A Further Note on Economic Growth and Manpower

by

C. E. V. Leser

Restricted Circulation
Not for Publication

January 1963
A Further Note on Economic Growth and Manpower

by

C. E. V. LESER

In a previous note [1], a possible pattern of economic growth was considered and its implications on manpower requirements were examined. If assumptions on labour force size, together with assumptions on productivity growth, are chosen as starting points instead of postulating a fixed overall growth rate, an optimisation problem solvable by linear programming is obtained. The assumptions made here are, of course, to a large extent, arbitrary, but they are consistent with the approach adopted in [1]. They are as follows:

(a) The volume of agricultural output may increase up to 2.25% per annum. This can be achieved while at the same time the agricultural labour force declines by 1% p.a. A decline to at least this extent is called for, since more labour cannot be profitably employed in agriculture. If a lower target is set for the growth in agricultural output, then for every 1% that the growth rate falls below 2.25% the agricultural labour force may decline by a further 0.5%; e.g. a 1.25% increase in output may be achieved with a 1.5% reduction in agricultural labour.

(b) The volume of industrial output may increase up to 6.5% annually; this calls for a 2% increase in the industrial labour force. A higher intake of labour and expansion of output is not possible, but they may be lower. For every 1% that the industrial growth rate falls below 6.5%, the man-
power expansion is reduced by .5%; e.g. a 5.5% increase in industrial output can be got with a 1.5% increase in the labour force.

(c) The service trades experience an annual growth in labour productivity of 1%; that is to say, their labour force may grow by 1% less than their output.

(d) The output growth rate of the service trades must equal the overall growth rate, thus keeping their share in the national income constant.

(e) Net emigration approximately cancels out the natural increase in population, but the work participation rate of the population may slightly rise. An increase in the size of the whole labour force by .5% p.a. is thus feasible.

White \( w_1, w_2 \) and \( w_3 \) for the percentage increases in the labour force of agriculture, industry and service trades respectively; \( y_1, y_2 \) and \( y_3 \) for the percentage growth rates in national income derived from these three sectors, and \( y \) for the overall growth rate. Furthermore note that in 1960, the three sectors in that order absorbed 37.1%, 24.9% and 38.0% respectively of the total labour force and contributed 26.0%, 29.4% and 44.6% respectively of the national income. These proportions are assumed to be applicable as weights for average annual growth rates.

The relationships described under the headings (a) to (e) may then be written as follows:
\[
\begin{align*}
y_1 &= 4.25 + 2w_1 \\
w_1 &\leq -1 \\
y_2 &= 2.5 + 2w_2 \\
w_2 &\leq 2 \\
y_3 &= 1 + w_3 \\
y_3 &= y \\
0.371w_1 + 0.249w_2 + 0.380w_3 &\leq 0.5 \\
0.260y_1 + 0.294y_2 + 0.446y_3 &= y
\end{align*}
\]

and the optimisation problem consists in finding the values of \(w_1, w_2,\) and \(w_3\) or \(y_1, y_2,\) and \(y_3\) which make \(y\) a maximum.

The only unusual feature about this problem lies in the fact that the variables may assume positive or negative values. By using the transformation

\[
x_1 = -1 - w_1, \quad x_2 = 2 - w_2
\]

the problem reduces to a conventional one in linear programming. If the 4 equalities are used to eliminate all variables other than \(x_1, x_2,\) and \(y,\) we obtain the conditions

\[
x_1 \geq 0, \quad x_2 \geq 0 \\
0.4031x_1 + 0.3614x_2 \geq 0.5313 \\
4.5054 - 0.9386x_1 - 1.0614x_2 &= y
\]
It is easily seen that $y$ is maximised when $x_1 = 1.32$ and $x_2 = 0$, i.e. when industrial production expands as much as possible. In terms of the original variables, the solution is

$$
\begin{align*}
  w_1 &= -2.32, \\
  w_2 &= +2, \\
  w_3 &= +2.27 \\
  y_1 &= -0.39, \\
  y_2 &= +6.5, \\
  y_3 &= y = +3.27
\end{align*}
$$

Thus with a slight reduction in agricultural output but a $6\frac{1}{4}\%$ increase in industrial output, a $3\frac{1}{4}\%$ overall growth rate would be possible. The total labour force increase must, of course, reach its maximum of $4\%$.

The solution may be compared with the "probable" growth pattern suggested in [1], implying

$$
\begin{align*}
  w_1 &= -1.5, \\
  w_2 &= +1, \\
  w_3 &= +1.97 \\
  y_1 &= +1.25, \\
  y_2 &= +4.5, \\
  y_3 &= y = +2.97
\end{align*}
$$

and, incidentally,

$$
.371w_1 + .249w_2 + .380w_3 = .44
$$

That is to say, a $1\frac{1}{4}\%$ increase in agricultural production and a $4\frac{1}{4}\%$ increase in industrial production was envisaged, yielding an overall growth rate of about $3\%$ with a manpower increase just below $4\%$.

It follows that by a drastic reallocation of resources, together with a slightly greater labour force increase, the annual average growth rate in real national income may be raised from $3\%$ to $3\frac{1}{4}\%$. A higher growth
rate is still not envisaged as feasible; and for its realisation one or other of the assumptions (a) to (e) would have to be violated. If a higher growth rate is believed to be obtainable, then it is necessary to consider which assumption should be modified and in what way.

REFERENCE

Memorandum No. 6, The Economic Research Institute

Note: Errata in [1]

p. 1, par. 2, line 1: for "grossed" read "gross and"
line 2: for "material" read "national"
line 8: for "trends" read "trades"
p. 2 line 1: for "combined" read "continued"
p. 3 line 2: delete "not"; after "that" insert "in".