An Analysis of Route Symmetry in an Urban Bus Public Transport Network Using Electronic Fare Collection Data

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ABSTRACT

Symmetry of a public transport network with regard to passenger journeys is often assumed. It is an assumption that may simplify the models or algorithms used to obtain public transport analysis results. This paper investigates to what level a public transport bus network provides symmetry on a route level. Analysis of a dataset of a bus network in a medium sized city in Europe shows that there is no perfect symmetry within the bus transport network. A set of equations is derived that quantify the level of symmetry which is calculated for two different types of journeys: [1] Single journeys where the number of passengers per route in one direction is compared to the number of passengers in the opposite direction ($R1_{Inbound}$ vs. $R1_{Outbound}$). [2] Transfer journeys with one transfer point where the route combination in one direction is compared to the route combination in the opposite direction (R1/R2 vs. R2/R1). Matrices are analysed and the level of symmetry is determined.

The paper aims to clarify that symmetry should not be assumed without considering an error term attached to the particular model or algorithm. It quantifies the level of symmetry, which may be used when comparing different networks, sub-networks or routes. It may further serve to optimise origin/destination analysis as it is often argued that each journey also has a return journey.

INTRODUCTION

The assumption that public transport networks have a symmetric behaviour is often used in models or algorithms that focus on public transport issues. This assumption is introduced to simplify models for ease of calculation. The assumption consists of the idea that each journey in one direction also has a corresponding journey in the opposite direction (return journey). The assumption is mostly used to justify the analysis in the first place or just to simplify the model or algorithm. That the assumption is not exactly true is widely accepted and has also been shown by Navick (*1*). It is however still used and considered as a valuable assumption for estimating numbers of alightings on routes.

It can be argued that for regular commuting passengers those who carry out a transfer journey in one direction will also transfer for the return journey. If this could be assumed then the total number of transfer journeys (F_{ij}) with one combination of routes (e.g. route combination R1/R2) will be similar to the total number of transfer journeys (F^T_{ij}) with the opposite combination of routes (e.g. route combination R2/R1). The analysis also considers single journeys where the number of passengers per route in one direction is compared to the number of passengers in the opposite direction ($R1_{Inbound}$ vs. $R1_{Outbound}$). Various sparse matrices are analysed and the level of symmetry is determined.

Considering only one route combination pair (e.g. R1/R2 and R2/R1), equation 1 can be used to analyse whether the total number of passengers travelling in one direction is equal to the total number travelling in the opposite direction on this particular route interchange with an acceptable error. Equation 1 only applies to individual transfer journey pair combinations. F_{ij} represents the transfer journey numbers for a particular route interchange combination (e.g. R24/R7) while F^T_{ij} is the number of transfer journeys that were recorded on the opposite direction of the route interchange combination (e.g. R7/R24).

$$F_{ij} = F^T_{ij} \tag{1}$$

F = Transfer route interchange matrix,
F^T = Transpose of F,
i = Route number of first boarding,
j = Route number of second boarding.

Perfect symmetry is defined as the situation where both route combination pairs have exactly the same number of transfers (e.g. R1/R2 recorded 1,200 transfer journeys and R2/R1 recorded 1,200 journeys too). Realistically this will only be the case for a few route combination pairs from an entire network. This raises the question to what degree one might expect the journeys in a transport network to be symmetric. This paper is an attempt to answer this question. A generic equation is proposed that quantifies the *'Degree of Symmetry'* to a system, route or route segment level. A similar set of equations is introduced for single journeys.

Substitutional routes are routes that mainly serve the same or a similar path of a different route offering the passenger the option of more than one choice to reach their final destination. Substitutional routes are not incorporated into this analysis, which means that each route is treated as an independent with no substitutes available. Better frequency or directness of other substitutional routes is therefore not included in the study. It is understood that car pooling and trip chaining exists however due to its complexity and unknown parameters these aspects could not be included into this analysis.

The aim is to provide a measure of symmetry when analysing or comparing routes or route segments. Using the symmetry assumption is one possibility to obtain Origin/Destination information of public transport passengers. However some routes do not have symmetry with regard to journey numbers as shown by this paper and the assumption of symmetry would therefore bias or even misrepresent the results. The measures developed in this paper should provide an indication of the bias caused by the non-symmetric characteristic of travel on a public transport network.

The method proposed in this paper aims to calculate a degree of symmetry on each route/route combination solely using electronic fare collection data. Tests on the current dataset of electronic fare collection data aim to set an approximate border value for the degree of symmetry when routes can be treated as symmetric. The second aim was to develop the equation in such a way that it can be applied to any route and any transport network regardless of size, number of routes and number of stops.

In accordance with a confidentiality agreement details of the dataset cannot be presented and where route numbers etc are mentioned they have been changed.

BACKGROUND

Details about the Project Database

The project database is based on data gathered from an urban bus operator on its entire transportation network. The vast amount of transactional data from 1998 and 1999 (160 million records) has been moved from text files (one file per day) into a large relational Oracle database (*2*). The initial transactional data has been enriched with additional datasets (bus stop locations, spatial information, ticket types, transfer journey identification), which contributes considerably to the application, capability and usability of the system.

The transfer journeys were identified using an iterative classification algorithm (*3*) which shows whether each individual passenger boarding is part of a transfer journey. This newly added data attribute facilitates the analysis presented in this paper. It is important to note that only journeys that have been carried out using a magnetic strip cards can be analysed. Cash transactions cannot be analysed.

The transfer journey identifier has been created for the months April '99, May '99, September '99 and October '99 as these months did not reflect any major abnormalities such as school breaks or summer holiday periods. All analysis is based on these four months unless stated differently. During this 4 month period 7,962,107 individual boardings were recorded. The classification algorithm proposed in this paper identified over 1.4 million transfer journeys. Approximately 12.5% of all transactions (including cash boardings) were carried out using magnetic strip cards. Approximately 35% of all magnetic strip card transactions were part of a transfer boarding.

Literature Review

Some research studies have used the assumption of network symmetry (*1, 4*). Both studies used the assumption of symmetry in their method to estimate performance measures based on electronic fare collection and automatic passenger counter (APC) data. Navick (*1*) assumed that *'the boarding pattern for a route in one direction is equivalent to the alighting pattern in the opposite direction'*. This assumption was tested with APC's that counted boardings and alightings at each stop. A Kolmorgorov-Smirnov test was applied to the dataset. The test statistic was created by comparing the cumulative distribution of eastbound boardings with westbound alightings, with the westbound alighting allocated to the eastbound stops. The authors however state that it is important to ensure that the data of the routes are correct before applying the assumption of symmetry on particular routes.

Other research (*4*) applies the assumption to estimate average distance travelled. Over a period of 52 days each passenger boarding any of the 38 routes was asked for his/her destination. The collected data were then compared to the estimates set by the assumption: passenger numbers in one direction of a particular route are equal to the boarding numbers in the opposite direction. Both studies concluded that the assumption of symmetric travel patterns on most routes can be applied although details of routes and data should be checked for each route.

DePalma (*5*) presented morning and evening peak travel using Vickrey's (*6*) bottleneck model. Although this study is not exclusively concerned with public transport journeys it is discovered that there is equilibrium between morning peak and evening peak journeys when analysing journeys of identical travellers. This equilibrium however breaks down when non-identical travellers are included in the study (*7, 8*). This is similar to the scenario that exists in public transport networks as it is often impossible to identify identical travellers. This paper will also conclude that the equilibrium with regard to journeys in one direction and then into the opposite direction on a route and network level does not exist, therefore perfect symmetry does not exist.

Horowitz (*9*) presented through trip tables for small urban areas. A method for quick response travel forecasting included the fact that the potential for travel has symmetry. Although this paper works with areas rather than routes their point is still valid for this paper. It suggests that the trip opportunities from one area *i* to area *j* should be equal to the number of trip opportunities from area *j* to area *i*. The trip opportunities in one direction are equal to the opportunities in the opposite direction in the urban bus public transport network presented in this paper.

INITIAL ANALYSIS OF SYMMETRY PATTERNS

In preparation, prior to the development of the degree of symmetry equation, it was necessary to test whether the transfer journey numbers for each route combination pair (e.g., R24/R7) were similar to the corresponding numbers of the inverse direction of the same combination pair (e.g., R7/R24). These results are shown in Table 1. The table illustrates the transfer journey numbers of the two combination pairs (e.g. R24/R7and R7/R24) and also the rank of each combination pair in relation to the transfer journey numbers of each individual combination pair to the overall route interchange matrix. For example, where i = R24 and j = R7 ➔ $F_{(R24,R7)}$= 4,538 and $F^T_{(R24,R7)}$= 4,113. This means that 4,538 passengers transferred from R24 to R7 and 4,113 passengers transferred from R7 to R24. Although there is a difference of 425 transfers (9.3%) the two numbers are still relatively close. The closer the numbers are to each other the higher is the degree of symmetry for that combination pair. Experience with the data showed that the example of R24 and R7 as combination pair (difference of 9.3%) has a high degree of symmetry. This will be further analysed in the following sections. The rank of $F_{(R24,R7)}$ is 1 because 4,538 recorded transfer journeys was the highest recorded number. $F^T_{(R24,R7)}$ produced 4,113 transfer journeys which was the second highest number recorded, hence rank 2.

Figure 1 is the graphical representation of Table 1 showing that there is an emerging pattern between two route combination pairs by displaying and comparing the total number of transfers of each combination pair. The label (e.g. R7/R12 - R12/R7) labels a pair of bars; the first one with R7/R12 and the second bar with R12/R7. The horizontal bar chart clearly indicates the tendency to symmetric behaviour of the route interchanges displayed in the graph. To establish this fact more quantitatively an analysis was needed to determine the degree of symmetry in a network including different parameters (e.g. transfer journey attributes such as date, time or specific routes).

DEVELOPMENT OF THE 'DEGREE OF SYMMETRY' EQUATION

The aim of this section is to see if there is a degree of correlation between the total number of transfers in one direction and the total number of transfers in the opposite direction (R1/R2 and R2/R1). Evaluating each route combination pair individually showed that there is a relatively high degree of similarity between most of the transfer route pairs. The next step was to show that this is true for all significant transfer route pairs of the entire transport network by calculating the '*Degree of Symmetry*' of the entire transfer route matrix. A significant transfer route combination pair is a pair where one of the total numbers of transfer journeys lies above a predefined cut-off-point referred to as parameter *a*.

The cut-off point '*a'* was introduced to exclude combination pairs with a relatively low number of total journeys and to reduce the bias this may cause. The cut-off point is a variable and can be determined by the user. A small number of transfer journey boarding records on a particular route combination may not be significant with regard to the symmetry of the network but may bias the results considerably. For example, the route combination R1/R2 recorded 10 transfer journeys while the combination R2/R1 only recorded 2. In this case, the two numbers 2 and 10 are only fractions of the numbers from other transfer journey boarding records (such as R7/R8 in Table 2). However, the degree of symmetry of the combination R1/R2 and R2/R1 may bias the overall symmetry of the network. The degree of symmetry equation focused on not favouring routes that run more frequently which may attract a larger number of passengers. The cut-off point of significance may change with the parameters the transfer matrix is based on. For example, should the cut-off point of a matrix that consists of one day's transfer journeys be chosen more carefully than for a matrix that includes data of several months due to the differences in journey numbers? As it has been introduced as a parameter of the equation it can be changed at any stage throughout the analysis.

When looking at the symmetry of the entire transport network with regard to transfer journeys the total numbers of a transfer journey combination pair are interchangeable without changing the degree of symmetry as there is no further information such as direction or time of travel attached. For example the combination 'R24/R7' with a total number of 4,538 is interchangeable with the total number of 4,113 which is assigned to the transfer route combination 'R7/R24' without interfering with the symmetry of the transfer route matrix. It was therefore not possible to apply a statistical technique such as a paired t-test or other methods to determine the degree of symmetry of the matrix. A one sampled t-test on the mean differences of each route combination pair showed that H_0 ($\mu_{Diff} = 0$) has to be rejected on statistical evidence. Table 1 and Figure 1 indicate some symmetry but show that there is no perfect symmetry present. However, knowing to what degree a transport network is symmetric would be very beneficial for transport modelling and decision making as many models and techniques rely on the assumption that the network is symmetric (*1*, *4*). The following equation defines the degree of symmetry for one route interchange *i/j* of matrix F:

$$S_{ij} = 1 - \frac{|(F_{ij} - F_{ij}^T)|}{(F_{ij} + F_{ij}^T)} \qquad (2)$$

S_{ij} Degree of Symmetry of Route interchange *i/j*,
F Transfer route interchange matrix,
F^T Transpose matrix of F,
i Route number of first boarding,
j Route number of second boarding.

The largest possible value for *S* is 1 and this represents perfect symmetry. As mentioned above, this may happen for some of the transfer route combination pairs but not for the entire transport network. The smallest number for *S* can be 0 although this can only occur when the absolute difference between the route combination in one direction (e.g. R1/R2) and the route combination in the opposite direction (e.g. R2/R1) is very large. A result of 0 for *S* may indicate an error in the dataset or refers to routes that only serve one direction. The smaller the degree of symmetry *S* the less symmetry is present among the transfer route combination pairs. It is worth mentioning that the equation produces a degree of symmetry of 0 if any of the two input values of transfer journeys (F_{ij} and F^T_{ij}) are 0 independent of the non-zero term. It is therefore recommended to set the cut-off point *a* >0.

Equation 3 shows the calculation of the total symmetry of a number of routes or the entire network. The cut-off point *a* has to be chosen in advance to define the significance level of particular route interchanges. The formula states that as long as one value of the two transfer journey numbers (F_{ij} or F^T_{ij}) is above the cut-off point *a* it has to be included as significant route interchange. The equation states that the symmetry *S* is the sum of all route combination pair symmetries divided by the total number of observations where $F_{ij} > a$ *or* $F^T_{ij} > a$ has to be adhered. Again, if $a = 0$ then the degree of symmetry of one route combination pair is zero when one of the two values F_{ij} and F^T_{ij} is zero. It is therefore recommended to choose a large enough cut-off point.

$$S = 1 - \frac{\sum_{i=1}^{n} \sum_{j=i+1}^{n} \left(\frac{|(F_{ij} - F_{ij}^T)|}{(F_{ij} + F_{ij}^T)} \right)}{N} \quad \text{where } F_{ij} > a \text{ or } F_{ij}^T > a \text{ for all } i, j. \qquad (3)$$

S Degree of Symmetry,
a cut-off point – Total number of transfers at route interchange,
n Number of routes,
N Total observations of F_{ij}'s > *a*.

RESULTS OF DEGREE OF SYMMETRY EQUATION FOR TRANSFER JOURNEYS

Three main matrices were generated in order to test Equation 3. The first matrix showed the total number of transfer journeys for the morning peak period (7.00 – 9.00), the second matrix showed the total number of journeys for the evening peak period (16.00 – 18.00) and the last matrix showed the total number of all transfer journeys. All three matrices were based on a dataset consisting of transfer journey records over a 4 month period. Due to the size of the matrices it was not possible to display them in this report. A subset of the matrix is however presented in Table 2. It was expected that a relatively high degree of symmetry would be obtained for matrix 3 as it showed all transfer journeys over the entire period of time. The degree of symmetry of matrices 1 and 2 was expected to be very low because of the shift of morning and evening peak time journeys. For simplicity the direction of the journeys was neglected for this part of the study and it is therefore unknown what percentage of the passengers travelled outbound and what percentage travelled inbound. Including the direction attribute into the study would add a third dimension to the matrix without a real contribution to the study.

Table 3 shows the results of the degree of symmetry equation after it was applied to all three matrices. The table shows three columns; one for each matrix symmetry analysis. Each column is subdivided into three further columns showing the number of transfer journeys, the number of significant route interchanges and the calculated degree of symmetry. The symmetry equation was applied with changing cut-off point *a* as indicated in the left column of Table 3. Analysing the 'All' columns in Table 3 indicates a comparatively high symmetry when considering all transfer journeys. The degree of symmetry extends from S = 0.573 to S = 0.930 for the cut-off points 0 and 1,500 respectively. Table 2 shows to what extent the degree of symmetry changes when the cut-off point is increased. The cut-off point 0 may not be representative as too many insignificant interchanges bias the result. Based on the analysis shown in Figure 2 to Figure 5 it could be argued that a route/network can be assumed to be symmetric when the degree of symmetry S is above 0.85. The value of 0.85 has been chosen for this study as the results indicated a degree of symmetry above this value. At S = 0.85 the slope of the 'All Transfer Journeys' function is changing to a lower rate of change in slope. Other studies may need to choose a higher or lower value, however, this figure is dependent on the individual data set and other studies may need to adjust this figure. It would be interesting to carry out the analysis on datasets of other cities to compare the results. The degree of symmetry S when morning peak and evening peak transfer journeys are examined separately is, as expected, very low. Figure 2 is a graphical representation of Table 3. The degree of symmetry among *all* transfer journeys slowly increases as the cut-off point increases. This is mainly due to the exclusion of routes with less passenger boardings. It seems that the larger the numbers of transfers the more symmetric is the route. Due to the smaller numbers of transfer journeys in morning and evening peak periods (compared to all day figures) the number of transfer journeys are below 1,100 which explains the missing values of S after the cut-off point surpasses 1,000.

DEGREE OF SYMMETRY OF SINGLE JOURNEYS

After analysing the symmetry of transfer journeys it was decided to analyse the symmetry of all single journeys within the network. This part of the study also concentrated on the months April 99, May 99, September 99 and October 99. A crosstabulation table was generated showing the routes, the direction of travel (0 – outbound, 1 – inbound) and the relevant frequencies for each instance. The table was based on over 5 million single journeys that took place in the above mentioned period. The route and the direction parameter (0 or 1) were used as combination pair for the notation (e.g. Route x in direction 0 or 1 was considered to be the combination pair Rx0 and Rx1). Equation 4 was used to calculate the degree of symmetry for each individual route (S_{Single}). The equation uses the absolute difference between the single journeys in one direction (e.g., outbound - 0) minus the single journeys in the opposite direction (e.g., inbound – 1) and divides this by the sum of both single journey numbers. The calculated value is then subtracted from 1 and results in the degree of symmetry for single journeys of a particular route.

$$S_{Single} = 1 - \frac{|(R_0 - R_1)|}{(R_0 + R_1)} \tag{4}$$

S_{Single} Symmetry within one route considering both directions
R_0 Total number of single journeys on route R in outbound direction (0)
R_1 Total number of single journeys on route R in outbound direction (1)

Equation 5 calculates the degree of symmetry for the entire network or for a number of predefined routes. The calculation sums the obtained degree of symmetry of each route and divides this by the total number of observations, where the total number of single journeys in at least one direction has to be greater than the predefined cut-off point *a*. This result is then subtracted from 1 which gives the total degree of symmetry.

$$S_{Total} = 1 - \frac{\sum_{i=1}^{n} \left(\frac{|(R_0 - R_1)|}{(R_0 + R_1)} \right)}{N} \text{ where } R_0 > a \text{ or } R_1 > a \text{ for all } i \tag{5}$$

S_{Total} Symmetry of all routes considering both directions
i Route number
N Total number of observations where $R_0 > a$ *or* $R_1 > a$

The results, after applying the equation to the single route/direction crosstabulation table, are shown in Table 4 and Figure 3. The analysis included data for the entire day, morning peak and evening peak over a four month period. The different time parameters produced a result that supports the assumption that there cannot be symmetry during peak time periods as evidenced by a small value for the degree of symmetry (S<0.85). The results for S for all cut-off points (a) during the morning and evening peak periods were between 0.429 and 0.6 which suggests no symmetry, on the basis of the definition of symmetry above. As observed throughout the symmetry analysis of transfer journeys, the degree of symmetry increases as the cut-off point a increases. The single journey analysis is almost linear for the 'All Day' parameter starting at S = 0.839 (a=0) progressing to a degree of symmetry of S = 0.916 (a=5000).

STATISTICAL SUMMARY OF RESULTS

The descriptive statistics provide a more detailed analysis of the degree of symmetry of transfer and single journeys (All Day). The lower and upper bound of the 95%

confidence interval of transfer journeys is much smaller than that from the single journey analysis. Single journeys are therefore more symmetric than transfer journeys which should be incorporated when applying the degree of symmetry within other transport models.

Single Journeys

The median of 0.93 indicates that ignoring outliers in the lower range of the results would lead to a much higher total degree of symmetry. Outliers are defined as route combinations with a very low degree of symmetry. This low degree of symmetry may be caused by the nature of the route combination. For example, one of the routes may only run in the morning but not in the evening. The skewness of the sample is -1.904 with a standard error of 0.191. This again underlines that outliers in the lower range may be introducing bias to degree of symmetry.

Figure 4 presents the data including some of the descriptive statistics in form of a histogram. The skewed graph underlines what has been stated in the previous paragraph. The very low degrees of symmetry may be caused by routes that do not have an equilibrium with regard to trip opportunities in one direction and trip opportunities in the opposite direction.

Transfer Journeys

The median of 0.8 is smaller than the median of single journeys (0.93) but still indicates that ignoring routes with a low degree of freedom would lead to a higher overall degree of freedom. This is also shown in the skewness statistic. The total number of observation N is much larger for transfer journeys than for single journeys as the focus was on route combinations, which results in smaller numbers of passengers for most observations. Figure 5 presents a histogram of all degree of symmetry results. The graphs Figure 4 and Figure 5 clearly show the difference between single and transfer journeys.

FINAL COMMENTS

It is important not to see the degree of symmetry as a number that can be included into other equations or models. It should be treated as an indicator of the degree of symmetry and might be useful as a way of comparing the symmetry of two routes or networks. The research of the degree of symmetry was carried out for this project with regard to the generation of origin destination (OD) pairs. One option when attempting to generate OD pairs from electronic fare collection data is to assume symmetry. However some routes do not have symmetry with regard to journey numbers as shown by this paper and the assumption of symmetry would therefore bias or even misrepresent the results. The symmetry equations could be applied using different values for the cut-off point. The OD matrices are then based on routes or route combinations that have a degree of symmetry which is higher than the predefined value (e.g. $S > 0.85$) ensuring that the assumption holds.

Therefore, the proposed set of equations provides a measure of the reliability of the assumption of network/route symmetry for transport analysis.

CONCLUSIONS

This paper analysed whether there is a symmetric behaviour among passenger journeys at a route level. Almost 8 million records were analysed, and with the use of a set of equations, a definition for degree of symmetry was proposed. The following main conclusions were drawn:

- As shown in the paper, electronic fare collection data is a suitable data source to analyse historical passenger journeys on route and system level.
- A one sampled t-test on the mean differences of each route combination pair showed that H_0 ($\mu_{Diff} = 0$) has to be rejected on statistical evidence. Initial analysis however indicates that there is a trend to symmetry between two routes. However, perfect symmetry only exists in a few cases.
- A cut-off point *a* was introduced to determine and reduce the level of bias caused by insignificant route combinations. The value of *a* has to be chosen carefully and with respect to the period of time that needs to be analysed.
- The set of equations can calculate the degree of symmetry for particular route combinations or for the entire matrix. The degree of symmetry *S* reaches from 0 to 1 where 0 defines the lowest value of symmetry and 1 defines the highest degree. In the case of the analysis S <0.85 was not classified as symmetric whereas all values above 0.85 were seen as symmetric with the attached error of *S*.
- The assumption that there cannot be symmetry throughout a peak time holds for all tests applied to the matrices. Therefore the assumption of symmetry cannot be used for any period less than one day.
- The analysis of transfer journeys showed that a satisfying degree of symmetry (S >0.85) is reached when the cut-off point a >=100. S >0.9 was reached when a >400.
- The degree of symmetry function of single journeys is almost linear, showing S=0.839 for a=0 and S=0.916 for a=5,000. As single journeys only consider one route and not a route combination (as used for transfer journeys) the degree of symmetry is more linear and more independent of *a* as the total number of journeys is much higher.
- The equations presented in this paper can be used as a tool to determine the degree of symmetry *S* on a route or system level. It may serve as an indicator of the reliability of the assumption that symmetry exists.

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TABLES AND FIGURES

TABLES:

FIGURES:

Table 1: Balance analysis of the transfer route matrix

Rank	1st	2nd	Boardings	Rank	1st	2nd	Boardings
1	R24	R7	4,538	**2**	R7	R24	4,113
3	R7	R159	3,347	**4**	R159	R7	3,219
5	R14	R48	3,036	**9**	R48	R14	2,591
6	R48	R67	2,832	**22**	R67	R48	2,099
7	R25	R7	2,794	**29**	R7	R25	1,866
8	R205	R14	2,609	**14**	R14	R205	2,464
10	R112	R26	2,569	**11**	R26	R112	2,559
12	R201	R48	2,483	**15**	R48	R201	2,378
13	R32	R24	2,473	**20**	R24	R32	2,165
16	R205	R49	2,313	**18**	R49	R205	2,232
17	R18	R7	2,292	**37**	R7	R18	1,783
19	R7	R48	2,183	**40**	R48	R7	1,761
21	R112	R18	2,104	**26**	R18	R112	1,961
23	R30	R159	2,053	**27**	R159	R30	1,947
24	R32	R7	2,001	**74**	R7	R32	1,456
25	R7	R175	1,972	**33**	R175	R7	1,809
28	R14	R175	1,932	**30**	R175	R14	1,832
31	R175	R159	1,822	**86**	R159	R175	1,356
32	R205	R175	1,816	**58**	R175	R205	1,566
34	R27	R112	1,808	**76**	R112	R27	1,443
35	R7	R70	1,807	**38**	R70	R7	1,774
36	R205	R7	1,803	**42**	R7	R205	1,728
39	R159	R24	1,771	**64**	R24	R159	1,515
41	R22	R7	1,738	**79**	R7	R22	1,408
43	R205	R67	1,712	**46**	R67	R205	1,664
44	R7	R115	1,690	**54**	R115	R7	1,608
45	R67	R7	1,666	**51**	R7	R67	1,626
47	**R205**	**R159**	**1,637**	**50**	**R159**	**R205**	**1,629**

Table 2: Subset of the transfer route matrix

		Second Boarding Route (B)													
First Boarding Route (A)		R1	R7	R214	R54	R55	R57	R8	R58	R59	R60	R24	R98	R99	Etc.
	R1		8		3			3	1						…
	R7	3		2	161	10	9	**1479**	29	42	1	**4113**	232	**847**	…
	R214		19		9	13		1		4			1		…
	R54		131	2		22		32	17	49			2	28	…
	R55		8	5	69			17	3	8			4	11	…
	R57		4					3							…
	R8	2	**1382**		31	24	6		13	15		**1109**	265	**748**	…
	R58		13		14	1		14		14				11	…
	R59		51		35	6		16	19		2	5	1	6	…
	R60		1		2					4		2			…
	R24		**4538**		1			5		16	1			1	…
	R98		212		5	6		280	3	2				129	…
	R99		**765**		13	3		**782**	18	11			116		…
	Etc.	…	…	…	…	…	…	…	…	…	…	…	…	…	

Table 3: Results of Degree of Symmetry – Transfer Journeys

	7:00 - 9:00			16:00 - 18:00			All Day		
Cut off point 'a'	**Number of Journeys**	**Number of significant route interchanges**	**Degree of Symmetry**	**Number of Journeys**	**Number of significant route interchanges**	**Degree of Symmetry**	**Number of Journeys**	**Number of significant route interchanges**	**Degree of Symmetry**
0	295,061	12,998	**0.573**	275,513	13,586	**0.439**	1,432,882	34,225	**0.573**
10	276,190	6,441	**0.492**	255,030	6,450	**0.624**	1,411,768	10,999	**0.754**
50	206,638	2,230	**0.536**	175,727	1,938	**0.686**	1,311,152	5,684	**0.833**
100	144,249	990	**0.537**	114,228	802	**0.680**	1,196,208	3,774	**0.858**
200	80,069	330	**0.520**	52,763	222	**0.674**	994,465	2,157	**0.879**
300	47,824	136	**0.542**	23,867	72	**0.645**	836,524	1,423	**0.890**
400	34,943	82	**0.556**	13,956	34	**0.668**	695,768	971	**0.894**
500	25,245	52	**0.541**	8,281	18	**0.648**	595,288	718	**0.905**
600	15,662	28	**0.511**	2,293	4	**0.610**	484,733	498	**0.911**
700	10,885	18	**0.433**	2,293	4	**0.610**	395,528	348	**0.910**
800	7,947	12	**0.447**	1,234	2	**0.558**	353,667	286	**0.915**
900	5,457	8	**0.360**				317,275	240	**0.914**
1000	4,041	6	**0.274**				270,775	186	**0.919**
1100							234,649	148	**0.926**
1200							219,687	134	**0.925**
1300							191,710	110	**0.926**
1400							169,834	92	**0.931**
1500							**155,933**	**82**	**0.930**

Table 4: Results of Degree of Symmetry - Single Journeys

	7:00 - 9:00			16:00 – 17:00			All Day		
Cut Off point	Number of Journeys	Number of significant routes	Degree of Symmetry	Number of Journeys	Number of significant routes	Degree of Symmetry	Number of Journeys	Number of significant routes	Degree of Symmetry
0	1,191,405	160	**0.480**	961,623	159	**0.527**	5,053,299	161	**0.839**
10	1,191,405	160	**0.456**	961,623	159	**0.527**	5,053,299	161	**0.839**
50	1,191,297	157	**0.441**	961,409	152	**0.529**	5,053,248	160	**0.839**
100	1,190,790	151	**0.438**	960,959	148	**0.530**	5,053,098	159	**0.839**
200	1,189,909	147	**0.435**	961,623	144	**0.532**	5,052,537	157	**0.840**
300	1,188,682	143	**0.432**	958,898	140	**0.542**	5,051,118	153	**0.846**
400	1,187,856	141	**0.431**	954,077	129	**0.558**	5,049,640	150	**0.852**
500	1,185,314	137	**0.429**	951,724	125	**0.567**	5,048,155	148	**0.854**
600	1,184,490	136	**0.434**	949,606	122	**0.571**	5,048,155	148	**0.854**
700	1,169,114	133	**0.430**	944,764	116	**0.582**	5,047,133	147	**0.855**
800	1,180,229	132	**0.439**	938,638	110	**0.587**	5,042,554	143	**0.861**
900	1,173,591	127	**0.441**	936,128	108	**0.585**	5,039,811	141	**0.863**
1000	1,167,629	122	**0.448**	929,335	103	**0.587**	5,038,218	140	**0.864**
1100	1,165,550	121	**0.449**	926,553	101	**0.589**	5,033,017	137	**0.866**
1200	1,162,625	119	**0.453**	924,068	99	**0.598**	5,031,725	136	**0.870**
1300	1,157,738	116	**0.450**	917,075	95	**0.599**	5,031,725	136	**0.870**
1400	1,152,653	113	**0.448**	915,453	94	**0.602**	5,029,671	135	**0.871**
1500	1,147,368	110	**0.455**	906,478	90	**0.602**	5,024,682	133	**0.872**
2000	1,118,958	98	**0.449**	879,731	80	**0.605**	5,006,042	126	**0.886**
2500	1,106,156	93	**0.449**	861,873	75	**0.600**	4,989,727	122	**0.886**
5000	968,970	64	**0.449**	737,633	52	**0.600**	4,843,455	97	**0.916**

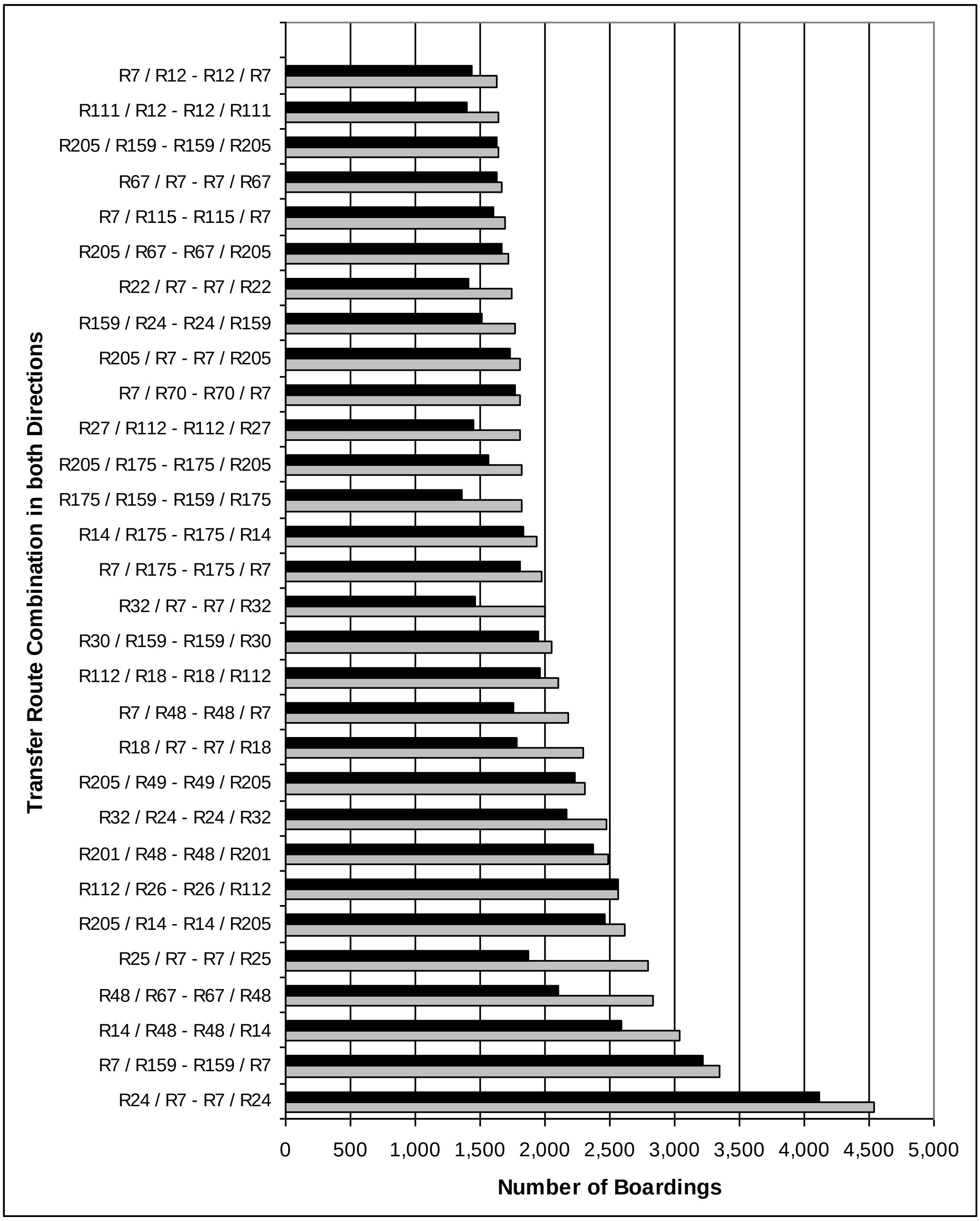

Figure 1: Relationship between the two Route Combination Pairs of Transfer Journeys

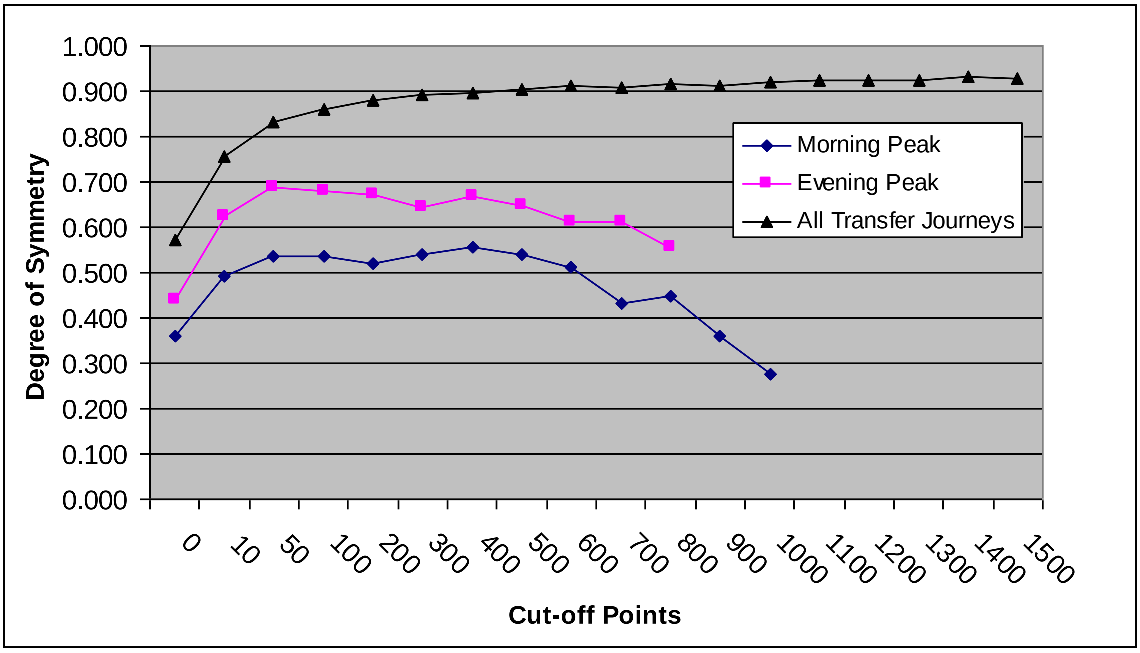

Figure 2: Degree of Symmetry Chart – Transfer Journeys

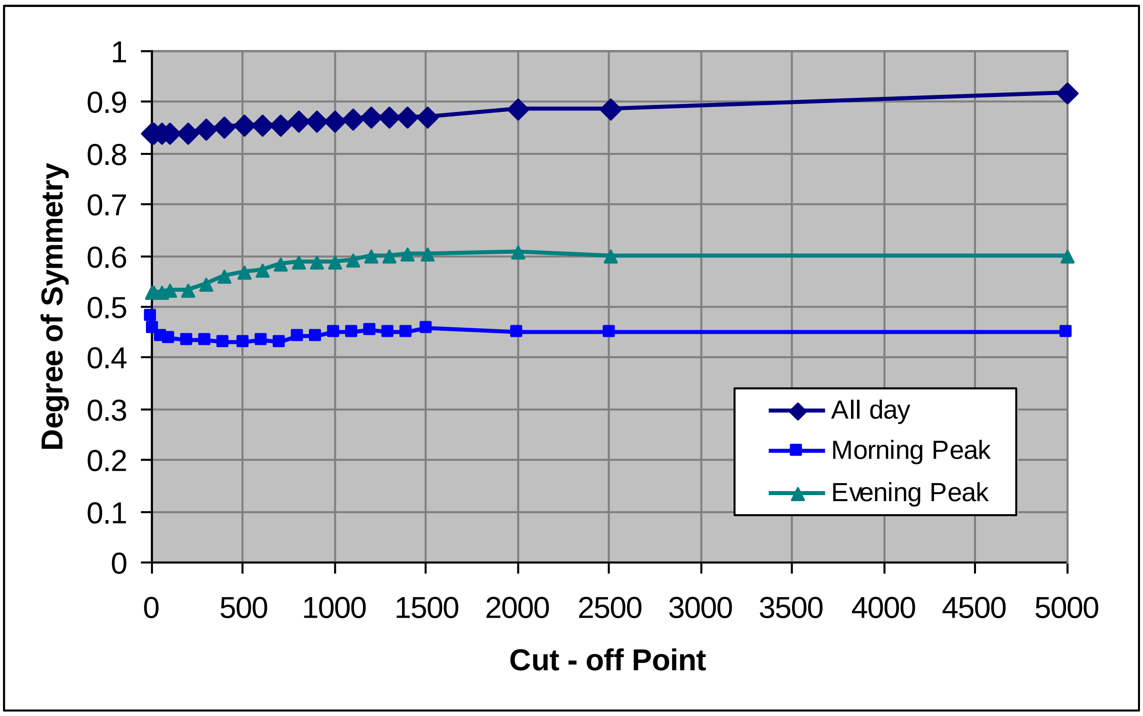

Figure 3: Degree of Symmetry Chart - Single Journeys

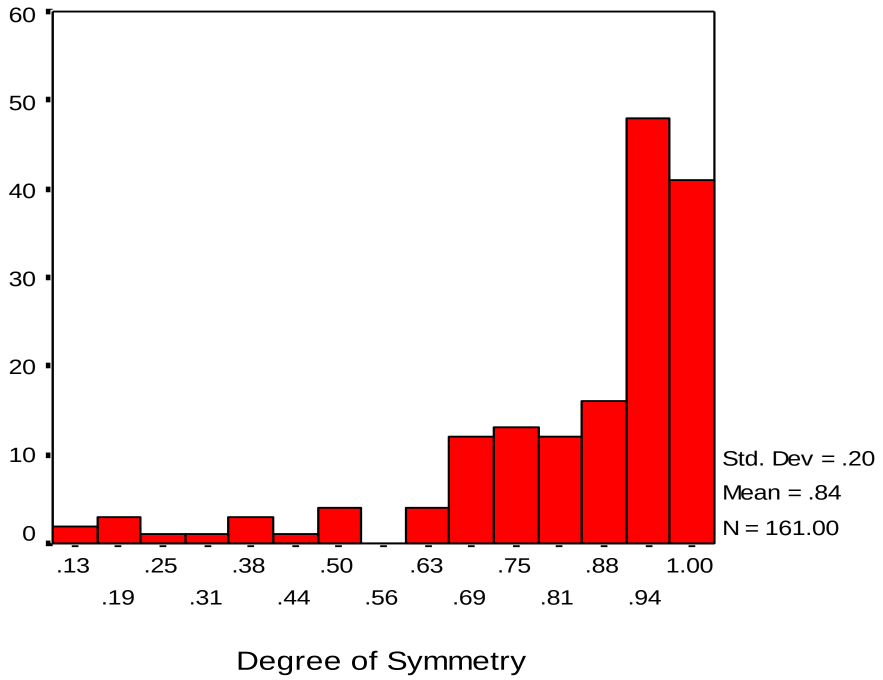

Figure 4: Histogram of Degree of Symmetry of all individual Routes – Single Journeys

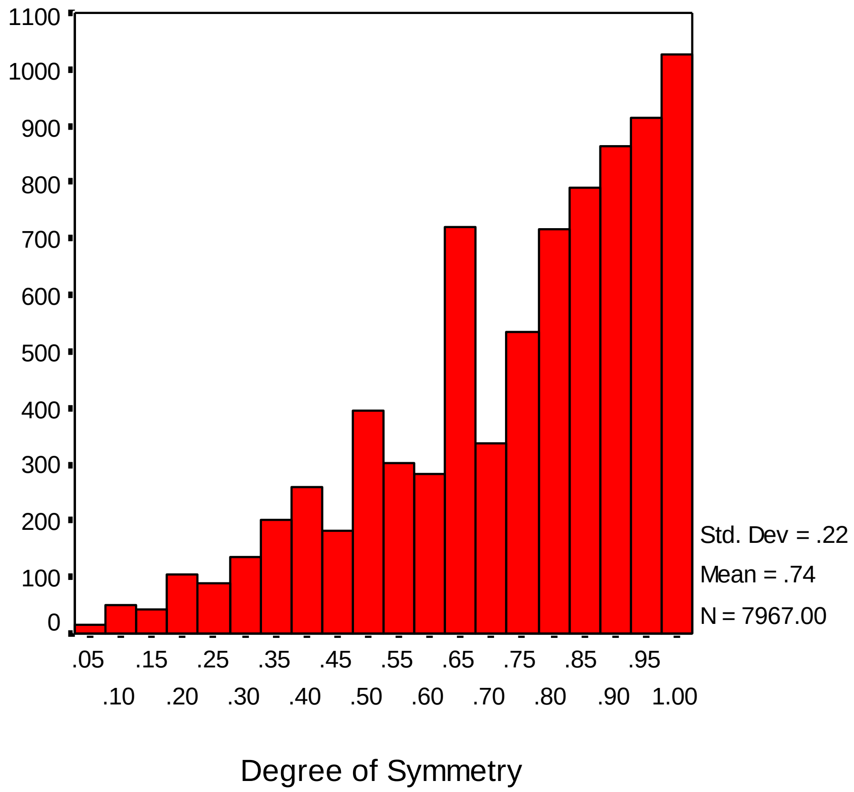

Figure 5: Histogram of Degree of Symmetry of all individual Routes – Transfer Journeys