On the Existence of the Simultaneous Occurrence of the Braess and Emission Paradoxes

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Abstract

Braess’ paradox is well-known and examined. However, an emission paradox, an analogue to Braess’ paradox for vehicular emissions, is not. Without considering the emission paradox, the road network improvement that mitigates congestion may increase harmful vehicular emissions. In this paper, we analytically examine the occurrence of the emission paradox and the simultaneous occurrence of the Braess and emission paradoxes in the classical Braess’ network. We ascertain that the occurrence of the emission paradox depends on the demand for travel, the parameters of link performance functions as well as link emission factors. We also find that the Braess and emission paradoxes do not always occur at the same time, and that the emission paradox is more likely to occur than the Braess paradox in some networks. More importantly, we discover that under some conditions of the parameters of link performance functions, the emission paradox does occur but Braess’ paradox does not. This implies that road network design for mitigating congestion alone may not be able to avoid the increase in vehicular emissions. A more comprehensive view for road network design is necessary to avoid the occurrence of both the Braess and emission paradoxes.

Keywords: Braess’ Paradox, Emission Paradox, Traffic Assignment
Introduction

Braess (1968) presented a remarkable example and demonstrated a counterintuitive phenomenon that adding a new link can increase the total system travel cost (TSTC). However, its impact in relation to vehicular emissions caused by network expansion is less often discussed. The road network improvement, including adding links, may be able to mitigate congestion but may increase vehicular emissions such as nitrogen oxide, carbon monoxide, nitrogen dioxide, sulphur dioxide, ozone and particulates. These emissions are harmful to human health. Up till now, only Nagurney (2000a, 2000b) raised and studied this important emission issue. Through specific numerical examples, Nagurney demonstrated that three distinctive paradoxical phenomena can occur as regards the total emissions generated. One of them revealed that, in the well-known Braess’ network, the total emissions also increase after adding the new link to the network. This paradoxical phenomenon is called an emission paradox, which is analogous to the classical Braess’ Paradox, and also calls attention to a careful road network expansion. It is because road network expansion is an expensive investment but the available budget is limited. Road network planners should carefully design and take into account some unwanted outcomes possibly arisen from road network expansion, such as an increase in overall vehicular emissions. Moreover, without considering the emission paradox, the road network improvement that mitigates congestion may increase harmful vehicular emissions. It is therefore pivotal to consider and study the simultaneous occurrence of both the Braess and emission paradoxes. However, to our best knowledge, no research considered both paradoxes simultaneously.

This paper analytically examines the simultaneous occurrence of both the Braess and emission paradoxes in the classical Braess’ network. The conditions of the occurrence of emission paradox in this network are also derived, which are not studied before. Compared with Pas and Principio (1997), we also examine more conditions of the parameters of the link performance functions in Braess’ network. The remainder of this paper is organized as follows: The next section analyses the conditions under which the emission paradox occurs in Braess’ network. The third section examines the conditions under which both the Braess and emission paradoxes occur at the same time in Braess’ network. Finally, the last section gives concluding remarks.

When Does Nagurney’s Emission Paradox Occur in Braess’ Network?

In this section we study the conditions under which the emission paradox occurs in a classical Braess’ network. The result will be useful in analyzing the simultaneous occurrence of both the Braess and emission paradoxes. The Braess’ network is shown in Figure 1. Before a new link is added to connect nodes 2 and 3 (Figure 1a), the network consists of four nodes, and four links, and one O/D pair. This four-link network has two paths between O/D pair (1,4): path 1 (1-2-4) and path 2 (1-3-4). After a new link is added (Figure 1b), this network has one more path, path 3 (1-2-3-4). Before and after adding the new link, the total travel demand is the same and fixed. The following notations are adopted throughout this paper: $c_{ij}$ is the travel...
cost on link $ij$; $\alpha_y$ is the free flow travel cost on link $ij$, and $\alpha_y > 0$; $\beta_y$ is the parameter of the performance function of link $ij$ in which a lower value of $\beta_y$ means a wider capacity on this link, and $\beta_y > 0$; $x_{ij}$ is the flow on link $ij$; $f_k$ is the path flow on path $k$; $h_y$ is the emission factor on link $ij$; $D$ is the total travel demand, and; $Q$ is the total vehicular emissions.

According to the emission models developed by the National Environmental Protection Agency (NEPA), the vehicular emissions from link $ij$ is equal to the product of the corresponding emissions factor $h_y$ and link flow $x_{ij}$ (Allen, 1995; Anderson et al., 1996; DeCorla-Souza et al., 1994), and the total vehicular emission $Q$ is the sum of the vehicular emissions from each link:

$$Q = \sum_y h_y x_{ij}.$$  \hspace{1cm} (1)

This equation will be used for deriving conditions for the occurrence of Nagurney’s emission paradox. Note that in the above equation, link flows are required for calculating total emissions, and are functions of path flows. Therefore, in the following, all the feasible path flow patterns in addition to the total emissions before and after adding the new link are determined. The path travel costs are also obtained next, which will be used to derive the conditions for the occurrence of each flow pattern.

Before the addition of link 2-3, two paths’ costs are equal and the flow on each path is the same when the User Equilibrium condition is reached. We can therefore obtain:

\begin{figure}[h]
\centering
\begin{subfigure}{.45\textwidth}
\centering
\begin{tikzpicture}
  \node (1) at (0,0) {1};
  \node (2) at (-2,-2) {2};
  \node (3) at (2,-2) {3};
  \node (4) at (0,-4) {4};
  \path
  (1) edge[<->] node {$c_{12} = \beta_1 x_{12}$} (2)
  (1) edge[<->] node {$c_{13} = \alpha_1 + \beta_2 x_{13}$} (3)
  (2) edge[<->] node {$c_{24} = \alpha_1 + \beta_2 x_{24}$} (4)
  (3) edge[<->] node {$c_{34} = \beta_3 x_{34}$} (4);
\end{tikzpicture}
\caption{Four-link network}
\end{subfigure}
\begin{subfigure}{.45\textwidth}
\centering
\begin{tikzpicture}
  \node (1) at (0,0) {1};
  \node (2) at (-2,-2) {2};
  \node (3) at (2,-2) {3};
  \node (4) at (0,-4) {4};
  \path
  (1) edge[<->] node {$c_{13} = \alpha_1 + \beta_3 x_{13}$} (3)
  (1) edge[<->] node {$c_{12} = \beta_1 x_{12}$} (2)
  (2) edge[<->] node {$c_{24} = \alpha_1 + \beta_4 x_{24}$} (4)
  (3) edge[<->] node {$c_{34} = \beta_3 x_{34}$} (4)
  (2) edge[<->] node {$c_{23} = \alpha_2 + \beta_2 x_{23}$} (3);
\end{tikzpicture}
\caption{Five-link network}
\end{subfigure}
\caption{Braess’ networks.}
\end{figure}
\begin{align*}
\eta_1 &= \eta_2 = \frac{D(\beta_1 + \beta_2)}{2} + \alpha_1, \quad \text{and} \\
f_1 &= f_2 = D/2.
\end{align*}

The total emissions in the network are then calculated as follows:

\begin{align*}
Q_{\text{four links}} &= h_{12}x_{12} + h_{24}x_{24} + h_{13}x_{13} + h_{34}x_{34}, \quad \text{or} \\
Q_{\text{four links}} &= D(h_{12} + h_{24} + h_{13} + h_{34})/2.
\end{align*}

After link 2-3 is added, there are only three mutually exclusive flow patterns:

1. **All three paths are used.** \((i.e., f_1, f_2, f_3 > 0.)\)

In this case, the User Equilibrium flow pattern must satisfy the following conditions:

\begin{align*}
f_1 &= f_2 = \frac{\alpha_2 - \alpha_1 + D(\beta_1 + \beta_2)}{\beta_1 + 3\beta_2}, \quad \text{and} \\
f_3 &= D - 2f_1.
\end{align*}

\(f_1 = f_2\) is due to the symmetrical characteristic of Braess’ network. The travel costs on the three paths are equal and expressed as:

\begin{equation}
\eta_1 = \eta_2 = \eta_3 = \alpha_1 + D\beta_1 + (\beta_2 - \beta_1) \left[ \frac{\alpha_2 - \alpha_1 + D(\beta_1 + \beta_2)}{\beta_1 + 3\beta_2} \right],
\end{equation}

The total emissions generated in the network are:

\begin{equation}
Q_{\text{five links}} = f_1(h_{24} + h_{34} - h_{12} - 2h_{13}) + D(h_{12} + h_{34} + h_{23}).
\end{equation}

2. **Only the new path (path 3) is used.** \((i.e., f_1 = f_2 = 0, f_3 > 0.)\)

In this case, paths 1 and 2 carry no flow and only the new path (path 3) carries flows. The User Equilibrium flow pattern and the corresponding path travel cost must satisfy:

\begin{align*}
f_1 &= f_2 = 0, \\
f_3 &= D, \quad \text{and} \\
\eta_1 &= \eta_2 = \alpha_1 + \beta_1 D \geq \eta_3 = D(2\beta_1 + \beta_2) + \alpha_2.
\end{align*}

We can then obtain the condition for the occurrence of flow pattern 2:

\begin{equation}
D \leq \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2},
\end{equation}

The total emissions, in this case, are as follows:

\begin{equation}
Q_{\text{five links}} = f_3(h_{12} + h_{23} + h_{34}) = D(h_{12} + h_{23} + h_{34}).
\end{equation}

3. **Only the new path (path 3) is not used.** \((i.e., f_1 = f_2 > 0, f_3 = 0.)\)

In this case, path 3 carries no flow and the total emissions are the same as described in equation (4). The User Equilibrium flow pattern and path travel cost must follow:

\begin{align*}
f_1 &= f_2 = D/2, \\
f_3 &= 0, \quad \text{and} \\
\eta_1 &= \eta_2 = \alpha_1 + \beta_1 D/2 + \beta_2 D/2 \leq \eta_3 = \alpha_2 + \beta_1 D/2 + \beta_2 D/2.
\end{align*}

From equation (12), we have:

\[ f_1 = f_2 = D/2, \]

\[ f_3 = 0, \quad \text{and} \]

\[ \eta_1 = \eta_2 = \alpha_1 + \beta_1 D/2 + \beta_2 D/2 \leq \eta_3 = \alpha_2 + \beta_1 D/2 + \beta_2 D/2. \]
which determines the occurrence of flow pattern 3. The total emissions, in this case, are

\[
Q_{\text{five links}} = Q_{\text{four links}}. \tag{14}
\]

From (4), (8), (11), (14), we can determine the change in total emissions. When all three paths are used (i.e., \( f_1, f_2, f_3 > 0 \)),

\[
Q_{\text{five links}} - Q_{\text{four links}} = (f_1 - D/2)(h_{24} + h_{13} - h_{12} - h_{34} - 2h_{32})
\]
\[
= (f_1 - D/2)A
\]
\[
= \left(f_1 - \frac{f_1 + f_2 + f_3}{2}\right)A
\]
\[
= -(f_3/2)A, \tag{15}
\]

where \( A = h_{24} + h_{13} - h_{12} - h_{34} - 2h_{32} \); \( f_3 \) must be non-negative and satisfy equation (6). When only the new path (path 3) is used (i.e., \( f_1 = f_2 = 0, f_3 > 0 \)),

\[
Q_{\text{five links}} - Q_{\text{four links}} = (D/2)(h_{12} + 2h_{23} + h_{14} - h_{24} - h_{13}) = (-D/2)A. \tag{16}
\]

When only the new path (path 3) is not used (i.e., \( f_1 = f_2 > 0, f_3 = 0 \)),

\[
Q_{\text{five links}} - Q_{\text{four links}} = 0. \tag{17}
\]

Based on the changes in total emissions (15)-(17), we can determine whether Nagurney’s emission paradox occurs in Braess’ network. The emission paradox occurs if and only if the change in total emissions is positive:

\[
Q_{\text{five links}} - Q_{\text{four links}} > 0. \tag{18}
\]

Conditions (15) and (18) implies that the emission paradox occurs if and only if \(- (f_3/2)A > 0\) and \( f_1, f_2, f_3 > 0 \). As \( f_3 > 0 \), \(- (f_3/2)A > 0\) can be simplified to \( A < 0 \) and hence the emission paradox occurs if \( A < 0 \) and \( f_1, f_2, f_3 > 0 \). Conditions (16) and (18) implies that the emission paradox occurs if and only if \(- (D/2)A > 0\) and \( f_1 = f_2 = 0, f_3 > 0 \). As demand must be non-negative, \(- (D/2)A > 0\) can be simplified to \( A < 0 \), and the emission paradox occurs if \( A < 0 \) and \( f_1 = f_2 = 0, f_3 > 0 \). Conditions (17) and (18) imply that when only the new path is not used, the emission paradox must not occur. These results are summarized in Table 1; the emission paradox occurs if \( A < 0 \) and the new path carries flows (but the old paths may or may not carry flows).
Table 1. Summary of the Occurrence of the Emission Paradox under Each Flow Pattern.

<table>
<thead>
<tr>
<th>Flow patterns</th>
<th>Conditions of parameters</th>
<th>1. ( f_1, f_2, f_3 &gt; 0 )</th>
<th>2. ( f_1, f_2 = 0, f_3 &gt; 0 )</th>
<th>3. ( f_1, f_2 &gt; 0, f_3 = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A &lt; 0 )</td>
<td>The emission paradox must happen</td>
<td>( A \geq 0 )</td>
<td>The emission paradox never happens</td>
<td></td>
</tr>
</tbody>
</table>

When \( A < 0 \)? We assume that the emission factor of each link is proportional to the travel distance on that link. According to the triangle formula, we get \((h_{ij} - h_{i2} - h_{i1}) < 0 \) and \((h_{i4} - h_{i3} - h_{i2}) < 0 \). By adding these two inequalities, we obtain \( A = (h_{i3} - h_{i2} - h_{i1}) + (h_{i4} - h_{i3} - h_{i2}) < 0 \). However, if emission factors depend on not only distance traveled but also some other factors, then \( A \) can be any real values (DeCorla-Souza et al. 1994).

When does the new path carry flows? Alternatively, when does the new path carry no flows and when does each of the three flow patterns occur? They all depend on the demand level and the parameters of the link performance functions. Table 2 summarizes the results and are obtained based on analyzing equations (10) and (13), the conditions of the parameters of the link performance functions, and the feasible demand range.
<table>
<thead>
<tr>
<th>Flow patterns</th>
<th>Conditions of Parameters</th>
<th>1. $f_1, f_2, f_3 &gt; 0$</th>
<th>2. $f_1, f_2 = 0, f_3 &gt; 0$</th>
<th>3. $f_1, f_2 &gt; 0, f_3 = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\beta_1 - \beta_2 &gt; 0$; $\alpha_1 - \alpha_2 &gt; 0$.</td>
<td>$\alpha_1 - \alpha_2 &lt; D &lt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)}$</td>
<td>$D \leq \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$</td>
<td>$D \geq \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)}$</td>
<td></td>
</tr>
<tr>
<td>2. $\beta_1 - \beta_2 &lt; 0$; $\alpha_1 - \alpha_2 &lt; 0$.</td>
<td>$D &gt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)}$</td>
<td>Never happen under any demand value</td>
<td>$D \leq \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)}$</td>
<td></td>
</tr>
<tr>
<td>3. $\beta_1 - \beta_2 &lt; 0$; $\alpha_1 - \alpha_2 &gt; 0$.</td>
<td>$D &gt; \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$</td>
<td>$D \leq \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$</td>
<td>Never happen under any demand value</td>
<td></td>
</tr>
<tr>
<td>4. $\beta_1 - \beta_2 = 0$; $\alpha_1 - \alpha_2 &gt; 0$.</td>
<td>Must happen under any demand value</td>
<td>Never happen under any demand value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. $\beta_1 - \beta_2 &lt; 0$; $\alpha_1 - \alpha_2 = 0$.</td>
<td>$D \leq \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$</td>
<td>Never happen under any demand value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. $\beta_1 - \beta_2 &gt; 0$; $\alpha_1 - \alpha_2 &lt; 0$.</td>
<td></td>
<td>Never happen under any demand value</td>
<td>Must happen under any demand value</td>
<td></td>
</tr>
<tr>
<td>7. $\beta_1 - \beta_2 = 0$; $\alpha_1 - \alpha_2 = 0$.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. $\beta_1 - \beta_2 &gt; 0$; $\alpha_1 - \alpha_2 = 0$.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. $\beta_1 - \beta_2 = 0$; $\alpha_1 - \alpha_2 &lt; 0$.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to Table 2, when the parameters of link performance functions can be described by condition 1, all three flow patterns occur but under different demand conditions. When $\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} < D < \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)}$, there are flows on all three paths; when $D \leq \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$, only path 3 carries flows; however, when $D \geq \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)}$, path 3 carries no flow. Under condition 2, there are only two flow patterns occurred; when $D > \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)}$, all three paths carry flows; when $D \leq \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)}$, the new path will not be used. Conditions 3 and 4 give the same demand condition for the occurrence
of flow pattern 1 and give the same demand condition for the occurrence of flow pattern 2. In addition, conditions 3 and 4 result in the case that under any demand value, flow pattern 3 does not occur. When the parameters of link performance functions fall into condition 5, under any demand value, only flow pattern 1 happens, in which all three paths are utilized. It is noticeable that under conditions 6, 7, 8 and 9, only flow pattern 3 occurs and flow patterns 1 and 2 do not occur. This implies that, under these parameters’ conditions, the new link will not be used after the construction.

According to Tables 1 and 2, we can summarize five conditions under which the emission paradox occurs:

1. \( A < 0, D < \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}, \beta_1 - \beta_2 > 0, \alpha_1 - \alpha_2 > 0. \)
2. \( A < 0, D > \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)}, \beta_1 - \beta_2 < 0, \alpha_1 - \alpha_2 < 0. \)
3. \( A < 0, \beta_1 - \beta_2 < 0, \alpha_1 - \alpha_2 > 0. \)
4. \( A < 0, \beta_1 - \beta_2 = 0, \alpha_1 - \alpha_2 > 0. \)
5. \( A < 0, \beta_1 - \beta_2 < 0, \alpha_1 - \alpha_2 = 0. \)

All these five conditions include \( A < 0. \) However, the inclusion of a particular demand condition depends on the relationship between \( \alpha_1 \) and \( \alpha_2 \) and that between \( \beta_1 \) and \( \beta_2. \)

It is worthwhile to mention that when the parameters of link performance functions satisfy conditions 6, 7, 8 and 9, the emission paradox must not occur, as the new link will not be used under any demand after construction. The new link should not be built under these conditions. In terms of cost and benefit, the road network planner should examine the network configuration carefully to avoid such redundant construction.

Here is an example. We adopt the parameters used in Nagurney’s example: \( \alpha_1 = 50, \alpha_2 = 10, \beta_1 = 10, \beta_2 = 1, D = 6, \) and \( h_{12} = h_{13} = h_{24} = h_{34} = h_{23} = 0.1. \) The objective is to determine whether Nagurney’s emission paradox occurs. As \( A = h_{24} + h_{13} - h_{12} - h_{34} - 2h_{23} = -0.2 < 0, \) and \( \beta_1 - \beta_2 > 0, \alpha_1 - \alpha_2 > 0, \) we have \( D = 6 < \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} = 8.89. \) The first of the five conditions described before is satisfied and the emission paradox therefore occurs.

When Do Both Braess’ and Emission Paradoxes Occur Simultaneously?

Pas and Principio (1997) only considered condition 1 when examining the occurrence of Braess’ paradox. Here, we further examine all other conditions of parameters of the link performance functions to analyze whether Braess’ paradox occurs. The results are summarized in Table 3. Table 3 also summarizes the conditions for the occurrence of the emission paradox.

According to this table, under condition 1, the Braess and emission paradoxes occur when the demand falls into the range described in Table 3. Under condition 2,
Braess’ paradox never happens, but the emission paradox may happen when \( D > \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} \). Under conditions 3, 4 and 5, the emission paradox must occur, but Braess’ paradox never happens. Under conditions 6, 7, 8 and 9, both the Braess and emission paradoxes never take place as the new path carries no flows.

### Table 3. Demand Conditions under which the Braess and Emission Paradoxes Occur.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Braess’ Paradox</th>
<th>Emission Paradox</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \beta_1 - \beta_2 &gt; 0, \alpha_1 - \alpha_2 &gt; 0; )</td>
<td>When ( \frac{2(\alpha_1 - \alpha_2)}{3\beta_1 + \beta_2} &lt; D &lt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} ), Braess’ paradox occurs</td>
<td>When ( D &lt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} ), emission paradox occurs</td>
</tr>
<tr>
<td>2. ( \beta_1 - \beta_2 &lt; 0, \alpha_1 - \alpha_2 &lt; 0; )</td>
<td>Braess’ paradox never occurs under any demand value</td>
<td>When ( D &gt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} ), emission paradox occurs</td>
</tr>
<tr>
<td>3. ( \beta_1 - \beta_2 &lt; 0, \alpha_1 - \alpha_2 &gt; 0; )</td>
<td></td>
<td>Emission paradox must occur under any demand value</td>
</tr>
<tr>
<td>4. ( \beta_1 - \beta_2 = 0, \alpha_1 - \alpha_2 &gt; 0; )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. ( \beta_1 - \beta_2 &lt; 0, \alpha_1 - \alpha_2 = 0; )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6, 7, 8 and 9.</td>
<td>Both congestion and emission paradoxes never occur under any demand value</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2 shows an example about the occurrence of two paradoxes in the classical Braess’ network. As we can see, under Braess-network’s parameters with Nagurney’s emission factors, there are three regions:

1. Both paradoxes do not occur (when the demand is larger than or equal to 8.89 units).
2. Both paradoxes occur at the same time (when the demand between 2.58 and 8.89 units).
3. The emission paradox occurs but Braess’ paradox does not (when the demand is less than 2.58 units).

The existence of the last region implies that road network design for mitigating congestion alone may not be able to avoid the increase in vehicular emissions. Simultaneous considerations of both paradoxes in road network design are necessary.

Figure 2 also shows that the range of demand for the occurrence of emission paradox includes the range for the occurrence of Braess’ paradox, which implies that the emission paradox is more likely to occur. However, we do not know whether this is always true for any network. This is left for future studies.
Figure 2. The occurrence of two paradoxes in Braess’ network, where $\beta_1 - \beta_2 = 10 - 1 > 0$ and $\alpha_1 - \alpha_2 = 50 - 10 > 0$.

Concluding Remarks

In this paper, we analytically examine the occurrence of the emission paradox and the simultaneous occurrence of the Braess and emission paradoxes in the classical Braess’ network. We discover the emission paradox does not always occur. The occurrence of the emission paradox depends on the demand for travel, the parameters of link performance functions as well as link emission factors. Moreover, the Braess and emission paradoxes do not always occur at the same time. More importantly, we find that under some conditions of the parameters of link performance functions, the emission paradox does occur but Braess’ paradox does not. This implies that road network design for mitigating congestion alone may not be able to avoid the increase in vehicular emissions. A more comprehensive view of road network design considering both congestion and emissions simultaneously is necessary to avoid the occurrence of both the Braess and emission paradoxes.

References


