Resilience of Traffic Networks to Extreme Weather Events: Analysis and Assessment

A thesis submitted to the University of Dublin, Trinity College in candidature for the Degree of

Doctor of Philosophy in Civil Engineering

By

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September 2017

Declaration

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Acknowledgement

Firstly, I would like to express my sincere gratitude to both of my supervisors, Prof. Alan O'Connor, and Prof. Maria Nogal for their constant guidance and encouragement during my Ph.D thesis.

Thank you Alan for your support during these three years, being there to encourage me in every decision, for your continuous optimism and of course for this enormous opportunity that for sure it is going to change my career. Thank you Maria for your guidance that truly helped me to develop this research, for your patience, really, I cannot thank enough the number of hours that you have dedicated, and of course for being there every single day, not only as a supervisor but also as a friend. So, a big thank you because without both of you this work would have not been possible.

Starting a Ph.D. thesis in a new country with a different language is a challenging experience, so special thanks to my colleges who were here when I started. I really appreciate your welcome, making the beginnings of this research experience, and also in Dublin easier.

Thanks as well, to my colleges during the three years for their moral support, for the endless conversations, and the good times that we have spent. Also I thank my friends, for understanding a Ph.D. life and being always there with support words, and of course, thank you all for all the fun that we have had in the last three years.

Furthermore, I would like to thank every single person in the University and outside that has also contribute to make this thesis possible. Thanks to Prof. Brian Caulfield who also helped me to be here. Thanks to all the Professors and staff of Trinity for their nice words, and support during these years. Also, thank you to the Dublin City Council for providing the traffic data that were used in this research.

And last but not least thank you to my parents, my sister, and my family for their unconditional support during all this process, and their encouragement in all my decisions. Thank you for always being there in every possible aspect.

Abstract

The increment of extreme weather events is creating larger, and more frequent problems among infrastructure systems worldwide. In this thesis, the focus is on traffic networks by the analysis of a recent concept, the resilience. This concept evaluates the impact that perturbations create on traffic networks from the beginning of the perturbation until the total recovery. Due to the novelty of the concept in the Transport area, this thesis aims to develop new mathematical tools in order to quantify, and understand this concept in traffic networks.

Following the previous objective, this thesis provides the reader with the following contributions:

Literature review. A literature review about existing definitions, and methodologies to evaluate resilience in transport networks is presented, together with an analysis of the existing traffic assignment models, and travel cost functions. In addition, a literature review of methodologies for the identification of critical, and vulnerable links is included.

A dynamic restricted equilibrium assignment model. A methodology to evaluate resilience in traffic networks is presented, based on a new dynamic restricted equilibrium assignment model, which evaluates not only the evolution of the cost level during the whole evolution of the perturbation, but also the stress suffered by the network users due to the changes in the travel conditions. In addition, formulations to evaluate the perturbation, and the recovery resilience are introduced.

A bounded link travel cost function. A new link travel cost function which explicitly considers the effects of weather events in traffic networks is introduced, including not only a parameter to determine the intensity of the hazard, but also a parameter to include the local vulnerability of each link when the perturbation occurs.

A mapping, and a bi-phase sensitivity analysis. A bi-phase sensitivity analysis of the parameters included in the proposed method to evaluate resilience, including local approach (OAT), and a global approach (Latin Hypercube) is presented. The statistical approach implemented allows its use in complex models by reducing the number of points needed for the analysis, with the consequent time saving.

Methodologies to identify critical, and vulnerable links. Novel methodologies to identify, and rank the links of traffic networks by their vulnerability, and also by their criticality are presented. The methods are based in the analysis of the Fisher Information Matrix , and its eigenvalues and eigenvectors. In addition, this methodology is extended for the identification of vulnerable, and critical areas of a traffic network .

Practical applications. The proposed methodologies are tested in examples, and real traffic networks in order to show their performance, and characteristics. In addition, the presented examples allow the validation of the results, and the associated computational requirements.

Contents

List of Figures

List of Tables

List of Symbols

c_a	travel cost associated with link a
c_{0a}	free travel cost associated with link a
d	uniform demand value used for the representation of network demand
d_{pq}	demand associated with origin-destination pq
$f_{i,j}$	The (i,j) entry of the FIM
h	hazard intensity
h_{max}	maximum intensity of the hazard during the studied period
h_{pq}	flow on routes with origin-destination pq
i	percentage used in the sensitivity analysis
m	shape parameter of the new travel cost function to define the upper bound
p_a	local vulnerability for link a
t	time interval
t_a	actual travel time for link a
t_{a0}	free flow travel time for link a
t_r	interval of time at which equilibrium is reached
$\overrightarrow{v}$	Eigenvector
$\overrightarrow{v}_{\lambda_{max}}$	Eigenvector associated with the largest eigenvalue
x	modified variable in the sensitivity analysis
C_0	initial total cost (t=0)
C_a	integral of the travel cost function of link a
C_{th}	cost threshold associated with the system break-down point
C_T	actual total cost
F	Fisher Information Matrix (FIM)
M	Number of areas
N	Number of links
R	model response calculated for the initial parameter set Z
R_i^+	model response when one parameter has been increased by the percentage i
R_{pq}	set of routes with origin-destination pq
S	congestion ratio
T_{th}	reference value associated with recovery time

Y	subset of variables which remain constant in the sensitivity analysis
Z	set of variables analysed in the sensitivity analysis
α	system impedance
β	shape parameter of the travel time function
γ	shape parameter of the travel time function
Δt_n	Variation of the travel time of link n
ΔT_n	Variation in the total travel times of the network when link n is being damaged
ΔC_m	Variation of the capacity of area m
ΔC_n	Variation of the capacity of link n
Δt_m	Variation of the travel time of area m
$\Delta T_A m$	Variation in the total travel times of the network when area m is being damaged
θ	Sensitivity vector
θ_k	normalized slope associated with state of perturbation k
λ	Eigenvalue
λ_{max}	Largest eigenvalue
ν_a	link flow associated with link a
ν_a^*	link flow associated with link a corresponding to the UE state
ν_a^{max}	link capacity to provide certain service of level
η_{pq}	number of routes with origin-destination pq
ρ_r	net flow variation among routes within state of perturbation k
ξ	sensitivity
σ_k	stress level of traffic network associated with state of perturbation k
τ_k	cost level of traffic network associated with state of perturbation k
χ_k^r	perturbation resilience associated with state of perturbation
ψ_k	exhaustion level of traffic network associated with state of perturbation k
$\mathcal{A}$	set of links
$\mathcal{D}$	subset of origin-destination pairs of nodes
$\mathcal{N}$	set of nodes

List of Acronyms

BPR	Bureau of Public Roads
$DDMLC$	Disaster Management Life-Cycle
DSO	Dynamic System Optimum
DTA	Dynamic Traffic Assignment
DUE	Dynamic User Equilibrium
DUO	Dynamic User Optimal
$D2D$	Day-to-day
EC	European Commission
EU	European Union
$FIFO$	First Input First Output
FIM	Fisher Information Matrix
$GBPR$	Generalization of BPR
KKT	Karush-Kunn-Tucker
LH	Latin Hypercube
LSM	Least Square Method
NRI	Network Robustness Index
NVI	Network Vulnerability Index
OAT	One-At-a-Time
OD	Origin-Destination
RI	Resilience Index
$RMSE$	Root Mean Square Error
SO	System Optimum
SP	System Performance
STA	Static Traffic Assignment models
SUE	Stochastic User Equilibrium
$TEN-T$	Trans-European Transport Networks
TRE	Total Recovery Effort
TSP	Targeted System Performance
UE	User Equilibrium
VI	Variational Inequality

VA Vulnerability attribute

Part I

Introduction

Chapter I

Introduction

Contents

I.1 Motivation

The occurrence of extreme weather events has become increasingly common in the last number of years. A variety of extreme weather events, including droughts, rain induced landslides, river floods, winter storms, wildfire, and hurricanes, have threatened and damaged many different regions worldwide. This situation largely affects infrastructure systems and results in risk situations for people.

Natural extreme events have caused huge losses, such perturbations as Hurricane Katrina (2005) with estimated cost of $75 billion in the New Orleans area and along the Mississippi coast, Hurricane Sandy (2012) which caused almost 150 deaths and damaged or destroyed an estimated 650.000 homes, or the Haiti earthquake (2010) which caused more than 100.000 deaths. More recently, Hurricane Harvey (2017) whose economic cost has been estimated at 81 billion to 108 billion, and Hurricane Irma (2017) with an estimated damage of more than $62 billion.

In Europe, events such as the last flooding in Germany in 2016, in France in 2016, and in Belgium in 2016 are increasingly often creating threatening scenarios. Between 1998 and 2004 in Europe, floods caused some 700 fatalities, the displacement of about half a million people and at least EUR 25 billion in insured economic losses, (European-Commission (2004)).

In addition, these situations occur not only more frequently but also reaching levels that were not previously foreseen. These kinds of perturbations often cause significant losses, which can last long period, from the moment when the perturbation starts until the total recovery.

In Figure I.1 , the increment in the number of occurrences of natural disasters is shown from 1984 until 2013. It is noted that this Figure also shows the direct damage (measured in USD billions) created by these perturbations.

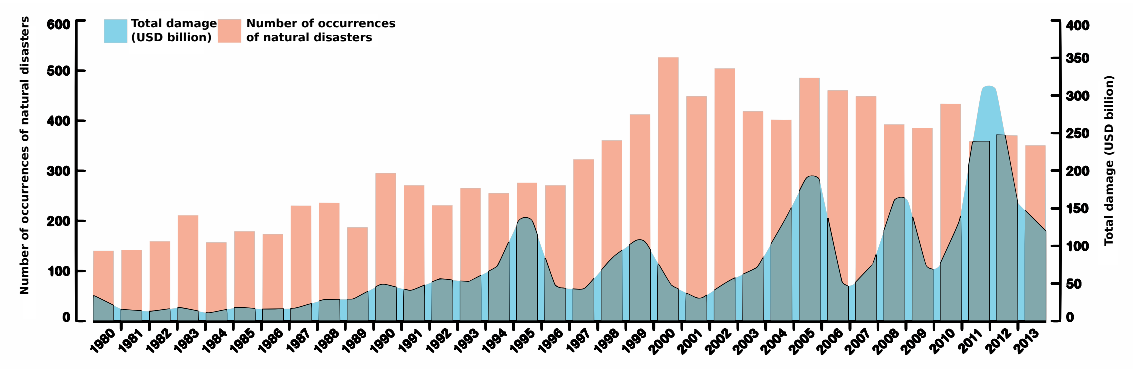

Figure I.1: Graphical representation of the occurrence of several weather events, and total damage measured in USD billions. *Source: The International Disaster Database, EM-DAT database.*

Therefore, governments and managers of the infrastructures need to handle these circumstances and, unfortunately, in the field of transport, the number of tools dealing with these threatening situations is reduced.

Transport networks are a key element of a modern society, since human activity revolves around it. Among their main features are the capacity to move people and goods from one location to another, with the ability to move goods a very crucial factor. Goods movements include the shipment of items such as (a) raw materials, including minerals, energy, food and other resources, (b) finished products, which need to be transported to the final clients and (c) also wastes, which is a vital role, with actions such as their removing and the prevention of their accumulation. This means that transport networks develop an essential function in our lives, making necessary their understanding and improvement.

An essential point about transport networks is their vulnerability. Since they are exposed to all sorts of perturbations, it is imperative to develop tools that explicitly consider the impacts of extreme weather events on such critical infrastructure.

In addition, these perturbations caused by natural hazards can be uncertain and vary during the lifetime. Despite of this fact, there are models to predict natural hazards, but these predictions could not be accurate. Also, there are cases where the perturbation is not predicted and the place, the intensity and the duration of the disturbance are totally unknown.

When an extreme weather event takes place, some important questions arise, such as how stressed the traffic network is, whether the system is able to respond to this situation, or how long the system needs to recover a new equilibrium position after suffering such a perturbation. The concept addressing all these questions is known as resilience.

Resilience is a relatively new concept, which was first defined around 40 years ago. Recently, this concept has become more widely studied and its relevance is seen in many fields.

Foster (1993) defined resilience as "the capacity to absorb shocks gracefully". Although this is only one definition of many which are presented in published literature.

The term resilience gains importance when a catastrophic event occurs in a traffic network. In such a situation resilience gives us important information about the system behaviour.

Analysing this concept is not only interesting to solve problems caused by extreme climatology, but also to prepare traffic networks for such hazards. Therefore, it will be possible to create more secure and qualified systems because a larger and more exhaustive knowledge about networks would have been obtained.

Therefore, the aim of this thesis is to advance the area of transport resilience, developing new mathematical tools to evaluate this concept.

I.2 Context and current practice

The resilience concept has been studied in different areas, i.e., ecology, socio-ecological systems, economics, urban infrastructure, telecommunication systems, water distribution systems, or internet protocol networks.

When resilience started to be analysed, the definition was mainly focused on the capacity of the system to absorb an impact. In this first approach, the concept of resilience was not including aspects which have been shown as crucial in its definition, such as the recovery process, and the time needed to reach again the normal operation of the network. As more studies were developing in this area, the definition has become more complex, including in its definition aspects of the whole perturbation process.

In recent times, the holistic concept of resilience evaluates concepts such as the ability of a system to prepare and to adapt to changes, and the recovery pattern of the system. However, there is not an established definition for resilience in the area of transport networks.

In transportation, some authors have been studying this concept. For example, according to Bruneau et al. (2003), resilience consists of four parameters: robustness, redundancy, resourcefulness, and rapidity. In a similar way, Murray-Tuite (2006) asserts that, resilience is defined in ten dimensions; redundancy, diversity, efficiency, autonomous components, strength, collaboration, adaptability, mobility, safety, ability to recover quickly. It is also noted than some authors, such as Institute (2010), and Ta et al. (2009) define resilience focusing on the capacity of the system to accommodate the impact, other authors, such as Henry and Ramirez-Marquez (2012), Miller-Hooks et al. (2012) define resilience based on the recovery process, and finally, only a few authors, such as Freckleton et al. (2012) highlight the importance of the time frame necessary for the whole process, from the perturbation until the total recovery.

When analysing the different methodologies to evaluate transport resilience that can be found in literature, it is noted that some authors have tried to measure this complex concept. However, this is not a straightforward task, due to all aspects included in the resilience concept. There is a large variety of parameters which constitute resilience, and many of them are subjective and difficult to measure.

When studying previous methodologies to evaluate resilience, a first division can be established. Firstly, the group of authors who analyse resilience in a qualitative manner, and secondly, those authors who present a quantitative methodology to evaluate it. The focus of this Ph.D. thesis is the quantitative methods, since a quantitative methodology allows a better understanding of some aspect of the resilience and a systematic comparison between different traffic networks.

Among the experts in resilience, those who present quantitative methods are scarce. In addition, some gaps in knowledge can be found among these methodologies.

- **Quantification of the concept.** It is difficult to find methodologies that quantify the whole concept of resilience. Some methods try to evaluate the concept by measuring many parameters that describe resilience, but at the end a large amount of resources are needed, and a total value of resilience is not reached. Also, sometimes the approaches are semi-quantitative, where the final results describe the resilience of the network as low, medium or high.

- **Dynamic evaluation of the concept.** Many methods presented in literature do not include the dynamic aspect of resilience during the whole process. Thus, some methodologies analyse the impact of the perturbation as a “punctual” or “static” damage, without measuring the probable changes in the perturbation.

- **Users domain.** When describing the concept of resilience, the impact caused by the perturbation in the users is highlighted by many authors. Also, qualitative methods note the importance of the behaviour of the users in the evaluation of resilience. However, when analysing the quantitative methodologies to evaluate resilience, this aspect is generally not included, and the methods focus on the damages created in the network.

These identified gaps are the main topic of research of the work presented in this thesis.

I.3 Objectives

The ultimate goal of this thesis is to help design efficient and resilient traffic networks, i.e. traffic networks that are designed to withstand distinct disturbances caused by catastrophic climatological events. This research aims to improve the current traffic networks and help design new networks more efficiently. For that reason in this research the following questions will be answered.

What does resilience mean on a traffic network?

The first objective of this Ph.D thesis is to create an original definition of resilience based on an extensive review of the previous definitions of this concept in the area of transport networks.

To reach a new method to quantify resilience, it is necessary to clearly identify the term that is going to be evaluated. So far, there is not a clear definition of resilience in transport networks, in the literature review very different approaches to the term can be found. Mainly, every author uses a different definition and there is not a common one, thus big variations between the definitions can be found. Thereby, this research will contribute with a definition of the concept, taking into account a wide review of the approaches that other authors have previously developed.

How to calculate resilience in a traffic network?

The second objective of this Ph.D thesis is the development of a tool capable of evaluating the resilience of a traffic network when it is affected by extreme weather conditions. This new methodology should be able to quantify resilience, including the dynamic nature of the problem.

Once the definition of resilience is clarified, a methodology to assess resilience will be implemented.

Due to the complexity of the concept, most of the methodologies found in the transport area are based on qualitative approaches or they only measure part of the concept. Also, there are few quantitative models to measure it and even less have a dynamic approach.

This new tool will contribute with a quantitative and dynamic approach to address the problem. The importance of a quantitative tool lies in the fact that an objective and precise result is obtained. Even though qualitative methodologies provide lots of information with the incorporation of a wide framework, sometimes these methodologies can obtain the same result for different problem. For example, in the case of a methodology whose results to evaluate resilience can be low, medium or high, the possibilities of having very different networks with different performance when affected by the hazard and the same final result is very probable. This makes extremely difficult the comparison between networks. Developing a quantitative method, this probability will be reduced, since the range of options is evidently wider.

Other strong point of this new tool is the dynamic nature of the approach, which means that it is possible to introduce the dynamic behaviour of the climatological event and also obtain the dynamic response of the traffic network. The dynamic component of a hazard is really relevant in the results, and for that reason, this methodology will allow the incorporation of the evolution of the event when affecting the traffic network.

In addition, since resilience is measured for the occurrence of climatological perturbations, a new cost function including the weather parameters will be implemented in the model.

The combination of the model and the travel cost function results in a powerful tool to understand and measure the resilience in a traffic network, allowing the complete simulation of the impact of extreme climatological events on traffic networks and permitting a quantitative evaluation of the system response.

How to improve the resilience of a traffic network?

Finally, once it is possible to measure the resilience, a study of the parameters involved in the methodology should be developed, since the profound knowledge of the variables involved will allow us to understand how to modify the traffic network to improve its resilience.

With the goal of improving the resilience of a network when affected by a hazard, the weaknesses of the system should be identified. Therefore, the knowledge of the most influential variables involved in the model to evaluate the resilience allows its enhancement. With this aim, a sensitivity analysis is carried out. A sensitivity analysis identifies the influence of each parameter on the outputs of the model. Thus, the definition of the inputs will be more efficient after studying how these parameters modify and influence the model.

In addition, since traffic networks are formed by multiple elements, which have particular characteristics, it is important to know that some of these elements will contribute to a worse performance of the networks when a perturbation occurs. This happens because these elements have a main role in the network. Among these elements, it is possible to differentiate between vulnerable and critical elements. The first ones will suffer a large impact when the network is affected. On the other hand, the critical links, when affected by the perturbation, will create a large impact in the global behaviour of the traffic network. The latter will be the ones affecting greatly the value of resilience.

Therefore, this thesis aims to develop a methodology which is able to detect, and rank both types of the links, the vulnerable and the critical elements.

I.4 Organization of the thesis

With the final goal of evaluating resilience in traffic networks, and providing a better understanding of what happens when a perturbation takes place, in this thesis, the following chapters are presented. In addition, in Figure I.2 an explanatory diagram about the organization of this thesis is introduced.

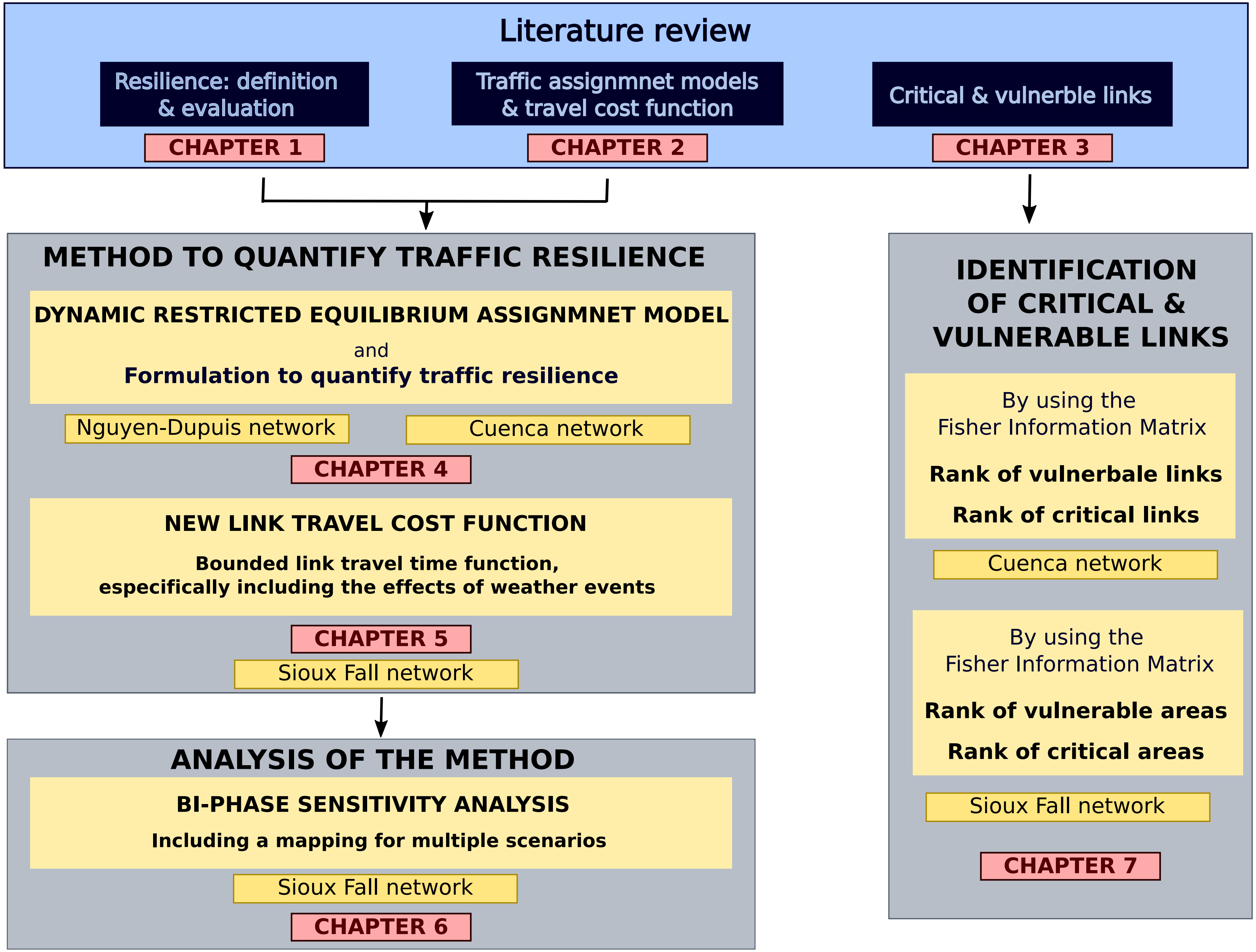

Figure I.2: Diagrammatic representation of the thesis structure.

The first three chapters of the thesis aim to present a review of the different methodologies found in the literature, in the area that is analysed. A description of each of the Chapters is introduced as follows:

- **Chapter 1:** ***Resilience in transport networks: definitions and methodologies***.

  This chapter is divided in two parts. The first part analyses the concept of resilience, multiple definitions of resilience in the area of transport are presented, and finally the definition of the concept used in this thesis is presented. In the second part of the chapter, the focus is on the evaluation of the concept. Therefore, several methodologies presented in literature are analysed, and some of the gaps found in these methods highlighted.

- **Chapter 2:** ***Traffic assignment models and travel cost functions***.

  A powerful approach to cover the requisites for the quantification of the concept, is through the use of a traffic assignment model. These models allow the understanding of the users behaviour when a perturbation occurs. Therefore, in this chapter, an analysis of the most relevant traffic assignment models presented in literature is introduced.

  One of the main parts of the traffic assignment models is the objective function used in the optimization problem. These functions are usually travel cost functions. Therefore, a review of the main travel cost functions presented in the literature is introduced in this chapter too.

- **Chapter 3:** ***Critical and vulnerable links in transport networks***.

  Having as a final goal of this thesis the identification of the critical and vulnerable elements of traffic networks, this chapter analyses the definitions presented in the literature for both concepts. In addition, some of the methodologies found in the literature for the identification of these elements are introduced, including those that analyse the vulnerable elements, the critical elements, and some methodologies that can identify both of them.

  It is noted that both terms, vulnerable and critical, are sometimes used as the same concept in the literature, and a clear differentiation between both of them is not presented. Therefore, after an analysis of the definitions used by several authors, this chapter presents the definitions for both concepts, which will be used in this thesis. Finally, several methods presented previously by some authors, are compared, and the main gaps of these methodologies are highlighted.

The following chapters present the original contributions of this Ph.D. thesis, following the goal of addressing the objectives previously presented. Therefore, a description of each chapter is presented as follows:

- **Chapter 4:** ***A dynamic equilibrium-restricted assignment model, together with a method to quantify resilience***.

  In this Chapter a model that captures the consequences of a perturbation in a traffic network, and provides a new tool to evaluate the resilience of a traffic system under a time-varying disruption are presented.

  The dynamic equilibrium-restricted assignment model is a macroscopic traffic model that simulates the dynamic response of the network when suffering a disruption. This model is based on the following assumptions: (a) the global behaviour of users is analysed on a day-to-day basis, that is, the problem of within-day dynamics is neglected. (b) Only negative perturbations are taken into account, i.e., those perturbations which imply a travel cost increment. (c) Users select their route choices that reduce their individual travel costs. This selection is based on the complete information about the past day's travel costs. (d) The capacity of adaptation of the users to the changes, the lack of knowledge of the new situation and the lack of information of the behaviour of other users impede the immediate response and recovery of the system.

  Therefore, the user response is analysed, providing the evolution of the overcosts generated, and the evolution of the stress suffered. This means that this method to quantify resilience integrates the evolution of the cost, and the stress to finally calculate the evolution of the exhaustion level of the network. This exhaustion level analyses how far the network is from total exhaustion.

  In addition, based on the values obtained for the exhaustion level, formulations to quantify two different types of resilience are presented. These types of resilience are termed the perturbation resilience, which evaluates the period of time while the perturbation lasts, and the recovery resilience, which evaluates the period once the perturbation has finished.

- **Chapter 5:** ***A bounded link travel cost function that explicitly considers the effect of climatological events***.

  A novel link travel time cost function is proposed, which allows the consideration of weather events in traffic networks, such as rainfall, snow or loss of visibility.

  The proposed function introduces the impact caused by climatological perturbations in traffic networks by the introduction of two new parameters in the formulation. These parameters are (a) the intensity of the hazard, h, which includes the magnitude of the damage from the part of the perturbation, and (b) the local link vulnerability, p_a, which incorporates the magnitude of the impact depending on the characteristics of the network.

  In addition, the new upper-bounded travel cost function is defined. This upper bound condition aims to reflect a more realistic definition of the travel time. Upper-unbounded travel cost functions allow the increment of travel times until infinite values, a fact that is not very realistic. Even in the case of extreme weather events where traffic networks may suffer large disruptions, the travel times of the network will increase up to a level, from which the link can be considered as "broken". It is noted that this characteristic will provide a greater computational efficiency when working with optimization problems, and even more when dealing with disrupted situations due to the large effects of these climatological events on the congestion ratios.

- **Chapter 6:** ***Mapping and Bi-phase Sensitivity analysis.***

  Sensitivity analyses identify the importance of the model parameters and variables, providing a profound knowledge of the model. Most of the sensitivity analyses imply a local evaluation of the variability of the parameters. However, in those cases where the model involves five or more degrees of freedom, these studies become highly time consuming and incapable to obtain conclusive solutions.

  Based on the complexity of the resilience model defined previously, a sensitivity analysis that reduces the number of studied points used in the analysis is required. This chapter presents a bi-phase approach, by integrating a local into a global sensitivity method. The methodology reduces the number of studied points needed due to its statistical approach. The proposed approach uses a global technique (Latin Hypercube), without losing efficiency, and a local technique (One-At- a-Time). This methodology is recommended in those multidimensional models that make other approaches inefficient, since the global phase is based on the statistical distribution of the variables, the methodology can provided a robust and reliable solution, with low computational cost.

  Prior to the sensitivity analysis developed in this chapter, an example where all the methods presented previously, such as the dynamic-restricted equilibrium model, the method to evaluate resilience, and the bounded link travel time function are tested together. Thus, in a well-known traffic network (Sioux Fall network), the dynamic restricted equilibrium assignment model, and the link travel cost function are applied, and the resilience of the traffic network evaluated. In addition, for a better understanding of the evaluation of resilience a mapping with multiple scenarios is developed.

- **Chapter 7:** ***A methodology to identify the vulnerable, and the critical links of a traffic network.***

  In this chapter, new methodologies to identify critical and vulnerable links in a traffic network are presented. It is noted that the proposed methods are able not only to identify the most vulnerable, and the most critical link, but also to rank all the links of the network accordingly to their vulnerability and their criticality.

These proposed methodologies use the Fisher Information Matrix, and the analysis of eigenvalues and eigenvectors, to systematically rank the links of a traffic network. Thus, the use of this technique allows the incorporation in the analysis of variables, such as the user demand, the travel time, and the flow of the network for the identification. Different aspects will be included in the Fisher matrix in order to identify the vulnerable, or the critical links.

For a global analysis of the elements, the methodology introduces damages in all the possible locations, which are incorporated in the Fisher Matrix, and through their analysis, a rank including all the links can be obtained as a result. This aspect is important, since it allows a global analysis without the necessity of a previous selection of the elements that apparently are more susceptible.

Finally, based on the same principles, both methods have been implemented to identify vulnerable and critical areas of the network. Sometimes when dealing with climatological perturbations, the impacts are affecting a wide area. Therefore, methodologies to rank areas, which can be defined as required by the users of these methodologies, are presented.

The general conclusions of this Ph.D. thesis are presented at the end of the document, in Chapter 8. Together with the conclusions, future lines of research that can be exploited are introduced. To conclude, details of the published work resulting from the development of this thesis, including journal papers, and conferences, are listed.

Part II

State of the art

Chapter 1

Resilience in transport networks: definitions and methodologies

Contents

1.1 Origin of the concept

The concept of resilience is defined by the Cambridge dictionary as the ability to quickly return to a previous good condition. The origin of the word resilience comes from the Latin word “resiliere” whose meaning is “bounce back”. Resilience has been introduced in different fields of knowledge over the last 40 years, however, the use of this concept in the area of transport networks has been very recent, and even its definition is not settled yet. For this reason, in this chapter, a review of the definitions and methodologies for the evaluation of resilience in transport networks is presented.

The presence of the term resilience has significantly increased during the last years. In Figure 1.1, the frequency of appearance of the word resilience in literature is shown, it is noted that the engine used analyses sources printed between 1500 and 2008 in Google's text corpora. Together with resilience, the words vulnerability, and climate change are shown in the graphic, both expressions are highly related in the literature. Vulnerability is included because in many cases it is analysed together with the system resilience, and climate change because it can be one of the reasons of the increment in the use of resilience, due to the increase in the number of extreme weather events in the last years.

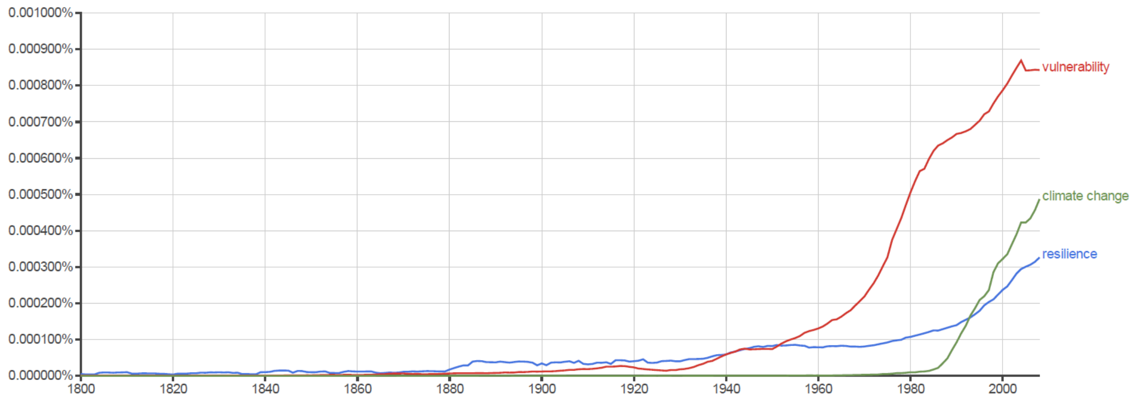

Figure 1.1: Frequency of appearance of the words resilience, vulnerability, and climate change. Source: ***Google Ngram Viewer***

Holling (1973), who can be considered as the first author who studied the concept of resilience applied to the area of ecology, introduced the concept of resilience to the scientific world, defining it as *"the ability of ecological system to absorb changes of environment variables"*.

After Holling (1973), the following studies began to differentiate between two types of resilience, the first one is related to the previous definition of resilience in the area of ecology, and a new one is defined as engineering resilience.

Holling (1996) describes the two different approaches as:

- **Engineering resilience** "concentrates on stability near an equilibrium steady state, where resistance to disturbance and speed of return to the equilibrium are used to measure the property".

- **Ecological resilience** "emphasizes conditions far from any equilibrium steady state, where instabilities can flip a system into another regime of behaviour, that is, to another stability domain".

The concept of ecological resilience is comparable to static resilience, which was later defined by Rose (2007) as "a system's capability to maintain its function". On the other hand, the engineering resilience can be compared with a dynamic resilience which refers to the rapidity with which a system returns to a state of normal function after a severe perturbation. This concept of dynamic resilience was understood by Pimm (1984) as "how fast the system returns towards equilibrium after a shock".

Nowadays, it can be considered as one of the most general definitions of resilience, the one given by Foster (1993), who defined resilience as ***"the capacity to absorb shocks gracefully"***.

1.1.1 Areas of application

As explained before, since the introduction of resilience in the area of ecology, this concept has been incorporated in multiple fields. The concept of resilience has been used in areas from ecology to psychology, and from psychiatry to internet networks. In Table 1.1, several approaches for resilience in different areas are shown.

Table 1.1: Understanding of resilience in different areas presented by Omer et al. (2009)

Area	**Resilience Approach**
Ecology	Resilience is defined as the process of moving from one stable domain to another in which the system develops evolution tolerance.
Psychiatry	Resilience is the process by which an individual learns how to be more resilient to future situations.
Material science	Resilience is the capacity of a material to absorb energy when it is elastically deformed.
Engineering	One definition of resiliency is the ability of the system to return to a stable state after a perturbation.

In addition, some definitions from different areas are presented below:

- **Socio-ecological systems**. Carpenter et al. (2001) defined resilience as: "the amount of interruption that can be mitigated before the need to restructure the system or the ability of the system to deal with unexpected events without losing its characteristics"; and Walker et al. (2004) as: "the capacity of a system to absorb disturbance and reorganize while undergoing change so as to still retain essentially the same function, structure, identity, and feedbacks".

- **Supply chains** [1]. Christopher and Peck (2004) understood resilience as "the ability of a system to return to its original state or move to a new, more desirable state after being disturbed".

[1] Supply chain is defined as: the network of organizations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services in the hands of the ultimate consumer,Christopher (1999)

- **Industrial safety**. Hollnagel et al. (2007) defined resilience as "the property of the system which gives the ability to recoup with system complication and sustaining its functionality under expected or unexpected event".
- **Urban infrastructure**. Attoh-Okine et al. (2009) adopted the definition proposed in Branscomb (2006), understanding resilience as "the ability of the system to recover and adapt to external shocks, which include natural intentional and technogenic disasters and failure due to poor design".
- **Telecommunication systems**. Omer et al. (2009) defined resilience as "the ability of the system to both absorb shock as well as to recover rapidly from a disruption so that it can return back to its original service delivery levels or close to it."
- **Water distribution systems**. Todini (2000) defined resilience as "the capability of the designed system to react and to overcome stress conditions".
- **Internet protocol networks**. Sterbenz et al. (2013) defined resilience as "the ability of the network to provide desired service even when challenged by attacks, large-scale disasters and other failures".

Some differences can be identified when resilience is implemented in each of the fields presented, however, expressions found in the previous definitions such as "the capacity of a system to absorb disturbance", "the ability of a system to return to its original state", "the ability to recoup with system complication", and "the ability of the system to both absorb shock as well as to recover rapidly" will remain in the definition of resilience when applied to the Transport area.

In the following sections, resilience is analysed in the area of transport networks, and the remainder of this chapter is structured as follows. In Section 1.2, a review of resilience definitions proposed by previous authors in the transport area is presented, and in Section 1.3, a definition of resilience for traffic networks is introduced which will be used for the development of this thesis. A review of methodologies that quantify resilience in transport networks is presented in Section 1.4, followed by a comparison of these methodologies (Section 1.4.2). Finally some conclusions are drawn in Section 1.5.

1.1.2 Policy context

In the policy context, special attention has been paid to the vulnerability of transport systems due to the effect of weather events, and their eventual increase due to change in climate. The interest on transport resilience has been growing in the policy area, increasing the number of initiatives developed to this end.

In an European level, multiple resources have been focused on adaptation to climate change, such as the European Commission's proposal for a 7^{th} Environmental Action Programme to 2020, see (EC). Also, multiple funds provided by the EU are taking into account the possible effects caused by the increment of extreme weather events, for example, in the period 2004-2020, 20% of the EU budget is foreseen for adaptation and mitigation strategies to fight climate change (European-Environmental-Agency (2014)).

Several documents can be highlighted in this section which describes the European policy in this area, some of these documents are described as follows:

- EU Strategy to climate change (European-Commission (2013)). The main objective of this document is to help in the development of a more climate-resilient Europe. In the presented document, different scales are highlighted, from local to national and EU levels, contributing in each of the levels to the preparedness and the capacity to respond to the weather impacts. In addition, this document remarks the importance of the coordination among the different actions, and the necessity of improvement of this coordination activities.

  Among the actions proposed by the EU strategy on adaptation to climate change, (EC), there are two that specifically include the necessity of implementing the concept of resilience, such as:

  - Action 7: Ensuring more resilient infrastructure.
  - Action 8: Promote insurance and other financial products for resilient investment and business decisions.

- TEN-T Regulation (European-Union (2013)). This document presents several references to adaptation to climate change and resilience to extreme weather events when TEN-T projects are planned. Thus, the development of this specific network should take into account the possible impacts due to any potential impact caused by extreme weather or man-made events. Also, the possible vulnerabilities of the transport network should be in consideration, and an adequate plan, development, and operation of the infrastructure in a resource-efficient way should be implemented.

  This guideline includes specific provisions for adaptation, such as:

  - Funding projects of common interest, through the inclusion of climate impacts within the compulsory socio-economic cost-benefit analysis.
  - Development of the comprehensive network. Climate and resilience are considered as a development priority; technological development and innovation should also address the improvement of climate resilience.
  - Core network corridors. The work plan for each corridor must include provisions on adaptation, with "proposed measures to enhance resilience to climate change".

- EU White Paper on transport (European-Comission (2011)). In this document, transport infrastructure projects that include adaptation strategies aims to be selected to be co-financed with EU funds. It is highlighted that these projects should include the development of infrastructures that are resilient to the possible impact of climate change events. In this way, the EU ensures the presence of research and innovation projects that refers to the adaptation of the transport system when extreme weather effects occur.

In addition to the EU level, at a national level, several strategies and plans have been developed in this regards by countries inside the European Union. Thus, countries, such as Austria, Belgium, Denmark, Finland, France, Germany, the Netherlands, Poland, Slovakia, Spain and the United Kingdom, specifically address adaption measures in the transport system. The national plans and strategies presented for each of the countries have a very different degree of detail. Those plans presented at an early stage, around 2005, are quite general, and they are more like a guideline document than a complete adaptation document with specific actions and goals. Also, many of these adaptations plans are focused on specific problems that are suffered on the transport system of the country.

For example, in the United Kingdom, the National Adaptation Programme, Governmnet (2013), includes the assessment of vulnerabilities and risks of the transport system, developing specific actions and objectives for the adaptation strategies. On the other hand, in Spain, the National Adaptation Plan was presented as a framework for guidance that can be applied for the different levels, local, regional and national, where specific actions and goals are not implemented. However, later on, a specific programme for the adaptation of the transport system in Spain was presented, for the development of measures in 2014-2020, OECC2014 (2014).

1.2 Resilience in transport networks

When analysing how transport resilience has been presented in the literature, multiple definitions can be found. It is noted that there is not an established definition of resilience yet. Some authors highlight this fact such as Cao (2015) who said that *"the different understand of its definition and connotation brings obstacle for practitioners to promote the transportation resilience"*, and Rashidy and Hassan (2014) who concluded that *"there is no common definition of resilience in the literature; each discipline has focused on resilience from one or more perspectives"*.

Recently, some review papers have been presented where resilience and other related concepts such as vulnerability are analysed for different systems, including transport networks, supply chains, maritime transportation, or port infrastructures system (see Cao (2015), and Reggiani et al. (2015)). In the following Section, the focus is on definitions presented for transport network, and in Table 1.2, a compendium of resilience's definitions from the last years can be found.

1.2.1 Compendium of resilience's definitions in transport networks

Table 1.2: Definitions of resilience in transport networks.

Author	Year	Resilience's definition
Bruneau et al. (2003)	2003	The ability of the system to **reduce the chances** of a shock, to **absorb** a shock if it occurs (abrupt reduction of performance) and to **recover quickly** after a shock (re-establish normal performance).
Subcommittee on Disaster Reduction (2005)	2005	Resilience is the ability of a community or system to **adapt** to hazards so as to **maintain an acceptable level of service**.
Murray-Tuite (2006)	2006	A characteristic that indicates system performance under unusual conditions, **recovery speed**, and the amount of outside assistance required for **restoration** to its original functional state.
Battelle (2007)	2007	A characteristic that enable the system to **compensate for losses** and allows the system to function even when infrastructure is damaged or destroyed.
Litman (2008)	2008	A system's ability to **accommodate** variable and unexpected conditions **without catastrophic failure.**
Ta et al. (2009)	2009	The ability for the system to **absorb** the consequences of disruptions to **reduce the impacts** of disruptions and **maintain** freight mobility.
Ip and Wang (2011)	2011	The ability of a system to **return to a stable state** following a strong perturbation caused by failure, disaster or attack.
Serulle et al. (2011a)	2011	The ability for the system to **maintain its demonstrated level of service** or to **restore** itself to that level of service in specified time frame.
Vugrin et al. (2011)	2011	Given the occurrence of a particular disruptive event (or set of events), the resilience of a system to that event (or events) is the ability to efficiently **reduce both the magnitude** and **duration** of the deviation from targeted system performance levels.

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Author	Year	Definition
Henry and Ramirez-Marquez (2012)	2012	Describes the ratio of **recovery** at **time** t to loss suffered by the system at some previous point in time.
Freckleton et al. (2012)	2012	The ability for a transportation network to **absorb** disruptive events gracefully and **return** itself to a level of service equal to or greater than the **pre-disruption level** of service within a reasonable **time frame**.
Miller-Hooks et al. (2012)	2012	Both the network's inherent ability to **cope with disruption** via its topological and operational attributes and potential actions that can be taken in the immediate aftermath of a disruption or disaster event.
Chen and Miller-Hooks (2012)	2012	A network's capability to **resist** and **recover** from a disruption or disaster.
Adams et al. (2012)	2012	The capacity to **absorb** the effects of a disruption and to **quickly return** to **normal operating levels**.
Lee et al. (2014)	2014	The system's ability to efficiently **reduce both the magnitude** and the **duration** of systemic impacts and **recovery efforts**.
Faturechi and Miller-Hooks (2014)	2014	The network's ability to **resist** and **adapt** to disruption.
The National Academy of Sciences (NAS)	2015	The ability to **plan and prepare** for, **absorb**, **recover** from, and **adapt** to adverse event.

Along with the fact that there is not an official definition for resilience, there is not a consensus about what the resilience term includes when analysing the resilience of transport networks. It is possible to differentiate between: (a) the definitions that mainly focus on the part of absorbing or accommodating the perturbation, i.e. Institute (2010), Ta et al. (2009), (b) other definitions that focus the definition of resilience on the recovery phase such as the one provided by Henry and Ramirez-Marquez (2012) who describes transport resilience as "the process of recovery after the hazard", and (c) other definitions, i.e. Murray-Tuite (2006) or Miller-Hooks et al. (2012) that understand resilience as a combination of both phases, incorporating in the same concept the ability of adaptation and the recovery process.

Finally, only a few of these definitions take into account the necessity of defining a time frame. In this case, authors such as Freckleton et al. (2012) or Serulle et al. (2011b) include the importance of the period that is used to recover from the perturbation, and to reach an acceptable level of service.

For a better understanding of the concept, some words have been highlighted in the definitions presented in Table 1.2. In Figure 1.2 each of the highlighted words has been included in a different area of resilience, including (a) pre-event and post-event, (b) perturbation, (c) recovery, and (c) time.

Inside the perturbation group, see Figure 1.2, words such as “absorb”, “resist”, “accommodate”, “adapt” , and “cope with disruption” can be found, this group of words represent the phase of disruption when the network is being damaged. On the recovery group, words such as “return to a stable state”, “restore”, “recover”, and “ reduce the impact” can be found. These words are related to the process of the network after the impact, where the objective is to minimize the damages of the event until normal operation is reached again.

Two additional groups can be found in Figure 1.2, the first one is referring to the time that is needed from the initial impact to the total recovery, including words such as “quickly return”, “reasonable time frame”, and “reduce the duration”. The other group highlight the pre, and post event actions, including words such as “pre-disruption level”, “ability to plan”, and “prepare”.

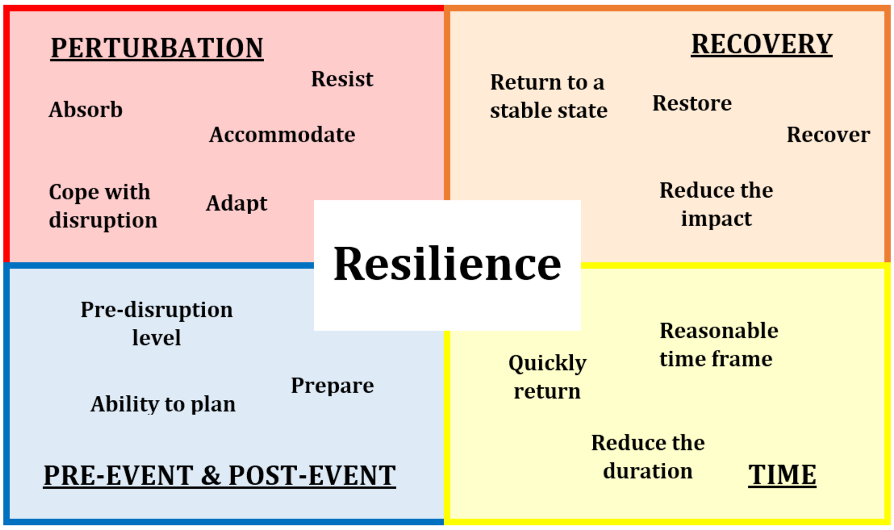

Figure 1.2: Resilience concept divided in four categories: pre-event, perturbation, recovery, and time.

1.3 Definition of resilience in transport networks

After the analysis of transport resilience definitions developed in Section 1.2, some important characteristics are obtained, and presented below in this Section. The conclusions here explained will establish the definition of resilience that will be applied in this thesis. In addition, in Figure 1.3 a graphical explanation of the understanding of resilience in traffic networks can be found.

An adequate resilience definition has to be based on the following items:

- The definition of resilience should specify the **perturbation stage**; ability to absorb, resist or accommodate a perturbation. This phase is a dynamic phase, and should not be considered as a "punctual" impact in the network, since most of the perturbations vary over the time. In addition, if the perturbation is very strong, the system can break down and this collapse point should be identified.

- Once the perturbation is finished, the definition of resilience should specify the **recovery stage**; the ability to restore, return or recover from the perturbation. This recovery action goes from the moment that the perturbation has stopped, until the moment that a new system equilibrium where the transport network is able to operate, is reached again. However, the level of service associated with the new equilibrium point could be equal, better or worse than the previous level of service of the transport network.

- Finally, both phases, i.e. the perturbation period and the recovery period, should be analysed in a specific **time frame**. The resilience of traffic networks will be highly dependent on the time frame that is necessary in the process. Thus, the evaluation of the impact, and the damages suffered in the network should be always evaluated with regard to the time spent in the whole performance.

In addition, two more points when dealing with resilience in transport networks are highlighted below:

- Even though many definitions of resilience are not specified, in this thesis, the analysis of this concept in transport network is associated with the disruption that is analysed. This means that the resilience of a transport network is evaluated for a specific event. Then, different values of resilience can be obtained depending not only on the characteristics of the network but also on the characteristics of the event, such as duration, intensity, and area affected. As an example, in order to highlight this fact, Vugrin et al. (2011) include in their definition of resilience, the sentence: " given the occurrence of a particular disruptive event (or set of events)".

- Pre-event and post-event actions are essential strategies when dealing with extreme events in transport networks. Sometimes, extreme weather events can be forecasted, and consequently, pre-event measures can be programmed to mitigate future damages in the network. Also, some events can cause permanent damages in the network, and for reaching the total recovery some external actions need to be implemented. However, when measuring the concept of resilience in transport networks, the "inherent" capacity of the network to absorb and recover from the perturbation will be the one measured in the concept of resilience, and external pre and post actions will not take part in the value of resilience.

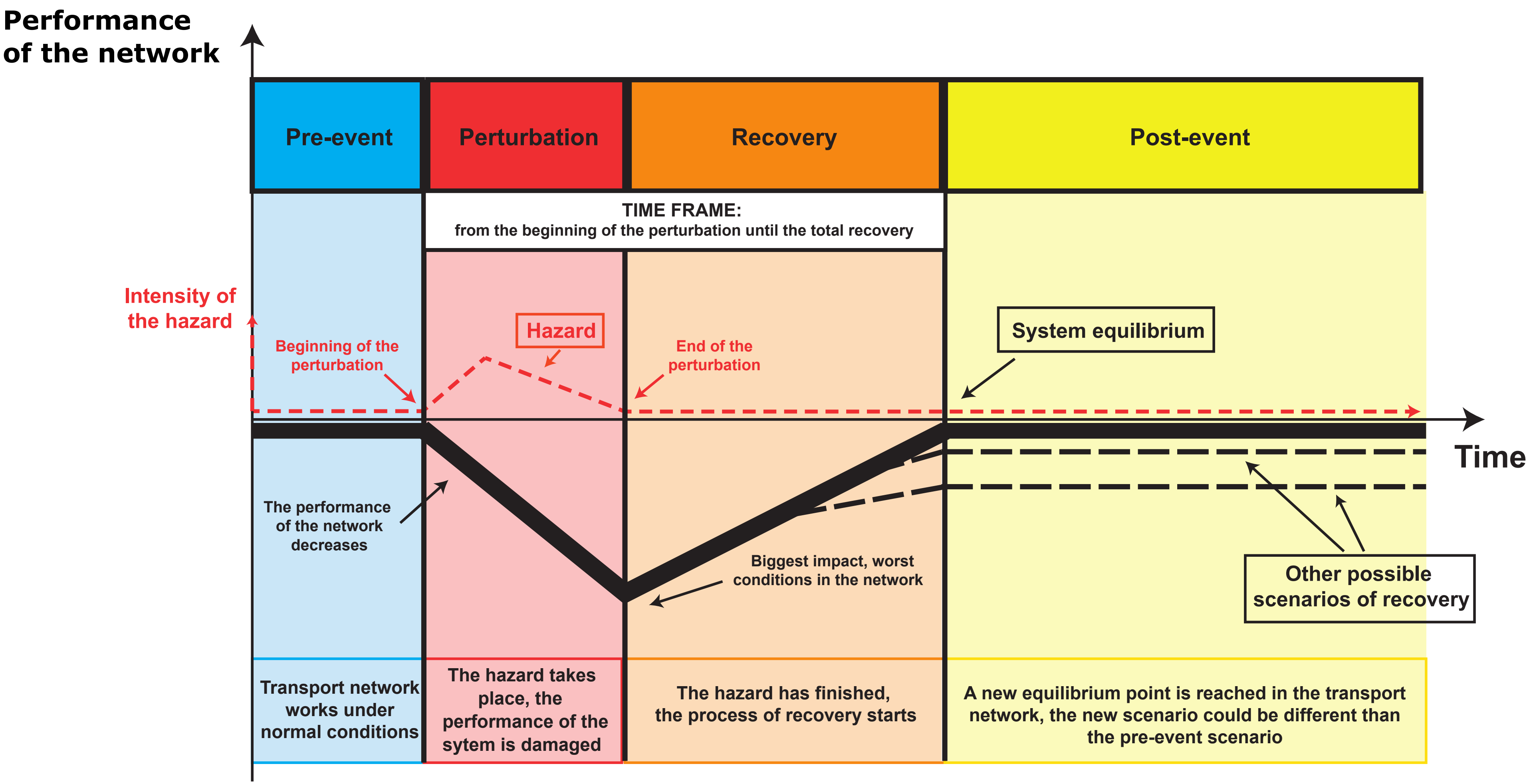

Figure 1.3: Definition of resilience

1.4 Methodologies for measuring resilience

Some authors have tried to measure the concept of resilience. This is not a straightforward task, due to all the parameters involved in the analysis of resilience. This fact is noted by authors such as Hosseini et al. (2016) who highlight that *" The term "resilience" is increasingly used in research journals, government documents, and the media, but work still remains on making resilience assessment usable"*, also, that *"methods for resilience planning are still a relatively unexplored area"*, Reggiani et al. (2015) who said that *" only a few contributions deal explicitly with empirical applications or simulations of real networks"*, and Henry and Ramirez-Marquez (2012) who noted two issues, (1) *" there is no consistent quantitative approach to resilience because there is no consistent treatment of the concept of resilience"*, and (2) *" the quantitative approaches available are limited in their scope and usability and hence are not amenable for use outside the discipline where they have been developed."*

When previous methodologies to evaluate resilience are studied, different approaches for its evaluation can be found. This includes authors who analyse resilience in a qualitative manner, others who present a semi-quantitative methodology, and finally those that introduce a quantitative methodology to evaluate it. In this thesis, and more precisely in the following section, the focus will be on those methodologies that have been presented in the literature examining this concept from a quantitative perspective.

1.4.1 Quantitative methodologies for measuring resilience in transport networks

Bruneau et al. (2003) analyse seismic resilience of a community. Despite the fact that this approach is applied to the case of earthquakes, the method presented by Bruneau et al. (2003) is explained in this Section due to its general applicability that can be easily extended to other systems. Several authors in the area of transport networks used this approach for the analysis of resilience.

The framework introduced by Bruneau et al. (2003) considered that the increment of the resilience of a system needs to be based on the reduction of failure probabilities, the reduction of consequences from failures, and the reduction of time to recovery. In this method for the evaluation of resilience, resilience is divided in four dimensions; namely, robustness, rapidity, resourcefulness, and redundancy, the definition of each of the four dimensions is given in Table 1.3.

The approach presented by Bruneau et al. (2003) is based on the **resilience triangle model** introduced in Figure 1.4.

Figure 1.4 represents a system impacted by a disruptive event, with t_0 being the moment where the perturbation takes place, and t_1 the time when the quality of the infrastructure is recovered. $Q(t)$ is the quality of the infrastructure during the time, and

Table 1.3: Definition of the four dimensions of resilience by Bruneau et al. (2003)

Dimension	Definition
Robustness	The strength of system, or its ability to prevent damage propagation through the system in the presence of disruptive event
Rapidity	The speed or rate at which a system could return to its original state or at least an acceptable level of functionality after the occurrence of disruption
Resourcefulness	The level of capability in applying material (i.e., information, technological, physical) and human resources (i.e., labor) to respond to a disruptive event
Redundancy	The extent to which carries by a system to minimize the likelihood and impact of disruption

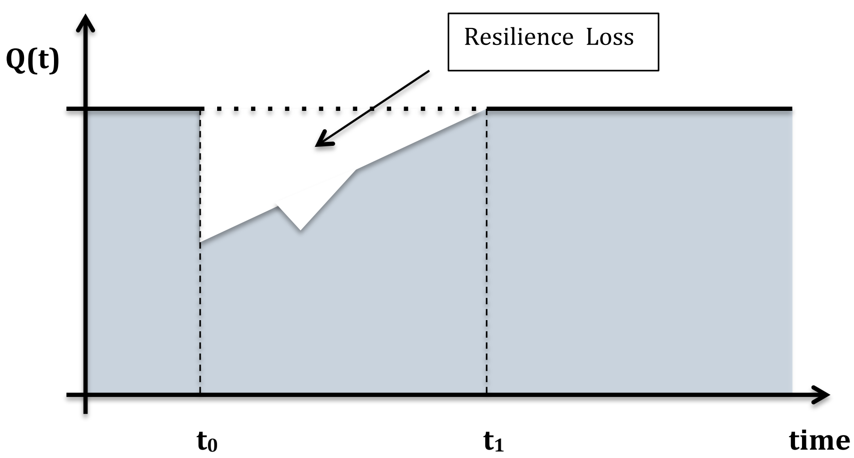

Figure 1.4: Resilience triangle model used by Bruneau et al. (2003)

it will be measured as a function of the four dimensions previously presented. In the method, a pre-event level is assumed with a 100 % of $Q(t)$, then the perturbation occurs in t_0 and a loss in the quality of the infrastructures, $Q(t)$, is obtained, this loss is recovered during the time until the pre-event level of $Q(t)$ is reached in t_1.

Therefore, the resilience loss (RL) is measured by Bruneau et al. (2003) using Equation 1.1.

$$RL = \int_{t_0}^{t_1} (100 - Q(t))dt, \tag{1.1}$$

where the remaining area after a reduction in the quality of the infrastructure, $Q(t)$, is calculated. This reduction of the quality of the infrastructure is determined by the dimensions presented in Table 1.3, but specific examples or case studies are not developed by the author in this paper.

In addition, resilience is also conceptualized by Bruneau et al. (2003) in four levels: technical, organizational, social, and economic. The authors highlight that these four dimensions cannot be evaluated in a unique measure, therefore multiple measures are necessary in order to assess the global system performance. This separation will be used for different authors to describe resilience. See Table 1.4 for a description of the four levels.

Table 1.4: Levels of resilience presented by Bruneau et al. (2003)

Resilience level	Definition
Technical	Physical systems perform when subjected to earthquake forces
Organizational	The ability to respond to emergencies, and carry out critical functions
Social	The capacity to reduce the negative social consequences of loss of critical services
Economic	The capacity to reduce both direct and indirect economic losses

Following the resilience triangle model, presented by Bruneau et al. (2003) for a conceptual definition of resilience, Bocchini and Frangopol (2012) analyse the resilience of networks of highway bridges when earthquakes occur. The authors focus on the restoration activities to maximize resilience, in this way the following equation is introduced to measure resilience, R:

$$R = \frac{\int_{t_1}^{t_2} F(t)dt}{t_2 - t_1}, \tag{1.2}$$

with F being the network performance, which is evaluated by (a) the total travel time (i.e., the sum of the time spent by all the users of the network to reach their destinations, considering the trips that start at a time t after the event), (b) the total travel distance, and (c) two balancing factors, one associated with the time spent by the network users, and another associated with the distance travelled. t_1, and t_2 are defined in Figure 1.5, being the time when the restoration activities start, and the time when these activities finish respectively. In this way, the authors implement the formulation proposed by Bruneau et al. (2003) normalizing the value of resilience over the time necessary for the recovery.

Bocchini and Frangopol (2012) introduce a two level optimization structure, where firstly the traffic flows are measured by a traffic assignment model, and the recovery decisions are analysed in the second level. To select the recovery strategies, the variables introduced are the time necessary for the reparation and the rate of expenditure. In

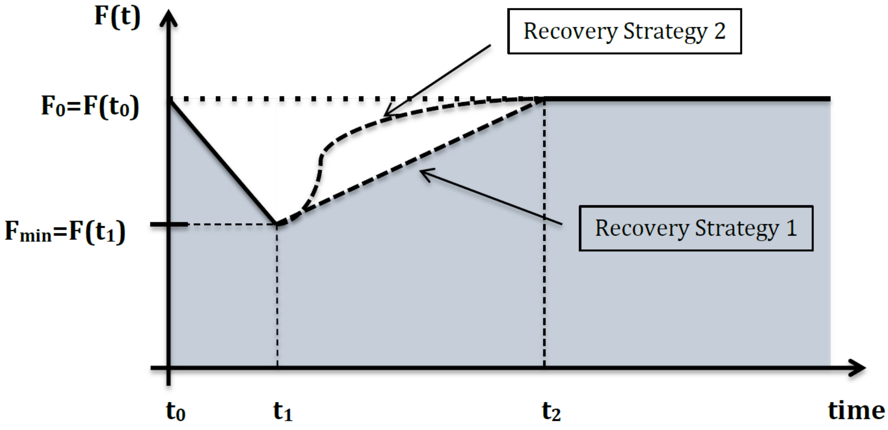

Figure 1.5: Approach used by Bocchini and Frangopol (2012) based on the resilience triangle model presented by Bruneau et al. (2003)

this way, and with a limited budget available for the restoration process, the restoration activities will be the ones that recover the capacities of the damaged bridges. In the paper, the disruption analysed is an earthquake, the impact is simulated as a reduction of the functionality of each of the elements of the network, and the beginning of the restoration activities is the starting point for the measurement of resilience.

It is noted that both research works previously described, Bruneau et al. (2003), and Bocchini and Frangopol (2012), address the concept of resilience focusing on the recovery process. Using the same perspective, Chen and Miller-Hooks (2012), and Henry and Ramirez-Marquez (2012) evaluate the recovery of transport network under disruption.

Henry and Ramirez-Marquez (2012) calculate resilience as a ratio of recovery to loss, at a given time. Using the formula presented in Equation 1.3, which was previously introduced by Rose (2007).

$$R(t_r|e_j) = \frac{F(t_r|e_j) - F(t_d|e_j)}{F(t_0) - F(t_d|e_j)}, \forall e_j \in D \tag{1.3}$$

where $R(t_r|e_j)$ indicates the proportion of delivery function that has been recovered from a disrupted scenario e_j, among the possible set of disrupted scenarios D. This value of resilience is measured for a specific figure-of-merit, $F(t_r|e_j)$ for a time t_r under a disruptive event, e_j. $F(t0)$ describes the value of the figure of merit for the original state (pre-disruption), and $F(t_d|e_j)$ for the disrupted state (the largest disruption), being t_d the time of the final disrupted scenario (see Figure 1.6). The figure of merit for a perturbation state, $F(\bullet)$, is measured by three characteristics, (a) shortest path (b) maximum flow, and (c) overall health of road network.

The authors present a time-dependent quantifiable metric where only one of the links can be repaired at each time. When the perturbation takes place in the network, some links are destroyed (no partial damage is considered), and the restoration process is measured in terms of the time necessary for repairing each of the links. The final result is a sequence of links that optimize the value of resilience. Figure 1.6 introduces an example of the results by using different strategies of restoration, where as time goes, more parts of the network are restored, and the resilience, as a ratio of the restoration, is recovered. In Figure 1.6, the first strategy (left graph) is better since it is able to recover high values of resilience at an earlier time, unlike, the second strategy (right graph) which does not reach high values until almost half of the recovery time.

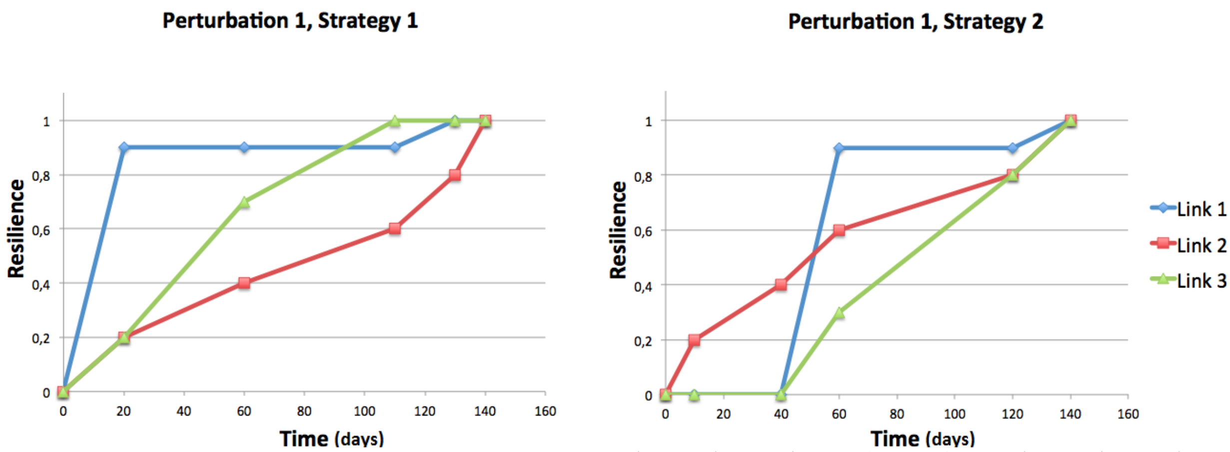

Figure 1.6: Approach used by Henry and Ramirez-Marquez (2012) to evaluate resilience in transport networks.

Also, analysing the recovery process after a disruption to assess resilience, Chen and Miller-Hooks (2012) introduce an optimization model for maximizing the quantity of demand that can be absorbed after disruption for a given time. In this case, the method is applied for intermodal freight transportation networks. In this paper, the links can be repaired simultaneously once the perturbation event takes place. The model will define the group of recovery strategies with the final goal of maximizing the demand at a given time. Contrary to Henry and Ramirez-Marquez (2012), the time necessary for the restoration activities in the method proposed by Chen and Miller-Hooks (2012), is not measured, but a global budget for the restoration activities is given.

Chen and Miller-Hooks (2012) evaluate the network resilience, R, as “the post-disaster expected fraction of demand that, for a given network configuration, can be satisfied within specified recovery costs (budgetary, temporal, and physical)”, and it is formulated as follows:

$$R = E\left[\frac{\sum_{w \in W} d_w}{\sum_{w \in W} D_w}\right], \tag{1.4}$$

where d_w is the maximum demand that can be satisfied for the origin-destination (OD) pair w during the post-disaster stage, and D_w is the demand that can be satisfied for the OD pair w during the pre-disaster stage, being W a set of OD pairs.

The authors formulate a stochastic mixed-integer program, including (a) integer variables that represents the different options of the recovery activities on corresponding links and the selection of routes carrying flow, and (b) continuous variables, representing the flow along each route and demand that cannot be satisfied for each OD pair. Multiple disaster scenarios are tested, and for each of them different recovery budgets are analysed. Then, the final value of resilience will depend on the budget used for the restoration activities, and the number of restoration activities implemented.

Pant et al. (2014), based on the deterministic metrics described by Henry and Ramirez-Marquez (2012) (see Equation 1.3), present an extended model to the stochastic case of resilience, introducing three stochastic measures of resilience, namely, (a) time to total system restoration, (b) time to full system resilience, and (c) time to α 100% resilience, with α being the percentage of restoration. A description of the three metrics is given in Table 1.5.

Table 1.5: Metrics to evaluate resilience introduced by Pant et al. (2014)

Metric	Definition
Time to total system restoration	It measures the total time spent from the point when recovery activities commence up to the time when all recovery activities are finalized, irrespective of whether components are repaired in series or parallel.
Time to full system resilience	It measures the total time spent from the point when recovery activities started up to the exact time when system service is completely restored.
Time to α 100% resilience	It measures the total time spent from the point of time when recovery commences until the time that α 100% of system functionality is restored.

In the same line of research, analysing the optimal recovery response for maximizing resilience of transport networks, Vugrin et al. (2010) formulated a framework to assess resilience. The authors consider two main terms, the restoration of system performance, and the required expenditure of resources. Vugrin et al. (2010) integrated the two concepts in Equation 1.5, as follows:

$$RI = SI + \alpha\ TRE, \tag{1.5}$$

where SI is described as “the impact that a disruption has on system productivity and is measured by evaluating the difference between a targeted system performance (TSP) level, and the actual system performance (SP) following the disruption”. TRE is evaluated as "the efficiency with which the system recovers from a disruption and is measured by analysing the amount of resources expended during the recovery process". Finally, α is a weighting factor, and RI is the resilience index.

When evaluating SI, the authors include the inherent capacities of the system when a perturbation occurs which are divided in (a) absorptive capacity, considering aspects that automatically manifest after the disruption, and (b) the adaptive capacity, considering internal aspects that manifest over time after the disruption. However, when analysing TRE, the authors consider the restorative capacity, including the external efforts that can be required. See Figure 1.7 for a graphic description.

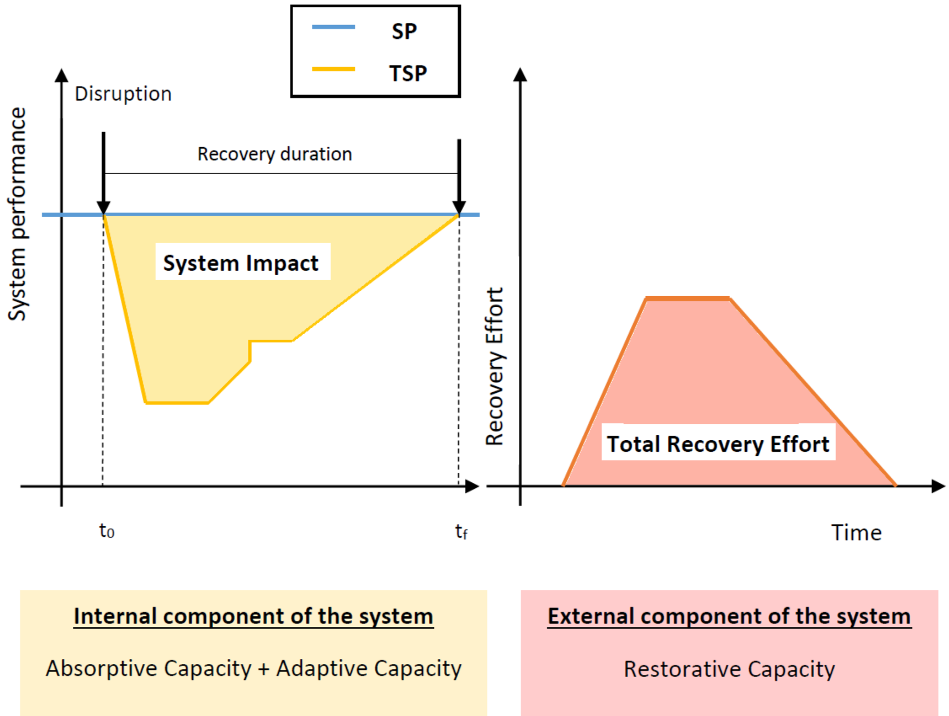

Figure 1.7: Description of System Impact(SI) and Total Recovery Effort(TRE) presented by Vugrin et al. (2010)

Finally, the results obtained in the paper are shown in metric form, where each of the variables, such as the absorptive capacity, the adaptive capacity, and the restorative capacity, are evaluated as low, medium or high.

The same framework to evaluate resilience is addressed in Vugrin et al. (2011), and Vugrin and Camphouse (2011). For example, in Vugrin et al. (2011), the resilience is evaluated quantitatively by using a resilience cost measurement methodology, which is formulated as follows:

$$\text{Resilience Cost} = \frac{\text{Systemic Impact} + \text{Market Recovery Effort} + \text{Transportation Recovery Effort}}{\text{Target Market Value of Production}}. \tag{1.6}$$

In addition, Lee et al. (2014) use the recovery-dependent resilience approach presented by Vugrin et al. (2011) (see Equation 1.5). Lee et al. (2014) quantify the SI, including variables such as the increment in travel times (VOT), and the operating costs (VOC), with both being calculated by using link volumes, and traffic speed with a traffic allocating software, EMME. The formulation for both variables is presented as follows:

$$VOTS = VOT_{after} - VOT_{before}, \tag{1.7}$$

where

$$VOT : \sum_{l} \sum_{k=1}^{3} (T_{kl} P_k Q_{kl} Duration(day)), \tag{1.8}$$

with VOTS being the travel times increment, T_{kl}, vehicle-travel time by vehicle type of links l, P_k, value of travel time by vehicle type, Q_{kl}, volume of link, and $Duration(day)$ the number of days to complete the recovery. The vehicle type includes the options of auto, bus and truck.

$$VOCS = VOC_{after} - VOC_{before}, \tag{1.9}$$

where

$$VOC : \sum_{l} \sum_{k=1}^{3} (D_{kl} VT_k Duration(day)), \tag{1.10}$$

with VOCS being the operating costs increment, $D_k l$, the vehicle-km by link, and VT_k, the operation costs/km based on travel speed of each vehicle type.

For the evaluation of the TRE, Lee et al. (2014) consider the rehabilitation expenses, including, for example, the time and the cost of the recovery.

Later on, Vugrin et al. (2014), using the approach presented by Vugrin et al. (2011) (see Equation 1.5), present a bi-level optimization approach for identifying optimal recovery responses that maximize resilience for disrupted transportation networks. In the lower level, the flows of the network are evaluated, and in the upper level, an analysis of the optimal recovery strategies is addressed. In this case, the recovery actions are developed as a sequence of discrete tasks, and a time limit is introduced in the process.

The main variables of the methodology, SI and TRE, are measured as follows:

$$SI = \sum_{t=1}^{T} \left[\sum_{i} (H_i[x_i(t)] - H_i^0(t)) + \sum_{rs} \gamma_{rs} e_{rs}(t) \right], \tag{1.11}$$

where $x_i(t)$ is the network flow for link i, $H_i[x_i(t)]$ is the total cost for link i, $H_i^0(t)$ is the total cost for link i in the non-disrupted state, and t is the time, analysed as a discrete variable, going from 1 to T, which is the length of the planning horizon. e_{rs} is the volume of travel from origin r to destination s, that cannot be accommodated at time t, and γ_{rs} is a penalty cost for the demand that cannot be accommodated. In this case, the cost is the performance measure for evaluating the SI, and may include aspects such as travel time, distance, fuel consumption, or other factors.

For the measurement of TRE, Vugrin et al. (2014) include the reparation tasks on the damaged parts of the network, and is formulated as:

$$TRE = \sum_i \sum_j \sum_m \sum_t C_{ijmt} \mu_{ijmt}, \tag{1.12}$$

where

$$\mu_{ijmt} = \begin{cases} 1 & \text{if task } j \text{ on link } i \text{ is initiated in mode } m \text{ in period } t; \\ 0 & \text{otherwise,} \end{cases} \tag{1.13}$$

with C_{ijmt} being the associated travel cost of link i for task j in mode m.

Both variables are combined into an objective function, see Equation 1.14.

$$MinZ = \sum_{t=1}^{T} \left[\sum_i (H_i[x_i(t)] - H_i^0(t)) + \sum_{rs} \gamma_{rs} e_{rs}(t) + \sum_i \sum_j \sum_m \sum_t C_{ijmt} \mu_{ijmt} \right], \tag{1.14}$$

The constrains of the optimization problem are related to the restoration tasks, including aspects such as:

- Task j for link i can be performed in only one selected mode and will not be scheduled more than once.
- Tasks j and l for link i may have precedence constraints.
- Link repairs require some physical resources in limited supply, and availability of these resources may constrain recovery scheduling.

The objective function proposed by Vugrin et al. (2014) considers network flows, costs of unmet demand, and recovery costs. The authors highlight that this approach can find better recovery sequences than others methodologies that only minimise the total time to complete the recovery tasks.

Freckleton et al. (2012) present a semi-quantitative methodology to evaluate resilience in transport networks. In this case, the methodology is based on a Fuzzy Interference System, which was previously introduced in Heaslip et al. (2009), Heaslip et al. (2010)

and Serulle et al. (2011a). Despite the different approach for the evaluation, using fuzzy logic, the authors based their definition of resilience in the idea of Bruneau et al. (2003), using as a starting point the resilience triangle model shown in Figure 1.4, and the concept of the resilience cycle, presented in Figure 1.8.

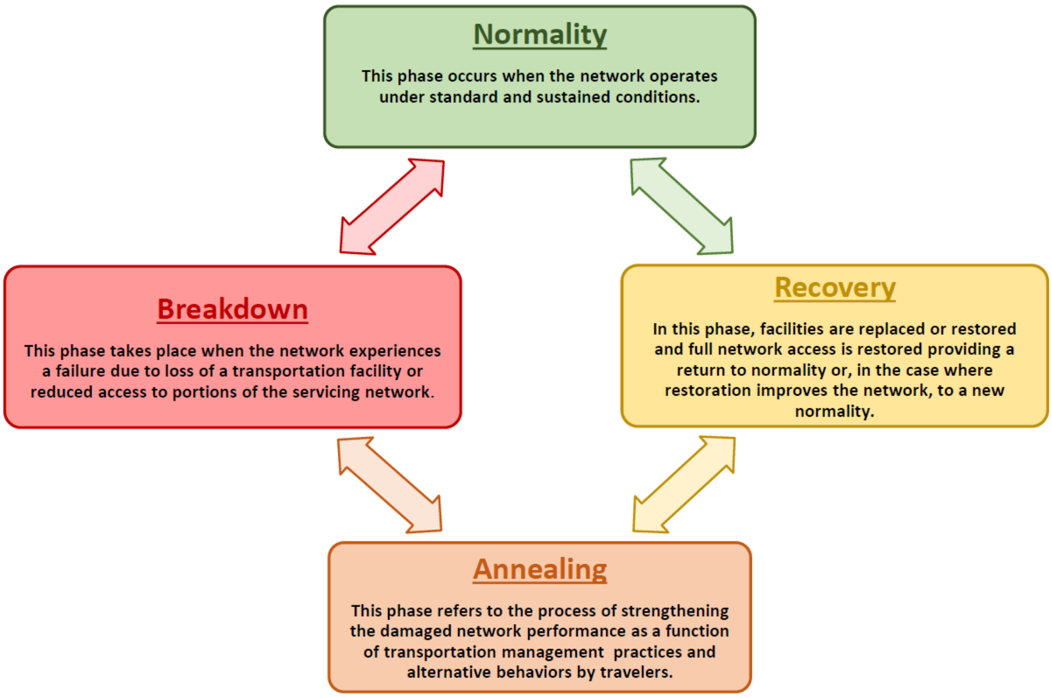

Figure 1.8: Transportation resilience circle, used by Freckleton et al. (2012)

The aim of this method is to obtain a value for the "Total Network Resiliency". For that purpose, the total network resilience is divided in four metric groups, with each being formed by multiple components, see Figure 1.9 for a description of the concept and hierarchy. Fuzzy logic is introduced by the authors in order to transform the relationship between components, and groups into mathematical relations. By applying fuzzy logic theory, the authors can combine the terms presented in Figure 1.9, and obtain a global value of resilience.

The 16 components presented by Freckleton et al. (2012) in Figure 1.9 are measured on a qualitative scale with ranging input values. Inside each metric group, each component receives a weight according the number of variables considered in each metric group. For example in the first metric group (Individual Metric Group), see Figure 1.9, there are 5 components, each of them have a weight of 1/5. In this way, the components that appear in more than one metric group will carry a larger weight in the final value of the global resilience.

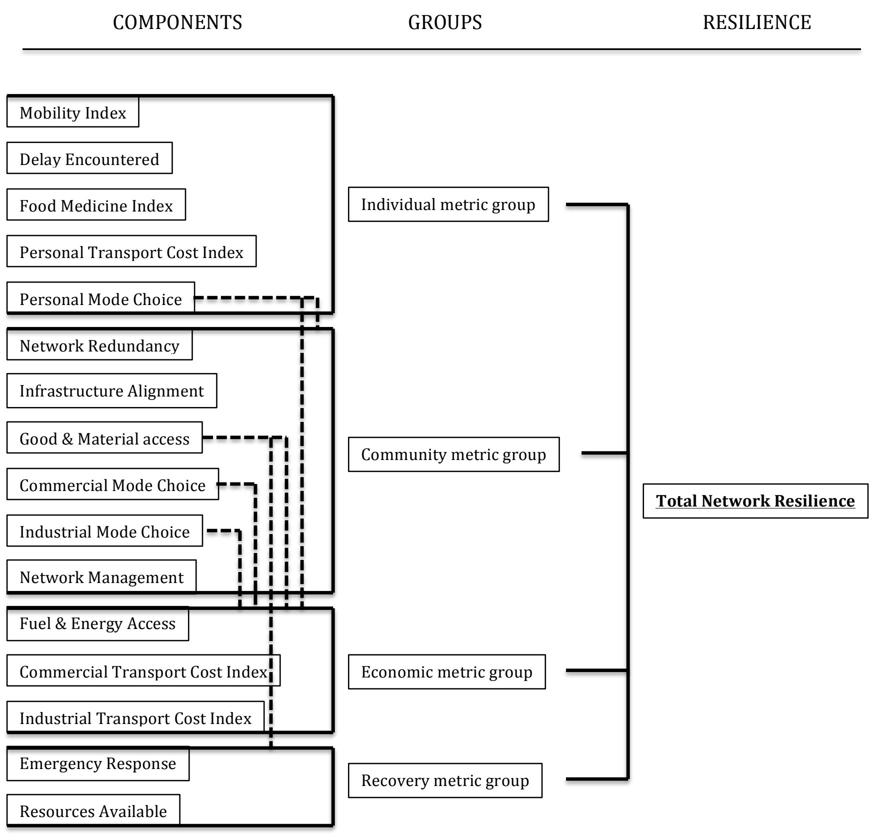

Figure 1.9: Metric to assess resilience, used by Freckleton et al. (2012)

It is noted that the division of the total resilience in four metric groups, namely, individual, community, economic and recovery, see Figure 1.9 may be inspired with the way that Bruneau et al. (2003) conceptualized resilience in four interrelated dimensions, such as technical, organizational, social, and economic. Without being the same metrics, the authors try to cover all the aspects of resilience presented by Bruneau et al. (2003) previously.

The value of each core metric group is obtained by the inputs of each of the group components. Finally, the outputs of each metric group, and also the global value of resilience are defined by a qualitative scale of nine levels, being extremely low, very low, low,medium low, medium, high, very high and extremely high.

Murray-Tuite (2006) introduces an approach to evaluate resilience, where this concept is divided in ten dimensions. The first six dimensions, namely, redundancy, diversity, efficiency, autonomous components, strength, adaptability, were defined by Godschalk (2003) (see Table 1.6 for description of the variables), and the other four dimensions are collaboration, mobility, safety, and the ability to recover quickly, also described in Table

1.6. In a previous paper, Murray-Tuite and Mahmassani (2004) developed a disruption index for transportation networks, where the terms of redundancy, mobility, and diversity, are combined into one measure, however the concept of resilience is not assessed. In Murray-Tuite (2006), the author presents all the dimensions necessary for the evaluation of resilience, but again a total value of resilience is not reached. The author highlights the complexity of the dimensions, and the difficulty to obtain a global value of resilience, and finally, in the paper, only four of the ten dimensions have a formulation for its evaluation.

Table 1.6: Definition of the ten dimensions of resilience presented in Murray-Tuite (2006)

Dimension	Definition
Redundancy	indicates that multiple components serve the same function
Diversity	means that the components are functionality different
Efficiency	indicates input-output ratio optimization
Autonomous components	components have the ability to operate independently
Strength	indicates the system's ability to withstand an event
Adaptability	implies that the system is flexible and elements are capable of learning from past experience
Collaboration	indicates that information and resources are shared among components or stakeholders
Mobility	indicates that the travelers are able to reach their chosen destinations at an acceptable level of service
Safety	suggest that the system does not harm its users or unduly expose them to hazards
Ability to recover quickly	means that an acceptable level of service can be restored rapidly and with minimal outside assistance after an event occurs

Faturechi and Miller-Hooks (2014) present a bi-level, three stage stochastic mathematical model to quantify travel time resilience, the objective of the model is to maximize resilience in terms of travel time. Travel time is the only variable considered in modelling users' behaviour, and a budget is restricting the restoration actions.

The upper level presented in the model includes the selection of the strategies for an increment of resilience, taking into account the Disaster Management Life-Cycle (DMLC) introduced by Waugh (1999), including three stages presented as follows:

- **Pre-event expansion and retrofit as mitigation options** to enhance the coping capacity of the road network. All links of the network except for bridges can have an increase in their capacities.

- **Pre-event preparedness** where resources are located shortly in advance of a predicted event occurrence to improve the response actions. In the case of an unpredicted event, no action in this stage will take place.

- **Post-event short-term response actions** taken post-disaster to restore the damages of the network, including increment in network capacity, minimize the extent of damage, and/or protect the remaining facilities.

The goal of the model is to maximize the resilience among the different damaging scenarios analysed, and at the same time minimizing the total travel time of the network. Resilience is obtained for multiples scenarios, and formulated as follows:

$$R_{T,B} = \frac{tt^{r-1}}{tt^{0-1}} \tag{1.15}$$

where tt^{r-1} is the total travel times at time, r, i.e. at the end of pre-event (see Figure 1.10), and tt^{0-1} is the total travel times at the end of the response stage.

In addition, in Figure 1.10, a description of the different stages proposed by Faturechi and Miller-Hooks (2014) when evaluating resilience is presented. Variables tt^{r-1}, and tt^{0-1} are also identified in Figure 1.10. With this methodology is possible to determine the best measures to enhance the resilience of the network in the three stages, that is, pre-event, response, and recover (see Figure 1.10).

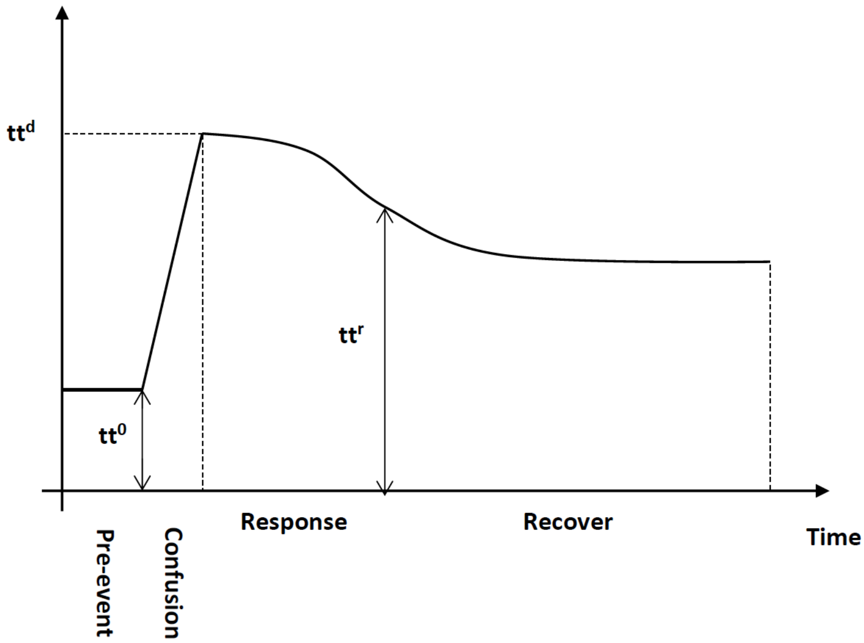

Figure 1.10: Approach used by Faturechi and Miller-Hooks (2014) for the identification of the different stages when a perturbation takes place.

It is noted that this model assumes that the users have perfect information on the damage and repair links.

Zhang and Wang (2016) present a novel metric based on system reliability, and network connectivity to measure resilience-based performance of transportation networks. The authors use the resilience concept previously introduced by Ip and Wang (2011) (see Table 1.2). The resilience-based performance metric (named WIPW), considers different variables, including the network topology, redundancy level, traffic patterns, structural reliability of network components, and functionality of the network during a community's post-disaster recovery.

Having a network $G = (V, A)$, where $V = 1, 2...n$ is the set of nodes, and $A = 1, 2...m$ is the set of links, the resilience-based performance metric of a road network, WIPW, is formulated as follows:

$$WIPW(G) = \sum_{i=1}^{n} w_i \frac{1}{n-1} \sum_{j=1, j \neq i}^{K_{(i,j)}} w_k(i,j) \cdot R_k(i,j), \tag{1.16}$$

where, $K_{(i,j)}$ is total number of independent pathways (IPW) between nodes i and j, and $R_k(i,j)$ is the reliability of independent pathways $P_k(i,j)$, and is defined as follows:

$$R_k(i,j) = \prod_{\forall l \in P_k(i,j)} q_l \tag{1.17}$$

where l is the individual road links, and q_l denotes the reliability of l. In this paper, the reliability can be estimated through Monte Carlo Simulation using a Gaussian Copula to model correlation.

In Equation 1.16, w_i, and $w_k(i,j)$ are two weighting factors. The first one measures the relative importance of a node, and the second one identifies the relative impact that this pathway has on people's normal life activities and the local economy.

The authors present this metric to measure resilience based on the evaluation of the network reliability. Thus, they highlight that resilience can be increased by: (a) incrementing reliabilities of critical network components through appropriate retrofitting, (b) optimizing network topology through new construction, (c) altering traffic flow patterns through appropriate routing policies, and (d) strategically siting emergency response facilities.

1.4.2 Comparison of methodologies to measure resilience in transport networks

In this section, the methodologies to evaluate resilience previously explained are summarized, highlighting for each of them the following aspects: (a) the main objective of the paper; this is included because, even though in some of the papers the final goal is not the evaluation of resilience, a methodology is presented; (b) how resilience is measured, and (c) the main disadvantages found in each of the methodologies.

Author	Methodology
Bruneau et al. (2003)	**Main objective**: presentation of a framework for defining seismic resilience. The authors introduce the resilience triangle model, see Figure 1.4, approach that has been used by many authors later, such as Bocchini and Frangopol (2012), Lee et al. (2014) and Vugrin et al. (2014). **Disadvantages**: This methodology presents a very wide framework for resilience assessment, including four different levels, and four dimensions, and numeric examples for its evaluation are not included.
Murray-Tuite (2006)	**Main objective**: presentation of multiple metrics for the evaluation of resilience in transport networks. A comparison of some of the proposed metrics using system optimum and user equilibrium assignment models is also included. **Resilience is measured** by the evaluation of ten dimensions: redundancy, diversity,efficiency, autonomous components, strength, collaboration, adaptability, mobility, safety, and the ability to recover quickly. **Disadvantages**: A large amount of resources is needed for the evaluation of all the metrics. In the paper, only four of the ten dimensions are evaluated, and finally a quantification of the total resilience of the network is not obtained.
Vugrin et al. (2010)	**Main objective**: presentation of a general framework for assessing the resilience of infrastructure and economic systems, including a quantitative model for measuring the resilience of systems. **Resilience is measured** through the evaluation of both the system impact (SI) and the total recovery effort (TRE). **Disadvantages**: this methodology includes the absorptive capacity and the adaptive capacity of the system, but the main focus is on the recovery strategies again. In addition even though the perturbation impact is considered, it is mainly determined as a punctual effect. Finally, the results obtained are not numerical. In this paper, the resilience is evaluated by giving a qualitative value (low, medium or high) to the different variables, such as the absorptive capacity, the adaptive capacity and the restorative capacity.

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Author	Methodology
Bocchini and Frangopol (2012)	**Main objective**: presentation of an optimization procedure for the restoration activities, maximizing the total network resilience. **Resilience is measured** as the area under the curve of the network performance. The performance of the network is evaluated depending on the restoration task. **Disadvantages**: This methodology analyses only the restoration process, the damages are simulated in the network as a loss in the functionality of each of the elements, and through the implementation of multiple recovery activities the functionality of the network increases. Therefore, the model considers exterior intervention for the restoration activities, and they are a key element in the measurement of resilience. Also, the damage is introduced as a "static" damage, without a dynamic evaluation of the perturbation process.
Chen and Miller-Hooks (2012)	**Main objective**: presentation of an indicator that accounts the impact of potential recovery activities. These activities might be taken in the immediate aftermath of the disruption to meet target operational service levels, and need to be adhered to a fixed budget. **Resilience is measured** as the post-disaster expected fraction of demand that, for a given network configuration, can be satisfied within specified recovery costs. **Disadvantages**: This methodology focuses on the optimization of the restoration activities for a given budget, considering exterior intervention for the restoration activities. Different disaster scenarios are tested, but the damage is not introduced as a dynamic action, and it is considers as a "punctual" disruption.
Henry and Ramirez-Marquez (2012)	**Main objective**: presentation of generic metrics and formulae for quantifying system resilience. **Resilience is measured** as the ratio of recovery at time t to loss suffered by the system at some previous point. Resilience value goes from 0, when none of the links damaged by the perturbation has been restored, until 1 when all the network has been restored. The resilience value increases with the time as the restoration activities progress.

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Author	Methodology
	Disadvantages: This methodology analyses only the restoration process, the disruption is considered as "static", being some of the links of the network completely destroyed (full closure of the link). The model considers exterior intervention for the restoration activities, and the resilience is measured as a function of the time, including only the progress in these recovery activities.
Freckleton et al. (2012)	**Main objective**: the definition of a conceptual framework to evaluate resilience in transport network, this approach is similar to the one presented by Serulle et al. (2011b). **Resilience is measured** by using a fuzzy inference approach in the framework presented by the authors, including a high number of metrics, previously described in Section 1.4. **Disadvantages**: this methodology includes a large number of metrics to describe the resilience of the network, for the evaluation of each of the metrics many resources are needed together with large amounts of data. However, after using the fuzzy logic approach for the combination of all the metrics into a total value of resilience, the results obtained for the value of resilience are qualitative, being extremely low, very low, low, medium low, medium, high, very high and extremely high.
Pant et al. (2014)	**Main objective**: based on the method proposed by Henry and Ramirez-Marquez (2012), a stochastic measure of resilience is introduced. **Resilience is measured** by three stochastic metrics of resilience, including (a) time to total system restoration, (b) time to full system service resilience, and (c) time to α%-resilience. **Disadvantages**: This methodology only analyses the system restoration process, including external planning efforts planned in advance or following the occurrence.
Lee et al. (2014)	**Main objective**: presentation of a resilience index to quantify the resilience of transport systems, based on the approach presented by Vugrin et al. (2010) to assess resilience. **Resilience is measured** by the evaluation of the system impact (SI), and the total recovery effort (TRE), including the increments caused by the perturbation in the travel times and the operating costs.

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Author	Methodology
	Disadvantages: this methodology quantifies the system impact only by the modifications in the travel times. A traffic allocating software (EMME) is used for the evaluation of travel times, and for the cost function a volume delay function is applied.
Vugrin et al. (2014)	**Main objective**: the formulation of a bi-level optimization model for network recovery that maximizes resilience for disrupted transportation networks. **Resilience is measured** by using the approach presented in Vugrin et al. (2010), see Equation 1.5. In this paper, the lower level analyses the network flows using a deterministic user equilibrium, and the upper level identifies the optimal recovery modes and sequences. **Disadvantages**: in this methodology the system impact is considered as a "punctual" event that takes place at t_0, in the example presented is considered as a reduction of the capacity in some of the links of the network.
Faturechi and Miller-Hooks (2014)	**Main objective**: quantifying and optimizing travel time resilience in transport networks under disaster scenarios. **Resilience is measured** by a ratio of the total travel time at the end of the pre-event stage, and at the end of the response stage. Resilience strategies are limited under a given budget for taking mitigation, preparedness, and response actions. Also, a given time is allotted for the implementation of response actions. Travel times are estimated with a partial user equilibrium. Link travel time function used is the Bureau of Public Roads. **Disadvantages**: External actions are included when measuring resilience of the network, pre and post the event. The moment that the perturbation occurs, the damage is introduced as a "punctual" impact, and a reduction in the characteristics of the links is applied. In addition, it is assumed that the users have perfect knowledge of the damaged links and when the restoration actions are completed.
Wang et al. (2016)	**Main objective**: the introduction of a metric based on system reliability and network connectivity to measure resilience in transport networks. In addition, the authors propose a resilience-based framework for risk mitigation in transportation networks.

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Author	Methodology
	Resilience is measured as a weighted average number of reliable independent routes between any network OD pairs. **Disadvantages** the authors present a method to assess resilience by evaluating the reliability of the bridges in a transport network, analysing the impact of earthquakes in the network. Then, the focus of this approach is mainly on the pre-disaster activities that can be conducted in the bridges, and on choosing new strategies to increase the resilience by modifying the topology of the network.

1.5 Conclusions

Methodologies to evaluate resilience in transport networks

When evaluating the impact of a disruption in a transport network or any other system, two phases can be identified; (a) perturbation stage when the hazard takes place in the network, and (b) recovery stage, when the hazard has finished but the network continues with the restoration process due to damages created by the disruption. Both phases have been evaluated by the authors in the literature using distinct approaches:

- Perturbation phase: when analyzing different methods to evaluate resilience in the literature, it is difficult to find methods that incorporate this stage as a proper study phase, where the evolution and the changes in the perturbation are included, and analysed during a period of time. Most of the methodologies consider the perturbation as a "punctual" damage, something that happens instantly or in very short period of time. For example, some authors include a reduction in the capacities of some links, such as Faturechi and Miller-Hooks (2014)). Other methods with this approach are the ones presented by Vugrin et al. (2014), and Lee et al. (2014).

- Recovery phase: this stage is included in the majority of the methodologies to evaluate resilience. It is noted that in the literature, resilience methods are seen as methods to select the best group of measures to reduce the impact of a perturbation. Therefore, many models focus their attention on an optimization of the possible recovery strategies after a disruption. In addition, some of these methodologies include external actions to help the reduction of the damage, such as Faturechi and Miller-Hooks (2014), Bocchini and Frangopol (2012), Chen and Miller-Hooks (2012), Henry and Ramirez-Marquez (2012), and Pant et al. (2014).

When analysing the approaches of several authors for the evaluation of resilience, different methods can be found. These methods include indices, evaluation of resilience through the assessment of multiple metrics, fuzzy-logic approaches, or a combination of some of these approaches. Below, these approaches are summarized:

- Metrics: some authors such as Bruneau et al. (2003), Murray-Tuite (2006), and Freckleton et al. (2012) evaluate resilience through very large metrics, i.e. including a large number of variables. In these models, each of the variables included, such as redundancy, robustness, or adaptability needs to be evaluated separately, creating a process that requires a large amount of data, which is generally very difficult to obtain. For example, Murray-Tuite (2006) does not analyse all the dimensions, and a final value of resilience is not obtained. Freckleton et al. (2012) combine the evaluation of the metrics with fuzzy logic to obtain a final value of resilience, but after quantifying each of the 16 variables, the value obtained for the total resilience is reduced to the following options, extremely low, very low, low,medium low, medium, high, very high and extremely high.

- Indices: this approach has been used by many authors to present a methodology to quantify resilience. Different types of indices have been presented, such as Henry and Ramirez-Marquez (2012) who introduce a ratio of recovery at time t, Vugrin et al. (2010, 2011, 2014) who use an index with two main elements; the system impact (SI) and the total recovery effort (TRE), and Faturechi and Miller-Hooks (2014) who evaluate resilience as a ratio of the total travel times measured at the end of the response stage, and at the end of the pre-event stage.

- Area: Bruneau et al. (2003) introduced the resilience triangle approach to measure the resilience loss as the area between the normal functioning of the network and the performance when the network is disturbed. Following this approach Bocchini and Frangopol (2012) evaluates resilience as an area measure, in this case the different recovery strategies can be compared evaluating the network performance.

When measuring resilience in transport networks, different variables have been incorporated in each of the methodologies from the literature, such as:

- Travel times. These are probably the main concern when a disruption happens in a transport network. For example, Lee et al. (2014) use a traffic allocation software combined with volume delay functions to assess the travel times in the network, Faturechi and Miller-Hooks (2014) incorporate the well-know BPR function to evaluate the travel times and other authors such as Bocchini and Frangopol (2012), and Vugrin et al. (2014) include in their formulation the total travel times and the cost of each of the link depending on the flow.

- Time to restoration. This variable is indispensable when analysing the resilience of a network. The time spent from the beginning of the perturbation until the total recovery will determine the level of resilience of the network. Some authors have included this variable, such as Pant et al. (2014) where resilience is evaluated by three metrics, including (a) time to total system restoration, (b) time to full system service resilience, and (c) time to α%-resilience, or Bocchini and Frangopol (2012) who normalise the value of resilience over the time necessary for the recovery.

- Cost of the restoration activities (budget). As previously mentioned, some authors such as Faturechi and Miller-Hooks (2014), and Vugrin et al. (2014), include external actions to deal with the recovery process. These authors optimise the implementation of the restoration activities according to a budget that, in most of the cases, has been previously set before the hazard.

- Users domain. A limitation in the quantitative methodologies for evaluating resilience is the lack of methods that incorporate the response of the users when dealing with a perturbation in the network. It is important to incorporate this perspective since, when a disruption is happening, the users' behaviour is unlikely to be the same than the one expected in normal conditions, thus it is a relevant aspect to be considered when analysing the system performance.

Chapter 2

Traffic assignment models and travel cost functions

Contents

2.1 Introduction

In the previous chapter, the concept of resilience in transport networks was analysed. Some methodologies have been presented for the evaluation of this concept, however, either a definition or a mathematical tool to assess the concept of resilience in traffic networks is not yet established. More efforts are needed in this area, in order to provide efficient tools for the evaluation of resilience. Especially, for the scenarios where extreme weather events take place, which will be the ones mainly evaluated in this thesis.

When analysing the effects of extreme weather events in transport networks, one of the major consequences will be the impact on traffic, and the users will be one of the main aspects affected. In the event of a hazard, the users will suffer consequences such as longer travel times, and the necessity of re-routing. In addition, the users may experience a degree of uncertainty during the period of disruption, since access to information can be scarce.

For that reason, prior to the evaluation of resilience, one of the main challenges will be the understanding of the users' behaviour in the network when a perturbation occurs. In order to overcome this problem, traffic assignment models will have a determining role in the evaluation of the behaviour of the network. In consequence, this chapter introduces traffic assignment models, their relevant role when evaluating transport resilience, and a review of their formulations.

Moreover, when a traffic assignment model is defined, an indispensable part of the problem is the objective function. This objective function is usually defined as a travel time cost function, and multiple options have been proposed in the literature. Hence, an analysis of different travel cost functions is presented in this Chapter.

The Chapter is structured as follows.

In Section 2.2, traffic assignment models are analysed, including:

- In Section 2.2.1, it is highlighted why traffic assignment models are necessary when evaluating resilience in transport networks.
- In Section 2.2.2, a classification of traffic assignment models is included, splitting them into Static Traffic Assignment models (STA), analysed in Section 2.2.3, and Dynamic Traffic Assignment (DTA) models, analysed in Section2.2.4, which are further divided into deterministic DTA, and stochastic DTA. Inside the deterministic DTA, different approaches are analysed, such as (a) mathematical programming formulations, (b) optimal control formulations, and (c) variational inequality formulations.
- In Section 2.2.5, a comparison of the main deterministic DTA models presented in the literature is introduced.
- Finally, in Section 2.2.6, the day to day traffic assignment models are analysed.

In Section 2.3, cost functions are reviewed, including:

- In Section 2.3.1, a detailed review of the main cost functions proposed in the literature is presented.
- In Section 2.3.2, it is analysed how the effects of extreme weather events have been introduced in cost functions in the literature.
- In Section 2.3.3, a presentation of the mathematical properties that an appropriate travel time cost function should have is provided.

- Finally, in Section 2.3.4, a compilation of travel cost functions presented in literature is given.

2.2 Traffic assignment models

A traffic assignment model is used to estimate the traffic flows on a network, i.e. it is able to determine how the users select their routes depending on the conditions of the traffic network. With this aim, these models take as inputs, (a) the network topology, (b) the link performance functions, which determine the relationship between travel time and traffic volume per unit time on each of the links of a network, and (c) the OD matrix, i.e. the matrix of flows, which indicates the volume of traffic between origin and destination OD pairs.

It is noted that from here on, a network is represented by a directed graph $G = (N, A)$, consisting of a set of nodes N, and a set of links A.

With these inputs, traffic assignment models will determine the traffic volumes and the travel times, based on specific traffic conditions. See Figure 2.1 for a description of the inputs and outputs of traffic assignment problems.

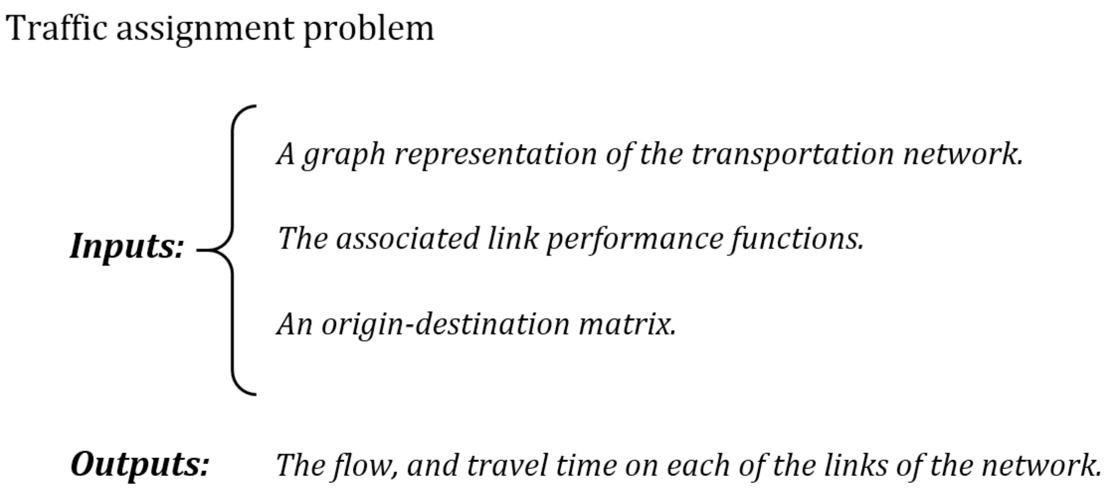

Figure 2.1: Description of inputs and outputs of a traffic assignment problem.

Therefore, the traffic assignment models have as an output, the users' behaviour, i.e., how they select the adequate paths to go from their origins to destinations, which will be determined in the problem. To reach this objective, these models use the concept of equilibrium, whose meaning is similar to the physical approach of equilibrium.

An **equilibrium state** is defined as "the state in which there are no forces that try to push the system to some other state. Furthermore, when the system is in disequilibrium, there are forces that tend to direct the system toward the equilibrium state", Sheffi (1985).

In economic theory, more precisely in the classical view of an economic market, it is possible to find the bases of the equilibrium applied in the traffic assignment models. When analysing the equilibrium in markets, two main interacting groups can be found, namely, the producers (supply) and the consumers (demand).

The first group will define the supply function, this function will indicate the quantity of goods that the producers are able to supply as a function of the price. The second group, which defines the demand function, will describe how the users behave, i.e. the quantity of product that will be demanded depending on the price.

An example is presented in Figure 2.2, where both functions are characterized, and the point where both of them intersect is defined as the **"equilibrium point"**. In this point, for a price of the product, P^{eq}, all the produced quantity, Q^{eq}, is consumed. If the price is higher than the equilibrium price, P^{eq}, there will be a gap between the quantity produced and the quantity consumed at that price, therefore part of the supply of product will remain unsold. In this situation, the remaining products will grow indefinitely, and at some point, this situation will produce a reduction in the price of the product, creating an increment in the consumption. A similar situation will happen when the price is smaller than P^{eq}, in this case the demand will be higher than the quantity of produce available. Again, this situation is an unstable situation, where at some point the price will increase to reduce the demand of products.

Therefore, the only stable situation of the market will occur when the equilibrium is reached and the supply and the demand intersect.

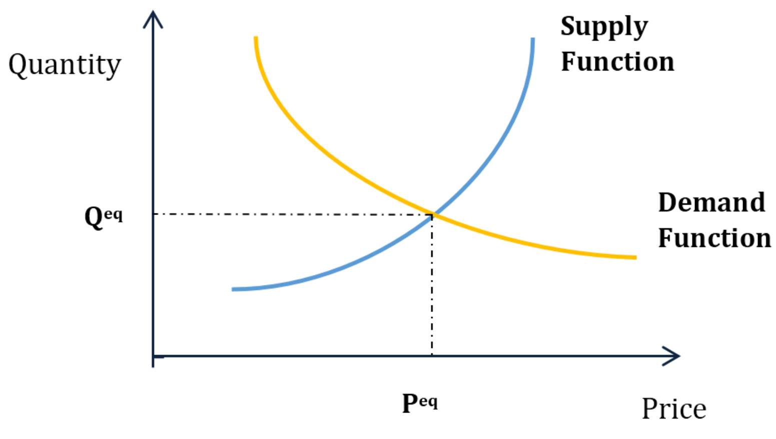

Figure 2.2: Demand/supply equilibrium in a perfectly competitive economic market

Translating this theory into the Transport area, it is possible to compare the supply function with the transport network, and the demand function with the users of the network. Thus, the equilibrium is determined by the flow, i.e. the quantity of travellers using each part of the network, and the travel times that the users need to reach their destinations. Both components have to be determined simultaneously by a system-based analytical approach.

2.2.1 Why traffic assignment models for the evaluation of resilience?

As explained before, the resilience of a transport network when a perturbation event takes place is determined depending on the reaction of the network to the hazard. On the one hand, this reaction will rely on the characteristics of the network, such as capacities of the links, and the topological characteristics. These characteristics are determined by the design of the network, and they are an important parameter on how resilient a network is. However, these characteristics are not easily modifiable during the perturbation.

On the other hand, a decisive role is played by the users of the network. Their behaviour during, and after the perturbation will define the type of response of the network against the hazard. Finally, the user's response will be a crucial variable to determine if the network is able to withstand the perturbation or not.

As previously explained in Chapter 1, when a perturbation is affecting a transport network, two phases can be identified, firstly a phase where the users experience a deterioration in the conditions of the network, this phase takes place when the hazard is affecting the network, and a second phase, when the hazard has finished and the network starts the process of recovery. It is important to highlight that in both stages the users need to adjust their behaviour to the changeable conditions.

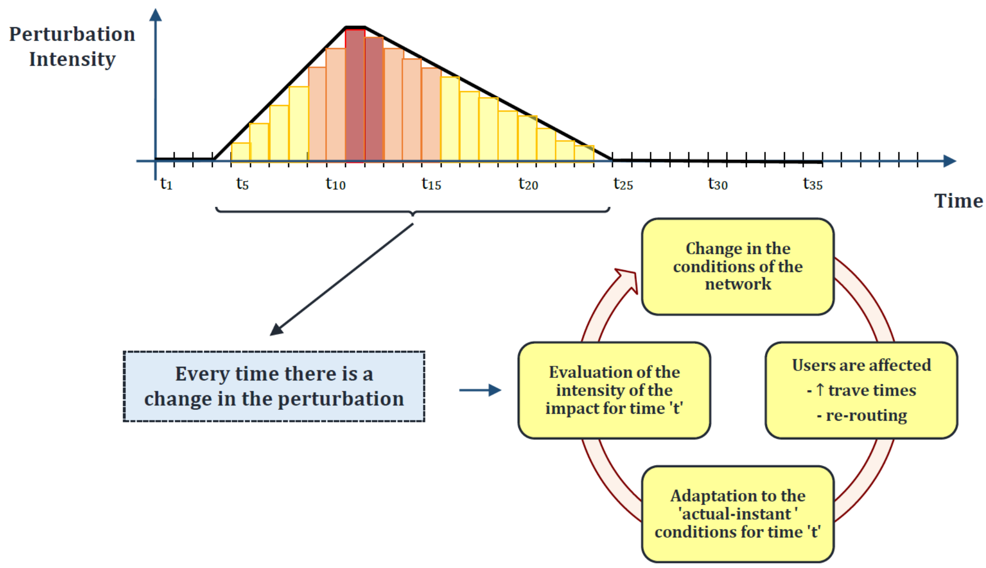

Figure 2.3: Description of the impact when a perturbation affects a traffic network.

In Figure 2.3 a description of the impact when a perturbation occurs in a traffic networks is presented. The transport network is assumed to start from an initial equilibrium state, which is considered the normal-operation state of the network. The initial equilibrium will be broken at the moment that the perturbation starts, forcing the users to adapt to the new conditions. This situation will take place over and over again while the external conditions change, during the hazard when there is a deterioration of the traffic conditions, and also during the recovery process with an improvement of the conditions. Finally, the network will reach a new equilibrium state after the process of perturbation and recovery, which can be the same as the initial one, better, or worse.

For these reasons, the resilience of a network will highly depend on the users' capacity to accommodate a disruption. Thus, traffic assignment models are essential for knowing these modifications in the behaviour of the users at any time. Therefore, traffic assignment models will make it possible to assess more accurately the impact of an extreme event in a traffic network, and therefore the resilience.

2.2.2 Typology of traffic assignment models

Due to the important role of traffic assignment models when analysing the network performance when a perturbation occurs, in this Section a review of the existing traffic assignment models is presented. Firstly, a general classification is shown in Figure 2.4.

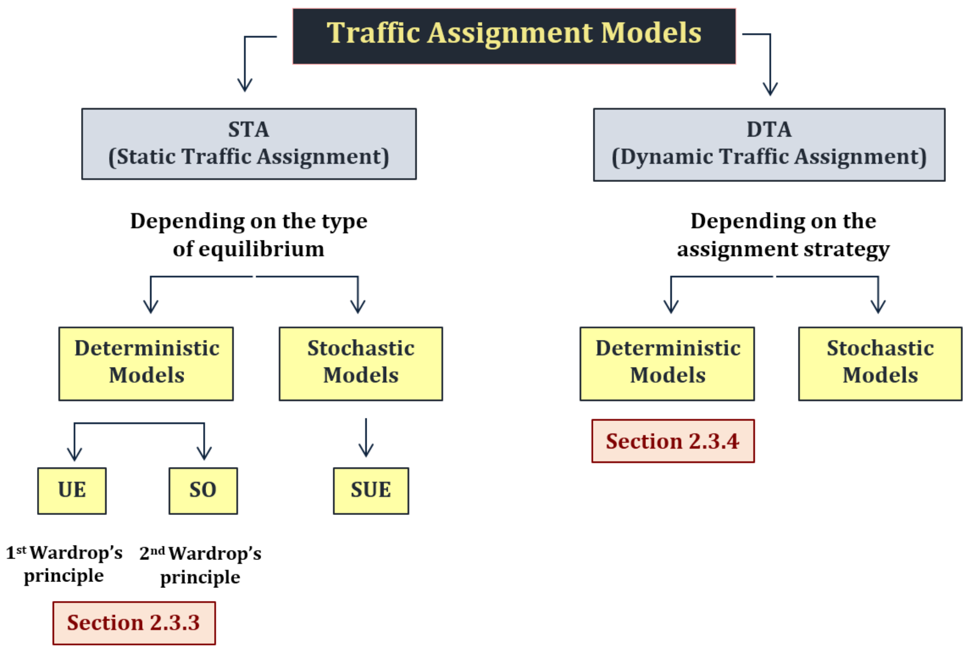

Figure 2.4: General classification of Traffic Assignment Models

In Figure 2.4, a main division of the traffic assignment models can be performed, differentiating between static traffic assignment models (STA) and dynamic traffic assignment models (DTA). These models can be briefly describe as follows:

Static traffic assignment models, STA, describe how the users select their routes assuming that the traffic conditions are stationary, then the aim of the model will be to obtain the mean values of interest.

Dynamic traffic assignment models, DTA, describe how the users select their routes assuming that the traffic conditions change with time, then the results obtained will change over the time.

Previously, Chapter 1 highlighted the dynamic component of resilience. A methodology to evaluate traffic resilience will need to incorporate a traffic assignment model that is able to deal with changing conditions. Therefore, the following sections will focus on the analysis of DTA.

Most of the DTA presented in the literature are based on the principles of equilibrium defined by the STA. Later on, the STA equilibria were transformed into dynamic models using different approaches. Therefore, for a better understanding of the DTA, a brief explanation of the STA will be firstly conducted (see Section 2.2.3).

Together with the classification of STA and DTA, other classification of the assignment models can be done. This classification will distinguish between deterministic and stochastic models, see Figure 2.4. These models, deterministic and stochastic, can be defined as follows:

Deterministic traffic assignment models: describe how the users select their routes assuming that every set of variables of these models are uniquely determined by parameters in the model, and by sets of previous states of these variables. Deterministic models always perform the same way for a given set of initial inputs.

Stochastic traffic assignment models: describe how the users select their routes assuming the randomness in the variables included, variable states are not described by unique values, but rather by probability distributions.

Due to the novelty of the concept of resilience in transport networks, and the lack of previous models to quantify this concept the approach of this thesis will be a deterministic one. This deterministic approach to the concept should be upgraded in future works to a stochastic approach. In following Sections the deterministic models are analysed, more precisely the deterministic DTA models. For a further description of the different traffic assignment models found in the literature see Nogal et al. (2011).

2.2.3 Static traffic assignment models

This Section reviews the main notions of the STA models, specifically the approaches to determine the equilibrium of the network. The types of equilibrium explained below will be the bases for the DTA models. More information about STA models can be found in Sheffi (1985).

When analysing STA models, a classification depending on the behavioural assumption governing route choice can be done, and two main groups can be identified. On one side, when users as individual elements try to minimize their own travel times. This approach is analysed in the widely-used method of the User Equilibrium (UE), see Section 2.2.3. On the other hand, when the objective is to minimize the total travel time of the network as a whole, the approach is defined as the System Optimum (SO), presented in Section 2.2.3.

In this Section, the UE and the SO are presented, both being deterministic approaches of STA.

Stochastic analysis of the equilibrium has been also presented for STA models, such as the Stochastic User Equilibrium (SUE). SUE is reached when no traveler believes that his travel time can be improved by individually changing route, and it is assumed that driver's perceptions and preferences are random. For more information about SUE see Sheffi (1985).

User equilibrium

The equilibrium concept previously described in Section 2.2 for the economic markets was transferred by Wardrop (1952) to the area of transport networks, introducing the Wardrop's first principle defined as:

Wardrop's first principle, Wardrop (1952): *"Journey times in all routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route"*.

Based on this principle, the UE can be defined as:

UE definition: For each OD pair, at user equilibrium, the travel time on all used paths is equal, and (also) less than or equal to the travel time that would be experienced by a single vehicle on any unused path (Sheffi (1985)).

This definition was transformed by Beckmann et al. (1956) into the following mathematical programming problem. Having a connected transport network, with a set of nodes N, and a set of links A. For certain origin-destination (OD) pairs of nodes, $pq \in D$, where D is a subset of NxN , connected by a set of routes R_{pq}, there are given positive demands d_{pq} which give rise to a link flow pattern $\mathbf{v} = (v_a)_{a \in A}$, and a route flow pattern

$\mathbf{h} = (h_{pqr})_{r \in R_{pq}, pq \in D}$, when distributed through the network.

$$\underset{\mathbf{h},\mathbf{v}}{Minimize} Z(v) = \sum_{a \in A} \int_0^{v_a} c_a(s)ds, \tag{2.1}$$

subject to:

$$\sum_{r \in R_{pq}} h_{pqr} = d_{pq}, \qquad \forall (p,q) \in D \tag{2.2}$$

$$\sum_{(p,q) \in D} \sum_{r \in R_{pq}} \delta_{apqr} h_{pqr} = v_a, \qquad \forall a \in A \tag{2.3}$$

$$h_{pqr} \geq 0, \qquad \forall r \in R_{pq} \forall (p,q) \in D \tag{2.4}$$

with

$$\delta_{apqr} = \begin{cases} 1 & \text{if route } r \text{ from node } p \text{ to node } q \text{ contains arc } a; \\ 0 & \text{otherwise,} \end{cases} \tag{2.5}$$

where ν_a is the link flow, $c_a(\nu_a)$ is the travel cost function for a link a, assuming that it is positive and strictly increasing.

System optimum

As shown before, UE aims to reach the best option for each of the users of the network individually. However, this does not imply that this is the best option for the network system as a whole, and trying to reach a global equilibrium of the network, Wardrop introduced the second principle. The definition is as follows:

Wardrop's second principle, Wardrop (1952): *"The average (or total) travel time should be minimized"*.

This definition is followed to reach the SO, whose mathematical formulation is the same as the system (2.1)-(2.5) but modifying the objective function to minimise, as shown below:

$$\underset{\mathbf{h},\mathbf{v}}{Minimize} Z = \sum_{a \in A} v_a c_a(v_a), \tag{2.6}$$

subject to:

$$\sum_{r \in R_{pq}} h_{pqr} = d_{pq}, \qquad \forall (p,q) \in D \tag{2.7}$$

$$\sum_{(p,q) \in D} \sum_{r \in R_{pq}} \delta_{apqr} h_{pqr} = v_a, \qquad \forall a \in A \tag{2.8}$$

$$h_{pqr} \geq 0, \qquad \forall r \in R_{pq} \forall (p,q) \in D \tag{2.9}$$

Comparison: UE vs SO

Once both options to address the equilibrium in STA have been presented, it is noted that important differences can be found between them. Hereafter, in Table 2.1, a comparison between UE and SO is presented.

Table 2.1: Comparison: UE vs SO

	UE	**SO**
Principle	1^{st} Wardrop's principle	2^{nd} Wardrop's principle.
Objective	Minimize individual users travel times	Minimize total travel time in the system.
Objective function	$\sum_{a\in A}\int_0^{v_a} c_a(s)ds$	$\sum_{a\in A} v_a c_a(v_a)$
Objective function for uncongested network	$\sum_{a\in A} v_a c_a(v_a)$	$\sum_{a\in A} v_a c_a(v_a)$
Constraints	$\sum_{r\in R_{pq}} h_{pqr} = d_{pq}$ $\sum_{(p,q)\in D}\sum_{r\in R_{pq}} \delta_{apqr} h_{pqr} = v_a$ $h_{pqr} \geq 0$	$\sum_{r\in R_{pq}} h_{pqr} = d_{pq}$ $\sum_{(p,q)\in D}\sum_{r\in R_{pq}} \delta_{apqr} h_{pqr} = v_a$ $h_{pqr} \geq 0$
Result	Any of the users of the network can improve their individual travel time	Some users may improve their travel times by unilaterally changing routes.
Behaviour	Noncooperative.	Cooperative.

After analysing the different characteristics of both approaches, it is noted that, when the behaviour of individual users in transport networks is evaluated, the most realistic approach will be the UE equilibrium. In the UE approach, users can select their routes individually, and they will try to reach their destinations by choosing their shortest route.

When a perturbation occurs in a network, and the behaviour of the users wants to be evaluated in order to assess the resilience, it is not realistic to assume that some users will prefer to increase their travel time for a reduction in the total travel times of the network. Therefore, it seems more accurate to use a UE approach.

The SO approach will be more useful when other types of situations need to be evaluated. For example, when modelling a group of users of the network that are governed by a unique organisation, such as vehicles of a particular company. In this case, the final goal is a reduction of the total travel times, and the SO will be a more suitable approach.

2.2.4 Dynamic traffic assignment models

After analysing the bases of the STA, a review of the DTA is developed in this Section.

Dehoux and Toint (1991) noted that: “Dynamic equilibrium models introduce the time dimension, and are based on generalizing the notion of Wardropian equilibrium in time: the network must be ”in equilibrium” at all moments during the design period”.

Dynamic models explicitly consider fluctuations in (a) link level variables, (e.g. users of the network using one link can be affected by the users of that link who arrive at an earlier time), (b) route level variables, (e.g. how the users choose their routes will depend on the route conditions at the time departure), and sometimes, (c) the fluctuations in demand level variables, (e.g. the way of choosing a departure time by the users) Bliemer (2001). In this way, for a set of time-varying origin-destination demands, a dynamic traffic assignment model will describe the flows over time and space on a transport network.

Inside DTA, two approaches can be found, (a) assignment strategy models, and (b) network loading models. The first ones evaluate the behaviour of the users, defining their route choices, and their departures times. Thus, these models will be the ones analysed in the following sections, since for the evaluation of resilience, the behaviour of the users is the variable to be analysed. On the other hand, the second type, network loading problems, calculate the flow distribution over a network with a given route inflow profile for each origin-destination pair.The latter can be further split into simulation-based models, and analytical-based models. For an extensive review of network loading problems, see Nogal et al. (2011).

Likewise, when analysing DTA, deterministic and stochastic models can be found as shown previously in Figure 2.4. As explained before this thesis will review the deterministic approaches. Specifically, those models presented in literature for the assignment strategy models, including (a) mathematical programming DTA, (b) optimal control formulations, and (c) variational inequality formulations.

Finally, a comparison of the deterministic approaches is presented in Section 2.2.5, this comparison is addressed according to the characteristics with more relevance when dealing with perturbations in a traffic network, including characteristics such as, how the models address equilibrium, how they consider time, and what is the information that the users have about the conditions of the network.

Mathematical programming DTA

Merchant and Nemhauser (1978a,b) proposed the first mathematical model for the dynamic traffic assignment. The authors use exit link functions to control the outgoing link flows during a time interval. The Merchant and Nemhauser (1978a,b) model provided a nonlinear optimization problem which is described for networks with a single destination. The optimization problem can be expressed as follows:

$$\underset{x_{a,j},u_{a,j}}{Minimize} \sum_{j=1}^{T} \sum_{a \in A} c_{a,j}(x_{a,j}) \tag{2.10}$$

subject to:

$$x_{a,j-1} - x_{a,j} - w_a(x_{a,j-1}) + u_{a,j} = 0 \tag{2.11}$$

$$\sum_{a \in E(i)} u_{a,j} - \sum_{a \in I(i)} w_a(x_{a,j-1}) = p_{i,j} \tag{2.12}$$

$$\sum_{a \in E(q)} u_{a,j} - \sum_{a \in I(q)} w_a(x_{a,j-1}) + s_j = 0 \tag{2.13}$$

$$u_{a,j} \geq 0 \tag{2.14}$$

$$x_{a,j} \geq 0 \tag{2.15}$$

where $I(i)$, and $E(i)$ are the set of links entering and emerging from node $i \in N$. $x_{a,j}$ is the number of vehicles in link a at the beginning of the time interval, j, $u_{a,j}$, the number of vehicles entering link a during the time interval, j, $w_a(.)$ are the exit link functions. These types of functions are defined as "exit link functions", being one of the main disadvantages of the model due to the difficulty of their definition. $p_{i,j}$ is the number of vehicles entering the network at node i during the time interval, j, with destination q, and s_j the number of vehicles leaving the network during time interval, j, at the destination node q. Finally $c_{a,j}(.)$ is the cost function of link a during the time interval j, where $1 \leq j \leq T$, being T the time intervals considered.

The constraints of the optimization problem represent (a) the conservation of vehicles in each link, Eq. 2.11 (b) the balance of the flows at the nodes, Eq. 2.12 , and (c) the balance of flows at the destination nodes, Eq. 2.13 . The main disadvantage of this method is the non-convexity of the constraints, which generates algorithmic difficulties.

After Merchant and Nemhauser (1978a,b), multiple authors have used the optimization model previously described, such as Ho (1980), Ho (1990), Carey (1986). In addition Carey (1987) modifies the optimization problem proposed by Merchant and Nemhauser (1978a,b) in a convex optimization method, improving the algorithmic difficulties that the model was presenting. Later on, Birge and Ho (1993) developed a stochastic case based on the DTA optimization problem of Merchant and Nemhauser (1978a).

On the other hand, inside the mathematical program-based formulation for DTA, one of the first approaches to develop a model for the UE DTA was proposed by Janson (1991b). The previous model explained, Merchant and Nemhauser (1978a,b), does not focus on the extension of Wardrop's principle, and the existence of an equilibrium. Janson (1991b) formulated a Dynamic user equilibrium (DUE) assignment model that can be defined as:

DUE is a temporal generalization of the static equilibrium assignment problem (UE). Following the definition of the UE, the routes with flow will be the ones with minimal travel cost when the network is in equilibrium. In DUE, together with the route choice presented in UE, the departure time choice is also included as a variable.

In addition, previous models were only considering instantaneous route choice, and Janson (1991b) assumed that the users choose their routes based on actual travel times (see Section 2.2.5 for a description of instantaneous and actual route choice). This model also incorporated exit time functions, rather than link exit functions that were included in the previous models.

This dynamic approach, DUE, is a generalization of the static approach of the UE previously presented in Section 2.2.3, and based on the formulation of Beckmann et al. (1956), the mathematical formulation of Janson's model is presented as follows:

$$\underset{\mathbf{h},\mathbf{v}}{Minimize} Z(v) = \int_t \sum_{a \in A} \int_0^{v_a(t)} c_a(w)dwdt \tag{2.16}$$

subject to:

$$\sum_{r \in R_{pq}} h_{pqr}(t) = d_{pq}(t), \qquad \forall (p,q) \in D, \forall t \tag{2.17}$$

$$v_a(t) = \int_s \sum_{(p,q) \in D} \sum_{r \in R_{pq}} h_{pqr}(s)\delta^t_{apqr}(s)ds, \qquad \forall a \in A, \forall t \tag{2.18}$$

$$h_{pqr}(t) \geq 0, \qquad \forall (p,q) \in D, \forall t \tag{2.19}$$

with

$$\delta^t_{apqr}(s) = \begin{cases} 1 & \text{if traffic on route } r \text{ of OD(p,q) departing at time } s \text{ is present on link} \\ & a \text{ at time } t; \\ 0 & \text{otherwise} \end{cases} \tag{2.20}$$

Following the notation introduced in the Beckman model, D is a subset of OD pairs (p,q), $\nu_a(t)$ is the link flow at time t, $c_a(t)$ is the cost function associated with link a for a time t, and $d_{pq}(t)$ are positive demands for OD at time t.

Following the model presented by Janson (1991b), Janson and Robles (1995) introduced a quasi-continuous DUE assignment model. This model is developed in two levels, an upper level that addresses the DTA, and a lower level that analyses the reachability of each node in each time interval, also, the variable time in this model is assumed as a discrete variable. Similar approaches were introduced by Jayakrishnan et al. (1995), and Romph (1994).

Some of the difficulties that can be found in the mathematical program formulation were presented by Peeta and Ziliaskopoulos (2001), being:

- The use of link performance and/or link exit functions.

- Holding-back of traffic[1]: this problem arises when link exit constrain are satisfied as strict inequalities (Bliemer (2001)).
- Efficient solution for real-time deployment in large-scale traffic networks.
- A clear understanding of solution properties for realistic problem scenarios.

Optimal control formulations

Optimal control formulations, in contrast with mathematical programming formulations, aim to obtain a continuous reformulation of the dynamic assignment problem, being based on optimal control theory, and assuming that:

- OD trips: are given as continuous functions of time.
- Link flows: are found out as continuous functions of time.
- Constrains: are similar to the ones described for the mathematical programming formulations but they are defined in a continuous-time setting.

Two different approaches can be found in the optimal control formulations, the ones using link exit functions, such as Friesz et al. (1989), and the ones without link exit functions, such as Ran et al. (1993).

In the first case, Friesz et al. (1989) provided approaches for both, the SO and the UE. The SO approach describes a continuous-time version of the model proposed by Merchant and Nemhauser (1978a,b) as previously presented in Section 2.2.4. The UE objective describes a time-dependant generalization of the model proposed by Beckmann et al. (1956), presented in Section 2.2.3 for the UE traffic assignment.

Extending the model based on UE presented by Friesz et al. (1989), Wie et al. (1990) included multiple destinations, since the model proposed by Friesz et al. (1989) was considering the single destination case. Also, Ran and Shimazaki (1989a) introduced an optimal control formulation using the instantaneous UE DTA model. For the SO approach presented by Friesz et al. (1989), Ran and Shimazaki (1989b) developed an approach with multiple OD, using linear exit functions.

Without using link exit functions, Ran et al. (1993) proposed an optimal control approach using an instantaneous DUE, in this case, they defined some control variables, such as link inflows, and link outflows.

Ran et al. (1993) defined an instantaneous continuous-time DUE traffic assignment model, which was defined as follows:

Dynamic User Optimal, DUO: "*If, at each instant of time t, and for each origin destination pair in the network, instantaneous unit travel costs for all routes being used*

[1]Often in SO models, some vehicles can be artificially delayed on a link for a time that exceeds what can be considered as a "reasonable"

are equal and minimal, then corresponding traffic volumes, evolving in time, follow a user-optimal equilibrium".

Using an optimal control approach, Ran et al. (1993) define the link inflows and the links outflows as control variables. In contrast with other models that do not account for dynamic queuing and congestion cost, this model splits the link travel cost function into two main components, (a) an instantaneous flow-dependent running time, f_{1a} over link a, and (b) an instantaneous queuing delay, f_{2a}. These functions are assumed to be non-negative, increasing and differentiable. In addition, this approach can be mathematically formulated as follows:

$$Minimize I = \int_0^T \sum_{a \in A} \left\{ \int_0^{e_a(t)} f_{1a}\Big[x_a(t), w\Big] dw + \int_0^{g_a(t)} f_{2a}\Big[x_a(t), w\Big] dw \right\} dt, \tag{2.21}$$

subject to:

$$\frac{d_{x_{apq}}}{dt} = e_{apq}(t) - g_{apq}(t) \qquad \forall a, p, q; \tag{2.22}$$

$$\frac{E_{pqr}(t)}{dt} = y_{pqr}(t) \qquad \forall r, p, q \neq p; \tag{2.23}$$

$$\sum_{a \in A} e_{apq}(t) = d_{pq}(t) \qquad \forall q, p \neq q; \tag{2.24}$$

$$\sum_{a \in \mathrm{B}(j)} g_{apq}(t) = \sum_{a \in \mathrm{A}(j)} e_{apq}(t) \qquad \forall q, p, j \neq p, q; \tag{2.25}$$

$$\sum_{a \in \mathrm{B}(j)} g_{apq}(t) = y_{pqr}(t) \qquad \forall p, q \neq p; \tag{2.26}$$

$$x_{apqr}(t) = \sum_{h \in \bar{\tau}} (x_{hpqr}(t + \bar{\tau}_a(t)) - x_{hpqr}(t)) + (Y_{pqr}(t + \bar{\tau}_a(t)) - Y_{pqr}(t))$$

$$\forall a \in \mathrm{B}(j), p, q, r, j \neq p; \tag{2.27}$$

$$e_{apq}(t) \geq 0 \qquad \forall a, p, q; \tag{2.28}$$

$$g_{apq}(t) \geq 0 \qquad \forall a, p, q; \tag{2.29}$$

$$x_{apqr}(t) \geq 0 \qquad \forall a, p, q, r; \tag{2.30}$$

$$\geq 0 \qquad \forall p, q, r; \tag{2.31}$$

$$Y_{pqr}(t) \geq 0 \qquad \forall p, q, r; \tag{2.32}$$

$$Y_{pqr}(t_0) = 0 \qquad \forall p, q, r; \tag{2.33}$$

$$x_{apqr}(t_0) = 0 \qquad \forall a, p, q, r, \tag{2.34}$$

where $x_a(t)$ is the number of users traveling on link a at time t, being T the time horizon, $Y_{pqr}(t)$ is the cumulative number of vehicles entering the network at origin p and exiting at destination q over route r at any time t; $y_{pqr}(t)$ denotes the instantaneous flows that arrives at destination q from origin p at time t. $A(j)$ is the set of links whose tail nodes is j (after j), and $B(j)$ is the set of links whose head nodes is j (before j); r is a section of route r from node j to destination q; and τ_a is the link a travel time that is estimated and updated in an iterative procedure. $e_{apq}(t)$, $g_{apq}(t)$ and $y_{pqr}(t)$ are control variables.

Advantages of optimal control formulations:

- The results are obtained in a continuous-time optimal control formulation, unless the mathematical program formulation uses a discrete-time approach.

Limitation of optimal control formulations:

- There are not specific constraints to ensure FIFO[2]
- Congestion is not included in the model in a realistic way.
- Drissi-Kaïtouni and Hameda-Benchekroun (1992) highlighted that the use of link exit functions does not account for the congestion which may exist at the road junctions.

Variational inequality formulations

The Variational Inequality (VI) is a different formulation platform to express the dynamic Wardrop principle. In this case, the DUE can be determined depending on the path inflows. To reach a DUE, the flow entering a route at a time t would be greater than zero only if the travel cost on the route is equal to the minimum travel cost at that time. Smith (1979)

Mathematically, this approach for the DUE can be formulated as follows:

$$\underset{\mathbf{h}^*}{Minimize} \int_t \underset{\mathbf{h}\in H(t)}{Max} (-[\mathbf{h} - \mathbf{h}^*(\mathbf{t})]^T).\mathbf{c}(\mathbf{t})dt \tag{2.35}$$

subject to

$$\mathbf{h}^*(t) \in H(t), \forall t, \tag{2.36}$$

where $\mathbf{h}$ is the path inflow vector, $\mathbf{h}^*$ is the path inflow vector at a time t, $\mathbf{c(t)}$ is the column vector of route cost incurred by travelers departing at a time t, and $H(t)$ determines the set of all routes flows at time t.

Using VI approach, Dafermos (1980) introduced a model in the area of static traffic assignment models. Later on, Friesz et al. (1993) formulated a continuous time VI model, where the path cost is defined by the travel cost obtained with a link performance function, and a penalty for early or late arrival that is produced along the path. This approach can

[2]FIFO (First Input First Output) rule (on the link level) means that users who enter the link earlier will leave it sooner. See Nogal et al. (2011) for more information about FIFO rules

contribute to a more realistic approach, but efficiency issues arise due to the complexity of the algorithm. A discrete-time VI approach is presented by Wie et al. (1995), including a heuristic algorithm that facilitate the computational issues. In this methodology a complete path enumeration is needed.

Ran et al. (1996) introduced a link-based VI approach. Chen and Hsueh (1998) used a link approach too, showing that only with the link inflows the travel time of a link a can be represented without losing generality. In this way, the model does not need to also use exit flow functions. Following the approach used by Chen and Hsueh (1998), Bliemer and Bovy (2003) presented VI methods with continuous, and discrete-time approaches.

Advantages of VI formulations:

- More general than previous approaches, giving a better analytical flexibility.

Limitation of VI formulations:

- Computationally more intense than optimization models.
- Path-based formulations in this area require an enumeration of all the possible paths.

2.2.5 Review of deterministic DTA

In Table 2.2, a comparison of deterministic DTA is addressed. In order to analyse the different models, the following criteria have been used, being described as follows:

- **Formulation of the DTA**: mathematical models can be formulated using different approaches, in this classification three types are included (a) mathematical programming formulation, (b) optimal control formulations, and (c) variational inequality formulations.
- **Equilibrium**: as explained before the equilibrium of a traffic assignment models is a key element in the problem, DTA, just like STA, can pursue (a) a system optimal, where the overall traffic system is optimized, in other words, the total travel cost is of the whole network is minimized, or (b) a user optimal equilibrium, where the goal is that all the users of the network choose their optimal route individually.
- **OD**: in this criterion, it is possible to refer to two options, that is, models that can only deal with single destinations, and models using multiple OD.
- **Time**: this variable has been introduced in the DTA models by mainly two approaches, (a) considering time a continuous variable, giving an approach closer to reality, and (b) analysing the variable time as a discrete variable. A priori, the first option seems more suitable, since analysing the time as a continuous variable

is closer to reality. However, it is noted that continuous-time models need to handle more difficulties when used.

- **Users' experience**: to analyse how the users acquire their knowledge of the conditions of the network, and therefore how the users choose their route, two approaches can be found in the DTA. Option (a) is assuming that the users have perfect information about the conditions of the network only at the moment that they have to take the decision, in this way they will choose the route with the lowest cost at the moment of their departure. However, the chosen route can vary the time and at the moment of their arrival may not be the cheapest one. Option (b) is assuming that the users of the network have perfect information about the current and the future times of the routes, in this way users will pick the route that minimizes their travel times also at the arrival.

Table 2.2: Review of deterministic dynamic traffic assignment models

DTA type	Author	Based on	Equilibrium	OD	Time	Users experience
Mathematical programming formulation	Merchant and Nemhauser (1978a,b)		DSO	single	Discretized time	Instantaneous
	Ho (1980)	Merchant and Nemhauser (1978a)	DSO	single	Discretized time	Instantaneous
	Carey (1986, 1987)	Merchant and Nemhauser (1978a)	DSO	multiple	Discretized time	Instantaneous
	Carey (1992)	Merchant and Nemhauser (1978a)	DSO	multiple	Discretized time	Instantaneous
	Carey and Subrahmanian (2000)	Merchant and Nemhauser (1978a)	DSO	multiple	Discretized time	Instantaneous
	Janson (1991a,b)		DUE	multiple	Discretized time	Actual
	Janson and Robles (1995)	Janson (1991a,b)	DUE	multiple	Discretized time	Actual
	Jayakrishnan et al. (1995)	Janson (1991a,b)	DUE	multiple	Discretized time	Actual

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DTA formulation	Author	Based on	Equilibrium	OD	Time	Users experience
	Drissi-Kaïtouni and Hameda-Benchekroun (1992)		DUE	multiple	Discretized time-setting	Actual
	Birge and Ho (1993)	M-N (stochastic)	SO	multiple	Discretized time-setting	Instantaneous
	Wardell and Ziliaskopoulos (2000)	Daganzo (1994)	SO	single	Discretized time-setting	Instantaneous
Optimal control formulations	Friesz et al. (1989)		DSO,DUE	single	Continuous	Instantaneous
	Wie et al. (1990)	Friesz et al. (1989)	DUE	multiple	Continuous	Instantaneous
	Ran and Shimazaki (1989b)		DSO	multiple	Continuous	Instantaneous
	Ran and Shimazaki (1989a)		DUE	multiple	Continuous	Instantaneous
	Ran et al. (1993)		DUE	multiple	Continuous	Instantaneous
	Lam and Huang (1995)		DUE	single	Continuous	Instantaneous
Variational inequality formulations	Dafermos (1980)		Introduce VI in static traffic equilibrium			
	Smith (1993)		DUE	multiple	Continuous	Actual

Continuous in the next page

From the previous page

DTA formulation	Author	Based on	Equilibrium	OD	Time	Users experience
	Friesz et al. (1993)		DUE	multiple	Continuous	Actual
	Wie et al. (1995)		DUE	multiple	Discrete	Actual
	Ran et al. (1996)		DUE	multiple	Continuous	Actual
	Chen and Hsueh (1998)		DUE	multiple	Discrete	Actual
	Bliemer and Bovy (2003)	Chen and Hsueh (1998)	DUE	multiple	Discrete and Continuous	Actual

2.2.6 Day-to-Day traffic assignment models

When approaching the problem of resilience for the first time, given its dynamic nature, the previously explained DTA models seem the best way of addressing the problem. However, to capture the disequilibrium caused by a perturbation over the time, these models may not be the best approach.

Day-to-day (D2D) dynamic models are able to deal with disequilibrium states that have an evolution over the time, approaching the dynamic problem in a different way than the previously explained DTA. D2D models analyse the evolution process of the equilibrium of the network more than the final equilibrium state which was the center of attention for the classical models, such as STA, and DTA. D2D models study the traffic fluctuations, and what is more interesting when dealing with resilience, the learning behaviour of the drivers. D2D models incorporate a learning model based on driver's past experiences to select their travel choices during the period of analysis. In Figure 2.5 a description of the problem is shown, where for every time unit a STA model is applied, and the temporal units are connected to provide a dynamic approach of the problem. Therefore, between two consecutive time units, the information of the state of the network is shared.

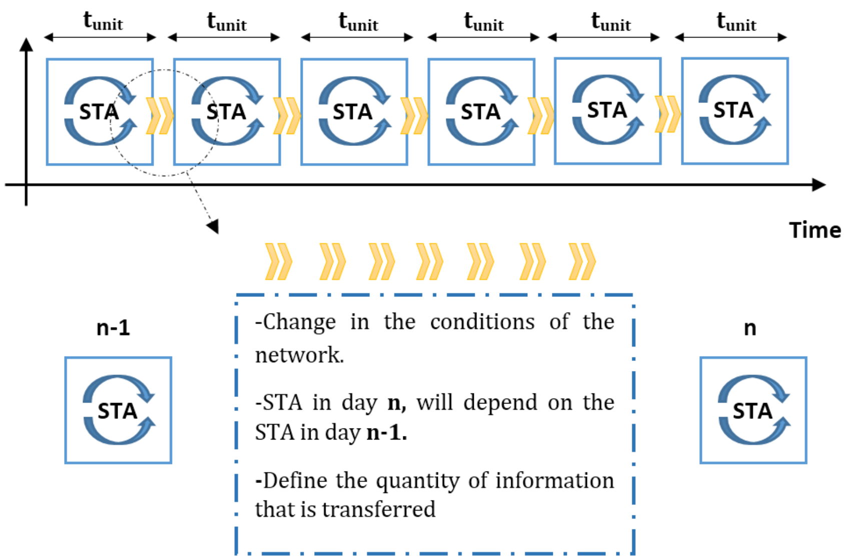

Figure 2.5: Description of the approach of D2D models

By using these types of models, the users' adaptation to new conditions of the network, as for example when a disruption occurs, can be incorporated with a great flexibility by using day-to-day approaches.

Some D2D models have been proposed previously in the literature, a classification can be done depending on how they deal with the variable time, having (a) continuous-time approaches, and (b) discrete-time approaches. The first ones, continuous-time approaches, define "sufficiently" small time steps which are able to simulate a continuous period of time. In this category, Smith (1984), and Zhang and Nagurney (1996) developed models where perfect information of the network conditions is used by the users. Also, after them, authors such as, Watling (1999), Cho and Hwang (2005), Mounce (2006) presented their approaches. One of the main disadvantages of the continuous-time approaches is that continuous adjustments of the user's choices is not a realistic approach to the problem, since in reality it is not very accurate to conduct re-routing at every instant. For the discrete-time approach, Friesz et al. (1994), Nagurney and Zhang (1997), Horowitz (1984), Cantarella and Cascetta (1995), and Watling (1999) presented deterministic approaches.

Watling and Hazelton (2003) identified some differences from previous traffic models, such as DTA, and D2D models, see Table 2.3.

Table 2.3: Comparison between DTA and D2D

DTA	**D2D**
The evaluation of equilibrium has been typically estimated by some optimization problems, such as mathematical formulation approaches or variational inequality problems. This makes the approach rather restrictive in terms of the generalisations that are feasible.	The day-to-day dynamic approaches have an important flexibility for perturbation stages.
Once a suitably constructed equilibrium solution algorithm is formulated in DTA models, the interpretation of model outputs is typically straightforward.	When applying a day-to-day approach, the interpretation stage can be often the most challenging part of the problem.
In traditional equilibrium models, the underlying hypothesis is the very notion of a market in equilibrium: a typically isolated, "self-consistent" state of the network which, if attained, would persist under certain rational rules of behaviour.	The underlying belief is in the behavioural dynamics, namely how the behaviour on day n is affected by behaviour, and the state of the network on days $n-1$ and earlier.

2.3 Cost Functions

Traffic assignment models require travel cost functions to analyse the behaviour of the traffic network, as it was explained previously. The objective function of these optimization problems usually involves a cost function, therefore, these functions will have an important role in the model.

Cost functions are applied in traffic problems to measure the cost of travelling throughout the network's links, in order to determine the user's costs from origin to destination. Different indicators can be used to determine these costs in traffic networks, such as travel time, travel distance, tolls, fuel cost, and road types. However, travel time is used in most of the traffic cost functions as the main or the only indicator to measure the cost. This is because travel time is usually the most relevant parameter in mobility problems. Hereinafter, the analysis will be focused on link travel time functions.

Link travel time functions describe how long it takes for a user of the network that enters a link a at a time t to reach the end of that link. When the performance of the link travel time is analysed, it is assumed that it depends on the link flows through the analysed link, or on the link flows of other links whose behavior is related to the analysed one. Thus, as the link flow increases the travel time will be higher.

Even though the traffic flow is one of the most used variables to determine the cost of a link, other variables can be used to define the performance of a link (see Section 2.3.4 for a compilation of link travel time functions). Therefore, the definition of some basic traffic variables used in cost functions is provided below.

Some basic traffic variables used in cost functions

The discussed variables are, traffic flow, average speed, and density:

- **Flow**, ν, is defined as the number of vehicles, n, passing some designated point in the network in an interval of time, t. This can be expressed mathematically as:

$$\nu = \frac{n}{t} \tag{2.37}$$

- **Average speed**, u, can be defined in two different ways, namely, (a) time - mean speed, $\bar{u}$, and (b) space - mean speed, u. The first one, $\bar{u}$, is the arithmetic mean of the speeds observed at a point of the network, and mathematically can be expressed as:

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i, \tag{2.38}$$

  where u_i is the speed of each of the vehicles when crossing the point where the speed is observed. The second one, u, is determined depending on the time needed by a vehicle to traverse some known length of network, l. When the speeds of the vehicles are measured over the same length, L, the space-mean speed is also known as the harmonic mean of speed, and it is the one normally used in the traffic assignment models, being expressed mathematically as:

$$u = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{L/t_i}} \tag{2.39}$$

- **Density**, k, is defined as the number of vehicles occupying some length of the network at a time, t, being mathematically expressed as:

$$k = \frac{n}{l} \tag{2.40}$$

Finally, these variables previously explained are connected with each other through the equation known as the **fundamental relationship of traffic flow**, defined as follows:

$$\nu = uk \tag{2.41}$$

2.3.1 Review of travel time cost functions

An extensive sample of link capacity functions can be found in literature. In Table 2.6, a selection of the most relevant cost functions is presented. Due to the high number of travel cost functions, in this Section, a review of the most used travel time functions is addressed. Part of this review has been also presented in Nogal et al. (2016a).

The most widely used congestion function is the well-known BPR function, proposed by Bureau of Public Roads, (see BPR (1964)), and its mathematical equation is presented as follows:

$$t_a(\nu_a) = t_a^0 \left(1 + \alpha \left(\frac{\nu_a}{c_a}\right)^{\beta}\right) \tag{2.42}$$

where t_a is the travel time for link a, t_a^0 is the free travel time for link a, ν_a is the flow through link a, c_a is the capacity of link a, and β and α are positive shape parameters which will be defined depending on the transport network analysed.

The main advantage of this formulation is its simple mathematical form, together with the minimal input requirements. However, this formulation has also some drawbacks when dealing with high congestions ratios, i.e. when the number of users in a link is over the capacity. In a realistic scenario, this ratio may become much larger than 1. However, when the equilibrium assignment is being computed the congestion ratio can reach very large values, which causes numerical problems, such as loss of precision and even overflow conditions.

To reduce this problem, Spiess (1990) suggested a new class of travel time functions, a conical volume-delay function. This function is described as:

$$t_a(\nu_a) = t_a^0 \left(2 + \sqrt(\beta^2(1 + S^2 + \alpha^2) - \beta(1 - S) - \alpha)\right) \tag{2.43}$$

where S is the congestion ratio computed as $\frac{\nu_a}{c_a}$ to provide a certain service level, $\beta = \frac{2\alpha-1}{2\alpha-2}$, and α is any number greater than 1.

Another of the most frequently employed travel time functions is the one presented by Davidson (1966), the formula is a simple rational expression to evaluate the link travel cost. This function became popular due to its flexibility and ability to cover a wide range of traffic conditions. Davidson (1966) understood the travel time of a link as the free flow travel time plus a quantity of time due to the iteration with other vehicles. The function presented by Davidson is formulated as:

$$t_a(\nu_a) = t_a^0 \left(1 + \alpha_a \frac{\frac{\nu_a}{c_a}}{1 - \frac{\nu_a}{c_a}} \right) \tag{2.44}$$

where α_a is a shape parameter to determine the curve.

The main disadvantage of this function is that travel time for link volumes exceeding the capacity were not defined. This could create computational problems in network loading models.

The previous function was improved by Tisato (1991), who presented the Modified-Davidson function, introducing a second equation, and formulated as follows:

$$t_a(\nu_a) = \begin{cases} t_a^0 \left(1 + J \frac{S}{1-S} \right) \\ t_a^0 \left(1 + J \frac{\mu}{1-\mu} + J \frac{S-\mu}{(1-\mu)^2} \right) \end{cases} \tag{2.45}$$

where μ is a user-selected proportion parameter, and J is a parameter associated with land use or area type surrounding the highway link.

A time-dependent modification of the Davidson formula is proposed by Akcelik (1991), and formulated as follows:

$$t_a(\nu_a) = t_a^0 \left(1 + \frac{u_a^0}{4} \left[S - 1 + \sqrt{(}S-1)^2 + \frac{8\tau S}{u_a^0 c_a} \right] \right) \tag{2.46}$$

where τ is a parameter to define the intersection delay. Some advantages of the formula proposed by Akcelik (1991) were highlighted by Singh and Dowling (2002), including a better convergence, and a more realistic estimation of the speed.

A comparison of cost functions previously explained is presented in Figure 2.6, and the parameters used can be found in Table 2.4. The values of the parameters presented in Table 2.4 are obtained from Mtoi and Moses (2014), who calibrated these functions for highways.

Table 2.4: Parameters assumed for the comparison of cost functions presented in Figure 2.6

Function	**Parameters**	
BPR	$\alpha = 0.263$	$\beta = 6.869$
Conical Delay	$\alpha = 1.029$	$\beta = 18.390$
Akcelik	$\tau = 0.100$	
Modified Davidson	$J = 0.009$	$\mu = 0.950$

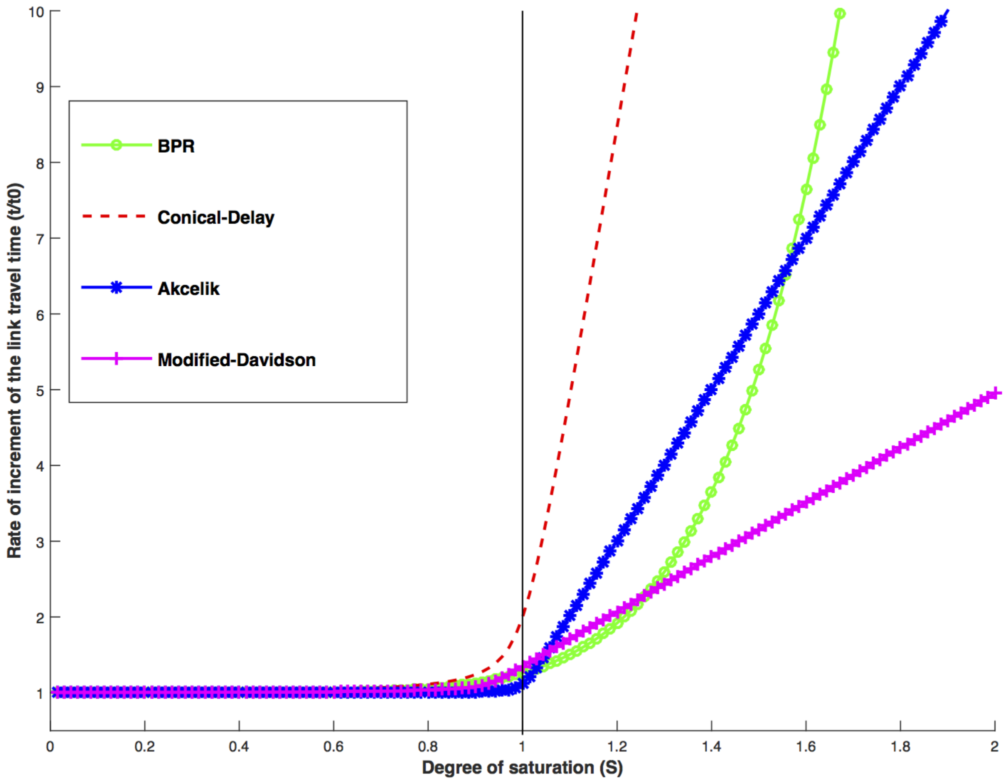

Figure 2.6: Graphical comparative of the cost functions analysed in Section 2.3.1.

In Figure 2.6, the analysed function, i.e. BPR, Conical Delay, Akcelik, and Modified Davidson, are shown. These function share a similar behaviour for values of the degree of saturation smaller than 1, however, their behaviour substantially diverges for higher values of this parameter. Finally, it is noted that the cost functions previously presented in Figure 2.6 share a common characteristic; they are upper unbounded. Thus, when the studied link reaches a certain level of congestion, the corresponding link travel time becomes infinite. In some stages of the iterative assignment procedure it is rather likely to obtain these high degrees of congestion. Furthermore, it is noted that, even considering the enormous travel costs reached when extreme perturbations occur in the network, travel times are finite unless a complete breakdown of the system occurs, as Branston (1976) states. Therefore, dealing with upper unbounded cost functions when analysing very perturbing scenarios does not seem very realistic.

2.3.2 Cost functions and the effect of extreme weather events in traffic networks

Although climatological phenomena such as heavy rainfall, dense fog, and frost, deeply affect the traffic response, these phenomena are rarely considered by the cost functions. When analysing the most frequently used cost function in traffic assignment models, none of them specifically considers the effects of weather events in their formulations.

The effect of climatological events on traffic network performance has been empirically demonstrated to cause up to 15% of reduction in speed-at-capacity due to moderate rainfall, Rakha et al. (2008), and up to 43% of reduction in free-flow speed with heavy snow, Ibrahim and Hall (1994). Therefore, the necessity of considering this effect when evaluating the travel costs is justified. Even more, when dealing with the evaluation of transport resilience when suffering weather perturbations, in this case, the presence of a cost function that explicitly considers the effects of extreme weather events becomes mandatory.

Moreover, neglecting this fact increases the uncertainties of the model. This aspect has been studied by Shahabi et al. (2015), whereas Ng and Waller (2012) analyse the user choice based on travel time reliability. But, despite the relevance, and the current importance of this topic, it is not easy to find in the literature cost functions that include weather parameters in their formulations.

One example of these cost functions is the one proposed by Lam et al. (2008). Lam et al. (2008) introduced a statistical generalization of the BPR to take into account the effect of the rainfall on the road physical performance. This generalization of BPR (GBPR) is formulated as follows:

$$t_a(V_a, I) = g_{t_a^0}(I)t_a^0 + \alpha_a \frac{1}{g_{c_a}(I)C_a}(V_a)^\beta \qquad \forall a \in A, \tag{2.47}$$

where C_a is the link capacity; ; I $(I \geq 0)$ is the hourly average of rainfall intensity; t_a^0 is the free flow travel time on link a, α_a and β, are the parameters in the conventional BPR function.

They include two statistical distribution functions to modify both, the free-flow travel time, $g_{t_a^0}(I)$, and the road capacity, $g_{c_a}(I)$, being I $(I \geq 0)$ the hourly average of rainfall intensity. These functions should follow the conditions presented in Table 2.5.

Nevertheless other functions considering the impact of the fog, the snow or even intense wind are difficult to find.

2.3.3 Mathematical conditions of a travel time cost-function

Link travel cost functions have as a main objective to closely reflect the real traffic behaviour. Travel time functions should be realistic, and they should have the possibility of being applied in different contexts. In addition, and in order to be useful when deal-

Table 2.5: Conditions for the scaled functions of the generalized BPR presented by Lam et al. (2008)

Scale function	Condition
$g_{t_a^0}(I) \geq 1$	Increasing function with respect to I meaning that the vehicle's free-flow travel time increases with the rainfall intensity.
$g_{c_a}(I) \leq 1$	Decreasing function with respect to I meaning that the link capacity decreases with the rainfall intensity.
$g_{t_a^0}(0) = 1$, and $g_{c_a}(1) = 1$	Meaning that when there is no rain the GBPR travel time function is equivalent to the conventional BPR function.

ing with traffic assignment problems, these function need to fulfil some mathematical requirements. Here below, the mathematical properties that an appropriate link travel cost function should have are outlined.

- The link travel cost function should be **strictly increasing**. Being a necessary condition to be able to converge to a unique solution in the traffic assignment problem.
- The link travel cost function should be **continuous and differentiable**. The existence of the derivative, and being strictly increasing ensure convexity of the congestion function. This is a desirable property but it is not necessary.
- Trying to **avoid overflow** when dealing with computer programs, the travel time function should not assign infinite travel time, even in the case that the flow is larger than the capacity.

In addition, Spiess (1990) highlighted the following conditions for volume-delay functions, being $f(x)$ a normalized congestion function, which can be expressed as:

$$t_a(\nu_a) = t_a^0 f\left(\frac{\nu_a}{c_a}\right). \tag{2.48}$$

were the saturation of the link, $\left(\frac{\nu_a}{c_a}\right)$ is the argument of the delay function.

In order to ensure compatibility with the BPR:

- $f(0) = 1$ and $f(1) = 2$.
- $f'(1) = \alpha$, with α being the exponential parameter of the BPR function, determines the way that the congestions effect impacts the traffic when the flow reaches the capacity.

To mitigate some BPR's drawbacks:

- $f'(x) < M\alpha$, being M a finite positive constant. This prevents the volume delay function to reach very high values when $\frac{\nu_a}{c_a}$ is larger than 1.
- $f'(0) > 0$, this condition ensures the uniqueness of link volumes.
- The computing times of a "new" cost function should not take more time than the time that is needed for the use of the BPR.

2.3.4 Compilation of travel time cost functions presented in the literature

A compilation of several cost function from the literature is included in Table 2.6. Among the functions, it is possible to find those that determine the link travel time (a) depending on the flow, such as BPR (1964), and Davidson (1966), (b) depending in the density, such as Ran et al. (1996), and (c) those that determine the speed on the link depending on the density, such as Greenshields et al. (1935), and Greenberg (1959).

The notation used in Table 2.6 is the following; for a link, a, ν_a is the flow the link, ν_a^0 is the link free flow speed, c_a is the capacity in the link, t_a is the link travel time, t_a^0 is the link free flow travel time, u_a is the link speed, u_a^0 is the free flow speed, S is the congestion ratio computed as $\frac{\nu_a}{c_a}$, k_a is the link density, k_a^{cri} is the link density for maximum flow, and J_a the link jam density. Other parameters are shape parameters specific from each of the functions, being nonnegative parameters.

Table 2.6: Review of cost functions

Author	Function
Greenshields et al. (1935)	$u_a(k_a) = u_a^0 \left(1 - \frac{k_a}{J_a}\right)$
Greenberg (1959)	$u_a(k_a) = \alpha_a ln \left(\frac{J_a}{k_a}\right)$
Underwood (1961)	$u_a(k_a) = u_a^0 exp \left(\frac{k_a}{k_a^{cri}}\right)$

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Author	Function
Smock (1962)	$t_a(\nu_a) = t_a^0 exp\left(\frac{\nu_a}{c_a}\right)$
Mosher Jr (1963) (1)	$t_a(\nu_a) = t_a^0 + ln(c_a) - ln(c_a - \nu_a)$
Mosher Jr (1963) (2)	$t_a(\nu_a) = \alpha_a - \frac{c_a(t_a^0 - \alpha_a)}{\nu_a - c_a}$
BPR (1964)	$t_a(\nu_a) = t_a^0\left(1 + \alpha\left(\frac{\nu_a}{c_a}\right)^\beta\right)$
Davidson (1966)	$t_a(\nu_a) = t_a^0\left(1 + \alpha_a\frac{\frac{\nu_a}{c_a}}{1 - \frac{\nu_a}{c_a}}\right)$
Soltman (1966)	$t_a(\nu_a) = t_a^0 2^{\frac{\nu_a}{c_a}}$
Overgaard (1967)	$t_a(\nu_a) = t_a^0\alpha_a^{\left(\frac{\nu_a}{c_a}\right)^{\beta_a}}$

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Author	Function
Spiess (1990)	$t_a(\nu_a) = t_a^0 \left(2 + \sqrt(\beta^2(1 + S^2 + \alpha^2) - \beta(1 - S) - \alpha)\right)$
Tisato (1991)	$t_a(\nu_a) = \begin{cases} t_a^0 \left(1 + J\frac{S}{1-S}\right) \\ t_a^0 \left(1 + J\frac{\mu}{1-\mu} + J\frac{S-\mu}{(1-\mu)^2}\right) \end{cases}$
Ran et al. (1996)	$t_a(k_a) = t_a^0 + \alpha_a(k_a)_a^\beta$
Akcelik (1991)	$t_a(\nu_a) = t_a^0 \left(1 + \frac{u_a^0}{4}\left[S - 1 + \sqrt(S-1)^2 + \frac{8\tau S}{u_a^0 c_a}\right]\right)$

2.4 Conclusions

Traffic assignment models

With the final goal of developing a methodology to evaluate the resilience of a transport network, an analysis of the traffic assignment models presented in literature has been performed in the first part of this chapter, and the following conclusions are drawn:

- Resilience has been introduced in the transport area in recent years, however due to the novelty of the topic it is difficult to find traffic assignment models that specifically address this problem. Most of the traffic assignment models have been designed to deal with problems related with traffic, such as congestion. However, when evaluating the behaviour of the users in a transport network affected by a perturbation new challenges arise. For example, in the case of an extreme event, the users may have limited information about the real conditions of the network.

- When analysing the impact of a perturbation in a transport network, it is common to obtain disequilibrium states during long periods of time in the network. A priori, a mathematical programming formulation of a dynamic traffic assignment models may seem as one of the most realistic approaches for simulating this phenomenon. However, Day-to-day (D2D) dynamic models are able to deal with disequilibrium states which have an evolution over time better than DTA. Therefore, D2D can be an interesting option for a formulation of a specific traffic assignment model that is able to capture the nature of this problem.

Cost functions

In the second part of this chapter, an analysis of the travel time cost functions presented in the literature has been performed. These functions are a main element in the optimization problem of traffic assignment models, and for the use of these functions in a method to evaluate resilience when extreme weather events occur some conclusions are drawn:

- It has been demonstrated by several authors that weather events can have a large impact on traffic conditions. However, it is difficult to find travel time cost functions that explicitly consider these effects in their formulation. When analysing the resilience of traffic network affected by hazards, it is crucial to implement a cost functions that can introduce variables related to the disruption, such as the intensity of the perturbation.

- It is noted that different parts of the network can have different vulnerabilities for the same disruption. For a better understanding of the response of the network against a perturbation, the introduction of variables that can identify the different levels of vulnerability in the network will improve the evaluation of resilience when a hazard takes place.

- In the cost functions analysed in this chapter, when a link reaches a certain level of congestion, the corresponding link travel time becomes infinite. This fact arises because the presented cost functions are upper unbounded. However, even considering enormous travel costs due to high levels of congestion, or extreme weather events, travel times are unlikely to obtain these high values in reality. Therefore, dealing with upper unbounded cost functions does not seem to be very realistic. In addition, this type of cost functions, when working with mathematical algorithms and iterative models may result in serious increases of the computational times.

Chapter 3

Critical and vulnerable links in transport networks

Contents

3.1 Introduction

The management of transport networks under crisis scenarios requires a profound knowledge of the network in order to implement the most efficient measures to reduce the impact of perturbations. One of the most powerful strategies to deal with these situations is creating preventive actions to adapt the network to future impacts, increasing the ability to prepare and improve the network before the real disruption takes place.

When a traffic network is affected by a perturbation, the critical and vulnerable elements of the network demand special attention. These elements will have a key role in the behaviour of the network, since they can create risky situations. When a reduction in the characteristics of these elements occurs, it can happen that not only the damaged link is impacted, but the whole network can be at risk.

The identification of these element is an essential part of adaptation strategies in the transport area. Also, when analysing the resilience of a traffic network, an early identification of vulnerable, and critical elements will help the development of strategies for the increment of the value of the resilience.

Therefore, this chapter analyses the approaches presented previously in the literature for the identification of critical and vulnerable elements in transport networks. In Section 3.2, a description of different approaches for the definition of vulnerable and critical elements in transport networks presented in the literature is given. In addition, in Section 3.2.1 the definitions for vulnerable and critical links in traffic networks used in this thesis are presented. In Section 3.3, approaches introduced by several authors to identify vulnerable and critical links are analysed, and in Section 3.3.1 a comparison of the different methodologies is included, identifying how the authors quantify the vulnerability, or the criticality, and the main disadvantages of the methodologies. Finally, in Section 3.4 some conclusions are drawn.

3.2 Understanding critical and vulnerable links in a transport network

Due to the relevance of the topic, different approaches have been presented previously in the literature. Nevertheless, there is still a long way ahead, starting from the definition of two concepts, such as, critical link and vulnerable link of a transport network. Nowadays, there is not an established definition for these two concepts in the literature. Different perspectives can be found among experts, and in some cases, both terms are not differentiated, and the same criterion is applied to both of them.

In 1994, Laurentius (1994) analysed vulnerability as a "susceptibility for rare, big risks". Later, Berdica (2002) understood the vulnerability of a link in a road transportation system as " a susceptibility to incidents that can result in considerable reductions in road serviceability", this last definition being one of the most frequently used in the Transportation literature when analysing vulnerability in road networks, and Holmgren (2004) defined vulnerability as "sensitivity to threats and hazards".

It is noted that the previous definitions of vulnerability describe this concept in a very wide perspective, where multiple interpretations on the part of the author can be included. Hereafter, other authors' approaches to the topic are analysed.

Holme et al. (2002) study the concept of vulnerability measuring the functionality of networks by analyzing two concepts, the average geodesic length, which is defined as 'the characteristic path length", and centrality measures such as the vertex betweenness centrality, and the edge betweenness centrality.

Husdal (2005) links the concept of vulnerability with reliability. According to the author, reliability describes "the operability of the network under varying strenuous conditions (i.e. the ability to continue to function)". The vulnerability describes "the non-operability of the network under varying strenuous conditions (i.e. the susceptibility to fail to function)".

Taylor et al. (2006) analyse this concept in terms of accessibility. Transport networks are created to give access to different locations and, for that reason, the vulnerability of the elements is measured depending on the capacity to provide access in different scenarios. Also, considering the accessibility as a main point to define the vulnerability is the definition given by Jenelius et al. (2006), who defined two concepts for the evaluation of vulnerability associated with the accessibility, the exposure, and the importance. The higher the values of these concepts the higher the vulnerability.

Taylor et al. (2012), also from the point of view of the accessibility, compares the levels of remoteness (or its opposite, accessibility) of different points of the area under analysis.

Knoop et al. (2012) consider robustness as the opposite term of vulnerability, whereas vulnerability describes the weakness of a network, the robustness describes the strength of a network. In the paper, robustness is defined as: " the ability of the network to maintain its functionality under conditions that deviate from the normal conditions."

Appert and Laurent (2007) based on the definition of Berdica (2002), determine the vulnerability by two factors, namely, the absolute severity of an incident, and the relative consequences of its occurrence on a given infrastructure or service (serviceability in the broad sense).

3.2.1 Definition of critical and vulnerable link in a transport network

Despite many authors having defined the vulnerable locations as critical, it is crucial to differentiate between critical links and vulnerable links. It is necessary to be able to identify both elements in a network because they can have different response when a perturbation occurs. In the Oxford dictionary, vulnerable is defined as "exposed to the possibility of being attacked or harmed, either physically or emotionally", and critical as "having the potential to become disastrous; at a point of crisis" or " having a decisive or crucial importance in the success, failure, or existence of something".

Both terms have a completely different perspective, in consequence this thesis will define vulnerable and critical links based on their meaning.

- **The most vulnerable link of a transport network** will be the one *suffering* the largest impact when a perturbation happens in any point of the network.

- **The most critical link of a transport network** will be the one that *creates* the largest impact in the network when damaged.

For a clearer understanding of both terms, in Figure 3.1, an explanatory diagram with different scenarios has been introduced.

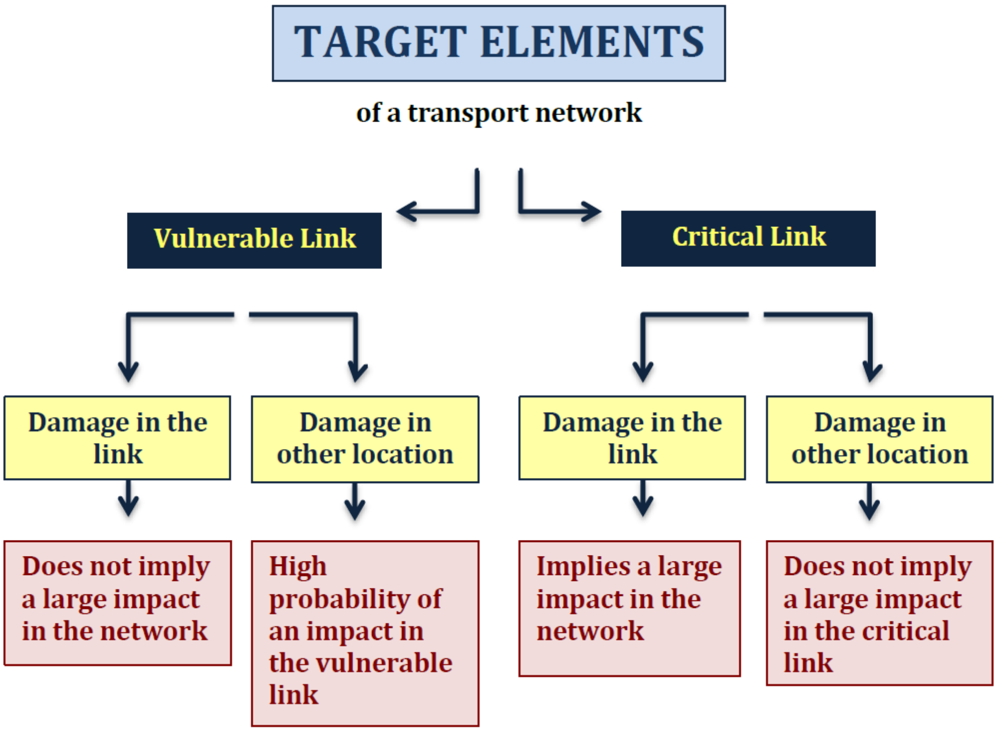

Figure 3.1: Explanatory diagram of the vulnerable and critical links of a transport network

3.3 Methodologies to identify critical and vulnerable links

Once both concepts are defined, a methodology to identify them in a transport network is required.

Some authors have presented methodologies to find the vulnerable and critical links in transport networks, in this section a selection of the methodologies to identify the vulnerability and criticality of the elements of transport networks has been analysed. Three categories have been presented: (a) authors that present vulnerability indices, (b) authors that present criticality indices, and (c) authors that analysed both concepts, including different methods to evaluate each of them. At the beginning of each section, a table can be found, where the methodologies explained in that section are summarized, including a brief explanation of the author's approach.

Vulnerability indices

In Table 3.1, the analysed methods in this section are briefly introduced, including the main concept that is measured in the vulnerability index, and the variables that are used by the author for the evaluation of vulnerability.

Table 3.1: Vulnerability index, including the main concept measured, and the variables used.

Vulnerability index	Main concept measured	Variables introduced in the index
Murray-Tuite and Mahmassani (2004)	Connectivity (redundancy)	Total number of paths connecting OD, number of alternate paths, flow, and utility of the alternate path.
Latora and Marchiori (2005)	Redundancy	Smallest sum of the links latency throughout all the possible paths in the graph from a OD.
El-Rashidy and Grant-Muller (2014)	Vulnerability attributes + Fuzzy Logic	Link capacity, flow, length, free flow, and traffic congestion density.
Balijepalli and Oppong (2014)	Serviceability	Flow and the travel time compared in scenario pre and post disruption.
Berdica and Mattsson (2007)	Vulnerability by the analysis of case studies	Base scenario compared with disrupted scenario by the modification in the travel time, trip length, and travel speed.

Murray-Tuite and Mahmassani (2004) present a bi-level method to identify vulnerable links in a transportation network. The authors present a game between two entities; In the lower level, the traffic management organisation that organises the vehicles according to a system optimal traffic assignment (Chapter 2), and in the upper level, an "evil entity" that has limited resources to create a damage in the network.

An index for the measurement of the link vulnerability is presented, this index is a measure of the importance of a specific link to the connectivity of an origin-destination pair. The formulation of the vulnerability index is presented below:

$$V_a^{r,s} = \begin{cases} 1.0 & \text{if } k^{r,s} > K^{r,s} \\ 1.0 - \sum_{j=1}^{k^{r,s}} g_j^{r,s} \frac{X_{a,j}}{x_a^{r,s}} & \text{otherwise,} \end{cases} \tag{3.1}$$

where $V_a^{r,s}$ is the vulnerability index for link a with respect to an OD, (r, s), $K^{r,s}$ is the total number of paths connecting the OD pair $r - s$ (this number of paths may be limited by user), $k^{r,s}$ is the number of alternate paths needed to accommodate $x_a^{r,s}$, with $x_a^{r,s}$ being the flow on link a from r to s. $X_{a,j}$ is the amount of flow on link a to be accommodated by the alternative path j, and $g_j^{r,s}$ is the utility of alternative path j. Utility is defined as "the combination of the relative capacity and the ratio of the free-flow path travel time to the marginal path travel time for alternate path j."

The vulnerability index proposed by Murray-Tuite and Mahmassani (2004) goes from

0 to 1, with 1 indicating that the analysed link is indispensable to the connectivity of the OD. The vulnerability index is appropriate for determining the importance of a link to OD connectivity.

For the analysis of the vulnerability at the network level, the vulnerability index, $V_a^{r,s}$, is multiplied by a weight factor that incorporates the proportion of the OD demand carried by the analysed link. Thus, the vulnerability indices for link a for each of the OD are included in a disruption index. The disruption index will evaluate the importance of a given link to the connectivity in a network level.

Once, the index to quantify the links vulnerability has been defined, the game approach is introduced. In this game, different disrupted scenarios are considered, including damages in different links of the network caused by the"evil entity". Some scenarios considered by the authors are: the management agency is not aware of a possible threat, the management agency suspects that the evil entity will take some action, and the agency perceives a general threat.

Latora and Marchiori (2005) analyse the critical locations of a network by measuring the difference between the optimal performance of the network, and the worst performance under a set of damages. In this paper, the authors aim to identify what they call critical locations, but using a vulnerability index presented below. For the identification of these locations, the authors characterize the performance of the network by analysing one single variable, the efficiency, $\Theta(S)$. This variable introduced by Latora and Marchiori (2001) is defined as "a measure of how efficiently the nodes of the network communicate if they exchange information in parallel", and it is formulated as follows:

$$\Theta(S) = E[S] = \frac{1}{N(N-1)} \sum_{i \neq j \in S} \frac{1}{d_{ij}}, \tag{3.2}$$

with d_{ij} being the smallest sum of the links' *latency* throughout all the possible paths in the traffic network from a node i to a node j, and N the number of nodes in the network. In the case of a transport network, the latency is measured as the time that it takes to go from one point to the next one.

Including the efficiency, the authors proposed the following vulnerability index:

$$V(S,D) = \frac{\Theta(S) - W(S,D)}{\Theta(S)}, \tag{3.3}$$

where $V(S,D)$ is the vulnerability of an infrastructure S, under a class of damages D, and whose range goes from 0 to 1. $W(S,D)$ is the worst performance of the infrastructure when suffering the damages, D, and is measured by the relative drop in the network performance, $\triangle\Theta^-/\Theta$, being $\triangle\Theta^- = \Theta(S) - \Theta(DAMAGE(S,d))$, and d, the damage analysed.

The type of disrupted scenarios analysed in this paper include: (a) damage in a link, (b) damage in a node (implying the damage of a number of links equal to the node's degree), and (c) damage in a couple of nodes. For each of the links, and the nodes of the network under a specific damage, the authors measure their importance in the network by analysing the reduction in the network's performance when the element has been eliminated.

Therefore, the authors analyse the *vulnerability* of a link or a node by analysing their redundancy, comparing the network's performance when the element is part of the network, and when it is not.

El-Rashidy and Grant-Muller (2014) present a link vulnerability index developed on the basis of several link vulnerability attributes, based on the work previously performed by Tampère et al. (2007). El-Rashidy and Grant-Muller (2014) assess the link vulnerability of a transport network, through a two stages process, firstly, different attributes are chosen to characterize the vulnerability, including aspects such as link capacity, flow, length, free flow and traffic congestion density, see Table 3.2 for a description of the vulnerability attributes.

VA_1 ; VA_2 and VA_3 attributes were presented by Tampère et al. (2007), and VA_4 and VA_5 attributes by Knoop et al. (2012).

Later on, these attributes are combined using fuzzy logic to obtain a link vulnerability index for each link a. To obtain a single vulnerability indicator, $VI(a)$, for each link based on the previously explained VAs, the authors assign a weight factor to each of the attributes. After applying fuzzy logic, the result is a fuzzy vector containing the values of the different attributes for each link. The authors transform this vector in a single value of $VI(a)$ by using the approach proposed by Ross (2009) that involves multiplying the fuzzy vector by a standardising vector to take into account the effect of different assessment levels.

Finally, including the vulnerability index of each link, $VI(a)$, previously calculated, two overall network vulnerability indexes are obtained, a physically-based aggregated vulnerability index, and an operational based aggregated vulnerability index.

The physical based aggregated vulnerability index VI_{Ph} is formulated as follows:

$$VI_{Ph} = \frac{\sum_a^e VI_a l_a n_a}{\sum_a^e l_a n_a}, \tag{3.4}$$

where e is the number of links in the road network, n_a is the number of lanes in link a and l_a is the length of link a.

The operational based aggregated vulnerability index VI_{op} is calculated as follows:

$$VI_{op} = \frac{\sum_a^e VI_a f_a}{\sum_a^e f_a}, \tag{3.5}$$

where f_a is the traffic flow in link a.

Balijepalli and Oppong (2014) introduce a network vulnerability index considering the serviceability, r_i; being serviceability of a link defined as "the possibility to use that link during a given period which then relates to the possibility of partial degradation of roads",

Table 3.2: Vulnerability attributes used by El-Rashidy and Grant-Muller (2014)

Vulnerability attribute	Definition	Formulation	Parameters
VA_1	It reflects the link traffic flow in relation to link capacity.	$VA_1 = \frac{f^i_{am}}{1-\frac{f^i_{am}}{C_{am}}}$	f^i_{am} is the flow on link a during period time i for a travel mode m, and C_{am} is the capacity of link a for a travel mode m.
VA_2	It identifies the direct impact of link flow with respect to link capacity.	$VA_2 = \frac{f^i_{am}}{C_{am}}$	
VA_3	It represents the inverse of the time needed for the tail of the queue to reach the upstream junction.	$VA_3 = f^i_{am}\frac{n_a kj_{am} - \frac{f^i_{am}}{V_{am}}}{l_a}$	n_a is the number of lanes that have been used by m, kj_{am} reflects congestion density for a, V_{am} is the free flow speed of a for m, and l_a is the length of a.
VA_4	It reflect the relative link importance by using the capacities	$VA_4 = \frac{C_{am}}{C_{max}}$	C_{am} is the capacity of link a, and C_{max} is the maximum capacity of all network links.
VA_5	It represents the level of importance of the link by link length	$VA_5 = l_a$	l_a is the length of link a.

and is calculated as the total available capacity of link i divided by the standard hourly link capacity per lane for a given type of road. This concept is closely related with the link vulnerability definition used in this thesis.

The Network Vulnerability Index (NVI) is formulated as follows:

$$NVI = \sum_{i=1}^{|A|}\left[\left(\frac{x_i^{before}}{r_i^{before}}t_i^{before}\right)\right] - \sum_{i=1}^{|A|}\left[\left(\frac{x_i^{after}}{r_i^{after}}t_i^{after}\right)\right], \tag{3.6}$$

where x_i is the flow in link i, t_i is the travel time in link i, and $|A|$ is the number of links on the network.

The flow and the travel time are evaluated for two different moments, before any disruption, and after, when one of the links of the network has been completely blocked or partially damaged, and both situations are compared to evaluate the vulnerability index of the network.

In Berdica and Mattsson (2007), a model-based case study to identify the vulnerability of road networks is presented, where the authors analyse different failures in a specific network, covering the entire Stockholm Region.

The scenarios with disruption are compared with a base scenario without "interferences", and inside the possible failures the authors consider disruptions such as lane or complete closures of the links of the network, reductions of the free flow speed of all the links or only of the secondary roads, and different percentages of reduction or increment of traffic in the network.

Once the damage is applied, link flows are obtained with user equilibrium models (see Chapter 2), and the different scenarios are compared in terms of travel time, trip length, and travel speed. Finally, the changes in these variables are transformed into economic terms, the total costs of the scenarios with disruptions is compared with the costs of the base scenario.

It is highlighted that the results are not completely realistic, since micro simulations models will be necessary to catch the effects of phenomena in the network such as queues due to a dense traffic.

Critical locations

In Table 3.3, the analysed methods in this section are briefly introduced, including the main concept that is measured in the criticality index, and the variables that are used in the literature for the evaluation of criticality.

Table 3.3: Criticality index, including the main concept measured, and the variables used.

Criticality index	**Main concept measured**	**Variables introduced in the index**
Scott et al. (2006)	Robustness Index	Comparison of scenarios pre and post disruption, using travel time, and flow of the links.
Wang et al. (2016)	Robustness (based on Scott et al. (2006))	Analysis of the effects on travel cost when each of the links is eliminated.
Demšar et al. (2008)	Topology	Graph theory, analysing how the links are connected to each other.

Scott et al. (2006) introduce a method to identify the critical links, considering network flows, link capacity and network topology. The Network Robustness Index is presented as "a new measure for evaluating the critical importance of a given link to the overall system as the change in travel-time cost associated with rerouting all traffic in the system can make that segment become unusable".

The authors describe the critical links as the ones creating the worse scenario when damaged, highlighting the importance of variables such as the flow of the links. To calculate the Network Robustness Index, firstly the travel-time cost of removing a link is calculated as follows:

$$c_a = \sum_a t_a x_a \delta_a, \tag{3.7}$$

where

$$\delta_a = \begin{cases} 1 & \text{if link } a \text{ a is not the link removed ;} \\ 0 & \text{otherwise,} \end{cases} \tag{3.8}$$

being x_a the flow in link a, and t_a the travel time.

Afterwards, this cost is compared with a scenario without disruptions in order to calculate the Network Robustness Index (NRI) as:

$$NRI = c_a - c, \tag{3.9}$$

where

$$c = \sum_a t_a x_a, \tag{3.10}$$

The travel times are calculated by using a UE assignment model. In addition, Ball et al. (1989) and Corley and David (1982) also study the removal of links to locate the critical links, named in these papers as "vital links".

In line with the definition of critical links presented by Scott et al. (2006), Wang et al. (2016) introduce a bi-level formulation for identifying the most critical links in a transportation network, and the most critical combination of links (including groups of two or three links). The authors present an optimization problem where the performance of the network is measured as the sum of travel costs associated with all flows on it, and the most critical links or combination of links, are identified as those whose closure would create the largest increase in the total travel cost.

The optimization problem is formulated as a bi-level mixed-integer non-linear programme with user equilibrium constraints, where the objective function is the maximization of the total travel cost of the network. Travel time is calculated through the Bureau of Public Roads (BPR) cost function.

One of the limitations of the methodology is a higher computational cost in large-scale network problems. The authors will try to overcome this limitation in future works.

Demšar et al. (2008) analyse the critical location of a network using graph theory. The authors study the topology of the network, i.e. how the links are connected to each other, to be able to identify which are the problematic elements.

A critical location in the paper is defined as "an object or an area whose removal or destruction changes the structure of the network in terms of flow and connectedness". Some aspects are highlighted to identify critical locations, such as being the only element that connects two subparts of the network (tunnels, bridges ...), elements included in a high number of routes (main roads), and highly dense parts of the network (city center). A real example is presented in the city of Helsinki.

Two topological graph measures are used in the paper to identify the critical locations, namely "betweenness" and "clustering coefficient". In the following definitions the concept of a graph, G, from graph theory is used, and it is defined as "an ordered pair $G = (V, E)$ comprising a set V of vertices or nodes or points together with a set E of edges or arcs or lines".

"Betweenness" is used to measure the centrality of a given vertex, v, in the graph, and it is defined as "the proportion of the shortest paths between every pair of vertices that pass through the given vertex v towards all the shortest paths". Being formulated as follows:

$$b(v) = \sum_{s \neq t \neq v} \frac{\sigma_v(s,t)}{\sigma(s,t)}, \tag{3.11}$$

where s and t are two distinct vertices of the graph analysed, and they are not equal to the analysed vertex, v, $\sigma_v(s,t)$ is the number of shortest paths from s to t that pass through v, and $\sigma(s,t)$ the total number of shortest paths from s to t.

The "Clustering coefficient" can determine the importance of the vertex in its immediate neighbourhood. The authors define the coefficient as "the number of edges between the vertices within the immediate neighbourhood of vertex v divided by the number of all possible edges between them", formulated as follows:

$$b(v) = \frac{2e(v)}{d(v)d(v) - 1}, \tag{3.12}$$

where $e(v)$ is the number of edges between neighbours of the analysed vertex v and $d(v)$ is the degree of vertex. The degree of a vertex is "the number of edges that connect to it, where an edge that connects a vertex to itself is counted twice".

Critical and vulnerable elements

In this section, the authors that analyse both concepts, vulnerable, and critical, are introduced. It is noted that in this section, the studied authors present different methods to evaluate each of the concepts, therefore they differentiate between vulnerable and critical elements of the network.

In Table 3.4, the methods analysed in this section are briefly introduced, including the main concept that is measured by the vulnerability and criticality indices, and the variables that are used by the authors for the evaluation of both concepts.

Table 3.4: Vulnerability index, including the main concept that is measured, and the variables used.

Criticality and vulnerability index	**Main concept measured**	**Variables introduced in the index**
Jenelius et al. (2006)	Vulnerability: "Exposure"	Comparison of scenarios pre and post disruption, using travel time, and a weight factor that reflects the importance of the OD pair.
Jenelius et al. (2006)	Criticality: "Importance"	Comparison of scenarios pre and post disruption, using travel time, and a weight factor that reflects the significance of the OD pair.
Rodriguez-Nuñez and Garcia-Palomares (2014)	'Criticality: "Importance" (nodes)	Comparison of scenarios pre and post disruption, using average overall travel time.
Rodriguez-Nuñez and Garcia-Palomares (2014)	'Vulnerability: "Exposure" (links)	Comparison of scenarios pre and post disruption, using average overall travel time, and the subset of all links with route alternatives when the analysed node is disrupted.

Jenelius et al. (2006) present an approach to analyse the vulnerability and criticality of roads networks under extreme weather events by studying links or group of links affected. With this aim, the authors divide the concepts of vulnerability, and criticality in two parts, (a) the probability of a hazardous event, and (b) the consequences of the event in a certain place.

To measure the consequences of an event in a certain place, the authors differentiate between two measures, the exposure, and the importance of an element. Following the definitions of vulnerable and critical elements introduced by D'este and Taylor (2003), the exposure is linked to the vulnerability of a node, and the authors suggest that a node is vulnerable "if loss (or substantial degradation) of a small number of links significantly diminishes the accessibility of the node, as measured by a standard index of accessibility".

On the other hand, the importance is linked to the criticality, and the authors note that "the criticality of a certain component (link, node, groups of links and/or nodes) in the network involves both the probability of the component failing and the consequences of that failure for the system as a whole". Therefore, the more critical the element that has been damaged is, the larger the consequences in the whole system. The criticality of an element is analysed in the paper as the "importance".

The damage introduced by the authors to simulate the perturbation is a link or a group of links being completely disrupted (closed), which forces all travellers on those links to take other routes which are less advantageous routes.

For the identification of both, the exposure and the importance, the consequences caused by the hazard are measured in terms of travel time. For the calculation of the travel times the principles of UE are applied.

Finally, the importance, and the exposure are defined, and quantified as follows.

The importance of link a with regard to the whole network is formulated as follows:

$$Importance(a) = \frac{\sum_i \sum_{j \neq i} w_{ij}(c_{ij}^{(a)} - c_{ij}^{(0)})}{\sum_i \sum_{j \neq i} w_{ij}}, \tag{3.13}$$

where $c_{ij}^{(a)}$ is the travel time from node i to node j when the link a has failed, $c_{ij}^{(0)}$ is the travel time without disruption from node i to node j, and w_{ij} is an assigned weight that reflects the significance of the OD pair in relation to the other OD pairs.

The exposure of a node to a certain event is formulated as follows:

$$Exposure(m) = \frac{\sum_{a \in E^{nc}} \sum_{i \in V_m^d} \sum_{j \neq i} w_{ij}(c_{ij}^{(a)} - c_{ij}^{(0)})}{L^{nc} \sum_{i \in V_m^d} \sum_{j \neq i} w_{ij}}, \tag{3.14}$$

where L_{nc} is the number of non-cut links (i.e. links that do not divide the network in parts when damaged), V_m^d is the set of demand nodes located in municipality m, and E^{nc} is the set of non-cut links.

In the case that the closed link is indispensable to go from the origin to the destination, the travel times will be infinite, and the network will be divided in different parts. For the evaluation of this scenario, the authors introduce the concept called "unsatisfied demand" which measures the number of trips from i that are unable to reach j due to the closed element e.

The consequences of a hazard are determined as previously explained by the exposure, and the importance, combined with the probability of occurrence of the hazardous events, will define the vulnerable and the critical element as shown in Table 3.5.

Table 3.5: Concepts in the analysis of vulnerability and criticality by Jenelius et al. (2006)

Concept , A*B	Concept A	Concept B
Vulnerability	Probability	**Exposure**, or conditional vulnerability.
Criticality	Weakness (probability)	**Importance**, or conditional criticality.

Rodriguez-Nuñez and Garcia-Palomares (2014) measure the vulnerability and criticality of public transport networks, estimating the criticality level of each link by the variation in the serviceability. The serviceability is understood as a parameter to estimate the importance, and evaluated as a weighted average travel time for each node of the network.

The methodology is based on the concept of importance of a link presented by Jenelius (2009), described by the author as "the importance of a link will become apparent if, when it fails to be operative, it has a significant effect on the overall performance of the system".

The importance (criticality) of each of the links is measured by the time lost by closure of the analysed link a, being mathematically formulated as:

$$I_a = \bar{T_a} - \bar{T_0}, \tag{3.15}$$

where $\bar{T_0}$ is the average overall travel time in the original scenario, and $\bar{T_a}$ is the impact of disruption in each link, which is measured through the average times without the link that is analysed.

The vulnerability is measured at a node level, and it is determined as the exposure of the node when a link is damaged. The exposure index tries to evaluate the expected average travel time increase for trips with origin at the node i when a link in the set A_w is damaged, and it is calculated as follows:

$$V_i = \frac{\sum_{a \in A_w} (\bar{T_{ia}} - \bar{T_i})}{N_w}, \tag{3.16}$$

where V_i is the exposure of the node i, A_w is the subset of all links with route alternatives when disrupted, and N_w is the number of links in A_w.

The authors apply the methodology in a case study, the Madrid Metro network, where some disruption scenarios are tested.

3.3.1 Comparison of methodologies to measure vulnerability and criticality in transport networks

In this section, in Table 3.6, the methodologies to evaluate the vulnerability and criticality previously explained are summarized, highlighting for each of them the following aspects: (a) the main objective of the paper; this is included because, some of the papers address only one of them; (b) how vulnerbaility and/or criticality are measured, and (c) the main disadvantages found in each of the methodologies.

Table 3.6: Review of methodologies to measure vulnerability and criticality in transport networks.

Author	Methodology
Murray-Tuite and Mahmassani (2004)	**Main objective**: introduction of a bi-level formulation to identify vulnerable links in transportation network.

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From the previous page

Author	Methodology
	Vulnerability: is measured as "the utility of the alternate paths required to accommodate the flow on the damaged link is multiplied by the proportion of flow that would be diverted to that path". **Disadvantages**: Only some disruptive scenarios are analysed, the authors do not introduce a systematic analysis of all the links of the network.
Latora and Marchiori (2005)	**Main objective**: presentation of a methodology to identify the "critical" components of a critical infrastructure network, (including nodes and links). The critical locations are identified by a vulnerability index. **Vulnerability**: of a link or a node is understood as the difference between the performance of the network when all the elements are functioning, and when the link or node analysed has been removed. The variable chooses to measure the performance of a network is the "efficiency", which measures the redundancy of the element. **Disadvantages**: the authors do not differentiate between critical and vulnerable elements. The redundancy is the only variable used to evaluate the vulnerability of a element. The authors only consider complete damage of a link, and partial damages are not considered.
Jenelius et al. (2006)	**Main objective**: presentation of indices to measure the exposure of links and the importance, to identify he vulnerable and critical links of a network, respectively. **Vulnerability and criticality**. Vulnerability is associated with a node, and it is measured by the "exposure". Criticality is associated with a link, and it is measured by the "importance" of the link. Both terms are measured by the increment in generalised travel cost when links are closed, and when the evaluation is done for nodes, it is assumed that the links arriving to the analysed node are closed. **Disadvantages**: vulnerability and criticality are differentiated but vulnerability is measured in nodes, and criticality in link. In addition, the only type of damage studied is the total disruption of the link, i.e. the link is removed from the network.

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From the previous page

Author	Methodology
Scott et al. (2006)	**Main objective**: the introduction of an approach for identifying critical links and evaluating network performance. In the context of identifying critical links, a measure for determining the value of a link within the overall traffic network is introduced, the Network Robustness Index. **Critical** links are defined as the ones creating the worse scenario when damaged, highlighting the importance of variables such as the flow of the links. For their identification, the authors present the Network Robustness Index, which is evaluated by the difference of the travel-time cost when the analysed link is removed from the network, and in normal operation. **Disadvantages**: in the methodology the type of damage that is considered is done by disabling links in the network. Only some scenarios are tested. In this paper a network is used as a base network, and from which three networks are created by removing a different number of links in each of them, reaching different levels of connectivity in each example studied.
Berdica and Mattsson (2007)	**Main objective**: the presentation of a model-based case study, to analyse the vulnerable links of a transport network, including the real case of the Stockholm road network. **Vulnerable** links are analysed comparing different scenarios of disruption with a normal scenario. The changes in the scenarios are included by analysing variables such as: link flow calculated with a UE assignment model, the travel time, trip length, and travel speed. **Disadvantages**: This paper addresses a methodology for specific cases, this fact makes necessary to reproduce the model in every case that needs to be analysed. This model gives important information about the effects of a specific damage, but it is not providing a global methodology that can be used in every type of network and analysing all the elements of the network.
Demšar et al. (2008)	**Main objective**: presentation of a mathematical method for modelling the vulnerability risk of network elements. The authors noted that this method can be used for identification of critical locations in a spatial network too.

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From the previous page

Author	Methodology
	Critical location is “defined as an object or an area whose removal or destruction changes the structure of the network in terms of flow and connectedness”. However, the authors develop a vulnerability risk measure to identify the critical locations, so both concepts are not really differentiated. For the vulnerability measure, they propose two measures, the “betweenness”, and the “clustering coefficient”, both of them determined by topological characteristics of the network. High values of “betweenness” correspond to main links of the network. **Disadvantages**: the method uses graph theory to identify the critical locations of the network, therefore, only the way that the links are connected in the network is analysed. Vulnerable and critical elements are not differentiated.
El-Rashidy and Grant-Muller (2014)	**Main objective**: the introduction of a methodology for assessing the level of vulnerability of road transport networks that reflects the importance of network links. **Vulnerable** links are identified by several attributes, including variables such as link traffic flow, the capacity of the link, and the link length. These attributes are combined through fuzzy logic, and one single value for each link is calculated. In addition, using the link vulnerability index, two aggregated vulnerability indicators are proposed i.e. a physically based aggregated vulnerability indicator, and an operational based aggregated vulnerability indicator. **Disadvantages**: each of the attributes presented in Table 3.2 needs to be evaluated for each link of the network, also each weight applied to each attribute has to be calculated for each of the networks analysed. For the development of the methodology, and for obtaining accurate results when fuzzy logic is applied, it is necessary a deep knowledge of the network.
Balijepalli and Oppong (2014)	**Main objective**: the introduction of a Network Vulnerability Index (NVI), which analyses the serviceability and the importance of each road link on the network. The serviceability is defined in terms of loss of capacity of road links relative to their base values adopted from the hierarchy of road system reflecting their importance.

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From the previous page

Author	Methodology
	Vulnerability index is evaluated by measuring the flow and the travel time for two different moments with and without disruption. **Disadvantages**: it is necessary the application of the method for each of the links that wants to be evaluated.
Rodriguez-Nuñez and Garcia-Palomares (2014)	**Main objective**: the introduction of a methodology for analysing the criticality and vulnerability of public transport networks. The criticality is measured at the level of a link, and it is evaluated by the serviceability. Once the critical links are obtained, the vulnerability can be assessed at node level. **Criticality** of a link is measured by the difference of the travel times with the link and without. **Disadvantages**: the authors only analyse the complete closure of the link, and its effects in the performance of the network. Partial damages are not considered.
Wang et al. (2016)	**Main objective**: the introduction of a bi-level formulation for determining the most critical links in a transportation network whose closure or failure would incur the worst deterioration of network performance. **Criticality** of a link is measured by using an optimization problem that maximises the total travel costs of the network when one link or groups of two or three links are removed from the network. Then, the links within the groups of links that obtain a higher cost in the optimization problem will be the most critical ones. **Disadvantages**: the authors only analyse the complete closure of the link, the optimization problem needs to be evaluated for each link. In addition some problems with the high computational costs are highlighted by the authors for large-scale networks.

3.4 Conclusions

Once the evaluation of resilience in a network under a disruptive event can be conducted, the next step will focus on the improvement of its value. The improvement of resilience will imply that the network is able to have a better performance under a future perturbation impact.

As a first step for the enhancement of the resilience, the identification of vulnerable and critical links is a problem that needs to be addressed. Some of these links can contribute to create a larger reduction of resilience when affected by a perturbation. Therefore, in this chapter, a review of several methodologies presented in the literature for the identification of critical and vulnerable elements has been conducted, and the followings conclusions are drawn:

- There is not a clear differentiation between vulnerable and critical elements of a network, even though this differentiation is essential when the final goal is the increment of the network resilience. Critical links will have an important impact in the value of the resilience when damaged. Then, the reduction of the impact or the improvement of the characteristics of these links will be part of the strategies for the enhancement of resilience. On the other hand, vulnerable links will have a smaller impact in the values of resilience, since the failure of these links does not necessary imply a large impact on the network, and the consequences of the impact might be concentrated on the given link. However, their identification may be crucial when a transport network needs to be upgraded as a whole.

- Different methodologies have been introduced for the identification of vulnerable and critical links in transport networks in the last years. Several approaches have been found, from those methods that study the identification of these elements by analysing only how the elements are connected; such as Murray-Tuite and Mahmassani (2004), and Latora and Marchiori (2005), analysing the connectivity, and Demšar et al. (2008), using graph theory, to methods that include traffic variables, such as Balijepalli and Oppong (2014), El-Rashidy and Grant-Muller (2014), and Wang et al. (2016), including mainly the analysis of the flow, and the travel times of the network. However, if the final goal is the enhancement of resilience of a network affected by a perturbation, it is noted that in the analysis of the vulnerable and critical links, traffic variables are indispensable, since the main impact that will be evaluated is the behaviour of the users, and their modifications due to the hazard.

- Among the authors that have analysed the vulnerable and critical links of a network when a perturbation occurs, most of them only introduce the total damage of the link, i.e. the link is considered as closed, and not part of the network anymore. The removal of a link, and the effects of this removal in the traffic variables has been studied by authors such as Jenelius et al. (2006),Rodriguez-Nuñez and Garcia-Palomares (2014), and Wang et al. (2016). However, when analysing the effects of weather events, such as intense rains, snow, and wind, the total damage of a link may not be the most realistic scenario. Many times the links are only partially affected, and they suffer a reduction in their characteristics. Therefore, in this context, it is important to introduce a methodology that can evaluate partial damages in the links.

- Some methods for the identification of vulnerable and critical links analyse only some scenarios, i.e. the authors select a group of disrupted situations which will be tested in the network. However, it is important to select methodologies that systematically analyse the whole network. Links that a priori may not seem critical or vulnerable might have a significant influence in the behaviour of the network.

Part III

Original Contributions

Chapter 4

Resilience in traffic networks: a dynamic restricted equilibrium model

Contents

4.1 Notation

c_a	travel cost associated with link a
c_{0a}	free travel time associated with link a

d_{pq}	demand associated with origin-destination pq
h_{pq}	flow on routes with origin-destination pq
η_{pq}	number of routes with origin-destination pq
t	time interval
t_r	interval of time at which equilibrium is reached
ν_a	link flow associated with link a
ν_a^*	link flow associated with link a corresponding to the UE state
ν_a^{max}	link capacity to provide certain service of level
C_a	integral of the travel cost function of link a
C_{th}	cost threshold associated with the system break-down point
C_T	actual total cost
C_0	initial total cost (t=0)
R_{pq}	set of routes with origin-destination pq
T_{th}	reference value associated with recovery time
$\mathcal{A}$	set of links
$\mathcal{D}$	subset of origin-destination pairs of nodes
$\mathcal{N}$	set of nodes
α	system impedance
β_a	saturation parameter of the cost function
γ	saturation parameter of the cost function
θ_k	normalized slope associated with state of perturbation k
χ_k^r	perturbation resilience associated with state of perturbation
ρ_r	net flow variation among routes within state of perturbation k
σ_k	stress level of traffic network associated with state of perturbation k
τ_k	cost level of traffic network associated with state of perturbation k
ψ_k	exhaustion level of traffic network associated with state of perturbation k

The research included in this chapter has been presented in Nogal et al. (2016b), Martinez-Pastor et al. (2015), Nogal et al. (2015b) and the I International Conference on Surface Transportation System Resilience to Climate Change and Extreme Weather Events, Washington D.C. (TRB) [2015].

4.2 Motivation

The aim of this chapter is to present a new methodology to evaluate traffic resilience when a perturbation occurs. There is an existing gap in this area due to the novelty of this concept when applied in traffic networks. Existing methodologies have been previously studied, and more efforts need to be done on the quantification of resilience. The main challenges are to develop a method able to include the dynamic nature of the problem, and the role of the network users during the process. In addition, the practical character of the methodology is an essential goal. There is a risk of developing an impractical method when trying to cover the multiple aspects included in the evaluation of this holistic concept.

4.3 Introduction

The management of traffic networks requires useful tools that help decision-making when a perturbation occurs. Nowadays, extreme weather events take place more frequently. This fact can be appreciated in the increment of the number of catastrophic events, damaging all kind of infrastructures around the world. For that reason, the evaluation of the impacts caused by these disruptions is an essential strategy to a robust, reliable development of a country's infrastructure systems. Among the different existing systems, this research focuses on traffic networks, which are key-elements in the progress, and well-being of a society.

The concept proposes to evaluate the behaviour of a traffic network when a perturbation takes place is known as resilience. This holistic concept studies the complete process, from the perturbation until the total recovery when a disruption occurs. Therefore, this Chapter aims to develop a method to evaluate the concept of resilience when a perturbation impacts a traffic network.

It is noted that when a perturbation takes place in a traffic network, two main effects occur: (i) user travel costs (generally time) increase and (ii) users become aware of these greater costs and try to reduce them by changing their route choices, generating a certain stress level in the network. When the perturbation stops and the initial state is recovered, the travel costs are recuperated and users eventually return to their initial route choices. On the other hand, if the alteration stops but the initial state is not recovered, users will find other route choices that minimize their costs, though these costs will be greater than before.

To evaluate this process, a dynamic restricted equilibrium traffic assignment model is developed. This model allows the simulation of the perturbation, by analysing the overcost generated, and also the stress level of the users when the network is perturbed.

It is highlighted that two different stages need to be considered, namely, the perturbation stage and the recovery stage. The former implies a modification in the initial network conditions, leading to a cost increment and a certain degree of user stress. In this stage it is important to analyse the network capacity to absorb the impact and to adapt to changes. In the recovery stage, when the perturbation has stopped, the system starts to recover, and tries to reach a new equilibrium state, i.e., a minimum travel cost compatible with the final network conditions. The critical parameter in this stage is the time necessary to achieve this equilibrium state.

Reflecting these two stages, two types of resilience are evaluated in the following sections, namely, the perturbation resilience and the recovery resilience, both of which are necessary to characterize a traffic network. It is noted that in both formulations, not only the impact on the travel cost due to the perturbation is included, but also the effect on the behaviour of the users, i.e., it evaluates the user stress level.

In addition, the proposed methodology includes the majority of the concepts analysed by previous authors when the resilience in transport networks is defined, such as the redundancy, the adaptability, and the robustness.

Finally, it is noted that this effective and straightforward methodology allows (a) the comparison of the resilience to a given perturbation for different traffic networks, (b) the identification of the traffic network weakness when different disruptions occur, and (c) the analysis of the effect of the user information about the state of the traffic network.

The chapter is organized as follows; Section 4.4 introduces the effects caused by a perturbation in a traffic network. Section 4.5 presents a new traffic assignment model, defined as "a dynamic equilibrium-restricted assignment model", which is applied to an illustrative example. Section 4.6 describes how to estimate the exhaustion level of the network, evaluated through the analysis of the cost level, and the stress level suffered by the traffic network when a perturbation occurs. Section 4.7 presents the formulation to evaluate the resilience of a traffic network, by measuring the perturbation resilience, and the recovery resilience, and in Section 4.8, a comparison with previous methodologies is performed. In Section 4.9, a real example of application to illustrate the performance of the proposed method is introduced. Finally, in Section 4.10, the main conclusions are drawn.

4.4 Traffic networks, and the effect caused by a perturbation

Throughout this thesis, the focus will be on the impact caused by perturbations on traffic networks. Even though when a perturbation occurs, multiple aspects of an infrastructure may be threatened, the tools provided in the following sections aim to assess the performance of a traffic network.

In addition, it is noted that only negative perturbations are taken into account, i.e., those perturbations which imply a negative effect on the conditions of the traffic performance.

Description of the impact of a perturbation on a traffic network

The direct effect of a perturbation upon a traffic network is the increment of the user travel costs, which are generally evaluated by using the travel times. Therefore, when a disruption is affecting a link, or a route of the network, it will imply an increment on the travel times of the affected elements.

When the users of the affected elements of the network become aware of the increments caused in their travel times by the perturbation, they will try to reduce them by selecting routes less affected by the perturbation.

It is noted that this strategy will depend on the options available for each particular case. In some cases, there may not be other options to go from one origin-destination (OD) pair of the network, and the users will need to maintain their original route assuming the travel time increment caused by the perturbation in their routes.

When the users are searching for a route, they always tend to choose the shortest route from their origin to their destination. However, the option of changing to a better route may not be always available. The following conditions need to happen to make the possibility of changing routes valid:

- The existence of less affected roads, which are connected with the most distressed areas, and can be an alternative option for the destinations that the users want to reach, i.e., the network is redundant allowing different routes for a same OD pair.
- A sufficient capacity of adaptation of the users of the network. This means that the users need to have enough knowledge of the new situation, and information of the behaviour of other users to choose other routes that can enhance their route travel cost.

However, even when the users have the option of choosing other routes to avoid the damaged parts of the network, and to improve their travel times, the necessity of changing their usual routes will generate a certain stress level in the network. This increment in the stress level is created because the users are in a certain way obliged to modify their behaviour due to the changes made in their usual routes. Therefore, this needs to be reflected in some way in the performance of the network.

When the users modify their routes, avoiding the ones more affected by the perturbation, the new selected routes become more saturated. This will cause an increase in the new selected route travel costs, and again they will try to search for less saturated routes.

The described process continues until an eventual new equilibrium is achieved. This equilibrium state is reached when users cannot improve their situation by modifying their choices. In such a case, their individual travel costs are the minimum costs associated with the existing situation.

Then, at the moment that the situation changes, the users will need to repeat the process again until a new equilibrium for the new situation is reached. This "adaptation

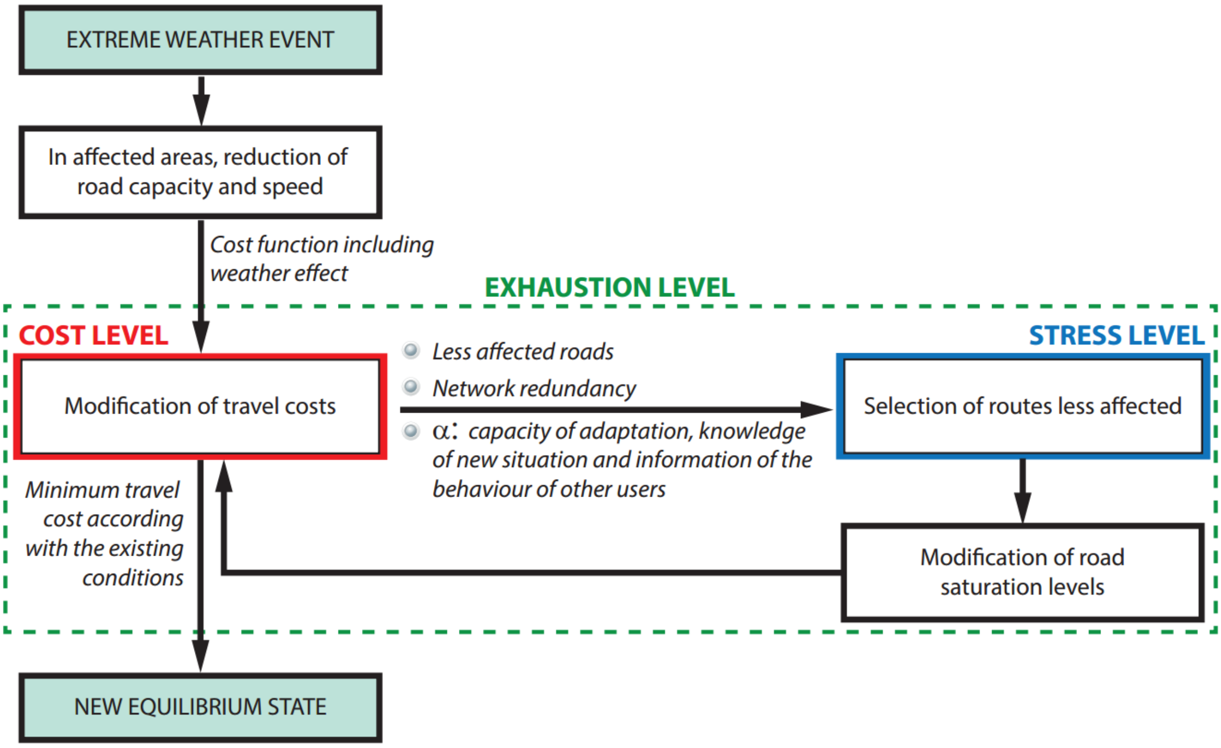

Figure 4.1: Diagrammatic representation of the traffic response of a traffic network suffering a weather impact. Nogal et al. (2016b)

process" is essential in the majority of the cases, since the perturbation may evolve with the time. Therefore, for each time interval, the users will need to adapt to the conditions of the network, as many times as necessary from the beginning of the perturbation until the total recovery of the network.

The process previously explained is represented in Figure 4.1, schematizing the response of a traffic network suffering a weather impact. It is noted that the impact caused by a perturbation in a traffic network can be evaluated through a cost level (measuring the impact in the travel cost, or travel times), and a stress level (measuring the modification of the initial routes of the users).

Travel costs can remain in low values after a perturbation if a certain degree of adaptability, redundancy, etc. exists in the network, this means that the users will have the option of using other routes with relatively good conditions when their initial route is affected. However, this behaviour will result in an increase in the stress level of the network.

Finally, the mechanism of cost-stress response is limited, the larger the disruption, the lower the remaining response capacity of the network. Therefore, the traffic network behaviour when suffering a disruption can be assessed by means of its exhaustion level, that is, the share allocated to the used resources.

4.5 A dynamic equilibrium-restricted assignment model

With the aim of assessing the effects previously described when a network is affected by a perturbation, this section presents a dynamic equilibrium-restricted assignment model. This model analyses the system performance before, during and after a perturbation.

This dynamic equilibrium-restricted assignment model will allow the estimation of traffic network resilience, allowing the simulation of the network behaviour when a disruptive event occurs.

Assumptions for the evaluation of resilience

First of all, this model is based on some assumptions, which are described below:

- The global behaviour of users is analysed on a day-to-day basis, that is, the problem of within-day dynamics (see Jagtman et al. (2005), and Bier and Hausken (2013)) is neglected.
- Only negative perturbations are taken into account ,i.e., those perturbations which imply a travel cost increment.
- Users select their route choices that reduce their individual travel costs. This selection is based on the complete information about the past day's travel costs.
- The capacity of adaptation of the users to the changes, the lack of knowledge of the new situation, and the lack of information of the behaviour of other users impede the immediate response and recovery of the system.

Introduction of the dynamic equilibrium-restricted assignment model

As explained in Chapter 2, the problem of traffic assignment models consists in describing how users with different OD select their routes according to traffic conditions. Traffic flows governs the network performance in terms of travel time. Thus, to understand how users behave, i.e. how users select the adequate paths to go from origins to destinations, an assignment model needs to be introduced.

These models can be understood as optimization models whose main objective is the minimization of the travel costs in the network, subjected to some restrictions, such as flow conservation and the link route OD compatibility.

In this particular case, when analysing the resilience of a traffic network, a progressive response of the traffic network is expected due to a time-varying impact (the perturbation). Therefore, a dynamic approach is required.

With this objective, this section is devoted to presenting this new model, which is defined as the "dynamic equilibrium-restricted assignment model".

Analysing the definition of the dynamic user-optimal equilibrium condition, the dynamic user equilibrium state is reached when, for each origin-destination pair, the actual route travel cost experienced by travellers entering during the same time interval is equal and minimal. This implies that:

"The travel costs incurred by traffic on all routes entered by traffic during the same time interval are equal or less than those that would be on any unused route at that time interval."

This definition is not entirely reflecting the reality when representing the behaviour of the users of a traffic network. When the users of a network try to adapt to the new conditions in a defined time period, a minimum equilibrium state may not be reached within the analysed period.

The reason of this phenomenon is that the behaviour of the traffic network is **restricted** by a system impedance. This means that the capacity of the system to alter its previous state is not infinite, and if the perturbation is large enough the system will need a longer period to recover the equilibrium state.

Therefore, the **impedance of the system** is caused due to:

- The capacity of adaptation that the users of the network have to the possible changes.
- The lack of knowledge of the new situation, when a perturbation occurs it is reasonable to think that precise information of the situation may not reach all the users.
- The lack of knowledge of the behaviour of other users, each of the users of the network will try to minimize their own travel times.

Following this description, a **dynamic "equilibrium-restricted"** state can be obtained when, for each origin-destination pair, the actual route travel cost experienced by travellers entering during the same time interval **tends to** be equal and minimal. This implies that:

"The travel costs incurred by traffic on all routes entered by traffic during the same time interval ***tend to*** *be equal or less than those that would be on any unused route at that time interval.*

In this "equilibrium-restricted approach" the system is not forced to reach the minimum equilibrium state in the defined period time, and could be unable to reach this state in such a time interval.

With the aim of introducing this new mathematical optimization program, let's consider a connected traffic network with set of nodes $\mathcal{N}$ and set of links $\mathcal{A}$. For certain origin-destination (OD) pairs of nodes, $pq \in \mathcal{D}$, where $\mathcal{D}$ is a subset of $\mathcal{N}$ x $\mathcal{N}$, connected by a set of routes R_{pq}, there are positive demands d_{pq} which give rise to a link flow pattern $\boldsymbol{\nu} = (v_a)_a \in A$, and a route flow pattern $\mathbf{h} = (h_{pqr})_{r \in R_{pq}, pq \in \mathcal{D}}$, when distributed through the network. Furthermore, for each link a there is a positive and strictly increasing travel cost function c_a.

Mathematically, this equilibrium-restricted state can be expressed as an optimization problem for each time interval t, that is:

$$\underset{\mathbf{h},\boldsymbol{\nu},\rho}{\text{Minimize}} \sum_{a \in A} C_a(\nu_a(t)), \tag{4.1}$$

subject to:

$$\sum_{r \in R_{pq}} h_{pqr}(t) = d_{pq}(t) : \lambda_{pq}, \qquad \forall pq \in \mathcal{D} \tag{4.2}$$

$$\sum_{pq \in D} \sum_{r \in R_{pq}} \delta_{apqr} h_{pqr}(t) = v_a(t) : \lambda_a, \qquad \forall a \in \mathcal{A} \tag{4.3}$$

$$h_{pqr}(t) = \rho_r(t) h_{pqr}(t - \Delta t) : \lambda_r, \quad \forall r \in R_{pq}, \forall pq \in \mathcal{D} \tag{4.4}$$

$$|\rho_r(t) - 1| \leq \alpha, \qquad \forall r \in R_{pq} \tag{4.5}$$

$$h_{pqr}(t) \geq 0 : \mu_{r,1}, \mu_{r,2} \quad \forall r \in R_{pq}, \forall pq \in \mathcal{D} \tag{4.6}$$

with

$$\delta_{apqr} = \begin{cases} 1, \text{if route } r \text{ from node } p \text{ to node } q \text{ contains arc } a; \\ 0, \text{otherwise,} \end{cases}$$

where $C_a(\cdot)$ is the integral of the travel cost function. Furthermore, $\rho = \rho_r \in \mathbb{R}^+$ measures the variation of route flows in two consecutive intervals of time, $t - \Delta t$ and t, and α represents the system impedance to alter its previous equilibrium state. The lower bound of α is zero, which implies the system is unable to reach a Wardropian equilibrium state and, the upper bound of α is infinite, which means the system reaches the Wardropian equilibrium immediately. It is noted that the system impedance in Eq. (4.5) can be replaced by a route impedance (α_r) or an OD impedance (α_{pq}) . λ_{pq}, λ_a, λ_r, $\mu_{r,1}$, $\mu_{r,2}$, and μ_{pqr} are the dual variables associated to the optimization problem, which are addressed in detail in the following section.

The presented equations included in the dynamic "equilibrium-restricted approach" are described as follows:

- The objective function, Eq. (4.1.): provides the set of link flows that minimises the sum of the integrals of the link costs subjected to the flow conservation conditions.

- Eqs. (4.2.) and (4.3.): represents the restriction of adaptation capacity in each time interval analysed. That is, users select those routes which minimize their individual travel costs.

- Eq. (4.4.): establishes that the route flow in a given time interval differs $|\rho_r - 1|$ with respect to the previous time interval. This restriction provides temporal continuity for the model.

- Eq. (4.5.): represents the variation of route flows between two consecutive intervals which is limited by the impedance.

- Eq. (4.6.): forces the non-negativity of route flows.

Given that both, the objective function, and the feasible region of the optimization problem, are convex, the solution of the model (4.1) − (4.6) for each interval of time analysed $t = \tau$, is guaranteed and presents uniqueness in terms of link flow. The optimal link flow pattern ν_a^* implies an optimal set of route travel cost for each interval of time.

A detailed mathematical explanation of the dynamic equilibrium-restricted assignment model is presented in the following section.

It is remarked that, in a given time interval, the system will reach a new equilibrium state when its adaptation capacity is larger than the degree of perturbation. In such a case, costs will be as small as the existing conditions allow them to be.

4.5.1 Mathematical explanation of the dynamic equilibrium-restricted assignment model

This section is devoted to the mathematical explanation of the dynamic equilibrium-restricted assignment model, by means of the associated Karush-Kunn-Tucker (KKT) conditions. The KKT conditions have been applied to the presented model to demonstrate that a feasible solution can be obtained, and the results obtained are presented below. The KKT condition include the primal feasibility conditions corresponding with Eqs (4.1)-(4.6) at the time interval $t = \tau$, and

$$c_a(\nu_a(\tau)) - \gamma_a = 0; \qquad \forall a \in \mathcal{A} \tag{4.7}$$

$$-\gamma_{pq} + \sum_a \gamma_a \delta_{apqr} + \gamma_r - \mu_{pqr} = 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}_{pq} \tag{4.8}$$

$$-\gamma_r H_r(\tau) + \mu_{r,1} - \mu_{r,2} = 0; \qquad r \in \mathcal{R}_{pq} \tag{4.9}$$

$$\mu_{pqr} h_{pqr}(\tau) = 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}_{pq} \tag{4.10}$$

$$\mu_{r,1}(\rho_r(\tau) - 1 - \alpha) = 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}_{pq} \tag{4.11}$$

$$\mu_{r,2}(\rho_r(\tau) - 1 + \alpha) = 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}_{pq} \tag{4.12}$$

$$\mu_{pqr} \leq 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}_{pq} \tag{4.13}$$

$$\mu_{r,1} \leq 0; \qquad \forall r \in \mathcal{R}_{pq} \tag{4.14}$$

$$\mu_{r,2} \leq 0; \qquad \forall r \in \mathcal{R}_{pq} \tag{4.15}$$

where λ_{pq}, λ_a, λ_r, μ_{pqr}, $\mu_{r,1}$, and $\mu_{r,1}$ are the KKT multipliers associated to the constrains (4.2)-(4.6) at $t = \tau$ respectively, and $H_r(\tau) = h_{pqr}(\tau - \Delta\tau)$.

Once the KKT conditions have been applied, and the equations obtained, the following conclusions can be obtained.

From Eqs. (4.7) and (4.8), it is known that

$$c_{pqr}(\tau) = \gamma_r + \gamma_{pq} + \mu_{pqr}; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}_{pq} \tag{4.16}$$

For those routes whose condition (4.5) is not activated, $\mathcal{R}^1_{pq} \in \mathcal{R}_{pq}$, it is immediate from Eqs. (4.9), (4.11), and (4.12) that $\mu_{r,1} = \mu r, 2 = \gamma_r = 0$. Therefore, the following conclusions are derived using Eqs. (4.10), and (4.16).

$$c_{pqr}(\tau) = \gamma_{pq} \qquad if \qquad h_{pqr}(\tau) \leq 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}^1_{pq} \tag{4.17}$$

$$c_{pqr}(\tau) \leq \gamma_{pq} \qquad if \qquad h_{pqr}(\tau) = 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}^1_{pq}. \tag{4.18}$$

In other words, γ_{pq} is the optimal route travel cost from origin p to destination q, which is independent of the route considered. Eq. (4.18) shows that users only select those routes that minimise their route travel costs. Moreover, this cost is reached when the capacity of adaptation α is larger enough to impede the system from reaching this state (recall condition (4.5.) is not active).

In the case that $\mathcal{R}^1_{pq} \equiv \mathcal{R}_{pq}$, the solution correspond to the Wardropian equilibrium, that is $\gamma_{pq} = c^*_{pq}$.

For those routes in $\mathcal{R}^2_{pq} \in \mathcal{R}_{pq}$, whose condition (4.5) is active, being $\rho_r(t) = 1 - \alpha$, it is known that $\mu_{r,2} \leq 0$ through the complementary slackness conditions (4.12), and (4.15). Beside, using (4.9), (4.11), and (4.14) it is obtained $\mu_{r,1} = 0$ and $\gamma_r = -\frac{\mu_{r,2}}{H_r(\tau)}$, and consequently,

$$c_{pqr}(\tau) = \gamma_{pq} + \frac{\mu_{r,2}}{H_r(\tau)} \qquad if \qquad h_{pqr}(\tau) \leq 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}^2_{pq}. \tag{4.19}$$

$$c_{pqr}(\tau) \leq \gamma_{pq} + \frac{\mu_{r,2}}{H_r(\tau)} + \mu_{pqr} \qquad if \qquad h_{pqr}(\tau) = 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}^2_{pq} \tag{4.20}$$

These equations imply that users selecting routes in $\mathcal{R}^2_{pq}$ will experience route travel costs larger than the optimal route travel cost because some users still have not adapted to the new situation, resulting in an additional cost of $\frac{\mu_{r,2}}{H_r(\tau)}$. Therefore, the term $\mu_{r,2}$ represents the costs per user in route $r \in \mathcal{R}^2_{pq}$ who has not selected the optimal route pattern.

Parallel, for those routes in $\mathcal{R}^3_{pq} \in \mathcal{R}_{pq}$ which fulfill $\rho_r(t) = 1 + \alpha$, the following expressions are obtained,

$$c_{pqr}(\tau) = \gamma_{pq} - \frac{\mu_{r,1}}{H_r(\tau)} \qquad if \qquad h_{pqr}(\tau) \leq 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}^3_{pq} \tag{4.21}$$

$$c_{pqr}(\tau) \leq \gamma_{pq} - \frac{\mu_{r,1}}{H_r(\tau)} + \mu_{pqr} \qquad if \qquad h_{pqr}(\tau) = 0; \qquad \forall (p,q) \in \mathcal{D}, \forall r \in \mathcal{R}^3_{pq} \tag{4.22}$$

That is, users selecting routes in $\mathcal{R}^3_{pq}$ will experience route travel costs smaller than the optimal route pattern.

Finally, Eqs. (4.20), and (4.22) imply that users only select those routes that minimise their route travel costs within their adaptation capacity.

Considering the Wardropian assignment problem, all users experience the same minimum travel cost, c^*_{pq}, for each OD pair when the UE is reached. Thus, the total travel costs of all users travelling form p towards q is $d_{pq}c^*_{pq}$. In this case of the restricted user equilibrium, the total travel cost of all users with the same OD pair in time τ is,

$$d_{pq}\gamma_{pq} - \sum_{r \in \mathcal{R}^2_{pq}} \mu_{r,2}(\alpha - 1) - \sum_{\mathcal{R}^3_{pq}} \mu_{r,1}(\alpha + 1) \geq d_{pq}c^*_{pq}, \tag{4.23}$$

and consequently,

$$\gamma_{pq} \geq c^*_{pq} + \frac{\sum_{r \in \mathcal{R}^2_{pq}} \mu_{r,2}(\alpha - 1) + \sum_{\mathcal{R}^3_{pq}} \mu_{r,1}(\alpha + 1)}{d_{pq}}. \tag{4.24}$$

In other words, for each time interval analysed, the optimal route travel cost of the restricted user equilibrium depends on (a) the capacity of adaptation, (b) the OD demand, and (c) the actual number of routes affected by the impedance.

4.5.2 Illustrative example

A summary of the characteristics of the illustrative example can be found in Table 4.1.

Table 4.1: Summary of the illustrative example.

Concept	Illustrative example
Network	**Nguyen-Dupuis**
Cost Function	**BPR**
Uniqueness	**Maximum entropy**
Perturbation	**Reduction of the capacity of some links**

Network: Nguyen-Dupuis traffic network

To facilitate the understanding of the concepts introduced in the previous section, the Nguyen-Dupuis traffic network is used in the following example. This traffic network consists of 13 nodes, 38 links, 66 routes, and 34 OD pair. See Figure 4.2 for a graphical description. The time interval considered is one day and the total time is 70 days.

Cost Function: BPR

For this example, the well-known BPR function (see Chapter 2) is proposed as the cost function, which is expressed as follows:

$$c_a(t) = c_{0a}(t)\left[1 + \beta_a\left(\frac{\nu_a(t)}{\nu_a^{max}(t)}\right)^{\gamma}\right], \tag{4.25}$$

where c_a is the link travel time, c_{0a}, β_a and γ are the free travel time and saturation parameters, respectively, ν_a and ν_a^{max} are the actual flow and the link capacity to provide a certain service level. Sub index a implies association with link a.

The BPR function have been chosen in this chapter because the new cost function presented in this thesis has not been presented yet, for following examples in Chapter 5, and Chapter 6, the chosen cost function will be the new cost function which allows the introduction of weather related parameters. In addition, among all the available cost functions presented in the literature (see Chapter 2), the BPR has been selected for its

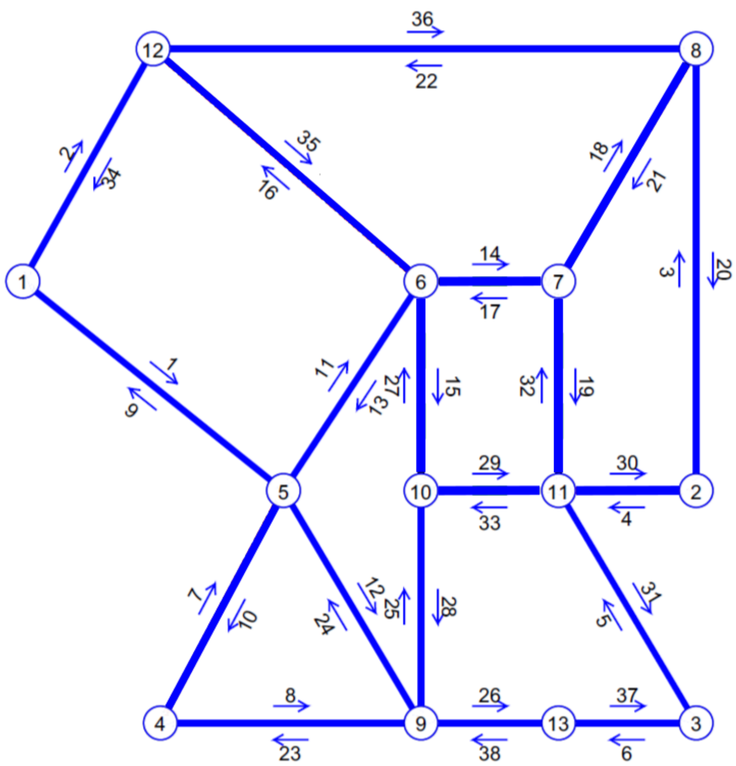

Figure 4.2: Nguyen-Dupuis traffic network, graphical description by nodes, and links.

simplicity, its popularity in the development of traffic problems, and well performance when traffic assignment models are used.

The network data used in this example are $c_{0a}(t) = 80\,km/h$, $\beta_a(t) = 0.7$, $\gamma = 2.1$ and $\nu_a^{max}(t_0) = 80$ users. The demand for different OD pairs is assumed to be constant for each time increment.

Uniqueness

When analysing the system, it is considered that the network, at the initial time, is in equilibrium, and the link flows are calculated for the initial equilibrium state by using the UE model proposed by Beckmann et al. (1956). It is noted that the use of a UE equilibrium approach for the evaluation of the flows provides uniqueness in terms of link flows but not in terms of route flows (see Nogal et al. (2011)). Therefore, the selection of routes flows, h_{pqr}, is carried out according to the criteria of **maximum entropy**, which is expressed as follows:

$$\underset{\mathbf{h}}{\text{Maximize}}\, Z = -\sum_{pq\in D, r\in R_{pq}} h_{pqr} log(h_{pqr}), \tag{4.26}$$

subject to

$$\sum_{pq\in D}\sum_{r\in R_{pq}} \delta_{apqr} h_{pqr} = \nu_a^*, \quad \forall a \in \mathcal{A}, \tag{4.27}$$

$$\sum_{r\in R_{pq}} h_{pqr} = d_{pq}, \quad \forall pq \in \mathcal{D}, \tag{4.28}$$

where $\nu_a^* = \nu_a(t=0)$ is the link flow pattern corresponding to the UE state.

In Table 4.2, and 4.3, the characteristics of the initial equilibrium state are shown. It is noted that, as expected, all routes sharing an OD pair have the same minimum route travel time, otherwise they do not have any flow.

Description of the perturbation

For the simulation of a disruption in the network, the capacity of some links of the network has been reduced. This perturbation can be due to road maintenance works for example, or any other event that can cause this effect in the network.

In this case, the affected links are $[4, 7, 10, 14, 15, 16, 17, 18, 19, 21, 27, 29, 30, 32, 33, 35]$, and the duration of the works will last 15 days, starting at day 10. See Figure 4.3 for a graphical description of the disruption, and Figure 4.4 for the location of the affected links in the network.

Table 4.2: Link characteristics at the UE

Route	**Links**	$h_{pqr}(users)$	$\lambda_a^*(h)$
1	1-11-14-18-20	15.01	26.80
2	2-35-14-18-20	12.61	26.80
3	2-36-20	15.01	26.80
OD 1-2		75.00	
4	1-11-14-19-31	8.74	20.81
5	1-11-15-29-31	13.36	20.81
6	1-12-25-29-31	0.00	21.87
7	1-12-26-37	109.32	20.81
8	2-35-14-19-31	7.34	20.81
9	2-35-15-29-31	11.23	20.81
OD 1-3		150.00	
10	1-11-14-18	15.01	19.96
11	2-35-14-18	12.61	19.96
12	2-36	47.38	19.96
OD 1-8		75.00	
13	3-21-17-13-9	13.96	28.84
14	3-21-17-16-34	14.14	28.84
15	3-22-34	46.90	28.84
OD 2-1		75.00	
16	3-21-17-13-10	0.00	21.66
17	3-21-19-33-28-23	0.00	24.90
18	4-33-28-23	75.00	11.53
OD 2-1		75.00	
21	5-32-17-13-9	10.92	21.33
22	5-32-17-16-34	11.07	21.33
23	5-33-27-13-9	11.58	21.33
24	5-33-27-16-34	11.73	21.33
25	5-33-28-24-9	0.00	24.96
26	6-38-24-9	104.70	21.33
OD 3-1		150.00	
31	7-11-14-18-20	0.00	19.09
32	8-25-29-30	75.00	10.28
33	8-25-29-32-18-20	0.00	21.94
OD 4-2		75.00	
38	21-17-13-9	13.96	19.94
39	21-17-16-34	14.14	19.94
40	22-34	46.90	19.94
OD 8-1		75.00	

Table 4.3: Link characteristics at the UE

Link	c_{0a}	$\nu_a^*(users)$	$\lambda_a^*(h)$	**Link**	c_{0a}	$\nu_a^*(users)$	$\lambda_a^*(h)$
1	1.86	186.44	9.56	20	2.46	125.00	6.85
2	1.48	163.56	6.14	21	1.50	129.38	4.39
3	2.46	150.00	8.90	22	3.02	170.62	13.41
4	0.77	75.00	1.23	23	1.45	125.00	4.05
5	1.50	120.30	3.98	24	1.50	154.70	5.70
6	0.77	154.70	2.91	25	1.29	75.00	2.08
7	1.46	50.00	1.85	26	0.77	184.32	3.86
8	1.45	125.00	4.05	27	1.16	74.04	1.86
9	1.86	180.12	9.02	28	1.29	100.00	2.74
10	1.46	50.00	1.85	29	0.77	154.83	2.91
11	1.39	102.12	3.02	30	0.77	75.00	1.23
12	1.50	109.32	3.53	31	1.50	115.68	3.79
13	1.39	100.43	2.97	32	1.16	71.26	1.80
14	0.77	162.11	3.13	33	0.77	174.04	3.51
15	1.16	54.83	1.53	34	1.48	169.88	6.53
16	1.97	124.26	5.46	35	1.97	139.82	6.44
17	0.77	175.64	3.56	36	3.02	173.74	13.81
18	1.50	126.26	4.25	37	0.77	184.32	3.86
19	1.16	35.85	1.31	38	0.77	179.70	3.70

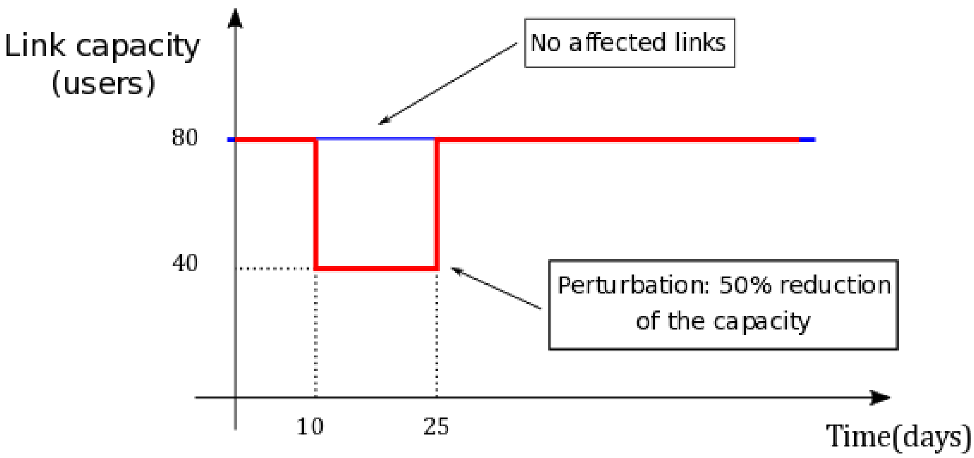

Figure 4.3: Description of the perturbation for the Illustrative example: reduction in the capacity of the affected links

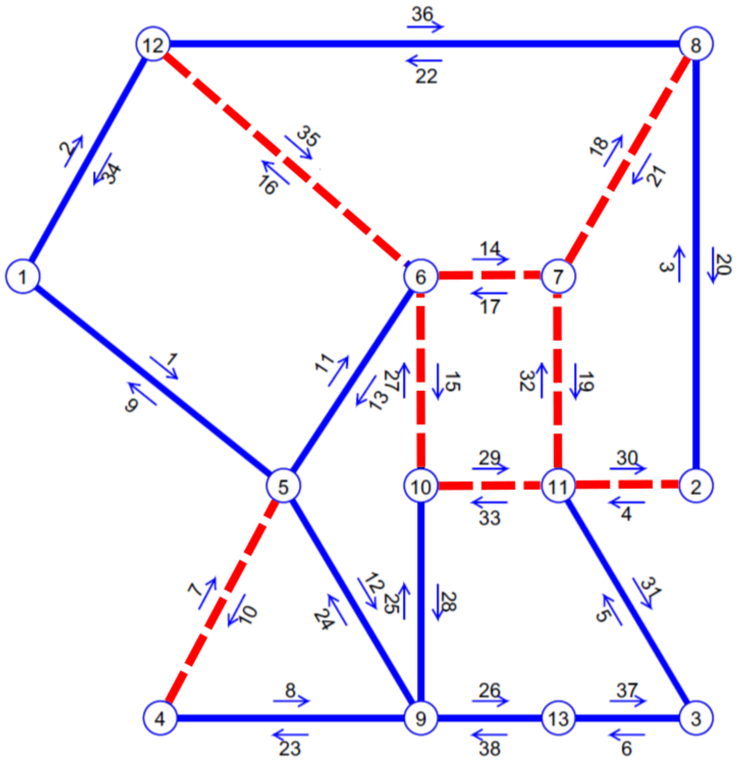

Figure 4.4: Nguyen-Dupuis network showing the links affected by the perturbation in the illustrative example

Results

Once the network is affected by the perturbation, the initial conditions change, and the system tries to reach a new equilibrium state. However, there is a limitation, and the quantity of flow that can be transferred from one link to another is limited by the system impedance, α. Thus, the equilibrium is not reached instantaneously, and the network will need some time until stable conditions are reached again.

In Figure 4.5, the iterative process to obtain the solution of the example is described. Thus, each day of the studied period, an evaluation of the dynamic restricted equilibrium model is performed. This daily evaluation is needed because due to the effects of the perturbation the conditions of the network will change, and after the network will need to adapt. The chosen studied period should be longer than the perturbation and recovery stage. It is highlighted in Figure 4.5, that the presented model is restricted by the capacity of the adaptation of the users, α, meaning that in each time interval (day), a limited number of users can change day routes. For this reason, a global equilibrium of the

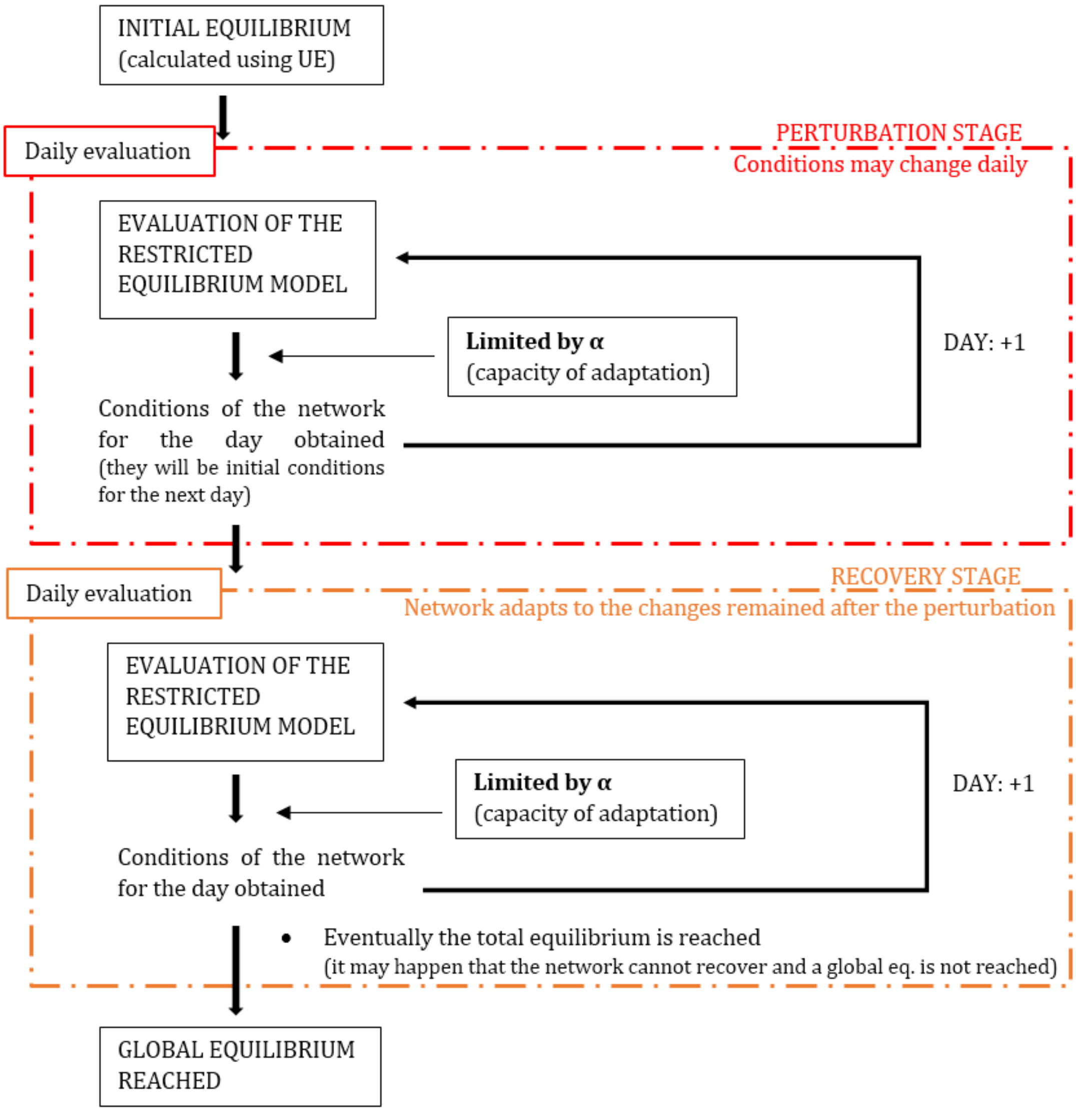

Figure 4.5: Explanatory diagram describing the iterative process followed to obtain the results of the example

network is not reached instantly. In addition, the conditions from the previous day will be the starting point for the next day and so on.

For this example, the capacity of adaptation of the users, α, is considered to be 0.1.

In Figure 4.6, the evolution of link flow and link capacity before, during, and after the perturbation are shown for some selected links of the network. It is noted that most of the links have important variations in their flows during the perturbation, with exception of link 7, which is affected in its travel time (see Figure 4.9), due to the selection of routes

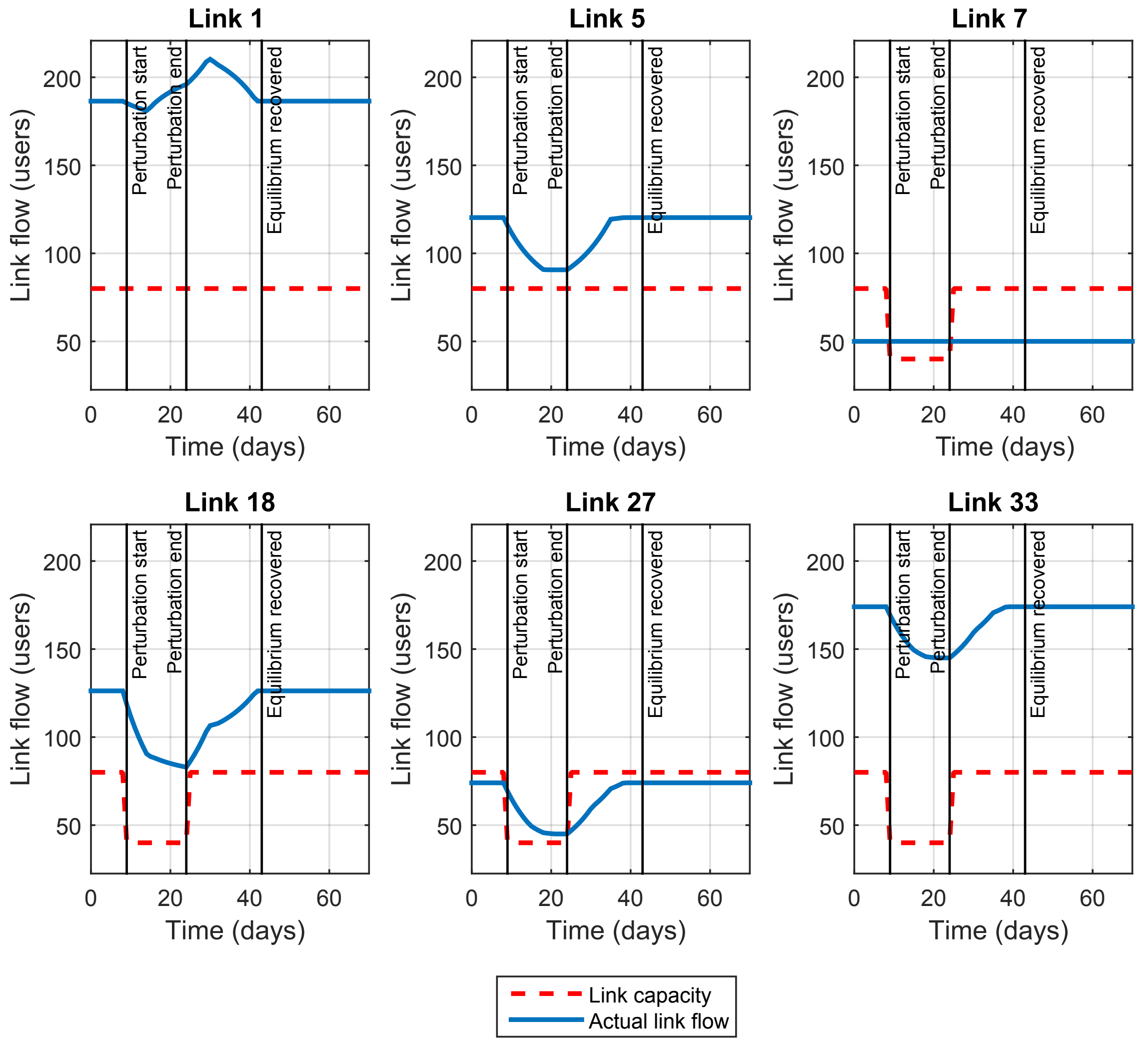

Figure 4.6: Evolution of link flow and capacity with time for some selected links ($\alpha = 0.1$).

used for this example.

In Figure 4.7, the evolution of route flows during, and after the perturbation is shown for some selected routes of the network.

In both Figures, 4.6, and 4.7, even though the perturbation starts the 10^{th} day, and last 15 days, until the 25^{th} day, the equilibrium of the system is not recovered until the 50^{th} day. Thus, the network needs 25 days after the end of the disruption for reaching the total recovery.

Finally, for a better comprehension of the problem, in Figure 4.8, the evolution of the link flows for all the links of the network is provided, and in Figure 4.9, the evolution of the link travel time for all the links of the network is given.

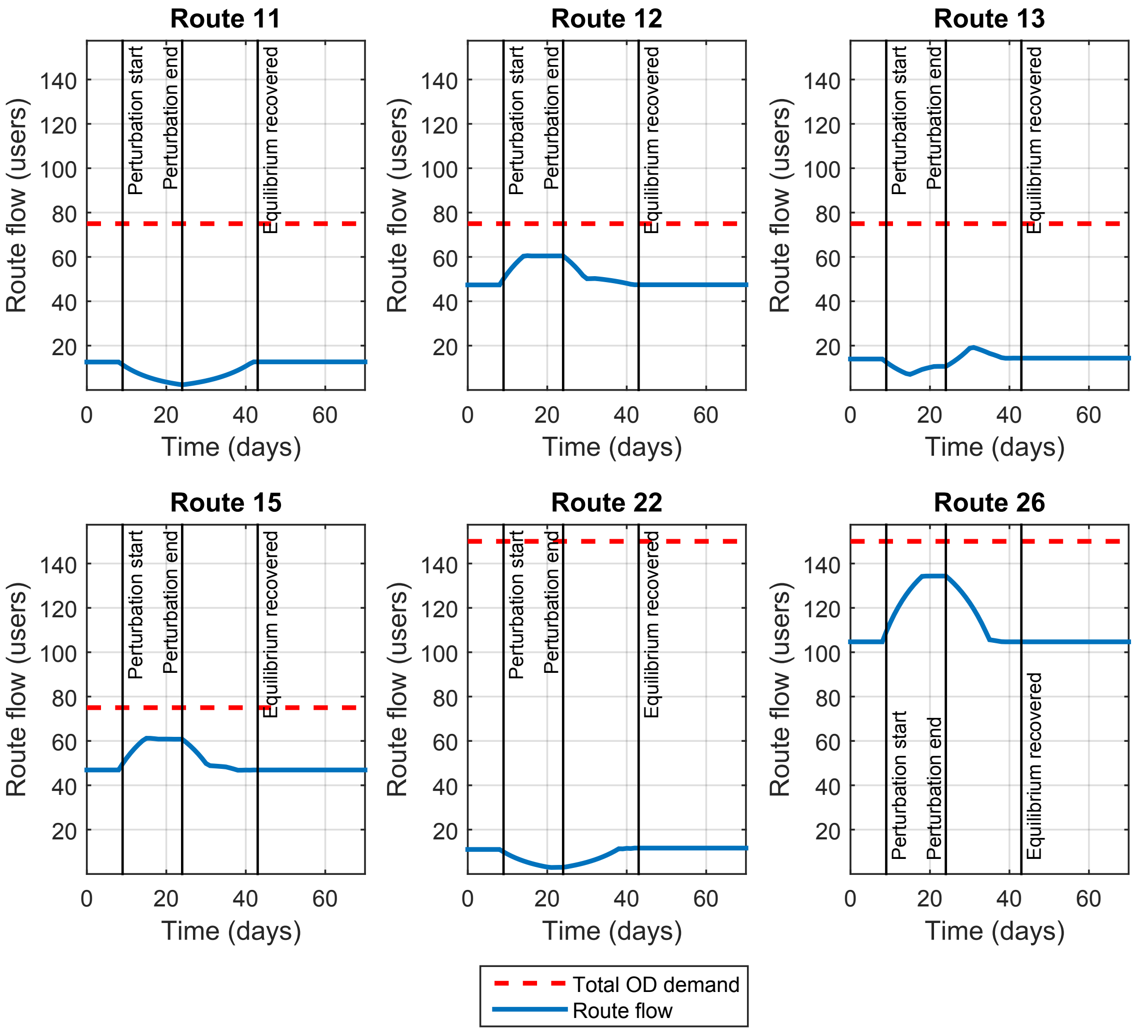

Figure 4.7: Evolution of route flow with time for some selected routes ($\alpha = 0.1$).

Both Figures, 4.8, and 4.9 are divided in three sub figures, in the left, and center graphs the evolution of the links directly affected by the perturbation of the network are shown, and in the right graph, the links that are not directly affected by the perturbation are illustrated.

Analysing this results, some conclusions can be drawn. The affected links, i.e. those that have been suffering a direct reduction in their capacities, experience an increment in their link travel times.

However, for the links that are not affected by the maintenance works, different tendencies can be observed. For example, some of the links will decrease their flows, such as link 2, 5 and 11, and others will increase their link flows, such as link 1, 9 and 6.

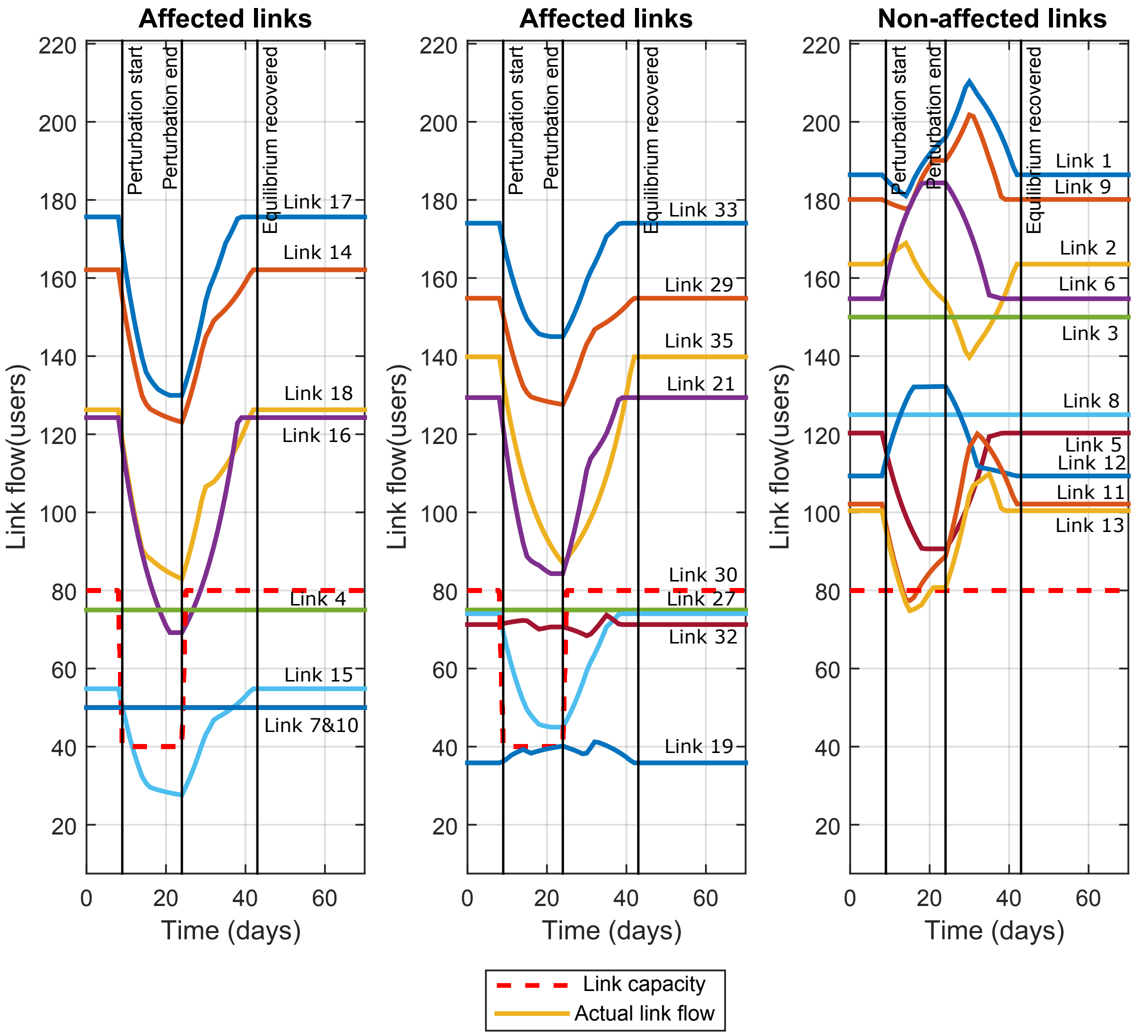

Figure 4.8: Evolution of link flow and link capacity for all links ($\alpha = 0.1$).

In the case of link 2, even though it is not affected directly by the perturbation, it is part of many routes (route 2, 3, 8, 9, 11 and 12), therefore this link becomes also affected in an indirect way, and users prefer to change to other options. The same conclusions can be drawn for link 5, which is also part of several routes (route 21, 22, 23, 24 and 25). Thus, in this link, the link flow is also decreased during the perturbation.

On the other hand, certain links increase their link flow, for example, link 6, which is not affected by the perturbation, and also is only part of one of the routes (route 26), and the case of link 12, which is part of two routes (route 5, and 6).

For the development of this example, the optimization system presented in Eq. (4.1)-(4.6), has been iteratively solved for each time interval, using the Optimization Toolbox

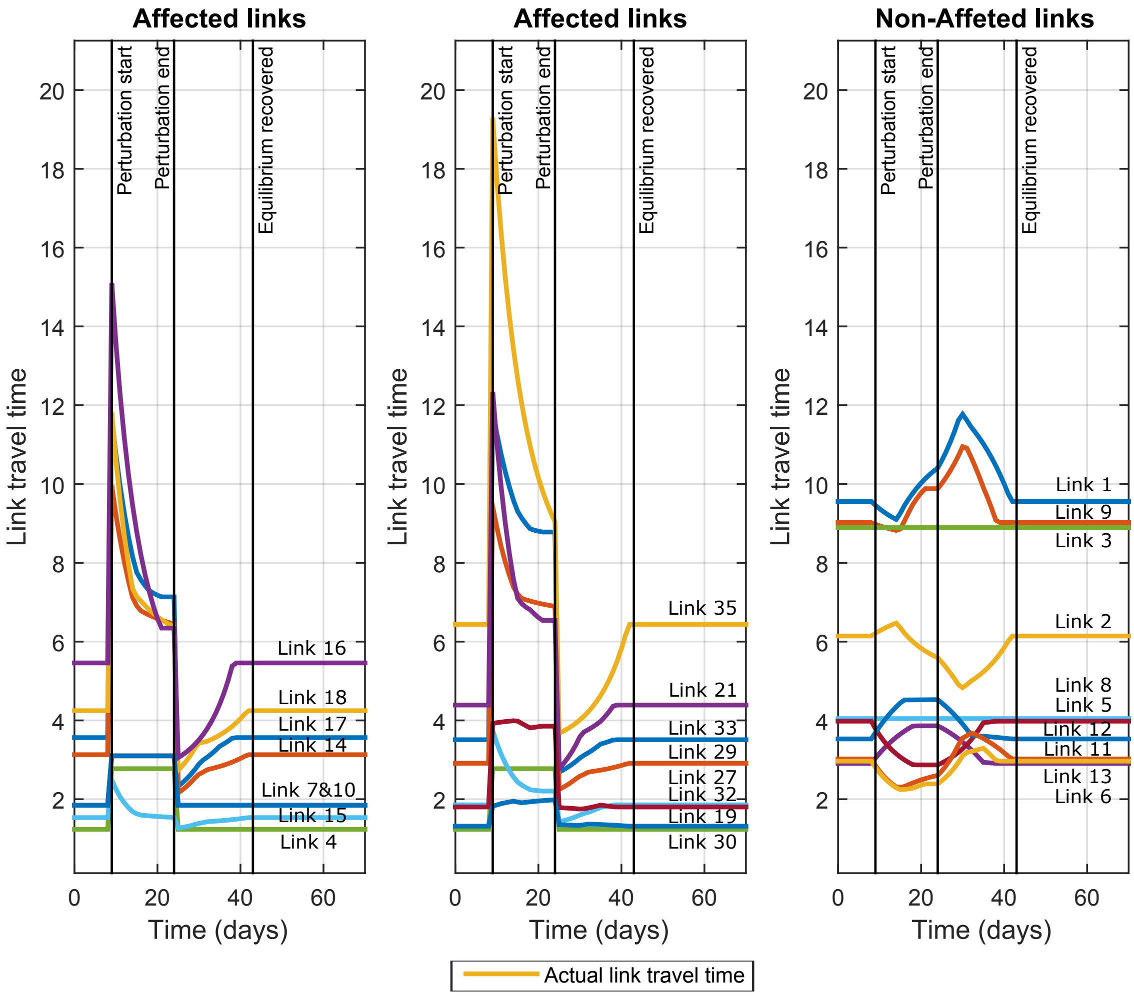

Figure 4.9: Evolution of link travel time for all links ($\alpha = 0.1$).

of Matlab 2014a. Thus, the flow pattern obtained in each iteration has been used as input for the following iteration.

4.6 Evaluation of the exhaustion level of the network

When a perturbation occurs in a traffic network, the users try to modify their behaviour to adapt to the new conditions (see Figure 4.10). As explained before, the adaptation implies modifications of the travel cost, which will be evaluated in the measurement of the evolution of the cost level in the network during the perturbation. In addition, due to the changes caused by the disruption, the users will suffer a degree of stress when they need to modify their routes, the evolution of the stress in the network will be measured by the stress level.

Finally, the exhaustion level of the network will be evaluated by a combination of both, the cost level, and the stress level. Below, a description of the evaluation of these levels is presented, namely, **the cost level**, **the stress level**, and **the exhaustion level**.

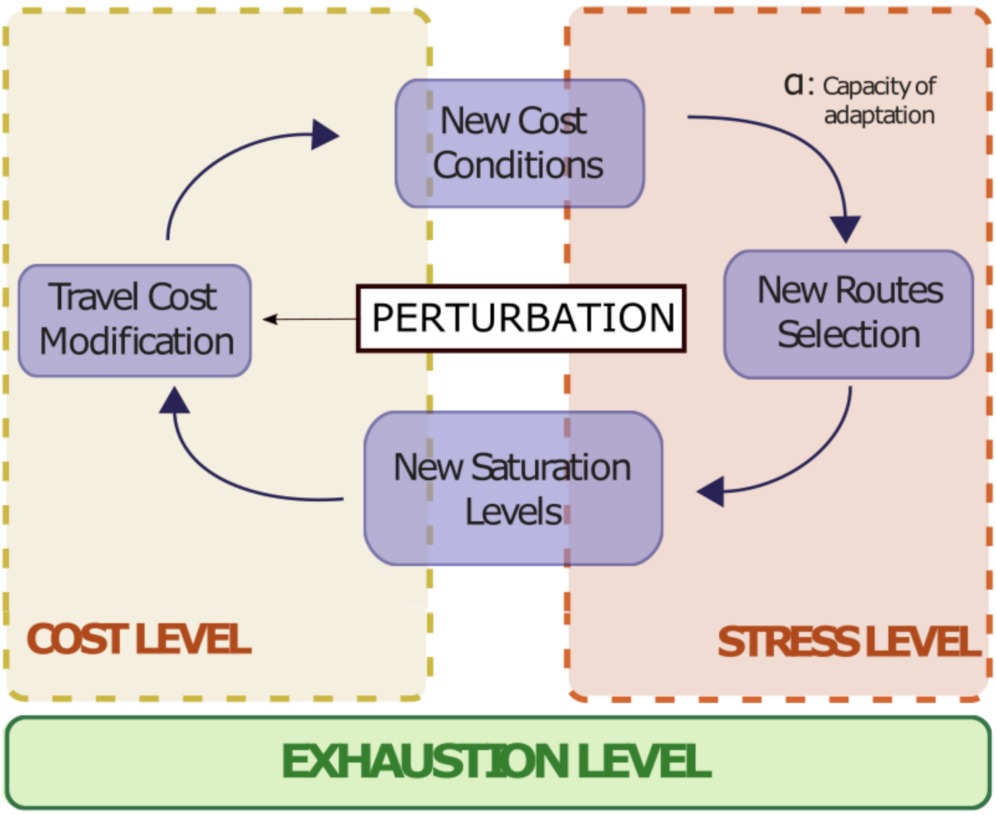

Figure 4.10: Description of the different levels evaluated in the network when a perturbation occurs: Cost level, Stress level, and Exhaustion level.

4.6.1 Cost level

In this section, a formulation for the measurement of a normalized cost level of the traffic network is presented. The actual total cost in the network can be computed with the data obtained from the dynamic equilibrium-restricted assignment model. In this case, the measurement of the links travel cost has been done with the BPR function. However, any other function to evaluate the travel times can be used instead, and more variables to represent the travel cost can be introduced together with the travel time.

The formulation to evaluate the cost level is presented as follows:

$$C_T(t) = \sum_{a \in A} c_a(t), \tag{4.29}$$

where $C_T(t)$ is the actual total cost for a time t of the traffic network.

Consequently, a normalized **cost level,** $\tau_\kappa(t)$, associated with a state of perturbation. k, is evaluated as follows:

$$\tau_\kappa(t) = \frac{C_T(t) - C_0}{C_{th} - C_0}, \tag{4.30}$$

where C_0 is the initial total cost ($t = 0$), and C_{th} is a cost threshold.

The cost threshold, C_{th}, is determined by the system breakdown point, and it is defined as the limit-state associated with the failure of the traffic network due to extreme overcost generated by a strong perturbation. This threshold is necessary because some perturbations can introduce an impact in the network which is enough strong to produce the break down of the system, and even though the network could theoretically recover (sometimes this recovery may imply external recovery actions), it would involve an unacceptable effort by the system.

The range of τ_κ is normally between $[0, 1]$, however it is noted that τ_κ is upper unbounded. This fact allows the consideration of brief rush situations when a perturbation occurs.

Finally, in Figure 4.11 (b), the cost level evolution associated with the illustrative example previously described in Section 4.5.2 is shown. It is noted that in the graph, when the perturbation starts (e.g. maintenance works that reduce the capacity of some links of the network), the cost level suffers a large increment.

Even though the disruption is constant during the perturbation period (see Figure 4.3), the cost level reduces its value with time. This fact is because, at the first time, the users can only partially adapt to the perturbation (in this case $\alpha = 0.1$), therefore, not all the users are aware, and not all of them can modify their routes in the first time interval. As the time goes, since the perturbation is kept constant, more users become aware at every time interval, and then, more users can choose better routes, reducing the total cost level.

4.6.2 Stress Level

For the evaluation of the stress level, two variables previously presented in the dynamic-equilibrium restricted assignment model are indispensable, ρ_r, and α. These variables will determine the response of the users when a perturbation occurs.

In the previous section, variable ρ_r was presented in a mathematical context. Nevertheless, it is worthwhile to pay attention to this informative variable because it allows the

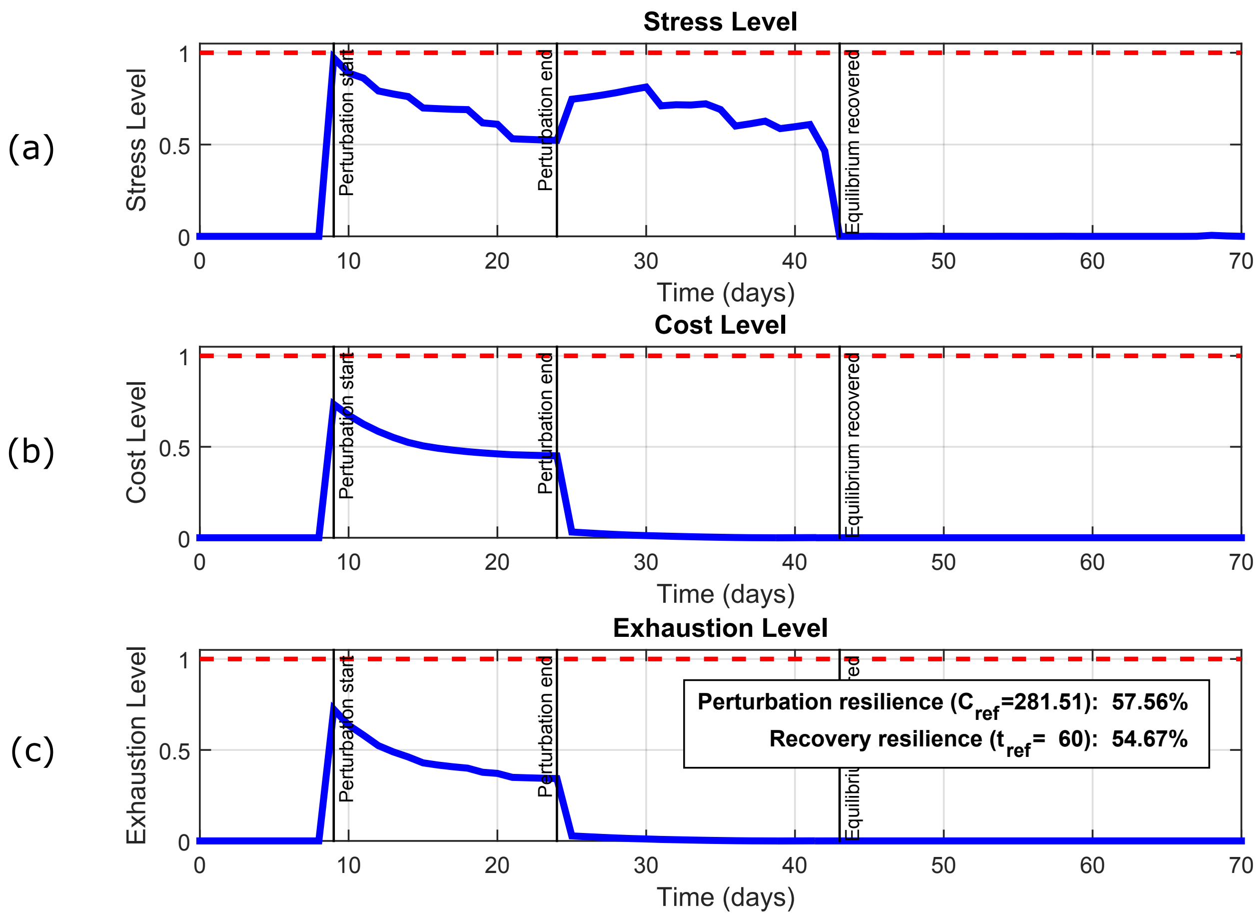

Figure 4.11: Resilience analysis for the first part of the illustrative example, perturbation of 15 days ($\alpha = 0.1$).

identification of the stress level of the system. Hereunder, both variables are explained.

Variable ρ_r, which is limited by the system impedance α, measures the net flow variation among routes within the same OD pair. The variable ρ_r is computed as the variation of route flow in two consecutive intervals of time, in this case, since the time interval is one day, ρ_r will determine the net flow variation between one day and the previous one. Therefore, the possible values of ρ_r are explained as follows:

- $\rho_r = 1$ implies that route r does not modify its state, i.e, that route has reached an equilibrium position compatible with the existing conditions. This means that it would not be a modification on the flow of the route in that time interval.

- $|\rho_r - 1| = \alpha$ implies that the associated route r is changing its state as much as its capacity of adaptation allows. Then, route, r is gaining, ($\rho_r - 1 = \alpha$), or losing, ($-\rho_r + 1 = \alpha$), the maximum flow allowed for the value of α. When the maximum values are reached, that route presents the maximum stress state.

- In the interval from $-\rho_r+1 = \alpha$ to $\rho_r < 1$, the route is trying to reach the equilibrium state by losing net flow.

- In the interval from $\rho_r > 1$ to $\rho_r - 1 = \alpha$, the route is trying to reach the equilibrium state by gaining net flow.

Variable $\boldsymbol{\alpha}$ is the system impedance value. The value of α can be calibrated for a real network by means of surveys of the network users. These surveys will identify the dissuasive effect of delays on the route choices of the users. Therefore, networks with high values of α will be those networks where the users change their routes easily, when the travel times of their usual routes suffers small increases, the users of these networks will try to find others. However, lower values of α will be applied to those networks where the users are resistant to change their usual routes, and larger increments of the travel times are needed for changing the usual routes of the users.

To illustrate these concepts, in Figure 4.12, the ρ_r evolution bounded by α for the illustrative example is shown for some links of the network. Thus, the maximum value of ρ_r in this case is 1.1, and the minimum is 0.9. This means that the capacity of adaptation of this network is quite small, and even with an increment of the costs, not many users will be able to change or will decide to change their routes. This fact can be due to several reasons, for example, (a) the users only have partial information of the situation, and because of an unknown situation they prefer to continue using their usual routes, or (b) some users may prefer to assume a small increment in their usual routes, than explore new, and unknown routes.

Analysing Figure 4.12, route 11, 13, and 22 reach maximum levels in the values of ρ_r during some periods of the time. Therefore, these routes are highly stressed during the perturbation, and all the users of the routes that are able to change their routes, will change them. On the other hand, routes 12, 15, and 26, are also affected by the perturbation but their values of ρ_r are smaller. Therefore, for these routes, only some of the users able to change their routes, will decide to do so.

For the evaluation of the total stress level of the network, it is assumed that a traffic network is no longer serviceable when at least one OD pair reaches its maximum adaptation capacity. Then, the stress level of a traffic network is defined by the most vulnerable OD pair.

Consequently, for a given state of perturbation κ, a **normalized stress level,** σ_κ

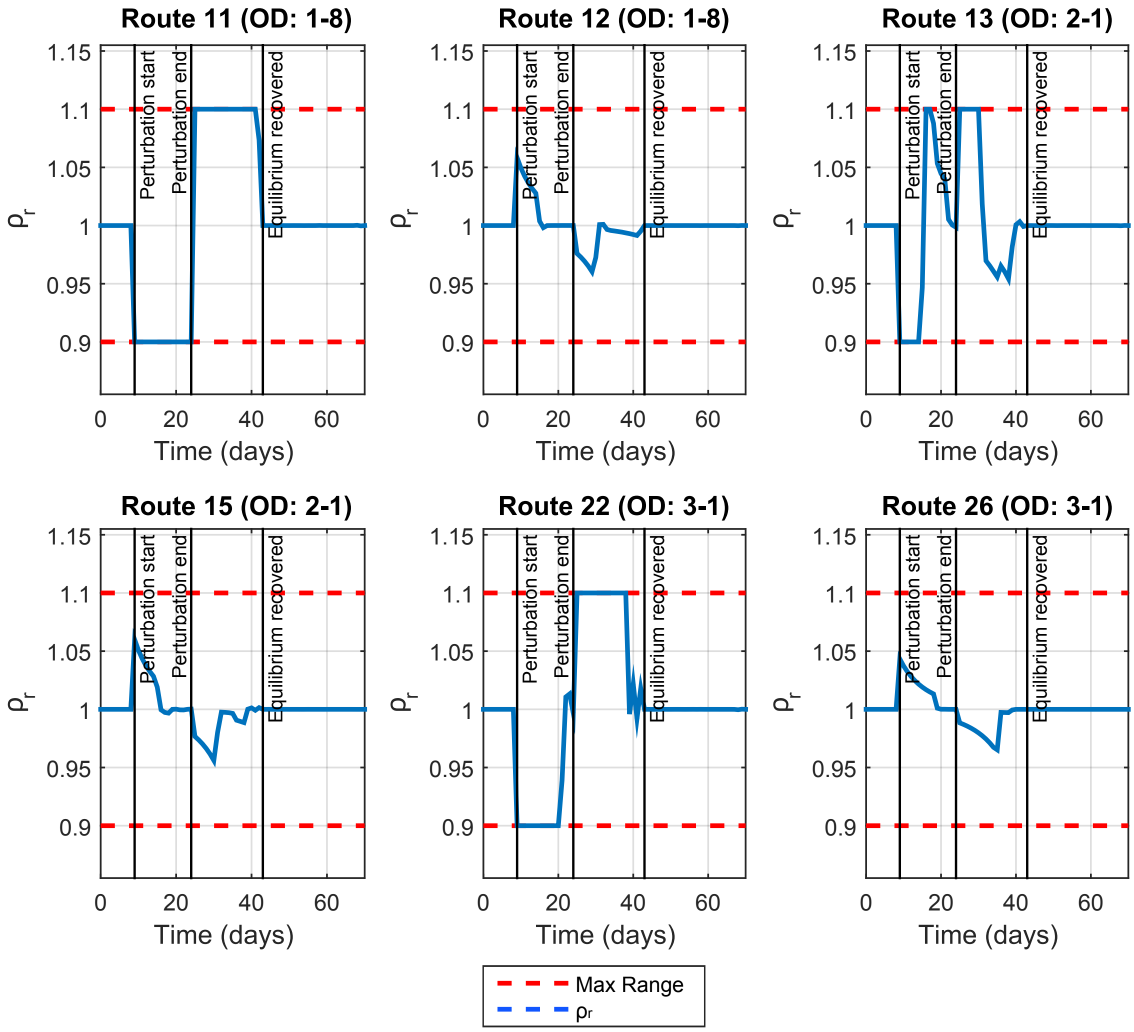

Figure 4.12: Evolution of ρ_r with time for some selected routes ($\alpha = 0.1$).

associated with the whole traffic network, is established as follows:

$$\sigma_\kappa(t) = \max_{pq \in D} \left(\frac{1}{\alpha} \frac{\sum_{r \in R_{pq}} |\rho_r(t) - 1|}{n_{pq}} \right), \tag{4.31}$$

where n_{pq} is the number of routes with OD pair pq. It is noted that σ_κ is taking into account the worst states in each time. The range of $\sigma_\kappa(t)$ is $[0, 1]$, where zero means that the system has achieved an equilibrium state and 1 implies that the adaptation degree is exhausted.

In Figure 4.11 (a), the stress level evolution associated with the illustrative example is shown. In the same way when analysing the cost level, at the first moment that the perturbation impact the network, the stress level suffers a large increment. In this case, the increment of the stress level is even larger than the one obtained for the cost level. This fact might be due to the small α ($\alpha = 0.1$), which creates a scenario where a small amount of the network users are able to change their routes for each period of time.

Similar to the cost level evolution, as the times goes, the stress level reduces its value. This happens for similar reasons as in the cost level. In the example the perturbation remains constant (see Figure 4.3), and during the impact (a) more users become aware of the situation, and decide to modify their routes, and (b) since ρ_r is limited by α, and only can take values from 0.9 to 1.1, therefore, only up to 10% of the users will modify their situation in one time interval. If the link flow associated with an equilibrium situation is smaller than the actual flow after the adaptation to the new routes, a longer period of time is needed to reach the equilibrium.

Unlike the case of the cost level that when the perturbation is finished can recover in a short period of time, the stress level keeps higher values even when the perturbation is finished. For example, at the exact time that the perturbation is finished (see Figure 4.11), the stress level suffers a significant increment. This fact is because the users were getting used to the disrupted situation (constant perturbation over the time), and in the moment that the perturbation finishes the users are exposed to a new scenario, and they will need to modify their routes again trying to adapt the new conditions (in this case better conditions, without the perturbation).

4.6.3 Exhaustion Level

Finally, both levels are combined for the evaluation of the exhaustion level of the network. This level identifies how far the network is from the system breakdown, i.e. the total exhaustion of the system. This level will be the one used for the evaluation of resilience in the following sections.

It is highlighted that when evaluating resilience, a combination of both levels is indispensable. When a perturbation occurs, the users of the network will try to minimize their travel times at any time. As a result, it can happen that after the perturbation the travel times remain in lower values due to the election of different routes on the part of the users. However, these changes need to be incorporated in the value of resilience, since they are causing disruption to the users. Thus, when the travel costs keep low values in the network, the necessity of changing routes will be reflected by an increment in the stress level of the network.

In addition, it is noted that when the redundancy of the network is low, and the users do not have, or have less options to change their usual route when affected by a disruption, the values of the stress level of the network will continue in low values. However, this effect will be reflected in a larger increment of the cost level of the network, and therefore this phenomena can be reflected when the resilience is evaluated.

With the aim of covering these effects in a single level, **the exhaustion level,** ψ_κ, associated with a given state of perturbation, κ, is introduced. Thus, this level includes the cost and stress level, and it is formulated as follows:

$$\psi_\kappa(t) = \frac{1}{2}\left(1+\sigma_\kappa(t)\right)\left(\tau_\kappa(t)\right)^b, \quad b \geq 1, \tag{4.32}$$

where b is a coefficient to penalize the cost level when it is larger than the cost threshold. Considering the possible ranges of the variables σ_κ and τ_κ, in normal situations ψ_κ is bounded between $[0, 1]$. Due to possible rush points during the impact of the perturbation in the cost level function, ψ_κ can take values larger than 1. In these cases, the system is assumed to break down.

This formulation of the equation ensures that in the case previously explained, the lower stress level do not result in low or null exhaustion levels.

In Figure 4.11 (c), the exhaustion level evolution associated with the illustrative example is shown. It is noted that when the perturbation finishes, the exhaustion level recovers almost totally; however, it is not until the stress level can reach an equilibrium that it is considered that the recovery process has finished.

4.7 Assessment of resilience

In Chapter 1, several conclusions were drawn about the concept of resilience, and multiple methodologies to measure it in the area of transport networks were analysed. One of the main conclusions that was found is the necessity of defining two stages when resilience is evaluated. These stages were defined as the perturbation stage, when the perturbation is directly affecting the network, and the recovery stage, when the perturbation has finished, but the network still needs to recover. This last stage will last until the network reaches a new equilibrium state. See Figure 4.13 for a graphical description of the two stages.

Therefore, the methodology proposed in this chapter defines two types of resilience, namely, perturbation resilience, and recovery resilience. The necessity of separating these two concepts is highlighted, since they need to quantify resilience from different approaches. Thus, perturbation resilience focuses more on how the network withstands the perturbation, this means if the network is able to keep working while the perturbation lasts. Aspects such as robustness, and strength, are the ones more relevant in this stage.

On the other hand, the recovery resilience needs to include how the network recovers after the perturbation is finished, and if the network is able to recover a normal operation after the perturbation. In this case, aspects, such as the ability to recover and the rapidity, are the ones specifically considered in the formulation.

Consequently, the formulations to measure these two types of resilience are presented as follows.

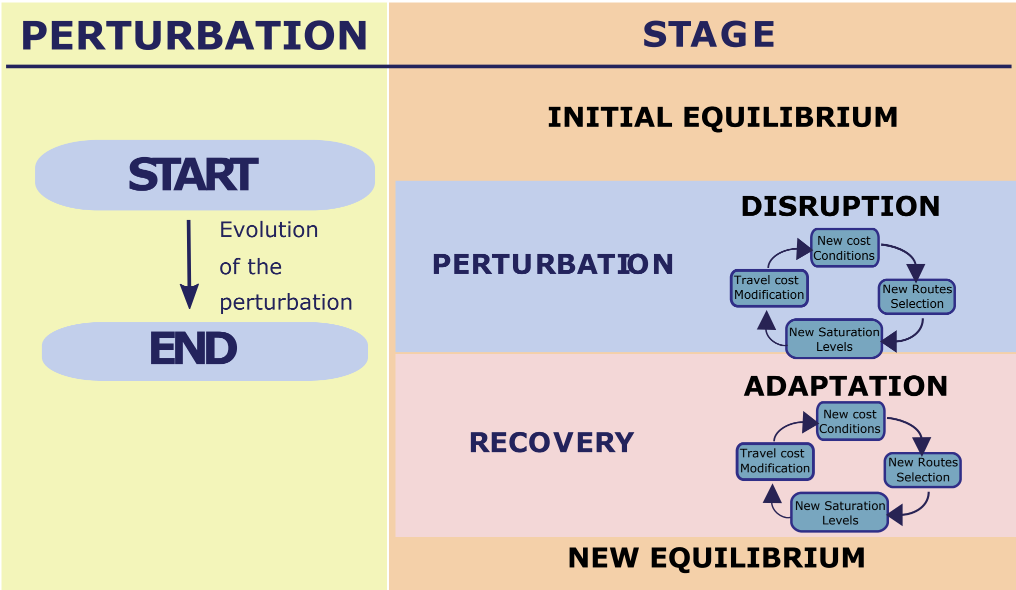

Figure 4.13: Diagram to define the perturbation, and the recovery stage of the definition of resilience.

4.7.1 Perturbation resilience

When analysing the perturbation stage, the concept of resilience aims to evaluate:

" the ability to absorb, resist or accommodate a perturbation".

Perturbation resilience rather than measure the damage, defines the capacity of the network to withstand a disrupted scenario. Consequently, the level that identifies how far the network is from the total exhaustion is the exhaustion level.

For that reason, the perturbation resilience can be evaluated as the normalized area over the exhaustion curve, as indicated as follows:

$$\chi_\kappa^p = \frac{\int_{t_{p0}}^{t_{p1}} (1 - \psi_\kappa(t))\, dt}{t_{p1} - t_{p0}}\, 100, \tag{4.33}$$

where t_{p0} and t_{p1} denote the initial and the final time of the disruptive event, respectively. Since $0 \leq \psi_k(t) \leq 1$, the perturbation resilience is defined between $[0, 100]$.

Analysing the perturbation resilience for the illustrative example, the value obtained is 57.56% (see Figure 4.11).

4.7.2 Recovery resilience

When analysing the recovery stage, the concept of resilience aims to evaluate:

" the ability to restore, return or recover from the perturbation"

Therefore, when evaluating the recovery resilience, the measurement of the area over the exhaustion level is not enough to include all the necessary aspects. For example, in the case that the exhaustion level remains in lower values but it takes a considerable time to reach a new equilibrium, the measurement of the area will give high values of the recovery resilience. However, the longer the time the lower should be the recovery resilience, and only with the evaluation of the area over the curve this aspect is not covered.

To include this aspect, a normalized slope associated with the exhaustion level curve between the perturbation end time, and the equilibrium recovery time, is considered. The value of this slope can be calculated as follows:

$$\theta_\kappa = \frac{2}{\pi} \arctan\left(\frac{\psi_\kappa(t_{p1})\, T_{th}}{t_r - t_{p1}}\right). \tag{4.34}$$

With the aim of normalizing, a reference value associated with the recovery time, T_{th}, is established. The range of θ_κ is $[0, 1]$.

In Figure 4.14, where the exhausted function in the recovery stage is represented, this angle is shown.

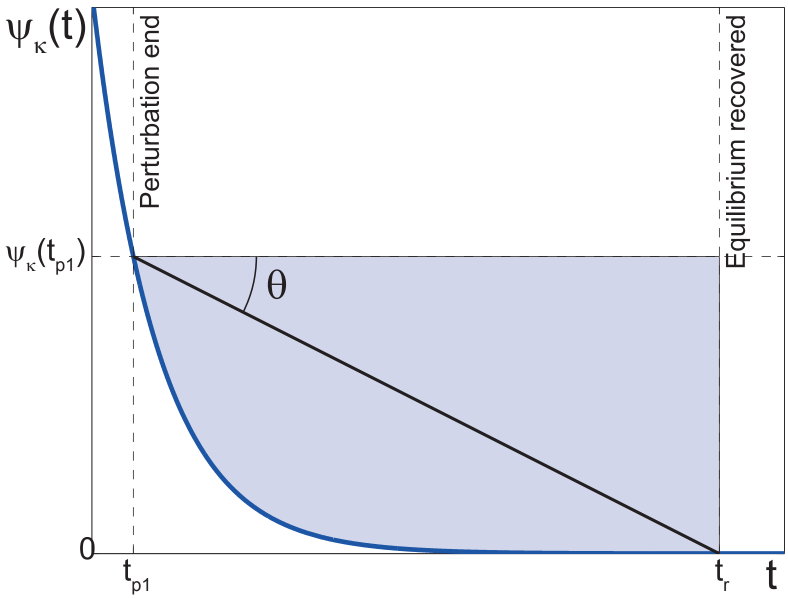

Figure 4.14: Exhausted function in the recovery stage.

In addition, when defining the recovery resilience, it is important to consider the system pattern during the perturbation state until the new equilibrium state is reached. In order to take these aspects into consideration, the following recovery resilience formulation is proposed:

$$\chi_{\kappa}^{r} = \theta_{\kappa} \frac{\int_{t_{p1}}^{t_r} \left(\psi_{\kappa}(t_{p1}) - \psi_{\kappa}(t)\right) dt}{\psi_{\kappa}(t_{p1})(t_r - t_{p1})} 100, \tag{4.35}$$

where the integral over the curve includes the effect of the different curvatures of the perturbation function, and the instant when complete equilibrium is reached is denoted by t_r.

It is noted that χ_{κ}^{r} takes values between $[0, 100]$. The lower bound implies that the system is enabled to reach an equilibrium state, i.e., the traffic network has a null recovery resilience. The upper bound implies that system recovers its equilibrium immediately.

Analysing the recovery resilience for the illustrative example, the value obtained is 54.67% (see Figure 4.11).

4.8 Comparison with previous methodologies

In Chapter 1, an analysis of different methodologies to assess resilience in transport networks has been developed. It was explained that some authors analyse this concept by the evaluation of multiple parameters, such as redundancy, rapidity, or robustness.

Therefore, in this section, it is explained how the proposed methodology to evaluate resilience in traffic network includes these concepts, without the necessity of a particular evaluation of each of them.

Below, each of the concepts is defined, and an explanation is provided as to how the presented methodology to evaluate resilience in traffic networks incorporates that concepts.

- **Robustness**

  The strength of system, or its ability to prevent damage propagation through the system in the presence of disruptive event. Bruneau et al. (2003)

  It is included when determining the exhaustion level of the network. The proposed exhaustion level of the new model includes the robustness of the system to withstand a perturbation. Therefore, a robust network against a specific perturbation will show a lower exhaustion level than if the network is less robust, in which the exhaustion level will be larger for the same perturbation. Finally, the exhaustion level will determine the perturbation resilience, therefore the robustness of the network is reflected in the final value of the perturbation resilience.

- **Rapidity**

  The speed or rate at which a system could return to its original state or at least an acceptable level of functionality after the occurrence of disruption. Bruneau et al. (2003)

It is included since the proposed model provides an evolution of the situation of the network when affected by a perturbation. Thus, it is easy to evaluate the time that is needed for the recovery of the network. In addition, when the recovery resilience is calculated, the time needed for the total recovery of the network plays an important role in its evaluation. Networks that can recover fast from the perturbation will obtain larger values in the recovery resilience.

- **Resourcefulness**

  The level of capability in applying material (i.e., information, technological, physical) and human resources (i.e., labor) to respond to a disruptive event. Bruneau et al. (2003)

  This concept is more focused on the external actions, and how the system responds to them when a perturbation occurs. The proposed model allows the simulation of improvements in the network, such as increase of the capacity, or increment on the information that the users have about the situation of the network. Therefore, different actions to reduce the impact of the perturbation can be tested, and their effectiveness can be objectively evaluated.

  For example, using the case presented for the illustrative example, the resilience of the network is 57.56% for the perturbation resilience, and 54.67% for the recovery resilience. Trying to increase these values, some actions can be performed. The maintenance works were supposed to last 15 days, and create a reduction of 50% of the capacity in the affected links. Trying to reduce the impact of these works, the managers of the network might try to restore the capacity as soon as possible, and one of the proposed options is to remain with a 50% reduction during the first 5 days, and in the next 10 days the affected link can be working at 75% of their capacity (instead of the previous 50%). Thus, the evaluation of resilience for this new maintenance works is shown in Figure 4.15.

  Therefore, if this increment of the link capacities can be achieved in the last 10 days, the values of resilience increase up to 74.65% for the perturbation resilience, and 46.21% for the recovery resilience.

  It is noted that the recovery resilience decreases. This is because, even though the impact created in the network is smaller, and the exhaustion level when the perturbation is finished is very small, the network needs quite a long time to completely recover from that small impact. Then, the network is a bit less efficient for the recovery process.

- **Redundancy**

  The extent to which carries by a system to minimize the likelihood and impact of disruption. Bruneau et al. (2003)

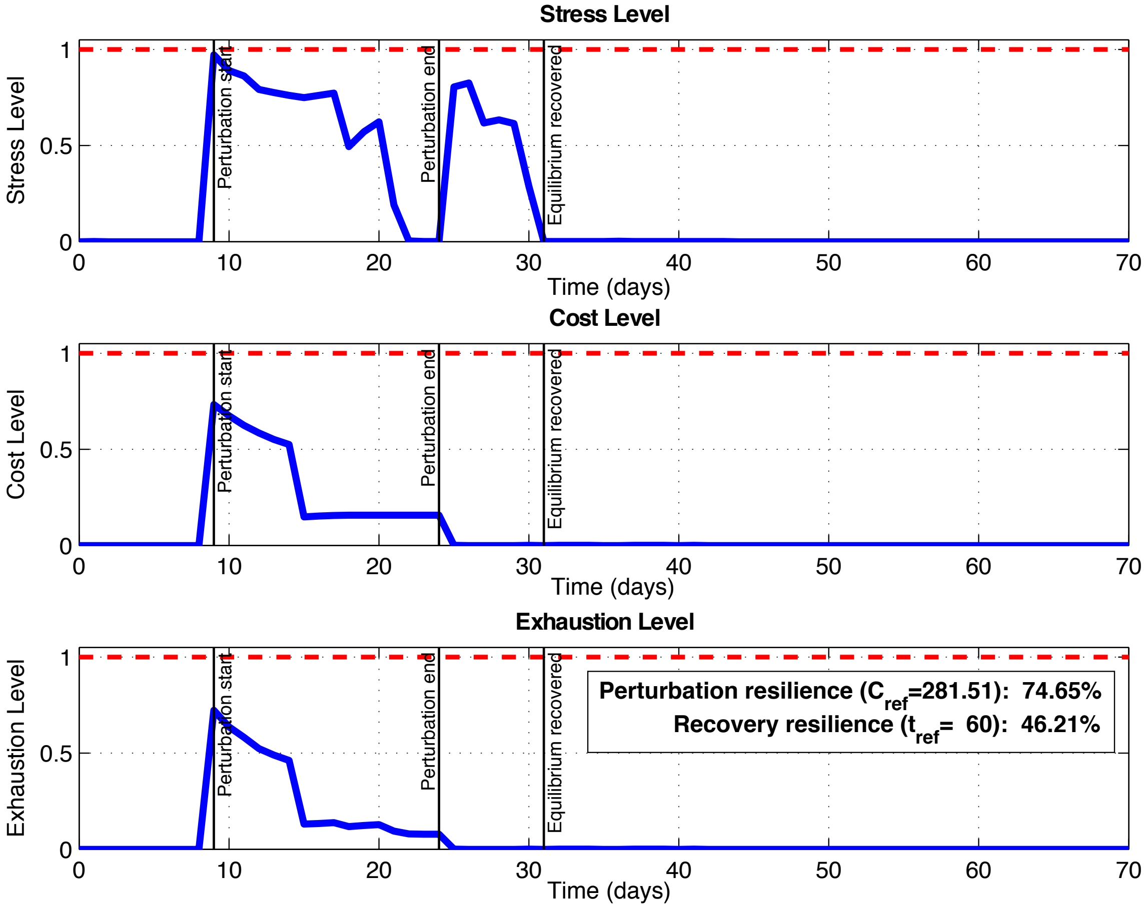

Figure 4.15: Evaluation of the resilience in the case of the illustrative example, when additional actions to reduce the impact of the perturbation are considered.

Indicates that multiple components serve the same function. Murray-Tuite (2006)

By using a traffic assignment model, the redundancy of the network is automatically included in the process. It is recalled that one of the inputs of the model is the number of routes per OD; the larger the redundancy, the greater the number of possible routes. Thus, multiple route options among OD pairs will help to increase the resilience.

- **Diversity**

  Means that the components are functionality different. Murray-Tuite (2006)

  When analysing traffic networks, each of the elements can be characterized independently, including aspects such as capacity, and maximum speed, to include the "diversity" into the evaluation of resilience. In addition, in the next chapter, a new parameter is introduced, the local vulnerability. This parameter determines how each of the links of the network responds when affected by a specific hazard. See Chapter 5, Section 5.5 for a complete explanation of this parameter.

- **Efficiency**

  Indicates input-output ratio optimization. Murray-Tuite (2006)

  This concept is briefly described by the author, and its meaning can be understood under multiple perspectives. For example, when analysing the resilience of a network, by this methodology, the efficiency of different solutions can be tested, see Section 4.9, where multiple solutions are applied for the same case, and the best performance can be chosen.

- **Autonomous components**

  Components have the ability to operate independently. Murray-Tuite (2006)

  This concept when analysing a road traffic network is out of the scope of the current work, since in a traffic network, elements operate in a system where the elements depend on each other. Therefore, this can be included for example when analysing the resilience of a system where multiple transport modes operate at the same time.

- **Strength**

  Indicates the system's ability to withstand an event. Murray-Tuite (2006)

  This concept proposed by Murray-Tuite (2006) is similar to the one previously explained proposed by Bruneau et al. (2003), i.e. the robustness. Therefore, the new model includes the strength of the system when the perturbation occurs.

- **Adaptability**

  Implies that the system is flexible and elements are capable of learning from past experience. Murray-Tuite (2006)

  This concept implies a continuous analysis of a network during several perturbations. When similar perturbations affect the same network, it can be considered that the users will have a better response to this kind of impact, and their adaptability for that specific type of perturbations will improve.

  The proposed model allows the introduction of an improvement of the adaptability through the parameter α. A better adaptability can be reflected with a higher value of this parameter, meaning that the users will have a better capacity of adaptation to the impact, and therefore, they will be able to modify their routes faster. Consequently the network can obtain a better resilience for the same perturbation, since the users would have learnt from the previous experiences.

For example, using the case of the illustrative example, it can be simulated that the users have previously experienced that maintenance works. Therefore they will have a better capacity of adaptation if it happens again. To evaluate this scenario, instead of $\alpha = 0.1$, the same problem is simulated with $\alpha = 0.5$, and the results are shown in Figure 4.16.

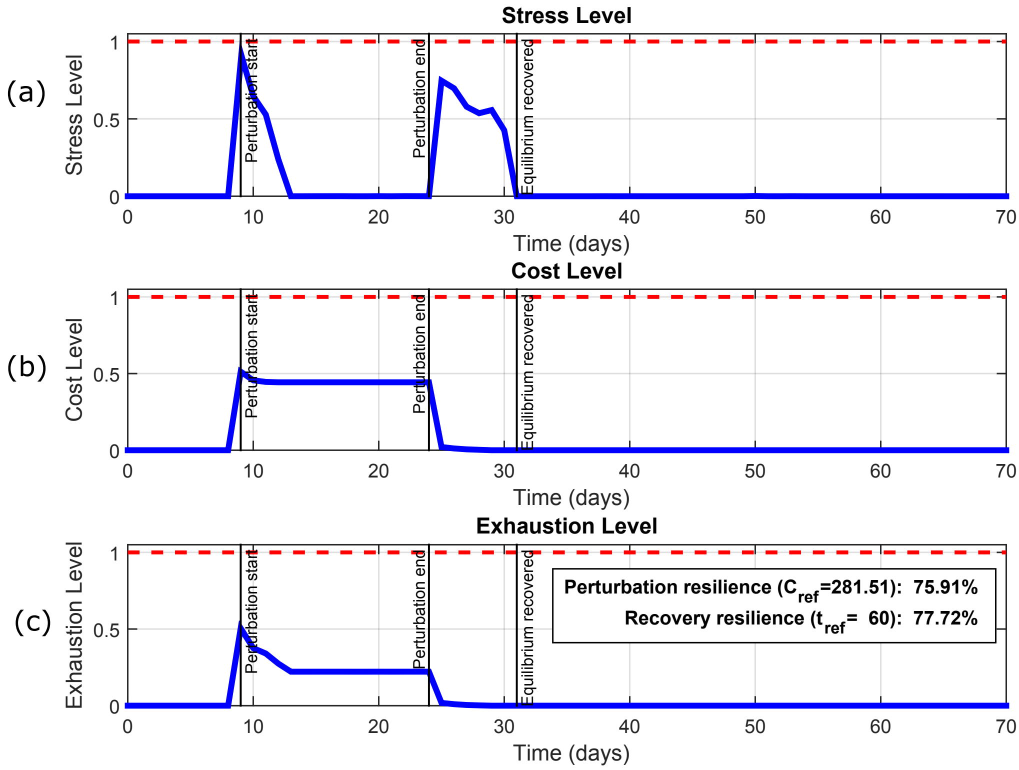

Figure 4.16: Evaluation of the resilience in the case of the illustrative example, when $alpha = 0.5$.

It is highlighted from Figure 4.16, that the users have a significantly better performance. Perturbation resilience goes from 57.56% in the previous example to 75.91%, and recovery resilience goes from 54.67% to 77.72%.

- **Collaboration**

  Indicates that information and resources are shared among components or stakeholders. Murray-Tuite (2006)

  This concept is out of the scope included in this model that analyses traffic networks. Its inclusion would necessitate a wider scope, including the different agents involved in a whole transport network, such as, owners, managers, and different authorities, to evaluate the importance of their collaboration when dealing with disrupted scenarios. However, in the scope analysed in this model, including only the users of the traffic network, the collaboration can be incorporated too, since the parameter of the capacity of adaptation is included. By using this parameter, one can evaluate how the information that the users have about the real conditions of the network affect their behaviour. Thus, the users of a network with perfect information of the new scenario will be more willing to change their routes for better ones. On the other hand, the users of those networks that cannot obtain real-time information, may prefer to use their usual routes if the conditions are not clear enough to change.

- **Mobility**

  Indicates that the travellers are able to reach their chosen destinations at an acceptable level of service. Murray-Tuite (2006)

  It is included in the proposed model by the use of a traffic assignment model, and by the evaluation of the evolution of the travel costs of the network. In addition, this model includes a cost threshold, which is able to determine the maximum cost level that is allowed for in the studied network. Therefore, if a very high mobility wants to be included, the cost threshold can be reduced. Then, any perturbation that goes over that limit means that the level of mobility that is necessary on this case will be reduced. On the other hand, if the impact of the perturbation does not go over the limit established, it means that the users, even with the effect of the perturbation, will maintain the level of service, or mobility required.

- **Safety**

  Suggests that the system does not harm its users or unduly expose them to hazards. Murray-Tuite (2006)

  This concept is introduced when measuring resilience in a wider level than the one applied here. Thus, when analysing traffic networks it is assumed that the network by itself does not expose or harm the users more than the perturbation does.

- **Ability to recover quickly**

  Means that an acceptable level of service can be restored rapidly and with minimal outside assistance after an event occurs. Murray-Tuite (2006)

  This concept is similar to the one previously proposed by Bruneau et al. (2003), i.e. rapidity. In addition, Murray-Tuite (2006), includes in the definition " with the minimal assistance after an event occurs", this methodology is able to evaluate how the traffic network recovers from a perturbation without external actions, i.e. by itself, and also simulating external actions that may improve the characteristics of the network before, during, and after the perturbation.

Table 4.4: Parameters used in the Cuenca example

BPR parameters			
$\tau_{0a}(t)$	$\beta_a(t)$	γ	$x_a^{max}(t_0)$
40 km/h	1	3	100 users/km
Perturbation characteristics			
Perturbation type	Residual Capacity & Speed reduction	Affected Links	
Constant	50 users/km & 30 Km/h	See highlighted links in Fig. 4.17	
Performance parameters			
α	Study period	Time interval	
0.10	90 days	One day	
Resilience analysis parameters			
b	C_{th}	T_{th}	
1	0.58 hours	30 days	

4.9 Case study: Cuenca network

In this section the applicability of the proposed method on a real network is presented. The Cuenca network (Spain) has been considered, which is shown in Figure 4.17. This network, which has been previously used by Castillo et al. (2012, 2013), consists of 232 nodes, 672 links and 207 routes, and a total demand of 12420 users. The parameters assumed to carry out the resilience analysis are summarized in Table 4.4.

In the central part of the city, some works to upgrade the exterior lane need to be performed. The works will affect the 22 central links (see Figure 4.17). It is expected that the capacity of the affected links is reduced by 50% during a period of 30 days. In addition, in the links affected, the velocity needs to be reduced for safety reasons, from 40 Km/h to 30 Km/h. The works are expected to start the 9^{th} day, and to be concluded the 39^{th} day. After the disrupted period, it is expected to recover the total capacity of the affected links.

To evaluate the effect that this perturbation will cause in the network, the resilience is evaluated, and the methodology presented previously applied.

As a result, Figure 4.18 shows the stress level, the cost level and the exhaustion level for the proposed perturbation. The values obtained for the resilience are, 73.53% for the perturbation resilience, and 16.97% for the recovery resilience. It is noted that to reach a new equilibrium state, the network needs more than a month after the perturbation.

Trying to reduce the impact of this disruption, the authorities ask for some strategies to minimize the effects of the works. Therefore, some possible solutions are tested hereunder to evaluate if they create better scenarios.

The fist solution to be tested is to perform the works in two stages, thus, a first group of the affected links will be upgraded during the first 30 days, from the 9^{th} day to the 39^{th}

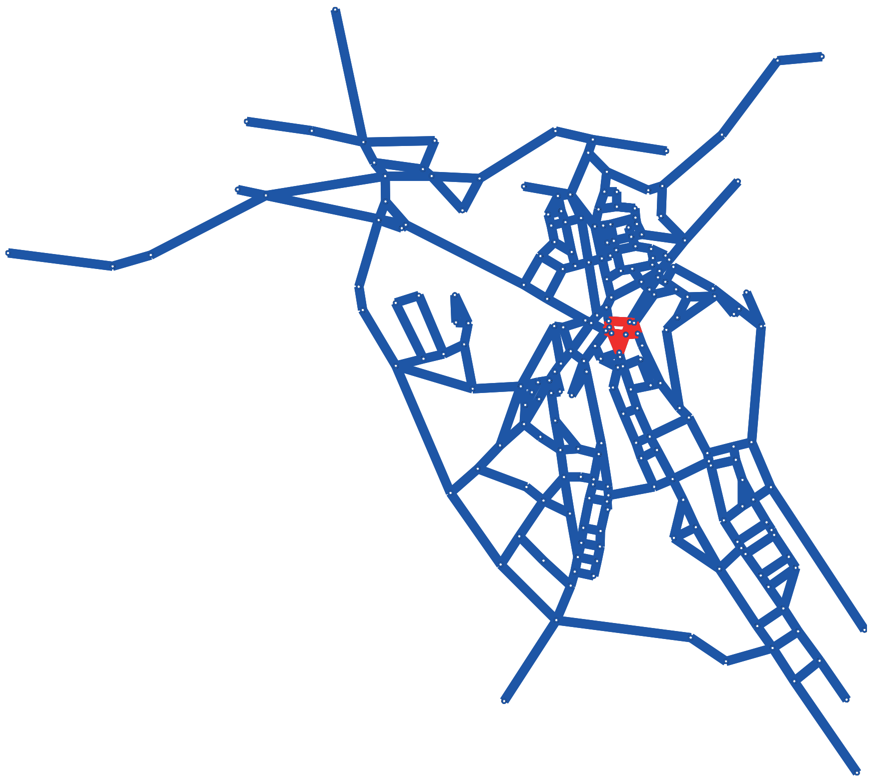

Figure 4.17: Cuenca network (Spain). The affected links are highlight in red colour.

day, and a second group of the links will be disrupted in the second 30 days, from the 40^{th} day to the 69^{th} day. The rest of the parameters remain the same as in the previous case. This solution aims to create a lower impact, even though the works need to be done in two periods.

The results for this solution are shown in Figure 4.19. It is noted that the perturbation resilience has incremented up to 86.66%, since the network can absorb easily the smaller impact. However, analysing the stress level, the levels are very similar, and during the recovery process they remain significantly high. For that reason, the recovery resilience is lower than the previous case, with a value of 5.65%. This fact is because in this case, when the perturbation is finished, the exhaustion level is smaller than in the previous case, i.e. the network has to recover from smaller damage, but the time needed is again more than a month.

The second solution to be evaluated is an improvement of the information that the users of the network have about the new situation. It is highlighted than the previous cases were conducted with $\alpha = 0.1$, meaning that the capacity of adaptation of the users

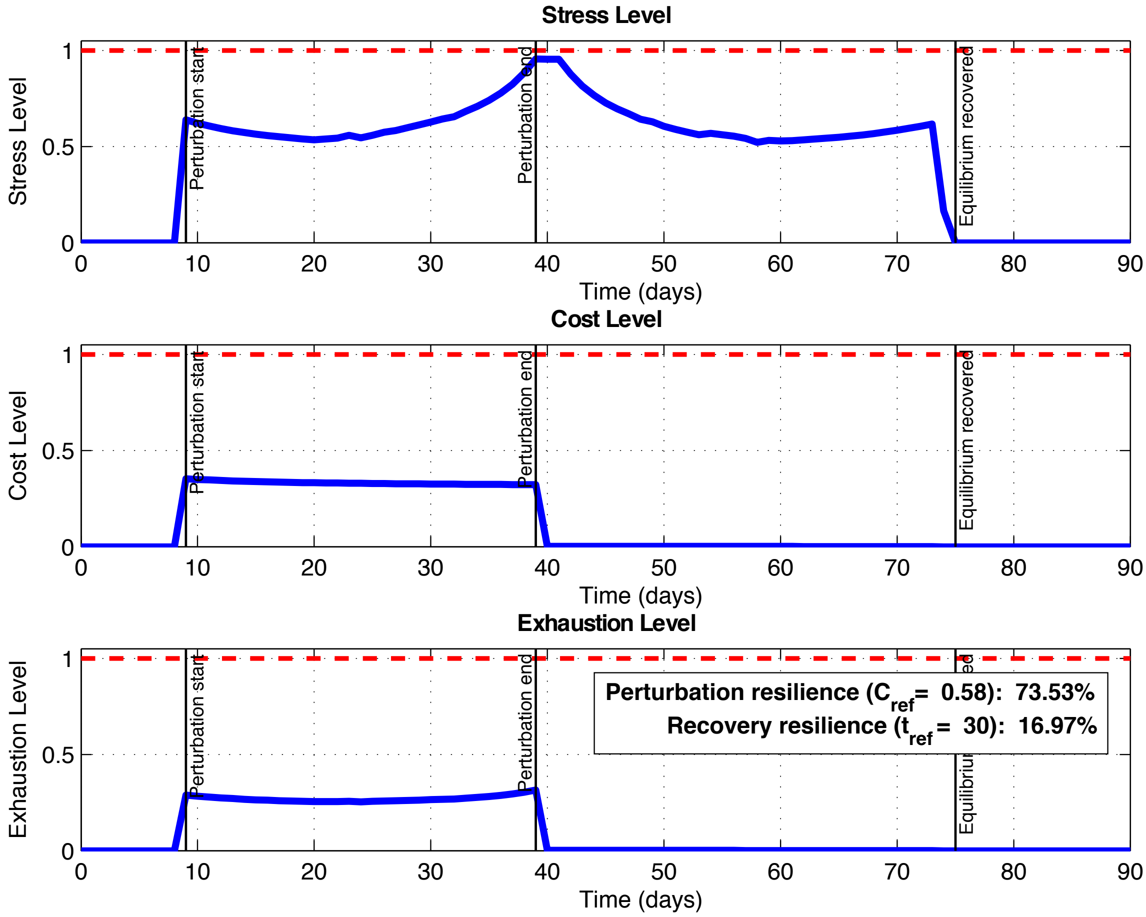

Figure 4.18: Resilience analysis of the Cuenca network.

is very small, and when the perturbation occurs most of them may not have the option of choosing a better route. However, if the authorities make an extra effort to disseminate more information about the works, the users will be able to select better routes, when the perturbation occurs, and they will be more willing to change. Therefore, the first case is now tested with $\alpha = 0.5$.

The results of this option are shown in Figure 4.20. In this case, a reasonable increment is achieved for both stages of resilience, obtaining 80.79% for the perturbation resilience, and 52.89% for the recovery resilience. It is highlighted that in this case, the recovery period needs less than 10 days, this fact is because when the network has a larger capacity of adaptation to the new situation, they can improve their travel times, and their stress level is smaller.

A summary of the results obtained for the cases developed in this section for the Cuenca network is presented in Table 4.5.

It is highlighted that the resilience analysis, which has been implemented in Matlab

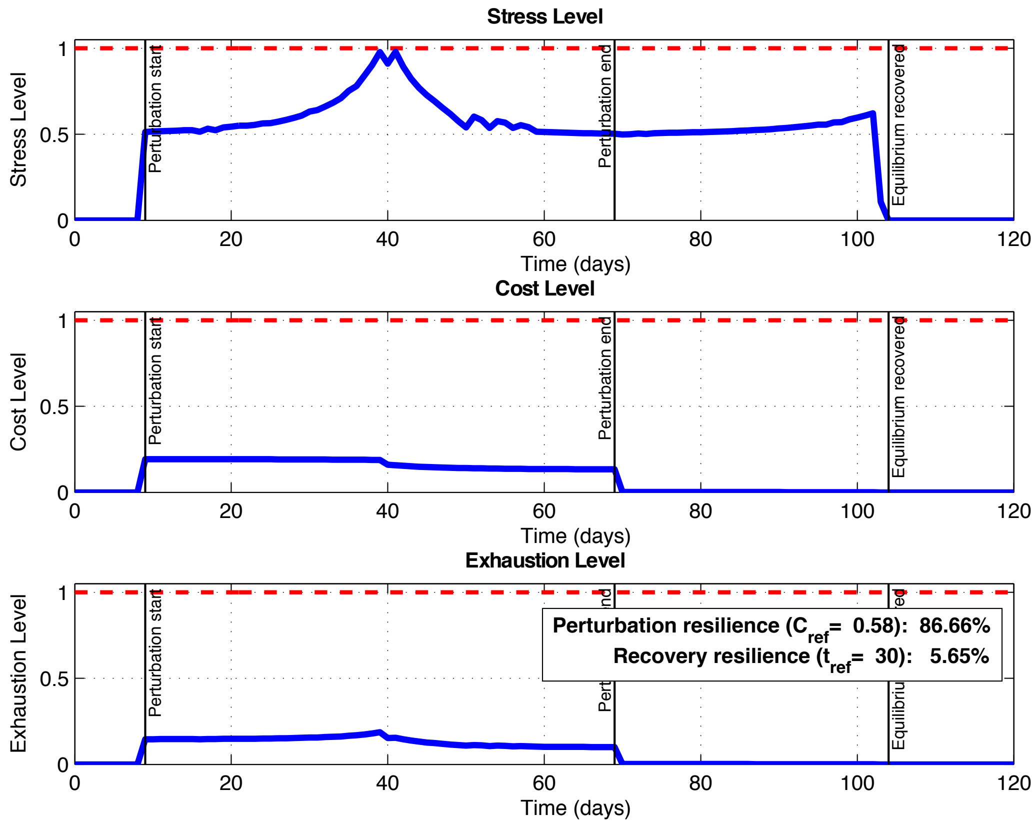

Figure 4.19: Resilience analysis of the Cuenca network, making the works in different periods.

Table 4.5: Summary of the results obtained for the general case, and the two strategies developed for the Cuenca example

Case	α	Duration (days): perturbation	**Perturbation resilience**	Duration (days): recovery	**Recovery resilience**
General case	0.1	30	73.53	36	16.97
1^{st} **strategy:** 2 stage impact	0.1	60	86.66	35	5.65
2^{nd} **strategy:** increment of α	0.5	30	80.79	6	52.89

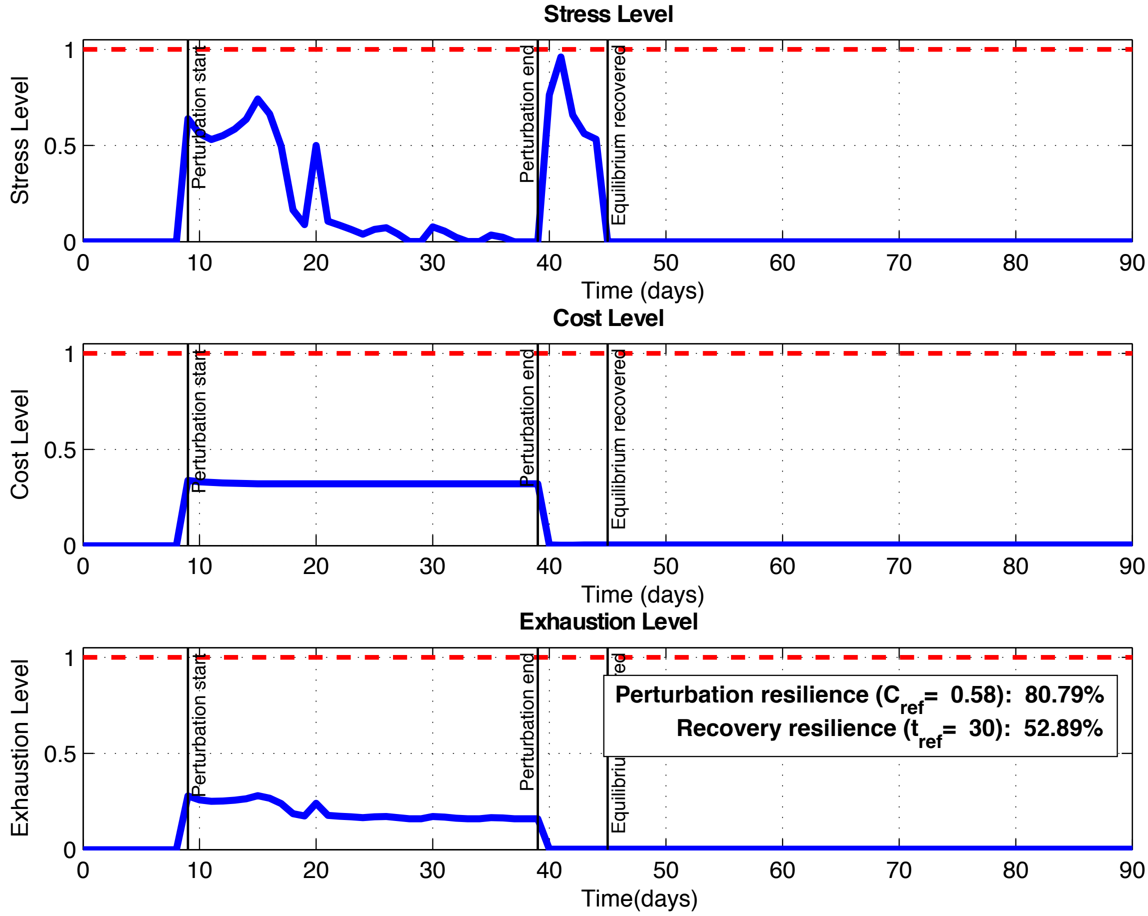

Figure 4.20: Resilience analysis of the Cuenca network, applying an increment of the capacity of adaptation of the users, $\alpha = 0.5$.

2014b, has required 54 sec. on a computer with processor Intel(R) Core(TM) i5 1.7GHz. This fact illustrates that the proposed methodology is a really useful tool to evaluate the perturbation and recovery resilience indices of a real traffic network.

4.10 Conclusions

In this chapter, a new methodology to quantify the resilience of traffic network is presented. This methodology allows a dynamic evaluation of the concept, including multiple aspects, such as the impact that the perturbation creates in the travel costs, but also the respond of the users during the whole process. In addition, the following conclusions can be drawn:

- A new dynamic assignment model has been presented. This model is defined as a dynamic equilibrium-restricted assignment model since the network performance is restricted by a system impedance to alter its previous state. This "equilibrium-restricted" model tries to reflect a more realistic scenario where users do not have perfect information of the conditions of the network at every moment, and they may be resistant to change when the situation is partially unknown. To reflect this situation, two variables are included in the model:

The system impedance, α, defining the capacity of adaptation to the changes, and the lack of knowledge of the behaviour of other users.

Net flow variation, ρ_r, allowing the analysis of the user response, and of the stress level in a network.

- Three different levels are presented to evaluate the behaviour of the network before, during, and after the perturbation. From the outputs obtained from the dynamic equilibrium-restricted assignment model, the cost level, and the stress level of the network are assessed. Finally, the exhaustion level is formulated as a combination of the cost, and the stress levels of the traffic network.

Cost Level describes the evolution during the studied period of the travel costs in the traffic network, in this case by the evaluation of the travel times.

Stress Level describes the evolution of the stress of the users, i.e. measures the necessity of the users for changing their usual routes due to the new conditions created by a perturbation.

Exhaustion Level describes the impact caused in the network by determining how far the traffic network is at each time from total exhaustion.

- A straightforward formulation for assessing the traffic network resilience after suffering a disruptive event has been proposed. Specifically, the perturbation resilience which is evaluated during the performance of a disruptive event in a traffic network, and the recovery resilience which is evaluated once the perturbation stops until a new equilibrium in the network is reached. Both parameters are evaluated by using the exhaustion level of the network considering:

Perturbation resilience quantifies the capacity of the network to absorb, resist or accommodate a perturbation. Evaluating how far the system is from the total exhaustion. Also, a cost threshold is included to assume the system break-down.

Recovery resilience measures the ability to restore, return or recover from the perturbation. Including aspects such as the recovery time, and the pattern followed in the restoration process.

- Results show that the proposed methodology is a useful tool to assess and compare the resilience capacity of a traffic network. Both values, perturbation, and recovery resilience are normalized, allowing the comparison of different resilience indices.

- It has been presented that the methodology covers multiple parameters than other authors have used for the evaluation of resilience, such as redundancy, robustness, and rapidity. Including these parameters automatically inside the methodology, a whole evaluation of the concept without the necessity of evaluating each of them separately is obtained.

- Finally, the methodology presented to evaluate resilience has been conducted on a real network, and the efficiency of the proposed methodology has been demonstrated.

Chapter 5

A bounded link travel cost function to include the weather effect on traffic network

Contents

5.1 Notation

t_a	actual travel time for link a
t_{a0}	free flow travel time for link a
m	shape parameter of the new travel cost function to define the upper bound
β	shape parameter of the new travel cost function
γ	shape parameter of the new travel cost function
p_a	local vulnerability for link a
h	hazard intensity
S	congestion ratio
ν_a	actual link flow for link a

ν_a^*	link flow associated with link a corresponding to the UE state
ν_a^{max}	link capacity for link a
h_{pq}	flow on routes with origin-destination pq

The research included in this chapter has been presented in Nogal et al. (2016a), Nogal et al. (2015a), and Nogal et al. (2016c).

5.2 Motivation

The aim of this chapter is to present a new link travel cost function that specifically considers the weather effect. Even though multiple travel cost function have been presented in the literature, there is an existing gap when looking for travel cost functions that include in their formulation parameters related to weather effects, and their impacts on traffic behaviour. Therefore, the main challenge is to introduce a travel cost function able to fill this gap, reflecting these effects, and helping in the development of new weather related tools for traffic networks. In addition, this cost function will be included in the objective function of the dynamic restricted equilibrium model presented in Chapter 4, to finally achieve a precise methodology to evaluate resilience in traffic networks suffering from extreme weather events.

5.3 Introduction

In the previous chapter, a new methodology for the evaluation of resilience in traffic networks has been developed. This methodology is based on a novel dynamic traffic assignment model. As explained in Chapter 2, an indispensable part of traffic assignment models is the travel cost function. These functions are presented in the objective function in the optimization problem, because the main goal is minimizing the travel time of the users in the traffic network. Traditionally, these travel cost functions are represented as link congestion functions, or volume-delay functions, i.e. the travel time is considered as a function of the traffic volume in a given link.

Due to the novelty of the resilience concept in traffic networks, there is a lack of specific travel cost functions which can incorporate the required aspects. In this case, the focus is on the impact caused by climatological events in traffic network, hence, travel cost functions that can evaluate these phenomena, and their impact in traffic are desired for a better evaluation.

It has been demonstrated that weather events deeply affect the traffic response, however, these phenomena are rarely included in these kinds of functions. Climatological events, such as rainfall, snowfall, dense fog, and frost, need to be incorporated in travel cost functions for a precise evaluation of resilience in traffic networks, including not only the intensity of these events, but also, it is necessary to characterise how the network responds to each event, since the response of one part of the network can significantly vary to the response obtained from another part.

Thus, with the final goal of assessing the resilience of traffic networks when extreme weather events occur, a new travel cost function is presented in this chapter. The proposed function exhibits a steady performance when the intensity of the hazard varies but it is also valid in the absence of a climatological event. In addition, with the aim of a better representation of the traffic reality, this cost function is upper-bounded. This fact significantly improves the computational efficiency of the formulation.

This chapter is organised as follows, in Section 5.4, an analysis of the influence that climatological events create on traffic flows is presented, including the review of several studies performed in the area. Section 5.5 introduces the novel bounded link travel cost function which incorporates the related weather parameters, also an illustrative example is presented for a better understanding of the function characteristics. In Section 5.6, a description of the main properties of the proposed travel cost function is presented. A comparison of the new travel cost function with the most common travel cost functions presented in literature, for the case of good climatological conditions, is addressed in Section 5.7, and in Section 5.8, a calibration of the weather travel cost function using real data is provided. Finally, in Section 5.9, some conclusions are drawn.

5.4 Influence of climatological events on traffic networks

The influence of climatological events on the performance of traffic networks has been widely studied in the literature. There are empirical studies that show the influence of phenomena such as rainfall, snowfall or reduction of the visibility on traffic networks characteristics, such as the speed, and the capacity of the elements, see Sršen et al. (2012). These reductions are caused due to the impact that weather events have in the normal behaviour of the network, for example, when intense rain affects a road, the users will need to reduce their speed to be able to circulate during the disrupted scenario, or when an intense fog affects a part of the network, it will be necessary to drive significantly slower than in good atmospheric conditions.

Therefore, when weather events occur the users will need to adapt to the new conditions, and this fact will be reflected in their travel times. Hereunder, some empirical studies are analysed to demonstrate the impacts caused by climatological events on traffic networks.

Koetse and Rietveld (2009) present a survey of the empirical literature on the effects of climate change and weather conditions on the transport sector. For road transport, the authors highlight that most studies show a reduction in traffic speed due to precipitation, and especially snow. This effect is particularly large during peak hours, and on congested roads. Thus, some studies are included in the research, such as Martin et al. (2000), who range from 10% speed reduction in wet conditions to 25% speed reduction in wet and slushy conditions, and Hranac et al. (2006), who range from 3% to 9% reduction in free-flow speed, and speed at capacity around, respectively, for light rain. These reductions will normally increase depending on the rain intensity, reaching values around 6-9% and 8-14%, respectively for a high intensity.

Koetse and Rietveld (2009) also note that larger effects exist for the case of snow in traffic networks. Hranac et al. (2006) range this case from $5-16\%$ reduction in free-flow speed and speed-at-capacity for light snow.

Maze et al. (2006) conclude that rain, snow and reduced visibility cause clear reductions in traffic speed; up to 6% for rain, up to 13% for snow, and up to 12% for reduced visibility.

Agarwal et al. (2005) range capacity reductions from $1\%-3\%$, $5\%-10\%$, and $10\%-17\%$, for trace, light, and heavy rain conditions, respectively, and speed reductions of $1\%-2\%$, $2\%-4\%$, and $4\%-7\%$ for trace, light, and heavy rain, respectively. For the case of snow, the authors find reductions in the capacity from $3\%-5\%$, $6\%-11\%$, and $7\%-13\%$ for trace, light, and moderate snow, respectively, and speed reductions of $3\%-5\%$, $7\%-9\%$, and $8\%-10\%$ for trace, light, and moderate snow, respectively.

It has been shown that the intensity of the hazard is the factor that the majority of studies highlight when describing the effects of climatological events on traffic behaviour. Together with the intensity of the hazard, other authors analyse the weather impacts on traffic behaviour depending on factors such as the hierarchy of road and the number of road lanes, Wang et al. (2006), the target area, geometric and traffic characteristics, Smith et al. (2004) and, cultural and socio-economic factors, Tsapakis et al. (2013).

In Table 5.1, a summary of the reductions of free flow speed, reductions of speed-at-capacity, and reductions of capacity due to weather events found in literature are presented. Table 5.1 is based on the conclusions previously addressed in this Section, and in the following studies, Lam et al. (2008), Ibrahim and Hall (1994), Agarwal et al. (2005), Maze et al. (2006), and Rakha et al. (2008).

Some conclusions can be obtained from the data presented in Table 5.1. The more severe the climatological event, the larger the reduction in the characteristics of the traffic behaviour is, being valid for the different types of perturbation. Also, when the network is working with flows near the capacity, the reductions produced by weather events are larger than when the degree of saturation is lower.

After the presentation of the effects caused in traffic networks, and being the aim of this chapter the presentation of a travel cost function that explicitly considers the effects of weather events in traffic networks, some requirements that need to be accomplished by the new function are introduced as follows:

Table 5.1: Weather impacts on traffic behaviour according literature.

	Reduction of free flow speed	**Reduction of speed-at-capacity**	**Reduction of capacity**
Rain			
Light rain (0.2 mm/h)	2 to 3.6%	8 to 10%	10 to 11%
Moderate rain (6 mm/h)	4 to 7%	-	10 to 17%
Heavy rain (16 mm/h)	4.4 to 9%	8 to 14%	10 to 17%
Snow			
Light snow (0.1 mm/h)	5 to 16%	5 to 20%	3 to 20%
Moderate snow (3 mm/h)	5 to 19%	-	6 to 20%
Heavy snow (12.0 mm/h)	11 to 43%	-	7 to 27%
Reduction of visibility			
Less than 0.4 km	10 to 12%	-	-

- **Intensity of the hazard.** Several authors, such as Koetse and Rietveld (2009), and Tsapakis et al. (2013), have concluded that the intensity of the weather event is a decisive factor when the impact of a hazard has to be evaluated in a traffic network. Therefore, a parameter that explicitly determines the intensity of the hazard should be included in the new cost function. This parameter will be defined as the **"hazard intensity"**, and is denoted as h. In this way, when there are increments in the intensity of the hazard the travel times will increase accordingly.

- **Local vulnerability of the link**. When analysing the effects of climatological events in traffic networks, the impacts will be determined not only by the characteristics of the hazard, but also by the specific sensibility of each of the links of the network. This means that a specific perturbation can have different impacts depending on the vulnerability that a link experiences against that specific hazard. For example, in the case of intense rain, the sensibility of a link will depend on characteristics such as the catchment area, the slope of the road, type of pavement, existence of element protection, or any characteristic that can modify the response of the link against a particular hazard. Therefore, a parameter which incorporates this local vulnerability of each link of the network against specific climatological events needs to be included in the travel cost function. Thus, this parameter will be denoted as p_a, and is defined as the local vulnerability of the link.

- As any of the cost functions previously presented in the literature (see Chapter 2), a new travel cost function has to reflect that when a perturbation occurs in a traffic network, the effects will be larger when the flows are close to capacity. In addition, in the specific case of climatological perturbations, when extreme weather events take place in traffic networks, their effects will have impacts also when the degree of saturation of the network is low. Thus, hazards with a high intensity will disrupt the network even though the conditions are close to the levels of free flow speed.

Finally, in Figure 5.1, a diagram for a better understanding of the parameters previously explained is presented. By introducing these new parameters in a travel cost function, the impact directly caused by the hazard in the traffic network can be simulated. The intensity of the hazard, h, includes the characteristics of the hazard, and the local link vulnerability, p_a, includes the characteristics of the network. Both parameters are able to recreate the disruption created in the traffic network.

5.5 Travel cost function including the weather effect

In this section, a new travel cost function is presented. The main goal of this function is to be able to reflect the effects caused by weather events in traffic networks. It is highlighted that one of the most convenient properties of the formulation of any cost function is the parsimoniousness, i.e. the maximum simplicity of a formulation trying to include the greatest number of variables involved in a given problem.

Taking this condition into consideration, hereunder the indispensable aspects for the definition of the new function are described:

- An important condition that any travel cost function needs to address to reflect real traffic conditions is that the congestion ratio penalizes the link travel cost. In this way, when the flows increase in a traffic network, and the degree of saturation of the links of the network is incremented, the travel times should increase.

- In most of the travel cost functions, the link travel time is lower bounded by the free-flow travel time, however, the travel time tends to be represented by upper unbounded functions. This means that travel cost function can reach an infinite value of travel time. This fact does not seem very realistic, since when travel times start to increase in a link due to congestion, or the effects of climatological events, there is a point where the link can be considered "broken", meaning that the link cannot be used. For example, when there is a large flooding that does not allow the

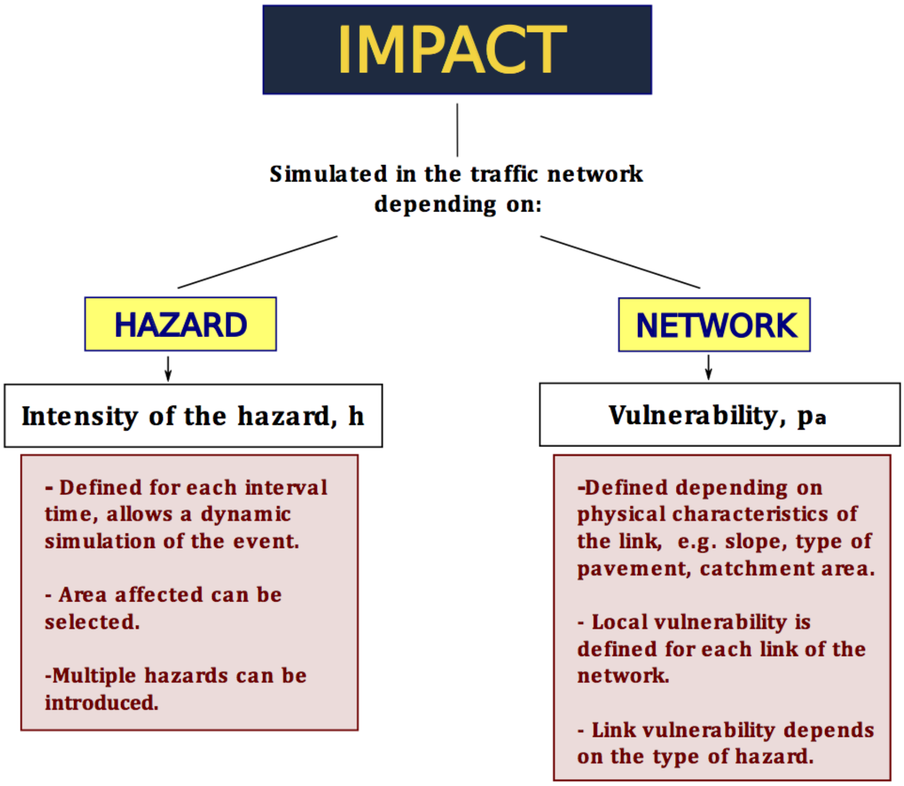

Figure 5.1: Explanatory diagram for the definition of the new parameters, the intensity of the hazard, h, and the local link vulnerability, p_a.

vehicles to cross. In addition, before the travel time of a link reaches an extremely high value, that link will not be used for more users. If a link with an average travel time of 1 hour is studied, it can be established that values of the travel time of for example 20 hours are almost impossible in reality. This means that a maximum travel time can be established since it is going to exist a value for the travel time that it is not going to be surpassed. Therefore, in order to represent the reality of traffic networks, the new proposed cost function is defined **upper bounded**, since travel times cannot reach infinite values in real networks.

Including the effects of weather events:

- The weather conditions affecting any part of the analysed network directly influence the traffic performance. The impact caused by the climatological event is introduced through the intensity of the hazard, h.
- Each link has specific features with regard to a given climatological event. Therefore,

the local vulnerability of each link can be evaluated, and incorporated into the new function with the parameter, p_a. This parameter can enhance or aggravate the user response, since links with a high vulnerability against a specific hazard can be avoided by the users when that hazard is forecasted.

Accordingly, the following link travel cost function is proposed:

$$t_a = t_{a0}\left\{1 + m\, exp\left[-\left(\frac{\beta S + p_a h}{1-h}\right)^{-\gamma}\right]\right\}, \tag{5.1}$$

where t_a and t_{a0} are the actual travel time and the free flow travel time, respectively; S is the congestion ratio computed as ν_a/ν_a^{max}, ν_a being the actual flow and ν_a^{max} the link capacity to provide a certain service level; m, β and γ are positive traffic parameters related to the link characteristics in the absence of weather events; the weather parameters h and p_a, are defined in the ranges $h \in [0, 1)$ and $p_a \in [0, 1]$. The calibration of Eq. (5.1) can be carried out by means of the maximum likelihood method, (see Section 5.8 for the calibration).

The new travel cost function can be also used in absence of climatological events, when $h = 0$. Thus, the formulation of the new travel cost function can be expressed as follows:

$$t_a = t_{a0}\left\{1 + m\, exp\left[-(\beta S)^{-\gamma}\right]\right\}. \tag{5.2}$$

In Equation 5.2 only the shape parameters are included, β, γ, and m, also in this case, when there is not a hazard ($h = 0$), the parameter p_a is not involved.

Figure 5.2 is presented for a better understanding of the possibilities of the new function, and how this function integrates the two parameters to represent the climatological events. In Figure 5.2, the shape that the travel cost function acquires for multiple values of p_a, and h is represented. Thus, there are six graphics, each of them for a different value of the local vulnerability of a link, from 0 to 1, also, inside each graphic, the travel cost function is represented for different values of h from 0 to 1. The shape parameters assumed for this graphic are $\beta = 0.83$, $\gamma = 5.2$, and $m = 9$, selected to give a representative example of the new function, see Section 5.7 for a comparison with other calibrated function presented in the literature.

Analysing Figure 5.2, it is noted that for smaller values of p_a, the different curves for different values of h are closer. This means that when increasing the intensity of the hazard, the travel times increase in small quantities. However, when observing the graphs for larger values of p_a, the curves for different intensities are more separated, therefore this implies that when increasing the intensity of h, the travel times are incremented much faster.

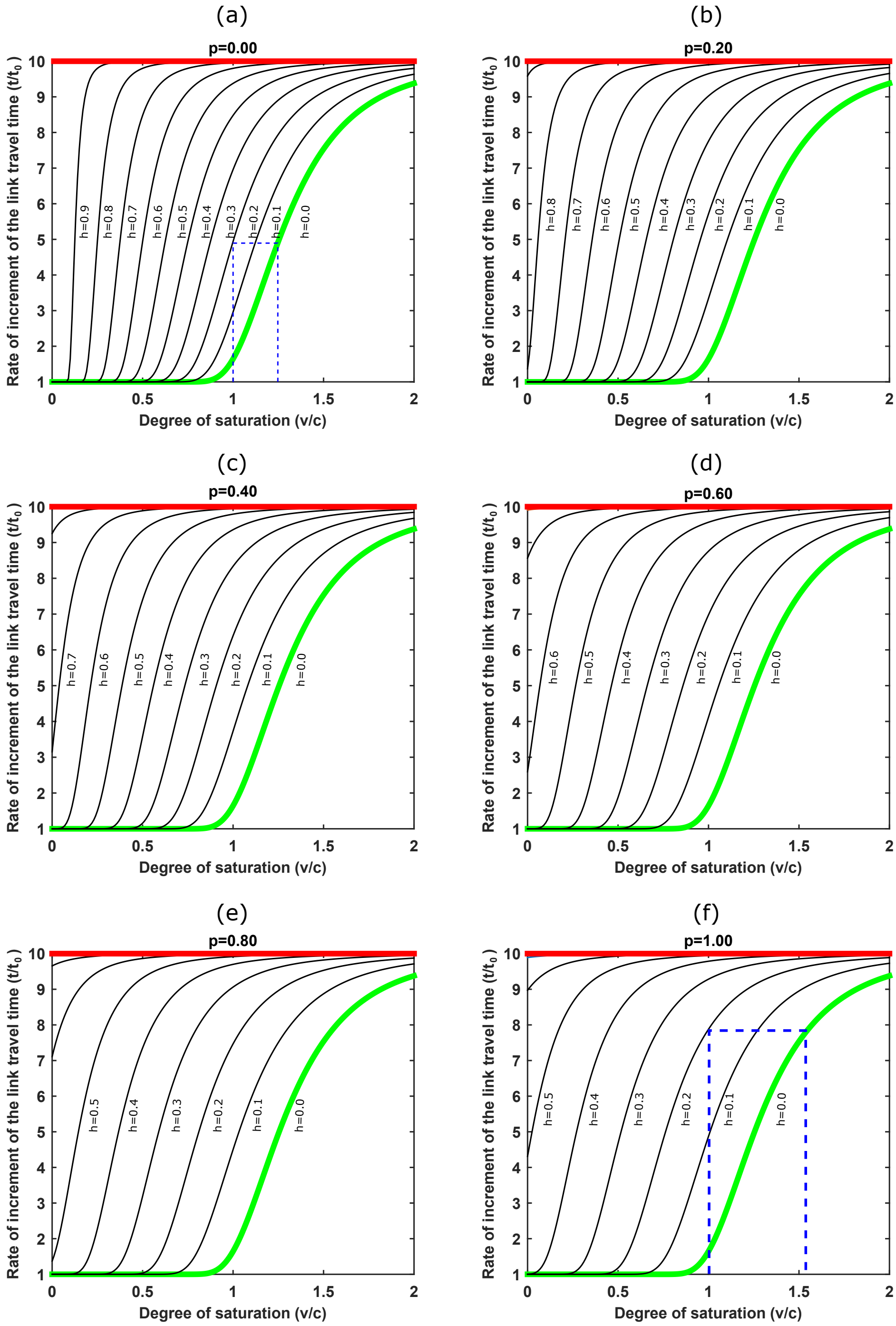

Figure 5.2: Proposed cost function with $m = 9$, $\beta = 0.83$, $\gamma = 5.2$, and different values of the parameters p_a, and h.

This fact reflects that when a link is highly vulnerable to the effects of a weather event, for example, a part of the road that is highly exposed when rainfalls occur, this link will experience higher travel times than if the hazard affects other link with a smaller vulnerability.

Previously, empirical data have been presented in Table 5.1 where it is shown that the impact of a weather event implies a reduction in the capacity. This fact can be corroborated in Figure 5.2 with the results presented of the new cost function.

By analysing the dashed-blue line (Figure 5.2), the performance of the link cost in the case of $h = 0.20$ and $p_a = 0.00$ ((a) graph) when the saturation ratio is $S = \nu_a/\nu_a^{max} = 1$, is equivalent to consider a reduction of the capacity of 1.25.

In Figure 5.2, for larger values of the link vulnerability, for example when $p = 1.00$ ((f) graph), the corresponding equivalent capacity is $1/1.56$ times the capacity in the absence of weather events. Therefore, the proposed formulation is coherent with the empirical reduction of capacity explained in the previous section.

As empirically demonstrated in the previous section, climatological events are reported to increase the travel times even in the case of free travel time conditions. This phenomena can be appreciated in Figure 5.2, for example when $p_a = 0.8$ ((e) graph) for a hazard of $h = 0.5$, the travel times are larger than free flow travel times, even for very small degree saturation. In addition, for smaller values of p_a this fact can be appreciated, for example, when $p_a = 0.2$ ((b) graph) for a hazard of $h = 0.6$, the travel times are also larger than free flow travel times for congestion ratios smaller than 0.5.

5.5.1 Illustrative Example

To facilitate the understanding of the function previously presented, an illustrative example is presented in this Section. A simple traffic network is considered, see Figure 5.3 for a graphic description of the network.

The network consists of 5 nodes, 16 links and 8 routes, whose characteristics are presented in Table 5.2, and 5.3. Table 5.2 introduces a description of the routes used in this example, and Table 5.3, a description of the links, including the link flow, $\nu_a^*(users)$, the actual link travel time, t_a, and the free flow travel time, t_{a0}, calculated with the proposed travel cost function at the User Equilibrium state.

In this example, the parameters used for the new travel cost function, are $\beta_a = 0.83$, $\gamma = 5.20$, and $m = 8$. The capacity of the links, ν_a^{max}, is 80 users, and for the links, 9, 10, 11, 12, 13, 14, 15 and 16, the capacity, ν_a^{max}, is 40 users.

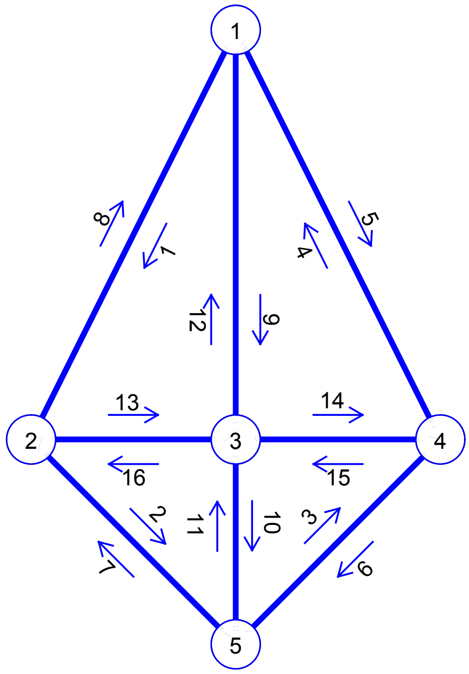

Figure 5.3: Graphic representation of the network used in the illustrative example.

Table 5.2: Routes given by the links and route flow under a UE scenario.

Route	**Links**	**OD**	$h_{pqr}(users)$
1	1-13-14	1-4	64.31
2	9-10-3	1-4	35.69
3	2-11-12	2-1	50
4	2	2-5	50
5	3-4	5-1	54.73
6	7-8	5-1	45.27
7	5-15-16	1-2	38.24
8	5-6-7	1-2	61.76

Once that the network has been calibrated for the UE conditions, and the system is in equilibrium, i.e., all users have selected those routes that actually minimize their route travel time, a disruption can be simulated in the network.

By using the new travel cost function, a hazard is introduced in the network, defining its intensity for every day. In this example, the simulated event is an intense rain, which will impact the whole network. The rain starts the 1^{st} day, and its intensity increases until reaching its maximum intensity, the 4^{th} day. After that, the intensity of the rain is reduced finishing the 8^{th} day. See Figure 5.4 for a description of the hazard.

Table 5.3: Link characteristics at the UE, i.e., link flow, link travel time, and free flow link travel time

Link	$\nu_a^*(users)$	t_a	t_{a0}
1	64.31	0.64	0.19
2	100.00	0.95	0.12
3	90.42	0.89	0.12
4	54.73	0.28	0.19
5	100.00	1.51	0.19
6	61.76	0.34	0.12
7	107.04	0.99	0.12
8	45.27	0.19	0.19
9	35.69	1.12	0.22
10	35.69	0.56	0.11
11	50.00	0.90	0.11
12	50.00	1.80	0.22
13	114.31	0.95	0.11
14	64.31	0.97	0.11
15	38.24	0.66	0.11
16	38.24	0.66	0.11

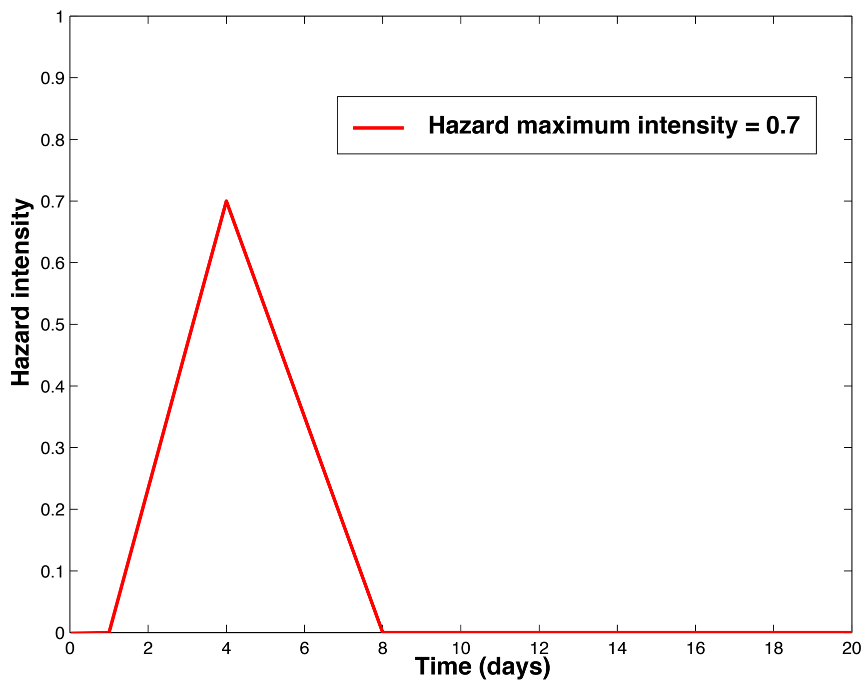

Figure 5.4: Definition of the hazard by the maximum intensity of the daily rain used in the illustrative example.

In addition, the local vulnerability of each link against this event can be introduced in the cost function, by using the parameter, p_a. Thus, in this case, the external links of the network, 1, 2, 3, 4, 5, 6, 7, and 8, present a lower vulnerability, with $p_a = 0.1$, i.e. these links are less exposed to the rain in this example. The interior links, 9, 10, 11, 12, 13, 14, 15, and 16, (see Figure 5.5), have a larger vulnerability, with $p_a = 0.5$, i.e. due to their position. They are assumed to be more exposed to the hazard,because the internal part of the network is in a lower area, therefore it will be more affected by the rain.

Once the network and the hazard have been defined, the methodology previously presented in Chapter 4 is used together with the new cost function proposed in this chapter to evaluate the resilience of the network when the rain impacts the network. Thus, the new cost function is included in the objective function of the dynamic equilibrium restricted model, and the results are shown as follows.

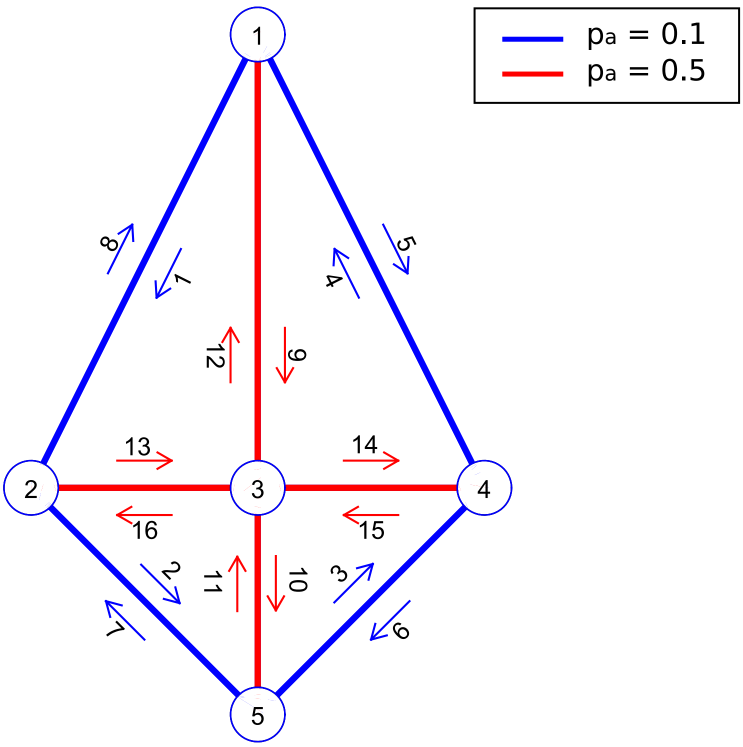

Figure 5.5: Identification of the different links vulnerabilities of the network used in the illustrative example.

For a better understanding of the effects caused in the network by the hazard, the evolution of the link travel times, and the link flows are shown in Figure 5.6 and Figure 5.7.

In Figure 5.6, all the links increase their travel times during the first part of the impact, when the intensity of the rain is increasing. Some of the links experience larger increments in their travel times, this fact is due to the role of each link in the functioning of the network, and their characteristics. In this case the capacities, and the local vulnerabilities also vary depending on the link.

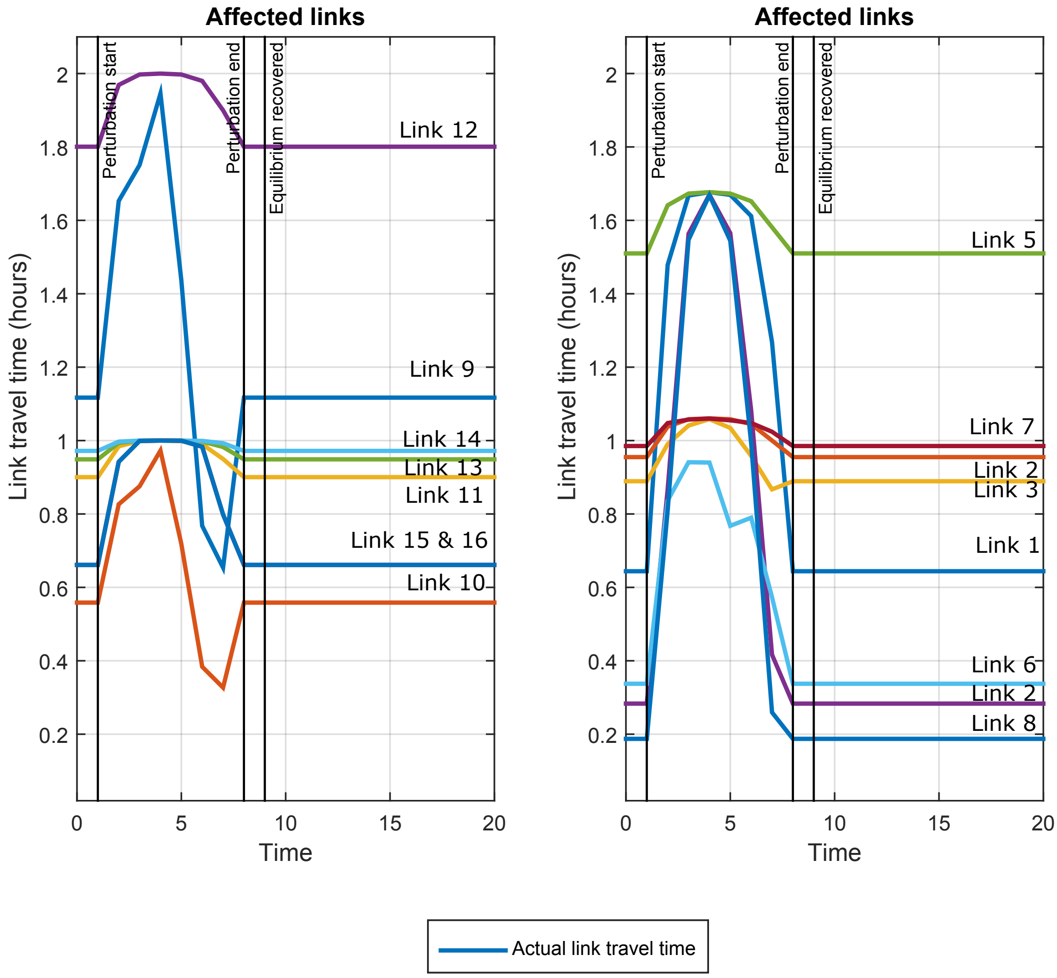

Figure 5.6: Evolution of the link travel times of the illustrative example network.

In Figure 5.7, instead of the link travel times, the link flows are shown. Thus, in this figure, it is possible to define how the users select different links when the rain affects the network. For example, in the case of link 2, this link is part of route number 4, (see Table 5.2) defined only by this link, and route number 3. These two routes have the special characteristic that they are the only ones defined for each OD, i.e, route 4 is the only route from node 2 to node 5, and route 3 from node 2 to 1. So, the flow in link 2 cannot change, since the users do not have other option for this OD. Analysing Figure 5.7, it is shown that the flow of link 2 is not changing with the perturbation. Therefore, the effect of the hazard is noted in the increment of travel time of link 2, see Figure 5.6.

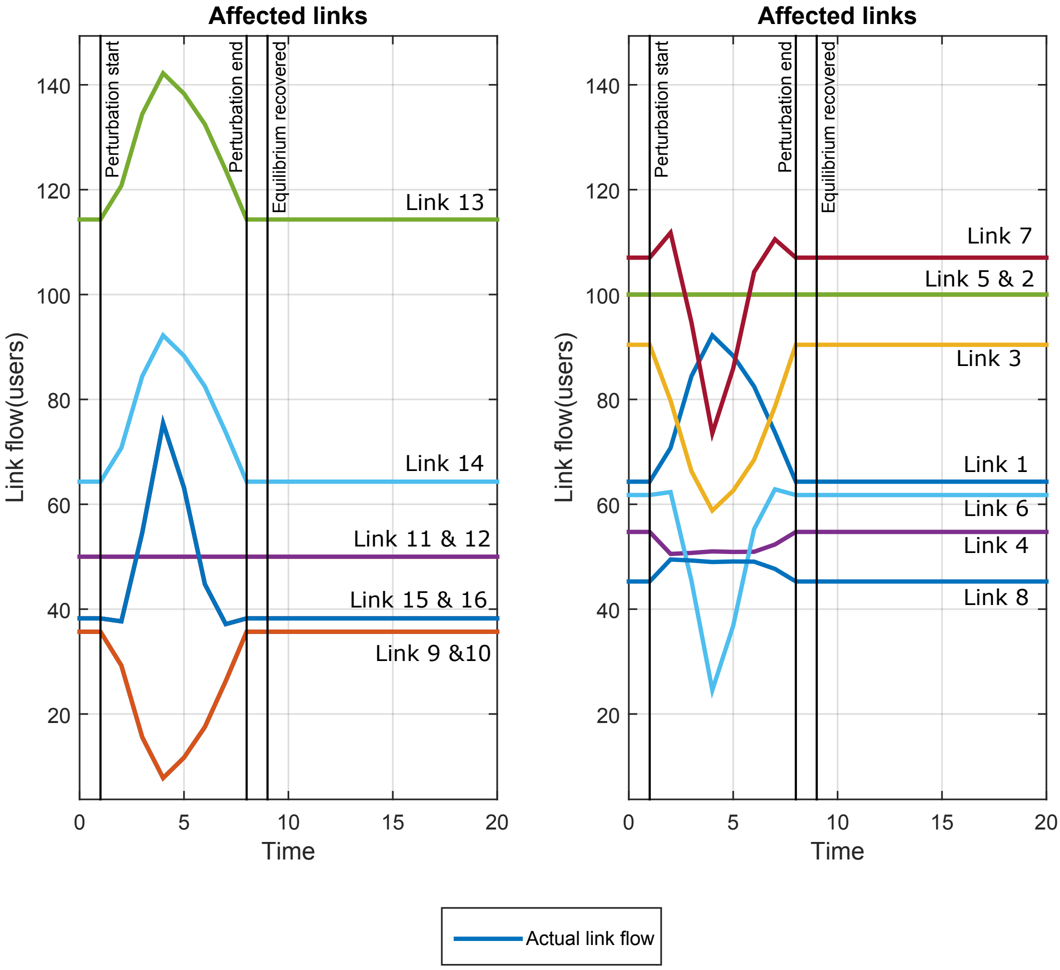

Figure 5.7: Evolution of the link flow of the illustrative example network.

In addition, Figures 5.8, and 5.9 show the performance of the new cost function in each of the network links during the occurrence of the hazard. Figure 5.8 presents the performance for the links 1 to 9, while Figure 5.9 presents links 10 to 16 of the network.

Both Figures show the shapes that the cost function acquires for different values of the hazard. The value of the function when the hazard is equal to zero is represented in green; in blue, the values of the cost function for the maximum intensity of the hazard, in this case 0.7, and in black, the values that the cost function takes for in-between values of the hazard.

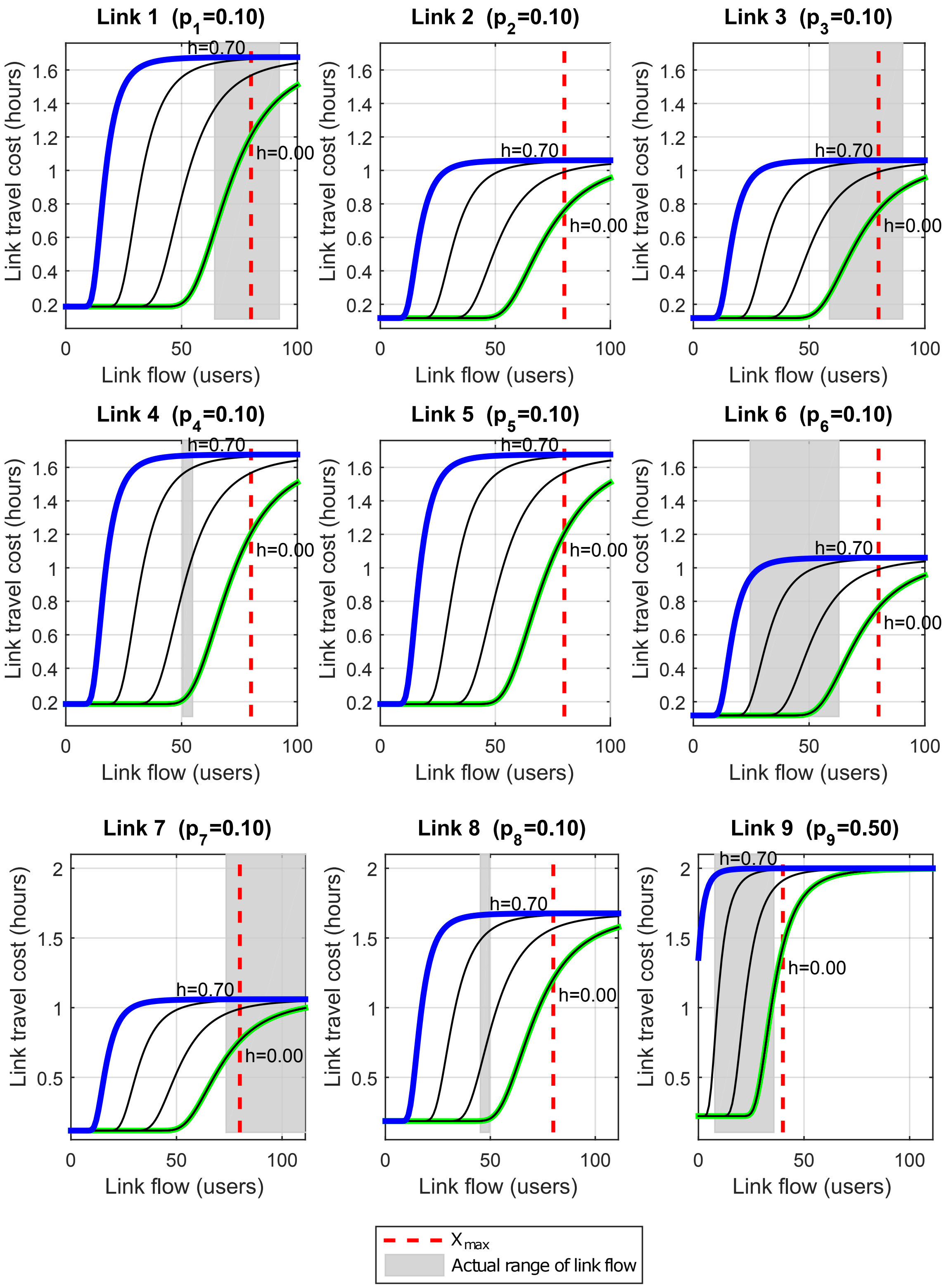

Figure 5.8: Performance of the new travel cost function for the illustrative example, links 1 to 9.

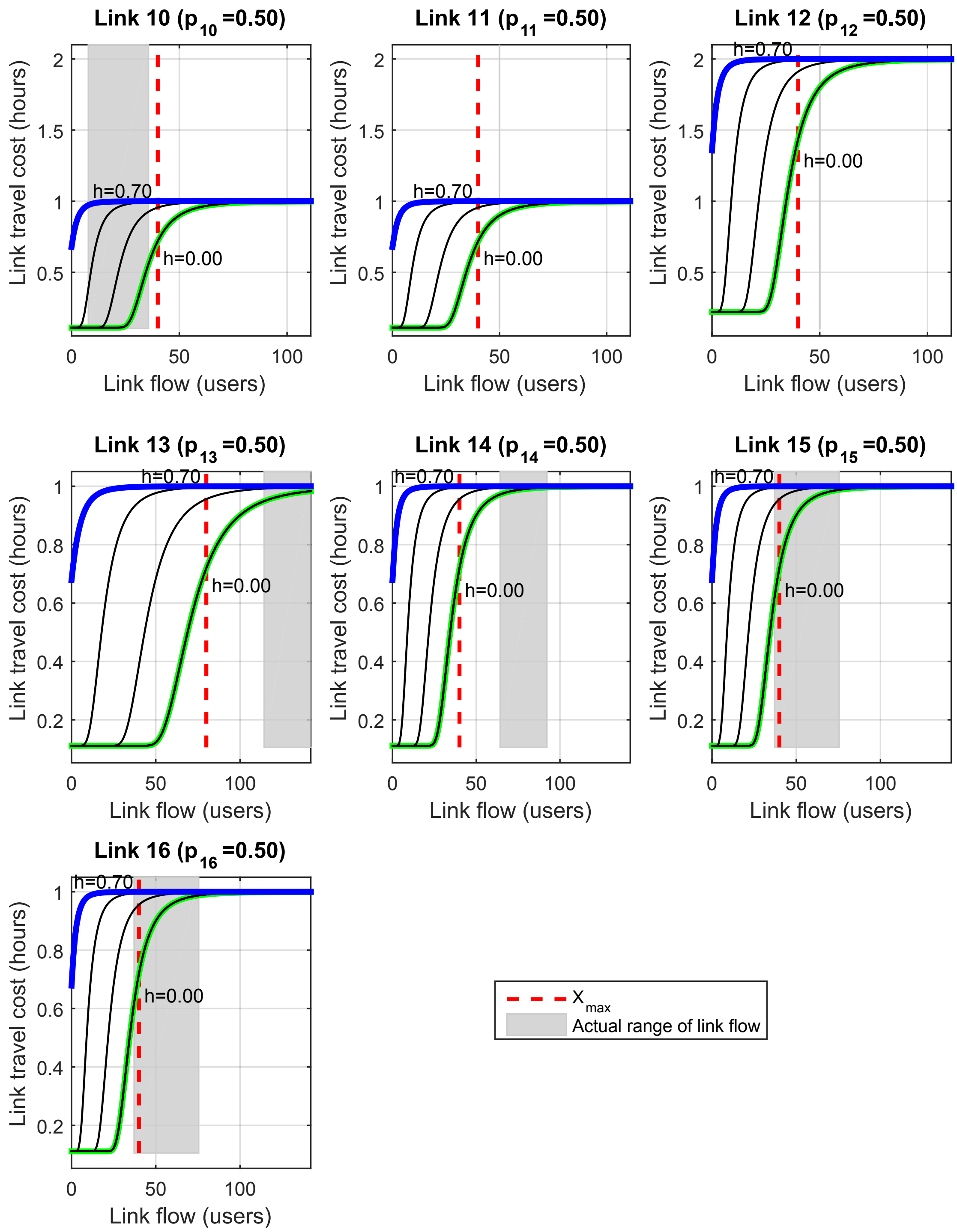

Figure 5.9: Performance of the new travel cost function for the illustrative example, links 10 to 16.

Figures 5.8 and 5.9 also illustrate the actual range of the link flows during the analysed period of time. This range is shown with a grey rectangle in each of the graphics, indicating the part of the cost function that is in use in every case. In those graphics where there is not a grey rectangle to define the variation of the link flows, it is because the link flow does not vary, and remains constant during the whole studied period, this is the case of link 2 ($\nu_a = 100$ users), link 5 ($\nu_a = 100$ users), link 11 ($\nu_a = 50$ users), and link 12 ($\nu_a = 50$ users).

Finally, in Figure 5.10, the final results of the evaluation of resilience for this example are shown. The stress, cost, and exhaustion levels of the network are shown in the continuous-blue lines. In addition, the evolution of the intensity of the rain is shown in a spot-red line in the last graphic. The values obtained for the resilience perturbation, and the recovery resilience are 55.78%, and 89.94%, respectively.

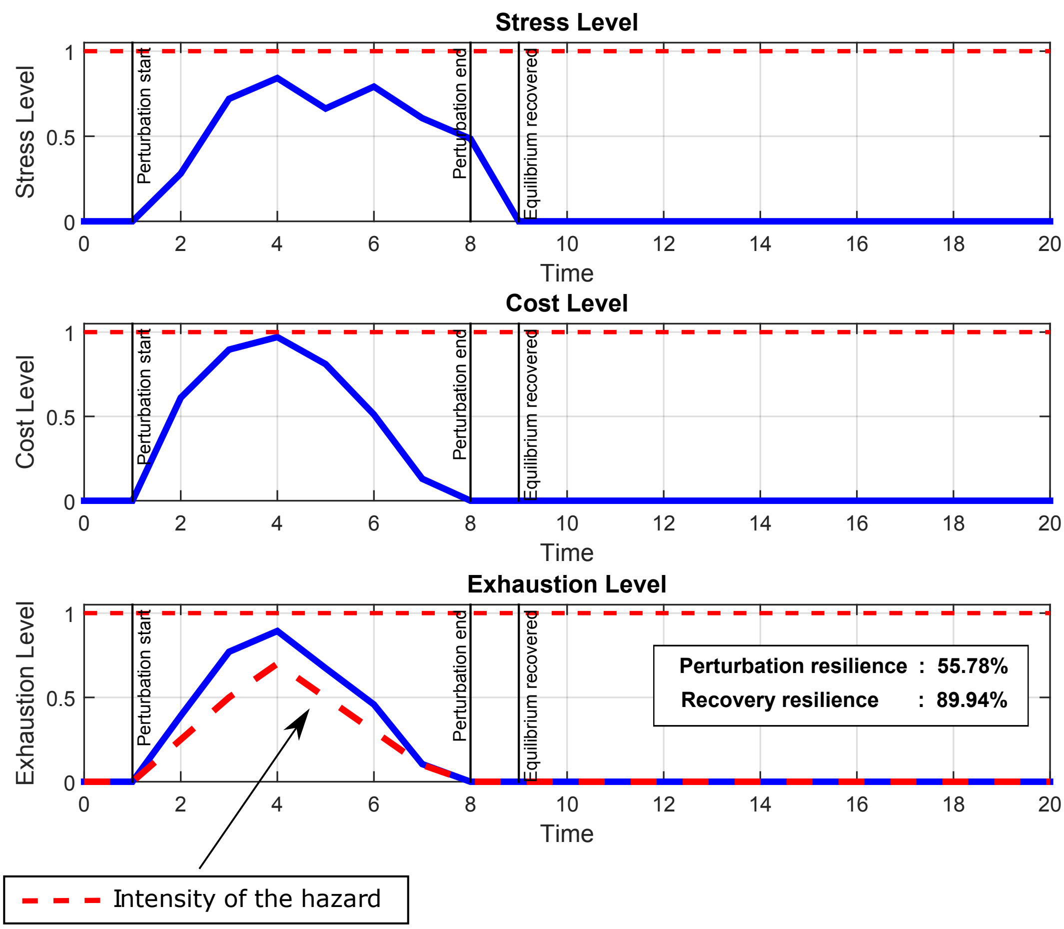

Figure 5.10: Evaluation of the resilience for the illustrative example, showing the stress level, cost level, and exhaustion level.

5.6 Properties of the proposed travel cost function

When a new travel cost function is defined, in addition to be able to reflect the real traffic behaviour of networks, some mathematical conditions have to be addressed. These mathematical conditions are indispensable when working with functions used in optimization problems, such as the traffic assignment models.

Therefore, the mathematical conditions of the proposed link travel time function are introduced as follows:

- The link travel cost function is dimensionally consistent. A detailed explanation of the implications of this property can be found in Castillo et al. (2014).

- The travel cost function is continuous and strictly monotonic w.r.t. the link flow.

  To demonstrate this fact, the partial derivative of the cost function w.r.t. ν, is defined in Eq. (5.3), and can be expressed as follows;

$$\frac{\partial t_a}{\partial \upsilon} = \left(\frac{\beta}{\nu_a^{max}}\right)\frac{mt_{a0}\gamma}{1-h}k^{-\gamma-1}exp(-k^{-\gamma}), \tag{5.3}$$

  with $k = \frac{\beta S + p_a h}{1-h}$. It is noted that the parameters involved in the function defined are always positive. Therefore, the derivative presented in Eq. (5.3) is always larger than zero. In addition, this property is often required when a unique solution in terms of link flows is necessary for the assignment problems.

- The integral of the proposed travel cost function is continuous. Hereunder, the expression for the integral is presented:

$$\int_{\upsilon>0} t_a d\upsilon = t_0\nu_a^{max}\left\{S + \frac{m(1-h)}{\beta\gamma}\Gamma\left[\frac{-1}{\gamma}, k^{-\gamma}\right]\right\}. \tag{5.4}$$

  The lower integration limit is defined by $\Gamma(\cdot)$, the upper incomplete gamma function, and its second parameter the lower integration limit.

- The proposed function is lower bounded, as the majority of travel time functions, by the free travel time, t_0. In addition, the new function includes an upper bounded limit, defined by the parameter m. Therefore, the range of the function is defined by $[t_0, t_0(1+m))$. The addition of the upper bound allows working in a limit range when traffic assignment problems are used. Therefore, this property avoids the loss of precision, and overflow problems when affected by extreme weather events. See Section 5.7 for the comparison of the efficiency of the function.

- As explained before, when applying the new cost function, the link travel times increase as the intensity of the hazard increases. Furthermore, this function is strictly monotonic w.r.t. h. By analysing the partial derivative of the new travel function w.r.t h, this fact is demonstrated. Therefore, as follows the derivative w.r.t. h is presented:

$$\frac{\partial t_a}{\partial h} = (p_a + k)\frac{m t_{a0}\gamma}{1-h} k^{-\gamma-1} exp(-k^{-\gamma}). \tag{5.5}$$

Given the positive range of the parameters involved, the derivative presented in Eq. (5.5) is always larger than zero. By using this property, the simulation of time-varying weather perturbations can be introduced, $h = h(\tau)$, with τ denoting the time.

- In addition, the proposed link travel cost function increases as the local vulnerability, p increases. Furthermore, this function is strictly monotonic w.r.t. p. By analysing the partial derivative of the new travel function w.r.t p, this fact is demonstrated. Therefore, as follows the derivative w.r.t. p is presented:

$$\frac{\partial t_a}{\partial p_a} = h\frac{m t_{a0}\gamma}{1-h} k^{-\gamma-1} exp(-k^{-\gamma}). \tag{5.6}$$

- Finally, when analysing the derivative w.r.t the link flow when the link flow is null, and considering climatological events, it is noted that this derivative is larger than zero, i.e. $\frac{\partial t_a}{\partial v} |_{v=0,h>0} > 0$. This property becomes relevant to avoid problems associated with the stability of the computational process when dealing with low levels of flow.

5.7 Comparison of the proposed cost function

Another important property, which is not necessary but highly recommendable, is compatibility with existing cost functions. This section presents a comparison of the proposed formulation with some of the most common existing functions, which were introduced previously in Chapter 2. In addition, this section demonstrates one of the main advantages of the new travel cost function, its computational efficiency.

Even though the proposed cost function has been designed to simulate the behaviour of traffic when weather events impact a network, it is noted that this formulation can be used in the absence of hazards. For this reason, in this section, the new cost function is compared with other travel cost functions proposed in the literature, the parameter to define the intensity of the hazard, h, is set equal to 0, and the proposed function presents the formulation introduced in Equation (5.2).

In Table 5.4, the functions used for the comparison are presented, together with the values used for their parameters. The parameters used to address this comparison have been obtained from Mtoi and Moses (2014), who calibrated the presented cost functions

Table 5.4: Parameters assumed for the comparison of cost functions presented in Figure 2.6

Function	**Formulation**	**Parameters**		
Proposed Function	$t_a = t_{a0}\left\{1 + mexp\left[-\left(\frac{\beta S + p_a h}{1-h}\right)^{-\gamma}\right]\right\}$	$\beta = 0.830$	$\gamma = 5.2$	m=9
BPR	$t_a(\nu_a) = t_a^0\left(1 + \alpha\left(\frac{\nu_a}{c_a}\right)^{\beta}\right)$	$\alpha = 0.263$	$\beta = 6.869$	
Conical Delay	$t_a(\nu_a) =$ $t_a^0\left(2 + \sqrt{(\beta^2(1 + S^2 + \alpha^2) - \beta(1-S) - \alpha)}\right)$	$\alpha = 1.029$	$\beta =$ 18.390	
Akcelik	$t_a(\nu_a) = t_a^0\left(1 + \alpha_a \frac{\frac{\nu_a}{c_a}}{1-\frac{\nu_a}{c_a}}\right)$	$\tau = 0.100$		
Modified David-son	$t_a(\nu_a) = \begin{cases} t_a^0\left(1 + J\frac{S}{1-S}\right) \\ t_a^0\left(1 + J\frac{\mu}{1-\mu} + J\frac{S-\mu}{(1-\mu)^2}\right) \end{cases}$	$J = 0.009$	$\mu = 0.950$	

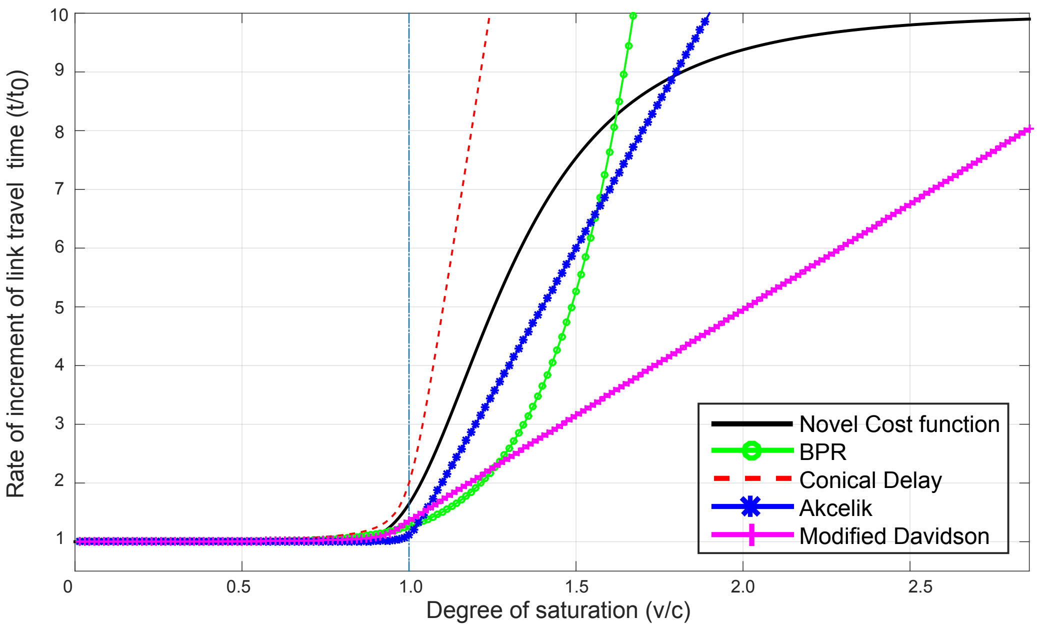

Figure 5.11: Comparison of the proposed cost function with the most used cost functions.

for the case of highways. Thus, the cost functions are the BPR, BPR (1964), the conical delay function presented by Spiess (1990), the cost function proposed by Akcelik (1991), and the modified Davidson function introduced by Tisato (1991).

In Figure 5.11, a graphical representation of the functions together with the new travel cost function is presented. A comparison of the functions obtained from the literature was already discussed in Chapter 2, see Section 2.3.1.The performance of the new function is comparable with the existing functions. For lower values of the degree of saturation, all the functions have a very similar behaviour, however when the degree of saturation increases the values obtained for each of the functions start to diverge. Nevertheless, the new cost function is within the range of values of the existing functions.

The main difference observed in the behaviour of the function for good weather conditions when compared with the rest of the functions is its upper bound. This characteristic is one of the main differences, and also one of its main advantages. Consecutively in this section, the advantages of this characteristic are presented.

Advantages of an upper bounded travel cost function:

- It allows a more **realistic representation** of the behaviour of travel times in traffic networks. The travel times have been traditionally represented with upper unbounded cost function, but this may not be the more realistic representation, since travel times can increment their values in large quantities but it is impossible that they reach infinite values. In the worst case, when a link or a route of a network experiences the highest possible degree of saturation, this link or route becomes impracticable, and it can be considered that the element has been "broken". Therefore, it seems more realistic to determine an upper bounded limit for the travel times than to include the possibility of infinite travel times.

- The new formulation, because of the upper limit, exhibits a better performance in terms of computational times when compared with others. This **computational efficiency** is demonstrated here below with an illustrative example, and it becomes decisive when:

  - Working with **optimizations problems**. Generally, cost functions are used in traffic assignment models to obtain the network travel times. Thus, defining a range of travel times with a lower and an upper bound, reduces the range of possible solutions analysed in the optimization process, and, therefore, the function will exhibit a more steady performance than other upper unbounded functions.

  - Introducing the effect of **extreme weather events**. When analysing the effects of weather perturbations on traffic networks, there are more probabilities of reaching unusually high travel times. The impacts of intense rainfalls, very dense fogs or other weather related phenomena can drive the network to large values on the degree of saturation. Therefore, when the optimization is performed, some peaks resulting in overflows can be obtained.

With the aim of studying the effect of the congestion degree, and proving the computational efficiency of the new travel cost function, an illustrative example is presented. In this case, the new travel cost function is compared with the well-known BPR. The BPR function has been elected for the computational comparison because is probably the most used travel cost function, and is known for its simplicity, and good performance.

In this example, the network used has been the Nguyen-Dupuis network, presented in Chapter 4 (see Section 4.5.2). This network has 13 nodes, 38 links, and 66 routes with 34 OD pairs.

In the first case, all the characteristics of the network have been kept the same than in the example presented in Chapter 4. For testing the computational efficiency, the computational times are measured when implementing the well-known user equilibrium model proposed by Beckmann et al. (1956), programmed with the $Matlab^R$ 2014b version.

For creating different scenarios, the capacities of 19 of the 38 links of the network have been linearly reduced. In each evaluation, the capacities of the selected links have been reduced by 2%, from the 98% of their initial values to the 2%. In those cases where the capacities reach smaller values, higher travel costs will be experienced by the network, and some peaks can be reached due to overflows. These cases will help to show the better performance of the new cost function in these scenarios. The links have been selected randomly for this example.

In Figure 5.12, the results of this comparison are presented. It is noted that the computational times associated with the BPR cost function exhibit a clear tendency to rise when incrementing the congestion. Moreover, it presents some peaks that can result in overflows when dealing with more complex traffic networks. On the other hand, the proposed formulation exhibits a more constant computational efficiency with a low range of variation, during all the evaluations.

This computational efficiency becomes more remarkable, as the network becomes more saturated. To demonstrate this fact, the computational times have been measured in a new example, where the capacity of all the links is more restricted. In the previous example, the capacity, ν_a^{max}, of the links was established in 80 users. For the new example, the capacity has been reduced to half, with $\nu_a^{max} = 40$ users, and the rest of the characteristics remain the same. The same process of reducing the capacities has been implemented, and the results are shown in Figure 5.13.

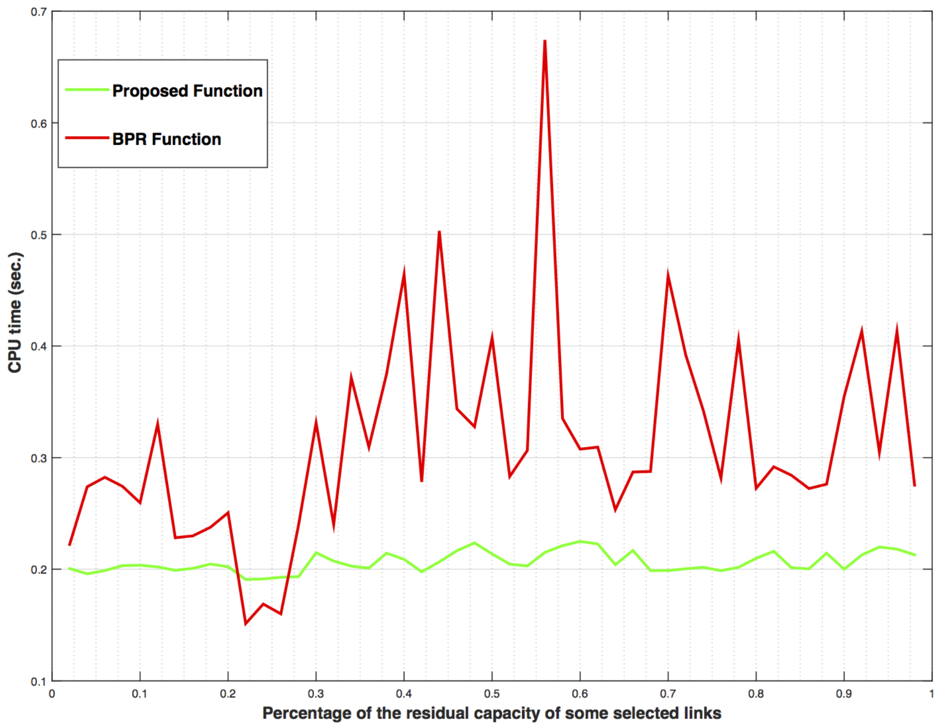

Figure 5.12: Comparison of computational times between BPR and the proposed travel cost function.

The results in Figure 5.13 confirm the computational efficiency of the new function. Thus, when the network is more saturated, the new cost function keeps the constant range of variation for the computational times, and the BPR exhibits a more variable behaviour, reaching even more peaks than in the previous case.

5.8 Calibration of the proposed cost function

In order to show the applicability of the proposed travel cost function, in this section, a calibration by using real data is performed. The function is calibrated with data from the Dublin traffic network, which were provided by the Dublin City Council (Ireland.)

The link used for the calibration is part of the Navan Road, analysing the length between the intersection with Ashtown Grove until Kinvara Ave intersection. The total length of the link analysed is 500m, and the free flow speed is 50 km/h. In Figure 5.14, the link selected for the calibration is indicated.

It is noted that the chosen link (see Figure 5.14) has a bus-only lane along the road. This type of lanes requires a particular analysis. Therefore, among the two possible lanes,

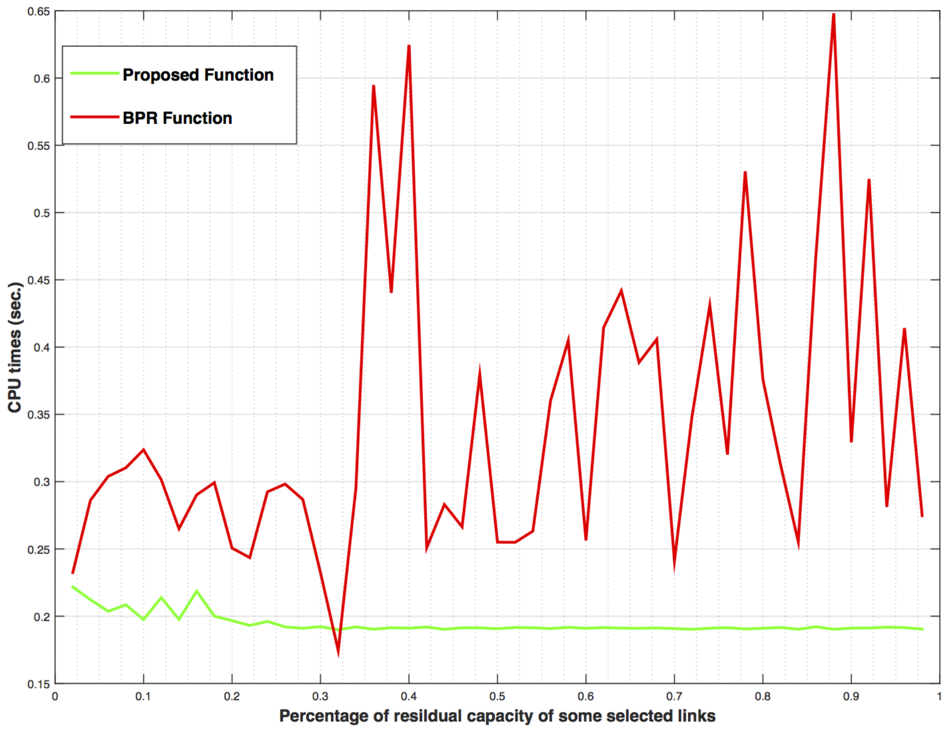

Figure 5.13: Comparison of computational times between BPR and the proposed travel cost function. Smaller capacity of the links.

the regular lane has been the one chosen for the analysis.

For obtaining the traffic data, the SCATS Traffic Reporter system has been used. This program is used by the Dublin City Council as an urban traffic control system, and the flow and speed of multiple intersections of the city can be obtained.

The analysed time period for the calibration has been from 6/11/2014 to 21/11/2014 and from 23/11/2014 to 13/12/2014 (the data from 6/12/2014 to 6/12/2014 are not available), and the traffic flow, and corresponding speed of this link has been obtained every 15 min.

For obtaining the climatological data, in the analysed period, the Irish Meteorological Service Station in the Phoenix Park has been used, and the rain fall intensity per hour has been obtained.

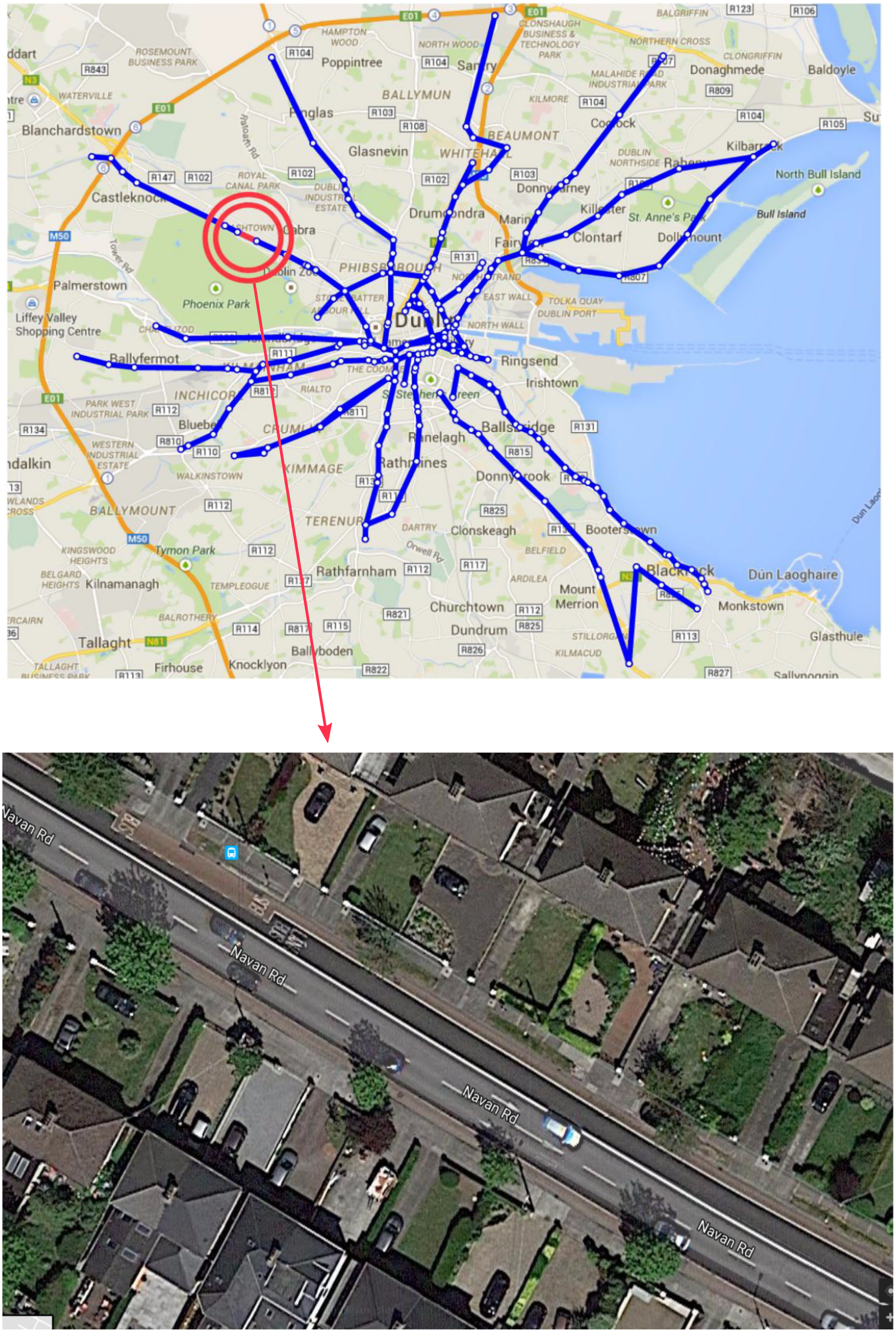

Figure 5.14: Dublin traffic network and selected link. (Source of background: https://maps.google.com)

Once, the rain intensities have been collected, the parameter, h, has to be defined. The returned period for 100 years is measured in 38.3 mm/h, and in order to avoid that possible future hazards exceed this value, making h overcome its maximum, the value to normalize the parameter h has been assumed as 50 mm/h. In this way, it is guaranteed that present, and future hazards do not reach values of h larger than 1.

For the possible identification of outliers among the collected data, the number of standard deviations that the traffic flow differs from the mean has been obtained, before the performance of the calibration. This selection has been done separately for the two analysed periods corresponding to the period of good weather conditions, and the period of rainy conditions.

Therefore, the chosen criteria for the selection has been that, if the number of standards deviations that differ from the mean is larger than 2.8, the corresponding data are considered an outlier. It is noted that the 2.8 value corresponds with a probability of 0.0025.

Once the outliers have been identified, the calibration is performed following the Least Square Method (LSM). This calibration is conducted by analysing the data associated with no weather disruptions, including the link travel times, and the corresponding link saturation degrees. Thus, this calibration is applied to the traffic parameters included in the formulation, such as m, β and γ.

After assessing the traffic parameters, the p_a parameter can be estimated by applying the LSM to the link travel time corresponding with different hazard intensities, h.

In Figure 5.15, the results of the calibration method for this link are presented. The data obtained for the analysed period have been represented by a colour scale indicating the rain intensity, and the shape obtained for the new cost function is represented in the figure for the different intensities of the hazard.

The values obtained for the traffic parameters of the new travel time cost function have been $m = 5.27$, $\beta = 1.02$, and $\gamma = 3.16$. Also, the calibration for the parameter has been introduced, obtaining a value of $p_a = 0.01$. This small value of p_a means that the local vulnerability of the link when exposed to this type of hazard (rain) is very small. It is noted that by analysing the topological characteristics of the link do not present a special sensibility to rain perturbation.

In addition, in Figure 5.15, the calibration for the BPR function has been added for comparison, represented by a dashed black line. For the BPR case, the data have been calibrated all together, since this function does not allow a differentiation among good or bad weather conditions.

To compare the fit of both functions, the Root Mean Square Error (RMSE) has been computed. The results are shown in Table 5.5, for the cases of non-rain data and the case of all data included. Analysing the results, it is noted that the proposed cost function obtains a slightly better fit in both situations. Thus, for the case of non-rainy data, the proposed function has an error of 17.85, and the BPR function of 18.16. In the case where

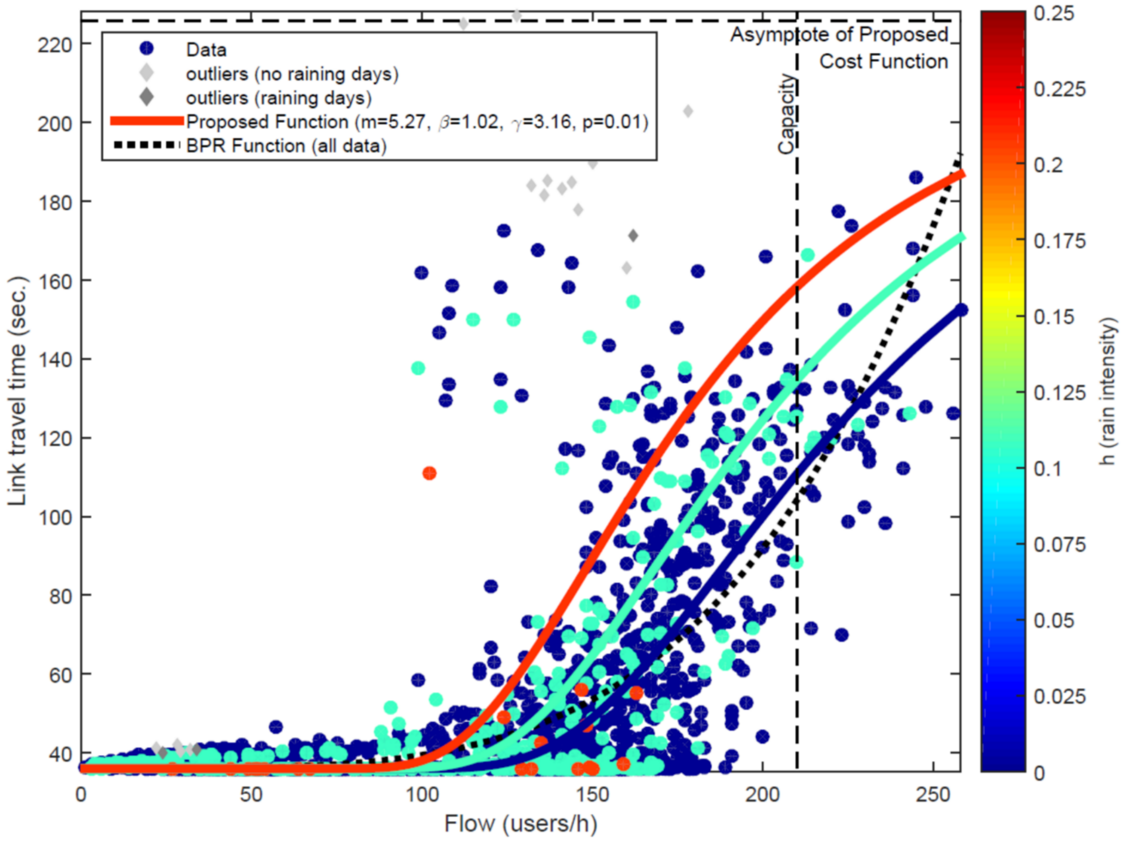

Figure 5.15: Estimation of parameters of the proposed cost function for Ashtown Grove to its intersection with Kinvara Avenue, Dublin (Ireland).

all the data are analysed, the new function has an error of 18.32, and the BPR of 18.42.

Table 5.5: RMSE of the fitting data.

Proposed Function (non-rain data)	**BPR** (non-rain data)	**Proposed Function** (all data)	**BPR**(all data)
17.85	18.16	18.32	18.42

5.9 Conclusions

In this chapter a new link travel time cost function has been presented, which allows the introduction of the effect of climatological events such as rain, snow, fog and wind, on traffic network performance. It is upper-bounded avoiding high computational times, and even possible overflows associated with high traffic congestion levels.

The implementation of this formulation will allow the development of new models to analyse the traffic response when important climatological hazards occur, and will result in a decrease of their associated model uncertainties. In the case of this thesis, the combination of the previous model presented, and this new travel cost function allows a more detailed assessment of the resilience of traffic networks when climatological events take place.

Additionally, the following conclusions can be drawn from this chapter:

Including the effects of the hazard:

- It has been determined by several authors that the intensity of the hazard is a decisive parameter in the impact of a weather event on the traffic behaviour. Therefore, a parameter has been incorporated in the new formulation which includes the intensity of the hazard, h. This parameter allows the simulation of the effect of different intensities of hazards in the performance of the network.

- The proposed formulation is able to reflect the empirical influence of climatological events observed by various studies, which noted reduction on the speed and capacity of the links affected. Thus, the weather parameters proposed in the new cost function represents the effects shown in the literature for extreme weather events on traffic networks.

- The presented cost function captures the fact that for a given hazard, the rate of reduction of speed increases with increments of the degree of saturation. Consequently, the impacts of weather events are larger when the network experiences high degrees of saturation.

- When dealing with extreme weather events, even with conditions near the free flow behaviour, the traffic can experience deterioration on its characteristics. Consequently, the proposed travel cost function, when suffering hazards with high intensities, is able to reflect their effects in the travel times of the network.

- Due to the mathematical properties of the proposed formulation, time-varying hazards can be introduced, which permits its implementation in dynamic analysis.

- It is noted that the number of available data for events that are designed as extreme weather events may not be very large, but the model admits the possibility of introducing climatological events that may not be identified as an extreme weather event but produce a relevant impact on the traffic conditions. Thus, the data that are needed in the application of the model can be also selected during periods where the network was suffering a weather-related event, and their normal behaviour was suffering significant worse conditions.

Including the response of the network:

- The impact of a weather event is determined not only by the hazard intensity, but also by the vulnerability of the network against a specific event. Therefore, the sensitivity of the network needs to be reflected in a formulation to assess the weather impacts. The proposed cost function includes the parameter p_a, the local vulnerability of the network, to reflect the specific sensitivity that has been reported in the literature when a hazard occurs.
- The parameter p_a is specific for each link of the network. This allows a detailed customisation of the network for a more realistic approach. In this way, when a hazard with the same intensity affects the whole network, different responses can be obtained from each of the links depending on their vulnerabilities to that hazard.

General characteristics

- The mathematical formulation presents important regularity properties, such as increasing with flow, strictly monotonic and continuously differentiable.
- In addition to the novelty of including parameters to reflect the impact of hazards, the new cost function is upper-bounded. This characteristic allows the simulation of more realistic scenarios when the traffic performance is analysed. In addition, the upper bounded formulation improves the computational efficiency of the function, which reduces the peaks, and the possible overflows problem when using optimization problems independently of the congestion ratios.

Using the new function with good weather conditions:

- It has been shown that the behaviour of the proposed travel cost function is similar to other cost functions presented in the literature in absence of climatological events.
- The proposed formulation has been calibrated using real data from the Dublin network. The behaviour of the calibrated cost function has been compared with the BPR function, showing that the proposed formulation provides a better fit in both cases, using LSM, in the absence of climatological events and in the presence of weather hazards.

Chapter 6

Mapping and Bi-phase Sensitivity analysis

Contents

6.1 Notation

t_a	actual travel time for link a
t_{a0}	free flow travel time for link a
ν_a^{max}	link capacity to provide certain level of service
p_a	local vulnerability for link a
h	hazard intensity
h_max	maximum intensity of the hazard during the studied period
d_{pq}	demand associated with origin-destination pq
d	uniform demand value used for the representation of network demand
t	time interval
ξ	sensitivity
Z	set of variables analysed in the sensitivity analysis
x	modified variable in the sensitivity analysis
Y	subset of variables which remain constant in the sensitivity analysis
i	percentage used in the sensitivity analysis
R_i^+	model response when one parameter has been increased by the percentage i
R	model response calculated for the initial parameter set Z
α	system impedance

The research included in this chapter has been presented in Martinez-Pastor et al. (2017b), Martinez-Pastor et al. (2016b), and Martinez-Pastor et al. (2016a).

6.2 Motivation

The aim of this chapter is to move one step forward in the understanding of resilience of traffic networks. Due to the novelty of the parameters, and methods previously introduced, a deep analysis of the parameters is developed in this chapter.

For this purpose, the concepts presented in previous chapters are incorporated in a case study, which will be the guiding thread of this chapter. For the parameters analysis, a mapping of multiple scenarios is presented, where it is possible to appreciate the influence of the variation of the parameters for a specific case. In addition, to cover all the possible scenarios, a sensitivity analysis is introduced. The methodology used to perform this analysis is an innovative bi-phase sensitivity analysis, which allows covering the entire range of the parameters with a small sample of points.

The final goal of this analysis is to be able to develop preventive actions when a hazard is forecasted in a traffic network. Therefore, knowing the effects that will cause the modification of the parameters, such as the demand, or the capacity of adaptation of the users on the resilience of the network, it will be possible to adjust these parameters for a smaller impact of the perturbation in the network.

6.3 Introduction

In previous chapters, a methodology to evaluate the resilience of a traffic network when a perturbation occurs (Chapter 4), and a new link travel time function (Chapter 5) have been presented. Both tools, when are used together, have the objective of assessing traffic resilience when a climatological disruption takes place.

Once the methodology has been developed, some questions arise, for example:

- What measures should be taken when a hazard is forecasted?
- What kind of measures will be the more efficient to improve the resilience of the threatened network?
- What are the variables with an important role in the value of resilience?

This chapter aims to answer these questions, and to continue the research line in the evaluation, and improvement of traffic resilience. With this goal, a detailed analysis of the combination of both, the dynamic methodology, and the new cost function, is addressed. The analysis will be focused on the parameters involved in the methodology.

Mapping, and sensitivity analysis are tools that are often used to understand complex models, and the parameters involved. However, in the area of resilience due to its novelty and even more due to the lack of quantitative methodologies to measure it, analyses of the parameters involved are scarce. Therefore, this chapter aims to move a step forward in the understanding of resilience of traffic networks by addressing both, a mapping and a sensitivity analysis.

Thus, a mapping of possible scenarios is introduced. This technique studies the evolution of resilience when some of the parameters involved are modified. This means, that when the results are obtained, and plotted in a graphic, it will be possible to analyse the effects on the value of resilience of the modifications of the studied parameters.

This will allow a better comprehension of the results of the model, since it will be possible to understand not only the modifications caused on the behaviour of the network for the variation of one parameter, but also for different parameters combinations.

It is also highlighted that mapping will help with the decision-making process for the implementation of actions to reduce the impact on the network.

Furthermore, and to be able to classify and determine the role of the parameters of the model in a wider scenario, this chapter introduces a sensitivity analysis. For the development of the sensitivity analysis, a bi-phase sensitivity analysis methodology is proposed. This methodology combines a local, and a global approach, to allow a profound analysis of the model to evaluate resilience. This methodology will be implemented in the model, and different case studies will be addressed to reach a better comprehension of the parameters involved.

It is noted that both methodologies are needed to fully understand the model, mainly due to its complexity. The presented model includes several variables, and when a sensitivity analysis is developed, statistical methodologies need to be used. For that reason, the results of the sensitivity analysis can help to determine the relevance of each of the variables in the results, and how they can be compared in a sensitivity scale. However, only with the results of the sensitivity analysis is not enough to understand in a local context how each of the variables is going to affect the resilience. This means that, if an analysis of a specific case is performed and decisions need to be done to increase the resilience of a traffic network, the results from the sensitivity analysis are not enough, and a mapping is needed to provide support in any decision-making process.

Finally, prior to the mapping, and the sensitivity analysis, an example is introduced, where both tools are applied to measure the resilience of a traffic network. This example will be the one used throughout the whole chapter, and all the methodologies proposed will be applied for this case study. Since, in previous chapters, different size networks have been used for the application of the concepts introduced, this chapter aims to put all together in the same network, looking for a better comprehension of the proposed concepts.

The chapter is organised as follow; Section 6.4 presents a case study where the methodology to quantify traffic resilience, and the proposed cost function to include the weather events are combined. In addition, a comparison with previous methodologies presented in the literature is addressed. In Section 6.5, a detailed study of the parameters involved in the model is developed, together with a mapping of a transport network when a specific area is affected by a hazard. Section 6.6 presents the bi-phase methodology for developing of the sensitivity analysis, and it is applied to the case study presented at the beginning of this Chapter. Finally, some conclusions are drawn in Section 6.7.

6.4 Sioux Fall case study: Evaluation of resilience in traffic networks.

In this section, a larger traffic network is introduced for the application of the previous methods presented. Resilience is evaluated by means of the dynamic restricted equilibrium model where the objective cost function is the new link travel cost function proposed previously. The formulations to assess the values of perturbation, and recovery resilience are the ones presented in Chapter 4.

6.4.1 Description of the network

The network chosen for this chapter is the well-known Sioux Fall network.

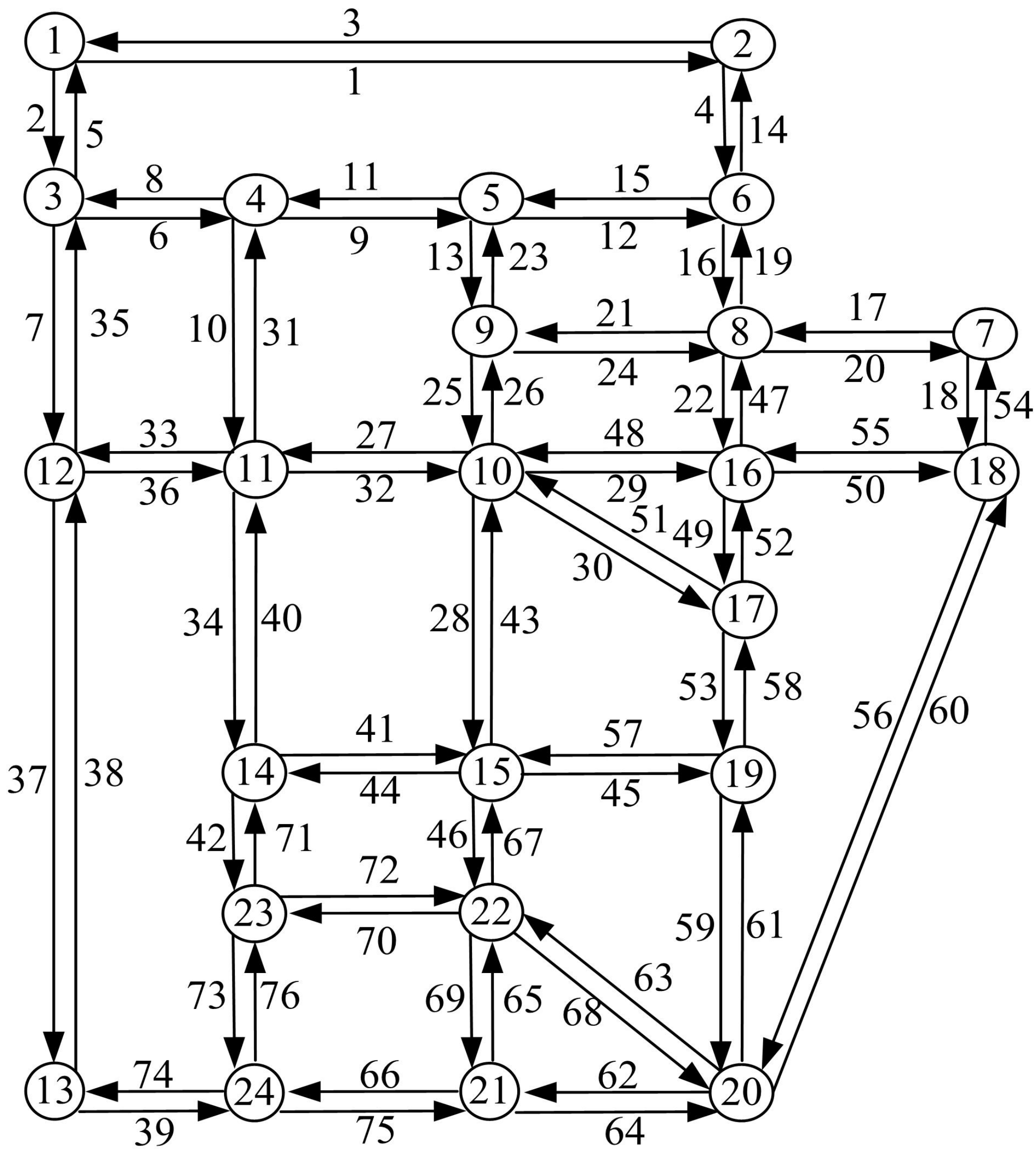

Figure 6.1: Sioux Fall Network, defined by nodes and links.

This network has been widely used in multiple papers, and research projects conducted in the Transport area. Thus, due to the novelty of the methods, a network which many experts are familiarized with may facilitate the study of these new methods and concepts.

The Sioux Fall network is composed by 24 nodes and 76 links, as shown in the graphic description of the network included in Figure 6.1. For this case study, 60 routes have been considered, joining 10 OD pairs. In Table 8.2 a description of the routes given by their links can be found. The demand of the network is 600 users/OD pair, and the capacities of the links, ν_a^{max}, of the network are described in Table 8.1.

Before the perturbation is introduced in the network, an equilibrium state is calculated by using the UE traffic assignment model proposed by Beckmann et al. (1956), and described in Section 2.2.3.

The results of the initial traffic assignment are shown in Table 8.3 and Table 8.4, see Appendix.

In Table 8.3, the link flow, link travel flow, and link free travel time are shown for each link of the network. In Table 8.4, the demand, the route travel time, and the route free travel time are presented for each route described in Table 8.2.

6.4.2 Description of the perturbation

In this case, the network is assumed to be affected by strong winds, which mainly affect the northern part of the network (see Figure 6.2). This can be for example, because the north part of the network is near the sea cost.

The described winds last 17 days, the intensity of the winds varies during this period of time, experiencing a fast increment during the first three days, reaching its maximum intensity the 14^{th} day. From the 14^{th} day, when the intensity of the wind is the highest, the wind starts to decrease gradually, until its disappearance the 28^{th} day. A description of the variation of the intensity of the perturbation is included in Figure 6.2, together with a description of the area affected by the hazard.

When analysing the vulnerability of the links of the network when strong winds occur, it has been considered, that all the links of the network have the same local vulnerability. For this case study, the local vulnerability, p_a, is assumed as 0.2.

Finally, the last parameter necessary is the capacity of adaptation of the users, α. For this example the value of α is 0.5. This means that the users of this network have a medium level of adaptation when the conditions of the network change.

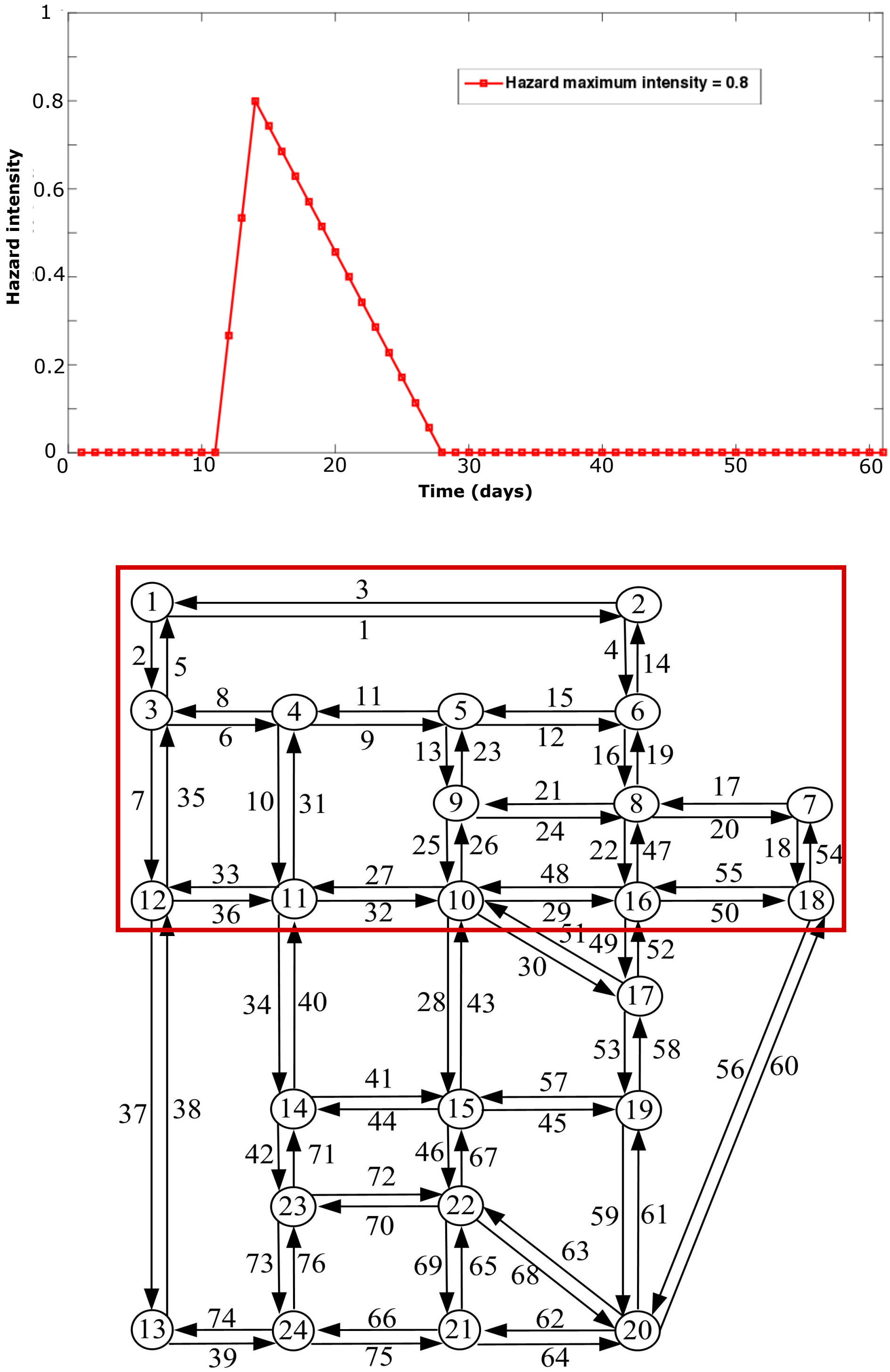

Figure 6.2: Graphical description of the perturbation, by their maximum intensities every day, and area of the network affected by the perturbation.

6.4.3 Results: Resilience of the traffic network.

Once the characteristics of the network, the initial equilibrium state, and the hazard have been defined, the methodology to evaluate resilience can be applied.

As general results of the behaviour of the network during the perturbation, in Figure 6.3, and Figure 6.4, the evolution of the link travel times, and the link flows during the studied period are shown.

In Figure 6.3, the link travel times are presented. Affected links (north part of the network) are shown in the (a), and (b) graph. In a majority of these links, the link travel times increase due to the impact of the intense wind. In the (c) graph, links that are not affected (south part of the network) are shown, and in this case, there is a combination, having links which suffer a decrease in their travel times, and other that increase it.

In Figure 6.4, the link flows are presented. In the same way that in the case of travel times, affected links are shown in the (a), and (b) graph, and no affected links are shown in the (c) graph.

Finally, in Figure 6.5, the evolution of the stress level, the cost level, and the exhaustion level are introduced, together with the final value of perturbation resilience, and recovery resilience.

In this example, even though the hazard reaches a high value of intensity (h=0.8), the perturbation resilience is equal to 83.42%, this means that the network is quite resilient to this type of perturbation. One of the reasons for the high value of perturbation resilience is that the Sioux Fall network has a large redundancy, i.e. there are many options to go from one origin to a destination, also, the network is very "square", therefore there will be route options with not very large differences, and when one of them is damaged, the users can use other options.

On the other hand, when analysing the results obtained in Figure 6.5, the value of the recovery resilience is very low (Recovery resilience= 2.41 %). This fact is due to the higher levels of stress suffered by the network during, and after the perturbation. It is noted that even when the perturbation is finished, and the cost level is near zero, the stress level remains in high values for a long period of time until the equilibrium is recovered. In addition, it is important to highlight that the demand used in this example is reasonably high with 600 users/OD, which will result in lower values of both the perturbation, and the recovery resilience.

Finally, a new case is evaluated for a better knowledge of the performance of this network. In this case, in addition to the forecasted wind with a maximum intensity of 0.8 the 14^{th} day, an increment of the wind will impact the network during the following days (see Figure 6.6). Thus, the new wind is expected to increase its intensity from day 19^{th}, and reach its maximum intensity the day 20^{th} with a value of 0.7. Finally the wind will gradually decrease its intensity until disappear the day 28^{th}.

The results obtained for the second case are shown in Figure 6.6, as expected, since

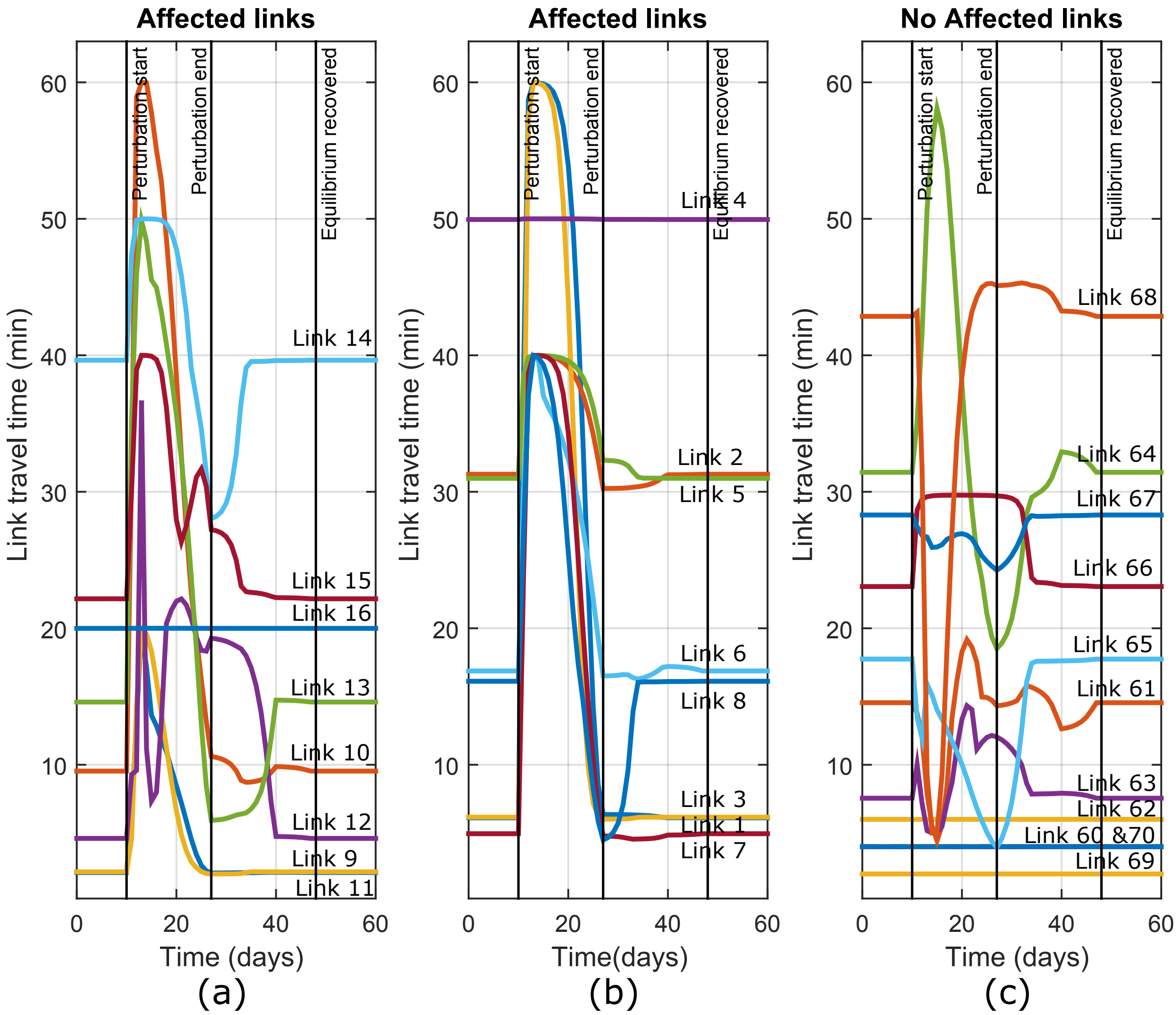

Figure 6.3: Evolution of the link travel times during the studied period for the Sioux Fall case study.

the combination of two hazards is affecting the network this time, both, the perturbation, and the recovery resilience have been reduced. In this case, the value for the perturbation resilience is 79.61%, and for the recovery resilience is 2.01%. It is noted that the network, when analysing the perturbation stage, reaches similar values as in the previous example for the exhaustion level, which does not go over 0.5. In this case, a second peak in the exhaustion level is shown when the second wind impacts the network with an intensity of 0.7, but since the intensity is smaller than in the previous peak, the exhaustion level reaches a lower value than the one obtained in the first peak.

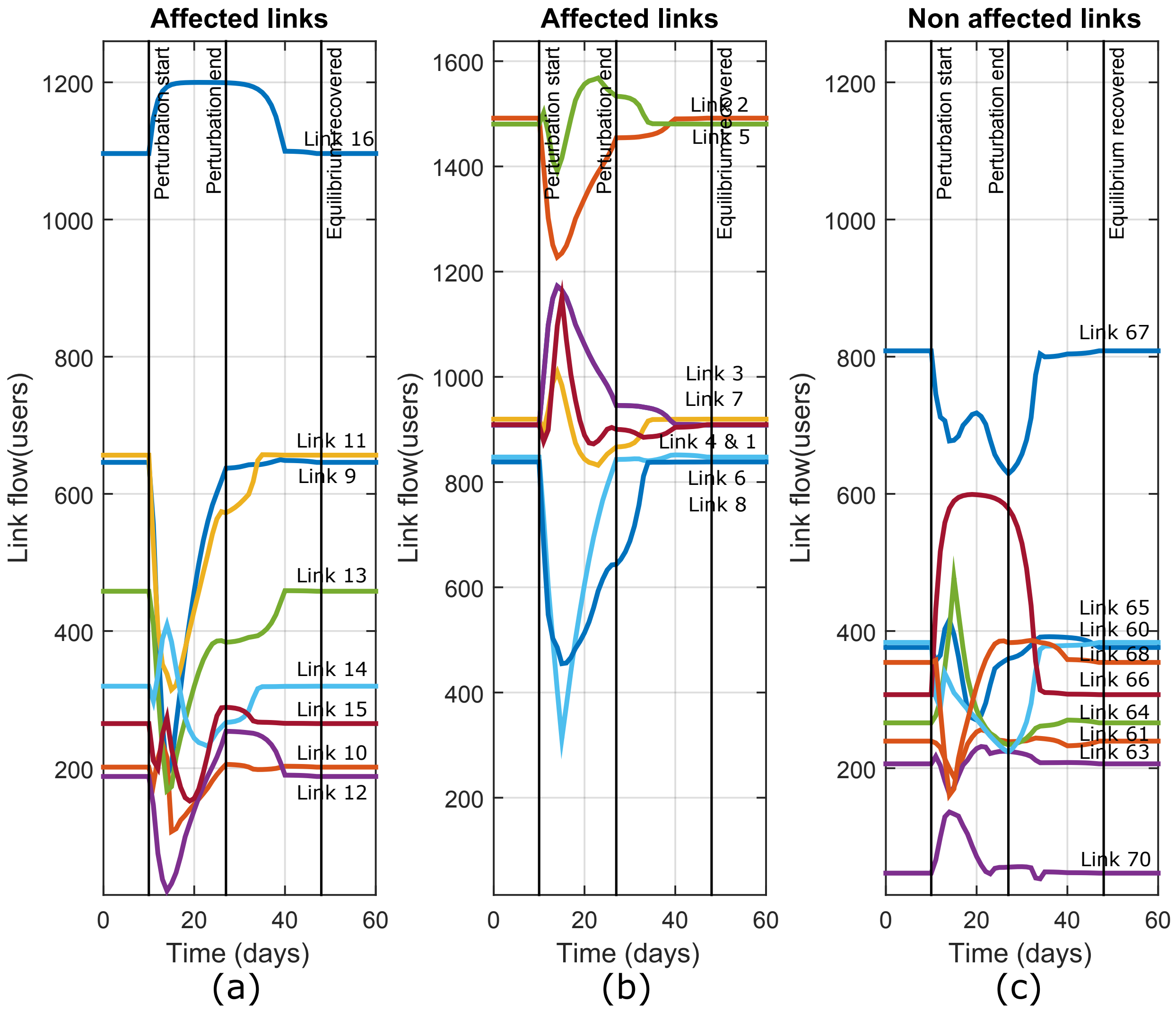

Figure 6.4: Evolution of the link flows during the studied period for the Sioux Fall case study.

Regarding the recovery resilience, the stress level is the most affected, as in the previous example. The fact that the stress level reaches very high values, and during a long period, occurs mainly due to the characteristics of the network, such as the high redundancy as previously explained.

With this case study, it has been shown that this methodology provides multiple options when the impacts of perturbations need to be evaluated. In addition, it is highlighted that the methodology can be applied at any traffic network by using data that are usually available for these networks.

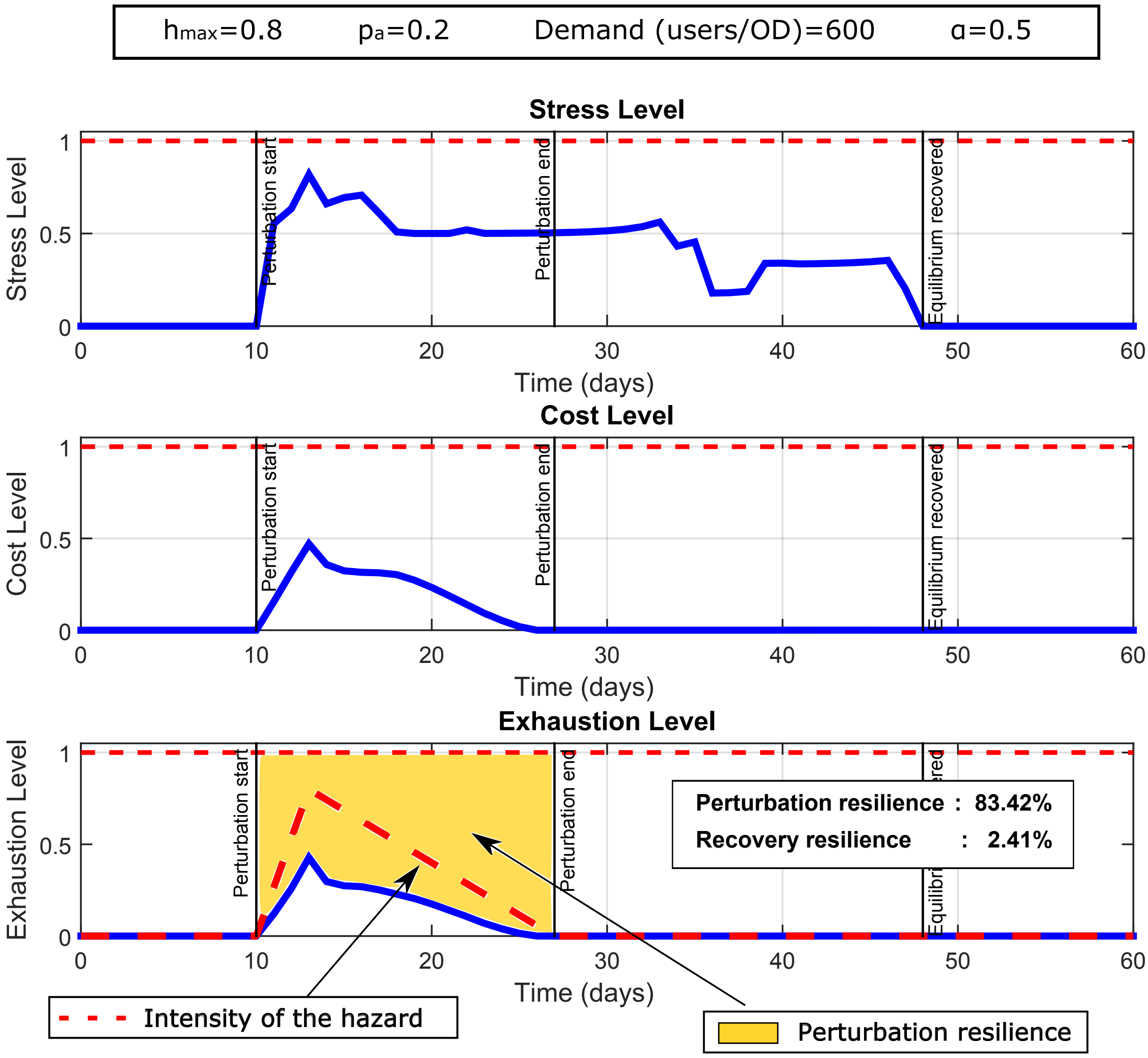

Figure 6.5: Resilience evaluation for the Sioux Fall case study. h = 0.8, Demand = 600 users/OD

6.4.4 Comparison of the new methodology to evaluate resilience with previous methodologies.

In this section, the presented methodology is compared with the methodologies to evaluate resilience presented in literature. In Chapter 1, several methodologies proposed by multiple authors were described, also, in Section 1.4.2, some of the disadvantages found in each of the methods were highlighted. Even though the main advantages of the model have been presented in the previous chapter, this section explains how the new model, including the new travel cost function, fills the gaps that previous models present.

In Table 6.1, the disadvantages found in Section 1.4.2 of Chapter 1, are included, and

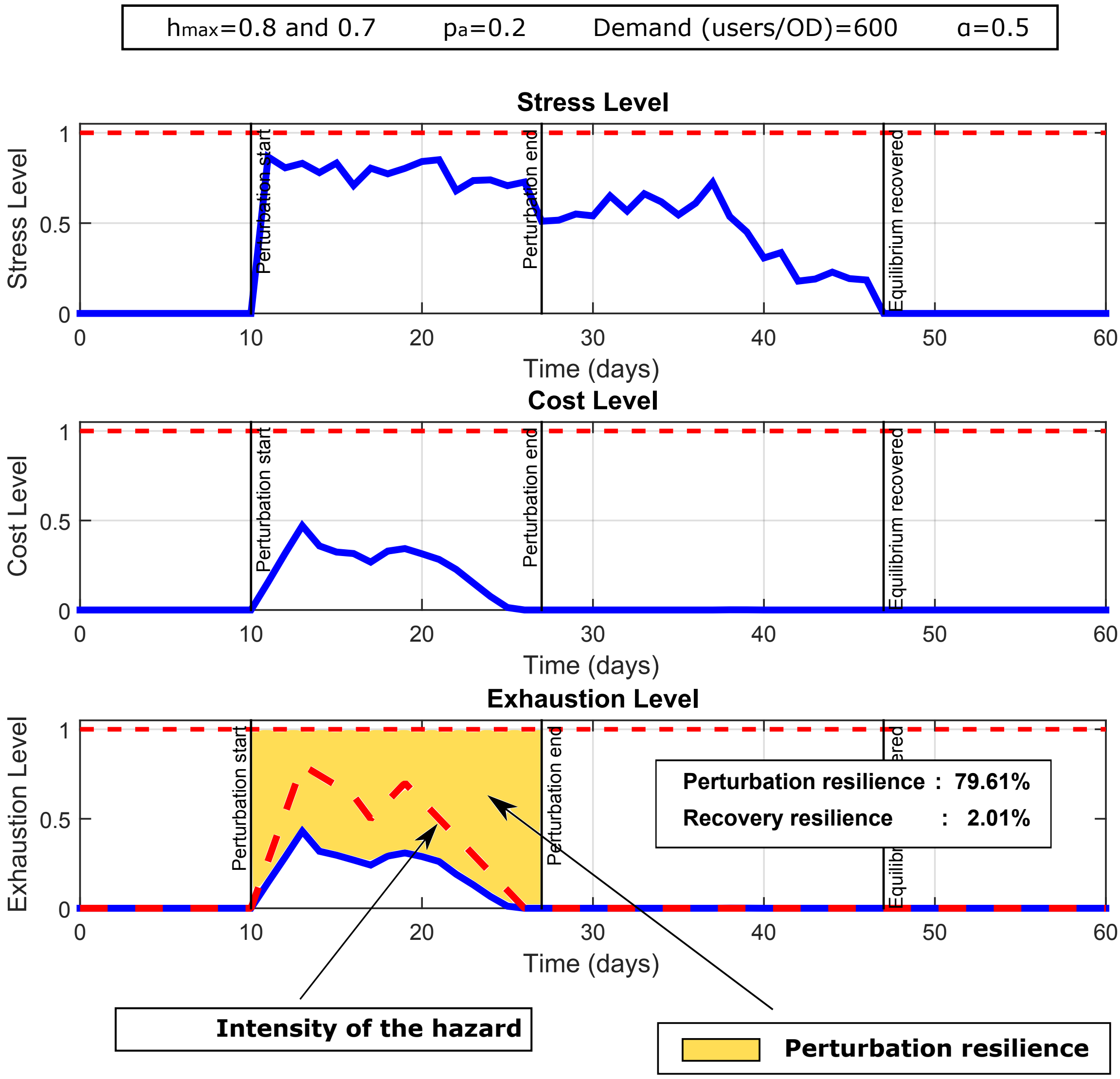

Figure 6.6: Resilience evaluation for the Sioux Fall case study. h = 0.8 and 0.7, Demand = 600 users/OD

how they are addressed by the new method explained.

Table 6.1: Comparison of the new methodology to evaluate resilience in traffic networks with previously methods presented in the literature.

Disadvantages found in previous methodologies	**New model approach**
1. **Semi-quantification of the concept.** Some authors present methodologies that do not quantify resilience.	The proposed method quantifies the perturbation, and recovery resilience, giving a number between 0 and 100 for each of them. At the same time, the evaluation for each time interval of the exhaustion level of the network is calculated.
2. **Evaluation of resilience by the quantification of different metrics.** Several approaches address the concept by the evaluation of multiple metrics, including concepts such as redundancy, robustness and rapidity.	It has been presented in Section 4.8 that the proposed methodology covers many of the aspects included in these metrics when evaluating resilience. In addition, the quantification becomes more efficient when these parameters are indirectly included than when each of them need to be evaluated separately.
3. **Evaluation only from the moment that the hazard is finished (recovery stage).** Some methods are focused only on the evaluation of the consequences after the disrupted event.	The proposed methodology allows the complete evaluation of the perturbation stage during the event, and the recovery stage, after the event until stable conditions are reached again. In addition, complete information of the network situation is provided at any time, that is, cost, stress, and exhaustion level.

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Disadvantages found in previous methodologies	**New model approach**
4. **Analysis of the perturbation as a "punctual" damage.** Many analysis describe the hazard as a damage that occurs in a short time interval (instantly) in the network, without being able to describe dynamic hazards that vary during over time.	The proposed method allows the simulation of a dynamic hazard, which is defined by its intensity at any time. In addition, multiple hazards can be simulated in the network for the same evaluation.
5. **Measuring the resilience by external actions.** Some methods evaluate resilience mainly depending on the external actions done to reduce the impact.	The proposed method allows the evaluation of traffic resilience when no external actions are included, i.e. the own resilience of the system. In addition, external actions to reduce the impact, and increase the resilience can be introduced, and their efficiency in terms of resilience evaluated.
6. **Necessity of a very wide framework to define resilience.** It was shown in Chapter 1 that some methodologies need a large amount of resources to evaluate the resilience of a network	The proposed method simplifies the evaluation of resilience in traffic networks, since the required data can be easier accessible than in other methods, such as topological information of the network, demand of each OD pair, and description of the link local vulnerability.

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Disadvantages found in previous methodologies	New model approach
7. **Lack of methods that include the stress caused in the users by the perturbation.**	The proposed method includes the stress that the users suffer when a perturbation takes place in the network. The modification of the usual routes chosen by the users will deteriorate the characteristics of the normal operation of the network.

6.5 Mapping of possible scenarios

In the previous section, the full methodology to evaluate traffic resilience has been tested in the Sioux Fall network, and two cases, where the network is impacted by different perturbations have been presented. For a better understanding of the model, a mapping where multiple possible scenarios are tested is developed in this section.

Mapping allows recreating in a more visual way the behaviour of the network, and hence the performance of the methodology. Mapping is a tool that highlights the way that a variation of the parameters can affect the results of the model, and can help the decision-making process at the moment of dealing with extreme weather events. For example, when a perturbation is forecasted and the managers of the transport network need to implement measures to reduce the impact.

Mapping becomes a relevant tool for all the different phases in which the traffic network is involved, including before, during, and after any kind of perturbation.

- **Before the weather event takes place**, mapping can analyse the behaviour of each parameter for the scenarios that have been chosen for the analysis. In this way, the real impact that these measures will have on the value of resilience can be tested. It is noted that multiple variations of the parameters that can be modified prior to the impact can be included in the analysis.

  Therefore, in this phase, mapping makes the simulation of the perturbation scenarios possible, understanding how the resilience will change if some conditions of the network are modified and therefore, allowing to develop adaptation measures before the hazard, minimizing its impact.

- **During the weather event**, mapping will facilitate decision-making for the managers of the infrastructure while the hazard is occurring. The model for the assessment of traffic resilience is a dynamic model, and the evolution of the hazard can change. For example, the way that the hazard is forecasted may vary considerably, with modifications not only between different cases but also within the same perturbation.

  Therefore, in this phase, mapping can provide crucial information even once the perturbation has started, helping to develop the mitigation strategies during the perturbation stage.

- **After the weather event**, mapping becomes a relevant tool for the improvement of traffic resilience in a network too. Once the perturbation has finished, an overall perspective of the harmful situation can be addressed more precisely, knowing at this point the real scenario, and the value of the variables involved. The overall approach after the hazard will allow a better understanding of the extreme weather events, and this learning will be used in the case of future perturbations, improving the decision-making and the knowledge of the reaction of the network against climatological impacts.

It is important to highlight that some changes in specific parameters may not produce a relevant improvement of the resilience of some traffic networks, and also, their modification can imply very high cost. Then, the more relevant information available, the more profitable is the management of the resources. The results of mapping enable the response of the model for the modification of more than one parameter, thus a variation in multiple combinations of parameters can also be analysed.

6.5.1 Parameter analysis

For the performance of the mapping, the parameters of the model included are those that can be modified before, during, or after the perturbation for a better behaviour of the network when the hazard takes place. These parameters are the normalised intensity of the hazard, h, the vulnerability of the link, p_a, the capacity of adaptation of the users, α, and the demand, d_{pq}.

The characteristics of each of these parameters are highlighted as follows:

- **The intensity of the hazard, $\boldsymbol{h}$**, is a dimensionless parameter whose range goes from 0, when there is not a hazard, to 1 for the maximum intensity. This parameter determines the evolution of the hazard with the time, the proposed dynamic model to evaluate the resilience allows the consideration of these temporal changes by introducing the corresponding intensity for each time step. The intensity of the hazard is a crucial parameter for the evaluation of resilience.

- **The demand,** d_{pq}, is the number of users travelling from origin p to destination q, p and q being different nodes of the traffic network. When this parameter is introduced into the dynamic traffic assignment model, the link flows over time are calculated. To assess the influence of the demand for the different case studies, without loss of generality, a uniform demand value is used for all the OD-pair analysed, denoted as d.

- **The capacity of adaptation of the users,** α, is a dimensionless parameter whose range goes from 0, when the users in the network have a null capacity of adaptation, to 1 when the capacity of adaptation is the largest, i.e. the adaptation is done immediately. This parameter allows the introduction of the user domain in the evaluation of resilience. For the quantification of α, aspects such as the quantity, and quality of information that the users have about the conditions of the network, and if the users are determined to change their usual route for small changes, or they need a large change to modify their routes, are evaluated.

- **The vulnerability of the link,** p_a, is a dimensionless parameter whose range goes from 0, when the link is not vulnerable to the climatological perturbation considered, to 1 when the link is completely vulnerable to the considered perturbation. This parameter can significantly change depending on the weather events that affect a link. For example, some links may be very vulnerable to intense rain, due to a high probability of flooding in the area of the link, however the same links may have a low vulnerability when intense winds affect the area because, for example, their location is more protected against winds.

6.5.2 Mapping of a transport network when a specific area is affected by a hazard

Based on the case study developed at the beginning of this chapter (Sioux Fall network), the mapping for multiple scenarios is developed in this section.

Therefore, several climatological hazards are simulated to impact upon the transport network, the impact of the perturbation is evaluated with a combination of the method to evaluate resilience proposed in Chapter 4, and the new travel cost function presented in Chapter 5.

Four different perturbations are analysed, with four different maximum intensities, that is, 0.1, 0.3, 0.6 and 0.8. In all the cases, the perturbation starts on the 11^{th} day and finishes on the 27^{th} day, having its maximum intensity on the 14^{th} day. In Figure 6.7, the evolution on time of the four different hazards is drawn. It is noted that the perturbations

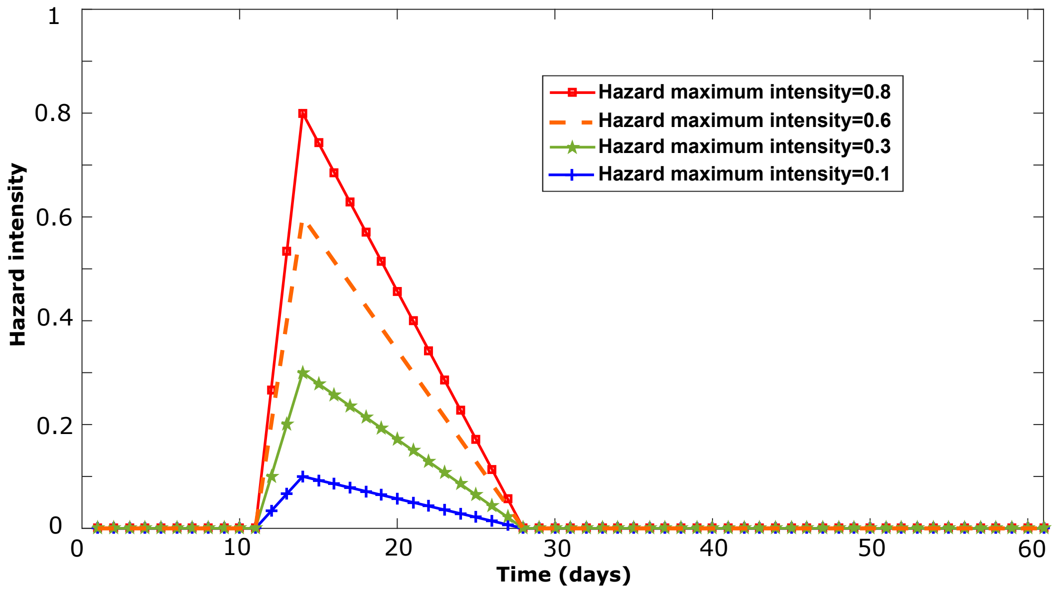

Figure 6.7: Evolution of the hazards for various intensities

have the same evolution than the one presented in the first example (see Figure 6.2), but in this case different values of h are tested.

Also, the area affected by the hazard is the same as the one used in the study case, see Figure 6.2, where the area is indicated by a red square.

In order to cover the entire range of definition of the variables involved, the values chosen for the mapping analysis are described as follows:

(a) The hazard intensity, with the values of 0.1, 0.3, 0.6 and 0.8, is selected, with the evolution previously described.

(b) The demand will be analysed for the cases of 120, 300, 600 and 900 users/OD. It is highlighted that larger values of the demand could imply unrealistic saturations levels, and lower values will not represent significant changes in the analysis.

(c) The capacity of adaptation, α, will adopt the whole range from 0 to 1 in incremnets of 0.1 in each of the cases presented.

(d) The local link vulnerability, p_a will adopt the values of 0.2 and 0.8 for each case.

In this way, 320 possible scenarios are going to be analysed, measuring for each possible combination of the described parameters the perturbation resilience. Thus, a complete picture of the situation can be evaluated, and a better understanding of the effect caused by a variation of the parameters in the value of resilience is obtained.

The results of the mapping are shown in Figure 6.8, each of the graphs represents perturbation resilience vs. α, for the two types of vulnerability considered, that is, 0.2 (represented with a blue line in the graphic) and 0.8 (represented with a red line in the graphic). In every column of Figure 6.8, a different value of d is evaluated, and each row is representing a different maximum intensity of the hazards.

In addition, a code of three colours have been applied to each of the graphics, in order to distinguish the impact of the variables of the graphic on the value of resilience. Therefore, the graphics in green have a combination of variables that does not reduce in a relevant quantity the value of resilience; the graphics in yellow have a combination of variables that creates an intermediate reduction of the resilience; and finally, the graphics in red have a combination of variables that produces a significant reduction of the resilience.

Firstly, it is shown that in the cases when either the intensity of the hazard or the demand is low, the values of the resilience are almost constant, that means that the network is able to absorb the hazard with a small disruption and the improvement of the rest of variables involved will not significantly increase the total value of the resilience.

On the contrary, for those cases when the demand and the intensity of the hazard are high, the performance of the perturbation resilience is more sensitive to variations of parameters such as the demand, d, the intensity of the hazard, h and α.

In addition, it is appreciated that in most of the cases, the variable p has a smaller role than the rest of the variables analysed, since the modification of this variable does not significantly affect the value of the resilience.

Considering for example that, for a forecasted hazard with $h = 0.8$, the demand in the network for the period analysed is 900 users/OD, α is 0.1 and p is 0.8, the resulting value of perturbation resilience obtained is 64.93% as shown in Figure 6.8, last subplot. Therefore, in this case, the resilience of the network is quite affected by the hazard.

In this case, some of the different strategies to improve the performance of the network when affected by the hazard can be (a) to reduce the demand of users in the network (moving to the left subplot of the last row), (b) to increment α (moving to the right through the x axis of the subplot), both of them applicable in a short-term basis, and (c) to reduce the vulnerability of the links. This last strategy requires a long term perspective.

Using the first strategy, moving from the predicted demand of 900 users/OD to a demand of 300 users/OD, the value of perturbation resilience increases from 64.93% up to 89.34%. This strategy can be achieved by means of measures such as limiting the number of vehicles that can use the network, for example allowing only the odd or even numbers depending on the day; promoting the public transport; and transferring the users to other transport modes such as underground or tram, where available.

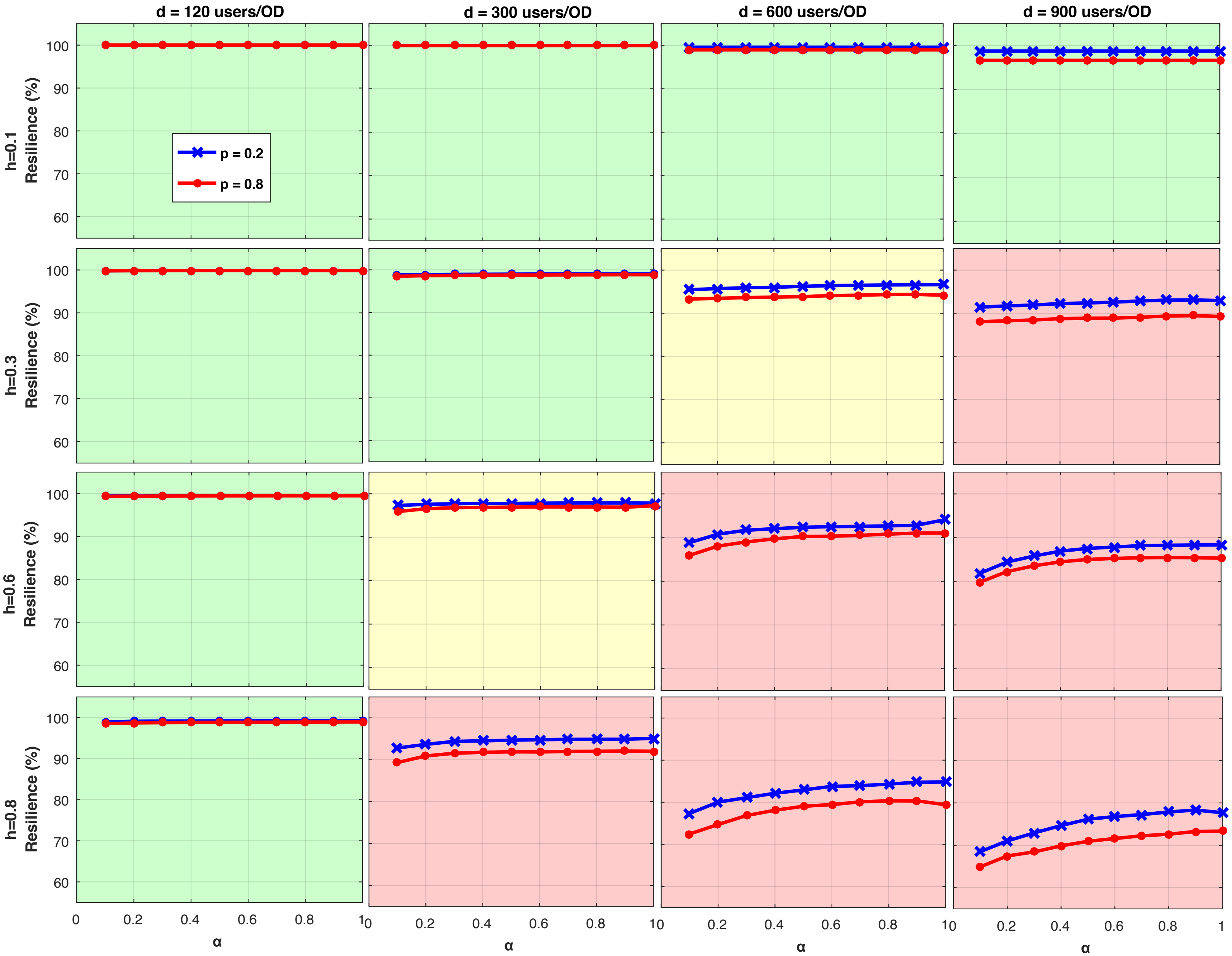

Figure 6.8: Mapping of possible scenarios for the case study

In the second strategy, increasing the capacity of adaptation of the users, from 0.1 to 0.7, the perturbation resilience increases up to 72.25%. This strategy can be achieved giving more information to the users about the real situation of network, this can be done through ATIS (Advance Traveller Information Systems), this will allow the users to choose better routes and hence reduce the stress of the situation.

In order to identify the most efficient measures, a mapping analysis should be addressed for a proper decision-making process, before, during and after the event. Since a mapping approach allows the managers of the network to have more information of the situation and to invest the resources in a more appropriate way. Deciding when, how many and what type of measures are needed for the improvement of the resilience of the network.

It is important to highlight that mapping provides a global vision of the problem, and the obtained solutions can be easily interpreted. However, it needs to be conducted for each situation, with an estimation of the value of the variables, such as the area affected by the hazard, the network analysed or the number of routes per OD in the network. Therefore, the conclusions about the relevance of the parameters are specific to each case study. To solve this problem, and with the objective of fully understanding the concept of resilience and the parameters involved in a wide perspective, it is necessary to develop a sensitivity analysis, which will be addressed in the following section of this chapter.

6.6 Sensitivity analysis: bi-phase approach

Sensitivity analyses identify the influence of each parameter in the outputs of the model. This kind of analysis allows for a profound knowledge of the methodology presented in this thesis. In addition, the definition of the inputs would be more efficient after knowing how these parameters can modify and influence the model.

It is also relevant to differentiate between important and sensitivity parameters, Hamby (1995) distinguishes by "important" those parameters whose uncertainty makes a substantial contribution to the *uncertainty* in the assessment results and "sensitive" those parameters which have a notable influence on the assessment results. An "important" parameter has to be "sensitive", however a "sensitive" parameter has not to be defined as "important", since the variability of a parameter will not appear in the outputs unless the model is sensitive to the uncertainty of the inputs.

Different methodologies to make sensitivity analysis have been developed previously. An initial classification can distinguish between global and local methodologies, based on two different levels to carry out a sensitivity analysis.

A sensitivity analysis can be defined as "the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input" (Saltelli et al. (2004)). Some of the main reasons to develop a sensitivity analysis are highlighted;

- The model parameters require additional research for strengthening the knowledge base, thereby the output uncertainty is reduced.

- Some model parameters might have a negligible contribution, therefore they can be eliminated from the final model. This would result in a reduction of the required computational time.

- Bigger effort should be made in defining those variables and parameters with larger contribution into the model. The sensitivity analysis allows the identification of those important variables and parameters.

- The consequences in the results can be determined when changing a given input parameter or variable.

A local sensitivity analysis evaluates sensitivity at one point in the parameter hyperspace. The local techniques aim to estimate the local impact of a parameter on the model output. A sensitivity coefficient is obtained, which is basically the ratio of the change in the output to the change in the input while all other parameters remain constant (Krieger and Albright (1977)). Some methodologies to develop a local analysis are, (a) differential sensitivity analysis, based on partial differentiation of the model in aggregated form; (b) one-at-a-time measures, which is one of the simplest methods to develop a sensitivity analysis, which is based on repeatedly varying one parameter at a time while holding the others fixed, and (c) the sensitivity index, which calculates the output percentage difference when varying one input parameter from its minimum value to its maximum value.

Global sampling methods scan in a random or systematic way the entire range of possible parameter values and possible parameter sets. These techniques analyse the whole parameter space at once. Some methodologies to develop a global analysis are, (a) simple random sampling, using Monte-Carlo analysis. This method works by generating a random value of the variable analysed and scaling this one to the target variable via its probability distribution. (b) Stratified sampling, which represents an improvement over simple random sampling by forcing the sample to conform to the whole distribution being analysed. Any reduction in the number of simulations required for a Monte-Carlo analysis will result in a reduction in computational effort. This is an example of how some techniques have evolved and can outperform the simple random sampling. (c) The Latin-Hypercube simulation is a method of sampling that can be used to produce input values for estimation of expectations of functions of output variables (McKay et al. (2000)). The method works by dividing the input into strata and then generating samples so that the value generated for each parameter comes from a different stratum (Helton and Davis (2003)).

6.6.1 Bi-phase sensitivity analysis

This section introduces an alternative methodology to develop a sensitivity analysis. The applied methodology is an integration of two phases, a local into a global sensitivity method. More precisely, a combination of One-At-a-Time (OAT) for the local sensitivity and Latin Hypercube (LH) sampling for a global approach (see Van Griensven et al. (2006)).

- **Upper level approach**. In order to address the first phase of this sensitivity methodology, a sampling strategy is carried out. The selected global sampling procedure is the LH that allows the reduction of the sample size. Due to the importance of the pairing procedure, the Translational Propagation algorithm proposed by Viana et al. (2010) has been implemented in this analysis. The main advantage of this methodology is that it requires virtually no computational time. When the sample is obtained, the local sensitivity analysis can be accomplished as follows. Considering that the total space is covered and the sample is a reliable and robust representation of the entire space, the model is evaluated for each point of the sample, using a local sensitivity analysis.

- **Lower level approach**. On the other hand, the second phase of this methodology is based on a local technique to evaluate the sensitivity. According to the OAT technique, the analysis is performed by modifying every variable in each sample point by a percentage to calculate the corresponding model response in that closed point. It is important to modify one variable each time to identify the behaviour of the modified variable only. Measuring the variation, according to the OAT methodology, the sensitivity in each point is captured. This local method is as simple as efficient, however this process can become quite computationally intensive with larger models. Therefore, instead of applying it in a large number of points to cover the entire range of the parameters, a global methodology has been chosen to obtain a sample of points that represents the different variables. The formulation to assess the sensitivity is based on the concept of derivative, that is

$$\xi = \frac{R_i^+(x,Y) - R(Z)}{i}, x, Y \in Z, \tag{6.1}$$

where Z is the set of variables involved in the model, x, is the modified variable and Y, the subset of variables which remain constant. R is the model response calculated for the initial parameter set Z and R_i^+ is the model response when one parameter has been increased by a percentage i. Sensitivity, denoted by ξ, is a dimensionless parameter. The percentage i, is also a critical point, as small values can exhibit the numerical instabilities in the model, being the obtained behaviour not according with the tendency of the model. However, if this value is too large, the derivative loses its meaning. Accordingly, in this example the percentage will be 5%, avoiding the effects mentioned above.

6.6.2 Study case: bi-phase sensitivity analysis

The sensitivity analysis is performed over the Sioux Fall network shown in Figure 6.2, characteristics are shown in Section 6.4.

Each variable has been divided in sets of 25 points according to the Latin Hypercube method described previously, covering the entire range of the parameters statistically. One of the main advantages of the approach is that, with the analysis of a reduced number of points, general conclusions can be drawn for all the variables. The analysis of the 25 points in a four dimensions space implies the evaluation of 390625 cases. With the pairing system proposed by Viana et al. (2010), 25 combinations of these points are analysed, which are the ones shown in Table 6.2.

Once the methodology has been applied, the obtained results are shown in Figure 6.9 and Table 6.3. These results will be used as a baseline for comparison with the different cases discussed below.

In Figure 6.9, the results are divided in 4 graphics, each of them for a variable, α, p, h, and d. Table 6.3 shows a summary of the results, identifying the highest value for each parameter and showing the average value of the sensitivity for three intervals. The chosen intervals are defined as: (a) low, defined from 0-0.333 in the case of α, p, and h, and defined from 0-480 users/OD, in the case of d, (b) medium, defined from 0.333-0.666 in the case of α, p, and h, and defined from 480-660 users/OD, in the case of d, and (c) high, defined from 0.666-1 in the case of α, p, and h, and defined from 660-840 users/OD, in the case of d. Thus, for each of the intervals defined, an average value for the sensitivity is obtained and shown in the second column in Table 6.3.

Those points distant from the tendency are due to the method used to select the points, the Latin Hypercube, which statistically covers the entire range of the variables involved. Each of the 25 points analysed has a value for α, p, h, and d, therefore, when one variable α is being analysed, large differences in the rest of the variables can occur. For example, the point selected with a red square in the figures, corresponding with the Point 20 in Table 6.2, with values $\alpha = 0.615$, $p = 0.345$, $h = 0.802$, and $d = 672$, is clearly out of the tendency. Additionally, the point selected with a green circle (Point 21 in Table 6.2), with $\alpha = 0.645$, has a value of h= 0.073, significantly smaller than in the previous case, which finally makes the sensitivity smaller. In consequence, this methodology is used to analyse the tendency of the parameters rather than local changes, since multiples scenarios are being analysed with a small number of points. On the other hand, this methodology allows the analysis of the parameters of large models or those that require a long time for each iteration, since they will be highly time consuming using other local methodologies to develop sensitivity analysis.

Table 6.2: Points selected by the Latin Hypercube method

Point	α	p_a	h	d(users/OD pair)
1	0.230	0.093	0.086	444
2	0.268	0.119	0.109	468
3	0.308	0.130	0.127	504
4	0.342	0.148	0.156	516
5	0.795	0.164	0.169	528
6	0.370	0.508	0.190	540
7	0.402	0.185	0.520	552
8	0.437	0.207	0.014	558
9	0.823	0.219	0.037	570
10	0.474	0.561	0.060	576
11	0.514	0.005	0.211	588
12	0.892	0.037	0.225	594
13	0.020	0.258	0.250	600
14	0.538	0.262	0.274	612
15	0.922	0.295	0.288	104
16	0.126	0.679	0.323	624
17	0.560	0.771	0.353	642
18	0.605	0.061	0.568	648
19	0.149	0.330	0.667	666
20	0.615	0.345	0.802	672
21	0.645	0.380	0.073	690
22	0.679	0.082	0.374	714
23	0.188	0.418	0.418	726
24	0.716	0.455	0.449	756
25	0.758	0.497	0.473	804

It is possible to identify two groups of variables, those that when the value is incremented, a positive increment in the total value of resilience is created, i.e. the sensitivity value for these variables will be positive. In this group is the variable α. In the other group, those variables that will experience a decrease in the value of resilience when its value is increased, that means that these variables will have negative values of sensitivity and the higher their values the lower the value of resilience, in this group are p, h, and d.

Figure 6.9 and Case 1 (Baseline case) in Table 6.3 show that two strong variables exist with a biggest impact upon the value of resilience, the intensity of the hazard and the demand. These variables have the biggest ranges with maximum absolute values of -29.326 for h and -48.405 for d. In addition, both variables for the average behaviour show a tendency that the higher the value of the variable, the higher the value of sensitivity.

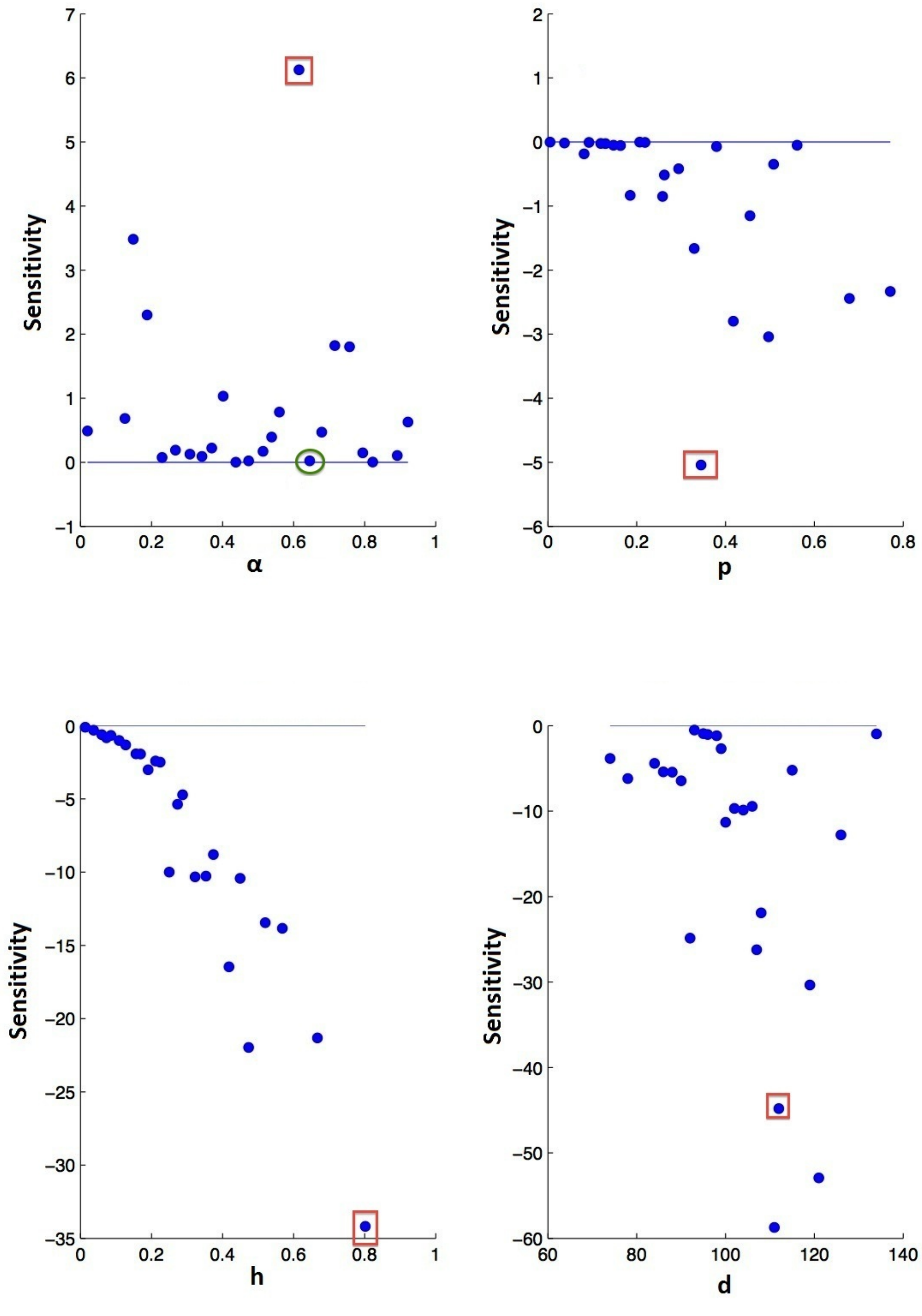

Figure 6.9: Bi-phase sensitivity analysis

This tendency is even stronger in the case of h, where the graphic shows a strong exponential tendency for this parameter. Therefore, the modification of these parameters will cause a high impact in the value of resilience inspite of the change in the rest of the variables.

In the following section, the bi-phase sensitivity analysis is developed in more examples, modifying other characteristics which are susceptible of a high impact on the results.

6.6.3 Influence of the area affected by the hazard

It has been shown than the intensity of the hazard, h, is a very sensitive parameter, however other variables related to the hazard can be very influential, for example, the area affected by the hazard. Therefore, the influence of the area affected is studied, considering the three areas chosen arbitrary to represent a small, medium, and large affected area.

Firstly, this area has been reduced, as represented in Figure 6.10, and the results of the sensitivity analysis for this case are shown in Case 2 in Table 6.3.

In Case 2 (Table 6.3), as expected, since the area affected by the hazard is smaller, the sensitivity for h has been significantly reduced. In Case 2 (Table 6.3), the highest value of h was -29.326 and in this case this value is reduced up to -6.584, which is an important decrease. But the area affected by the hazard not only influences the sensitivity of the variable h. If the biggest value obtained for the sensitivity of d is compared, it is shown that changes from -48.405 in Case 1 to a value of -7.889 in this example (Case 2). Therefore important changes occur in the sensitivity of d. The explanation of this effect is because the network is less affected by the hazard. Consequently, the network is less exhausted, therefore other variables such as the demand will be less sensitive to the changes.

Secondly, the area affected by the hazard has been amplified; in this case, the hazard is affecting the whole network. The results of the sensitivity analysis are shown in Case 3 in Table 6.3.

It is noted that the increment in the sensitivity of h, the maximum value reached is -36.888 (Case 3), in the intervals for low values, the sensitivity of h increases from -2.114 to -4.719 (see Table 6.3, Case 1 and 3), and in the medium values, the sensitivity increases from -11.906 to -26.926, (see Table 3, Case 1 and 3). The sensitivity obtained for the interval of high values, i.e. (0.666-1), the sensitivity is zero. This result is because the network is completely exhausted and a critical level has been reached, for that reason the value of resilience goes to zero, meaning that the network cannot absorb the perturbation due to the intensity of the hazard. Therefore, for these values of h, the network is not able to absorb this perturbation without a complete failure.

Table 6.3: Results for the initial case study

Parameter	Interval	Case 1	Case 2	Case 3	Case 4	Case 5
		Baseline	Influence of the extent of the hazard		Influence of the redundancy	
α		Sensitivity	Sensitivity	Sensitivity	Sensitivity	Sensitivity
Maximum absolute value	0 - 1	5.411	1.866	3.951	23.098	0.930
Average value for low interval	0 - 0.333	0.946	0.461	1.225	1.680	0.192
Average value for medium interval	0.333 - 0.666	0.680	0.171	0.601	1.566	0.104
Average value for high interval	0.666 - 1	0.546	0.257	0.799	4.610	0.153
p	Interval	Sensitivity	Sensitivity	Sensitivity	Sensitivity	Sensitivity
Maximum absolute value	0 - 1	-4.392	-3.004	-5.196	-12.259	-2.446
Average value for low interval	0 - 0.333	-0.266	-0.322	-0.200	-0.978	-0.121
Average value for medium interval	0.333 - 0.666	-1.590	-1.069	-1.653	-4.221	-0.699
Average value for high interval	0.666 - 1	-1.972	-2.013	-2.923	-1.426	-0.888
h	Interval	Sensitivity	Sensitivity	Sensitivity	Sensitivity	Sensitivity
Maximum absolute value	0 - 1	-29.326	-6.584	-36.888	-33.211	-22.122
Average value for low interval	0 - 0.333	-2.113	-0.727	-4.719	-3.792	-0.635
Average value for medium interval	0.333 - 0.666	-11.906	-4.897	-26.926	-18.151	-4.445
Average value for high interval	0.666 - 1	-25.202	5.262	0	-23.188	-17.470
d	Interval	Sensitivity	Sensitivity	Sensitivity	Sensitivity	Sensitivity
Maximum absolute value	0 - 140	-48.405	-7.889	-43.232	-36.082	-11.803
Average value for low interval	0 - 480	-3.035	-0.337	-3.657	-3.226	-0.285
Average value for medium interval	480 - 660	-7.540	-1.499	-6.643	-15.505	-2.263
Average value for high interval	660 - 840	-23.700	-4.417	-11.303	-22.089	-5.746

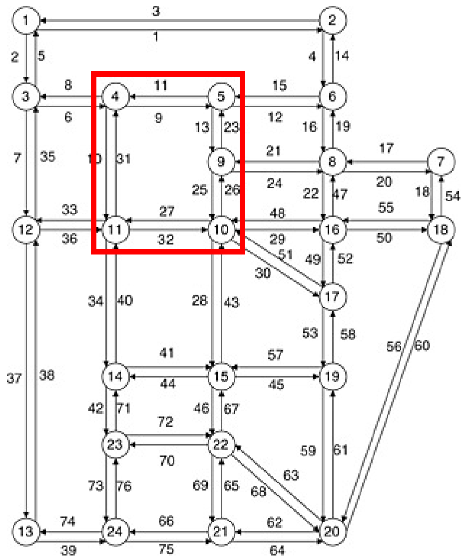

Figure 6.10: Smaller area affected by the hazard for a second case study

6.6.4 Influence of the redundancy of the network

As well as the area affected by the hazard, the number of routes between each OD is a key factor. Since the phenomenon analyzed is the perturbation of an extreme weather event in a transport network, the availability of more or less routes between the OD pairs will have a big role in the system performance.

In the first example, 6 routes per OD were analyzed (see Table 8.2). With the goal of studying a higher redundancy, two more routes per each OD have been added to the previous routes described in Table 8.2. The routes added are shown in Table 6.4. The area affected by the hazard corresponds with the first example (baseline case).

As expected, the results in Case 4 in Table 6.3 show the big influence of the redundancy in the sensitivity of α. Its value has increased from 5.411 to 23.098, when 8 routes per OD are introduced. This phenomenon is due to the increment of possibilities in the movement of the users, therefore the capacity of adaptation becomes more relevant in the final value of the resilience.

Finally, to study the effect of a reduction of the redundancy, the number of routes per OD is reduced to 4, then the chosen routes for this case have been the same than in Table 8.2 but eliminating the two last routes from each OD. In Case 5 in Table 6.3 the results for the sensitivity analysis when the number of routes from OD is reduced to 4 are shown. In the same way, in the previous example, a major influence of the redundancy on the value of the sensitivity of α is observed. Its value decreases to 0.930, because the possibilities to change the routes are reduced, then the parameter α reaches lower values of sensitivity.

Table 6.4: Description of the routes added to the initial case

Route Id.	OD	Links
61	1-18	2 6 9 12 16 20 18
62	1-18	2 6 9 13 24 20 18
63	1-20	2 6 10 34 41 46 69 64
64	1-20	2 7 36 34 42 73 75 64
65	24-2	76 72 67 43 29 47 19 14
66	24-2	76 72 67 43 26 23 12 14
67	2-20	4 16 21 25 28 46 68
68	2-20	4 16 21 25 28 46 69 64
69	2-13	4 15 11 8 7 37
70	2-13	4 16 22 49 53 57 44 42 73 74
71	7-12	18 55 49 51 27 33
72	7-12	18 55 49 53 57 43 27 33
73	13-7	38 35 6 9 12 16 20
74	13-7	38 35 5 1 4 16 20
75	21-1	64 61 58 52 47 19 14 3
76	21-1	64 60 54 17 19 14 3
77	24-8	75 64 61 53 51 26 24
78	24-8	75 65 67 43 29 47
79	20-1	60 55 47 19 14 3
80	20-1	60 55 47 19 15 11 8 5

6.7 Conclusions

The goal of this chapter is to provide more information about the parameters involved in the new method to evaluate resilience when a climatological event affects a traffic network, and hence about the behaviour of the new model. In this way, in addition to the evaluation of resilience previously presented, a profound understanding of the performance of the model is obtained, to finally develop strategies for the increment of the resilience in traffic networks.

For that reason, a mapping and a sensitivity analysis have been conducted, and the following conclusions are obtained:

- **Regarding the methodology.**
  - Mapping and sensitivity analysis allow understanding the parameters involved in a model. However, each of them covers a very different scale. Mapping recreates multiple scenarios where in each evaluation, only one of the parameters changes. With mapping, the evolution of the value of resilience can be obtained for the studied cases, and this data can be easily shown in a graphic

for a decision-making process. However, to cover the total range of the parameters, and all the possible combinations, a huge amount of evaluations will be necessary. Therefore, to have general information about the impact of the parameters in the method, a sensitivity analysis needs to be developed. Given the complexity of the model, statistical strategies are introduced into the sensitivity analysis to reduce the sample of points, and at the same time to cover the entire space.

  - Regarding the methodology introduced to develop the sensitivity analysis, a powerful methodology in two levels has been implemented. The main advantage of this bi-phase method is that it requires minimal computational time, due to the statistical selection of points. The statistical approach used for the selection of points has been the Latin Hypercube, which by studying a small set of points, a clear overview of the sensitivity analysis can be obtained.
  - The bi-phase methodology includes local (One-At-a-Time), and global techniques (Latin Hypercube). This kind of methodology is justified when complex models are analysed or a large number of variables are involved, because local methods are very efficient but they do not cover the entire space; whereas, global methods provide a robust and reliable approach but the computational cost could be too high in complex models.

- **Regarding the results obtained for the parameters of the model.**
  - The results of the bi-phase sensitivity analysis confirm that the increment of the intensity of the hazard, the link vulnerability and the demand, produces a reduction of the resilience, meanwhile the increment of the capacity of adaptation of the users increases the resilience of the system.
  - The demand, d, and the intensity of the hazard, h, are shown to have a key role, as expected, and the link vulnerability, p, a smaller influence, obtaining low sensitivity values. These parameters, d and h, show a strong tendency, regardless of the rest of the parameters. In the case of the intensity of the hazard is even stronger when the area affected covers a significant part of the network, since the graph shows an exponential tendency when the value of the intensity increases.

- **Regarding the results obtained for other variables.**
  - The increment of the redundancy of a network can be used as a strategy to improve the value of resilience. Incrementing the number of possible routes or creating new links in the network, will have a positive impact in the value of the resilience, and the influence of the perturbation of the hazard on the network will be reduced.

– In addition, redundancy has been shown as a relevant parameter with a large impact on the sensitivity the capacity of adaptation of the users. When the capacity of adaptation of the users, α, is high, the redundancy of the network can be largely used by the users, because the users will quickly select better routes when their usual routes are affected by the perturbation. On the contrary, when the value of α is small, the redundancy will have a smaller effect, since the users will be less open to change their usual routes. It is noted that a large reduction in the redundancy of the network can change the sensitivity ranking of the parameters, converting α into the one with the lowest sensitivity values.

– The area affected by the hazard has been also analysed. The modification of this variable can modify the sensitivity ranking of the parameters, by reducing significantly the sensitivity of the intensity of the hazard when the area affected by the hazard is small. On the contrary, when the hazard is covers an extensive area, the intensity of the hazard becomes the most relevant parameter in the model with the largest sensitivity.

Chapter 7

Critical and vulnerable links

Contents

7.1 Notation

F	Fisher Information Matrix (FIM)
$f_{i,j}$	The (i,j) entry of the FIM
θ	Sensitivity vector
N	Number of links
M	Number of areas

λ	Eigenvalue
λ_{max}	Largest eigenvalue
$\vec{v}$	Eigenvector
$\vec{v}_{\lambda_{max}}$	Eigenvector associated with the largest eigenvalue
ΔC_n	Variation of the capacity of link n
Δt_n	Variation of the travel time of link n
ΔT_n	Variation in the total travel times of the network when link n is being damaged
ΔC_m	Variation of the capacity of area m
Δt_m	Variation of the travel time of area m
$\Delta T_A m$	Variation in the total travel times of the network when area m is being damaged
tr	trace of a matrix

The research included in this chapter has been presented in Martinez-Pastor et al. (2017a).

7.2 Motivation

As discussed earlier in this thesis, traffic networks are constantly threatened by a wide range of perturbations, from natural to man-made hazards. In the last years, the concept of resilience is helping in this area, analysing the whole performance of the traffic network when hazards occur, from the beginning of the perturbation until total recovery. In addition to the evaluation of resilience developed in previous chapters, the presence of elements which are more susceptible to perturbations increases the challenge in the performance of traffic networks under damaging situations. These elements have a key role in the behaviour of traffic networks against extreme events, therefore this chapter focuses on their identification in order to create more resilient networks.

The following sections aim to provide tools for the identification of the target elements of traffic networks, differentiating between vulnerable and critical elements. In this way, stakeholders of traffic networks will be able to detect these components prior to perturbations.

7.3 Introduction

In previous chapters, a complete methodology to evaluate the recent concept of resilience in traffic network has been introduced. The methodology presented in this thesis combines the novel dynamic equilibrium restricted traffic assignment model presented in Chapter 4, with the new bounded travel cost function which specifically includes the parameters of the weather events, see Chapter 5. In addition, for a total understanding of the methodology and the parameters involved, a mapping and a sensitivity analysis has been implemented in Chapter 6.

Once the concept of resilience has been analysed, new goals arise, one being the improvement of this indicator. Since transport networks are complex and dynamic systems, defined by multiple components, the challenge of improving resilience becomes an issue of significant relevance. To addressing this goal, this chapter focuses on the localization of the elements that contribute the most to a worse performance of the network under a perturbation. By doing so, it will be possible to improve the performance of the weak elements, and consequently, the traffic network behaviour and the value of resilience.

In order to achieve the presented objective, these elements are differentiated in two groups, critical and vulnerable links, according to the definitions presented in Chapter 3. These definitions are used for the development of the methodologies presented in this Chapter. In addition, for a wider explanation of the concepts see Chapter 3.

The methodologies introduced in the following sections were primarily developed to identify the most critical and the most vulnerable elements, but they are also able to rank all the links of the network from the most to the least critical, and from the most to the least vulnerable. This feature of the methodologies provides information regarding the whole network in the same analysis, making these methodologies advantageous for the improvement of resilience, since the performance of all the links are simultaneously evaluated.

In the first part of this chapter, the network is analysed in a link-based approach. The methodologies presented in this part study the behavior of the links when affected by a perturbation, and the results provide information about the link vulnerability and the link criticality. In the second part of the chapter, the method is extended to an area analysis, therefore groups of links in areas affected by a perturbation are analysed. The results present a classification of the areas selected for the analysis, depending on their performance against damaging events, and rankings of the most vulnerable areas and the most critical areas are obtained. See Figure 7.1 for an explanatory diagram of the methodologies presented in this chapter.

The introduction of both types of methodologies is necessary. The performance of a single link affected by a hazard, and the performance of an area of the network affected by a hazard can vary significantly. The availability of both tools addresses the issue more accurately, giving the option of choosing the more suitable tool in each case.

In addition, three examples are presented, exhibiting different perspectives of the methodology and particular cases. Finally, for an analysis of the methodologies in real scenarios, two complex networks are tested.

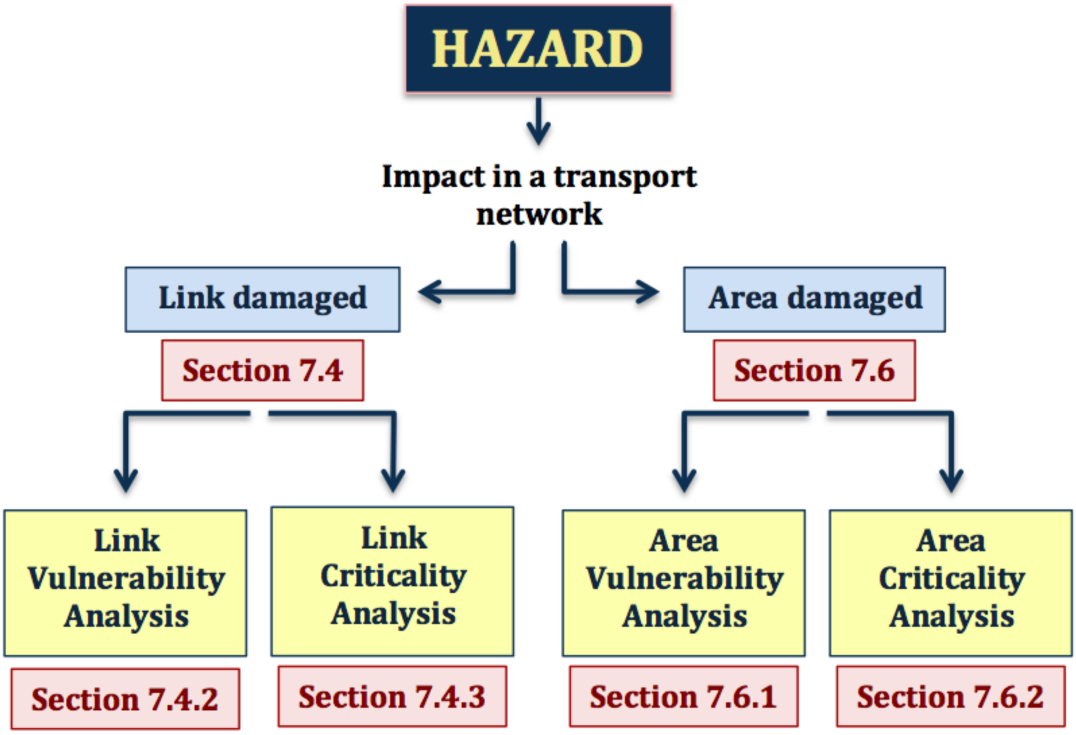

Figure 7.1: Explanatory diagram of the methodologies of the chapter

7.4 Analysing the vulnerable and critical links of traffic networks

In the presented framework, this section introduces, and develops two new methodologies; one to identify and rank the vulnerable links, and the other to identify and rank the critical links of traffic networks. Both methods are based on the Fisher matrix and the analysis of its principal components, such as eigenvalues and eigenvectors. The Fisher matrix is a well-known methodology in areas, such as, statistics, economics, mathematics, and structural engineering. However, this is the first time that this methodology is presented in the area of traffic networks to identify the critical and vulnerable links when a network is affected by a perturbation.

7.4.1 Fisher Matrix for the identification of vulnerable and critical links in traffic networks

In statistics, the Fisher Information Matrix (FIM) is "a way of measuring the amount of information that an observable random variable X carries about an unknown parameter

ϕ upon which the probability of X depends on".

For a N-variate multivariate normal distribution, $X \sim N(\mu(\phi), \Sigma(\phi))$, let the K-dimensional vector of parameters be $\phi = [\phi_1, \phi_2, ..., \phi_K]^T$, and the vector of random normal variables be $X = [X_1, X_2, ..., X_N]^T$. Assuming that the mean of values of these random variables are $\mu(\phi) = [\mu_1(\phi), \mu_2(\phi), ..., \mu_N(\phi)]^T$, and let $\Sigma(\phi)$ be the covariance matrix. Then, the (i,j) entry, for $1 \leq i, j \leq K$, of the FIM is defined as follows:

$$f_{i,j} = \frac{\partial \mu^T}{\partial \phi_m} \Sigma^{-1} \frac{\partial \mu}{\partial \phi_n} + \frac{1}{2} tr \left(\Sigma^{-1} \frac{\partial \Sigma}{\partial \phi_m} \Sigma^{-1} \frac{\partial \Sigma}{\partial \phi_n} \right), \tag{7.1}$$

where $tr(\cdot)$ denotes the trace of a square matrix, and $(\cdot)^T$ denotes the transpose of a vector.

Simplifying this formulation for the deterministic analysis developed in this chapter, it is assumed that $\Sigma(\phi) = I$, I being the identity matrix, and the (i,j) entry of the FIM can be simplified, obtaining the following expression:

$$f_{i,j} = \frac{\partial \mu^T}{\partial \phi_m} \frac{\partial \mu}{\partial \phi_n}, \tag{7.2}$$

where the $f_{i,j}$ denotes the matrix element for the row i, and the column j. Following the previous definition of the $f_{i,j}$ element, the FIM can be re-written as a sum of matrices, as follows:

$$F = \sum_{n=1}^{N} \frac{\partial \mu_n}{\partial \phi} \frac{\partial \mu_n^T}{\partial \phi}, \tag{7.3}$$

and, since the analysis is deterministic the mean of the variables, μ, can be expressed as the value of X, obtaining:

$$F = \sum_{n=1}^{N} \frac{\partial X_n}{\partial \phi} \frac{\partial X_n^T}{\partial \phi}. \tag{7.4}$$

This approach to build the FIM analyses the variation of an expected value X, as a function of a variable ϕ. It is noted that this way of expressing the matrix is very relevant, since it allows the definition of the matrix by a sensitivity vector. Thus, the sensitivity vector is defined as $\theta_n = [\frac{\partial X_n}{\partial \phi_1}, \frac{\partial X_n}{\partial \phi_2}, ..., \frac{\partial X_n}{\partial \phi_K}]$, and, the FIM is expressed as follows:

$$F = \sum_{n=1}^{N} \theta_n^T \theta_n, \tag{7.5}$$

where F is the FIM, and θ_n is the sensitivity vector, which can be re-written as $\theta_n = [\theta_1, \theta_2, \theta_3, ..., \theta_K]$. This approach has been used in Dowski (2002), and it was also implemented in the area of bridges for the placement of sensors by Meo and Zumpano (2005).

In order to use the FIM matrix for the identification of the vulnerable and critical links of the network, a sensitivity vector, θ_n must be defined for each of the n-links of the network, for $1 \leq n \leq N$, with N being the total number of links of the network.

For each sensitivity vector, k components are defined, for $1 \leq k \leq K$, with K being the total number of links of the network. This is to represent that each of the components is associated with one of the links of the network. Thus, from now on, N will be used to refer also to the variable K, because in the case of traffic networks K will be equal to N.

In following sections, different methodologies are presented to identify the vulnerable or the critical links of a network. Therefore, the sensitivity vector will be defined depending on whether the vulnerable or the critical links are being analysed. Thus, in Sections 7.4.2 and 7.4.3, the sensitivity vectors are introduced.

7.4.2 Methodology to identify vulnerable links in a traffic network

When the vulnerable links of a network need to be identified, the FIM will be designed in the way that each component will carry the information of how vulnerable that link is when the network is suffering a perturbation.

One sensitivity vector is defined for each link of the network. To identify the vulnerability of the links as described in the introduction section of this chapter and widely explained in Chapter 3, a damage in each link is introduced by a reduction of the capacity of the link. This reduction of the capacity simulates the impact, that can be created due to different perturbations, such as climatological hazards, man-made events, maintenance works or any other impact in the traffic network. In this way, each sensitivity vector is associated with one link, and it will contain the information of the effects of the damage of that link on the rest of the links of the network.

The effect of this damage is measured in terms of cost by implementing a user equilibrium traffic assignment together with the well-known BPR travel cost function. The cost increment produced by the new conditions of the network is evaluated by analyzing the previous and the disturbed situation of the network. In this chapter, the variation of the cost is measured in terms of travel time.

For this chapter, the selected cost function has been the BPR because the variables analysed for the identification of the vulnerable and critical links are the travel time and the capacity of the links. Thus, the impacts of possible hazards are simulated by reductions in these variables, and the intensity of the hazard or the resilience are not introduced in the formulation at the moment. In future works, the methodology can be implemented with the new travel cost function presented in Chapter 5.

Therefore, in this methodology to identify the vulnerable links, the sensitivity vector θ_n is defined as follows;

$$\theta_n = [\frac{\Delta t_1}{\Delta C_n}, \frac{\Delta t_2}{\Delta C_n}, ..., \frac{\Delta t_n}{\Delta C_n}, ..., \frac{\Delta t_N}{\Delta C_n}], \tag{7.6}$$

where ΔC_n is the variation of the capacity for each link, and Δt_n is the variation produced in the travel time of each link when the capacity of link n is reduced.

Building the FIM with the sensitivity vectors of all the links, using equation (7.5), a matrix with all the vulnerability information of the network is developed. Then, the eigenvalues and eigenvectors of FIM are calculated. The eigenvalues will be associated with the level of vulnerability of each of the links. The most vulnerable link will be identified by selecting the largest eigenvalue, λ_{max}, of the FIM, and the eigenvector associated with this eigenvalue, $\overrightarrow{v}_{\lambda_{max}}$. Thus, the maximum value in the eigenvector will determine the position of the link, as below:

$$v_a = \max(\overrightarrow{v}_{\lambda_{max}}), \tag{7.7}$$

where a is the index of the maximum value of the eigenvector, and as such will indicate the position of the most vulnerable link.

This approach for the identification of vulnerable links can be established because of the following mathematical assumptions.

Knowing that eigenvalues and eigenvectors are connected following the equation,

$$F\overrightarrow{v}_n = \lambda_n \overrightarrow{v}_n, \tag{7.8}$$

where $\overrightarrow{v}_n$ is the eigenvector associated with the eigenvalue λ_n, and F is FIM.

It is possible to associate the eigenvalue with one of the elements of the FIM, when the eigenvectors obtained have the form $\overrightarrow{v_n} = [1, 0, 0, ..., 0]$, with one of the components 1 or a value near 1, and the rest of the components of the vector with a value of 0.

Therefore, when analysing the largest eigenvalue obtained, the eigenvector associated with the form $\overrightarrow{v_n} = [1, 0, 0, ..., 0]$, determines that the value of the eigenvalue is given by only one element of the FIM. In this case, if the eigenvector has the term of 1 in the first position, applying Equation (7.8), the element selected is the $F_{1,1}$, (first row, first column). Based upon the definition of the FIM as shown in Equation (7.5), this term is:

$$F_{1,1} = \frac{\Delta t_1'^2}{\Delta C_1^2} + \frac{\Delta t_1''^2}{\Delta C_2^2} + \frac{\Delta t_1'''^2}{\Delta C_3^2} + ... + \frac{\Delta t_1'^{N^2}}{\Delta C_n^2}. \tag{7.9}$$

Analysing this element of the matrix, it is noted that it is the definition of the link vulnerability of the selected row. Equation (7.9) determines the impact that is suffered by one link, link 1 in this case, when each of the links of the network are damaged, i.e. the vulnerability of link 1 is assessed in that element of the matrix. Therefore, it is possible to conclude that with eigenvectors with the form $\overrightarrow{v_n} = [1, 0, 0, ..., 0]$, the eigenvalues associated identify the vulnerability of each link.

For other eigenvectors, the position of the value 1 in the eigenvector will give the location of the vulnerable link associated with an eigenvalue. Therefore, for the largest eigenvalue, the position of the value of 1 in the eigenvector will determine the most vulnerable link of the network. Subsequent eigenvalues will determine the rank of the vulnerable links.

7.4.3 Methodology to identify critical links in a traffic network

In this section a methodology to identify the critical links of the network is presented, based on the same principles as the previous methodology, and using Equation (7.5) to build the FIM. For the identification of the critical links of a traffic networks, the sensitivity vector, θ_n, is defined in a different way to include the level of criticality of each link in the FIM instead of the vulnerability. For this reason, the sensitivity vector for each link is determined as follows,

$$\theta_n = [\frac{\Delta t'_n}{\Delta C_1}\Delta T_1, \frac{\Delta t''_n}{\Delta C_2}\Delta T_2, \frac{\Delta t'''_n}{\Delta C_3}\Delta T_3, ..., \frac{\Delta t'^n_n}{\Delta C_N}\Delta T_n], \tag{7.10}$$

where $\Delta t'^n_n$ is the variation in the travel time of link n when the capacity of link n is reduced, and ΔT_n is the variation in the total travel times of the network when the link n is damaged.

There are two main differences between this vector and the vector previously presented to analyse the vulnerable links. The first difference is that in the vulnerability method, each vector was defined by reducing the capacity of one of the links of the network, keeping that value constant in the denominator, and in the numerator introducing the impact in the travel time in the different links, see Equation (7.6). However, in the case of analysing the criticality, the denominator of the vector varies, because in each element of the vector a reduction of the capacity of each of the links of the network is measured, and in the numerator, the effect of these damages in each of the links is measured in the link associated to the vector, as shown in Equation (7.10).

The second difference is the introduction of ΔT_i, the variation of the total travel times of the network. This parameter is necessary because the global performance of the network needs to be introduced to be able to identify the critical links. In this way, it is possible to incorporate the effects of the global behavior of the network when the link analysed is damaged.

Therefore, following the same mathematical assumptions presented in the previous section, the eigenvalues will determine the level of criticality of each link of the network, and the eigenvectors associated will be the ones able to locate the link associated with each eigenvalue. The most critical link is identified by analysing the maximum eigenvalue, λ_{max}, and its associated eigenvector, $\overrightarrow{v}_{\lambda_{max}}$, as follows:

$$v_a = \max(\overrightarrow{v}_{\lambda_{max}}), \tag{7.11}$$

where a is the index of the maximum value of the eigenvector, and as such will indicate the position of the most critical link.

When analysing the largest eigenvalue with an associated eigenvector with the format,

$[1, 0, 0, 0, ...]$, the element of the FIM matrix selected is

$$F_{1,1} = \frac{\Delta t_1^2}{\Delta C_1^2} \Delta T_1^2 + \frac{\Delta t_2^2}{\Delta C_1^2} \Delta T_2^2 + \frac{\Delta t_3^2}{\Delta C_1^2} \Delta T_3^2 + ... + \frac{\Delta t_N^2}{\Delta C_1^2} \Delta T_N^2. \tag{7.12}$$

This element quantifies how critical is a link in a network. Analysing Equation (7.12), it is possible to conclude that the selected element of the FIM determines the impact of a perturbation in all the links of the network when damaging one link. In the example shown in Equation (7.12) the criticality of link 1 is assessed.

As explained in the methodology to identify vulnerable links, the eigenvectors, and the eigenvalues associated with the FIM are able to rank from the most to the least critical all the links of the network.

7.4.4 Example 7.1: identifying vulnerable and critical links

An example is developed to understand the advantages of the method for the identification of vulnerable and critical links in a traffic network.

The network, depicted in Figure 7.2, is used for the example. This network has 6 nodes and 14 links. A description of the capacities and free flow travel times are shown in Table 7.1, and the routes considered in Table 7.2.

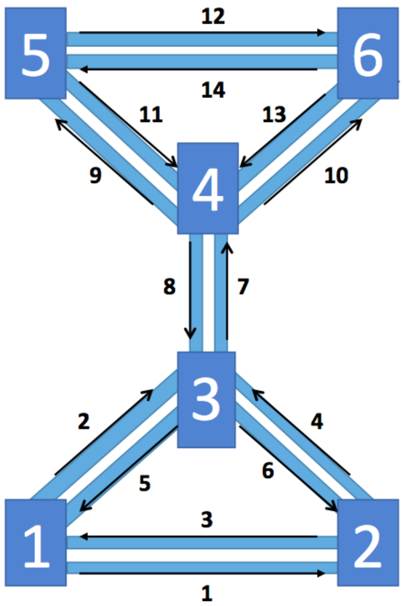

Figure 7.2: Example 7.1: Network described by nodes and links.

Applying the methodology to identify the vulnerable links of the network the results obtained are presented in Table 7.3, also a graphical representation of the results is shown in Figure 7.3.

Table 7.1: Example 7.1.: Characteristics of the network

Link	Capacity (users)	Free flow travel time (min)
1	50	2
2	50	1.42
3	50	2
4	50	1.42
5	50	1.42
6	50	1.42
7	50	1
8	50	1
9	50	1.42
10	50	1.42
11	50	1.42
12	50	2
13	50	1.42
14	50	2

Table 7.2: Example 7.1: Routes selected

Route Id	OD pair	Links	Demand (users/route)
1	1-6	2-7-10	50
2	1-6	1-4-7-10	50
3	1-6	2-7-9-12	50

In Figure 7.3, links that are not used in this example are shown in grey, thus these links will not appear in the rankings. The links with a certain level of vulnerability are represented with different tonalities of red. Finally the ones in blue are those without a vulnerability level obtained. The results indicate that link 7 is the most vulnerable of the network, the eigenvalue associated is 53.14, see Table 7.3. In second and third position of the ranking are links 10 and 2, both with an associated eigenvalue of 0.06.

Comparing the eigenvalues for the first and the second position of the rank a large difference is found, meaning that the level of vulnerability is significantly more relevant in the first position than in the rest of the positions of the rank. Therefore, link 7 is the most vulnerable of the network with a relevant difference from the rest. Finally, in fourth and fifth position, links 9 and 4 with an eigenvalue of 0.01, which means that their level of vulnerability is low. The rest of the links of the network have an eigenvalue of zero.

Once the vulnerable links have been analysed, the method to identify the critical links is applied. The results obtained are shown in Table 7.4, and in Figure 7.4.

Table 7.3: Example 7.1: Results of the methodology to identify vulnerable links

Rank position	Eigenvalue	Vulnerability Link Rank
1	53.14	7
2	0.06	10
3	0.06	2
4	0.01	9
5	0.01	4
6	0.00	-
7	0.00	-
8	0.00	-
9	0.00	-
10	0.00	-
11	0.00	-
12	0.00	-
13	0.00	-
14	0.00	-

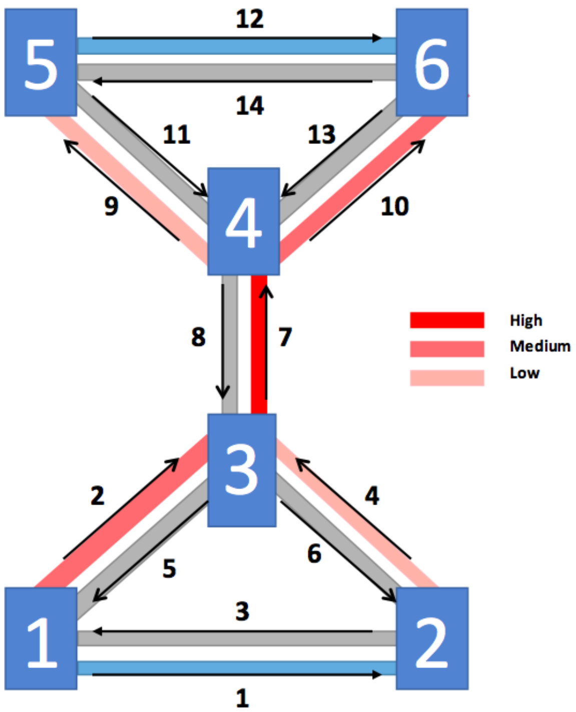

Figure 7.3: Example 7.1: vulnerable links represented in the network

Table 7.4: Example 7.1: Results of the methodology to identify critical links

Rank position	Eigenvalue	Eigenvector (largest value)	Critical Link Rank	Resilience
1	3815.84	1.00	7	93.73
2	0.0023	0.94	10	99.78
3	0.0023	0.94	2	99.78
4	0.00	-	-	100.00
5	0.00	-	-	100.00
6	0.00	-	-	100.00
7	0.00	-	-	100.00
8	0.00	-	-	100.00
9	0.00	-	-	100.00
10	0.00	-	-	100.00
11	0.00	-	-	100.00
12	0.00	-	-	100.00
13	0.00	-	-	100.00
14	0.00	-	-	100.00

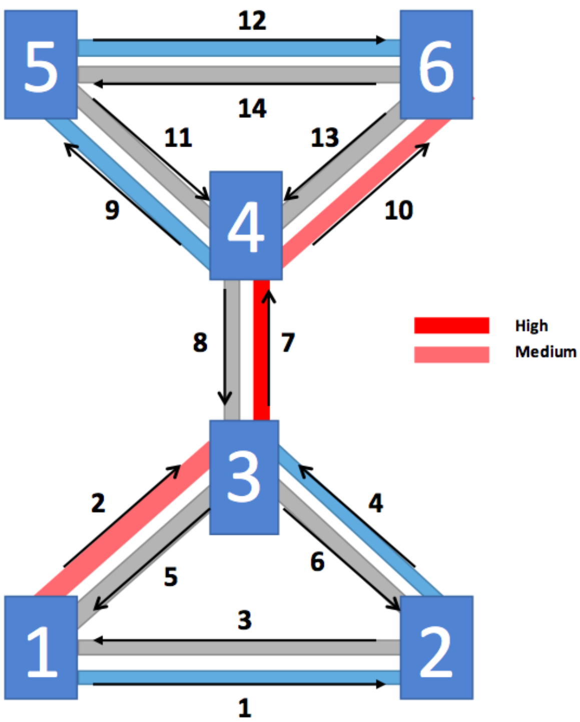

Figure 7.4: Example 7.1: critical links represented in the network

The results show that link 7 is also the most critical link of the network. It is noted that there is a large difference in the level of criticality of this link compared with the rest of the links of the network, even more than in the case of vulnerability. The eigenvalue associated with link 7 is 3815.84, see Table 7.4, and the next eigenvalue of the ranking is 0.0023, highlighting again the relevance of link 7 in terms of criticality.

In this example, the results are expected due to the characteristics of the network, link 7 is easily identifiable as an element which can create an important impact in the network when damaged, i.e. a critical link. Also, link 7 can be the one suffering a deterioration in its performance when other parts of the network are damaged, i.e. a vulnerable link.

Following the criticality ranking obtained in Table 7.4, in second and third position, links 10 and 2 have a smaller level of criticality, being identified with a medium level in Figure 7.4. The rest of the links, as shown in Table 7.4, do not have a relevant level of criticality, because they are not involved in the routes selected in this example, shown in grey color in Figure 7.4, and others, in blue color in Figure 7.4 because the users have better routes to choose when moving from one node to another, and a deterioration of the capacities of these last ones is easily absorbed due to the redundancy presented in the network.

In addition, in the methodology for the identification of critical links, and to validate the results, the resilience of the network is evaluated. The methodology used to evaluate the resilience of the network is the one presented in Chapter 4. Perturbations in different areas will be introduced to corroborate the results, in each iteration one of the links will be damaged to study the variation in the resilience. Then, when a critical link is damaged the resilience will be more reduced than when a less critical link is damaged. For this example, the damage introduced is a reduction of the capacity to half of its initial value.

In the last column of Table 7.4 the results of the evaluation of resilience are shown. When the capacity of link 7 is reduced to half, the resilience obtained is 93.73, which means a significant reduction in the value of resilience for an impact affecting only one link. However when the second link of the ranking, (link 10), is impacted by the perturbation, the value of resilience is 99.78. The same resilience, 99.78, is obtained when link 2 is impacted, since they have the same level of criticality. Finally when the impact is applied to other links of the network, the value of resilience remains as 100, because a small impact in these links will not affect the whole network. Consequently, the ranking obtained for the criticality will provide valuable information for the improvement of resilience.

7.4.5 Example 7.2: particular case, symmetric network

In the example previously presented, the network was designed to highlight one of the links as the most critical of the network due to its location. The central link, link 7 in Figure 7.2, connects both parts of the traffic network, and necessarily all the users cross this link to reach the destination node. In addition, this link was also identified as the most vulnerable of the network, since perturbations in other parts of the network have a large impact in this link. Therefore, the identification of the critical and vulnerable links of the network presented was expected to be arrived at this result, as shown in the results.

However, what happens when the analysed network has links with same levels of criticality or vulnerability? In these cases, both methodologies are able to identify groups of links with same levels.

To demonstrate this phenomenon, a simple and symmetric network is introduced, see Figure 7.5. This network has 4 nodes, 4 links, and 2 routes with the same OD pair, 1-4, and the features of the links are the same. See Table 7.5 and Table 7.6 for a description of the network. Based upon the assigned characteristics, the users will have exactly the same conditions choosing any of the routes, and the vulnerability and criticality of the links should be the same for all.

The results obtained applying both methodologies are shown in Table 7.7 for the methodology to identify the vulnerable links, and for the method identifying the critical links.

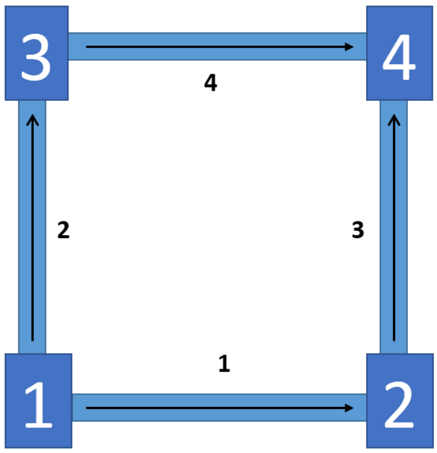

Figure 7.5: Example 2: symmetric network.

Table 7.5: Characteristics of the network, Example 7.2

Link	Capacity (users)	Free flow travel time (min)
1	50	1
2	50	1
3	50	1
4	50	1

Table 7.6: Routes selected for Example 7.2

Route Id	OD pair	Links	Demand (users/route)
1	1-4	1-3	100
2	1-4	1-4	100

Table 7.7: Example 7.2, symmetric network results

Method	Largest eigenvalue	Eigenvector associated
Vulnerable links method	0.35	(0.5, 0.5, 0.5, 0.5)
Critical links method	8.1	(0.5, 0.5, 0.5, 0.5)

As expected, the results show that all the links have the same level of vulnerability and criticality. The eigenvector obtained in both methodologies has all its components equal to 0.5, therefore all the links contribute in the same proportion to the eigenvalue, and there are not links more critical or vulnerable than others in the network.

7.4.6 Difference between vulnerable and critical links

In both examples previously studied the results for the critical and vulnerable links were very similar. This situation can happen in traffic networks, because, despite the clear difference between both definitions, many times due to the topology of the network and the characteristics of the link both concepts coincide in the same links. Therefore, it can happen that one link of the network causes the largest impact in the whole network when impacted (the most critical link), and at the same time, the same link can be the one that suffers the largest impact when other parts of the network are impacted (the most vulnerable link).

Nevertheless, the differentiation between both kind of links is crucial, since the consequences of an impact in these links are completely different, as explained before.

A specifically designed network to highlight the differences between vulnerable and critical links is presented in Figure 7.6. The network has 4 nodes and 4 links, which are defined as shown in Figure 7.6. Different thicknesses are used in the links to highlight the different capacities, see Table 7.8 for a description of different capacities and free flow travel times. Link 1 has the largest capacity, being 50 users, following by links 3 and 4 with a capacity of 30 users, and finally, link 2 with a capacity of 20 users. The routes analysed in this network are the ones detailed in Table 7.9.

This basic network is introduced in this case, because when complex examples are studied, it can be extremely difficult to previously identify the most vulnerable or the most critical link, and the results can be complex to predict. The same example will be used to identify the vulnerable and critical links of the network, and both results can be compared.

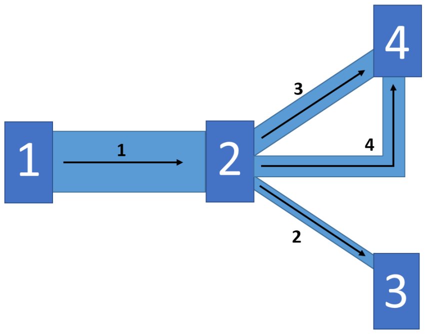

Figure 7.6: Simple network.

Table 7.8: Characteristics of the network

Link	Capacity	Free flow travel time
1	50	1
2	20	1
3	30	1
4	30	1.2

Table 7.9: Routes selected for the simple example

Route Id	OD	Links	Demand (users/ OD)
1	1-3	1-2	20
2	1-4	1-3	20
3	1-4	1-4	20

Before applying the methodology, some details about the composition of the network are highlighted, in order to facilitate the understanding of the results of the model.

This network has a main link, link 1, between node 1 and 2, which has the largest capacity. This link is a key link in the network, giving access to the rest of the nodes, such as nodes 2, 3 and 4. Thus, link 1 will have a main role in the traffic performance, and any damage in link 1 is expected to have a large impact in the network behaviour. For these reasons, link 1 is expected to be the most critical link of the network in this example.

Link 2 appears in route 1, connecting node 1 and node 3, the users using route 1 need to use firstly the link 1, and after the link 2 to reach their destination. Also, link 2 is the end of route 2, and does not give access to other nodes, for that reason, link 2 has a smaller capacity, and a smaller relevance in the network.

Despite a minor relevance, it is highlighted that link 2 is the only one that gives access to node 3. This is a relevant fact when measuring vulnerability, since node 3 will have a big dependency of link 2, because it is the only option to reach this node. Thus, any change in link 2 will have a large impact on route 2. However, the damages in link 2 will not affect the rest of the routes. It is important to note the difference with the case of a damage in link 1, where not only one route but all them are expected to be impacted, including route 1, 2 and 3.

Therefore, link 2 will represent a vulnerable link in the network, since changes in other links of the network, for example a damage in link 1, will affect it; but a damage in the link 2 will create problems mainly in the same link, rather than in other links of the network. Moreover, it has the smaller capacity, which is a relevant characteristic when analysing the vulnerability.

Finally, route 2 and route 3 connect node 1 with node 4, in this OD pair, the users have two options to reach their destination, thus, the redundancy of the network is larger and this will contribute to reduce the criticality of both links, 3 and 4. When one of the routes of 1-4 OD pair is damaged the users can choose the other one, and easily avoid the affected part of the network.

In conclusion, and before applying the methodology, the most vulnerable link is expected to be link 2, and link 1 is expected to be the most critical in this example. Consequently, the methodology is applied to the example, and the results are shown in Table 7.10, with a visual representation of the vulnerability level of the links is depicted in Figure 7.7.

Table 7.10: Results of Example 7.3: Vulnerable links

Rank position	Eigenvalue	Vulnerability Link Rank
1	1.667	2
2	0.9976	1
3	0.0033	3-4

The eigenvalues obtained for the example are shown in Table 7.10, a rank from the most to the least vulnerable link is also presented. As expected, the results agree with the assumptions previously described in this section, obtaining as the most vulnerable link, link 2. The value of the eigenvalue is used to measure the difference between the vulnerable level of the links, when two links have similar eigenvalues it means that their vulnerability level is similar. On the contrary, when the eigenvalues have very different values there exists a large difference between the vulnerability level of two links. Then, following link 2 is link 1 with a medium level of vulnerability, and finally link 3 and 4, whose vulnerability level is significantly lower with an eigenvalue close to zero.

This example is extremely simple, and the performance of the network can be easily understood without a methodology. However this example is significantly relevant to understand how the method works, and the meaning of a vulnerable link.

The results of the methodology are shown in Table 7.11, and as expected, the most critical link of the network is link 1, followed by link 2.

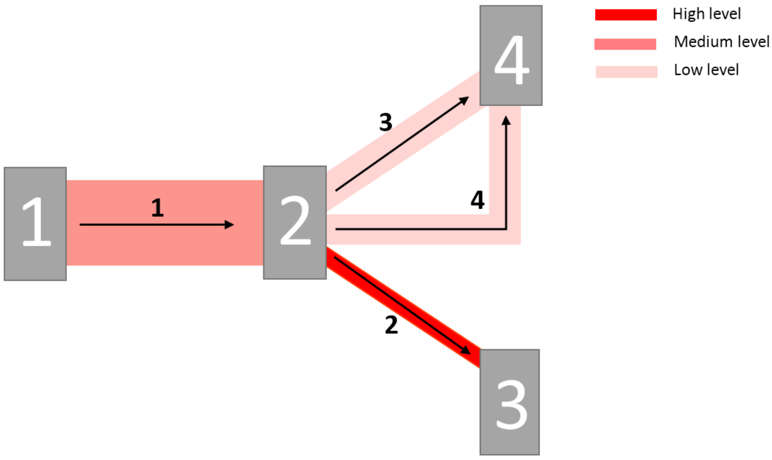

Figure 7.7: Vulnerability level of the links, Example 7.3

Table 7.11: Results of Example 7.3: Critical links

Rank position	Eigenvalue	Critical Link Rank
1	0.4872	1
2	0.0046	2
3	0	3-4

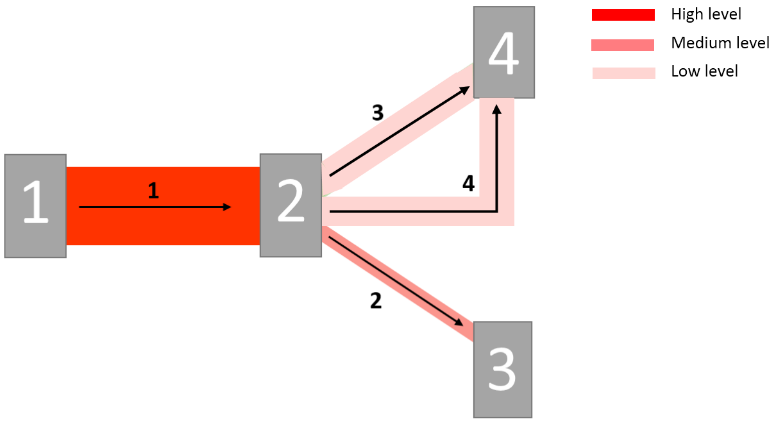

Figure 7.8: Critical level of the links, Example 7.3

In addition and in the same way as in example 7.1, the residence of the network is calculated when each of the links is damaged, and the results are shown in the last column of Table 7.12. As expected, when the most critical link of the network is damaged, the largest impact is obtained on the value of resilience, in this case the resilience is reduce

Table 7.12: Results of Example 7.3: Critical links

Critical Rank position	Link damaged	Resilience
1	1	92.99
2	2	97.08
3	3	99.99
3	4	99.99

from 100 to 92.99. The impact on the value of resilience is reduced as less critical links are damaged. Confirming that perturbations in the most critical link of the network will create the worst performance of the whole network, and reinforcing the results obtained with the methodology presented.

7.5 Case study: Cuenca Network, identifying critical and vulnerable links

Finally, the presented methodologies are implemented in a real case. The selected network is the Cuenca network, in Spain. See Figure 7.9 for a description of the network. The Cuenca network has 232 nodes, 672 links, and 207 routes have been chosen for this example with a total demand of 20700 users. In this example, the free flow travel time of the links has been calculated assuming that the velocity is the same for all the links, therefore it will depend on the length of the link, i.e the distance between nodes. The capacity used is 700 users for each link of the network.

For a better understanding of the results in this complex case, a figure of the Cuenca network has been modified to visualize the number of routes that cross each link. In Figure 7.10, the thickness of the links has been modified depending on the quantity of routes passing across each link. Thus, the thicker the link, the higher the number of routes passing that link. When analysing the vulnerable and critical links of a network, the saturation degree of the link will be a determining variable. Using the methodologies previously described, the vulnerable and the critical links of the network are obtained and ranked from the most vulnerable/critical to the least. Table 7.13 shows the results for the vulnerable links, and Table 7.14, the results for the critical links.

With the objective of confirming the rank of the critical links, the resilience of the network is evaluated damaging one link each time, in the same way as in the previous example. The values obtained for the resilience are shown in column 3, Table 7.14. The damage applied to simulate a perturbation is a reduction of the capacity of the selected link. The disruption lasts 8 days, during the first 4 days the capacity is reduced, reaching the lowest level in the 4^{th} day, with 35% of the initial capacity. In the last 4 days, the value of the capacity of the link gradually increases to 100 % of the initial capacity.

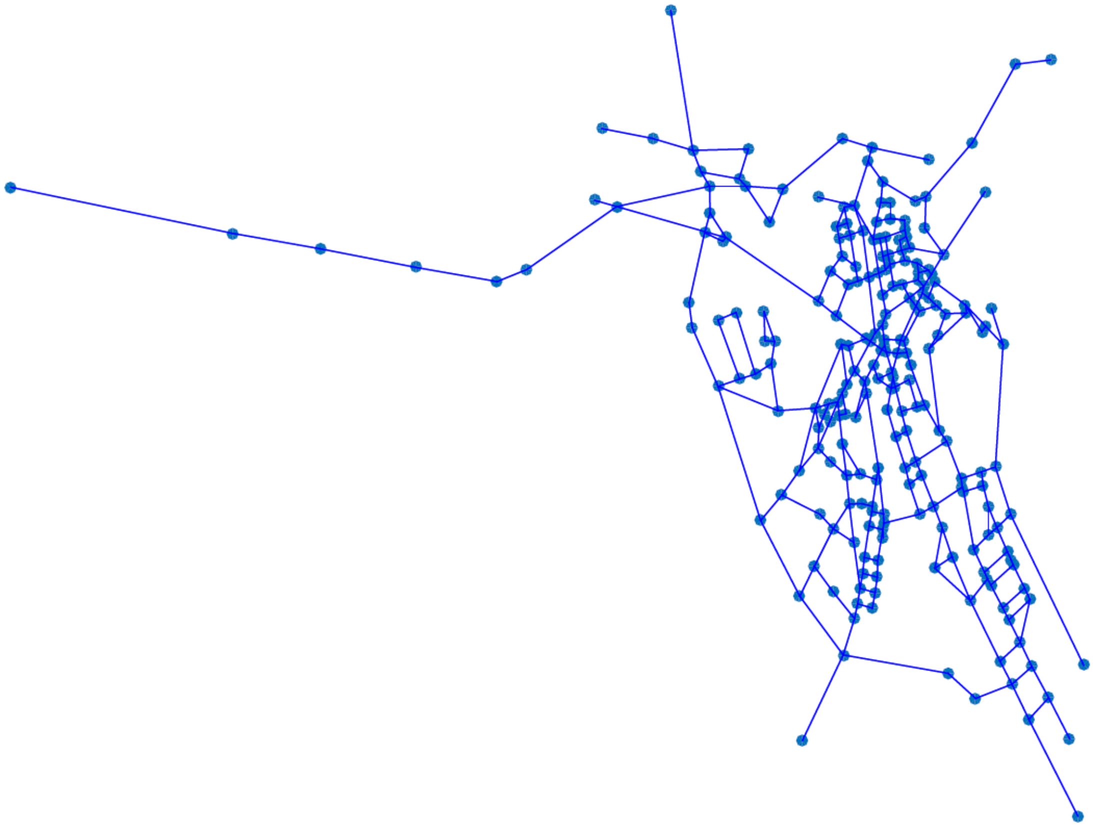

Figure 7.9: Cuenca Network represented by nodes and links

The value of resilience in the first row, 85.25, is obtaining by damaging only the most critical link, 438. In the second row, the second link of the rank is the only one being damaged, obtaining a resilience of 92.01, and in the same way for the following values.

Significant differences in the values of resilience are found, which exhibits the relevance of the methodology. In the case that the perturbation is affecting the most critical link, link 438 the resilience of the network is reduced by almost 15%, which represents a considerable impact given that only one link is damaged. However, if the same perturbation is affecting the following link in the route, link 72, ranked in 9 position, the reduction of resilience is only approx 2%.

When resilience is calculated, its value is expected to increase as less critical links are affected. Observing the results in Table 7.14 that expected tendency is obtained. However, small variations can be found in links ranked in the positions 7^{th}, and 8^{th}, where decimal reductions in the resilience are noted with respect to link 6. These small variations can happen for links with similar values of their eigenvalues, which means that the criticality level is similar, as in the case of the links 6, 7 and 8. It is noted that the analysed matrix

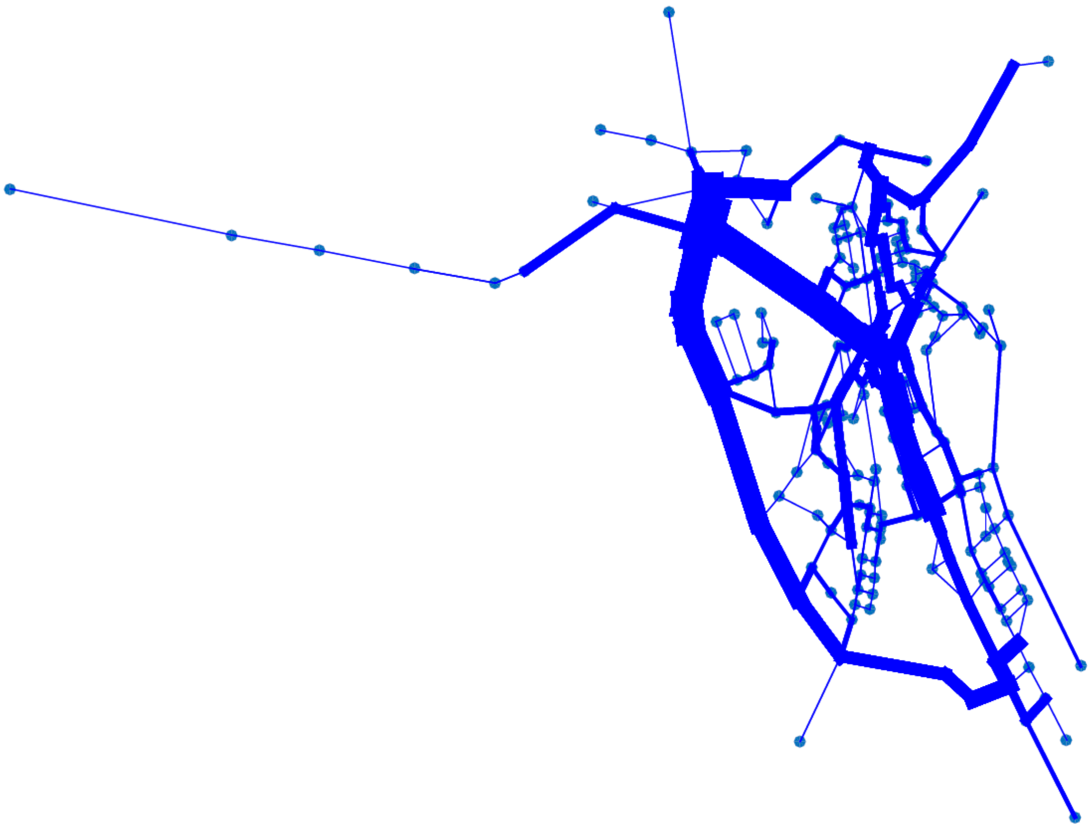

Figure 7.10: Cuenca Network, representing number of the routes crossing each link with the thicknesses of the links.

has a size of 672x672, so small inaccuracies might occur.

In addition, the most critical links are graphically located in the Cuenca network, see Figure 7.11, 7.12, and 7.13. Due to the difficulty of showing the results in a real network with a large number of links, three levels have been chosen depending on critical rank position, and the reduction of resilience when the critical link is damaged. Figure 7.11 represents the most critical links, those ones that reduce the resilience more than 4.5%, critical rank positions 1 to 4; Figure 7.12 represents those ones that reduce the resilience between 2.5% and 4.5%, rank positions from 5 to 8; and in Figure 7.13, the links shown decrease the resilience from 1% to 2.5%, ranked from 9 to 11. The rest of the links will have a lower impact, smaller than 1% of the resilience, thus they are considered as non-critical in this analysis.

As expected, the most critical links of the network are located in the areas where high number of routes are supplied. In this network, two main corridors can be identified, the first one is circular surrounding the city, and the other one crosses the city in the middle, see Figure 7.10, therefore special attention is needed for these areas, since all the critical

Table 7.13: Results of the Cuenca Network: Vulnerable links

Rank position	Eigenvalue	Vulnerable Link
1	6098996.23	438
2	2204742.12	349
3	1476374.54	622
4	827637.44	348
5	382234.04	619
6	340788.03	416
7	278927.21	459
8	256336.92	352
9	194851.37	72
10	86929.72	221
11	71804.45	557
12	59233.74	265
13	58654.96	409
14	40991.14	383
15	38495.99	385
16	17119.53	229

Table 7.14: Results of the Cuenca Network: Critical links

Rank position	Eigenvalue	Critical Link	Resilience (Link damaged)
1	49459990.89	438	85.25
2	6916707.13	349	92.01
3	5241596.87	622	93.72
4	789494.34	348	95.56
5	221984.87	619	97.21
6	93498.83	352	97.80
7	83151.44	459	97.37
8	73446.31	416	97.39
9	47061.34	72	98.14
10	8076.53	221	98.85
11	6390.87	557	98.98
12	2934.51	265	99.09
13	1762.36	409	99.09
14	1062.63	383	99.27
15	937.20	385	99.30
16	185.35	229	99.57

links are situated there.

Comparing the ranks for vulnerable and critical links, it is appreciated that there is not a significant difference between them. Showing that in real networks both types of links can have the same location. The main links of the network, i.e. the ones obtained in the results of this case, can be the critical ones, creating the largest impact when damaged. Also the main links can be identified as the vulnerable ones, suffering a large impact when other parts of the network are damaged.

Nevertheless, the difference between critical and vulnerable links is fundamental. Sometimes, it will be necessary to focus on the critical links, for example when a hazard is forecasted and a collapse of the network needs to be avoided, in this case, critical links need to be identified. In other occasions, the identification of the vulnerable links becomes more relevant, for example, to avoid isolated nodes, or nodes, where the accessibility is very reduced, even when the number of affected users is reduced.

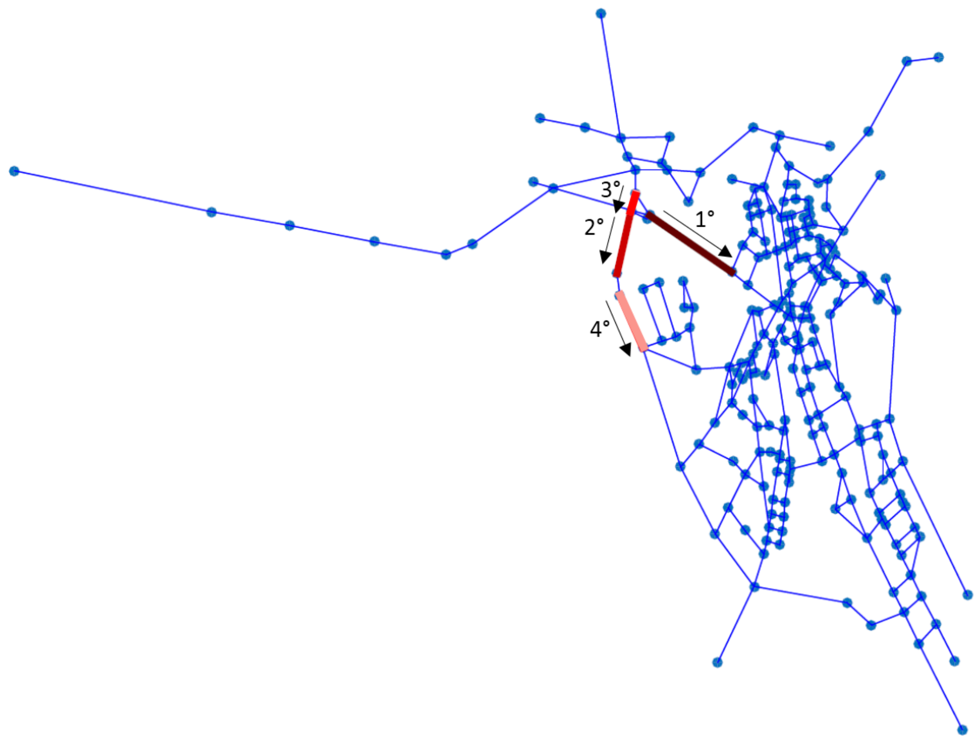

Figure 7.11: Critical links that reduce the resilience of the network more than a 4.5%

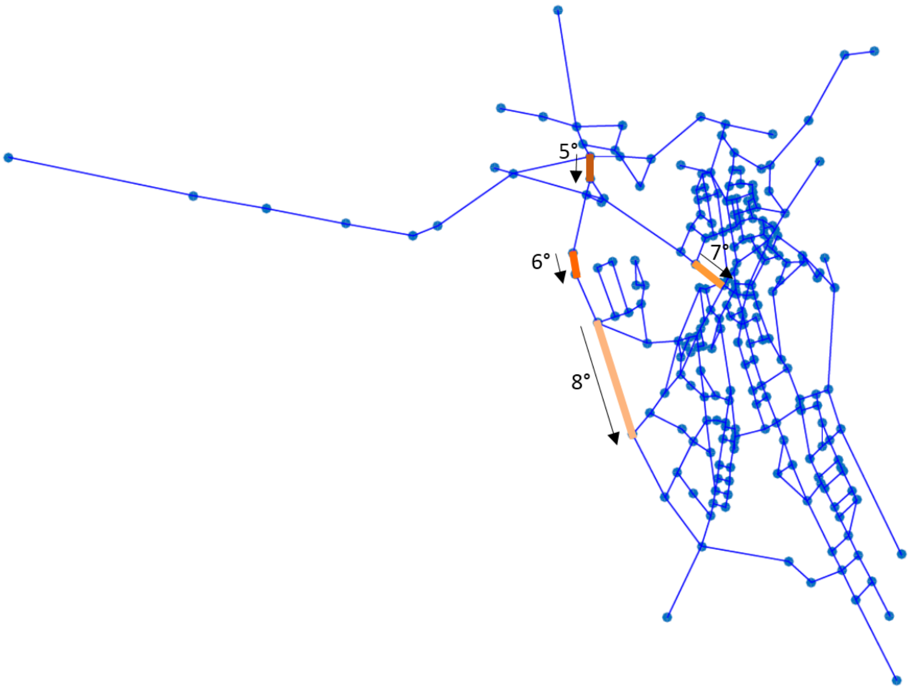

Figure 7.12: Critical links that reduce the resilience of the network from 2.5% to 4.5%

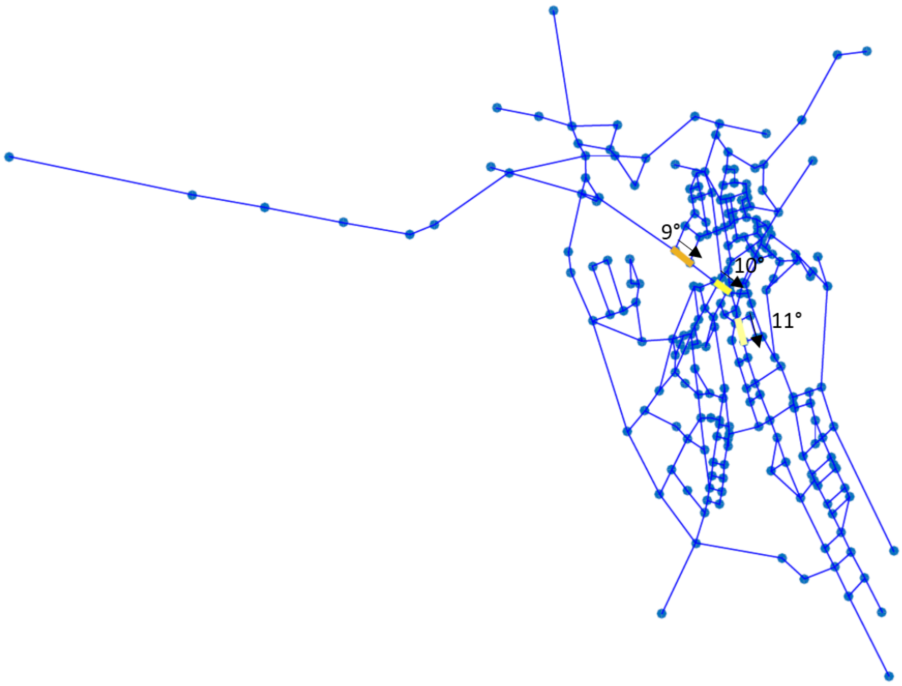

Figure 7.13: Critical links that reduce the resilience of the network from 1% to 2.5%

It is noted that in this example the cost function used is the BPR, therefore the capacity will be one of the main factors to identify the vulnerability. In this case, the capacity is assumed as equal for all of them, so the flow will be one of the main factors to detect the critical and vulnerable links, because when applying the UE equilibrium, the more saturated links will be the ones with less alternative options for each OD (small redundancy). When other cost functions are considered, other parameters can become more relevant.

7.6 Analysing vulnerable and critical areas in a traffic network

The methodologies previously introduced were developed to analyse specific links. Each of the links of the network was tested, and the results presented a rank from the most to the least critical and vulnerable links of a traffic network. The perturbations to simulate the damaging scenarios were affecting only one link each time. These methodologies based on a link analysis become relevant when the information required is about links, for example, works in one specific road, lane closures in one street, and small flooding affecting part of a link, these case have impacts in the network that require a method based on a link analysis.

However, when climatological perturbations are analysed, i.e. intense rain, coastal or river flooding, and strong winds, the damaged area usually affects more than one link. Also, other kind of perturbations such as road works or closures, can also affect extended areas of the network, therefore area analysis is necessary. Based on the previous methods to analyse critical and vulnerable links, methodologies to analyse the critical and vulnerable areas of a traffic network are presented in this section.

Firstly, the traffic network is divided in areas, which will be the ones analysed in the methodology. The division of the network can be made depending on different criteria. The criteria can cover aspects of the characteristic of the network, including the proximity to natural elements such as rivers or coastal zones, lower zones where the water is more probable to remain stagnant, and city center areas where the density of the traffic can be higher and a possible impact of a perturbation can cause worse consequences. On the other hand, a systematic division of the network can be made too, defining as many equal areas as wanted.

These methodologies provide a new approach. When extended parts of the network are damaged, the response of the network can be different that when one link is affected. Therefore, the consequences of perturbations affecting an extension of the network need to be analysed, these methodologies will identify and rank the weak areas of the network.

The FIM is defined following the same principles as in the previous methodologies.

For this methodology analysing extended zones, the FIM is formulated as follows,

$$F = \sum_{m=1}^{M} \theta_m^T \theta_m, \tag{7.13}$$

where F is the FIM, and $\theta_m = [\theta_1, \theta_2, \theta_3, ..., \theta_M]$ is the sensitivity vector which carries the information about each of the areas of the network, with M being the number of areas, in which the traffic network has been divided.

In the same way that two methodologies where defined to identify critical and vulnerable links in a traffic network, two methodologies are presented to analyse the critical and vulnerable areas of the network.

7.6.1 Methodology to identify and rank vulnerable areas of a traffic network

Based on the same principles as in the previous methodologies for link analysis, the sensitivity vectors are defined measuring the effects of an impact in the area analysed. In this methodology to identify the vulnerable areas, the vulnerability of the area is defined in the same way as the vulnerability of a link. Thus a vulnerable area will be the one that suffers the largest impact when other parts of the network are damaged.

The impact is introduced through a reduction of the capacity of one of the areas analysed, and the effect is measured by the variation of the travel times. In the same way as in the previous methods, a user equilibrium traffic assignment method is used to measure the variation of the travel times when a reduction of the capacity occurs.

For the methodology to identify vulnerable areas of a traffic network, a sensitivity vector is defined for each of the areas selected as follows,

$$\theta_m = [\frac{\Delta t_1}{\Delta C_m}, \frac{\Delta t_2}{\Delta C_m}, ..., \frac{\Delta t_m}{\Delta C_m}, ..., \frac{\Delta t_M}{\Delta C_m}], \tag{7.14}$$

where ΔC_m is the variation of the capacity for each area, and Δt_m is the variation produced in the travel time of each area when the capacity of the area m is reduced.

7.6.2 Methodology to identify and rank critical areas of a traffic network

As in the link analysis, a methodology to identify critical areas of a traffic network is presented. In this case, a critical area will be defined as the area that creates the largest impact when suffering a perturbation. Therefore, the sensitivity vector is defined for each of the areas as follows,

$$\theta_m = [\frac{\Delta t'_m}{\Delta C_1} \Delta T_1, \frac{\Delta t''_m}{\Delta C_2} \Delta T_2, ..., \frac{\Delta t'''_m}{\Delta C_m} \Delta T_m, ..., \frac{\Delta t'^m_m}{\Delta C_M} \Delta T_M], \tag{7.15}$$

where $\Delta t'^{m}_{m}$ is the variation in the travel times of area m when the capacity of the area m is reduced, and ΔT_m is the variation in the total travel times of the network when the area m is damaged.

Once the FIMs are defined for both methodologies, the vulnerable and the critical, the analysis of the eigenvalues and eigenvector follows the same process as in the link analysis. The eigenvalues being the ones identifying the level of vulnerability or criticality, and the eigenvectors, those selecting the area associated with the eigenvalue.

Due to the similarity with the previous methodologies, the characteristics explained in other sections for the link analysis, such as, what happens when a network is symmetric, and the differences between vulnerable and critical, can be comparable. Therefore the small example will not be presented in this section and the methodologies will be applied in a real case directly.

7.7 Case study: Sioux Fall, identifying critical and vulnerable areas.

To demonstrate the applicability of the methodologies, the methods proposed for the identification of critical and vulnerable elements are used in a real network. These methodologies are applied in the well-known Sioux Fall network, see Figure 7.14 for a description of the network. The initial variables of the links, such as capacity and free flow travel time, are defined in Table 7.15.

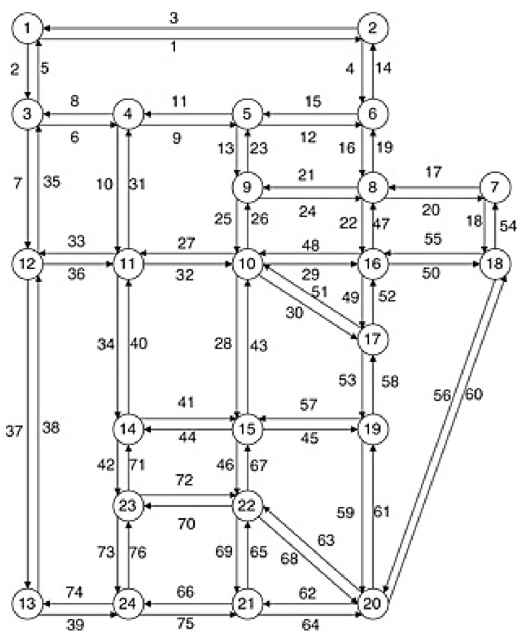

Figure 7.14: Sioux Falls network, defined by links and nodes

Table 7.15: Link characteristics of the Sioux Fall network.

Link	Capacity (users)	Free flow travel time (min)	Link	Capacity (users)	Free flow travel time (min)
1	1554.01	6	39	305.48	4
2	1404.21	4	40	292.59	4
3	1554.01	6	41	307.65	5
4	297.49	5	42	295.49	4
5	1404.21	4	43	810.72	6
6	1026.63	4	44	307.65	5
7	1404.21	4	45	873.89	3
8	1026.63	4	46	575.95	3
9	1066.97	2	47	302.75	5
10	294.53	6	48	291.30	4
11	1066.97	2	49	313.79	2
12	296.88	4	50	1180.79	3
13	600.00	5	51	299.61	8
14	297.49	5	52	313.79	2
15	296.88	4	53	289.44	2
16	293.92	2	54	1404.21	2
17	470.51	3	55	1180.79	3
18	1404.21	2	56	1404.21	4
19	293.92	2	57	873.89	3
20	470.51	3	58	289.44	2
21	303.01	10	59	300.16	4
22	302.75	5	60	1404.21	4
23	600.00	5	61	300.16	4
24	303.01	10	62	303.59	6
25	834.95	3	63	304.54	5
26	834.95	3	64	303.59	6
27	600.00	5	65	313.79	2
28	810.72	6	66	293.12	3
29	291.30	4	67	575.95	3
30	299.61	8	68	304.54	5
31	294.53	6	69	313.79	2
32	600.00	5	70	300.00	4
33	294.53	6	71	295.49	4
34	292.59	4	72	300.00	4
35	1404.21	4	73	304.71	2
36	294.53	6	74	305.48	4
37	1554.01	3	75	293.12	3
38	1554.01	3	76	304.71	2

All the possible routes for the OD pairs shown in Table 7.16 were obtained, with a total of 67622 routes. The large number of routes is due to the high redundancy of the network, however most of the routes from the 67622 have an unreasonable travel time and the users of the network will never choose them as an option. For this reason, and to facilitate the calculations, the number of routes has been reduced without losing the goal of representing all the possible routes of the network. Between each OD pair, only the routes with the smallest travel time, and in consequence the ones more likely to be used are selected. In this case, the threshold used is a 170% increment of the minimal travel time required to cover the distance between each OD pair for the free travel flow conditions. Therefore, the routes with smaller or equal travel time than the threshold are the ones chosen for the case study, obtaining a total of 484 routes. The demand (users/OD) introduced in this case study is shown in Table 7.16.

Table 7.16: OD pairs chosen for the Sioux Fall case study.

OD pair	Initial Node	Final Node	Demand(users/OD pair)
1	1	13	10
2	1	18	140
3	1	20	880
4	2	13	50
5	2	18	20
6	2	20	50
7	7	12	650
8	7	13	610
9	7	20	10
10	12	7	650
11	13	1	10
12	13	2	50
13	13	7	610
14	18	1	140
15	18	2	20
16	20	1	880
17	20	2	50
18	20	7	10

Four areas are selected for the analysis, being Area 1 (A1) in North-West part of the network, Area 2 (A2) in North-East, Area 3 (A3) in South-West, and finally Area 4 (A4) in South-East, as shown in Figure 7.15.

Applying the methodologies defined in Section 7.6.1 and 7.6.2 the results obtained are shown in Table 7.17 for the identification of the vulnerable areas, and in Table 7.18 for the identification of the critical areas.

The results show that the most critical and vulnerable area is Area 2 (North-East).

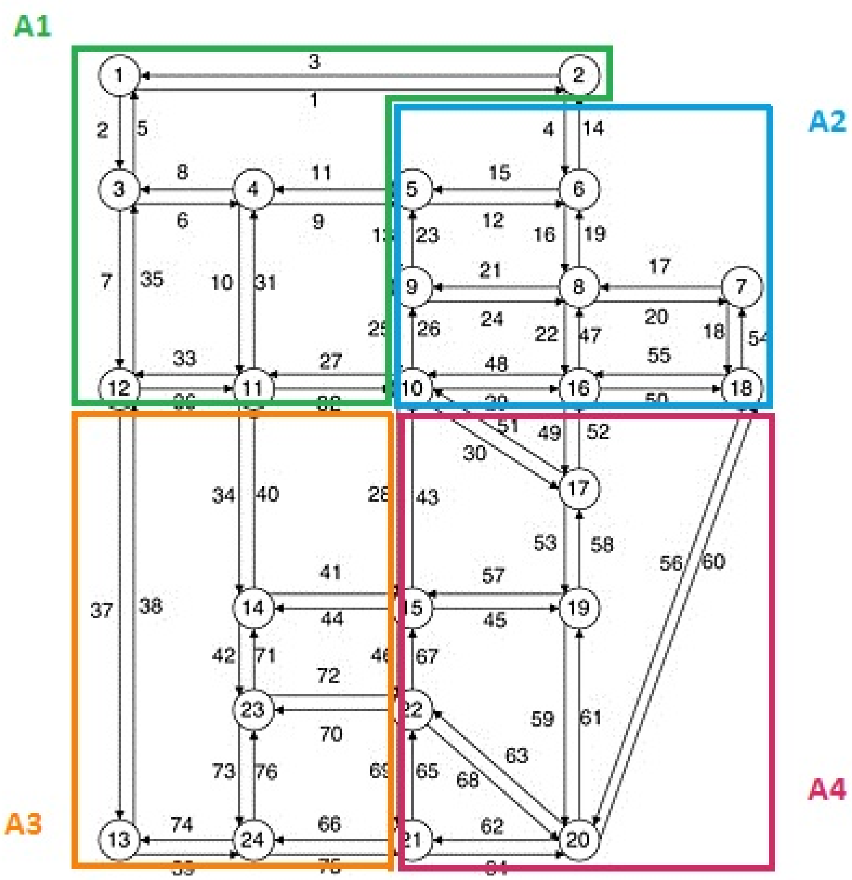

Figure 7.15: Sioux Falls network, areas selected for the vulnerability and criticality analysis.

Table 7.17: Vulnerable areas of the Sioux Fall network.

Rank position	Vulnerable Area	Eigenvalue
1	Area 2	16.43
2	Area 4 and Area 3	8.38
3	Area 1	2.28

In the case of Sioux Fall, due to the high number of routes analysed, and the high redundancy of the network, it is not possible to identify a priori which area is the most critical/vulnerable. For a better understanding of the results two more figures are introduced, where two of the main variables related to the criticality and the vulnerability of

Table 7.18: Critical areas of the Sioux Fall network.

Rank position	Critical Area	Eigenvalue
1	Area 2	2.1
2	Area 3	0
3	Area 4	0
4	Area 1	0

the links are illustrated.

Figure 7.16 is a representation of the number of routes crossing each link, the darker the color of the link, the larger the number of routes. In Figure 7.16, it is possible to identify the part of the network with more affluence of routes. Figure 7.17 represents the capacity of the links, in the way that the darker the color of the link the larger the capacity. This variable is very relevant in the identification of weak links because those links with small capacity tend to be more vulnerable.

Analysing Figure 7.16 and Figure 7.17, it is possible to notice that a combination of low capacity links with a high number of routes exits in Area 2 (links [16, 17, 19, 20, 22, 47, 48, 39]), making this area a reasonable solution for both of the methodologies applied.

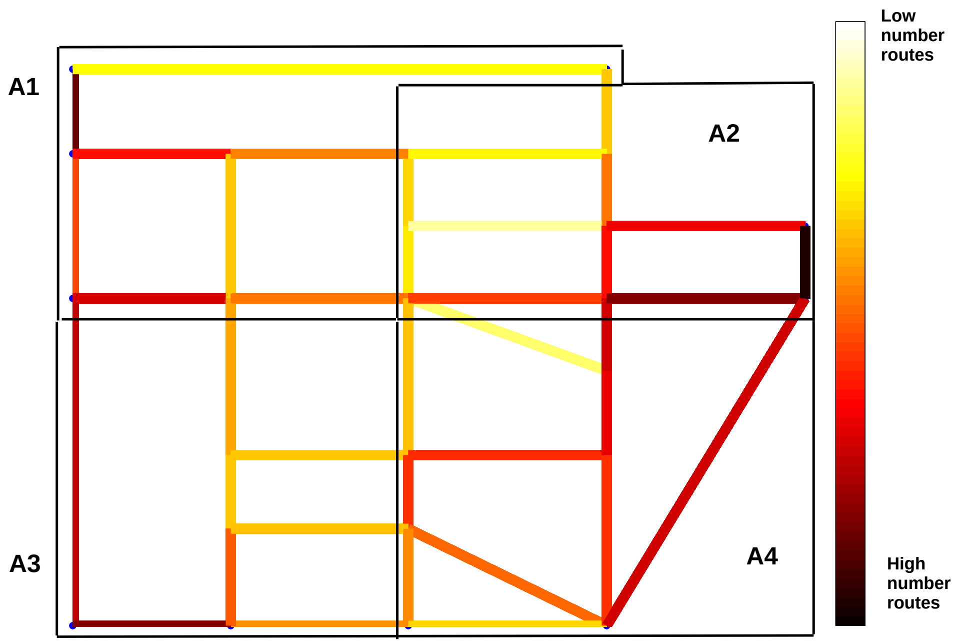

Figure 7.16: Sioux Falls network, showing the number of routes per link.

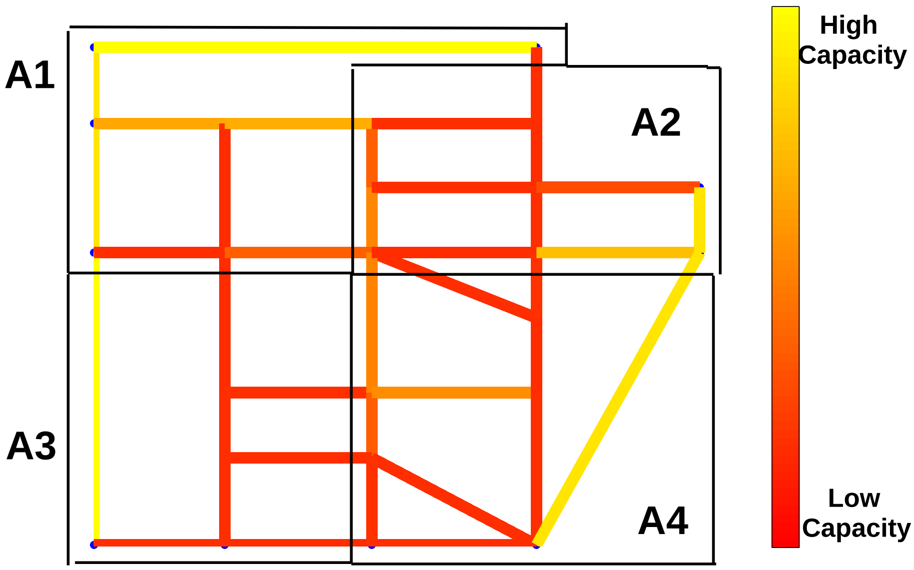

Figure 7.17: Sioux Falls network, showing the capacity of the links

Finally, the resilience is calculated to confirm the results, and to demonstrate that when the most critical area is affected by a perturbation, the lowest value of resilience is obtained, unlike when other less critical areas are affected. The results of the evaluation of resilience are shown in Table 7.19.

Table 7.19: Sioux Fall case study: values of resilience

Rank position	Critical Area	Resilience
1	Area 2	94.39
2	Area 3	97.18
3	Area 4	98.69
4	Area 1	99.87

The results prove that Area 2 is the most critical one, since the obtained value for resilience is 94.39 when damaging this area. However, when other areas are damaged the value of resilience is higher, for example when Area 3 is perturbed, the resilience obtained is 97.18.

7.8 Conclusions

This chapter presents two methodologies, one to identify and rank the vulnerable links of a traffic network, and the other one to identify and rank the critical links of a network. Both methodologies use the Fisher Matrix Information (FIM) for the identification, being defined in a particular way for each methodology. In addition, two more methods are introduced, analysing instead of specific links, areas of the network affected by a perturbation, therefore one methodology analyses the vulnerable areas of a traffic network, and the other identifies the critical areas.

- It is proven that using the eigenvector associated to each eigenvalue of the matrix, it is possible to identify and rank the vulnerable and the critical links of the network. Also, the same assumptions are used in the localization of vulnerable and critical areas.

- The eigenvalue determines the position in the rank, with the largest the one that identifies the most vulnerable or critical element, and going down in the rank as the value of the eigenvalue decreases. In this way, in one analysis, all the links of the network are studied.

- This chapter distinguishes between vulnerable and critical links, identifying both of them. In this way, specific strategies can be implemented depending on the desired objectives, focusing on either the vulnerable elements of the network or the critical locations.

- The importance of the critical links when the resilience of the network is analysed is highlighted. The improvement of the conditions of the most critical links can be used as a strategy to increase the resilience of a network. Therefore, the proposed methods also provide relevant information to improve the performance of the whole network when affected by a perturbation.

- The presented methodologies analyse the performance of traffic networks when suffering a perturbation, to systematically identify the vulnerable and critical links. Therefore, these methodologies uses variables such as the flow, the capacity, the travel time, and the demand. This methodology can be extended to other relevant variables.

- The impacts are evaluated in terms of travel time, using a user equilibrium traffic assignment model to measure the variation in the travel times when the perturbation takes place. However, this fact is easily convertible to economical cost, including other parameters such as road tolls.

- These methodologies allow the introduction of partial impact on the link, unlike others methodologies, the complete closure or removal of the link is not mandatory. In this way, it is possible to incorporate any degree of damage, from a small reduction of the characteristics of the link to complete failure.

- The introduction of the methodologies that analyse the vulnerable and critical areas of the network makes more realistic the analysis of extreme weather events in traffic networks. Usually these events affect more than one link in a network. The methodologies are adaptable and flexible when defining the areas to be analysed. Sometimes particular areas of the network may require specific analysis because of their characteristics or location, for example areas near a river are more probable to suffer flooding. The selected areas can be changed and multiple analyses can be developed for better comprehension of the behavior of the network.

- Finally, the results obtained with these methods can be combined with others obtained from other tools, and more viewpoints can be included. For example, when a particular network is being analysed, parameters such as crucial locations, proximity to natural elements such as rivers, sea, and mountains, and critical infrastructures such as bridges and tunnels, can be specifically incorporated in the results, since these elements inherently included a certain degree of vulnerability by their very nature.

Part IV

Conclusions, Future Work and Publications

Chapter 8

Conclusions, future works, and publications

Contents

8.1 Conclusions

For a better understanding of this section, the conclusions are divided in three parts, each of them related to one of the questions presented in the introduction of the research objectives of the thesis. Therefore, the sections will focus on the definition, evaluation, and improvement of resilience, and the conclusions are presented as follows:

8.1.1 Conclusions with respect to the definition of traffic resilience

The definition of resilience of traffic networks when a perturbation occurs needs to include the whole process, from the beginning of the perturbation, until the total recovery of the network. The following aspects should be part of a definition of this concept:

- The incorporation of the perturbation phase, including the ability of the network to absorb, resist or accommodate a perturbation, understanding the perturbation stage as a dynamic process where the disruption may suffer multiple changes.

- The incorporation of the recovery stage, including the ability of the network to restore, return or recover from the perturbation. This phase lasts until the network has reached a new equilibrium, and normal operation conditions.

- The consideration of a time frame necessary to develop the performance of the whole process, including the perturbation, and the recovery stage. This parameter is crucial when the resilience of a network is evaluated.

In this thesis, the resilience of a traffic network is evaluated for a particular perturbation. Therefore, if the resilience is evaluated for the same traffic network when a different disruption takes place, the results may change. Even so, the values obtained give a relevant indication of the performance of the network, and even more, they can help to identify those aspects for the development of more resilient traffic networks.

8.1.2 Conclusions with respect to the quantification of traffic resilience

A complete methodology to evaluate resilience in traffic networks when an extreme weather event occurs has been presented in this thesis, by the combination of the tools presented in Chapter 4, and 5 of the original contributions. This methodology to evaluate resilience in traffic networks combines a dynamic equilibrium-restricted assignment model, and a new bounded link travel cost function which allows a complete analysis of the impact created by the hazard. The dynamic assignment model assesses on a day to day approach, the variations created by the perturbation on the exhaustion level of the total network. It is noted that in the presented exhaustion level, several parameters are analysed, including not only the effects created on the travel times but also how the disruption affect to the users. Thus, the stress suffered by the users of the network during the perturbation and recovery process is measured and included as a parameter of the global exhaustion level of the network.

In addition, in order to create a specific model that can describe the impact of weather events on a traffic network, the new cost function explicitly considers the effect of climatological events, including on its formulation parameters such as the intensity of the hazard, and the local vulnerability of each of the links of the network. Introducing this new cost function in the presented assignment model allows to simulate on a traffic network the effect of these weather-related parameters, since with common cost functions, for example the BPR, the impacts of the hazard needs to be introduced with reductions in the capacity or the travel time, and cannot be introduced directly.

Hereunder, the conclusions obtained for each of these chapters are shown.

Summary of the conclusions from Chapter 4. *A dynamic equilibrium-restricted assignment model, together with a method to quantify resilience.*

- A method to quantify the resilience of traffic networks when a perturbation takes place is presented. This methodology is based on a novel dynamic restricted equilibrium assignment model, which allows a dynamic simulation of the impact of the disruption on traffic behaviour, and a dynamic evaluation of the results.

- The dynamic equilibrium restricted assignment model allows the simulation of the network behaviour when a disruptive event occurs. This approach permits the evaluation of the extra cost generated by the perturbation, and the assessment of the stress level of the system together. This model proposes that the network behaviour is restricted by a system impedance to alter its previous state, which is limited by the capacity of adaptation of the users.

- By the combination of the evolution of the cost level, and the evolution of the stress level obtained from the dynamic equilibrium restricted assignment model, the exhaustion level of the network is calculated. The exhaustion level of the network determines how far the network is from total exhaustion.

- Two different resilience indices are evaluated in the model, the perturbation resilience, and the recovery resilience. Each of them analyses a different stage of the network, the perturbation stage, which takes place during the impact of the disruption, and the recovery stage, which occurs once the perturbation has finished until the network reaches a new equilibrium stage with normal conditions.

- The **perturbation resilience** defines the capacity of the network to withstand a disrupted scenario. It is calculated by the normalized area over the exhaustion level.

- The **recovery resilience** includes in its evaluation not only the area, but also a normalized slope associated with the exhaustion level, which allows introducing in the assessment of resilience the time necessary to reach a new equilibrium.

- It has been shown that the proposed methodology includes in the evaluation most of the indicators that other authors use to describe resilience, such as redundancy, robustness, rapidity, and resourcefulness. Therefore, this method allows a more efficient approach to assess the concept, without the necessity of evaluating each of the parameters separately. It has been noted that the individual evaluation of the indicators may need a large amount of resources, and the value of total resilience index is not easy to reach through this type of evaluation.

Summary of the conclusions from Chapter 5. *A bounded link travel cost function that explicitly considers the effect of climatological events.*

- A new bounded link travel cost function has been presented, which allows the inclusions of the effect of weather events, such as intense rain, fog, or wind, in traffic networks. The implementation of this new function helps the development of new tools to deal with the impact caused by any climatological perturbation affecting the performance of traffic networks.

- A parameter that determines the intensity of the hazard, has been included in the new travel cost function. The intensity of the hazard has been highlighted by several authors as the characteristic of the hazard that best describes the impact caused on the traffic flow.

- In addition, the local vulnerability has been introduced in the function to reflect the impact caused in the network. This parameter determines the specific sensibility that each link of the network experiences when a particular hazard occurs.

- When analysing the effects of extreme weather events in traffic networks, deteriorations near the free flow conditions of the network can be experienced. Thus, when strong perturbations affect the network, increments in the travel times can be obtained for low flows. The proposed travel cost function reflects this aspect.

- The presented formulation, unlike most of the travel cost functions presented in the literature, is upper-bounded. This fact has two main contributions for the performance of the new cost function. Firstly, it can better represent the reality of travel times. Travel times have been usually analysed with functions where infinite values can be reached, even though this is not very realistic. Secondly, this characteristic gives the new formulation an efficient behaviour when working with optimization problems. Thus, the proposed function avoids high computational times, and even some possible overflows.

- Due to the mathematical properties of the new function, time-varying perturbations can be simulated. Therefore, this function can be implemented when dynamic analyses are performed.

- The proposed cost function has been specially designed for the assessment of climatological impact in traffic networks. However, it has been demonstrated that the function can be used in normal conditions, and its behaviour is comparable to other functions presented in the literature.

8.1.3 Conclusions with respect to the improvement of traffic resilience

Once a methodology to evaluate resilience has been introduced, the next goal is the improvement of the resilience of traffic network. In this thesis, part of this goal has been performed by two different tools. Firstly, an innovative bi-phase sensitivity analysis has been presented in Chapter 6. The sensitivity analysis has been performed with an innovative methodology specifically designed for complex models, where two phases are developed, including a global phase to reduce the number of selected sampling points, and a local phase to analyse the sensitivity on each of these selected points. The global phase has been developed by using a statistical approach that allows covering the entire space of the parameters with a significant reduction of the studied points. In addition, since the results obtained from the sensitivity analysis are complex, and they do not give answer to all the questions, such as the selection of the best adaptation measures when a hazard is forecasted, a mapping for different scenarios has been introduced. This mapping analysis allows the simulation of particular situations of the network and the analysis of variations in each of the variables. In this way, it is possible to define which adaptation measures can be performed before, and during the perturbation to reduce the impact of the hazard.

Secondly, the importance of the critical links when the resilience of the network is detailed. It is noted that the identification of these elements can contribute to a powerful strategy to increase the resilience of a network by improving the conditions of the most critical links. Therefore, in Chapter 7, new methods to rank the critical and vulnerable links of a traffic network are presented. The main advantage of these methodologies is that allows a systematic analysis of each of the links of the network, identifying not only the most vulnerable and the most critical link, but also ranking each of the links of the network, depending on their degree of vulnerability or criticality.

Hereunder, the conclusions obtained for each of these chapters are shown.

Summary of the conclusions from Chapter 6: *Mapping and Bi-phase Sensitivity analysis*

- The methodologies presented in the previous chapter are tested together in a large network. In addition, a mapping of multiples scenarios is presented, in order to understand how the resilience changes depending on the parameters. Therefore, values of resilience are obtained for different demands, intensities of the hazard, local vulnerabilities, and capacities of adaptation.

- A bi-phase sensitivity analysis has been conducted for a better understanding of the parameters presented in the methodology to evaluate the traffic resilience. The sensitivity analysis presented introduces an statistical approach to the problem, being able to reduce the number of points needed in the performance of the analysis to cover the entire space.

- The two levels methodology to analyse the sensitivity includes a local (One-At-a-Time), and a global techniques (Latin Hypercube). This kind of methodology is justified when a large number of variables are involved, because local methods are very efficient but they do not cover the entire space; whereas, global methods provide a robust and reliable approach but the computational cost could be too high in complex models.

- The results obtained for the sensitivity analysis show that the demand of the network, and the intensity of the hazard reach high sensitivity values. This means that these parameters have a key role in the analysis of resilience.

- Unlike the intensity of the hazard, the local vulnerability, and the demand, which produce a decrease in the value of resilience when their values increment, the capacity of adaptation of the users, creates an increment of the resilience when its value is increased.

- In addition to the main parameters of the method, other variables that are not modeled by means of a parameter in the formulation, but have an important role in the evaluation have been analysed. These variable are the redundancy of the traffic network, and the area affected by the perturbation.

- The area affected by the hazard can significantly modify the results of the sensitivity analysis. Therefore, when the zone exposed to the hazard is reduced, the values of sensitivity obtained for the intensity of the hazard may not be as high as expected. On the contrary, when the area covers a relevant extension of the network the sensitivity value of this parameter rises.

- The redundancy of the network highly affects the sensitivity values obtained for the capacity of adaptation of the users. Thus, when incrementing the redundancy of the network, the users have more options to reach their destinations. If the capacity of adaptation has a high value, the users can take advantage of this redundancy, which makes this parameter very relevant when the sensitivity is analysed. On the other hand, if the redundancy of the network is very low, even if the users have a high capacity of adaptation, the sensitivity values of this parameter remain low, since the users do not have options to choose.

Summary of the conclusions from Chapter 7. *A methodology to identify the vulnerable, and the critical links of a traffic network.*

- In this chapter two methodologies, one to identify and rank the vulnerable links of a transport network, and the other one to identify and rank the critical links of a network are presented. Both methodologies use the Fisher Matrix Information (FIM) for the identification, being defined in a particular way for each methodology.

- This method differentiates between vulnerable and critical links, defining and identifying both of them. In this way, specific strategies can be implemented depending on the desired objectives, either the vulnerable elements of the network or the critical locations.

- It is proven that using the eigenvector associated with each eigenvalue of the matrix, it is possible to identify and rank the vulnerable/critical links of the network. Then, the value of each eigenvalue determines the position in the rank, being the largest, the one that identifies the most vulnerable/critical link, and going down in the rank as the value of the eigenvalue decreases. Therefore, in one analysis, all the links of the network are analysed.

- The presented methodologies include the performance of the traffic, and the disruptions affecting the traffic. Therefore, this methodology systematically identifies the vulnerable/critical links using variables such as the flow, the capacity, the travel time, and the demand. The impacts are evaluated in terms of travel time, but this is easily convertible to economical cost, or any other cost to be considered.

- In addition, unlike other methodologies, the complete closure or removal of the link is not mandatory, a partial impact in the link with the desired intensity can be incorporated.

- The results obtained with these methods can be combined with others obtained from other tools, and more viewpoints can be included. For example, parameters such as crucial locations, proximity to natural elements such as rivers, sea, and mountains, and critical infrastructures such as bridges and tunnels, can be specifically incorporated in the results.

- These methodologies have been implemented to detect vulnerable, and critical areas of the network. Based on the same principles, these methodologies allow the analysis of wide extensions of the network, since when evaluating the effects of climatological perturbations, it is probable that these disruptions may affect more than a particular link.

- Finally, the methodologies introduced in this chapter can have a major contribution in the area of resilience of traffic networks. Being able to rank the vulnerable, and critical links contributes to a better knowledge of the element of the network when perturbations occur, and hence of its resilience, not only in a scientific level, but also in a practical level.

8.2 Future works

Several tools have been presented for the evaluation, and improvement of resilience on traffic networks when perturbations take place. However, they are just the starting point, and many works based on the ones presented in this thesis will be developed in future researches. Hereunder, some of the areas for future work are presented.

- Stochastic approach of the methodology to evaluate resilience. The methodology presented in this thesis for the evaluation of traffic resilience has been developed with a deterministic approach. Due to the novelty of the topic, it has been considered to firstly create a deterministic methodology for the evaluation. However, it is expected that implemented in a stochastic methodology could be beneficial.

- Implementation of the methodology to evaluate resilience for new types of users. The proposed method has being developed by the principles of the user equilibrium, i.e, it has been considered that each of the users of the network tries to minimize their personal travel time. This is what happens when "private" users are analysed. However, the proposed methodology can be useful for companies or public entities, who manage a large group of users/vehicles. In this case, the approach needs to be different, since a company may look for an optimization of the total number of users more than a particular optimization. Therefore, the method can be implemented by using this type of approach which has its base in the system optimum principles.

  In a near future, with the increment of tools to manage the information of traffic networks, it may become more interesting the use of approaches based on the system optimum principles. Nowadays, the information that the users have about the real state of the network is only partial, and far from perfect. Therefore, the users tend to look for their personal interest. However, this may change when more advance and real information of the state of the traffic is available. Then, the users may prefer to adapt their behaviour in order to minimize the total travel times, and hence their personal travel times. In addition, from a policy perspective, the implementation of a system optimum could better satisfy the priorities of transport planners and managers addressing the challenges of extreme weather events and the adaptation of the transport system. As explained before, when the traffic network is managed from a global perspective, the approach based on a system optimum will be the one achieving this goal.

- Application of the methodology to different transport modes. The proposed methodology for the evaluation of resilience has been specifically designed for traffic networks, mainly evaluating the behaviour of cars on roads. However, the resilience concepts developed in this thesis can be very useful when applied to other transport modes, for example, in the case of rail networks. Even though, the traffic assignment approach is not valid in other systems such as trains, the approach defined for the concept can be applicable by defining an exhaustion level according to the rail system characteristics. Therefore, the concept presented for the evaluation of resilience can be introduced specifically for the behaviour of other transport modes.

In addition, with the growth of real-time information that may be available for users of transport, some of the new concepts included in the presented traffic assignment method can be introduced in models for other transport modes, such as buses. These transport modes require a different approach for their models but the inclusion of concepts such as the capacity of adaptation when a disruption takes place can be advantageous. When real-time information is available to all users, bus routes may be subjected to some modification to avoid eventual disruptions on the network. In this way, transport modes such as buses can incorporate a degree of capacity of adaptation that can help to reduce the impact of events on the network. In the case of systems such as rail, these concepts would require a completely different approach, since the lines used for this transport mode are fixed, and the options for modification are extremely limited.

- Stochastic approach of the proposed bi-phase sensitivity analysis. In the same way that the resilience methodology can be developed including an stochastic approach of the problem, the sensitivity analysis can be implemented using this approach too.

- Identification of the critical links by using the evaluation of resilience inside the Fisher matrix. As a first approach to the problem, the methodology has been developed by using variables, such as the travel times, and the capacity of the links of the network. However, in the case of the critical links, a methodology to identify them by using the resilience as a parameter for their identification can be developed.

- Stochastic approach of the method to identify the critical and vulnerable links. The new method presented for the identification of these elements have been developed with the use of a deterministic approach of the Fisher matrix. It is noted that the Fisher matrix can be upgraded with an stochastic procedure.

8.3 Publications

The results of this thesis have been presented in the following journal papers, and conferences:

Journal Papers:

- *Nogal, M., O'Connor, A., Caulfield, B., and Martinez-Pastor, B.* (2016). Resilience of Traffic Networks: From Perturbation to Recovery via a Dynamic Restricted Equilibrium Model. Reliability Engineering & System Safety, 156, 84-96

- *Nogal, M., O'Connor, A., Martinez-Pastor, B., and Caulfield, B.* (2017). Novel Probabilistic Resilience Assessment Framework of Transportation Networks against Extreme Weather Events. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 3(3), 04017004.

- *Nogal, M., Martinez-Pastor, B., O'Connor, A.and Caulfield, B.* (2017). A bounded link travel cost function to include the weather effect on a traffic network. Submitted.

- *Martinez-Pastor, B., Nogal, M., and O'Connor, A.* (2017). Novel Probabilistic Resilience Assessment Framework of Transportation Networks against Extreme Weather Events. Reliability Engineering & System Safety. Submitted.

- *Martinez-Pastor, B., Nogal, M., and O'Connor, A.* (2017). Identifying critical and vulnerable links in transport networks, new approach using the Fisher Matrix. Reliability Engineering & System Safety. Submitted.

Conferences:

- *Martinez-Pastor, B., Nogal, M., O'Connor, A.and Caulfield, B.* A Dynamic Restricted Model to Evaluate the Traffic Resilience Under Extreme Weather Events. International Conference on Transportation System Resilience to Climate Change and Extreme Weather Events, Washington, September 2015.

- *Martinez-Pastor, B., Nogal, M., O'Connor, A.and Caulfield, B.* Evaluation of Resilience in Traffic Networks: Models and Characteristics. The Irish Transport Research Network Conference, Galway, August 2015.

- *Martinez-Pastor, B., Nogal, M., O'Connor, A.and Caulfield, B.* A Sensitivity Analysis of a Dynamic Restricted Equilibrium Model to Evaluate the Traffic Network Resilience. Annual Meeting Transportation Research Board, Washington, January 2016.

- *Nogal, M., Martinez-Pastor, B., O'Connor, A.and Caulfield, B.* Dynamic Restricted Equilibrium Model to Determine Statically the Resilience of a Traffic Network to Extreme Weather Events. 12th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP12 Vancouver, Canada, July 12-15, 2015.

- *Nogal, M., O'Connor, A., Martinez-Pastor, B.,and Caulfield, B.* Statistical Tools to Help the Decision-Making Process within the Risk Analysis of Transportation Networks in Response to Extreme Weather Events. International Forum on Engineering Decision Making, IFED 2015, Japan, May 6 - 9, 2015.

- *Martinez-Pastor, B., Nogal, M. and O'Connor, A.* Bi-phase methodology for sensitivity analysis of complex models.CERI Conference, Galway, August 2016.

- *Olmsted, S., O'Connor, A., Samaras, C., Cook, L. and Martinez-Pastor, B.* A Climate Engineering Assessment for Transportation Assets - Incorporating Probabilistic Analysis into Extreme Weather and Climate Change Design Engineering. Annual Meeting Transportation Research Board, Washington, January 2017.

Part V

Appendix

Appendix

Table 8.1: Capacity of the links of the Sioux Fall case study.

Link	ν_a^{max} (users)	Link	ν_a^{max} (users)	Link	ν_a^{max} (users)	Link	ν_a^{max} (users)
1	1554,01	20	470,51	39	305,48	58	289,44
2	1404,21	21	303,01	40	292,59	59	300,16
3	1554,01	22	302,75	41	307,65	60	1404,21
4	297,49	23	600,00	42	295,49	61	300,16
5	1404,21	24	303,01	43	810,72	62	303,59
6	1026,63	25	834,95	44	307,65	63	304,54
7	1404,21	26	834,95	45	873,89	64	303,59
8	1026,63	27	600,00	46	575,95	65	313,79
9	1066,97	28	810,72	47	302,75	66	293,12
10	294,53	29	291,30	48	291,30	67	575,95
11	1066,97	30	299,61	49	313,79	68	304,54
12	296,88	31	294,53	50	1180,79	69	313,79
13	600,00	32	600,00	51	299,61	70	300,00
14	297,49	33	294,53	52	313,79	71	295,49
15	296,88	34	292,59	53	289,44	72	300,00
16	293,92	35	1404,21	54	1404,21	73	304,71
17	470,51	36	294,53	55	1180,79	74	305,48
18	1404,21	37	1554,01	56	1404,21	75	293,12
19	293,92	38	1554,01	57	873,89	76	304,71

Table 8.2: Routes of the Sioux Fall case study, given by the links.

Route Id	OD	Links	Route Id	OD	Links
1	1-18	2 7 36 32 29 50	31	7-12	17 21 25 27 33
2	1-18	2 6 10 32 29 50	32	7-12	17 22 48 27 33
3	1-18	14 16 22 50	33	7-12	18 55 48 27 33
4	1-18	2 6 9 13 24 22 50	34	7-12	17 19 14 3 2 7
5	1-18	2 6 9 12 16 22 50	35	7-12	18 55 49 53 57 44 40 33
6	1-18	1 4 16 20 18	36	7-12	17 19 15 11 8 7
7	1-20	2 7 37 39 75 64	37	13-7	38 36 32 29 50 54
8	1-20	2 7 36 34 42 72 68	38	13-7	39 76 71 40 32 29 47 20
9	1-20	2 7 36 34 41 45 59	39	13-7	39 76 71 41 43 29 50 54
10	1-20	2 6 10 32 30 53 59	40	13-7	39 75 65 67 43 29 47 20
11	1-20	2 6 9 13 25 28 46 68	41	13-7	39 75 65 67 45 58 52 50 54
12	1-20	2 6 9 12 16 22 49 53 59	42	13-7	39 75 64 61 58 52 47 20
13	24-2	75 64 60 54 17 19 14	43	21-1	66 74 38 35 5
14	24-2	74 38 35 5 1	44	21-1	66 76 71 40 31 8 5
15	24-2	76 71 41 45 58 52 47 19 14	45	21-1	65 67 44 40 31 8 5
16	24-2	76 71 40 32 26 24 19 14	46	21-1	65 67 43 27 31 8 5
17	24-2	75 65 67 45 58 52 47 19 14	47	21-1	65 67 43 26 23 11 8 5
18	24-2	75 64 61 58 52 47 19 14	48	21-1	65 70 71 40 33 35 5
19	2-20	4 16 22 49 53 59	49	24-8	75 64 61 58 52 47
20	2-20	4 16 20 18 56	50	24-8	75 65 67 45 58 52 47
21	2-20	4 15 13 25 28 46 68	51	24-8	76 72 67 45 58 52 47
22	2-20	4 15 13 25 30 53 59	52	24-8	76 71 41 45 58 52 47
23	2-20	4 16 22 50 56	53	24-8	76 71 41 43 29 47
24	2-20	3 2 7 37 39 75 64	54	24-8	76 71 40 32 29 47
25	2-13	3 2 7 37	55	20-1	54 17 19 14 3
26	2-13	4 16 22 49 53 59 62 66 74	56	20-1	63 67 43 26 23 11 8 5
27	2-13	4 15 11 10 34 42 73 74	57	20-1	63 70 71 40 31 8 5
28	2-13	4 15 13 25 28 44 42 73 74	58	20-1	61 58 51 26 23 11 8 5
29	2-13	4 16 21 25 28 46 69 66 74	59	20-1	61 58 51 27 31 8 5
30	2-13	4 16 22 49 53 57 46 69 66 74	60	20-1	60 54 17 21 23 11 8 5

Table 8.3: Initial equilibrium state for the Sioux Fall case study; link flow, link flow travel time, link free travel time.

Link	Link Flow (users)	Travel Time (min)	Free Travel Time (min)	Link	Link Flow (min)	Travel Time (min)	Free Travel Time (min)
1	908.28	6.11	6	39	275.17	22.73	4
2	1491.72	31.29	4	40	201.85	6.63	4
3	919.56	6.16	6	41	252.69	20.65	5
4	908.28	49.95	5	42	0	4	4
5	1480.44	30.98	4	43	505.01	6.63	6
6	847.54	16.87	4	44	154.87	5	5
7	909.24	4.95	4	45	468.68	3	3
8	838.27	16.12	4	46	354.24	3.23	3
9	645.95	2.1	2	47	600	49.52	5
10	201.58	9.54	6	48	267.59	23.95	4
11	656.41	2.16	2	49	67.35	2	2
12	187.94	4.6	4	50	891.72	8.27	3
13	458.01	14.6	5	51	17.56	8	8
14	319.56	39.64	5	52	690.62	19.89	2
15	265.06	22.16	4	53	268.93	12.32	2
16	1096.23	19.99	2	54	976.07	3.46	2
17	641.13	28.03	3	55	334.94	3	3
18	1243.22	10.81	2	56	600	4	4
19	584.62	19.81	2	57	67.35	3	3
20	908.28	29.67	3	58	708.17	19.93	2
21	56.51	10	10	59	201.58	5.78	4
22	291.72	33.4	5	60	376.07	4	4
23	391.35	6.36	5	61	239.49	14.55	4
24	103.77	10	10	62	0	6	6
25	354.24	3	3	63	206.37	7.55	5
26	334.84	3	3	64	266.11	31.43	6
27	314.93	5	5	65	383.38	17.74	2
28	354.24	6	6	66	307.23	23.06	3
29	509.38	39.26	4	67	808.52	28.3	3
30	201.58	11.66	8	68	354.24	42.85	5
31	181.85	6.51	6	69	0	2	2
32	570.59	32.49	5	70	46.99	4	4
33	334.94	50.46	6	71	299.67	29.3	4
34	0	4	4	72	265.76	21.65	4
35	907.23	4.91	4	73	0	2	2
36	369.01	54.01	6	74	907.23	39.95	4
37	644.18	3	3	75	356.73	26.55	3
38	1276.24	12.39	3	76	518.44	19.57	2

Table 8.4: Equilibrium state for the case study, route demand, route travel time, and free route travel time.

Routes	Demand (users)	Route Travel Time (min)	Free Travel Time (min)	Routes	Demand (users)	Route Travel Time (min)	Free Travel Time (min)
1	0	170.27	26	31	0	96.48	27
2	0	137.71	26	32	0	140.84	23
3	0	117.72	21	33	267.59	93.22	20
4	103.77	116.53	33	34	0	129.87	24
5	187.94	116.53	24	35	67.35	93.22	27
6	308.28	116.53	18	36	265.06	93.22	19
7	44.18	119.95	24	37	369.01	149.87	23
8	0	162.75	31	38	0	229.17	31
9	0	123.68	30	39	140.38	149.87	30
10	201.58	119.95	33	40	0	220.4	30
11	354.24	119.95	32	41	90.62	149.87	24
12	0	128.36	29	42	0	214.26	29
13	0	152.92	25	43	307.23	111.29	18
14	600	94.34	21	44	0	132.18	27
15	0	221.32	30	45	87.52	111.29	28
16	0	160.45	35	46	46.19	111.29	30
17	0	224.39	27	47	159.06	111.29	29
18	0	221.32	29	48	0	144.03	28
19	0	123.44	20	49	221.93	161.87	22
20	600	114.42	16	50	0	164.93	20
21	0	141.79	31	51	265.76	161.87	21
22	0	119.46	31	52	112.31	161.87	23
23	0	115.61	19	53	0	164.93	26
24	0	126.11	30	54	0	176.77	24
25	600	45.4	17	55	319.56	101.1	22
26	0	192.45	33	56	159.38	101.1	32
27	0	133.75	31	57	46.99	101.1	31
28	0	146.66	38	58	16.4	101.1	32
29	0	157.19	38	59	1.15	101.1	33
30	0	188.91	31	60	56.51	101.1	34

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