Optical characterisation of plasmonic nanostructures on planar substrates using second–harmonic generation

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Abstract: Off-normal, polarization dependent second–harmonic generation (SHG) measurements were performed ex situ on plasmonic nanostructures grown by self–assembly on nanopatterned templates. These exploratory studies of Ag nanoparticle (NP) arrays show that the sensitivity of SHG to the local fields, which are modified by the NP size, shape and distribution, makes it a promising fixed wavelength characterization technique that avoids the complexity of spectroscopic SHG. The off–normal geometry provides access to the out–of–plane SH response, which is typically an order-of-magnitude larger than the in–surface–plane response measured using normal incidence, for example in SHG microscopy. By choosing the plane of incidence orthogonal to the NP array direction, it was shown that the \( p \)–polarized SH response, as a function of input polarization, is very sensitive to NP morphology, with a change of 20% in the aspect ratio of the NPs producing a variation of a factor of 30 in the easily measurable ratio of the \( p \)–polarized SH field strength for \( s \)– and \( p \)–polarized input. The results show that such a fixed geometry could be used for the in situ characterization of anisotropic nanostructure morphology during growth by self–assembly, which could be particularly useful in situations where rotating the sample may be neither desirable nor easily accomplished.

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1. Introduction

Plasmonic nanostructures are being used as building blocks for a variety of nanophotonics applications. When metal nanoparticles (NPs) are illuminated, localized surface plasmon resonances (LSPRs) are excited, which are extremely sensitive to interactions with the environment. If several NPs are located in close proximity to one another, such that their induced evanescent fields overlap, the interactions can be so strong that the time varying charge distribution and the associated spectral response of the system will differ drastically from that of the isolated constituents [1]. Optical interactions between plasmonic NPs can produce many new phenomena, including strong local field enhancement, broadened spectral response and directional emission, which can be utilized for numerous applications, including waveguiding [2], biosensing [3] and improving the efficiency of solar cells [4]. The dramatic amplification of the optical response has also led to the development of LSP–enhanced spectroscopies, such as a variation of surface enhanced Raman scattering (SERS) that allows single molecule detection [5].

For these enhancement effects, strong coupling between nanoparticles is desired and the required length scales are challenging for conventional lithographic techniques, especially if large patterned areas are required. In these cases, self-assembly methods are an attractive alternative solution. Recently, an experimental approach for producing strongly coupled, aligned NP arrays in a reproducible fashion was developed by a number of groups, using similar methodologies based on glancing angle deposition on patterned templates. For example, the controlled annealing of vicinal single crystal Al2O3 substrates leads to faceted templates, which can be adjusted to control the periodicity and the growth of aligned nanoparticle (NP) arrays [6, 7]. Alternatively, low energy ion beam irradiation can produce self-organized, nanoscale periodic patterns on various solid substrates [8–14]. NPs deposited on such templates align themselves with respect to the ripple or facet direction, as shown in Fig. 1.

Monitoring the formation, in real time, of nanostructured templates and the evolution of NP arrays during growth, in order to ensure the required functionality, is challenging. Surface and interface sensitive optical techniques offer advantages in this area over conventional surface science techniques. Specifically, all pressure ranges of gas–solid interfaces are accessible, the material damage and contamination associated with charged particle probes is eliminated, and insulating substrates can be studied without the problem of charging effects [15]. For example, it has been shown recently that reflection anisotropy spectroscopy (RAS), a surface sensitive form of polarized reflectance, is a particularly useful in situ probe of Ag NP growth on faceted Al2O3 [7,16], while spectroscopic ellipsometry combined with analytic modelling of the optical
response also appears promising [17, 18].

The linear optical properties of such NP arrays and plasmonic nanostructures are now well understood. Progress is also being made in modelling the more complex nonlinear optical response, particularly regarding the enhancement effects of local fields and the sensitivity of second–harmonic generation (SHG) to NP shape [19]. The experimental characterization of plasmonic nanostructures using SHG has been reviewed recently, with an emphasis on SHG microscopy [20]. Other experimental work has explored methods of increasing the size of the SHG response from nanostructured arrays [21, 22]. Most of the characterization work has used normal incidence excitation. While this restricts the SHG response to the smaller in-surface-plane tensor components, polarization-dependent studies at normal incidence have been shown to be very sensitive to NP morphology [23–26].

The potential of SHG for monitoring the growth of anisotropic NP arrays appears to be largely unexplored. In a very recent study, the seeded growth of Au nanoshells in solution showed significant changes in the SHG response as the reaction proceeded [27]. In this paper, it is shown that polarization–dependent, off–normal–incidence measurements, which can access both the in-surface-plane and the larger out-of-surface-plane SHG tensor components [28], allow a simple non–resonant, fixed wavelength approach, compatible with in situ growth monitoring, to be used for characterizing NP morphology. The systems explored are Ag NPs grown on faceted Al2O3 and rippled Si(001).

![Fig. 1. (a) SEM image of Ag clusters on faceted Al2O3. Inset: SEM image of Ag clusters on unannealed Al2O3, where the Ag is deposited perpendicularly to the sample surface (scale bar 100 nm). (b) SEM image of Ag clusters on rippled Si(001) prior to capping with amorphous Si. (c) High magnification cross–sectional TEM image of Ag along the step edge of a faceted template, with an 8 nm scale bar (after [29]). (d) Cross–sectional TEM image of Ag NPs on a native–oxide–covered rippled Si substrate, with a 20 nm scale bar (after [30]). The larger particles are located in the ripple valleys.](image)

2. Experiment

Anisotropic Ag nanostructures were grown by self–assembly using two techniques on different substrates. The Ag/Al2O3 system was produced by glancing angle deposition of Ag, using a technique described by Verre et al [16, 17], on vicinal α–Al2O3 under high vacuum conditions (base pressure of 2×10−8 mbar). The sapphire substrate is annealed for 24 hours at 1400°C, producing an ordered and faceted structure. The substrate is then tilted relative to a collimated Ag flux, resulting in arrays of anisotropic Ag NPs decorating the facets [Fig. 1(a)], and strong plasmonic resonances. The dimensions of the NPs were characterised using SEM and TEM, and the average NP size was 18 nm × 15 nm × 12 nm.

Ag islands on rippled nanostructures on a Si(001) substrate were grown under high vacuum conditions at the Helmholtz–Zentrum Dresden–Rossendorf, using a procedure that has been described in detail previously [31, 32]. The rippled structures have a nanoscale periodicity of 30 nm and an amplitude of ~1 nm, as measured by AFM. The templates were prepared by ir-radiating native–oxide–covered Si(001) at room temperature with a collimated beam of 500 eV...
Ar$^+$ ions at incidence angle of 67° with respect to the surface normal, aligned with the ⟨100⟩ azimuth. After ex situ characterization, Ag was deposited orthogonally to the ripples on the native–oxide–covered surface by electron beam evaporation at a grazing angle of 70° to the surface normal. SEM was used to characterize the Ag NP layer structure and Table 1 shows the average dimensions and island separations for the samples under investigation. Finally, a capping layer of 20 nm of amorphous silicon ($\text{a-Si}$) was deposited to limit tarnishing of the NPs. The penetration depth of the optical probe allows the NP layer to be characterized through the capping layer. Figures 1(c) and 1(d) show cross-sectional TEM images of the NP layers. It can be seen that the Ag NPs are separated from the substrates by $\sim$1 nm, due to the native oxide layer in the case of rippled Si, and the tarnishing of the uncapped Ag NPs on the faceted Al$_2$O$_3$.

Optical measurements of Ag NPs on rippled Si and Al$_2$O$_3$, were performed in ambient conditions. Measurements on Ag on rippled Si substrates were reproducible after several months, due to the protection of the capping layer, unlike the Ag on Al$_2$O$_3$ samples, where experiments were performed within hours of exposure to ambient in order to reduce the effects of tarnishing of the NP layer on the SH response. The SHG measurements were made in reflection using a femtosecond laser tuned to a wavelength of 800 nm. Unamplified 130 fs Ti:sapphire laser pulses of an average power of 200 mW at the samples were used, at a repetition rate of 76 MHz. An off–normal geometry was chosen to allow access to the p–polarized SH response, which involves the larger out–of–plane tensor components, [28]. The input beam of 60 µm diameter at the sample was incident at 45°. A half–wave plate in the input beam was used to rotate the polarization vector of the linearly polarized light and the $p$– and $s$–polarized SH responses were measured as a function of wave plate angle, using a Pellin–Broca prism to spatially separate the SH beam from the fundamental. The diode–pumped laser system is very stable and it was sufficient to normalise the signal using the square of the laser output power.

### Table 1

<table>
<thead>
<tr>
<th>Sample</th>
<th>$y$ nm</th>
<th>$x$ nm</th>
<th>Aspect ratio</th>
<th>$l_y$ nm</th>
<th>$l_x$ nm</th>
<th>NP density 10$^{15}$ m$^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag/flat Al$_2$O$_3$</td>
<td>15(4)</td>
<td>15(4)</td>
<td>1</td>
<td>30(11)</td>
<td>26(7)</td>
<td>3.0(3)</td>
</tr>
<tr>
<td>Ag/Al$_2$O$_3$</td>
<td>18(5)</td>
<td>15(3)</td>
<td>1.20</td>
<td>24(6)</td>
<td>120(35)</td>
<td>0.65(4)</td>
</tr>
<tr>
<td>S1</td>
<td>28(10)</td>
<td>16(3)</td>
<td>1.75</td>
<td>34(7)</td>
<td>30(4)</td>
<td>1.1(2)</td>
</tr>
<tr>
<td>S2</td>
<td>24(9)</td>
<td>15(3)</td>
<td>1.60</td>
<td>29(7)</td>
<td>30(4)</td>
<td>1.1(2)</td>
</tr>
<tr>
<td>S3</td>
<td>22(7)</td>
<td>15(3)</td>
<td>1.47</td>
<td>27(4)</td>
<td>29(6)</td>
<td>1.4(2)</td>
</tr>
</tbody>
</table>

### 3. Theory and SHG phenomenology

In general, local fields at interfaces increase the intensity of the SHG response by a factor of $|L(2\omega)L^2(\omega)|^2$ [33], where $L(\omega)$ and $L(2\omega)$ are the field enhancement factors at the fundamental and SH wavelengths, respectively. $L(\omega) \sim |E_{loc}/E_0|$, where $E_{loc}$ is the local field and $E_0$ is the incident field. The plasmonic structures enhance nonlinear effects by local field enhancement, near the metal–dielectric interface, associated with the excitation of LSPs.

Nonlinear effects are governed by the symmetry of the material. Within the electric dipole approximation, SHG is forbidden in the bulk of a centrosymmetric material and it is the breaking of symmetry at the interface of such materials that makes SHG surface and interface sensitive. For NPs located at the interface between centrosymmetric materials, a SH response is
expected due to the reduced symmetry of the local fields acting on the NP surface response, the non-local bulk response from centrosymmetric metallic NPs being much smaller [34]. For the experimental approach used here, the variation of the $s-$ and $p-$ polarized SH intensity is of the form,

$$I_p^\alpha \propto |A \cos^2 \alpha + B \sin^2 \alpha + C \sin 2\alpha|^2$$

$$I_s^\alpha \propto |F \cos^2 \alpha + G \sin^2 \alpha + H \sin 2\alpha|^2$$

where $\alpha$ is the polarization vector angle of the linearly polarized light with respect to the plane of incidence [15]. The SH response is fitted to Eqs. (1) and (2), by varying $A$ to $H$, which will be complex when the exciting field or the SH field is close to a resonance of the system. These phenomenological parameters depend on Fresnel coefficients and the dipolar second order susceptibility tensor components $\chi^{(2)}_{ijk}$. Higher order terms, arising from the bulk quadrupolar contribution and the effects of strain and static electric fields may also contribute to a smaller extent to the response [28]. The presence of symmetry elements at the interface reduces the number of independent tensor components, but the data were fitted assuming no symmetry. The allowed tensor components according to each parameter are shown in Table 2. For a $p-$ polarized output, $A$ corresponds to a pP geometry and $B$ corresponds to an sP geometry, where the first and second letters refer to the input and output polarizations, respectively. For an $s-$ polarized output, $F$ corresponds to p$s$ and $G$ corresponds to s$s$.

Measurements were made with the plane of incidence along the NP array ($yz$ plane, $y-$azimuth) and across the NP array ($xz$ plane, $x-$azimuth), as shown in Fig. 2. The presence of any symmetry elements at the interface will simplify the response. For example, the allowed electric dipole $ijk$ terms are $zzz$, $zxx = zyy$, and $xzx = yzy$ for a Si(001) interface with 4mm symmetry. For the $s-$ polarized output, Table 2 shows that only $H$ contributes via the $xzx = yzy$ component, resulting in a simple $\sin^2(2\alpha)$ behaviour for this high symmetry interface. More generally, if the plane of incidence is a mirror plane of the interface, then only $H$ contributes to the $s-$ polarized SH response [35].

### 4. Results and discussion

Selected samples were analysed using linear optics to assess reflectivities and resonance energies: some of these results have been published elsewhere [18]. SHG was then performed: $\alpha-$p and $\alpha-$s measurements were made on Ag on Al$_2$O$_3$ and on rippled Si samples, along both $x-$ and $y-$azimuths.
Fig. 2. Geometry of SHG measurements, where the polarization vector of the input beam is rotated and either the $s$– or $p$–polarized SH intensity is measured. The plane of incidence is aligned either along ($y$–azimuth) or across ($x$–azimuth) the NP arrays.

Fig. 3. Absorbance spectra of Ag NPs arrays [Fig. 1(a)] on flat and faceted alumina probed by spectrophotometry. A correction was applied to the UV region of the spectrum to eliminate an increased background signal due to scattering. Polarization dependent absorbance parallel (black line) and perpendicular (red line) to the islands on faceted Al$_2$O$_3$ is shown. The response from Ag islands on flat Al$_2$O$_3$ (blue line) is also shown. The fundamental and SH wavelengths are marked on the figure.

4.1. Ag on Al$_2$O$_3$

The polarized absorbance spectra from Ag on flat and on faceted Al$_2$O$_3$ are shown in Fig. 3, where a correction has been applied in the UV region to account for a monotonically increasing background signal due to scattering. Polarization dependent absorbance features are observed when the electromagnetic field is aligned parallel and perpendicular to the NP array. The LSPRs along the arrays ($y$–azimuth) appear in the visible range, where the difference in peak width and position is due to island separation, shape, and shape dispersion. The figure also shows that the fundamental wavelength is off–resonance, while any resonance enhancement at the SH wavelength is likely to be small.

4.1.1. Ag on flat Al$_2$O$_3$:

Figure 4 shows polar plots of the $\alpha$–p and $\alpha$–s response measured along each azimuth. The response of the uncoated surface was negligible under these conditions, and so the measured
SHG response is due entirely to the NP layer. The signal is approximately an order of magnitude larger than that from native-oxide-covered silicon using 800 nm excitation, and is easily detected. The size of the response is also comparable for the NP arrays on the rippled Si substrates (see later), indicating that the SHG response from NP arrays grown on semiconductor and insulating substrates can be easily detected using non–resonant conditions. The inset in Fig. 1(a) shows that deposition of Ag on a flat substrate produces more symmetric NPs and does not result in the formation of NP arrays. We observe very similar lineshapes from both azimuths, consistent with the roughly isotropic distribution of NP shapes and arrangement on the surface. Table 3 shows the parameter values extracted from the fits. Both the figure and the table show that the two azimuths are very similar. For the $\alpha$–s response, there is a very small departure from the $\sin^2(2\alpha)$ behaviour (lobes of equal intensity at 45° and related angles) that would be expected if the NP distribution were fully isotropic. Some of this departure arises from a slight imperfection in the half–wave plate and its sensitivity to alignment, which has been discussed in a previous publication [36], however the small but significant difference between the azimuths of the $\alpha$–s response, and also in the $A$ parameter value of the $\alpha$–p response (Table 3), shows that a very small anisotropy in the NP growth on flat Al$_2$O$_3$ cannot be ruled out. As well as confirming the largely isotropic NP morphology, Fig. 4 shows that the $p$–polarized SH response, which is accessible when an off–normal geometry is used, is roughly an order–of–magnitude larger than the $s$–polarized response. It is well known that $p$–polarization, which accesses out–of–plane, $z$–dependent tensor components, produces a much larger SH response at interfaces [28], with the maximum signal generally being obtained for the pP configuration ($\alpha = 0^\circ$), as seen in Fig. 4.
Table 3. Fitted parameter values for $\alpha$-p and $\alpha$-s measurements from Ag NPs on flat Al$_2$O$_3$. Estimated errors are given in parenthesis.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$x$-azimuth</th>
<th>$y$-azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag on flat Al$_2$O$_3$</td>
<td>311(1)</td>
<td>287(2)</td>
</tr>
<tr>
<td>B</td>
<td>62(3)</td>
<td>60(1)</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>-7(1)</td>
<td>-8(1)</td>
</tr>
<tr>
<td>H</td>
<td>138(1)</td>
<td>118(1)</td>
</tr>
</tbody>
</table>

4.1.2. Ag on faceted Al$_2$O$_3$:

Figure 5 shows the $\alpha$–p and $\alpha$–s response from aligned Ag NPs on faceted sapphire. The polar plots from each azimuth now differ dramatically, with the $\alpha$–px plot showing lobes rotated by 90° from the isotropic response. Bearing in mind that $\alpha$ is defined with respect to the plane of incidence, both $\alpha$–p plots show a dominant SH response in the $y$ direction along the array. SHG in this off–normal geometry is clearly extremely sensitive to the morphology of the NP layer. It has been shown previously that deposition on the faceted substrate produces truncated ellipsoidal NPs distributed in arrays at the facets, together with some small isotropic NPs on the terraces, as shown in Fig. 1(a) [7, 16]. Figure 5 shows that the anisotropic NP response

![Fig. 5. $\alpha$–p and $\alpha$–s SH response plotted as a function of polarizer angle along both azimuths, for Ag NPs on faceted Al$_2$O$_3$ together with fits (black line) using Eqs. 1 and 2. Note the change of scale by a factor of 10 in the $\alpha$-s measurements. The NP arrays are aligned along the $y$-azimuth.](image)
Table 4. Fitted parameter values for $\alpha$-p and $\alpha$-s measurements from Ag NPs on faceted Al$_2$O$_3$. Estimated errors are given in parenthesis.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$x$-azimuth</th>
<th>$y$-azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag on faceted sapphire</td>
<td>12(1)</td>
<td>132(1)</td>
</tr>
<tr>
<td>B</td>
<td>76(1)</td>
<td>11(2)</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>38(1)</td>
</tr>
<tr>
<td>G</td>
<td>$-6(1)e^{i51}$</td>
<td>11(1)</td>
</tr>
<tr>
<td>H</td>
<td>29(1)</td>
<td>12(1)</td>
</tr>
</tbody>
</table>

dominate. The maximum of the $\alpha$-p response is roughly 3 times larger along the $y$-azimuth than the $x$-azimuth, which is consistent with the higher NP density in the $yz$-plane (see Fig. 1). In addition, Table 1 shows an average spacing between NPs in the $y$ direction of 6 nm, a separation where dipole–dipole interactions will have a significant effect on the local fields [18]. Table 1 also shows a NP aspect ratio of 1.2 and thus aligned NPs with an average 20% departure from isotropy in the $y$ direction, combined with the dipole–dipole interaction between the NPs in arrays aligned in the same direction, produce the dramatic change in the SH response observed. These results are consistent with recent theoretical work emphasizing the sensitivity of SHG to NP shape [26].

The $|A/B|$ ratio for the $x$-azimuth, across the aligned arrays, appears to be promising as a quantitative measure. From Table 3, this ratio is 5.0 for the isotropic morphology, while from Table 4 this ratio is 0.16, a dramatic change of a factor of 30. Table 2 helps in interpreting these results. The $A$ parameter can include up to 6 tensor components, with the isotropic $zzz$ component often dominating the interface response. This is not the case here, because the factor of 10 change in the $A$ parameter in Table 4 between the $x$- and $y$- azimuths shows that $zzz$ can make only a small contribution. The larger response from out-of-plane, $z$- dependent tensor elements allows the $zyy$ component of the $B$ parameter for the $x$- azimuth to be identified as the dominant term, the minor component being $xyx$. For the $y$- azimuth, the $zyy$ component appears in the $A$ parameter (Table 2), which is very large (Table 4). Normal incidence geometry, as used in SHG microscopy, does not allow access to the $|A/B|$ ratio.

The dominance of the $zyy$ component is consistent with stronger local fields along the $y$- azimuth, where the NPs are closely packed, combined with the larger susceptibility normal to the surface. The $|A/B|$ ratio for the $x$-azimuth is related to the relative strengths of the local field in the $x$- and $y$- directions, which in turn depends on the in-plane aspect ratio of the NPs and also the dipole-dipole coupling, if the NPs are closely spaced. The equivalent ratio for the $s$- polarized SH output involves much smaller quantities. However, the $\alpha$-s response from each azimuth, although roughly an order of magnitude smaller, is also highly anisotropic. The $x$-azimuth response looks similar to the $\alpha$-s response from the flat substrate. As discussed above, if the $xz$-plane were a mirror plane of the surface, a simple $\sin^2(2\alpha)$ response would be measured. Figure 5 shows that the $xz$-plane of the NP arrays approximates to a mirror plane, with lobes at 45º and related angles. The very large departure from this behaviour for the $y$-azimuth is consistent with the removal of the $yz$ mirror plane due to the faceting and subsequent growth of aligned NPs decorating the facets. Absolute values of $A$-$F$ are lower than for the NPs on the flat substrate, because the coverage is significantly less (see Fig. 1(a) and Table 1). Finally, there is only one complex parameter value in Table 4, supporting the argument that the SH response is not significantly resonantly enhanced at the fundamental and SH wavelengths.

The results show that SHG is sensitive to these anisotropic local fields and can identify anisotropy in NP shape and distribution. A simple $pP$ measurement along the two azimuths measures the $A$ parameter and is sufficient to identify NP anisotropy, while rotating the input

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polarization gives access to the |A/B| ratio, corresponding to the ratio of pP to sP SH field strengths, which may provide quantitative information (see below). An α–s scan along each azimuth can detect the presence of mirror plane symmetry in the NP response. The α–s results are consistent with previous polarized SHG microscopy at normal incidence, [23–25].

4.2. Ag on rippled Si(001)

Figure 6 shows polar plots of the α–p and α–s response of the rippled nanostructures with arrays of Ag NPs of varying aspect ratio (Table 1), and Table 5 and 6 show the parameter values extracted from the fits. A detailed study of the SHG response from the native–oxide–covered rippled Si(001) substrate at this excitation wavelength has been published previously [37]. The rippled Si(001) response is comparable in size to that of native–oxide–covered Si(001), while the NP response is roughly two to five times larger at these NP densities (Table 1). Thus, in contrast to the faceted Al₂O₃ substrate, there will be a small contribution from the rippled substrate in the response. Figure 6 shows that the NP response is highly anisotropic, with each azimuth being similar in shape to that of NP response on faceted Al₂O₃, see Fig. 5. There is a small but significant difference in the p–x plot in Fig. 6 around α ~0° and ~180°, where small lobes can be seen. Comparison of Table 4 and Table 5 shows that the A parameter for the x–azimuth has increased in size and has changed sign, relative to the B parameter. The overall size of the signal from each azimuth is now similar, reflecting the comparable NP density in each direction, in contrast to the faceted sample, as shown in Fig. 1. The change in the A parameter
Table 5. Fitted parameter values for $\alpha$-p measurements from Ag NPs on rippled Si. Estimated errors are given in parenthesis.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$x$-azimuth</th>
<th>$y$-azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>A -52(5)</td>
<td>246(2)</td>
</tr>
<tr>
<td></td>
<td>B 188(1)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C 0</td>
<td>0</td>
</tr>
<tr>
<td>S2</td>
<td>A -45(4)</td>
<td>237(2)</td>
</tr>
<tr>
<td></td>
<td>B 152(1)</td>
<td>10(5)</td>
</tr>
<tr>
<td></td>
<td>C 0</td>
<td>0</td>
</tr>
<tr>
<td>S3</td>
<td>A -15(4)</td>
<td>184(2)</td>
</tr>
<tr>
<td></td>
<td>B 122(1)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C 0</td>
<td>0</td>
</tr>
</tbody>
</table>

is consistent with the increase in the NP density in the $x$ direction, as the tensor components of $A$ depend on $x$ and $z$ fields only.

Overall, the SHG response increases with increasing NP aspect ratio in both directions, S1 having the largest aspect ratio and S3 having the smallest. The general similarity to the faceted results is further evidence for the SHG response being sensitive to anisotropic local fields. This is useful qualitative information, but the number of samples is too small and the variation in the NP size too large to provide definitive quantitative information. For example, the spacing between NPs in the $x$ and $y$ directions is $15 \pm 1$ nm and $5 \pm 1$ nm, respectively, for all three samples (Table 1), indicating that the influence of dipolar coupling between NPs should be similar and leaving the NP aspect ratio as the main difference to be explored. As discussed above, the $|A/B|$ ratio for the $x$-azimuth (across the arrays) is promising as a quantitative measure and values of $0.28(3)$, $0.30(3)$ and $0.12(3)$ for S1, S2 and S3, respectively, are obtained (Table 5). While S3 is clearly distinguished as having the smallest $|A/B|$ ratio, consistent with having the smallest aspect ratio, S1 and S2 cannot be distinguished within error due to the dispersion in NP size.

Turning now to the $s$--polarized SH response, as with the $\alpha$--p measurements the NPs with larger aspect ratios give larger responses in both directions. There is a clear departure from mirror plane symmetry along both azimuths, as shown by the large values of the $G$ parameter and the rotation of the lobes away from $45^\circ$ and related angles. The contrast between the $s$--$x$ response for the faceted substrate shown in Fig. 5 and the rippled substrate shown in Fig. 6 shows that the $s$--polarized SH response provides qualitative information on the departure from mirror plane symmetry in these NP arrays. The more dominant $H$ parameter in the Al$_2$O$_3$ sample is consistent with the high quality facets having minimal “snaking” compared to the rippled Si samples.

5. Conclusion

These exploratory ex situ studies have shown that the sensitivity of SHG to local fields makes it a promising fixed wavelength characterization technique for anisotropic NP arrays grown by self–assembly. The anisotropic SH response using off–normal geometry is easily detected under non–resonant conditions, avoiding the complexity and expense of spectroscopic SHG. Out–of–plane nonlinear susceptibilities produce SHG intensities an order of magnitude larger than the in–surface–plane response measured using normal incidence, for example in SHG microscopy. Measurements along and across the NP array directions show dramatic differences compared to the SH response of isotropic NPs grown on a flat surface. A simple pP measurement along the two azimuths is sufficient to identify anisotropy in the NP morphology, while measuring the $s$--polarized SH output along each azimuth as the input polarization is rotated can detect...
Table 6. Fitted parameter values for $\alpha$-s measurements from Ag NPs on rippled Si. Estimated errors are given in parenthesis.

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<thead>
<tr>
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departure from mirror plane symmetry. By choosing the plane of incidence orthogonal to the NP array direction, it was shown that the $p-$polarized SH response, as a function of input polarization, is very sensitive to NP morphology, with a change of 20\% in the inplane aspect ratio of the NPs producing a variation of a factor of 30 in the ratio of the easily measureable $pP$ to $sP$ SH field strengths. The results show that a simple fixed geometry could be used for the in situ characterization of anisotropic nanostructure morphology during growth by self-assembly, which could be particularly useful in situations where rotating the sample may be neither desirable nor easily accomplished. While linear optical techniques, particularly RAS [7, 16], are likely to remain the preferred choice for in situ NP growth monitoring in real time, these results indicate that SHG is promising as a complementary technique.

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