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INFLUENCE OF SPATIAL VARIABILITY ON WHOLE LIFE MANAGEMENT OF REINFORCED CONCRETE BRIDGES

By

Omran Mohamed Keshel

BSc, MSc

Thesis submitted to the University of Dublin, Trinity College,

for the Degree of Doctor of Philosophy

August 2009
DECLARATION

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Omran Mohamed Kenshel

August 2009
ABSTRACT

The number of deteriorating bridges due to chloride-induced corrosion increases annually as does the cost of inspection, maintenance, repair and where necessary replacement. Meanwhile, budgets made available to bridge owners/managers for repair and maintenance of these bridges are reducing. To optimise and manage their budget spend, bridge owners/managers need to rely more on rational decision making methods rather than on subjective engineering judgment. In this thesis, the author has developed a probabilistic-based model which aims to predict the lifetime performance of Reinforced Concrete (RC) structures exposed to chloride corrosive environment and consequently to optimise their lifetime management.

The optimal time to repair/maintenance of deteriorating structures can be predicted by relative comparison of their selected performance criteria to its corresponding target limit. The probabilistic model developed in this thesis considers modelling the performance of a structure against corrosion-induced deterioration in terms of both; its surface (visual) condition and in terms of its safety. The surface condition is related to the Serviceability Limit State (SLS) while the safety is related to the Ultimate Limit State (ULS) of the structure. This approach allows for a fair comparison between the times to repair/maintenance predicted in terms of SLS and ULS to be made.

The model developed in this thesis takes into consideration the spatial variability of the deterioration properties which have been neglected in many reliability based studies dealing with the service life prediction of structures exposed to chloride-induced corrosion. The thesis provides a site specific data on the Scale of Fluctuation (θ), a parameter which is essential for spatial variability modelling, for two key deterioration parameters; the Surface Chloride Content (Cv) and the Diffusion Coefficient (D_{app}). To obtain values for θ, for Cv and D_{app}, the author performed an experimental study on 45 concrete cores extracted from an aging structure (Ferrycarrig bridge) located in a marine environment. Important recommendations were made concerning the measurement plans to ensure accurate prediction of the parameter θ of the two investigated properties.

For the beam structure considered in this work, it was found that the inclusion of spatial variability is fundamental for modelling of the visual condition of the structure. In the case of predicting the safety lifetime performance, the inclusion of spatial variability was
found to influence the failure probability of the beam only when the flexure resistance is controlling. When the beam failure probability was governed by its shear resistance, spatial variability had little effect on the predicted safety performance.

This thesis also investigated the effect of pitting (localised) corrosion on the predicted safety performance of the Reinforced Concrete (RC) beam girder considered, as opposed to the effect of general (uniform) corrosion. It was found that pitting corrosion causes much more severe reduction to the beam shear resistance than to the beam flexure resistance. As a result, the beam failure probability was governed by the effect of pitting corrosion on the shear links.

When comparing the times to first repair/maintenance predicted for the beam considered, it was found that the beam can achieve its intended design life in terms of SLS. However, the beam was considered unsafe (i.e. in terms of ULS) after only a few years of its construction as results of the severe effect of pitting corrosion caused to the shear links. The load model used for assessing the safety of the beam girder under consideration was based on the analysis of site-specific Weigh-In-Motion traffic data using Extreme Value theory.

Sensitivity analysis was performed in this thesis to measure the relative influence of the considered random variables on the beam probability of failure. It was found that the parameter that can have the most significant impact on the beam reliability is the Concrete Cover Depth \( (C_d) \). It was therefore recommended that accurate statistical information regarding this parameter should be obtained, particularly its \( \theta \) value, if the safety of the considered structure is to be estimated with a reasonable accuracy.
To the memory of my dearest Father,

to my beloved Mother and

to my dear family

I dedicate this thesis
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I would like to take this opportunity to express my great appreciation and sincere thanks to all of those who supported me and helped to whatever extend, to bring this work to an end.

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Chapter 1:

Introduction
Chapter 1: Introduction

1.1 INTRODUCTION

Bridge structures are a key component of highway infrastructure and their long-term performance in service is clearly of great social, environmental and economical importance. While traffic volumes and loadings are mounting, the number of deteriorating bridges increases annually as does the cost of inspection, maintenance, repair and replacement (Costa and Appleton 1999; Frangopol et al. 2001; Stewart and Rosowsky 1998a). For example, in the United Kingdom, some 40,000 bridges have failed to meet the European Union requirements that by 1999 all European highways must be capable of carrying 44 ton vehicles (Jeppsson and Thelandersson, 2003). In the United States, nearly 27% of the 594,000 bridges, listed in the inventory in 2004, are classified as deficient for either structural or functional reasons (FHWA, 2006). The given classification implies that these bridges cannot carry the increasingly heavier traffic loads and have to be taken out of use or be subjected to weight restrictions. The estimated cost to maintain, repair or replace the increasing number of deficient bridge structures constitutes a substantial portion of the national budget of many of the developed countries. For example, the estimated cost in 2001 to eliminate the increasing number of deficient bridges in the United States is about $80 billion (Estes and Frangopol, 2001). In 2002, Japan allocated approximately 13.5 trillion yen that is about 21.5% of the total construction budget to repair and maintenance works (Sancharoen et al., 2006). The annual expenditure on maintenance and repair on national bridges in England is in the order of €180 million, in France is about €50 million, in Norway €30 million and in the Netherlands is about €100 million (Li, 2004b). It is therefore reasonable to assume that with such huge expenditures, any realised and efficient performance prediction methodology can result in significant savings.

The vast majority of the deteriorating Reinforced Concrete (RC) bridges located in marine environments deteriorate due to chloride-induced corrosion (Mallet, 2007). Chlorides in marine environments penetrate the concrete cover, reach the imbedded reinforcement and initiate corrosion. As a result, the reinforcement cross-sectional area reduces leading to deterioration in the bridge load carrying capacity (i.e. Ultimate Limit State, ULS). Meanwhile, the expansion of the corrosion products generates pressure on the concrete surrounding the reinforcement leading to cracking, spalling and delamination of the concrete cover (i.e. Serviceability Limit State, SLS). The visual damage caused by corrosion activity, i.e. cracking, spalling and delamination, necessitates extensive and costly repair/maintenance and in some cases replacement to be implemented.
To optimise and manage their budget spend while keeping bridges safe and functional, bridge owners/managers need to rely more on rational decision making methods rather than on subjective engineering judgment. It is important that such methods should allow for the current and future condition of bridge structures to be predicted and, therefore, facilitate bridge owners/managers to make optimal repair/maintenance or replacement decisions and hence optimise their budget spend. In traditional Bridge Management Systems (BMS), maintenance and repair of deteriorating bridges is mainly planned based on the information obtained through visual inspections and the experience of local professionals (Vu and Stewart, 2005). One of the most severe limitations that have characterised the traditional BMS approach, as pointed out by Frangopol et al. (2001), is that bridge reliability (safety) is not directly incorporated in BMS and their ability to predict future deterioration of the structure surface and safety conditions is very limited. Becoming increasingly aware of these limitations, researchers/institutions have recognised that future BMS’s will have to depend more on probabilistic and reliability-based methods (e.g. Val and Melchers, 1997; 1998; Frangopol et al., 2001). Reliability-based BMS permits the inclusion of uncertainty of all parameters and models associated with the deterioration process. In addition, reliability-based BMS have the advantage of employing a rational indicator such as the Reliability Index ($\beta$), defined in Chapter 3, which provides clearer and better defined safety performance criteria.

A major shortcoming in the work carried out to date in this regard (e.g. Akgul and Frangopol 2005a; Akgul and Frangopol 2005b; Enright and Frangopol 1998a; Val and Melchers 1997) is the neglect of the uncertainty associated with spatial variability of the deterioration parameters. Spatial variability work carried out to date has been mainly focused on predicting the lifetime visual (surface) condition of the corroding structure (e.g. Li et al. 2004; Vu and Stewart 2005). Another major shortcoming is the separate modelling of the surface condition (serviceability) based lifetime performance and the safety based (ULS) lifetime performance of the deteriorating bridge structures. Due to the common parameters (material, environmental properties, etc) positive correlations are expected between the two performance criteria. Therefore, it would be more appropriate if the surface condition and the load carrying capacity (Safety) performances are both predicted using the same approach and where possible the same physical/empirical deterioration models and parameters. The advantage of this approach allows fair and rational comparison between the times to first repair/maintenance intervention predicted in terms of
both Safety and Surface condition performance criteria. The approach will also help identifying the most important deterioration parameters and, therefore, provide designers of future generation of structures with clues to design durable and long lasting structures.

1.2 OBJECTIVES AND SCOPE

This thesis focuses on three main issues which can be considered as the prime objective of the research work carried out here in and these are:

(i) To develop a probabilistic-based performance prediction tool that can be used to predict the optimal time for repair/maintenance intervention of RC structures exposed to aggressive chloride environments using a bridge or bridge component as an illustrative example.

(ii) The proposed model should take into consideration the inclusion of spatial variability of the deterioration models involved in the estimation of time for repair/maintenance intervention.

(iii) The proposed probabilistic-based model should consider the dual modelling of the Serviceability Limit State (SLS) (i.e. deterioration in the visual condition) and the Ultimate Limit State (ULS) (i.e. the deterioration in the load carrying capacity) of the investigated structure/member.

In the context of achieving these prime objectives, the literature review has revealed some shortcomings, in the research preceded the work of this thesis, which were identified as a secondary objectives for this thesis to address:

1. Data on the Scale of Fluctuation ($\theta$), a parameter which is necessary for modelling of the spatial variability of the deterioration parameters, hardly existed. Therefore it was decided that an experimental investigation need to be performed to obtain values for this parameter.

2. When assessing the performance of a given structure, the corrosion activity was often assumed to have already started and the influence of the corrosion initiation parameters on the lifetime safety performance was hardly assessed. The initiation stage and propagation stage of the deterioration process were rarely considered within the same framework, which undermines studying the influence of the
corrosion initiation parameters on the lifetime safety performance. The developed model should consider including both the initiation and prorogation models.

3. Despite the few and limited studies on the effect of pitting (localised) corrosion, which indicated the severity of this form of corrosion on the load carrying capacity of corroding structures, the effect of variability of pitting corrosion on the safety performance of structures has not been fully investigated. Hence, the developed model should consider the effect of pitting corrosion on the lifetime safety of the structure under consideration.

4. The load models used to assess the load carrying capacity of corroding structures were either oversimplified or estimated from conservative standards or codes of practices and not from actual traffic data. In this thesis, it was decided that the load model should be based on a realistic site-specific load data for the uncertainty associated with the loading to be considered.

The author believes that safety/condition based bridge maintenance is an attractive bridge management tool that permits optimal planning of bridge management and facilitates bridge owners/managers in postponing or reducing costly repair works. In the case of limited funding, such a tool will ensure that the available funding is allocated to deteriorating bridges which are in most need of maintenance/repair from the safety viewpoint. For the same bridge structure, this approach will help pre-identify the structural component that if repaired the overall level of safety of the bridge is ensured to be above the minimum requirement. The methodology presented in this thesis can be generalised and used as a tool by structural engineers and asset owners/managers in assessing other types of structures and help them make rational decisions with regard to the optimal time to the maintenance and rehabilitation of their structures.

1.3 THESIS LAYOUT

The thesis is organised in eight chapters and five appendices. Chapter 1 provides an introduction to the problem under investigation and highlights the general objectives of the current work and indicates the organization of the thesis.

Chapter 2 presents the material deterioration models that can be used to describe the performance of RC structures exposed to marine environments and affected by chloride-induced corrosion. These models are presented in such a way so they can be used to
quantify the different stages of corrosion-induced deterioration of RC bridges and hence to predict the end of the structure service life. The variability of the key deterioration parameters is discussed. The selected models will then be used in the subsequent chapters to describe the structure's lifetime performance where the impact of the corrosion-induced damage on the structure performance can be quantified. In this chapter criteria used by bridge owners/managers/engineers for deciding on the time of maintenance and repair were also discussed. This chapter also investigates the experiences and practices revealed by the literature to propose maximum allowable deterioration levels which can be used for predicting the optimal time to maintenance and repair intervention.

Chapter 3 describes a different type of uncertainty that can affect the predicted performance of a structural system. Methods for calculating the reliability index are described. Among these methods is the Monte Carlo (MC) simulation technique which is adopted by this thesis for the calculation of the Probability of Failure ($P_f$) hence evaluating the temporal safety performance. In addition, the minimum allowable safety level specified by a number of international standards is discussed. This chapter focus on presenting the methods that can be used to incorporate the uncertainty associated with spatial variability into the reliability analysis. The Random Field (RF) theory which is the field of statistics that deals with the spatial variability modelling is discussed in detail. In this chapter, key parameters necessary for RF-based modelling are identified. The parameter Scale of fluctuation ($\theta$) is given special attention being very important for describing the spatial variability of a deteriorating property. The chapter also presents a review of previous works that have dealt with spatial variability modelling in the subject of RC corrosion. The review highlights a number of shortcomings and limitations. Among the most significant is the lack of reliable data regarding the parameter $\theta$ for key deteriorating variables identified in Chapter 2, the Surface Chloride Content ($C_s$) and the Diffusion Coefficient ($D_{app}$).

Chapter 4 presents a background to the structure studied in this work (Ferrycarrig Bridge) to overcome the shortcomings indicated in Chapter 3 with respect to the lack of reliable experimental data for RF modelling. The RC bridge selected is located in a marine environment on the Wexford-Dublin road, Ireland. The experimental work carried out mainly intends to obtain statistical information on the two parameters $C_s$ and $D_{app}$ which have been identified in Chapter 2 to be key deteriorating variables and, therefore, of a significant importance to service life modelling. The testing procedure and methods for determining chloride contents in the concrete sample are discussed in this chapter. The
experimental work also intends to provide measurements that will enable obtaining information on the parameter $\theta$ for the two aforementioned deterioration variables.

Chapter 5 applies methods available in the literature to analyse the data measurements collected in chapter 4 to determine values of $\theta$ for $C_s$ and $D_{app}$. Two methods are presented in this chapter, the Maximum Likelihood Method (MLM) and the autocorrelation Curve Fitting Method. The reliability of both methods is discussed. The Curve Fitting method requires the available data measurements to be increased; this led to introducing a statistical interpolation method called Kriging. The kriging method is a statistical interpolation method that can be used to interpolate the data at locations where the investigated property was not measured, i.e. ‘missing’ data points, for the purpose of increasing the number of observations to obtain a reliable prediction of the parameter $\theta$. In this chapter, the $\theta$ values obtained in this thesis is compared to the corresponding values obtained or assumed by other researchers. This chapter also proposes a procedure by which the number of measurements and the optimal relative location can be used to achieve a reliable estimate for the parameter $\theta$ for future field investigations.

Chapter 6 demonstrates how the RF-based probabilistic model used in this thesis is developed using a RC beam girder as an illustrative example. Formulation of the limits state functions which are essential for calculating the probability of failure and hence the reliability of the beam girder is discussed in detail. The model consists of two parallel stages; the first stage intends to predict the surface condition of the beam as a result of the progressing chloride attack. At this point the model is benchmarked against existing results from the literature. In the second stage, the model is further analysed to predict the lifetime safety performance of the RC beam girder under investigation. This chapter uses real traffic data to extract information regarding the maximum load effect expected to be imposed on the beam girder under investigation. The chapter explains how Extreme Value (EV) theory can be used to estimate the maximum load effect from traffic data collected through the use of Wight-in Motion system.

In Chapter 7, the statistical information obtained in Chapter 4 and 5 are used as an input parameters to the RF-based model developed in chapter 6. Results describing the performance of the investigated beam girder against corrosion are presented in this chapter in terms of both the Condition and Safety profiles. The influence of spatial variability on both profiles is discussed. The relative influence of the two types of corrosion (General and
Pitting) is also investigated. Comparison between the optimal times to maintenance based on the condition and safety profiles are made. Finally, a sensitivity analysis is performed in this chapter to measure the relative importance of each of the random variables from the safety view point.

Chapter 8 presents the summary of the work carried out in this thesis and indicates final conclusions of the current work and recommendations for future research.
1.1 INTRODUCTION

Bridge structures are a key component of highway infrastructure and their long-term performance in service is clearly of great social, environmental and economical importance. While traffic volumes and loadings are mounting, the number of deteriorating bridges increases annually as does the cost of inspection, maintenance, repair and replacement (Costa and Appleton 1999; Frangopol et al. 2001; Stewart and Rosowsky 1998a). For example, in the United Kingdom, some 40,000 bridges have failed to meet the European Union requirements that by 1999 all European highways must be capable of carrying 44 ton vehicles (Jeppsson and Thelandersson, 2003). In the United States, nearly 27% of the 594,000 bridges, listed in the inventory in 2004, are classified as deficient for either structural or functional reasons (FHWA, 2006). The given classification implies that these bridges cannot carry the increasingly heavier traffic loads and have to be taken out of use or be subjected to weight restrictions. The estimated cost to maintain, repair or replace the increasing number of deficient bridge structures constitutes a substantial portion of the national budget of many of the developed countries. For example, the estimated cost in 2001 to eliminate the increasing number of deficient bridges in the United States is about $80 billion (Estes and Frangopol, 2001). In 2002, Japan allocated approximately 13.5 trillion yen that is about 21.5% of the total construction budget to repair and maintenance works (Sancharoen et al., 2006). The annual expenditure on maintenance and repair on national bridges in England is in the order of €180 million, in France is about €50 million, in Norway €30 million and in the Netherlands is about €100 million (Li, 2004b). It is therefore reasonable to assume that with such huge expenditures, any realised and efficient performance prediction methodology can result in significant savings.

The vast majority of the deteriorating Reinforced Concrete (RC) bridges located in marine environments deteriorate due to chloride-induced corrosion (Mallet, 2007). Chlorides in marine environments penetrate the concrete cover, reach the imbedded reinforcement and initiate corrosion. As a result, the reinforcement cross-sectional area reduces leading to deterioration in the bridge load carrying capacity (i.e. Ultimate Limit State, ULS). Meanwhile, the expansion of the corrosion products generates pressure on the concrete surrounding the reinforcement leading to cracking, spalling and delamination of the concrete cover (i.e. Serviceability Limit State, SLS). The visual damage caused by corrosion activity, i.e. cracking, spalling and delamination, necessitates extensive and costly repair/maintenance and in some cases replacement to be implemented.
To optimise and manage their budget spend while keeping bridges safe and functional, bridge owners/managers need to rely more on rational decision making methods rather than on subjective engineering judgment. It is important that such methods should allow for the current and future condition of bridge structures to be predicted and, therefore, facilitate bridge owners/managers to make optimal repair/maintenance or replacement decisions and hence optimise their budget spend. In traditional Bridge Management Systems (BMS), maintenance and repair of deteriorating bridges is mainly planned based on the information obtained through visual inspections and the experience of local professionals (Vu and Stewart, 2005). One of the most severe limitations that have characterised the traditional BMS approach, as pointed out by Frangopol et al. (2001), is that bridge reliability (safety) is not directly incorporated in BMS and their ability to predict future deterioration of the structure surface and safety conditions is very limited. Becoming increasingly aware of these limitations, researchers/institutions have recognised that future BMS’s will have to depend more on probabilistic and reliability-based methods (e.g. Val and Melchers, 1997; 1998; Frangopol et al., 2001). Reliability-based BMS permits the inclusion of uncertainty of all parameters and models associated with the deterioration process. In addition, reliability-based BMS have the advantage of employing a rational indicator such as the Reliability Index ($\beta$), defined in Chapter 3, which provides clearer and better defined safety performance criteria.

A major shortcoming in the work carried out to date in this regard (e.g. Akgul and Frangopol 2005a; Akgul and Frangopol 2005b; Enright and Frangopol 1998a; Val and Melchers 1997) is the neglect of the uncertainty associated with spatial variability of the deterioration parameters. Spatial variability work carried out to date has been mainly focused on predicting the lifetime visual (surface) condition of the corroding structure (e.g. Li et al. 2004; Vu and Stewart 2005). Another major shortcoming is the separate modelling of the surface condition (serviceability) based lifetime performance and the safety based (ULS) lifetime performance of the deteriorating bridge structures. Due to the common parameters (material, environmental properties, etc) positive correlations are expected between the two performance criteria. Therefore, it would be more appropriate if the surface condition and the load carrying capacity (Safety) performances are both predicted using the same approach and where possible the same physical/empirical deterioration models and parameters. The advantage of this approach allows fair and rational comparison between the times to first repair/maintenance intervention predicted in terms of
both Safety and Surface condition performance criteria. The approach will also help identifying the most important deterioration parameters and, therefore, provide designers of future generation of structures with clues to design durable and long lasting structures.

1.2 OBJECTIVES AND SCOPE

This thesis focuses on three main issues which can be considered as the prime objective of the research work carried out here in and these are:

(i) To develop a probabilistic-based performance prediction tool that can be used to predict the optimal time for repair/maintenance intervention of RC structures exposed to aggressive chloride environments using a bridge or bridge component as an illustrative example.

(ii) The proposed model should take into consideration the inclusion of spatial variability of the deterioration models involved in the estimation of time for repair/maintenance intervention.

(iii) The proposed probabilistic-based model should consider the dual modelling of the Serviceability Limit State (SLS) (i.e. deterioration in the visual condition) and the Ultimate Limit State (ULS) (i.e. the deterioration in the load carrying capacity) of the investigated structure/member.

In the context of achieving these prime objectives, the literature review has revealed some shortcomings, in the research preceded the work of this thesis, which were identified as a secondary objectives for this thesis to address:

1. Data on the Scale of Fluctuation (\( \theta \)), a parameter which is necessary for modelling of the spatial variability of the deterioration parameters, hardly existed. Therefore it was decided that an experimental investigation need to be performed to obtain values for this parameter.

2. When assessing the performance of a given structure, the corrosion activity was often assumed to have already started and the influence of the corrosion initiation parameters on the lifetime safety performance was hardly assessed. The initiation stage and propagation stage of the deterioration process were rarely considered within the same framework, which undermines studying the influence of the
corrosion initiation parameters on the lifetime safety performance. The developed model should consider including both the initiation and prorogation models.

3. Despite the few and limited studies on the effect of pitting (localised) corrosion, which indicated the severity of this form of corrosion on the load carrying capacity of corroding structures, the effect of variability of pitting corrosion on the safety performance of structures has not been fully investigated. Hence, the developed model should consider the effect of pitting corrosion on the lifetime safety of the structure under consideration.

4. The load models used to assess the load carrying capacity of corroding structures were either oversimplified or estimated from conservative standards or codes of practises and not from actual traffic data. In this thesis, it was decided that the load model should be based on a realistic site-specific load data for the uncertainty associated with the loading to be considered.

The author believes that safety/condition based bridge maintenance is an attractive bridge management tool that permits optimal planning of bridge management and facilitates bridge owners/managers in postponing or reducing costly repair works. In the case of limited funding, such a tool will ensure that the available funding is allocated to deteriorating bridges which are in most need of maintenance/repair from the safety viewpoint. For the same bridge structure, this approach will help pre-identify the structural component that if repaired the overall level of safety of the bridge is ensured to be above the minimum requirement. The methodology presented in this thesis can be generalised and used as a tool by structural engineers and asset owners/managers in assessing other types of structures and help them make rational decisions with regard to the optimal time to the maintenance and rehabilitation of their structures.

1.3 THESIS LAYOUT

The thesis is organised in eight chapters and five appendices. Chapter 1 provides an introduction to the problem under investigation and highlights the general objectives of the current work and indicates the organization of the thesis.

Chapter 2 presents the material deterioration models that can be used to describe the performance of RC structures exposed to marine environments and affected by chloride-induced corrosion. These models are presented in such a way so they can be used to
Chapter 1: Introduction

quantify the different stages of corrosion-induced deterioration of RC bridges and hence to predict the end of the structure service life. The variability of the key deterioration parameters is discussed. The selected models will then be used in the subsequent chapters to describe the structure's lifetime performance where the impact of the corrosion-induced damage on the structure performance can be quantified. In this chapter criteria used by bridge owners/managers/engineers for deciding on the time of maintenance and repair were also discussed. This chapter also investigates the experiences and practices revealed by the literature to propose maximum allowable deterioration levels which can be used for predicting the optimal time to maintenance and repair intervention.

Chapter 3 describes a different type of uncertainty that can affect the predicted performance of a structural system. Methods for calculating the reliability index are described. Among these methods is the Monte Carlo (MC) simulation technique which is adopted by this thesis for the calculation of the Probability of Failure ($P_f$) hence evaluating the temporal safety performance. In addition, the minimum allowable safety level specified by a number of international standards is discussed. This chapter focus on presenting the methods that can be used to incorporate the uncertainty associated with spatial variability into the reliability analysis. The Random Field (RF) theory which is the field of statistics that deals with the spatial variability modelling is discussed in detail. In this chapter, key parameters necessary for RF-based modelling are identified. The parameter Scale of fluctuation ($\theta$) is given special attention being very important for describing the spatial variability of a deteriorating property. The chapter also presents a review of previous works that have dealt with spatial variability modelling in the subject of RC corrosion. The review highlights a number of shortcomings and limitations. Among the most significant is the lack of reliable data regarding the parameter $\theta$ for key deteriorating variables identified in Chapter 2, the Surface Chloride Content ($C_s$) and the Diffusion Coefficient ($D_{app}$).

Chapter 4 presents a background to the structure studied in this work (Ferrycarrig Bridge) to overcome the shortcomings indicated in Chapter 3 with respect to the lack of reliable experimental data for RF modelling. The RC bridge selected is located in a marine environment on the Wexford-Dublin road, Ireland. The experimental work carried out mainly intends to obtain statistical information on the two parameters $C_s$ and $D_{app}$ which have been identified in Chapter 2 to be key deteriorating variables and, therefore, of a significant importance to service life modelling. The testing procedure and methods for determining chloride contents in the concrete sample are discussed in this chapter. The
Chapter 1: Introduction

experimental work also intends to provide measurements that will enable obtaining information on the parameter $\theta$ for the two aforementioned deterioration variables.

Chapter 5 applies methods available in the literature to analyse the data measurements collected in chapter 4 to determine values of $\theta$ for $C_s$ and $D_{app}$. Two methods are presented in this chapter, the Maximum Likelihood Method (MLM) and the autocorrelation Curve Fitting Method. The reliability of both methods is discussed. The Curve Fitting method requires the available data measurements to be increased; this led to introducing a statistical interpolation method called Kriging. The kriging method is a statistical interpolation method that can be used to interpolate the data at locations where the investigated property was not measured, i.e. 'missing' data points, for the purpose of increasing the number of observations to obtain a reliable prediction of the parameter $\theta$. In this chapter, the $\theta$ values obtained in this thesis is compared to the corresponding values obtained or assumed by other researchers. This chapter also proposes a procedure by which the number of measurements and the optimal relative location can be used to achieve a reliable estimate for the parameter $\theta$ for future field investigations.

Chapter 6 demonstrates how the RF-based probabilistic model used in this thesis is developed using a RC beam girder as an illustrative example. Formulation of the limits state functions which are essential for calculating the probability of failure and hence the reliability of the beam girder is discussed in detail. The model consists of two parallel stages; the first stage intends to predict the surface condition of the beam as a result of the progressing chloride attack. At this point the model is benchmarked against existing results from the literature. In the second stage, the model is further analysed to predict the lifetime safety performance of the RC beam girder under investigation. This chapter uses real traffic data to extract information regarding the maximum load effect expected to be imposed on the beam girder under investigation. The chapter explains how Extreme Value (EV) theory can be used to estimate the maximum load effect from traffic data collected through the use of Wight-in Motion system.

In Chapter 7, the statistical information obtained in Chapter 4 and 5 are used as an input parameters to the RF-based model developed in chapter 6. Results describing the performance of the investigated beam girder against corrosion are presented in this chapter in terms of both the Condition and Safety profiles. The influence of spatial variability on both profiles is discussed. The relative influence of the two types of corrosion (General and
Pitting) is also investigated. Comparison between the optimal times to maintenance based on the condition and safety profiles are made. Finally, a sensitivity analysis is performed in this chapter to measure the relative importance of each of the random variables from the safety viewpoint.

Chapter 8 presents the summary of the work carried out in this thesis and indicates final conclusions of the current work and recommendations for future research.
Chapter 2:

Corrosion-Induced Materials

Deterioration Models
2.1 INTRODUCTION

The length of service life of Reinforced Concrete (RC) structures in many standards is defined as "the period of time in which the structure maintains its design requirements of: safety, functionality and aesthetics without unexpected costs of maintenance" (Andrade et al., 2006). The explicit definition of length of service life may have technical as well as legal implications. Therefore, there is a need to develop a calculation method to aid in the making of reliable predictions for the service life of RC structures located in aggressive environments in order to achieve cost effectiveness in whole life asset management.

For RC bridge structures, different deterioration mechanisms have been recognised, e.g. carbonation-induced corrosion, freeze/thaw, alkali-silica reaction, sulphate attack, etc. The majority of RC bridge structures in marine environments, however, are mainly affected by chloride-induced corrosion (Mallet, 2007). Furthermore, for corrosion affected RC bridge structures in marine environments, repairing costs due to concrete cracking and spalling exceed those from other forms of deterioration by a substantial margin (Zheng et al., 2005). It is, therefore, RC bridge structures in marine environments deteriorating due to chloride-induced corrosion which will be the focus of this thesis. However, the methodology to be presented by the research carried out can indeed be applied to other forms of deterioration mechanisms or to material other than RC such as steel structures wood and masonry etc.

The primary objective of this thesis is to develop a rational method to predict the optimum time to maintenance/repair of deteriorating RC bridges located in marine environments. Central to achieving this objective is knowledge of the existing material deterioration models which can be employed to describe the structure performance over time. The main objective of this chapter, therefore, is to review and identify the most reliable material deterioration models that have been reported in the literature to quantify the different corrosion-induced deterioration stages. This also includes assessing the variability of the main parameters involved in the description of the selected models and identifies shortcomings and possible research areas for further improvements. The selected models will be subsequently used to describe the lifetime performance of the corroding structure where the relative impact of each model/parameter on the structure performance can be quantified. In the upcoming sections, the material deterioration models, variability of the deterioration parameters and the encountered research shortcomings will be presented.
2.2 FORMULATION OF SERVICE LIFE MODELS

The material deterioration models often described in the literature in the context of structure service life modelling where each stage of the deterioration process is quantified in terms of time and the sum of these times makes the total service life. For RC structures, it is postulated that during the hydration of cement a highly alkaline pore solution (pH between 13 and 13.8), principally of sodium and potassium hydroxides, is gained (Bertolini, 2004). In this alkaline environment a protective oxide layer, a few nanometres thick, is formed on the reinforcing steel bar embedded in concrete. In spite of its attested protective property against mechanical damage of the steel surface, the formed layer can be destroyed by carbonation of concrete or by the presence of chloride ions leading to the depassivation of the reinforcing steel. This stage of the service life of RC structures affected by corrosion-induced deterioration is referred to as the Initiation stage Figure 2.1(a). The second distinguished service life stage begins when the steel reinforcement is depassivated and the corrosion process begins its activity and finishes when an undesired limit state is reached prompting a rehabilitation action to be taken. This stage is referred to as the Propagation Stage.

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Figure 2.1. (a) Tuutti's simple model of corrosion-affected structure service life, (b) Detailed service life model.

2.2.1 Serviceability-based service life formulation

Although other SLS performance criteria, i.e. Vibration, Deflection etc, exist, they are not directly relevant when describing the service life of the chloride affected structure.
Therefore, the serviceability-based service life formulation indicated here describes the service life of the corroding structure in terms of its surface condition and therefore is directly related to quantifying its performance in terms of the Serviceability Limit State (SLS) criteria. Tuutti (1982) presented a simplified ‘two-stage’ service life model for corrosion-affected RC structures, Figure 2.1(a). Ever since then, researchers have introduced several modifications to Tuutti’s ‘two-stage’ model by subdividing the propagation stage into several sub-stages, Figure 2.1(b). The modification attempts were mainly motivated by the desire to provide practical engineering criteria which can be employed to quantitatively define the end of the propagation stage, hence the end of structure service life.

Because it is the corrosion activity that directly leads to the structural deterioration of its serviceability and load carrying capacity, much of the effort of improving Tutti’s model was directed towards the propagation stage (e.g. Bazant, 1979; Molina et al., 1993; Liu and Weyers, 1998; Duracrete, 2000; Vu et al., 2005; Bhargava et al., 2006b; El Maaddawy and Soudki, 2007). Two distinct propagation sub-stages were recognised; (i) the generation of corrosion-induced cracks also referred to as first or initial cracking, (ii) the propagation of the crack to a maximum limiting width. In some cases, a third sub-stage maybe identified when a change of the slope of sub-stage II is produced; possibly due to an easier diffusion of oxides through the generated crack (Alonso et al., 1998) leading to spalling and delamination of the concrete surrounding the corroding reinforcements.

Sub-stage I, Figure 2.1(b), starts when depassivation of the steel, due to the presence of chlorides or carbonation at the reinforcement level, is confirmed and ends when a hair-size crack (typically assumed in the order of 0.05 mm) appears on the surface of the RC structure. The propagation Sub-stage II starts from when Sub-stage I ends and ends when the crack width reaches a maximum limiting crack size often used as an indicator for rehabilitation and repair intervention which maybe different from standard to standard and from agency to agency (Vu and Stewart, 2005). In general, as will be indicated in Section 2.5.1, the time period for Sub-stage I depends on several factors such as the cover depth, bar diameter, tensile strength of the concrete, corrosion current density among others and it is expected to be comparatively short, i.e. 2-5 years (Liu and Weyers, 1998). According to Alonso et al. (1998) Sub-stage I is controlled by the cover/diameter ratio and the porosity of the concrete, whereas sub-stage II and III are controlled mainly by the corrosion current density and the concrete porosity.
What constitutes the end of service life of an RC structure affected by corrosion has been the subject of discussion for several decades. For example; several authors have defined the end of service life of an RC structure as the time when depassivation of the reinforcement due to the presence of a threshold value of chlorides takes place (Thoft-Christensen, 1997; Bertolini, 2004). This consideration has been mainly motivated by the difficulty in modelling the corrosion activity and its deterioration mechanism (Costa and Appleton, 1999). However, some good results from numerical models supported by experimental works showed that it is possible to predict the time it takes from the steel depassivation to the appearance of corrosion-induced visual crack on the surface of the concrete (Sub-stage 1). The end of Sub-stage 1 was used by some researchers (e.g. Liu and Weyers, 1998) to define the end of the service life of the structure and hence the time to first repair/maintenance. More recently, the time for the corrosion-induced hair size crack developed on the concrete surface to propagate and reach a maximum limiting crack size (in the range of 0.3 to 1.0 mm) was modelled by fitting experimental results to theoretical models (e.g. Duracrete, 2000; Vu et al., 2005). This allowed for further extension of the service life of the structure that can be numerically quantified.

2.2.2 Strength-based service life formulation

This formulation concerns the structure service life in terms of its load carrying capacity and hence it is directly related to the Ultimate Limit state (ULS) performance criteria. During the formation of the initial crack on the concrete surface due to stresses generated by the corrosion product expansion, and while the crack is increasing in size, the reinforcement cross-sectional area is reduced due to the ongoing corrosion, as will be explained further in Section 2.5.3. This process has a direct implication on the load carrying capacity of the RC section. Depending on the loading imposed, the remaining cross-sectional area of the steel reinforcement will reach a point where the safety requirements of the RC structure will no longer be acceptable. The corrosion-induced reduction of the reinforcement cross-sectional area has been used to model the time-dependent load carrying capacity of corroding RC structures mainly as a function of the corrosion current density as will be explained in Section 2.5.3.

It has been recognised that deterioration in the load carrying capacity of an RC structure affected by corrosion does not happen due to the reduction in the reinforcement cross-
Chapter 2: Materials deterioration models

sectional area alone. The deterioration of the load carrying capacity and hence the structure safety can be further affected by (Palsson and Mirza, 2002; Cairns et al., 2005):

(i) loss of bond between the steel reinforcement and concrete as a result of longitudinal cracking caused by corrosion.
(ii) loss of the concrete cross-section as a result of corrosion-induced spalling and/or delamination of the concrete cover.
(iii) the loss of the ductility of the reinforcement due to the uneven distribution of the cross-sectional area along the length of the reinforcing bar (as a result of localised corrosion) and the stress concentrations associated with the abrupt change in geometry of the reinforcing bar.

Consideration of these three factors in this thesis will be discussed in the remaining part of this section.

According to the available experimental data, after the initiation of corrosion the bond strength between reinforcement and concrete initially increases slightly due to the improvement in the friction between the steel/concrete interface caused by the corrosion product (Auyeung et al., 2000). As soon as cracking of the concrete occurs, the bond strength decreases rapidly as corrosion propagates. A typical relationship between the bond strength and the level of corrosion (the percentage loss of the reinforcement weight) obtained by number of researchers from pullout tests are shown in Figure 2.2. The level of corrosion at which the bond strength start to decrease is still a matter of disagreement with values from 1-4% have been reported (Chung et al., 2004; Bhargava et al., 2007).

In a reliability-based study carried out by Val et al., (1998) on a deteriorating simply supported RC slab bridge affected by corrosion, they found that the complete loss of bond between the reinforcement and concrete due to corrosion-induced cracking had a relatively insignificant effect on the bridge reliability in flexure. The term ‘Reliability’ is an indicator of the structure safety and defined as the complement of the probability of failure and will be discussed in detail in Chapter 3. However, the loss of bond between steel and concrete due to corrosion is still lacking reliable models that can be used for service life prediction. Furthermore, loss of bond and loss of the concrete cross-sectional area are of a complicated nature and therefore require further computational details that will add to the complexity of the reliability problem to be formulated in this thesis. To investigate the influence of loss
of bond on the reliability of the corroding RC structures, the resistance model of the structure needs to be redeveloped from first principles where the mechanism of loss of bond can be incorporated at the derivation stage of the code formulas. This is another research area and worth further investigation but was considered beyond the scope of this thesis.

Figure 2.2 Bond strength versus percentage of corrosion (Val et al., 1998)

Regarding the effect of the corrosion on the mechanical property of the steel reinforcement, the mode of failure of corroding reinforcing bars is not clear. While many studies reported some reduction of ductility of the steel reinforcement due to pitting corrosion (localized form of corrosion, Section 2.5.3.2), others found complete loss of ductility and fracture in the corroded bars after only 20% loss of section (Stewart and Al-Harth, 2008). Palsson and Mirza (2002) have also reported that when pitting corrosion reaches 50% of the original bar section, bar behaviour becomes very brittle. For the current study, the mode of failure of the corroded reinforcing bars is assumed to be yielding. Hence, reinforcing bar capacity is directly proportional to yield strength which is equal to the product of the yield stress and cross-sectional area in which only the reinforcement cross-sectional area is affected by corrosion.

In light of the above discussion, the impact of loss of bond, loss of the concrete cross-sectional area and the impact of the change in the failure mode of the corroding reinforcement bars due to the ongoing corrosion activity will not be considered in this thesis.
Due to the absence of reliable predictive models to describe sub-stage III, only models which describes the initiation stage, Sub-stages I and II of the propagation stage will be discussed further in the remaining of this chapter.

It can be seen from the previous discussion that due to the common deterioration parameters, there exists a positive correlation between the deterioration of structures in terms of SLS, i.e. cracking, and in terms of ULS, i.e. loads carrying capacity. For the service life of a corroding structure to be rationally estimated, the SLS and ULS based performances need to be estimated simultaneously so a valid comparison can be made between the times to repair/maintenance intervention predicted in terms of both criteria. The approach of dually modelling the service life of RC structures with respect to SLS and ULS has been a major shortcoming in the research carried out to date and therefore this task has been highlighted as one of the objectives of this thesis. The dual modelling concept introduced here, its advantages and significance will be elaborated further in Chapter 6.

It has already been established that this thesis focuses on RC bridge structures deteriorating due chloride-induce corrosion. The initiation stage of this deterioration process is mainly governed by the availability of chloride ions on the concrete surface and the mechanisms by which chloride ions transport through the concrete cover and reach the reinforcement in enough quantities to initiate corrosion (Bertolini, 2004). It is, therefore, reasonable to assume that the prediction of service life of RC structures exposed to marine environments and hence the prediction of the time to maintenance/repair will be influenced by the source of chlorides and chlorides transportation mechanism. The following section will give brief insight into these two factors.

2.3 CHLORIDE PENETRATION

2.3.1 Sources of chlorides

The presence of chloride ions in concrete structures can be attributed to many different sources. In the past for example, in some cases, chlorides were added to the fresh concrete in order to accelerate the hydration process especially in cold countries (Neville, 1995; Ahmad, 2003). In some other cases they are originated from the use of sea dredged sands,
contaminated aggregates or contaminated mixing water, especially when saline water was used in the mix in coastal or marine construction (Ahmad, 2003). In countries with cold climates, a major source of chlorides may be the de-icing salts frequently applied over roads and highway bridges during the winter seasons (Papadakis et al., 1996). However, a large number of RC structures in coastal areas, which are the focus of this thesis, receive chlorides directly from sea water in direct contact with air or mist rich in chloride salts. Significant chlorides were reported to have been carried by winds and deposited on structures as far as 3 km away from the sea costs (McGee, 1999). From the surface of the concrete structure, chlorides dissolve in the water of the pore system of the concrete and penetrate inwards by any of the mechanisms to be discussed in the following section.

2.3.2 Chlorides transportation mechanisms

Chloride ions can transport from the surface of the concrete towards the reinforcements in several ways, Bertolini (2004) has mentioned three major ways by which chloride ions can penetrate the concrete cover and reach the reinforcements:

1. Permeation; due to pressure gradients
2. Capillary absorption; due to capillary action inside capillaries
3. Diffusion; due to concentration gradients

If there is applied hydraulic head on one face of the concrete and chlorides (dissolved in water) are present, permeation of chlorides into the concrete cover may take place (Stanish et al., 2000). However, the situation where a hydraulic head is maintained on highway structures is rare.

In the second mechanism, due to the drying and wetting cycles which concrete structures often undergo, the chlorides ions will be drawn into the pore structure of the concrete cover through capillary suction. It was postulated, e.g. by Stanish et al. (2000), that the depth of drying is very small and therefore this mechanism will not by itself bring the chlorides to the level of reinforcement unless the concrete quality is extremely poor or the reinforcement placed at very shallow depths. However, it does serve to quickly bring chlorides to a closer distance from the reinforcement where then can penetrate further by diffusion.
Of the three transport mechanisms described above the most familiar mechanism for chloride ions penetrating through concrete as considered by most researchers is diffusion (Bentur et al., 1997; Broomfield, 1997; Stanish et al., 2000; Bertolini, 2004). As the pores of the concrete remain saturated the chloride ion movement is controlled by diffusion. The rate of chloride penetration due to diffusion is quantified by the use of the Diffusion Coefficient \( (D) \) which describes the quantity of chlorides passing through a unit area per unit time (i.e. \( \text{mm}^2/\text{year}, \text{cm}^2/\text{sec} \) or \( \text{m}^2/\text{sec} \)). Therefore, it will be assumed in this thesis that chloride’s transport through the concrete cover happens solely due to diffusion which implies that the initiation stage can be quantified in terms of the parameter \( D \).

The experience on RC structures subjected to chloride attack has shown that chloride penetration produces a profile in the concrete which can be approximated by Fick's 2
\(^{nd} \) law of diffusion is based on a theoretical relationship that describes the unidirectional non-steady-state flow in the \( x \) direction as follows:

\[
\frac{\partial C}{\partial t} = - \frac{\partial}{\partial x} \left( D \frac{\partial C}{\partial x} \right)
\]

Equation 2.1

Equation is solved assuming that the concentration of the diffusing ions \( C \), measured on the surface of the concrete, is constant throughout the exposure time \( t \), and is equal to \( C_s \), and that \( D \) does not vary in time, concrete is homogeneous, so that \( D \) does not vary through the thickness of the concrete, and the concrete has an initial chloride content that is constant with time and is equal to \( C_i \).

Solving the above equation, using these assumptions as boundary conditions led to the following common expression:

\[
C(x,t) = C_i + (C_s - C_i) \left[ 1 - \text{erf} \left( \frac{x}{2 \sqrt{D_{app} t}} \right) \right]
\]

Equation 2.2

where: \( C(x, t) \) is the total chloride content (Cl\% by mass of cement or concrete) at time \( t \) (years) of exposure, and at depth \( x \) (mm) from the surface of the concrete; \( D_{app} \) is the apparent diffusion coefficient \( \text{(mm}^2/\text{year)} \); \( C_s \) is the surface chloride content (Cl\% by mass
of cement or concrete); and $C_i$ is the initial chloride content (Cl% by mass of cement or concrete), often neglected. $D_{app}$ and $C_s$ are calculated by curve-fitting the experimental data obtained from structure or laboratory tests to Equation.

Figure 2.3 shows a typical chloride profile fitted to Fick's 2nd law of diffusion (Equation). In this figure it can be seen that chloride content decreases with increasing depths from the surface of the concrete except for the outer region. This is because the concrete skin has a different matrix composition compared to the internal concrete due to phenomena such as contact with the moulds, segregation of aggregates or dielectric reaction between the concrete surface and chloride environment (Andrade et al., 1997). Moreover, chlorides at the outer layer of concrete cover can often be washed out by the rain or by the cooling water used during the sample extraction operation. More discussion regarding such figures and the sensitivity of the fitted parameters will be presented later in Section 4.4.1. Due to the expected variation in the environmental condition, concrete properties, etc, between structures and within the same structure, it is expected that the shape of such profile will also vary and hence the fitted parameters will vary accordingly.

![Figure 2.3 Typical Chloride content profile fitted to Fick's 2nd law of diffusion](image)

However, as indicated by Bertolini (2004), the assumptions regarding Fick's 2nd law are rarely met in practice and penetration of chloride ions may not occur due to diffusion alone and other transportation mechanisms may contribute to the chloride penetration. For instance, when the concrete surface is dry, initial mechanism appears to be suction; salt
water is rapidly absorbed by dry concrete. Another example in which chloride penetration may be attributed to a mechanism other than diffusion, is when thermal or load-induced cracks are present on the concrete surface and their size is significantly large (i.e. greater than 0.1 mm), in this case the transportation mechanism may become more dominated by permeation (Li, 2002). These observations have caused researchers to introduce several modifications to Fick’s 2nd law as demonstrated by Liang et al. (2002). However, the lack of the statistical data for the input parameters of these models seems to justify the use of the simplified diffusion model given by Equation for service life predictions, provided that $C_s$ and $D_{app}$ are estimated from large number of measurements (Duprat, 2007). In statistics, a sample is considered large if the number of observations is greater than 30 (Ang and Tang, 1975). The increasing sample size may give good information regarding variation across the sample; however, for variation of properties in space (i.e. across structure), samples may need to be taken at systematic locations to obtain information regarding spatial variation if such variation is to be considered. The importance of considering this type of variability (spatial variability) on the predicted service life of a structure will be explained in detail in Chapter 3.

2.4 MODELLING TIME TO CORROSION INITIATION

The first step towards a practical quantification of the service life (as defined in Section 2.1) of an RC structure exposed to a chloride rich environment is to predict the time it takes for the chloride ions to penetrate the concrete cover and reach the reinforcement in enough quantity to depassivate the reinforcement, and hence initiate corrosion. Traditionally, the time for chloride ions to penetrate through the concrete cover from the surface and reach a critical (threshold) value $C_{cr}$ at the level of reinforcement, has been modelled using an expression derived from Fick’s 2nd law of diffusion obtained by rearranging Equation and introducing $x=C_d$ where $C_d$ is the reinforcement cover depth:

$$T_i = \frac{C_d^2}{4D_{app}} \left[ \text{erf}^{-1}\left(\frac{C_{cr} - C_s}{C_t - C_s}\right)\right]^2$$

Equation 2.3

where $T_i$ is time to corrosion initiation (years); $D_{app}$ is in $(\text{mm}^2/\text{year})$; $C_s$, $C_t$ and $C_{cr}$ are in $(\text{Cl}\% \text{ per mass of cement or concrete})$ and $C_d$ is in (mm).
Chapter 2: Materials deterioration models

Thus the time to corrosion initiation for reinforcement placed at any cover depth $C_d$ depends on:

(i) how fast chloride ions penetrate the concrete cover represented by $D_{app}$.
(ii) the chloride content that is available at the surface of the concrete structure, $C_s$.
(iii) the critical chloride concentration needed for depassivation of the steel reinforcement, $C_{cr}$.
(iv) the initial chloride content that already existed in the concrete mix, $C_i$.

It can be seen now that the uncertainties on the input parameters of Equation ($D_{app}, C_s, C_{cr}, C_i$ and $C_d$) will influence the estimation of the time to corrosion initiation and hence the predicted time to maintenance and repair of the RC structure. While $C_i$ is often neglected and $C_d$ is not affected by the environmental conditions or type of materials, only the first three parameters will be discussed in more details in the upcoming sections.

2.4.1 The diffusion coefficient ($D_{app}$)

The Diffusion Coefficient ($D_{app}$) represents the concrete property towards its resistance to chloride ions penetration. It is influenced by the mix proportions (water/cement ratio, cement type, the amount of the fine materials e.g. silica fume etc), curing, compaction, environment (e.g. relative humidity and temperature) and time (Bertolini, 2004). The source of chlorides (e.g. de-icing salts, airborne sea spray) doesn’t seem to have a significant influence on $D_{app}$ (Vu and Stewart 2000).

Sometimes, as in the case for some of the examples which will be presented in this thesis, information on $D_{app}$ is not available to be used for the purpose of service life prediction for a given structure with a given concrete characteristics. Therefore, several mathematical and experimental models have been proposed to predict $D_{app}$ (e.g. Papadakis et al., 1996; Stewart and Rosowsky, 1998a; Thoft-Christensen, 2003; Han, 2007). Most of these models considered the impact of time, water/cement ratio, curing temperature, relative humidity and concrete mixture on the predicted value and distribution of $D_{app}$. Several researchers have already reviewed the proposed models and investigated their suitability by comparing them to an accumulated data of $D_{app}$ (e.g. Vu, 2003). Vu (2003) found that the existing $D_{app}$ data can be best described by the prediction model which was proposed by Papadakis et al. (1996) as can be seen from Figure 2.4.
2.4.1.1 Papadakis et al. (1996) model

For the development of this model, researchers applied diffusion and adsorption theory to model chloride penetration into concrete. A mathematical model that relates $D_{app}$ to the concrete properties was proposed considering interrelated mechanism of both absorption and diffusion of chloride ions in concrete. The equation proposed to calculate $D_{app}$ is as follows:

$$D_{app} = 0.15 \frac{1 + \rho_c wc}{1 + \rho_c wc + \frac{\rho_c}{\rho_a} ac} \left( \frac{\rho_c wc - 0.85}{1 + \rho_c wc} \right)^3 D_{H_2O}$$

Equation 2.4

where $\rho_c$ and $\rho_a$ are the specific gravity of cement and aggregates, respectively, and $wc$, $ac$ are the water/cement ratio and the aggregate/cement ratio, respectively, and $D_{H_2O}$ is the diffusion coefficient of chloride in an infinite solution ($=1.6 \times 10^{-9} \text{ m}^2/\text{s}$) at 25 °C. $D_{app}$ values determined by Equation found to be far more sensitive to the parameter $wc$ than for example to $ac$ considering that $\rho_c$ and $\rho_a$ fairly constant.

![Graph](image)

Figure 2.4 Influence of water/cement ratio ($wc$) on the diffusion coefficient (Vu, 2003).

Han (2007) noticed that the expression given by Equation was applied to fully saturated, fully hydrated Ordinary Portland Cement (OPC) concrete and curing temperatures from 20 to 25 °C, therefore he suggested that values obtained from Equation may need to be
modified by including factors that take account of humidity, ageing, and curing
temperature that is different from the those used by the model developers. However, there
was no reliable information/models with regard to the effect of curing temperature and
relative humidity that can be used for the purpose of probabilistic modelling intended for
the current study. The influence of ageing however has been reasonably covered in the
literature and consideration of the impact of this factor has been taken account of in this
thesis as will be seen from the following section.

In the current thesis, values and statistical information of $D_{app}$ will be determined from a
real bridge structure exposed to marine environments as will be shown in Chapter 4. The
use of Equation will be limited to situations where $D_{app}$ values for concrete characteristics
other than that specified for the investigated structure are not available and need to be
estimated. Using Equation to estimate $D_{app}$ will ensure that the natural positive correlation
that exists between the $D_{app}$ and $w_c$ parameters, i.e. Figure 2.4, are maintained. For
example, concrete of a high $w_c$ ratio is expected to have a high value of $D_{app}$ when
compared with concrete with a low $w_c$ ratio.

2.4.1.2 Time-dependent diffusion coefficient

It can be seen from Equation that the time necessary to initiate corrosion is inversely
proportional to $D_{app}$ and it is, therefore, of a great interest to obtain a good estimate of $D_{app}$
for accurate prediction of the service life of the RC structure. In many service life
prediction studies, and due to lack of long term time-dependant data of $D_{app}$, the general
practice was to assume that $D_{app}$ is constant over time (e.g. Val and Stewart, 2003).
However, many research studies have shown that $D_{app}$ has a very strong time dependency
attributed mainly to the continuing densification of the concrete microstructure as a result
of the continuing hydration of the cement (e.g. Mangat and Molloy, 1994; Costa and
Appleton, 1999). The reduction of $D_{app}$ with time was found to be more significant in
concrete containing slowly reacting puzzolans such as Pulverised Fly Ash (PFA) or
Ground Granulated Blast Slag (GGBS) than in OPC concrete (Thomas and Bamforth,
1999). The effect of time on the $D_{app}$ under different exposure conditions is shown in
Figure 2.5 for results obtained by Costa and Appleton (1999). It is clear from the figure
that $D_{app}$ is a property that tends to reduce over time due to the ongoing densification of the
concrete composition. The significance of this reduction and its influence on the predicted
time to corrosion initiation hence on the predicted time to first maintenance will be investigated in the rest of this section.

Figure 2.5: A typical relation between diffusion coefficient and time for concrete specimen exposed to various environmental conditions (Costa and Appleton 1999).

The following power function expression was proposed by Mangat and Molloy (1994) and has been used by several service life modelling and reliability researchers to describe the relation between $D_{app}$ and time (e.g. De Schutter, 1999; Duracrete, 2000; Goltermann, 2003; Ferreira, 2004; Li, 2004b; Polder and de Rooij, 2005):

$$D_{app}(t) = D_{app0} \left( \frac{t_0}{t} \right)^m$$

Equation 2.5

where $D_{app}(t)$ is the diffusion coefficient after exposure time of $t$ (years), and $D_{app0}$ is the reference diffusion coefficient at a reference year $t_0$ (years) and $m$ is the age reduction
factor which depends mainly on the mix proportion of the concrete material and can be determined by curve-fitting field or experimental data of $D_{\text{app}}$ collected over time to Equation.

The importance of the ageing factor can be directly related to estimating $T_i$ and hence to estimating time to first maintenance/repair. For example, if the calculation of $T_i$ is to be made based on a value of $D_{\text{app}}$ which is obtained at a reference year $t_0$, the calculated $T_i$ value will depend on whether the reduction of $D_{\text{app}}$ over time were or were not considered. A key parameter for the description of this reduction, as can be seen from Equation, is the ageing factor $m$.

Values and statistical information which have been proposed by different researchers for the factor $m$ are listed in Table 2.1. Looking at the proposed values of $m$, it seems reasonable to assume that $m$ can be represented by a Normal distribution with a mean value of 0.2, 0.4 and 0.6 for OPC, PFA and GGBS concretes respectively with a coefficient of variation $\text{COV}=0.15$. The selection of these values was also motivated by the findings of McPolin et al. (2005) who indicated, based on an experimental study, that OPC concrete showed little change in its diffusivity property over time followed by PFA followed by GGBS concrete.

Table 2.1 values and statistical distribution/parameters proposed by different researchers for the again factor $m$

<table>
<thead>
<tr>
<th>Reference</th>
<th>Value of $m$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nokken et al. (2006)</td>
<td>0.55-0.77</td>
<td>Based on 3 years laboratory data</td>
</tr>
<tr>
<td>Ferreira (2004)</td>
<td>Normal (μ=0.37-0.60, COV=0.07)</td>
<td>Used for probabilistic modelling</td>
</tr>
<tr>
<td>Li (2004b)</td>
<td>Beta (μ=0.5, COV=0.14)</td>
<td>Used For probabilistic modelling</td>
</tr>
<tr>
<td>Goltermann (2003)</td>
<td>Constant 0.75</td>
<td>For concrete with wc&lt;0.55 and exposed to spray and atmospheric zone based on analysis of 102 chloride profiles</td>
</tr>
<tr>
<td>Bentz (2003)</td>
<td>Normal (μ=0.2 &amp; 0.6, COV=0.25)</td>
<td>Used for probabilistic modelling, 0.2 for OPC and 0.6 for PFA or GGBS</td>
</tr>
<tr>
<td>Thomas and Bamforth (1999)</td>
<td>0.1 for OPC 0.7 for PFA 1.2 for GGBS</td>
<td>Based on 8 years data</td>
</tr>
<tr>
<td>Costa and Appleton (1999)</td>
<td>0.36-0.60</td>
<td>Based on an experimental study of 52 concrete panels exposed to marine environment for 3-5 years.</td>
</tr>
</tbody>
</table>
The influence of age of the structure at the time of testing is demonstrated in Figure 2.6. Two cases are demonstrated; (a) when $D_{app0}$ were obtained at a reference age $t_0=27$ years (i.e. the age of the bridge at the time when $D_{app0}$ is obtained), and (b) when $D_{app0}$ were obtained at 28 days. The 27 years reference age and the corresponding $D_{app0}$ value indicated in the Figure 2.6(a) are belonging to the bridge which was investigated in this thesis, see Chapter 4. It is evident from Figure 2.6(a) that as the factor $m$ increases $D_{app}$ varies significantly throughout the exposure period. This case demonstrates how prior knowledge of the concrete mix would influence predicting the lime variant $D_{app}$ and hence predicting $T_D$. On the other hand, as the factor $m$ decreases, i.e. $m \leq 0.2$, the change in $D_{app}$ over time becomes less dramatic. Figure 2.6(b) indicates a situation where a value of 74.05 mm$^2$/year is assumed to have been measured from a structure at 28 days and the reduction of $D_{app}$ is predicted for three concrete mixes. It has to be noticed that the value of $D_{app0} = 74.05$ mm$^2$/year was projected from Figure 2.6(a) for GGBS concrete for demonstrating purposes. This value was then assumed to have been measured from a structure of unknown concrete mix at the reference age $t_0=28$ days. If this value is to reduce over time depending on the concrete mix (i.e. on $m$), it can be seen that the reduction rate in $D_{app}$ with time is higher if the concrete mix was assumed to be made of GGBS or PFA than if it was made of OPC concrete. In this particular example, the reduction in $D_{app}$ over time after 27 years of construction was 48%, 73% and 86% for OPC, PFA and GGBS concretes respectively.

For the work of this thesis, the use of constant or time variant $D_{app}$ will obviously depend on the type of concrete mix assumed for the structure under consideration. However, some
analysis will be carried out to investigate the implication of neglecting the time variant property of $D_{app}$ on the estimating $T_i$ hence on estimating the time to maintenance/repair intervention. In order to calculate $T_i$ based on the evidence that $D_{app}$ decreases over time, $D_{app}$ in Equation was often replaced by the time-dependent expression given by Equation (e.g. Costa and Appleton, 1999; Ferreira, 2004; Li, 2004b). However, this substitution is mathematically inconsistent as it was shown by Visser et al. (2002) and Luping and Gulikers (2007). $D_{app}$ is a function of time that cannot be directly put into the error function solution, $erf(\cdot)$, without time integration. Having considered this common misunderstanding, Visser et al. (2002) have proposed the following revised form of Equation:

$$C(x,t) = C_i + (C_s - C_i) \left[ 1 - erf \left( \frac{x}{2D_{app} \left( \frac{t_0}{m} \right)^{1-m}} \right) \right]$$

Equation 2.6

The use of Equation for curve-fitting of a measured chloride profile requires a value $m$, before the relevant model parameters ($C_s, D_{app}$) can be determined. While mathematically it is possible to determine all parameters including $m$ by curve-fitting, for this procedure it is necessary to have multiple measured chloride profiles from the same structure taken at different points in time. Due to the lack of time dependent data for $D_{app}$ for most cases, the sequential use of Equation and Equation to find the ageing factor $m$ seems justifiable (de Rooij and Polder, 2004). In the context of this thesis, this means that Equation 2.2 can be used to curve-fit the chloride profile to determine $D_{app0}$ and $C_s$ at time $t_0$ and Equation 2.6 to estimate $T_i$.

By rearranging Equation, the time to corrosion initiation can be calculated as follows:

$$T_i = \left[ \frac{C_i^2}{4D_{app} \left( \frac{1-n}{t_0^m} \right)} \left( \frac{erf^{-1} \left( \frac{C_{cr} - C_i}{C_s - C_i} \right)^{-2}}{1-m} \right) \right]^{1/m}$$

Equation 2.7

The weakness of using a constant diffusion coefficient can be demonstrated by attempting to make predictions of $T_i$ using Equation. Assuming that values of $C_i = 0.264$ Cl% per mass of concrete and $D_{app0} = 16.34$ mm$^2$/year were obtained by curve-fitting Equation to a
chloride profile extracted from a 27 years old RC structure (see profile P3-N3 in Table 4.5). If the value of $D_{app}$ were assumed to be constant throughout the exposure period and the value is then used in predicting $T_i$ using Equation, the analysis will yield the solid line (constant $D_{app}$) in Figure 2.7 (assuming $C_{cr}=0.07$ Cl% per mass of concrete). Using the time-dependant model, Equation, the predicted $T_i$ for different concrete covers was found to be as indicated by the dotted lines for different concrete types. It can be seen that ignoring the time dependency of the parameter $D_{app}$ will result in a significantly overestimated $T_i$, the error depends on the concrete quality to reduce diffusivity over time seen by the impact of the ageing factor $m$. As the ageing factor approaches a value that is less than 0.2 (i.e. OPC concretes) the results obtained by the time-dependant model approaches that predicted by the time-invariant model. This is very significant observation considering the amount of published works that have not taken into account the time-dependent diffusivity property of the concrete. Therefore, in this thesis, it is postulated that if the material of the investigated structure was made of OPC, then the time-invariant model will be used in the calculation of $T_i$, otherwise the time-dependant $D_{app}$ will be considered.

![Figure 2.7 Time to corrosion initiation as a function of concrete cover $C_d$ for different concretes made of different mixtures using time-dependent and constant $D_{app}$](image)

### 2.4.2 Surface chloride content ($C_s$)

As indicated in Section 2.3.2, $C_s$ is obtained by mathematically curve-fitting Equation to the profile of chloride contents obtained from chemical analysis of the concrete sample.
taken at incremental depths from the concrete surface, Figure 2.3. It is well documented in the literature (e.g. Bamforth, 1996; McGee, 1999; Bertolini, 2004) that the value of $C_s$ will vary from a structure to a structure and within a structure due to number of factors among which:

(i) the composition of the concrete.
(ii) the position of the structure with regard to the sea level and sea coast.
(iii) the orientation of the structure surface with regard to prevailing winds and rain.
(iv) the chloride concentration in the environment (i.e. source of chlorides; de-icing salts, airborne salts, direct contact with the sea water).

It has also been observed from various field data reported in the literature (e.g. Enright and Frangopol, 1998) that $C_s$ exhibits the highest level of variation among the random variables influencing the time to corrosion initiation. This is expected given that $C_s$ is the variable that is most affected by the exposure conditions which is inherently variable. Prior to reviewing some of the measured $C_s$ field data reported in the literature, it is important to distinguish between some of the major conditions that is shown to have a significant influence on this parameter. The aim of the review about to be presented in the following sub-sections is to study if it is justifiable to use statistical information for $C_s$ from the literature as input parameters for the probabilistic model to be developed in this thesis.

2.4.2.1 $C_s$ in bridges exposed to marine environments

For marine structures, four different exposure zones has been recognised (e.g. Bertolini, 2004). The influence of the structure being in one of these zones on the measured value of $C_s$ will be briefly discussed next.

*The submerged zone:* due to the continuous presence of water in this zone, oxygen is not available and the risk of corrosion is low. Therefore, $C_s$ rarely measured in such zones as the risk of corrosion in these zones considered to be negligible.

*The splash and the tidal zones:* both zones are often treated as one zone, concrete surfaces in these zones are exposed to cyclic wetting and drying so that chlorides accumulates on the concrete surface by process of wetting with seawater, evaporation and salt crystallization (Val and Stewart, 2003). Therefore it is expected that the deposited amount of chlorides will be in its highest concentration at these zones. This also results in a
higher $C_s$ values than $C_S$ values measured from structures located in the atmospheric zone as was confirmed by data collected by Wood and Crerar (1997) and Salta et al. (2008) and plotted in Figure 2.8. However, it has been reported that for heights in excess of 4 m above sea water level $C_s$ does not change significantly (McGee, 1999).

Figure 2.8 Influence of location above the sea level on the surface chloride contents measured at different locations above the sea level.

The atmospheric zone: in this zone concrete is not directly in contact with the sea water, chlorides are carried out to the concrete surface by winds. The amount of chlorides deposited on the concrete surface will depend on the speed of the wind and the distance of the structure from coastline as well as the orientation and sheltering of the surface of the structure (McGee, 1999).

Influence of distance from the coastline
As stated in the previous section, the distance of the structure exposed to airborne chlorides from the coast line have a significant impact on the amount of chlorides to be deposited on the concrete surface, hence causing the measured $C_s$ to increase. A typical drop of the surface chloride concentrations with distance from the coast line is shown in Figure 2.9. As can be seen from the figure, the first 100 m from the coast line shows a high level of the measured $C_s$ values. As the distance increases beyond 100 m, the measured $C_s$ value tends to decrease rapidly. Similar observations have been reported elsewhere (e.g. Meira et al., 2007).

Some $C_s$ data has been collected by Vu (2003) and illustrated in Table 2.2 and Figure 2.10(a) which both show the large scatter in the measured values.
Figure 2.9 Influence of the distance from the sea coast on $C_s$ as concluded by McGee (1999) based on a field study of 1158 bridges in the Australian state of Tasmania.

Table 2.2 Surface Chloride Contents $C_s$ data for airborne chlorides (Vu, 2003)

<table>
<thead>
<tr>
<th>Reference</th>
<th>$C_s$ kg/m$^3$ [Cl% per mass of concrete$^a$]</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morinaga (1992)</td>
<td>0.3 [0.013] (mean)</td>
<td>Data from 46 specimens exposed to various sea environments in Japan</td>
</tr>
<tr>
<td>Browne (1982)</td>
<td>10 [0.431] (for coastal building)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.76 [0.248] (for offshore platform)</td>
<td></td>
</tr>
<tr>
<td>Sorensen and Maahn (1982)</td>
<td>3.4 to 10.2 [0.147 to 0.440] (mean)</td>
<td>Regression from field data of the Langeland bridge (7.2 m above sea level in Denmark)</td>
</tr>
<tr>
<td></td>
<td>1.5 [0.065] (mean for 23 years)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.7 [0.073] (mean for 32 years)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.6 [0.241] (mean for 55 years)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.8 [0.164] (mean for 58 years)</td>
<td></td>
</tr>
<tr>
<td>Uji et al. (1990)</td>
<td>mah</td>
<td>Offshore structures in Japan</td>
</tr>
<tr>
<td></td>
<td>3.7, 4.1, 3.4, 4.9, 2.8, 1.0 DNA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.159, 0.177, 0.147, 0.211, 0.121, 0.143]</td>
<td>Bridge decks in Kentucky, USA</td>
</tr>
</tbody>
</table>

$^a$ values in [] were converted to this units using concrete density =1320 kg/m$^3$
Chapter 2: Materials deterioration models

Figure 2.10 Scatter of data of $C_s$ collected from bridges exposed to (a) Airborne chlorides, (b) De-icing source of chlorides (Vu, 2003).

2.4.2.2 $C_s$ in bridges exposed to de-icing salts

For RC bridges in cold regions, de-icing salts (which contain sodium or calcium chlorides) are frequently applied to free the roads from ice and snow. For example, the amount of de-icing salts applied to a typical bridge deck in the Snow Belt states of the Unite States is about 1.2 kg/m$^2$ each winter season with some bridges receive as much as 4.99 kg/m$^2$ (Vu, 2003). As such, the de-icing application of salts has been recognised as a major source of chlorides which leads to the corrosion-induced deterioration of RC bridge structures. The level of chloride attack within an RC bridge varies from element to element (e.g. decks, columns, piers or abutments). For example, bridge piers may be exposed to salt spray from passing vehicles and deck soffits may be exposed to salt water leaking through joints in the deck. This means that $C_s$ varies even from location to location within the same structure component which implies that some parts of the bridge/bridge element would deteriorate earlier and hence require an earlier repair intervention.

In his research, Vu (2003) collected a larger number of data from the literature and presented Figure 2.10(b) which indicates the scatter of the $C_s$ values obtained from bridges subjected to de-icing application. As can be seen from the figure, $C_s$ tends to be uniformly distributed between values of 0.043 to 0.259 Cl% per mass of concrete (1 to 6 kg/m$^3$) with some values reaching as high as 0.9 Cl% (21 kg/m$^3$). Based on Figure 2.10, $C_s$ values for the case of de-icing salts exposed structures can be assumed to exhibit a wider variation.
than $C_s$ values measured in the atmospheric marine environment. However, there is no strong evidence to suggest that the overall mean value of $C_s$ is higher for any of the two cases.

### 2.4.2.3 Time-dependent surface chloride content

Some field data collected from several investigations has shown the tendency of $C_s$ to increase nonlinearly with time which suggests that the assumption of $C_s$ being constant may not be valid. For example, data collected from 15 bridge decks in North America were presented by Kassir and Ghosn (2002), Figure 2.11, clearly shows that the average value of $C_s$ increases over time. Kassir and Ghosn (2002) proposed that this trend can be represented by an exponential function of the form:

$$C_s(t) = C_s \left(1 - e^{-\alpha t}\right)$$  

Equation 2.8

where $\alpha$ is a factor represents the rate of increases and can be obtained by curve-fitting data of $C_s$ collected over time to Equation and $t$ is the time elapsed since of the start of the accumulation of chlorides at the surface of the structure in (years).

![Figure 2.11. Typical trend for surface chloride increase with time (Kassir and Ghosn, 2002).](image)

A more popular expression for the description of the time-dependent $C_s$ is that proposed by Costa and Appleton (1999) which takes the following form:
where $C_{s0}$ is the reference $C_s$ value that is measured at reference year $t_0$, and $n$ is a factor can be determined by curve-fitting data of $C_s$ collected over time to Equation.

While structures in tidal and splash zones expected to have their $C_s$ built up quickly, $C_s$ for structures exposed to airborne chlorides or seasonal application of de-icing salts is expected to take some time. The values reached and when it is reached will depend mainly on (Meira et al., 2007):

(i) the height from the sea level.
(ii) the distance from the sea coast.
(iii) orientation of the structure with regard to wind.
(iv) protection and sheltering of the structure surface from the rainfall.

According to Kassir et al. (2002), the time needed for $C_s$ to reach a constant value, will take between 10 to 15 years.

While there is some evidence that $C_s$ varies with time, Bamforth (1996), Thomas et al. (1999) and McGee (1999) among others all advocate that there is no tendency for values either to increase or decrease and therefore suggested that after the first six months of exposure it would be reasonable to assume a constant average value of $C_s$ for predictive purposes. Modelling $C_s$ as a time-invariant property is a common practice among many researchers in this field (e.g. Val et al., 1998; Akgul and Frangopol, 2005; Vu and Stewart, 2005; Duprat, 2007). This is understandable, as the predicted time to corrosion initiation is proportional to $C_s$, assuming a high value will reduce the estimated time to corrosion initiation warning and repair intervention would be much earlier and hence unduly conservative. Based on this, the time-dependent $C_s$ will not be considered in this thesis.

In a parametric study carried out by Enright and Frangopol (1998), they found that the mean value of $T_i$ is more sensitive to the change of COV of $C_s$ than to changes in COV of any of the other main variables ($C_d$, $C_i$, $C_{cr}$, $D_{app}$). This means that for an accurate prediction of $T_i$ for a particular structure, the statistical parameters of the variable $C_s$ is better estimated from the structure under investigation or from structures that shares
similar exposure conditions. This approach has also been emphasised by a number of other researchers. For example, Akgul and Frangopol (2005) recommended that for the assessment of a particular existing structure, an appropriate approach is to determine the value of the chloride ingress parameters with the aim of customizing the values reported in the literature to the material characteristics and salt-exposure conditions of the considered structure.

It can be seen from the preceding review that parameter $C_s$, which is a key parameter for estimating $T_i$ and hence for estimating the time to maintenance and repair, is a highly variable property. It was shown that $C_s$ varies not just across samples but also varies in space, i.e. between locations within the same structure. This is mainly due to the variation of chloride concentrations sprayed on different parts of the structure. Therefore, and based on the review study carried out here in with regard to the scatter of the $C_s$ data reported in the literature, it was decided that for the probabilistic analysis to be performed in this thesis, the statistics and spatial variation information of $C_s$ will better be determined from a site specific investigation. Ferrycarrig Bridge was therefore selected to extract a suitable number of $C_s$ data as will be seen in Chapter 4.

### 2.4.3 Critical chloride content ($C_{cr}$)

The critical (threshold) chloride content is one of the key parameters needed for determining $T_i$. It is therefore important to investigate the variability of such an important parameter considering the expected influence it would have on the uncertainty associated with the predicted time to maintenance and hence on the decisions to be made in this regard. $C_{cr}$ represents the chloride ions concentration that must be present at the steel concrete interface for depassivation to occur and corrosion to initiate. As discussed by Alonso et al. (2000), there are many variables that affect the $C_{cr}$ value. Among those variables are: concrete mix proportions, cement type, $C_3A$ content of cement, blended materials, water/cement ratio, temperature, relative humidity, pH level, steel surface conditions, source of chloride penetration, the availability of oxygen at the steel interface and many others. The great number of variables involved in the chloride-induced corrosion process explains the reason why the $C_{cr}$ values reported in the literature present such wide range of variability.
In addition to the influence of the amount of variables involved, Alonso et al. (2000) has pointed out that the variation in the detection method used to define $C_{cr}$ value are equally blamed for the large scatter of $C_{cr}$ reported in the literature. For example; some researchers consider that depassivation is produced when a certain shift in the corrosion potential is produced; others use the visual inspection and identify depassivation with the appearance of rust spots on the steel surface whereas some others choose to relate depassivation with a certain level in the corrosion current.

Numerous studies have been conducted to determine the $C_{cr}$ value for different concrete mixes and under various environmental conditions. As a result, a large scatter in the proposed values was reported in the literature as can be seen from Table 2.3. However, most of design code and national standards seems to agree to some extent on the maximum permitted chloride content value as can be seen from Table 2.4.

The values of $C_{cr}$ listed in Table 2.3, Table 2.4 and Table 2.5 (or the centre value whenever $C_{cr}$ was given in a range format) are all plotted in Figure 2.12 to show the dispersion of the means of the collected data. Whenever the data were given in a range format, a mean value was used. Normally, 350–400 kg/m$^3$ of cement are typical for bridge decks, and deck concrete density was typically assumed to be 2,320 kg/m$^3$ (Cady and Weyers, 1992). Akgul and Frangopol (2005) used a cement content value of 16% to convert data of $C_{cr}$ from Cl% per mass of cement to Cl% per mass of concrete. This corresponds very well with a value of 16.4% found from laboratory testing on concrete samples taken from 7 crosshead beams of the then 24 years old Ferrycarrig Bridge as will be shown later in Chapter 4. Therefore, whenever the value of $C_{cr}$ was given in Cl% per mass of concrete units, the cement content value of 16% was used to convert values of $C_{cr}$ to Cl% per mass of concrete units.

When the $C_{cr}$ data listed in Table 2.3, Table 2.4 and Table 2.5 were plotted and fitted to the Lognormal distribution, Figure 2.13, a mean value and COV values of 0.07 (Cl% per mass of concrete) and 0.64 were respectively obtained. The obtained mean value corresponds very well with a value proposed by Polder (2005) for probabilistic modelling of service life prediction in which he indicated that about 0.07 (Cl% per mass of concrete) is a good estimate of the value at which the probability of corrosion is about 50%. The COV value of 0.30 reported by Polder (2005) is smaller than the value obtained here in (COV=0.64). This is understandable since Polder (2005) recommendation for the value of COV
Chapter 2: Materials deterioration models

proposed for concrete made of OPC whereas the data reported here covers wide range of materials and test conditions. This value (i.e. COV=0.64) is therefore too high to be assumed appropriate to model the variability of the parameter $C_{cr}$.

Table 2.3. Critical chloride values as reported by several authors (Glass and Buenfeld, 1997; Price and Emerson, 1997; Dhir and McCrthy, 1999; Alonso et al., 2000; Oh et al., 2003; Bertolini, 2004).

<table>
<thead>
<tr>
<th>Date</th>
<th>Reference</th>
<th>Measured value of critical chloride concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\text{Cl}% \text{ per mass of concrete}$</td>
</tr>
<tr>
<td>1982</td>
<td>Sorensen and Maahn, Fischer et al.</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1884</td>
<td>Vassie; derived from UK bridges, Hanson and Sorenson</td>
<td>0.2-1.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>Lukas; derived from Austrian bridges and the following references:</td>
<td>1.8-2.2</td>
</tr>
<tr>
<td></td>
<td>-Richartz</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Van Daveer, US bridges</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Browne, Domone, Geohegan</td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>Takewake and Mastumotos</td>
<td>0.03-0.1</td>
</tr>
<tr>
<td>1990</td>
<td>Funahashi; based on the following nine literature:</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>-Lewis</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Hausmann</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Berman</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Clear and Hay</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Clear</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Stratfall et al.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Cady</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Browne</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Pfeifer et al.</td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>Glass and Buenfeld Review</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Bamforth and Andrews</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Schiessel and Raupach</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Henriksen</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Elsener and Böhnii</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Locke and Sirnan</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>Dhir Review</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Knoefel</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Page and Lambert</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Everett and Treadaway</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Weigler and Segmuller</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>Alonso et al. Review</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Gouda and Halaka</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Hussain et al.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Schiessel and Breit</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Thomas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Thomas et al.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Hope and Ip</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>Oh et al.</td>
<td></td>
</tr>
</tbody>
</table>

46
Table 2.4: Permitted total chloride content of RC according to international guidelines (Berit, 1998).

<table>
<thead>
<tr>
<th>Country</th>
<th>Guideline/year</th>
<th>Permitted chloride content (max)</th>
<th>Cl_total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cl/cement (%)</td>
<td>Cl/concrete (kg/m³)</td>
</tr>
<tr>
<td>Germany</td>
<td>pr EN 206 (1997)</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>UK</td>
<td>BS 8110 (1985)</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>Norway</td>
<td>NS 3420 (1986)</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>France</td>
<td>RILEM TC 124 (1994)</td>
<td>0.3 - 0.5</td>
<td>-</td>
</tr>
<tr>
<td>USA</td>
<td>ACI 222 (1985)</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>ACI 318 (1989)</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>Australia</td>
<td>AS 3600 (1988)</td>
<td>-</td>
<td>0.8</td>
</tr>
<tr>
<td>*China</td>
<td>CNS 3090 (1998)</td>
<td>-</td>
<td>0.15 - 0.6</td>
</tr>
</tbody>
</table>

*Reference (Liang et al., 2002)

Table 2.5. Statistical data for $C_{cr}$ (Cl% per mass of concrete) used by different researchers in reliability based analysis.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Distribution</th>
<th>Mean</th>
<th>COV (Range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akgul and Frangopol (2005)</td>
<td>Lognormal</td>
<td>0.037</td>
<td>0.15</td>
</tr>
<tr>
<td>Thoft-Christensen (1998)</td>
<td>Normal</td>
<td>0.048</td>
<td>0.17</td>
</tr>
<tr>
<td>Polder and de Rooij (2005)</td>
<td>Normal</td>
<td>0.080</td>
<td>0.30</td>
</tr>
<tr>
<td>Enright and Frangopol (1998)</td>
<td>Lognormal</td>
<td>0.040</td>
<td>0.10</td>
</tr>
<tr>
<td>Vu and Stewart (2005)</td>
<td>Normal</td>
<td>0.103</td>
<td>0.20</td>
</tr>
<tr>
<td>Li (2004b)</td>
<td>Normal</td>
<td>0.072</td>
<td>0.16</td>
</tr>
<tr>
<td>Duprat (2007)</td>
<td>Good concrete</td>
<td>Uniform</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>Ordinary concrete</td>
<td>Uniform</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td>Poor concrete</td>
<td>Uniform</td>
<td>0.043</td>
</tr>
<tr>
<td>Stewart and Rosowsky (1998b)</td>
<td>Uniform</td>
<td>0.039</td>
<td>(0.026-0.052)</td>
</tr>
<tr>
<td>Val and Trapper (2008)</td>
<td>Normal</td>
<td>0.080</td>
<td>0.20</td>
</tr>
<tr>
<td>Bentz (2003)</td>
<td>Normal</td>
<td>0.050</td>
<td>0.10</td>
</tr>
<tr>
<td>Li and Melchers (2005)</td>
<td>Uniform</td>
<td>0.082</td>
<td>(0.078-0.085)</td>
</tr>
<tr>
<td>fib Bulletin 34 (2006)</td>
<td>Beta</td>
<td>0.096</td>
<td>0.25(0.032-0.32)</td>
</tr>
</tbody>
</table>

Note: when data in the original reference were given in (Cl% per mass of cement) or kg/m³ units, Cement/Concrete ratio=0.16 and concrete density=2320 kg/m³ where used for conversion to (Cl% per mass of concrete) units.
As can be seen from Table 2.5, a COV value between 0.10 and 0.30 is frequently used by researchers in the literature for service life prediction modelling. In this thesis and in light of the proceeding discussion, it seems reasonable to assume a lognormal distribution with a mean value of 0.07 (Cl% per mass of concrete) and COV=0.25 to describe the variability of the $C_{cr}$ parameter.

Figure 2.12. Scatter of mean values of $C_{cr}$

Figure 2.13 (a) Histogram of $C_{cr}$ data collected from the literature ($\mu=0.07\%$, COV=0.64), (b) Lognormal probability plot fitted to the $C_{cr}$ data.
2.5 PROPAGATION MODELS

As indicated in Section 2.1, the main objective of this chapter is to investigate the available deterioration models and the variability associated with the key deterioration parameters employed by these models. In the previous section (Section 2.4) these objectives were applied to the initiation stage of the corrosion-induced deterioration process. In this section models and parameters describing the propagation stage (see Figure 2.1) will be investigated. The deterioration models that are available to describe the propagation period can be divided into two distinct groups. The first group are those models which are concerned with the formation of the corrosion-induced cracks on the surface of the concrete. These models therefore are more related to the SLS of the structure. The second group of models are those concerned with reduction of the cross-sectional area of the flexure and shear reinforcements due to the ongoing corrosion. These models therefore are directly related to the load carrying capacity and hence the ULS. Both groups of models however are not independent and they are closely related due to the common parameters they both employ to describe the corrosion-induced deterioration of the structure performance over time.

As briefly described in Section 2.2 and in terms of the first group of models, the literature recognises two distinct stages that aim to describe the propagation stage (Alonso et al., 1998). The first group of models try to describe sub-stage I or the time period from which depassivation of the reinforcement has taken place due to the chloride attack up to the point where a hair size cracks appears on the concrete surface. The second group of models try to predict the length of Sub-stage II or the time period from when corrosion-induced hair size cracks start to propagate to reach a maximum crack width that can be considered as a critical.

Regarding the second group of corrosion–induced propagation models which is of a direct concern to the load carrying capacity, two distinct models have been recognised. The first model assumes a uniform loss of the cross-sectional reinforcement that is subjected to corrosion activities and referred to as the General corrosion. The second assumes that the corrosion is localized and causes far more loss of section at that specific location than generally caused by the uniform (general corrosion). This later form of corrosion referred to as the Pitting corrosion.
2.5.1 Crack initiation models

The process of corrosion-induced concrete cracking can be described as follows: after corrosion has initiated, the reinforcement corrosion propagates in concrete and produces expansive rust (mainly ferrous and ferric hydroxide, Fe(OH)$_2$ and Fe(OH)$_3$) that occupies a much larger volume than the original reinforcement, see Table 2.6. This results in the generation of radial pressure on the surrounding concrete at the interface between the reinforcement and the concrete (Broomfield, 1997). Eventually, the generated stresses exceed the tensile capacity of the concrete cover leading to cracking of the concrete cover.

<table>
<thead>
<tr>
<th>Name of corrosion products</th>
<th>FeO</th>
<th>Fe$_3$O$_4$</th>
<th>Fe$_5$O$_9$</th>
<th>Fe(OH)$_2$</th>
<th>FeO(OH)$_3$</th>
<th>Fe(OH)$_3$.3H$_2$O</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.777</td>
<td>0.724</td>
<td>0.699</td>
<td>0.622</td>
<td>0.523</td>
<td>0.347</td>
</tr>
<tr>
<td>$\alpha'$</td>
<td>1.80</td>
<td>2.00</td>
<td>2.20</td>
<td>3.75</td>
<td>4.20</td>
<td>6.40</td>
</tr>
</tbody>
</table>

Note: $\alpha$ is the ratio of molecular weight of iron to the molecular weight of the corrosion products; $\alpha'$ is the ratio of volume of expansive corrosion products to the volume of iron consumed in the corrosion process.

A number of analytical models have been proposed to model the stresses required to initiate cracking of the concrete cover due to corrosion and the time it takes for these stresses to be generated (Bazant, 1979; Liu and Weyers, 1998; Pantazopoulou and Papouli, 2001; Wang and Liu, 2004; Bhargava et al., 2006a; El Maaddawy and Soudki, 2007). Among all proposed models, models by Liu and Weyers (1998) and El Maaddawy and Soudki (2007) seems to have generated wide acceptance among researchers in this field, therefore these two models will be described with some detail. Once a model for describing the time to first cracking is selected, it will be used within the probabilistic framework to be developed in this thesis for the prediction of the service life of the structure under consideration hence for the estimation of the time to maintenance and repair.

2.5.1.1 Liu and Weyers (1998) model

In this model, the concrete around a corroding reinforcing bar was treated as a thick-walled cylinder (Bazant, 1979), which is subjected to internal pressure due to formation of...
corrosion products having larger volume than that of the original steel. Stresses in the cylinder wall are calculated using the solution provided by isotropic linear elasticity theory and it is assumed that cracking occurs when the stresses reach the concrete tensile strength, Figure 2.14.

![Figure 2.14. Schematic of corrosion induced concrete cover cracking](image)

From the figure, the inner radius of the hypothetical thick-walled cylinder \( a = (D + 2d_o)/2 \), and outer radius \( b = C_d + (D + 2d_o)/2 \). Where \( D \) is the bar diameter, \( d_o \) is the thickness of pore band around the steel/concrete interface, \( C_d \) is concrete cover depth. It is assumed that until corrosion products have filled the porous zone no pressure develops between the reinforcing bar and concrete. According to Liu and Weyers (1998), if the concrete material can be considered as a homogenous elastic material with \( (v_c) \) *poison ratio* and \( (E_{eff}) \) *effective modulus of elasticity* which is related to the *modulus of elasticity* of concrete \( (E_c) \) and the *creep coefficient* of the concrete \( (\varphi_{cr}) \) via \( E_{eff} = E_c / (1 + \varphi_{cr}) \). The radial stress \( (\sigma) \) at the concrete/rust products interface can be estimated as:

\[
\sigma = \frac{2E_{eff}d_s}{(D + 2d_o)\left(\frac{b^2 + a^2}{b^2 - a^2 + v_c}\right)} \quad \text{(N/mm}^2\text{)}
\]

*Equation 2.10*

where \( d_s \) is the thickness of corrosion products needed to generate radial stresses, which was determined by equating the expression in Equation to the tensile capacity of the concrete cover which is in turn was related to the *tensile strength* of concrete \( (f_t) \) and \( C_d \) as follows (Bazant, 1979):
\[ \sigma = \frac{2C_d f_i}{D + 2d_s} \quad \text{(N/mm}^2\text{)} \]  

Equation 2.11

By equating Equation and Equation, \( d_s \) found to be:

\[ d_s = \frac{C_d f_i}{E_{\text{eff}}} \left( \frac{a^2 + b^2}{b^2 - a^2} \right) \quad \text{(mm)} \]

Equation 2.12

The next step was to determine the critical amount of rust products that is needed to initiate the cracking. Liu and Weyers (1998) estimated \( W_{cr} \) from:

\[ W_{cr} = \rho_{\text{rust}} \left( \pi D \left[ d_s + d_o \right] + \frac{W_{st}}{\rho_{st}} \right) \]

Equation 2.13

where \( W_{cr} \) is the critical mass of the corrosion products needed to induce cracking of concrete cover in (mg/mm); \( \rho_{\text{rust}} \) is the density of the corrosion products; \( \rho_{st} \) is the density of the parent steel; \( W_{st} \) is the mass of corroded steel and it is related to \( W_{cr} \) so that \( W_{st} = \alpha W_{cr} \). Where \( \alpha \) is the ratio of the molecular weight of iron to the molecular weight of the corrosion product and its value, which depends on the type of the corrosion product, can be obtained from Table 2.1. The value of \( d_o \) was estimated to be in the order of 12.5 \( \mu \text{m} \). For more details on the model see (Liu and Weyers, 1998).

The time to first cracking was then estimated as follows:

\[ T_{\text{1st}} = \frac{W_{cr}^2}{2k_p} \quad \text{(years)} \]

Equation 2.14

where \( k_p \) is the rate of rust production which is related to the rate of the metal loss and is expressed as a function of the corrosion rate \( (i_{\text{corr}}) \quad \mu \text{A/cm}^2 \) as follows:

\[ k_p = 0.098 \left( \frac{1}{\alpha} \right) \pi D i_{\text{corr}} \]

Equation 2.15
It can be seen that according to Liu-Weyers's model, that the time to first corrosion-induced cracking, \( T_{1st} \), is mainly determined by the corrosion rate, tensile strength of concrete, concrete cover depth and the type of corrosion product.

### 2.5.1.2 El-Maaddawy and Soudki (2007) model

In this model, the concrete around a corroding reinforcing steel bar is modelled as a thick-walled cylinder with the wall thickness equal to the concrete cover, Figure 2.14(a). The concrete ring is assumed to crack when the tensile stresses in the circumferential direction at every part of the ring have reached the tensile strength of concrete, Figure 2.14(b). The internal radial pressure at the point of cracking is then determined and related to the steel mass loss. Faraday's law was then employed to predict the time from corrosion initiation to corrosion-induced cracking (Andrade et al., 1993). The model accounts for the time required for corrosion products to fill the porous zone before they start inducing expansive pressure on the concrete surrounding the steel reinforcing bar.

The approach followed by the authors in their development of this model was very similar to that of Liu and Weyers (1998). However, El-Maaddawy and Soudki (2007) stated that in Liu–Weyers’s model, the rate of steel mass loss caused by corrosion was assumed to decrease as time progresses. The rate of steel mass loss was assumed to be directly proportional to the square root of the product of the corrosion current and the time of corrosion exposure. They found that for the same time of corrosion exposure, this assumption significantly underestimates the amount of steel weight loss compared with that obtained using Faraday’s law. Underestimating the rate of steel loss caused by corrosion would result in overestimating the time to corrosion cracking. Based on this they have proposed the following alternative model:

\[
T_{1st} = \frac{7117.5(D + 2d_o)(1 + v_c + \psi)}{i_{corr}E_{eff}} \left[ \frac{2C_d f_i}{D} \frac{2d_o E_{eff}}{(1 + v_c + \psi)(D + 2d_o)} \right]
\]

Equation 2.16

where \( T_{1st} \) (years), \( D, d_o, v_c, C_d, f_i, E_{eff}, i_{corr} \), as defined in the previous section (\( d_o = 10-20 \) \( \mu\text{m} \)). \( \psi \) is a factor dependent on \( D, C_d \) and \( d_o \).

\[
\psi = D^2 / 2C_d (C_d + D); \quad D' = D + 2d_o
\]

Equation 2.17
To compare the two models, $T_{1st}$ were calculated for a range of typically measured corrosion rate values (from 0.1 to 10 $\mu A/cm^2$) using the relevant input parameters listed in Table 2.7. The results were plotted in Figure 2.15. It can be seen from the figure that for a low corrosion rate values, Liu-Weyers’s model produces more than twice as much longer $T_{1st}$ period than that obtained from El-Maaddaway-Soudki’s model. However, although the ratio between the two results stays the same, as the corrosion rate value increases (i.e. $i_{corr} \rightarrow 10 \mu A/cm^2$) $T_{1st}$ is significantly reduced that it doesn’t matter much which of the two models is used.

Table 2.7 Input parameters for calculating $T_{1st}$ as used by (Liu and Weyers, 1998)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_d$</td>
<td>mm</td>
<td>30</td>
</tr>
<tr>
<td>$D$</td>
<td>mm</td>
<td>16</td>
</tr>
<tr>
<td>$d_o$</td>
<td>$\mu m$</td>
<td>12.5</td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>-</td>
<td>0.18</td>
</tr>
<tr>
<td>$f'_c$</td>
<td>MPa</td>
<td>25</td>
</tr>
<tr>
<td>$f_c = 0.53\sqrt{f'_c}$</td>
<td>MPa</td>
<td>2.65</td>
</tr>
<tr>
<td>$E_c = 4600\sqrt{f'_c}$</td>
<td>GPa</td>
<td>23</td>
</tr>
<tr>
<td>$\varphi_{cr}$</td>
<td>-</td>
<td>2.0</td>
</tr>
<tr>
<td>$\rho_{rust}$</td>
<td>mg/mm$^3$</td>
<td>3.6</td>
</tr>
<tr>
<td>$\rho_{st}$</td>
<td>mg/mm$^3$</td>
<td>7.85</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Although Liu-Weyers’s model have been used frequently by researchers (e.g. Wang and Liu, 2004; Vu and Stewart, 2005; Bhargava et al., 2006a; Sancharoen et al., 2006; Val, 2007; Kenshel and O’Connor, 2008) in the estimation of the time to crack initiation, Chernin and Val (2008) have recently criticized this model. They pointed out that in this model the value of $d_o = 12.5 \mu m$ used was determined based on the incorrect estimation of $k_p$ (see Equation). According to Chernin and Val (2008) it seems that in the process of finding the value of $d_o$, the $k_p$ value was erroneously replaced by the value of 0.098 appeared in Equation. This mistake was not repeated in the derivation of El-Maaddaway-Soudki’s model in which the value of $d_o$ was found to range from 10 to 20 $\mu m$. When these values were used by (El Maaddawy and Soudki, 2007), results yielded by Equation were
compared with the times to first cracking observed in a number of accelerated corrosion tests with a good agreement observed.

Figure 2.15 Comparison between Liu-Weyers (1998) and El Maaddawy-Soudki (2007) $T_{1st}$ models.

Based on this discussion, El Maaddawy-Soudki’s model will be used in this thesis to estimate the time to first cracking.

2.5.2 Crack propagation models

For the crack propagation, (i.e. sub-stage II of the propagation stage, Figure 2.1(b)), two distinctive models have been reported in the literature, the Duracrete (2002) model and Vu et al. (2005) model.

2.5.2.1 Duracrete (2000) model

The Duracrete final report (Duracrete, 2000) describes the propagation period as follows:

$$T_p = \frac{\left( w_{lim} - 0.05 \right) + a_1 + a_2 \left( \frac{C_d}{D} \right) + a_3 f_{opt}}{\beta_{pD} w_{c} \alpha_{pD} i_{corr}} \text{ (years)}$$

Equation 2.18

where;
where $T_p$ is the propagation time years (i.e. $T_p = T_{1st} + T_{cp}$) and the input parameters are as defined in Table 2.8. Typical values of the listed parameters can be obtained from Li et al. (2004).

Table 2.8 Input parameters for the Duracrete crack propagation model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{lim}$</td>
<td>Is the limiting crack width (0.3-1.0 mm)</td>
</tr>
<tr>
<td>$\beta_D$</td>
<td>Parameter that controls the propagation</td>
</tr>
<tr>
<td>$\alpha_D$</td>
<td>The pitting factor</td>
</tr>
<tr>
<td>$a_1, a_2$ and $a_3$</td>
<td>Regression parameters</td>
</tr>
<tr>
<td>$t_{corr}$</td>
<td>mean corrosion rate in (mm/year)</td>
</tr>
<tr>
<td>$w_i$</td>
<td>is the wetness period of the year</td>
</tr>
<tr>
<td>$f_{spl}$</td>
<td>is the tensile splitting strength of concrete in MPa</td>
</tr>
<tr>
<td>$f_{co}$</td>
<td>Basic concrete compressive strength in MPa</td>
</tr>
<tr>
<td>$t_{age}$</td>
<td>is the age of the concrete at the time of inspection (years)</td>
</tr>
<tr>
<td>$Y_1$</td>
<td>is a variable representing additional variation due to the special placing, curing and hardening conditions of in situ concrete</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>Is a variable representing variations due to factors not well accounted for by concrete compressive strength (e.g. gravel type size)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Is a factor taking into account systematic variation of in situ compressive strength and strength of standard tests</td>
</tr>
</tbody>
</table>

2.5.2.2 Vu et al. Model (2005) model

Vu et al. (2005) proposed an empirical model to estimate the period of time in which the corrosion-induced crack to propagate from a hair size (typically assumed in the order of 0.05 mm) and reach a maximum size of $w_{lim}$ (mm) (Sub-stage II in Figure 2.1(b)). The model is based on accelerated corrosion tests conducted in the laboratory involving eight RC slabs with 16 mm diameter reinforcing bars and varying water/cement ratios and cover depths. Assuming a constant corrosion rate, the time it takes for the corrosion-induced crack to propagate from a hair size to reach a limiting width of $w_{lim}$ was estimated as follows:
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\[ t_{cp} = k_R \frac{0.0114}{t_{corr}} \left[ A \left( C_d / w_c \right)^B \right] \]  
(years)  
\[ 0.33 \text{ mm} \leq w_{lim} \leq 1.0 \text{ mm} \]  
Equation 2.20

where:
\[ k_R \approx 0.95 \left[ \exp \left( -\frac{0.3 i_{corr} \text{(exp)}}{i_{corr}} \right) - \frac{i_{corr} \text{(exp)}}{2500 i_{corr}} + 0.3 \right] \]  
Equation 2.21

where \( C_d \) is the reinforcement cover depth in (mm) and \( w_c \) is the water/cement ratio, \( i_{corr} \) is the measured corrosion rate (\( \mu A/cm^2 \)) assumed constant with time. \( A \) and \( B \) are empirical constants and \( k_R \) is a rate of loading corrosion factor where \( i_{corr}(\text{exp}) \) is the accelerated corrosion rate used in the experiment. The developers of the model used an impressed corrosion rate of 100 \( \mu A/cm^2 \) in their testing therefore \( i_{corr}(\text{exp})=100 \mu A/cm^2 \). \( A=65 \) and \( B=0.45 \) for \( w_{lim}=0.3 \), \( A=700 \) and \( B=0.23 \) for \( w_{lim}=1.0 \) mm. The factor \( k_R \) was developed to allow for the high rate of loading resulted from the use of high corrosion rate in the accelerated corrosion tests.

As suggested by Liu and Weyers (1998), the formation of rust products on the steel surface will reduce the diffusion of the iron ions away from the steel surface resulting in reduced corrosion rate with time. To take into account the time-dependant corrosion rate, Vu and Stewart (2000) proposed the following expression for determining the time to crack propagation \( T_{cp} \):

\[ T_{cp} = \left[ \frac{\lambda_{cp} + 1}{\alpha_{cp}} \times \left( t_{cp} - 1 + \frac{\alpha_{cp}}{\lambda_{cp} + 1} \right) \right]^{\frac{1}{\lambda_{cp} + 1}} \]  
\( t_{cp} > 1 \text{ year}, w_{lim} \leq 1.0 \text{ mm} \)  
Equation 2.22

where \( t_{cp} \) is the time for a crack to propagate to a limit crack width in years for a time-invariant corrosion rate (can be calculated from Equation), \( \alpha_{cp} \) and \( \lambda_{cp} \) are constants for describing the reduction of the corrosion rate with time their proposed values are \( \alpha_{cp}=0.85 \) and \( \lambda_{cp}=-0.3 \).

Due to the lack of statistical information regarding the input parameters for the Durarcrete model, the time to crack propagation which was defined in Section 2.2 and again in the current section, will be modelled using Vu et al. (2005) model. It has to be noticed that this
model is only relevant in the case of SLS service life modelling and therefore will only affect the time to first maintenance predicted in terms of surface condition of the structure under consideration. However, in terms of the ULS, and as mentioned earlier in Section 2.2.2, propagation models are described as a function of the loss of reinforcement cross-sectional area which is directly influences the load carrying capacity and hence the safety of the structure. The propagation models which provide quantitative estimate the loss in the load carrying capacity over time thus of a concern to the ULS will be discussed in the following section.

2.5.3 Loss of cross-sectional area models

As indicated in Section 2.2.2, two types of corrosion, general and pitting, are possible. General corrosion affects the reinforcement by casing a uniform loss of its cross-sectional area. Pitting corrosion, in contrast to general corrosion, concentrates over small areas of the reinforcement. Models quantifying of the magnitude of such losses will be described here.

2.5.3.1 Due to general corrosion

As the corrosion process progresses, the cross-sectional area of the reinforcement of an RC member will be reduced leading to reduction of the capacity of individual elements and by implication of the structure as a whole. If the corrosion is assumed to be of a uniform type, Figure 2.16, the loss of reinforcement diameter can be described by the use of Faraday’s law of electrochemical equivalence (Andrade et al., 1993).

\[ \Delta D(t)/2 = \frac{AD(t)/2}{I} \]

Figure 2.16 General (uniform) Corrosion

Faraday’s law indicates that a constant corrosion rate of 1.0 µA/cm² corresponds to a uniform metal loss of bar diameter of 0.0232 mm per year (or 1.0 µA/cm² = 11.6 µm/year
metal loss of the bar radial) (Andrade et al., 1993). If the corrosion rate is assumed to be constant over time, then the remaining cross-sectional area of corroding main reinforcement after \( t \)-years \( A_i(t) \) can thus be estimated as:

\[
A_i(t) = \sum_{i=1}^{n} \frac{\pi}{4} \left( D_o - \Delta D(t) \right)^2 
\geq 0 \quad \Delta D(t) = 0.0232 \frac{i_{corr}}{2} \left(t - T_i \right) 
\]

where \( T_i \) is the time to corrosion initiation (years), \( n_b \) is the total number reinforcing bars and \( D_o \) is the original bar diameter and \( \Delta D(t) \) is the reduction of bar diameter at time, \( t \).

2.5.3.2 Due to pitting corrosion

Pitting (localized) corrosion is very intense form of corrosion in which a small area over the reinforcement length may suffer much greater loss of section than the rest of the reinforcing bar, Figure 2.17. For that reason, the measurements of the corrosion rate \( i_{corr} \) cannot be directly translated into the loss of cross-sectional area of the corroding bar in the same way indicated by Equation.

According to Gonzalez et al. (1995) the maximum penetration depth caused by pitting corrosion \( P_{max} \) can be 4 to 8 times of that caused by general corrosion. This conclusion was derived from results obtained from tests made on specimens of 125 mm long and have a bar diameter of 8 mm. The corrosion rate \( i_{corr} \) for general corrosion can be related to \( P_{max} \) at any time \( t \) via the ratio \( R = P_{max}/P_{av} \). \( P_{av} \) is the average penetration depth expected from general corrosion \( (P_{av} = \Delta D/2) \). Therefore, the maximum pitting depth in (mm) at any time \( (t) \) may be estimated as follows (Gonzalez et al., 1995):

\[
P_{max}(t) = 0.0116 i_{corr} \left( t - T_i \right) R
\]

In order to be able to use values of \( P_{max}(t) \) to estimate the loss of cross-sectional area of the reinforcing bar due to pitting corrosion, hence estimating the residual cross-sectional area of the reinforcement, an assumption regarding the shape of the pit has to be employed. Two such configuration has been provided by the literature, Rodriguez et al. (1997), Figure 2.18(a), and Val and Melchers (1997), Figure 2.18(b).
Figure 2.17 Reinforcing steel bar affected by pitting corrosion (Reis et al., 2005)

Figure 2.18 Pit configuration according to: (a) Rodriguez et al. (1996), (b) Val and Melcher (1997)

The residual cross-sectional area of a single corroding bar at any time $t$ for both configurations can be calculated as follows:

(a)

$$A_{\text{bar}}(t) = \pi (D_o - P(t))^2 / 4$$  \hspace{1cm} \text{Equation 2.25}

(b)

$$A_{\text{bar}}(t) = \begin{cases} 
\frac{\pi D_o^2}{4} - A_1 - A_2, & P(t) \leq \frac{D_o}{\sqrt{2}} \\
A_1 - A_2, & \frac{D_o}{\sqrt{2}} < P(t) \leq D_o \\
0 & P(t) > D_o 
\end{cases}$$  \hspace{1cm} \text{Equation 2.26}
where:

\[ A_1 = \frac{1}{2} \left[ \alpha_1 \left( \frac{D_o}{2} \right)^2 - a \left\{ \frac{D_0}{2} - \frac{P(t)^2}{D_o} \right\} \right] \]

\[ A_2 = \frac{1}{2} \left[ \alpha_2 P(t)^2 - a \frac{P(t)^2}{D_o} \right] \]

\[ a = 2 p(t) \sqrt{1 - \left( \frac{P(t)}{D_o} \right)^2} \]

\[ \alpha_1 = 2 \arcsin \left( \frac{a}{D_o} \right) \]

\[ \alpha_2 = 2 \arcsin \left( \frac{a}{2P(t)} \right) \]

Finally the remaining total cross-sectional area of the reinforcement subjected to pitting corrosion, after \((t-T)\) years of active corrosion, \(A_s(t)\) can be estimated as follows:

\[ A_s(t) = \sum_{j=1}^{n_{b0}} A_{bar(i)}(t) \geq 0 \]

\[ \text{Equation 2.27} \]

\[ \text{Equation 2.28} \]

where \(n_{b0}\) is the total number of longitudinal reinforcement bars.

As can be seen from the preceding presentation, estimating the residual reinforcement cross-sectional area in the case of pitting corrosion is highly dependant on the variation of the pitting factor \(R\). This indicates that the uncertainty associated with estimating the end of service life and hence the predicted time to first maintenance/repair for a structure in terms of the ULS will depend significantly on the assumed pit sizes. Considering the limited research studies dealing with the effect of variability of pitting corrosion on the safety of corroding structures in marine environments (Val, 2005), this thesis will consider investigating the influence of variability and spatial distribution of corrosion-induced pits (i.e. pitting corrosion) on the structure safety as one of its objectives. More information regarding statistics and distribution of corrosion pits will be given in Chapter 6. In this regard, this thesis needed to adopt a pit configuration for the calculation of the residual cross-sectional area of the reinforcement. The hemispherical pit configuration proposed by Val and Melchers (1997), Figure 2.18(b), was adopted. The reason for this was that, as shown by Figure 2.18(a), Rodriguez et al. (1997) configuration seems very conservative as a large proportion of the cross-sectional area of the bar is discarded as a result of the simplified approach. On the other hand, Val and Melchers (1997) configuration tends to calculate the exact area of the pit.
2.5.4 Corrosion rate ($i_{corr}$)

As can be seen from most of the relations and models given in Section 2.5, the corrosion rate is a key parameter for the plotting of the propagation stage and for determination of the residual cross-sectional area of both, the flexure and shear reinforcements and hence the residual capacity of the deteriorating structural member. Usually the corrosion rate is governed by the availability of water and oxygen at the steel concrete interface, the concrete quality and cover, temperature and humidity (Vu and Stewart, 2000). Andrade and Alonso (1996) have accumulated and studied corrosion rate data measured in the laboratory and on site. They observed that values of the measured corrosion rate that is below 0.1 μA/cm² are insignificant and does not seem to have a major effect on the service life of the structure. They also concluded that values between 0.5 to 5 μA/cm² causes about 5 to 25% loss of cross-sectional area of the reinforcement during 20-50 years of active corrosion. Values between 0.5 to 5 μA/cm² have been frequently measured on site with values in excess of 10 μA/cm² rarely reported (Gonzalez et al., 1995; Andrade and Alonso, 1996; Rodriguez et al., 1997; Alonso et al., 1998). Based on their observations, Andrade and Alonso (1996) proposed a classifications of the corrosion rate levels, shown in Table 2.9. Similar classifications proposed by other researchers are also indicated in the same table.

Regarding the variability of $i_{corr}$ Enright and Frangopol (1998a) provided some statistics on $i_{corr}$ based on measurements performed onsite. The information they provided are given in Table 2.10 where can be seen that the measured values are consistent with those reported by Andrade and Alonso (1996). However, while these statistics (mean value, standard deviation and the distribution type etc) are useful in describing the random nature of $i_{corr}$, information regarding systematic spatial variation has not been reported (Vu and Stewart, 2005).

Considering the importance of $i_{corr}$ as the key parameter which can influence the rate by which the reinforcements cross-sectional area is reduced, several attempts have been made to predict the corrosion rate where field data on the parameter are not available. In this regard, for a typical environment of an ambient relative humidity of 75% and temperature of 20 °C, Vu and Stewart (2000) suggested an empirical formula for the estimation of corrosion rate at the start of the corrosion activity $i_{corr(1)}$. The proposed model relates the corrosion rate to $w_c$ and $C_d$ as follows:
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\[ i_{\text{corr}(t)} = \frac{37.8(1 - w_c)^{-1.64}}{C_d} \]

Equation 2.29

According to Duprat (2007), results obtained from Equation found to be in agreement with corrosion rate measurements obtained from experiments performed by Gonzales et al. (1995) and with the average corrosion rate field measurements suggested by Enright and Frangopol (1998a), shown in Table 2.10.

Table 2.9. Reported corrosion rate classes

<table>
<thead>
<tr>
<th>Classification</th>
<th>Corrosion rate, ( i_{\text{corr}} ) (( \mu \text{A/cm}^2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negligible</td>
<td>-</td>
</tr>
<tr>
<td>Low</td>
<td>0.1</td>
</tr>
<tr>
<td>Moderate</td>
<td>1.0</td>
</tr>
<tr>
<td>High</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2.10 Expected corrosion rate (\( i_{\text{corr}} \)) values based on data from existing bridges (Enright and Frangopol, 1998a).

<table>
<thead>
<tr>
<th>mm/year</th>
<th>(( \mu \text{A/cm}^2 ))</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.013</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>0.025</td>
<td>1.0</td>
<td>0.2</td>
</tr>
<tr>
<td>0.076</td>
<td>3.0</td>
<td>0.3</td>
</tr>
<tr>
<td>0.127</td>
<td>5.0</td>
<td>0.4</td>
</tr>
<tr>
<td>0.254</td>
<td>10</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Meanwhile, and as indicated by Liu and Weyers (1998), there is strong evidence to suggest that corrosion rate reduces over time due to the formation of rust products which slow down the diffusion of irons away from the steel surface. To account for the time-dependant reduction of the corrosion rate, Vu and Stewart (2000) suggested that corrosion rate values obtained from Equation can be modified so that the corrosion rate time-dependent can be obtained as follows:
where $a_{cp}$ and $\lambda_{cp}$ are constants and their values are as given in Section 2.5.2.2.

Vu and Stewart (2000) corrosion rate model has been used by several other researchers to predict the life time safety performance of RC structures (Duprat, 2007; Stewart and Mullari, 2007; Val, 2007; Stewart and Suo, 2009).

In a real case assessment, it is highly desirable that values of $i_{corr}$ are obtained from site specific measurements taken from the structure that is under investigation. However, in many cases, as in this thesis, field data on $i_{corr}$ measurements may not be available, therefore, Equation and Equation maybe used to estimate the $i_{corr}$ for a given structure with a set of environmental conditions and material properties. In this thesis, Vu and Stewart (2000) corrosion rate model will be used to produce values of $i_{corr}$ that correlate well with the concrete quality and the chosen cover depth used for the considered structure. Care will have to taken when employing Equation and Equation so that produced values of $i_{corr}$ correspond well with the commonly field measured $i_{corr}$ data reported by the literature. The $i_{corr}$ produced in the way explained in this section will then be used to estimate the propagation stage and hence the end of service life of the structure under consideration. It is, therefore, obvious that neglecting the variability of the $i_{corr}$ will influence the estimated time to maintenance and repair.

2.6 CRITERIA FOR REPAIR AND MAINTENANCE

The service life of a structure was defined in Section 2.1. The models that can be employed to quantify each stage of the deterioration stages for structures affected by chloride-induced corrosion were described in the proceeding sections. However, the end of service life of a bridge or its component cannot be determined accurately unless the end of service life is clearly defined. The lack of quantifying definition to the end of service life hinders the engineer’s ability to evaluate and priotorise bridges/bridge components for maintenance/repair based on cost requirements. Traditionally, maintenance and repair is carried out in order to ensure that the structure is able to fulfil some design requirements throughout its lifetime (Engelund, 1997). The most important of these requirements is to
ensure that the bridge load carrying capacity does not deteriorate below a certain level often characterised by a reliability performance indicator which was introduced in Section 2.2.2 and will be further discussed in Chapter 3. However, usually in practice, repair or replacement of concrete structures is primarily based on visual inspection that records the signs of deterioration such as cracking, spalling rust stains etc (Li et al., 2004). Therefore, the time for maintenance can be related to the surface condition of the structure, e.g. when a given percentage of the surface of the structure shows signs of deterioration. This criterion is more related to the visual condition of the structure than to its ability to carry the imposed loads. The following sub-sections are dedicated to discuss these criteria. Criteria related to the load carrying capacity of the structure and hence to its ULS will be discussed in Chapter 3.

2.6.1 Corrosion initiation criteria

Repair of RC structures exposed to chloride environments can be deemed necessary if the chloride content at the level of reinforcement exceeds the $C_{cr}$ value and thus corrosion is expected to start soon after. For example, Weyers (1998) used criteria by which when 30% of the reinforcement steel have initiated corrosion (i.e. $C_{cr}$ is exceeded at the reinforcement level) then repair action must be taken. Similarly, Li (2004b) referred to criteria in which if 20% of the concrete area have shown that it has corrosion initiated, then repair may have to be considered. Li (2004b) anticipated that if 20% of the concrete surface area has its corrosion initiated, this is maybe an equivalent to saying that 1% of the concrete surface area has shown a sign of corrosion (i.e. cracks). This is inline with the 0.5-1.5% criterion which is being used in the Netherlands to decide on the repair of RC structures affected by chloride-induced corrosion (van Beek et al., 2003). However, this criterion may be considered too conservative as corrosion initiation in itself does not affect the load carrying capacity immediately. In addition, for a large RC structure/component, even the active corrosion at a single location may not affect the structure load carrying capacity as such. Therefore these criteria can be used to alert engineers/bridge owners to plan for repair/maintenance intervention for the future but no immediate action is expected to be undertaken. In this thesis an average value of 25% (i.e. an average value between the two values proposed by Weyers (1998) and Li (2004b)) will be used to define the time to first repair if the criterion of corrosion initiation is to be used.
2.6.2 Percentage of delaminated/spalled surface area criteria

As already mentioned in the previous section, for many agencies the criteria for time to repair/maintenance intervention is presented in terms of percentage of concrete surface area that is shown some corrosion-induced visible damage (i.e. cracking, delamination, spalling). Based on a survey response of 30 bridge and material engineers regarding maintenance treatments for bridge decks, according to Weyers et al. (1991), the survey responses indicated that the overlay of the entire surface of the bridge deck is appropriate when 2% to 4% of the deck surface has suffered from spalling. Based on the same survey results, another study showed that it is likely that the end of the functional service life for concrete bridge decks is reached when the percentage of the whole deck surface area that is delaminated, spalled and patched with asphalt ranges from 5.8% to 10% or that of the worst traffic lane ranges from 9.3% to 13.6% (Fitch et al., 1995). Based on the results of the survey, Fitch et al. (1995) found that this level varies from engineer to engineer and agency to agency then he suggested that delamination of 10% to 20% of a bridge deck area can be assumed to constitute a maximum acceptable level of deterioration for SLS.

It has to be mentioned that there have been cases where RC crosshead beams were replaced because the corrosion has caused 50% delamination of the concrete surface area (Reid, 2009). It was considered that if the crosshead beam was repaired with conventional repair methods this would had resulted in 90% removal of the concrete surface area which was deemed to be both costly and impractical for that particular case. Another reason for the replacement was that the reinforcements were severely damaged.

In the Netherlands, practical experience of the engineers involved in inspection, maintenance and repair of concrete bridges stated that bridges are repaired when 0.5 to 1.5% of the surface area of the bridge exhibits a visual sign of corrosion related concrete damage (van Beek et al., 2003).

It can be seen that there is a lack of agreement among institutions, practical engineers and researchers on the value of the percentage of spalled, delaminated or cracked surface area which can be used as a criteria to define the end of service life of the structure and thus the time for repair/maintenance intervention.
2.6.3 Crack width criteria

Cracks in concrete structures can have negative effects on important parameters such as permeability, rate of chloride ingress and thus on protection against reinforcement corrosion. Thus in many cases, the criteria for time to repair intervention are given in terms of the maximum corrosion-induced crack width. There may be a variety of opinions of what may constitute a maximum acceptable cracks width for a concrete structure. For example, most design codes and standards, such as the American Concrete Institute ACI 318 (1999), the British Standard BS 8110:1 (1997) and the Canadian Standard CSA A23.3 (1994), prescribe the maximum accepted crack width up to 0.4 mm for concrete structures (Newhook et al., 2002; Li et al., 2008). A maximum crack width between 0.3 and 0.4 mm deemed to be appropriate for durability LS by ACI 209 (1978) whereas a limit crack width of 0.8 mm has been recommended by Sakai et al. (1999) for serviceability (aesthetics) requirements.

Experimental results indicated that cracks due to reinforcement corrosion were joined together to create longitudinal cracking at crack widths between 0.3 and 0.5 mm (Vu et al., 2005). Andrade et al. (1993) suggested that the service life of a structure may be considerably reduced only if crack widths in excess of 0.3-0.5 mm are not repaired. This is in line with the previously mentioned maximum accepted crack widths recommended by varies standards. Clearly, the maximum acceptable limit for crack width depends on a number of factors including the local regulation, the owner policies and the intended function of the structure. In general, for the purpose of service life modelling, it may be reasonable to consider the maximum crack limit of 1.0 mm caused by corrosion activity as limiting serviceability intervention criteria. Thus for this thesis the end of the crack propagation stage (Sub-stage II indicated in Figure 2.1), the end of the structure service life in terms of serviceability requirements will be considered to have been reached when the width of corrosion-induced crack reaches 1.0 mm.

Based on the preceding review, the criteria proposed in this thesis for serviceability requirements maybe is better expressed as the surface area of the RC structure ($A_s/\%$) that is experiencing corrosion-induced crack greater than 1.0 mm in size. In order to be able predict this proportion; the structure needs to be hypothetically discretised into small sections, then each section is checked if it has experienced the maximum allowed crack width at any point in time. Eventually, the number of cracked elements (sections) is
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counted and the proportioned to the total number of element to determine $A_x \%$. Such procedure is possible by employing the Random Field theory which will be discussed in Chapter 3.

2.7 CONCLUSIONS

The main objective of this chapter was to review the available deterioration models and identify the appropriate models that can be used to model the service life of a corroding structure and thus for estimating the optimal time to maintenance and repair. Part of the task was to investigate the variability associated with the key deterioration parameters employed by these models. RC structures in chloride environments undergo two recognised deterioration stages, *Initiation* and *Propagation*. While there is a wide acceptance to the use of Ficks 2\textsuperscript{nd} law of diffusion to estimate the initiation stage, the propagation stage has been subjected to extensive research. Two sub-stages have been recognised for the propagation stage, crack initiation and the crack propagation.

The key parameters to describe the initiation stage are the $D_{app}$, $C_s$, $C_{cr}$, $C_d$ and $C_i$. The two variables $D_{app}$, $C_s$ have been shown to vary significantly with mean values differ from a structure to a structure, within a structure and from an environment to another. It was decided, therefore, that the statistical information of both parameters are to be obtained from a real a structure exposed to a marine environment. Ferrycarrig Bridge was, therefore, chosen to study the variability of the two parameters which were shown to be highly structure-dependant $D_{app}$, $C_s$, as will be seen in Chapter 4.

The time-dependant property of $D_{app}$ needs to be considered if the original material properties were not made from OPC concrete. The investigation carried out showed that the time to corrosion initiation is highly affected if $D_{app}$ was considered to vary over time for concretes made of PFA or GGBS. Values for the ageing factor $m$ which allows for the inclusion of the reduction of $D_{app}$ with time were proposed so that $m=0.2$, 0.4 and 0.6 for OPC, PFA and GGBS respectively.

Another key parameter for the estimation of the initiation period is $C_{cr}$. Again this parameter also showed a great variation. The wide scatter was attributed to the different methods by which this parameter is measured. It may be accepted that for the same material type and the same environmental condition, the variation of this parameter is
reduced. Based on the collected data, a Lognormal distribution with a mean value of 0.07 (Cl% per mass of concrete) and COV=0.25 were proposed for probabilistic modelling intended for this thesis.

For the propagation stage, several models were reviewed. Most of these models were mainly function of $i_{corr}$, $w_c$, $f'_c$ and $C_d$. Two models were chosen to represent the propagation stage. El-Maaddaway and Soudki (2007) model was selected to estimate the time to first cracking. For the second Sub-stage of the propagation, Vu et al. (2005) model was selected. This model is to describe the time period from the formation of the hair size crack until the size of the corrosion-induced crack reaches a 1.0 mm size.

Following the initiation of corrosion and during the formation of the cracks, the reinforcement cross-sectional area is being reduced by two mechanisms; by General (uniform) corrosion or by pitting (localised) corrosion. The pitting corrosion is more severe and causes the loss of the reinforcement cross-sectional area of several orders of magnitude as that caused by general corrosion. Both mechanisms will be considered in this thesis with investigating the effect of variability of the corrosion-induce pitting on the structure safety was highlighted as on of the tasks to be considered in this thesis. The pitting configuration proposed by Val and Melchers (1997) will be used to relate the loss of the reinforcement cross-sectional area caused by pitting corrosion to that caused by the general corrosion.

The literature has also shown the importance of the corrosion rate ($i_{corr}$) as the main parameter affecting the modelling of the propagation stage. It is desirable that values of this parameter are obtained from site specific measurements, however, in the case when such data are unavailable, the model proposed by Vu and Stewart (2000) can be used. The corrosion rate values predicted by the proposed model should be used in light of the typically measured values in real structures which have been reported to range from 0.1 to 10 $\mu$A/cm$^2$.

The criteria by which RC structures exposed to chloride-induced corrosion are repaired were investigated and found to differ from engineer to engineer and from agency to agency. However, for serviceability requirements, the percentage of the surface area that shows sign of corrosion related damage seems an appropriate quantitative indicator. For example, if the damage was defined such that corrosion is to be initiated, then the repair of the structure may be deemed necessary if 25% of the surface area has indicated corrosion.
initiation. On the other hand, if the damage was defined such that a significantly large cracks (>1.0 mm wide) to appear on the concrete surface, the indication for the time to first repair may be that when 1.5% of the surface area of the structure showed cracking of that size. The determination of these percentages can be made by using the Random Field (RF) theory which will be discussed in Chapter 3.
Chapter 3:

Random Field Theory
3.1 INTRODUCTION

In Chapter 2, models that have been proposed by various researchers to describe the deterioration process of Reinforced Concrete (RC) structures exposed to chloride induced corrosion were discussed. In the probabilistic approach, the inherent variability of the model parameters is considered by describing each model parameter as a random variable characterised by its Probability Density Function (PDF). However, by modelling each parameter as a random variable with a specified PDF mean ($\mu$) and standard deviation ($\sigma$), the spatial variability, i.e. the fluctuation of properties in space, of the model parameters is ignored. It may be accepted that some model parameters, such as the yield strength of the reinforcing steel, would exhibit very little spatial variability due to the high quality control that is implemented by the manufacturer. However, many material and geometrical properties, e.g. cover depth, diffusion coefficient, are expected to show considerable spatial variability due to the effect of environmental conditions and the inconsistency of the workmanship. Neglecting such sources of uncertainty is demonstrated in this thesis to have a significant impact on the evaluated safety and assumed whole life durability performance of the structure. Investigating the magnitude of this impact has been one of the main objectives of this thesis.

This chapter describes the fundamental concept of the theory that facilitates the inclusion of spatial variability in the assessment of a structural system. The area of knowledge that deals with modelling spatial variability is known in statistics as Random Field (RF) Theory. For that reason, the terms RF and Spatial Variability are used synonymously in the literature. Prior to introducing the RF theory, a brief description of the different types of uncertainty and probabilistic assessment approach in a structural system will be presented to provide background to RF-based modelling.

Also, in this chapter, cases from the literature which have dealt with the impact of including spatial variability on modelling the lifetime performances of RC structures against corrosion will be presented. Shortcomings and possible works that can lead to further improvements of the research carried out in this regard will then be identified to be addressed in this thesis.
3.2 TYPES OF UNCERTAINTIES

Generally, uncertainty in a structural system can be classified into three types, these are; inherent uncertainty, statistical and modelling uncertainties (Haldar and Mahadevan, 2000b). The uncertainty associated with spatial variability may in principle be classified as inherent uncertainty. Alternatively, it may be viewed as a fourth type of uncertainty due to its different modelling approach. The aforementioned types of uncertainties can be considered as of a quantitative type. Qualitative uncertainties are those who may come from: (1) the definitions of certain parameters, such as the definition of structural performance; quality; skill; experience of workers or environmental impact of projects, (2) human errors or (3) definitions of the interrelationships among parameters of the problem. Qualitative types of uncertainty are often dealt with using methods other than reliability analysis (i.e. Fuzzy Set Theory) which will not be considered in this thesis. A brief description of these quantitative uncertainties will be presented next.

3.2.1 Inherent uncertainty

Inherent uncertainty arises when a repeated observation of the same physical quantity does not give the same value, due to fluctuations in the environment, test procedure, instruments, observer etc. This type of uncertainty is often reduced by collecting a large number of data measurements; this provides good information about the variability of the measured quantity and establishes a quantifiable level of confidence in the value used in the design/assessment. However, in many cases the number of data measurements to be collected to reduce this type of uncertainty is limited by the availability of resources such as money and time.

3.2.2 Statistical uncertainty

As mentioned earlier, the number of data measurements may not be large enough to provide the precise information that is needed to describe the variability of the physical quantity of interest. In addition, different sample data sets will usually produce different statistical estimators. This type of uncertainty is classified as the Statistical Uncertainty. Statistical Uncertainty can be incorporated into a reliability-based analysis by treating the statistical parameters (i.e. mean (μ) and standard deviation (σ)) or parameters describing the
PDF) as random variables. Alternatively, the reliability analysis is repeated several times using different values of the parameters to investigate the sensitivity.

### 3.2.3 Modelling uncertainty

Computational models attempt to imitate the essential characteristics of a particular system's behaviour through the use of idealised mathematical formulations or numerical procedures. In the process, some aspects of the system behaviour are either ignored or simplified. For example, the effect of correlation between some of the variables may be ignored due to the lack of understanding of such correlation. In another example, for simplification, linear equations are used to describe some behaviour that is actually nonlinear. Such simplifications will then lead to differences in the results between the computational prediction and the actual behaviour of the response system. This is known as the Model Uncertainty or the Modelling Error. To include this form of uncertainty in a probabilistic assessment, past experiences or measurements quantifying the difference between the computational and the actual (experimental) results can be used to develop a statistical description of the model error (Haldar and Mahadevan, 2000b). The model error is typically included in the analysis as an additional random variable described by its PDF type (i.e. Normal, Lognormal, etc) and distribution parameters, i.e. $\mu$ and $\sigma$.

### 3.2.4 Spatial variability

Material and geometrical properties within a structural component are often considered homogeneous (perfectly correlated) (e.g. Val and Melchers, 1997) or randomly distributed (spatially uncorrelated) while in reality such properties usually exhibit some limited spatial correlation, Figure 3.1. That is to say, two samples taken very close to each other can have highly correlated properties and as the distance separating the two samples is increased, the correlation of their properties will decrease. Once the essential characteristics of such fluctuation are obtained, the uncertainty associated with spatial variability of the property of interest (i.e. surface chloride content, concrete compressive strength, cover depth, etc) can be accounted for by dividing the structure surface into number of elements (Vu and Stewart, 2005). Each element will be assigned a value for each of the modelled properties so that the correlation between different elements will depend on the distance separating them. The size of each element (hence the number of elements required) will depend on the intensity of the spatial fluctuation of the modelled property.
3.3 PROBABILISTIC APPROACH

In the context of structural engineering, the purpose of adopting a probabilistic approach is to develop models which can be used to help decision makers (i.e. structure owners, designers and consultants) to make optimal decisions regarding management of their network of assets by providing consistent/robust tools for the structure performance. Probabilistic models are generally based on physical models of the problem (i.e. Load vs. Resistance) and facilitate the inclusion of the uncertainty associated with the chosen models/model parameters. For example; Figure 3.2 shows two histograms for time to corrosion initiation \( T_i \) and time to crack propagation \( T_{cp} \). In this particular example, it can be shown that both \( T_i \) and \( T_{cp} \) follow a lognormal distribution and that the mean value of \( T_i \) is twice as much as \( T_{cp} \). Furthermore, \( T_i \) has greater variability around its mean value than \( T_{cp} \). It is, therefore, more useful to present these two results in this way rather presenting them as single deterministic values.

3.3.1 Reliability analysis

Reliability of an engineering system refers to the probability of survival or the complement of probability of failure (Melchers, 1999). The concept of a ‘Limit State (LS)’ is used to define ‘Failure’ in the context of structural reliability analysis. The term ‘Failure’ in the reliability analysis does not necessarily imply structural collapse, but in most cases it refers
to a situation when the performance of the structure exceeds a predefined limit. For example; if the LS to be considered is the initiation of reinforcement corrosion, then ‘Failure’ in this case may be defined as when the chloride content at the reinforcement depth exceeds a critical value. Thus the ‘LS’ is a boundary between desired and undesired performance of a structure as indicated in Figure 3.3 (Nowak & Collins 2000). In a standard case, when the LS function is not time-dependent, it may be written as (Melchers, 1999):

$$ Z = G(R, S) = R - S $$  

Equation 3.1

where: $Z$ is the LS margin, $G$ is the LS function, $R$ is the random variable representing the resistance and $S$ is the random variable representing the corresponding load effect or action.

![Figure 3.2 Histograms for: (a) time to initiation $T_i$ and (b) time to crack propagation $T_{cp}$.](image)

![Figure 3.3 Limit state Concept](image)
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$R$ and $S$ may be functions of other variables, deterministic or random. Both $R$ and/or $S$ may also be functions of time as schematically shown in Figure 3.4.

$$Z(t) = R(t) - S(t)$$  \hspace{1cm} \text{Equation 3.2}

Figure 3.4 Schematic representation of time-variant reliability problem (Stewart and Rosowsky, 1998a)

If in the LS functions $Z < 0$, then the failure state is reached. The probability that the LS has been violated ($Z < 0$) is referred to as the probability of failure ($P_f$) and can be obtained by solving the following equation (Melchers, 1999):

$$P_f = P[G(X) \leq 0] = \int \cdots \int_{G(X) \leq 0} f_x(x) \, dx$$  \hspace{1cm} \text{Equation 3.3}

where: $X$ is the vector of all relevant basic variables, $G(X)$ is the LS function which expresses the relationship between the LS and the basic variables, $f_x(x)$ is the joint PDF for the n-dimensional vector $X$ of basic variables.

In structural reliability, the probability of failure $P_f$ is also represented by the reliability index ($\beta$), which is expressed by (Melchers, 1999):

$$\beta = -\Phi^{-1}(P_f)$$  \hspace{1cm} \text{Equation 3.4}

where $\Phi(.)$ is the standard normal distribution function.
3.3.2 Calculation of the probability of failure ($P_f$)

The region of integration $G(X) \leq 0$ denotes the space of LS violation. Theoretically, the solution to Equation can be obtained through one of the following three methods (Melchers, 1999):

1. Direct analytical integration
2. Numerical Integration, such as simulation methods
3. By transforming the integrand into a multi-normal joint PDF for which some special results are readily available.

Except for some special cases, the integration of Equation over the failure domain $G(X) \leq 0$ cannot be performed analytically (Melchers, 1999). In addition, the LS equation contains functions of the basic variables that are too complicated for calculus to be used in the evaluation of their integrals. Methods 2 and 3 in this case become very practical choices for the evaluation of the failure probability. One common technique of type 2 methods is Monte Carlo (MC) simulation. As for type 3 methods, there are several classical techniques such as the First Order Second Moment method (FORM) and the Second Order Reliability Method (SORM), etc. These methods have been described in detail in many classic references (e.g. Ang and Tang, 1984; Melchers, 1999; Haldar and Mahadevan, 2000a).

The reliability problem which incorporates RF models (i.e. takes into consideration the uncertainty associated with spatial variability) usually results in a large number of random variables as will be shown in the course of this chapter. Furthermore, many of the involved model parameters are statistically and/or physically dependant, time-dependant and their mathematical models are of a nonlinear nature. Consequently, the LS function and the failure domain cannot be easily expressed or approximated by an analytical model that can be practically solved using type 1 or type 3 methods (Papadrakakis and Lagaros, 2002; Vu et al., 2005). For small scale problems FORM and SORM have been proved very efficient. However, as the number of random variables increases and the problem becomes more complex, MC simulation based methods are found to be more reliable in particular when the resulting LS margin, $Z$, is not normally distributed (Papadrakakis and Lagaros, 2002). MC simulation has been used by number of researchers to solve reliability problems where spatial variability is considered and has been demonstrated be a very powerful tool (e.g. Li,
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2004b; Vu et al., 2005). Therefore, the MC simulation method was chosen to solve the reliability problem to be formulated in this thesis. The basic principle of the MC method will be described in the following section.

3.3.3 Monte Carlo (MC) simulation

MC simulation is a technique which involves random sampling of variables to artificially produce a large number of experiments (or solutions of an algebraic equation) and observe the results. In the context of structural reliability analysis, this means, each basic random variable is randomly sampled from a specified PDF (Normal, Lognormal, Gumbel, etc). The LS function $G(X)$ is then checked; if the LS is violated (i.e. $G(X) \leq 0$), then the system has failed. The experiment is repeated many times, each time with a randomly chosen vector of values for the involved basic random variables. If $N$ trials are performed, the probability of failure is approximated by:

$$P_f = \frac{n[G(X) \leq 0]}{N}$$  \hspace{1cm} \text{Equation 3.5}

where the expression $n[G(X) \leq 0]$ denotes the number of trials $n$ for which $G(X) \leq 0$.

The ability of Equation to accurately estimate the probability of failure depends on the number of simulations $N$. Theoretically, the estimated $P_f$ will reach the true value as $N \to \infty$. However, the number of simulations $N$ that can be performed will be limited by the speed of the computer processor that is used. It has been reported (e.g. by Haldar and Mahadevan, 2000a) that the probability of failure obtained using MC simulation is almost the same as that obtained from the FORM method when the number of simulation is relatively large. One has to accept that there should be a ‘Trade-off’ between the accuracy desired and the time it takes for the computational problem to be solved.

3.3.4 Target reliability ($\beta_1$)

For the calculated probability of failure to be of significant engineering value it has to be compared to criteria by which the performance of the structural system under investigation can be rated. The criterion introduced to fulfil this requirement is known as the Target
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Reliability ($\beta_T$). Values for the minimum acceptable target performance level, $\beta_T$, for the design/assessment of different types of structure components have been explicitly specified by various national and international codes (e.g. ISO 2394, 1998; EN 1990:2002, 2002). These values were derived by a process of probabilistic calibration to different design codes. The specified $\beta_T$ values were intended to be applicable to a wide range of structural components. In most cases the selection of a target performance value depends on parameters such as importance of the structure, possible failure mode, etc. Table 3.1 presents values of the allowable maximum probability of failure specified by ISO code (ISO 2394-1998).

Table 3.1 Lifetime target Probability of Failure ($P_f$) according to ISO 2394 (1998).

<table>
<thead>
<tr>
<th>Relative costs of safety measures</th>
<th>Consequence of failure</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
<td>Some</td>
</tr>
<tr>
<td>High</td>
<td>0.5</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>Moderate</td>
<td>$10^{-1}$</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Low</td>
<td>$10^{-2}$</td>
<td>$10^{-3}$</td>
</tr>
</tbody>
</table>

The target reliability ($\beta_T$) indices specified by the Eurocode are given in Table 3.2. These values refer to a situation where the LS function is time-variant. Table 3.3 provides definitions for the reliability classes indicated in Table 3.2. The $\beta_T$ values are specified for structures with High, Moderate, and Low consequence for loss of human life. Once the structural probability of failure (or $\beta$) is computed, it can be compared with the $\beta_T$ value specified in Table 3.2 for the considered LS and the consequence determines compliance or violation.

Table 3.2 Minimum acceptable safety levels specified by Eurocodes (EN 1990-2002)

<table>
<thead>
<tr>
<th>Reliability Class</th>
<th>Minimum acceptable $\beta$ values ($\beta_T$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 year reference period</td>
</tr>
<tr>
<td></td>
<td>(=associated $P_f$)</td>
</tr>
<tr>
<td>CC3 (RC3)</td>
<td>5.2 ($1.0 \times 10^{-7}$)</td>
</tr>
<tr>
<td>CC2 (RC2)</td>
<td>4.7 ($1.3 \times 10^{-6}$)</td>
</tr>
<tr>
<td>CC1 (RC1)</td>
<td>4.2 ($1.3 \times 10^{-5}$)</td>
</tr>
</tbody>
</table>
### Table 3.3 Reliability classes specified by Eurocodes (EN 1990:2002)

<table>
<thead>
<tr>
<th>Consequence Class (Reliability Class)</th>
<th>Description</th>
<th>Examples of buildings and civil engineering works</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC3 (RC3)</td>
<td>High consequence for loss of human life</td>
<td>Grandstands, public buildings</td>
</tr>
<tr>
<td>CC2 (RC2)</td>
<td>Medium consequence for loss of human life</td>
<td>Residential and office buildings</td>
</tr>
<tr>
<td>CC1 (RC1)</td>
<td>Low consequence for loss of human life</td>
<td>Agricultural buildings (i.e. people do not normally enter)</td>
</tr>
</tbody>
</table>

### 3.4 CONCEPT OF RANDOM FIELD (RF) THEORY

In a classical reliability analysis problem, the basic model parameters are represented as single value random variables in which; for example, a variable such as the Surface Chloride Content \( (C_v) \) is assumed to have many values that vary around its corresponding mean value, variation across samples, independent of the area under consideration. Its variation in space, however, is often not considered. There are, therefore, two types of random variability need to be considered; variation across samples (random variables) and variation over space (spatial variability). Many existing studies (e.g. Val and Melchers, 1997; Val et al., 1998; Frangopol et al., 2001; Duprat, 2007) have neglected the random spatial variation within a structure or a structure component and take the structure or member as perfectly correlated with respect to the modelled parameters. This means that the material and geometrical properties are treated as being homogeneous across the structure. In this way, the result at one point may be applied to the entire structure or its member. In the context of maintenance, such a model would only suggest a total repair or replacement strategy of the structure or member, thereby ruling out, for example, the use of local repair as a possible rehabilitation option. An alternative approach is to treat random variables as Random Fields (RF) (Vanmarke, 1983). In this approach, the RF is represented by several random variables corresponding to several locations within the structure; two random variables whose locations are separated by a shorter distance are expected to have higher correlation than two random variables whose locations are separated by a longer distance. The method in which this can be achieved involves subdividing the structure into a series of elements and assigning a random variable to each element. The statistical correlation between any pair of random variables describing various elements can be represented through the use of some form of mathematical
function. Methods used for discretisation of the structure into RF elements and means for specifying the statistical correlation between elements will be presented next.

3.4.1 RF discretisation methods

The RF modelling requires the discretisation of the corresponding RF into sets of spatially correlated random variables. Therefore, the structure/structure member is divided into small elements in which spatial variation within an element is neglected. If the space of interest is of a large surface area (e.g. slab deck or large abutment wall) then the spatial variability can be represented by the use a Two-Dimensional RF model. In the case of a beam element (e.g. girder, crosshead beam) a One-Dimensional RF model may be more appropriate. Several methods have been proposed for the discretisation of RF into random variables (Der Kiureghian and Ke, 1988; Matthies et al., 1997). These methods may be classified based on their approach into three methods: (i) the Midpoint method, (ii) the Spatial Averaging method and (iii) the Series Expansion method. Due to their popularity among researchers, only the first two methods will be described with detail in the upcoming sections.

3.4.1.1 The midpoint method

Introduced by Der Kiureghian and Ke, (1988), in this method the value of RF over an element is represented by its value at the centre (midpoint) of that element. If the random variable was defined based on values at the nodes of the element rather than at its centre, the method is referred to as the Nodal-point method.

Assuming the RF \( X(s) \) is represented over the element \( i \) by a value taken at the centre of that element, Figure 3.5, so that:

\[
X_i = X(s_i) \quad \text{Equation 3.6}
\]

where \( s_i \) is the location of the centre of the \( i \)th element. The mean of the resulting random variable \( X_i \) is obtained as follow:

\[
E[X_i] = E[X(s_i)] = \mu \quad \text{Equation 3.7}
\]

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The correlation between any two elements $i$ and $j$ separated by distance $\tau$ can be obtained from the autocorrelation function $\rho(\tau)$ as follow:

$$\rho_{ij} = \rho(X_i, X_j) = \rho(\tau)$$  \hspace{1cm} \text{Equation 3.8}

The covariance between two midpoint values $X_i$ and $X_j$ for the RF $X(s)$ is expressed in terms of the autocorrelation function as follow:

$$\text{Cov}(X_i, X_j) = \sigma^2 \times \rho(\tau)$$  \hspace{1cm} \text{Equation 3.9}

where $\sigma$ is the standard deviation of the RF $X(s)$ which measures the intensity of the fluctuation or the degree by which $X(s)$ deviates from $\mu$, Figure 3.5.

The following advantages and limitations of the Midpoint method have been pointed out by Matthies et al., (1997):

1. The autocorrelation function is only a function of the distance between the centre of elements therefore the autocorrelation matrix can be easily computed
2. The resulting autocorrelation matrix is \textit{Positive Definite}, a matrix is said to be \textit{Positive Definite} if all of its 'eigenvalues' are positive (Wothke, 1993); this is important due to the fact that the autocorrelation function will need to be decomposed when generating random variables that are spatially correlated as will be shown in Section 3.4.4.
3. The size of each element has to be small enough to ensure that random properties are constant within an element, in other words the element size has to be smaller in comparison to the length where strong correlation between elements persist. This leads to a larger number of RF elements, hence an increase in the number of random variables resulting in a larger CPU computational time.
4. All RF elements need to be of an equal shape and size.
5. The method tends to over-represent the variability of the RF within each element.
3.4.1.2 The spatial average method

In this method, suggested by Vanmarcke (1983), the random variable is defined for each element using the local average of the random variable over an element. Consider the one-dimensional RF $X(s)$ in relation to two spatial segments $S$ and $S'$ as indicated in Figure 3.6, the RF for an element $i$ is given by:

$$X_i = \frac{1}{S} \int_{S/2}^{S/2} X(s) \, ds$$

Equation 3.10

where $S$ is the averaging interval. The expected value of $X_i$ is not affected by the averaging operation:

$$E[X_i] = E[X(s)] = \mu$$

Equation 3.11

where $\mu$ is the mean of the RF $X(s)$. The variance of the moving average process $X_i$ will be expressed as follows:

$$\text{Var}[X_i] = \sigma_i^2 = \gamma(S) \sigma^2$$

Equation 3.12

where $\gamma(S)$ is defined as the variance function of $X(s)$, which measures the reduction of the point variance $\sigma^2$ under local average. The variance function is dimensionless and is characterised by the following property:
\[
\begin{align*}
\gamma(0) &= 1 \\
\gamma(-S) &= \gamma(S) = \gamma(|S|)
\end{align*}
\]

Equation 3.13

where the covariance between any two local spatial averages \(X_i\) and \(X_j\) has been given by Vanmarcke and Grigoriu (1983) as follows:

\[
\text{Cov}(X_i, X_j) = \frac{\sigma^2}{2S} \left[ S_0^2 \gamma(S_0) - S_1^2 \gamma(S_1) + S_2^2 \gamma(S_2) - S_3^2 \gamma(S_3) \right]
\]

Equation 3.14

where the spatial intervals \(S_0, S_1, S_2\) and \(S_3\) are as defined in Figure 3.6.

In contrast to the Midpoint method, the spatial averaging method tends to under-represent the variability within each element (Der Kiureghian and Ke, 1988). The difficulties involved in this method can be summarised in two major points (Matthies et al., 1997):

1. The approximation of the non-rectangular elements may lead to a non-positive definite autocorrelation matrix.
2. The distribution function of the random variable \(X_i\) is difficult or even impossible to obtain except for Gaussian RFs.

The advantage of the method however, is that it yields accurate results even when a coarse mesh (larger elements size) is used (Der Kiureghian and Ke, 1988).
3.4.2 Models for the variance and the autocorrelation functions

The variance function $\gamma(S)$ is related to the autocorrelation function $\rho(\tau)$ as follows (Vanmarcke, 1977):

$$\gamma(S) = \frac{2}{S} \int_0^S \left(1 - \frac{\tau}{S}\right) \rho(\tau) d\tau \quad \text{Equation 3.15}$$

Vanmarcke (1977) presented several models of the autocorrelation function and the corresponding variance functions:

**Model (1):** The Triangular autocorrelation function:

$$\rho(\tau) = \begin{cases} 
1 - \frac{\tau}{a} & \text{for } |\tau| \leq a \\
0 & \text{for } |\tau| > a 
\end{cases} \quad \text{Equation 3.16}$$

Its variance function is:

$$\gamma(S) = \begin{cases} 
1 - \frac{|S|}{a} & \text{for } S \leq a \\
\left(\frac{a}{S}\right) \left(1 - \frac{a}{3S}\right) & \text{for } S > a 
\end{cases} \quad \text{Equation 3.17}$$

**Model (2):** The Exponential autocorrelation function:

$$\rho(\tau) = \exp\left(-\frac{\tau}{b}\right) \quad \text{Equation 3.18}$$

Its variance function is:

$$\gamma(S) = 2 \left(\frac{b}{S}\right)^2 \left(\frac{S}{b} - 1 + \exp\left(-\frac{S}{b}\right)\right) \quad \text{Equation 3.19}$$
Model (3): Autocorrelation function associated with a second order Autoregressive process:

\[ \rho(\tau) = \left[1 + \frac{|\tau|}{c}\right] \exp\left(-\frac{|\tau|}{c}\right) \]  
Equation 3.20

Its variance function is:

\[ \gamma(S) = 2\frac{c}{S} \left[2 + \exp\left(-\frac{S}{c}\right) - 3\frac{c}{S} \left(1 - \exp\left(-\frac{S}{c}\right)\right)\right] \]  
Equation 3.21

Model (4): The Gaussian (Square exponential) autocorrelation function:

\[ \rho(\tau) = \exp\left[-\left(\frac{\tau}{d}\right)^2\right] \]  
Equation 3.22

Its variance function is:

\[ \gamma(S) = \left(\frac{d}{S}\right)^2 \sqrt{\pi} \frac{S}{d} \text{erf}\left(\frac{S}{d}\right) + \exp\left(-\left(\frac{S}{d}\right)^2\right) - 1 \]  
Equation 3.23

A comparison between the proposed four models in terms of both the autocorrelation function and the variance function is shown in Figure 3.7 for \( \theta=2.0 \) m. As can be seen from the figure and as stated by Karimi (2001), there is no evidence to suggest the superiority of one particular model over the others in representing the actual computed correlation coefficients; each model therefore can be made to produce a reasonably good fit to the actual computed correlation coefficients.

### 3.4.3 The scale of fluctuation (\( \theta \))

Vanmarcke (1977) observed that \( \gamma(S) \) becomes inversely proportional to \( S \) at large values of \( S \) for each model and named the proportionality constant as the Scale of Fluctuation \( \theta \):

\[ \theta = \lim_{S \to \infty} S\gamma(S) \]  
Equation 3.24

As \( S \) becomes relatively large, \( S \to \infty \) the variance function will take the following form:
\[ \gamma(S) = \frac{\theta}{S} \]  

Equation 3.25

\[ \gamma(S) = \frac{2}{S} \left( \int_0^S \rho(\tau) d\tau - \frac{1}{S} \int_0^S \tau \rho(\tau) d\tau \right) \]  

Equation 3.26

Figure 3.7 (a) The autocorrelation models plotted against the lag distance (\( \theta = 2.0 \) m)  
(b) Variance functions plotted against the normalised averaging interval \( S/\theta \) (\( \theta = 2.0 \) m).

Therefore, for \( S \) much larger than \( \theta \), the \( \theta \) value can be estimated from the product of \( \gamma(S) \) and \( S \) as schematically indicated by Figure 3.8. For the four models listed earlier with regard to the autocorrelation function and its corresponding variance function, the previous observation was applied and the scale of fluctuation was determined in terms of the model parameters (by employing Equation) as follows:

- **Model (1):** \( \theta = a \)
- **Model (2):** \( \theta = 2b \)
- **Model (3):** \( \theta = 4c \)
- **Model (4):** \( \theta = \sqrt{\pi d} \)

One necessary condition for the existence of the scale of fluctuation can be defined by considering and rearranging Equation as follow:
If the definition of $\theta$ given by Equation (3.27) is applied, the condition for $\theta$ to exist is fulfilled when:

$$\lim_{S \to \infty} \frac{1}{S} \int_{0}^{S} \tau \rho(\tau) d\tau = 0$$

Equation 3.27

From Equation (3.27) and Equation (3.28) the scale of fluctuation $\theta$ can now be defined as:

$$\theta = 2 \int_{0}^{S} \rho(\tau) d\tau$$

Equation 3.28

Which also means that $\theta$ is equal to the area under the correlation function $\rho(\tau)$.

In a physical sense, the scale of fluctuation ($\theta$) represents the distance within which the material property shows relatively strong correlation from point to point (Vanmarcke 1977, 1983). For two points that lie within the distance $\theta$ the corresponding values of the material property are likely to be either both above or both below the mean $\mu$. In other words, the distance over which the material property is likely to stay either above or below its mean value is in the order of $\theta$. Materials with a large value of $\theta$ have very uniform properties across space, while materials with a small value of $\theta$ have more random properties across space.
For a continuous record of data, such as that can be obtained from a Cone Penetration Test (CPT) of soil property over the depth, Vanmarcke (1977) has proposed a simple but approximated method of determining the scale of fluctuation:

$$\theta = 0.8 \bar{d}$$  \hspace{1cm} \text{Equation 3.29}

In which $\bar{d}$ is the average distance between intersections of the fluctuating property and its mean as shown in Figure 3.9:

$$\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i$$  \hspace{1cm} \text{Equation 3.30}

While such a simple approximation method may be a good starting point when dealing with a continuous record of data, the method would not be useful when only a small set of data points is available, which is often the case for RC related data. Therefore, more rigorous and practical estimation methods are required. Other proposed methods for the estimation of $\theta$ will be covered in detail in Chapter 5 when data on key deterioration parameters are made available from the experimental work carried out as part of the this thesis, Chapter 4.

Figure 3.9 Approximation of the fluctuation scale of soil property (Vanmarke 1977)
3.4.4 Generation of random variables for RF modelling

The Midpoint method described in Section 3.4.1.1 is chosen for the study of this thesis to model the RF due to its ease of computation and, numerically stable results and its applicability to Gaussian and non-Gaussian RFs (Vu and Stewart, 2005).

It can be seen that the RF variable can be characterised by its mean ($\mu$), standard deviation ($\sigma$) and the autocorrelation function, $\rho(r)$. The mean and standard deviation of the RF variables can be obtained preferably from samples taken across the investigated area of the structure or from data that were collected from a structure of a similar nature. Data on the parameter $\theta$ are very scarce in the literature. Hence, such a parameter is essential for describing the autocorrelation function and hence for modelling the spatial variability of the investigated property. Whenever reported, data on $\theta$ for RC properties were either assumed based on engineering judgment (Vu and Stewart, 2005) or computed from poor and limited site measurements (Li, 2004b) as will be indicated in Chapter 6. For this reason, computing $\theta$ for the identified key deterioration parameters (see Chapter 2) from measurements taken from a real structure has been highlighted as a principal objective of this thesis.

When the MC simulation technique is used, the non-correlated standard Gaussian field is obtained through a procedure consisting of two steps:

1. random numbers uniformly distributed between 0 and 1 are generated and stored in a vector $U$. (Note that the number of elements of vector $U$ is equal to the number of RF elements).

2. in the second step, the non-correlated standard Gaussian field is obtained with:

$$Y = \Phi^{-1}(U)$$

Equation 3.31

where $\Phi(.)$ is the standard normal distribution function.

The randomly generated variables (vector $Y$) are non-correlated; therefore, they need to be transformed in such a way so that the resulting vector possesses a certain correlation between its elements. Therefore, a transformation operation is needed to convert the generated uncorrelated random variables into correlated random variables. The
transformation process can be carried out using decomposition matrix to be described in Sections 3.4.4.1 and 3.4.4.2.

The procedure for generating correlated RFs can be described as follow:

1) Assign an index to each location of the target observation.
2) Generate matrix $\tau$ where $\tau_{ij}$ is the distance between point $i$ and point $j$.
3) Compute the autocorrelation matrix $\rho$ from matrix $\tau$ using one of the autocorrelation functions $\rho(\tau)$ listed in Section 3.4.2.
4) Decompose matrix $\rho$ into matrix $C$ so that:

\[
\rho = CC^T \quad \text{(Choleski decomposition)} \quad \text{Equation 3.32}
\]

or

\[
\rho = \Theta \Lambda \Theta^T \quad \text{(Eigen decomposition)} \quad \text{Equation 3.33}
\]

where both Choleski decomposition and the Eigen decomposition will be introduced in Section 3.4.4.1 and 3.4.4.2 respectively.

5) Generate the uncorrelated Gaussian RF $Y$ (as by Equation)
6) Calculate the correlated Gaussian RF $V$:

\[
V = C \cdot Y \quad \text{Equation 3.34}
\]

or

\[
V = \Theta \Lambda^{1/2} \Theta \quad \text{Equation 3.35}
\]

3.4.4.1 Choleski decomposition method

Let $\rho$ be the matrix of correlation coefficients:

\[
\rho = \begin{bmatrix}
1 & \rho_{12} & \cdots & \rho_{1n} \\
\rho_{21} & 1 & \cdots & \rho_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{n1} & \cdots & \cdots & 1
\end{bmatrix} \quad \text{Equation 3.36}
\]

Since $\rho$ must be symmetric and positive definite, it can be factored into matrices that are the transpose of each other (see Equation). Where $C$ is a lower triangular matrix and its
transpose $C^T$ is an upper matrix. The procedure of how to compute $C$ is found in the literature (e.g. Nash, 1990; Weisstein, 1999) and is summarized as follow (Li, 2004b):

$$C_{11} = \rho_{11} = 1$$  
Equation 3.37

$$C_{ii} = \rho_{ii} \quad \text{(for } 2 \leq i \leq k \text{)}$$  
Equation 3.38

$$C_{ii} = \left( \rho_{ii} - \sum_{k=1}^{i-1} C_{ik}^2 \right)^{1/2} \quad \text{(for } 2 \leq i \leq n \text{)}$$  
Equation 3.39

$$C_{ij} = \rho_{ij} - \sum_{k=1}^{j-1} C_{ik} C_{jk} \quad \text{(for } 1 < j < i \leq n \text{)}$$  
Equation 3.40

$$C_{ij} = 0 \quad \text{(for } j > i \text{)}$$  
Equation 3.41

### 3.4.4.2 Eigen decomposition method

Also called the ‘Model’ decomposition (Gomes and Awruch, 2002), the Eigen decomposition takes the form indicated by Equation, where $\Lambda$ is the diagonal matrix containing the eigenvalues in descending order and the corresponding eigenvectors forming the columns of a matrix $\Theta$. Equation is valid when $\Theta$ is nonsingular; a square matrix is considered nonsingular if its determinant is nonzero (Weisstein, 1999).

Many computer mathematical applications such as MATLAB are now equipped with built-in functions that compute the decomposition matrices which allows users to compare the different decomposition methods or verify their results in the case of self-written computational codes.

### 3.4.5 Size of RF element

In general, when the reliability problem is formulated in terms of RF, the two following question need to be addressed (Haldar and Mahadevan, 2000b):

1. Should all the random variables be considered as RFs (spatially variable)?
2. If a random variable is considered as a RF, what is the best RF element size to be used?
The answer to the first question can be made through sensitivity analysis, the variables that have insignificant influence on the evaluated probability of failure can be modelled as random variables whereas variables that have a significant impact on the results may need to be modelled as a RF. For example, Mahadevan and Haldar (1991) recommended, based on results of limited number of numerical examples, that only random variables of a sensitivity index (with respect to a change in the computed safety index $\beta$, Equation) equal or greater than 0.3 should be considered as random variables. Such consideration should result in minimisation of the computational cost while maintaining a sufficient level of accuracy.

The size of RF element to be chosen depends on the magnitude of the scale of fluctuation and the relative correlation coefficient between two adjacent elements. In order to choose a meaningful size of RF element, the correlation coefficient between two adjacent elements should be small enough and considered statically independent for those two elements. The larger the element sizes the lower the correlation coefficient between two adjacent elements. Two adjacent elements are said to be perfectly correlated if the coefficient of correlation is equal to 1.0, i.e. element size is very small; they are considered to be uncorrelated if the coefficient of correlation is equal to zero, i.e. very large element size. Der Kiureghian and Ke (1988) and Engelund (1997) suggested that the size of element should be in the range of one-fourth to half of $\theta$. If the element size is very large, this leads to an underestimation of the influence of random variability, meanwhile too small elements sizes lead to a larger number of RF elements, hence an increase in the number of random variables resulting in a larger CPU computational time. It can be seen therefore that the selection an appropriate RF element size is a matter of ‘Trade-off’ between the accuracy desired and the computational time involved.

3.5 RF IN CORROSION OF REINFORCED CONCRETE

As indicated in Section 3.4, RF is a modelling technique which facilitates the inclusion of the uncertainty associated with spatial variability of properties involved in the analysis of a system performance. Ever since Vanmarcke (1977) published some work on the application of RF to soil property and later in 1983 presented a comprehensive review of the historical development of the theory behind the method, the RF method has found its way to many engineering disciplines including Fluid mechanics, Soil and Geotechnical mechanics and more recently to Structural engineering. However, the lack of reliable
experimental data on the parameter $\theta$ for the key deterioration parameters, which is necessary for modelling their spatial variability, has been the main cause behind the scarcity of research works in the area of RC concrete structures (Vu and Stewart, 2005). For this reason, obtaining site specific measurements on the parameter $\theta$ has been identified to be one of the principal objectives for the work of this thesis. This section will focus on presenting the limited RF related works that have been carried out in the subject of RC structures affected by chloride-induced corrosion.

### 3.5.1 Engelund (1997), Engelund and Sorensen (1998)

Engelund (1997) and Engelund and Sorensen (1998) implemented RF theory to evaluate the probability of the onset of corrosion in a RC concrete pier of a costal bridge. Key variables such as the apparent diffusion coefficient ($D_{app}$), the surface chloride content ($C_s$), the critical chloride content ($C_{cr}$) and the concrete cover depth ($C_c$) were all considered as a spatially variable and thus modelled as RF. The authors investigated the probability that corrosion has been initiated in a 1x1 m RC area representing a portion of a bridge pier located in a Danish marine environment. Applying the RF principles discussed earlier in this chapter, the authors divided the 1x1 m$^2$ area into small and equal RF elements so that the element size was smaller than $\frac{1}{2} \theta$ assumed for the property investigated ($\theta = 0.35$ m). By comparing the probability that corrosion has initiated in a 1x1 m$^2$ RC area to that initiated somewhere on the entire structure thus they concluded that the mean value for the time to corrosion initiation was found to be about 40% higher for the first case. This suggests that the size of the given structure plays an important role in the estimation of the probability of the onset of corrosion at any arbitrary location in the structure. The drawback of the work carried out by Engelund (1997) and Engelund and Sorensen (1998) with respect to spatial variability modelling is that the values for $\theta$ were not extracted from measurements obtained from a real structure. A second drawback was that the propagation period was simplified by assuming it follows a Normal distribution with a mean value of 15 years and COV of 0.25 and assumed independent for each RF element. This means that the propagation period was not predicted as a function of the material properties and the environmental conditions assumed for the case study and therefore the predicted time to maintenance may not be considered reliable.
3.5.2 Karimi (2001)

Karimi (2001) has presented a historical record of studies from different engineering disciplines on the application of RF theory and estimated the scale of fluctuation of $C_s$ from data collected from three different crosshead beams located in a coastal area. For the scale of fluctuation of the variable $D_{app}$, Karimi (2001) assumed its value based on suggestions by Engelund and Sorensen (1998) as will be indicated in Section 5.3.4. Karimi (2001) then used the obtained values of the $\theta$ to generate spatially correlated data as a part of the reliability analysis that aimed to estimate the probability of the onset of the corrosion. He found that the probability of the onset of corrosion was significantly underestimated when $C_s$ and $D_{app}$ were treated only as random variables and not as RFs (spatially variable).

The drawback of the research carried out by Karimi (2001) with respect to the parameter $\theta$, is that $\theta$ for the variable $C_s$ were determined based on only 3 sets of data. Furthermore, the parameter $\theta$ for $D_{app}$ was not estimated from the same sets of data, instead, the value suggested by Engelund and Sorensen (1998) was used. Clearly, the results of the RF analysis that does not consider reliable estimate of the parameter $\theta$ may themselves suffer the same lack of reliability.

3.5.3 Li et al. (2004) and Li (2004b)

Li et al. (2004) and Li (2004b) used RF methods to estimate the probability of the onset of corrosion and the percentage of the structure surface that showed sign of distress which can be the initiation of corrosion or the appearance of corrosion-induced cracks with a specified size limit. They applied three levels of spatial correlation of the variables which were considered as spatially variable; Low Correlation, Median Correlation and Full Correlation. They then applied a number of different repair strategies to assess the impact of each strategy on the time-dependant damaged surface area of the structure. Li (2004b) however, wrongly gave the term the scale of fluctuation (also called by Li the Correlation Length) to the model parameter $d$ (see Equation). Although this had no significant impact on his findings, clearly this has caused some confusion among researchers who they tried to quote and compare values of the fluctuation scale for spatially variable properties obtained by different authors, as discussed in Chapter 5. Li (2004b) also concluded that if the RF element size is less than $d/2$ ($d=\theta/\sqrt{\pi}$) largely variable and unstable results were
obtained. (Li used the Midpoint method and the Gaussian Autocorrelation to specify the correlation coefficient between the RF elements). Li (2004b) estimated the scale of fluctuation for two variables; $C_v$ and $D_{app}$ based on limited number of data points obtained from widely and irregularly spaced locations within the investigated Netherlands-based bridge structures. As a result, a wide range of $\theta$ values were obtained as will be shown in Chapter 5. It has to be mentioned that the data used by Li (2004b) were not extracted with the intention of estimating the parameter $\theta$, the implication of this will be explained in more detail in Chapter 5.

3.5.4 Stewart (2004)

Stewart (2004) applied RF methodology to investigate the effect of spatial variability of pitting corrosion on the reliability of a simply supported RC beam. The analysis considered various beam spans, code derived values for live to dead load ratios, bar diameters and number of bars in a given cross-section. Stewart found that probabilities of failure considering spatial variability of pitting corrosion were up to three times higher than probabilities of failure obtained from a non-spatial analysis (e.g. probabilities of failure based on the mid-span limit states only). He then concluded that the inclusion of spatial variability of pitting corrosion can lead to significant decreases in structural reliability for flexural RC members. It has to be mentioned that Stewart (2004) assumed values of the scale of fluctuation of $\theta=3.5$ m ($d=2.0$ m) for all the RF variables based on the limited work published by the earlier researchers.

3.5.5 Karimi et al. (2005)

Karimi et al. (2005) studied the sensitivity of the probability of onset of chloride-induced corrosion to the spatial variability of the diffusion coefficient $D_{app}$ using a 5 m long RC beam as an example. The beam was divided into 4, 6, 8 and 10 RF elements and for each case the probability of onset of corrosion was evaluated using MC simulation. They found that as the number of RF elements increased, the failure probability tended to converge to a limiting value. However, as the scale of fluctuation reduced, the number of the RF elements used in the analysis must be increased significantly for this convergence to occur. For example for $\theta=2.5$ m, the number of RF elements was 100 for the conversion to occur, and as $\theta$ was reduced to 0.1 m, the conversion did not take place with the same number of RF elements. It is recommended that such an approach be employed to estimate the
number of RF elements that should be used when RF based modelling is performed. This also indicates that as the scale of fluctuation becomes larger, a smaller number of RF elements will be required, hence less computational effort as a result. They also found that by neglecting the spatial variability associated with $D_{app}$, the estimated probability of onset of corrosion is significantly underestimated. Furthermore, the probability of onset of corrosion was found to be very sensitive to the correlation parameters assumed for $D_{app}$.

3.5.6 Vu and Stewart (2005)

Vu and Stewart (2005) used RF theory to predict the likelihood and extent of RC concrete corrosion-induced cracking. In their work they developed a methodology by which the probability that a certain percentage of a concrete surface will be subject to corrosion-induced cracking at any time during the service life can be quantified. They treated the concrete cover ($C_d$), water cement ratio $w_c$, corrosion rate density $i_{corr}$, chloride diffusion coefficient ($D_{app}$) and the surface chloride content ($C_s$) as spatially variable. Any other variable that depends on any of the earlier variables were also considered as spatially variable. In their conclusion they have demonstrated that the spatial variability of $C_d$ has lesser contribution to the final results than the spatial variability of the concrete compressive strength (which was treated as a dependant variable on the $w_c$ ratio) and $C_s$.

The surface area of the considered concrete structure was divided into $N$ statistically independent areas of size $A_i$, each of which was subdivided into $k$ number of square RF elements of size $\Delta$ as shown in Figure 3.10. The performance of the single independent area was obtained from RF analysis using the Midpoint Method and the Gaussian (Square Exponential) Autocorrelation Function (Equation) to generate the correlation coefficients between elements. Vu and Stewart (2005) then suggested that the damage for the larger area ($A$) can be obtained from the summation of the damages of the $N$ independent areas ($A_i$).

Vu and Stewart (2005) also proposed a formula by which the probability that at least $x\%$ of the concrete surface area has cracked at time $T$ years:

$$\text{Pr} \left[ d_{\text{crack}}(T) \geq x\% \right] = 1 - \Phi \left( \frac{x - \mu_{d_{\text{crack}}}(T)}{\sigma_{d_{\text{crack}}}(T)/\sqrt{N}} \right)$$

Equation 3.42

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where $\Phi(.)$ is the standard normal cumulative distribution function, $\mu_{d_{\text{crack}}}(T)$ and $\sigma_{d_{\text{crack}}}(T)$ are the mean and standard deviation of the proportion of the concrete surface with crack widths exceeding the limit crack width at time $T$ (years) for an independent area $A_i$ respectively and both can be obtained as follows:

$$\mu_{d_{\text{crack}}}(T) = \sum_{j=1}^{k} \frac{1}{k} \text{Pr}\left(t > T_{ij} + T_{cpj}\right) \times 100\%$$  \hspace{1cm} \text{Equation 3.43}

and

$$\sigma_{d_{\text{crack}}}(T) = \sqrt{\sum_{j=1}^{k} \left(\frac{1}{k} \mu_{d_{\text{crack}}}(T) - \text{Pr}\left(t > T_{ij} + T_{cpj}\right)\right)^2} \times 100\%$$  \hspace{1cm} \text{Equation 3.44}

where $T_{ij}$ and $T_{cpj}$ are the time to corrosion initiation and time to crack propagation of element $j$, respectively (i.e. Equation 2.3 or 2.7 for $T_{ij}$ and Equation 2.22 for $T_{cpj}$).

---

**Figure 3.10** Discretisation of total and independent area (Vu and Stewart 2005)

---

### 3.5.7 Darmawan and Stewart (2007)

Darmawan and Stewart (2007) investigated the influence of including spatial variability of pitting corrosion on the reliability of a prestressed concrete bridge girder. The limit states considered were flexural strength and serviceability (i.e. mid-span deflection). By
including the spatial variability of pitting corrosion in the reliability analysis, they showed that the probability of strength failure had increased by 10% when compared with a mid-span sectional analysis assuming pitting corrosion and by about 20% when assuming general corrosion, Figure 3.11(a). The comparison between a spatial and a mid-section analysis for the serviceability LS, Figure 3.11(b), showed that the effect of the spatial variability of pitting corrosion is more significant on the serviceability LS (i.e. mid-span deflection). This was attributed by the authors to the reduction on the beam stiffness as a result of the concrete cracking caused by the pitting-induced loss of strands.

Figure 3.11 Probability of failure for a spatial and a mid-section analysis. (a) strength probability of failure (b) serviceability probability of failure (Darmawan and Stewart, 2007).

3.5.8 Val (2007a)

Val (2007a) followed the same approach described by Stewart (2004) and studied the effect of corrosion of reinforcement on the flexural and shear strengths of an RC beam. Pitting and general corrosion were both considered with different corrosion rate values ranging from 0 to 3.0 \( \mu \text{A/cm}^2 \) representing different intensities of corrosive environments. Val (2007a) particularly focused on the influence of pitting corrosion of the shear links on the performance of the beam in shear. He ignored the initiation period and assumed that the 10 m long RC beam, which he used to demonstrate his approach, had already been subjected to corrosion activity.

By including the spatial variability of material and geometrical properties Val (2007a) investigated the possibility of the beam failure at a number of cross sections. He found that
the reduction of shear resistance due to corrosion of stirrups, in particular that caused by pitting corrosion, had a major adverse effect on the beam reliability.

3.5.9 Research shortcomings

From the review of the limited published works on the influence of spatial variability on the performance of RC structures affected by chloride corrosion, the following points can be observed:

1. The absence of data on the scale of fluctuation of major and key material deterioration parameters. The available data were obtained from the analysis of chloride profiles measured at locations that were not planned initially for this purposes. The available data on the scale of fluctuation were too little to be considered statistically sufficient.

2. The available literature indicated that focus was mainly given to the estimation of the probability of corrosion initiation or predicting the percentage of the structure that exhibited some level of corrosion-induce damage (e.g. SLS) with little work on the effect of spatial variability on the load carrying capacity (e.g. ULS) with the exception of Stewart (2004), Darmawan and Stewart (2007) and Val (2007a).

3. When the ULS was considered, in all cases, the initiation period was ignored and the structure assumed to have started its corrosion activity. This simplification may have a considerable influence on the estimated reliability of the structure under investigation. In the context of spatial variability, a RF element would have its corrosion started when the chloride content in that element reaches a critical value ($C_{cr}$), hence different RF elements will have different time to corrosion initiation. The simplified approach such as that used by Stewart (2004), Darmawan and Stewart (2007) and Val (2007a) would suggest that all RF elements would start corroding at the same time, and the spatial variability of the corrosion rate parameter ($i_{corr}$) is modelled independently from the time to corrosion initiation. Clearly this leads to misrepresenting the probability of failure by not including the spatial variability of the variables influencing the time to corrosion initiation in the process.

4. In the very few occasions when the influence of spatial variability of the material properties on the load carrying capacity was investigated, code-derived or deterministic load models were employed. This implies that the uncertainty
Chapter 3: Random Field theory

associated with load effect (the $S$ part of the basic reliability problem) had not been properly considered. A better approach is to use load models that are derived from real traffic data.

3.6 CONCLUSIONS

In this chapter the fundamental concept of RF theory and its application to structural engineering were described. Brief description of different types of uncertainty, probabilistic approaches and reliability methods were given at the start of the chapter. Sources of spatial variability in RC structures were also highlighted. Two major RF discretisation methods which are fundamental within the procedure of RF reliability based modelling were explained in detail including advantages and disadvantages of both methods. As a result, the Midpoint method was selected. In this chapter the role of the autocorrelation function, variance function and the scale of fluctuation with relation to RF modelling were demonstrated.

The chapter also presented the major published works which have dealt with modelling the influence of spatial variability on the performance of RC structures exposed to chloride-induced corrosion environment. A number of significant shortcomings of the available literature were highlighted. Among these shortcomings was the scarcity of information regarding the scale of fluctuation ($\theta$), which is an essential for modelling spatial variability of a physical quantity, for key deteriorating parameters such as $C_s$ and $D_{app}$.

Based on the findings of this chapter, the following were identified as important objectives for the work to be carried out for the current thesis.

- Perform an experimental study using site specific data to estimate values for the parameter $\theta$ particularly for two of the most influential deterioration parameters ($C_s$ and $D_{app}$) as identified in Chapter 2. Significantly, a real bridge, Ferrycarrig Bridge discussed in Chapter 4, was available for measurement.
- In contrast to the majority of published work with respect to spatial variability modelling in the field of RC corrosion, the work to be performed in this thesis will focus on the effect of spatial variability on both, the surface condition of the structure as well as on the load carrying capacity of the assessed structure. It was
also decided that both the corrosion initiation and the corrosion propagation be considered in the probabilistic model to be developed.

- It was decided that for a reliable modelling of the load effect to be considered, the estimation of the load effect in this thesis will have to be made based on realistic traffic data-derived load models. This will be discussed in Chapter 6.
Chapter 4: Experimental Work
Chapter 3 highlighted the importance of including the uncertainty associated with spatial variability (Random Field, RF) of the deterioration variables in service life modelling of structures exposed to chloride-induced corrosion environment. Furthermore, the information presented in Chapter 2 with regard to the parameters used to model material deterioration due to chloride ingress, have shown widespread variations, in particular the Surface Chloride Content ($C_s$) and the Diffusion Coefficient ($D_{app}$). This indicates that there is a need to rely more on site specific statistical data if a reliable service life performance prediction is to be made for a particular structure or a group of structures of the same condition. The statistical information necessary for any analysis that intends to consider spatial variability is: the mean value ($\mu$), standard deviation ($\sigma$), and the Scale of Fluctuation ($\theta$). While data on the first two statistical parameters ($\mu$ and $\sigma$) for $C_s$ and $D_{app}$ may be available in the literature for use with caution due to their widespread variation, data on $\theta$ is very scarce (Vu and Stewart 2005). This is due to the lack of knowledge with regard to the influence of spatial variability of the concrete material properties on the prediction of structure service life performance. In the few existing spatial variability-based published works, values for the scale of fluctuation of the deterioration variables, were assumed either based on widely spaced and limited data points (i.e. Li, 2004b), or entirely based on engineering experience (Vu and Stewart, 2005). The main objective of the testing undertaken in this thesis was to obtain site specific data on the scale of fluctuation and statistical parameters for $C_s$ and $D_{app}$ variables.

In the experimental study, values for $C_s$ and $D_{app}$ were determined by curve fitting Fick's 2\textsuperscript{nd} law of diffusion to experimentally obtained chloride profiles as discussed in Section 2.3.2. The chloride profiles were obtained from 45 concrete cores extracted from the 27 year old Ferrycarrig Bridge; a Reinforced Concrete (RC) bridge located in a marine environment on the Wexford-Dublin N11 road in the south east coast of Ireland, Figure 4.1. The concrete cores were extracted from the bridge crosshead beams prior to its rehabilitation which took place in the summer of 2007. Five chloride profiles and hence five values for each variable, $C_s$ and $D_{app}$, were obtained from each crosshead beam face. At the end of this chapter the statistical parameters for $C_s$ and $D_{app}$, including their statistical distribution type, will be presented. These will be used as input parameters for the reliability based model to be developed in Chapter 6. The distances between the
locations of the extracted cores were recorded so the fluctuation scale of both variables can be studied using methods to be described in Chapter 5.

Figure 4.1 Ferrycarrig Bridge; (a) Map view, (b) Photograph indicating piers locations

This chapter provides details of Ferrycarrig Bridge, results from previous inspection reports, the testing procedure for the chloride content determination using Potentiometric titration as well as a detailed discussion of the results obtained from the 45 chloride profiles. At the end of this chapter, the following information will be presented; (i) statistical parameters ($\mu$, $\sigma$ and probability distribution types) for $C_s$ and $D_{app}$ variables to be used as input parameters for the RF model which will be developed in Chapter 6; (ii) data on $C_s$ and $D_{app}$ accompanied by distances between their locations (within the crosshead beams) so the parameter $\theta$ for both variables can be determined using methods
to be discussed in Chapter 5. Due to the lack of actual field data on the parameter $\theta$ for $C_s$ and $D_{app}$ properties, the data to be obtained from the current experimental work is very significant with respect to predicting the lifetime performance of deteriorating structures.

4.2 HISTORY OF FERRYCARRIG BRIDGE

4.2.1 Structure description

Ferrycarrig Bridge carries the N11 National Primary road over the River Slaney at Ferrycarrig in County Wexford on the south east coast of Ireland, Figure 4.1(a). The bridge was constructed in 1980 and is located about 3.5 km away from the nearest main coast line. Its east side is in direct contact with the estuary part of the river. The bridge consists of eight spans of precast, prestressed beams with a reinforced in-situ concrete infill, supported on intermediate piers with abutments at both ends. The bridge is continuous over all the piers except the middle pier where an expansion joint has been provided in the deck. The bridge has an out-to-out width of 15.8 m and its eight spans are regularly spaced at 15.7 m which makes up an overall bridge length of 125.6 m. The bridge piers are made up of two separate concrete walls capped by reinforced concrete crosshead beams on which the bridge deck and the precast beams rest as shown in Figure 4.2. The bridge crosshead beams are 1.2 m deep, 1.0 m wide and 15.24 m long, Figure 4.2(b). The size of the crosshead beams makes them an ideal choice for the study of one-dimensional fluctuation properties of the concrete deterioration variables due to their relatively large length/height ratio. In 2004 the bridge was subjected to its first Special Inspection following the findings of an earlier Principal Inspection which was carried out in the year 2002 (Counihan et al., 2005). With regard to the volume of traffic, the Special Inspection report indicated that, on average, the bridge carries an annual daily traffic of 9127 vehicles of which 91% are light vehicles and 9% are heavy vehicles.

4.2.2 Bridge environment

Due to its location at the River Slaney and being in direct contact with the estuary part of the river, the bridge has been exposed to an atmospheric marine environment with high concentrations of both wind borne and river borne chlorides. The inspection report ruled out the possibility of the extensive use of the de-icing salts being the main source of
chlorides. It is likely that using salt for road de-icing purposes would not have been very common practice up until the mid 1990's (Enright and Frangopol, 2000). This would tend to indicate that most of the chlorides would have originated from the atmospheric marine environments and are not from the use of de-icing salts. This was confirmed by the fact that the level of chloride concentration determined during the special inspection survey carried out in 2004 on the bridge deck and infill concrete, were too low to be attributed to the extensive use of de-icing salts (Counihan et al., 2005).

Figure 4.2 Ferrycarrig Bridge, (a) Elevation view, (b) Cross section A-A

The results presented by the special inspection report (i.e. Counihan et al., 2005) indicated that all the concrete powder samples collected during the inspection survey, with the exception of few, showed a distinct concentration gradient decreasing rapidly through the depth of the concrete, as it will be shown later in Section 4.2.3.1. This would suggest that the measured chlorides have been introduced from an external source and are not part of
the original design mix and hence are not chemically bound within the concrete matrix. This means that the chloride ions are available to move freely towards the reinforcement and cause the initiation of corrosion. These findings were later supported by evidence from the chloride analysis results obtained from the current experimental study as will be seen from Section 4.4.

4.2.3 Condition of the bridge

In 2002, a Principal Inspection of the bridge was undertaken in accordance with the Eirspan Bridge Management System procedures (Counihan et al., 2005) adopted by the National Road Authority (NRA) of Ireland. All bridge components were visually examined with all defects noted. In general, the bridge was found to be in fair condition with few significant defects recorded. An exception to this was the crosshead beams and the south abutment where extensive cracking was observed. These cracks varied in width from hairline to 3.5 mm with the larger cracks being located at the exposed ends of the crossheads. The cracking was found to form a definite pattern, but on initial inspection could not be attributed to any specific structural or deterioration mechanism. There was extensive staining and leaching of the concrete associated with this cracking, Figure 4.3. Due to the presence of extensive cracking in the crosshead beams, a Special Inspection of the substructure was deemed to be required. Some of the relevant findings of the Special Inspection report will be presented in the following sections.

4.2.3.1 Bridge special inspection results

Following the recommendation of the Principal Inspection report, a Special Inspection of Ferrycarrig Bridge was carried out in the year of 2004. This was the first detailed inspection to be carried out on the bridge since its construction. The aim of the Special Inspection was to determine the cause and extent of cracking, leaching and staining on the crosshead beams and abutments. With more detailed survey information available, the Special Inspection report re-established that water staining, leachates and algae growth were observed on all crosshead beams and were concentrated on the outer cantilever exposed section of the crossheads, Figure 4.3. The unexposed internal sections of the crosshead beams were found to be relatively dry. The report also indicated that the most severe deterioration of the bridge is coincident with the un-waterproofed footways which
were exposed to the extremes of the weather. Some observations and test results provided by the Special Inspection report are discussed next.

Figure 4.3 Cracking, leaching and staining on the crosshead beams.

**Crack mapping survey**

Substantial cracking of the concrete in the crosshead beams was observed and measured to range between 0.1 mm and 3.5 mm. The cracking was found not to follow the line of the reinforcing bars which would suggest that reinforcement corrosion was not the cause of the cracking. A number of 50 mm diameter cores were extracted from the face of the crosshead beams to determine the depth of the cracks. The cores were drilled to a depth of 90 mm and found that the cracking did not taper and was evident for the full depth of the core. Initially, it was tabulated that the restraint provided by the tubular steel piles to the shrinkage of the in-situ concrete in the pier crossheads in the transverse direction could be a possible source of the observed cracking. At this stage, a deterministic structural assessment of the bridge was performed. Finite Element-based deterministic assessment of the bridge was carried out for two purposes: (a) the calculation of ultimate load effects for superimposed dead and live loads in order to determine the ultimate load effects on the structure and (b) the calculation of the serviceability and working load effects for superimposed dead loads, live loads, creep and shrinkage, temperature and differential
settlement. The latter was used to develop an understanding of the possible reasons for development of cracking observed at the pier crosshead beams.

The creep and shrinkage analysis of the structure concluded that the observed cracking in the pier crosshead beams was due to a lack of reinforcement to resist the Serviceability Limit State (SLS) stresses (shrinkage, thermal, creep). In addition, there was insufficient reinforcement to resist the applied Ultimate Limit State (ULS) torsional moments.

**Concrete dust samples**

A number of concrete dust samples were taken from the bridge for chloride ion and cement content analysis. The dust samples were extracted using a 25 mm diameter masonry bit and were collected in sealable plastic bags. The dust samples were collected at depths of 5-30 mm, 30-55 mm and 55-80 mm from 22 different locations within the crosshead beams. The chloride content value at these depths showed significant variation in particular at depths closer to the concrete surface, as can be seen from Figure 4.4. This was expected considering that the concrete zone closer to the surface is more affected, than the internal zones, by environmental factors known to influence the concentration of chlorides in concrete such as the wash-down effect caused by the driving rain as will be discussed in Section 4.4.1.

![Figure 4.4 Chloride contents at three different depth ranges obtained from 22 locations within crosshead beams (test undertaken in 2004).](image-url)
While mathematically possible, it is not very meaningful to obtain the values of $C_s$ and $D_{app}$ by curve fitting to Fick's 2nd law using only three data points of chloride contents as such result will not be very reliable. Another setback with regard to making beneficial use of the reported chloride content data is that the dust samples were extracted from a maximum of two locations per crosshead beam face, which are not enough data points to provide information on the fluctuation behaviour of the chloride content at different depths along the beam. This is a good example of how spatial variability has not been recognised as a possible source of uncertainty by the current assessment and testing engineering practices.

The report has also indicated that the more exposed west facing surfaces of the crosshead beams exhibited higher chloride content than the east facing ones; this was expected given the bridge orientation with regard to the prevailing south westerly winds as will be discussed in Section 4.4.5. As for the cement content, the result of the chemical analysis indicated that the cement content in the concrete dust samples taken from the crosshead beams was found to be around 16.4%. This value suggests that a higher concrete strength than that specified for the crosshead beams concrete was used. The concrete specified on the available drawings for the crosshead beams was of grade 30 which is typically produced using cement content in the order of 14% (Counihan et al., 2005).

**Carbonation survey**

As indicated in Section 2.2, the protective film around the reinforcement can be destroyed by the presence of chlorides or carbonation. Therefore, in many investigation practices the depth of carbonation front would be determined to establish if the breakdown of the protective film would have been caused by carbonation, chlorides or both (Bertolini, 2004). For the Ferrycarrig Bridge, the carbonation penetration depth was measured at several locations in the bridge, mainly in the vicinity of the holes drilled for the dust samples. The concrete surface was chipped away at four locations around each hole to expose the 'fresh' hardened concrete as required by the carbonation depth test procedure (BS 1881:201, 1986). The broken surface was then cleaned with an air pump and bottle brush and sprayed with the chemical indicator Phenolphthalein. In this test, noncarbonated areas of the concrete turn purple while carbonated areas remain colourless. The maximum penetration depth was measured as a distance from the concrete surface to the boundary of the uncoloured zone at each location around the hole. In general the level of carbonation penetration were found to be very low and well below the typical cover to reinforcement...
Chapter 4: Experimental work

with a maximum measured penetration of 17 mm determined in the crosshead beam in Pier 6. Based on these results, the risk of corrosion due to carbonation-induced depassivation for the bridge condition (at the time) was ruled out.

**Covermeter survey**

A covermeter was used to locate and record the depth of cover and assess the spacing and size of reinforcement. The recorded results of the cover meter showed that the reinforcement cover was ranging from 22-63 mm with an average cover of 48 mm. For the prestressed precast beams however, the survey indicated that the cover to the shear links varies from 0 to 25 mm with an average cover depth of 17 mm. This was well below the values specified on the drawings of the prestressed beams of 33 mm. This indicates that the cover depth ($C_d$) depends highly on the quality of the workmanship and varies significantly within the structure or its component (spatially variable) which implies that the time to corrosion initiation ($T_i$) also spatially variable.

4.2.4 Bridge rehabilitation (2007)

Based on the findings of the Special Inspection and the Structural Assessment reports, the rehabilitation of Ferrycarrig Bridge was deemed to be necessary. A contract was awarded to a qualified contractor to carry out the following tasks: (i) extensive concrete repair to all crosshead beams, (ii) re-waterproof the existing bridge deck including footways, (iii) replace the existing bearings at the central pier and (iv) repaint the existing parapet system. With regard to the crosshead beams, each was assigned a different repair strategy with a monitoring system to be installed so that their relative efficiency could be studied. Prior to the repair and instrumentation work which took place in the summer of 2007 and to obtain field data to achieve the objectives highlighted in Chapters 2 and 3 for this thesis, 70 concrete cores (50 mm in diameter) were extracted from Ferrycarrig bridge 7 crosshead beams. Ten concrete cores, five from each face, were extracted with their coordinates within the crosshead beam face recorded so that the fluctuation properties of the deterioration variables of interest can be studied in the future. The procedure used for the concrete cores processing, concrete grinding and chloride determination will be described in the upcoming section.

Adjacent to the locations of the extracted cores, concrete dust samples (similar to those taken from the bridge in 2004) were taken for chloride content analysis. In total, 60 holes
were drilled for the purpose of taking dust samples at three major depths (0-30 mm, 30-55 mm and 55-80 mm) for chloride content determination. The dust samples were sealed in plastic bags and sent to a specialised accredited laboratory for processing. The average values of the chloride contents are plotted in Figure 4.5 alongside the results obtained from the tests carried out in 2004 (i.e. the average values in Figure 4.4). As can be seen from the figure, the variation of the data is larger in the case of the 2007 test results as compared with that of the 2004 results. This could be attributed to the larger sample size in the case of the first (22 samples in 2004 vs. 60 samples in 2007).

![Figure 4.5 Average chloride contents at three different depths undertaken in 2004 (22 samples) and in 2007 (60 samples).](image)

To investigate whether there is a significant difference between the averages of the chloride contents (2004 vs. 2007 test results) for each depth; a statistical t-test was performed (Ang and Tang, 1975). The null hypothesis that there is a significant statistical difference between the two averages was not rejected (at a significance level of $\alpha = 0.05$) for all three depth ranges with a significance value (p-value) of 0.72, 0.40 and 0.19 respectively (if p-value > $\alpha$, the null hypothesis cannot be rejected). This means that the difference between the average of the data collected in 2004 and the data collected in 2007 is not statistically significant for any of the three depths. Furthermore, a Paired t-test (Ang and Tang, 1975) was performed to investigate if the increase in the chloride contents at all three depths due to the effect of age of the structure at the time of testing is significant. The result of the Paired t-test indicated that this increase is not statistically significant at a
significance level of $\alpha = 0.05$ (p-value $= 0.145$). This implies that the observed increase in the average chloride content values may not be attributed to the effect of time as the main influencing factor. This is expected, since a significant change in the chloride content at a certain depth from the surface of the concrete is usually observed either at a very early age of the structure when the rate of chloride penetration is typically high, or when the time lag between the two tests is more than just a few years.

In addition to the concrete cores extracted for the purpose of chloride analysis and the dust samples which were taken at the indicated three major depths from varies locations within the crosshead beams, another 14 concrete cores (one from each crosshead face), 100 mm in diameter, were extracted to establish the compressive strength of the concrete. The compressive strength test results, Table 4.1, showed that the crosshead beams have an average concrete compressive strength of 69 MPa. The measured strengths may initially indicate that the crosshead beams concrete are of higher strength than what was originally specified ($f_c'(28\text{ days}) = 30$ MPa). However, it is expected that the concrete strength would increase over time, e.g. Al-Khaiat and Fattuhi (2001) showed that the 30-years strength of OPC concrete was 2.3 times the 28-days strength which is closely inline with the current case. The measured concrete strengths can be useful indicator of the concrete quality with respect to its resistance to chloride penetration. For example, $D_{app}$ found to be inversely proportional to the concrete compressive strength; because concrete with higher strength has smaller pore volume and size due to their low water/cement ratio and/or higher cementitious material contents so that chlorides penetrate at a much slower rate (Jau and Tsay, 1998).

### 4.3 SAMPLE PREPARATION AND CHLORIDE ANALYSIS

The 50 mm diameter concrete cores were diamond drilled from the 7 crosshead beams using a drill mounted bit. The concrete cores were immediately sealed in plastic bags to avoid moisture evaporation, stored and later brought to the Trinity College Dublin (TCD) Civil Engineering laboratory for processing and chemical analysis. Some of these cores have suffered mechanical damage during (before or after) the extraction process and it was not operationally possible to use them for chloride profile determination. Furthermore, if one or more of the cores were damaged, the other cores belonging to the same crosshead beam face were also excluded from the testing program, as it was already decided that 4 data points or less will not be reliable enough to carry out the scale of fluctuation analysis.
Eventually, only 45 cores out of the 70 cores were identified and later processed for chloride profile determination at the TCD laboratory.

Initially, the cores were planned to be drilled along the crossheads at about the same distance from the bottom soffit of the crosshead beams, however, this was not always manageable due to on-site operational difficulties. The distances between the cores locations, their location with regard to both ends of the crossheads and with regard to the soffit of the crossheads were recorded in the manner shown in Figure 4.6. The exact location for each core is indicated in figures plotted in Appendix B. For identification, all cores were marked to indicate their location and orientation with respect to the crosshead beam surface they were extracted from. For this purpose, the following symbolizing procedure was adopted for the cores marking; Pi-Nj and Pi-Sj. Where P stands for ‘Pier’, i is for the pier (crosshead) number (i=1:7), N for North face, S for South face, and j is for the core number within the same crosshead face, (j=1:5). In Figure 4.6; the notations dis (j, j+1) refer to the horizontal distance between core j and core j+1 in metres.

Table 4.1 Compressive strength of concrete cores extracted from Ferrycarrig bridge in 2007

<table>
<thead>
<tr>
<th>Core extracted from crosshead beam</th>
<th>Compressive strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1-N</td>
<td>76</td>
</tr>
<tr>
<td>P1-S</td>
<td>73.5</td>
</tr>
<tr>
<td>P2-N</td>
<td>85</td>
</tr>
<tr>
<td>P2-S</td>
<td>65</td>
</tr>
<tr>
<td>P3-N</td>
<td>56</td>
</tr>
<tr>
<td>P3-S</td>
<td>73</td>
</tr>
<tr>
<td>P4-N</td>
<td>67.5</td>
</tr>
<tr>
<td>P4-S</td>
<td>66</td>
</tr>
<tr>
<td>P5-N</td>
<td>70.5</td>
</tr>
<tr>
<td>P5-S</td>
<td>67.5</td>
</tr>
<tr>
<td>P6-N</td>
<td>62</td>
</tr>
<tr>
<td>P6-S</td>
<td>67</td>
</tr>
<tr>
<td>P7-N</td>
<td>68.5</td>
</tr>
<tr>
<td>P7-S</td>
<td>71.5</td>
</tr>
</tbody>
</table>
As a result of excluding the group of sound concrete cores that were taken from the same crosshead beam face if they were less than 4, concrete cores which were taken from crosshead beam faces P1-N, P5-N, P5-S, P6-N and P6-S were not analysed for chloride contents.

4.3.1 Concrete sample powdering

Upon arrival to the TCD Civil Engineering laboratory, each concrete core was sliced into approximately 8 mm thick slices using an angle grinder with a 2 mm thick dry-cut diamond blade, Figure 4.7. As recommended by the Eurocode (EN 14629, 2007), the outer few millimetres of the concrete cores were discarded, this was recommended due to the fact that the surface layer of the concrete would generally have a different composition and texture matrix compared to the internal concrete for number of reasons which were given by Andrade et al. (1997) and will be discussed later in Section 4.4.1. Therefore, the first 5 mm of each core was excluded from the chloride analysis. The concrete slices were dried in an oven at 105 °C over night, as required by the EN 14629 standard. The concrete slices were then crushed using a manual Jaw Crusher to break them into smaller segments that are of a suitable small size for the machine used for the grinding operation (Disc Mill grinder), Figure 4.7. The crushed concrete samples were finally ground into powders using the Disc Mill grinder, which took about 30 to 40 seconds to pulverise each sample so all its particles could pass through the 1.18 mm sieve as required by the EN14629 standard for acid-soluble chloride determination adopted for the current analysis. It has to be noticed that the sieving operation were carried out only a number of times to establish the time needed to achieve the aforementioned target maximum particle size. Finally, each concrete sample would weigh between 18 to 25 g which allowed for repetitive testing when it was required. The slicing, crushing and grinding operations were carried out in a span of 2
working days for each set of cores that were taken from the same crosshead beam face. Each set of concrete cores were processed within the same 2-days session to minimize handling error within the same group of profiles to be analyzed for the determination of the scale of fluctuation of $C_s$ and $D_{app}$ (profiles from the same crosshead beam face).

When the sample powders were obtained for all eight incremental depths for the five cores extracted from the same crosshead face, they were stored in sealable plastic bags to avoid any atmospheric interference and the chloride analysis would have usually been carried out the following day. The chloride analysis of the 8 x 5 concrete powder samples in most cases took 3 to 4 working days. At the early stage of the chloride analysis, a duplicate test
was carried out for each concrete sample. Later; when the necessary expertise was acquired and a high level of confidence regarding the error associated with sample handling was established, the duplicate test were carried out only for one or two samples out of the 8 samples taken from the same concrete core.

4.3.2 Acid-soluble chloride determination

The acid–soluble chloride test aims at extracting the bound and free chlorides from the concrete sample (total chloride content) as opposed to the water-soluble chloride test which only targets the chloride ions that are free in the concrete pores (Free chloride content). The fact that whether it is only the free chloride ions that cause the corrosion to initiate or part of the bound chloride could be freed under certain conditions is still a matter of debate among researchers (Glass et al., 1996; Glass and Buenfeld, 1997; Mohammed and Hamada, 2003). It was tabulated that bound chlorides may participate in corrosion initiation when the pH level required to sustain passivation of the reinforcement (i.e. pH>12.5) is reduced. However, as indicated by Reddy et al. (2002), most of the bound chlorides will be released by a relatively small reduction in the pH level (i.e. pH=11). The release of bound chloride at a relatively high pH value suggests that the corrosion risk presented by bound chlorides may be very similar to that presented by free chlorides in concrete (Reddy et al., 2002). This seems to justify why the total chloride content (the acid-soluble chloride) has been widely used for service life prediction. Therefore, the acid-soluble chloride determination method was used in this study to determine the chloride content across the concrete cover region (chloride profile).

With regard to methods of extracting the total chloride from a concrete sample for chloride content determination, different codes and material testing standards offer different sample preparation procedures and test protocols. Table 4.2 summarises some of the most popular test methods provided by international standards and the major differences between their test protocols. However, recent advances in ion analysis technology (the use of chloride or silver ion selective electrode for Potentiometric titration) has not been recognized by all standards, therefore, and due to the availability of the new technology in the TCD Civil Engineering laboratory, it was decided that it would be more appropriate if the test protocol which recognizes this new technology should be used for the sample preparation. The test protocols recommended by the Eurocode (EN 14629) for Potentiometric titration was therefore used in this study for the sample preparation and determination of chloride
content. The Metrohm made (785 DMP Titrino) apparatus was used to carry out the chloride content analysis.

4.3.3 Potentiometric titration

In Potentiometric titration, the standard Silver Nitrate (the titrant), is added incrementally at a specified dosing rate. The titration procedure is then monitored by observing the change of the potential between the indicating electrode and a suitable reference electrode (the two electrodes are available as one combined electrode in the apparatus system used in this study). As the standard silver nitrate is added to the sample solution that contains chlorides, the change in the chloride ion activity produces a change in the observed potential. The equivalence point of the titration occurs when the change in the observed potential per incremental volume of the added titrant is at a maximum. The results of the determined chloride content are readily available from the computer connected to the apparatus or directly from the apparatus interface screen. The steps of sample preparation and the chloride extraction used in this study are described in Appendix A: Determination of Chloride Content by Potentiometric Titration.

Table 4.2 Major steps for sample preparation according to different material standards.

<table>
<thead>
<tr>
<th>Method</th>
<th>Sample size (g)</th>
<th>Sample must pass sieve size ≤ (µm)</th>
<th>Added Nitrate Acid (ml)</th>
<th>Added distilled water (ml)</th>
<th>Boiling duration (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RILEM TC178</td>
<td>1</td>
<td>160</td>
<td>50 (1:2)</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>BS 1881:124</td>
<td>5</td>
<td>150</td>
<td>10 (1.4)</td>
<td>50 cold + 50 hot</td>
<td>4 - 5</td>
</tr>
<tr>
<td>EN 14629</td>
<td>1 - 5</td>
<td>1180</td>
<td>10 (5 mol/l)</td>
<td>50 cold + 50 hot</td>
<td>3</td>
</tr>
<tr>
<td>NT BUILD 208</td>
<td>5</td>
<td>1000</td>
<td>10 (conc.)</td>
<td>20 cold + 50 hot</td>
<td>No boiling</td>
</tr>
<tr>
<td>AASHTO T-260</td>
<td>3</td>
<td>300</td>
<td>3 (conc.)</td>
<td>10 cold + 40 hot</td>
<td>1</td>
</tr>
<tr>
<td>ASTM C1152</td>
<td>5 - 10</td>
<td>315</td>
<td>25 (1:1)</td>
<td>75</td>
<td>Just to boil</td>
</tr>
</tbody>
</table>

120
4.3.4 System calibration

To make sure that the titration apparatus system was working properly, i.e. the true chloride concentration was being detected, the system had to be calibrated and checked against a standard solution with a known chloride concentration. This check was carried out every time an analysis was performed, in particular when the silver nitrate (the titrant) was being replaced or when the apparatus had not been used for sometime. For that reason, a standard concrete sample with a known chloride concentration produced for this purpose by the Swedish National Testing and Research Institute (SNTRI) was used. It has to be noticed that the chloride contents of the standard samples (labelled on the product bottles) were determined in the SNTRI laboratory using the AASHTO T-260 method for acid-soluble chloride determination. The advantage of using a standard concrete sample with a known chloride concentration over for example the use of a normal Sodium Chloride solution of a known concentration is that with the standard sample the process of sample preparation and chloride extraction (Appendix A) can be applied and any source of handling error that results in a significant change in the expected results can be identified.

At the earliest chloride analysis test trials of the current experimental study, a Manual Filtration (MF) procedure was used; as a result, the error obtained ranged from 12% to 47.5% depending on the sample chloride content, as can be seen from Figure 4.8. However, as soon as this large difference in the results between the TCD lab results and the chloride content labelled on the product bottles was obtained, the time consuming and inefficient MF procedure was replaced by a Vacuum Filtration (VF) procedure and excellent results were immediately obtained. The VF reduced the error to no more than 6.3% in its extreme case as can be seen from Table 4.3. The improvement in the results for the case of using VF can be explained by the efficient washing of the concrete sample which allowed most of the chlorides to be washed out (using hot distilled water) from the concrete sample to the glass beaker for titration. This had been very difficult to achieve within a practical time frame in the case of MF. Due to the very small variation within the same sample, it was difficult to show this variation on the relevant figure, Figure 4.8; therefore Table 4.3 was presented to show the variation within the sample for both results obtained from both laboratories.

The calibration against the standard concrete sample not only benefited checking the accuracy of the system used for detecting the chloride concentration in the concrete
sample, but also allowed the acid-soluble chloride determination method used in this study (EN 14629) to be checked against a different method of the same principle but from a different material standard (AASHTO T-260). The results shown in Figure 4.8 and Table 4.3 indicate that both methods give the same results with an error that does not exceed ± 6.3%.

![Figure 4.8 TCD lab results vs. Standard sample chloride concentration (SNTRI) and the effect of MF and VF on the determined chloride content results.](image)

Table 4.3 TCD lab results vs Standard sample chloride contents (Cl% per mass of concrete).

<table>
<thead>
<tr>
<th>Standard sample (AASHTO T-260)</th>
<th>TCD Lab (EN 14629)</th>
<th>Mean to mean error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.039 ± 0.0017</td>
<td>0.039 ± 0.001</td>
<td>0.0</td>
</tr>
<tr>
<td>0.040 ± 0.0029</td>
<td>0.038 ± 0.001</td>
<td>-5.0</td>
</tr>
<tr>
<td>0.042 ± 0.0021</td>
<td>0.041 ± 0.001</td>
<td>-2.4</td>
</tr>
<tr>
<td>0.101 ± 0.0021</td>
<td>0.105 ± 0.001</td>
<td>+4.0</td>
</tr>
<tr>
<td>0.117 ± 0.0034</td>
<td>0.117 ± 0.001</td>
<td>0.0</td>
</tr>
<tr>
<td>0.127 ± 0.0022</td>
<td>0.119 ± 0.002</td>
<td>-6.3</td>
</tr>
<tr>
<td>0.366 ± 0.0133</td>
<td>0.368 ± 0.002</td>
<td>+0.5</td>
</tr>
</tbody>
</table>
4.4 FERRYCARRIG BRIDGE RESULTS

4.4.1 Chloride profile curve fitting

The values of chloride contents at each incremental depth from the core surface were determined as described in Section 4.3.3 and expressed as a percentage of Chlorides (Cl%) per mass of concrete. The values were then plotted against depths from the core surface to give the chloride profile (Bertolini, 2004). The variables $C_s$ and $D_{app}$ were estimated by curve fitting the measured total chloride profiles to the analytical solution of Fick's 2nd law of diffusion given by Equation 2.2, (Bertolini, 2004). The curve fitting was performed using the non-linear Least Square regression method which is commonly used for this purpose (Thomas and Bamforth, 1999). Although the process of curve fitting to the measured chloride profiles seems straightforward, the choice of data points to be excluded as outliers from the fitting process can have a significant influence on the outcome of the obtained $C_s$ and $D_{app}$ results as will be shown in the following discussion.

Chloride profiles obtained in this study and in previous literature (e.g. Andrade et al., 1997; Sandberg et al., 1998; McPolin et al., 2005; Meira et al., 2007) tend to show that the outer region of the concrete contains less chloride nearer to the surface than slightly deeper in the concrete. This is because the concrete skin has a different matrix composition compared to the internal concrete due to phenomena such as a contact with the moulds, segregation of aggregates or dielectric reaction between the concrete surface and chloride environment (Andrade et al., 1997). Moreover, chlorides at the surface of cover concrete can often be washed out by the rain or by the cooling water used during the sample extraction operation. The thickness of the outer region to be discarded depends on the distance between the incremental measurements (e.g. the thickness of the concrete slices).

Due to operational reasons, and to avoid excessive fracture of the concrete slice and hence loss of sample; the minimum concrete slice thickness which can be obtained by the angle grinder configuration used in this study was 8 mm. Use of the profile grinder apparatus, to extract concrete powder from concrete cores at incremental depths of 1-2 mm, was not possible in this case due to the relatively small diameter of the concrete cores (i.e. < 50 mm) and due to its time consuming nature considering the large number of concrete cores to be tested.
The appropriateness of the fit for each chloride profile was demonstrated by the Regression Coefficient ($R^2$) which indicates the goodness of the fit, the closer to 1.0 for the $R^2$ value the better the fit. As a result, sometimes the first, the first two, and in fewer cases the first three data points close to the concrete surface needed to be excluded from the fitting procedure. This is because they are inconsistent with the nature of the remaining points of the chloride content data, i.e. they are outliers with regard to the shape of the expected decreasing chloride profile through the depth of the concrete as explained in Section 2.3.2. If such outliers were not excluded from the fitted data, this will adversely affect the fit results and the value of the regressor variables (i.e. $C_i$, $D_{app}$) with consequences for modelling the time to corrosion initiation using inappropriate values for the input parameters. The chloride profiles and their Fick’s law theoretical solution best fit for all the 45 concrete cores are plotted in Appendix C.

### 4.4.2 Initial chloride content ($C_i$)

The Initial Chloride content, $C_i$, is often assumed to be zero when the chloride profile is fitted to Fick’s 2nd law of diffusion. However, this is sometimes not the case (Polder and de Rooij, 2005), chloride measurements at the tail of the chloride profile may be considerably higher and for a better fitting result, $C_i$ may need to be assumed to be non-zero. In the current study this was confirmed by the chloride content results obtained from concrete powders taken from depths of 120 mm from nine randomly selected concrete cores. An average value of 0.011 (Cl% per mass of concrete) was obtained. This value is identical to the average chloride content reported in the Special Inspection report for depths 55-80 mm and only slightly lesser than the 0.012% average value obtained from similar range of depths during the 2007 rehabilitation testing programme as shown by Figure 4.5. Furthermore, an average $C_i$ value of a similar magnitude (0.010 Cl% per mass of concrete) has been reported by Polder and de Rooij (2005) based on data from field investigation carried out on six marine structures in the Netherlands.

Figure 4.9 shows a curve fitting procedure on chloride profile measurements of core P3-S4. The corresponding obtained parameters are presented in Table 4.4. It can be seen from the visual observation of the figure that the curve looks better fitted (ignoring the first point on the chloride profile as an outlier) when $C_i$ is assumed to be non-zero (i.e. Fit-2) than assuming $C_i=0$ (i.e. Fit-1). This has been confirmed by the improvement of the $R^2$ value from 0.926 (for Fit-1) to 0.997 (for Fit-2). Further, both fits were assessed by plotting the
95% confidence limits, the wider interval between the upper and lower bounds associated with Fit-1 indicates there is higher uncertainty to predict a new value of chloride content outside the range of data. On the other hand, the interval is much reduced in the case of Fit-2.

Figure 4.9 (a) Chloride profile for core sample P3-S4 fitted assuming $C_i=0$ (Fit-1) and $C_i>0$ (Fit-2), (b) Fit with 95% prediction bounds.

Table 4.4 Fitting parameters obtained from Fit-1 and Fit-2 of Figure 4.9

<table>
<thead>
<tr>
<th>Variable</th>
<th>Fit-1</th>
<th>Fit-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i$ (Cl%)</td>
<td>0.089%</td>
<td>0.128%</td>
</tr>
<tr>
<td>$C_r$ (Cl%)</td>
<td>0.0%</td>
<td>0.012%</td>
</tr>
<tr>
<td>$D_{app}$ (mm³/year)</td>
<td>19.88</td>
<td>7.13</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.926</td>
<td>0.997</td>
</tr>
</tbody>
</table>
Due to the lack of information regarding the original material properties used for the bridge construction, the source of $C_i$ cannot be explained. It is generally accepted however that up until late 1970s, concrete technology still had not fully recognized the scale of damage which could result from using chloride contaminated material in the reinforcement concrete construction. For example, the ACI Committee 201-1977 were among the first to recommend limitations on the presence of chlorides in reinforced concrete (Gaynorl, 1987). Therefore, limitations on the use of chloride contaminated concrete mix ingredients were not as strict as it is nowadays, hence it is reasonable to assume that the $C_i$ found in the current study may have originated from the mixing water or from the aggregate used in the original concrete mixes. However, it is worth mentioning though that the average value of $C_i=0.011$ (Cl% per mass of concrete) is still less than the maximum acceptable value stated by most of the standards listed in Table 2.4 which ranges from 0.03 to 0.06 (Cl% per mass of concrete).

The choice of whether or not to neglect $C_i$ will have a significant influence on the determined values of the deterioration variables $C_s$ and $D_{app}$ as can be seen from Table 4.4. However, although choosing Fit-2 over Fit-1 has resulted in 44% increase and 64% decrease in $C_s$ and $D_{app}$ respectively, this is not a quite true reflection of the influence of the choice of fit on the result of the predicted time to corrosion initiation. The reason for this is that for a given chloride profile, at a certain point in time, the best fit results in a coupled $C_s$ and $D_{app}$. Therefore, any change in one of the parameters will be combined by the change in the other; and the change in the predicted chloride content at any point in time at the level of the reinforcement will not be as high as such. For example, using $C_s$ and $D_{app}$ values obtained from Fit-1 and Fit-2 fitting procedures, to predict the chloride content at cover depth of 50 mm, Figure 4.9(a), after 27 years of service life (using Equation 2.2), have resulted in 22% increase in the predicted chloride content from Fit-1 to Fit-2. On the other hand, when the cover depth was assumed 45 mm, only 0.8% decreases from Fit-1 to Fit-2 was observed.

After studying all the chloride profiles (in Appendix C) in a similar way to the study of the chloride profile presented in Figure 4.9, it was found that a better fit was achieved when the $C_i$ was not neglected (i.e. $C_i>0$). Therefore, all chloride profiles obtained in this study were fitted using the Least Square method assuming that $C_i>0$, hence $C_i$, $C_s$, and $D_{app}$ were all treated as a fitting parameters in the curve fitting process.
The mean value and the standard deviation (or coefficient of variation) as well as the parent probabilistic distribution of the \( C_i \), \( C_j \) and \( D_{app} \) were estimated from 40 chloride profiles out of possible 45 chloride profiles. The remaining 5 profiles were excluded from the analysis for one of the following two reasons:

(i) The chloride profile did not show a typical decreasing profile that can be fitted to Fick’s 2\textsuperscript{nd} law of diffusion, as in the case of P4-N1, P2-N1 and P3-S2, Figure 4.10.

(ii) The chloride profile was excluded because its chloride profiles were interrupted by the presence of steel bars that were unintentionally cut during the concrete cores extraction, as in the case of P2-S1 and P2-N4, Figure 4.11.

The excluded (outliers) chloride profiles will be discussed in the following section.

![Figure 4.10 Outliers chloride profiles for cores P4-N1, P2-N2 and P3-S3](image-url)
4.4.3 Outliers chloride profiles

The chloride profiles obtained from concrete core samples P4-N1, P2-N1 and P3-S2, Figure 4.10, did not show the decreasing chloride content typically demonstrated by chloride profiles obtained from concrete structures exposed to external chloride penetration. The chloride content values obtained in the cover zone of these cores are in fact very low as compared with chloride contents obtained from neighbouring cores for the same depths. Considering these cores were located at the non protected cantilever part of the crosshead beams and directly underneath the unwaterproofed footpath, it is highly likely that the very high levels of water penetration have caused the chlorides to be
repeatedly washed out over time. It was then not possible to fit these chloride profiles to Fick’s 2\textsuperscript{nd} law equation.

The chloride profiles obtained from concrete core samples P2-S1 and P2-N4 are shown in Figure 4.11(a); as indicated in the figure, both have contained a steel bar at depths of 40 mm and 45 mm respectively. The steel bars were 14 mm in diameter which suggests that they were cut from the shear links of the crosshead beams. Interestingly, the presence of steel bars at the aforementioned depths had affected both chloride profiles as a whole and not only the chloride contents at depths adjacent to the reinforcement. Perhaps the chloride ions have been attracted by the presence of iron and that can be explained by the higher chloride content at depths near the location of the steel bar as compared to the chloride content of concrete samples taken at the same depth from the neighbouring cores as can be seen from Figure 4.11.(b). Due to their inconsistency with the rest of the results, $C_s$ and $D_{app}$ values obtained from the two chloride profiles will be excluded from the calculation of the average, standard deviation, and the estimation of the scale of fluctuation of both variables to be carried out in Chapter 5.

4.4.4 Statistical analysis of data results

The results obtained for all three variables, $C_s$, $D_{app}$ and $C_i$, are listed in Table 4.5. The data presented in this table provides very valuable information with regard to both random and spatial variation of three of the most important deterioration variables. Regarding spatial variability, the unique importance of the data presented in Table 4.5 can be attributed to the fact that such data are hardly ever reported in the literature in company with the distance separating the sample locations within the experimented structure. As indicated by a number of researchers (e.g. Li, 2004b; Vu and Stewart, 2005), the absence of such information hinders attempts to characterise the spatial variability of the deterioration variables.

The data presented in Table 4.5 for variables $C_s$, $D_{app}$ and $C_i$ was fitted to statistical distributions to determine their statistical properties. It can be seen from Figure 4.12 that data obtained for all the three variables can be assumed to have come from a Normal distribution (truncated at zero value). The suitability of the distributions to the fitted data was further assessed by plotting the data on a Probability plotting paper where linearity confirms the appropriateness of the assumed distribution, Figure 4.12(b).
### Chapter 4: Experimental work

Table 4.5 Initial chloride content ($C_i$), Surface chloride content ($C_s$) and Diffusion coefficient ($D_{app}$) data obtained from curve fitting to 45 chloride profiles.

<table>
<thead>
<tr>
<th>Pier face</th>
<th>Core number</th>
<th>$C_i$ (Cl% per mass of concrete)</th>
<th>$C_s$ (Cl% per mass of concrete)</th>
<th>$D_{app}$ (mm²/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pier 1 South</td>
<td>P1-S1</td>
<td>0.011</td>
<td>0.202</td>
<td>7.58</td>
</tr>
<tr>
<td></td>
<td>P1-S2</td>
<td>0.011</td>
<td>0.416</td>
<td>3.09</td>
</tr>
<tr>
<td></td>
<td>P1-S3</td>
<td>0.012</td>
<td>0.31</td>
<td>4.29</td>
</tr>
<tr>
<td></td>
<td>P1-S4</td>
<td>0.009</td>
<td>0.183</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>P1-S5</td>
<td>0.009</td>
<td>0.189</td>
<td>7.63</td>
</tr>
<tr>
<td>Pier 2 North</td>
<td>P2-N1*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P2-N2</td>
<td>0.01</td>
<td>0.226</td>
<td>10.45</td>
</tr>
<tr>
<td></td>
<td>P2-N3</td>
<td>0.012</td>
<td>0.238</td>
<td>9.57</td>
</tr>
<tr>
<td></td>
<td>P2-N4</td>
<td>0.011</td>
<td>0.242</td>
<td>27.24</td>
</tr>
<tr>
<td></td>
<td>P2-N5</td>
<td>0.01</td>
<td>0.169</td>
<td>10.21</td>
</tr>
<tr>
<td>Pier 2 South</td>
<td>P2-S1</td>
<td>0</td>
<td>0.281</td>
<td>24.36</td>
</tr>
<tr>
<td></td>
<td>P2-S2</td>
<td>0.004</td>
<td>0.143</td>
<td>14.9</td>
</tr>
<tr>
<td></td>
<td>P2-S3</td>
<td>0.006</td>
<td>0.186</td>
<td>6.81</td>
</tr>
<tr>
<td></td>
<td>P2-S4</td>
<td>0.011</td>
<td>0.148</td>
<td>7.61</td>
</tr>
<tr>
<td></td>
<td>P2-S5</td>
<td>0.008</td>
<td>0.12</td>
<td>12</td>
</tr>
<tr>
<td>Pier 3 North</td>
<td>P3-N1</td>
<td>0.013</td>
<td>0.279</td>
<td>6.67</td>
</tr>
<tr>
<td></td>
<td>P3-N2</td>
<td>0.007</td>
<td>0.228</td>
<td>13.78</td>
</tr>
<tr>
<td></td>
<td>P3-N3</td>
<td>0.005</td>
<td>0.264</td>
<td>16.34</td>
</tr>
<tr>
<td></td>
<td>P3-N4</td>
<td>0.005</td>
<td>0.205</td>
<td>16.03</td>
</tr>
<tr>
<td></td>
<td>P3-N5</td>
<td>0.015</td>
<td>0.543</td>
<td>10.32</td>
</tr>
<tr>
<td>Pier 3 South</td>
<td>P3-S1</td>
<td>0.01</td>
<td>0.259</td>
<td>14.33</td>
</tr>
<tr>
<td></td>
<td>P3-S2*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P3-S3</td>
<td>0.011</td>
<td>0.12</td>
<td>13.84</td>
</tr>
<tr>
<td></td>
<td>P3-S4</td>
<td>0.012</td>
<td>0.128</td>
<td>7.13</td>
</tr>
<tr>
<td></td>
<td>P3-S5</td>
<td>0.012</td>
<td>0.446</td>
<td>6.91</td>
</tr>
<tr>
<td>Pier 4 North</td>
<td>P4-N1*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P4-N2</td>
<td>0</td>
<td>0.225</td>
<td>13.73</td>
</tr>
<tr>
<td></td>
<td>P4-N3</td>
<td>0.013</td>
<td>0.262</td>
<td>11.13</td>
</tr>
<tr>
<td></td>
<td>P4-N4</td>
<td>0</td>
<td>0.105</td>
<td>15.7</td>
</tr>
<tr>
<td></td>
<td>P4-N5</td>
<td>0.02</td>
<td>0.122</td>
<td>7.326</td>
</tr>
<tr>
<td>Pier 4 South</td>
<td>P4-S1</td>
<td>0.013</td>
<td>0.053</td>
<td>10.25</td>
</tr>
<tr>
<td></td>
<td>P4-S2</td>
<td>0.015</td>
<td>0.076</td>
<td>5.250</td>
</tr>
<tr>
<td></td>
<td>P4-S3</td>
<td>0.021</td>
<td>0.086</td>
<td>14.41</td>
</tr>
<tr>
<td></td>
<td>P4-S4</td>
<td>0.013</td>
<td>0.032</td>
<td>2.03</td>
</tr>
<tr>
<td></td>
<td>P4-S5</td>
<td>0.016</td>
<td>0.043</td>
<td>22.82</td>
</tr>
<tr>
<td>Pier 7 North</td>
<td>P7-N1</td>
<td>0.017</td>
<td>0.076</td>
<td>9.52</td>
</tr>
<tr>
<td></td>
<td>P7-N2</td>
<td>0.003</td>
<td>0.512</td>
<td>13.9</td>
</tr>
<tr>
<td></td>
<td>P7-N3</td>
<td>0.007</td>
<td>0.326</td>
<td>9.84</td>
</tr>
<tr>
<td></td>
<td>P7-N4</td>
<td>0.007</td>
<td>0.285</td>
<td>14.15</td>
</tr>
<tr>
<td></td>
<td>P7-N5</td>
<td>0.014</td>
<td>0.383</td>
<td>8.17</td>
</tr>
<tr>
<td>Pier 7 South</td>
<td>P7-S1</td>
<td>0.013</td>
<td>0.238</td>
<td>8.15</td>
</tr>
<tr>
<td></td>
<td>P7-S2</td>
<td>0.013</td>
<td>0.175</td>
<td>8.46</td>
</tr>
<tr>
<td></td>
<td>P7-S3</td>
<td>0.015</td>
<td>0.296</td>
<td>6.78</td>
</tr>
<tr>
<td></td>
<td>P7-S4</td>
<td>0.011</td>
<td>0.209</td>
<td>9.16</td>
</tr>
<tr>
<td></td>
<td>P7-S5</td>
<td>0.011</td>
<td>0.196</td>
<td>10.73</td>
</tr>
</tbody>
</table>

* Chloride profiles which were not possible to fit to Fick's 2nd law due to their unusual trend.

**Bold characters** indicate that values were determined from the analysis of chloride profiles which were considered outliers due to the influence of the presence of steel bars (see Section 4.4.3).
The Chi-Square statistical hypothesis tests were performed to confirm, for each variable, if the data comes from a Normal/Lognormal distribution. From statistics, the hypothesis is accepted if the Chi-Square value, $\chi^2$, is less than the critical Chi-Square, $\chi^2_{0.05}$, value obtained from the relevant table for a specified significant level $\alpha$ (Ang and Tang, 1975).

![Histograms and Normal PDF plots](image1)

![Probability plots](image2)

Figure 4.12 (a) Histograms, Probability Density and (b) Probability plots of $C_s$, $D_{app}$, and $C_i$.

The results of the Chi-Square test, Table 4.6, confirmed that the data, for all three variables, can be assumed to have come from a Normal distribution for a significance level $\alpha = 0.05$. While the hypothesis that the data comes from a Lognormal distribution were rejected for both variables, $C_s$ and $C_i$, the hypothesis was accepted for $D_{app}$ with preference to the Normality assumption. This is indicated by observing that the difference between $\chi^2$ and $\chi^2_{0.05}$ values, Table 4.6, is smaller in the case of the Normal distribution than in the case of the Lognormal distribution.

Table 4.6 Chi-Square goodness of fit test results for $C_s$, $D_{app}$ and $C_i$ data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Chi-Square test $\chi^2 &lt; \chi^2_{0.05}$</th>
<th>Reject Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>Lognormal</td>
</tr>
<tr>
<td>$C_s$</td>
<td>2.21 &lt; 11.07</td>
<td>11.13 &gt; 9.49</td>
</tr>
<tr>
<td>$D_{app}$</td>
<td>1.64 &lt; 7.81</td>
<td>1.69 &lt; 9.49</td>
</tr>
<tr>
<td>$C_i$</td>
<td>2.89 &lt; 9.49</td>
<td>25.42 &gt; 7.81</td>
</tr>
</tbody>
</table>
The literature tends to report that data for $C_s$ and $D_{app}$ follow a Lognormal distribution (e.g. Vu and Stewart, 2000; Duprat, 2007). However no statistical evidence was provided to indicate the superiority of the Lognormal distribution over the Normal distribution. Perhaps researchers favour the Lognormal distribution over the Normal distribution because such variables cannot physically have a negative value, and it would be easier to ensure that a negative value will not be generated when a random data for the variables are sampled from a Lognormal distribution. However, many modern computer simulation packages (i.e. MATLAB-2006a) have made it possible to generate data from a Normal distribution that is truncated at a desired minimum or maximum value without affecting the statistical properties of the parent normal distribution.

The mean value and the coefficient of variation (C.O.V) final results for all three variables are presented in Table 4.7. It can be seen from the table that the $C_s$ have the largest variation among the three parameters with a coefficient of variation value of 0.56. However, this value is consistent with the published data which ranges from 0.50 to 0.83 reported for structures in marine environments (e.g. Vu and Stewart, 2000; Duprat, 2007).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>$(\mu, \sigma)$</th>
<th>C.O.V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_s$ (Cl % per mass of concrete)</td>
<td>Normal</td>
<td>$(0.218, 0.121)$</td>
<td>0.56</td>
</tr>
<tr>
<td>$D_{app}$ (mm²/year)</td>
<td>Normal</td>
<td>$(10.25, 4.13)$</td>
<td>0.40</td>
</tr>
<tr>
<td>$C_t$ (Cl % per mass of concrete)</td>
<td>Normal</td>
<td>$(0.011, 0.005)$</td>
<td>0.45</td>
</tr>
</tbody>
</table>

### 4.4.5 Influence of the bridge orientation on $C_s$ and $D_{app}$

The average values of the chloride penetration parameters $C_s$ and $D_{app}$ for each crosshead face were calculated and plotted in Figure 4.13 to investigate if there is a significant difference in the results obtained from the north faces and the south faces. The figure shows that there is a notable increase for the results obtained from the north face as compared to those obtained from the south face for both variables, $C_s$ and $D_{app}$, for all tested crossheads (the north face of crosshead beam No 1 was not tested for the reasons given in Section 4.3). The noted increase was in the range of 28-208% for the case of $C_s$ and in the range of 9-28% for the case of $D_{app}$. A statistical t-test analysis confirmed that this increase is statistically significant for both variables at a significance level $\alpha=0.05$ with a p-value of 0.007 and 0.008 for $C_s$ and $D_{app}$ respectively. It is postulated that this can be
attributed to one or a combination of the two following reasons; the north side is more exposed to the area of the river that is in direct contact with the open sea, i.e. the estuary, which naturally has a higher chloride concentration than the main river waters. This positioning of the crosshead beams allows the north faces to receive more airborne chlorides than the slightly sheltered southern faces. The second explanation is that the southern face of the crossheads directly faces the prevailing wind (windward face); they act as obstacles to the wind, decreasing their speed and creating turbulence behind the crosshead, as a result more chlorides end up deposited on the leeward side of the crossheads (the north face) than the windward side. In addition, the surface chlorides on this windward faces can be reduced by the wash-down effect caused by the driving rain. The phenomena explained in this section have also been reported in a study carried out on Hadsel Bridge in Norway (Bamforth, 1996). The survey of the Hadsel Bridge showed that the $C_s$ values measured on the leeward face were more than six times higher than those measured on the windward face. Vu (2003) studied the literature data of $C_s$ in which the source of measured data with regard to the direction of the wind were given (i.e. windward or leeward). He then concluded that $C_s$ value measured on the leeward face of the structure maybe expected to be 3.5 times that measured on the windward faces.

These results may carry a significant importance for the service life prediction and maintenance planning at the very early stage of the structure design. For example; studying the structural orientation and its position with regard to the direction of the prevailing winds or the exposure to the source of airborne chlorides, may give some insight on which side of the structure, or structure component, should have a particular protection to delay
the deterioration process on that side (i.e. larger concrete cover or applying surface coating), or should be prioritized for special inspection.

4.5 CONCLUSIONS

A chloride analysis was performed on a total of 45 concrete cores extracted from nine crosshead beam faces of Ferrycarrig Bridge. The aim of the testing was to obtain site specific statistical information, which is not available in the literature, on the two most important deterioration parameters that concerns the initiation stage of the structure service life $C_s$ and $D_{app}$. This chapter provided background information on the bridge under investigation with a particular emphasis on the exposure condition, the level and the cause of the deterioration of the crosshead beams from which cores where extracted. The chloride analysis was carried out using a sample preparation protocol adopted by the EN 14629 standard for the acid-soluble chloride determination using Potentiometric titration. The method and the apparatus used were calibrated using a standard sample with a predetermined chloride concentration.

The $C_s$ and $D_{app}$ results obtained from the current experimental study showed that the crosshead beams of the bridge under investigation have a very good resistance to chloride penetration as can be seen from the average value of $D_{app}$. The results showed that data obtained for both variables $C_s$ and $D_{app}$ follow a Truncated Normal distribution. The current study also showed that initial chloride content $C_i$ does exist and cannot be neglected. Neglecting $C_i$ can result in inaccurate estimation of $C_s$ and $D_{app}$ parameters which will consequently affect the estimated time to corrosion initiation. The $C_i$ data results were also found to follow a Truncated Normal distribution. Among the three parameters, the $C_s$ parameter exhibits the highest variation with a coefficient of variation of 0.56; where $D_{app}$ and $C_i$ have a coefficient of variation of 0.40 and 0.45 respectively which is consistent with the literature. According to a proposed exposure class classification system put forward by Bamforth (1996), the range of $C_s$ values obtained in this study, correspond to Mild to Moderate exposure.

The results also showed that there is a significant increase (up to 3 times higher) in the average values for both parameters $C_s$ and $D_{app}$ obtained from the north side face (windward) of the crosshead beams as compared with results obtained from the south side face (leeward) due to crosshead orientation with regard to the prevailing wind. Particularly
for the variable $C_s$, the windward side of the crosshead beams showed values for $C_s$ up to 3 times higher than those measured on the leeward sides. These findings can be used to identify the area of the structure/component that should be prioritized for special inspection or extra protection as early as the design stage and could in this context form the basis of performance-based design.

$C_s$ and $D_{app}$ data of special interest to spatial variability modelling have been obtained from an aging structure located in marine environments. The data, which will be analysed in the following chapter, are of a unique importance as they have been taken at frequent distances from nine different crosshead faces and should provide valuable information with regard to the fluctuation properties of the deterioration variables. The statistical parameters obtained from this chapter will be used, in conjunction with the scale of fluctuation to be determined in Chapter 5, as an input for the spatial-temporal reliability based model which is developed in Chapter 6.
Chapter 5:

Estimation of the Scale of Fluctuation
5.1 INTRODUCTION

In Chapter 3 the fundamental concepts of Random Field (RF) theory have been discussed and the importance of the Scale of Fluctuation parameter, $\theta$, was highlighted. As discussed in Chapter 3 and at the start of Chapter 4, there is a scarcity of available data on the parameter $\theta$ for most of the deterioration related variables. For this reason, spatial variability of the deterioration variables has been rarely considered in reliability based assessment of deteriorating Reinforced Concrete (RC) structures.

The experimental works carried out in Chapter 4 were essentially aimed at determining $\theta$ values using site specific data for two of the most influencing corrosion deterioration variables $C_s$ and $D_{app}$. The aim of this chapter is to analyse the data provided by Chapter 4 to estimate $\theta$ for both variables. The importance of the data provided by Chapter 4 is that the data for both variables were reported with the corresponding location within each of the crosshead beam faces. Locations (or distance between samples) are rarely reported in the literature which hinders any attempt to obtain information regarding the spatial fluctuation of the investigated properties. In addition to its significance as being taken from a real structure located in a marine environment, it is important to point out that the data were collected from nine different crosshead beam faces. The significance of this can be appreciated, given that the most reliable and most referenced estimation of $\theta$ for $C_s$ and $D_{app}$ variables, to date, were based on a set of data that was taken from three crosshead beam faces (Ramachandran et al., 2001).

The $\theta$ values to be estimated from the current analysis will be compared to those reported in the literature and recommendations will be made on the value of $\theta$ to be used as input parameter for the RF model which will be developed in Chapter 6.

The use of $\theta$ values based on site specific data will have a significant contribution to the quality of the research works that attempt to include the effect of spatial variability of the concrete properties in reliability-based service life modelling. It is anticipated that the finding of the current experimental study, with regard to the estimated values of the parameter $\theta$ for the investigated properties, will highlight the importance of adequate pre-selection of the location and the number of measurements to be taken from structures once such opportunity becomes available to research engineers. The justification of making multiple measurements for a particular deterioration variable at one continuous side of the
structure/structure component and recording the distance between their locations can only be made when the influence of the spatial variability of that variable on the prediction of life time performance is established; this will be discussed in Chapter 7.

5.2 SPATIAL FLUCTUATION OF CHLORIDE CONTENTS

To demonstrate the degree of physical fluctuation of chloride contents at different depths of the concrete cover along the crosshead beams, the results of the chloride contents obtained from crosshead P1S was plotted against the distance from the left end of the crosshead as shown in Figure 5.1. Similar plots for results obtained from crossheads 2, 3, 4 and 7 are provided in Appendix B. It can be seen from this figure, and from the similar figures in Appendix B, that the chloride content at a certain depth from the concrete surface is not a constant property along the beam. This is, as discussed in Section 3.2.4, due amongst other things to the inconsistency of workmanship, the inconsistency of the concrete mix proportions, and the variation of the exposure conditions. However, the fluctuation intensity of the chloride content along the beam decreases gradually as the typical reinforcement depth is approached, i.e. at depths $\geq 35$ mm. This may indicate that the largest proportion of the chloride content variation along the beam can be attributed to the exposure condition rather than to the inconsistency in the workmanship or the original material properties.

It is anticipated that the fluctuation pattern would be described in more detail as the number of measurements along the beam increases, for example; if more concrete cores were drilled and the distance between their locations were reduced. However, this is not always operationally or economically feasible. For example, the number of concrete cores that can be extracted from an individual structure may be restricted to a very small number and only to specific locations that are not considered structurally critical by the owner. Furthermore, the budget necessary for extracting and processing a sufficient number of concrete cores may not be available.

The methods used in this thesis to calculate $\theta$ for $C_\text{s}$ and $D_{\text{app}}$ from the data collected in Chapter 4 will be described in the following section.
Figure 5.1 (a) Chloride content of five concrete cores at different depths obtained from Crosshead Pl-S (b) Location of the concrete cores extracted from Crosshead Pl-S.

5.3 ESTIMATION THE SCALE OF FLUCTUATION (θ)

As mentioned in Section 3.4.3, the parameter θ of a spatially variable property can be obtained from a record of observations (data measurements) in which the distance between the location of these observations are already recorded. To determine a value for the parameter θ for a spatially variable property from a digitised record consisting of k observations, an analytical model that describes the correlation coefficients between any two observations separated by distance τ needs to be employed. The Autocorrelation Function (ACF), introduced in Section 3.4.2, is a mathematical tool by which the spatial correlation between observations can be measured. As discussed in Section 3.4.2, several
analytical models have been proposed to describe the correlation behaviour between observations as a function of the separating distance. For convenience, the ACFs which were proposed by Vanmarcke (1977, 1983) and presented in Section 3.4.2 have been reproduced in Table 5.1 in addition to some other ACFs found in the literature.

Table 5.1 Autocorrelation Functions (ACF) and the corresponding Scale of Fluctuation (\( \theta \)).

<table>
<thead>
<tr>
<th>ACF Name</th>
<th>ACF Model</th>
<th>Scale of fluctuation ( \theta )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Triangular</td>
<td>( \rho(\tau) = \begin{cases} 1 - \frac{r}{a} &amp; \text{for }</td>
<td>\tau</td>
<td>\leq a \ 0 &amp; \text{for }</td>
</tr>
<tr>
<td>2 Exponential</td>
<td>( \rho(\tau) = \exp \left( -\frac{\tau}{b} \right) )</td>
<td>( 2b )</td>
<td>(Vanmarcke 1983)</td>
</tr>
<tr>
<td>3 Second-order autoregressive</td>
<td>( \rho(\tau) = \left[ 1 + \frac{</td>
<td>\tau</td>
<td>}{c} \right] \cdot \exp \left( -\frac{</td>
</tr>
<tr>
<td>4 Square exponential (Gaussian)</td>
<td>( \rho(\tau) = \exp \left( -\left( \frac{</td>
<td>\tau</td>
<td>}{d} \right)^2 \right) )</td>
</tr>
<tr>
<td>5 Cosine exponential</td>
<td>( \rho(\tau) = \exp \left( -\frac{</td>
<td>\tau</td>
<td>}{e} \right) \cdot \cos \left( \frac{</td>
</tr>
<tr>
<td>6 Sinusoidal</td>
<td>( \rho(\tau) = \sin \left( -2.2 \frac{</td>
<td>\tau</td>
<td>}{f} \right) )</td>
</tr>
</tbody>
</table>

Two major procedures have been reported in the literature for the estimation of \( \theta \) for a spatially variable property from a digitized record of data. In the first procedure, reported by Li (2004b), the Maximum Likelihood Method (MLM) is used in which different values for the model parameter of the proposed ACF model is assumed and the value that maximises the corresponding ML function is taken as the model parameter. In the second procedure, proposed by Vanmarcke (1983), a proposed ACF model (from Table 5.1) can be adjusted to provide the best fit to the actual sample correlation coefficients \( \rho_{\text{exp}}(\tau) \) thereby providing estimates of the corresponding model parameter (i.e. \( a, b, c, d, e \) or \( f \) in Table
5.1). For both procedures, the resulting scale of the fluctuation $\theta$ can be estimated from the theoretical relationships between $\theta$ and the model parameters as described in Table 5.1.

It is noted that both procedures require the use of the analytical ACF model, i.e. $\rho(\tau)$. The choice of which of the analytical models listed in Table 5.1 to use would depend on how best the proposed ACF model fits the correlation coefficients calculated from the measured data $\rho_{\text{Exp}}(\tau)$. The procedure for calculating $\rho_{\text{Exp}}(\tau)$ will be discussed in Section 5.3.2.1. In most cases, a large number of data points taken at frequent distances are not available to produce a distinct pattern of $\rho_{\text{Exp}}(\tau)$ similar to that illustrated in Figure 5.2. Therefore, the choice of an analytical model for $\rho(\tau)$ is somewhat arbitrary. However, the Square Exponential ACF model, ACF # 4 in Table 5.1, also known as Gaussian ACF, has been the most frequently used by researchers in the field of reinforced concrete (Engelund and Sorensen, 1998; Gomes and Awruch, 2002; Li et al., 2004; Vu and Stewart, 2005; Keshel and O'Connor, 2009). Therefore the Gaussian ACF was used in this thesis to determine values for the parameter $\theta$ and for the generation of spatially correlated data as will be shown in Chapter 6.

![Figure 5.2 The autocorrelation functions (ACF) fitted to experimental correlation coefficients of $C_s$ data generated using $\theta=3$ m.](image)

The associated model parameter $d$, which is related to $\theta$ as described in Table 5.1 and is referred to by some researchers as the Correlation Length, for $C_s$ and $D_{\text{app}}$ variables can be obtained using the MLM or the Curve Fitting method. Both methods will be applied to the data collected in Chapter 4 in the upcoming sections for the estimation of the parameter $\theta$. 

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5.3.1 The maximum likelihood method (MLM)

The steps for the estimation of the parameter $d$, hence $\theta$, using MLM as reported by Li (2004b) are described as follows:

1- For each variable $C_s$, and $D_{app}$, there is $k$ number of observations $x_i$, ($i=1, 2, \ldots, k$)

2- Find the mean ($\mu_x$) and standard deviation ($\sigma_x$) of the $k$ observations;

3- Normalize the $k$ observations: $u_i = (x_i - \mu_x)/\sigma_x$;

4- Assume a value for the parameter $d$;

5- Calculate the correlation matrix $[\rho]$ where each element $\rho_{ij}$ represents the correlation coefficient between every pair $(u_i, u_j)$ of the $k$ observations by measuring the real distance between each two observations (e.g. location of concrete cores) and using Equation # 4 in Table 5.1.

6- Transform the dependent $u$-values into independent $v$-values by transformation matrix $[C]$ introduced in Section 3.4.4.1 ($k$ is the number of observations):

$$
\begin{bmatrix}
   v_1 \\
   v_2 \\
   \vdots \\
   v_k
\end{bmatrix} =
\begin{bmatrix}
   C_{11} & 0 & \ldots & 0 \\
   C_{21} & C_{22} & \ldots & 0 \\
   \vdots & \vdots & \ddots & \vdots \\
   C_{k1} & C_{k2} & \ldots & C_{kk}
\end{bmatrix}
\begin{bmatrix}
   u_1 \\
   u_2 \\
   \vdots \\
   u_k
\end{bmatrix}
$$

Equation 5.1

7- Solve for $[v]$ so that:

$$
[v] = [C]^{-1} [u]
$$

Equation 5.2

8- The Likelihood function:

$$
L = \prod \left( \frac{1}{\sqrt{2\pi}} \exp\left\{ -\frac{(v_i)^2}{2} \right\} \right) = \left( \frac{1}{\sqrt{2\pi}} \right)^k \exp\left\{ -\frac{\sum v_i^2}{2} \right\}
$$

Equation 5.3

9- Let $L_i = \sum v_i^2$, find $d$ which maximise $L$, that is to say $L_i$ is minimal.
Chapter 5: Estimation of the Scale of Fluctuation

In this method the corresponding likelihood values \((L_1)\) for different assumed values for the model parameter \(d\) are plotted in Figure 5.3 (North faces) and Figure 5.4 (South faces) for both variables, \(C_s\) and \(D_{app}\). In some crosshead face cases, the number of observations were reduced from five to four either because (i) the chloride profile could not be fitted to Fick’s 2\textsuperscript{nd} law equation (Equation 2.2), and hence results for \(C_s\) and \(D_{app}\) were not obtained, or (ii) the obtained values for the two fitted variables were considered outliers for the reasons indicated in Section 4.4.3. The results for the two variables that were reduced to four observations are those obtained from crosshead beam faces P2-N, P2-S, P3-S and P4-N.

Figure 5.3 The likelihood values \((L_1)\) versus the Correlation Length \((d)\) for \(C_s\) and \(D_{app}\) for pier crossheads North faces, P2-N, P3-N, P4-N and P-7N.
Chapter 5: Estimation of the Scale of Fluctuation

It is expected that the scale of fluctuation results obtained from the analysis that is based on five observations would be considered more reliable than the results obtained from the analysis based on four observations. This is because the more observations (data points) available the better the information regarding the fluctuation of the investigated property.
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will be. The estimated \( d \) (and \( \theta \)) values are summarised in Table 5.2. To distinguish between the results that were obtained based on four and five observations, the second are typed with a bold font in Table 5.2 whereas the results obtained based on the analysis of four observations were typed with a regular font. It has to be mentioned that concrete cores taken from crosshead beam faces P1-N, P5-N, P5-S, P6-N and P6-S were not processed for the reasons given in Section 4.3 and thus results for the aforementioned crosshead beam faces are not determined.

Table 5.2 Results of parameters \( d \) and \( \theta \), for \( C_s \) and \( D_{app} \) variables (Values with bold fonts are based on analysis of 5 observations).

<table>
<thead>
<tr>
<th>Pier face</th>
<th>Surface Chloride Content, ( C_s )</th>
<th>Diffusion Coefficient, ( D_{app} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( d ) (m)</td>
<td>( \theta ) (m)</td>
</tr>
<tr>
<td>P1-S</td>
<td>2.1</td>
<td>3.72</td>
</tr>
<tr>
<td>P2-N</td>
<td>0 - 0.8</td>
<td>0 - 1.3</td>
</tr>
<tr>
<td>P2-S</td>
<td>0 - 1.1</td>
<td>0 - 1.9</td>
</tr>
<tr>
<td>P3-N</td>
<td>0 - 1.1</td>
<td>0 - 1.9</td>
</tr>
<tr>
<td>P3-S</td>
<td>0 - 0.9</td>
<td>0 - 1.7</td>
</tr>
<tr>
<td>P4-N</td>
<td>2.5</td>
<td>4.4</td>
</tr>
<tr>
<td>P4-S</td>
<td>0 - 0.9</td>
<td>0 - 1.6</td>
</tr>
<tr>
<td>P7-N</td>
<td>0 - 1.2</td>
<td>0 - 2.1</td>
</tr>
<tr>
<td>P7-S</td>
<td>0 - 1.1</td>
<td>0 - 1.9</td>
</tr>
</tbody>
</table>

As can be seen from Table 5.2, the MLM was not able to capture a distinct value for the parameter \( \theta \) for \( C_s \) and \( D_{app} \) for most of the crosshead beam cases. As such, results of \( \theta \) were obtained in a range format with the exception of \( \theta \) for \( C_s \) from P1-S and P4-N and \( \theta \) for \( D_{app} \) from P1-S, P3-S and P7-S. The reason for the method failing to produce a single value for \( \theta \) was attributed to the following factors: (i) the limited number of observations, i.e. data points are too few to describe the fluctuation of the investigated property, (ii) the influence of the grid distance between observations, i.e. data points are widely spaced and therefore could not give enough information to describe the spatial fluctuation of the investigated property, or (iii) the influence of the shape of data fluctuation across the beam (i.e. the order of the observations). Each one of these possible reasons was investigated closely as will be shown next.
5.3.1.1 Influence of the number of observations

Initially, the reason for the MLM failure to capture a single value for $\theta$ was attributed to the limited number of observations (i.e. $k$ in steps 1-9) which are being used to calculate the maximum likelihood function, a reason which has also been pointed out by Li (2004b). However, this cannot be the case as the results for $C_s$ from P3-N, P4-S and P7-N and for $D_{app}$ from P7-N, P4-S and P3-N all have five observations but a single value for $\theta$ could not be obtained as opposed to the cases of P1-S for $C_s$, P1-S for $D_{app}$ and P7-S for $D_{app}$ which also had five observations but single values of $\theta$ were obtained. Furthermore, P4-N for $C_s$ and P3-S for $D_{app}$ both had four observations and the MLM produced single $\theta$ values of 4.4 and 3.9 for both variables respectively, Table 5.2. Therefore the number of observations obtained here was ruled out as the possible cause for the instability of the MLM.

5.3.1.2 Influence of the grid distance between observations

The influence of the grid distance between the data samples has also been investigated as a possible cause of the method inconsistency. Grid distances belonging to samples that were taken from several other crosshead faces (P3-N, P7-S and P7-N) were assumed to have been the grid distance for the data of $D_{app}$ collected from P1-S for which a distinct value of $d=2.3$ m ($\theta=4.1$ m) was already determined, as shown in Figure 5.5. This means that $d$ and $\theta$ were estimated again for the $D_{app}$ data collected from P1-S but this time the grid distance of P3-N, P7-S and P7-N were employed instead of the original grid distance of P1-S. These grid distances were randomly selected from among those which have five observations so the results of those which have similar grid distances can be compared. Estimation of the model parameter $d$ and hence $\theta$ was then carried out by following steps 1-9 outlined at the start of this section. As can be seen from the figure, grid distance of P1-S and P3-N are almost identical, and yet there is a large difference between the $d$ values obtained in both cases (2.3 m compared with 1.6 m). The grid distance of P7-N, which is not very different from the grid distance of P1-S and P3-N, has resulted in $d$ values that ranges from 0-0.4 m. This demonstrates that the calculated $d$ values are more influenced by the data than by the grid distance between their locations and hence the grid distance between the observations could not be identified as the reason for the instability of the MLM.
5.3.1.3 Influence of the shape of spatial fluctuation of data

The previous sub-sections have demonstrated that the number of observations and the grid distance between observations could not be the reason (or the only reason) for the instability of the MLM and its failure to capture a distinct value for \( \theta \). Given the importance of the parameter \( \theta \) for the reliability analysis to be carried out in this thesis, it was decided that the reason for the irregularity of the MLM should be identified before the method can be accepted to calculate \( \theta \) values. Therefore, the focus was then shifted to investigate the influence of the shape of data fluctuation across the beam (i.e. the order of the observations) as a possible cause for the irregularity of the MLM. To do this, the \( d \) values were calculated using different orders of the same observations which were taken from the same crosshead beam face (i.e. measurements made at location 1 is replaced by measurements made at location 3 and that of location 4 is replaced by 2, etc, Table 5.3). By changing the order of the observations, the statistics of the set of observations, i.e. the mean (\( \mu \)) and the standard deviation (\( \sigma \)), are kept unchanged and hence eliminating the influence of the data statistics as a possible cause of the inconsistency of the MLM. As can be seen from Figure 5.6, when \( D_{app} \) data collected from P7-N were used as an illustrative example, the calculated \( d \) values were found to vary from 0 to 3.2 m depending on the order of the data observations employed. Due to the strong correlation between data separated by distances that are smaller than \( \theta \) (i.e., as explained in Section 3.4.3, data with a large \( \theta \) value are expected to lie either above or under the mean value of the data) it may be accepted that the shape of the data fluctuation across the beam would influence the
determined $d$ values. However, this still could not explain why the MLM has failed to give a single value for the model parameter $d$ for some of the cases (i.e. Order 1 and Order 5) where the order of the data was changed as indicated in Figure 5.6.

Table 5.3 $D_{app}$ data measurements obtained from crosshead beam face P7-N randomly rearranged in several orders to obtain values for $\theta$ using the MLM.

<table>
<thead>
<tr>
<th>Location</th>
<th>Order 1*</th>
<th>Order 2</th>
<th>Order 3</th>
<th>Order 4</th>
<th>Order 5</th>
<th>Order 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P7-N1</td>
<td>9.52</td>
<td>13.9</td>
<td>14.15</td>
<td>8.17</td>
<td>9.52</td>
<td>9.84</td>
</tr>
<tr>
<td>P7-N2</td>
<td>13.9</td>
<td>14.15</td>
<td>13.9</td>
<td>9.52</td>
<td>14.15</td>
<td>9.52</td>
</tr>
<tr>
<td>P7-N3</td>
<td>9.84</td>
<td>9.52</td>
<td>9.84</td>
<td>9.84</td>
<td>8.17</td>
<td>14.15</td>
</tr>
<tr>
<td>P7-N4</td>
<td>14.15</td>
<td>8.17</td>
<td>9.52</td>
<td>13.9</td>
<td>13.9</td>
<td>13.9</td>
</tr>
<tr>
<td>P7-N5</td>
<td>8.17</td>
<td>9.84</td>
<td>8.17</td>
<td>14.15</td>
<td>9.84</td>
<td>8.17</td>
</tr>
</tbody>
</table>

* The original order of the data

Figure 5.6 The corresponding $d$ value estimated using MLM for $D_{app}$ data obtained from P7-N with data being randomly rearranged in several orders.

Based on the preceding analysis and discussion, the MLM in the form given in this section was considered inconsistent and, therefore, their results could not be relied on to draw a final conclusion on the values of the parameter $\theta$ for the two deterioration variables under investigation ($C_s$ and $D_{app}$). It has to be mentioned that the MLM was used by some researchers, i.e. Li (2004b), to propose values of $\theta$ for variables $C_s$ and $D_{app}$ but no investigation such as that performed here was performed to investigate the reliability of the method and its applicability to a small set of data points. This raises doubts about the
reproducibility and repeatability of the methods used to produce the scarcely reported $\theta$ values and hence on the reliability of $\theta$ values reported in the literature.

### 5.3.2 Curve fitting of $\rho(\tau)$ method

According to Vanmarcke (1983), if there are sufficient data measurements, and an analytical model for $\rho(\tau)$ has been chosen, the nonlinear regression methods, such as the Least Squares method, can be used to estimate the parameters of the chosen model (i.e. $d$ in the case of Gaussian ACF). By curve-fitting the analytical model $\rho(\tau)$ to the correlation coefficients determined from the experimental data $\rho_{\text{Exp}}(\tau)$, the value for the model parameter $d$ which produces the best fit can be obtained. The values for $\theta$ can then be obtained in terms of the model parameter $d$, as given in Table 5.1, The method used to calculate $\rho_{\text{Exp}}(\tau)$ will be described in the following section.

#### 5.3.2.1 The experimental correlation coefficients $\rho_{\text{Exp}}(\tau)$

Consider $n(u)$ number of data observations of random variable $Z(u)$ with mean $m$ and standard deviation $\sigma$, the Autocovariance Function $C(\tau)$ for any separation distance $\tau$ and the corresponding experimental correlation coefficients $\rho_{\text{Exp}}(\tau)$ are given by Ramachandran et al. (2001) as follows:

\[
C(\tau) = E\left[Z(u) - m\right] \left[Z(u + \tau) - m\right] \quad \text{Equation 5.4}
\]
\[
\rho_{\text{Exp}}(\tau) = \frac{C(\tau)}{\sigma^2} \quad \text{Equation 5.5}
\]

Where $E\{\}$ denotes the expectation of the expression $\{\}$, and $Z(u + \tau)$ is the magnitude of the variable $Z$ at distance $\tau$ away from $Z(u)$. Hence, for $C_s$ and $D_{\text{app}}$ data sets collected in Chapter 4, Equation and Equation can be used to plot the experimental correlation function $\rho_{\text{Exp}}(\tau)$. However, the number of data sets collected in this study, and in most studies related to the two investigated variables, were not sufficient to allow plotting a complete and clear trend of $\rho_{\text{Exp}}(\tau)$ as can be seen from Figure 5.7. In this figure, $\rho_{\text{Exp}}(\tau)$ for $C_s$ data obtained from Pl-S were plotted and an attempt was made to obtain the best fit using the Gaussian ACF model. If the correlation coefficients were determined for different lag intervals, most importantly for lag intervals that correspond to the declining part of the
analytical ACF model, i.e. more $\rho_{\text{Exp}}(\tau)$ values for the lag interval $<3.0$ m in Figure 5.7, a better fit can be obtained. Therefore, some statistical interpolation method must be used to estimate the ‘missing’ data if a reliable estimate of $\theta$ is to be obtained using the curve fitting method. The term ‘missing’ in this context refers to data at locations where it should have been measured, and therefore, needs to be predicted so a reliable estimate of $\theta$ can be obtained. The interpolation method should ensure that the spatial statistics (variability) of the original data remain unaffected. In the current study, the measured $C_s$ and $D_{app}$ data will be used to predict values of the same variables at other required locations in order to increase the number of data measurements. The technique by which this can be carried out is well known in the field of Mining Engineering, Geology and more recently in Hydrology and Soil sciences and is referred to as Kriging in recognition of the pioneering work of the Mining engineer Danie Krige in the year 1951 (Goovaerts, 1997). The technique will be introduced in the following section.

![Figure 5.7 Actual correlation coefficients $\rho_{\text{Exp}}(\tau)$ fitted using the Gaussian autocorrelation function (ACF) for $C_s$ data collected from P1-S.](image)

5.3.2.2 Kriging

Consider the problem of estimating the value of property $z$ at any unsampled location $u$ using $z$-data available over the concrete slab area in which number of data measurements of the same property has been obtained from locations $\alpha =1, 2, \ldots, 6.$, Figure 5.8. Most interpolation algorithms (such as inverse distance squared, splines, radial basis functions, triangulation, etc) estimate the value at a given location as a weighted sum of data values at
surrounding locations. In most cases, they assign weights according to functions that give a decreasing weight with increasing separation distance. On the other hand, Kriging assigns weights according to a data-driven weighting function, rather than an arbitrary separation distance-driven function. The kriging approach and terminology described by Goovaerts (1997) are employed here to describe Kriging approach of evaluating the ‘missing’ data. Or the additional data required for obtaining a reliable estimate of $\theta$.

\[
Z^*(u) = m(u) = \sum_{\alpha=1}^{n(u)} \lambda_{\alpha}(u) [Z(u_{\alpha}) - m(u_{\alpha})] \tag{Equation 5.6}
\]

where:

$\lambda_{\alpha}(u)$ is the ‘kriging’ weight assigned to datum $z(u_{\alpha})$ for estimation location $u$; where the same datum will receive different weights for different estimation locations. $m(u)$ and $m(u_{\alpha})$ are the expected values (means) of the random variable $Z(u)$ and $Z(u_{\alpha})$. $n(u)$ is the number of data points in local neighbourhood used for estimation of $Z^*(u)$.

The estimation error was defined as a random variable $Z^*(u)-Z(u_{\alpha})$. The goal was then set to determine weights, $\lambda_{\alpha}$, that minimize the variance of the estimator.
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\[ \sigma_E^2(u) = \text{Var}\{Z^*(u) - Z(u)\} \]  
Equation 5.7

Under the unbiasedness constraint:

\[ E\{Z^*(u) - Z(u)\} = 0 \]  
Equation 5.8

The RF \( Z(u) \) is decomposed into residual and trend components, \( Z(u) = R(u) + m(u) \), with residual component treated as a RF with stationary mean of 0 and a stationary covariance (a function of lag, \( \tau \), but not of position, \( u \)):

\[ E\{R(u)\} = 0 \]  
Equation 5.9
\[ \text{Cov}\{R(u), R(u + \tau)\} = E\{R(u) \cdot R(u + \tau)\} = C_R(\tau) \]

The residual covariance function is generally derived from the input *semivariance function* \( \gamma(\tau) \) as follow:

\[ C_R(\tau) = C_R(0) - \gamma(\tau) \]  
Equation 5.10

The kriging method in its simplest form assumes the mean \( m(u) \) to be known and constant throughout the study area so that \( m(u) = m \) where \( m \) is the mean of the sample data. Only this form of kriging will be discussed here. For more complex form of kriging, the author recommends the classic books by Cressie (1993) and Goovaerts (1997) for further reading.

If \( m(u) = m \) then Equation can be rearranged to obtain the following expression:

\[ Z^*(u) = \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u) [Z(u_\alpha) - m] + m \]
\[ = \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u) Z(u_\alpha) + \left[ 1 - \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u) \right] m \]  
Equation 5.11
This is the unbiased estimate, since \( E\{Z'(u)-Z(u)\}=m-m=0 \). The estimation error, \( Z'(u)-Z(u) \) can be considered as a linear combination of random variables representing residuals at the data points, \( u_a \), and the estimation point, \( u \):

\[
Z^*(u)-Z(u)=\sum_{\alpha=1}^{n(u)} \lambda_\alpha(u) R(u_\alpha) - R(u) = R^*(u) - R(u)
\]

Equation 5.12

where \( R(u_a)=Z(u_a)-m \) and \( R(u)=Z(u)-m \). Using rules for the variance of a linear combination of random variables, the error variance is then given by:

\[
\sigma_E^2(u) = \text{Var}\{R^*(u)\} + \text{Var}\{R(u)\} - 2\text{Cov}\{R^*(u), R(u)\}
\]

\[
= \sum_{\alpha=1}^{n(u)} \sum_{\eta=1}^{n(u)} \lambda_\alpha(u) \lambda_\eta(u) C_R(u_\alpha-u_\eta) + C_R(0) - 2 \sum_{\alpha=1}^{n(u)} \lambda_\alpha(u) C_R(u_\alpha-u)
\]

Equation 5.13

To minimize the error variance given by Equation, we take the derivative of the above expression with respect to each of the kriging weights and set each derivative to zero. This leads to the following system of equations:

\[
\sum_{\eta=1}^{n(u)} \lambda_\eta(u) C_R(u_\alpha-u_\eta) = C_R(u_\alpha-u); \quad \alpha = 1,...,n(u)
\]

Equation 5.14

The system Equation of \( n(u) \) linear equations is known as the system of equations or the simple kriging system.

Because of the constant mean, the covariance function for \( Z(u) \) is the same as that for the residual component:

\[
C_R(\tau) = E\{R(u):R(u+\tau)\} = E\{[Z(u)-m][Z(u+\tau)-m]\} = C(\tau)
\]

Equation 5.15

So that the simple kriging system can be written directly in terms of \( C(\tau) \):
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\[
\sum_{\eta=1}^{n(u)} \lambda_{\eta}(u) C(u_{\alpha} - u_{\eta}) = C(u_{\alpha} - u) \quad \alpha = 1, \ldots, n(u) \quad \text{Equation 5.16}
\]

Using matrix notation, the simple kriging system (Equation) can be expressed as:

\[
K\lambda(u) = k \quad \text{Equation 5.17}
\]

where \( K \) is the \( n(u) \times n(u) \) matrix of covariances between data points:

\[
K = \begin{bmatrix}
    C(u_1 - u_1) & \cdots & C(u_1 - u_{n(u)}) \\
    \vdots & \ddots & \vdots \\
    C(u_{n(u)} - u_1) & \cdots & C(u_{n(u)} - u_{n(u)})
\end{bmatrix} \quad \text{Equation 5.18}
\]

\( \lambda(u) \) is the vector of simple kriging weights for the surrounding data points, \( k \) is the vector of covariances between the data points and the estimation point:

\[
\lambda(u) = \begin{bmatrix}
    \lambda_1(u) \\
    \vdots \\
    \lambda_{n(u)}(u)
\end{bmatrix} \quad \text{Equation 5.19}
\]

\[
k = \begin{bmatrix}
    C(u_1 - u) \\
    \vdots \\
    C(u_{n(u)} - u)
\end{bmatrix} \quad \text{Equation 5.20}
\]

If the covariance model \( C(\tau) \) is permissible (the data covariance matrix is positive definite) and no two data points are collocated (i.e. \( u_\alpha \neq u_\eta \) for \( \alpha \neq \eta \)) then the kriging weights can be solved by rearranging Equation:

\[
\lambda(u) = K^{-1}k \quad \text{Equation 5.21}
\]

Once the kriging weights \( \lambda(u) \) are determined, the kriging estimate can be computed from Equation so that:
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Equation 5.22

\[ Z^* (u) = \sum_{\alpha=1}^{n(u)} \lambda_{\alpha} (u) Z(u_\alpha) + \lambda_m (u) m \]

where:

\[ \lambda_m (u) = 1 - \sum_{\alpha=1}^{n(u)} \lambda_{\alpha} (u) \]

Equation 5.23

5.3.2.3 Modelling the semivariogram \( \gamma(\tau) \)

The Semivariance Function \( \gamma(\tau) \) used in Equation can be obtained empirically from a set of data points as follows:

\[ \gamma_{\text{Exp}} (\tau) = \frac{1}{2N(\tau)} \sum_{\alpha=1}^{N(\tau)} [z(u_\alpha + \tau) - z(u_\alpha)]^2 \]

Equation 5.24

where \( \gamma_{\text{Exp}} (\tau) \) denotes the empirical semivariance function, \( N(\tau) \) denotes the number of pairs of data that are separated by a lag distance \( \tau \). When the semivariance function is plotted against the corresponding lags, the plot is referred to as the semivariogram. In practice, data may not be sampled at regular intervals and estimating the empirical semivariance function (Equation) can be very tricky due to the need to pool data into lag bins.

When performing kriging to predict the ‘missing’ data, the empirical semivariogram, \( \gamma_{\text{Exp}} (\tau) \), needs to be replaced with an acceptable analytical semivariogram model, \( \gamma(\tau) \). The reason for this is that the kriging algorithm needs access to semivariogram values for lag distances other than those provided by the empirical semivariogram. Therefore, several analytical semivariogram models have been proposed as a function of the lag distance. Parameters of the analytical model can be obtained by curve-fitting of the proposed analytical semivariogram model to the empirical semivariogram. Five such analytical models are listed in Table 5.4 with models number 2, 3 and 4 being the most frequently used. The parameters \( c \) and \( r \) in the models listed in Table 5.4 are the model parameters that need to be estimated so the selected semivariogram model can be characterised. In geostatistics, \( c \) is referred to as the Sill and \( r \) as the Practical Range as illustrated by Figure
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5.9. Both parameters are obtained by curve fitting the proposed analytical semivariogram model to the empirical (experimental) semivariances $\gamma_{\text{Exp}}(\tau)$. Figure 5.9 demonstrates that the values of $\gamma_{\text{Exp}}(\tau)$ in the $r$ region fits closely with the analytical model of $\gamma(\tau)$. Beyond the Range region, the $\gamma_{\text{Exp}}(\tau)$ shows significant scatter which indicates that weak correlation exists between data too far apart. Such behaviour is typical for most semivariogram plots of natural properties (Goovaerts, 1997).

<table>
<thead>
<tr>
<th>Name</th>
<th>Semivariogram model $\gamma(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Nugget effect model</td>
<td>$\begin{cases} 0 &amp; \text{if } \tau = 0 \ c &amp; \text{otherwise} \end{cases}$</td>
</tr>
<tr>
<td>2 Spherical model</td>
<td>$\begin{cases} \frac{c}{1.5 - 0.5r} &amp; \text{if } r \leq r \ c &amp; \text{otherwise} \end{cases}$</td>
</tr>
<tr>
<td>3 Exponential model</td>
<td>$\gamma(\tau) = c \left[ 1 - \exp\left(\frac{-3r}{r}\right) \right]$</td>
</tr>
<tr>
<td>4 Gaussian model</td>
<td>$\gamma(\tau) = c \left[ 1 - \exp\left(\frac{-3r^2}{r^2}\right) \right]$</td>
</tr>
<tr>
<td>5 Power model</td>
<td>$\gamma(\tau) = c \cdot \tau^\omega$ with $0 &lt; \omega &lt; 2$</td>
</tr>
</tbody>
</table>

Figure 5.9 Typical semivariogram plot obtained from porosity data, (Bohling, 2005).
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Figure 5.9 indicates that the fitting procedure hence the estimated model parameters \((r\) and \(c)\) are controlled by the semivariances which corresponds to lag distances within the Range region. The significant of this is that data measurements should be taken at lag distances that are less than the Range region if a better fit to the analytical semivariogram model to be obtained. This issue will be discussed further in Section 5.4.

The decision on which of the analytical semivariogram models should be selected is subjective. Sometimes the number of data pairs is very small, therefore, the produced \(\gamma_{Exp}(\tau)\) may be noisy (i.e. widely and unsystematically scattered) and it would be difficult to distinguish which of the available analytical models would fit the experimental semivariogram best. Subjective judgement is often used in such cases to choose between the models. In the current study, the Gaussian semivariogram model was chosen due to its popular use by researchers (Goovaerts, 1997).

In the current study, initially, the experimental semivariogram was determined for each \(C_s\) and \(D_{app}\) data set belonging to each crosshead face. The experimental semivariogram of both variables for P1-S are indicated in Figure 5.10. As can be seen from the figure, the experimental semivariogram is quite noisy; this is mainly due to the small number of data pairs that are separated by the same lag interval (plus or minus lag tolerance). For example, as indicated in Figure 5.11, the first semivariance value (point # 1) was based on four pairs of data in which their corresponding lags ranged from 2.87 m to 3.48 m. The second semivariance value (point # 2) is based on three pairs of data in which their lags ranged from 6.29 m to 6.34 m. The third semivariance value (point # 3), in both graphs, was determined based on two lag distances that were 9.28 m and 9.77 m. The last semivariance value (point # 4) in the figure was based only on one pair of data.

If data were taken at regular grid locations, semivariance values can be estimated for each corresponding lag interval rather than for a window of lag intervals that are within an acceptable lag tolerance. The acceptable lag tolerance refers to the range of lags that can be represented by a middle lag value and often estimated based on subjective judgment (Goovaerts, 1997). Furthermore, as the number of data pairs increases, more square differences (see Equation) will be used for calculating the semivariance value and the less noisy the semivariogram becomes. This kind of analysis reveals details that are of extreme practical importance for future planning of data sampling that aims to investigate the
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Spatial variability parameters. Similar graphs were plotted in Appendix E for $C_s$ and $D_{app}$ data collected from all other crosshead faces.

![Graph](image-url)

Figure 5.10 Semivariograms constructed using data collected from crosshead beam P1-S for (a) $C_s$ and (b) $D_{app}$, (the individual semivariogram).

![Graph](image-url)

Figure 5.11 Square differences of data separated by lags ($\tau$) vs. corresponding lags ($\tau$) calculated for $C_s$ data obtained from P1-S.

To indicate the importance of having sufficient data points for each lag distance, semivariance values were estimated from $C_s$ and $D_{app}$ data for all crosshead beams faces assuming that all data belonging to the same crosshead face and the corresponding semivariograms were then constructed as shown in Figure 5.12. By assuming that all data have come from the same crosshead beam face, the number of data points that correspond
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to each lag interval increases and, as a result, the semivariogram becomes less noisy. For example, the experimental semivariances indicated in Figure 5.12 are scattered closer to the analytical Gaussian model as compared with those indicated in Figure 5.10. The semivariograms obtained in this way (the average semivariogram) may be considered as an approximation for all the individual semivariograms. The fitting parameters of the ‘average’ semivariogram may be used to construct a semivariogram that is representative of the data spatial dependence for all crosshead beam faces.

Figure 5.12 Semivariograms constructed using data collected from all tested crosshead faces for $C_s$ (a) and $D_{app}$ (b), (the average semivariogram).

The semivariogram parameters, $c$ and $r$, which were obtained from fitting the ‘average’ experimental semivariances to the analytical Gaussian semivariogram model, were then used as an input to the analytical semivariance function employed to perform kriging interpolation. The complete set of the interpolated data (obtained by kriging) was superimposed on the actual data for both variables, $C_s$ and $D_{app}$, for all eight crosshead faces as can be seen from Figure 5.13(a) to Figure 5.20(a). It can be noted that results of P2-N were excluded because there were only three data points. Parameters $d$, and hence $\theta$, were then determined by fitting the Gaussian ACF to the actual (experimental) correlation coefficients, $\rho_{Exp}(\tau)$, calculated from the complete set of data for both variables, $C_s$ and $D_{app}$, Figure 5.13(b) to Figure 5.20(b). It can be seen from these figures that the interpolated data forms a smoothly fluctuating curve that passes through the original (actual) data for all crosshead cases. It is, therefore, reasonable to assume that as the number of the original observations increases, more reliable prediction of the ‘missing’ data can be achieved. The spatial statistics of the original data was maintained when
predicting the new data due to the use of kriging interpolation. It can be noted from Figure 5.13(b) to Figure 5.20(b) that the predicted data are spatially correlated and the correlation decreases as the distance separating the samples increases. This proves that the predicted data are spatially correlated. The second observation that can be made from these figures is that the number of data measurements at lag distances corresponds to the declining part of the Gaussian ACF is more significant than measurements made at distances beyond the correlation region. This is because the first can contribute to improving the goodness of the fitted ACF model and hence better and more reliable estimate of the parameter $\theta$. Therefore, more measurements should have been made within these lag intervals if a reliable prediction of the fitted parameters to be obtained. However, it is not always possible to know in advance what might be the length of correlation region for a certain property. For this reason, the analysis performed in this chapter provides significantly important information regarding the correlation length (and $\theta$) expected for $C_s$ and $D_{app}$ and their dependant variables and hence helps for planning future field measurements intended for measuring $\theta$ as will be seen from the discussion of Section 5.4.

Figure 5.13 (a) Complete set of data after kriging, (b) the experimental correlation function fitted to Gaussian ACF for $C_s$ (left) and $D_{app}$ (right) data collected from P1-S.
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Figure 5.14 (a) Complete set of data after kriging, (b) the experimental correlation function fitted to Gaussian ACF for $C_s$ (left) and $D_{app}$ (right) data collected from P2-S.

Figure 5.15 (a) Complete set of data after kriging, (b) the experimental correlation function fitted to Gaussian ACF for $C_s$ (left) and $D_{app}$ (right) data collected from P3-S.
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Figure 5.16 (a) Complete set of data after kriging, (b) the experimental correlation function fitted to Gaussian ACF for \( C_s \) (left) and \( D_{app} \) (right) data collected from P3-N.

Figure 5.17 (a) Complete set of data after kriging, (b) the experimental correlation function fitted to Gaussian ACF for \( C_s \) (left) and \( D_{app} \) (right) data collected from P4-S.
Figure 5.18 (a) Complete set of data after kriging, (b) the experimental correlation function fitted to Gaussian ACF for $C_s$ (left) and $D_{app}$ (right) data collected from P4-N.

Figure 5.19 (a) Complete set of data after kriging, (b) the experimental correlation function fitted to Gaussian ACF for $C_s$ (left) and $D_{app}$ (right) data collected from P7-S.
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Values for the model parameter $d$ estimated using parameters of the individual semivariogram and parameters of the semivariogram that have been constructed using data from all crosshead faces (the average semivariogram) are listed in Table 5.5. It can be seen that the difference in the estimated $d$ values between the two cases is not very large, with the exception of P3-S for $D_{app}$. More discussion on the values presented in Table 5.5 will be provided in the coming sections.

5.3.3 The scale of fluctuation for $C_s$

The values of $\theta$ obtained from the current study for $C_s$, which are presented in Table 5.5, ranges from 1.8 m to 3.7 m with an average of 2.8 m ($d=1.0$ to 2.1 m and $d$ average = 1.6 m). These values correspond to using $r$ and $c$ parameters determined from employing the individual semivariogram models. When $r$ and $c$ were determined from employing the average semivariogram models, $\theta$ was found to range from 1.9 m to 3.5 m with an average value of 2.7 m ($d=1.1$ to 2.0 m and $d$ average = 1.5 m). The value reported by Karimi (2001) for $\theta$, which was determined using the Curve Fitting method, was based on the analysis of $C_s$ data collected from three crosshead beams exposed to de-icing source of chlorides. For one of the three crosshead beams, the reported values of $\theta$, Table 5.6, were
estimated using 11 samples collected over a length of 15 m. The number of samples and the size of area by which the samples were collected from, were not provided in the original reference for the other two crosshead beams. Both values, the average value (i.e. \( \theta = 2.7 \) m) and the value proposed by Karimi (2001), compare well as the latter is within less than one standard deviation of the first.

Table 5.5 The model parameter, \( d \), for \( C_s \) and \( D_{app} \) variables (Values with bold fonts are based on analysis of 5 observations).

<table>
<thead>
<tr>
<th>Pier face</th>
<th>( d (\theta) ) for ( C_s ) in m</th>
<th>( d (\theta) ) for ( D_{app} ) in m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>**P1-S</td>
<td>2.1(3.7)</td>
<td>2.0(3.5)</td>
</tr>
<tr>
<td>**P2-N</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>**P2-S</td>
<td>1.6(2.8)</td>
<td>1.5(2.7)</td>
</tr>
<tr>
<td>**P3-N</td>
<td>1.0(1.8)</td>
<td>1.1(1.9)</td>
</tr>
<tr>
<td>**P3-S</td>
<td>1.5(2.7)</td>
<td>1.3(2.3)</td>
</tr>
<tr>
<td>**P4-N</td>
<td>1.9(3.4)</td>
<td>1.9(3.4)</td>
</tr>
<tr>
<td>**P4-S</td>
<td>2.0(3.5)</td>
<td>1.5(2.7)</td>
</tr>
<tr>
<td>**P7-N</td>
<td>1.2(2.1)</td>
<td>1.1(1.9)</td>
</tr>
<tr>
<td>**P7-S</td>
<td>1.7(3.0)</td>
<td>1.4(2.5)</td>
</tr>
<tr>
<td>Average</td>
<td>1.6(2.8)</td>
<td>1.5(2.7)</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.4(0.6)</td>
<td>0.3(0.6)</td>
</tr>
</tbody>
</table>
* \( d \) values estimated based on parameters of the individual semivariogram.
** \( d \) values estimated based on parameters of the average semivariogram.

Table 5.6 Results for the Scale of Fluctuation reported in the literature.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Scale of fluctuation, ( \theta ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( C_s )</td>
</tr>
<tr>
<td>Kenshel (2009)</td>
<td>2.7</td>
</tr>
<tr>
<td>Karimi (2001)</td>
<td>1.96</td>
</tr>
<tr>
<td>Engelund (1997)</td>
<td>1.8 (( d=1.0 ))</td>
</tr>
<tr>
<td>*Li (2004b)</td>
<td>Case 1</td>
</tr>
<tr>
<td></td>
<td>Case 2</td>
</tr>
<tr>
<td></td>
<td>Case 3</td>
</tr>
<tr>
<td></td>
<td>Case 4</td>
</tr>
</tbody>
</table>
* Li (2004b) wrongly gave the name 'the scale of fluctuation' to the parameter \( d \)
Li (2004b) collected $C_s$ and $D_{app}$ data from four different cases for the study of $\theta$, Figure 5.21. In the first case, Li (2004b) estimated $\theta$ for both variables $C_s$ and $D_{app}$, based on six data points collected from two rows within 1.5 m x 0.5 m concrete slab, three from each row, the distances between sample locations were 0.5 m in the longitudinal direction and 0.2 m in the transverse direction.

In the second case, the analysis was carried out on data collected from an aging bridge at locations that are widely spaced. The distances between core locations in this case, as indicated in Figure 5.21, were: 90 m, 0.5 m, 135 m and 0.5 m respectively. In the third case, the data for both variables were collected from locations that are also widely spaced, with the distance separating the six samples being 0.5 m, 360 m, 0.5 m, 90 m, 0.5 m respectively. Using the MLM, Li (2004b) found that $d$ value ranges from 0.4 m to 50 m for both variables for the second case and between 0 to 100 m for $C_s$ for the third case. Clearly, the spacing between the cores locations in both cases make the results less reliable, as was acknowledged by Li (2004b) who concluded that the distance between the samples has to be in the range of 0.5 m to 10 m for the parameter $\theta$ to be reliably estimated from the sample data for the properties of interest.

In the fourth case, Li (2004b) estimated values for $d$ for variable $C_s$ based on the analysis of chloride contents at three different concrete depths, namely at 0-10 mm, 10-30 mm and 30-50 mm. The chloride contents were obtained from samples that were taken from a six year old tunnel that had been exposed to a chloride environment. A total of twelve samples were taken at locations as indicated in Figure 5.21. The value for $d$ estimated by Li (2004b) from this case for $C_s$, which have been indicated in Table 5.6, was based on the analysis of the chloride contents at depth 0-10 mm. It can be seen that the fluctuation parameters estimated in the fourth case does not represent the fluctuation parameter of the variable $C_s$. This is due to the fact that $C_s$ is influenced by the chloride profile through out the cover region and can not be represented by the chloride content at the outer region alone. Furthermore, similar to the proceeding two cases, the distances between samples are very large, therefore, any estimation of the fluctuation parameters that are based on such large distances between samples cannot be considered as reliable.

Based on the available literature, the value of $\theta$ for $C_s$ can be assumed to be in the range of 0.2 m to 3.5 m, Table 5.6. The results of the parameter $\theta$ for the variable $C_s$ based on the fluctuation analysis of the chloride profiles obtained from the eight crosshead faces
considered in the current study are within this range, therefore it can be assumed that $\theta$ for $C_s$ is in the order of 2.7 m.

![Diagram of concrete cores arrangements](image)

Figure 5.21 Concrete cores arrangements that have been used by Li (2004b) for the study of $\theta$ for $C_s$ and $D_{app}$, distance between samples is in (mm) except for Case 4 in meters.

### 5.3.4 The scale of fluctuation for $D_{app}$

The values of $\theta$ reported by Karimi (2001) for the variable $D_{app}$ was not based on the analysis of the field data in the same way the $\theta$ value for $C_s$ was estimated. The $\theta$ value indicated in Table 5.6 for $D_{app}$ was proposed based on the suggestions by Engelund and Sorensen (1998) that the spatial variability of $D_{app}$ is closely associated with the microstructure of the concrete and that the $\theta$ of such variable exhibits a random spatial

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fluctuation at interval lengths of about 0.2 m to 0.5 m. The average of the two values was then used by Karimi (2001) to represent the $\theta$ for $D_{app}$ as indicated in Table 5.6.

The values of $\theta$ for $D_{app}$ which have been reported by Li (2004b) were 0.7 m ($d=0.4$ m) from Case 1, 0.7 to 89 m ($d=0.4$ to 50 m) from case 2 and 0-255 m ($d=0-140$ m) for case 3. While $\theta$ values reported by Li (2004b) from Case study 2 and 3 cannot be considered reliable for the same reasons indicated in the previous section with respect to $C_v$, the value reported from Case 1 is within the range suggested by Engelund and Sorensen (1998) and later adopted by Karimi (2001), Table 5.6.

The results of parameter $\theta$ of $D_{app}$ obtained from the current study, and presented in Table 5.5, range from 1.6 m to 4.8 m ($d=0.9$ m to 2.7 m) when the *individual* semivariogram parameters were used. When the *Average* semivariogram parameters were used, $\theta$ found to range from 1.4 to 3.0 m ($d=0.8$ to 1.7 m). The value of $\theta=4.8$ m ($d=2.7$ m) may be considered as an outlier if compared with the rest of the results. If this is the case, the average value of $\theta$, when the *individual* semivariogram model parameters were used, becomes 2.5 m, about 32% higher than that obtained when the *average* semivariogram parameters were used ($\theta=1.9$ m).

It can be noticed that the average $\theta$ value for $D_{app}$ obtained from the current study is about five times that reported by Engelund and Sorensen (1998) and Li (2004b). While the value reported by Li (2004b) from Case study 1 was estimated with the MLM which have been shown to be inconsistent, Engelund and Sorensen (1998) have not shown how they have estimated their value. Furthermore, in his reliability based analysis, Li (2004b) finally ignored that value and used a value of $\theta=3.5$ m ($d=2.0$ m) for both variables $D_{app}$ and $C_v$. Therefore, it would be reasonable to assume that the value estimated from the current study is within the minimum and the maximum value that had been reported in the literature with regard to variable $D_{app}$.

Considering the available literature estimates of the parameter $\theta$ for $D_{app}$, and based on the preceding discussion, the value estimated from the current study can be regarded as the most reliable to the time of writing this thesis. Value of $\theta=1.9$ m ($d=1.1$ m) for $D_{app}$ will be used as an input for the RF model to be developed in Chapter 6.
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5.4 RECOMMENDATIONS FOR SAMPLING LOCATIONS

The procedure for modelling the semivariogram from the available field data has highlighted the importance of sampling at frequent distances rather than from arbitrary locations when investigating the parameter $\theta$. The implication of not sampling at frequent distances has been presented in Section 5.3.2.3. As explained in the relevant section, sampling at frequent intervals will ensure that multiple semivariance values for the same lag interval are obtained. Therefore, there will be no need to pool lags that have close values together when constructing the semivariogram model from the sampled data.

Based on the current investigation, for the purpose of obtaining a distinct semivariogram model that can be fitted by the proposed analytical mode, a minimum of two lag intervals have to be smaller than the expected \textit{practical range} parameter, $r$. From the current investigation, value for $r$ can be assumed to be in the order of $r=1.81$ m for the case of $C$, and $r=1.14$ m for the case of $D_{app}$ as already indicated in Figure 5.12. Arrangement for the samples location and the distances between them proposed for future work are shown in Figure 5.22. The value of $x_1$ is found to be better between 0.2-0.5 m and $x_2$ between 0.4-0.8 m. To further explain the usefulness of specifying values within the range already given, Table 5.7 indicates the frequency of each possible lag interval that corresponds to chosen $x_1=0.3$ m and $x_2=0.7$ m. This arrangement was designed to produce as many lag intervals within the expected range of sample correlation which will help in describing the mounting part of the semivariogram model, Figure 5.23. The number of samples proposed here should be taken as a minimum. It has to be kept in mind that some samples may not yield any results that can be used in the analysis, as a result, the frequency that corresponds to some lag intervals will be reduced and hence a ‘noisier’ semivariogram plot will be obtained. If resources allow, more sample pairs should be taken. As the number of sample pairs increase, the frequency that corresponds to the same lag interval will also increase and as result a smoother semivariogram plot will be obtained.

The measurement planning procedure proposed here can be employed when the instrumentation of new and existing structures is performed. For example, instrumentation of structures should be designed in a way not only to provide statistical information of the related deterioration parameters but also to provide information which is necessary for the modelling of their spatial variability.
Chapter 5: Estimation of the Scale of Fluctuation

Figure 5.22 Proposed sample locations for investigating the scale of fluctuation.

Table 5.7 Frequency of lag intervals correspond to choosing $x_1 = 0.3$ m, $x_2 = 0.7$ m

<table>
<thead>
<tr>
<th>Lag, $\tau$ (m)</th>
<th>0.3</th>
<th>0.7</th>
<th>1.0</th>
<th>1.3</th>
<th>2.0</th>
<th>2.3</th>
<th>3.0</th>
<th>3.3</th>
<th>4.0</th>
<th>4.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 5.23 Schematic representation of the possible semivariogram plot that can be achieved by using sampling configuration as proposed by Figure 5.21 and Table 5.6.

5.5 CONCLUSIONS

The main objective of the experimental work carried out in this study was to estimate values for the scale of fluctuation, $\theta$, of $C_e$ and $D_{app}$ variables from data collected from an aging structure exposed to marine environment. Once $\theta$ values for the two deterioration variables are estimated, they will be used as input parameters to generate spatially correlated data as part of the RF model which will be developed in Chapter 6. Two estimation methods have been applied to the data collected in Chapter 4 for the two
investigated variables. Eight groups of results each consisting of four or five observations were analysed using the MLM described by Li (2004b) and the Curve Fitting to $\rho(\tau)$ Method proposed by Vanmarcke (1983). The MLM was found to be inconsistent and did not yield a unique value for the investigated fluctuation parameter in most cases.

The curve fitting method (Vanmarcke, 1983) requires the ‘missing’ data to be predicted. The Kriging method (Ramachandran et al., 2001) of statistical interpolation was proposed to predict the missing data at locations where samples were not taken. Unlike most of the ordinary interpolation algorithms which assign weights according to functions that give a decreasing weight with increasing separation distance between the data samples, Kriging assigns weights according to a data-driven weighting function. The theory and the concept of the method were presented in Section 5.3.2.2. The method has been used in other fields of science such as Mining Engineering, Geology and more recently in Hydrology and Soil sciences. Its use however in the field of structural engineering is almost non existent with the exception of work by Ramachandran et al. (2001) and Karimi (2001). To use Kriging for the prediction of the missing data, knowledge of the associated semivariance models which describes the spatial correlation between data is essential. Parameters of such model can be obtained from curve fitting a proposed analytical model to that obtained from the experimentally collected data as discussed in Section 5.3.2.3.

Values of $\theta$ for the two investigated deterioration variables obtained using the curve-fitting method have been compared to values reported in the literature. When compared to the literature, the $\theta$ value obtained from the current study for the variable $C_s$, was found to be 1.5 times that reported in the literature. As for $D_{app}$, the $\theta$ value obtained from the current study was found to be 5 times that reported in the literature. Based on the investigation carried out in the this chapter with respect to the estimation methods, the number of sample data, distance separating samples and the number of group of samples that have been used by other researchers for the estimation of $\theta$, the results obtained from the current study for both variables and in particular for $D_{app}$, can be regarded as the most reliable up to the date of writing this thesis. The means and standard deviations of the parameter $\theta$ obtained from the current study for $C_s$ and $D_{app}$ variables are summarised in Table 5.8.
Table 5.8 Mean and standard deviation of the Scale of Fluctuation $\theta$ (in m) obtained from the current study for $C_s$ and $D_{app}$.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_s$</td>
<td>2.7</td>
<td>0.6</td>
</tr>
<tr>
<td>$D_{app}$</td>
<td>1.9</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Finally, based on the obtained $\theta$ values for both variables, $C_s$ and $D_{app}$, sample location for future experimental investigation of the fluctuation property of these two variables have been proposed. The proposed arrangement for sampling location was designed in such a way so that a distinctive experimental semivariogram plot can be obtained from the minimum possible number of sample data. The proposed sampling procedure aims to improve the quality of the empirical semivariogram plot by achieving: (i) higher frequency for the same lag interval which will reduce the variability of the semivariances that correspond to the same lag interval and therefore results in much smoother semivariogram plot; (ii) more lag intervals within the expected range of sample correlation which will help in describing the mounting part of the semivariogram model. The measurement planning procedure proposed here can be employed when instrumentation of structures is carried out (i.e. by Health Monitoring System organisations) so that data obtained can be of a full usefulness for present and future assessment.
Chapter 6:

Development of the Random Field-Based Reliability Model
6.1 INTRODUCTION

As explained in Chapter 2, Reinforced Concrete (RC) structures in chloride environments undergo a series of deterioration stages that affect both the visual condition of the structure as well as its load carrying capacity and hence its safety. The surface condition of the structure during its lifetime may be termed as the ‘Condition’ profile whereas the lifetime performance of the structure with regard to its load carrying capacity may be referred to as the ‘Safety’ profile (Frangopol et al., 2001; Neves and Frangopol, 2005). Due to the common parameters associated with the limit states evaluation (material properties, environmental and exposure conditions, geometrical arrangements, etc.), positive correlations are expected to exist between the level of deterioration for both profiles (Condition and Safety) at any given time during the life of the structure. Therefore, it would be more appropriate if both profiles were predicted using the same approach and where possible the same physical/empirical deterioration models and parameters. The first major objective of this chapter is to develop a stochastic model that allows investigation of the possibility of such integration using a hypothetical RC structure affected by chloride-induced corrosion as an illustrative example.

The lifetime performance of the structure in terms of its ‘Condition’ profile can be represented by the proportion of the concrete surface area that is showing signs of corrosion-induced damage ($A_x\%$). This damage may be defined in terms of the area of steel bars where corrosion has been initiated, the appearance of cracks with a specified limiting width on the surface of the structure or in terms of surface spalling of the concrete cover due to the ongoing reinforcement corrosion. Existing works which considered the structure or its components as homogenous (not considering the spatial variability of the random variables) could not predict $A_x\%$ over time (i.e. $A_x\%$ is either 0% or 100%). This is because the available methods do not require that the surface of the assessed structure/component to be discretised into number of elements of an appropriate size so the percentage of $A_x\%$ can be computed as discussed in Chapter 3. The concept of computing $A_x\%$ was made possible by using the Random Field (RF) theory introduced in Chapter 3. A number of researchers (e.g., Engelund, 1997; Li, 2004b; Malioka and Faber, 2004; Vu and Stewart, 2005; Kenshel and O'Connor, 2009) have shown that by applying RF theory, it is possible to predict the percentage of the surface area of the structure that is affected by chloride-induced corrosion at any given year during the structure service life provided that models to compute times for different deterioration stages are available.
Chapter 6: Development of the RF-based model

As was mentioned in Chapter 3, the procedure of using RF to predict $A_{y\%}$ at any given year involves the discretisation of the structure or structural component into small elements and treating the random variables (material properties, geometrical dimensions, etc.) within each element as a spatially constant. The statistical correlations between the random variables for different RF elements are specified based on the correlation characteristics provided by the Scale of Fluctuation ($\theta$) and the Autocorrelation Function $\rho(\tau)$ (Vanmarcke, 1983). Such correlation characteristics have not been available in the literature which required the extensive experimental work (Chapter 4) and the statistical analysis (Chapter 5) to be performed to provide the unavailable information. Once such information was obtained and the stochastic RF is defined, the Monte Carlo (MC) simulation technique, explained in Section 3.3.3, can be used to randomly generate values of the random variables for each of the discretised RF element following the procedure which was described in Section 3.4.4.

The RF based stochastic model developed in this chapter has the advantage of using site-specific material and deterioration properties which were collected from Ferrycarrig Bridge (Chapter 4 and Chapter 5) as input parameters. The model was initially aimed at predicting $A_{y\%}$ based on works carried out by Li et al. (2004) and by Vu and Stewart (2005) and was further developed here to predict the loss in the load carrying capacity (Safety profile) of the corroding structure using the Reliability Index ($\beta$) as a performance indicator. The effect of pitting corrosion, a phenomena discussed in Section 2.5.3.2, was also taken into consideration. The second major objective of this chapter is to apply the principles of RF theory discussed in Chapter 3 to model the ‘Condition’ profile and investigate how spatial variability influences the ‘Safety’ profile.

Once the maximum acceptable surface area damage $A_{\text{max}}\%$ (discussed in Section 2.6) or a minimum target reliability index $\beta_T$ (discussed in Section 3.3.4) is reached, then the two profiles (Condition & Safety) can be used to determine the optimum time to first repair/maintenance intervention. This approach provides a rational tool by which two different criteria can be applied to determine the optimum time to first repair/maintenance as illustrated in Figure 6.1. The figure schematically illustrates the approach followed in this thesis to predict the time to first repair/maintenance intervention. It also shows how material properties, deterioration models and statistical parameters can all be shared by the two prediction models, the Condition and the Safety. The criteria used for deciding the time to first repair based on the condition state ($T_{R1C}$) is the $A_{\text{max}}$, while the criteria used to
Chapter 6: Development of the RF-based model

decide on the time to first repair based on the safety state \( T_{RIS} \) is controlled by \( \beta_T \). The approach developed in this chapter, which has not been considered before in the literature, offers an opportunity where the Serviceability Limit State (SLS) and Ultimate Limit State (ULS) related performance criteria are being dually modelled for the purpose of investigating the optimum time to repair/maintenance intervention.

![Structural Lifetime Performance Diagram](image)

**Figure 6.1 Optimum time for repair/maintenance intervention flowchart.**

For the ‘Safety’ profile to be realistically modelled, a probabilistic modelling of the load effect based on real traffic data should be considered. In this chapter real truck load data obtained from a Weight in Motion (WIM) data collection system was analysed using Extreme Value (EV) statistics to obtain the maximum load effect for the remaining ‘serviceable life’ of the structure. This is another addition to the work done to date with regard to the service life modelling of corrosion affected RC structures. In previous works, researchers used either codified deterministic load effect models or predictive probabilistic models that are not based on real traffic data. EV statistics and its application to predicting the maximum load affect of highway bridges will also be discussed in this chapter.
6.2 CONDITION PROFILE MODELLING

Modelling the Condition profile for an RC structure is necessary due to the fact that the criterion mostly used by researchers, local agencies, assessment engineers, etc. to decide on the time and the type of the repair/maintenance intervention is directly related to the condition state. For example, and as discussed in Section 2.6, maintenance and repair of corroding RC structures is often decided when a certain percentage $A_{\text{max}}\%$ of the structure surface area has shown signs of corrosion related damages. Values of these percentages were found to vary significantly from agency to agency and from engineer to engineer. However, based on the review carried out in Section 2.6 and in terms of initiation of corrosion as the damage type, a value of $A_{\text{max}} = 25\%$ was proposed for the work of this thesis, and in terms of the appearance of 1.0 mm wide cracks as the damage type, a value of $A_{\text{max}} = 1.5\%$ was proposed. It is, therefore, important that the Condition profile is modelled accurately for these performance indicators to be identified for a given structural assessment case.

It was shown in Chapter 3 that RF modelling is a powerful tool that can be used to predict the deterioration of the surface condition of the corroding structure over time. To demonstrate application of RF theory to the field of RC structures, a hypothetical RC T-beam girder was considered and assumed to have been exposed to chloride attacks since its construction. Following the RF approach described in the literature (e.g., Engelund, 1997; Haldar and Mahadevan, 2000; Malioka and Faber, 2004; Vu and Stewart, 2005) which was further explained in Chapter 3, the beam surface was discretised into number of small elements. The discretisation of the beam will be discussed later in Section 6.2.1. The chloride-induced deterioration models discussed in Chapter 2 were then employed to describe the deterioration process of the concrete surface condition with time for each individual RF element.

As discussed in Section 2.2 and indicated by Figure 6.2, four distinguished phases of the deterioration process were recognised. However, due to the lack of quantification models that can be used to describe the fourth stage (Time to concrete spalling $T_{sp}$), only the first three stages were considered in this thesis. The three recognised deterioration stages are: (i) the corrosion initiation stage ($T_{i}$), (ii) the crack initiation stage (first cracking) ($T_{1st}$) and (iii) the crack propagation stage ($T_{cp}$). The propagation period ($T_{p}$) thus consists of $T_{1st}$ and $T_{cp}$. Models describing each of the deterioration stages have already been discussed in
Chapter 2 and will be briefly revisited in the upcoming sections to show how they have been employed in the current model for the formulation of the limit states considered.

Figure 6.2 Initiation and propagation stages.

6.2.1 Structural discretisation

The hypothetical RC beam used in this study to demonstrate the influence of considering/neglecting spatial variability on predicting the optimal time to first repair/maintenance intervention was taken from Enright and Frangopol (1998a). The RC T-beam highway bridge is located near Pueblo, Colorado and is designated as ‘Colorado Highway Bridge L-18-BG’. The bridge consists of three 9.1 m simply supported spans where each span has five girders @ 2.6 m centres. The cross-section of the beam girder is shown in Figure 6.3(a). It is assumed in this study that the considered beam girder is exposed to chloride ions penetration from all three exposed surfaces as indicated in Figure 6.3(b).

It was already explained in Section 3.2.4 that in an RF model, material and geometrical properties are considered not to be perfectly correlated (i.e. spatially constant) within a structure or a component, but rather vary across the structure with some limited field correlation. For this spatial variation to be considered and for the extent of damage to be predicted, the structure surface needs to be discretised into a number of small square/rectangular elements so values for the random variables can be assigned for each
RF element with correlation between the elements taken into account during the random variables generation process.

In the beam girder under consideration, for the two vertical sides of the beam, a Two-Dimensional RF model that would take into account the fluctuation of the random variables in both directions can be used. If the fluctuation of random variables in one direction of the beam (e.g. the transverse direction) can be neglected compared with the longitudinal direction, a simple One-Dimensional model can be applied. In the current case, in the One-Dimensional RF model the beam is discretised into strips (rectangular elements) of a width $\Delta_x$ (m) and a height that is equal to height of the beam web ($h_w$) as shown in Figure 6.4(a). In the Two-Dimensional RF model the vertical faces of the beam were divided into multiple equal segments with a vertical size $\Delta_y = h_w/k_y$ (m) where $k_y$ is the number of RF elements specified for the vertical direction, Figure 6.4(b). The same meshing principle could be applied to the bottom face of the beam; however, due to the relatively smaller width of the beam bottom ($b_w=0.4$ m) as compared with the length of the beam (9.6 m), only One-Dimensional meshing model was considered for the bottom face, Figure 6.4(c). The size of the RF elements will be discussed in Section 6.2.1.2.
6.2.1.1 The autocorrelation function

The role of the autocorrelation function in RF modelling was demonstrated in Section 3.4.2 and Section 5.3. Several types of autocorrelation functions that have been reported in the literature were discussed in the aforementioned sections.

The autocorrelation function is needed to specify the correlation coefficients between any two neighbouring RF elements separated by distance, $\tau$. To date, no specific autocorrelation function has been favoured for the type of analysis that is similar to the one carried out in this study. However, the Square Exponential autocorrelation function, Equation 3.21, is the most frequently used by researchers in the field of RC corrosion (Li et al., 2004; Malioka and Faber, 2004; Vu and Stewart, 2005) and therefore was used in the current thesis for the analysis of $\theta$ as indicated in Chapter 5 and will also be used in this chapter to generate the correlation coefficient matrix. The Two-Dimensional form of the Square Exponential autocorrelation function is expressed as follows:
\[ \rho(\tau) = \exp \left[ -\left( \frac{\tau_x}{d_x} \right)^2 - \left( \frac{\tau_y}{d_y} \right)^2 \right] \]

Equation 6.1

where \(d_x\) and \(d_y\) are the model parameters (correlation lengths) for a Two-Dimensional RF in x and y direction respectively which is related to \(\theta\) through the relation \(\theta = \sqrt{\pi d}\) (see Section 3.4.3), and \(\tau_x = x_{j+1} - x_j, \tau_y = y_{j+1} - y_j\) is the distance between centre of elements \(j\) and \(j+1\) in x and y directions respectively. If a One-Dimensional RF model is considered the y component is neglected.

From Equation, it can be observed that the degree of correlation between the RF elements is dependant upon two main parameters; correlation length \((d)\), hence \(\theta\), and the distance \((\tau)\) which is directly dependant on the element size \(\Delta_x\) and \(\Delta_y\). In order to obtain an appropriate \(d\) value for a RF variable (i.e. a random variable that is also a spatially variable), data sets consisting of sample measurements taken at frequent distances are needed. In practice, as discussed in Chapter 5, such measurements are rarely taken at frequent distances; consequently, data on \(d\) are scarce and usually assumed based on engineering judgment. However, in the current study values for the parameter \(d\) for two key deteriorating variables, namely the Surface Chloride Content (\(C_{sc}\)) and the Diffusion Coefficient (\(D_{app}\)), were obtained following extensive experimental (Chapter 4) and numerical/statistical analysis (Chapter 5). The values of the parameter \(d\) were determined for both variables by performing spatial correlation analysis on the data collected from the ageing RC Ferrycarrig Bridge as shown in Chapter 5.

Based on the spatial correlation analysis performed in Chapter 5, values of \(d\) (hence \(\theta\)) were found to be as indicated in Table 6.1. Due to the positive correlation between \(D_{app}\) and other concrete properties such as the concrete compressive strength \((f'c)\), water/cement ratio \((w/c)\) and the corrosion rate density \((i_{corr})\), it was reasonable to assume that these later variables have similar fluctuation properties as that of their associated variable, \(D_{app}\). Therefore, all variables which are dependant on or related to \(D_{app}\) were assumed to have the same \(\theta\) value as that found for \(D_{app}\). For all other RF variables, values of \(\theta\) that have been used by other researchers in the field (e.g. Li et al., 2004; Vu and Stewart, 2005) indicated in Table 6.1 will be used.
Table 6.1 The Scale of fluctuation ($\theta$) and the corresponding Correlation Length ($d$) to be used in the analysis.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\theta$ (m)</th>
<th>$d$ (m)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_s$</td>
<td>2.7</td>
<td>1.5</td>
<td>Kenshel, (2009)</td>
</tr>
<tr>
<td>$D_{app, f_c, wc, i_{corr(1)}}$</td>
<td>1.9</td>
<td>1.1</td>
<td>Kenshel, (2009)</td>
</tr>
<tr>
<td>Other RF variables</td>
<td>3.5</td>
<td>2.0</td>
<td>Li et al., (2004) &amp; Vu and Stewart (2005)</td>
</tr>
</tbody>
</table>

6.2.1.2 Size of the random field element

As discussed in Section 3.4.5, the size of the discretised RF element to be chosen depends on the parameter $\theta$ of the random variable of interest and the calculated correlation coefficients between the two neighbouring elements. If the size of the RF element is too large, this implies that the random variable is constant within each element according to the Midpoint method, which may result in underestimation of the effect of spatial variability of the random variable particularly when the value of $\theta$ is small relative to the RF element size. On the other hand, a small element size leads to the generation of a very fine mesh that causes the random variables in elements close to each other to have high correlation with each other resulting in numerical difficulties in the decomposition of the correlation coefficient matrix discussed in Section 3.4.4 (Gomes and Awruch, 2002). Therefore, the RF element size has to be chosen in such a way to avoid high correlations among the random variables specified for neighbouring elements. A number of proposed element sizes were discussed in Section 3.4.5 where an elements size between $\frac{1}{4} \theta$ and $\frac{1}{2} \theta$ was recommended. In Chapter 7, a sensitivity analysis will be performed to define the optimal element size for the beam example under consideration.

6.2.2 Formulation of SLS for the condition profile

Following the generation of the RF mesh (i.e. beam discretisation), the desired LS functions needs to be formulated and evaluated at the centre of each RF element. The properties assigned to the centre of the element are representative of the whole element according to the ‘Midpoint’ discretisation method discussed in Section 3.4.1.1. Only one LS is considered here for the Condition profile modelling; for example, an element will be considered to have failed if the crack size initiated in that element due to chloride-induced corrosion has exceeded $w_{lim}$ (mm). The proportion of concrete surface, $A_s\%$, which
exhibited a maximum crack width $w_{\text{lim}}$ (mm) will then be estimated by counting the number of RF elements which have violated the corresponding LS. This LS was selected based on the fact that the performance criterion often used by bridge owners/managers to define the end of the service life of a deteriorating RC bridge is related to the extent of corrosion-induced crack damage observed on the surface of the structure surface (Li, 2004b). The time to repair/maintenance intervention was taken to occur when $A_{\text{c}}\%$ reaches a threshold value of $A_{\text{xmax}}\%$.

Once the desired LS is formulated, it will later be evaluated at the centre of each RF element each time a single realization of Monte Carlo (MC) simulation is performed.

The crack width LS function, $G_w(t)$, can thus be expressed as follows:

$$G_w(t) \leq 0; \quad \text{where} \quad G_w(t) = w(t) - w_{\text{lim}}$$  \hspace{1cm} \text{Equation 6.2}$$

where $w(t)$ is corrosion-induced crack width at time $t$. For computational convenience, the above LS function, $G_w(t)$, can also be expressed in terms of time, that is to say the LS will be violated when the time elapsed since construction to the occurrence of maximum limiting crack width $(T_i + T_p)$ is less than the time $t$.

$$G_w(t) \leq 0; \quad \text{where} \quad G_w(t) = (T_i + T_p) - t$$  \hspace{1cm} \text{Equation 6.3}$$

6.2.2.1 Calculation of $A_{\text{c}}(t)\%$

The proportion of the concrete surface with crack widths exceeding $w_{\text{lim}}$ at time $t$ for a single MC simulation realisation can be calculated according to Stewart and Mullard (2006) as follows:

$$A_{\text{c}}(t) = \frac{n\left[\left(T_i + T_p\right) \leq t\right]}{k} \times 100\%$$  \hspace{1cm} \text{Equation 6.4}$$

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where \( n[\] \) denotes the number of RF elements that satisfy the inequality \((T_i + T_p) \leq t\). The results of Equation will be in the form of a Probability Density Function (PDF) that changes its shape and parameters with time. The annual mean value of \( A_s\% \) can then be inferred from these PDF's to represent the Condition profile as schematically shown in Figure 6.5. The advantage of having the results in the form of a histogram or a PDF allows the results of \( A_s(t)\% \) to be presented in terms of fractiles other than the mean (50% fractile), i.e. the 10% and 90% fractiles. It is significant to note that the PDF of \( A_s(t)\% \) may not have a normally distributed shape similar to that presented in Figure 6.5 at year 60, in that case, the choice of the mean value to represent the average annual value may be misleading (Kenshel and O'Connor, 2009). This issue will be discussed further in Section 7.3 of Chapter 7.

![Figure 6.5 Influence of time on the proportion of damaged surface area](image)

### 6.3 SAFETY PROFILE MODELLING

As the propagation stage starts, the cross-sectional area of longitudinal reinforcement of the RC beam, which provides its flexural capacity, will be reduced due to the ongoing corrosion activity, leading to rupture at the critical cross-section of the RC beam. Similarly, the shear links, which provide the beam with a substantial proportion of its shear capacity, lose some of their cross-sectional area as corrosion progresses. Consequently, the structural safety of the RC beam will be reduced over time. In this thesis, the structural safety of the considered RC beam girder is determined with respect to the flexural and the shear...
strengths although other effects (e.g. torsion, fatigue, etc) can equally be considered. Models describing the flexure and shear capacities and their deterioration over time due to the chloride-induced corrosion will be covered in this section. These models are vital for formulating the LS functions which will be employed to estimate the Probability of Failure \( (P_f) \) and hence the Reliability Index \( (\beta) \), which indicate the performance of the structural safety of the assessed beam at any given time during its service life. The relationship between \( P_f \) and \( \beta \) are as given by Equation 3.4. Before proceeding to describe the models used for determination of the residual flexure and shear capacities of the RC beam due to general and pitting corrosion, the resistance models will be presented first.

### 6.3.1 Beam flexure and shear resistance models

As mentioned in Section 6.2.1, the RC T-beam girder forms a component of an RC bridge deck that is located in the North American state of Colorado; the bridge is likely to have been designed according to an American Standard (i.e. AASHTO). Therefore, the flexure and shear capacities of the beam under investigation at any section (i.e. centroid of each RF element) were calculated using formulas provided by AASHTO-LRFD (1994). Since the codified flexure and shear capacity were developed for deterministic design and assessment practices, they are often referred to in the original document as the ‘Nominal’ capacity, in which a specified factor of safety (<1.0) is multiplied by to determine the ‘Ultimate’ capacity. Here, due to the probabilistic usage of these formulas, no factors of safety will be applied; hence the term ‘Ultimate’ capacity will be used throughout.

#### 6.3.1.1 Flexural resistance model

In AASHTO-LRFD (1994), the computation of the flexural capacity is based on the Whitney’s rectangular approximation, Figure 6.6(a), of the parabolic stress distribution (Wang and Salmon, 2002). In order to determine the flexure capacity of an RC T-beam section, the depth of the idealized rectangular stress block \( a \) that replaces the parabolic stress block must be determined. Figure 6.6 shows the two cases where \( a \leq h_f \) (the flange thickness of the T-beam), and where \( a > h_f \), respectively. To determine if \( a \leq h_f \), the distance \( x \) shown in Figure 6.6(b) (the distance from the extreme compression fibre to the neutral axis NA) must be found. The distance \( x \) can be obtained from equating the forces acting on the section in Figure 6.6(c) and it is as follows:
\[ x = \frac{A_s f_y - 0.85 \eta_1 f'_c (b_f - b_w) h_f}{0.85 \eta_1 f'_c b_w}; \quad a = \eta_1 x \]  

Equation 6.5

If \( x \leq h_f \), then \( a \leq h_f \) and \( x \) should be recalculated with \( b_w = b_f \). The value of \( \eta_1 \) is given in Table 6.2.

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Figure 6.6 Flexural capacity of RC T-beam section with tension reinforcement only; (a) forces on the section for rectangular RC section (b) \( a \) is in the flange, (c) \( a \) is in the web after Barker and Puckett (1997).
Table 6.2 Values for $\eta_1$ given in (Barker and Puckett, 1997).

<table>
<thead>
<tr>
<th>$\eta_1$</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>$f_c' \leq 28 \text{MPa}$</td>
</tr>
<tr>
<td>0.65</td>
<td>$f_c' \geq 56 \text{MPa}$</td>
</tr>
<tr>
<td>$0.85 - 0.05 \left( f_c' - 28 \right)/7$</td>
<td>$28 \text{MPa} \leq f_c' \leq 56 \text{MPa}$</td>
</tr>
</tbody>
</table>

The computation for the flexure capacity at any time $M_u(t)$ of a T-section can be carried out for the two cases (assuming only the reinforcement cross-sectional area is reducing with time due to the effect of corrosion, no bond loss or anchorage slip is considered as discussed in Section 2.2.2) as follows:

**Case 1: $a \leq h_f$, Figure 6.6(a).**

$$M_u(t) = A_s(t) f_y \left[ d_{ef} - \frac{A_s(t) f_y}{2 \times 0.85 f_c'(t) b_f} \right]$$  

**Equation 6.6**

**Case 2: $a > h_f$, Figure 6.6(b).**

$$M_u(t) = \left[ A_s(t) f_y - 0.085 \eta_1 f_c'(t) (b_f - b_w) h_f \right] \times \left[ d_{ef} - \frac{A_s(t) f_y - 0.085 \eta_1 f_c'(t) (b_f - b_w) h_f}{2 \times 0.85 f_c'(t) b_w} \right] + 0.85 \eta_1 f_c'(t) (b_f - b_w) h_f \left( d_{ef} - \frac{h_f}{2} \right)$$  

**Equation 6.7**

where all variables involved in the formulation of Equation and Equation are defined in Figure 6.6 and Table 6.3.
Table 6.3 Random variables for the RC T-beam girder.

<table>
<thead>
<tr>
<th>Variable (units)</th>
<th>Description</th>
<th>Distribution</th>
<th>(Mean, COV)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{om}$ (mm)</td>
<td>Initial diameter of flexure reinforcement</td>
<td>Lognormal</td>
<td>(35.8, 0.02)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$D_{ov}$ (mm)</td>
<td>Initial diameter of shear reinforcement</td>
<td>Lognormal</td>
<td>(12.7, 0.02)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$A_y(t)$ (mm$^2$)</td>
<td>Time-dependant cross-sectional area of flexure reinforcement</td>
<td>Lognormal</td>
<td>Eq. 2.28</td>
<td>-</td>
</tr>
<tr>
<td>$A_s(t)$ (mm$^2$)</td>
<td>Time-dependant cross-sectional area of shear reinforcement</td>
<td>Lognormal</td>
<td>Eq. 2.28</td>
<td>-</td>
</tr>
<tr>
<td>$d_{eff}$ (mm)</td>
<td>Effective depth of flexure reinforcement</td>
<td>Lognormal</td>
<td>(687, 0.03)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$C_{CM1}$ (mm)</td>
<td>Cover depth of flexure reinforcement, layer 1</td>
<td>Lognormal</td>
<td>(50, 0.10)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$C_{CM2}$ (mm)</td>
<td>Cover depth of flexure reinforcement, layer 2</td>
<td>Lognormal</td>
<td>(137, 0.10)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$C_{DV}$ (mm)</td>
<td>Cover depth of shear links</td>
<td>Lognormal</td>
<td>(38.1, 0.10)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$b_f$ (mm)</td>
<td>Effective flange width</td>
<td>Fixed</td>
<td>2600</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$b_n$ (mm)</td>
<td>Web width of the beam</td>
<td>Fixed</td>
<td>400</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$h_f$ (mm)</td>
<td>Flange thickness</td>
<td>Fixed</td>
<td>190</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$h_n$ (mm)</td>
<td>Web height</td>
<td>Fixed</td>
<td>600</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$s$ (mm)</td>
<td>Shear links spacing</td>
<td>Lognormal</td>
<td>(100, 0.15)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$f_y$ (MPa)</td>
<td>The specified Steel reinforcement yield strength</td>
<td>Lognormal</td>
<td>(460, 0.12)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$f_{ck}$ (MPa)</td>
<td>The specified (characteristic) 28 days concrete compressive strength</td>
<td>Lognormal</td>
<td>(40, 0.18)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$f_c(t)$ (MPa)</td>
<td>Time-dependant compressive strength</td>
<td>Lognormal</td>
<td>Eq. 6.12</td>
<td>-</td>
</tr>
</tbody>
</table>

6.3.1.2 Shear resistance model

Similarly, the time-dependant ultimate shear resistance of the beam at any given section is calculated by simply combining the contributions of concrete and shear links to the shear resistance of the section provided by AASHTO-LRFD (2004) as follows:

$$V_s = V_c + V_s$$

Equation 6.8
where all parameters in Equation have been defined in Table 6.3. Equation was derived from an expression that is based on the variable-angle truss model for a uniformly loaded beam in which the beam is treated as a truss with a diagonal crack in which the local stresses at the crack (indicated in Figure 6.7) must be in equilibrium. In the original derivation of Equation the vertical forces acting on the diagonally cracked section were set to be in equilibrium, the distance between the tension and the compression reinforcement, \(d_v\), were approximated by \(d_{\text{eff}}\) and the angle \(\phi\) was taken as \(\phi=45^\circ\) whereas \(V_{ci}\) was experimentally related to \(f_c\) so that \(V_{ci}=1/6\sqrt{f_c}\) (MPa).

### Figure 6.7 Shear strength of RC section with shear reinforcements after Barker and Puckett (1997).

#### 6.3.2 Modelling the concrete compressive strength

In design, the characteristic strength \((f_{ck}')\), rather than the mean strength, is used (i.e. \(f_{ck}'=f_{ck}\) in all previous code-provided equations). This strength is defined as the level below which only a small proportion (usually 5%) of all the results are likely to fall (Narayanan and Beeby, 2001). When concrete is ordered, it is a concrete with some specified characteristic strength that will be asked for. To ensure this, the producer has to provide a concrete with an average strength that is well above the specified characteristic strength. The amount by which the average exceeds the characteristic value depends on the effectiveness of the producer’s control methods. Eurocode 2 (EC2) relates the concrete
mean cylinder compressive strength to the specified characteristic strength for concrete up to 50 MPa as follows:

\[ f_{cyl} = f_{ck} + 8 \]  

Equation 6.10

Bearing in mind that poor concreteing will result in low concrete compressive strengths, Stewart (1997) introduced a workmanship reduction factor \( k_w \) that will take into account the influence of workmanship quality (i.e. curing and compaction) on the actual structure concrete compressive strength. Based on worker performance survey data, Stewart (1997) performed a probabilistic analysis in which he then proposed that the actual concrete compressive strength mean \( \mu \) and coefficient of variation (COV) of the assessed structure may be related to the compressive strengths obtained from the standard test cylinders, which are cured and compacted under standard conditions, as follow:

\[
\mu\left(f_c\right) = \mu(k_w) \times \mu(f_{cyl}); \\
\text{COV}\left(f_c\right) = \sqrt{\left[\text{COV}(k_w)\right]^2 + \left[\text{COV}(f_{cyl})\right]^2}
\]

Equation 6.11

The statistical parameters for \( k_w \) were determined for three concrete quality classes; Poor, Fair and Good and are presented in Table 6.4.

<table>
<thead>
<tr>
<th>Worker performance</th>
<th>Minimum curing times</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 days</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Poor</td>
<td>0.53</td>
</tr>
<tr>
<td>Fair</td>
<td>0.72</td>
</tr>
<tr>
<td>Good</td>
<td>1.0</td>
</tr>
</tbody>
</table>

To allow for the influence of time-dependant increase in concrete compressive strength to be considered in the current analysis, the parameter \( f_{c}^\prime \), which represents the 28 concrete compressive strength in Equation, Equation and Equation can be replaced by a time-dependant compressive strength \( f_{c}^\prime(t) \). The following expression proposed by ACI 209 (1978) has been used in reliability based assessment of corroding structures (e.g. Stewart...
and Rosowsky, 1998a) and therefore it was used here in to model the evolution of concrete compressive strength with time.

\[
f'_c(t) = \frac{t}{\gamma + \omega t} f'_c(28)
\]

Equation 6.12

where \( t \) is the time elapsed since the beam construction in days, \( \gamma = 4.0 \) and \( \omega = 0.85 \) for moist cured Ordinary Portland Cement (OPC).

### 6.3.3 Model error of the resistance models

Based on a study conducted on 1146 RC beams aimed at comparing experimental shear strengths with those obtained from predictive models provided by a number of national standards and codes (e.g. ACI, AASHTO, BS 8110 and EC2), Somo and Hong (2006) found that predicting the shear capacity of a RC beam with shear links using Equation, may lead to underestimation of the shear capacity of the RC beam. They recommended a model error (bias factor) with a mean value of 1.3 and a coefficient of variation that is larger than 0.3 to account for the uncertainty associated with the use of the predictive model proposed by codes and standards used for estimation of the shear capacity. No similar experimental-based study has been reported in the literature with regard to the flexure capacity. However, based on simulation of moment-curvature relationship performed by Tabsh and Nowak (1991) a mean model error of 1.14 and a coefficient of variation of 0.13 were proposed to account for the uncertainty associated with the flexure resistance model determined according to AASHTO.

### 6.3.4 Loss of flexure capacity

As indicated in Section 2.5.3, two forms of corrosion mechanisms were identified to cause the reduction in the reinforcement cross-sectional area. These are the General corrosion and the Pitting corrosion. The calculation of the residual cross-sectional area of the reinforcement due to any of the two types of corrosion was already explained in the aforementioned section of Chapter 2. It was shown that in the case of general corrosion, the corrosion rate \( i_{corr} \) can be used to estimate the loss of reinforcement diameter by the use of Faraday’s law of electrochemical equivalence which indicates that a constant
corrosion rate of 1.0 |μA/cm²| corresponds to a uniform loss of bar diameter of 0.0232 mm per year (Andrade et al., 1993).

If the corrosion rate is assumed to be constant over time, then the remaining cross-sectional area of corroding longitudinal reinforcement after \( t \) years of construction can be estimated from Equation 2.23. If the corrosion rate was assumed to be a time-variant, Equation 2.30 may be used to estimate values for the corrosion rate parameter in Equation 2.23. The mean value of the constant corrosion rate at the start of corrosion activity (i.e. \( i_{\text{corr}(1)} \)) were estimated from Equation 2.29 for concretes with different Durability Design Specifications (DDS) and presented in Table 6.5. The three types, i.e. Poor/Fair/Good, were introduced by Vu and Stewart (2000) to describe RC structures with different characteristics with regard to chloride ingress resistance. The two main parameters identified to make the difference between Poor/Fair/Good quality concrete were \( f_{ck}' \) and \( C_d \). As can be seen from the table, the proposed corrosion rate values, which was calculated as a function of \( f_{ck}' \) and \( C_d \) according to Equation 2.29, correspond well with the typically measured values from real structures reported in Section 2.5.4.

<table>
<thead>
<tr>
<th>Concrete quality (DDS)</th>
<th>( f_{cyt}(f_{ck}') ) MPa</th>
<th>( C_d ) mm</th>
<th>( i_{\text{corr}(1)} ) μA/cm²</th>
<th>Corrosion level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor</td>
<td>35 (28)</td>
<td>25</td>
<td>7.1</td>
<td>High</td>
</tr>
<tr>
<td>Fair</td>
<td>47 (40)</td>
<td>50</td>
<td>2.1</td>
<td>Moderate</td>
</tr>
<tr>
<td>Good</td>
<td>53 (46)</td>
<td>75</td>
<td>1.0</td>
<td>Low</td>
</tr>
</tbody>
</table>

For pitting corrosion, the pit configuration given by Val and Melchers (1997) and discussed in Section 2.5.3.2 was used to estimate the area of the pit and hence the residual cross-sectional area of the corroding bar. Therefore, Equations 2.26-2.28 were used to calculate the overall residual reinforcement cross-sectional area for a given beam section (i.e. at the centre of each RF element) at any given year after corrosion has been initiated. The maximum pit depth at any time \( t \), \( P_{\text{max}}(t) \), is related to the average penetration depth \( P_{\text{av}}(t) \) caused by the general corrosion through the relation given by Equation 2.24. In the given relation, \( P_{\text{max}}(t) \) depends on the factor \( R \) introduced in Section 2.5.3.2 (\( R=P_{\text{max}}/P_{\text{av}} \)). In general, the factor \( R \) is frequently reported in the range of 4 to 8, however, in the following section the probabilistic property of this factor will be discussed in more detail.
6.3.4.1 Probabilistic model of pitting corrosion

Because pitting corrosion causes a loss of the cross-sectional area of the reinforcement that is several orders of magnitude greater than that caused by general corrosion \((R=4-8)\), it is expected that pitting corrosion will have a significant influence on the safety of the corroding structure. Therefore, the variability associated with the maximum pitting depth needs to be accurately quantified. In this regard, Stewart and Al-Harthy (2008) conducted an accelerated corrosion tests on reinforcing bars with diameters of \(16\) mm \((\Phi 16)\) and \(27\) mm \((\Phi 27)\) embedded in concrete slabs with the intention to characterise the pitting statistics, the results of which are indicated in Table 6.6. They concluded that the maximum pitting depth can be modelled using EV distribution type I (Gumbel EVD). To predict the distribution for the maximum pit depth for a bar length or surface area \((A)\) other than the ones used in their experiments, the researchers suggested that Gumbel statistical parameters can be modified as follows:

\[
\lambda = \lambda_o + \frac{1}{\delta_o} \ln \left(\frac{A}{A_o}\right), \quad \delta = \delta_o
\]

Equation 6.13

where \(\lambda_o\) and \(\delta_o\) are Gumbel parameters obtained from plotting the corrosion test results for reinforcing bars of surface area \(A_o\) on Gumbel probability paper. The evaluation of pitting statistics was carried out in terms of the pitting factor \(R\) which implies that the predicted distribution has to be truncated at value of \(R=1.0\) as the factor \(R\) is physically impossible to be less than 1.0. Hypothesis test showed that the differences in pitting factor for \(\Phi 16\) and \(\Phi 27\) steel bars are statistically significant, the pitting factors for \(\Phi 27\) were found to be higher than those for \(\Phi 16\) which is in line with Equation. The applicability of Equation to steel bars greater than \(\Phi 27\) has not been confirmed. However, in the present analysis for the lack of data on the statistics of the factor \(R\), Equation were used to extrapolate the Gumbel distribution parameters required for randomly generating values of factor \(R\) using parameters given by Stewart and Al-Harthy (2008) for steel bar of \(\Phi 16\) \((\lambda_o=5.56, \delta_o=1.16)\). As indicated in Table 6.6, the pitting factor \(R\) is expected to increase with the increase of the bar diameter. This observation was supported by a statistical hypothesis test performed by the relevant authors on their results to conclude that the difference between values of \(R\) for both bar diameters were statistically significant.
Table 6.6 Statistics of pitting corrosion obtained from accelerated corrosion tests performed by (Stewart and Al-Harthy, 2008).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( \bar{i}_{corr} ) (( \mu \text{A/cm}^2 ))</th>
<th>Length (mm)</th>
<th>Diameter (mm)</th>
<th>Time (Days)</th>
<th>No. of Samples</th>
<th>Pitt factor ( R )</th>
<th>Gumbel parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean   COV</td>
<td>( \lambda_o )</td>
</tr>
<tr>
<td>Specimen-1</td>
<td>160-185</td>
<td>100</td>
<td>16</td>
<td>78</td>
<td>32</td>
<td>6.2     0.18</td>
<td>5.56</td>
</tr>
<tr>
<td>Specimen-2</td>
<td>125-150</td>
<td>100</td>
<td>27</td>
<td>78</td>
<td>32</td>
<td>7.1     0.17</td>
<td>6.55</td>
</tr>
</tbody>
</table>

By applying Equation the Gumbel parameters for factor \( R \) were found for the bar \( \Phi35 \) used for the flexure reinforcement with a length that is equal to the RF element size of 310 mm (corresponds to using 31 One-Dimensional RF elements used for the calculation of the safety profile as will be described in Chapter 7). The Gumbel parameters for this case were calculated using the parameters for \( \Phi27 \) (from Table 6.1) because they produced a higher value than that produced by parameters of \( \Phi16 \) so therefore are more conservative \((\lambda_{35}=7.9, \delta_{35}=1.07)\). Similarly, for the shear reinforcing bars the properties corresponds to \( \Phi16 \) were used because the two bars are more related in terms of size, the Gumbel parameters in this case were found to be \((\lambda_{12}=6.8, \delta_{12}=1.16)\) considering the bar length to be approximately equal to that of the beam web high \( (h_w) \).

### 6.3.5 Loss of shear capacity

As shear links are generally located much closer to the concrete surface than the flexure reinforcements, they tend to be the first steel to experience the full corrosive effect of the chloride environment and are likely to be more heavily damaged than the flexural steel. Moreover, the relative effect of pitting on the reinforcement cross-sectional area seems to increase with the decrease of the bar diameter (Enright and Frangopol, 2000b; Val and Stewart, 2003) which indicates that shear reinforcement is more vulnerable to corrosion attack since shear reinforcement bars generally have smaller diameter than the flexure reinforcement bars. This is particularly true for structures where a significant amount of shear resistance is provided by the shear links (e.g. the support zone of a simply supported RC beam). The reduction in the stirrups cross-sectional area caused by both general and pitting corrosion can eventually lead to a sudden failure as shear failure is generally perceived to be of a brittle type (Higgins et al., 2003).

The estimation of residual cross-sectional area of the shear links after \((t-T_i)\) years of active corrosion is similar to that of the flexure reinforcement and therefore can be estimated in
the same manner using the same formulas indicated previously for the flexure case. The corrosion rate and the time variant corrosion rate models employed for modelling the time-dependent loss of the beam flexure capacity, indicated in Section 6.3.4, are equally applicable for modelling the time-dependent loss of the shear links cross-sectional area and hence the time-dependent shear capacity of the beam.

6.3.6 Formulation of the ULS functions

In order to calculate the annual failure probabilities of the beam under consideration and hence its safety profile, a limit state function, which depends on a set of basic random variables (see Section 3.3.1), in terms of each failure mode needs to be formulated. Two ULS functions were considered for the beam problem at hand; the flexure and the shear limit states.

6.3.6.1 Flexure failure LS

The corresponding LS function for beam failure in flexure at any given time \( t \) during the service life of the beam \( G_M(t) \) is as follows:

\[
G_M(t) \leq 0; \quad \text{where} \quad G_M(t) = M_u(t) - M_b(t)
\]

Equation 6.14

where: \( M_u(t) \) is the ultimate bending moment capacity of the RC section at time \( t \) (years) and can be calculated according to the relevant design code, in this case \( M_u(t) \) estimated as in Section 6.3.1.1, and \( M_b(t) \) is the induced bending moment at the same section at the same year and it will be estimated later from Section 6.3.7.

6.3.6.2 Shear failure LS

For the beam failure in shear, the LS function \( G_V(t) \) is expressed as follows:

\[
G_V(t) \leq 0; \quad \text{where} \quad G_V(t) = V_u(t) - V_b(t)
\]

Equation 6.15

where: \( V_u(t) \) is the ultimate shear capacity of the RC section at time \( t \) and can be calculated according to the relevant design code, \( V_u(t) \) is estimated from Section 6.3.1.2, and \( V_b(t) \) is
the induced shear force at the same section at the same year and will be estimated from the following section.

6.3.7 Load modelling using extreme value (EV) theory

In traditional structural assessments, the extreme load effect (e.g. induced bending moment or shear) is calculated using conservative deterministic loading models provided by standard/code specifications (O'Connor and O'Brien, 2005). The deterministic loading models have been derived based on practical experience or in some cases from model calibration studies (e.g., O'Connor, 2001). In both cases the parameters of the model are selected in such a way so that they will overrate the predicted maximum loading effects which a broad range of structures may be expected to experience during their design life. The requirement that the deterministic model should be applicable to a wide range of bridge structures often results in conservative deterministic load models. This approach may be acceptable when used for the design of new structures, however, when such conservative deterministic load models are used for the assessment of existing bridge structures, the result may lead to unnecessary replacement or repair of the investigated structure. This issue was highlighted in Chapter 3 as a major shortcoming in the research carried out in the field of spatial variability modelling of RC corrosion, Section 3.5. A preferable approach in the case of structural assessment is to rely more on the site-specific traffic data for the determination of the maximum load effects. Site-specific traffic data can be obtained using the Weight-in-Motion technique (WIM) which will be discussed in the upcoming sections.

It has to be mentioned that the load modelling is out of the scope of this thesis; the details which will be presented in the upcoming sections, are only to provide an insight on methods used to generate the EVs load effects from site-specific traffic data obtained using WIM. The EVs load effects, i.e. \( M_{\delta}(t) \) and \( V_{\delta}(t) \), are those which will be used for predicting the safety profile of the beam under consideration.

6.3.7.1 Weigh-in-Motion (WIM)

Weigh-in-Motion (WIM) is the process of taking weight measurements of trucks travelling at a full highway speed while crossing the monitored bridge. The measurements are often made using sensors embedded in the pavement or strain gauges attached to the sofitt of the
Chapter 6: Development of the RF-based model

bridge deck which can provide measurements on stresses in the bridge deck and hence the weight of the truck and its axles (Dempsey and O'Brien, 1995; O'Connor and Eichinger, 2007). The traffic data is usually collected over a short period (several days/weeks), this implies that the recording period may be too short to give reliable information on the traffic flow. Therefore, Monte Carlo (MC) simulation is used to increase the amount of data that is recorded (e.g. from one week data to 10 weeks data) using the information provided by the traffic data collected over a short period through the WIM system.

The WIM traffic records used in this study for the MC simulation were provided by O'Connor (2001). The WIM data were recorded during a 7-days monitoring period in 1997 on a main motorway that is connecting Paris to Lille in France. A total of 86455 trucks were recorded, where only trucks that weighed >3500 kg were considered for the analysis. The MC simulation was performed by the aforementioned author for bridges with varies span lengths ranging from 5 m to 200 m. The approach adopted by O'Connor (2001) to predict the maximum load effects needed for the current stochastic model will be described in the upcoming sections.

6.3.7.2 Monte Carlo (MC) simulation of traffic data

As indicated in the previous section, MC simulation aims to increase the amount of traffic data when the period of time is relatively short for WIM records to provide representative information on traffic loading. MC simulation can also be used to generate different loading scenarios, i.e. congested flow, free flow and flow following an accident and or lane closure. That may not have been captured during the short period of monitoring. MC simulation of traffic requires knowledge of the mathematical models that describe the traffic characteristics, these models may be broken into two main categories (O'Connor and O'Brien, 2005):

1. Those describing the traffic as a whole, i.e. proportion of the vehicles in each lane, proportion of vehicles in each vehicle class, vehicles spacing, etc.
2. Those describing the vehicles within each class, i.e. gross vehicles weight (GVW), axel weight, vehicles geometry, speed, etc.

The most important models mentioned in points 1 and 2 will be explained next.
Chapter 6: Development of the RF-based model

Vehicle proportions and classification

Before the MC generation of the traffic record can be performed, vehicles need to be classified into different categories, i.e. based on the number of axles and axle spacing and configuration so that the proportion of each class of vehicles can be determined through a traffic counting exercise or from available WIM records (O'Connor and O'Brien, 2005). Table 6.7 illustrates the vehicle classification system employed by O'Connor (2001) in his calibration study for the normal load model of the EC1 Part 3. The same classification system was used to generate the traffic data used in the current study to determine the maximum load effect on the beam girder under consideration.

Inter-vehicle spacing and vehicle speed

During the process of generating the traffic flow, it was necessary to specify the statistical distribution of the inter-vehicle spacing (i.e. headway) and the vehicle speed. O'Connor and O'Brien (2005) used a Gamma distribution for the modelling of the inter-vehicle spacing and a normal distribution to model the vehicle speed.

Table 6.7 Vehicle classification system (O'Connor and O'Brien, 2005).

<table>
<thead>
<tr>
<th>2-Axle</th>
<th>3-Axle</th>
<th>4-Axle</th>
<th>5-Axle</th>
<th>6-Axle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A11</td>
<td>A12</td>
<td>A22</td>
<td>A113</td>
<td>A123</td>
</tr>
<tr>
<td>A111</td>
<td>A112</td>
<td>A122</td>
<td>A1212</td>
<td></td>
</tr>
<tr>
<td>A11-11</td>
<td>A11-12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>A12-11</td>
</tr>
</tbody>
</table>

Vehicle characteristics

When generating the traffic flow using MC simulation, the random variables of the vehicle characteristics within each vehicle class such as gross weight, axle weight, axle spacing, length, speed etc need to be accurately described. Parameters of the statistical distribution of the aforementioned characteristics can be obtained from the WIM records. Figure 6.8
demonstrates the distribution of the gross vehicle weight (GVW) for class A113 defined in Table 6.7. The bi-modal distribution shown in Figure 6.8 is a simple combination of two normal distributions, the first mode describes the partially loaded trucks, whereas the second describes the fully loaded trucks. The PDF of the bi-modal distribution takes the following form:

\[
f_w(x) = \rho_1 f_{N(m_1, \sigma_1)} + \rho_2 f_{N(m_2, \sigma_2)}
\]

where:

\[
\rho_1 + \rho_2 = 1 \quad \text{and} \quad f_{N(m_i, \sigma_i)} = \frac{1}{\sqrt{2\pi\sigma_i}} \exp\left(-\frac{(x - m_i)^2}{2\sigma_i^2}\right)
\]

The distribution parameters \(\rho_1\) and \(\rho_2\), reflect the characteristics of the two modes and corresponding population properties.

\[\text{Figure 6.8 Gross weight distribution class A113 (O'Connor and O'Brien, 2005).}\]

**Traffic flow simulation**

When modelling traffic flow using simulation methods, depending on the number of lanes, different traffic flow scenarios may be considered, i.e., jammed, free-flow or mixed flow. For two lane short span bridges (span<30 m), such as that considered in this thesis, the
Chapter 6: Development of the RF-based model

Jammed and free-flow scenarios are the most important (O’Connor 2001). The same reference has showed that for two lane short span bridges, the maximum load effects will be induced by free-flowing traffic as the structure is too short for significant jammed queues of vehicles to form and cause a maximum load affect. This is explained by the effect of considering the dynamic load of the moving vehicle through the inclusion of the Dynamic Amplification Factor (DAF) when considering the free-flowing scenario in the traffic simulation. In the jammed scenario, the vehicles are assumed to move too slowly to induce any significant dynamic effects.

6.3.7.3 Prediction of the extreme load effects

Having generated the desired amount of traffic data, which is often larger than that originally obtained from the WIM record, the resulting load effect data, i.e. bending moment and shear force, is obtained using influence lines/surfaces technique. The calculated bending moments and shear forces then is fitted to an EV distribution such as Gumbel type I distribution or Weibull (Gumbel type III) distribution using probability paper. The selection of the appropriate distribution is based upon linearity of the data plotted. In the present study, the MC simulation was carried out to generate 4 weeks of traffic data, based on the information provided by the 7 days WIM record. The data is obtained from the simulation were extrapolated to determine the extreme load effects for a desired reference period of 100 years (i.e. the bridge design life).

**Return period (R_p)**

For the extrapolation to be performed, the desired reference period will have to be expressed in terms of ‘Return Period’ \( R_p \). The Return Period is a statistical tool by which the reference period (i.e. the structure design life \( T \)) is linked to a specified probability of exceedance to account for the effects of model and statistical uncertainty inherent in a process, which employs measured data recorded over a limited period to determine characteristic effects (O’Connor, 2001). Furthermore, specifying a return period greater than the reference period, may ensure that the future traffic growth in some way been accounted for. The return period related to the design life of the structure as follows:

\[
R_p = \frac{T}{\alpha}; \quad 0 < \alpha << 1.0
\]

Equation 6.18
Where \((1-a)\) is the specified fractile. For example to predict the characteristic extremes during a structure design life of \(T=100\) years with a 0.9 fractile \((1-a, \alpha=0.10)\), from Equation a return period \(R_p=1000\) is needed. However, in the case of assessment of existing bridge structure, the remaining service life of the structure is less than the design life; therefore, the calculation of \(R_p\) should consider this issue to avoid overestimation of the EV load effect. For example, if the structure to be assessed is a 25 years old, and the intended design life is 100 years, \(R_p\) should be determined only for the remaining 75 years \((T=75\) years).

**The use of the influence line/surface**

The maximum load effects to be considered in this case study are the mid-span moment and the shear force at the support for a simply supported beam. To evaluate the bridge behaviour under varying load scenarios, influence lines/surface can be employed. In structural analysis, the load effect produced by the live load (i.e. vehicle load) in a given part of the structure varies with the position of the live load on the structure. In the present case, the bending moments and shear force (load effects) values were obtained using the influence lines approach. Two influence line functions were defined; the bending moments and shear forces at points along the bridge due to a unite axel load moving along the bridge. The influence line functions used in the simulation are presented in Table 6.8.

**Influence of data sample**

When dealing with EV problems, a critical step in the estimation process is the selection of the fraction of the sample data to be used. According to Castillo (2005), the main reason for using a small fraction of the sample data and not the whole sample data is that the parent population of the data generally does not follow an EV model as can be observed from Figure 6.9(a). However, it is known that the tail behaves as one EV tail, asymptotically. The two most important factors affecting the selection of the fraction of the sample data to be used for the fitting process are the sample size and speed of convergence. Castillo (2005) has proposed a fraction of \(2\sqrt{n}\) to be used for sample data of size \(n\), for a medium and high speed of convergence and for a low speed of convergence even a lower value e.g. \(\sqrt{n}\) may be used. In the current procedure, \(\sqrt{n}\) of the data sample was used. To illustrate the behaviour of the tail, the upper \(\sqrt{n}\) fraction of the truck load data was plotted in Figure 6.9(b). It can be seen from the figure that the linearity of the data has improved which allows for better fitting to the selected EV distribution than for example when the whole \(n\) data is used.
Table 6.8 Influence lines used in MC simulation.

<table>
<thead>
<tr>
<th>Description of the influence line</th>
<th>Influence line representation</th>
</tr>
</thead>
</table>
| Bending moment of a simply supported span | \[ M = \frac{x}{2} \quad \text{for} \quad 0 < x \leq L/2 \]
| \[ M = \frac{L}{4-x/2} \quad \text{for} \quad L/2 < x \leq L \] | ![Bending moment of a simply supported span](image) |
| Shear force of a simply supported span | \[ V = \frac{1-x}{L} \quad \text{for} \quad 0 \leq x \leq L \] | ![Shear force of a simply supported span](image) |

Choice of EV Distribution (EVD)

As stated earlier, the EVs of load effects may be modelled using a Gumbel type I or Weibull extreme value distribution (EVD). A Weibull distribution results when the maximum values are sampled from a parent frequency distribution having a finite upper bound. On the other hand, the Gumbel I distribution results when the maximum values are sampled from a parent distribution with no upper bound. Both Gumbel I and Weibull distribution are expressed mathematically as follow respectively:

\[
G(x) = \exp \left[ -\exp \left( -\frac{x - \lambda}{\delta} \right) \right] \quad -\infty < x < \infty; \quad \delta > 0 \]

Equation 6.19
The parameters of both distributions can be estimated using the Least Square method or the Maximum Likelihood method (O'Connor, 2001). As the tail region is of prime importance in the extrapolation, the Weibull distribution is considered more appropriate in this case as it can be seen from comparing Figure 6.10 with Figure 6.11. The definition of the \( x \) and \( y \) axes of the Weibull and Gumbel probability plots are indicated in Table 6.9. Where \( p \) is the plotting position and in the current analysis was calculated according to the following equation (Castillo, 2005):

\[
p_{i,s} = \frac{i - 0.44}{N + 0.12}; \quad i = 1,...,N
\]

Equation 6.21

where \( N \) is the number of sample data used for the fitting procedure (e.g. \( N=\sqrt{n} \)) and \( n \) is the total recorded/simulated data.

Figure 6.10 Weibull EVD approximation to midspan moment extremes for simple span of 9 m.
Chapter 6: Development of the RF-based model

The maximum load effect (Extreme Value) results obtained from data fitted to Weibull distribution and extrapolated for different return periods (reference periods of 25, 50, 75 and 100 years) for moment and shear to be used for the current analysis are summarised in Table 6.10.

Table 6.10 The maximum load effect (Extreme Values) results obtained from the simulated data fitted to Weibull distribution and extrapolated for different reference periods.

<table>
<thead>
<tr>
<th>Reference period (years)</th>
<th>EV for Bending Moment (kN.m)</th>
<th>EV for Shear Force (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>1975</td>
<td>434</td>
</tr>
<tr>
<td>50</td>
<td>1993</td>
<td>438</td>
</tr>
<tr>
<td>75</td>
<td>2027</td>
<td>446</td>
</tr>
<tr>
<td>100</td>
<td>2062</td>
<td>453</td>
</tr>
</tbody>
</table>
6.3.7.4 Girder distribution factors (GDF)

Having determined the maximum load effect for the desired return period, the value of the moment/shear obtained from the extrapolation does not represent the maximum bending moment/shear acting on a single RC beam girder; this value thus far represents the predicted maximum moment/shear induced by the presence of the heaviest trucks on the bridge deck as a whole in the longitudinal direction. The proportion of this value that is resisted by a single RC beam girder can be determined by multiplying the obtained value by a specified GDF. In the current study, the GDFs for both flexure and shear in interior girders are provided by AASHTO-LRFD (1994) and presented respectively as follows:

*For Multiple lanes loading:*

\[
GDF^M = 0.075 + \left( \frac{S}{2900} \right)^{0.6} \left( \frac{S}{L} \right)^{0.2} \left( \frac{K_g}{L t_s^2} \right)^{0.1}
\]
Equation 6.22

\[
GDF^v = 0.2 + \frac{S}{3600} - \left( \frac{S}{10700} \right)^2
\]
Equation 6.23

*For Single lane loading:*

\[
GDF^M = 0.06 + \left( \frac{S}{4300} \right)^{0.4} \left( \frac{S}{L} \right)^{0.3} \left( \frac{K_g}{L t_s^2} \right)^{0.1}
\]
Equation 6.24

\[
GDF^v = 0.36 + \frac{S}{7600}
\]
Equation 6.25

where \( S \) is the girder spacing (mm); \( L \) is the span length (mm); \( t_s \) is the thickness of the slab (mm) (\( t_s = h_f \) in Figure 6.3 in this case), \( k_g = n \left( I + A_{girder} e_g^2 \right) \), \( n \) is the modular ratio \( (n=E_{girder}/E_{deck}) \), \( I \) is the moment of inertia of the girder (mm\(^4\)), \( A_{girder} \) is the girder cross-sectional area (mm\(^2\)) and \( e_g \) is the distance between the centres of gravity of the girder and the slab (mm).
For the beam girder example under consideration, the mean values for the GDFs were calculated for the two loading cases (multiple and a single lane loading) and the result are presented in Table 6.11.

The GDFs presented in Table 6.11 are the mean values that any beam girder can have, however, several experimental and finite element analysis studies, (e.g., Kim and Nowak, 1997; Eom and Nowak, 2001), has confirmed that the GDFs predicted using Equation are somewhat conservative for short span bridges as can be seen from Figure 6.12. Furthermore, Eom and Nowak (2001) suggested, based on field testing and finite element analysis, that for simply supported bridges the AASHTO-LRFD GDFs for one lane loading (Equation and Equation 6.25) is more realistic for estimating the design load effect.

<table>
<thead>
<tr>
<th>Loading scenario</th>
<th>Calculated GDF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bending</strong></td>
<td></td>
</tr>
<tr>
<td>Two lane loaded</td>
<td>0.761</td>
</tr>
<tr>
<td>One lane loaded</td>
<td>0.558</td>
</tr>
<tr>
<td><strong>Shear</strong></td>
<td></td>
</tr>
<tr>
<td>Two lane loaded</td>
<td>0.866</td>
</tr>
<tr>
<td>One lane loaded</td>
<td>0.702</td>
</tr>
</tbody>
</table>

Figure 6.12 Measured GDF’s for moments vs. GDF’s calculated using AASHTO standard and AASHTO LRFD for (a) 10 girders and (b) 6 girders bridges (Kim and Nowak, 1997).
Chapter 6: Development of the RF-based model

The uncertainty in the GDFs may be expressed in terms of the model error (bias factor). For GDFs based on simplified code methods such as that provided by AASHTO-LRFD (1994), a normally distributed bias factor with a mean value of 0.93 and coefficient of variation of 0.12 were reported in the literature for the case of bending (Nowak et al., 2001). No such information was reported with regard to GDF for shear, therefore, the bias factor mean and coefficient variation for the bending moment GDF’s are assumed to be valid for the case of shear.

6.3.8 Calculation of the reliability index $\beta(t)$

Having defined the individual LS functions and assigned the probability distribution to the set of basic random variables and the correlation coefficients between the RF elements, the failure probability for each element with respect to each failure mode can be determined for each year of the structure service life. It has to be noted that when calculating the probability of failure ($P_f$) for each period over the service life of the structure, the discretised periods have to be long enough for the correlation between periods to be negligible (Durprat, 2007). For example, Durprat (2007) mentioned that for an industrial warehouse loading, the length of independent periods can be estimated at 2 years. Structural assessments of bridges are often based on a limited reference period of 2-5 years and after the end of this period the bridge is normally re-assessed as its structural capacity is likely to change (Vu and Stewart, 2000). Thus, it would be more logical and appropriate to compare probabilities of failure for relatively short reference periods. However, too short discretised periods, i.e. one year long, can result in a very long computational time. Therefore, in the present study, due to the lack of reliable data on the correlation between incremental periods, and for the sake of simplicity, the probability of failure will be assumed independent for each incremental time period of 5 years which is within the figures indicated by Vu and Stewart (2000). The procedure followed in this thesis to calculate the time-dependent reliability (Safety Profile) of the beam girder under investigation as follows:

For a series reliability system consists of $k$ RF elements, the critical limit state occurs when the actual load effects exceeds the resistance at the mid-point (centre) of any element. The critical moment and shear limit state for a One-Dimensional RF model consisting of $k$ elements at any year $t_i$ can be expressed as follows:
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\begin{align*}
G^M_i (X) &= \min_{j=1,k} \left[ R^M_j (t_i) - S^M_j (t_i) \right] \tag{6.26} \\
G^V_i (X) &= \min_{j=1,k} \left[ R^V_j (t_i) - S^V_j (t_i) \right] \tag{6.27}
\end{align*}

where \( G^M_i (X) \) and \( G^V_i (X) \) are the flexure and shear LS functions respectively, \( R^M_j (t_i) \) and \( R^V_j (t_i) \) are the distribution for moment and shear resistance respectively, for element \( j \) evaluated at its centre at time \( t_i \), \( S^M_j (t_i) \) and \( S^V_j (t_i) \) is the corresponding load effects at the centre of the same RF element due to the load acting at time \( t_i \).

The annual probability of failure of the beam in terms of bending moment (flexure) or in terms of shear can be computed respectively as follows:

\begin{align*}
P_f^M (i) &= \Pr [ G^M_i (X) \leq 0 ] \tag{6.28} \\
P_f^V (i) &= \Pr [ G^V_i (X) \leq 0 ] \tag{6.29}
\end{align*}

The total annual probability of failure of the beam then can be calculated by combining the probability of failure of the beam in moment and in shear.

\[ P_f (i) = \Pr [ G^M_i (X) \leq 0 \cup G^V_i (X) \leq 0 ] \tag{6.30} \]

In general, and as indicated by Stewart (2004), if it is assumed that \( m \) load events \( S_j \) occur within the time interval \((0,T)\) at times \( t_i \) where \( i=1,2,3,\ldots,m \), the cumulative probability of failure any time during the time interval \( 0 \) to \( T \) for \( m \) events is:

\[ P_f (T) = 1 - \Pr [ G^M_i (X) > 0 \cap G^V_i (X) > 0 \cap \ldots \cap G^M_i (X) > 0 ] \tag{6.31} \]

where:

\[ \Pr [ G^M_i (X) > 0 ] = 1 - \Pr [ G^M_i (X) \leq 0 \cup G^V_i (X) \leq 0 ] = 1 - P_f (i) \tag{6.32} \]
If the failure events are assumed independent events, then Equation can be approximated by:

\[ P_f(T) = 1 - \prod_{i=1}^{n} [1 - P_f(i)] \]  
Equation 6.33

where \( P_f(i) \) is obtained from Equation.

The reliability of the structure is then assessed by using the conditional probability of failure which integrates the survival period of the structure prior to the time at which the reliability is estimated (Vu and Stewart, 2000; Duprat, 2007). To calculate the conditional probability that the beam will fail in \( t \) subsequent years given that it has survived \( T \) earlier years, the following expression can be used:

\[ P_f^c(t|T) = \frac{P_f(T+t) - P_f(T)}{1 - P_f(T)} \]  
Equation 6.34

where \( P_f(T+t) \) and \( P_f(T) \) are calculated using Equation.

Finally, the probability of failure can then be translated into safety reliability index \( \beta \) through the relationship given by Equation 3.3 \( (\beta = -\Phi^{-1}(P_f)) \).

### 6.4 IMPORTANT ASSUMPTIONS

This section provides further clarifying information regarding some of the assumptions that have been made during the model development:

- In the Midpoint discretisation method, random variables are considered constant for a single RF element and represented by a value that is evaluated at the centre of that element; this implies that when corrosion is initiated in a RF element all reinforcing bars in the same layer in that element are assumed to start corroding roughly at the same time.
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• The maximum pitting depths for each bar (or shear link) or for different bars within an element are treated as statistically independent (uncorrelated). This is because of the lack of knowledge on statistical correlation between the corrosion pits.

• After corrosion-induced concrete cracking has taken place, the beam section is still assumed to be physically sound when evaluating the section moment and shear capacities, and only corrosion-induced reduction of the reinforcement cross-sectional area is taken into account. In addition, bond strength between concrete and reinforcement are assumed not to be affected by corrosion; therefore, Equation, Equation 5.11 and Equation were used throughout the lifetime of the beam to estimate flexure and shear capacities.

• If a random variable is assumed to be also RF (i.e. spatially variable), all variables which depend on that RF will also be considered as a RF. For example water/cement ratio ($w/c$) and diffusion coefficient ($D_{app}$) are dependant variables on the concrete compressive strength ($f_c'$), therefore, they are also spatially variable.

• Although several mechanisms exist as discussed in Section 2.3.2, chlorides penetration through the concrete cover in the current case was assumed to happen solely due to diffusion. Furthermore, the presence of cracks (e.g. due to load-induced stresses, shrinkage or corrosion product expansion) of a size larger than 0.3 mm, may expose the reinforcement to the direct influence of the environment. While it is recognised, this situation has not been considered in the present analysis and its consideration is beyond the scope of this thesis.

6.5 MODEL BENCHMARKING

In order to benchmark the RF-based reliability model developed in this chapter, results obtained using this model were compared to results published by Vu and Stewart (2005). Therefore, for the comparison to be reliable, all model input parameters were adjusted so that all data used to predict the time-variant surface damaged surface area $A_d(t)\%$ (i.e. condition profile) are identical to that used in the aforementioned publication. This requires that the same concrete surface geometry should be used. Vu and Stewart (2005) presented results of $A_d(t)\%$ for three cases representing Poor, Fair and Good DDS concrete structures (see Section 6.3.4 for definition of DDS). The three types of concrete quality were introduced to describe RC structures with different characteristics with regard to chloride ingress resistance as function of the $f_c'$ and $C_d$ as can be seen from Table 6.12.
Table 6.12 Durability Design Specifications suggested by Vu and Stewart (2005).

<table>
<thead>
<tr>
<th>DDS</th>
<th>$f'c$ (MPa)</th>
<th>$C_d$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Fair</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Good</td>
<td>46</td>
<td>75</td>
</tr>
</tbody>
</table>

The benchmarking results are shown in Figure 6.13. It can be seen from the figure that the results of the current model agree to a large extent with the results of the benchmark model for Good DDS concrete. For Fair DDS concrete, a 13% decrease, 8% decrease and 5% increase were observed for the case of $w_{lim}=0.05$ mm, 0.3 mm and 1.0 mm respectively after 120 years of service. The results however show a significant difference for concrete with a Poor DDS (almost 30% increase for all $w_{lim}$ cases after 120 years). Poor DDS concrete is expected to suffer a higher percentage of damage after the same period of time when it is compared with Fair or Good DDS concrete, as a result, a higher associated margin error is also expected.

Figure 6.13 A, % model results (colourful) versus results published by Vu et al., (2005) (black).

It has to be mentioned that the results produced using the RF model developed in this thesis and those provided by Vu and Stewart (2005) in terms of the condition profile (i.e. $A_s(t)$% in Figure 6.13) are both produced by employing the same RF principles discussed in Chapter 3. If the same input parameters (e.g. structural, environmental, statistical parameters, etc) were used, both models are expected to yield the same results. However,
simplifications during the development of each model and the code writing approach used by the developers of each model, as well as the computational round-off error, are all expected to contribute to the margin error found above. Due to the lack of similar experimental results to be used for comparison, there was no evidence to suggest that one of the two models is more accurate in predicting the condition profile than the other. However, the results obtained using the RF model developed in this thesis is more conservative when compared with profiles produced by the benchmark model except for the case of Fair DDS concrete when $w_{lim} = 0.05$ mm where the benchmark model results at $t = 120$ years were slightly more conservative (13% increase).

Having benchmarked the results obtained using the RF-based reliability model developed in this thesis in terms of the condition profile, the model can now be used to produce results in terms of both, the condition and the safety profiles of the beam girder under consideration using the input data provided by Chapter 4 and Chapter 5. The results produced using the RF–based reliability model developed in this thesis will be presented and discussed in Chapter 7 with the aim of predicting the optimal time to repair/maintenance intervention.

6.6 CONCLUSIONS

The prime objective of the work carried out in this chapter is to develop a probabilistic-based model to serve as a rational tool that can help bridge owners/managers/engineers in their decision regarding the optimal time for repair/maintenance intervention of their structures. To achieve this objective, a methodology to predict the lifetime performance of a given structure or a network of structures in terms of its visual condition and in terms of its load carrying capacity was developed. The structure lifetime performance in terms of its visual condition (i.e. related to SLS) is termed in this thesis as the ‘Condition’ profile. On the other hand, the structure lifetime performance in terms of its load carrying capacity (i.e. related to ULS) was termed here in as the ‘Safety’ profile.

A comprehensive spatial/temporal reliability based model was then developed to predict both profiles (Condition and Safety). A corroding RC beam girder was used as an illustrative example in which existing chloride-induced corrosion material deterioration models discussed in Chapter 2 were employed in conjunction with the Random Field (RF) theory discussed in Chapter 3. Employing the RF principles, discussed in Chapter 3, to
facilitate the inclusion of the spatial variability of the deteriorating parameters in the developed model, has been identified as the second main objective of this chapter.

The RF-based model was first developed to predict the Condition profile of the RC beam girder which was assumed to have been exposed to a corrosive marine environment. At this point, the model was benchmarked against published results of a similar model which had been developed using RF principles explained in Chapter 3 to predict the condition profile of a given structure. The models produced very similar results for the case of Good DDS concrete. For the case of Fair DDS concrete, the model developed in this thesis was conservative by about 13% for the outmost case, i.e. at $t=120$ years. For Poor DDS concrete the current model was conservative by about 30% for the outmost case, i.e. at $t=120$ years. It was concluded that these margin error are acceptable considering the difference in the models developers' computer programming approach, possible simplification, and the round-off error. There was no evidence to back any of the models to claim that it is more accurate with the lack of similar experimental data. It was therefore concluded that the model developed in this thesis is capable of predicting the condition profile with reasonable accuracy with reference to the benchmark model.

Having benchmarked the RF-based model developed in this chapter, it was concluded that the developed model can now be used to produce the Condition and Safety profiles using the site-specific data collected in Chapter 4 and Chapter 5. The model was further developed to predict the Safety profile (i.e. lifetime reliability) of the considered beam girder. This is the first time that such an attempt has been made to predict the Condition and Safety profiles of the same structure using the same deterioration models/parameters, thus achieving one of the prime objectives stated in Chapter 1 of this thesis.

Another addition to the work being carried out in the context of spatial variability modelling of deteriorating RC structures, it is the first time when the maximum load effect used for the calculation of time-dependant reliability (safety profile), were determined from the analysis of site specific WIM data using EV statistics. This ensures that the uncertainty associated with the maximum load affect due to traffic loading is being accounted for while avoiding the use of conservative deterministic load models which may lead to unnecessary replacement or repair of the investigated structure. By employing a site-specific load model instead of using a conservative code model, the objective outlined in Chapter 3 in this regard is achieved.
Chapter 7:

Analysis and Discussions of the Random Field Model Results
7.1 INTRODUCTION

The Random Field (RF)-based probabilistic model developed in Chapter 6 can be used to produce results of significant importance to bridge owners/mangers seeking to optimise their budget spend through predicting the optimal time to first bridge repair/maintenance. By predicting the optimal time to first and subsequent repair/maintenance of deteriorating structures, bridge owners/mangers can optimise/manage their budgets, allocate resources to those in most need and plan repair/maintenance works for years ahead. As indicated in Chapter 6, the time to repair/maintenance intervention can be predicted in terms of the Serviceability Limit State (SLS) related criteria, i.e. 'Condition' profile, or in terms of the Ultimate Limit State (ULS) related performance criteria, i.e. 'Safety' profile, see Figure 6.1. The main objective of this chapter therefore is to compare the predicted times to first repair/maintenance intervention of the corroding RC structure determined based on condition and safety profiles (i.e. SLS vs. ULS). The development of the RF-based stochastic model was carried out in terms of a practical example of an RC T-beam girder exposed to marine environment.

One evident advantage which characterises the results to be provided in this chapter is that both the SLS and ULS performance indicators (Condition and Safety profiles) were determined using the same material properties, deterioration models/parameters and RF parameters. This allows a fair and valid comparison between the times to first repair intervention to be determined from both criteria. In addition, most of previous works of the service life probabilistic modelling of RC structures affected by chloride-induced corrosion have considered the two major corrosion stages, initiation and propagation, separately. For example, they assume that the time to corrosion initiation \(T_i\) is zero and all bars would start corroding at the same time (e.g. Li, 2004b). In reality however, reinforcing bars or different parts of the same bar start corroding at different times due to the fluctuation of the parameters influencing the onset of corrosion. This simplification was indicated in Chapter 3 as being one of the shortcomings of the previous studies that have considered the effect of spatial variability of input parameters on the performance of RC structures affected by chloride-induced corrosion.

Also in Chapter 6, the term concrete Durability Design Specification (DDS) was introduced based on previous works of Vu and Stewart (2000) to describe RC structures with different characteristics with regard to chloride ingress resistance. The two main
parameters identified to make the difference between Poor, Fair and Good quality concrete were the compressive strength ($f'_c$) and the reinforcement cover depth ($C_d$). Other parameters which are dependant on any of the two parameters would also vary depending on the DDS considered. In this chapter, the three proposed DDS categories, i.e. Poor/Fair/Good, are used to demonstrate the influence of concrete quality on the predicted results of the Condition and the Safety profiles.

The discussion provided in this chapter takes the following sequence:

1. The influence of the Scale of Fluctuation ($\theta$) on the spatial fluctuation of the time to damage initiation is demonstrated in Section 7.2.
2. Prediction of the time to repair/maintenance in terms of the Condition profile of the beam under consideration is discussed in Section 7.3.
3. Prediction of the time to repair/maintenance in terms of the Safety profile of the beam under consideration is performed in Section 7.4.
4. In Section 7.5, Comparison between the times to repair/maintenance of the beam under consideration predicted in terms of both, the Condition and Safety, profiles is discussed.
5. The sensitivity of Safety profiles towards the input random variables is the discussed in Section 7.6.

In due course of this chapter and in the context of the above mentioned tasks, the following are discussed in detail:

1. The influence of spatial variability and its model parameters, on the condition and safety lifetime performances of the beam structure under consideration.
2. The influence of the General and Pitting corrosion and variability of the corrosion pits on the lifetime safety performance of the beam structure under consideration.

### 7.2 INFLUENCE OF THE SCALE OF FLUCTUATION ON SPATIAL FLUCTUATION OF THE TIME TO CORROSION INITIATION

To demonstrate the role of spatial variability in predicting the damage characteristics for the RC beam girder under consideration, $T_i$ was used as an indicator. Consequently, $T_i$ was determined for varies values of the parameter $d$, the results of which are shown in Figure 7.1-Figure 7.4.
Chapter 7: Analysis & Discussions

All graphs in Figures 7.1-Figures 7.4 were produced using the input data listed in Table 6.3 and Table 6.4. The predicted values of $T_i$ were plotted for one Monte Carlo (MC) realisation for values of $d$ ranging from 0 to 5 m ($d=0$, 0.5, 2 and 5 m) to indicate (graphically) the influence of $d$ on the predicted fluctuation of $T_i$ along the beam. It can be seen from Figure 7.1, where $d=0$ m, that all properties involved in the calculation of $T_i$ and $T_i$ fluctuate rapidly and randomly along the beam (spatially random or uncorrelated). This state is equivalent to saying that all properties, including $T_i$, are spatially uncorrelated. This is a special case, for example, $d=0$ m means that there is no correlation between the generated data along the beam for the same RF variable. The input parameters in this case can be generated randomly from the specified PDF distributions without having to apply the mathematical transformation (Matrix decomposition) which is necessary to convert the randomly generated data into a spatially correlated data as indicated in Section 3.4.4 and illustrated by Table 7.1. The elimination of the matrix decomposition step results in a less computational time which can be useful in the case when the RF mesh is very dense and require massive computational effort.

Table 7.1 Simplified procedure of generating RF variables

<table>
<thead>
<tr>
<th>Randomly generated data of a RF variable</th>
<th>Applying Matrix decomposition</th>
<th>Spatially correlated data of a RF variable</th>
</tr>
</thead>
</table>

On the other hand, when the parameter $d$ was assigned values that are greater than zero, e.g. Figure 7.2, Figure 7.3 and Figure 7.4, more organised shapes of all properties and $T_i$ were produced, indicating that they exhibit some degree of correlation along the beam (spatially correlated). The graphs show that the correlation between the predicted values of $T_i$ for different RF elements along the beam (spatial correlation of $T_i$) is strongly dependent on the value of $d$ used for the generation of the RF variables (i.e., random variables that are also spatially variable). From Figures 7.2-7.4, it can be observed that the fluctuation intensity of $T_i$ along the beam is inversely proportional to $d$. This is expected as when $d\to\infty$ this suggests that the beam deterioration parameters are homogeneous along the beam which implies that the calculated $T_i$ is constant along the beam. Therefore, the fluctuation of $T_i$ along the beam is expected to decrease as the value of $d$ increases until reaches a constant state for very large values of $d$ (i.e. $d\gg$ the beam length). This is also a special case in the context of RF modelling. For example, when $d\to\infty$, this is equivalent to
saying that properties are constant along the beam and therefore the traditional reliability approach which considers the whole structure as homogeneous, as discussed in Section 3.4 and by Kenshel and O'Connor (2009), is applicable and the structure safety may be evaluated at sections where they are considered to be structurally most critical.

The fact that different parts of the beam will have different $T_i$ values implies that different parts of the beam will initiate corrosion-induced damage (e.g. cracking, severe loss of reinforcement cross-sectional area) at different beam ages. From the structure safety viewpoint, this also means that structurally critical parts of the structure, i.e. midspan for flexure and end supports for shear in the case of the simply supported beam under consideration, can be exposed to a higher levels of chloride attack than the rest of the beam, for example, as a result of salt water leaking through deck joints. Therefore, considering the different sections of the structure (i.e. RF-based modelling) is more realistic scenario than for example considering the entire structure as one homogenous element. This would have been impossible to illustrate if spatial variability was not considered. Further discussion in this regard will be given in Section 7.4.1.

Figure 7.1 Influence of neglecting the spatial variability on the time to corrosion initiation, $d=0$ m ($\theta=0$ m) (Spatially Random or Uncorrelated).
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Figure 7.2 Influence of the Scale of Fluctuation on the time to corrosion initiation, $d=0.5$ m ($\theta=0.9$ m).

Figure 7.3 Influence of the Scale of Fluctuation on the time to corrosion initiation, $d=2.0$ m ($\theta=3.5$ m).
Figure 7.4 Influence of the Scale of Fluctuation on the time to corrosion initiation, \( d = 5.0 \) m (\( \theta = 8.9 \) m).

The influence of \( \theta \) on the spatial variability of other damage characteristics, e.g., the time to first cracking (\( T_{1c} \)) or the time to crack propagation (\( T_{cr} \)) is similar in principle to that demonstrated for the case of \( T_i \).

### 7.3 PREDICTION OF THE CONDITION PROFILE, \( A_x(t) \)

To predict the time for repair/maintenance intervention based on the surface condition of the structure, the condition profile, i.e. \( A_x(t) \) needs to be predicted. As indicated in Section 6.2.2.1, the proportion of the surface area of the beam (\( A_x \% \)) that is exhibiting a specified damage characteristic, i.e. crack width of 1.0 mm or greater, is obtained in the form of a Probability Density Function (PDF). In general, if the shape of the annual \( A_x \% \) PDF approximated to be of a 'Normal' distribution, the mean value of \( A_x \% \) can be represented in terms of the 50% fractile (the average), as was schematically shown in Figure 6.5 and as discussed by Kenschel and O'Connor (2009). The condition profile can then be plotted by connecting the mean values of the annual \( A_x \% \) PDFs. However, as can be seen from Figure 7.5, the annual PDF of \( A_x \% \) for most years is demonstrated to be highly 'non-Normal', the shape of the distribution is skewed either to the left (positively skewed) as in the early years of the structure service life or to the right (negatively skewed) as in the late years of...
the structure service life. This behaviour is expected, for example, at an early age of the structure service life (i.e. \( t<30 \) years) it is highly likely that the structure’s surface would not show signs of corrosion-induce cracking, specially for Fair or Good DDS concretes, thus very small values of \( A_x\% \) (i.e. \( A_x\%<10\% \) as indicated in Figure 7.5) are expected to be obtained more frequent leading to a skewed shape of the constructed PDF. The same principle applies to the structure at a very late age (i.e. \( t>80 \) years) where it would be reasonable to expect that most of the structure surface, if not repaired, would exhibit corrosion-induced cracking of size 1.0 m or greater leading to more frequent high \( A_x\% \) values. Therefore, the mean value is not the proper measurement of the central tendency (the average) of the quantity under investigation, i.e. \( A_x\% \).

Figure 7.5 Evolution of \( A_x\% \) with time (for Fair concrete and \( d=1.5 \) m, \( \Delta=0.48 \) m, One-Dementional RF model).

Because the mean value can easily be affected by outliers and extreme data, when measuring the central tendency of a given skewed distribution the median value is often used in such cases (Ang and Tang, 1975). In the context of the current study, the appropriateness of using the median, to measure the mean value of \( A_x\% \), can be checked by assuming all RF variables to be homogenous across the beam, i.e. constant along the beam. This can be achieved by choosing a very large value of \( d \) to represent the spatial variability of each RF variable (\( d \gg \) the beam length). By selecting a large value of \( d \), the generated RF variables will be constant along the beam (for the same MC realisation). Hence, the
beam is expected to behave as one homogenous element, for example, $A_x\%$ is either 0% or 100%. If the average of $A_x\%$ PDFs were represented in terms of the mean instead of the median, the expected 0%-100% results of $A_x\%$ could not be obtained, as can be seen from Figure 7.6 (the dash lines). For the two investigated DDS concrete (Poor and Fair), the figure shows that only when the average of $A_x\%$ PDFs was represented in terms of the medians, the expected 0-100% results (the continuous lines) were achieved. This means that the mean values (i.e. Li et al., 2004; Vu and Stewart, 2005) cannot be considered accurate in representing the average $A_x\%$, i.e. the condition profile. Therefore, for the purpose of this work, the condition profile for the beam girder under investigation will be presented in terms of the median.

![Figure 7.6 A_x% represented in terms of its mean and its median for Poor and Fair DDS concrete with \( d \approx \infty \).](image)

It worth mentioning that none of the previous studies which have used RF modelling methodology to predict the condition profile for a corroding RC structure, e.g. Li (2004b) and Vu and Stewart (2005), have investigated the validity of using the Mean instead of the Median to estimate the average of $A_x\%$ and by doing so they have assumed that $A_x\%$ PDFs to be ‘Normally’ distributed.

### 7.3.1 Influence of the scale of fluctuation ($\theta$)

Before using the values of $\theta$ obtained in Chapter 5 in the analysis to be carried out in this chapter, the influence of the parameter $\theta$ on the predicted condition profile $A_x(t)$ was
investigated. For convenience, the investigation was carried out in terms of the parameter $d$ which is related to $\theta$ as indicated in Table 5.1 (i.e. $\theta=\sqrt{d}$). Values for $d$ ranging from 0 to $\infty$ were assumed and the results of the condition profile were plotted in terms of the $A_x$% medians, Figure 7.7. The condition profile in this case was plotted in terms of the 1.0 mm crack size criterion for a Fair DDS concrete. This means that $A_x$% was expressed such that the proportion of the concrete surface area that have corrosion-induced cracks widths in excess of 1.0 mm.

Figure 7.7 shows that as $d$ increases, the time to initiate the corrosion damage increases (the horizontal part of the curves in Figure 7.7 with a zero $A_x$%). For very large values of $d$, e.g. $d \geq 50$ m, the profile becomes steeper and moves towards fulfilling the condition of 0%-100% as explained earlier. By assuming that $d = \infty$, it is suggested that the RF variables are homogenous across the beam which implies that the beam should behave as a more homogenous element and therefore is expected to either exhibit 0% or 100% damage level throughout its service life as indicated at the start of this section.

Figure 7.7 also indicates the significance of considering the spatial variability of the deteriorating variables when predicting the time to damage initiation and hence the possible time to first repair/maintenance intervention. Assuming that the time of first repair/maintenance intervention was to be made based on $A_x$% has reached value of 1.5%,
as suggested in Chapter 2, if spatial variability were not considered, the time to first
intervention was found to be 95 years after the beam construction. Meanwhile, if the
spatial variability were considered, for a very small $d$ value (e.g. $d \approx 0$), the time to first
intervention was about 32 years after the beam construction. As $d$ increases, the time to
first repair also increases within the range of 32 to 95 years. It can be seen therefore that by
including the uncertainty associated with spatial variability, the time to first repair/
maintenance intervention can be much shorter than that predicted when spatial variability
associated uncertainty is ignored. For bridge owners/managers, this indicates the
significance of considering the uncertainty associated with spatial variability in any
structure performance prediction tool.

Figure 7.7 also shows that for a conservative assessment it might be reasonable to use a
zero value for $d$ to obtain a conservative estimate for the time to damage initiation or the
time for a specified value of $A_x\%$ (i.e. $A_{x_{\max}}\%$) to be reached. This means that the matrix
decomposition step needs not to be included which reduces the computational time reduces
significantly.

Figure 7.7 also shows that if $A_{x_{\max}}\%$, in terms of the $w_{\lim}=1.0$ mm criterion, was assumed
to be 50%, the influence of the parameter $d$ becomes insignificant. This means that for a
preliminary upper limit assessment by which $A_{x_{\max}}=50\%$, the analysis can be carried out
with the randomly generated spatial data which requires less computational time when
compared with the spatially correlated data as explained earlier.

### 7.3.2 Influence of the RF element size

For the RF analysis (analysis that considers spatial variability), the optimal size of the RF
element ($\Delta$) needs to be specified. In Section 3.4.5, the significance of the RF element size
was discussed. It was stated that the element size should not be too large as this will lead to
underestimation of the influence of spatial variability by assuming that deterioration
properties are constant within the same RF element. For a property to be considered
constant within an RF element, physically, this assumption is only reasonable if the RF
element size is relatively smaller than $\theta$, i.e. $\Delta<1/2 \theta$ as suggested by Der Kiureghian and
Ke (1988) and Englund (1997), see Section 3.4.5. On the other hand, if the element size
was too small, numerical difficulties associated with decomposition of a large
autocorrelation matrix arise.
Results of the condition profile for Fair DDS concrete are shown in Figure 7.8 for $\Delta_x=2.4$, 0.96, 0.48, 0.32, and 0.24 m which corresponds to using 4, 10, 20, 30 and 40 One-Dimensional RF elements respectively. The effect of choosing a small One-Dimensional RF element width can be observed from the condition profile becoming smoother. The reason for the course trend produced in the case of the larger element size can be explained as follows; e.g. if the case where 4 RF elements were used, the available alternatives are 1, 2, 3 or 4 elements can fail (i.e. to have a crack size $\geq 1.0$ mm) out of possible 4. This corresponds to 25%, 50%, 75% or 100% as it is evidently seen from the relevant figure. The condition profile therefore becomes smoother as the number of RF elements increases (hence the size of each RF element decreases) as more possible fractions of the percentages given above becomes available.

![Figure 7.8 Influence of the element size ($\Delta_x$) on the predicted condition profile for Fair DDS and a scale of fluctuation $\theta=1.3$ m.](image)

Another observation can be made from Figure 7.8 that $A_x$% converges rapidly when the element size $\Delta_x \leq 0.32$ m. i.e. profiles produced by $\Delta_x=0.32$ m and $\Delta_x=0.24$ m are almost identical. Therefore, the optimal size was used for the analysis which was carried out for the remaining of this chapter was $\Delta_x=0.32$ m for One-Dimensional RF model which corresponds to using 30 One-Dimensional RF elements.
7.3.3 Influence of the RF meshing configuration

In the previous section, the influence of the RF element size was investigated for a One-Dimensional meshing configuration. In the model development procedure discussed in Chapter 6, One-Dimensional and Two-Dimensional meshing configurations were proposed, Section 6.2.1. The influence of using One-Dimensional or Two-Dimensional RF model on the results of the condition profile was also investigated. Figure 7.9 presents results of the condition profile for Fair DDS concrete with the vertical side of the beam was hypothetically divided into \( k_y = 1, 2 \) and \( 3 \) RF elements so that \( \Delta_y = h_w / k_y \) m where \( h_w \) is the heights of the beam web in m as indicated in Figure 6.3 and Figure 6.4, \( k_y \) is the number of RF elements in the vertical direction of the beam. The number of RF elements in the horizontal (longitudinal) direction was fixed at 30 (i.e. \( \Delta_x = 0.32 \) m). As can be seen from the graph, almost no change in the results can be observed. This suggests that the height of the beam web (\( h_w = 0.6 \) m) is too small relative to \( \theta (\theta = 1.3 \) m in this case) for the RF variables to show any improvement in the predicted result of the condition profile.

![Figure 7.9 Influence of RF meshing configuration (One-Dimensional and Two-Dimensional) on the predicted condition profile.](image)

When the computational times were compared for the three meshing cases, it was found that if the number of vertical RF elements was doubled or tripled so did the computational time with almost no improvement observed for the predicted condition profiles. The computational times associated with each of the meshing configuration were recorded and
are shown in Table 7.2. As expected, the computational time increases as the size of the RF element decreases due to the increase in the number of random variables to be generated and due to the increases in the size of the autocorrelation matrix need to be decomposed. Therefore, it was decided that the One-Dimensional RF meshing configuration is sufficient to accurately represent the spatial variability of the beam properties with $\Delta_x = 0.32$ m for the plotting of the condition and the safety profiles. For convenience, and to ensure that there would be a RF element representing the exact midspan of the beam when calculating the safety profile, an odd number of RF elements had to be used, thus 31 RF elements (i.e. $\Delta_x = 0.31$ m) were used instead of 30.

<table>
<thead>
<tr>
<th>RF meshing (No. of Horizontal elements x No. of Vertical elements)</th>
<th>Computational time* (minutes: seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D (31 x 1)</td>
<td>07:10</td>
</tr>
<tr>
<td>2-D (31 x 2)</td>
<td>13:17</td>
</tr>
<tr>
<td>2-D (31 x 3)</td>
<td>18:03</td>
</tr>
</tbody>
</table>

* The machine used was Dell Precision T3400 Intel(R) Core(TM)2 Quad CPU Q6700 @ 2.66 GHz and 7.93 GB of RAM.

7.3.4 Influence of the concrete DDS

The condition profile was plotted for the three DDS concretes which were proposed in Section 6.3.4. Figure 7.10 shows that for the concrete with a Good DDS there was a negligible damage for the 100 years of service life. For the concrete with a Poor DDS however, 20% of the surface area of the beam will exhibit corrosion-induced cracks of 1.0 mm wide or greater within only 6 years of the beam construction. For concrete with a Fair DDS, the 20% damage is reached just after 63 years of the beam construction. This means that there is 50% probability that 20% of the beam surface area will exhibit corrosion-induced cracks of size 1.0 mm or greater after 63 years of construction.

To indicate how Ferrycarrig Bridge condition profile fits within the condition profiles for the three proposed concrete DDSs, the properties obtained from the bridge were used with the mean values of $f'_c$, $C_d$, and the Diffusion Coefficient ($D_{app}$) being the key parameters that were replaced by those obtained from the experimented bridge. The condition profile showed that Ferrycarrig Bridge can be classified to have DDS category comparably similar to that of the concrete with Fair DDS in terms of its resistance to chloride-induced damage.
This corresponds well with the results of the inspection report of the bridge summarised in Chapter 4 which indicated that signs of corrosion were not observed on the face of the crosshead beams.

![Graph showing influence of Durability Design Specifications (DDS) on the predicted condition profile in terms of $w_{lim}=1.0$ mm criterion.]

**Figure 7.10 Influence of Durability Design Specifications (DDS) on the predicted condition profile in terms of $w_{lim}=1.0$ mm criterion.**

### 7.3.5 Influence of the damage criterion

As explained in Section 2.6, the criterion for the onset of maintenance and repair in practice is usually based on a maximum acceptable proportion of the surface area of the inspected structure showing signs of corrosion-induced damage. The damage criterion, to be considered when assessing structures which should be repaired or maintained, varies from agency to agency and from engineer to an engineer. For example, the damage criterion may be given in terms of the initiation of corrosion (the chloride content at the level of reinforcement is exceeding the critical chloride content $C_{cr}$), Criterion 1 in Table 7.3. Alternatively, the damage criterion may be given in terms of the appearance of hair size cracks on the concrete surface, Criterion 2, or in terms of the appearance of significantly large corrosion-induced cracks, on the concrete surface, Criterion 3. In the current chapter, the condition profile was predicted in terms of all three damage criteria of which the results are presented in Figure 7.11.
Table 7.3 Definitions of criterion used for plotting the condition profiles

<table>
<thead>
<tr>
<th>Criterion name</th>
<th>Criterion definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion 1</td>
<td>Cl% at $C_d$ level $\geq C_{c%}$</td>
</tr>
<tr>
<td>Criterion 2</td>
<td>$0.05 \text{ mm} \leq w_{\text{lim}} &lt; 1.0 \text{ mm}$</td>
</tr>
<tr>
<td>Criterion 3</td>
<td>$w_{\text{lim}} \geq 1.0 \text{ mm}$</td>
</tr>
</tbody>
</table>

Figure 7.11 Influence of Damage criterion on the proportion of the damaged surface area $A_x \%$ for concrete with Fair DDS.

The implication of selecting one of the three criteria listed in Table 7.3 is evident in Figure 7.11. For example, if the condition profile was predicted in terms of Criterion 1, the 25% limit, which was proposed in Section 2.6.1, is reached at approximately 31 years for concrete with Fair DDS. If the condition profile was predicted in terms of Criterion 2, the same limit would be reached after just about 33 years for the same DDS concrete. Meanwhile, if the condition profile was predicted in terms of Criterion 3, the 1.5% limit which was proposed in Section 2.6.2 for this type of damage would be reached at about 43 years after the beam construction. At this time ($t=43$ years), the beam would have 41% of its surface area initiated corrosion or 38% covered with hair-size corrosion-induced cracks.

It can also be observed from Figure 7.11 that the condition profile plotted based on Criterion 2 does not differ significantly from the condition profile predicted based on
Criterion 1. This is due to the fact that the time to first cracking is comparatively short when compared to the time to crack propagation as explained in Section 2.2.1.

When using the Ferrycarrig Bridge DDS, the time to first maintenance intervention predicted in terms of Criterion 1 was found to be about 47 years after the beam construction. This is equivalent to 5% of the beam surface area showing corrosion-induced 1.0 mm wide cracks, Criterion 3. The relation between the levels of damage in terms of both criteria is of significant practical importance for engineers concerned with the prediction of the structures service life. For example, it is much easier to evaluate the percentage of the damaged surface area in terms of the visible cracks than in terms of the initiation of corrosion. The later requirements imply that concrete dust samples need to be taken at the reinforcement depths for chemical analysis to investigate if the chloride contents at these levels have exceeded $C_{cr}$. This is both costly and time consuming process and thus may not be practical. The alternative is to perform routine inspection and evaluate the structure based on the visible signs of the corrosion activity such rust stains or the appearance of corrosion-induced cracks on the surface of the structure.

The chemical analysis of the concrete cores extracted from Ferrycarrig bridge (see Chapter 4) showed that, 27 years after the bridge construction, only 2 out of 40 concrete cores (about 5%) have indicated that chloride contents at $C_d=40$ mm had exceeded the critical chloride content ($C_{cr}=0.07 \text{Cl}\% \text{per mass of concrete}$). This may indicate that about 5% of the reinforcement has its corrosion initiated. Interestingly, Figure 7.12 shows that the same percentage (5%) of the surface area of the beam under consideration is expected to have its corrosion initiated after 26 years which matches closely with the Ferrycarrig Bridge. There is therefore a strong evidence to suggest that the model developed in this thesis was able to accurately predict the extent of damage ($A_x\%$) in terms of Criterion 1 for the Ferrycarrig Bridge.

If the condition profile was defined in terms of Criterion 3, the time to first repair/maintenance intervention is predicted to be about 42 years. This is still inline with the findings of the inspection report which was carried out on the bridge prior to its 2007 summer renovation which reported that no corrosion-induced cracks were observed on the surface of the crosshead beams.
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Considering the objective outlined in Chapter 1 with regard to comparing the time to first repair/maintenance obtained in terms of the condition profile (i.e. SLS) with that obtained in terms of the safety profile (i.e. ULS), the later needs to be determined first. This is the task of the following section.

7.4 PREDICTION OF THE SAFETY PROFILE, $\beta(t)$

To predict the time to first repair/maintenance for a given structure from the safety viewpoint, the safety profile, which is presented in terms of the Reliability Index ($\beta$) introduced in Chapter 3, needs to be plotted. The advantage of using $\beta$ as a safety performance indicator is that it can be directly compared to the minimum (Target) Reliability Index ($\beta_T$) recommended by the design codes for the ULS. For example, as indicated by Table 3.2, the Eurocode (EC2) recommends that the actual reliability index at the end of the design life-cycle of 50 years should remains greater than a minimum acceptable value of $\beta_T = 3.8$ for the ULS for structures with a medium consequence for loss of human life. The procedure describes the calculation of the annual $\beta$ values and hence the Safety profile was discussed in Section 6.3.8.

Two cases of the beam deterioration were considered; (i) due to general corrosion and (ii) due to pitting corrosion. The loss of the beam resistance and thus its safety with time was
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estimated in terms of the loss of its reinforcement flexure and shear cross-sectional area for both cases (i.e. due to general and pitting corrosion). The loss of the reinforcement cross-sectional area due to pitting corrosion was related to that caused by the general corrosion through the pitting factor \( R \) which was treated as a random variable with a mean value depends on the diameter and the length of the reinforcing bar as explained in Section 6.3.41. Before proceeding to the results of the safety profile calculated for the beam under investigation, the importance of spatial variability modelling in the context of calculating \( f_i(t) \) will be explained with the help of the simulated data.

7.4.1 Role of spatial variability in the predicted safety profile

To demonstrate how spatial variability is expected to influence the safety profile of the RC beam under consideration, two cases were considered. In the first case, the deterioration properties were assumed to be constant along the beam which is equivalent to the state of full spatial correlation (i.e. \( d = \infty \)). This is also similar to the conventional structural reliability analysis which tends to evaluate the failure probability of the Limit State (LS) only at sections within the structure where the highest load effect is expected to occur (Darmawan and Stewart, 2007). For example, for a simply supported beam, these sections are the midspan for the flexure LS and the end support for the shear LS. In this way, spatial variability is ignored and the failure probability is determined based on evaluating the LS functions at a single RF element.

In the second case, the spatial variability of the deteriorating properties along the beam was considered. Each RF element was considered as an individual component and its individual failure probability was used to form a system reliability problem for the whole beam. In this case, the governing LS was not always violated at sections (i.e. centre of RF element) within the structure where the highest load effect is induced as mentioned earlier. Other sections along the beam may experience the LS violation first as will be shown later.

A MC realisation for each of the two cases was plotted for the two failure modes (flexure and shear) considered for estimating the combined (Total) reliability of the beam under investigation, Figure 7.13. On the figure, the load effects induced on the beam (i.e. \( M_b \) and \( V_b \)) were also indicated. Both the induced load effect and the beam resistance in terms of flexure \( (M_a) \) and shear \( (V_a) \) were calculated at the centre of each RF element along the beam using Equation 6.6/6.7 and Equation 6.9 respectively. It is clear that for the first case,
as a result of using homogeneous material/geometrical and environmental properties along the beam (No spatial variability NSV), the calculated resistance model is also homogeneous along the beam. This means that the LS function is ensured to be violated at locations within the beam that are structurally critical, i.e. section 1 in Figure 7.13.

![Graph](image1.png)

Figure 7.13 Role of spatial variability (SV) on the predicted reliability of the RC beam (a) in terms of the bending moment (b) in terms of shear.

On the other hand, if the spatial variability (SV) of the deterioration properties were taken into account when determining the beam resistance, fluctuating resistances were obtained. Having a fluctuating resistance along the beam increases the chances that the LS violation
at locations within the beam other than those predefined to be structurally the most critical. For example, in this particular MC realisation, section 2 of the beam is shown to be the part of the beam where the LS is more likely to experience the LS violation when compared with the conventional midspan/end-support critical sections (section1). This demonstrates clearly the advantage of considering spatial variability over the traditional reliability approach when calculating the safety profile of a given structure.

Another observation can be made from Figure 7.13(b) that in addition to the exhibited fluctuation of the shear resistance due to the natural fluctuation of the material properties along the beam, other forms of irregularity may also be accounted for. For example, an extensive presence of chlorides (e.g. due to leakage of water from the expansion joints) or increased diffusivity and/or very low cover depth as a result of a poor workmanship at certain parts of the beam can lead to faster chloride penetration and/or higher corrosion rate and hence to a higher loss of reinforcement cross-sectional area. It is possible therefore that the significant drop in the shear resistance at the part of the beam which is represented by RF elements number 17 to 25 Figure 7.13(b) can be assumed to have been caused by one of the aforementioned reasons. The influence of such phenomena is only possible by considering different parts of the structure when modelling the reliability profile of a given structure, i.e. RF based reliability modelling.

7.4.2 Influence of pitting corrosion

Investigating the influence of pitting corrosion and the influence of variability and spatial distribution of corrosion-induced pitting on the safety of structures exposed to chloride attack has been severely neglected in previous reliability studies (e.g. Vu and Stewart, 2000). For this reason, this task has been highlighted, i.e. in Chapter 2 and Chapter 3, to be one of objectives of this thesis. This objective will be addressed in this section.

In the current analysis, the deterioration of the beam girder over time was estimated considering two forms of corrosion; (i) due to general (uniform) and (ii) due to pitting corrosion. As discussed in Section 6.3.4 and by Kenshel and O'Connor (2009), in the case of general corrosion, the deterioration occurs due to the reduction of the cross-sectional area of the reinforcement. In the case of pitting corrosion, the maximum pitting depths was specified through the use of the factor $R$ which relates the maximum pitting depth to the average penetration caused by the general corrosion. The analysis carried out in this thesis
assumes statistical independence between the pitting depths for each RF element and between reinforcing bars within the same RF element. The concept of having fully correlated or totally independent pitting depths is illustrated by Figure 7.14. In the first case, (a) fully correlated, for the same MC realisation; all corrosion-induced pits would have the same depth. In the second case, (b) independent, total independency between pits depths was assumed resulting in different pitting depths to be generated for each reinforcing bar within the same RF element or for the same bar expanding along different elements.

Figure 7.14 A single reinforcing bar and beam cross-section showing (a) Fully correlated and (b) Independent corrosion-induced pitting depths.

In reality, there may exist some correlation between the pitting depths of the neighbouring reinforcing bars or between pitting depths of the same bar. However, it is highly unlikely that all corrosion-induced pits would have the same depth value (fully correlated pitting depths) at the same stage of the corrosion activity. Therefore, and due to the lack of the statistical data to describe the likely correlation between pits depths, the pitting factors (hence pitting depths) were randomly (independently) generated from the Gumbel distribution as described in Section 6.3.4.1.
The subject of investigating the possible correlation between corrosion-induced pits requires an experimental/field study and therefore is outside the scope of this thesis. However, initial results obtained from the analysis carried out in this chapter indicated that the time-dependant loss of cross-sectional area due to pitting corrosion was not affected by modelling pitting depths as fully correlated or totally independent.

### 7.4.2.1 Influence of pitting corrosion on the beam resistance

The results indicating the influence of general and pitting corrosion on the beam flexure and shear performances were plotted in terms of the residual reinforcement cross-sectional area, Figure 7.15(a), and in terms of the residual resistance, Figure 7.15(b). It is clear from both graphs that pitting corrosion did not cause the flexure reinforcement cross-sectional area to reduce significantly when compared with the case of general corrosion. It was only after 85 years of service that pitting corrosion starts to become slightly more critical than general corrosion. For example, after 100 years, it can be observed that pitting corrosion reduced the cross-sectional area of the flexure reinforcement by about 7%, versus 5% reduction caused by the general corrosion. In terms of resistance, the same proportions of losses apply. These percentages correspond to about 40% increase in losses caused by pitting corrosion over that caused by the general corrosion in terms of both, cross-sectional area and resistance.

As observed, the overall loss of the beam flexure resistance as a result of pitting corrosion after 100 years of service was only about 7%. This loss may not be very significant, for example, it has been reported in the literature that if the loss of the structural resistance was less than 10-20% the safety of the structure is not considered to be significantly affected (Stewart and Mullard, 2007). In some other cases, the literature report that it is only when 30% of the reinforcement resistance have been lost, the corroding structure would be considered unsafe and therefore would require an immediate action (Li, 2004a).

The small percentage losses of the beam flexure cross-sectional area and hence the beam resistance in terms of flexure can be attributed to a number of reasons among which:

(i) High $T_i$ values as a result of the larger $C_d$ provided to the flexure reinforcement which implies that the corrosion activity starts at a late age.
(ii) The small $i_{corr}$ values as a result of the increased $C_d$ value in accordance with Equation 2.29.

(iii) Due to the large bar diameter which makes the proportion of the corroding reinforcement insignificant when compared with the original bar diameter. This is line with similar remarks by Stewart and Al-Harty (2008) who indicated that the relative effect of pitting on the cross-sectional area of the reinforcing bar reduces with increasing bar diameter.

Figure 7.15 Predicted beam deterioration versus time in terms of (a) reinforcement cross-sectional area and (b) beam resistance.
However, the significance of these losses can only be reasonably assessed if the uncertainty associated with imposed load is taken into account as will be discussed in the following section.

In the case of shear, pitting corrosion showed to be more critical than in the case of flexure. For example, after 50 years of service, the loss of the shear links cross-sectional area caused by pitting corrosion was 41% higher than that caused by the general corrosion versus 0% in the case of flexure. In terms of shear resistance, the loss caused by pitting corrosion (after 50 years of service) was 55% higher than that caused by general corrosion versus 0% in the case of flexure. If the reference age was 100 years, the overall losses of the beam shear resistance caused by both pitting and general corrosion were 36% and 28% (as compared with 7% and 5% for flexure) respectively. This indicates that the relative effect of pitting corrosion on the reinforcement cross-sectional area and hence on the beam resistance, seems to increase with the decrease of the bar diameter, this is inline with the literature (e.g. Enright and Frangopol, 2000b; Val and Stewart, 2003; Stewart and Al-Harthiy, 2008).

The slight increase in the normalized resistance observed at the early age of the beam indicated in Figure 7.15(b), which is more notable in the case of shear, was attributed to the evolution of the compressive strength, i.e. in accordance with Equation 6.12.

7.4.2.2 Influence of pitting corrosion on the safety profile

For the results indicated earlier with regard to the percentage losses of the reinforcement cross-sectional area and the beam resistance to be of a practical engineering value, the influence of both forms of corrosion had to be expressed in terms of a performance criteria which takes into account the uncertainty of loading imposed on the structure. Therefore, for both failure modes, flexure and shear, and for each of the two forms of corrosion, general and pitting, the annual reliability indices corresponding to the beam annual failure probabilities were calculated for 100 years of the beam service life (in 5 years increments). The results are indicated in Figure 7.16 for two cases; (a) spatial variability was not considered (b) spatial variability was considered. The influence of pitting corrosion on the safety profile will be discussed in terms of Flexure versus Shear and spatial variability versus no spatial variability analysis. The results of $\beta(t)$ indicating Flexure, Shear and
Total were produced employing Equations 6.28, 6.29 and 6.30 respectively in Equations 6.31 and 6.34 as discussed in Section 6.3.8.

The reason for not having a complete trend that shows the values for the reliability indices at the very early age is that probability of failure is too small (i.e. $P_f \approx 0$) to be captured by the MC simulation method with a number of iterations that are feasible to performed using commercially available computers. The number of iterations were used to perform the reliability analysis carried out in this thesis was 1,000,000 iterations for each incremental year. The entire analysis to produce one safety profile would normally take about 48 hours to complete. Since the focus of this thesis is to show the relative influence of spatial variability with reference to the No spatial variability scenario and the relative influence of pitting corrosion as compared with the general corrosion case, the 1,000,000 iterations were considered to be sufficient. Furthermore, with this number of simulation iterations, it was possible to produce stable results of the safety profile in which the target reliability can be clearly identified (i.e. $\beta_T=3.8$).

**Flexure vs. Shear**

Figure 7.16 indicates that the beam reliability decreases with time. This is due to the reduction of the cross-sectional area of flexure and shear reinforcements. For both cases, general and pitting corrosion, the reduction of the beam reliability over time can be seen to be governed by shear rather than by flexure for both spatial and no spatial analysis. Because of their relatively smaller cover depths, $C_d$, than the main flexure reinforcements, shear links are expected to have a shorter $T_f$ period (according to Equation 2.1) and a higher value of $i_{corr}$ (according to Equation 2.29). It is therefore expected that shear links are more vulnerable to corrosion attack than the flexure reinforcement. Furthermore, the shear links have a smaller diameter (than flexure reinforcements) which implies that the percentage loss in the cross-sectional area is more prominent in the case of shear than in the case of flexure. This agrees well with the literature (e.g. Val, 2005) which indicates that the influence of shear failure on the beam reliability increases when higher diameter bars are used for the longitudinal (flexure) reinforcement.

Figure 7.16 indicates the severe influence that pitting corrosion can impose on the beam reliability, particularly when the reliability of the beam is governed by the shear LS. This agrees well with the literature, for example, the results shown by Figure 7.16 confirms the concluding remarks by Val (2005) who indicated that reliability of the corroding RC
structures may be significantly overestimated if pitting corrosion of the shear links was not considered. It can therefore be concluded that the reduction of the beam shear resistance due to pitting corrosion has a major effect on reliability of the beam under consideration.

Figure 7.16 Influence of General (G) and Pitting (P) corrosion on the beam safety profile for (a) No spatial variability (NSV) and (b) spatial variability (SV).
Spatial Variability vs. No Spatial Variability

To investigate the influence of considering spatial variability on the safety profile of the beam girder under consideration, results presented in Figure 7.16 for flexure and shear were re-plotted on a single graph, Figure 7.17. The first observation can be made from this new figure that spatial variability has no influence on the predicted reliability indices in terms of shear for both cases general and pitting corrosion. This means that the violation of the shear LS was governed by the end support RF element (i.e. section 1 in Figure 7.13(b)) where the induced shear force is expected to be at peak.

![Figure 7.17](image.png)

Figure 7.17 Time dependant Flexure and Shear reliability indices for General (G) and Pitting (P) corrosion, considering Spatial variability (SV) and No spatial variability (NSV), Reproduced from Figure 7.16(a) and (b).

The second observation which can be made from Figure 7.17 is that the influence of spatial variability on the beam reliability in terms of flexure is more evident in the case of pitting corrosion than in the case of general corrosion. For example, after 50 years of service, the inclusion of spatial variability (SV) has caused the flexure failure probability predicted in terms of pitting corrosion to increase by 12% over that predicted for the case when spatial variability was not considered (NSV). In the case of general corrosion, the increase in the flexure failure probability was only about 2% for the SV scenario. After 100 years of service, the inclusion of spatial variability has caused the flexure failure probability predicted in terms of pitting corrosion to increase by 40% in comparison with the NSV scenario. In the case of general corrosion, this increase was only 7% for the SV scenario. It
can be concluded therefore that ignoring spatial variability can lead to overestimation of the beam reliability, more evidently in the case of pitting corrosion, when the reliability of the beam is governed by the flexure mode of failure.

The reason why the influence of spatial variability showed to be more significant in the case of flexure than in the case of shear was attributed to the fact that the critical zone by which the LS experiences violation is wider in the case of flexure than in the case of shear. For example, for the 31 RF elements, there were more elements that are likely to govern the flexure LS than elements which are likely to govern the shear LS. To support this conclusion, a histogram was constructed, Figure 7.18, to show the frequency of RF elements that has governed the LS for the two failure modes, flexure and shear. Figure 7.18 shows that the governing LS is not always at the midspan in the case of flexure or at the end support in the case of shear. However, the figure shows that the midspan RF element (RF #16) has governed the LS about 25% of the times. The remaining 75% were shared by all other elements with the element adjacent to the midspan element have higher proportion than those further away. Meanwhile, in the case of shear, the end support element (RF #1) has governed the shear LS about 47% of the times. It is clear that the remaining RF elements in this case governed the LS violation fewer times than that in the case of flexure. This explains that why the influence of spatial variability on the reliability of the beam is expected to be more prominent in the case of flexure than in the case of shear.

Figure 7.18 Distribution of spatial position of governing limit state after 50 years of service due to pitting corrosion for (a) Flexure (b) Shear.

Figure 7.19 (reproduced from Figure 7.16) shows the safety profile predicted in terms of the combined (Total) reliability for general (G) and pitting (P) corrosion with the inclusion
of spatial variability (SV) and without spatial variability (NSV). The figure indicates that the influence of spatial variability is not significant because (as explained earlier) the combined safety profile in this case is governed by shear rather than flexure. However, it is evident from the figure that pitting corrosion has a stronger effect on the beam reliability than general corrosion. For example, after 50 years of service, the combined reliability of the beam due to pitting corrosion was 1.16 verses 2.35 due to the general corrosion. Thus, it can be said that after 50 years of service, the reduction in the beam reliability due to pitting corrosion is 51% higher than that caused by general corrosion.

![Figure 7.19](image.png)

Figure 7.19 The safety profile presented in terms of the combined (Total) probability of failure (reproduced from Figure 7.16(a) and (b)) for General (G) and Pitting (P) with Spatial Variability (SV) and without Spatial Variability (NSV).

If the safety profile is shown to be governed by pitting corrosion, as in the current case, the assumption which neglects the effect of loss of bond between the reinforcement and concrete as a result of excessive cracking or spalling can therefore be justified. For example, since pitting corrosion is localised, it is less likely to cause the disruption of the concrete cover and hence no reduction is expected for the bond strength around the pits (Val and Melchers, 1997).

When compared with the target reliability specified by EC2 for the ULS, the beam safety has violated the $\beta_f$ just after 11 years of the beam construction in the case of pitting corrosion.
corrosion and after 25 years in the case general corrosion. Based on this result, if the time to first repair/maintenance intervention is to be decided based on the ULS criteria and only the general corrosion was considered this would result in overestimating the time for intervention by about 14 years. At this time \( t=25 \text{ years} \), the beam failure probability would have exceeded the target failure probability by about 41%.

As indicated at the end of Section 7.3 and in the context of the prime objective of this thesis, the time to first repair/maintenance intervention determined based on the safety performance (i.e. ULS) will have to be compared with those predicted in terms of the condition profile (i.e. SLS). This is the subject of the following section.

### 7.5 TIME TO REPAIR/MAINTENANCE INTERVENTION

One of the main objectives of this thesis was to develop a rational decision-making tool to assess the time to first repair/maintenance intervention by relative comparison of each performance profile to its corresponding target performance as specified by the relevant codes of practice or as required by the structure’s owner/manager. In this regard, the results presented earlier, in terms of the condition profile (Section 7.3) and in terms of the safety profile (Section 7.4), were used to infer the optimal times to first repair/maintenance intervention for the beam case under investigation, Figure 7.20.

![Graphs showing times to first repair/maintenance based on SLS and ULS criteria](image)

Figure 7.20 Times to first repair/maintenance based on (a) SLS criteria (b) ULS criteria.

Figure 7.20(a) indicates that if the decision on the time to first repair/maintenance is to be made in light of the predicted condition profile (i.e. SLS) and criteria related to it, the time
to first repair/maintenance intervention can be expected at about 47 years of the beam construction if Criterion 1 is to be considered (i.e. $A_{x_{\text{max}}}=25\%$). If Criteria 3 is of interest (i.e. $A_{x_{\text{max}}}=1.5\%$), the time to first repair/maintenance would be required after about 42 years of construction. Criterion 2 was neglected because it is closely related to Criterion 1 as explained in Section 7.3.5. The results indicate that both criteria are closely related with the difference between the predicted times to first repair is only 5 years. Furthermore, if the intended minimum designed service life of the structure is taken as 50 years (Li, 2004b), then the results shows that the beam cannot meet its intended design service life. However, failing to meet the design service life was only by 3 years in terms of Criterion 1 and by 7 years in terms of Criterion 3; the later is more relevant to practical engineers concerned with making decisions regarding repair/maintenance as discussed in Section 7.3.5. It is therefore accepted to assume that the beam structure under consideration and under the specified environmental circumstances have almost maintained the intended design service life in terms of the visual condition criteria (i.e. in terms of SLS).

Meanwhile, if the decision on the time to first repair/maintenance is to be made based on the safety profile (i.e. ULS) and its related $\beta_1$ as specified by the relevant code of practice (i.e. $\beta_1=3.8$ as per the EC2), the time to first repair/maintenance was found to be about 25 years of the beam construction in the case of general corrosion, Figure 7.20(b). When pitting corrosion was considered, the time to first repair/maintenance would be required after only 11 years of the beam construction. In both cases, the beam has failed to maintain its intended design service life (50 years) by a significant margin. The case is more critical when considering pitting corrosion which is in contrast with what some researchers (e.g. Vu and Stewart, 2000) had postulated. The view of the mentioned researchers was that pitting corrosion would not significantly influence the structural capacity of the corroding structure because it is unlikely that many bars will be affected by pitting. The results presented in this thesis, Figure 7.20(b), have shown that pitting corrosion is more critical than general corrosion from the safety viewpoint and hence from the safety-based bridge (and bridge networks) management viewpoint.

The result indicates that the time to first repair/maintenance, which was determined based on the predicted beam condition profile considering Criterion 3 (42 years), are longer than the times to first repair/maintenance predicted in terms of the Safety profile by 31 years and 17 years for general and pitting corrosion respectively. Criterion 3 was chosen to represent the condition profile in the current comparison because it is more relevant to
practical engineers than Criterion 1 as explained in Section 7.3.5. It can be seen therefore that the decision on the time to first repair/maintenance intervention can be governed by the safety criteria of the structure rather than by the commonly practiced and traditionally used visual condition criteria.

Moreover, if the performance criteria to be considered do not take into account pitting corrosion (i.e. pitting corrosion is not relevant when modelling the condition profile) the predicted time to first repair/maintenance intervention may be too permissive. For example, if the decision is governed by the safety criteria considering general corrosion, the beam under consideration would have been scheduled for repair 14 years later than it should. If the decision on first repair is governed by the visual condition considering Criterion 3, the beam would have been scheduled for repair 31 years later than it should. It has to be mentioned that the violation of the target reliability specified by the code does not imply an immediate collapse of the structure; it only means that the structure under the current loading/deterioration conditions does not meet the acceptable (target) safety criteria and, therefore, action has to be made to bring its safety to an acceptable level.

The results presented here indicate that the time to first repair/maintenance of chloride affected bridge structures should consider the reliability/safety of the structure (i.e. ULS) and should not only consider the surface (visual) condition (i.e. SLS). This viewpoint have been shared by other researchers (e.g. Enright and Frangopol, 2000a; Onoufriou and Frangopol, 2002) who called for the need that repair/maintenance of deteriorating bridge structures should be based on the safety rather than on the visual condition of the bridge elements.

It has to be stressed that the results presented in this thesis, which indicated that the time to repair/maintenance is governed by the structure Safety and not by its surface condition, should not be generalised. For example, if the same structure was assessed using different (but lower) traffic load data, the optimal time to first repair/maintenance may be governed by the condition profile results rather than by the Safety profile. The analysis performed in this thesis should be viewed such that it only indicates the importance that the time to repair/maintenance of bridge/network of bridges should be made based on the relative comparison between the ULS and SLS related criteria and not on subjective judgment.
7.6 SENSITIVITY ANALYSIS

In order to assess the relative importance of each random variable involved in the calculation of the safety performance of the RC beam under consideration, a sensitivity analysis was performed. The sensitivity analysis helps identifying which of the random variables has the greatest influence on the calculated beam reliability. In this thesis, the sensitivity of the reliability results (Safety profile) were assessed by recording the change in the reliability indices that corresponds to increasing the mean value (μ) or the value of the standard deviation (σ) of the modelled random variable by 10% (O'Connor et al., 2004). The results of the sensitivity analysis are presented in terms of both general and pitting corrosion and for time periods of \( t = 25, 50 \) and 75 years of the beam age, Figure 7.21 and Figure 7.22.

It was found, as seen from Figure 7.21 and Figure 7.22, that the single most important parameter which has the greatest impact on the reliability index is the mean value of \( C_d \). For example, in this case of general corrosion, the mean value of \( C_d = 50 \) mm was increased by 10%; as a result, changes of +0.7, +0.49 and +0.39 in the value of \( \beta \) were noted for \( t = 25, 50 \) and 75 years respectively. In the case of pitting corrosion, the corresponding changes in the value of \( \beta \) were +0.42, +0.32 and +.29 for \( t = 25, 50 \) and 75 respectively.

The figures also indicate that the mean value of the specified concrete compressive strength \( (f_{ck}') \), the beam effective depth \( (d_{eff}) \), the Model Error (ME) of the girder distribution factor ME\( (GDF) \), ME of the corrosion rate \( (i_{corr}) \), ME of the beam shear resistance \( (V_u) \) all have comparatively similar influence in magnitude on the change of \( \beta \). However, ME\( (GDF) \) and ME\( (i_{corr}) \) have an adverse affect on \( \beta \) in contrast with \( f_{ck}', d_{eff}, \) and ME\( (V_u) \). All other random variables, which have been identified in Table 6.3, are shown to have minor influence on the predicted reliability for both general and pitting corrosion cases.

It can also be seen from both figures that the reliability indices and hence the safety profile is more sensitive to the change in the mean value of the modelled parameters than to the change in their standard deviation.
Figure 7.21 Change in $\beta$ for 10% increase in the value of the mean ($\mu$) and the standard deviation ($\sigma$) of the modelled parameters (General corrosion).
Figure 7.22 Change of $\beta$ for 10% increase in the value of the mean ($\mu$) and the standard deviation ($\sigma$) of the modelled parameters (Pitting corrosion).
It is also observed from the figures that the reliability indices, for both general and pitting corrosion, are insensitive to the changes in the mean or standard deviations values of the ME specified for the beam flexure resistance $M_E(M_a)$. This is expected considering that the safety profiles have been shown, i.e. in Section 7.4, to be governed by shear rather than by flexure for the beam case under consideration. This situation can change for structural cases where the failure probability (hence reliability) is governed by flexure.

The significance of the sensitivity analysis performed in this chapter can be seen by its ability to pinpoint which of the deterioration variables has to be modelled as spatially variable by considering its influence on the predicted reliability. For example, a variable which is shown to have a considerable impact on the reliability, the uncertainty associated with its spatial variability should not be ignored and therefore should be modelled as RF (i.e. spatially variable). The basic requirements for considering the spatial variability of a RF variable is knowledge of its $\theta$ value which has to be determined accurately from experimentally collected data using methods presented in Chapter 5. Thus, based on the results presented here with respect to the sensitivity of the safety profiles to the mean value of $C_{f1}$, $\theta$ for this parameter should be estimated accurately if the uncertainty associated with this important deterioration parameter is to be fully considered.

### 7.7 CONCLUSIONS

The main objective of this chapter is to compare the predicted times to first repair/maintenance intervention of a structure determined based on condition and safety profiles (i.e. SLS vs. ULS). In this context, results presented in this chapter indicate that if only the general corrosion is considered, the decision of time to first repair/maintenance intervention can be governed by the safety criteria of the structure rather than by the commonly practiced and traditionally used visual condition criteria. For example, for the beam case considered in this thesis, the time to first repair/maintenance, which was predicted in terms of the beam condition profile, are found to be 42 years as compared with 31 years and 17 years predicted in terms of terms of the Safety profile for general and pitting corrosion respectively. Therefore, it was concluded that if the performance criteria to be considered for deciding on the time to first repair/maintenance do not take into account pitting corrosion, i.e. condition profile, the predicted time to first repair/maintenance intervention may be too permissive. These findings strongly points to
the necessity of having a bridge management system tool that considers lifetime safety of the structure as viable indicator for maintenance and repair interventions.

In previous works, researchers used the Mean to describe the central tendency of expected damage proportion ($A_t$%); in this chapter, however, it was shown that the Median is more reliable central tendency measurement tool. This chapter has also demonstrated how considering spatial variability should be expected to have some influence on the lifetime reliability (the Safety profile) of corroding RC structures.

The influence of the Scale of Fluctuation ($\theta$), which describes the degree of spatial correlation of a specified property, on the predicted condition of the structure, was also investigated. It was found that as $\theta$ increases, the time to first repair/maintenance predicted in terms of SLS also increases. Therefore, it was concluded that studies that have not calculated the $\theta$ based on a site-specific data, their prediction of the time to first repair/maintenance may inherent a considerable error.

In the context of safety profiles, spatial variability has shown to be only important if the reliability of the structure was governed by the flexure LS. In this case, the influence of spatial variability on the reliability of the beam was more evident in the case of pitting corrosion than in general corrosion. For the case when the reliability of the structure is to be governed by the shear LS, as in the current example, results showed that spatial variability has insignificant influence. This was explained by demonstrating that the end support RF element has governed the LS 47% of the times as oppose to 25% by the midspan element in the case of flexure.

The reduction of the beam reliability over time was shown to be governed by shear for both spatial and non spatial analysis. This was attributed to the fact that the shear links are often placed at relatively smaller cover depths, $C_{d_s}$ than the main flexure reinforcements. The shear links are therefore expected to have a shorter $T_r$ period and a higher value of $i_{corr}$. This indicates that, for the current beam case, shear links would have suffered larger cross-sectional/resistance losses than the flexure reinforcement at the same point in time.

For the two types of corrosion studied in this thesis, general and pitting corrosion, results showed that pitting corrosion potentially has far more aggressive effect on the reliability of corroding structures than general corrosion. The results also suggest that pitting corrosion
affects shear resistance far more severely than it would affect flexure resistance. The literature reports that many of today’s deteriorating RC bridge structures, including the bridge taken as an example in this thesis, deteriorate due to corrosion caused by chloride-contaminated water leaking through the deck joints. This means that more intense form of deterioration can take place in location where high shear stresses are expected (e.g. beam girders at the supports). It was therefore concluded that pitting corrosion at location of high shear stresses can have major impact on the reliability/safety of structures. Thus for the safety assessment of RC beams in corrosive marine (or de-icing salts) environments, the effect of pitting corrosion of shear links on shear resistance should be considered, otherwise the reliability of the beam may be considerably overestimated.

Finally, the sensitivity analysis carried out in this chapter with respect to the safety profile has indicated that the single most important parameter that has the greatest impact of the calculated reliability index is the mean value of the concrete cover depth $C_d$. This shows that increasing the cover depth at the design stage not only benefits structures located in chloride environments in terms of durability but also contributes to improving the structure safety mainly by delaying the initiation of corrosion. Based on the results presented in this work, with respect to the sensitivity of the safety profiles to the mean value of $C_d$, this thesis recommends that $\theta$ for this parameter should be estimated accurately from a field data if the uncertainty associated with this important deterioration parameter is to be fully considered.
Chapter 8:

Conclusions &

Recommendations
8.1 INTRODUCTION

The subject of Reinforced Concrete (RC) corrosion in the field of bridge engineering has been a matter of discussion and research from different viewpoints. The intention, however, is one, and that is how to keep RC bridge structures safe and functional with the minimum cost possible. The increasing number of deteriorating RC bridges worldwide demands the use of rational methods to enable bridge owners/managers/engineers to make optimum decisions regarding the management of their assets. The research carried out in this thesis focused on developing a methodology that can be used to predict the deteriorating condition and safety of bridges to and hence help in predicting the optimal time to repair/maintenance intervention. Such a tool is vital considering the large number of deteriorating bridges and the limited budget/resources available for their repair. The developed methodology facilitates prioritisation of bridges/bridge components that are of most need for repair/maintenance intervention within the network of bridges. In this way, bridges are ensured to be kept safe and functional while the limited available budget and resources are wisely managed. The prime objectives/tasks of this thesis were as follows:

(i) To develop a probabilistic-based performance prediction tool that can be used to predict the optimal time for repair/maintenance intervention of RC structures exposed to aggressive chloride environments using a bridge or bridge component as an illustrative example.

(ii) The proposed probabilistic model should take into consideration the inclusion of spatial variability of the deterioration parameters involved in estimating the time for repair/maintenance intervention.

(iii) The proposed probabilistic model should consider the dual modelling of the Serviceability Limit State (SLS) (i.e. deterioration in the visual condition) and the Ultimate Limit State (ULS) (i.e. the deterioration in the load carrying capacity) of the investigated structure/member.

In the context of achieving these prime objectives, the literature review has revealed some shortcomings in the research preceded the work of this thesis and therefore these were identified as secondary objectives for this thesis to address:

1. Data on the Scale of Fluctuation (θ), a parameter which is necessary for modelling the spatial variability of the deterioration parameters, hardly existed in the
literature. Therefore, it was decided that an experimental investigation needs to be performed to obtain values for such parameter.

2. When assessing the performance of a given structure, the corrosion activity was often assumed to have already started and the influence of the corrosion initiation parameters on the lifetime safety performance was hardly assessed. The initiation stage and propagation stage of the deterioration process were rarely considered within the same framework, which undermines studying the influence of the corrosion initiation parameters on the lifetime safety performance. The developed model should consider including both the initiation and propagation models.

3. Despite the few and limited studies on the effect of pitting (localised) corrosion, which indicated the severity of this form of corrosion on the load carrying capacity of corroding structures, the effect of variability of pitting corrosion on the safety performance of structures has not been properly investigated. Hence, the developed model should consider the effect of pitting corrosion on the lifetime safety of the structure assessed.

4. The load models used to assess the load carrying capacity of corroding structures were either oversimplified or estimated from conservative standards or codes of practises and not from actual traffic data. In this thesis, it was decided that the load model should be based on a realistic site-specific load data for the uncertainty associated with the loading to be considered.

In light of the above, this thesis developed a probabilistic-based reliability model which addresses the aforementioned shortcomings with the ultimate aim is to predict the optimal time to repair/maintenance intervention for RC bridges exposed to marine environments using an RC beam girder as an illustrative example.

8.2 MATERIAL DETERIORATION MODELS

In order to be able to quantify the end of the service life of a corroding RC structure, and hence estimate the time to its repair/maintenance, models describing the deterioration process of the structural materials due to chloride-induced corrosion were needed. The task of identifying the available material deterioration models and the uncertainty associated with their key parameters was undertaken in Chapter 2. Chapter 2 indicated that RC structures in chloride environments undergo two distinguished deterioration stages, *Initiation* and *Propagation*. The literature revealed that there is a wide acceptance of the
use of Ficks 2nd law of diffusion to estimate the Initiation stage. The Propagation stage was described by two sub-stages; Crack Initiation and the crack propagation.

The key parameters to describe the initiation stage are; the Diffusion Coefficient \( D_{app} \), the Surface Chloride Content \( C_s \), the Critical Chloride Content \( C_{cr} \), the Reinforcement Cover Depth \( C_d \) and the Initial Chloride Content \( C_i \). The two variables \( D_{app}, C_s \) were shown to vary significantly with their mean values differing from structure to structure and from one environment to another. It was decided therefore that the statistical information of both parameters should be obtained for a real structure exposed to marine environment. Ferrycarrig bridge was therefore chosen to study the variability of parameters \( D_{app} \) and \( C_s \). Parameters \( C_d \) and \( C_i \) were found not to depend on the environmental condition and thus it was concluded that their variability could be estimated either from the literature or from the structure under investigation.

Chapter 2 also concluded that the time-dependant property of \( D_{app} \) needs to be considered if the original material properties were made from Pulverised Fly Ash (PFA) or Ground Granulated Blast-furnace Slag (GGBS) concretes. However, for Ordinary Portland Cement (OPC) concretes, the investigation carried out showed that the ageing factor \( m \), which permits the inclusion of the reduction of \( D_{app} \) with time, to be \( \leq 0.2 \). With such values, the time to corrosion initiation was found not to be affected significantly. It was thus concluded that it would be reasonable to assume that \( D_{app} \) is constant with time for OPC concrete. The beam example considered in this thesis was assumed to have been made of OPC concrete; therefore, the time-invariant model of \( D_{app} \) was used.

Another key parameter for the estimation of the initiation stage was \( C_{cr} \). Again, this parameter has also shown great variation. The wide scatter was attributed to the different methods by which this parameter is measured. It was concluded that for the same material type, the same environmental condition and the same measurement method, the variation of this parameter is reduced. Based on the literature, a Lognormal distribution with a mean value of 0.07 (Cl% per mass of concrete) and Coefficient of Variation (COV) of 0.25 were proposed for probabilistic modelling intended for this thesis.

For the propagation stage, several models were reviewed. Most of these models were mainly functions of the corrosion rate density \( i_{cor} \), water cement ratio \( w/c \), the concrete compressive strength \( f_c \) and \( C_d \). The model proposed by El-Maaddaway and Soudki
Chapter 8: Summary and Conclusions

(2007) was selected to estimate the time to first cracking, i.e. hair-size cracking. For the second sub-stage, the model proposed by Vu et al. (2005) was selected to describe the time period for the hair-size crack to grow and reaches 1.0 mm in width.

Following the start of the corrosion activity and during the formation of the cracks on the concrete surface, the reinforcement cross-sectional area is being reduced by two mechanisms; by General (uniform) corrosion or by Pitting (localised) corrosion. Pitting corrosion is more severe and causes loss of cross-sectional area several orders of magnitude (i.e. 4-8) of that caused by the General corrosion. Both forms of corrosion were considered in this thesis. The spatial variation of the corrosion-induced pits was also considered in line with the objectives indicated in Section 8.1.

The literature has demonstrated that \( i_{\text{corr}} \) is the main parameter affecting the modelling of the propagation stage. It was concluded that it is desirable that data of this parameter are obtained from site specific measurements. However, for the lack of such data, the empirical model proposed by Vu and Stewart (2000) was used in this thesis. Values of \( i_{\text{corr}} \) predicted by the proposed model were used in light of the typically measured values in real structures which were reported to range from 0.1 to 10 \( \mu \text{A/cm}^2 \).

In order to be able to predict the time to repair/maintenance, the minimum acceptable level (target) of structure performance against deterioration needs to be defined. It was found that the target performance differs from engineer to engineer and from agency to agency. For serviceability requirements (SLS), the percentage of the surface area that shows sign of corrosion related damage \( (A_x \%) \) can be an appropriate quantitative indicator. For example, if the damage was defined such that corrosion is initiated, then the repair of the structure may be deemed necessary if 25% (i.e. \( A_{\text{max}} = 25\% \)) of the surface area has indicated corrosion initiation. On the other hand, if the damage was defined such that a significantly large cracks (\( >1.0 \text{ mm wide} \)) to appear on the concrete surface, the indication for the time to first repair may be that when 1.5% (i.e. \( A_{\text{max}} = 1.5\% \)) of the surface area of the structure showed cracking of the mentioned size. It was concluded that for the determination of such percentage the Random Field (RF) theory would have to be used.

In terms of safety, the Target Reliability Index \( (\beta_t=3.8) \) specified by the EC2 for ULS failure was used as minimum acceptable level of deterioration in the safety of the structure.
If the $\beta_T$ is reached, the structure is assumed to have reached the stage where a corrective intervention measures (repair/maintenance) must be taken.

## 8.3 Random Field (RF) Analysis

Modelling spatial variability has been highlighted in Section 8.1 as one of the three major objectives of this thesis. The literature review carried out in Chapter 3 has demonstrated how the Random Field (RF) theory can be employed to facilitate the inclusion of the uncertainty associated with the spatial variability of the deteriorating parameters into the intended probabilistic analysis. RF-based modelling also allows the prediction of the proportion of the structure that exhibits a certain damage characteristic, i.e. cracking. The fundamental concept of RF theory and its application to structural engineering were described in Chapter 3. In a RF-based analysis, the structure is hypothetically divided (discretised) into $k$ square/rectangular RF elements and random variables are used to represent the RF over each element. The size of each RF element depends on the intensity of fluctuation of the RF variable. Two major RF discretisation methods which are fundamental for the procedure of RF-based modelling were explained in detail including advantages and disadvantages of each method (the Midpoint method, and the Spatial Averaging method).

This thesis described the major published works which have dealt with the subject of spatial variability modelling in the field of RC corrosion. A number of significant shortcomings of the available literature pointed towards the lack of reliable data on the Scale of Fluctuation ($\theta$) parameter. The parameter $\theta$ is a key parameter for describing the spatial variability of a physical quantity. Based on the findings of the literature review performed in Chapter 3, it was concluded that the following tasks should be considered in the work of this thesis:

1. To perform an experimental study to obtain a site specific data of the parameter $\theta$ for two of key deterioration parameters identified in Chapter 2, $C_s$ and $D_{app}$. Ferrycarrig bridge was selected perform this experimental study.
2. To estimate the maximum load effect using a realistic traffic load data instead of using the conservative load models provided by the design codes. Weight-In-Motion (WIM) traffic data were used to obtain the maximum load effects that are expected to be
imposed on the structure under investigation using Extreme Value (EV) statistics. This task was carried out in Chapter 6.

8.4 SIGNIFICANCE OF THE EXPERIMENTAL RESULTS

Considering the wide scatter of key deterioration parameters presented by Chapter 2 and for the lack of reliable data regarding parameter \( \theta \), chloride analysis was performed on a total of 45 concrete cores extracted from nine crosshead beam faces of Ferrycarrig Bridge. Background information on Ferrycarrig bridge was given in Chapter 4 with a particular emphasis on the exposure condition, the level and the cause of the deterioration of the crosshead beams from which the cores where extracted. The chloride analysis was carried out using a sample preparation protocol adopted by the EN-14629 standard for the acid-soluble chloride determination using Potentiometric titration. The method and the apparatus used were calibrated using a standard sample with predetermined chloride concentrations.

The \( C_v \) and \( D_{upp} \) results obtained from the experimental study performed in this thesis showed that the crosshead beams under investigation have a very good resistance to chloride penetration as indicated by the low values of \( D_{app} \) which were found to range from 2.03-22.82 mm²/year. The results showed that data obtained for both variables \( C_v \) and \( D_{app} \) follow a Truncated Normal distribution. The current study also showed that \( C_i \) does exist and cannot be neglected. Neglecting \( C_i \) can result in inaccurate estimation of \( C_v \) and \( D_{app} \) parameters which will consequently affect the estimated time to corrosion initiation. Results of \( C_i \) were also found to follow a Truncated Normal distribution. Among the three parameters, it was found that \( C_v \) exhibits the highest variation with a COV of 0.56; where COV of \( D_{app} \) and \( C_i \) were 0.40 and 0.45 respectively, which is consistent with the literature.

The results also showed that there was a significant increase in the average values for both parameters \( C_v \) and \( D_{app} \) obtained from the north side face (windward) of the crosshead beams as compared with results obtained from the south side face (leeward) due to crosshead orientation with regard to the prevailing wind. These findings can be used to identify the area of the structure or structure component that should be prioritised for special inspection or extra protection as early as the design stage.
In addition to that they were taken from a real aging structure exposed to marine environment, the unique importance of the \( C_s \) and \( D_{app} \) data presented in this thesis is that they were taken at frequent distances from nine different crosshead faces. This enabled the analysis of these data to obtain valuable information regarding the parameter \( \theta \) which is essential for spatial variability modelling. The methods used for estimating \( \theta \) were discussed in Chapter 5 and summary and concluding remarks regarding the methods used and the obtained values of \( \theta \) will be presented in the following section.

8.5 VALUES FOR THE SCALE OF FLUCTUATION

The data collected in Chapter 4 for \( C_s \) and \( D_{app} \) variables were analysed in Chapter 5 to estimate values for \( \theta \). Estimated values of \( \theta \) were needed for the generation of spatially correlated data as part of the RF probabilistic model developed in this thesis. Two estimation methods were applied to the data collected from the experimental program (Chapter 4) for the two investigated variables, \( C_s \) and \( D_{app} \). Eight groups of results each consisting of either four or five observations were analysed using the Maximum Likelihood Method (MLM) as described by Li (2004b) and the Curve Fitting of the Autocorrelation Function \( \rho(r) \) Method proposed by Vanmarcke (1983). It was concluded that the MLM cannot be used for the estimation of \( \theta \) because it was found to be inconsistent and in most cases gave a range of values rather than a single value of \( \theta \).

The Curve Fitting method required the missing data of the two variables to be predicted for the theoretical autocorrelation function \( \rho(r) \) to be fitted to the experimental autocorrelation function \( \rho_s(r) \) to obtain values for \( \theta \). The Kriging method of statistical interpolation was used to predict the missing data at locations where samples were not taken. Unlike most of the ordinary interpolation algorithms which assign weights according to functions that give a decreasing weight with increasing distance between the data samples, Kriging assigns weights according to a data-driven weighting function. The theory and the concept of kriging method were presented in Chapter 5. The method has been used in other fields of science such as Mining Engineering, Geology and more recently in Hydrology and Soil sciences. Its use however in the field of structural engineering is almost non existent with the exception of work by Ramachandran et al. (2001) and Karimi (2001).

Values of \( \theta \) for the two variables (\( C_s \) and \( D_{app} \)), which were obtained using the Curve Fitting method, were found to be slightly larger than those reported in the literature. The
results obtained from the current study for both variables and in particular for $D_{app}$, can be regarded as the most reliable up to the date of writing this thesis. The mean values of $\theta$ obtained from the current study for $C_s$ and $D_{app}$ variables were 2.7 m and 1.9 m respectively with standard deviation of 0.6 m for both.

Based on the obtained values of $\theta$ for $C_s$ and $D_{app}$, a measurement plan for future experimental investigation of the fluctuation property of these two variables was proposed. The proposed measurement plan was designed in a way so that a distinctive experimental semivariogram can be plotted from the minimum number of observations. The proposed sampling plan aims at improving the quality of the empirical semivariogram plot by achieving: (i) higher frequency for the same lag interval which will reduce the variability of the semivariances that correspond to the same lag interval and therefore results in much smoother semivariogram plot; (ii) more lag intervals within the expected range of sample correlation which will help in describing the mounting part of the semivariogram model.

The fact that the $\theta$ values used for the analysis performed in this thesis was derived from an RC bridge exposed to marine environment, adds a great deal of credibility to the results presented herein.

The sensitivity analysis carried out in Chapter 7 with respect to the safety profile has indicated that the single most important parameter that has the greatest impact on the calculated reliability index is the mean value of the concrete cover depth ($C_d$). Based on this, it was concluded that values of $\theta$ for $C_d$ need to be estimated accurately from field measurements if the uncertainty associated with this variable is to be considered fully.

### 8.6 MODEL DEVELOPMENT AND ASSUMPTIONS

The task of developing the probabilistic model was carried out in Chapter 6 of this thesis. In this chapter, a methodology was introduced in which the time-dependant deterioration of the considered structure in terms of its surface (visual) condition and in terms of its load carrying capacity (Safety) can be dually modelled. The surface condition performance, an SLS performance indicator that is often used by bridge owners/managers to decide on the time for repair/maintenance intervention, was termed in this thesis as the ‘Condition’ profile. On the other hand, the time-dependant performance of the considered structure in
terms of its load carrying capacity which is related to the ULS was termed here in as the ‘Safety’ profile.

The RF-based reliability model developed in Chapter 6 can predict both the Condition and Safety profiles of a given structure under a given environmental condition and hence allows for predicting the optimal time to first maintenance/repair based on the agreed target limits. A corroding RC beam girder were used as an illustrative example in which existing chloride-induced corrosion material deterioration models identified in Chapter 2 were employed in conjunction with the RF theory discussed in Chapter 3. The advantage of the developed model; it incorporates material, loading, geometrical and environmental uncertainties in one probabilistic model that is capable of identifying the most important deterioration parameters for further investigation.

The model was first developed to predict the extent of damage that the concrete surface area of the beam girder is expected to endure due to corrosion-induced cracking (Condition profile, $A_s(t)$). The model was further developed to predict the time-dependant reliability of the deteriorating beam girder under traffic loading (Safety profile, $\beta(t)$). This is the first time when such an attempt have been made to predict both, the Condition profile and the Safety profile using the same deterioration models/parameters in which both the initiation stage and propagation stage of corrosion has been dealt with collectively. In this way, objectives 1, 2 and 3 indicated in Section 8.1 were considered. Regarding Sub-objective 3, the model considered the variability of the pitting corrosion by treating the pitting factor $R$ as an RF variable follows an extreme value distribution. Two cases of correlation between pits depths were considered, fully correlated pits depths and totally independent pits depths. The analysis carried out in this thesis indicated that the time-dependant loss of cross-sectional area due to pitting corrosion was not affected by the type of correlation between pits depths.

In the context of spatial variability modelling in the field of RC corrosion, the work of this thesis considered for the first time the use of site-specific Weigh-in-Motion traffic data when determining the maximum load effects (i.e. design flexure, shear etc) to be used for the calculation of the Safety profile. Using a realistic traffic loading ensures that the uncertainty associated with the maximum load affects is being accounted for while avoiding the use of conservative deterministic load models which may lead to unnecessary replacement or repair of the investigated structure. This is directly related to achieving
Sub-objective 4 indicated in Section 8.1. The Eurocode (EN1990-2002) requires that that the minimum acceptable safety level be assessed for 50 years reference period, considering that the beam under investigation was 25 years old, the maximum load effect was estimated for the remaining 25 years only.

The RF-based probabilistic model developed in this thesis was benchmarked using results published in the literature. It was found that the model produced very similar results to those published in the literature in terms of the condition profile with a margin error that did not exceed 13% for the concrete quality similar to that used for the structure example of this thesis.

8.7 TIME TO FIRST REPAIR/MAINTENANCE

In Chapter 7 of this thesis, it was shown how in previous published works, researchers used the Mean to describe the central tendency of expected damage proportion (A_x %); in this chapter, however, it was shown that the Median is more reliable central tendency measurement tool. This chapter has also demonstrated how considering spatial variability should be expected to influence the lifetime reliability (the Safety profile) of corroding RC structures.

One of the prime objectives of the probabilistic-based model developed in Chapter 6 was to facilitate a rational comparison between the predicted times to repair/maintenance intervention of a structure determined based on Condition and Safety profiles (i.e. SLS vs. ULS). In this context, results presented in Chapter 7 indicated that if only the general corrosion is considered, the decision of time to repair/maintenance intervention can be governed by the safety criteria rather than by the commonly practiced and traditionally used surface condition criteria. For example, for the beam case considered in this thesis, that the optimal time repair/maintenance, which was predicted in terms of the beam condition profile, were found to be much larger than that predicted in terms of the Safety profile. However, although this cannot be generalised to all structural conditions, the importance of the relative comparison between ULS and SLS performance indicators when determining the optimal time to repair/maintenance has been demonstrated. It was concluded that if the performance criteria to be considered for deciding on the time to first repair/maintenance do not take into account pitting corrosion (i.e. condition profile) the predicted time to first repair/maintenance intervention maybe too permissive. These
findings strongly points to the necessity of having a bridge management system tool that considers lifetime safety of the structure as a viable indicator for maintenance and repair interventions.

The influence of the Scale of Fluctuation ($\theta$) on the predicted lifetime performance of the structure was also investigated. It was found that as $\theta$ increases, the time to repair/maintenance predicted in terms of SLS also increases. Therefore, it was concluded that studies that have not calculated values of $\theta$ based on a site-specific data, their prediction of the time to repair/maintenance may result in a considerable error.

In terms of the safety performance, spatial variability was shown to be only important if the beam reliability was governed by the flexure Limit State (LS). In this case, the influence of spatial variability on the reliability of the beam was more evident in the case pitting corrosion than in general corrosion. For the case when the reliability of the structure is to be governed by the shear LS, results showed that spatial variability has no influence. This was explained by demonstrating that the end support RF element has governed the LS 47% of the times as oppose to 25% by the mid-span element in the case of flexure.

The reduction of the beam reliability over time was shown to be governed by shear for both spatial and non spatial analysis. This was attributed to the fact that the shear links are often placed at relatively smaller cover depths, $C_d$, than the main flexure reinforcements. The shear links are, therefore, expected to have a shorter $T_i$ period and a higher value of $i_{corr}$. This indicates that, shear links could have suffered larger cross-sectional/resistance losses than the flexure reinforcement at the same point in time.

For the two types of corrosion studied in this thesis, general and pitting corrosion, results showed that pitting corrosion potentially has far more aggressive effect on the reliability of corroding structures than general corrosion. The results also suggested that pitting corrosion affects shear resistance far more severely than it would affect flexure resistance. For example after 50 years of service, pitting corrosion has caused the shear resistance to reduce by 55% more than that caused by general corrosion. In the case of flexure, no difference between the reductions caused by pitting and general corrosion could be observed. The literature reported that many of today’s deteriorating RC bridge structures, including the bridge taken as an example in this thesis, deteriorate due to corrosion caused by chloride-contaminated water leaking through the deck joints. This means that an intense
form of deterioration can take place in locations where high shear stresses are expected (e.g. beam girders at the supports). Therefore, pitting corrosion at locations of high shear stresses can have a severe impact on the reliability/safety of structures. This had led to conclude that the assessment of the safety of RC beams in corrosive marine environments should consider the effect of pitting corrosion of shear links on shear resistance of the beam, otherwise reliability of the beam may be considerably overestimated.

Results presented in this thesis has also indicated that if only the general corrosion was considered, the decision of the time to repair/maintenance intervention can be governed by the safety criteria of the structure rather than by the visual condition criteria. Furthermore, if the performance criteria to be considered for the time to first repair/maintenance decision do not take into account pitting corrosion, i.e. condition profile, the predicted time to maintenance intervention maybe too permissive. These findings strongly points to the necessity of having a bridge management system tool that consider lifetime safety of the structure as a viable indicator for maintenance and repair interventions.

Finally, although the methodology presented in this thesis was applied to RC beams affected by chloride-induced corrosion, the concept of RF-based reliability modelling presented herein may be generalised to problems with alternate materials and deterioration mechanism (e.g. timber, steel, masonry).

8.8 RECOMMENDATIONS FOR FUTURE WORKS

The research carried out in this thesis has provided important insight into number of issues related to the prediction of the service life of RC bridges affected by chloride-induced corrosion. However, the author has highlighted the following areas for future research and further improvement of the work carried out in this thesis:

- If chance presents itself to a researcher who needs to carry out an experimental study similar to that carried out in this thesis, it is highly recommended that procedure for sampling with regard to relative location of samples should follow that proposed in Chapter 5 of this thesis. This will ensure that a distinctive semivariogram plot can be obtained and hence an accurate prediction of the parameter θ can be achieved.
- Based on the results presented in this thesis with respect to the sensitivity of the safety profiles to the mean value of $C_d$, this thesis recommends that the θ for this
parameter should be estimated accurately from a field data if the uncertainty associated with this important deterioration parameter is to be fully considered.

- MC simulation technique proved to be very efficient in the case of predicting the condition profile of a given deteriorating structure. However, for the safety profile calculation MC technique was shown to be a time consuming. The possibility of considering spatial variability in a reliability analysis using alternative reliability calculation methods needs to be investigated. One possibility is the use of Stochastic Finite Element method introduced by Haldar and Mahadevan (2000b). Another possibility is the use advanced of MC simulation techniques where the number if iterations can be significantly reduced by techniques such Importance Sampling.

- Pitting corrosion showed to cause severe reduction in the reinforcement cross-sectional area in the case of shear links and hence affects the reliability of the structure to decrease dramatically after corrosion has initiated. Modelling pitting corrosion, therefore, requires further investigation. The author suggests that the research carried out in this thesis should be continued with more focus on the modelling of pitting corrosion.
REFERENCES


4. ACI 318. (1999). Building code requirements for reinforced concrete. American Concrete Institute, Farmington Hills, Michigan, USA.


Appendixes
APPENDIX A: DETERMINATION OF CHLORIDE CONTENT BY POTENTIOMETRIC TITRATION

A.1 Procedure for the Extraction of Acid Soluble Chloride

1. Weight into a stoppered 500mL conical flask (2.5 gm) of the concrete sample.
2. Disperse with (20mL) of distilled water and add 5mL of concentrated nitrate acid (69%)  
3. Add (25mL) of hot distilled water, heat the conical flask on a hot plate at medium heat to boiling, then boil for 3 minutes  
4. Remove the conical flask from the hot plate and allow it cool sufficiently to handle.  
5. Cool to room temperature and filter into 250 Flask Buchner using chloride-free filter* paper (make sure you wet the filter paper with distilled water beforehand). the Flask Buchner is connected to a small vacuum bump for vacuum filtration to achieve maximum chloride extraction.  
6. when filtering is complete, rinse what is left of the sample minimum of 3 times and empty it into the filter, wash the filter paper and the inside walls of the Funnel Buchner with distilled water and wait until all has been filtered into the Flask Buchner, then empty the solution into a 125 ml beaker glass and wash the Flask Buchner internal walls into the beaker  
7. make sure u fill the beaker up to approximately 75-100 ml to ensure that the silver electrode is adequately covered by the solution (you can have enough quantity by keeping washing up the sample, filter paper and the flask Buckner)  
8. if you are expecting low chloride concentration you can use 0.02 or 0.01 mol/L SILVIR NITRATE(AgNO₃) to titrate against which will be pumped automatically from the burette to the dispensers in the auto-titration system.  
9. Calculation of the chloride content Cl% as a percentage of the concrete to the nearest 0.001% will be carried out automatically by the auto-titration device according to the expression described in the following section.

* Hardened Ashless chloride-free filter paper is suitable for acid based titration.
A.2 Determination of Chloride Content

Chloride content is determined by potentiometric titration using a chloride or silver selective electrode. The silver nitrate solution of 0.01 mol/l is used as titrant. The titration is carried out with an increment of titrant volume 0.1 ml until the significant equivalent point is reached.

The chloride content in a sample is calculated by the following pre-stored equation:

\[
(\text{Cl}\ %) = \frac{EPI \times C30 \times C01 \times 100}{C00}
\]

Equation a.1

Where;

- Cl % → chloride concentration that needs to be determined (% per mass of concrete sample)
- EPI → volume of AgNO₃ will be consumed for titration (mL)
- C30 → Molarity of AgNO₃ (mol/L) e.g. 0.01, .02 or 0.05
- C01 → Molecular weight of chloride (35.453 g/mol)
- C00 → mass of concrete sample in mg (e.g. 2500 mg)

A.3 Chloride content results

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### Table A.2 Chloride contents from core P2-N

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### Table A.3 Chloride contents from core P2-S

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<td>0.0145</td>
</tr>
<tr>
<td>61</td>
<td>0.041</td>
</tr>
</tbody>
</table>

### Table A.7 Chloride contents from core P4-S

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>Cl% per mass of concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P4-S1</td>
</tr>
<tr>
<td>9</td>
<td>0.041</td>
</tr>
<tr>
<td>16</td>
<td>0.033</td>
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<tr>
<td>24</td>
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<tr>
<td>31</td>
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<td>0.0145</td>
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<tr>
<td>61</td>
<td>0.041</td>
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</table>
**Table A.8 Chloride contents from core P7-N**

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>P7-N1</th>
<th>P7-N2</th>
<th>P7-N3</th>
<th>P7-N4</th>
<th>P7-N5</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0.021</td>
<td>0.045</td>
<td>0.028</td>
<td>0.046</td>
<td>0.041</td>
</tr>
<tr>
<td>16</td>
<td>0.038</td>
<td>0.124</td>
<td>0.123</td>
<td>0.127</td>
<td>0.081</td>
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<tr>
<td>24</td>
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<td>0.145</td>
<td>0.102</td>
<td>0.114</td>
<td>0.107</td>
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<tr>
<td>31</td>
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<td>0.133</td>
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<td>0.047</td>
<td>0.019</td>
<td>0.034</td>
<td>0.023</td>
</tr>
<tr>
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<td>0.018</td>
<td>0.026</td>
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<td>0.018</td>
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<td>0.019</td>
<td>0.011</td>
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**Table A.9 Chloride contents from core P7-S**

<table>
<thead>
<tr>
<th>Depth (mm)</th>
<th>P7-S1</th>
<th>P7-S2</th>
<th>P7-S3</th>
<th>P7-S4</th>
<th>P7-S5</th>
</tr>
</thead>
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<td>9</td>
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<td>0.021</td>
<td>0.022</td>
<td>0.04</td>
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<td>0.067</td>
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<td>0.055</td>
<td>0.074</td>
<td>0.065</td>
<td>0.068</td>
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<td>0.043</td>
<td>0.047</td>
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<tr>
<td>54</td>
<td>0.014</td>
<td>0.017</td>
<td>0.017</td>
<td>0.014</td>
<td>0.017</td>
</tr>
<tr>
<td>61</td>
<td>0.013</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
</tr>
</tbody>
</table>
APPENDIX B: CORES LOCATIONS AND CHLORIDE CONTENTS

Cores locations with respect to each crosshead beam face as extracted from the Ferrycarrig bridge and chloride contents at these locations at varies depths from the concrete surface.

Figure B.1 (a) Chloride content of five cores at different depths obtained from P1-S crosshead beam face. (b) Cores locations within the beam.
Chloride content ( % per mass of concrete)

APPENDIX B

Figure B.2 (a) Chloride content of five cores at different depths obtained from P2-N crosshead beam face. (b) Cores locations within the beam.
Figure B.3 (a) Chloride content of five cores at different depths obtained from P2-S crosshead beam face. (b) Cores locations within the beam.
Figure B.4 (a) Chloride content of five cores at different depths obtained from P3-N crosshead beam face. (b) Cores locations within the beam.
Figure B.5 (a) Chloride content of five cores at different depths obtained from P3-S crosshead beam face. (b) Cores locations within the beam.
Figure B.6 (a) Chloride content of five cores at different depths obtained from P4-N crosshead beam face. (b) Cores locations within the beam.
APPENDIX B

Chloride content of five cores at different depths obtained from P4-S

Figure B.7 (a) Chloride content of five cores at different depths obtained from P4-S crosshead beam face. (b) Cores locations within the beam.
Chloride content of five cores at different depths obtained from P7-N

(a)

Chloride content (% per mass of concrete)

Distance from the crosshead end (m)

(b)

Figure B.8 (a) Chloride content of five cores at different depths obtained from P7-N crosshead beam face. (b) Cores locations within the beam.
Chloride content of five cores at different depths obtained from P7-S

Figure B.9 (a) Chloride content of five cores at different depths obtained from P3-N crosshead beam face. (b) Cores locations within the beam.

Table B.1 Distance of the Ferrycarrig crosshead beams soffit from the river High and Low water levels.

<table>
<thead>
<tr>
<th>Crosshead beam #</th>
<th>H.W.OST (m)</th>
<th>L.W.OST (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.09</td>
<td>3.92</td>
</tr>
<tr>
<td>2</td>
<td>1.77</td>
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<td>4</td>
<td>1.15</td>
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<tr>
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<td>7</td>
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</table>
Figure C.1 Chloride profile for cores P4-N1, P4-N2, P4-N3, P4-N4 and P4-N5
Figure C.2 Chloride profile for cores P+1, S+1, P+2, S+2 and S+3.
Figure C.3: Chloride profile for cores P3-N1, P3-N2, P3-N3, and P3-N5.
Figure C.4: Chloride profile for cores P3, S1, P3, S2, P3, S3, P3, S4 and P3, S5.
Figure C.5 Chloride profile for cores P7-N1, P7-N2, P7-N3, P7-N4 and P7-N5.

Cl % per mass of concrete

Depth (mm)

C = 0.383 % D = 8.17 mm/year R² = 0.998

C = 0.285 % D = 14.15 mm/year R² = 0.996

C = 0.326 % D = 9.54 mm/year R² = 0.999

C = 0.0756 % D = 9.52 mm/year R² = 0.994

Appendix C.
Figure C.6. Chloride profile for cores P7-S1, P7-S2, P7-S3, P7-S4 and P7-S5.
Figure C.7 Chloride profile for cores P2-N1, P2-N2, P2-N3, P2-N4 and P2-N5
Figure C.8: Chloride profile for cores P2-S1, P2-S2, P2-S3, P2-S4 and P2-S5.
Figure C.9 Chloride profile for cores PI-S1, PI-S2, PI-S3, PI-S4 and PI-S5

Appendix C
APPENDIX D: THE INDIVIDUAL SEMIVARIOGRAMS

Semivariograms estimated for $C_s$ and $D_{app}$ parameters from data collected from each crosshead beam face (the individual semivariograms).

Figure D.1  Semivariograms constructed using data collected from crosshead beam P1-S for (a) $C_s$ and (b) $D_{app}$.

Figure D.2  Semivariograms constructed using data collected from crosshead beam P2-S for (a) $C_s$ and (b) $D_{app}$.
Figure D.3 Semivariograms constructed using data collected from crosshead beam P3-S for (a) $C_s$ and (b) $D_{app}$.

Figure D.4 Semivariograms constructed using data collected from crosshead beam P3-N for (a) $C_s$ and (b) $D_{app}$.
Figure D.5 Semivariograms constructed using data collected from crosshead beam P4-S for (a) $C_s$ and (b) $D_{app}$.

Figure D.6 Semivariograms constructed using data collected from crosshead beam P4-N for (a) $C_s$ and (b) $D_{app}$. 
Figure D.7 Semivariograms constructed using data collected from crosshead beam P7-S for (a) $C_s$ and (b) $D_{app}$.

Figure D.8 Semivariograms constructed using data collected from crosshead beam P7-N for (a) $C_s$ and (b) $D_{app}$.
APPENDIX E: PUBLICATIONS


*See the following pages
Assessing chloride induced deterioration in condition and safety of concrete structures in marine environments

Omran Kenshel — Alan O'Connor

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Trinity College Dublin, Dublin 2, Ireland
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ABSTRACT. Prediction of the present and future state of Reinforced Concrete (RC) structures suffering from chloride-induced corrosion is important if proper planning for inspection and maintenance is to be made. The majority of research studies have thus far focused on the diffusion process of chloride ions through the concrete cover, the time to corrosion initiation and on the prediction of the surface condition of the structure. However, practical evidence and theoretical analysis suggests that many structures can tolerate considerable corrosion damage without serious reduction to their load carrying capacity. Therefore, visual impression-based maintenance is not an optimum solution particularly when financial resources are limited. To support this notion, accurate models are needed to predict the deterioration rate of the structural load carrying capacity over time. This paper uses existing empirical RC deterioration models to predict the loss in the load carrying capacity of a typical RC T-beam using a reliability based approach. The approach takes into consideration the spatial variability of the deterioration parameters, thereby demonstrating the importance of its inclusion in any such analysis.

RéSUMÉ. L'évaluation de l'état actuel et la prédiction de l'état futur des structures en béton armé dégradées du fait de la corrosion provoquée par les chlorures sont importantes si l'on souhaite planifier correctement l'inspection et la maintenance. La majorité des études ont focalisé jusqu'à présent sur les processus de diffusion des ions chlorures dans la couche d'endrobage, le temps d'amorçage et la prédiction des conditions de surface. Cependant, le constat pratique et les analyses théoriques suggèrent que de nombreuses structures peuvent tolérer un niveau considérable d'endommagement par la corrosion sans réduction sérieuse de leur capacité portante. Dès lors, la maintenance reposant sur l'inspection visuelle n'est pas une solution optimale quand les ressources financières sont limitées. Optimiser la maintenance requiert que l'on dispose de modèles corrects pour prédire le taux de perte de la capacité structurale au cours du temps. Cet article utilise des modèles empiriques de détérioration du béton armé pour prédire la perte de capacité structurale d'une poutre en T en utilisant une approche fiabiliste. L'approche considère la variabilité spatiale des paramètres de détérioration et démontre l'importance de sa prise compte dans une telle analyse.

KEYWORDS: corrosion, deterioration, monte carlo simulation, reliability, spatial variability.

MOTS-CLEFS: corrosion, détérioration, fiabilité, simulation de Monte-Carlo simulation, variabilité spatiale.

DOI: 10.3166/EJECE.13.593-613 © 2009 Lavoisier, Paris
1. Introduction

The number of bridges deteriorating due to chloride-induced corrosion increases annually as does the cost of inspection, maintenance, repair and where necessary replacement (Costa and Appleton, 2002; Frangopol et al., 2001; Stewart and Rosowsky, 1998). Meanwhile, budgets made available to the majority of bridge owners/managers for maintenance of these bridges are reducing year on year (Lauridsen et al., 2007). To optimise and manage budget spend, bridge owners/managers need to rely more on rational decision making methods rather than on subjective engineering judgment. In traditional Bridge Management Systems (BMS), decisions regarding the time/type of maintenance intervention are often made in light of information obtained through visual inspection. Although many of these systems may claim to be capable of optimising and selecting the most cost-effective options for given budgets over the planning time frame through their Life-Cycle Cost Analysis approach, they have serious limitations. One of the most severe of which, as outlined by Frangopol et al. (2001), is that the bridge reliability (i.e. a measure of the probability of failure associated with a specified limit state), is not directly incorporated into the BMS and as such the ability of the BMS to predict future deterioration of the structure safety and serviceability conditions is very limited. In an attempt to overcome these limitations, it has been recognised that future BMS’s will have to depend more on probabilistic and reliability-based methods. Reliability-Based BMS (RBBMS) permit inclusion of the uncertainty associated with parameters of significance into analysis models which attempt to represent temporally the deterioration process. In addition, RBBMS have the advantage of using the reliability index “β” as a relative safety indicator, thereby providing (i) a more robust and better defined safety performance evaluation criteria and (ii) detailed information concerning the sensitivity of the result, i.e. β, to modelled parameters thereby facilitating “smart” condition assessment and, where necessary, optimised repair (O’Connor and Enevoldsen, 2008).

A highlighted shortcoming in the work done to date in this regard, (e.g. Akgul and Frangopol, 2005a; Akgul and Frangopol, 2005b; Enright and Frangopol, 1998; Val and Melchers, 1997) is the neglect of the spatial variability of the deterioration parameters. Research on the significance of spatial variability to date has been mainly focused on the prediction of the visual condition of the corroded structure over time (e.g. Li et al., 2004; Vu and Stewart, 2005). Due to the common parameters considered at limit state evaluation (i.e. material and environmental properties, etc.) positive correlations are expected between the times of damage initiation of the visual condition of the structure and its safety. Therefore; it would be more appropriate if the lifetime visual condition and safety performances were both predicted using the same approach and where possible the same physical/empirical deterioration models and parameters. The main objective of this paper is to investigate the possibility of such integration using a bridge structure as a demonstrative example. For the purpose of analysis the lifetime visual condition performance is taken to be represented by the proportion of concrete surface area
that is showing signs of distress. This distress can be defined as the level of chloride contamination, limiting crack size or spalling etc. Theoretically, spatial variability based modelling has been shown to be a suitable tool to determine this proportion if models to compute times for different deterioration stages are available. Results for the level of deterioration are obtained in the form of a time dependent probability distribution function (PDF), i.e. the parameters of which change as a function time. The concept and theory of spatial variability based modelling (SVM) of lifetime visual condition has been covered elsewhere and will not be covered in this paper (e.g. Li et al., 2004; Vu and Stewart, 2005). It is demonstrated herein that SVM can be equally applied to determine the lifetime safety performance of the structure represented by the reliability index $\beta$ where models for relating the time-dependant loss of reinforcement cross-sectional area to the level of corrosion are available.

2. Deterioration modelling

Three phases of deterioration are considered in this paper; corrosion initiation, crack initiation and crack propagation, Figure 1. The material deterioration models employed to simulate these phases are briefly discussed in the following sections, for further detail on the models and their derivation the reader is encouraged to consult the literature as referenced.

Figure 1. Schematic representation of deterioration process of RC structure
2.1. **Time to corrosion initiation, \( T_i \)**

The corrosion initiation period is assumed to refer to the time during which passivation of steel reinforcement is lost and corrosion begins its activity. It is assumed that for chloride-induced corrosion, depassivation takes place as soon as a critical threshold value of chloride concentration at the level of reinforcement is reached. The time to corrosion initiation is commonly derived from Fick's 2nd law of diffusion and is expressed as follows:

\[
T_i = \frac{C_i^2}{4 D_{app}} \left[ \text{erf}^{-1} \left( \frac{C_c - C_{cr}}{C_c} \right) \right]^2
\]

where \( T_i \) is time to corrosion initiation (years); \( D_{app} \) is the apparent diffusion coefficient (mm²/year); \( C_i \) is the surface chloride concentration (% per weight of cement or concrete), \( C_{cr} \) is the critical chloride concentration (% per weight of cement or concrete) and \( C_c \) is the concrete cover (mm). \( D_{app} \) and \( C_i \) are often determined by fitting data of chloride concentration obtained from chemical analysis of concrete dust samples taken across the depth of the structure to Tick's 2nd law of diffusion. In this paper \( D_{app} \) was estimated from a formula proposed by (Papadakis et al., 1996):

\[
D_{app} (\text{m}^2/\text{sec}) = 0.15 \left(1 + \frac{\rho_c wc}{\rho_c wc + ac} \left[ \frac{\rho_c wc - 0.85}{1 + \rho_c wc} \right] \right)^3 D_{H_2O} \]

where \( \rho_c \) (=3.16) and \( \rho_a \) (=2.6) are the specific gravity of cement and aggregates, respectively, and \( wc, ac (=2) \) are the water/cement ratio and the aggregate/cement ratio, respectively, and \( D_{H_2O} \) is the diffusion coefficient of chloride in an infinite solution at 25°C (=1.6x10⁻⁹ m²/sec). It has to be mentioned that Equation [2] is applicable to a fully hydrated OPC (Ordinary Portland Cement) concrete and curing temperatures from 20 to 25°C. Due to the well known positive correlation between \( wc \) ratio and the concrete compressive strength, in this paper the \( wc \) ratio has been estimated from the concrete compressive strength through a formula which have been reported by Vu and Stewart (2000): \( wc=27/[f_{cyt}+13.5])\).

2.2. **Time from corrosion initiation to corrosion cracking, \( T_{1st} \)**

The time to first cracking, termed \( T_{1st} \), refers to the period of time it takes from corrosion initiation to the stage where the corrosion products build-up starts to cause hair-size cracking at the concrete surface. Many authors use the model developed by Liu and Weyers (1998) to describe this stage of the deterioration process, however, it has been recently demonstrated by Chernin and Val (2008) that this model is incorrect. As such the Liu and Weyers (1998) model should not be used to represent
this critical stage of the process. The model proposed by El Maaddawy and Soudki (2007) is employed in this paper:

$$T_{i_{cr}} = \left[ \frac{7117.5(D + 2\delta_c)(1 + \nu + \psi)}{i_{corr} E_{ef}} \right] \left[ \frac{2C_f}{D} \left( 1 + \nu + \psi \right) \left( D + 2\delta_c \right) \right]$$  \[3\]

where $T_{i_{cr}}$ (years) models the time from corrosion initiation to corrosion first cracking (defined by the appearance of hair-size crack on the concrete surface), $D$ is the diameter of the steel reinforcing bar, $C$ is the clear concrete cover (mm), $\delta_c$ is the thickness ($=12.5 \times 10^{-3}$ mm) of the porous zone around the steel bar which will have to be filled first before stresses between the steel bar and concrete interface due to rust expansion can be generated, $\psi$ is a factor dependent on $D$, $C$ and $\delta_c$, $i_{corr}$ is the corrosion rate density ($\mu A/cm^2$) which depends on many factors such as the availability of oxygen, moisture and often obtained from field measurements, $E_{ef}$ is the effective elastic modulus of the concrete which is equal to $[E_c/\nu_\epsilon C]$, $E_c$ is the elastic modulus of the concrete (N/mm$^2$), $\nu_\epsilon$ (=2.0) is the creep coefficient, $\nu$ (=0.18) is the Poisson's ratio for the concrete and $f_{ct}$ is the tensile strength of the concrete (N/mm$^2$). For more information on the model see El Maaddawy and Soudki 2007.

### 2.3. Time to crack propagation, $T_{cp}$

For a crack to propagate from a hair size (typically assumed of the order of 0.05mm) to a maximum size of $w_{lim}$, Vu and Stewart (2005) proposed the following empirical model as a function of the corrosion rate density, water/cement ratio ($w_c$) and reinforcement cover ($C$) in mm:

$$T_{cp} = 0.0167 i_{corr}^{-1.1} \left[ 42.9 \left( \frac{w_c}{C} \right)^{-0.54} + \left( \frac{w_{lim} - 0.3}{0.0062} \right)^{1.5} \right]$$  \[4\]

where $T_{cp}$ is in (years). This model was validated for $w_{lim}$ ranging from 0.3mm to 1.0mm. The corrosion rate density used in this paper to evaluate both Equations [3] and [4], were estimated from an expression proposed by Vu and Stewart (2000). The proposed expression, Equation [5], gives the corrosion rate value at any time $t$ in $\mu A/cm^2$ as a function of concrete water/cement ratio ($w_c$) and the reinforcement cover ($C$, mm). For more details on the derivation of the model and its limitations readers are encouraged to consult the literature.

$$i_{corr}(t) = \frac{32.13 (1 - w_c)^{-1.64}}{C} \left( t - T_{i_{cr}} \right)^{-0.3}$$  \[5\]
3. Reduction in reinforcement area

As the corrosion process progresses, the cross sectional area of the reinforcement of an RC member will be reduced leading to reduction of the capacity of individual elements and by implication of the structure as a whole. If the corrosion is assumed to be of a uniform type, as illustrated in Figure 2, the loss of reinforcement diameter can be described by the use of Faraday’s law of electrochemical equivalence (Andrade et al., 1993).

![Figure 2. General corrosion](image)

The law indicates that a constant corrosion rate of 1μA/cm² corresponds to a uniform loss of bar diameter of 0.0232mm per year. If the corrosion rate is assumed to be constant over time, then the remaining cross-sectional area of corroding main reinforcement after t-years $A_r(t)$ can thus be estimated as:

$$A_r(t) = \sum_{i=1}^{n_b} \frac{\pi}{4} \left( D_o - \frac{\Delta D(t)}{2} \right)^2 \geq 0; \quad \Delta D(t) = 0.0232 i_{corr} (t - T_i)$$  \[6\]

where $T_i$ is the time to corrosion initiation (years), $n_b$ is the total number reinforcing bars and $D_o$ is the original bar diameter and $\Delta D(t)$ is the loss of bar diameter at time, $t$.

Pitting corrosion was also considered in the analysis performed in this paper. The methodology proposed by Val and Melchers (1997), illustrated in Figure 3, was employed to calculate the net cross sectional area of the corroding bar, whereby:

$$A_{bar}(t) = \begin{cases} \frac{\pi D^2}{4} - A_1 - A_2, & P(t) \leq D_o \sqrt{2} \\ \frac{D_o}{\sqrt{2}} < P(t) \leq D_o \\ 0 & P(t) > D_o \end{cases}$$  \[7\]
Assessing chloride induced deterioration

Figure 3. Pitting Corrosion (Val and Melchers, 1997)

with:

\[
A_i = \frac{1}{2} \left[ \alpha_i \left( \frac{D_o}{2} \right)^2 - a \left( \frac{D_o}{2} - \frac{P(t)}{D_o} \right)^2 \right]
\]

\[
A_j = \frac{1}{2} \left[ \alpha_j P(t)^2 - a \left( \frac{P(t)}{D_o} \right)^2 \right]
\]

\[
a = 2 P(t) \sqrt{1 - \left( \frac{P(t)}{D_o} \right)^2}
\]

\[
\alpha_i = 2 \arcsin \left( \frac{a}{D_o} \right)
\]

\[
\alpha_j = 2 \arcsin \left( \frac{a}{2 P(t)} \right)
\]

where \(P(t)\) is the depth of the corrosion-induced pit (mm) and is related to the uniform loss of the bar diameter due to the general corrosion (Figure 2) through the dimensionless ratio factor \(R\) which ranges from 4 to 8 (Stewart, 2004):

\[
P(t) = R \cdot \Delta D(t)
\]

4. Resistance modelling

Resistance models describing the ultimate moment, shear, torsion and axial capacities etc. of structural elements can be derived from first principles or adopted with reference to design/assessment codes applicable to the structure under consideration. For the purpose of this paper, models for moment and shear resistance
provided by the Bridge Design Specification (AASHTO-LRFD, 1994) were considered. Considering a rectangular non-prestressed member (where a T-section can be treated as a rectangular section as long as the neutral axis is verified to be within the flange region) for which the strength of the compression steel is neglected, the time variant nominal flexure resistance of an RC beam (assuming only the reinforcement cross section is changing with time due to the effect of corrosion) is given as follows:

\[
M_n(t) = A_y(t) f_y \left[ d_{ef} - \frac{A_y(t) f_y}{1.7 f' c b_f} \right]
\]

where \(A_y(t)\) is the area of non-prestressed tension reinforcement at time \(t\), \(f_y\) is the specified yield strength of reinforcing bars, \(d_{ef}\) is the effective beam depth, \(f' c\) is the specified 28 days compressive strength and \(b_f\) is the width of the compression face of the member.

Similarly, the time variant nominal shear resistance of the beam at any given section is calculated by combining the contributions of the concrete and shear links to the shear resistance of the section as follows:

\[
V_n(t) = \frac{1}{6} \sqrt{f' c} b d_{ef} + \frac{A_y(t) f'_c d_{ef}}{s}
\]

where \(b\) is the effective web width, \(A_y(t)\) is the area of shear reinforcement at time \(t\) and \(s\) is the shear stirrups spacing. The effect of bond loss is not considered in this analysis.

5. Spatial variability modeling

Early probabilistic analysis of RC deterioration typically modeled the material and geometrical uncertainty using a single random variable for the whole structure or component without considering the spatial variability of the modeled parameters. In this approach, material and geometrical properties are considered to be perfectly correlated within a structure or component. In reality, these properties exhibit some limited field correlation. That is to say, two samples taken very close to each other will have highly correlated properties and as the distance separating the two samples is increased, the level of correlation of their properties may be expected to decrease. At a certain distance there will no longer be any significant correlation between the properties of the two samples. This distance is known as the Correlation Length. To account for this field variation, the surface of the structure can be discretised into a number of smaller elements, Figure 4, following a Random Field (RF) approach described by Halder and Mahadevan (2000) and Vu and Stewart (2005).
To determine the correlation coefficient between any two neighbouring elements separated by distance \( r \), an auto correlation function is needed. The auto correlation function \( \rho(r) \) specifies the correlation coefficient between two elements separated by the distance \( r \), i.e. the autocorrelation function represents the spatial correlation between two elements. If the distance between the two elements is very small, the auto correlation function will be very close to unity where as the distance increases the autocorrelation function reduces. Standard forms of different proposed autocorrelation function are given elsewhere in the literature (i.e. Vanmarcke, 1983). Lack of available field data, makes it difficult to judge which model can best represent the fundamental nature of spatial variability of the parameters concerning reinforced concrete deterioration. However the square exponential autocorrelation function Equation [12] has been widely used in the literature to represent the spatial variability of material properties and loading (Vu & Stewart, 2005; Li et al., 2004) and as such it is adopted here.

In Equation [12] \( d_x \) and \( d_y \) are the correlation length for a two dimensional random field in \( x \) and \( y \) direction respectively; and \( r_x = x_i - x_j \), \( r_y = y_i - y_j \) are the distances between centroid of elements \( i \) and \( j \) in \( x \) and \( y \) direction respectively. If a one-dimensional RF is considered the \( Y \) component is neglected. A non-isotropic field may be introduced by simply using a different correlation length in each direction e.g. in the \( x \) and \( y \) directions.

\[
\rho(r) = \exp \left( - \left( \frac{r_x}{d_x} \right)^2 - \left( \frac{r_y}{d_y} \right)^2 \right) \quad [12]
\]
To demonstrate how the autocorrelation function can describe the correlation coefficients between elements that are separated by a distance $t$, the autocorrelation function given by Equation [12] were evaluated using a correlation length of $dx=dy=d=2.0 \text{m}$ and plotted in Figure 5. It is apparent from this figure that a high correlation ($i.e., >0.75$) exists if the distance between the elements is less than $(d/2) \text{ m}$.

![Square Exponential autocorrelation function](image)

**Figure 5. Plot of the Square Exponential autocorrelation function**

It is apparent that the degree of correlation between elements will be dependant upon two main parameters, (i) the correlation length and (ii) the element size. In order to obtain an appropriate correlation length for a RF parameter, available data sets consisting of sample measurements taken at frequent distances are needed. In practice, such measurements are rarely taken at frequent distances; therefore data on Correlation Lengths are scarce and usually assumed based on engineering judgment. Reported correlation length values for some of the RF parameters related to corrosion-induced deterioration points to a value of approximately 2.0m (Vu and Stewart, 2005). Regarding the RF element size, Englund (1997) has suggested that the size of elements should be in the range of one-fourth to one-half of the Correlation Length. However it is noted that if the element size is very large, this leads to an underestimation of the influence of random field variation (i.e. spatial variability), while too small an elements size leads to a larger number of RF elements, and hence an increase in the number of random variables resulting in a larger CPU computational time.

6. Structural reliability assessment

Once the structure/element has been discretised into an RF mesh, the desired Limit State (LS) functions need to be formulated and evaluated at the centroid of each RF element following the “Mid-Point” discretization method described by
Haldar and Mahadevan (2000). Three (time-variant) LS’s are considered in this paper; one SLS criterion concerning the structure surface area condition (e.g. proportion of concrete surface area that exhibits crack widths above a specified limiting value) and the two ULS criteria concerning the apparent safety of the structure (e.g. with respect to the computed flexural and shear capacities). These LS’s were chosen to be representative of performance criteria used by bridge owners/managers to define the end of service life of deteriorating structures and/or the time for repair and maintenance intervention, e.g. (i) the damaged structure surface area has reached a threshold value of \( A_°/° \) (SLS) or (ii) the computed probability of failure, \( p_\nu \), is greater than that specified/permitted by the applicable code for the chosen limit state, consequence and mode of failure. Table 1 presents acceptable probabilities of failure specified by the ISO standard (ISO 2394 1998).

**Table 1. Values for the allowable \( p_\nu \) based on the ISO standard (ISO 2394 1998)**

<table>
<thead>
<tr>
<th>Relative costs of safety measures</th>
<th>Consequence of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
</tr>
<tr>
<td>High</td>
<td>0.5</td>
</tr>
<tr>
<td>Moderate</td>
<td>10^-3</td>
</tr>
<tr>
<td>Low</td>
<td>10^-2</td>
</tr>
</tbody>
</table>

In many applications, the probability of failure is presented in terms of the safety index, \( \beta \), with:

\[
\beta = -\Phi^{-1}(p_\nu)
\]  

[13]

for which \( \Phi^{-1}(\cdot) \) is the inverse function of the standardised normal distribution. Table 2 presents acceptable \( \beta \) values as specified by the Eurocode (EN1990-2002). More informations on the reliability classes specified by the Eurocode are available in the cited literature.

**Table 2. Minimum acceptable safety levels specified by Eurocodes (EN1990-2002)**

<table>
<thead>
<tr>
<th>Reliability Class</th>
<th>Minimum acceptable ( \beta ) values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 year reference period (( =)associated ( p_\nu ))</td>
</tr>
<tr>
<td>CC3 (RC3)</td>
<td>5.2 (1.0x10^-5)</td>
</tr>
<tr>
<td>CC2 (RC2)</td>
<td>4.7 (1.3x10^-6)</td>
</tr>
<tr>
<td>CC1 (RC1)</td>
<td>4.2 (1.3x10^-5)</td>
</tr>
</tbody>
</table>
In performing a structural safety or reliability assessment the computed $\beta$ is compared to the target value, $\beta_{\text{target}}$, for the considered limit state, and consequence, to determine compliance or violation (O'Connor and Enevoldsen 2008).

In the following, the LS's formulated and evaluated at the centroid of each RF element are presented.

### 6.1. Serviceability limit state – concrete cracking

The proportion of the concrete surface area with crack widths exceeding $w_{\text{lim}}$ at time $t$ for a single MC simulation realisation is calculated as (Vu and Stewart, 2005):

$$A_s(t) = \frac{n[G_w(t) \leq 0]}{k} \times 100\%$$  \[14\]

where $G_w(t) = (T_f + T_{\text{st}} + T_{\text{cp}}) - t$, $T_f$, $T_{\text{st}}$, and $T_{\text{cp}}$ are given by Equations [1], [3] and [4] respectively. $n[.]$ represents the number of elements (from a total of $k$) that satisfy the inequality and $k$ is the total number of RF elements. It has to be noted that Equation [14] gives a single value (between 0 and 100%) for each single MC simulation realisation. When the total number of MC simulation realisation is performed (1 million realisations was performed for each year, assuming a service life of 100 years), a histogram of $A_s$ for each year $t$ is obtained as demonstrated in Section 7.1.

### 6.2. Ultimate limit state – flexure & shear

Exceedance of the ultimate limit states for flexure and shear, at time $t$, are defined as $G_f(t) \leq 0$ and $G_p(t) \leq 0$ respectively, where:
Assessing chloride induced deterioration

\[ G_M(t) = M_a(t) - M_b(t) \]  \[ G_V(t) = V_a(t) - V_b(t) \]

with \( M_a(t), V_a(t), M_b(t) \) and \( V_b(t) \) the moment and shear capacities and the induced moment and shear forces at time \( t \), at the centre of each RF element respectively. The probability of failure of each RF element with regard to each LS function can be calculated using Monte Carlo simulations. While the probability of failure of the beam at any year \( t \) (at ULS) of a series system comprising \( k \) RF elements can be calculated using Equations [17] and [18], the cumulative probability of failure of \( m \) independent events (considering each year as an independent event) can be calculated from Equation [19] (Stewart, 2004):

\[ G_i(X) = \min_{j=1,k} \min \{G_M, G_V\} \]  \[ P_f(t_i) = \text{Pr}\{G_i(X) < 0\} \]  \[ P_f(T) = 1 - \prod_{i=1}^{m} \left[1 - P_f(t_i)\right]; \quad \beta(t) = -\Phi^{-1} \left(P_f(T)\right) \]

where \( G_i(X) < 0 \) is the limit state function of the beam system at year \( t_i \), \( P_f(t_i) \) is the probability of failure of the beam system at year \( t_i \). Where the conditional probability that the beam will fail in \( t \) subsequent years given that it has survived \( T \) earlier years is determined as (Stewart 2004):

\[ P_f(t|T) = \frac{P_f(T+t) - P_f(T)}{1 - P_f(T)}; \quad \beta(t) = -\Phi^{-1} \left(P_f(t/T)\right) \]

with \( P(T+t) \) and \( P(T) \) calculated using Equation [19].

6.3. Sample analysis

By way of example to demonstrate the methodology outlined a simply supported RC T-beam girder, representing an internal component of a bridge superstructure, is considered. Three types of loads are considered, dead load including self weight \((g_d)\), lane live load \((q_l)\) and the truck live load \((W)\). The truck load is modelled by three vertical concentrated loads separated by 4.27m representing the HS20 design.
truck load as shown in Figure 6 (AASHTO-LRFD 1994). Statistical parameters and probability distribution of modelled random variables are provided in Table 4.

![Figure 6](image)

**Figure 6.** a) discretisation of the RC bridge beam and position of loads, and b) beam cross section

<table>
<thead>
<tr>
<th>Random Variable (units)</th>
<th>PDF</th>
<th>Mean &amp; (C.O.V.)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_s$ (% wt. cem.)</td>
<td>N/F</td>
<td>1.7(0.25)</td>
<td>Li (2004)</td>
</tr>
<tr>
<td>$C_{cr}$ (% wt. cem.)</td>
<td>N/F</td>
<td>0.5(0.2)</td>
<td>Li (2004)</td>
</tr>
<tr>
<td>$D_{ap}$ (Model Err)</td>
<td>N/F</td>
<td>1.0(0.2)</td>
<td>Vu (2000)</td>
</tr>
<tr>
<td>$C$ (mm)</td>
<td>N/F</td>
<td>50(0.18)</td>
<td>Vu (2000)</td>
</tr>
<tr>
<td>$f_{cd}$ (MPa)</td>
<td>LN/F</td>
<td>19(0.18)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$i_{sum}$ (Eq. Model Err)</td>
<td>N/F</td>
<td>1.0(0.2)</td>
<td>Vu (2000)</td>
</tr>
<tr>
<td>$d_{eff}$ (mm)</td>
<td>N/F</td>
<td>802(0.05)</td>
<td>Vu (2000)</td>
</tr>
<tr>
<td>$D_{sh}$ (mm)</td>
<td>LN/F</td>
<td>35.8(0.02)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$D_{sh}$ (mm)</td>
<td>LN/F</td>
<td>12.7(0.02)</td>
<td>Enright (1998)</td>
</tr>
<tr>
<td>$b$ (mm)</td>
<td>Fixed</td>
<td>400</td>
<td>-</td>
</tr>
<tr>
<td>$b_1$ (mm)</td>
<td>Fixed</td>
<td>2600</td>
<td>-</td>
</tr>
<tr>
<td>$h_t$ (mm)</td>
<td>Fixed</td>
<td>200</td>
<td>-</td>
</tr>
<tr>
<td>$W$ (kN)</td>
<td>N</td>
<td>287.5(0.4)</td>
<td>Stewart (1998)</td>
</tr>
<tr>
<td>$q_e$ (kN/m')</td>
<td>N</td>
<td>9.3(0.4)</td>
<td>-</td>
</tr>
<tr>
<td>$g_s$ (kN/m')</td>
<td>N</td>
<td>1.05(0.2)</td>
<td>-</td>
</tr>
<tr>
<td>$w_{fin}$ (mm)</td>
<td>Fixed</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>$R$</td>
<td>Gumbel</td>
<td>5.65(0.22)</td>
<td>Stewart (2004)</td>
</tr>
</tbody>
</table>

N=Normal; LN= Lognormal; F= Random Field
skewed particularly at the early stages of the service life which is particularly significant in lifetime performance modelling. It is postulated therefore that the mean value may not be an appropriate representation of the true average. The median value in this case seems to be a more realistic and accurate representation. The appropriateness of the use of the median is demonstrated in the case of full correlation (no spatial variability), i.e. where the beam is expected to behave as one homogenous element and as such $A_s$ is either 0% or 100%. If the average of the distribution was represented by the mean instead of the median, the expected 0% or 100% results of $A_s$ cannot be obtained numerically. Only when the median value is used, are results obtained for the main bars and links without SV as illustrated in Figure 8.

![Figure 8. Influence of concrete cover and spatial variability (SV) on proportion of damaged beam surface area](image)

For the example considered, considering SV, it is apparent from Figure 8 that after 25 years of ageing approximately 8% of the beam bottom surface area has demonstrated corrosion-induced cracks of a size greater than 1.0mm (if corrosion of the shear links is ignored and corrosion of main bars only is considered). When corrosion of the shear links is also considered, for the same time period, i.e. 25 years, the percentage of corrosion-induced cracks of a size greater than 1.0mm is more than doubled. However, most of the experimental studies carried out to predict the time to concrete cracking due to corrosion have used a bar diameter of 16 mm (Alonso et al., 1998; El Maaddawy and Soudki, 2007; Vu and Stewart, 2005), and corrections are required to account for the change in bar diameter. Assuming that 1.5% of concrete spalling, which is often used as a criterion for first repair and is taken to be equivalent to 20% of the concrete surface cracked, according to Figure 8 the first repair is required after approximately 35 years, considering the main reinforcement only. It is apparent from Figure 8 that depending on whether the
corrosion of the shear links or the main reinforcements is considered as the reason for spalling, the time to first repair will vary significantly.

7.2. Reinforcement cover

The influence of reinforcement cover on the extent of surface cracking can also be seen from Figure 8. It is apparent from the figure that because shear reinforcement is generally placed closer to the surface than the main reinforcement (i.e. less cover), the corrosion initiation time is shorter than that of the main reinforcements, hence, as might intuitively be expected, the limit for cracking damage caused by the corrosion of shear links has been reached in a shorter period of time when compared to that caused by the main longitudinal reinforcement. The significance of this result is apparent when the concentration of shear links at beam ends, i.e. traditional joint locations, is considered in the context of the durability issues now associated with leaking joints in older structures.

7.3. Safety profile results - ULS

The effect of general and pitting corrosion on moment and shear capacities are presented in Figure 9. It is apparent that pitting corrosion causes a significant reduction in the beam moment capacity after 50 years of the bridge construction. For the earlier years however, general corrosion seems to be only slightly more critical. In the case of shear capacity, pitting corrosion was demonstrated to be more critical than general corrosion right from the start of the corrosion activity, leading to a higher rate of strength deterioration. Figure 9 demonstrates that the reduction in shear capacity caused by pitting corrosion after 50 years of construction can be 15% higher than that caused by general uniform corrosion.

Assessment and design codes require that the actual reliability index should remain greater than a minimum acceptable value of e.g. $\beta_{\text{target}}=3.8$, Table 2 for CC2 (EN1990-2002; Duprat, 2007) at all stages during the life of the structure. Figure 10 presents the temporal nature of $\beta$ for the structure considered in this paper. For the purpose of comparison, results are presented for deterioration due to General and Pitting Corrosion where spatial variability was (SV) and was not (No SV) modelled. It is apparent from the figure that ignoring spatial variability leads to overestimation of the reliability index, $\beta$. For example the 50 years reliability index for the general corrosion case obtained by the 'no spatial variability' scenario is greater by 22% than the reliability index obtained from the spatial variability scenario. Assuming that the time to first maintenance will be based on the target reliability being reached (i.e. $\beta<\beta_{\text{target}}$), this means that the structure will be scheduled for maintenance about 12 years later than it would be were spatial variability included in the analysis. The significance here for structural safety management and budget planning is apparent.
Figure 9. Influence of general (G) and pitting (P) corrosion and on the temporal beam load carrying capacity

Figure 10. Influence of general, pitting corrosion and spatial variability (SV) on the beam reliability

Deterioration in the beam safety over time due to pitting corrosion does not seem to have been as greatly affected by the inclusion of spatial variability as can be seen from Figure 10. It worth mentioning though, that the spatial variability of the maximum penetration depth within RF elements has not been considered in the present analysis due to a lack of relevant data. Therefore, to draw a final conclusion on the effect of spatial variability on lifetime structural reliability due to pitting
corrosion, further research is required. Figure 10 also demonstrates that pitting corrosion has a profoundly negative impact on the lifetime reliability of the beam under investigation. For example, 50 years after construction, pitting corrosion will have caused a 43% decrease in the beam's reliability index value as compared to the general corrosion case with spatial variability and up to a 53% decrease in the computed reliability index in the case no spatial variability.

8. Conclusions

A comprehensive spatial/temporal model for condition and safety deterioration prediction of corroding RC structures has been developed using existing chloride-induced corrosion deterioration models. The model incorporates material, loading, geometrical and environmental uncertainties to identify the deterioration parameters most influencing the maintenance decision making process. Spatial variability, which has been neglected in many reliability based studies, has been shown to be of great importance when predicting the lifetime condition and safety profiles. The present study has mainly focused on integrating the lifetime condition performance of deteriorating structures with lifetime reliability performance. Results presented in this paper have shown that the decision for the time to first maintenance intervention can be robustly assessed by relative comparison of the computed structural reliability/safety to the criteria required by the structure owner/manager as specified in relevant codes of practice. The present analysis has shown that pitting corrosion has a significantly negative and more aggressive impact on the structural reliability index than general uniform corrosion in particularly with respect to shear resistance. This leads to the conclusion that pitting corrosion at locations of high stress, particularly shear stress due to the low cover to reinforcement and typically small bar diameter, can have a major impact on the predicted safety of structures/structural elements. Further research in this area is required. Finally, although the methodology presented in this paper was applied to chloride-induced corrosion of RC beam, the concept of SV presented herein may be generalised to problems with alternate materials and deterioration mechanism (i.e. timber, steel, etc.).

9. References


