Correcting for Seasonality: II Correction of Moving Averages for Trend, especially at High or Low Turning Points

by

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It is known that at monotonic sections of the time curve (ideally when the true trend is linear to time), the unweighted moving average (m.a.) gives a good representation of the true trend, though it may eliminate cycles of length less than the moving average period. At high and low turning points of the trend the moving average (graphed at its mid-time point) must modify the true trend, i.e. at a low turning point the m.a. will run above the trend line and at high turning point below it. This is clear from the fact that at e.g. a high turning point the m.a. is the centre of gravity of a curve section whereas the top point of the trend is on its periphery.

The object of this note is to correct the m.a. at these turning points so as to yield the true trend value.

Let the time unit be the calendar quarter and let a turning point, e.g. a minimum, be near $t = 0$. Then the individual time points of reference for the original data near $t = 0$ will be marked $-\frac{5}{2}$, $-\frac{3}{2}$, $-\frac{1}{2}$, $+\frac{1}{2}$, $+\frac{3}{2}$, $+\frac{5}{2}$, and the centred four-quarter by m.a. points of reference will be $-2$, $-1$, $0$, $+1$, $+2$. It is assumed that the true time trend (with seasonality eliminated) will yield the same moving average as does the original series (without seasonality correction). Suppose that near the origin the true trend can be represented by
(1) \[ Y_t = a_0 + a_1 t + a_2 t^2 \]

and the corresponding m.a. point \( y_t \). Our object in the first place is to compute the coefficients \( a_0, a_1 \) and \( a_2 \) from the m.a. We have

\[
y_{-1} = \frac{1}{4}(Y_{-5} + \frac{Y_{-3}}{2} + \frac{Y_{-1}}{2} + \frac{Y_{+1}}{2})
\]

and analogous expressions for \( y_0 \) and \( y_{+1} \). Then, from (1), we have the following three equations to determine the coefficients from the known m.a. values \( y_{-1}, y_0, y_{+1} \) ...

(2) \[
y_{-1} = a_0 - a_1 + \frac{9}{4} a_2 \quad \text{Note: } \frac{9}{4} = \frac{1}{4}\left[(-\frac{5}{2})^2 + (-\frac{3}{2})^2 + (-\frac{1}{2})^2 + (+\frac{1}{2})^2\right] \text{ etc}
\]

\[
y_0 = a_0 + \frac{5}{4} a_2
\]

\[
y_{+1} = a_0 + a_1 + \frac{9}{4} a_2
\]

From (2) we find the following values of \( a_0, a_1, a_2 \):

\[
a_0 = \frac{1}{6}(-5y_{-1} + 18y_0 - 5y_{+1})
\]

(3) \[
a_1 = \frac{1}{2}(-y_{-1} + y_{+1})
\]

\[
a_2 = \frac{1}{2}(y_{-1} - 2y_0 + y_{+1}).
\]

Now let the m.a. graph near the low point be

(4) \[ y_t = a_0 t + a_1 t + a_2 t^2 \]

The coefficients are found from the values of \( y_t \) at \( t = -1, 0, +1 \):
\[ y_{-1} = a_0' - a_1' + a_2' \]

(5) \[ y_0 = a_0' \]

\[ y_{+1} = a_0' + a_1' + a_2' \]

whence

\[ a_0' = y_0 \]

(6) \[ a_1' = \frac{1}{2}(-y_{-1} + y_{+1}) \]

\[ a_2' = \frac{1}{2}(y_{-1} - 2y_0 + y_{+1}) \]

Comparing (3) and (6) we see that

\[ a_1' = a_1 \]

(7) \[ a_2' = a_2 \]

Hence

(8) \[ y_0 - y_o = a_0' - a_o = y_0 - \frac{1}{8}(-5y_{-1} + 18y_0 - 5y_{+1}) \]

(from (6) and (3))

\[ = \frac{5}{8}(y_{-1} - 2y_0 + y_{+1}) \]

or the true \( Y_c \) is found from the (known) m.a. \( y_0 \) by deducting

(9) \[ \frac{5}{8}(y_{-1} - 2y_0 + y_{+1}) \]

The latter expression will clearly be positive at a low point on the m.a. curve. Obviously at a high point an
identical expression with sign changed is added to the m.a.
maximum to give the true high point. These turning points
will be systematically computed, marked on the chart, and
the moving average graph adjusted free-hand near the turn-
ing points to give the true trend. Other guide points can
be computed as desired, even though these are not maxima
or minima. The adjustment (9) can be applied at any point
on the m.a. graph, with \( y_0 \) as the m.a. ordinate of reference
and \( y_{-1} \) and \( y_{+1} \) the ordinates to left and right. Clearly
this adjustment is strictly necessary only wheter the m.a.
graph is markedly curved: the adjustment is zero where
the graph is linear. It would not really be onerous to
make the correction at every point when one's period is,
say, 5 years so that there are only 20 observations to be
dealt with.

The correction at (9) applies to the four-
quarterly m.a. The corresponding correction at \( t = 0 \)
for the twelve-monthly m.a. is

\[
(10) \quad \frac{572}{96}(y_{-1} - 2y_0 + y_{+1})
\]

or, to a close approximation,

\[
(10)' \quad 6(y_{-1} - 2y_0 + y_{+1})
\]

It will be borne in mind that at (9) the \( y_{-1} \), \( y_{+1} \) refer
to quarterly but at (10) and \( (10)' \) to monthly data. That
a quadratic approximation to trend (an assumption implied
in the method used) extending over 14 time observations
(only 6 in the case of quarterly data), may be made, may
seem somewhat implausible.
As an application of (10)' consider the m.a.'s relating to miscellaneous freight-car loadings given in an article by H. C. Barton. There are three well-marked turning points as follows:

<table>
<thead>
<tr>
<th></th>
<th>Low (November 1932)</th>
<th>High (April 1937)</th>
<th>Low (June 1938)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{-1}$</td>
<td>69.8</td>
<td>116.7</td>
<td>88.1</td>
</tr>
<tr>
<td>$Y_0$</td>
<td>69.4</td>
<td>116.8</td>
<td>88.0</td>
</tr>
<tr>
<td>$Y_{+1}$</td>
<td>69.7</td>
<td>116.2</td>
<td>88.6</td>
</tr>
<tr>
<td>Correction (10)'</td>
<td>4.2</td>
<td>-4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>Corrected value</td>
<td>65.2</td>
<td>121.0</td>
<td>83.8</td>
</tr>
<tr>
<td>Mr Barton's freehand value</td>
<td>70.5</td>
<td>119.9</td>
<td>83.3</td>
</tr>
</tbody>
</table>

The identity (except for sign) of the corrections is fortuitous. In two cases the corrected value agrees well with Mr Barton's. The discrepancy at the low for November 1932 is mainly due to the fact that about this time Mr Barton took account of a short-term cycle which was accorded a "peak" of 74.1 a month before (i.e. in October 1932): the present method can do nothing about minor cycles. Actually Mr Barton reaches a freehand value of 65.8 (near the 65.2 shown above) three months later, i.e. in February 1933, his value of 65.8 being nearly 8 below the corresponding m.a. of 72.6

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