Macro-economic model building
for Ireland
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Macro-models describing the working of a nation's economy have been constructed for a number of countries, differing from each other in the number of variables used, the extent of disaggregation introduced, and the complexity of interrelationships. At one end of the scale, there are models consisting of a few equations only, like the one developed for the Federal Republic of Germany by Gehrig (1965); at the other end, there are very detailed structures like the Social Science Research Council model for the United States of America (Klein, 1964). A new econometric model is not necessarily of general interest merely because it adds a new country to the list of those for which models have been constructed. However, a number of problems arise in model building which have to be answered anew in each context, in the light of the available data and the main purpose for which the model is constructed. It is hoped that the very simple model presented here still offers a few points of general interest.

The use of econometric models in conjunction with national accounts statistics formed the subject of a number of papers which were discussed at a conference at Chapel Hill, North Carolina, in 1962 and which were recently published (Conference on Research in Income and Wealth, 1964). The present model may also be described as a "model of income determination"; it is in fact designed for the main purposes of studying the relationships between the year-to-year changes in
some of the main aggregates appearing in the national accounts, and of forecasting the changes one year ahead. Some of its characteristics are as follows:

a) The model is recursive, and single-equation least-squares estimation is not only permissible but with the usual assumptions equivalent to maximum likelihood and therefore optimal; the main estimation problem which arises is the avoidance of multicollinearity. This means that it may prove possible to add on further equations, and in particular to explain a variable which at present is treated as exogenous, without having to recompute the established equations.

b) First differences are used throughout the main part of the analysis, and almost all variables appearing in the equations are differences between volume or price index numbers for successive years. This implies that the sum of several variables appears as a weighted average of the individual series with weights of the base year 1953. It also means that the coefficients may approximately, though not exactly, be interpreted as elasticities.

The conventions used in the notation of variables will be to describe the original data at constant or current prices and the implied price index numbers by capital letters, the first differences between volume or price index numbers by ordinary letters. Furthermore, a prime will denote a price term and a dot a current value term, a variable without prime or dot describing a term at constant prices. Thus, the various symbols used in connection with each concept are as follows:
Personal expenditure: 
Government expenditure: 
Gross fixed capital formation: 
Exports of goods and services: 
Imports of goods and services: 
Gross national product: 
Final demand: 
Market supplies: 
Stock changes: 

The only other variables used are changes in wholesale price index numbers for imports and for home production, denoted by \( m' \) and \( y' \), respectively to be consistent with the notation above; these include the effect of changes in customs duties.

The variables \( C, C', G, G' \ldots \) are those for which forecast values are ultimately to be derived, whilst the equations center around the variables \( c, c', g, g' \ldots \). Obviously

\[
\begin{align*}
C' &= 100 \frac{C'}{C} \\
c &= 100 \frac{(C - C_{-1})}{\bar{C}} \\
c' &= \frac{C'}{C} \cdot \bar{C} - C'_{-1}
\end{align*}
\]

where \( \bar{C} \) refers to the base year 1953 and \( C_{-1}, C'_{-1} \) to the year prior to the current year; similarly for the other variables.

Furthermore, final demand as defined here excludes stock changes; thus

\[
\begin{align*}
D &= C + G + I + X \\
S &= M + Y \\
I_s &= S - D
\end{align*}
\]

and similar relations exist for \( D, S \) and \( I_s \). Changes in the volume of final demand and of market supplies, \( d \) and \( s \), may be derived from \( D \) and \( S \) or as weighted
averages of other differences between volume index numbers. With the data for Ireland used here, the identities for \( d \) and \( s \) are

\[
d = 0.5293c' + 0.0855g' + 0.1115i' + 0.2737x' \\
s = 0.2808m + 0.7192y'
\]

The seven endogenous variables in the model as constituted at present are:

\( c, c', g', i', x', m, y \)

while the following ones are treated as exogenous

\( i, x, m', y', m''_w, y''_w \)

In addition, some lagged variables are used. \( g \) does not appear explicitly in the model, but only implicitly with a time lag as component of \( d_{-1} \).

Data for the years 1953-62 inclusive as published by the Central Statistics Office (1963) were used, and thus nine first differences are available as observations. Data for 1952 were also utilised to give the required first observation for lagged variables. Wholesale price index data are available from other sources.

The seven equations, not counting the identities for \( d \) and \( s \), are of three different kinds: there are four price relationships, two decision functions for imports and home production, and a consumption function.

The price relations were intended to express the price changes in final demand components \( c' \), \( g' \), \( i' \) and \( x' \) as linear regressions through the origin on price changes of imports \( m' \) and gross national product \( y' \). In the export price equation the coefficient of the import price change was found to be
negative, though small and not significant. The term with \( m' \) was therefore dropped from the equation for \( x' \); but to indicate the long-term tendency for import and export prices to rise to a similar extent, an adjustment term containing the difference between import and export price levels at the beginning of the period, \( M'_{-1} - X'_{-1} \), was introduced.

The variable \( M'_{-1} - X'_{-1} \) was also introduced, together with \( m' \) and \( y' \), into the other price relations, but the coefficients of \( M'_{-1} - X'_{-1} \) were not estimated; instead, the same coefficient giving an adjustment in the opposite direction to that for \( x' \) was inserted into the equations for \( c', g' \) and \( i' \). The coefficients of \( m' \) and \( y' \) in these equations were estimated.

The equations for the changes in volume of imports and gross national product contain four explanatory variables; changes in investment \( i' \); changes in exports \( x' \); the difference between changes in final demand and market supplies in the previous period \( d_{-1} - s_{-1} \); and the difference between changes in wholesale prices of home produced and imported goods \( y'_w - m'_w \). The third variable provides an adjustment for the position with regard to stocks, which do not enter the equations in any other way.

Some restrictions were put on the parameters. These were effected by estimating directly the coefficients in equations for \( s \) and \( m - y \), and thus obtaining indirectly the coefficients in the equations for \( m \) and \( y \). It was assumed that the stock position does not affect the shares of imports and home production in total market supplies and that the price relationship does not affect the total of market supplies. Hence the variables used in the equation for \( s \) are \( i, x \) and \( d_{-1} - s_{-1} \); the variables used in the equation for \( m - y \) are \( i, x \) and \( y'_w - m'_w \). The constant
term in the equation for \( m - y \) was negligible, and a
regression through the origin was estimated. \( R^2 \) was
.711 in the equation for \( s \) and .825 in the equation
for \( m - y \).

The version of the consumption function which
was chosen after some experiments expresses changes in
real personal expenditure \( c \) in terms of changes in real
gross national product \( y \), the difference between changes
in consumption and national product in the previous
period \( y_{-1} - c_{-1} \), and the changes in the price index for
personal expenditure \( c' \). Apart from the identities it
is the only equation containing more than one endogenous
variable. It was directly estimated.

The set of equations obtained is as follows:

\[
\begin{align*}
c' &= .3815m' + .8186y' - .0784 (M'_{-1} - X'_{-1}) \quad (R^2 = .928) \\
g' &= .3841m' + .0267y' - .0784 (M'_{-1} - X'_{-1}) \quad (R^2 = .641) \\
i' &= .7169m' + .7081y' - .0784 (M'_{-1} - X'_{-1}) \quad (R^2 = .710) \\
x' &= .3942y' + .2079 (M'_{-1} - X'_{-1}) \quad (R^2 = .591) \\
m &= 1.223 + .5427i + .2420x + .4635 (d_{-1} - s_{-1}) + .0863 (y'_{w} - m'_{w}) \quad (R^2 = .838) \\
y &= 1.223 + .1901i + .1942x + .4635 (d_{-1} - s_{-1}) - .4241 (y'_{w} - m'_{w}) \quad (R^2 = .565) \\
c &= 2.350 + .5596y + .5777 (y_{-1} - c_{-1}) - .4530c' \quad (R^2 = .540)
\end{align*}
\]

The price relations reflect the high dependence of
the price for investment goods on the import price, the high
sensitivity of the cost of government expenditure to the
internal price level, and the slow increase in export prices
compared with domestic prices during the period under
consideration.
The constant term in the import and home production decision function may be interpreted as showing the autonomous growth arising in the absence of any increase in investment and exports; this growth rate is estimated as being little above 1% per annum. An increase in investment has a substantial, an increase in exports a more moderate effect on imports; the effect of both factors on home production is about equal. Changes in final demand of all kinds also have a substantial delayed effect on imports and gross national product. Furthermore, the difference between the coefficients of $y'_w - m'_w$ in the equations for $y$ and $m$ may be interpreted as a price elasticity for the ratio between home production and imports, and the value of this elasticity is estimated at about -1.5.

The consumption function states that with an increase in the price index for consumption by 2.5 percentage points, which was the annual average observed over the period 1953-62, and with a static gross national product, personal expenditure would tend to increase by about 1%. The short-term income elasticity of consumption as measured lies in the neighbourhood of .5 and .6, and the adjustment for differences between last year's expansion rates in gross national product and personal expenditure is of the same order of magnitude. Finally, the short-term price elasticity for all consumption is estimated at about .4 or .5; people compromise between keeping nominal and real expenditure unchanged in response to price movements, other things being equal.

The fit is better for the consumption price equation than for the other price equations, and better
for the import function than for the home production and consumption functions; but this is not unconnected with the fact that imports show relatively larger fluctuations than national product and consumption.

The model was developed before even provisional national accounts data for 1963 became available. With the help of these data (Central Statistics Office, 1964) it became possible to test the model by substituting the values of the predetermined variables for 1962-63 into the equations. Some minor revisions had been made meanwhile in the data for 1962 and earlier years, and the revised data were used to obtain the variables for 1962-63; but the coefficients in the equations were not recalculated.

The values substituted are:

\[ m' = 2.1 \]
\[ y' = 2.6 \]
\[ M'_{-1} - X'_{-1} = 109.7 - 111.0 = -1.3 \]
\[ i = 16.1 \]
\[ x = 9.6 \]
\[ d_{-1} - s_{-1} = 3.1 - 3.8 = -0.7 \]
\[ y'_{w} - m'_{w} = 1.5 - 1.9 = -0.4 \]
\[ y_{-1} - c_{-1} = 2.9 - 3.9 = -1.0 \]

Denoting the "predicted" or "theoretical" values by the suffix \( p \), we obtain
The model gives a fairly good explanation for the movement in volume of imports, gross national product and personal consumption, as well as for the change in the consumption price index. For the other implied price changes the agreement is less close; but of course, the actual figures are themselves rounded and approximate data which cannot lay claim to great accuracy.

Of the national accounts data at 1953 prices, the model does not predict G, I and X but gives predictions for M, Y and C. Results for 1962 and 1963 are, for the former variables

\[
\begin{align*}
g_{-1} &= 70 \\
I_{-1} &= 97 \\
X_{-1} &= 264
\end{align*}
\]

and for the latter variables

\[
\begin{align*}
M_{-1} &= 279 \\
Y_{-1} &= 623 \\
C_{-1} &= 462
\end{align*}
\]
For the accounts in current prices, theoretical values can be obtained based on actual quantum components and predicted price components in the case of \( \hat{G}, \hat{i} \) and \( \hat{x} \); based on actual price and predicted quantum components in the case of \( \hat{M}, \hat{y} \); and based on predictions for both components in the case of \( \hat{C} \). Results for 1962 and 1963 are, for the first set of variables

\[
\begin{align*}
\hat{G}_{-1} &= 91 & \hat{G}_p &= 98 & \hat{G} &= 96 \\
\hat{i}_{-1} &= 119 & \hat{i}_p &= 139 & \hat{i} &= 136 \\
\hat{x}_{-1} &= 293 & \hat{x}_p &= 316 & \hat{x} &= 319
\end{align*}
\]

for the second set

\[
\begin{align*}
\hat{M}_{-1} &= 306 & \hat{M}_p &= 339 & \hat{M} &= 341 \\
\hat{y}_{-1} &= 774 & \hat{y}_p &= 831 & \hat{y} &= 823
\end{align*}
\]

and finally

\[
\begin{align*}
\hat{C}_{-1} &= 566 & \hat{C}_p &= 599 & \hat{C} &= 601
\end{align*}
\]

The differences in current prices are, of course, virtually linear functions of the differences in constant prices and in price index numbers.

The derived figures for final demand, market supplies and stock changes in 1963 at 1953 prices are

\[
\begin{align*}
\hat{D}_p &= 943 & D &= 944 \\
\hat{S}_p &= 958 & S &= 954 \\
\hat{I}_{sp} &= 15 & \hat{I}_s &= 10
\end{align*}
\]

and at current prices

\[
\begin{align*}
\hat{D}_p &= 1,152 & \hat{D}_p &= 1,152 \\
\hat{S}_p &= 1,170 & \hat{S}_p &= 1,164 \\
\hat{I}_{sp} &= 18 & \hat{I}_s &= 12
\end{align*}
\]
The model does not give a particularly good explanation for stock changes. This is to be expected as it was not designed for that purpose, and stock changes merely appear as residuals. Otherwise, the results for 1963 appear to be sufficiently encouraging to permit further use of the model for the time being.

At the time of writing this paper, the model is being used to forecast the national accounts data specified here for 1964, both at constant and current prices. The straightforward procedure is, of course, to estimate the values of \( m' \), \( y' \), \( i \), \( x \) and \( g \) from outside information and the endogenous variables from the model. In view of the fact that outside information is also available about the endogenous variables, it may be possible to improve on this procedure.

The most difficult practical problem is the prediction of exports; even though figures are available for part of the year by this time there is still considerable uncertainty about their course during the remainder of the year. Alternative assumptions may therefore be appropriate. Other things being equal, the assumption made about exports has, among other things, a substantial effect upon the value obtained for stock changes, a higher export volume being accompanied by a smaller theoretical increase in stocks; this is at any rate a sensible result.

The limitations of the model are obvious. The fit is not as good as might be desired, for some of the equations at any rate, though the model does not pretend to be more accurate than it is; there is that danger when using actual time series instead of first differences or ratios. It is not practicable
to add many more explanatory variables without incurring a serious risk of multicollinearity. The model is highly aggregated, and moderate disaggregation might improve the fit. The practical difficulty arising in this context is to obtain price index numbers and constant price data for series published in current price terms only. Nor does the model, with the data available for Ireland, easily lend itself to conversion into a quarterly model; for the purpose of really short-term forecasting it might be necessary to choose different variables and to construct a different model.

There are, however, possibilities of extending the model by building on it both at the bottom and at the top, as it were. On the one hand, some of the variables presently treated as exogenous might be explained by others. For example, it would be desirable to obtain an investment function; and it would be of interest to explain the price index for the gross national product by an index of wage rates and other variables. On the other hand, the movements in variables not so far studied might be explained by the movements in national accounts data.

One interesting possibility in this direction would be to explain the changes in population size of the country. In the past, there has been considerable net emigration from the Republic of Ireland, notably to Britain, offsetting or more than offsetting the natural population increase; the number of emigrants has fluctuated, to some extent in response to economic conditions in the two countries.

A simple formulation would be to express changes in population size as a linear function of
the difference between the growth rates in real gross national product, using the already encountered variable \( y \) for Ireland, and changes \( y_u \) in the index of gross domestic product at factor cost in 1958 prices for the United Kingdom. Using data for 1953-62 as before, this approach yields the equation

\[
 n = -0.390 + 0.1193 (y-y_u) \ (R^2 = 0.469)
\]

This would suggest that a difference in favour of Ireland by 3 percentage points in the growth of gross national product, presented in fixed base index form, would be required to ensure stability of the Irish population total. However, it may be possible to improve on this explanation, which is presented here as showing a line of thought rather than an established result.
References.


Gehrig, G. (1963), *Ein makroökonomisches Modell für die Bundesrepublik Deutschland*, Schriftenreihe des IFO - Instituts für Wirtschaftsforschung Nr. 56, Berlin-München.