MULTIVARIATE SHORT-TERM TRAFFIC FLOW FORECASTING USING TIME-SERIES ANALYSIS

Bidisha Ghosh, Biswajit Basu, and Margaret O’Mahony

Department of Civil, Structural and Environmental Engineering
Trinity College, Dublin

ABSTRACT: (156 words)
The existing time series models used for short-term traffic condition forecasting are mostly univariate in nature. Generally the extension of the existing univariate time-series models to a multivariate regime involves huge computational complexities. A different class of time-series model called structural time-series model (STM) (in its multivariate form) has been introduced in this paper to develop a parsimonious and computationally simple multivariate short-term traffic condition forecasting algorithm. The different components of a time-series dataset such as the trend, seasonal, cyclical and calendar variations can be modelled separately in STM methodology. A case study in Dublin city centre with serious congestion is performed to test the effectiveness of the forecasting strategy. The results indicate that the proposed forecasting algorithm is an effective approach to predict the real-time traffic flow at multiple junctions within an urban transport network where the junctions are not strictly on the same route or share the same path with the network.
INTRODUCTION

Implementation of Intelligent Transportation Systems (ITS) to provide dynamic traffic control requires continuous forecasting of traffic conditions in near (short-term or less than 1 hour, (Smith et al., 2002)) future. Short-term traffic forecasting is an important tool to follow evolution of traffic conditions over time in a transport network. This type of advanced forecasting methodologies having a time horizon of 15 minute or less (Smith et al., 2002) can provide information to support short-range operational modifications to improve the efficiency of the network at a finer scale. With the increasing need to develop more adaptive (site and time specific) traffic management systems, considerable research attention has been focussed on short-term traffic forecasting.

The well-known short-term forecasting algorithms can broadly be classified into univariate and multivariate approaches. The univariate approach is based on modelling traffic condition related variables (such as speed, flow or occupancy etc.) utilising observations from any single site, whereas developing a single model considering several sites for input and output is termed a multivariate approach. Unlike univariate models, these models are capable of capturing the temporal as well as the spatial evolution of traffic conditions over time in a transportation network. But due to ease of computation, univariate models are more common in short-term traffic forecasting literature (Kamarianakis and Prastacos, 2003).

Both multivariate and univariate models can be developed using different empirical and theoretical techniques (Van Arem et al., 1997). The empirical approaches (non-
parametric and parametric) employ a fairly standard statistical methodology and/or a heuristic method for traffic flow forecasting without referring to the actual traffic dynamics. The non-parametric techniques include non-parametric regressions (e.g. Davis and Nihan, 1991) and neural networks (e.g. Smith and Demetsky, 1994; Vlahogianni et al., 2005). Due to an intrinsic multi-input nature neural network models are often favoured in the space-time or multivariate models (Zhang et al., 1998). The parametric techniques include different time-series models such as, linear and non-linear regression, historical average algorithms (e.g. Smith and Demetsky, 1997), smoothing techniques (e.g. Smith and Demetsky, 1997; Williams et al., 1998) and autoregressive linear processes (Ahmed and Cook, 1979; Levin and Tsao, 1980; Hamed et al., 1995; Williams et al., 1998; Williams, 2003). Of all autoregressive linear processes, seasonal auto regressive integrated moving average (SARIMA) models (e.g. Williams et al., 2003 and Ghosh et al., 2005) perform better than other time-series techniques (Chung and Rosalion, 2001; Smith et al. 2002).

With a few exceptions (Whittaker et al., 1997; Kamaranakis and Prastacos, 2002; Williams, 2003; Kamaranakis and Prastacos, 2003) most of the literature in short-term traffic forecasting focus on univariate. The available multivariate empirical models in the short-term traffic forecasting literature are mainly multivariate variations on the existing univariate parametric statistical models, e.g. the multivariate ARIMA model (Kamaranakis and Prastacos, 2003), space-time ARIMA model (Kamaranakis and Prastacos, 2002). These models can account for the dimension of space in a transport network. But the models are computationally demanding as the multivariate nature involves estimation of a large number of parameters. Multivariate
time-series models based on state-space methodology were introduced as a short-term traffic forecasting technique by Stathopoulos and Karlaftis (2003).

In this paper, for the first time in traffic flow related studies, multivariate structural time-series (MST) models are applied to model traffic flow observations from multiple intersections within an urban signalized transport network. In a structural time-series model (STM), the evolution of different components of time-series data such as the trend, seasonal, cyclical and calendar variations with time can be modelled separately. Classical time-series analysis using the ARMA (SARIMA, ARIMAX) class of models is based on the theory of stationary stochastic processes. Hence the time-series dataset to be modelled using the SARIMA technique is always required to be checked for its stationarity. If the dataset is not stationary, then transformations are required to be performed to achieve weak stationarity. As STM is not based on this theory, no such transformations or checks are required for the application of this methodology. The MST models are computationally inexpensive as compared to other multivariate time-series models. Missing observations and inclusion of exogenous variables like traffic flow observations of other upstream junctions can be incorporated comparatively easily in the MST model (Harvey, 1989; West and Harrison, 1997; Durbin and Koopman, 2001).

THEORETICAL BACKGROUND

The STM methodology is a particular time-series analysis technique which is set up in terms of components which have a direct physical interpretation (Harvey, 1989). The different components of STM are the (deterministic and stochastic) trend, seasonal,
cyclical and calendar variation together with the effect of explanatory variables and interventions (outlier and structural breaks). The basic principle behind a STM is similar to that of the Holt Winters Exponential Smoothing (HWES) model, but more complex. Multivariate STMs are straightforward extension of the univariate STMs and involve less computational complexities than the other existing multivariate time-series techniques. An overview of the univariate and multivariate STM model definitions is given in this section. A detailed discussion on this subject is available in Harvey (1989) and Durbin and Koopman (2001). A software package called STAMP 6.0 (Structural Time-Series Analyser, Modeller and Predictor) is used in this study for modelling traffic flow observations using STM.

**Univariate STM Methodology**

A univariate structural time-series model is formulated based on the unobserved components which have a direct interpretation in terms of the temporal variability of a time series dataset. Consequently the evolution of the components such as trend or seasonality over time and their contribution to the final predictions can be observed clearly. A univariate STM for a time series dataset \( y \) can be described by the following general equation involving all possible types of temporal components in its form:

\[
\begin{align*}
    y_t &= \mu_t + \gamma_t + \psi_t + \nu_t + \varepsilon_t, \\
    \varepsilon_t &\sim \text{NID} (0, \sigma^2) \\
    t &= 1, \ldots, T
\end{align*}
\]  

(1)
where, $\mu_t$ is the trend, $\gamma_t$ is the seasonal, $\psi_t$ is the cycle, $\nu_t$ is the first-order AR component and $\epsilon_t$ is the irregular or the random error. For the purpose of traffic flow modelling, the univariate and multivariate STM are considered to be comprised of three components; stochastic trend, seasonality and irregular. Hence, the equation 1 reduces to the following form:

\[
\begin{align*}
    y_t &= \mu_t + \gamma_t + \epsilon_t, \\
    \epsilon_t &\sim \text{NID}(0, \sigma^2_\epsilon) \\
\end{align*}
\]

The stochastic trend component $(\mu_t)$ represents the long-term movement in a time-series which can be extrapolated into the future. In the case of traffic flow observations over a few weeks from a developed urban transport network, this long-term movement does not show any significant gradient and should be modelled for the local fluctuations. A Markov model of the stochastic trend can be considered in this purpose.

\[
\begin{align*}
    \mu_t &= \mu_{t-1} + \eta_t \\
    \eta_t &\sim \text{NID}(0, \sigma^2_\eta) \\
\end{align*}
\]

(Change of slope is not considered)

The irregular disturbance/variance $\sigma^2_\epsilon$ and the stochastic trend (level) variance $\sigma^2_\eta$ are mutually uncorrelated. The $\mu_t$ process collapses to a linear trend if $\sigma^2_\eta = 0$. The periodic nature of the time-series dataset is chosen to be modelled using a
trigonometric specification of the seasonal component as this ensures smooth changes in the seasonal as observed in traffic flow time-series data.

\[ \gamma_t = \sum_{j=1}^{\lfloor s/2 \rfloor} \gamma_{j,t} \]  

(4)

where each \( \gamma_{j,t} \) is generated by

\[
\begin{bmatrix}
\gamma_{j,t} \\
\gamma_{j,t}^*
\end{bmatrix} = 
\begin{bmatrix}
\cos \lambda_j & \sin \lambda_j \\
-\sin \lambda_j & \cos \lambda_j
\end{bmatrix} 
\begin{bmatrix}
\gamma_{j,t-1} \\
\gamma_{j,t-1}^*
\end{bmatrix} + 
\begin{bmatrix}
\omega_{j,t} \\
\omega_{j,t}^*
\end{bmatrix}, 
\quad j = 1, \ldots, \frac{s}{2},
\]

(5)

Where, \( \lambda_j = \frac{2\pi j}{s} \) is the frequency, in radians, and the seasonal disturbances \( \omega_t \) and \( \omega_t^* \) are mutually uncorrelated random normal disturbances with zero mean and common variance \( \sigma_{\omega}^2 \). When \( s \) is even, the equation 5 at \( j = \frac{s}{2} \) collapses to

\[ \gamma_{j,t} = \cos \lambda_j \gamma_{j,t-1} + \omega_{j,t} \]  

(6)

Equations 1 to 5 define the STM used in this study. The disturbances of the individual components of the STM (\( \sigma_e^2, \sigma_\eta^2 \) and \( \sigma_\omega^2 \)) mentioned in these equations are all mutually uncorrelated. The variances \( \sigma_e^2, \sigma_\eta^2 \) and \( \sigma_\omega^2 \) denote the extent to which the individual components, such as the trend or seasonal component will vary with time and are called hyperparameters (Lenten and Moosa, 2003).
The equations 1 to 5 are generally solved in state-space form using Kalman filter based algorithms (Kalman, 1960; Harvey, 1989). The hyperparameters and the components are estimated using maximum likelihood estimation method.

In some cases, the time series observations to be modelled have dynamic relationships with some other independent variables. They are called explanatory or exogenous variables and inclusion of these variables in STM, may improve the forecasting precision of the models. The inclusions of the explanatory variables change the first part of equation 1 to the following form:

\[
y_t = \mu_t + \gamma_t + \psi_t + \nu_t + \sum_{i=1}^{k} \sum_{\tau=0}^{q} \Delta_{i\tau} x_{i,t-\tau} + \varepsilon_t
\]  

(7)

where, \(x_{i,t-\tau}\) is an exogenous variable, \(k\) is the total number of exogenous variables, \(\tau\) is the time lag and \(\Delta_{i\tau}\) is a set of unknown constants. The significance of \(\tau\) is that in some cases, the lagged value of the dependent variable can be considered as an exogenous variable in STM.

**Multivariate STM Methodology**

Multivariate time-series data can be chiefly classified into two distinct types, Panel data and interactive data (Harvey, 1989). In the case of panel data, the time-series variables are subjected to the same or similar influences but the individual elements do not interact with each other. As the variables follow a similar temporal nature, they can be modelled jointly. In contrast, multivariate interactive time-series data consists
of a set of variables which have some behavioural relationships among themselves and interact dynamically with each other. This distinction is important to find out the suitable type of multivariate structural time-series analysis technique. The multivariate traffic flow observations (i.e. observations from different stations in the same transport network) modelled in this paper are considered to be subjected to similar influences but not to have any dynamic interaction (detailed explanation in next section) and are modelled as panel data.

The panel data can be modelled by a multivariate structural time-series (MST) technique where the various components of the different time-series variables are allowed to be contemporaneously correlated. Such a type of MST model is referred to as, *seemingly unrelated time-series equations* (SUTSE) (Harvey, 1989). The univariate equations described in the previous subsections can easily be extended to SUTSE model.

\[
\begin{align*}
\mathbf{y}_t &= \mathbf{\mu}_t + \mathbf{\gamma}_t + \mathbf{\epsilon}_t, \\
\mathbf{\epsilon}_t &\sim \text{NID}(\mathbf{0}, \mathbf{\Sigma}_\epsilon) \\
\end{align*}
\]  

where, \( \mathbf{y}_t \) is a vector of \( N \times 1 \) time-series observations which depends on the unobserved trend component, \( \mathbf{\mu}_t \), seasonal component \( \mathbf{\gamma}_t \), and irregular component \( \mathbf{\epsilon}_t \), which are also vectors. \( \mathbf{\Sigma}_\epsilon \) is the \( N \times N \) variance matrix of the irregular disturbances. In multivariate regime, the equations 3, 4, 5 and 6 change in a similar manner as in the case of equation 2. The various unobserved components of the univariate STM now
become vectors in the MST model and the disturbances of these components become $N \times N$ variance matrices.

The inclusion of explanatory variables in the MST model is simple and similar to the univariate approach described in equation 7.

$$y_t = \mu_t + \gamma_t + \sum_{r=0}^{q} \delta_r x_{t-r} + \epsilon_t$$  \hspace{1cm} (9)

where, $x_t$ is a vector of $K \times 1$ explanatory variables. Elements of the unknown parameter matrix $\delta_r$ can be specified to be zero, thereby excluding certain explanatory variables from particular equations.

A vector process $y_t$ is said to be homogenous if all linear combinations of its $N$ elements have the same stochastic properties. In a multivariate homogenous system, the disturbance matrices are required to satisfy the following equation:

$$\Sigma_k = q_k \Sigma_*$$

and

$$\Sigma_e = h \Sigma_*.$$  \hspace{1cm} (10)

where, $q_k$, $k = 1, \ldots, g$ and $h$ are non-negative scalars and $\Sigma_*$ is an $N \times N$ matrix.

In these cases possible assumption of homogeneity in a SUTSE model decreases the computational complexity to a great extent.
One of the major advantages of STM methodology is its transparency (Durbin and Koopman, 2001). In STM, the different components which make up the time-series are modelled separately unlike the SARIMA methodology (e.g. Williams and Hoel, 2003 and Ghosh et al., 2005) where the trend and seasonal components are eliminated by differencing. Hence, in STM it is easy to understand the evolution and contribution of each of the components in the final results. In a multivariate regime, MST models are straightforward vector extensions of the univariate STM. The SUTSE models do not involve estimation of huge co-variance matrices as Vector ARMA (VARMA) models. Due to the recursive nature of the STM models any known structural change over time is easy to implement, whereas SARIMA models cannot include such structural changes since they are homogenous and stationary in form. Treatment for missing values in a time-series is also very simple in STM equations. Explanatory variables, outliers, structural breaks etc. can also be easily modelled in a STM framework as described before. Introduction of the same variables in SARIMA models require tedious computational efforts. STM is more general, flexible and can be easily transferred from a univariate to multivariate regime than the ARIMA class of models.

**METHODOLOGY**

A multi-input multi-output (where the number of input intersections are more than number of output intersections) short-term traffic flow simulation and forecasting model is proposed in this study for efficient modelling of traffic in a congested urban transport network. The proposed model is developed for a set of 15 minute aggregate
traffic volume observations from different approaches and different intersections of the transport network.

Unlike the previous multivariate traffic flow models developed for urban transport networks (Stathopoulos and Karlaftis, 2003; Kamarianakis and Prastacos, 2002), the locations of the sites of data collection within the transport network are not required to be considered in the proposed methodology. The aim of this approach is to develop a multivariate traffic flow simulation and forecasting model for multiple intersections within a transport network which may not be situated on the same route. As the intersections or stations of observations are not situated on the same route it is highly unlikely that the same platoon of vehicles will pass through different intersections at different time instants. Hence, information about the directions of traffic flow is not essential. A SUTSE model is ideal for modelling such multivariate time-series observations as the behavioural relationship among the variables are not considered.

As an improvement to the panel data modelling methodology a spatial dimension is introduced to the proposed multivariate traffic flow model. The traffic flow observations from the nearest available upstream intersection of each of the modelled data collection stations are included as explanatory variables to the MST model equations. This ensures that the effect of any abrupt change occurring in the upstream junction can be accounted for in the model.

**APPLICATION OF THE PROPOSED MST MODEL**

*Traffic Flow Observations from Urban Transport Network*
The proposed multivariate traffic flow forecasting methodology is applied to a congested urban transportation network in the city centre of Dublin to test the effectiveness of the forecasting strategy. A small network of ten intersections within the transport network is chosen for this purpose.

The time interval of traffic flow observation data collection is unique to the data collection system of the existing urban traffic control system of any city. The data interval can vary from a few seconds to one hour. Short-term forecasting algorithms applicable to a traffic management system should have a prediction horizon of 15 minutes or less (Smith and Demetsky, 1997). The univariate traffic flow observations obtained over each 15 minute interval from the inductive loop-detectors situated at these ten intersections and their nearest available upstream junctions are modelled using the proposed multivariate traffic flow model. There are two important points which are required to be checked before applying the proposed methodology.

1. The location and the distance of the sites can be chosen at random. The only criteria for the choice is that the average travel time between two sites on the same route within the network should not be more than 15 minutes. Considering a 30 km/hr free flow speed within a congested urban transport network, the radius of the simulation network should not be more than 7.5 km.

2. The second and the most important point is that the univariate traffic flow observations from different data-collection sites should not have behavioural or physical relationships among themselves. If and only if the
multivariate traffic flow observations behave as a panel data set, the SUTSE model can be applied.

In figure 1, a map of the chosen urban transport network at the city-centre of Dublin is given. The ten junctions where the multivariate time-series model is applied for short-term traffic volume simulation and prediction are shown with numbered yellow squares in the map. In the figure, the direction of the univariate traffic movement at each intersection is shown with a pink arrow. The origin of the pink arrow is marked with a numbered dark brown circle which signifies the nearest upstream junction to each intersection from which traffic volume data can be obtained. The length of the pink arrow signifies the distance between an intersection and its nearest available upstream junctions. If this distance is considerably high then it is possible that the changes in traffic flow at the upstream junction may not directly influence the traffic flow at the downstream intersection.

It is evident from the figure, that the choice of the site locations at which 15 minute traffic volume is modelled is random. The chosen intersections are not on the same route within the transport network and none of the two stations have a distance of more than 7.5 km between them?. Hence, the chosen network of ten intersections conforms to the conditions mentioned in point one. The ten intersections at which the proposed multivariate short-term traffic flow model is applied for simulation and forecasting are termed as output intersections in the rest of the text.
The traffic flow observations used for modelling from all the chosen intersections were recorded from 3rd November 2003 6:30 a.m. to 26th November 2003 6:30 a.m., excluding the weekends. A cross-section time-series plot of the traffic flow observations from the ten output intersections during 4th and 5th of November in 2003 is given in figure 2. The plot shows that there is a definite temporal similarity among the curves. The output junctions at which the direction of the univariate traffic flow fall on the routes towards the city-centre have high traffic volumes during the morning peak hours whereas the junctions for which the same fall on the routes away from the city-centre have higher traffic volumes during evening peak hours than the morning peak. Consequently, the peak hourly volumes from all these ten output junctions may not have high positive correlations, but the time of occurrence of the maximum traffic volumes passing through the junctions are very similar. Considering this contemporaneous correlation among the ten output traffic volume time-series datasets, they can be modelled as panel data using SUTSE models.

In table 1 further details about the ten output intersections are given along with the name of the nearest upstream intersections at which traffic flow observations are available. The 15 minute aggregate univariate traffic volumes from the mentioned loop-detectors in the upstream junctions are used as explanatory variables in the SUTSE model equations. In equation 5.9, the elements of the matrix $\delta_i$ are so chosen that the forecasts from each intersection is affected only by the changes at its upstream junction and not by the changes at other upstream intersections.

*Time-Series Model*
All of the ten series of traffic flow observations are modelled using homogenous SUTSE models with equation 9 and equations 3, 4, 5 and 6 in their vector forms. The estimated values of the trend/level component and the standard deviations of the disturbances of the components are provided in table 2. The elegance of the STM lies in the meaningful depiction of the components as shown in figure 3. In the figure, trend, seasonality and the random error components (obtained from the traffic flow observations collected from output station 2, as an example) are shown individually in three different subplots. The subplot (A) shows the original traffic flow data series along with a trend component as simulated and predicted from the proposed multivariate model. The subplots (B) and (C) of the figure individually show the seasonal and the irregular components respectively, simulated and predicted from the MST model. The hyperparameter estimates (table 2) and the plot of the seasonal component show that the seasonality is deterministic in nature. On the other hand the trend component is stochastic and depicts the within-day local fluctuations in the data. The trend component varies about a zero mean value validating the assumption that there is no slope component latent within the traffic flow dataset.

For all of the ten output junctions, 50 points in the future are forecasted (figure 4). The traffic flow data obtained on the 26th November 2003 from 6:30 a.m. to 8:00 p.m., i.e. the data collected in the next 12.5 hours (50x15 = 750 minute = 12.5 hours) are compared with these forecasts. The forecasting precision (MAPE) from the proposed SUTSE model for each of the ten output junctions with and without considering the influence of upstream junctions are given in figure 5 in the form of a bar diagram. Among the ten output intersections the upstream junction is situated directly upstream of the output station in six cases. These six cases have been
indicated in the bar diagram by a coupling sign shown on the bars. It is observed from figures 4 and 5 that the MAPE values for the forecasts at these output stations improve significantly when the traffic flow observations from the nearest available upstream junctions are incorporated in the SUTSE model as explanatory variables. The forecasting precision for the remaining four output junctions do not improve when the traffic flow observations from the nearest available upstream junction are incorporated in the model. These are the junctions not having any upstream junction nearby from where the loop-detector observations can be available. In such cases, the traffic flow at the nearest upstream junction do not directly influence the traffic at the output station due to the presence of excessive merging and diverging manoeuvres in between the upstream and the output junction. Hence, the inclusion of the traffic flow at far away upstream junctions as exogenous variables in the proposed model negatively affects the performance of the model. Thus it is preferable to ignore the influence of the traffic volume changes at an upstream junction if it is not located at an immediate vicinity of the output station. I think it would be important to try to justify why this is.

The MAPE values from the univariate SARIMA (2,0,1)(0,1,1)$_96$ (Ghosh et al. 2005) model for the traffic flow observations at the ten output intersections is shown in table 3 in comparison to the MAPE values from the proposed MST model for the same junction. In most of the cases, the proposed multivariate traffic flow time-series model prove to be more accurate than the ordinary univariate SARIMA model for short-term simulation and forecasting of traffic volume in a congested urban network.

CONCLUSIONS
In this paper a structural time-series methodology is applied to develop a multivariate short-term traffic flow forecasting model for an urban signalized transport network. This is the first instance of applying STM to short-term traffic condition related studies. I think you would need to include how the results from STM might compare with other models here as the reader will be looking for comparisons. This will help justify the use of this technique and its advantages. The model developed in the paper is observed to have achieved some distinct advantages over the existing well-known univariate SARIMA time-series model. These are:

- The model is capable of simultaneous simulation and modelling of traffic conditions at multiple intersections in an urban signalized transport network where it is difficult to model the existing paths and turning movements.

- The multivariate short-term traffic condition forecasting model developed here is computationally much simpler and performs more accurately than the most of the existing multivariate models.

- In structural time-series model the evolution of each individual component (trend, seasonality etc.) of the traffic flow data over time can be traced separately. Consequently, the deterministic nature of the seasonal component of the traffic volume observations from junctions at urban signalized arterials has been established.
The MST model can include the effect of changes in traffic conditions at one or more immediate upstream junctions to improve the predictions at the downstream output junction.

The distance of the nearest available upstream junction from the output intersection influences the forecasting precision to a certain extent. Consequently, for developing comparatively more efficient and robust multivariate short-term traffic flow forecasting algorithms further studies can be performed to incorporate the movement of traffic between the upstream junctions and the forecasting sites.

REFERENCES


LIST OF FIGURES

1. Map of the Chosen Transport Network
2. Plot of Two day Traffic volumes from Ten Output Intersections
3. Plot of Individual Components of STM Model
4. Forecasts from Ten Output Intersections from the SUTSE model
5. Forecasting Errors from Ten Chosen Intersections
Figure 1 Map of the chosen Transport Network.
Figure 2 Plot of Two day Traffic volumes from Ten Output Intersections.
Figure 3 Plot of Individual Components of STM Model.
Figure 4 Forecasts from Ten Output Intersections from the SUTSE model.
Figure 5 Forecasting Errors from Ten Chosen Intersections.
LIST OF TABLES

1. Details of the Ten Output Sites
2. Estimates of Parameters and Hyperparameters
3. Comparison of Univariate SARIMA and MST model
4.

<table>
<thead>
<tr>
<th>Station in Map</th>
<th>Intersection Name</th>
<th>Data Collecting Loop-Detectors</th>
<th>Upstream Junction &amp; Loop-Detectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TCS 26</td>
<td>4, 5, 6</td>
<td>TCS 182 (1, 2)</td>
</tr>
<tr>
<td>2</td>
<td>TCS 196</td>
<td>10, 11, 12, 13</td>
<td>TCS 193 (5, 6, 7)</td>
</tr>
<tr>
<td>3</td>
<td>TCS 17</td>
<td>5, 6, 7, 8</td>
<td>TCS 183 (5, 6, 7)</td>
</tr>
<tr>
<td>4</td>
<td>TCS 183</td>
<td>1, 2, 3, 4</td>
<td>TCS 26 (4, 5, 6)</td>
</tr>
<tr>
<td>5</td>
<td>TCS 232</td>
<td>1, 2, 3, 4</td>
<td>TCS 146 (1, 2, 3, 4)</td>
</tr>
<tr>
<td>6</td>
<td>TCS 49</td>
<td>1, 2</td>
<td>TCS 48 (1, 2)</td>
</tr>
<tr>
<td>7</td>
<td>TCS 166</td>
<td>1, 2, 3</td>
<td>TCS 188 (3, 4)</td>
</tr>
<tr>
<td>8</td>
<td>TCS 193</td>
<td>1, 2, 3, 4</td>
<td>TCS 232 (6, 7)</td>
</tr>
<tr>
<td>9</td>
<td>TCS 439</td>
<td>1, 2, 3</td>
<td>TCS 196 (6, 7, 8, 9)</td>
</tr>
<tr>
<td>10</td>
<td>TCS 269</td>
<td>1, 2, 3</td>
<td>TCS 196 (10, 11, 12, 13)</td>
</tr>
</tbody>
</table>

Table 1 Details of the Ten Output Sites.
<table>
<thead>
<tr>
<th>Station in Map</th>
<th>$\mu_t$</th>
<th>$\sigma_c$</th>
<th>$\sigma_\eta$</th>
<th>$\sigma_{\omega}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>318.61</td>
<td>33.718</td>
<td>9.9832</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>206.92</td>
<td>31.621</td>
<td>9.3628</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>144.88</td>
<td>19.336</td>
<td>6.5408</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>322.08</td>
<td>28.386</td>
<td>13.090</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>312.33</td>
<td>25.763</td>
<td>11.202</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>154.74</td>
<td>17.406</td>
<td>13.413</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>155.9</td>
<td>12.705</td>
<td>12.241</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>224.77</td>
<td>23.216</td>
<td>9.9464</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>162.13</td>
<td>17.536</td>
<td>4.8608</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>111.05</td>
<td>14.446</td>
<td>2.233</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2 Estimates of Parameters and Hyperparameters.
<table>
<thead>
<tr>
<th>Station in Map</th>
<th>MAPE of SARIMA (%)</th>
<th>MAPE of MST (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.02</td>
<td>5.89</td>
</tr>
<tr>
<td>2</td>
<td>12.7</td>
<td>10.9</td>
</tr>
<tr>
<td>3</td>
<td>11.24</td>
<td>12.66</td>
</tr>
<tr>
<td>4</td>
<td>7.07</td>
<td>7.4</td>
</tr>
<tr>
<td>5</td>
<td>8.6</td>
<td>6.52</td>
</tr>
<tr>
<td>6</td>
<td>15.56</td>
<td>7.96</td>
</tr>
<tr>
<td>7</td>
<td>8.32</td>
<td>4.94</td>
</tr>
<tr>
<td>8</td>
<td>7.12</td>
<td>6.2</td>
</tr>
<tr>
<td>9</td>
<td>8.45</td>
<td>7.4</td>
</tr>
<tr>
<td>10</td>
<td>10.94</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Table 3 Comparison of Univariate SARIMA and MST model.