLUDE 'SPEC.CMN'
DIMENSION CA(l),RT(NEQ,NROOT),MAXA(l),MHT(l),SIGMZZ(NROOT,NROOT),
1 CADIAG(NROOT),CAS(NEQ,l),CASTAR(NROOT,l),CBARJ(ISPT,NUME),
1 BR(NNC),BBR(NROOT,1),RT(NEQ),TT(NEQ),CAP(NWK),
1 SIGR22(ISPNUM),SIGR33(ISPNUM),SIGR23(ISPNUM)
C
NUME=NPAR(2)
NC=NROOT
II=0
DO 160 J=1,NROOT
CALL Mult(TT,CA,R(U),MAXA,NEQ,NWK)
DO 180 I=JJ«100T
BRT=0.
DO 190 K=1,NEQ
BRT=BRT+R(K,I)*rr(K)
190 CONTINUE
BRR(I,J)=BRT
180 CONTINUE
160 CONTINUE
DOIP=lJJC
DOJP=l,NC
BBR(IP,JP)=BBR(JPJP)
END DO
END DO
C
WRITE(IOUT,*)'DIAGONALISED DAMPING MATRIX;'
DOIC=lJ4ROOT
SUM=0.
DO ID=l,NROOT
IF(ID.EQ.IC)THEN
SUM=SUM
ELSE
SUM=SUM+BBR(ICJD)*SIGMZZ(ICJ,SIGMZZaC.IC)
END IF
CADIAGaC)=BBR(IC,IC>*SUM
C
WRITE(IOUT,*)'CADIAG=',CADIAG(IC)
END DO
CALL FORCE(A(N13)JSPT,NUME,CDBARJ,
1 A(N16),CADIAG,A(N17),SIGMZZ,
1 A(N122),A(N123),A(N124),
1 A(N120),A(N121),A(N127),A(N128),A(N129),
1 IA(IN3),A(IN8),A(IN12),IA(IN4),NC,NNC,
1 ALPHACAP,ISPNUM,SIGR22,SIGR33,SIGR23)
C
PRINT*,'CALCS PERFORMED IN MODAL.FOR'
RETURN
END
SUBROUTINE FORCE(XYZ,ISPT,NUME,CDBARJ,PHI,CADIAG,W,
1 SIGMZZ,TRPHI,R1,R2,
1 F1G,F2G,PHI1,PHI2,PHI3,L,M,ATOT,DTOT,MATP,
1 NC,NNC,ALPHA,CAP,ISPNUM,SIGR22,SIGR33,
1 SIGR23)

INCLUDE 'SPEC.CMN'
DIMENSION XYZ(6,1),F1G(NEQ),F2G(NEQ),
1 FS(4),FC(4),GT(12,4),OJ(8),RJ(8),
1 TWMS(4,4),TRANS(12,12),ATOT(1),DTOT(1),MATP(1),
1 CDBARJ(ISPT,NUME),TQC(4),TQS(4),GQC(12),GQS(12),
1 FORCOS(12),FORSIN(12),PHI(NEQ,NROOT),
1 TRPHI(NROOT,NEQ),R1(NROOT),R2(NROOT),CADIAG(NROOT),
1 W(NROOT),I W(I) ARE THE NAT. FREQ'S MH 11/5/95 DELETED Z1 Z2
1 LM(12,1),CAP(NWK)

DIMENSION B1(nume,30),B2(nume,30),B3(nume,30),B4(nume,30)

DIMENSION TORCOS(12),TORSIN(12),H1L(12,JROOT),
1 PHI2(10,JROOT),PHI3(8,JROOT),ZEL1(12),ZEL2(12),
1 TZ11(10),TZ21(10),TD1(8,8),ZEL31(8),ZEL32(8),
1 GS1(2,12),GS2(2,10),GS3(2,8),UMJ1(2),UMJ2(2),
1 SIGMZZ(NROOT,NROOT),TZ31(8),TZ32(8),TWM(3,3),
1 SIGR22(ISPNUM),SIGR33(ISPNUM),SIGR23(ISPNUM)

C INITIALISATION

OPEN(10,'FILE=’WL.DAT,STATUS=’UNKNOWN’)
PRINT*,’CALCULATING IN SUB FORCE’
STEP=0.04
DO IS=1,NROOT
DO JS=1,NROOT
SIGMZZ(IS,JS)=0.0
END DO
END DO
ICOUNT=0
 DO IV=1,NUME
 DO JY=1,ISPT
 ICOUNT=ICOUNT+1
 SIGR22(ICOUNT)=0.
 SIGR33(ICOUNT)=0.
 SIGR23(ICOUNT)=0.
 END DO
 END DO

D=-XMUD
Pi=3.1415927
ICOUNT=0
 DO IF=1,NEQ
 DO IY=1,ISPT
 ICOUNT=ICOUNT+1
 TRPHI(IJ)=PHI(IJ)
 END DO
 END DO
WL=1000.
ICOUN=0

C

DO FR=1.,WLST
IFR=FR
WF=FR*STEP
WRITE(10,111)WF,WL
111 FORMAT(1X,F10.5)

C INITIALISATION

DO NN=1,NEQ
 F1G(NN)=0.
 F2G(NN)=0.
 END DO

C

DO N=1,NUME
MTYPE=MATP(N)

C

IF(XYZ(3,N).LE.0.0.AND.XYZ(3,N).GE.XMUD)THEN
IF(XYZ(6,N).LE.0.0.AND.XYZ(6,N).GE.XMUD)THEN
DO IT=1,4

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DO IT=1,4
TWMS(IT,IT)=0.
END DO
END DO
DO IO=1,12
DO IO=1,4
GT(IO,JO)=0.
END DO
END DO
C
READ(11,REC=N)E1W
READ(12,REC=N)E2W
READ(13,REC=N)E3W
C
READ(21,REC=N)TWM(1,1)
READ(22,REC=N)TWM(1,2)
READ(23,REC=N)TWM(1,3)
READ(24,REC=N)TWM(2,1)
READ(25,REC=N)TWM(2,2)
READ(26,REC=N)TWM(2,3)
READ(27,REC=N)TWM(3,1)
READ(28,REC=N)TWM(3,2)
READ(29,REC=N)TWM(3,3)
C
TWMS(1,1)=TWM(1,1)
TWMS(1,2)=TWM(1,2)
TWMS(1,3)=TWM(1,3)
TWMS(2,1)=TWM(2,1)
TWMS(2,2)=TWM(2,2)
TWMS(2,3)=TWM(2,3)
TWMS(3,1)=TWM(3,1)
TWMS(3,2)=TWM(3,2)
TWMS(3,3)=TWM(3,3)
C
READ(41,REC=N)XWA
READ(42,REC=N)XWB
XL2=0.
DO IQ=1,3
DX=XYZ(IQ,N)-XYZ(IQ+3,N)
XL2=XL2+DX**2.
END DO
XL=SQRT(XL2)
SPTM1=SPT-1.
DELX=XL/SPTM1
ISPT=SPT
C
DO IC=1,12
FORCOS(IC)=0.
FORSIN(IC)=0.
END DO
C
DO IC=1,ISPT
RI=I
DELTAX=(RI-1.)*DELX
XW1=XWA+TWM(1,1)*DELTAX
XW3=XWB+TWM(1,3)*DELTAX
C
TW=2.*3.1415927/WF \*WAVE PERIOD (HERTZ). FREQUENCY (RADIANS)
DWL=D/WL
IF(DWL.GT.0.5)THEN
C DEEP WATER
WL=9.81*(TW**2.)/(2.*PI)
CHWL=-WL/2
IF(XYZ(3,N).LE.CHWL.AND.XYZ(6,N).LE.CHWL)THEN
GOTO 100
ELSE
CONTINUE
END IF
DK=2.*3.1415927/WL
B1(N,J)=Whit**2*pi*(EXP(DK*XW3))*COS(DK*XW1)/TW
B2(N,J)=Whit**2*pi*(EXP(DK*XW3))*SIN(DK*XW1)/TW
B3(N,J)=Whit**2*pi*(EXP(DK*XW3))*SIN(DK*XW1)/TW
B4(N,J)=Whit**2*pi*(EXP(DK*XW3))*COS(DK*XW1)/TW
C
ELSE
DO IK=1,10
DO JK=1,NROOT
ZEL21(IK) = PHI2(IKJK)*Z1(JKIFR) + ZEL21(IK)
ZEL22(IK) = PHI2(IKJK)*Z2(JKIFR) + ZEL22(IK) !Z2 SAME MH 11/5/95
END DO
END DO
DO IK=1,10
DO JK=1,10
TZ21 (IK) = TD(IKJK)*ZEL21 (JK) + TZ21 (IK)
TZ22 (IK) = TD(IKJK)*ZEL22 (JK) + TZ22 (IK)
END DO
END DO

C ELSE IF(XYZ(3,I).LT.XMUD.AND.XYZ(6,I).LT.XMUD)THEN
DO ID=1,2
DO JD=1,2
TD1(ID, JD)=T(ID+1, JD+1)
TD1(ID+2, JD+2)=T(ID+4, JD+4)
TD1(ID+4, JD+4)=T(ID+7, JD+7)
TD1(ID+6, JD+6)=T(ID+10, JD+10)
END DO
END DO
C
DO IZ=1,8
ZEL31(IZ) = 0.
ZEL32(IZ) = 0.
TZ31(IZ) = 0.
TZ32(IZ) = 0.
END DO
C
DO IK=1,8
DO JK=1,NROOT
ZEL31(IK) = PHI3(IKJK)*Z1(JKIFR) + ZEL31(IK) !MH CHANGED Z1 Z2 11/5/95
ZEL32(IK) = PHI3(IKJK)*Z2(JKIFR) + ZEL32(IK)
END DO
END DO
DO IR=1,8
DO JR=1,8
TZ31(IR) = TD1(IR, JR)*ZEL31(JK)+TZ31(IR)
TZ32(IR) = TD1(IR, JR)*ZEL32(JK)+TZ32(IR)
END DO
END DO
C
ELSE
CONTINUE
END IF
C
C LOOP OVER ALL SIMPSONS POINTS IN THE ELEMENT
SPTM1=SPT-1
ISPT=SPT
DELX=XL/SPTM1
READ(41, REC=I)XWA
READ(42, REC=I)XWB
C
READ(11, REC=I)E1W
READ(12, REC=I)E2W
READ(13, REC=I)E3W
DO IS=1,ISPT
ICNT=ICNT+1
SS=IS
XIM=(SS-1.)*DELX
IF(XYZ(6,I).GT.XMUD)THEN
DO IR=1,2
DO IC=1,12
GS1(IR, IC)=0.
END DO
END DO
GS1(1,2)=G3(XIM, XL)
GS1(1,6)=G4(XIM, XL)
GS1(1,8)=G5(XIM, XL)
GS1(1,12)=G6(XIM, XL)
GS1(2,3)=G3(XIM, XL)
GS1(2,5)=G7(XIM, XL)
END IF
GS1(2,9) = GS5(X1MJ, XL)
GS1(2,11) = GS8(X1MJ, XL)

DO IU = 1, 2
UMJ1(IU) = 0.
UMJ2(IU) = 0.
END DO

DO IR = 1, 2
DO IC = 1, 12
UMJ1(IR) = GS1(IR, IC) * TZ11(IC) + UMJ1(IR)
UMJ2(IR) = GS1(IR, IC) * TZ12(IC) + UMJ2(IR)
END DO
END DO

ELSEIF (XYZ(3J) .EQ. XMUD .AND. XYZ(6J) .LT. XMUD) THEN
DO IR = 1, 2
DO IC = 1, 10
GS2(IR, IC) = 0.
END DO
END DO

GS2(I, 6) = GS3(X1MJ, XL)
GS2(I, 7) = GS5(X1MJ, XL)
GS2(I, 9) = GS8(X1MJ, XL)
GS2(2, 3) = GS3(X1MJ, XL)
GS2(2, 5) = GS7(X1MJ, XL)
GS2(2, 8) = GS8(X1MJ, XL)

DO IU = 1, 2
UMJ1(IU) = 0.
UMJ2(IU) = 0.
END DO

DO IR = 1, 2
DO IC = 1, 10
UMJ1(IR) = GS2(IR, IC) * TZ21(IC) + UMJ1(IR)
UMJ2(IR) = GS2(IR, IC) * TZ22(IC) + UMJ2(IR)
END DO
END DO

ELSEIF (XYZ(3J) .LT. XMUD .AND. XYZ(6J) .LT. XMUD) THEN
DO IU = 1, 2
UMJ1(IU) = 0.
UMJ2(IU) = 0.
END DO

DO IR = 1, 2
DO IC = 1, 8
GS3(IR, IC) = 0.
END DO
END DO

GS3(1, 1) = GS3(X1MJ, XL)
GS3(1, 4) = GS4(X1MJ, XL)
GS3(1, 5) = GS5(X1MJ, XL)
GS3(1, 8) = GS6(X1MJ, XL)
GS3(2, 2) = GS3(X1MJ, XL)
GS3(2, 3) = GS7(X1MJ, XL)
GS3(2, 6) = GS5(X1MJ, XL)
GS3(2, 7) = GS8(X1MJ, XL)

DO IR = 1, 2
DO IC = 1, 8
UMJ1(IR) = GS3(IR, IC) * TZ31(IC) + UMJ1(IR)
UMJ2(IR) = GS3(IR, IC) * TZ32(IC) + UMJ2(IR)
END DO
END DO

ELSE
CONTINUE
END IF

B11 = B1(IJS)
B12 = B2(IJS)
B13 = B3(IJS)
B14 = B4(IS)

H1 = B11 * E1W * (E1W * B11 + E3W * B13)
H2 = B12 * E1W * (E1W * B12 + E3W * B14)
H3 = E2W * (E1W * B11 + E3W * B13)
H4 = E2W * (E1W * B12 + E3W * B14)
H5 = B13 * E3W * (E1W * B11 + E3W * B13)
H6 = B14 * E3W * (E1W * B12 + E3W * B14)

H7 = TWM(2,1) * H1 + TWM(2,2) * H3 + TWM(2,3) * H5
H8 = TWM(2,1) * H2 + TWM(2,2) * H4 + TWM(2,3) * H6
H9 = TWM(3,1) * H1 + TWM(3,2) * H3 + TWM(3,3) * H5
H10 = TWM(3,1) * H2 + TWM(3,2) * H4 + TWM(3,3) * H6

H11 = UMJ2(1) * WF
H12 = UMJ1(1) * WF
H13 = WF * UMJ2(2)
H14 = WF * UMJ1(2)

H15 = H7 - H11
H16 = H8 - H12
H17 = H9 - H13
H18 = H10 - H14

C CALCULATE NEW SIGR22, SIGR33, SIGR23
IF (FR EQ. 1 OR FR EQ. WLST) THEN
  RULL = 1.
  IIR = 2.
ELSE IF (IIR EQ. 2) THEN
  RULL = 4.
  IIR = 1.
ELSE
  RULL = 2.
  IIR = 2.
END IF

C DO IP = 1, WLST
DI = IP
STEP = DI * STEP
IF (STEP < GE. WF) GO TO 101
END DO

101 CONTINUE
DIFF = WF - STEP *(DI - 1.)
DIFSPC = SPECTR(IP) - SPECTR(IP - 1)
SPE = SPECTR(IFR + 1)
IF (IP EQ. 1) SPE = 0.
IF (FR GT. 200) SPE = 0.

C SIGR22(ICNT) = (H15 ** 2. + H16 ** 2.) * SPE * STEP * RULL / 3. +
1 SIGR22(ICNT)
SIGR33(ICNT) = (H17 ** 2. + H18 ** 2.) * SPE * STEP * RULL / 3. +
1 SIGR33(ICNT)
SIGR23(ICNT) = (H15 * H17 + H16 * H18) * SPE * RULL * STEP / 3. +
1 SIGR23(ICNT)
END DO !LOOP OF SIMPS POINTS

C ELSE
DO IST = 1, ISPT
ICNT = ICNT + 1
SIGR22(ICNT) = 0.
SIGR33(ICNT) = 0.
SIGR23(ICNT) = 0.
END DO
ENDIF
ELSE
DO IST = 1, ISPT
ICNT = ICNT + 1
SIGR22(ICNT) = 0.
SIGR33(ICNT) = 0.
SIGR23(ICNT) = 0.
END DO
ENDIF
END DO !LOOP OF ELEMENTS
C COMPUTE SIGMZZ(IJ)
    DO IRT=1,NROOT
    DO JRT=1,NROOT
        SIGMZZ(IRT,JRT)=(WF**2.)*(Z1(IRT,IFR)*Z1(JRT,IFR)+
                     Z2(IRT,IFR)*Z2(JRT,IFR))*STEP*RULL*SPE/3.+
                     SIGMZZ(IRT,JRT)  \ MH CHANGED Z1 Z2
    END DO
    END DO
C END DO LOOP OVER FREQUENCIES
    END DO
C CALL CBARJ(CDBARJ,NUME,NC,ALPHA,ISP,T,SIGMZZ,
            A(N13),NNC,CAP,DTOT,MATP,ISPNUM,SIGR22,
            SIGR33,SIGR23)
    RETURN
    END
SUBROUTINE RESULTS(PHI)

INCLUDE 'SPEC.CMN'
DIMENSION PHI(NEQ,NROOT), Ul(neq),U2(neq),IDF(10),SIGR22(10)

CHARACTER *1 EE(IO)
CHARACTER NAME*10,DD*1
DATA EE/'r,'2','3','4' ,' 5',' 6' .7 ' ,'8',' 9' ,'0V

OPEN(30,FILE='DOF.DAT',STATUS='OLD')
READ(30,100)NDF,NTFLG
100 FORMAT(2I10)
DF=NDF
DIV=DF/8.
IDL=AINT{DIV)+1
DO IR=1,IDL
IRM1=(IR-1)*8
READ(30,200)(IDF(K+IRM1),K=1,8)
END DO

CLOSE(30)
DO IK=1,NDF
DD=EE(IK)
NAME='SPE'//DD/AOUT
INUM1=65+IK
OPEN(INUM1 ,FILE=NAME,STATUS='UNKNOWN')
INUM2=85+IK
NAME=TRN'//DD//' .OUT'
OPEN(INUM2,FILE=NAME,STATUS='UNKNOWN')
WRITE(INUM2,*)THE TRANSFER FUNCTION FOR DEGREE OF FREEDOM:'
WRITE(INUM2,305)roF(IK)
WRITE(INUM2,*)' Iteration number = 'JTERFLG
WRITE(INUM2,*)'  Ratio = '^TIO(ITERFLG)
IF(NTFLG.EQ.1)THEN
WRITE(INUM2,*)' ALPHA w(Hz)'
ELSE
WRITE(INUM2,*)' ALPHA w(rad/s)'
ENDIF
END DO

DO 202 FFF=1,10
SIGR22(FFF)=0.
202 CONTINUE

WLST=200
ICOUN=0
DO 301 F=1,WLST

IF(F.EQ.1.0.OR.F.EQ.WLST)THEN
RULL=1.
ELSE IF(IIR.EQ.2)THEN
RULL=4.
ELSE
RULL=2.
ENDIF
ENDIF

IFR=F
WF=F*STEP
DO IC=1,NROOT
ICOUN=ICOUN+1
END DO

DO 203 NN=1,NEQ
U1(NN)=0.
U2(NN)=0.
203 CONTINUE

DO N=1,NEQ
DO I=1,NROOT

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U1(N) = PHI(N,I)*Z1(I,IFR)+U1(N)
U2(N) = PHI(N,I)*Z2(I,IFR)+U2(N)
END DO
END DO

C DO IP=1,WLST
DI=IP
STEP=DI*STEP
IF(STEP.LT.WF)GO TO 101
END DO
101 CONTINUE
DIFF=WF-STEP*(DI-1.)
DIFSPC=SPECTR(IP)-SPECTR(IP-1)
SPE=SPECTR(IFR+1)
IF(IP.EQ.1)SPE=0.
IF(F.GT.200)SPE=0.
C
DO IP=1,NDF
NI=IDF(IP)
NUM1=65+IOP
NUM2=85+IOP
SP1=SPE*(U1(NI))**2.+(U2(NI))**2.
SP2=(U1(NI))**2.-(U2(NI))**2.
C
This is to output data in Hz(not RADS) and (mm2/Hz)
(not m2s/rad), depending on the flag NTFLG in DOF.DAT
C
PI=3.1415927
AMIL=1000000
SP1H=2*PI*AMIL*SP1
WFH=WF/(2*PI)
C IF(NTFLG.EQ.1)THEN
WRITE(NUM1,300)SP1H,WFH
ELSE
WRITE(NUM1,300)SP1,WF
END IF
C IF(RATIO(ITERFLG).LT.0.03.OR.ITERFLG.EQ.ITEFP1)THEN
IF(NTFLG.EQ.1)THEN
WRITE(NUM2,301)SP2,WFH
ELSE
WRITE(NUM2,311)SP2,WF,SPE
END IF
311 FORMAT(F20.17,F20.5,F20.17)
300 FORMAT(F20.17,F20.5)
SIGR22(IOP)=SP1*RULL*STEP/3.+SIGR22(IOP)
WLSTAB=ABS(WLST)
IF(F.EQ.WLSTAB)THEN
SG22RT=SQRT(SIGR22(IOP))
WRITE(NUM1,*)' SIGR22 = ',SG22RT
ELSE
CONTINUE
END IF
END DO
I0EFP1=+ITEFLG+1
PRINT*, 'CONVERGENCE REACHED'
OPEN(83,FILE='RATIO.OUT',STATUS='OLD')
DO 302 RAT=1,ITERFLG
WRITE(83,303)RATIO(RAT)
302 CONTINUE
CLOSE(83)
303 FORMAT(F10.4)
304 FORMAT(I5)
305 FORMAT(/10XJ[5/)
PRINT*, 'PROGRAM FINISHED'
CALL EXIT
RETURN
END
SUBROUTINE ALPHAU(XYZ,C,LM,DTOT)

THIS PROGRAM CALCULATES:

- VARIANCES OF WATER PARTICLE VELOCITIES
- ELEMENT HYDRODYNAMIC DAMPING MATRIX
- GLOBAL HYDRODYNAMIC DAMPING MATRIX

INCLUDE 'SPEC.CMN'

DIMENSION XW(6),XYZ(6,1),DTOT(150),Q(12,12),CA(12,12),TR(12,12), ICDD(144),GG1(12,12),C(NWK),LM(12,1)

CALL ZERO(C)

SPT=ISPT
JCOUNT=0
PIE=SQRT(8/PI)
ND=12

NOTE: FOR NONLINEAR CASE ALPHA IS 0 - SAME COORDINATE SYSTEM FOR WAVE AND STRUCTURE.

INITIALISE DAMPING MATRIX TO ZERO
DO ID1=1,NWK
C(ID1)=0.0
END DO

LOOP OVER ALL ELEMENTS AND COMPUTE FOR HORIZONTAL MEMBERS (IE MEMBERS WITH IELFLG(N)=1)

D=-XMUD
DO 100 N=1,NUME
IFLAG=IELFLG(N)
IF(IFLAG.EQ.1)THEN ! note only calculate hydrodynamic damping for submerged vertical members.
INK=INK+1
XL=0.
DO IP=1,3
XL=XYZ(IP,N)-XYZ(IP+3,N)
END DO
XL2=XL2+XL*XL
XLN=SQRT(XL2)
SPTM1=SPT-1.
DEL=XLN/SPTM1
DELX=0.0
EXPRESS THE NODAL CO-ORDS IN TERM OF THE WAVE CO-ORD SYSTEM.
XW(1)=XYZ(1,N)
XW(2)=XYZ(2,N)
XW(3)=XYZ(3,N)
XW(4)=XYZ(4,N)
XW(5)=XYZ(5,N)
XW(6)=XYZ(6,N)

DO 310 IP=1,JSPT ! NO. OF SIMPS POINTS
CH=0
IEW=IEW+1
SIP=IP
WL=1000.
JCOUNT=JCOUNT+1
DELX=(SIP-1)*DEL
XW1=XW(1)
IF(XW(3).GT.0)XW(3)=-XW(3)
XW3=XW(3)-DELX

INITIALISE SIGVH, THE INTEGRATION PARAMETER FOR STANDARD DEVIATION OF HORIZONTAL WATER PARTICLE VELOCITY FOR SIMPSON'S POINT j
SIGVH=0.0

C LOOP OVER ALL FREQUENCIES OF INTEREST
DO 320 IXN=1,NFRQ
XNF=IXN
DNF=XNF
C
WF=DELTAW*DNF
C
TW=2.3*14.15927/WF !WAVE PERIOD (HERTZ). FREQUENCY (RADIANS)
DWL=WF/WL
IF(DWL.GT.0.5)THEN
C DEEP WATER
WL=9.81*(TW**2)/(2*PI)
DK=2*3.1415927/WL
BI=2*PI*(EXP(DK*XW3))*COS(DK*XW1)/TW
B2=2*PI*(EXP(DK*XW3))*SIN(DK*XW1)/TW
ELSE
DK=waveno(WF,D)
WL=2.*3.1415927/DK
B1=BB1(TW,DK,XW3,D,XW1,WL)
B2=BB2(TW,DK,XW3,D,XW1,WL)
END IF
C
C COMPUTE APB
APB=BI**2 + B2**2
C
IF(XNF.EQ.1.0.OR.XNF.EQ.NFRQ)THEN
RULL=1.
IIR=2
ELSE IF(IIR.EQ.2)THEN
RULL=4.
IIR=1
ELSE
RULL=2.
IIR=2
END IF
C
C CALCULATE THE SPECTRUM ORDINATE FOR THE PARTICULAR
C FREQUENCY
SPE=SPECTR(IXN)
IF(IP.EQ.1)SPE=0.0
SIGVH=APB*SPE*NULL*DELTAW/3 + SIGVH
320 CONTINUE
C
C END OF FREQUENCY LOOP
IF(IP.EQ.1.OR.IP.EQ.ISPT)THEN
RULL1=1.
IIR1=2
ELSE IF(IIR1.EQ.2)THEN
RULL1=4.
IIR1=1
ELSE
RULL1=2.
IIR1=2
END IF
C
C COMPUTE HYDRODYNAMIC DAMPING COEFFICIENT FOR EACH ELEMENT, CH(I)
CH=SIGVH*RULL1*DEL/3
C
C COMPUTE REMAINING TERMS IN ELEMENT INTEGRAL
RIL=IP-1
XIMJ=RIL*DEL
KD=WATDEN*CD*DTOT(N)*CH
GG1(2,2)=G3(XIMJ,XL)*G3(XIMJ,XL)*KD*PIE + GG1(2,2)
GG1(2,6)=G3(XIMJ,XL)*G4(XIMJ,XL)*KD*PIE + GG1(2,6)
GG1(2,8)=G3(XIMJ,XL)*G5(XIMJ,XL)*KD*PIE + GG1(2,8)
GG1(2,12)=G3(XIMJ,XL)*G6(XIMJ,XL)*KD*PIE + GG1(2,12)
GG1(6,2)=G4(XIMJ,XL)*G3(XIMJ,XL)*KD*PIE + GG1(6,2)
GG1(6,6)=G4(XIMJ,XL)*G4(XIMJ,XL)*KD*PIE + GG1(6,6)
A100
GG1(6,8) = G4(X1MJ, XL) * G5(X1MJ, XL) * KD * PIE + GG1(6, 8)
GG1(6,12) = G4(X1MJ, XL) * G6(X1MJ, XL) * KD * PIE + GG1(6, 12)
GG1(8,6) = G3(X1Mr, XL) * G5(X1MJ, XL) * KD * PIE + GG1(8, 6)
GG1(8,8) = G5(X1MJ, XL) * G5(X1MJ, XL) * KD * PIE + GG1(8, 8)
GG1(8,12) = G5(X1MJ, XL) * G6(X1MJ, XL) * KD * PIE + GG1(8, 12)
GG1(12,2) = G3(X1MJ, XL) * G6(X1MJ, XL) * KD * PIE + GG1(12, 2)
GG1(12,6) = G6(X1MJ, XL) * G4(X1MJ, XL) * KD * PIE + GG1(12, 6)
GG1(12,8) = G6(X1MJ, XL) * G5(X1MJ, XL) * KD * PIE + GG1(12, 8)
GG1(12,12) = G6(X1MJ, XL) * G6(X1MJ, XL) * KD * PIE + GG1(12, 12)

C
310 CONTINUE
C
C END OF SIMPSONS LOOP
C
C NEXT WE MUST PRE- AND POST- MULTIPLY
C BY THE TRANSFORMATION MATRICES TO
C CONVERT TO GLOBAL CO-ORDS

NN=0
DO 150 IP=1,12
DO 150 JP=1,12
Q(IP,JP)=0.0
CA(IP,JP)=0.0
TR(IP,JP)=T(IP,JP,N)
150 CONTINUE
DO 160 IT=1,12
DO 160 JT=1,12
Q(IT,JT)=GG1(IT,IR)*T(IR,JT,N)+Q(IT, JT)
160 CONTINUE
DO 170 IA=1,12
DO 170 IB=1,12
DO 170 IC=1,12
CA0IA, IB)= TR(IA,IC)*Q(IC, IB)+CA0IA, IB)
170 CONTINUE
K=0
DO 180 LF=1,12
DO 180 LK= LF, 12
K=K+1
CDD(K)=CA(LF, LK)
180 CONTINUE
C
C NOTE: HERE WE MUST ENSURE TO ADD THE
C ADDED DAMPNESS TO THE CORRECT
C DAMPNESS ELEMENT
C
ELSE
K=0
DO 185 LF=1,12
DO 185 LK= LF, 12
K=K+1
CDD(K)=0.0
185 CONTINUE
C
C END IF
C
C NOW ASSEMBLE CA(I,J)

CALL ASSDAM(C(1), IA(IN5), CDD, LM(1,N), ND)

C
100 CONTINUE
C
C END OF ELEMENT LOOP
CALL ZERO(C)
RETURN
END

C
FUNCTION WAVENO(WF,D)
C
C THIS CALCULATES THE WAVE NUMBER USING THE DISPERSION RELATIONSHIP
C AND CAN BE USED TO CALCULATE THE WAVELENGTH FOR A FREQUENCY
C INITIALISE DK
WNO=0.6
TOL=1.E-3
ITMAX=40
DO I=1,ITMAX
FK=9.81*WNO*TANH(WNO*D)-WF**2.
FKD=9.81*TANH(WNO*D)+9.81*WNO*D*(COSH(WNO*D))**(-2)
FACT=FK/FKD
WN1=WNO*FACT
C C CHECK FOR CONVERGENCE
CHECK=FACT/WN1
IF(ABS(CHECK),LT,TOL)GO TO 100
WNO=WN1
END DO
100 CONTINUE
WAVENO=WN1
RETURN
END
C FUNCTION BB1(TW,K,D,XW3,D,XW1,WL)
BB1=9.81*TW*COSH(DK*(XW3+D))*COS(DK*XW1)/(WL*COSH(DK*D))
RETURN
ENTRY BB2(TW,D,XW3,D,XW1,WL)
BB2=9.81*TW*COSH(DK*(XW3+D))*SIN(DK*XW1)/(WL*COSH(DK*D))
RETURN
ENTRY BB3(TW,D,XW3,D,XW1,WL)
BB3=9.81*TW*SINH(DK*(XW3+D))*SIN(DK*XW1)/(WL*COSH(DK*D))
RETURN
ENTRY BB4(TW,D,XW3,D,XW1,WL)
BB4=9.81*TW*SINH(DK*(XW3+D))*COS(DK*XW1)/(WL*COSH(DK*D))
RETURN
END
C C
FUNCTION G1(X1M,XL)
G1=1.-X1M/XL
RETURN
C ENTRY G2(1M,XL)
G2=X1M/XL
RETURN
C ENTRY G3(X1M,XL)
G3=1.-3.*(X1M/XL)**2.+2.*(X1M/XL)**3.
RETURN
C ENTRY G4(X1M,XL)
G4=X1M-2.*(X1M**2.)/XL+(X1M**3.)/(XL**2.)
RETURN
C ENTRY G5(X1M,XL)
G5=3.*(X1M/XL)**2.-2.*(X1M/XL)**3.
RETURN
C ENTRY G6(X1M,XL)
G6=-X1M+2.*(X1M**2.)/XL-X1M**3./XL**2.)
RETURN
C ENTRY G7(X1M,XL)
G7=X1M+2.*(X1M**2.)/XL-X1M**3./XL**2.)
RETURN
C ENTRY G8(X1M,XL)
G8=(X1M**2.)/XL-(X1M**3.)/(XL**2.)
RETURN
END
SUBROUTINE ASSDAM (AX,MAXA,S,LM,ND)

C ....................................................................
C . PROGRAM
C . ASSEMBLES UPPER TRIANGULAR ELEMENT STIFFNESS TO COMPACTED
C . GLOBAL STIFFNESS
C
INCLUDE 'SPEC.CMN'
DIMENSION S(144),AX(NWK),MAXA(1),LM(1)

C
NDI=0
DO 200 I=1,ND
II=LM(I)
IF (II) 200,200,100

100 MI=MAXA(II)
   KS=I
   DO 220 J=1,ND
      JJ=LM(J)
      IF (JJ) 220,220,110
   110 IJ=II-JJ
      IF (IJ) 220,210,210
   210 KK=MI+IJ
      KSS=KS
      IF (IJ.GE.I) KSS=J+NDI
      AX(KK)=AX(KK)+S(KSS)
   220 KS=KS+ND-J
200 NDI=NDI+ND-I
   II=LM(IR)
   IF(II.EQ.0) GO TO 202
   MI=MAXA(II)
202 CONTINUE
300 CONTINUE
RETURN
END
SUBROUTINE CGEN(SIGMZZ,C,EIGV,RJD,Z,MATP)

C

C .............................................................................................
C THIS PROGRAM CALCULATES THE GENERALISED DAMPING MATRIX (CTOT)
C FROM BOTH THE STRUCTURAL DAMPING & HYDRODYNAMIC DAMPING
C READ IN SIGMZZ FROM THE LINEAR ANALYSIS
C .............................................................................................

INCLUDE 'SPEC.CMN'

DIMENSION SIGMZZ(NROOT,NROOT),C(NWK),PCP(NROOT,NROOT)
C ...............................................................
1,CTOT(NROOT),EIGV(NROOT),CDIAGG(NROOT),
1CHELM(12,12),CHELG(12,12),CHGT(600,600),CHG(NEQ,NEQ),TGM1(3,3),
3TGM(12,12),TGMT(12,12),TEMP(12,12),R(NEQ,NROOT),TEMP2(NEQ,NROOT),
4RTRAN(NROOT,NEQ),SIGMU(NUMNP),ID(6,NUMNP),XYZ(6,NUME),MATP(600),
4SMZZ(NROOT,NROOT)

C

C THIS ROUTINE FIRST COMPUTES THE GLOBAL HYDRODYNAMIC DAMPING MATRIX
C

CD=1.0/2.0
WATDEN=10000  IWATER DENSITY
SQRT8=SQRT((8/PI))
OPEN(10,FILE='TD.DAT',STATUS='OLD')
DO INUM=1,NUMNP
READ(10,1110)X ID(JNUM;DNUM)JNUM=1,6)
END DO
1110 FORMAT (615)

C

C MUST READ IN SIGMZZ'S FROM LINEAR ANALYSIS
PRINT*, NUMNP='NUMNP
WRITE(IOUT,'*') DETAILS OF ID ARRAY'
DO IE2=1,NUMNP
WRITE(IOUT,1110) ID(JNUM,IE2)JNUM=1,6)
END DO

OPEN(41,FILE='SIGMZ.DAT',STATUS='OLD')
DO IN1=1,NROOT
READ(41,1000)SMZZ(IN1,IN2),IN2=1,NROOT)
END DO
1000 FORMAT(2F10.3)

C

C MUST READ IN SIGMU'S FROM LINEAR ANALYSIS
C
OPEN(21,FILE='SIGMU.DAT',STATUS='OLD')
DO IN3=1,NUMNP
READ(21,1100)SIGMU(IN3)
END DO
1100 FORMAT(F15.3)

C

C INITIALISE MATRICES
DO IP1=1,3
DO IP12=1,3
TGM1(IP1,IP12)=0.0
END DO
END DO
DO IP1=1,12
DO IP2=1,12
CHELM(IP1,IP2)=0.0
CHELG(IP1,IP2)=0.0
TGM(IP1,IP2)=0.0
TGMT(IP1,IP2)=0.0
END DO
END DO
DO IP3=1,600
DO IP4=1,600
CHGT(IP3,IP4)=0.0
END DO
END DO
DO IP5=1,NEQ
DO IP6=1,NEQ
CHQ(IP5(IP6)=0.0

A104
C DEFINE TRANSFORMATION MATRIX - CONSTANT FOR ALL MEMBERS
TGM1(1,3)=-1
TGM1(2,2)=1
TGM1(1,3)=1
DO IP7=1,3
DO IP8=1,3
TGM(IP7,IP8)=TGM1(IP7,IP8)
END DO
END DO
TGM(IP7,IP8)=TGM1(IP7,IP8)
END DO
END DO
DO IP9=1,12
DO IP10=1,12
TGMT(IP9,IP10)=TGMT(IP10,IP9)
END DO
END DO
C LOOP OVER ALL ELEMENTS
DO I1=1,JNUME
MTYPE=MATP(I1)
IEL=INUME(I1) !NODE NO. FOR NODE 1 OF ELEMENT
JEL=JNUME(I1) !NODE NO. FOR NODE 2 OF ELEMENT
C CHECK IF MEMBER IS HORIZONTAL & IF SO IGNORE IT
IF(IELFLG(I1).EQ.1)THEN
C CHECK IF ELEMENT IS IN WATER
IF(XYZ(3,I1).LT.0.0.OR.XYZ(6,I1).LT.0.0)THEN
C CHECK IF ELEMENT IS A FOUNDATION MEMBER
IF(XYZ(3,I1).NE.XMUD.AND.XYZ(6,I1).NE.XMUD)THEN
C CALCULATE ELEMENT LENGTH XL
XL2=0.0
DO II=1,3
DX=XYZ(II+1,1)-XYZ(II+4,1)
XL2=XL2+DX**2.
END DO
XL=SQRT(XL2)
C COMPUTE DRAG FORCE COEFFICIENT
SIGU=(SIGMU(IEL)+SIGMU(JEL))/2
DRAG=WATDEN*CD*SQRT8*SIGU
C DEFINE ENTRIES IN LOCAL DAMPING MATRIX
CHELM(2,2)=13*XL2/35
CHELM(2,6)=58*XL2/105
CHELM(2,8)=XL2/10
CHELM(2,12)=13*XL2/420
CHELM(6,2)=58*XL2/105
CHELM(6,6)=2*XL2/105
CHELM(6,8)=13*XL2/420
CHELM(6,12)=XL2*XL2/140
CHELM(8,2)=XL2/10
CHELM(8,6)=13*XL2/420
CHELM(8,8)=12*XL2/35
CHELM(8,12)=11*XL2/210
CHELM(12,2)=13*XL2/420
CHELM(12,6)=XL2*XL2/140
CHELM(12,8)=11*XL2/210
DO IC1=1,12
DO IC2=1,12
CHELM(IC1,IC2)=CHELM(IC1,IC2)*DRAG
END DO
END DO
C EXPRESS HYDRODYNDAMIC DAMPING MATRIX IN GLOBAL COORDINATES
  TEMP=MATMUL(CHELM,TGM)
  CHELG=MATMUL(TGMT,TEMP)
C
C DEFINE GLOBAL STRUCTURE HYDRODYNAMIC DAMPING MATRIX
  J=6*(IEL-1)+1
  K=6*(JEL-1)+1
  JP5=J+5
  KP5=K+5
C
  JC1=0
  JC2=0
  DO J1=1,JP5
    JC1=JC1+1
    DO J2=1,JP5
      JC2=JC2+1
      CHGT(J1,J2)=CHELG(JC1,JC2)+CHGT(J1,J2)
      END DO
    END DO
  END DO
  JC1=0
  JC2=0
  DO K1=1,KP5
    JC1=JC1+7
    DO K2=1,KP5
      JC2=JC2+7
      CHGT(K1,K2)=CHELG(JC1,JC2)+CHGT(K1,K2)
      END DO
    END DO
  END DO
  JC1=0
  JC2=0
  DO J1=1,JP5
    JC1=JC1+1
    DO K2=1,KP5
      JC2=JC2+7
      CHGT(J1,K2)=CHELG(JC1,JC2)+CHGT(J1,K2)
      END DO
    END DO
  END DO
C
  CONTINUE
C
  ELSE     !MEMBER IS HORIZONTAL
  CONTINUE
C
  ELSE     !MEMBER IS IN FOUNDATION
  CONTINUE
C
  END IF
C
C END ELEMENT LOOP
C
  END DO
C
C DELETE FIXED ROWS AND COLUMNS OF DAMPING MATRIX
  ICOUNT=0
  DO IDI=1,NUMNP
    END
DO ID2=1,6

IF(ID(ID2,ID1),NE.,1) THEN
  CONTINUE
ELSE
  ICOUNT=ICOUNT+1
  JCOUNT=0
  DO ID3=1,NUMNP
    DO ID4=1,6
      IF(ID(ID4,ID3),EQ.,1) THEN
        CONTINUE
      ELSE
        JCOLINT=JCOUNT+1
        CHG(ICOUNT,JCOUNT)=CHGT(ID1,ID2)
        END IF
      END DO
    END DO
  END IF
END DO
END DO
END DO

DEFINE TRANSPOSE OF R
DO IR1=1,NEQ
  DO IR2=1,NROOT
    RTRAN(IR2,IR1)=R(IR1,IR2)
  END DO
END DO

TEMP2=MATMUL(CHG,R)
PCP=MATMUL(RTRAN,TEMP2)

COMPUTE EQUIVALENT DIAGONALISED HYDRODYNAMIC DAMPING
DO IZ1=1,NROOT
  DO IZ2=1,NROOT
    CDIAGG(IP1)=PCP(IP1,IZ2)*SMZZ(IP1,IZ2)/SMZZ(IP1,IP1)
    CDIAGG(IP1)=CDIAGG(IP1)
  END DO
END DO

COMPUTE MODAL STRUCTURAL DAMPING FOR MODE i (MSDI)
WHERE ETA IS MODAL DAMPING RATIO
DO KV2=1,NROOT
  RTEIG=SQRT(EIGV(KV2))
  RMSDI=2*ETA*RTEIG
END DO

TOTAL DECOUPLED DAMPING TERMS ARE
CTOT(KV2)=RMSDI+CDIAGG(KV2)
END DO

CALL RGEN(A(N16),A(N17),CTOT)
RETURN
END
SUBROUTINE RGEN(R,EIGV,CTOT)
C ...................................................................
C . THIS PROGRAM CALCULATES THE RECEPTANCES OF THE GENERALISED & ACTUAL
C . COORDINATES
C ...................................................................
C INCLUDE 'SPEC.CMN'
DIMENSION R(NEQ,NROOT),EIGV(NROOT),CTOT(NROOT),
  ZZ1(NROOT,NEQ,NFRQ),ZZ2(NROOT,NEQ,NFRQ),ZTEMP1(NROOT),
  ZTEMP2(NROOT),HTEMP(NEQ),H2TEMP(NEQ),REC(NEQ,NFRQ)
DIMENSION ZZA1(NROOT,NEQ,NFRQ),ZZA2(NROOT,NEQ,NFRQ),TEMPA1(NROOT),
  TEMP(NEQ),A1TEMP(NEQ),A2TEMP(NEQ)
OPEN(20,FILE='RECP.OUT,STATUS='UNKNOWN')
C LOOP OVER ALL FREQUENCIES
DO IFRQ=1,NFRQ
  FREQ=DELTAW*FRQ1
  FREQ2=FREQ**2
  C PUT A UNIT LOAD AT ALL DEGREES OF FREEDOM IN TURN,
  ie LOOP OVER ALL DOF'S I=1,NEQ
  DO IRT1=1,NEQ
    C COMPUTE RECEPTANCES AT ALL DOF'S DUE TO LOAD AT IRT1
    DO IRT2=1,NROOT
      W2=EIGV(IRT2)
      DENOM=(W2-FREQ2)**2 + FREQ2*CTOT(IRT2)*CTOT(IRT2)
      ZZ1(IRT2,IRT1,IFRQ)=(R(IRT1,IRT2)*(W2-FREQ2))/DENOM
      ZZ2(IRT2,IRT1,IFRQ)=-R(IRT1,IRT2)*FREQ*CTOT(IRT2)/DENOM
      ZZA1(IRT2,IRT1,IFRQ)=(R(IRT1,IRT2)*(W2-FREQ2))/DENOM2
      ZZA2(IRT2,IRT1,IFRQ)=-R(IRT1,IRT2)*FREQ*CTOT(IRT2)/DENOM2
    END DO
  END DO
C NEXT COMPUTE ACTUAL RECEPTANCES
C DO I1=1,NEQ   !LOOP FOR EACH APPLIED LOAD
C DO I2=1,NFRQ  !LOOP OVER ALL FREQUENCIES
C DO I3=1,NROOT !LOOP OVER ALL EIGENVECTORS
  ZTEMP1(I3)=ZZ1(I3,I1,I2)
  ZTEMP2(I3)=ZZ2(I3,I1,I2)
  TEMPA1(I3)=ZZA1(I3,I1,I2)
  TEMPA2(I3)=ZZA2(I3,I1,I2)
END DO
H1TEMP=MATMUL(R,ZTEMP1)
H2TEMP=MATMUL(R,ZTEMP2)
A1TEMP=MATMUL(R,TEMPA1)
A2TEMP=MATMUL(R,TEMPA2)
C PUT VALUES FROM TEMPORARY VECTORS INTO MATRICES OF RECEPTANCES
DO I4=1,NEQ
  H1(I4,I2)=H1TEMP(I4)  !REAL COMPONENT OF RECEPTANCE
  H2(I4,I2)=H2TEMP(I4)  !IMAGINARY COMPONENT OF RECEPTANCE
  HA1(I4,I1,I2)=A1TEMP(I4) !REAL COMPONENT OF RECEPTANCE
  HA2(I4,I1,I2)=A2TEMP(I4) !IMAGINARY COMPONENT OF RECEPTANCE
END DO
C H1(I4,I1,I2) REPRESENTS THE REAL RECEPTANCE AT DOF I4 DUE TO
C AN APPLIED LOAD AT DOF I1 AT FOR AN APPLIED LOAD FREQUENCY OF I2
END DO  !FREQUENCY LOOP
C END DO !APPLIED LOAD LOOP
C PRINT OUT SOME RECEPTANCE FUNCTIONS
DO 15=1,NFRQ
  REC(1,15)=0.0
  REC(4,15)=0.0
  REC(5,15)=0.0
  REC(6,15)=0.0
  REC(7,15)=0.0
  REC(8,15)=0.0
  REC(9,15)=0.0
DO 1=1,NEQ
  DO J2=1,NEQ
      REC(1,15)=(H1(1,J1,15)*H1(1,J2,15)+H2(1,J1,15)*H2(1,J2,15))
      1REC(1,15)
      REC(4,15)=(H1(4,J1,15)*H1(4,J2,15)+H2(4,J1,15)*H2(4,J2,15))
      1REC(4,15)
      REC(7,15)=(H1(7,J1,15)*H1(7,J2,15)+H2(7,J1,15)*H2(7,J2,15))
      1REC(7,15)
      REC(10,15)=(H1(10,J1,15)*H1(10,J2,15)+H2(10,J1,15)*H2(10,J2,15))
      1REC(10,15)
  C REC(9,15)=(H1(9,J1,15)*H1(9,J2,15)+H2(9,J1,15)*H2(9,J2,15))
  C REC(9,15)
  END DO
  END DO
  REC(1,15)=(H1(1,J1,15)*H1(1,J2,15)+H2(1,J1,15)*H2(1,J2,15))
  1REC(1,15)
WRITE(20,100)REC(1,15),REC(4,15),REC(10,15)
END DO
CLOSE(20)

CALL FORCE(IA(1),A(N13),A(N8),A(N12),IA(IN4))
RETURN
100 FORMAT(6E10.3)
END
SUBROUTINE FORCE(ID,XYZ,ATOT,DTOT,MATP)
C
C SUBROUTINE WR TO COMPUTE THE COEFFICIENTS OF
C THE DRAG AND INERTIA TERMS OF THE FORCE VECTOR
C
INCLUDE 'SPEC.CMN'
DIMENSION XYZ(6,NUME),DTOT(NUME),ATOT(NUME),CFMM(12),
1 CFDM(12),MATP(NUME),ID(6,NUMNP)
C
C SET INERTIA AND DRAG COEFFICIENTS
CM1=1.0
CM2=1.0
CMT=CM1+CM2 !TOTAL INERTIA COEFFICIENT
CD=1.0/2.0
WATDEN=1000 !WATER DENSITY
C
C INITIALISE MATRICES
DO IP2=1,12
CFMM(IP2)=0.0
CFDM(IP2)=0.0
END DO
C
C INITIALISE CFMGT & CFDGT
DO K1=1,NEQ
CFMGT(K1)=0.0
CFDGT(K1)=0.0
END DO
C
C LOOP OVER ALL ELEMENTS
DO II=1,NUME
M TYPE=MATP(II)
IEL=NUME(II) !NODE NO.FOR NODE 1 OF ELEMENT
JEL=JNUME(II) !NODE NO.FOR NODE 2 OF ELEMENT
C
C CHECK IF MEMEBER IS HORIZONTAL & IF SO IGNORE IT
IF(IELFLG(II).EQ.1)THEN
C
C CHECK IF ELEMENT IS IN WATER
IF(XYZ(3,II).LT.0.0.OR.XYZ(6,II).LT.0.0)THEN
C
C CHECK IF ELEMENT IS A FOUNDATION MEMBER
IF(XYZ(3,II).NE.XMUD.AND.XYZ(6,II).NE.XMUD)THEN
C
C CALCULATE ELEMENT LENGTH XL
XL2=0.0
DO II=1,3
DX=XYZ(II,II)-XYZ(II+3,II)
XL2=XL2+DX**2.
END DO
XL=SQRT(XL2)
C
C INITIALISE COEFFICIENTS
DO I2=1,12
CFMM(I2)=0.0
CFDM(I2)=0.0
END DO
C
C COMPUTE INERTIA FORCE COEFFICIENT TERMS
CFMM(2)=WATDEN*ATOT(MTYPE)*CMT*XL/2
CFMM(6)=WATDEN*ATOT(MTYPE)*CMT*XL2/12
CFMM(8)=WATDEN*ATOT(MTYPE)*CMT*XL/2
CFMM(12)=-WATDEN*ATOT(MTYPE)*CMT*XL2/12
C
C COMPUTE DRAG FORCE COEFFICIENT TERMS
CFDM(2)=WATDEN*DTOT(MTYPE)*CD*XL/2
CFDM(6)=WATDEN*DTOT(MTYPE)*CD*XL2/12
CFDM(8)=WATDEN*DTOT(MTYPE)*CD*XL/2
CFDM(12)=-WATDEN*DTOT(MTYPE)*CD*XL2/12
ELSE  !MEMBER IN AIR AND THUS NO FORCE
C
DO I2=1,12
CFMM(I2)=0.0
CFDM(I2)=0.0
END DO

A110
END IF

ELSE  !MEMBER IS HORIZONTAL

DO I12=1,12
   CFMM(I12)=0.0
   CFDM(I12)=0.0
END DO

END IF

ELSE  !MEMBER IS IN FOUNDATION

DO I13=1,12
   CFMM(I13)=0.0
   CFDM(I13)=0.0
END DO

END IF

C NEXT INSERT COEFFICIENTS INTO CORRECT LOCATION IN THE GLOBAL FORCE COEFFICIENT MATRICES FOR THE TOTAL STRUCTURAL SYSTEM (CFMGT + CFDGT)

IELT1=(IEL-1)*3+1
JELT1=(JEL-1)*3+1
IELT3=(IEL-1)*3+3
JELT3=(JEL-1)*3+3

C FIRST NODE OF ELEMENT
ID1=ID(1,IEL)
ID3=ID(5,JEL)
IF(ID1.NE.0)THEN
   CFMGT(ID1)=CFMM(2)+CFMGT(ID1)
   CFMGT(ID3)=-CFMM(6)+CFMGT(ID3)
   CFDGT(ID1)=CFDM(2)+CFDGT(ID1)
   CFDGT(ID3)=-CFDM(6)+CFDGT(ID3)
ELSE
   CONTINUE
END IF

C SECOND NODE OF ELEMENT
ID2=ID(1,JEL)
ID4=ID(5,XEL)
IF(ID2.NE.0)THEN
   CFMGT(ID2)=CFMM(8)+CFMGT(ID2)
   CFMGT(ID4)=-CFMM(12)+CFMGT(ID4)
   CFDGT(ID2)=CFDM(8)+CFDGT(ID2)
   CFDGT(ID4)=-CFDM(12)+CFDGT(ID4)
ELSE
   CONTINUE
END IF

C END MEMBER LOOP

DO I14=1,12
   CFMM(I14)=0.0
   CFDM(I14)=0.0
END DO

END

CALL SUBROUTINE TO COMPUTE CROSS SPECTRA AND CONVOLUTIONS OF NODE WATER VELOCITIES AND ACCELERATIONS
CALL SPECTRA(A(N1),A(N3))

RETURN
SUBROUTINE SPECTRA(X,Z)

C SUBROUTINE TO COMPUTE SPECTRA & CROSS SPECTRA OF RELEVANT VELOCITIES & ACCELERATIONS OF
C WATER PARTICLES WHICH ARE REQUIRED TO COMPUTE THE ZEROTH ORDER FORCE SPECTRUM
C INCLUDE 'SPEC.CMN'
C DIMENSION X(NUMNP),Z(NUMNP),STEMP(NFRQ),CONTEM(NFRQ)

C COMPUTE STANDARD DEVIATIONS AND CROSS SPECTRA OF VELOCITIES AT EACH
C SUBMERGED NODE POINT ABOVE THE MUDLINE
C INITIALISE STANDARD DEVIATIONS, CROSS SPECTRA AND CONVOLUTIONS
C OF CROSS SPECTRA TO ZERO
C
C OPEN(40,FILE=’SOUT.DAT’,STATUS=’UNKNOWN’)
C
PI2=2*PI
PI3=PI*3
NF2=NFRQ/2
DO INT=1,NUMNP
SD(INT)=0.0
END DO

C D=XMUD
SQRTP=SQRT((8/PI))

C LOOP OVER ALL ELEMENTS
DO I1=1,NUMNP
X11=X(I1)
Z11=Z(I1)

C CHECK TO ENSURE NODE IS SUBMERGED AND NOT ON THE FOUNDATION
IF(Z(I1).LE.0.AND.(I1).GT.XMUD)THEN
DO I3=1,NUMNP
X22=X(I3)
Z22=Z(I3)

C CHECK TO ENSURE THAT NODE IS SUBMERGED BUT NOT ON FOUNDATION
IF(Z(I3).LE.0.AND.(I3).GT.XMUD)THEN

C CHECK IF IN DEEP OR SHALLOW WATER
IF(DWL.GT.0.5)THEN
DEEPWATER
WL=9.81 *(TW*2)*((2*PI)*DK
DK=2*PI/WL
ELSE
C SHALLOW/TRANSITIONAL WATER
DK=AVENO(WF,D)
WL=2.*3.1415927/DK
END IF
DKD=DK*D
Z111=D+Z11
Z222=D+Z22
DKZ1=DK*Z111
DKZ2=DK*Z222

C COMPUTE CROSS SPECTRA OF VELOCITIES AT EACH NODE
C
SVV(I1,I3,J2)=(WF2)*(COSH(DKZ1))*(COSH(DKZ2))*SPECTR(I2)*
1COS(DK*(X11-X22))/(SINH(DK*D))**2

C COMPUTE CROSS SPECTRA OF VELOCITIES/ACCELERATIONS AT EACH NODE
C NOTE SVA WHEN I1=I3 IS ZERO BECAUSE OF SIN TERM
C
SVA(I1,I3,J2)=(WF*WF2)*(COSH(DKZ1))*(COSH(DKZ2))*SPECTR(I2)*
ISIN(DK*(X11-X22))/(SINH(DK*D))**2

SAV(I1,I3,I2)=SVA(I1,I3,I2)

END DO !FREQUENCY LOOP
ELSE
CONTINUE
END IF
END DO !NODE LOOP
ELSE
CONTINUE
END IF
END DO

C NOW COMPUTE THE THIRD CONVOLUTIONS OF THE CROSS SPECTRA OF VELOCITIES
C
C LOOP OVER ALL ELEMENTS
DO I4=1,NUMNP
X4=X(I4)
Z4=Z(I4)

C CHECK TO ENSURE NODE IS SUBMERGED AND NOT ON THE FOUNDATION
IF(Z4.LE.0.AND.Z4.GT.XMUD)THEN
DO I5=1,NUMNP
X5=X(I5)
Z5=Z(I5)
IF(Z5.LE.0.AND.Z5.GT.XMUD)THEN
DOI6=1,NF2
STEMP(I6)=SVV(I4,I5,I6)
END DO
CALL CONSPE(STEMP,CONTEM)
DOI7=1,NF2
CONVV(I4,I5,I7)=CONTEM(I7)
ELSE
CONTINUE
END IF
ELSE
CONTINUE
END IF
END DO

C COMPUTE STANDARD DEVIATIONS
DO K1=1,JMNP
SDSQ=0.0
DO K3=1,NF2
IF(K3.EQ.1.OR.K3.EQ.NF2)THEN
RULL=1.
ELSE IF(IIP.EQ.2)THEN
RULL=4.
ELSE
RULL=2.
ENDIF
SDSQ=STEP*RULL*SW(K1,K1,K3)+SDSQ
END DO
SD(K1)=SQRT(SDSQ)

C PRINT OUT SPECTRA AND CONVOLUTIONS OF WATER PARTICLE VELOCITIES
IF(K1.EQ.2)THEN
PRINT*, "STANDARD DEVIATION VEL AT NODE 2 = ' ,SD(2)
DO K2=1,NF2
WRITE(40,*),(SPECTR(K2),SW(K1,K1,K2),SW(5,5,K2))
WRITE(40,*),(SVV(K1,K1,K2),CONVV(K1,K1,K2))
END DO
CALL SPECOUT
END IF
ELSE
CONTINUE
ENDIF
END DO
CALL SPECOUT
RETURN
END
SUBROUTINE CONSPE(XIN,XOUT)
USE MSIMSL

PARAMETER (NX=256,NFRQ=128)

COMPLEX CX(NX),CZ1(NX),CZ2(NX),CXOUT(NX),CXOUT1(NX)
DIMENSION Z5(NX*2),X1(NX*2),XX(NX*2),WFFTR(2*2*NX+15),
Z7(NX*2),ZHAT(NX*2),XX(NX*2),XOUT(NFRQ),XOUT1(NX*2),
Z1(NX*2),XIN(NFRQ)

DO 11=1,NX
  X1(I1)=0.0
  CZ1(I1)=0.0
  CX(I1)=CMPLX(X(I1))
END DO

DO 12=1,NX/2
  CX(I2)=CMPLX(XIN(I2))
END DO

C COMPUTE FFT OF INPUT SPECTRUM
CALL FFTCB(NX,CX,CXOUT)

C COMPUTE PRODUCTS
DO 13=1,NX
  XX(I3)=REAL(CXOUT(I3))*REAL(CXOUT(I3))*REAL(CXOUT(I3))
  CZ1(I3)=CMPLX(XX(I3))
END DO

C COMPUTE IFT OF XX - THUS XOUT1 IS THE CONVOLUTION
CALL FFTCF(NX,CZ1,CXOUT1)

C PRINT RESULTS OF CONVOLUTION
RNX=NX
RNX1=NFRQ
SRNX=SQRT(RNX1)
SRNX1=SQRT(RNX)
DO 17=1,NFRQ
  XOUT(I7)=REAL(CXOUT1(I7))^4*(RNX^ SRNX)!**3
  Z5(I7)=REAL(CXOUT1(I7))^4*(RNX^ SRNX)!**2
END DO

100 FORMAT(F8.5)
C100 FORMAT(16X,F15.9)
200 FORMAT(5X,I5,2F12.5)
RETURN
END
SUBROUTINE SPEC1
C
C SUBROUTINE TO COMPUTE SPECTRA OF RESPONSE FOR THE ZEROTH ORDER EQUATION FROM THE SPECTRA
C OF FORCE AND THE RECEPTANCES
C
C INCLUDE 'SPEC.CMN'
DIMENSION SOUT(NEQ,NFRQ),SOUT1(NEQ,NFRQ),FSOUT(NEQ,NFRQ),
1CONOUT(NEQ,NFRQ),CONOUT1(NEQ,NFRQ),CO(CNEQ),CO1(NEQ),
2KFORCE(NEQ,NFRQ)
C
C OPEN OUTPUT FILE
OPEN(20,FILE='SPEC.OUT,STATUS='UNKNOWNN')
OPEN(30,FILE='FORSPE.OUT,STATUS='UNKNOWNN')

C
C Pi=8/PI
SPI8=SQRT(PI8)
PI34=1/(3*PI)
C
C INITIALISE OUTPUT SPECTRUM TO ZERO
DO K1=1,NEQ
DO K2=1,NFRQ
SOUT(K1,K2)=0.0
END DO
END DO
C
C DO I1=1,Q
NF2=NFRQ/2
C
C LOOP OVER ALL FREQUENCIES
DO I3=1,NF2
SOUT(I3,I3)=0.0
RI3=I3
WF=RI3*DELTAW
WF2=WF**2
C
C DOUBLE LOOP OVER ALL APPLIED DEGREES OF FREEDOM
DO I4=1,NEQ
R4=I4-1
RI4=I4/3
INT4=INT(RI43)+1
C
C FIND NODE NO. ASSOCIATED WITH DOF I4
R43=INT4-1
RI43=INT(RI43)
DIF=R43-RI43
IF(I5.EQ.2.OR.DIF.EQ.0)THEN
SF=0
ELSE
SF=CMFMT(I4)*CMFMT(I5)*(WF2*SVV(INT4,INT5,I3))+
1PI8*CFDGT(I4)*CFDGT(I5)*SD(INT4)*SD(INT5)*SVV(IV4,INT5,I3)+
2SPI8*(SD(INT4)*CFDGT(I4)*CFMGT(I5))*SVV(IV4,INT5,I3)+
3SD(INT5)*CFDGT(I5)*CMFMT(I4)*SAY(IV4,INT5,I3)+
4PI34*CFDGT(I4)*CFDGT(I5)*CONVV(IV4,INT5,I3)
5/(SD(INT4)*SD(INT5))
C
C ELSE
SF=CMFMT(I4)*CMFMT(I5)*(WF2*SVV(IV4,INT5,I3))+
1PI8*CFDGT(I4)*CFDGT(I5)*SD(INT4)*SD(INT5)*SVV(IV4,INT5,I3)+
2SPI8*(SD(INT4)*CFDGT(I4)*CFMGT(I5))*SVV(IV4,INT5,I3)+
3SD(INT5)*CFDGT(I5)*CMFMT(I4)*SAY(IV4,INT5,I3)+
4PI34*CFDGT(I4)*CFDGT(I5)*CONVV(IV4,INT5,I3)
5/(SD(INT4)*SD(INT5))
C
C END IF
C
C R52=I5-2
R423=INT4-R523
IF(I5.EQ.2.OR.DIF.EQ.0)THEN
SF=0
ENDIF
n=0
ELSE
CONTINUE
END IF
C COMPUTE OUTPUT SPECTRA
C
SOUT(11,I3)=SF*(H1(11,14,I3)*H1(11,15,I3)+
1H2(11,14,I3)*H2(11,15,I3))+SOUT(11,I3)
C COMPUTE SPECTRA OF FORCE
C
IF(I4.EQ.I5) THEN
FSOUT(I4,I3)=SF
CONOUT(I4,I3)=PI34*CFDGT(I4)*CFDGT(I5)*CONVV(INT4,INT5,I3)/
1(SD(INT4)*SD(INT5))
C
CONOUT1(I4,I3)=PI8*CFDGT(I4)*CFDGT(I5)*SD(INT4)*SD(INT5)*
1 SVV(INT4,INT5,I3)
RIFORCE(I4,I3)=CFMG(T(I4)*CFMG(T(I5)*(WF2*SVV(INT4,INT5,I3))
C
CO(I4)=PI34*CFDGT(I4)*CFDGT(I5)/
1 (SD(INT4)*SD(INT5))
C01(I4)=PI8*CFDGT(I4)*CFDGT(I5)/
1 (SD(INT4)*SD(INT5))
ELSE
CONTINUE
END IF
ELSE
CONTINUE
END IF
END DO
END DO !END DOUBLE LOOP OF APPLIED DEGREES OF FREEDOM
END DO !END FREQUENCY LOOP
END DO !LOOP II
C
DO J=1,NF2
WRITE(30,100)FSOUT(4,J),CONOUT1(4,J),CONOUT(4,J),FSOUT(19,J),
1SVV(3,3,J),CONVV(3,3,J),SVV(5,5,J),CONVV(5,5,J)
WRITE(20,*)SOUT(1,J),SOUT(16,J),SOUT(7,J)
END DO
100 FORMA(T(9E10.4)
RETURN
END
APPENDIX VIII

CUMULANT-NEGLECT CLOSURE
Cumulant-Neglect Closure

When solving multi-dimensional non-linear problems in the frequency domain, approximate solution techniques are generally needed. The most frequently used approximation scheme is the equivalent linearisation procedure in which the original system is replaced by an equivalent linear system.

Another closure procedure focuses on the properties of the cumulants of random processes and is known as the cumulant-neglect closure scheme. Consider the random variables $X_j$, $X_k$, $X_i$, and $X_m$, then the following relationship exists between cumulants and statistical moments, Strantonovich, (1963)

\[
\begin{align*}
E[X_j] &= \kappa_1[X_j] \\
E[X_jX_k] &= \kappa_2[X_jX_k] + \kappa_1[X_j]\kappa_1[X_k] \\
E[X_jX_kX_i] &= \kappa_3[X_jX_kX_i] + 3\{\kappa_1[X_j]\kappa_2[X_k,X_i]\}_s + \kappa_1[X_j]\kappa_1[X_k]\kappa_1[X_i] \\
E[X_jX_kX_iX_m] &= \kappa_4[X_jX_kX_iX_m] + 3\{\kappa_2[X_jX_k]\kappa_2[X_i,X_m]\}_s + 4\{\kappa_1[X_j]\kappa_3[X_k,X_i,X_m]\}_s \\
&\quad + 6\{\kappa_1[X_j]\kappa_1[X_k]\kappa_2[X_i,X_m]\}_s + \kappa_1[X_j]\kappa_1[X_k]\kappa_1[X_i]\kappa_1[X_m]
\end{align*}
\]

where

$\kappa_1[X_j]$ denotes the first cumulant of the random variable $X_j$

$\{ \}_s$ denotes a symmetrising operation with respect to its arguments, Lin and Cai (1995)
Lin and Cai (1995) state that the physical significance of a cumulant decreases as the order increases and that the most important properties of a random process are contained in the lower order cumulants. Lin and Wu (1984) show that by using a second-order cumulant-neglect closure scheme (that is, neglecting those cumulants above the second order) that good results were obtained against an exact solution when applied to a duffing oscillator under random excitation. Other researchers have also suggested the second-order cumulant neglect closure scheme gives good results in practical applications, Soong, (1973).

The average $E[X_jX_kX_l]$ contains cumulants of order 1, 2 and 3. From above the first order cumulant is equal to the time average of the random process. By re-arranging the second equation above the second-order cumulant may be written as

$$
\kappa_2[X_jX_k] = E[X_jX_k] - \kappa_1[X_j]\kappa_1[X_k] = E[X_jX_k] - E[X_j]E[X_k]
$$

When cumulants of higher order than two are ignored, the following may be written.

$$
E[X_jX_kX_l] = 3\{\kappa_1[X_j]\kappa_2[X_k,X_l]\} + \kappa_1[X_j]\kappa_1[X_k]\kappa_1[X_l] = 3\{E[X_j](E[X_k,X_l] - E[X_l]E[X_k])\} + E[X_j]E[X_k]E[X_l]
$$

Now if the processes $X_j$, $X_k$, and $X_l$ are zero mean stochastic processes then

$$
E[X_jX_kX_l] = 0
$$

This result is employed in Chapter 5 in developing the first order response terms.
Again, if cumulants of higher order than two are ignored, the average of the product of four zero mean stochastic processes may be written as

\[
E[X_jX_kX_lX_m] = 3\{\kappa_2[X_jX_k]\kappa_2[X_lX_m]\} + 6\{\kappa_1[X_j][\kappa_1[X_k]\kappa_2[X_lX_m]\]_S \\
+ \kappa_1[X_j]\kappa_1[X_k]\kappa_1[X_l]\kappa_1[X_m]\}
\]

Since variables \(X_j, X_k, X_l,\) and \(X_m\) have zero mean values, we may write the last expression as

\[
E[X_jX_kX_lX_m] = 3\{\kappa_2[X_jX_k]\kappa_2[X_lX_m]\} \\
= \kappa_2[X_jX_k]\kappa_2[X_lX_m] + \kappa_2[X_jX_l]\kappa_2[X_kX_m] + \kappa_2[X_jX_m]\kappa_2[X_kX_l]
\]

Substituting for the second-order cumulants from equation as defined above and setting the mean values of the individual processes to zero leads to

\[
E[X_jX_kX_lX_m] = E[X_jX_k]E[X_lX_m] + E[X_jX_l]E[X_kX_m] + E[X_jX_m]E[X_kX_l]
\]

This result is employed in Chapter 5 in developing the second order response terms.
DO IRT=1,NROOT
II=(IRT-1)*NEQ
DO JRT=1,6
IA1=6*(ND1-1)-IX*2+JRT+II+N16
PHI2(JRT,IRT)=A(IA1)
END DO
END DO
ELSE IF(IFLGX.NE.0.AND.JFLGX.NE.0)THEN
DO IRT=1,NROOT
II=(IRT-1)*NEQ
DO JRT=1,4
IA2=6*(ND2-1)-JX*2+JRT+II+N16
PHI3(JRT,IRT)=A(IA2)
END DO
END DO
ELSE
CONTINUE
END IF
ELSE
CONTINUE
END IF

C FIND U2MJ AND U3MJ

IF(XYZ(6,J).GE.XMUD)THEN
C
DO IZ=1,12
ZEL11(IZ)=0.
ZEL12(IZ)=0.
TZ11(IZ)=0.
TZ12(IZ)=0.
END DO
C
DO IK=1,12
DO JD=1,12
TD(ID,JD)=T(ID,JD)
END DO
END DO
DO IK=1,12
DO JK=1,12
TZ11(IK)=T(IK,JK)+TZ11(IK)
TZ12(IK)=T(IK,JK)+TZ12(IK)
END DO
END DO
C
ELSE IF(XYZ(3,J).EQ.XMUD.AND.XYZ(6,J).LT.XMUD)THEN
DO ID=1,6
DO JD=1,6
TD(ID,JD)=T(ID,JD)
END DO
END DO
DO IK=1,12
DO JK=1,12
TD(IK,JK+6)=T(IK+6,JK+6)
TD(IK+8,JK+8)=T(IK+8,JK+8)
END DO
END DO
C
DO IZ=1,10
ZEL21(IZ)=0.
ZEL22(IZ)=0.
TZ21(IZ)=0.
TZ22(IZ)=0.
END DO
DO IK=1,10
DO JK=1,NROOT
ZEL21(IK)=PHI2(IK,JK)*Z1(JK,IFR)+ZEL21(IK) !Z1 IFR
ZEL22(IK)=PHI2(IK,JK)*Z2(JK,IFR)+ZEL22(IK) !Z2 SAME MH 11/5/95
END DO
END DO
DO IK=1,10
DO JK=1,10
TZ21(IK)=TD(IK,JK)*ZEL21(JK)+TZ21(IK)
TZ22(IK)=TD(IK,JK)*ZEL22(JK)+TZ22(IK)
END DO
END DO
ELSE IF(XYZ(3,J).LT.XMUD.AND.XYZ(6,J).LT.XMUD)THEN
DO ID=1,2
DO JD=1,2
TD1(ID,JD)=T(ID+1,JD+1,1)
TD1(ID+2,JD+2)=T(ID+4,JD+4,1)
TD1(ID+4,JD+4)=T(ID+7,JD+7,1)
TD1(ID+6,JD+6)=T(ID+10,JD+10,1)
END DO
END DO
DO IZ=1,8
ZEL31(IZ)=0.
ZEL32(IZ)=0.
TZ31(IZ)=0.
TZ32(IZ)=0.
END DO
ELSE
CONTINUE
END IF
C
C LOOP OVER ALL SIMPSONS POINTS IN THE ELEMENT
SPTM1=SPT-1
ISPT=SPT
DELX=XL/SPTM1
READ(41,REC=I)XWA
READ(42,REC=I)XWB
READ(11,REC=I)E1W
READ(12,REC=I)E2W
READ(13,REC=I)E3W
DO IS=USPT
ICNT=ICNT+1
SS=IS
XI1=(SS-1.)*DELX
IF(XYZ(6,J).GE.XMUD)THEN
DO IR=1,2
DO IC=1,12
GS1(DUIC)=0.
END DO
END DO
END DO
GS1(IR,JC)=0.
END DO
END DO
GS1(1,2)=G3(XI1,1)
GS1(1,6)=G4(XI1,1)
GS1(1,8)=G5(XI1,1)
GS1(1,12)=G6(XI1,1)
GS1(2,3)=G3(XI1,1)
GS1(2,5)=G7(XI1,1)
DO IU=1,2
UMJ1(IU)=0.
UMJ2(IU)=0.
END DO
DO IR=1,2
DO IC=1,12
UMJ1(IR)=GS1(IR,IC)*TZ11(IC)+UMJ1(IR)
UMJ2(IR)=GS1(IR,IC)*TZ12(IC)+UMJ2(IR)
END DO
END DO
ELSE IF(XYZ(3,J).EQ.XMUD.AND.XYZ(6,J).LT.XMUD)THEN
DO IR=1,2
DO IC=1,10
GS2(IR,IC)=0.
END DO
END DO
GS2(U)=G3(X1MJ,XL)
GS2(1,6)=G4(X1MJ,XL)
GS2(1,7)=G5(X1MJ,XL)
GS2(1,10)=G6(X1MJ,XL)
GS2(2,3)=G3(X1MJ,XL)
GS2(2,5)=G7(X1MJ,XL)
GS2(2,8)=G5(X1MJ,XL)
GS2(2,9)=G8(X1MJ,XL)
DO IU=1,2
UMJ1(IU)=0.
UMJ2(IU)=0.
END DO
DO IR=1,2
DO IC=1,10
UMJ1(IR)=GS2(IR,IC)*TZ21(IC)+UMJ1(IR)
UMJ2(IR)=GS2(IR,IC)*TZ22(IC)+UMJ2(IR)
END DO
END DO
ELSE IF(XYZ(3,J).LT.XMUD.AND.XYZ(6,J).LT.XMUD)THEN
DO IU=1,2
UMJ1(IU)=0.
UMJ2(IU)=0.
END DO
DO IR=1,2
DO IC=1,8
GS3(INUC)=0.
END DO
END DO
GS3(1,1)=G3(X1MJ,XL)
GS3(1,4)=G4(X1MJ,XL)
GS3(1,5)=G5(X1MJ,XL)
GS3(1,8)=G6(X1MJ,XL)
GS3(2,2)=G3(X1MJ,XL)
GS3(2,3)=G7(X1MJ,XL)
GS3(2,6)=G5(X1MJ,XL)
GS3(2,7)=G8(X1MJ,XL)
DO IR=1,2
DO IC=1,8
UMJ1(IR)=GS3(IR,IC)*TZ31(IC)+UMJ1(IR)
UMJ2(IR)=GS3(IR,IC)*TZ32(IC)+UMJ2(IR)
END DO
END DO
ELSE
CONTINUE
END IF
B11=B1(IJS)
B12=B2(IJS)
B13=B3(IJS)
B14 = B4(1,15)

H1 = B11 - E1W*(E1W*B11 + E3W*B13)
H2 = B12 - E1W*(E1W*B12 + E3W*B14)
H3 = -E2W*(E1W*B11 + E3W*B13)
H4 = -E2W*(E1W*B12 + E3W*B14)
H5 = B13 - E3W*(E1W*B11 + E3W*B13)
H6 = B14 - E3W*(E1W*B12 + E3W*B14)

H7 = TWM(2,1)*H1 + TWM(2,3)*H3 + TWM(2,5)*H5
H8 = TWM(2,1)*H2 + TWM(2,3)*H4 + TWM(2,5)*H6
H9 = TWM(3,1)*H1 + TWM(3,2)*H3 + TWM(3,3)*H5
H10 = TWM(3,1)*H2 + TWM(3,2)*H4 + TWM(3,3)*H6

H11 = UMI(2,1)*WF
H12 = UMI(1,1)*WF
H13 = WF*UMI(2,2)
H14 = WF*UMJ(2,2)

H15 = H7 - H11
H16 = H8 - H12
H17 = H9 - H13
H18 = H10 - H14

C CALCULATE NEW SIGR22, SIGR33, SIGR23
IF (FR.EQ.1 .OR. FR.EQ. WLST) THEN
   RULL = 1.
   HR = 2
ELSE IF (HR.EQ.2) THEN
   RULL = 4.
   HR = 1
ELSE
   RULL = 2.
   HR = 2
END IF

DO IP = 1, WLST
DI = IP
STEP = DI*STEP
IF (STEP.GE.WF) GO TO 101
END DO
CONTINUE
DIFF = WF - STEP*DI
DIFF = SPECTR(IP) - SPECTR(IP-1)
SPE = SPECTR(IP)
IF (FR.EQ.1) SPE = 0.
IF (FR.GT.200) SPE = 0.
SIGR22(IP) = (H15**2 + H16**2) + SPE*STEP*ROLL/3.
SIGR33(IP) = (H17**2 + H18**2) + SPE*STEP*ROLL/3.
SIGR23(IP) = (H15*H17 + H16*H18) + SPE*ROLL*STEP/3.
END DO ! LOOP OF SIMPS POINTS
ELSE
IST = 1
ICNT = ICNT + 1
SIGR22(IP) = 0.
SIGR33(IP) = 0.
SIGR23(IP) = 0.
END DO
END IF
END DO ! LOOP OF ELEMENTS

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C COMPUTE SIGMZZ(IJ)
    DO IRT=1,NROOT
    DO JRT=1,NROOT
        SIGMZZ(IRT,JRT) = (WF**2.)*(Z1(IRT,IFR)*Z1(JRT,IFR) +
                         Z2(IRT,IFR)*Z2(JRT,IFR)) +
                         STEP*RULL*SPE/3. +
                         SIGMZZ(IRT,JRT) ! MH CHANGED Z1 Z2
    END DO
    END DO
END DO LOOP OVER FREQUENCIES
END DO

C CALL CBARJ(CDBARJ,NUME,NC,ALPHA,ISP,SIGMZZ,
              A(N13),NNC,CAPJDTOT,MATPJlSPNUM,SIGR22,
              SIGR33,SIGR23)
RETURN
END
SUBROUTINE RESULTS(PHI)
C
INCLUDE 'SPEC.CMN'
DIMENSION PHI(NEQ,NROOT),
U1(neq),U2(neq),IDF(10),SIGR22(10)
C
CHARACTER *1 EE(IO)
CHARACTER NAME* 10,DD*1
DATA EE/* '1','2','3','4','5','6','7','8','9','0'/
OPEN(30,FILE='DOF.DAT',STATUS='OLD')
READ(30,100)NDF,NTFLG
100 FORMAT(2I10)
DF=NDF
DIV=DF/8.
IDL=AINT(DIV)+1
DO IR=1,IDL
IRM1=(IR-1)*8
READ(30,200)(IDF(K+IRM1),K=1,8)
END DO
200 FORMAT(8I10)
CLOSE(30)
DO IK=1,NDF
DD=EE(IK)
NAME='SPE'/DD/AOUT
INUM1=65+IK
OPEN(INUM1,FILE=NAME,STATUS='UNKNOWN')
INUM2=85+IK
NAME=TRN'/DD'/OUT'
OPEN(INUM2,FILE=NAME,STATUS='UNKNOWN')
WRITE(INUM2,*)THE TRANSFER FUNCTION FOR DEGREE OF FREEDOM,'
WRITE(INUM2,305)IDF(IK)
WRITE(INUM2,*)Iteration number = 'ITERFLG
WRITE(INUM2,*) Ratio = 'RATIO(ITERFLG)
IF(NTFLG.EQ.1)THEN
WRITE(INUM2,*) ALPHA w(Hz)
ELSE
WRITE(INUM2,*) ALPHA w(rad/s)
END IF
END DO
C
DO 202 FFF=1,10
SIGR22(FFF)=0.
202 CONTINUE
C
WLST=200
ICOUN=0
DO 301 F=1,WLST
C
IF(F.EQ.1.0.OR.F .EQ. WLST)THEN
RULL=1.
ELSE IF(IIR.EQ.2)THEN
RULL=4.
ELSE
RULL=2.
END IF
C
IFR=F
WF=F*STEP
DO IC=1,NROOT
ICOUN=ICOUN+1
END DO
C
DO 203 NN=1,NEQ
U1(NN)=0.
U2(NN)=0.
203 CONTINUE
C
DO N=1,NEQ
DO I=1,NROOT
U1(N)=PHI(N,J)*Z1(I,J,IFR)+U1(N)
U2(N)=PHI(N,J)*Z2(I,J,IFR)+U2(N)
END DO
END DO

DO IP=1,WLST
    D=IP
    STEPI=D*STEP
    IF(STEPI.GE.WF)GO TO 101
END DO

CONTINUE

DIFF=WF-STEP*(DI-1.)
DIFFPC=SPECTR(IP)-SPECTR(IP-1)
SPE=SPECTR(IFR+1)
IF(IP.EQ.1)SPE=0.
IF(F.GT.200)SPE=0.

C

DO IP=1,NDF
    NI=IDF(IP)
    NUM1=65+IO
    NUM2=85+IO
    SP1=SP*((U1(NI))**2.+(U2(NI))**2.)
    SP2=(U1(N1))**2.+(U2(NI))**2.
END DO

C

This is to output data in Hz(not RADS) and (mm2/Hz) (not m2s/rad), depending on the flag NTFLG in DOF.DAT

PI=3.1415927
AMIL=1000000
SP1H=2*PI*AMIL*SP1
WFH=WF/(2*PI)

C

IF(NTFLG.EQ.1)THEN
    WRITE(NUM1,300)SP1H,WFH
ELSE
    WRITE(NUM1,300)SP1,WF
END IF

C

IF(RATIO(ITERFLG).LT.0.03.OR.ITERFLG.EQ.ITEFP1)THEN
    IF(NTFLG.EQ.1)THEN
        WRITE(NUM2,300)SP2,WFH
    ELSE
        WRITE(NUM2,311 )SP2,WF,SPE
    END IF
END IF

311 FORMAT(F20.17,F20.5)
300 FORMAT(F20.17,F20.5)
301 FORMAT(F20.17,F20.5)
302 FORMAT(F10.4)
303 FORMAT(I5)
304 FORMAT(I5)
305 FORMAT(10X,15)

SIGR22(IOP)=SPl*RULL*STEP/3.+SIGR22(IOP)
WLSTAB=ABS(WLST)
IF(F.EQ.WLSTAB)THEN
    SG22RT=SQRT(SIGR22(IOP))
    WRITE(NUM1,*)' SIGR22 = ',SG22RT
ELSE
    CONTINUE
END IF
END DO

301 CONTINUE

DO IOPP=1,NDF
    NUM1=65+IOPP
    NUM2=75+IOPP
    CLOSE(NUM1)
    CLOSE(NUM2)
END DO

ITEFP1=ITEFLG+1
PRINT*,"CONVERGENCE REACHED"
OPEN(83,FILE="RATIO.OUT",STATUS="OLD")
DO 302 RAT=1,ITERFLG
    WRITE(83,303)RATIO(RAT)
302 CONTINUE
CLOSE(83)

303 FORMAT(F10.4)
304 FORMAT(I5)
305 FORMAT(10X,15)

PRINT*,"PROGRAM FINISHED"
CALL EXIT
RETURN
END
SUBROUTINE ALPHAU(XYZ,C,JM,DTOT)

THIS PROGRAM CALCULATES:
- Variance of water particle velocities
- Element hydrodynamic damping matrix
- Global hydrodynamic damping matrix

INCLUDE 'SPEC.CMN'

DIMENSION XW(6),XYZ(6,1),DTOT(150),Q(12,12),CA(12,12),TR(12,12),ICDD(144),GGI(12,12),C(NWK),LM(12,1)

CALL ZERO(C)

SPT=SPT
ICOUNT=0
JCOUNT=0
PIE=SQRT(8/PI)
ND=12

NOTE: FOR NONLINEAR CASE ALPHA IS 0 - SAME COORDINATE SYSTEM FOR
WAVE AND STRUCTURE.

INITIALISE DAMPING MATRIX TO ZERO
DO ID=1,NWK
C(ID)=0.0
END DO

LOOP OVER ALL ELEMENTS AND COMPUTE FOR HORIZONTAL MEMBERS
(IE MEMBERS WITH IELFLG(N)=1)

D=XMUD
DO 100 N=1,NUME
IFLAG=IELFLG(N)
IF(IFLAG.EQ.1)THEN
! note only calculate hydrodynamic damping
INK=INK+1
XL=0.
XL2=0.
DO IP=1,3
XL=XYZ(IP,N)-XYZ(IP+3,N)
XL2=XL2+XL*XL
END DO
XLN=SQRT(XL2)
SPTM1=SPT-1.
DEL=XLN/SPTM1
DELX=0.0

INITIALISE HYDRODYNAMIC DAMPING FOR ALL ELEMENTS TO ZERO

EXPRESS THE NODAL CO-ORDS IN TERM OF THE
WAVE CO-ORD SYSTEM.
XW(1)=XYZ(1,N)
XW(2)=XYZ(2,N)
XW(3)=XYZ(3,N)
XW(4)=XYZ(4,N)
XW(5)=XYZ(5,N)
XW(6)=XYZ(6,N)

DO 310 IP=1,SPT
! NO. OF SIMPS POINTS
CH=0
IEW=IEW+1
SIP=IP
WL=1000.
JCOUNT=JCOUNT+1
DELX=(SIP-1)*DEL
XW1=XW(1)
IF(XW(3).GT.0)XW(3)=-XW(3)
XW3=XW(3)+DELX

INITIALISE SIGVH, THE INTEGRATION PARAMETER FOR STANDARD DEVIATION OF
HORIZONTAL WATER PARTICLE VELOCITY FOR SIMPSON'S POINT j
SIGVH=0.0

C LOOP OVER ALL FREQUENCIES OF INTEREST
DO 320 IXN=1,NFRQ
 XNF=IXN
 DNF=XNF

C WF=DELTAW*DNF

C TW=2.*1.414527/WF !WAVE PERIOD (HERTZ). FREQUENCY (RADIANS)

DWL=D/WL

IF(DWL.GT.0.5)THEN

C DEEP WATER

WL=9.81*(TW**2)/(2*PI)
DK=2*3.1415927*AVL
B1=2*PI*(EXP(DK*XW3))*COS(DK*XW1)/TW
B2=2*PI*(EXP(DK*XW3))*SIN(DK*XW1)/TW
ELSE

DK=WAVENO(WF,D)
WL=2.*3.1415927/DK
B1=BB1(TW,DK,XW3,XW1,WL)
B2=BB2(TW,DK,XW3,XW1,WL)

END IF

C COMPUTE APB

APB=B1**2 + B2**2

C IF(XNF.EQ.1.0.OR.XNF.EQ.NFRQ)THEN

RULL=1.
IIR=2.
ELSE IF(IIR.EQ.2)THEN

RULL=4.
IIR=1.
ELSE

RULL=2.
IIR=2.
END IF

C CALCULATE THE SPECTRUM ORDINATE FOR THE PARTICULAR
C FREQUENCY

SPE=SPECTR(IXN)
IF(IP.EQ.1)SPE=0.0
SIGVH=APB*SPE*RULL*DELTAW/3 + SIGVH

320 CONTINUE

C END OF FREQUENCY LOOP

IF(IP.EQ.1.0.OR.IP.EQ.ISPT)THEN

RULL1=1.
IIR1=2.
ELSE IF(IIR1.EQ.2)THEN

RULL1=4.
IIR1=1.
ELSE

RULL1=2.
IIR1=2.
END IF

C COMPUTE HYDRODYNAMIC DAMPING COEFFICIENT FOR EACH ELEMENT, CH(I)

CH=SIGVH*RULL1*DEL/3

C COMPUTE REMAINING TERMS IN ELEMENT INTEGRAL

RIL=IP-1
X1MJ=RIL*DEL

KD=WATDEN*CD*DTOT(N)*CH
GG1(2,2)=G3(X1MJ,XL)*G3(X1MJ,XL)*KD*PIE + GG1(2,2)
GG1(2,6)=G3(X1MJ,XL)*G4(X1MJ,XL)*KD*PIE + GG1(2,6)
GG1(2,8)=G3(X1MJ,XL)*G5(X1MJ,XL)*KD*PIE + GG1(2,8)
GG1(2,12)=G3(X1MJ,XL)*G6(X1MJ,XL)*KD*PIE + GG1(2,12)
GG1(6,2)=G4(X1MJ,XL)*G3(X1MJ,XL)*KD*PIE + GG1(6,2)
GG1(6,6)=G4(X1MJ,XL)*G4(X1MJ,XL)*KD*PIE + GG1(6,6)
GI(6,8) = G4(X1MJ,XL)*G5(X1MJ,XL)*KD*PIE + GI(6,8)
GI(6,12) = G4(X1MJ,XL)*G6(X1MJ,XL)*KD*PIE + GI(6,12)
GI(8,2) = G3(X1MJ,XL)*G5(X1MJ,XL)*KD*PIE + GI(8,2)
GI(8,6) = G4(X1MJ,XL)*G5(X1MJ,XL)*KD*PIE + GI(8,6)
GI(8,8) = G5(X1MJ,XL)*G5(X1MJ,XL)*KD*PIE + GI(8,8)
GI(8,12) = G5(X1MJ,XL)*G6(X1MJ,XL)*KD*PIE + GI(8,12)
GI(12,2) = G3(X1MJ,XL)*G6(X1MJ,XL)*KD*PIE + GI(12,2)
GI(12,6) = G6(X1MJ,XL)*G4(X1MJ,XL)*KD*PIE + GI(12,6)
GI(12,8) = G6(X1MJ,XL)*G5(X1MJ,XL)*KD*PIE + GI(12,8)
GI(12,12) = G6(X1MJ,XL)*G6(X1MJ,XL)*KD*PIE + GI(12,12)

CONTINUE
END OF SIMPSONS LOOP

NEXT WE MUST PRE- AND POST- MULTIPLY
BY THE TRANSFORMATION MATRICES TO
CONVERT TO GLOBAL CO-ORDS

NN=0
DO 150 IP=1,12
DO 150 JP=1,12
Q(IP,JP)=0.0
CA(IP,JP)=0.0
TR(IP,JP)=T(JP,IP,N)
150 CONTINUE
DO 160 IT=1,12
DO 160 JT=1,12
DO 160 IR=1,12
Q(IT,JT)=GI(IT,IR)*T(IR,JT,N)+Q(IT,JT)
160 CONTINUE
DO 170 IA=1,12
DO 170 IB=1,12
DO 170 IC=1,12
CA(IA,IB)=TR(IA,JC)*Q(IC,IB)+CA(IA,IB)
170 CONTINUE
K=0
DO 180 LF=1,12
DO 180 LK=LF,12
K=K+1
CDD(K)=CA(LF,LK)
180 CONTINUE

NOTE: HERE WE MUST ENSURE TO ADD THE
ADDED DAMPNESS TO THE CORRECT
DAMPNESS ELEMENT

ELSE
K=0
DO 185 LF=1,12
DO 185 LK=LF,12
K=K+1
CDD(K)=0.0
185 CONTINUE
CONTINUE
END IF

NOW ASSEMBLE CA(I,J)

CALL ASSDAM(C(1),IA(IN5),CDD,LM(1,N),ND)

END OF ELEMENT LOOP
CALL ZERO(C)
RETURN

FUNCTION WAVENO(WF,D)

THIS CALCULATES THE WAVE NUMBER USING THE DISPERSION RELATIONSHIP
AND CAN BE USED TO CALCUL THE WAVELENGTH FOR A FREQUENCY
C INITIALISE DK
WNO=0.6
TOL=1.E-3
ITMAX=40
DO I=1,ITMAX
FK=9.81*WNO*TANH(WNO*D)-WF**2.
FD=9.81*WNO*TANH(WNO*D)+9.81*WNO*D*((COSH(WNO*D))**(-2))
FACT=FK/FD
WN1=WNO-FACT
C
C CHECK FOR CONVERGENCE
CHECK=FACT/WN1
IF(ABS(CHECK).LT.TOL)GO TO 100
END DO
100 CONTINUE
WNO=WN1
RETURN
END
C
FUNCTION BB1(TW,DK,XW3,D,XW1,WL)
BB1=9.81*TW*COSH(DK*(XW3+D))*COS(DK*XW1)/(WL*COSH(DK*D))
RETURN
FUNCTION BB2(TW,DK,XW3,D,XW1,WL)
BB2=9.81*TW*COSH(DK*(XW3+D))*SIN(DK*XW1)/(WL*COSH(DK*D))
RETURN
FUNCTION BB3(TW,DK,XW3,D,XW1,WL)
BB3=9.81*TW*SINH(DK*(XW3+D))*SIN(DK*XW1)/(WL*COSH(DK*D))
RETURN
FUNCTION BB4(TW,DK,XW3,D,XW1,WL)
BB4=-9.81*TW*SINH(DK*(XW3+D))*COS(DK*XW1)/(WL*COSH(DK*D))
RETURN
END
C
FUNCTION G1(X1MJ,XL)
G1=1.-X1MJ/XL
RETURN
C
ENTRY G2(X1MJ,XL)
G2=X1MJ/XL
RETURN
C
ENTRY G3(X1MJ,XL)
G3=1.3.*(X1MJ/XL)**2.+2.*(X1MJ/XL)**3.
RETURN
C
ENTRY G4(X1MJ,XL)
G4=X1MJ-2.*(X1MJ**2.)/XL+(X1MJ**3.)/{XL**2.}
RETURN
C
ENTRY G5(X1MJ,XL)
RETURN
C
ENTRY G6(X1MJ,XL)
G6=-(X1MJ**2.)/XL+(X1MJ**3.)/(XL**2.)
RETURN
C
ENTRY G7(X1MJ,XL)
G7=-X1MJ+2.*(X1MJ**2.)/XL-X1MJ**3/(XL**2.)
RETURN
C
ENTRY G8(X1MJ,XL)
G8=(X1MJ**2.)/XL-(X1MJ**3.)/(XL**2.)
RETURN
END
SUBROUTINE ASSDAM (AX,MAXA,S,LM,ND)
C ........................................................................
C . PROGRAM
C . ASSEMBLES UPPER TRIANGULAR ELEMENT STIFFNESS TO COMPACTED
C . GLOBAL STIFFNESS
C
C INCLUDE'SPEC.COM'
DIMENSION S(144),AX(NWK),MAXA(I),LM(I)
C
NDI=0
DO 200 I=1,ND
II=LM(I)
IF (II) 200,200,100
100 MI=MAXA(II)
KS=I
DO 220 J=1,ND
JI=LM(J)
IF (JI) 220,220,110
110 IJ=II-JJ
IF (IJ) 220,210,210
210 KK=MI+IJ
KSS=KS
C
IF (J.GE.I) KSS=J+NDI
AX(KK)=AX(KK)+S(KSS)
220 KS=KS+ND-J
200 NDI=NDI+ND-1
DO 300 IR=1,12
II=LM(IR)
IF(II.EQ.0) GO TO 202
MI=MAXA(II)
202 CONTINUE
300 CONTINUE
RETURN
END
SUBROUTINE CGEN(SIGMZZ, C, EIGV, R, ID, XYZ, MATP)

C
C ..........................................................
C ! THIS PROGRAM CALCULATES THE GENERALISED DAMPING MATRIX (CTOT)
C ! FROM BOTH THE STRUCTURAL DAMPING & HYDRODYNAMIC DAMPING
C ! READ IN SIGMZZ FROM THE LINEAR ANALYSIS
C ..........................................................
C
INCLUDE 'SPEC.CMN'
DIMENSION SIGMZZ(NROOT,NROOT), C(NWK), PCP(NROOT,NROOT)
I.CTOT(NROOT), EIGV(NROOT), CDIAGG(NROOT),
2 CHELM(12,12), CHELG(12,12), CHGT(600,600), CHG(NEQ,NEQ), TGM1(3,3),
3 TGM(12,12), TGMT(12,12), TEMP(12,12), R(NEQ,NROOT), TEMP2(NEQ,NROOT),
4 RTRAN(NROOT,NEQ), SIGMU(NUMNP), ID(6,NUMNP), XYZ(6,NUM), MATP(NUM),
5 SSMZZ(NROOT,NROOT)
C
C THIS ROUTINE FIRST COMPUTES THE GLOBAL HYDRODYNAMIC DAMPING MATRIX
C
CD=1.0/2.0
WATDEN=1000
SQRT8=SQRT((8/PI))
OPEN(10, FILE='TD.DAT', STATUS='OLD')
DO I=1, NUMNP
READ(10, 1110) ID(I), J(I), JNUM=1, 6
END DO
1110  FORMAT (6I5)
C
C MUST READ IN SIGMZZ'S FROM LINEAR ANALYSIS
PRINT*; NUMNP = 'NUMNP
WRITE(IOUT, *) 'DETAILS OF ID ARRAY'
DO IE2=1, NUMNP
WRITE(IOUT, 1110) ID(JNUM, IE2), JNUM=1, 6
END DO
OPEN(41, FILE='SIGMZ.DAT', STATUS='OLD')
DO MI=1, NROOT
READ(41, 1000) SMZZ(MI, IN2), IN2=1, NROOT
END DO
1000  FORMAT (2F10.3)
C
C MUST READ IN SIGMU'S FROM LINEAR ANALYSIS
OPEN(21, FILE='SIGMU.DAT', STATUS='OLD')
DO IN3=1, NUMNP
READ(21, 1100) SIGMU(IN3)
END DO
1100  FORMAT (F15.3)
C
C INITIALISE MATRICES
DO IP1=1, 13
DO IP2=1, 13
TGM1(IP1, IP2)=0.0
END DO
END DO
DO IP1=1, 12
DO IP2=1, 12
CHELM(IP1, IP2)=0.0
CHELG(IP1, IP2)=0.0
TGM(IP1, IP2)=0.0
TGMT(IP1, IP2)=0.0
END DO
END DO
DO IP3=1, 600
DO IP4=1, 600
CHGT(IP3, IP4)=0.0
END DO
END DO
DO IP5=1, NEQ
DO IP6=1, NEQ
CHG(IP5, IP6)=0.0
END DO
END DO
C

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DEFINE TRANSFORMATION MATRIX - CONSTANT FOR ALL MEMBERS
TGM1(1,3)=-1
TGM1(2,2)=1
TGM1(1,3)=1
DO IP7=1,3
  DO IP8=1,3
    TGM(IP7,IP8)=TGM1(IP7,IP8)
  END DO
END DO

LOOP OVER ALL ELEMENTS
DO I1=1,NUME
  MTYPE=MATP(I1)
  IEL=INUME(I1)  ! INODE NO. FOR NODE 1 OF ELEMENT
  JEL=INUME(I1)  ! INODE NO. FOR NODE 2 OF ELEMENT

  CHECK IF MEMBER IS HORIZONTAL & IF SO IGNORE IT
  IF(FELFLG(I1),EQ.1)THEN
    CALL CHECK IF ELEMENT IS IN WATER
    IF(XYZ(3,I1),LT.0.0.OR.XYZ(6,I1),LT.0.0)THEN
      CALL CHECK IF ELEMENT IS A FOUNDATION MEMBER
      IF(XYZ(3,11),NE.XMUD.AND.XYZ(6,11),NE.XMUD)THEN
        CALL CALCULATE ELEMENT LENGTH XL
        XL2=0.0
        DO II=1,3
          DX=XYZ(II+3,I1)-XYZ(II,I1)
          XL2=XL2+DX**2.
        END DO
        XL=SQRT(XL2)

        COMPUTE DRAG FORCE COEFFICIENT
        SIGU=(SIGMU(I1)+SIGMU(J1))/2
        DRAG=WATDEN*CD*SQRT8*SIGU

        DEFINE ENTRIES IN LOCAL DAMPING MATRIX
        CHELM(2,2)=13*XL2/35
        CHELM(2,6)=58*XL2/105
        CHELM(2,8)=XL2/10
        CHELM(2,12)=13*XL2/420
        CHELM(6,2)=58*XL2/105
        CHELM(6,6)=2*XL2/105
        CHELM(6,8)=13*XL2/420
        CHELM(6,12)=XL2/10
        CHELM(6,12)=XL2/10
        CHELM(8,2)=XL2/10
        CHELM(8,6)=13*XL2/420
        CHELM(8,8)=12*XL2/35
        CHELM(8,12)=11*XL2/210
        CHELM(12,2)=13*XL2/420
        CHELM(12,6)=XL2/10
        CHELM(12,12)=11*XL2/210
        DO IC1=1,12
          DO IC2=1,12
            CHELM(IC1,IC2)=CHELM(IC1,IC2)*DRAG
          END DO
        END DO
      END IF
    END IF
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C EXPRESS HYDRODYNDAMIC DAMPING MATRIX IN GLOBAL COORDINATES
TEMP=MATMUL(CHELM,TGM)
CHELG=MATMUL(TGMT,TEMP)

C DEFINE GLOBAL STRUCTURE HYDRODYNAMIC DAMPING MATRIX
K=6*(IEL-1)+1
JP5=J+5
KP5=K+5

JC1=0
JC2=0
DO J1=J,JP5
   JC1=JC1+1
   DO J2=J,JP5
      JC2=JC2+1
      CHGT(J1,J2)=CHELG(JC1,JC2)+CHGT(J1,J2)
      END DO
   END DO

DO K1=K,KP5
   JC1=JC1+7
   DO K2=K,KP5
      JC2=JC2+7
      CHGT(K1,K2)=CHELG(JC1,JC2)+CHGT(K1,K2)
      END DO
   END DO

DO J1=J,JP5
   JC1=JC1+1
   DO K2=K,KP5
      JC2=JC2+7
      CHGT(J1,J2)=CHELG(JC1,JC2)+CHGT(J1,K2)
      END DO
   END DO

END DO

C END ELEMENT LOOP

C DELETE FIXED ROWS AND COLUMNS OF DAMPING MATRIX
ICOUNT=0
DO ID1=1,NUMNP
   A106
DO ID2=1,6
   IF(ID(ID2,ID1).NE.1)THEN
      CONTINUE
   ELSE
      ICOUNT=ICOUNT+1
      JCOUNT=0
      DO ID3=1,NUMNP
         DO ID4=1,6
            IF(ID(ID4,ID3).EQ.1)THEN
               CONTINUE
            ELSE
               JCOUNT=JCOUNT+1
               CHG(ICOUNT,JCOUNT)=CHGT(ID1,ID2)
            END IF
         END DO
      END DO
   END IF
END DO
END DO

C DEFINE TRANSPOSE OF R
DO IR1=1,NEQ
   DO IR2=1,JROOT
      RTRAN(IR2,IR1)=R(IR1,IR2)
   END DO
END DO

C COMPUTE EQUIVALENT DIAGONALISED HYDRODYNAMIC DAMPING
DO IZ1=1,NROOT
   DO IZ2=1,NROOT
      CDIAGG(IZ1)=PCP(IZ1,IZ2)*SMZZ(IZ1,IZ2)/SMZZ(IZ1,IZ1)
      1+CDIAGG(IZ1)
   END DO
END DO

C COMPUTE MODAL STRUCTURAL DAMPING FOR MODE i (MSDI)
C WHERE ETA IS MODAL DAMPING RATIO
DO KV2=1,JROOT
   RTEIG=SQRT(EIGV(KV2))
   RMSDI=2*ETA*RTEIG
   C TOTAL DECOUPLED DAMPING TERMS ARE
   CTOT(KV2)=RMSDI+CDIAGG(KV2)
   END DO

CALL RGEN(A(N16),A(N17),CTOT)
RETURN
END
SUBROUTINE RGEN(R,EIGV,CTOT)
C
C ..................................................................
C . THIS PROGRAM CALCULATES THE RECEPTANCES OF THE GENERALISED & ACTUAL
C . COORDINATES
C ..................................................................
C
INCLUDE 'SPEC.CMN'
DIMENSION R(NEQ,NROOT),EIGV(NROOT),CTOT(NROOT),
1 ZZ1(NROOT,NEQ,NFRQ),ZZ2(NROOT,NEQ,NFRQ),ZTEMP1(NROOT),
2 ZTEMP2(NROOT),H1TEMP(NEQ),H2TEMP(NEQ),REC(NEQ,NFRQ)
DIMENSION ZZ1A(NROOT,NEQ,NFRQ),ZZ2A(NROOT,NEQ,NFRQ),TEMPAI(NROOT),
4 TEMPAP(NROOT),AITEMP(NEQ),A2TEMP(NEQ)

OPEN(20,FILE='RECP.OUT',STATUS='UNKNOWN')
C
C LOOP OVER ALL FREQUENCIES
DO IFRQ=1,NFRQ
FRQ1=IFRQ
FREQ=DELTAW*FRQ1
FREQ2=FREQ**2
C
C PUT A UNIT LOAD AT ALL DEGREES OF FREEDOM IN TURN,
C ie LOOP OVER ALL DOF'S 1=1,NEQ
DO IRT1=1,NEQ
C COMPUTE RECEPTANCES AT ALL DOF'S DUE TO LOAD AT IRT1
C AT FREQUENCY IFRQ
C
DO IRT2=1,NROOT
W2=EIGV(IRT2)
DENOM=(W2-FREQ2)**2 + FREQ2*CTOT(IRT2)*CTOT(IRT2)
DENOMA=(W2-FREQ2)**2 + FREQ2*CTOT(IRT2)*CTOT(IRT2)
ZZ1(IRT2,IRT1,IFRQ)=R(IRT1,IRT2)*(W2-FREQ2))/DENOM
ZZ2(IRT2,IRT1,IFRQ)=-R(IRT1,IRT2)*FREQ*CTOT(IRT2)/DENOM
ZZA1(IRT2,IRT1,IFRQ)=(R(IRT1,IRT2)*(W2-FREQ2))/DENOMA
ZZA2(IRT2,IRT1,IFRQ)=-R(IRT1,IRT2)*FREQ*CTOT(IRT2)/DENOMA
C
END DO
C
END DO
C
END DO
C
C NEXT COMPUTE ACTUAL RECEPTANCES
C
DO I1=1,NEQ !LOOP FOR EACH APPLIED LOAD
C
DO I2=1,NFRQ !LOOP OVER ALL FREQUENCIES
C
DO I3=1,NROOT !LOOP OVER ALL EIGENVECTORS
ZTEMP1(I3)=ZZ1(I3,I1,I2)
ZTEMP2(I3)=ZZ2(I3,I1,I2) !TEMPORARY FILES FOR MATRIX MULTIPLICATION
TEMPAI(I3)=ZZA1(I3,I1,I2)
TEMPA2(I3)=ZZA2(I3,I1,I2) !TEMPORARY FILES FOR MATRIX MULTIPLICATION
END DO
H1TEMP=MATMUL(R,ZTEMP1)
H2TEMP=MATMUL(R,ZTEMP2)
AITEMP=MATMUL(R,TEMPAI)
A2TEMP=MATMUL(R,TEMPA2)
C
PUT VALUES FROM TEMPORARY VECTORS INTO MATRICES OF RECEPTANCES
DO I4=1,NEQ
H1(I4,I1,I2)=H1TEMP(I4) !REAL COMPONENT OF RECEPTANCE
H2(I4,I1,I2)=H2TEMP(I4) !IMAGINARY COMPONENT OF RECEPTANCE
HA1(I4,I1,I2)=AITEMP(I4) !REAL COMPONENT OF RECEPTANCE
HA2(I4,I1,I2)=A2TEMP(I4) !IMAGINARY COMPONENT OF RECEPTANCE
END DO
C
H1(I4,I1,I2) REPRESENTS THE REAL RECEPTANCE AT DOF I4 DUE TO
C AN APPLIED LOAD AT DOF I1 AT FOR AN APPLIED LOAD FREQUENCY OF I2
END DO !FREQUENCY LOOP

A108
C END DO  !APPLIED LOAD LOOP
C PRINT OUT SOME RECEPANCE FUNCTIONS
DO IS=1,NFRQ
RECI5(I5)=0.0
RECI5(I5+1)=0.0
RECI5(I5+2)=0.0
RECI5(I5+3)=0.0
RECI5(I5+4)=0.0
RECI5(I5+5)=0.0
RECI5(I5+6)=0.0
DO J1=1,NEQ
DO J2=1,NEQ
RECI5(I5)=(H1(J1,J1,I5)*H1(J1,J2,I5)+H2(J1,J1,I5)*H2(J1,J2,I5)) + 
RECI5(I5)
RECI5(I5+1)=(H1(J1,J1,I5)*H1(J1,J2,I5)+H2(J1,J1,I5)*H2(J1,J2,I5)) + 
RECI5(I5+1)
RECI5(I5+2)=(H1(J1,J1,I5)*H1(J1,J2,I5)+H2(J1,J1,I5)*H2(J1,J2,I5)) + 
RECI5(I5+2)
RECI5(I5+3)=(H1(J1,J1,I5)*H1(J1,J2,I5)+H2(J1,J1,I5)*H2(J1,J2,I5)) + 
RECI5(I5+3)
RECI5(I5+4)=(H1(J1,J1,I5)*H1(J1,J2,I5)+H2(J1,J1,I5)*H2(J1,J2,I5)) + 
RECI5(I5+4)
RECI5(I5+5)=(H1(J1,J1,I5)*H1(J1,J2,I5)+H2(J1,J1,I5)*H2(J1,J2,I5)) + 
RECI5(I5+5)
RECI5(I5+6)=(H1(J1,J1,I5)*H1(J1,J2,I5)+H2(J1,J1,I5)*H2(J1,J2,I5)) + 
RECI5(I5+6)
END DO
END DO
RECI5(I5)=(H1(1,1,I5)*H1(1,1,I5)+H2(1,1,I5)*H2(1,1,I5))
WRITE(20,100)RECI5(RECI5(I5),RECI5(I5),RECI5(I5))
END DO
CLOSE(20)
CALL FORCE(IA(l),A(N13),A(N8),A(N12),IA(IN4))
RETURN
100 FORMAT(6E10.3)
END
SUBROUTINE FORCE(ID,XYZ,ATOT,DTOT,MATP)
C
C SUBROUTINE WR TO COMPUTE THE COEFFICIENTS OF
C THE DRAG AND INERTIA TERMS OF THE FORCE VECTOR
C
C INCLUDE 'SPEC.CMN'
DIMENSION XYZ(6,NUME),DTOT(NUME),ATOT(NUME),CFMM(12),
CFDM(12),MATP(NUME),ID(6,NUMNP)
C
C SET INERTIA AND DRAG COEFFICIENTS
CM1=1.0
CM2=1.0
CMT=CM1+CM2 !TOTAL INERTIA COEFFICIENT
CD=1.0/2.0
WATDEN=1000 !WATER DENSITY
C
C INITIALISE MATRICES
DO IP2=1,12
CFMM(IP2)=0.0
CFDM(IP2)=0.0
END DO
C
C INITIALISE CFMGT & CFDGT
DO K1=1,NEQ
CFMGT(K1)=0.0
CFDGT(K1)=0.0
END DO
C
C LOOP OVER ALL ELEMENTS
DO II=1,NUME
MSTYPE=MATP(II)
IEL=NUME(II) !NODE NO.FOR NODE 1 OF ELEMENT
JEL=NUME(II) !NODE NO.FOR NODE 2 OF ELEMENT
C
C CHECK IF MEMBER IS HORIZONTAL & IF SO IGNORE IT
IF(IELFLG(II).EQ.1)THEN
C
C CHECK IF ELEMENT IS IN WATER
IF(XYZ(3,II).LT.0.0.OR.XYZ(6,II).LT.0.0)THEN
C
C CHECK IF ELEMENT IS A FOUNDATION MEMBER
IF(XYZ(3,II).NE.XMUD.AND.XYZ(6,II).NE.XMUD)THEN
C
C CALCULATE ELEMENT LENGTH XL
XL2=0.0
DO II=1,3
DX=XYZ(II,II)-XYZ(II+3,II)
XL2=XL2+DX**2.
END DO
XL=SQRT(XL2)
C
C INITIALISE COEFFICIENTS
DO I2=1,12
CFMM(I2)=0.0
CFDM(I2)=0.0
END DO
C
C COMPUTE INERTIA FORCE COEFFICIENT TERMS
CFMM(2)=WATDEN*ATOT(MSTYPE)*CMT*XL/2
CFMM(6)=WATDEN*ATOT(MSTYPE)*CMT*XL2/12
CFMM(8)=WATDEN*ATOT(MSTYPE)*CMT*XL/2
CFMM(12)=-WATDEN*ATOT(MSTYPE)*CMT*XL2/12
C
C COMPUTE DRAG FORCE COEFFICIENT TERMS
CFDM(2)=WATDEN*DTOT(MSTYPE)*CD*XL/2
CFDM(6)=WATDEN*DTOT(MSTYPE)*CD*XL2/12
CFDM(8)=WATDEN*DTOT(MSTYPE)*CD*XL/2
CFDM(12)=-WATDEN*DTOT(MSTYPE)*CD*XL2/12
ELSE !MEMBER IN AIR AND THUS NO FORCE
C
DO I2=1,12
CFMM(I2)=0.0
CFDM(I2)=0.0
END DO
A110
END IF

ELSE  !MEMBER IS HORIZONTAL

DO 112=1,12
CFMM(I12)=0.0
CFDM(I12)=0.0
END DO

END IF

ELSE  !MEMBER IS IN FOUNDATION

DO 113=1,12
CFMM(I13)=0.0
CFDM(I13)=0.0
END DO

END IF

C NEXT INSERT COEFFICIENTS INTO CORRECT LOCATION IN THE GLOBAL FORCE
C COEFFICIENT MATRICES FOR THE TOTAL STRUCTURAL SYSTEM (CFMGT + CFDGT)

IELT1=(IEL-1)*3+1
JELT1=(JEL-1)*3+1
IELT3=(IEL-1)*3+3
JELT3=(JEL-1)*3+3

C FIRST NODE OF ELEMENT
ID1=ID(1,IEL)
ID3=ID(5,IEL)
IF(ID1.NE.0)THEN
CFMGT(ID1)=CFMM(2)+CFMGT(ID1)
CFMGT(ID3)=-CFMM(6)+CFMGT(ID3)
CFDGT(ID1)=CFDM(2)+CFDGT(ID1)
CFDGT(ID3)=-CFDM(6)+CFDGT(ID3)
ELSE
CONTINUE
END IF

C SECOND NODE OF ELEMENT
ID2=ID(1,JEL)
ID4=ID(5,JEL)
IF(ID2.NE.0)THEN
CFMGT(ID2)=CFMM(8)+CFMGT(ID2)
CFMGT(ID4)=-CFMM(12)+CFMGT(ID4)
CFDGT(ID2)=CFDM(8)+CFDGT(ID2)
CFDGT(ID4)=-CFDM(12)+CFDGT(ID4)
ELSE
CONTINUE
END IF

C END MEMBER LOOP

DO 114=1,12
CFMM(I14)=0.0
CFDM(I14)=0.0
END DO

C CALL SUBROUTINE TO COMPUTE CROSS SPECTRA AND CONVOLUTIONS OF NODE WATER
C VELOCITIES AND ACCELERATIONS
CALL SPECTRA(A(N1),A(N3))

RETURN
END
SUBROUTINE SPECTRA(X,Z)
C
SUBROUTINE TO COMPUTE SPECTRA & CROSS SPECTRA OF RELEVANT VELOCITIES & ACCELERATIONS OF
WATER PARTICLES WHICH ARE REQUIRED TO COMPUTE THE ZERO TH ORDER FORCE SPECTRUM
INCLUDE 'SPEC.CMN'
DIMENSION X(NUMNP),Z(NUMNP),STEMP(NFRQ),CONTEM(NFRQ)
C
C COMPUTE STANDARD DEVIATIONS AND CROSS SPECTRA OF VELOCITIES AT EACH
SUBMERGED NODE POINT ABOVE THE MUDLINE
C
INITIALISE STANDARD DEVIATIONS, CROSS SPECTRA AND CONVOLUTIONS
OF CROSS SPECTRA TO ZERO
C
OPEN(40,FILE='SOUT.DAT',STATUS='UNKNOWN')
PI2=2*PI
PI3=PI*3
NF2=NFRQ/2
DO INT=1,NUMNP
SD(INT)=0.0
END DO
D=-XMUD
SQRT8P=SQR T((8/PI))
C LOOP OVER ALL ELEMENTS
DO I1=1,NUMNP
X11=X(I1)
Z11=Z(I1)
C
C CHECK TO ENSURE NODE IS SUBMERGED AND NOT ON THE FOUNDATION
C
IF(Z(I1),LE.0.AND.Z(I1),GT,XMUD)THEN
DO I3=1,NUMNP
X22=X(I3)
Z22=Z(I3)
C
C CHECK TO ENSURE THAT NODE IS SUBMERGED BUT NOT ON FOUNDATION
C
IF(Z(I3),LE.0.AND.Z(I3),GT,XMUD)THEN
WL=1000
C LOOP OVER ALL FREQUENCIES
SDVEL=0.0
DO I2=1,NF2
n2=I2
WF=rI2*STEP
WF2=WF**2
TW=2*PIAVF
DWL=D/WL
C
C COMPUTE CROSS SPECTRA OF VELOCITIES AT EACH NODE
C
SW(I1,I3,I2)=(WF2)*(COSH(DKZ1))*(COSH(DKZ2))*SPECTR(I2)*
1COS((SINH(DK*Z11))**2)
C
C COMPUTE CROSS SPECTRA OF VELOCITIES/ACCELERATIONS AT EACH NODE
C
NOTE SVA WHEN I1=I3 IS ZERO BECAUSE OF SIN TERM
C
SVA(I1,I3,I2)=(WF*WF2)*(COSH(DKZ1))*(COSH(DKZ2))*SPECTR(I2)*
1SIN((SINH(DK*Z11))**2)
SAV(I1,I3,I2)=SVA(I1,I3,I2)

END DO IFREQUENCY LOOP
ELSE
CONTINUE
ENDIF
END DO INODELOOP
ELSE
CONTINUE
ENDIF
END DO

NOW COMPUTE THE THIRD CONVOLUTIONS OF THE CROSS SPECTRA OF VELOCITIES
C
C LOOP OVER ALL ELEMENTS
DO I4=I,NUMNP
X4=X(I4)
Z4=Z(I4)
C CHECK TO ENSURE NODE IS SUBMERGED AND NOT ON THE FOUNDATION
IF(Z4.LE.0.AND.Z4.GT.XMUD)THEN
DO I5=I,NUMNP
X5=X(I5)
Z5=Z(I5)
IF(Z5.LE.0.AND.Z5.GT.XMUD)THEN
DOI6=I,NF2
STEMP(I6)=SW(I4,I5)
END DO
CALL CONSPE(STEMP,CONTEM)
DOI7=I,NF2
CONV(K4,I5,I7)=CONTEM(I7)
ELSE
CONTINUE
ENDIF
END DO
ELSE
CONTINUE
ENDIF
END DO
C
C COMPUTE STANDARD DEVIATIONS
DO K1=1,NUMNP
SDSQ=0.0
DO K3=1,NF2
IF(K3.EQ.1.OR.K3.EQ.NF2)THEN
RULL=1.
ELSE IF(IIP.EQ.2)THEN
RULL=4.
ELSE
RULL=2.
ENDIF
ENDIF
SDSQ=STEP*RULL*SW(K1,K1,K3)+SDSQ
END DO
SD(K1)=SQRT(SDSQ)
C
C PRINT OUT SPECTRA AND CONVOLUTIONS OF WATER PARTICLE VELOCITIES
IF(K1.EQ.2)THEN
PRINT"StandardDeviationVelAtNode2=",SD(2)
DO K2=1,NF2
WRITE(40,"(5F20.10)"
WRITE(40,"(4F20.10)"
C
END DO
ELSE
CONTINUE
ENDIF
END DO
CALL SPECOUT
RETURN
END
SUBROUTINE CONSPE(XIN,XOUT)
USE MSIMSL

PARAMETER (NX=256,NFRQ=128)

COMPLEX CX(NX),CZ1(NX),CZ2(NX),CXOUT(NX),CXOUT1(NX)

DIMENSION Z5(NX*2),X1(NX*2),X2(NX*2),XOUT(NFRQ),XOUT1(NX*2),
1 Z(NX*2),ZHAT(NX*2),X(NX*2),XOUT(NFRQ),XOUT1(NX*2),
2 Z1(NX*2),XIN(NFRQ)

DO I1 = 1,NX
  CX(I1)=0.0
  CZ1(I1)=0.0
  CX(I1)=CMPLX(X(I1))
END DO

DO I2 = 1,256
  CX(I2)=CMPLX(XIN(I2))
END DO

CALL FFTCB(NX,CX,CXOUT)

CALL FFTCF(NX,CZ1,CXOUT1)

RETURN
END
SUBROUTINE SPEC1
C
C SUBROUTINE TO COMPUTE SPECTRA OF RESPONSE FOR THE ZEROTH ORDER EQUATION FROM THE SPECTRA
C OF FORCE AND THE RECEP'TANCES
C
INCLUDE 'SPEC.CMN'
DIMENSION SOUT(NEQ,NFRQ),SOUT1(NEQ,NFRQ),FSOUT(NEQ,NFRQ),
1 CONOUTI(NEQ,NFRQ),CONOUT(NEQ,NFRQ),CO(NEQ),COI(NEQ),
2 Riforce(NEQ,NFRQ)
C
C OPEN OUTPUT FILE
OPEN(20,FILE='SPEC.OUT',STATUS='UNKNOWN')
OPEN(30,FILE='FORSPEC.OUT',STATUS='UNKNOWN')
C
PI8=8/PI
SPI8=SQRT(PI8)
PI34=4/(3*PI)
C
INITIALISE OUTPUT SPECTRUM TO ZERO
DO K1=1,NEQ
DO K2=1,NFRQ
SOUT(K1,K2)=0.0
END DO
END DO
C
DO I1=1,NEQ
NF2=NFRQ/2
C LOOP OVER ALL FREQUENCIES
DO I3=1,NF2
sout(I1,I3)=0.0
RI3=I3
WF=RI3*DELTAW
WF2=WF**2
C
DOUBLE LOOP OVER ALL APPLIED DEGREES OF FREEDOM
DO I4=1,NEQ
C FIND NODE NO. ASSOCIATED WITH DOF I4
RI4=I4-1
RI43=RI4/3
INT4=INT(RI43)+1
C
DO I5=1,NEQ
C FIND NODE NO. ASSOCIATED WITH DOF I5
RI5=I5-1
RI53=RI5/3
INT5=INT(RI53)+1
IF(SD(INT4).NE.0.AND.SD(INT5).NE.0)THEN
C
C COMPUTE CROSS SPECTRUM OF FORCE SF(I4,I5) AT FREQUENCY I3
IF(I4.EQ.I5)THEN
SF=CFMGT(I4)*CFMGT(I5)*(WF2*SVV(INT4,INT5,I3))+
1PI8*CFDGT(I4)*CFDGT(I5)*SD(INT4)*SD(INT5)*SVV(INT4,INT5,I3)+
2SPI8*(SD(INT4)*CFDGT(I4)*CFMGT(I5)*SVA(INT4,INT5,I3))+
3SD(INT5)*CFDGT(I5)*CFMGT(I4)*SVA(INT4,INT5,I3)+
4PI34*CFDGT(I4)*CFDGT(I5)*CONVY(INT4,INT5,I3)
5/(SD(INT4)*SD(INT5))
C
else
SF=CFMGT(I4)*CFMGT(I5)*(WF2*SVV(INT4,INT5,I3))+
1PI8*CFDGT(I4)*CFDGT(I5)*SD(INT4)*SD(INT5)*SVV(INT4,INT5,I3)+
2PI34*CFDGT(I4)*CFDGT(I5)*CONVY(INT4,INT5,I3)
5/(SD(INT4)*SD(INT5))
end if
C
RI52=I5-2
RI42=I4-2
IF(RI52.EQ.2.OR.RI42.EQ.0)THEN
SF=0
C
DO J=1,NF2
WRITE(30,100)FSOUT(4,J),CONOUT(4,J),CONOUT(4,J),FSOUT(19,J),
1SVV(3,3,J),CONVV(3,3,J),SVV(5,5,J),CONVV(5,5,J)
WRITE(20,*)SOUT(1,J),SOUT(16,J),SOUT(7,J)
END DO

100 FORMAT(9E10.4)
RETURN
END

END DO !END DOUBLE LOOP OF APPLIED DEGREES OF FREEDOM
END DO !END FREQUENCY LOOP
END DO !LOOP 1

DO J=1,NF2
WRITE(30,100)FSOUT(4,J),CONOUT(4,J),CONOUT(4,J),FSOUT(19,J),
1SVV(3,3,J),CONVV(3,3,J),SVV(5,5,J),CONVV(5,5,J)
WRITE(20,*)SOUT(1,J),SOUT(16,J),SOUT(7,J)
END DO

100 FORMAT(9E10.4)
RETURN
END
APPENDIX VIII

CUMULANT-NEGLECT CLOSURE
Cumulant-Neglect Closure

When solving multi-dimensional non-linear problems in the frequency domain, approximate solution techniques are generally needed. The most frequently used approximation scheme is the equivalent linearisation procedure in which the original system is replaced by an equivalent linear system.

Another closure procedure focuses on the properties of the cumulants of random processes and is known as the cumulant-neglect closure scheme. Consider the random variables $X_j$, $X_k$, $X_i$, and $X_m$, then the following relationship exists between cumulants and statistical moments, Stratonovich, (1963)

\[
E[X_j] = \kappa_1[X_j]
\]

\[
E[X_jX_k] = \kappa_2[X_jX_k] + \kappa_1[X_j]\kappa_1[X_k]
\]

\[
E[X_jX_kX_i] = \kappa_3[X_jX_kX_i] + 3\{\kappa_1[X_j]\kappa_2[X_k,X_i]\} + \kappa_1[X_j]\kappa_1[X_k]\kappa_1[X_i]
\]

\[
E[X_jX_kX_iX_m] = \kappa_4[X_jX_kX_iX_m] + 3\{\kappa_2[X_jX_k]\kappa_2[X_i,X_m]\} + 4\{\kappa_1[X_j]\kappa_3[X_k,X_i,X_m]\} + 6\{\kappa_1[X_j]\kappa_1[X_k]\kappa_2[X_i,X_m]\} + \kappa_1[X_j]\kappa_1[X_k]\kappa_1[X_i]\kappa_1[X_m]
\]

where

$\kappa_1[X_j]$ denotes the first cumulant of the random variable $X_j$

$\{ \}^s$ denotes a symmetrising operation with respect to its arguments, Lin and Cai (1995)
Lin and Cai (1995) state that the physical significance of a cumulant decreases as the order increases and that the most important properties of a random process are contained in the lower order cumulants. Lin and Wu (1984) show that by using a second-order cumulant-neglect closure scheme (that is, neglecting those cumulants above the second order) that good results were obtained against an exact solution when applied to a duffing oscillator under random excitation. Other researchers have also suggested the second-order cumulant neglect closure scheme gives good results in practical applications, Soong, (1973).

The average $E[X_jX_kX_l]$ contains cumulants of order 1, 2 and 3. From above the first order cumulant is equal to the time average of the random process. By re-arranging the second equation above the second-order cumulant may be written as

$$
\kappa_2[X_jX_k] = E[X_jX_k] - \kappa_1[X_j]\kappa_1[X_k] \\
= E[X_jX_k] - E[X_j]E[X_k]
$$

When cumulants of higher order than two are ignored, the following may be written.

$$
E[X_jX_kX_l] = 3\{\kappa_1[X_j]\kappa_2[X_k,X_l]\} + \kappa_1[X_j]\kappa_1[X_k]\kappa_1[X_l] \\
= 3\{E[X_j](E[X_k,X_l] - E[X_l]E[X_k])\} + E[X_j]E[X_k]E[X_l]
$$

Now if the processes $X_j$, $X_k$, and $X_l$, are zero mean stochastic processes then

$$
E[X_jX_kX_l] = 0
$$

This result is employed in Chapter 5 in developing the first order response terms.

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Again, if cumulants of higher order then two are ignored, the average of the product of four zero mean stochastic processes may be written as

\[
E[X_j X_k X_l X_m] = 3\{\kappa_2[X_j X_k]\kappa_2[X_l X_m]\} + 6\{\kappa_1[X_j]\kappa_1[X_k]\kappa_2[X_l X_m]\} + \kappa_1[X_j]\kappa_1[X_k]\kappa_1[X_l]\kappa_1[X_m]
\]

Since variables \(X_j, X_k, X_l\), and \(X_m\) have zero mean values, we may write the last expression as

\[
E[X_j X_k X_l X_m] = 3\{\kappa_2[X_j X_k]\kappa_2[X_l X_m]\} = \kappa_2[X_j X_k]\kappa_2[X_l X_m] + \kappa_2[X_j X_l]\kappa_2[X_k X_m] + \kappa_2[X_j X_m]\kappa_2[X_k X_l]
\]

Substituting for the second-order cumulants from equation as defined above and setting the mean values of the individual processes to zero leads to

\[
E[X_j X_k X_l X_m] = E[X_j X_k]E[X_l X_m] + E[X_j X_l]E[X_k X_m] + E[X_j X_m]E[X_k X_l]
\]

This result is employed in Chapter 5 in developing the second order response terms.