Terms and Conditions of Use of Digitised Theses from Trinity College Library Dublin

Copyright statement

All material supplied by Trinity College Library is protected by copyright (under the Copyright and Related Rights Act, 2000 as amended) and other relevant Intellectual Property Rights. By accessing and using a Digitised Thesis from Trinity College Library you acknowledge that all Intellectual Property Rights in any Works supplied are the sole and exclusive property of the copyright and/or other IPR holder. Specific copyright holders may not be explicitly identified. Use of materials from other sources within a thesis should not be construed as a claim over them.

A non-exclusive, non-transferable licence is hereby granted to those using or reproducing, in whole or in part, the material for valid purposes, providing the copyright owners are acknowledged using the normal conventions. Where specific permission to use material is required, this is identified and such permission must be sought from the copyright holder or agency cited.

Liability statement

By using a Digitised Thesis, I accept that Trinity College Dublin bears no legal responsibility for the accuracy, legality or comprehensiveness of materials contained within the thesis, and that Trinity College Dublin accepts no liability for indirect, consequential, or incidental, damages or losses arising from use of the thesis for whatever reason. Information located in a thesis may be subject to specific use constraints, details of which may not be explicitly described. It is the responsibility of potential and actual users to be aware of such constraints and to abide by them. By making use of material from a digitised thesis, you accept these copyright and disclaimer provisions. Where it is brought to the attention of Trinity College Library that there may be a breach of copyright or other restraint, it is the policy to withdraw or take down access to a thesis while the issue is being resolved.

Access Agreement

By using a Digitised Thesis from Trinity College Library you are bound by the following Terms & Conditions. Please read them carefully.

I have read and I understand the following statement: All material supplied via a Digitised Thesis from Trinity College Library is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of a thesis is not permitted, except that material may be duplicated by you for your research use or for educational purposes in electronic or print form providing the copyright owners are acknowledged using the normal conventions. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone. This copy has been supplied on the understanding that it is copyright material and that no quotation from the thesis may be published without proper acknowledgement.
Dynamics of Wind Turbines including Soil-Structure Interaction

Michael Harte

Trinity College Dublin

A thesis submitted in fulfillment of the requirements for the degree of

Doctor of Philosophy

2012
Declaration

The author hereby declares that, except where reference has been given, this thesis is entirely the author's own work, and has not been submitted, in whole or part, to any other University as an exercise for a degree.

The author confirms that the library may, for academic purposes, lend or copy this thesis upon request.

Michael Harte
September 2012
Acknowledgements

This research was carried out in the Department of Civil, Structural and Environmental Engineering, Trinity College Dublin. The work was funded by the Irish Research Council for Science, Engineering and Technology.

Firstly, I would like to thank my supervisor Professor Biswajit Basu. His guidance, support, and expert advice has been invaluable throughout these last few years and I feel extremely lucky to have had the chance to work with him. His positive outlook and confidence in my research was a constant source of inspiration throughout my time at Trinity college.

I am grateful to Professor Søren Nielsen for his contribution to the work. I would like to thank all staff at Plaxis, especially Claire Heaney, Ronald Brinkgreve and Vahid Galavi for their help, it has been greatly appreciated. In addition, I would like to thank anyone who has helped me in any way towards this thesis.

I would like to thank my fellow apprentices to Professor Basu: Andrea Staino, Breiffni Fitzgerald and Omosola Fifo. Especially Breiffni who has always been there to discuss any issues with me, be they technical or political. Last but far from a “Last Resort” a big thanks to my fellow researcher Aidan Quilligan who has walked with me along this journey of self discovery and also for the Craic.

I would like to thank my long term house mate Cathal Dalton, we made it. A word of thanks also to Shane “Always be Closing” Mullarkey for the late night motivational talks. I would also like to thank all the post-grad gang and all the lads from the Trinity Hurling and Pool teams. They have all helped to make my post-grad years truly enjoyable and unforgettable.

Finally, my thanks go to my family: Billy, Norah, Marie, Pierce, Aidan, Anne, Maya and Nicole for all their support and encouragement throughout the years, especially my brother Aidan for his editorial help.
Summary

This thesis investigates the dynamic response of Horizontal Axis Wind Turbines (HAWT) including Soil-Structure Interaction (SSI) effects. Multi-Degree-of-Freedom (MDOF) HAWT models are proposed for dynamic analysis using an Euler-Lagrangian approach including foundation interaction. Two distinct models are developed: an out-of-plane model and a coupled in-plane out-of-plane model. The out-of-plane model comprises of a rotor blade system, a nacelle and a flexible tower connected to a foundation system using a substructuring approach. The rotor blade system consists of three rotating blades and includes the effect of centrifugal stiffening due to rotation. The coupled in-plane out-of-plane model is extended to include in-plane motion for the blade, tower and foundation and for the roll, tilt, yaw and drivetrain nacelle DOFs.

Aerodynamic loading is generated using the modified Blade Element Momentum (BEM) theory. Turbulence is simulated using both a Kaimal spectrum and a rotationally sampled spectrum. Wave loads are simulated in accordance with Airy linear wave theory, the JONSWAP spectrum and Wheeler Stretching theory.

The Finite Element Method (FEM) package Plaxis 3D dynamic is used to formulate impedance functions for several soil-foundation models. A procedure is developed to obtain the impedance functions for a surface and embedded footing. The generated impedance functions are compared to results from known analytical formulas and the wave propagation based Cone method. Following this, impedance functions for three HAWT foundations are given: an onshore Gravity based Foundation (GBF), an offshore Suction Caisson Foundation (SCF) and an offshore MonoPile Foundation (MPF).

Parametric studies are carried out using the developed out-of-plane HAWT model for an onshore and offshore turbine. The stiffness of the soil is al-
tered while holding all other soil, foundation, turbine and loading parameters constant. The soil-foundation system, resting on linear-elastic half-space, is modelled using both a static and frequency dependant Coupled-Spring (CS) model. For the turbine and soil-foundation system considered, SSI is found to have little impact on the tower’s relative displacement but a detrimental effect on the total displacements, especially for softer soils. The soil is observed to reduce the tower’s modal frequencies but have little effect on the blade’s modal frequencies. The static CS models are observed to give comparable results to the frequency dependant CS models for the soil, foundation and turbine configuration considered.

The dynamic response of the coupled in-plane out-of-plane offshore HAWT model, founded on the SCF and MPF embedded in non-linear soil, is analysed. Again, the tower’s modal frequencies are seen to reduce due to the effects of the flexible soil. SSI is observed to add some damping to the system and to make the tower more susceptible to wave excitation.

A FEM model of an offshore wind turbine, founded on a SCF embedded in non-linear soil is built using Plaxis to examine the effects of unaligned wind and wave loading on the dynamic response of the turbine tower, support structure and soil-foundation system. The displacement of the turbine in the side-to-side direction is seen to increase as the alignment of the wave load is changed with respect to the wind load.

The NREL disturbed FAST code is used to analyse an offshore HAWT coupled with soil-foundation models derived using Plaxis. The turbine’s control system is seen to have a marked impact on the shear force and bending moment at the mudline. The inclusion of the foundation is shown to lower the tower’s modal frequency and increase the bending moment at the mudline. The foundation interaction and loading conditions are observed to affect the fatigue life of the turbine support structure.
Contents

1 Introduction ......................................................... 1
  1.1 Wind Energy, Past and Present ............................... 1
    1.1.1 Wind energy beginnings ................................. 1
    1.1.2 Windmills early history ................................. 2
    1.1.3 Middle Ages ............................................... 2
    1.1.4 First wind turbines ....................................... 2
    1.1.5 Energy crises .............................................. 3
    1.1.6 Wind energy present ..................................... 4
  1.2 Research aims ................................................. 6
  1.3 Organization of the thesis ................................. 7

2 Literature review .................................................. 9
  2.1 Introduction ..................................................... 9
  2.2 Wind turbine general properties ......................... 9
    2.2.1 Introduction ............................................... 9
    2.2.2 Rotor .......................................................... 11
    2.2.3 Nacelle/Yaw system ..................................... 12
    2.2.4 Towers ......................................................... 13
  2.3 Description of the foundations .............................. 13
    2.3.1 Gravity based .............................................. 14
    2.3.2 Monopile ..................................................... 15
    2.3.3 Multi-pile ..................................................... 16
    2.3.4 Suction buckets .......................................... 16
    2.3.5 Jackets ......................................................... 18
## CONTENTS

2.3.6 Floating ................................................. 18

2.4 Properties of the foundations ......................... 19

2.4.1 Strength ..................................................... 19

2.4.2 Stiffness .................................................... 19

2.4.3 Stability ..................................................... 19

2.4.4 Durability .................................................... 20

2.4.5 Scour .......................................................... 20

2.5 Dynamics of wind turbines .............................. 20

2.5.1 General structural dynamic theory ................. 20

2.5.1.1 Equations of motion ................................. 21

2.5.1.2 Modal analysis ......................................... 24

2.5.1.3 Eigenvalue analysis ................................. 24

2.5.2 Fourier domain ........................................... 26

2.5.3 Design options for support structures .............. 26

2.5.4 Campbell diagram ....................................... 28

2.5.5 Power generation ......................................... 29

2.5.5.1 Introduction ............................................. 29

2.5.5.2 Betz limit ................................................. 30

2.5.5.3 Power capture ........................................... 32

2.5.5.4 Power curve ............................................. 33

2.5.6 Fatigue ......................................................... 35

2.5.6.1 S-N curve and Miner rule ......................... 35

2.5.7 Damping .................................................... 36

2.6 Wind ............................................................ 37

2.6.1 Introduction ................................................. 37

2.6.2 Wind shear ................................................... 37

2.6.3 Turbulence ................................................... 39

2.6.4 Rotational sampled turbulence ................. 40

2.7 Wave .......................................................... 41

2.7.1 Introduction .................................................. 41

2.7.2 Sea surface description .............................. 42

2.7.3 Linear wave theory ................................... 43
## CONTENTS

2.7.4 Wave loads on structures .......................................................... 44  
  2.7.4.1 Morison's equation .......................................................... 44  
  2.7.4.2 Diffraction ................................................................. 45  
2.7.5 Unaligned wind and wave loads ....................................................... 45  
2.8 Soil-structure interaction ................................................................. 47  
  2.8.1 Introduction ........................................................................... 47  
  2.8.2 SSI background ..................................................................... 47  
  2.8.3 SSI for wind turbines ......................................................... 51  
2.9 Soil-foundation system ................................................................. 52  
  2.9.1 Introduction ........................................................................... 52  
  2.9.2 Background ........................................................................... 53  
  2.9.3 DNV/Riso design standards .................................................. 54  
  2.9.4 Cone method ........................................................................ 56  
    2.9.4.1 One-dimensional strength of material approach ............ 56  
    2.9.4.2 Foundation embedded in layered half space .............. 57  
    2.9.4.3 Implementation ......................................................... 60  
  2.9.5 Plaxis finite element analysis .................................................. 60  
    2.9.5.1 Soil models ................................................................... 61  
    2.9.5.2 Drained and Undrained Behaviour ............................... 63  
3 Model formulation for a HAWT ......................................................... 65  
  3.1 Introduction .............................................................................. 65  
  3.2 MDOF HAWT model ................................................................. 65  
    3.2.1 HAWT model formulation ................................................ 66  
      3.2.1.1 Out-of-plane model .................................................. 68  
      3.2.1.2 Coupled in-plane out-of-plane model ..................... 72  
      3.2.1.3 Structural damping ............................................... 76  
  3.3 Wind excitation ........................................................................... 77  
    3.3.1 Blade wind loading .......................................................... 77  
    3.3.2 Tower wind loading ......................................................... 81  
    3.3.3 Turbulence ....................................................................... 82  
      3.3.3.1 Homogeneous turbulence ........................................ 82  
      3.3.3.2 Rotationally sampled turbulence ............................. 83  

ix
CONTENTS

3.4 Wave excitation .................................................. 84
3.5 Modelling of load by virtual work ...................... 87
3.6 Soil-foundation models ...................................... 91
   3.6.1 Apparent Fixity model ................................. 92
   3.6.2 Coupled-Spring model ............................... 93
      3.6.2.1 Uncoupled-Spring model ..................... 96
      3.6.2.2 Regressed CS model ......................... 96
   3.6.3 Static CS model damping ........................... 98
   3.6.4 FEM model damping ................................. 100
   3.6.5 Bending moment and Shear forces .............. 101

4 Finite Element analysis of the foundation using Plaxis 103
   4.1 Introduction ................................................. 103
   4.2 Impedance functions .................................... 103
      4.2.1 Surface footing ....................................... 105
         4.2.1.1 Plaxis soil model .............................. 105
         4.2.1.2 Static Stiffness ............................... 106
         4.2.1.3 Dynamic Stiffness ......................... 108
         4.2.1.4 Low Frequency Estimates ................. 111
         4.2.1.5 Surface footing on soil with stiffness linearly increasing with depth .......... 113
      4.2.2 Embedded Foundation ............................. 115
         4.2.2.1 Static Stiffness ............................... 116
         4.2.2.2 Dynamic stiffness ......................... 119
   4.3 Wind turbine foundations ............................... 124
      4.3.1 Impedance functions .............................. 125
      4.3.2 Discussion .......................................... 138
   4.4 Regressed CS models .................................... 138
      4.4.1 Soil model ......................................... 138
      4.4.2 Regression models ............................... 139
      4.4.3 Analysis ........................................... 142
   4.5 Conclusions ............................................... 144
CONTENTS

5 Analysis of wind turbines with foundation interaction 147

5.1 Introduction ..................................................... 147
5.2 Benchmarking the developed model ....................... 147
  5.2.1 Fixed base ................................................... 148
  5.2.2 Flexible foundation ........................................ 149
5.3 Onshore HAWT model ......................................... 150
  5.3.1 Model properties ........................................... 150
  5.3.2 Soil foundation model ...................................... 151
    5.3.2.1 Foundation design .................................... 152
    5.3.2.2 CS models .............................................. 156
  5.3.3 Numerical results .......................................... 159
    5.3.3.1 Transfer Functions .................................... 160
    5.3.3.2 Time history response ............................... 164
    5.3.3.3 Foundation movement ................................ 169
    5.3.3.4 Bending moment and shear forces ............... 169
  5.3.4 Discussion ................................................... 170
5.4 Offshore, out-of-plane HAWT model ......................... 173
  5.4.1 Model properties ........................................... 173
  5.4.2 Soil foundation model ...................................... 177
  5.4.3 Numerical results .......................................... 179
    5.4.3.1 Tower modal frequencies with foundation coupling 179
    5.4.3.2 Time history response ............................... 185
  5.4.4 Discussion ................................................... 189
5.5 Offshore, coupled in-plane out-of-plane HAWT model ...... 191
  5.5.1 Model properties ........................................... 191
  5.5.2 Soil-foundation model ..................................... 192
  5.5.3 Modal frequencies ........................................... 193
    5.5.3.1 Eigenvalue analysis ................................... 193
    5.5.3.2 Coleman transformation ............................. 193
    5.5.3.3 Campbell diagram ..................................... 195
    5.5.3.4 Modal frequencies considering the flexible foundation 197
  5.5.4 Time history response ...................................... 201
CONTENTS

5.5.5 Discussion ................................................................. 204

5.6 Plaxis based offshore HAWT model ........................................... 206
  5.6.1 Model properties ...................................................... 206
    5.6.1.1 Support structure and turbine tower .................... 206
    5.6.1.2 Wind and wave loading ................................. 206
    5.6.1.3 Mesh generation ........................................... 207
    5.6.1.4 Soil model .................................................. 208
  5.6.2 FEM model results ................................................... 209
  5.6.3 Discussion ............................................................... 211

5.7 Conclusions ................................................................. 211

6 FAST modelling and analysis ................................................. 215
  6.1 Introduction ............................................................... 215
  6.2 FAST ................................................................. 216
    6.2.1 Inflow Turbulence and Aerodynamic Loads ............. 217
    6.2.2 Hydrodynamic Loads ........................................... 217
    6.2.3 Control .......................................................... 218
  6.3 Model definition ........................................................ 218
    6.3.1 FAST inputs ....................................................... 219
    6.3.2 Soil ............................................................... 219
    6.3.3 Control systems ............................................... 220
  6.4 Foundation models ...................................................... 221
    6.4.1 CS model ........................................................ 222
      6.4.1.1 Iterative CS model ...................................... 224
      6.4.1.2 Damping .................................................. 226
    6.4.2 AF model ......................................................... 226
      6.4.2.1 Implementation ........................................ 227
  6.5 Tower mode shapes ..................................................... 228
  6.6 Numerical results ........................................................ 228
    6.6.1 Modal frequencies ............................................... 228
    6.6.2 Turbine Load Statistics ...................................... 233
    6.6.3 Fatigue .......................................................... 238
  6.7 Conclusion ................................................................. 241

xii
## CONTENTS

7 Conclusions ................................................................. 243

7.1 Introduction .......................................................... 243

7.2 Summary and Conclusions ....................................... 243

7.3 Outlook ...................................................................... 248

References .................................................................... 262

A Fourier Transforms ...................................................... 263

A.1 Introduction .......................................................... 263

A.2 Fundamentals ........................................................ 265

A.3 Nyquist criteria ....................................................... 266

A.4 Matlab Model ........................................................ 270

A.5 Dynamic analyses in the time/frequency domain ........ 273

B Integrated Structural/Plaxis model .............................. 281

B.1 Method ................................................................. 281

B.2 Validation case ........................................................ 282

B.2.1 Integrated Struct/Plaxis model .............................. 282

B.2.2 Complete Plaxis Model ........................................ 285

B.3 Discussion ............................................................. 286

C BEM algorithm ............................................................ 289

D NREL HAWT models ................................................ 291

D.1 NREL 1.5 MW onshore HAWT ................................. 291

D.2 NREL 5 MW offshore HAWT ................................. 294

E System matrices for the MDOF out-of-plane HAWT model 303

E.1 4 DOF HAWT model ............................................... 303

E.2 6 DOF HAWT model ............................................... 304

F System matrices for the MDOF 1.5 MW out-of-plane HAWT model 307

G System matrices for the MDOF 5 MW out-of-plane HAWT model 311
CONTENTS

H  System matrices for the MDOF 5 MW coupled in-plane out-of-plane HAWT model  317
I  Example CS model  331
J  Example FAST Input File (Plalfrom.dat) for the MPF CS model  335
K  Example FAST User-Defined Subroutine (UserPtfmLd) for the MPF CS model  337
L  Example FAST Input File (Plalfrom.dat) for the MPF AF model  341
M  Example FAST Input File (Tower.dat) for the MPF AF model  343
N  Example BMODES Input File for the MPF CS model  345
## List of Figures

1.1 Global Cumulative Installed Wind Capacity 1996-2011, sourced from GWEC (2010) ................................................................. 4
1.2 Increase in power capacity and rotor diameters of wind turbines from 1980-2010 ................................................................. 5

2.1 HWAT with main component identified ..................................................... 10
2.2 Aerofoil section showing the principle of lift ........................................... 11
2.3 Wind turbine development, sourced from NREL (2009) ...................... 15
2.4 Offshore HAWT founded on a suction caisson foundation and a monopile foundation ................................................................. 17
2.5 SDOF mass-spring-damper system .......................................................... 21
2.6 SDOF response: (a) Quasi-static (b) Resonant (c) Inertia dominated response ................................................................. 22
2.7 (a) DAF versus normalised frequency (b) Phase lag versus normalised frequency ................................................................. 23
2.8 Structural design frequency intervals for a three bladed constant rotational speed wind turbine ................................................................. 27
2.9 Structural design frequency intervals for a three bladed variable speed wind turbine ................................................................. 28
2.10 Sparse and dense Campbell diagrams for an offshore wind turbine ........ 29
2.11 Actuator disc with stream tube ............................................................. 30
2.12 Variation of efficiency with the ratio of downstream to upstream velocity ................................................................. 32
2.13 Power coefficient versus tip speed ratio ................................................ 33
LIST OF FIGURES

2.14 Power output as function of rotational speed for different wind speed classes .................................................. 34
2.15 Power curve .................................................................................................................................................. 34
2.16 Wind profile .................................................................................................................................................. 37
2.17 Wind shear according to log and power law model: (a) mean wind speed at 90 m of 11.4 m/s (b) mean wind speed at 10 m of 9.5 m/s ................................................................................................................... 38
2.18 Wind speed spectrum over a broad range of frequencies .............................................................................. 39
2.19 (a) Normalised time varying wind speed (b) Wind velocity PSD ........................................................................... 40
2.20 Rotationally sampled wind velocity spectrum, after Winkelarr (1992) .................................................. 41
2.21 (a) Single point time recording of sea surface elevation (b) Wave spectrum of sea surface elevation ................................................................. 43
2.22 Absolute value of the misalignment between wind and waves as function of wind speed (shown from 0-30 m/s) and wind speed probability (colour scale), sourced from Fischer (2010) ........................................................................... 46
2.23 Simple physical models to represent the unbounded soil, after Wolf & Song (2002) .................................................. 54
2.24 Shear modulus and damping ratio versus strain level, sourced from DNV/Risø (2001) ........................................................................... 55
2.25 Wave propagation in cones: a) Initial cone with outward wave propagation b) Reflected and refracted waves at a material discontinuity propagating in their own cones, after Wolf & Deeks (2004b) ........................................................................... 57
2.26 Stack of disks with redundants and free-field motion to represent embedded cylindrical foundation, after Wolf & Deeks (2004b) ........................................................................... 58
2.27 Enforcement of rigid-body displacement and excavation of trapped mass, after Wolf & Deeks (2004b) ........................................................................... 60
2.28 The Mohr-Coulomb field surface in principal stress space, sourced from Plaxis (2011) ................................. 62
2.29 Reduction of secant shear modulus with shear strain ................................................................................. 62
3.1 Sketch of model orientation and coordinate axes ......................................................................................... 67
3.2 HAWT model ................................................................................................................................................... 70
3.3 Schematics of the combined wind turbine model ......................................................................................... 74
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4</td>
<td>Blade model according to BEM theory approach, after Staino <em>et al.</em> (2012a)</td>
<td>79</td>
</tr>
<tr>
<td>3.5</td>
<td>Schematic of a blade section showing local forces and velocities according to the BEM model</td>
<td>81</td>
</tr>
<tr>
<td>3.6</td>
<td>Turbulent wind velocity time history, with mean removed</td>
<td>83</td>
</tr>
<tr>
<td>3.7</td>
<td>(a) Turbulence at blade section 35 (NREL 5MW blade) in flapwise direction for blade 1,2,3, (b) Spectrum of the rotational sampled turbulence</td>
<td>84</td>
</tr>
<tr>
<td>3.8</td>
<td>Gravity load acting on the blade</td>
<td>89</td>
</tr>
<tr>
<td>3.9</td>
<td>CS model</td>
<td>93</td>
</tr>
<tr>
<td>3.10</td>
<td>Applied force/moment to obtain uncoupled lateral/rotational springs</td>
<td>96</td>
</tr>
<tr>
<td>3.11</td>
<td>Underdamped SDOF system</td>
<td>99</td>
</tr>
<tr>
<td>3.12</td>
<td>Underdamped SDOF system at different damping ratios</td>
<td>100</td>
</tr>
<tr>
<td>3.13</td>
<td>Material damping formulation</td>
<td>101</td>
</tr>
<tr>
<td>4.1</td>
<td>DOF for a rigid surface footing: (a) displacements and rotations and (b) forces and moments</td>
<td>104</td>
</tr>
<tr>
<td>4.2</td>
<td>Screen shot from Plaxis of the soil-foundation model used to generate the static stiffness values</td>
<td>109</td>
</tr>
<tr>
<td>4.3</td>
<td>Vertical forcing function (1 KN) operating at a frequency of 2 Hz and the resulting vertical soil displacement response</td>
<td>110</td>
</tr>
<tr>
<td>4.4</td>
<td>Vertical dynamic stiffness of a surface footing, generated using Plaxis, Cone method and analytical formulas</td>
<td>111</td>
</tr>
<tr>
<td>4.5</td>
<td>Horizontal and rotational dynamic stiffness of a surface footing, generated using Plaxis, Cone method and analytical formulas</td>
<td>112</td>
</tr>
<tr>
<td>4.6</td>
<td>Plot from Plaxis 3D output of dynamic time versus soil vertical displacement response (directly under the center of the footing), generated by a 1 KN dynamic vertical force at an excitation frequency of 0.01 Hz</td>
<td>113</td>
</tr>
<tr>
<td>4.7</td>
<td>Vertical dynamic stiffness of a surface footing on soil with SLIWD, generated using Plaxis, Cone method and analytical formulas</td>
<td>114</td>
</tr>
<tr>
<td>4.8</td>
<td>Horizontal and rotational dynamic stiffness of a surface footing on soil with SLIWD, generated using Plaxis, Cone method and analytical formulas</td>
<td>115</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

4.9 Cylindrical embedded foundation ................................................. 116
4.10 Screen short from Plaxis showing the soil-foundation model loaded vertically for embedment ratio of $D_e/R_F = 2$ ........................................ 117
4.11 Horizontal translation and rotation of a monopile foundation under a horizontal force and applied moment ........................................ 118
4.12 Static stiffness terms for a cylindrical embedded foundation, at several embedment ratios ......................................................... 119
4.13 Normalised coupling static stiffness $K_{C}^{0}/G_sR_F^2$ for a cylindrical embedded foundation, at several embedment ratios .................. 120
4.14 Screen shot from Plaxis showing the soil-foundation model for the full and symmetric model ......................................................... 121
4.15 Vertical dynamic stiffness of a embedded footing ($D_e/R_F = 2$), generated using Plaxis (using a full and half model) and Cone method ..... 122
4.16 Dynamic stiffness of a embedded footing ($D_e/R_F = 2$), generated using Plaxis (using a full and half model) and Cone method .......... 123
4.17 GBF dynamic stiffness for $c_s = 40$ m/s ........................................ 126
4.18 GBF dynamic stiffness for $c_s = 100$ m/s .................................... 127
4.19 GBF dynamic stiffness for $c_s = 200$ m/s .................................... 128
4.20 GBF dynamic stiffness for $c_s = 500$ m/s .................................... 129
4.21 SCF dynamic stiffness for $G_s = 1$ MPa ..................................... 130
4.22 SCF dynamic stiffness for $G_s = 10$ MPa .................................. 131
4.23 SCF dynamic stiffness for $G_s = 100$ MPa ................................. 132
4.24 SCF dynamic stiffness for $G_s = 500$ MPa ................................. 133
4.25 MPF dynamic stiffness for $G_s = 1$ MPa .................................. 134
4.26 MPF dynamic stiffness for $G_s = 10$ MPa .................................. 135
4.27 MPF dynamic stiffness for $G_s = 100$ MPa ............................... 136
4.28 MPF dynamic stiffness for $G_s = 500$ MPa ............................... 137
4.29 Static stiffness terms calculated for two multi-variable regression models compared to static stiffness terms computed using an exact CS model ......................................................... 144
5.1 BEM generated wind load: (a) Blade modal load (b) Nacelle modal load 152
5.2 Soil profiles for the GBF .......................................................... 153
LIST OF FIGURES

5.3 Forces and soil properties for foundation design ........................................ 154
5.4 Spread sheet for design of the 1.5 MW turbine foundation ......................... 155
5.5 Screen shot of CONAN for Site B with a shear wave velocity of 100 m/s 158
5.6 Example of the input text file needed for CONAN, for Site B with a shear wave velocity of 100 m/s, with the name of each column superimposed at the bottom of the text file ............................................................. 159
5.7 Impedance functions, produced from CONAN implementing Cone method, for Site B with a shear wave velocity of 100 m/s ........................................ 160
5.8 Structural fundamental modal frequency versus shear wave velocity. Results generated using Cone method, Plaxis and DNV/Riso coefficient to represent the soil-foundation system, Site A ........................................ 161
5.9 Nacelle total and relative response ............................................................. 162
5.10 Transfer functions: (a) Nacelle relative displacement (b) Nacelle total displacement (c) Foundation horizontal translation (d) Foundation rotation .............................................................................................................. 163
5.11 Frequency range of 1P and 3P, with structural fundamental modal frequency range due to SSI ............................................................. 165
5.12 Blade response time-histories ...................................................................... 166
5.13 Response time-histories: (a) Nacelle total displacement response (b) Nacelle relative displacement response .......................................................... 167
5.14 Response time-histories: (a) Foundation horizontal translation response (b) Foundation rotation response .............................................................. 168
5.15 Close up of nacelle relative displacement response ...................................... 169
5.16 Rotation of the foundation versus shear wave velocity: (a) Site A (b) Site B and Site C .......................................................................................... 170
5.17 Transfer functions: (a) Foundation shear force (b) Foundation Bending moment (c) Tower base shear force (d) Tower base bending moment 171
5.18 Response time-histories: (a) Foundation shear force (b) Foundation Bending moment (c) Tower base shear force (d) Tower base bending moment ................................................................................................................................. 172
5.19 Wind load: (a) Blade modal load (b) Tower modal load ............................ 174
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.20</td>
<td>Wave load for moderate sea conditions: (a) Tower modal load (b) Foundation modal load</td>
<td>175</td>
</tr>
<tr>
<td>5.21</td>
<td>Wave load for extreme sea conditions: (a) Tower modal load (b) Foundation modal load</td>
<td>176</td>
</tr>
<tr>
<td>5.22</td>
<td>Wave spectrum for the moderate and extreme sea environment</td>
<td>177</td>
</tr>
<tr>
<td>5.23</td>
<td>Free vibration response of the SCF for horizontal translation and rotation and SDOF decay curves (for damping ratios between 40-50%)</td>
<td>179</td>
</tr>
<tr>
<td>5.24</td>
<td>Tower displacement response transfer function for the SCF, modelled using coupled static stiffness CS model: (a) Nacelle relative displacement response (b) Nacelle total displacement response</td>
<td>182</td>
</tr>
<tr>
<td>5.25</td>
<td>Tower displacement response transfer function for the SCF, modelled using the frequency dependant CS foundation model (Plaxis): (a) Nacelle relative displacement response (b) Nacelle total displacement response</td>
<td>184</td>
</tr>
<tr>
<td>5.26</td>
<td>Response time-histories for a steady load, soil-foundation system modelled using a frequency dependant CS model (Cone method): (a) Nacelle relative displacement response (b) Nacelle total displacement response</td>
<td>186</td>
</tr>
<tr>
<td>5.27</td>
<td>Response time-histories, soil-foundation system modelled using a frequency dependant CS model (Cone method): (a) Nacelle relative displacement response (b) Nacelle total displacement response</td>
<td>187</td>
</tr>
<tr>
<td>5.28</td>
<td>Response time-histories, soil-foundation system modelled using an uncoupled static CS model: (a) Nacelle relative displacement response (b) Nacelle total displacement response</td>
<td>188</td>
</tr>
<tr>
<td>5.29</td>
<td>Nacelle relative displacement response for various wave loading conditions</td>
<td>189</td>
</tr>
<tr>
<td>5.30</td>
<td>Response spectrum: (a) Blade response (b) Tower response</td>
<td>190</td>
</tr>
<tr>
<td>5.31</td>
<td>Campbell diagram with modal frequencies (Hz) plotted against rotation speed $\Omega_b$ (rpm)</td>
<td>196</td>
</tr>
<tr>
<td>5.32</td>
<td>Tower first modal frequencies in the fore-aft direction plotted against rotation speed $\Omega_b$ (rpm)</td>
<td>200</td>
</tr>
</tbody>
</table>
5.33 Relative blade tip displacement response for the turbine founded on SCF (RD = 25%) compared to FB conditions: (a) Flapwise vibrations (b) Edgewise vibrations ................................................................. 201
5.34 Tower top relative displacement response for the turbine founded on SCF (RD = 25%) compared to FB conditions: (a) Fore-aft (b) Side-to-side .......................................................................................... 202
5.35 Close up of the tower top relative displacement response for the turbine founded on SCF (RD = 25%) compared to FB conditions: (a) Fore-aft (b) Side-to-side ........................................................................................................ 203
5.36 Tower relative displacement response transfer function for turbine founded on the SCF (RD = 25%) and FB conditions .......................................................... 204
5.37 SCF (RD = 25%) response fore-aft and side-to-side direction: (a) Horizontal translation (b) Rotation ........................................................................................................ 205
5.38 Screen shot from Plaxis showing the turbine tower model founded on the suction caisson, with wind and wave loading .......................................................... 207
5.39 Input time histories to Plaxis: (a) Wind load (b) Wave load ........................................................................................................ 208
5.40 Tower top displacement: (a) Fore-aft (b) Side-to-side (c) Magnitude ........................................................................................................ 210
5.41 Displacement at wave application point: (a) Fore-aft (b) Side-to-side (c) Magnitude ........................................................................................................ 210
5.42 Foundation translation response: (a) Fore-aft (b) Side-to-side (c) Magnitude ........................................................................................................ 211
5.43 Foundation rotational response: (a) Fore-aft (b) Side-to-side (c) Magnitude ........................................................................................................ 212
5.44 Time history of normalised displacement in the fore-aft direction for aligned wind and wave load (phase = 0) for each soil profile: (a) Tower top (b) Wave application point ........................................................................ 213
5.45 Spectrum of the free vibration response of the turbine for each soil profile and FB conditions ................................................................................... 214
6.1 Soil profile with two HAWT founded on: (a) SCF (b) MPF .......... 220
6.2 Foundation DOFs and parameters as defined in FAST, shown for a MPF ........................................................................................................ 223
6.3 Iterative scheme CS model ........................................................................................................ 225
6.4 Steps to achieve an AF model ........................................................................................................ 226
LIST OF FIGURES

6.5 Tower mode shape calculated using BMODES: (a) Fixed base model (b) CS model MPF (for the uncontrolled case) .............................................................. 229
6.6 Fore-aft bending moment at the mudline for the various foundation models, for the uncontrolled case .......................................................... 232
6.7 Fore-aft bending moment at the mudline for the various foundation models, for the controlled case .......................................................... 233
6.8 Power spectral density function: (a) Wind (b) Wave ........................................... 234
6.9 Time history: (a) Wind speed (b) Sea surface elevation ........................................ 236
6.10 Foundation response for the MPF CS model, for the uncontrolled case: (a) Horizontal translation and (b) Rotation ........................................... 236
6.11 Foundation response for the MPF CS model, for the controlled case: (a) Horizontal translation (b) Rotation ..................................................... 237
6.12 Forces at the mudline for the MPF CS model, for the uncontrolled case: (a) Bending moment (b) Shear force .......................................................... 237
6.13 Forces at the mudline for the MPF CS model, for the controlled case: (a) Bending moment (b) Shear force .......................................................... 238

A.1 Time history ........................................................................................................ 263
A.2 Time history overlain with sinusoidal waves ...................................................... 264
A.3 Sample spectra .................................................................................................... 266
A.4 Sine wave ........................................................................................................... 267
A.5 Sine waves at different frequencies ..................................................................... 267
A.6 Sampling the signal ........................................................................................... 268
A.7 Spectra showing the Nyquist Frequency (NF) .................................................. 269
A.8 Spectra zero centred .......................................................................................... 270
A.9 Half spectra ........................................................................................................ 271
A.10 Spectra outputs: (a) Normal spectra (b) Zero centred (c) Half spectra .............. 272
A.11 SDOF system .................................................................................................... 273
A.12 Displacement response, time domain solution ................................................ 276
A.13 Displacement response, for the time domain and frequency domain solutions .......................................................... 279

B.1 Framed structure ................................................................................................. 284
B.2 Integrated Struct/Plaxis model iterative scheme .............................. 284
B.3 Complete Plaxis model ....................................................................... 285
B.4 Complete Plaxis model, deformed mesh ......................................... 286

D.1 First two blade flapwise mode shapes, for the NREL 1.5 MW HAWT (MODES) ................................................................. 293
D.2 First three tower longitudinal mode shapes, for the NREL 1.5 MW HAWT (MODES) ................................................................. 293
D.3 Diagram of the offshore 5 MW HAWT model with two different foundation solutions, showing dimensions of interest ......................... 296
D.4 First two flapwise and first edgewise blade mode shapes for the NREL 5 MW offshore HAWT (MODES) ........................................... 301
D.5 First three tower mode shapes for the NREL 5 MW offshore HAWT (MODES) ................................................................. 301
D.6 First two tower mode shapes in the fore-aft and side-to-side direction for the NREL 5 MW offshore HAWT (BMODES) ......................... 302
Chapter 1

Introduction

1.1 Wind Energy, Past and Present

1.1.1 Wind energy beginnings

Wind has been used by mankind for millennia. As early as 4000-3500 BCE (Thomas & Sydenham, Retrieved 2012), the first sailing ships were developed by the Egyptians with the realisation that the wind’s energy could be used to propel vessels. This allowed mankind to regularly travel distances impossible or prohibitively expensive with muscle power alone. The very first sail boats used square sails that were “pushed” by the wind. This worked as long as the wind was blowing in the right direction; if the wind changed sailors had few options other than to let down their sail and wait.

The trick of sailing into the wind appeared around two thousand years ago. It was discovered that with proper orientation, triangular sails could convert wind from any direction into forward thrust. Although there was no physical understanding of the pulling force, the advantages were obvious.

Only in the 18th century, was it discovered that “lift” was generated by the way fluid flows over a curved surface, an airfoil (Naughton, Retreived 2012). The triangular shape sails causes the fluid to flow over the front faster than the back, reducing the pressure. In essence “sucking” the sails and boat forward.
1. INTRODUCTION

1.1.2 Windmills early history

Windmills were first used for tasks such as grinding grain and water pumping. Although windmills and sailing ships seem very different, they rely on the same natural phenomena to drive them: wind.

The earliest written reference to a windmill appears in the 1st century CE, the windwheel of the Greek engineer Heron of Alexandria, although scholars are unsure of the machine's purpose and suspect it was a toy (Lohrmann, 1961). However, stronger evidence exists for the use of windmills in Mesopotamian as early as 700 BCE - 900 CE. According to the Persian geographer Estakhri in the 9th century CE. They used a vertical axis design known as a Panemone, which was not particularly efficient as it turned on the principal of drag.

Though China is sometimes claimed as the birthplace of the windmill, the earliest documented Chinese windmill appears in 1219 CE (Dodge, Retrieved 2012).

1.1.3 Middle Ages

The earliest certain reference to a European windmill dates from 1185, in Yorkshire, England, although earlier dates have been suggested (Whitworth, 2002). It is sometimes argued that the Crusaders were inspired by windmills in the Middle East, but this is unlikely since the European horizontal-axle windmills were of significantly different design (Dodge, Retrieved 2012). The horizontal axis design meant a much more efficient conversion of kinetic to mechanical energy.

The technology spread across Europe during the Middle Ages, the earliest Irish record is from 1281 CE (Kilsanlon, Co. Wexford).

Over the next 700 years, the Dutch took windmill technology to new heights, developing new sail designs that increased efficiency, including using a leading edge on the sail to create aerodynamic lift.

1.1.4 First wind turbines

By the 19th century, windmills had all the major features crucial to the performance of modern wind turbines. In 1850 there were 9,000 windmills in the Netherlands alone (Stremke & Dobbelsteen, 2012). In the United States at the same time a new style of
windmill was developed that comprised of a small rotor set on top of a skeletal steel structure. A large amount, approximately six million, were installed on frontier farms to operate irrigation pumps (Righter, 1966).

The first windmill used to produce electricity was built in Scotland by Prof James Blyth in 1887 (Dodge, Retrieved 2012). Across the Atlantic, Charles F. Brush in 1888, built a 17 m rotor wind turbine in Ohio, USA (Righter, 1966). The term turbine originates from the Latin word “turbo” meaning spinning top and whirlwind.

The 19th century brought changing attitudes to energy generation. The Industrial Revolution, a unprecedented period of innovation for most of modern mechanics, left wind energy in the doldrums.

This was perhaps inevitable; fossil fuels, coal and later oil, had obvious advantages: A Porter-Allen high-speed steam engines (as used in Pearl Street Station New York in 1882 (192, 1922)) could produce 130 kW of power; compare that to a wind turbine designed by Porter-Allen’s contemporary Brush, which could at best only produce 12 kW of power and suffered from the same reliability issues that continue to effect modern day turbines. Besides this greater output, fossil fuels could also be transported easily and used to generate energy whenever required.

With a growing population - for the first time living mostly in cities - the demand for energy was vast and urgent. The advantages of steam engines over traditional methods of generating energy (wind, hydro, animals) were so dramatic that traditional methods were largely abandoned. By the end of the century, fossil fuels dominated the emerging market of electricity production.

1.1.5 Energy crises

The re-emergence of wind energy came about for two reasons: the 1970’s oil crisis and growing international concern over carbon emissions.

During the oil crisis, it became apparent how vulnerable, both politically and economically, the West’s dependency on oil left them. This has been seen again recently with the steady run-up in oil prices since 2003 due to political instability and gathering fears of peak oil.

Peak oil is the point in time when the maximum rate of petroleum extraction is reached, after which the rate of production will enter terminal decline.
1. INTRODUCTION

Therefore, both fear and disenchantment with fossil fuels has increased interest in commercial wind power, resulting in a so-called “wind rush”.

1.1.6 Wind energy present

According to the Global Wind Energy Council (GWEC), the global total of power generated by wind at the end of 2011 was just shy of 238 GW.

Europe as a whole installed 10.3 GW in 2011 and the total European capacity now stands at 96.6 GW. Offshore installations account for about 9% of the European market. The majority of new installations offshore in 2011 were in the UK, cementing the UK’s position as the European (and global) leader in offshore deployment, passing 2,000 MW installed in 2011.

Europe has a target of supplying 14-16% of its electricity through wind by 2020. On the back of this, total installations in Europe from 2012-2016 are expected to increase by about 65 GW, bringing total installed capacity to just over 160 GW. In Ireland, as of early 2009, there was 1,320 MW of installed wind energy capacity. In order for Ireland to reach 2020 targets, another 6,480 MW will be required to be installed (IWEA, 2010).

![Figure 1.1: Global Cumulative Installed Wind Capacity 1996-2011, sourced from GWEC (2010)](image)

Over the last 15-20 years, huge advances have been made in wind energy technology, addressing issues such as cost and reliability. Wind energy is now the fastest growing renewable energy source, leading the way as an affordable, reliable and environmentally friendly alternative to traditional fossil fuels. Furthermore, with offshore
wind energy growing in popularity, the potential for wind energy is vast. It is fitting, since the first wind technology was maritime that the future of wind energy should be offshore.

The current trend in the wind industry is to have larger and more powerful units, reaching higher into the atmosphere to obtain greater and more stable wind speeds. Figure 1.2 shows the increase in power capacity and rotor diameters of wind turbines from 1980 to 2010. The diameter of the rotors has doubled each decade.

Germany’s Enercon currently possess the world’s largest wind turbine which is rated to 7.5 MW and has a hub height of 135 m, a rotor diameter of 126 m and a total height of 198 m.

![Figure 1.2: Increase in power capacity and rotor diameters of wind turbines from 1980-2010](image)

As wind turbines have grown in size, there has been a parallel increase in flexibility, which creates problems of its own. Understanding the dynamic response of MW wind turbines is essential in controlling their reliability and efficiency.
1. INTRODUCTION

1.2 Research aims

The aim of this thesis is to investigate the structural dynamic response of Horizontal Axis Wind Turbines (HAWT) including Soil-Structure Interaction (SSI). To carry out the analysis several key areas are investigated:

Wind turbine models

Formulation of Multi-degree of freedom (MDOF) HAWT models using an Euler-Lagranian approach including soil-foundation interaction, capable of analysing the dynamic response of the turbine.

Realistic onshore and offshore loads

Wind loads are generated using the Blade Element Momentum (BEM) theory coupled with turbulence. Turbulence is generated using both the Kaimal spectrum, to create an isotropic homogeneous turbulent field, and the more realistic rotationally sampled spectrum. Wave loads are generated using Airy linear wave theory, the JONSWAP spectrum and Wheeler Stretching theory.

Soil-foundation models

Development and validation of both static and frequency dependant Coupled-Spring (CS) models using the Finite Element Method (FEM) geotechnical software package Plaxis 3D dynamic. The developed models are compared to results from analytical formulas, DNV/Risø standards and the wave prorogation based Cone method.

Numerical analysis

Analyse the dynamic response of onshore and offshore HAWT including SSI using the developed MDOF models, wind and wave loading and soil-foundation models.

FEM models

Development of Plaxis models to study the effect of unaligned wind and wave loading on the dynamic response of the turbine tower, support structure and soil-foundation system.
FAST models

The NREL distributed aeroelastic code FAST is coupled with an Apparent Fixity (AF) and CS foundation models to further investigate the effects of SSI on offshore HAWT including the impact of controllers for fatigue.

1.3 Organization of the thesis

This thesis has seven chapters outlined below:

Chapter 2 provides a detailed review of the literature relevant to the topics dealt with throughout this thesis. The concept and principles of SSI are outlined and its importance in wind turbine design is discussed.

In Chapter 3 the formulation of the Euler-Lagrange HAWT models used throughout this thesis are proposed. An out-of-plane and coupled in-plane out-of-plane model with foundation interaction are developed. The load models applied to the turbine are derived. BEM theory is used to simulate turbulent aerodynamic loading. Turbulence is described and simulated by both a Kaimal spectra and a rotationally sampled spectra. Airy linear wave theory, a JONSWAP spectra and Wheeler Stretching are used to describe a turbulent wave loading environment.

In Chapter 4 the FEM package Plaxis 3D dynamic is used to developed impedance functions for various soil-foundation system. A procedure is first presented to obtain the impedance of a surface and an embedded footing. The generated impedance functions are compared to known analytical formulas and the wave propagation based Cone method. Impedance function for an onshore Gravity based Foundation (GBF), an offshore Suction Caisson Foundation (SCF) and an offshore MonoPile Foundation (MPF) are then presented. Finally, multi-variable linear regression CS models are presented for a MPF.

In Chapter 5 the dynamic response of an onshore and an offshore wind turbine are analysed for a number of different soil-foundations models. The MDOF HAWT model with foundation coupling formulated in Chapter 3 is first validated. Next, numerical analysis is presented for the out-of-plane model for both an onshore and offshore turbine coupled with static and frequency dependant CS models. The coupled
1. INTRODUCTION

In-plane out-of-plane turbine model founded on a SCF and MPF embedded in non-linear soil is then examined. The effects of the SSI on systems (in terms of modal frequencies and displacement response) is analysed. Finally, the effect of unaligned wind and wave loading on the dynamic response of the turbine tower, support structure and soil-foundation system are examined using FEM.

In Chapter 6 the NREL developed aeroelastic code FAST is used to further investigate the effects of SSI on wind turbines. FAST is coupled with AF and CS models derived using Plaxis. The flexible foundations effects on the modal frequencies, load statistics and fatigue life are examined.

Chapter 7 concludes the thesis. The work is summarized and conclusions drawn. Suggestion for areas requiring further research are outlined.
Chapter 2

Literature review

2.1 Introduction

This chapter aims to give an overview of current published literature and basic theory relevant to the topics dealt with in this thesis. The general properties and design of wind turbines are briefly summarized to provide an insight into the area. This is followed by a discussion on the topics relevant to this work.

2.2 Wind turbine general properties

2.2.1 Introduction

A wind turbine is a machine that converts kinetic energy in the wind into mechanical power and then usually into electricity. Modern wind turbines are generally classified according to several properties:

- The orientation of the rotor (upwind or downwind)
- The rotor control system (pitch or stall)
- The hub design (rigid or teetering)
- The alignment with the wind (free yaw or active yaw)
- The number of blades
Wind turbines come in many different types and shapes. However the most common type of design is presently the Horizontal Axis Wind Turbine (HAWT), where the axis of rotation is parallel to the ground, with an upwind rotor and three blades. The main components of the HAWT are described below and shown in Figure 2.1. This thesis only deals with the type most commonly used in present day wind energy production, the upwind 3 bladed HAWT.

Figure 2.1: HAWT with main component identified
2.2 Wind turbine general properties

2.2.2 Rotor

The rotor of the turbine consists of the blades and supporting hub which converts wind energy into mechanical power.

The hub is connected to the nacelle and is usually conical. The blades stretch out radially from the hub. Their function is to collect kinetic energy imparted by the wind and transfer this energy to the hub and subsequently to the generator. The blades, the most important turbine components in cost and efficiency terms, are a constant research area. Both the hub and the blades are generally made from glass fibre reinforced plastics, though sometimes wood/epoxy laminates are used. The most common blade arrangement is three, the main reason being is that when all three blades are undergoing vibration, there is theoretically a zero bending moment at the hub.

The outermost part of the blade resembles an aircraft wing, a so-called aerofoil. When immersed in a fluid flow it behaves similarly, being subject to the same aerodynamic forces – namely drag and lift. The shape of the wind turbine blades is constantly being researched, as the shape determines the efficiency of the conversion of kinetic energy to mechanical energy. Figure 2.2 shows a typical blade section and demonstrates the principle of lift. The wind flow separates on contact with the blade and, due to the geometric shape of the blade, the flow must move faster over the top curved surface than over the bottom flat surface. Energy is conserved, so the pressure energy and the kinetic energy stays constant. On the top of the blade, the air is moving faster, so it has more kinetic energy and therefore less pressure energy. This implies a suction force to the blade i.e. lift, compelling the blade to move upwards. Thus the blades are set in rotary motion by a combination of lift and drag forces.

![Aerofoil section showing the principle of lift](image)

Figure 2.2: Aerofoil section showing the principle of lift
2. LITERATURE REVIEW

The power output from a wind turbine is controlled by two main methods; stall controlled or pitch controlled blades. Stall control involves increasing or decreasing the angle of attack of the blades. This results in the blades creating more or less aero-dynamic lift which determines the power generated by the turbine. Thus the power generated is regulated through stalling the blades after rated speed is achieved. As wind speed increases the blades begin to stall, lift drops and drag increases thereby reducing the driving torque. Pitch control involves changing the pitch angle of the blades. This also serves to control the torque generated. Variable pitch provides more control than a stall mechanism and is therefore more popular, especially in the larger multi-megawatt turbines. However, the required pitch bearings makes these hubs more expensive.

There are two main types of hubs: rigid and teetering. Rigid hubs prevent movement of the blades in and out of the plane of rotation, known as the edgewise and flapwise directions. A teetering hub is one mounted on bearings which allow the hub to teeter back and forth in both the edgewise and flapwise directions.

2.2.3 Nacelle/Yaw system

The nacelle houses all the electrical and mechanical equipment inside a protective casing. The drive train connects the rotor blades to electrical generator via the gearbox. Generally the rotor will turn relatively slowly, especially in large multi-mega watt turbines. The rotational energy created is transferred to the gearbox via the low speed drive shaft. The gearbox accepts this rotation and steps up the frequency by a factor of approximately thirty to fifty. This rotational energy is then transmitted to the electrical generator via the high speed shaft.

The generator then converts this mechanical energy into electrical energy. The majority of wind turbines use induction generators due to their durability, cost effectiveness and the ease with which they can be connected to the electrical grid. These generators generally require a near constant rotational speed. The use of variable speed generators is however increasing due to advantages: reduced wear and tear and increased energy capture over a wider range of wind speeds.
2.3 Description of the foundations

The nacelle also contains a yaw orientation system, which keeps the rotor aligned with the wind. If the rotor system is not perpendicular to the wind a “yaw error” is said to have arisen. Yaw errors can subject the rotor system to increased fatigue loads.

Upwind turbines generally have an active yaw drive that use motors to accurately orientate the rotor. Self-aligning (or free yaw) systems are usually used on downwind turbines. When wind speed becomes too high for the specific wind turbine system, the yaw control system will deflect the rotor system out of the wind, into a parked position. This prevents excessive loads, vibrations and rotational speeds occurring but does decease in operational efficiency.

2.2.4 Towers

The tower supports the nacelle and rotor system at an elevated height and transfers loads to the foundation. Wind speeds generally increase with elevation and are less turbulent. Therefore, the higher the tower the better. The main types are free standing tubular steel towers, lattice or truss towers and concrete towers. Tower height is typically 1.5 times the rotor diameter. The stiffness of the tower is an important aspect of design, due to the unwelcome possibility of coupled tower-rotor or tower-foundation vibrations.

For downwind rotors, an effect known as ‘tower shadow’ occurs (Tong, 2010). This is a wake that forms downwind of the turbine tower which periodically affects the airflow around the blades as they pass by the tower. This can cause complex tower dynamics, power fluctuations and noise generation. Upwind rotor configurations avoid this, although there is still a drop in wind speed as the blade passes in front of the tower.

2.3 Description of the foundations

The type of foundation used for a wind turbine depends on several factors:

- Imposed loading
- Ground conditions
- Particular performance requirement
2. LITERATURE REVIEW

- Choice of codes
- Economics
- Installation methods
- Water depth for offshore foundations

A good design should determine site conditions and select the option most economically appropriate. Design analysis should include evaluation of bearing capacity, amount of settlement, stress levels, deformation levels, overall stability, ground improvement, and dynamic ground behaviour.

Offshore turbines, due to the more demanding environment, have unique technical requirement. Sediment movement and local scour are important phenomena in coastal waters due to high currents and shallow water wave effects. The performance of offshore foundations can be seriously undermined by erosion therefore effective means of scour mitigation is an important design issues.

Several foundation concepts are used for both onshore and offshore turbines. They can roughly be divided into 6 categories: gravity based, monopile, multi-pile, suction bucket, jackets and floating. The first two can be applied onshore and offshore were as the last four concept are used mainly in offshore construction. Many of the proposed concepts utilize designs from the oil and gas industry.

2.3.1 Gravity based

This is the most common type of foundation onshore. The foundation relies on its own self weight and soil overburden to resist overturning of the structure. The mass distribution of the gravity based foundation must be balanced with the diameter and the height of the foundation to ensure overall stability. This type of foundation can be used with most soil strengths.

For offshore applications, gravity based foundation costs tend to be unsustainable beyond water depths of around 10 m. They also tend to be vulnerable to erosion and scour.
2.3 Description of the foundations

2.3.2 Monopile

Monopile foundations can be used for both onshore and offshore applications. The foundation consists of a large diameter monopile installed in the soil. This monopile is a welded steel pile with the same diameter as the lower section of the wind turbine tower. However, the diameter may vary depending on the type of connection between the pile and tower and the stiffness requirements of the foundation. The load from the wind turbine is transferred to the surrounding soil by lateral earth pressure on the monopile. The length of the pile is governed by the lateral resistance of the surrounding soil, as the critical design constraints (due to the rotor) here are the deflections and rotations at the mudline. The design depth (depth below seabed) of the monopile is typically between 20-30 m depending on water depth (offshore), wind turbine size and soil conditions. Each monopile carries a single turbine. For offshore applications, no preparations of the seabed is necessary. Scour protection may however be required when the concept is used at sandy locations. For very large offshore turbines, monopiles are not suitable as they get too big and heavy to handle with normal offshore equipment.
2. LITERATURE REVIEW

2.3.3 Multi-pile

This concept is used for very large offshore turbines. Several smaller piles are joined using a connection/transition piece and connected to a wind turbine tower. This distributes the loads over all the piles. Therefore the piles can be much smaller and thus easier to install.

One popular multi-pile configuration, originating from the offshore oil and gas industry, is the tripod. This is a welded steel structure with a centre column supported by three piles in separate pile sleeves. The design criterion for the piles in a tripod construction is usually the axial bearing capacity, contrary to the monopile, due to arrangement of forces.

However there are several alternative solutions. Suction caissons may be used instead of piles, and the main structure may be constructed as a jacket or as a concrete section, with the possibility of ballasting the foundation.

2.3.4 Suction buckets

In the search for cheaper alternatives to piled and gravity based foundations, the offshore industry is examining alternative shallow foundation solutions. One such foundation a suction caisson, which has a flat foundation with skirts around the periphery, rather like an upturned bucket. Suction caissons have been used in the oil and gas industry in platform construction (Sparrevik, 2002). Their application to offshore wind turbines is discussed in Houlsby & Byrne (2000), Byrne & Houlsby (2003) and Houlsby et al. (2005). Currently two main structural configurations are being considered: a monopod consisting of a single large caisson and a tetrapod in which the load is transferred through a truss structure to three or four smaller caissons.

When installed as monopod foundation, the suction caisson must sustain significant horizontal loads and moments, but relatively low vertical loads. The monopod is therefore generally seen as the more attractive option. However, at greater water depths a foundation with three or four smaller suction caissons may become appropriate. The overturning moment is then stabilized by the opposing vertical reactions of the suction caissons.
2.3 Description of the foundations

The suction caisson is installed using suction as the driving force and therefore does not require heavy installation equipment. The suction lowers the pressure in the cavity between the caisson and the soil surface, and generates a water flow. This flow reduces the effective stress around the tip of the skirt allowing installation as the penetration resistance of the soil is lowered.

As yet, this technology is not proven for large water depths. A fully operational 3.0 MW offshore wind turbine was installed on a prototype of the suction caisson foundation at a test field in Frederikshavn Denmark (Ibsen et al., 2005.).
2. LITERATURE REVIEW

2.3.5 Jackets

Jackets are basically underwater lattice towers that are mounted on piles or suction buckets. Jacket construction is already an established foundation system in the oil and gas industry. Compared to a monopile foundation, 40-50% saving in steel can be made. Since the single components are relatively small, they can be produced by automation and easily transported and installed.

These foundations are suitable for turbines in excess of 5 MW rated capacity and ideal for the water depths between 15 m to 80 m.

2.3.6 Floating

A floating wind turbine is an offshore wind turbine mounted on a floating structure that allows the turbine to generate electricity in water depths where bottom-mounted towers are not feasible.

Current fixed-bottom technology has been limited to deployment in water depths of less than 30 m thus far. Although this technology may be extended to deeper waters, eventually floating platforms may be the most economical for wind turbines beyond the continental shelf. There are three main concepts currently under development:

A first concept is a free floating structure, with the centre of gravity near or above the surface of the water.

The second concept is a free floating spar structure. This is basically a long, vertical beam on top of which the turbine is placed.

The third concept is a tension-cabled option. The support structure provides more lift than necessary to lift the turbine, and is attached to the ocean floor with tension cables. The tension in the cables must be sufficiently large to overcome the toppling moment caused by the rotor thrust and wave excitation.

Hywind, the first large-capacity, 2.3 MW, floating wind turbine became operational in the North Sea off the coast of Norway in September 2009 as a demonstration project.
2.4 Properties of the foundations

Typical consideration in the design of HAWT foundations are outlined below (Bonnett, 2005; DNV/Risø, 2001; Hau, 2006; IEC, 2005).

2.4.1 Strength

The loading transmitted to the foundation depends on the wind characteristics, the size of the wind turbine and the power regulations. For design approval and safety certification, wind turbine foundations are required to have sufficient strength to withstand extreme static loads. Typically the highest loads occur under stand-still conditions, and are calculated using an assumed highest wind speed for 50 years. The highest loads during operation should however also be checked.

A dynamic analysis is traditionally not required, even though wind turbines are, by definition, moving machines. This may be due to lack of understanding of dynamic soil-structure interaction coupled with a lack of viable methods to complete a dynamic analysis in the required design time framework. Most wind turbines for the utility market are designed according to the loads specified by the International Electrotechnical Commission (IEC). The IEC standards specifies 17 different ultimate load cases and five fatigue load case that must be considered in design. The design of the foundation is generally done manually using several simplifications and assumptions using static loads.

2.4.2 Stiffness

Wind turbine manufacturers generally specify the required minimum foundation stiffness, in order to ensure that the overall system’s natural frequency stays above the main excitation loads (depending on the design philosophy). Typically the dynamic characteristics of the soil-structure interaction are not considered.

2.4.3 Stability

Design for stability compares overturning moments and horizontal forces with resisting moments and sliding resistance. Wind turbine manufacturers usually require certain
2. LITERATURE REVIEW

serviceability limit checks to be carried out to ensure the settlement and rotation of the foundation (for the given static design loads) are under a certain minimum allowance. Consideration is also given for short-term (recoverable, elastic) and long term (plastic, consolidation) differential settlement.

2.4.4 Durability

The typical design life of a wind turbine is 20 years. Wind tribunes should be durable in order to maximise operational life and minimise maintenance. Concrete fatigue, reinforcing corrosion, concrete chemical attack, and freeze/thaw cycling should be considered in design. Durability is more critical for offshore conditions.

2.4.5 Scour

Waves and currents routinely move soil underwater, in a process called scour. Scour is of particular concern when designing offshore wind turbines as soil removed from foundations will result in diminished capacity, thus affecting the natural frequency of the structure. The three most common types of scour are local scour, which occurs around the foundation, global scour which occurs in the overall wind farm due to the effects of multiple foundations, and overall seafloor movements which include sand waves ridges and shoals.

2.5 Dynamics of wind turbines

2.5.1 General structural dynamic theory

A common situation in structural mechanics is that a structure is only affected by static forces i.e. gravity loads. If the structure is affected by a dynamic force, i.e. a force that is varied in time, it may have a different response compared with the response of a static force. Properties of a dynamic force that have an essential influence on the structural response are the amplitude of the force and the relation of the frequency of the load to the natural frequency of the structure.
In many cases, the load frequency is so far from the natural frequency of the structure that the force can be expressed as a static force by just applying a dynamic amplification factor to the maximum value of the force. However, in cases where the load frequency is close to the natural frequency this procedure may not suffice and a dynamic analysis may become necessary.

### 2.5.1.1 Equations of motion

The importance of proper modelling of the structural dynamics can be most conveniently illustrated by considering a Single Degree Of Freedom (SDOF) mass-spring-damper system as shown in Figure 2.5.

![SDOF mass-spring-damper system](image)

The dynamic response of a structure is found by solving the fundamental differential Equation Of Motion (EOM), expressed in standard form as,

\[ m \ddot{x}(t) + c \dot{x}(t) + kx(t) = F(t) \]  \hspace{1cm} (2.1)

When a harmonic excitation force \( F(t) \) is applied to the system, the magnitude and phase of the resulting displacement \( x(t) \) is strongly depend on the frequency of excitation \( \omega \). Three steady state response regions can be distinguished, as shown in Figure 2.6.

1. Quasi-static
2. Resonant
3. Inertia dominated
2. LITERATURE REVIEW

For frequencies of excitation well below the natural frequency of the system, the response is quasi-static: the displacement of the mass follows the time varying force almost instantaneously, as if it was excited by a static load. Figure 2.6b shows a typical response for frequencies of excitation within a narrow region around the system's natural frequency. In this region, the spring force and the inertia force (almost) cancel, producing a response that is several times larger than it would be statically. The resulting amplitude is governed by the damping present in the system. For frequencies of excitation well above the natural frequency, the mass cannot follow the excitation. As a result the response is lower and in almost counter phase—in this case the inertia of the system dominates the response.

Figure 2.6 illustrates the general fact that, in steady state, a sinusoidal input applied to a linear system generates a sinusoidal output at the same frequency, which differs in magnitude and phase.

A dynamic load can have a significantly larger effect than a static load of the same magnitude due to the structure's inability to respond quickly to the loading (by deflecting). The magnitude and phase modifying property of linear systems can be summarized by a plot of the Dynamic Amplification Factor (DAF) and the related phase lag. The DAF depicts the ratio between the dynamic response magnitude and the static re-
2.5 Dynamics of wind turbines

response magnitude due to the same magnitude of loading. Figure 2.7 shows the DAF and phase lag plot for a SDOF system.

![Figure 2.7: (a) DAF versus normalised frequency (b) Phase lag versus normalised frequency](image)

The peak in Figure 2.7 corresponds to the system's natural frequency. The height of the peak is determined by damping. Therefore any resonant problem can be countered with adequate damping. In dynamics, the frequency of excitation is at least as important as its magnitude. Resonant behaviour can cause severe load cases, fatigue, even failure. For structures where dynamics are expected to be problematic, detailed knowledge is vital of the expected frequencies of excitation and the natural frequencies of the structure or part of the structure.

A complete wind turbine system can be thought of as being constructed of a number of coupled multi degree of freedom mass-spring-damper systems (Molenaar & Dijkstra, 1999). Therefore the conclusion that can be drawn from this review is that the response of a wind turbine system subjected to time-varying loads needs to be carefully assessed.
2. LITERATURE REVIEW

2.5.1.2 Modal analysis

Natural frequencies and modes are a vibration property of a structure. A structure normally has an infinite number of degrees of freedom but it is possible to approximate the structure with a system having a finite number of degrees of freedom, a so-called Multi Degree Of Freedom (MDOF) system. A MDOF system has as many natural frequencies as the number of Degrees Of Freedom (DOFs) of the system. Resonance appears when the frequency of the loads corresponds to or is a multiple of a natural frequency of the structure. The vibration displacements of the structure is strongly amplified when resonance occurs, but the resonance effect is reduced with the complexity of the mode shape. This means that the lowest modal frequencies, which have less complex shapes, are most important to study. It is therefore possible to describe an approximate dynamic response with just a few modes.

2.5.1.3 Eigenvalue analysis

Considering a time invariant SDOF system, equation 2.1, the homogeneous solution of the system can be written as,

\[ x = X e^{\lambda t} = X e^{(\alpha + i \omega_0) t} = X e^{\alpha t} e^{i \omega_0 t} \]  

(2.2)

Insertion into the governing equation yields the eigenvalue problem,

\[ (m \lambda^2 + c \lambda + k) X = 0 \]  

(2.3)

Introducing the natural frequency \( \omega_0 = \sqrt{\frac{k}{m}} \) and a mass normalized damping coefficient \( \beta = \frac{c}{2k} \), we can rewrite the eigenvalue problem as,

\[ (\lambda^2 + 2 \beta \lambda + \omega_0^2) X = 0 \]  

(2.4)

For non-trivial solutions \( X \neq 0 \),

\[ \lambda = -\beta \pm \sqrt{\beta^2 - \omega_0^2} \]  

(2.5)
given the fact that $\xi = \frac{b}{\omega_0} = \frac{c}{2\sqrt{km}}$, we can rewrite,

$$\lambda = -\beta \pm i\omega_0 \sqrt{1 - \xi^2} \quad (2.6)$$

This implies,

$$x = Xe^{\lambda t} = Xe^{-\beta t}e^{i\omega_0 \sqrt{1 - \xi^2} t} \quad (2.7)$$

where the modal frequency is given as $\omega_d = \omega_0 \sqrt{1 - \xi^2}$.

Considering a MDOF system the governing equation for $N$ DOFs can be written as,

$$[M]\ddot{x} + [C]\dot{x} + [K]x = 0 \quad (2.8)$$

Rearranging in a state space form, with $2N$ equations, gives,

$$\{\dot{y}\} = [A] \{y\} \quad (2.9)$$

where the system matrix $A$ and the vector $\dot{y}$ are given as,

$$\{y\} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad [A] = \begin{bmatrix} [0] \\ -[M]^{-1}[K] - [M]^{-1}[C] \end{bmatrix} \quad (2.10)$$

where $I$ is the identity matrix. Insertion of the solution used in tradition eigenvalue analysis $y = Ye^{\lambda t}$ yields,

$$([A] - \{\lambda\}[I])\{Y\} = 0 \quad (2.11)$$

and since $\{Y\} \neq 0$, we can let $([A] - \{\lambda\}[I]) = 0$ and solve for $\lambda$, thus obtaining the eigenvalues and eigenvectors for the system.

This can be easily carried out in MATLAB using the `eig` function which takes in the system matrix $A$ and returns the eigenvectors and eigenvalues $\lambda$ of the system. For purely under-damped systems, all eigenvalues have a real part (damping) and an imaginary part (modal frequency) (see equation 2.7) and the eigen-solutions come in complex conjugated pairs,

$$\lambda = \{\lambda_1, \bar{\lambda}_1, \lambda_2, \bar{\lambda}_2, \ldots, \lambda_N, \bar{\lambda}_N\} \quad (2.12)$$
2. LITERATURE REVIEW

with corresponding pairs of eigenvectors in the $2N \times 2N$ modal matrix,

$$\phi = [\phi_1, \bar{\phi}_1, \phi_2, \bar{\phi}_2, \ldots, \phi_N, \bar{\phi}_N]$$

(2.13)

2.5.2 Fourier domain

The response and the loads on a wind turbine vary in time. For analysis of time histories in the time domain some generalities can be distinguished: maximum, minimum, mean etc. Time histories, however, can be transformed into the frequency domain (plotted as a spectrum) in order to make the data more accessible. A Fourier transformation can be used for this.

The basis of Fourier transformations is the assumption that signals can be represented by the sum of a number of sinusoids, each with a specific amplitude, frequency and phase angle. There are many algorithms used to implement the Fourier transforms. One of the most common is the Fast Fourier Transform (FFT) which can be implemented in MATLAB. A more detailed discussion on the FFT, including its implementation in MATLAB, can be found in appendix A.

2.5.3 Design options for support structures

Wind turbines cannot be viewed as static structures as they are in continuous vibration due to the rotation of the rotor. The four main loads on wind turbines are aerodynamics, gravity, dynamic interactions and mechanical control. Structural resonances can lead to large amplitude stresses and subsequent accelerated fatigue. For this reason, the rotor blades and support structure are designed to avoid resonance.

The current practice is to design the support structure such that the tower’s fundamental resonance does not coincide with the fundamental rotational (1P) or blade passing (3P for three-bladed turbines) frequencies of the rotor. These forcing frequencies are the dominant vibration sources and are associated with rotor imbalance and non-uniform flow over the blades as they rotate i.e rotational sampled turbulence (Petersen et al., 2010). This resonance avoidance approach has significant consequences for the structural design of offshore wind turbines (Petersen et al., 2010).
2.5 Dynamics of wind turbines

To avoid resonance, the structure should be designed such that its fundamental natural frequency does not coincide with either 1P or 3P excitation. This leaves three possible intervals. A very stiff structure with its first natural frequency above 3P, called a stiff-stiff structure. A medium stiff structure with its first natural frequency between 1P and 3P, a soft-stiff structure. And a very soft structure with its first natural frequency below 1P, called a soft-soft structure.

For offshore wind turbines, resonance is typically avoided by using the soft-stiff design approach, ensuring that the tower’s fundamental resonance frequency lies in the frequency band between the rotor and blades passing rates over the operating speed of the turbine. This approach has major implication for structural design, requiring very stiff foundations. It is also sensitive to the levels of damping in the design and requires soil characteristics within a particular range, limiting potential sites for offshore wind turbines and introducing an implicit reliance on static soil properties to achieve resonance avoidance (Petersen et al., 2010).

Most wind turbines now have variable speed generators, as they offer higher energy capture and lower dynamic excitation. Variable speed turbines have an increased frequency operation zones. This implies the interval for a soft-stiff design is narrower, as shown in Figure 2.9.
2. LITERATURE REVIEW

Figure 2.9: Structural design frequency intervals for a three bladed variable speed wind turbine

2.5.4 Campbell diagram

The Campbell diagram is a classical way of representing the dynamics of rotary machinery; it shows the relationship between forcing mechanisms as a function of the rotation rate, relative to important resonances over the operating range of the system. Resonance coincidence is represented by a system forcing mechanism crossing a resonance line.

A sparse and a dense Campbell diagram are shown in Figure 2.10, for the NREL 5 MW offshore wind turbine.

The operating range of the turbine is represented by the vertical black lines, with a cut-in speed of 6.9 rpm, a design speed of 12.1 rpm. Between the cut-in speed and 15 rpm, 1P and 3P do not cross the support structure's fundamental mode 0.34 Hz. Note also that the support structure's fundamental mode is between 1P and 3P throughout the operating range shown, which is consistent with a soft-stiff support structure design approach.

The dense Campbell diagram shows resonance coincidence among several sources and resonances above the cut-in speed, which implies the potential for large displacement responses. It should be noted that some of these coincidences are with high frequency resonances, which have the potential to contribute to the cumulative fatigue damage of the wind turbine system. In addition, even if resonance avoidance is formally achieved, there still may be dynamic amplification of the system's structural
2.5 Dynamics of wind turbines

vibrations in off-resonance conditions. This can be important for the limit state assessments (Petersen et al., 2010).

It is therefore necessary for the interactions among all resonances and forcing mechanisms to be explicitly considered in the designed process. A resonance avoidance strategy that only treats the 1P and 3P sources and their interaction with the fundamental support structure resonance is insufficient (Krolis et al., 2007).

2.5.5 Power generation

2.5.5.1 Introduction

A wind turbine extracts energy from moving air by slowing it down, and transferring its energy into a spinning shaft, which usually turns an alternator or generator to produce electricity. The power that is available depends on both the wind speed and the area swept by the turbine blades (Hansen, 2001).

\[
Power = \frac{1}{2} \rho_a A_b U^3
\]
Equation 2.14 is perhaps the most important formula in wind power, as it states the amount of energy in the wind. The term \( \rho_a \) is the density of the air, \( A_b \) is the swept area and \( \bar{v} \) is the wind speed. Equation 2.14 shows that power from wind is proportional to the area swept by rotor. Power available from a wind turbine is related to the square of the blade diameter. Doubling the diameter will cause the machine to sweep 4 times the area. The wind speed is to the 3\(^{rd}\) power, therefore doubling the wind speed generates 8 times the energy.

All the power in the wind could only be captured if its velocity was reduced to zero. Realistically this is impossible, as the captured air must also leave the turbine. Using the concept of stream tube, the maximum amount of power that can be extracted from the wind by the wind turbine is 59% of the total theoretical available. This is called the Betz limit.

### 2.5.5.2 Betz limit

The simplest model of a wind turbine is the so-called actuator disc model. In this model the turbine is replaced by a circular disc through which air flows with a velocity \( \bar{v}_t \) and across which there is a pressure drop from \( P_1 \) to \( P_2 \) as shown Figure 2.11. The power developed by the wind turbine is,

\[
\text{Power} = (P_1 - P_2)A_t \bar{v}_t \tag{2.15}
\]

where \( A_t \) is the turbine disc area. Volume flow continuity gives,

\[
A_u \bar{v}_u = A_d \bar{v}_d = A_t \bar{v}_t \tag{2.16}
\]
2.5 Dynamics of wind turbines

From momentum conservation, the force exerted on the turbine is equal to the momentum change between the flow far upstream of the disc to the flow far downstream of the disc. Thus we can write,

\[(P_1 - P_2)A_t = \text{Mass flow} \times \text{Velocity difference} = \rho_a A_u \bar{v}_u (\bar{v}_u - \bar{v}_d) \quad (2.17)\]

Bernoulli's equation can be applied upstream and downstream of the actuator disc to give,

\[P_a + \frac{1}{2} \rho_a \bar{v}_u^2 = P_1 + \frac{1}{2} \rho_a \bar{v}_u^2 \quad (2.18)\]

\[P_a + \frac{1}{2} \rho_a \bar{v}_d^2 = P_2 + \frac{1}{2} \rho_a \bar{v}_d^2 \quad (2.19)\]

where \(P_a\) is the ambient pressure in the flow both far upstream and far downstream of the actuator disc. Combining equations 2.16, 2.17, 2.18 and 2.19 gives,

\[P_1 - P_2 = \frac{1}{2} \rho \left( \bar{v}_u^2 - \bar{v}_d^2 \right) = \rho_a \frac{A_u}{A_t} \bar{v}_u (\bar{v}_u - \bar{v}_d) = \rho_a \bar{v}_l (\bar{v}_u - \bar{v}_d) \quad (2.20)\]

where,

\[\bar{v}_l = \frac{1}{2} (\bar{v}_u + \bar{v}_d) \quad (2.21)\]

thus the velocity through the actuator disc is the mean of the upstream and downstream velocities in the stream tube. The efficiency can be written as,

\[\eta = \frac{\text{Power}}{\frac{1}{2} \rho_a A_t \bar{v}_u^2} = \frac{1}{2} \left( 1 - \frac{\bar{v}_d}{\bar{v}_u} \right) \left( 1 + \frac{\bar{v}_d}{\bar{v}_u} \right)^2 \quad (2.22)\]

Figure 2.12 shows the variation of efficiency (often referred to as the power coefficient, \(C_p\)) with the ratio of downstream to upstream velocity. By differentiating equation 2.22, we can show that the maximum efficiency occurs when \(\bar{v}_d/\bar{v}_u = 1/3\) (i.e. when \(A_d/A_u = 3\)). The efficiency is then \(\eta_b \simeq 59\%\).
2. LITERATURE REVIEW

Figure 2.12: Variation of efficiency with the ratio of downstream to upstream velocity

2.5.5.3 Power capture

Although power extraction from the wind is constrained by the Betz limit, in practice only about 20% to a maximum of around 40% is harvested by the best modern HAWT at rated wind speed. At all other wind speeds, the efficiency will be worse, and since wind speeds are never constant, the turbine will generally operate at lower than its best efficiency.

Several parameters account for this loss 19% shortfall. If we take an optimised blade configuration (optimised to aerodynamic power capture capabilities through its airfoils and twist along the blade to make the angle of attack optimal per airfoil), the power still depends on the relation of rotor to wind speed. The ratio between the speed of the blade tip, \( \bar{v}_{tip} \), and the wind speed, \( \bar{v}_{o} \), is called the tip speed ratio \( \lambda_{b} \) and is given as,

\[
\lambda_{b} = \frac{\bar{v}_{tip}}{\bar{v}_{o}} = \frac{\Omega_{b}R_{b}}{\bar{v}_{o}}
\]  

(2.23)

For each tip speed ratio, the aerodynamic conditions at each blade section need to be determined. From these, the performance of the total rotor can be determined. The
results are usually presented as a graph of power coefficient, $C_p$ (defined as the ratio between the rotor power and the dynamic power in the wind) versus the tip speed ratio $\lambda_b$. To capture maximum power at every wind speed, the rotation speed should be changed to keep the $C_p - \lambda_b$ curve at its maximum.

![Figure 2.13: Power coefficient versus tip speed ratio](image)

Variable speed turbine, unlike their constant speed predecessors, can step up the tip speed ratio in relation to the incoming wind speed and hence keep the $C_p - \lambda_b$ curve at its maximum. Thus power captured by variable speed turbines continuously increases with higher wind speeds, as shown in Figure 2.14.

### 2.5.5.4 Power curve

One of the problems of wind power is that the power output is reliant on the vagaries of the weather. At very low wind speeds, there is insufficient torque exerted by the wind to make the turbine blades rotate. The speed at which the blades first starts to rotate and generate power is called the cut-in speed and is typically between 3-4 m/s. As the wind speed rises above the cut-in speed, electrical power output rises rapidly. However, typically somewhere between 12-17 m/s, the power output reaches the limit of the electrical generator. This limit is called the rated power output. The wind speed at which it is reached is called the rated output wind speed.
At higher wind speeds, turbines are designed to prevent any rise of this maximum rotation level. How this is done varies from design to design but two basic options exist, stall and pitch. As the speed increases above the rated output wind speed, the forces on the turbine structure continue to rise. To prevent the rotor being damaged a braking system is employed to bring the rotor to a standstill in these conditions. This is called the cut-out speed and is usually around 25 m/s. A typical power curve is shown in Figure 2.15.

Figure 2.14: Power output as function of rotational speed for different wind speed classes

Figure 2.15: Power curve
2.5 Dynamics of wind turbines

2.5.6 Fatigue

Fatigue is the phenomenon of slow deterioration of a material due to continuously varying loads over time. Wind turbines are vulnerable to fatigue because they are relatively slender and flexible, subject to vibration and resonance, acted upon by loads that are often non-deterministic, operated continuously in all types of weather with a minimum of maintenance, and constantly competing with other energy sources on the basis of life-cycle costs.

Progress in the development of modern wind turbines has progressed with our understanding of fatigue loads and modelling of structural-dynamic responses to unsteady winds (Spera, 2009).

2.5.6.1 S-N curve and Miner rule

To account for fatigue, an empirical design method using the S-N curve and Miner rule is commonly used. The S-N curve for the structural connection can be created by counting the number of cycles required for a component to fail for an array of stress ranges. For welded specimens, only stress variation counts. Mean stress has been shown not to influence the number of cycles to failure. Therefore fatigue loading is expressed by the range rather than level of stress.

When the S-N curve for the detail under consideration is known, calculations of the stresses that the detail will experience during its lifetime can be performed. The stress variations is the number of variations \( n_i \) per stress range. Taking the associated maximum variations \( N_i \) for each stress range from the S-N curve and applying Miner rule, the fatigue damage can be estimated. Miner’s rule states that the cumulative fatigue damage \( D_{fat} \) is equal to the sum of \( n_i \) over \( N_i \) for all stress ranges,

\[
D_{fat} = \sum_i \frac{n_i}{N_i}
\]  

(2.24)

The connection will not fail due to fatigue if \( D_{fat} < 1.0 \). To count the stress range cycles, several methods exist. One of the most popular is the rainflow method. This method reduces a spectrum of varying stress to a set of stress range cycles with number of occurrences which can be used in combination with the S-N curve to calculate the Miner sum.
2. LITERATURE REVIEW

2.5.7 Damping

Damping is an effect that reduces the amplitude of oscillations in a system. In a vibrating structure, damping appears due to energy losses from friction and viscous deformations. Since damping properties have a large influence on the total response of a dynamically loaded structure, it is important that they are accurately determined.

Damping has a significant influence on the turbines vibrations. The overall damping of support structures consists of aerodynamic damping, structural damping, damping due to vortex shedding and damping due to conservative devices. Offshore support structure have additional damping from the soil (as it tends to be softer) and hydrodynamic damping effects.

The material damping of steel comes from internal friction and is often taken as $\zeta = 1\%$. Soil damping consists of internal and geometric soil damping, and is discussed in more detail in section 2.8.

The basics of aerodynamic damping can be illustrated by considering a tower top in motion (Salzmann & van der Tempel, 2005). When the tower top is moving forward, the blades experience a small increase of wind speed and respond to it aerodynamically. The tower top response is such that an extra aerodynamic force will counteract the motion, so the eventual excursion due to the induced velocity will be less. When the tower top moves backward the aerodynamic force decreases, again reducing the tower top’s motion. As this effect is linked to the velocity term in the equation of motion, it is comparable to damping—hence the term aerodynamic damping.

The tower top displacement and the total fatigue damage are reduced by the wind loading on the rotor, however this damping is not present when the turbine is not producing energy (i.e. when blades are idling or parked). It has been calculated by Tempel, J. van der (2000) that, compared to a parked turbine, the fatigue life of the Opti-OWECS support structure (Kühn et al., 1997) will be doubled when the turbine is in operation.
2.6 Wind

2.6.1 Introduction

Winds are movements of air masses in the atmosphere, mainly caused by temperature differences due to uneven solar heating. Wind is a combination of mean wind and turbulent fluctuations about the mean flow. The wind is characterised by its speed and direction, which are affected by several factors, e.g. geographic location, climate characteristics, height above ground, and surface topography.

![Figure 2.16: Wind profile](image)

2.6.2 Wind shear

Wind velocity measured in the field shows variations in space, time and direction. In the lower 2 km of the earth's atmosphere, the atmospheric boundary layer, wind speed is affected by friction with the earth's surface. This effect, known as wind shear, reduces the wind speed from its undisturbed value at 2 km to nearly zero at the surface. From Figure 2.16 it is clear that the mean wind speed increases with height. Furthermore, the actual wind speed at any location varies in time and direction around its mean value due to the effects of turbulence.

To describe the shear effect on the mean wind speed at a certain elevation, two models are commonly used: the logarithmic profile and the power law profile. Both
models are fitted curves to measure wind shear effects. The logarithmic and the power law profiles are given in equations 2.25 and 2.26 respectively,

\[ \bar{v}_m(z) = \bar{v}_{m,r} \left( \frac{l_n \frac{z}{z_0}}{l_n \frac{z_0}{z_r}} \right) \tag{2.25} \]

\[ \bar{v}_m(z) = \bar{v}_{m,r} \left( \frac{z}{z_r} \right)^\beta \tag{2.26} \]

where \( \bar{v}_m \) is the mean wind speed at a height \( z \), \( \bar{v}_{m,r} \) is the mean wind speed at a reference height \( z_r \), \( z_0 \) is the surface roughness and \( \beta \) is the power law coefficient. A comparison of the log and power law profiles is shown in Figure 2.17, for open water conditions with a surface roughness length \( z_0 = 0.001 \) and a power law coefficient \( \beta = 0.11 \)

Figure 2.17: Wind shear according to log and power law model: (a) mean wind speed at 90 m of 11.4 m/s (b) mean wind speed at 10 m of 9.5 m/s
Design programs for turbine load calculations usually demand the input of the mean wind speed at hub height. The wind profile is then adjusted according to the shear theory selected and the terrain where the turbine is to be placed.

### 2.6.3 Turbulence

Wind speed is constantly changing, therefore its main feature is its mean, either over short intervals called gusts (3-10 second), medium intervals (10 minute means), or of longer periods (daily, monthly or yearly). When taking a longer measurement period, the time varying character of the wind can be captured in a spectrum, covering frequency ranges from years to seconds as shown in Figure 2.18.

![Spectral gap](image)

**Figure 2.18: Wind speed spectrum over a broad range of frequencies**

When wind is measured in the field, a time varying wind speed can be found as shown in Figure 2.19a. The turbulence intensity is defined as the standard deviation of the time varying wind speed divided by the mean wind speed, or expressed as a percentage,

\[
I_t = \frac{\sigma_v}{\bar{v}_m} \%
\]

The turbulence's intensity depends on height and the roughness of the terrain. Rougher terrain and lower altitude give higher turbulence intensities. Design standards give descriptions of the turbulence intensity based on these parameters. For design cases fixed turbulence levels can be selected for specific sites.
2. LITERATURE REVIEW

Several models have been fitted to the turbulence spectrum. The von Karman and Kaimal spectra are the most commonly used models (Burton et al., 2001). These models are based on the mean wind speed, the turbulence intensity and a length scale. Figure 2.19 was generated using a Kaimal spectra.

![Figure 2.19: (a) Normalised time varying wind speed (b) Wind velocity PSD](image)

2.6.4 Rotational sampled turbulence

The wind velocity spectrum experienced by rotating blades however is somewhat different to that represented in Figure 2.19. Due to the rotation of the blades, energy is shifted from the lower frequency range to multiples of the rotational frequency of the blades, giving rise to what is known as rotationally sample spectrum. This phenomenon is illustrated in Figure 2.20, taken from Winkelarr (1992). The rotational spectrum is that as experienced by the blades, the redistribution of energy being clearly visible.
2.7 Wave

2.7.1 Introduction

A wave is defined as a disturbance or oscillation that travels through spacetime, accompanied by a transfer of energy. Ocean waves are generated by the action of the wind. Waves in the oceans can travel thousands of miles before reaching land and range in size from ripples to 30 m high swells.

As explained by Stewart (2005), if we begin with a mirror-smooth sea and the wind suddenly begins to blow steadily, three different physical processes begin:

1. The turbulence in the wind produces random pressure fluctuations at the sea surface, which produces waves with wavelengths of a few centimetres.

2. Next, the wind acts on the wave producing pressure differences along the wave profile causing the wave to grow. The process is unstable because, as the wave gets bigger, so does the pressure differences. The instability causes the wave to grow exponentially.
3. Finally, the waves begin to interact among themselves to produce longer waves. The interaction transfers wave energy from short waves generated by Miles mechanism to waves with frequencies slightly lower than the frequency of waves at the peak of the spectrum. Eventually, this leads to waves going faster than the wind, as noted by Pierson & Moskowitz (1964).

To capture this random process numerous models have been developed over the years.

2.7.2 Sea surface description

The motion of the sea’s surface can be studied by isolating a single point on it. By measuring the surface elevation in time at this point, it is possible to represent the random sea motion, as in Figure 2.21a. The time varying signal can be transformed to an power density spectrum as shown in Figure 2.21b.

There are several standard spectra which try to reproduce the actual measured sea spectra at a certain location under certain circumstances. One frequently used is the Pierson-Moskowitz wave spectrum. This shape was fitted to measurements taken in the Atlantic Ocean during long periods of constant environmental conditions and is therefore based on the input of one single parameter: average wind speed. The spectrum describes the sea surface elevation due to the wind speed for a fully developed sea at infinite fetch.

Further measurements of wave spectra were done in the Joint North Sea Wave Project from which the JONSWAP spectrum originated (Hasselmann et al., 1973). This spectrum represents sea states that are not fully developed under a given wind condition. The wave spectrum shape is therefore much more peaked. The JONSWAP spectrum is actually an extended version of the Pierson-Moskowitz spectrum, incorporating a peak enhancement factor, which is controlled by a peak shape parameter $\gamma_w$. When the shape parameter is taken as $\gamma_w = 1$ the JONSWAP spectrum is equal to the Pierson-Moskowitz spectrum. A typical value for non fully developed seas is $\gamma_w = 3.3$.

Although Pierson-Moskowitz and JONSWAP are the most common, other descriptions of wave behaviour exist. By applying an inverse FFT to the generated wave spectrum, the sea surface’s elevation in the time domain can be found.
2.7 Wave

Figure 2.21: (a) Single point time recording of sea surface elevation (b) Wave spectrum of sea surface elevation

2.7.3 Linear wave theory

For harmonic waves, the water particle motion can be described by Airy linear wave theory (Wilson, 1984).

This theory can be used to predict the horizontal velocity $v_w(z, x, t)$ and acceleration $a_w(z, x, t)$ of a water particle as,

$$v_w(z, x, t) = \frac{H_w}{2 \omega_w} E(z) \cos(\omega_w t - \psi_w - kx)$$

(2.28)
2. LITERATURE REVIEW

\[ a_w(z, x, t) = \frac{H_w}{2} \omega^2_w E(z) \sin(\omega_w t - \psi_w - kx) \]  \hspace{1cm} (2.29)

with

\[ E(z) = \frac{\cosh(k(z + d))}{\sinh(kd)} \]  \hspace{1cm} (2.30)

where \( z \) is the vertical coordinate (measured positive upwards from the mean water level), \( d \) is the depth of mean water surface, \( k \) is the wave number, \( \omega_w \) is the wave frequency, \( \psi_w \) is the wave phase lag, \( H_w \) is the wave height and \( x \) is the distance downstream from the wave origin.

Linear wave theory in principle only applies to very small waves. It does not predict kinematics for points above the mean water level since they are not in water. The theory therefore needs to be 'stretched' to cover such points thus extending linear Airy wave theory to provide predictions of fluid velocity and acceleration (kinematics) at points above the mean water level.

The term \( E(z) \) is a scaling factor. However for \( z > 0 \) (i.e. above the mean water level) \( E(z) \) is greater than 1 so it amplifies the velocity. This gives particle velocity predictions that are unrealistically large (especially for high frequency waves). To deal with this problem the Wheeler Stretching method (Wheeler, 1970) is employed. This method stretches (or compresses) the water column linearly into a height equivalent to the mean water depth. This is done by replacing \( z \) by \( z' \),

\[ z' = \frac{d(z + d)}{(d + \eta)} - d \]  \hspace{1cm} (2.31)

where \( \eta \) is the instantaneous water surface elevation. This formula essentially shifts \( z \) linearly to be in the range \(-d\) to 0.

2.7.4 Wave loads on structures

2.7.4.1 Morison’s equation

After selecting the appropriate wave theory, the wave force can be calculated by the Morison’s equation (Wilson, 1984). Morison’s equation is an empirical formula to calculate the hydrodynamic loads on slender members per unit length. The equation is used extensively to estimate the wave loads in the design of oil platforms and other
offshore structures. It is applicable if the ratio of horizontal dimension (of the offshore structure) to wave length is smaller than 0.05, which is usually the case for slender wind turbine towers. The relative velocity of the structure can be incorporated but is ignored here as its magnitude is insignificant compared to the water particle velocities.

The equation is the sum of two force components: an inertia force in phase with the local flow acceleration and a drag force proportional to the square of the instantaneous flow velocity. The equation contains two empirical hydrodynamic coefficients: an inertia coefficient $C_m$ and a drag coefficient $C_d$, which are both determined from experimental data. For a wave with a flow velocity $v_w$, the inline force parallel to the flow direction is given by Morison’s equation as,

$$F_w = \rho C_m V_T a_w + \frac{1}{2} \rho C_d A_T v_w |v_w|$$  \hspace{1cm} (2.32)

where the inertia force is $\rho C_m V_T a_w$ and the drag force is $\frac{1}{2} \rho C_d A_T v_w |v_w|$, $A_T$ is the reference area and $V_T$ is the volume of the body.

### 2.7.4.2 Diffraction

The basic assumption of the Morison’s equation is that the submerged members on which the wave loads are calculated do not affect the waves. As long as the cylinder diameter is relatively small compared to the wave length this assumption is valid. For large diameter structures, like the monopile support structures for offshore wind turbines, placed in relatively shallow water with consequently reduced wave length, the validity of the Morison’s equation can be compromised. The effect a structure has on the wave field is called diffraction. To incorporate this effect in Morison’s equation, the MacCamy-Fuchs correction is introduced (Chakrabarti, 1987). This correction reduces the magnitude of the inertia coefficient. The correction factor is dependent on the ratio of tower diameter over wave length, $d_T/\lambda_w$.

### 2.7.5 Unaligned wind and wave loads

Wind and waves often act from distinct directions. In Figure 2.22, wind-wave-misalignments are shown as absolute values for an example site in the Dutch North Sea (Fischer, 2010). Figure 2.22 shows that small misalignments appear at all wind speeds and
large misalignments appear at lower wind speeds. This because that the wind-wave-correlation at high wind speeds is often combined with fully developed sea states and weather regimes (Fischer, 2010). As a result, for large misalignments the wave periods are closer to the first modal frequency of the support structure, resulting in higher dynamic amplification. This is also amplified by the fact that the side-to-side modes are less damped than the fore-aft ones, as nearly no aerodynamic damping exists in these modes.

In the wind industry, the general trend is for larger rotors that require larger support structures, which as a result tend to have lower first modal frequencies. This leads to an ever closer gap between the wave frequencies and the support structure modal frequencies. Therefore the excitation of the side-to-side modes by wave loading (out-of phase with the wind) is becoming a design issue for offshore multi-megawatt turbines.

Figure 2.22: Absolute value of the misalignment between wind and waves as function of wind speed (shown from 0-30 m/s) and wind speed probability (colour scale), sourced from Fischer (2010)
2.8 Soil-structure interaction

2.8.1 Introduction

It is widely recognized that the dynamic response of a structure on soft soil may be different from the response of a similarly excited structure supported on firm soil (Veletsos & Meek, 1974). When the external forces act on these systems the structural displacements and the ground displacements are linked to each other. The process in which soil and structure influence each other motion is called Soil-Structure Interaction (SSI).

2.8.2 SSI background

The major effects of SSI are flexibility arising from soil compliance and an energy feedback in the form of wave propagation into the soil during vibrations of the structure under investigation. It is especially important to consider interaction effects for massive, stiff, and lightly damped structures.

As Banerjee (1987) shows the main factors that influence the interaction between soil, foundation and the structure are:

- Layering and stiffness characteristics of the soil profile

- Size and flexural rigidity of the foundation

- Depth of foundation embedment

- The inertia characteristics; slenderness and first few natural periods of the structure

Two physical phenomena comprise the mechanisms of SSI: inertia and kinematic interaction. Inertia interaction is developed in the structure due to its own vibrations which gives rise to base shear and moment. These forces transmitted to the foundation produce horizontal and rotational dynamic deformations which affect the structure’s response. Kinematic interaction is due to the presence of stiff foundation elements on or in soil which cause foundation motion to deviate from the free field motions.
2. LITERATURE REVIEW

SSI is ignored when the response of a structure is computed assuming the ground motion at the base of the structure is the same as the free-field motion. Free-field motion is defined as the ground motion that would occur at the level of the foundation if no structure was present. Such an idealization works only if the supporting medium is rigid (inflexible ground). For structures supported on soft soils, the foundation motion is generally different from the free-field motion. The motion delivered at the level of foundation includes rocking and lateral (translational) components of foundation movement. Rocking motion of the foundation can have significant effect on the behaviour of tall structures. For flexibly supported structures significant amount of vibrational energy is dissipated into supporting medium by:

- Radiation of elastic stress waves as they travel away from the foundation. These waves are produced by the deformation of soil due to dynamic structural forces acting on the foundation. This effect is termed as radiation damping and is purely a geometric phenomenon.

- Inelastic behaviour of the supporting soils. This dissipation of energy due to hysteretic action of soils (termed as material or internal damping) depends on the level of strain induced in the soil during vibration. It is usually much smaller than radiation damping. It is analogous to structural damping and can be treated as viscous damping. It is usually assumed to be 5% of critical.

There are two main approaches to solving SSI problems, the direct method and sub-structure approach.

In the direct method, the idealised soil-structure system is analysed in a single step. The direct method of analysis proceeds by applying a consistent free-field ground motion to the boundaries of a discrete model and computing the response of the combined soil-structure system. Hence, a direct method determines the response of the soil and structure simultaneously. Non-linear analyses are possible using the direct approach because the assumptions of superposition are not required. However, non-linear analyses can be sensitive to poorly defined soil parameters and can be computationally intensive.
In practice, substructure analyses are more commonly used to approximate the effects of soil non-linearity with linear methods (Kausel, 2010). The substructure approach divides the SSI problem into a series of simpler problems, which are solved independently, and the results are then superposed to yield the response of the structure. The substructure approach is most often implemented in the frequency domain to account for the frequency dependence of the foundation impedance functions. The major argument for using substructure methods for analysis is that they are simpler. Once the scattering and impedance problems have been solved, they do not have to be repeated, even if the properties of the structure are changed in the design process. The critical step in the substructure approach is the determination of the impedance functions (dynamic-stiffness coefficients) of the soil-foundation system (Jaya & Meher Prasad, 2002). The foundation impedance functions depend upon the geometry of the foundation-soil contact area, the properties of the soil beneath the foundation and frequency of excitation.

Kausel (2010) describes the three steps required in the substructure approach:

• Evaluate the kinematic interaction. This depends on the stiffness and geometry of the soil

• Determine the impedance function. This impedance function describes the stiffness and damping characteristics of soil-foundation interaction

• Evaluate the inertial interaction, where the structure is supported on the impedance determined in Step 2 and is subjected at the base to the motions found in Step 1 (for earthquake loading)

Analytical studies of SSI requires suitable modelling of the soil which should adequately represent the soil stiffness, material and radiation damping. Various models over the years have been developed for foundation vibration analysis (Pradhan et al., 2004) such as:

1. Single degree of freedom mass-spring dashpot model

2. Elastic half-space theory

3. Cone model using the physical concept of wave propagation
2. LITERATURE REVIEW

4. Analytical solutions based on integral transform techniques

5. Semi-analytical and boundary element formulations requiring discretization of top surface only

6. Dynamic finite element methods using special wave transmitting boundaries

7. Hybrid methods combining analytical and finite element techniques

Throughout this work, several of the above mentioned methods (namely methods 3, 4 and 6) are used to analysis the soil-foundation system.

According to Veletsos & Verbic (1973), SSI is usually characterized by the following two major effects:

- Increase in the fundamental period of the fixed base structure which directly follows from considering the flexibility of the foundation soil.

- Change (usually increase) in the effective damping of the fixed base structure which is due to the energy dissipation capacity of soils through radiation and material types of damping.

These effects are observed comparing the dynamic response of rigidly and elastically supported structural systems.

SSI will have negligible effect on the design of most buildings and structures. SSI need only be considered for certain soil conditions and building types. Veletsos & Meek (1974) showed that SSI is warranted (for buildings) if \( \frac{c_s}{f h} < 20 \), where the term \( c_s \) is shear wave velocity of the soil, and the terms \( f \) and \( h \) are natural frequency and height of the fixed-based structure, respectively. If height of the structure is measured in meters, then natural frequency of framed and shear wall buildings can be approximately given by \( \approx \frac{30}{h} \) and \( \approx \frac{45}{h} \) respectively. Using these values, it can be shown that SSI will be important if shear wave velocity of the supporting soil is less than 600 m/s for framed buildings and 900 m/s for shear wall buildings. Luco (1986) in his study on the seismic response of tall chimneys showed that SSI had an effect only for softer soils \( (c_s < 750 \text{ m/s}) \) and could lead to reductions or increases in response, depending on the characteristics of the chimney and the seismic excitation. This has been confirmed in a recent study by Moghaddasi et al. (2011) in which a Monte Carlo simulation was
carried out for a range of SDOF structures and soil conditions excited by a series of seismic excitations. A full review of the development of SSI can be found in Kausel (2010).

A static SSI problem is investigated in Appendix B, whereby a structural analysis code is integrated with a soil-foundation analysis code.

2.8.3 SSI for wind turbines

The use of SSI in design is usually restricted to buildings in seismic zones (Ghosh & Basu, 2004, 2005; Mylonakis & Gazetas, 2000; Seed et al., 1977). However wind turbines contain moving parts and must sustain continuous vibration-induced forces throughout their operational life. Novak & Hifnawy (1983, 1988) showed that the response of a structure when subject to a dynamic wind loading can be affected by SSI. Murtagh et al. (2005b) showed that flexible soil-foundation system \( (c_s < 500 \text{ m/s}) \) can have the effect of reducing a wind turbine’s fundamental natural frequency and introduces a considerable amount of damping to the system. Kühn (2001) showed that the difference between frequency dependent and frequency independent models is small for a specific case of a gravity base foundation for a 3 MW turbine. Bush & Manuel (2009) compared fixed-base and flexible-based foundation models and showed that inclusion of the soil-foundation system can affect the dynamic response of the turbine. Larsen et al. (2005) showed that when pile length exceeds a certain minimum length any further elongation does not contribute to increased stiffness. The OWTEES technical report (Zaaijer, 2005) carried out sensitivity analysis of the predicted natural frequencies of wind turbine support structure for various soil-foundation models, paying particular attention to the uncertainty of several key geotechnical design parameters. The report showed the sensitivity of gravity base foundations to soil parameters is higher than for piled foundations, lending to the presumption that gravity base foundations cannot be uniform within a wind farm, but rather designed for local soil conditions at the location of the foundations. Following this, Zaaijer (2006) investigated the sensitivity of the support structure’s natural frequency to variation in models for pile foundations. However, the literature on the dynamic interaction of wind turbines that takes into account SSI is still somewhat sparse.
2. LITERATURE REVIEW

Wind turbines have increased tremendously in both size and performance in recent years. In order to reduce costs, the overall weight of components is minimized. This results in a more flexible wind turbine sensitive to dynamic excitation even at low frequencies (Liingaard et al., 2007). Modern wind turbines are typically installed with variable speed systems, so the rotational speed of the rotor varies from around 5-15 rpm. Therefore the first excitation frequency interval is around 0.08-0.25 Hz and is referred to as the 1P frequency interval. Since the first resonance frequency of a modern three bladed wind turbine is often placed below or between 1P and 3P, it is important to be able to evaluate the resonance frequencies of the wind turbine structure accurately (Liingaard, 2006).

Usually wind turbine foundations are modelled as fully fixed (soil support is not taken into consideration) or by extending the tower down to an equivalent fixed point (apparent fixity) (Larsen et al., 2005) or by static uncoupled springs (resulting in the foundation stiffness being frequency independent). These idealized assumptions of the boundary conditions can underestimate the damping of the system and lead to overestimation of stiffness and thus the system’s natural frequency. Consequently, unless the separation between the operational and natural frequencies is large, these assumptions of fixity should not be used and SSI needs to be considered in design (Satari & Hussain, 2008). It is necessary to know the structure’s overall natural frequency to allow sufficient separation of its natural frequency from the turbine’s operational frequencies to avoid potentially catastrophic failures (Hamaydeh & Hussain, 2011). Due to the increased variety of excitation frequencies at offshore locations and the larger influence of the foundation on the wind turbine response, modelling of the foundations dynamic behaviour becomes an even more pronounced issue (Zaaijer, 2006).

2.9 Soil-foundation system

2.9.1 Introduction

The soil-foundation system is modelled in this thesis using both analytical and numerical techniques and then coupled to the superstructure using a sub-structure approach.
2.9 Soil-foundation system

To model the soil-foundation system, three methods are used: FEM geotechnical software package Plaxis 3D dynamic, Cone method (developed by Wolf & Deeks (2004b)) and DNV/Risø standards. A brief background on the subject is presented and the three methods discussed.

2.9.2 Background

The classical methods for analysing the vibrations of foundations are based on analytical solutions for massless circular foundations resting on an elastic half-space. The fundamental solution for a homogeneous half-space subjected to a dynamic vertical load on its surface was presented by Lamb (1904). Following on from this, the classical solutions by Reissner (1936), Quinlan and Sung (1954) were obtained by integration of Lamb's solution for a vibrating point load on a half-space (Richart et al., 1970).

Veletsos & Wei (1971) presented the steady state response of a rigid circular disk supported on an elastic half-space excited by a harmonic horizontal and overturning moment. Luco & Westmann (1971) presented the dynamic compliances of a circular footing resting on an elastic half-space for a wide range of dimensionless frequency for the torsional, vertical, rocking and horizontal oscillations of a rigid disc, as well as for the coupling terms between the rocking and horizontal oscillations. Veletsos & Tang (1987) detailed a comprehensive analysis for the harmonic response of vertically excited, massless, rigid ring foundations supported at the surface of an homogeneous elastic halfspace. Wolf & Song (2002) reviewed dynamic SSI and assembled the current simple physical models for determining the impedance.

Whereas analytical solutions may be formulated for surface footings with simple geometries, numerical analysis is required in the case of flexible embedded foundations with complex geometry. During the last decade, with development of high performance computers, various approaches of different accuracy have been proposed. These can be classified as either numerical methods, analytical methods or semi-empirical methods with coupling procedures for modelling of unbounded media, and have been used to investigate the dynamic response of vibrating foundations founded on deformable soil region (Çelebi et al., 2006).

The Finite Element Method (FEM) is very useful for the analysis of structures with local inhomogeneities and complex geometries. However, only a finite region can
2. LITERATURE REVIEW

Figure 2.23: Simple physical models to represent the unbounded soil, after Wolf & Song (2002)

be discretized. Hence, at the artificial boundaries of the unbounded domain, e.g. soil, transmitting boundary conditions must be applied. Liingaard et al. (2007) evaluated the impedance of suction caissons foundation (for offshore wind turbines) using a dynamic three-dimensional coupled boundary element/finite element (BE/FE) program.

2.9.3 DNV/Risø design standards

The DNV/Risø ‘Guidelines for Design of Wind Turbines’ standards not only give details for the design of wind turbine foundations (both spread and pile) but also on the stiffness and damping characteristic of the soil-foundation system.

When estimating the stiffness of the soil-foundation system (especially the dynamic stiffness) it is common to deal with the shear modulus of the soil $G_s$. The standards recommend that this shear modulus be related to the initial shear modulus $G_o$ as a function of the soil shear strain $\gamma_{st}$ as indicated in Figure 2.24.

Under dynamic wind and wave loading, moderate strains in the range of $10^{-2} - 10^{-3}$ are achieved, typically $10^{-3}$. This equates to a shear modulus ratio of $\frac{G_s(\gamma_{st})}{G_o} = 0.4$ and a damping ratio of about $\xi_F(\gamma_{st}) = 0.125 = 12.5\%$, as shown in Figure 2.24.
The standards recommends selecting an initial shear modulus value $G_0$ and then, using Figure 2.24, selecting a design shear modulus value $G_d$ and a damping ratio $\xi_F$. With these parameters established, formulas are then given for the determination of static spring stiffness for the (wind turbine) foundation, in a variety of different soil and embedment conditions.

Only static stiffness formulas are given in the standards which state that “the dynamic stiffness may deviate from the static stiffness...however, for wind and wave loading of wind turbine foundations, on land as well as offshore, the induced vibrations will be of such a nature that the static stiffness will be representative for the dynamic stiffness that are required in structural analyses”. Thus a static spring should give an adequate representation of the foundation for use in aeroelastic wind turbine analyses. This assumption will be tested in this thesis.
2. LITERATURE REVIEW

2.9.4 Cone method

Rigorous procedures for calculating foundation dynamic stiffness exist, including finite element and boundary elements methods. However, these methods require significant computational time and experience, while the Cone model (Wolf & Deeks, 2004b) is simple to use and provides physical insight with conceptual clarity, all within acceptable engineering accuracy. The only approximations made are that of the 1-dimensional strength of materials based on wave propagation in cones. The use of Cone models does lead to some loss of precision compared to rigorous methods based on 3-D elastodynamics. However the accuracy of any geotechnical analysis is limited as many uncertainties can never be eliminated e.g. soil parameters.

Some of the first work carried out on Cone models was done by Ehlers (1942). Here, Ehlers used the Cone method to model the surface of a homogeneous liner elastic half-space for vertical and horizontal motions. Rocking and later torsional motions were examined by Meek & Veletsos (1974) and Veletsos & Nair (1974) respectively. Meek & Wolf (1992) explored the treatment of material discontinuities at the interface of a layer to a half-space. In the work, reflected and refracted waves at the boundaries of layers to a half-space were traced by their own cones and embedded foundations were treated using stacks of embedded disks modelled with double cones. Wolf & Deeks (2004b) proposed a more detailed approach, using the Cone method, to develop formulations of impedance functions of massless rigid foundations, and this method is used in subsequent chapters to calculate impedance functions for several wind turbine foundations.

2.9.4.1 One-dimensional strength of material approach

Cone models are referred to as the one-dimensional strength of material approach. The method is illustrated by considering a circular foundation (disk) on the surface of a layered half-space. Applying a load or moment to the disk leads to stresses that increase with depth in the half space acting on an area. The soil half-space is therefore modelled as a truncated semi-infinite bar with varying area i.e. a cone. The properties of the cone are taken to be the same as those of the half space. A wave propagates downwards
away from the loaded disk. This initial cone is shown in Figure 2.25a. When a discontinuity of the material properties is encountered, the incident wave creates a reflected wave propagating upwards and a refracted wave propagating downwards. Each wave propagates in their own cone, with the cross-sectional area increasing in the direction of wave propagation, as shown in Figure 2.25b.

Subsequently, the reflected and refracted waves act as incident waves, which generate reflected and refracted waves when a discontinuity of the material properties is encountered. The wave pattern can be established by tracking the reflection and refraction of each incident wave sequentially. This complicated wave pattern radiates energy towards infinity outside the dynamic system, therefore the unbounded soil acts as an energy sink, producing damping (radiation damping).

Figure 2.25: Wave propagation in cones: a) Initial cone with outward wave propagation b) Reflected and refracted waves at a material discontinuity propagating in their own cones, after Wolf & Deeks (2004b)

2.9.4.2 Foundation embedded in layered half space

In order to model an embedded foundation, rigid massless disks are placed in the layered half-space in the region which will later be excavated for an embedded cylindrical foundation. This is shown in Figure 2.26.

57
A sufficient number of disks should be chosen to accurately model the dynamic behaviour of the system. At least ten disks per wavelength are recommended. Therefore the maximum vertical distance between two neighbouring disks is given by,

$$\Delta e \leq \frac{\lambda}{10} = \frac{\pi c}{5\omega}$$  \hspace{1cm} (2.33)

where $\omega$ represents the highest frequency the dynamic model must be able to represent and $c$ designates the appropriate wave velocity. In addition disks should coincide with the interface of the half-space.

Figure 2.26: Stack of disks with redundants and free-field motion to represent embedded cylindrical foundation, after Wolf & Deeks (2004b)

The dynamic system consisting of $p$ disks embedded in the layered half-space without excavation (free field) can be regarded as the primary system. The primary system is used for the dynamic analysis based on the force method of structural analysis.

In the centre of the disks, forces acting on each DOF are present. Note that only one type of body wave exists for each DOF. Figure 2.26 shows the horizontal redundant with amplitudes $P_i(\omega)$ leading to horizontal displacements with amplitudes $u_i(\omega)$. The loaded disk on the surface is represented as a single cone whereas each embedded disk is modelled as a double cone, discussed in Wolf & Deeks (2004b).
2.9 Soil-foundation system

The relationship between the redundant forces and moments acting on the disks and displacement and rotations of the disks can therefore be expressed as,

\[ \{u(\omega)\} = [G(\omega)][P(\omega)] \]  \hspace{1cm} (2.34)

where \([G(\omega)]\) represents the dynamic-flexibility matrix of the free field, the matrix is complex and regular. Therefore it can be inverted to obtain the dynamic-stiffness matrix \([S(\omega)]\),

\[ \{P(\omega)\} = [S(\omega)][u(\omega)] \]  \hspace{1cm} (2.35)

In the free field, each disk displaces independent but for a rigid embedded foundation the disks and soil trapped between them are constrained to execute rigid body motion. This constraint is enforced by equating the motion of all disks,

\[ \{u(u_0)\} = \{A(\omega)\}\{u_0(\omega)\} \]  \hspace{1cm} (2.36)

where \(\{A(\omega)\} = \{1 \ 1 \ ... \ 1\}^T\) and \(u_0\) is the initial amplitude.

This results in the force acting on the rigid foundation being equal to the sum of all the forces acting on the disks,

\[ \{Q_o(\omega)\} = \{A(\omega)\}^T[P(\omega)] \]  \hspace{1cm} (2.37)

Thus combining equations 2.35, 2.36 and 2.37 gives,

\[ \{Q_o(\omega)\} = \{A(\omega)\}^T[S(\omega)]\{A(\omega)\}\{u_o(\omega)\} \]  \hspace{1cm} (2.38)

The trapped soil can then be analytically excavated by subtracting the mass of the soil multiplied by the acceleration of the rigid interior cylinder of soil from the force,

\[ P_o(\omega) = \{A(\omega)\}^T[S(\omega)]\{A(\omega)\}\{u_o(\omega)\} - m\ddot{u}_o(\omega) \]  \hspace{1cm} (2.39)

As illustrated in Figure 2.27 for the vertical DOF.

Thus the method works by placing an excitation force on the foundation, and then by tracking the reflected and refracted waves, in order to determine the resultant displacement amplitude of the foundation. The dynamic stiffness can then be found by dividing the amplitude of the applied force by the amplitude of the displacement. A full review and detailed discussion of the theory, implementation and applications of Cone method is given in Wolf & Deeks (2004b) and Wolf & Deeks (2004a).
2. LITERATURE REVIEW

![Diagram](image)

Figure 2.27: Enforcement of rigid-body displacement and excavation of trapped mass, after Wolf & Deeks (2004b)

2.9.4.3 Implementation

In Wolf & Deeks (2004b), where Cone model is developed from scratch, an executable program called “CONAN” and the corresponding MATLAB routines are given. CONAN, given the required input data for the foundation and soil profile, can generate the dynamic stiffness coefficient, for each DOF, for a surface or embedded foundations. The chosen half-space is modelled as a linear elastic material and with hysteretic damping (i.e. constant for all frequency levels), the half-space can consist of any number of horizontal layers $j$ of depth $d_{s,j}$, shear modulus $G_{s,j}$, Poisson’s ratio $\nu_{s,j}$, mass density $\rho_{s,j}$ and material damping $\xi_{s,j}$.

2.9.5 Plaxis finite element analysis

The Plaxis software can be used for 2-Dimensional and 3-Dimensional geotechnical analysis of deformation and stability of soil structures, as well as groundwater and heat flow, in geengineering applications such as excavation, foundations, embankments and tunnels. The 3D Dynamics module enables the effective analyse of the propagation of waves through the soil and their influence on structures. This allows for the analysis
of seismic loading as well as vibrations of the superstructure.

FEM is useful for the analysis of structures with complex geometries. One shortcoming of FEM is that only a finite region can be discretised i.e. the model must have boundaries. To counteract reflections at the boundaries due to stress waves caused by the dynamic loading, absorbent boundaries can be specified at the model boundaries. These act like dampers ensuring that an increase in stress on the boundary is absorbed without rebounding which would interfering in the computed results.

2.9.5.1 Soil models

In FEM, the mechanical behaviour of soils is simulated by means of constitutive models. Where FEM is used in soil-foundation modelling, the right selection of soil model parameters is essential in order to make good predictions in geoengineering projects. Plaxis offers a selection of soil models, three of which have been used extensively in this thesis: “linear-elastic”, “Mohr-Coulomb” and “Hardening soil model with small-strain stiffness”. The linear-elastic soil model simplifies the soil into a material in which there is a linear relation between load and deflection. This soil model is the same as that used in the Cone method.

The Mohr-Coulomb soil model follows the principle of elastoplasticity, in that stresses and strains are decomposed into an elastic part and a plastic part. For stress states within the yield surface, the behaviour is elastic and obeys Hooke’s law and beyond the yield surface, the behaviour is plastic.

In the Hardening soil model with small-strain stiffness, in contrast to an elastic perfectly-plastic model, the yield surface of a hardening plasticity model is not fixed in principal stress space, but can expand due to plastic straining. The model contains two main types of hardening: shear hardening used to model irreversible strains and compression hardening used to model irreversible plastic strains Plaxis (2011). The model has the advantage over the Mohr-Coulomb model in its use of a hyperbolic stress-strain curve instead of a bi-linear curve, and also offering greater control of stress level dependency. When using the Mohr-Coulomb model, the user has to select a fixed value of Young’s modulus whereas for real soils this stiffness depends on the stress level. The small-strain stiffness formulation involves the degradation of the shear
2. LITERATURE REVIEW

Figure 2.28: The Mohr-Coulomb field surface in principal stress space, sourced from Plaxis (2011)

stiffness with shear strain (Brinkgreve et al., 2007). The secant modulus decays non-linearly depending on the amount of shear strain, as depicted in Figure 2.29. The reduction curve is characterized by the small-strain modulus, \(G_0\), and the shear strain at which the secant shear modulus has reduced to \(0.7 \times G_0\).

A detailed discussion of each soil model is given in Plaxis (2011).

Figure 2.29: Reduction of secant shear modulus with shear strain
2.9 Soil-foundation system

2.9.5.2 Drained and Undrained Behaviour

In carrying out any analysis in geotechnical engineering, it is usually necessary to distinguish between drained and undrained loading. In Plaxis it is possible to carry out a drained or undrained analysis.

Drained condition occurs when there is no change in pore water pressure due to external loading. The pore water under excess pressure can drain out of the soil easily, causing volumetric strains in the soil. Undrained condition occurs when the pore water is unable to drain out of the soil. The rate of loading is much quicker than the rate at which the pore water is able to drain out of the soil. As a result, most of the external loading is taken by the pore water leading to an increase in the pore water pressure. The tendency of soil to change volume is suppressed during undrained loading, thus Poisson’s ratio of the soil is assumed to remain at 0.5 throughout loading.

The existence of either drained or undrained condition in a soil depends on: soil type (e.g. fine-grained or coarse-grained), geological formation (fissures, sand layers in clays, etc.) and rate of loading.

Drained analysis appropriate when:

- Soil permeability is high
- The rate of loading is low
- Short term behaviour is not of interest

Undrained analysis appropriate when:

- Soil permeability is low
- The rate of loading is high
- Short term behaviour has to be assessed

For a rate of loading associated with a normal construction activity, saturated coarse-grained soils (e.g. sands and gravels) experience drained conditions and saturated fine-grained soils (e.g. silts and clays) experience undrained conditions. Usually when designing a geotechnical structure, both undrained and drained conditions must be considered to determine which one is more critical.
2. LITERATURE REVIEW
Chapter 3

Model formulation for a HAWT

3.1 Introduction

In this chapter a MDOF HAWT model is formulated. Excitation models are also presented for wind and wave loading, the excitation which a wind turbine will generally be subjected to. Finally, as the aim of this research to investigate the SSI effect on wind turbines, the relevant soil-foundation models are also discussed.

3.2 MDOF HAWT model

A wind turbine is made up of numerous flexible and rigid bodies which interact dynamically through the rotation of the rotor and the vibration of the whole system. The most important aspect in modelling the dynamics of a wind turbine is the accurate coupling of the various components. In terms of modal analysis, the most common approach (to account for the coupling between the separate component), is to formulate the Equations Of Motion (EOM) with an energy method, such as the Lagrangian method.

Many design codes and simulation software packages are available which model the structural behaviour of wind turbines, examples include: FAST, developed by the NREL, Bladed, developed by Garrent Hassen and HAWK 2, developed by Risø.

Lee et al. (2005) outlines the various techniques employed to solve wind turbine dynamic problems.
3. MODEL FORMULATION FOR A HAWT

The mode acceleration approach has been used by Colwell & Basu (2009); Murtagh et al. (2004, 2005a) to develop simplified flapwise and edgewise wind turbine structural dynamic models. In this work, the transfer of shear force from the base of the blades to the tip of the tower provides the basic coupling. Thus the system is represented by 2 DOFs and the resulting equations solved by the mode acceleration method.

The most common approach used by researchers, however, is to formulate the EOM by the Lagrangian approach. This method facilitates the accurate representation of all coupling for the numerous flexible bodies (blades, tower, rotor shaft, etc.) and rigid bodies (i.e. yaw mechanism and control actuators) in two or even three dimensional space, by directly minimising the total energy functions of the dynamical system (Hansen & Kallesøe, 2011). The complexity in the model can be altered depending on the requirements of the analysis carried out.

Hansen (2003) outlines a detained Lagrangian formulation which provides the basis for the models presented in this study. Arrigan et al. (2010) employed a simplified version of this model in an investigation on the flapwise vibration control of wind turbine blades and for edgewise vibration control in Arrigan (2010). Staino et al. (2012b), in examining edgewise vibrations control, used a Lagrangian approach to model the rotor and nacelle of the wind turbine with dampers inside the blade. Quilligan et al. (2012) presented a Lagrangian formulation including coupling between the blade and tower vibrations and in Quilligan & O’Connor (2012) extend this model to included edgewise vibration. Harte et al. (2012) coupled the blade, tower and foundation in the flapwise direction using a Lagrangian formulation.

Here two models are presented: an out-of-plane model, capturing the fore-aft motion of the wind turbine and a more complex coupled in-plane out-of-plane model, capturing the fore-aft and side-to-side motion of the wind turbine.

3.2.1 HAWT model formulation

The MDOF wind turbine model is formulated using the Euler-Lagrange equations, as defined in Clough & Penzien (1995), and expressed in equation 3.1,

$$\frac{d}{dt} \left( \frac{\delta[K.E.]}{\delta q_e} \right) - \frac{\delta[K.E.]}{\delta q_e} + \frac{\delta[P.E.]}{\delta q_e} = Q_e$$  \hspace{1cm} (3.1)
3.2 MDOF HAWT model

$K.E$ is the kinetic energy and $P.E.$ is the potential energy of the conservative forces in the system. The term $q$ is the modal coordinate and $Q_e$ is the generalized loading for DOF $e$.

By formulating the EOM using equation 3.1, the coupling of the system is automatically included in the overall dynamic model.

A graphical representation of the proposed system is shown in Figure 3.1. The model includes two coordinate frames of reference, a local co-rotating system $(x, y, z)$ for each blade and a global system $(X, Y, Z)$ for the combined elements includes the nacelle, tower and foundation.

At the root of each blade exists the origin of the local blade system. The $z$-axis is
3. MODEL FORMULATION FOR A HAWT

the blade axis, while the x and y axes are directed in the flapwise (out-of-plane) and edgewise (in-plane) directions respectively. Similarly for the global coordinate system, the Z-axis runs up along the centroid of the tower with the X and Y axes acting in the fore-aft (transverse) and side-to-side (lateral) directions respectively.

Flapwise blade vibration and fore-aft tower vibration occur in the local “x” and global “X” directions respectively (out-of-plane motion) while edgewise blade vibration and side-to-side tower vibration correspond to the “y” and “Y” axes respectively (in-plane motion).

To apply modal analysis to the blade and tower components, a set of shape functions and natural frequencies must first be derived for free vibration conditions. This is achieved using the NREL disturbed MODES (Buhl, 2005) or BMODES (Bir, 2012), these packages can be used to carry out an eigenvalue analysis of the individual flexible components.

3.2.1.1 Out-of-plane model

A MDOF HAWT model is here developed for horizontal motion corresponding to flapwise blade vibrations. Thus the nacelle/tower can translate in the fore-aft direction. The foundation, resting on flexible soil, is modelled by the introduction of two DOFs allowing it to translate and rotate in the fore-aft direction. Figure 3.2 shows the model along with all the generalized DOFs.

The motion of the tower and blades in the proposed model can thus be considered as a summation of the products of the calculated mode shapes \( \Phi(z) \) and the corresponding temporal tip displacements \( q(t) \) for each particular mode \( e \). The general form is given as,

\[
{u(t,z)} = \sum_{e=1}^{E} \{q^e(t)\Phi^e(z)\} \tag{3.2}
\]

where \( E \) is the total number of modes. The motion of the blades relative to the hub can thus be described by a truncated modal expansion. The absolute displacement of the
rotor blade number $i$ (considering the hub and foundation motion) is given by,

$$ u_{b,i}(z, t) = \sum_{n=1}^{N} q_{b,i}^{n}(t) \Phi_{b}^{n}(z) + \sum_{k=1}^{K} q_{k}^{T}(t) \Phi_{T}^{k}(H_{b}) + q_{F_{H}}(t) + H_{b} q_{F_{p}}(t) \tag{3.3} $$

where $q_{b,i}^{n}$ is the modal blade coordinates describing the motion of blade $i$, relative to the hub, for the $n^{th}$ mode. The notation $N$ and $K$ indicates the number of modes taken for the blades and the tower respectively. The terms $\Phi_{b}^{n}(z)$ and $\Phi_{T}^{k}(Z)$ are the blade and tower mode shape functions respectively and $q_{k}^{T}$ is the modal tower coordinate describing the motion of $k^{th}$ tower mode relative to the foundation. The terms $q_{F_{H}}$ and $q_{F_{p}}$ are the horizontal translation and rotation of the foundation respectively. The hub height is denoted by $H_{b}$. The absolute motion of the tower at a height $Z$ can be described by the following expression,

$$ u_{T}(Z, t) = \sum_{k=1}^{K} q_{T}^{k}(t) \Phi_{T}^{k}(Z) + q_{F_{H}}(t) + Z q_{F_{p}}(t) \tag{3.4} $$

The kinetic and potential energies of the system can be written as,

$$ K.E. = \frac{1}{2} \left[ \sum_{i=1}^{3} \int_{0}^{R_{b}} m_{b}(z)(v_{b,i}(z, t))^{2} dz + M_{v_{T}}(v_{T}(H_{b}, t))^{2} ight] $$

$$ + \int_{0}^{H_{b}} m_{T}(Z)(v_{T}(Z, t))^{2} dz + M_{F} q_{F_{H}}(t)^{2} + I_{F} q_{F_{p}}(t)^{2} \right] \tag{3.5} $$

$$ P.E. = \frac{1}{2} \sum_{i=1}^{3} \left[ \int_{0}^{R_{b}} E I_{b}(z) \left( \frac{\delta^{2} \bar{u}_{b,i}}{\delta z^{2}} \right)^{2} dz + V_{c_{i},i}(z) \right] $$

$$ + \frac{1}{2} \int_{0}^{H_{b}} E I_{T}(Z) \left( \frac{\delta^{2} \bar{u}_{T}}{\delta z^{2}} \right)^{2} dz \tag{3.6} $$

where,

$$ \left( \frac{\delta^{2} \bar{u}_{b,i}}{\delta z^{2}} \right) = \sum_{n=1}^{N} (\Phi_{b}^{n}(z))'' q_{b,i}^{n}(t) \tag{3.7} $$
In equations 3.5 and 3.6, the term \( m_h(z) \) is the mass per unit length of the entire blade-root-hub assembly, \( R_b \) is the length of the blade and \( EI_h(z) \) is the flexural rigidity of the blade in the flapwise direction. The terms \( v_{R_b} \) and \( v_T \) denote the velocity of the rotor blade and tower respectively and the terms \( \bar{u}_{b,i} \) and \( \bar{u}_T \) denote the relative motion of the blade and tower respectively. The mass per unit length of the tower and the flexural rigidity of the tower are represented by \( m_T(z) \) and \( EI_T(z) \) respectively. The notation \( M_{t_{nac}} \), \( M_{F} \) and \( I_{F} \) represent the total lumped mass of the nacelle, foundation and the mass moment of inertia of the foundation about the tilt axes of rotation respectively. The overdots represent differentiation with respect to time and the primes represent differentiation with respect to position.

A centrifugal stiffening term is added into the blade stiffness matrix, resulting in blade stiffness increasing with the rotational speed, \( \Omega_b \). After considering infinitesimal elements along the blade of length \( dz \) and integrating them over the length, the tensile
3.2 MDOF HAWT model

force (in the blade) due to centrifugal body forces on blade \(i\), acting at the point \(\zeta\) can be expressed as,

\[
F_{c,i} = \Omega_b^2 \int_\zeta^{R_b} m_b(z) zdz
\]  

(3.9)

The resulting potential energy, due to centrifugal stiffening of blade \(i\), is given by,

\[
V_{c,i} = \left[ \frac{1}{2} \Omega_b^2 \int_0^{R_b} \left( \sum_{n=1}^{N} q_{b,i}^n \left( \frac{d\Phi_b^m(z)}{dz} \right)^2 \right) \right] \int_\zeta^{R_b} m_b(z) zdz
\]  

(3.10)

The effect of gravity on the vibration of the blades in the flapwise direction is negligible and therefore not included.

The reaction forces due to the soil-foundation interaction \(H(t)\) and \(M(t)\) are non-conservative and cannot be included in equation 3.6, because a potential energy does not exist. The effect of the reaction forces is therefore introduced in the formulation as non-conservative forces in the time domain and can be written in terms of the following convolution integral,

\[
\begin{bmatrix} H(t) \\ M(t) \end{bmatrix} = \int_{-\infty}^{t} \kappa(t) \begin{bmatrix} q_{F_u(t)} \\ q_{F_\theta(t)} \end{bmatrix} d\tau
\]  

(3.11)

where the impulse matrix function \(\kappa(\bullet)\) represents the memory affect and the coupling between the horizontal translation and the rotational DOF of the foundation.

Equation 3.5 and 3.6 can be substituted back into equation 3.1 and the EOM derived for flapwise motion.

The EOM for the coupled foundation/nacelle/blade wind turbine model can be expressed, in standard form, as,

\[
[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{Q(t)\} + [K_1(t)]
\]  

(3.12)

where \(M\), \(C\) and \(K\) are the mass, damping and stiffness matrices of the system respectively. The term \(K_1(t)\) is the non-conservative reaction forces for the soil-foundation system included on the right hand side of equation 3.12. The terms \(\ddot{x}(t)\), \(\dot{x}(t)\) and \(x(t)\) are the acceleration, velocity and displacement vectors and \(Q(t)\) is the generalised loading.
3. MODEL FORMULATION FOR A HAWT

The effect of the dynamic interaction with the foundation, given in equation 3.11, is accounted for by using a coupled horizontal translation and rotation impedance matrix. This is achieved by carrying out the analysis in the frequency domain (i.e. by Fourier transforming the EOM and incorporating the frequency dependent impedance matrix). The impedance matrix is obtained by a Fourier transform of equation 3.11. The response in the time domain will then be obtained by applying an inverse Fourier transform.

3.2.1.2 Coupled in-plane out-of-plane model

The combined model is formulated in a similar fashion to the model presented in the previous section with the inclusion of a number of additional DOFs. These include: side-to-side tower motion, edgewise (in-plane) blade vibration, nacelle tilt, roll and yaw, as well as rotor shaft rotation. An additional two DOFs, allowing the foundation to translate and rotate in the side-to-side direction, are also introduced. A rotational DOF, in the nacelle, describes the elastic torsion of the drivetrain (DT) at the rotor center \( \theta_{x,y} \), relative to the generator which is fixed to rotate at constant speed \( \Omega_f \).

As illustrated in Figure 3.1, the extended model includes two coordinate frames of reference: a local co-rotating system and a global ground-fixed system. Two new subscript notations, \( x \) and \( y \), are added to indicate the fore-aft and side-to-side direction respectively. In Figure 3.1, the orientation of the blade local axes, but not the origin, are shown at the tip of the blade.

Two DOFs describe each blade's individual bending along its length \( z \). This is given by the modal expansion,

\[
\begin{bmatrix}
    u_{b,i,x}(z,t) \\
    u_{b,i,y}(z,t)
\end{bmatrix} = \sum_{n=1}^{N_x} \begin{bmatrix}
    \Phi_{b,x}^n(z) \\
    0
\end{bmatrix} q_{b,i,x}^n(t) + \sum_{n=1}^{N_y} \begin{bmatrix}
    0 \\
    \Phi_{b,y}^n(z)
\end{bmatrix} q_{b,i,y}^n(t)
\]

In the ground fixed frame of reference, with the origin at the base of the foundation,
the position vector of the blade \(i\) can be written as (Hansen, 2003),

\[
\begin{align*}
    r_i &= \left\{ u_{T,x}(H_b,t) + u_{F_{T,x}}(t) + u_{F_{b,x}}(t) \\
    &\quad u_{T,y}(H_b,t) + u_{F_{T,y}}(t) + u_{F_{b,y}}(t) \\
    &\quad 0 \right\} \\
    &+ T_{nac}(t) \left\{ \begin{pmatrix} -L_x \\
    0 \\
    0 \end{pmatrix} + T_{DT}(t)T_{\varphi_i}(t) \begin{pmatrix} u_{b,i,x}(z,t) \\
    u_{b,i,y}(z,t) \\
    z \end{pmatrix} \right\}
\end{align*}
\]

(3.14)

where \(L_x\) is the distance from tower top (or drive-train intersection) to the rotor center. The transformation matrices are given as,

\[
T_{nac}(t) = \begin{bmatrix}
    1 & 0 & 0 \\
    0 & \cos \theta_{T,y} & -\sin \theta_{T,y} \\
    0 & \sin \theta_{T,y} & \cos \theta_{T,y}
\end{bmatrix} \times
\begin{bmatrix}
    \cos \theta_{T,z} & -\sin \theta_{T,z} & 0 \\
    \sin \theta_{T,z} & \cos \theta_{T,z} & 0 \\
    0 & 0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
    \cos \theta_{T,x} & 0 & \sin \theta_{T,x} \\
    0 & 1 & 0 \\
    -\sin \theta_{T,x} & 0 & \cos \theta_{T,x}
\end{bmatrix}
\]

(3.15)

\[
T_{DT}(t) = \begin{bmatrix}
    \cos \theta_{DT,x} & 0 & \sin \theta_{DT,x} \\
    0 & 1 & 0 \\
    -\sin \theta_{DT,x} & 0 & \cos \theta_{DT,x}
\end{bmatrix}
\]

(3.16)

\[
T_{\varphi_i}(t) = \begin{bmatrix}
    \cos \varphi_i & 0 & \sin \varphi_i \\
    0 & 1 & 0 \\
    -\sin \varphi_i & 0 & \cos \varphi_i
\end{bmatrix}
\]

(3.17)

where the term \(\varphi_i = \Omega_b(t) + \frac{2\pi}{3}(i - 1)\) is the mean azimuth angle to blade number \(i\). Matrix \(T_{nac}\) handles the tilt, yaw, and roll of the nacelle, matrices \(T_{DT}\) and \(T_{\varphi_i}\) handle the drive-train torsion and rotor rotation of blade number \(i\). The terms \(\theta_{T,x}, \theta_{T,y}, \theta_{T,z}\) and \(\theta_{DT,x}\), as shown in Figure 3.3, signify nacelle roll, tilt, yaw and drive-train shaft torsion. Note that the use of Euler angles is valid due to the following linearization about the zero steady state: \(\theta_{T,x} = \theta_{T,y} = \theta_{T,z} = \theta_{DT,x} = \theta_{\varphi_i} = 0\), (Hansen & Kallesøe, 2011).
3. MODEL FORMULATION FOR A HAWT

The absolute motion of the tower can be described by the following expression,

\[
\begin{align*}
\begin{cases}
    u_{T,x}(Z,t) \\
    u_{T,y}(Z,t)
\end{cases}
    &= \sum_{k=1}^{K} \left\{ \Phi_{T,x}^{k}(Z)q_{T,x}^{k}(t) + u_{F_{H,x}}(t) + u_{F_{y}}(t) \right\} \\
    &+ \sum_{k=1}^{K} \left\{ \Phi_{T,y}^{k}(Z)q_{T,y}^{k}(t) + u_{F_{H,y}}(t) + u_{F_{y}}(t) \right\}
\end{align*}
\]

The total kinetic and potential energies of the system can be written as,

\[
KE = \frac{1}{2} \left( M_T q_{T,x}^2(t) + I_T q_{T,y}^2(t) + M_T q_{T,y}^2(t) + I_T q_{T,y}^2(t) \right)
+ \frac{1}{2} \int_{0}^{H_b} m_T(Z) \left[ v_{T,x}^2(Z,t) + v_{T,y}^2(Z,t) \right] dZ
+ \frac{1}{2} I_x \dot{\theta}_{T,x}^2 + \frac{1}{2} I_y \dot{\theta}_{T,y}^2 + \frac{1}{2} I_z \dot{\theta}_{T,z}^2
+ \frac{1}{2} M_{tac} \left[ v_{T,x}^2(H_b,t) + v_{T,y}^2(H_b,t) \right]
+ \frac{1}{2} \sum_{i=1}^{3} \left\{ \int_{0}^{R_b} m_b(z) \times |\dot{r}_i|^2 \, dz \right\}
\]

Figure 3.3: Schematics of the combined wind turbine model
3.2 MDOF HAWT model

\[ P E = \frac{1}{2} \int_{0}^{H_b} \left( E I_{T,x}(Z) \left( \frac{\delta^2 \ddot{u}_{T,x}}{\delta z^2} \right)^2 + E I_{T,y}(Z) \left( \frac{\delta^2 \ddot{u}_{T,y}}{\delta z^2} \right)^2 \right) dZ \]

\[ + \frac{1}{2} G_x \theta_{T,x}^2 + \frac{1}{2} G_y \theta_{T,y}^2 + \frac{1}{2} G_z \theta_{T,z}^2 \]

\[ - g_{xy} \theta_{T,y} u_{T,x}(H_b, t) + g_{xy} \theta_{T,x} u_{T,y}(H_b, t) + \frac{1}{2} G_s \theta_{DT,x}^2 \]

\[ + \frac{1}{2} \sum_{i=1}^{3} \left\{ \int_{0}^{R_b} \left[ E I_{b,x}(z) \left( \frac{\delta^2 \ddot{u}_{b,i,x}}{\delta z^2} \right) + E I_{b,y}(z) \left( \frac{\delta^2 \ddot{u}_{b,i,y}}{\delta z^2} \right) \right] dz + V_{c,i} + K_i \right\} \]

(3.20)

where the terms \( I_x, I_y \) and \( I_z \) are the mass moments of inertia of the nacelle about the \( X, Y \) and \( Z \) axes. The terms \( G_x, G_y \) and \( G_z \) are the roll, tilt and yaw stiffness of the nacelle support, \( g_{xy} \) is the coupling stiffness of the nacelle support and \( G_s \) is the torsional stiffness of the drive-train. Gravity acting on the blade contributes to its stiffness and is included in the derivation in the term \( K_i \).

\[ K_i = \frac{1}{2} g \cos(\varphi_i) \int_{0}^{R_b} \left( \sum_{n=1}^{N_y} d_{b,i,x}^{n} \left( \frac{d\Phi_{b,x}^{n}(z)}{dz} \right) \right)^2 + \sum_{n=1}^{N_y} d_{b,i,y}^{n} \left( \frac{d\Phi_{b,y}^{n}(z)}{dz} \right)^2 \right) \int_{\zeta}^{R_b} m_{b}(z) zdz \]

(3.21)

where the subscript \( g \) stand for gravity. The potential energy due to centrifugal stiffening of blade \( i \), is given as,

\[ V_{c,i} = \frac{1}{2} \Omega^2 \int_{0}^{R_b} \left( \sum_{n=1}^{N_x} q_{b,i,x}^{n} \left( \frac{d\Phi_{b,x}^{n}(z)}{dz} \right) \right)^2 + \sum_{n=1}^{N_y} q_{b,i,y}^{n} \left( \frac{d\Phi_{b,y}^{n}(z)}{dz} \right)^2 \right) \int_{\zeta}^{R_b} m_{b}(z) zdz \]

(3.22)

This is an extension of equation 3.10.

The reaction force due to the soil-foundation interaction is included in the potential energy term using the method discussed in the previous section (equation 3.11).
3. MODEL FORMULATION FOR A HAWT

Evaluating the expressions for the potential and kinetic energy of the system, equations 3.19 and 3.20, and then substituting them into the Lagrangian formulation from equation 3.1, produces a dynamic model of the system. Linearisation of the model around an operating point results in a set of linear time varying equations. Thus the EOM for the vibration system can be expressed in standard form as,

\[
\begin{bmatrix}
[M_b] & 0 & 0 & [M_{bt}(\varphi_1)] \\
0 & [M_b] & 0 & [M_{bt}(\varphi_2)] \\
0 & 0 & [M_b] & [M_{bt}(\varphi_3)] \\
[M_{tb}(\varphi_1)] & [M_{tb}(\varphi_2)] & [M_{tb}(\varphi_3)] & [M_{TF}] \\
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\end{bmatrix}
\]

\[
+ 
\begin{bmatrix}
[C_b] & 0 & 0 & [G_{bt}(\varphi_1)] \\
0 & [C_b] & 0 & [G_{bt}(\varphi_2)] \\
0 & 0 & [C_b] & [G_{bt}(\varphi_3)] \\
[G_{tb}(\varphi_1)] & [G_{tb}(\varphi_2)] & [G_{tb}(\varphi_3)] & [G_t] + [C_T] \\
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
[K_b] & 0 & 0 & 0 \\
0 & [K_b] & 0 & 0 \\
0 & 0 & [K_b] & 0 \\
[K_{tb}(\varphi_1)] & [K_{tb}(\varphi_2)] & [K_{tb}(\varphi_3)] & [K_T] \\
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\end{bmatrix}
\]

where \( G \) represents the system gyroscopic forces and the term \( K_I(t) \) is the non-conservative reaction forces of the soil-foundation system. The terms \( M_b, K_b \) and \( C_b \) are the blade mass, stiffness and damping matrices respectively, the terms \( M_{TF}, K_{TF} \) and \( C_{TF} \) are the tower-foundation mass, stiffness and damping matrices respectively and the terms \( M_{bt} \) and \( K_{bt} \) are the azimuth dependent mass and stiffness coupling matrices.

Note that some terms (due to the inclusion of edgewise vibrations) depend on the azimuth angle i.e. are time dependent, further complicating the EOM. Therefore the EOM for the coupled model can not be transformed into the frequency domain and must be solved in the time domain. This has implications on the soil-foundation models that can be used.

3.2.1.3 Structural damping

The Lagrangian formulation does not account for damping, thus the terms in the damping matrix \( C \), used to model the internal energy dissipation, are added separately. Structural damping is specified as stiffness proportional damping, for the particular tower
3.3 Wind excitation

and blades being modelled. The blade or tower, for different turbine systems will have its own associated damping ratio $\zeta$. The damping for the blades is given as,

$$
[C_b] = \frac{2(\xi_b + \xi_{AD_b})}{\omega_{o,b}} [K_b]
$$

(3.24)

where the terms $\xi_b$ and $\xi_{AD_b}$ are the structural and aerodynamic damping ratio of the blade and $\omega_{o,b}$ is the natural frequency of the blade. The damping for the tower is given as,

$$
[C_T] = \frac{2(\xi_T + \xi_{AD_T})}{\omega_{o,T}} [K_T]
$$

(3.25)

where the terms $\xi_T$ and $\xi_{AD_T}$ are the structural and aerodynamic damping ratio of the tower and $\omega_{o,T}$ is the natural frequency of the turbine tower.

3.3 Wind excitation

As wind passes through the turbine’s rotor field it imparts both lift and drag on the blade. The lift is utilised as the main driving force of the blades and acts in their plane of rotation whereas the drag acts in the plane parallel to the primary wind flow i.e. flapwise direction. The wind also imparts a drag force on the turbine tower.

A realistic estimate of the wind loading, to which the rotor is subjected, can be obtained from the Blade Element Momentum (BEM) theory (Hansen, 2001). The model allows for a detailed quantitative description of the rotor behaviour based on the aerodynamic properties of the blade section airfoil, the geometrical characteristics of the rotor, as well as the wind speed and the rotational velocity of the blades.

3.3.1 Blade wind loading

Modal wind drag force time-histories are simulated in accordance with the BEM theory which couples the Momentum theory with local events taking place at the actual blades. BEM is currently the most popular tool for determining aerodynamic loads on a rotating rotor blade (Hansen et al., 2006), as satisfactory results can generally be obtained given good aerofoil data. Here a corrected BEM model, accounting for Glauert correction and Prandtl’s tip loss factor, is used. The dynamic loads are calculated by a quasi-static
3. MODEL FORMULATION FOR A HAWT

aerodynamic assumption so that changes in the effective angle of attack, $\alpha_{AOA}$, are instantly felt in the aerodynamic loads. This means that the time scale for adjustment of the non-stationary flow is assumed to be small compared to the fundamental eigen-period of the blade. Using BEM, it is possible to calculate steady wind loads and thus the thrust and power with respect to wind speed, rotational speed, pitch angle, number of blades and actual geometry of the aerofoil (in terms of blade profile, twist, pitch and chord distribution). The wind loads are calculated following an approach given by Hansen (2001), which assumes all sections are independent along the rotor, so that the aerofoil can be divided into several elements and the flow at each element calculated separately.

The wind inflow, $\vec{v}(t) = \vec{v} + \vec{w}(t)$, is modelled as a stochastic process with a fluctuating component $\vec{w}(t)$, and a mean component, $\vec{v}$, which includes the effects of wind shear. Wind shear is significant for MW turbines where the rotor diameters can extend beyond 100m. The effect of wind shear is accounted for by the power law (Tong, 2010),

$$\vec{v}(Z) = \vec{v}(H_b) \left\{ \frac{Z}{H_b} \right\}^{\beta}$$

in which $\vec{v}(H_b)$ is the mean wind velocity at height $H_b$ and $\vec{v}(Z)$ is the mean wind velocity at a height $Z$ above the ground surface. The exponent $\beta$ is an empirically derived coefficient that according to atmospheric stability, typically taken as 0.11 for offshore applications and 0.143 for open land surfaces.

As the blade rotates, its velocity component experiences a sinusoidal variation in magnitude. The frequency of this sinusoid is equal to the rotational frequency. Therefore the instantaneous wind speed may be expressed as,

$$V_{\omega,t}(r, t) = \vec{v}(H_b) + \Delta v \left( \frac{r}{R_b} \right) \cos(\Omega_b t + \frac{2\pi}{3}(i - 1))$$

where $r$ is the radial distance along the blade to the hub and $\Delta v$ is the change in wind speed between the hub and the top of the blade in an upright position, calculated from the wind shear. The scaling factor $(r/R_b)$ is used to calculate the required amplitude at each point along the blade in order to represent the sinusoidally varying wind velocity above and below the hub.

Therefore the blade is assumed to be discretised into $N$ sections, each element with a chord length $c(r)$ and width $dr$ located a radial distance $r$ from the hub, as shown
3.3 Wind excitation

Figure 3.4: Blade model according to BEM theory approach, after Staino et al. (2012a)

in Figure 3.4. To describe the BEM algorithm for calculating quasi-static aerodynamic wind loads, the following quantities are defined:

\[ V_{rel,i}(r, t) = \sqrt{[V_{o,i}(r, t)(1 - \bar{a}) + \bar{w}(r, t)]^2 + [\Omega_br(1 + \bar{a}')]^2} \] (3.28)

\[ \phi_{FA}(r, t) = \tan^{-1}\left( \frac{(1 - \bar{a})V_{o,b,i}(r, t) + \bar{w}(r, t)}{\Omega_br(1 + \bar{a}')} \right) \] (3.29)

\[ \alpha_{AOA}(r, t) = \phi_{FA}(r, t) - \beta_p(t) - \kappa(t) \] (3.30)

where \( V_{rel,i} \) is relative wind speed, \( \phi_{FA} \) is the flow angle, \( \alpha_{AOA} \) is the instantaneous local angle of attack, \( \beta_p \) is the pitch angle and \( \kappa \) is the local pre-twist of the blade (see Figure 3.5). The stochastic (turbulent) component, \( \bar{w} \), of the wind flow on the rotor plane is added to the steady wind field impacting on the rotor, thus turbulence is included only in the normal direction.

Assuming no radial dependency for the annular sections, i.e. no aerodynamic interactions between different elements, and assuming that the forces on the blade elements depend only on the lift and drag characteristics of its shape, BEM theory provides a method to estimate the axial and tangential induction factors, \( \bar{a} \) and \( \bar{a}' \), respectively. Once these parameters are known, the local loads on each segment can be determined.
3. MODEL FORMULATION FOR A HAWT

The local lift and drag forces can be respectively computed as,

\[ p_{L,i}(r, t) = \frac{1}{2} \rho_a V_{rel,i}^2(r, t) C_{l,b}(\alpha_{AOA}) c(r) \]  
\[ p_{D,i}(r, t) = \frac{1}{2} \rho_a V_{rel,i}^2(r, t) C_{d,b}(\alpha_{AOA}) c(r) \]

where \( \rho_a \) is the density of air, \( C_{l,b}(\alpha_{AOA}) \) and \( C_{d,b}(\alpha_{AOA}) \) are the lift and drag coefficients respectively, whose values are calculated from tables based on the blade aerofoil properties and angle of attack. The aerodynamic forces normal to and tangential to the rotor plane (corresponding to flapwise and edgewise direction) can be obtained by projecting the lift and the drag along the normal and the tangential plane, as shown in Figure 3.5. Therefore the local flapwise and edgewise loads are given, for blade number \( i \), by,

\[ p_{N_{b,i}}(r, t) = p_{L,i}(r, t) \cos(\phi_{FA}) + p_{D,i}(r, t) \sin(\phi_{FA}) \]  
\[ p_{T_{b,i}}(r, t) = p_{L,i}(r, t) \sin(\phi_{FA}) - p_{D,i}(r, t) \cos(\phi_{FA}) \]

The total force acting on the blade can then be computed by performing numerical integration along the length of the blade span. The generalised modal load, in the flapwise and edgewise direction, for blade number \( i \) and the \( n^{th} \) blade mode can be given as,

\[ Q_{b,x,i}^n(t) = \int_0^{R_b} p_{N_{b,i}}(z, t) \Phi_{b,x}^n(z) dz \]  
\[ Q_{b,y,i}^n(t) = \int_0^{R_b} p_{T_{b,i}}(z, t) \Phi_{b,y}^n(z) dz \]

where the position vector \( z \) and \( r \) have been interchanged.

A method for calculating the BEM algorithm for the quasi-static aerodynamic wind loads for each blade element, as given in Staino et al. (2012a), is described in Appendix C. In the method Prandtl tip loss factor and Glauert correction have been applied. The former corrects the assumption, used in the classical blade element momentum theory, of an infinite number of blades, while the latter has been applied in order to compute the induced velocities more accurately when the induction factor \( \bar{a} \) is greater than a critical value \( a_c \).
3.3 Wind excitation

Figure 3.5: Schematic of a blade section showing local forces and velocities according to the BEM model

3.3.2 Tower wind loading

The wind turbine tower acts as a bluff body with regard to the wind flow. Therefore the wind force on the tower can by calculated by estimating the total drag force on the tower. The drag force transferred to a structure in the path of wind is given by (Tong, 2010),

\[ F_{T,wd}(Z, t) = \frac{1}{2} C_{d,T} \rho_a A_T(Z) v_T^2(Z, t) \]  

(3.37)

where \( C_{d,T} \) is the drag coefficient for the tower, \( A_T(Z) \) is the area of the tower as a function of tower height and \( v_T(Z, t) \) is the velocity of the air hitting the tower, as a function of tower height (given by the wind power law) and the time. Thus the generalised modal wind load on the tower, for the \( k^{th} \) mode, can be given as,

\[ Q_{T,x,wd}^k(t) = \int_0^{H_b} F_{T,wd}(Z, t) \Phi_{T,x}^k(Z) dZ \]  

(3.38)

The wind load on the tower is only considered in the fore-aft direction.
3. MODEL FORMULATION FOR A HAWT

3.3.3 Turbulence

3.3.3.1 Homogeneous turbulence

The wind loading on any structural member may be decomposed into a quasi-static mean wind velocity \( \bar{v} \), and a fluctuating turbulent component \( \tilde{w} \). The generation of this fluctuating turbulent component on the rotor plane, is the basis of any aerodynamic simulation and may be obtained through the use of a Power Spectral Density Function (PSDF), as shown by Murtagh et al. (2005a). Nodal fluctuating velocity time-histories can be simulated by virtue of the fact that an arbitrary time-history, with zero mean, may be represented by a Discrete Fourier Transform (DFT) with a discretised version of a continuous frequency content. This is given as,

\[
\tilde{w}(t) = \sum_{k=1}^{\infty} a_k \cos(\omega_k t) + \sum_{k=1}^{\infty} b_k \sin(\omega_k t)
\]

(3.39)

where \( a_k \) and \( b_k \) are the Fourier coefficients, \( \omega_k \) is the discretised circular frequency and \( t \) is the time instant. This fluctuating velocity time-history is generated in conjunction with a wind velocity PSDF. Time-histories can thus be simulated using equation 3.40, a modified version of the spectrum offered by Kaimal et al. (1972), expressed as,

\[
\frac{S_{v}(H_b, \omega)}{\sigma_{v}^2} = \frac{100c}{3\omega(1 + 50c)^{5/3}}
\]

(3.40)

where \( S_{v}(H_b, \omega) \) is the one sided PSDF of the fluctuating wind velocity as a function of the hub elevation and circular frequency, \( \sigma_{v}^2 \) is the variance (related to the turbulence intensity) and \( c \) is known as the Monin coordinate which may be obtained from,

\[
c = \frac{\omega H_b}{2\pi \bar{v}(H_b)}
\]

(3.41)

Given the turbulence intensity and mean wind speed at the hub, the above formulation can be used to generate a turbulence component at the hub height i.e. \( \tilde{w}(t) \). This turbulence may be then applied across the entire rotor to create an an isotropic, homogeneous turbulence field. A sample turbulent wind velocity time history, with the mean removed, is shown in Figure 3.6, for a mean wind speed of 11.4m/s and a turbulence intensity of \( I_t = 10\% \).

However, due to blade rotation, the turbulence spectra will be non-homogeneous in nature leading to a rotation sampled spectra i.e. \( \tilde{w}(r, t) \).
3.3 Wind excitation

3.3.3.2 Rotationally sampled turbulence

According to Rasmussen et al. (2003), the main achievement with respect to wind field modelling for wind turbines was the recognition of the principle of rotational sampling of wind turbulence. A point on a rotating wind turbine blade encounters turbulence whose characteristics are quite different from turbulence measured by a stationary anemometer.

The issue of rotationally sampled turbulence and its influence on the rotating blades was first discussed in Madsen & Frandsen (1984) and Madsen et al. (1987). Connell (1982) noted that the spectrum of the observed turbulence is distorted in several subranges of frequency in characteristic ways. The mid frequency region is depleted and the removed energy is distributed into the high frequency end of the spectrum. The resulting two-peaked continuous spectrum contains narrow-band spikes of turbulent energy centred on the frequency of rotor rotation and multiples of that frequency, as shown in Figure 3.7.

Veers (1988) developed the Sandia method for generation of turbulence fields in the time domain and introduced an improved coherence model. This method is still the basis of the most widely used turbulence generation models (Rasmussen et al., 2003).

In order to generate rotationally sampled turbulence, the wind speed is first synthesized by applying an inverse Fourier transform to the power spectral density, via the Shinozuka technique (Shinozuka & Jan, 1972), to generate a stochastic time series.
3. MODEL FORMULATION FOR A HAWT

Figure 3.7: (a) Turbulence at blade section 35 (NREL 5MW blade) in flapwise direction for blade 1, 2, 3, (b) Spectrum of the rotational sampled turbulence

Then a model of the cross-spectral density is used to estimate the wind that the blade would experience as it rotates through the turbulence field (Manwell & McGowan, 2010). An example turbulent wind velocity time history on a section of blade \( r \), experienced due to rotationally sampled turbulence, i.e. \( \tilde{w}(r, t) \), is shown in Figure 3.7.

3.4 Wave excitation

Ocean waves are produced by the wind. The faster the wind, the longer the wind blows, and the bigger the area over which the wind blows, the bigger the waves. Irregular
3.4 Wave excitation

Ocean waves are often characterised by a wave spectrum that describes the distribution of wave energy (height) with frequency. One such spectrum is the JONSWAP (Joint North Sea Wave Project) spectra. This empirical relationship defines the distribution of energy with frequency within the ocean.

Hasselmann et al. (1973), after analysing data collected during JONSWAP, found that the wave spectrum is never fully developed (as suggested by Pierson and Moskowitz (1964) idealized wave spectra) but continues to develop through non-linear wave-wave interactions even for very long times and distances. Hasselmann et al. (1973) therefore proposed a spectrum in the form,

\[ S_m(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left[ -\frac{4}{5} \left( \frac{\omega_m}{\omega} \right)^4 \right] \exp \left[ -\frac{\omega - \omega_m}{2\sigma^2} \right] \]  

(3.42)

where the term \( \gamma_w \) is the peak enhancement factor (taken as 3.3 for the North Sea conditions), \( g \) is the acceleration of gravity and \( \omega \) is the circular wave frequency. Equation 3.42 defines a stationary Gaussian process of standard deviation equal to one. Wave data collected during JONSWAP were used to determine the values for the constants as follows,

\[ \alpha = 0.076 \left( \frac{U_{10}^2}{gF_{ch}} \right)^{0.22} \]  

(3.43)

\[ \omega_m = 22 \left( \frac{g^2}{U_{10}^2 F_{ch}} \right)^{1/3} \]  

(3.44)

and

\[ \sigma = \begin{cases} 
0.07 & \omega \leq \omega_m \\
0.09 & \omega > \omega_m 
\end{cases} \]  

(3.45)

where \( U_{10} \) is the wind velocity 10 m above sea level. The term \( F_{ch} \) is the fetch, the distance over which the wind blows with constant velocity. The fetch varies in its non-dimensional form as follows (Ditlevsen, 2002),

\[ 10^{-1} < \frac{gF_{ch}}{U_{10}^2} < 10^4 \]  

(3.46)

The JONSWAP wave spectrum can be combined with Airy linear wave theory to
predict the horizontal velocity and acceleration of a water particle as,

\[ v_w(Z, t) = \frac{H_w}{2} \omega_w \frac{\cosh(k(z' + d))}{\sinh(kd)} \eta(t) \]  

\[ a_w(Z, t) = \frac{H_w}{2} \omega_w^2 \frac{\cosh(k(z' + d))}{\sinh(kd)} \eta(t) \]

where \( z' \) is the vertical coordinate (measured positive upwards from the mean water level) corrected using Wheeler Stretching method, \( d \) is the depth of mean water surface, \( k \) is the wave number, \( \omega_w \) is the wave frequency and \( H_w \) is the wave height. The term \( \eta(t) \) is the instantaneous water surface elevation which is a function of the JONSWAP spectrum. The wave frequency can be calculated from the wave period \( T_w \) which in turn can be estimated by the wave height using the following relationship (Chakrabarti, 1987), valid for North Sea conditions,

\[ T_w = 2.48629 \sqrt{H_w} \]

The wave number \( k \) can be determined from the linear dispersion relationship,

\[ \omega_w^2 = (gk) \tanh(kd) \]

and the wave length is given as,

\[ \lambda_w = \frac{2\pi}{k} \]

With the horizontal velocity and acceleration of the water particle known, the wave force acting on the monopile can be calculated by Morison's equation (Wilson, 1984),

\[ F_{T,w}(Z, t) = \frac{1}{2} C_{d,T} \rho_w d_e |v_w(Z, t)| v_w(Z, t) + \frac{1}{4} C_{m,T} \rho_w \pi d_e^2 a_w(Z, t) \]

where \( \rho_w \) is the fluid (sea water) density, \( C_{d,T} \) is the drag coefficient, \( C_{m,T} \) is the inertia coefficient and \( d_e \) is the equivalent diameter of the turbine monopile.

Integrating over the submerged section of the tower, the total modal wave force acting on the offshore structure, for the \( k^{th} \) tower mode, is given by,

\[ Q_{T,x,w}^k(t) = \int_0^{d+\eta(t)} F_{T,w}(Z, t) \Phi_{T,x}^k(Z) dZ \]
3.5 Modelling of load by virtual work

The wave force is only taken in the fore-aft direction. A wind and wave joint distribution can be established by relating the wind speed at a height of ten meters, \( U_{10} \), to that at hub height. This is done via the wind power law given in equation 3.26.

### 3.5 Modelling of load by virtual work

In the formulation of the model, wind, wave and gravity loading have been considered. These excitation forces are modelled as external modal loads.

The total virtual work done by the external wind load on the blade in the flapwise direction is given by,

\[
\delta W_{b,x,wd} = \sum_{i=1}^{3} \int_{0}^{R_b} p_{N_b,i}(z,t) \left( \sum_{n=1}^{N} \Phi_{b,x}^{n} \delta q_{b,x,i}^{n} + \delta q_{T,x} + \delta q_{F_H,x} + \delta q_{F_g,x} H_b \right) dz 
\]

(3.54)

Differentiating the virtual work, \( \delta W_{b,x,wd} \), with respect to the generalized coordinates gives,

\[
Q_{b,x}(t) = \frac{\delta W_{b,x,wd}}{\delta q} 
\]

(3.55)

Therefore the generalized aerodynamic load on the blade, tower and foundation is computed as,

\[
Q_{b,x,i,wd}^{n} = \frac{\delta W_{b,x,wd}}{\delta q_{b,x,i}} = \int_{0}^{R_b} p_{N_b,i}(z,t)\Phi_{b,x}^{n}(z)dz 
\]

(3.56)

\[
Q_{B_{T,x,wd}} = \frac{\delta W_{b,x,wd}}{\delta q_{T,x}} = \sum_{i=1}^{3} \int_{0}^{R_b} p_{N_b,i}(z,t)dz 
\]

(3.57)

\[
Q_{B_{F_H,x,wd}} = \frac{\delta W_{b,x,wd}}{\delta q_{F_H,x}} = \sum_{i=1}^{3} \int_{0}^{R_b} p_{N_b,i}(z,t)dz 
\]

(3.58)

\[
Q_{B_{F_g,x,wd}} = \frac{\delta W_{b,x,wd}}{\delta q_{F_g,x}} = H_b \times \left( \sum_{i=1}^{3} \int_{0}^{R_b} p_{N_b,i}(z,t)dz \right) 
\]

(3.59)
3. MODEL FORMULATION FOR A HAWT

Due to the rotation of the blade in the edgewise direction, the derivation of the modal loads is more complex. The total virtual work done by the external load in the edgewise direction, on blade number \( i \), is given by,

\[
\delta \tilde{W}_{b,y,wd} = \sum_{i=1}^{3} \left[ \int_{0}^{R_b} \rho r_{b,i}(z,t) \right]
\]

\[
\sum_{n=1}^{N} \Phi_{b,y}^{n} \delta q_{b,y,i} + (\delta q_{T,y} + \delta q_{F_H,y} + \delta q_{F_g,y} H_b \cos(\varphi_i)) \right] dz
\]

Differentiating the virtual work, \( \delta \tilde{W}_{b,y,wd} \), with respect to the generalized coordinates gives,

\[
Q_{b,y}(t) = \frac{\delta \tilde{W}_{b,y,wd}}{\delta q}
\]

The generalized aerodynamic load on the blade, tower and foundation, due to the load on the rotor blade, is computed as,

\[
Q_{b,y,i,wd}^{n} = \frac{\delta \tilde{W}_{b,y,wd}}{\delta q_{b,y,i}^{n}} = \int_{0}^{R_b} \rho r_{b,i}(z,t) \Phi_{b,y}^{n}(z) dz
\]

\[
Q_{B_{T,y,wd}} = \frac{\delta \tilde{W}_{b,y,wd}}{\delta q_{T,y}} = \sum_{i=1}^{3} \int_{0}^{R_b} \rho r_{b,i}(z,t) dz \cos(\varphi_i)
\]

\[
Q_{B_{F_H,y,wd}} = \frac{\delta \tilde{W}_{b,y,wd}}{\delta q_{F_H,y}} = \sum_{i=1}^{3} \int_{0}^{R_b} \rho r_{b,i}(z,t) dz \cos(\varphi_i)
\]

\[
Q_{B_{F_g,y,wd}} = \frac{\delta \tilde{W}_{b,y,wd}}{\delta q_{F_g,y}} = H_b \times \left( \sum_{i=1}^{3} \int_{0}^{R_b} \rho r_{b,i}(z,t) dz \cos(\varphi_i) \right)
\]

The component of the gravity force acting in the edgewise direction on an element of length \( dz \) of the blade \( i \), as shown in Figure 3.8, can be given as,

\[
dQ_{b,y,g} = m_b(z) dz \sin(\varphi_i)
\]
3.5 Modelling of load by virtual work

The total virtual work done due to the gravity is obtained as,

$$\delta \hat{W}_{b,y,g} = \sum_{i=1}^{3} \int_{0}^{R_b} m_b(z)gsin(\varphi_i)dz$$

(3.67)

$$\left( \sum_{n=1}^{N} \delta q_{b,y,i}^{n} + \delta q_{T,y} + \delta q_{F_H,g} + \delta q_{F_b,g} H_b cos(\varphi_i) \right) dz$$

since $$\sum_{i=1}^{3} sin(\varphi_i)cos(\varphi_i) = 0$$, differentiating with respect to the generalized vector gives,

$$Q_{b,y,g}(t) = \frac{\delta \hat{W}_{b,y,g}}{\delta q}$$

(3.68)

$$Q_{b,y,i,g}^{n} = \frac{\delta \hat{W}_{b,y,g}}{\delta q_{b,y,i}^{n}} = g \int_{0}^{R_b} m_b(z)\Phi_{b,y,i}^{n}(z)sin(\varphi_i)dz$$

(3.69)

$$Q_{T,y,g} = Q_{F_H,g} = Q_{F_b,g} = 0$$

(3.70)

where $$Q_{b,y,i,g}^{n}$$, $$Q_{T,y,g}$$, $$Q_{F_H,g}$$ and $$Q_{F_b,g}$$ are the components of $$Q_{b,y,b}$$ and represent the generalized load on the blade $$i$$ for the $$n^{th}$$ mode and on the tower and foundation respectively. Thus, the total generalized load in the for the $$n^{th}$$ mode is given as,

$$Q_{b,y}^{n}(t) = Q_{b,y,i,w}^{n} + Q_{b,y,i,g}^{n}$$

(3.71)
Similarly, total virtual work done by the external wind load on the wind turbine tower is given by,

$$
\delta \dot{W}_{T,x,wd} = \int_0^{H_b} F_{T,wd}(Z, t) \left( \sum_{k=1}^{K} \Phi_T^k \delta q_T^k + \delta q_{F_H,x}^k + Z \delta q_{F_w,x}^k \right) dZ \quad (3.72)
$$

Differentiating the virtual work, $\delta \dot{W}_{T,x,wd}$, with respect to the generalized coordinates gives,

$$
Q_{T,x,wd}(t) = \frac{\delta \dot{W}_{T,x,wd}}{\delta q} \quad (3.73)
$$

The total virtual work done by the external wave load on the wind turbine tower is given by,

$$
\delta \dot{W}_{T,x,w} = \int_0^d F_{T,w}(Z, t) \left( \sum_{k=1}^{K} \Phi_T^k \delta q_T^k + \delta q_{F_H,x}^k + Z \delta q_{F_w,x}^k \right) \quad (3.77)
$$

Differentiating the virtual work, $\delta \dot{W}_{T,x,w}$, with respect to the generalized coordinates gives,

$$
Q_{T,x,w}(t) = \frac{\delta \dot{W}_{T,x,w}}{\delta q} \quad (3.78)
$$

The total virtual work done by the external wave load on the wind turbine tower is given by,

$$
\delta \dot{W}_{T,x,w} = \int_0^d F_{T,w}(Z, t) \left( \sum_{k=1}^{K} \Phi_T^k \delta q_T^k + \delta q_{F_H,x}^k + Z \delta q_{F_w,x}^k \right) \quad (3.77)
$$

Differentiating the virtual work, $\delta \dot{W}_{T,x,w}$, with respect to the generalized coordinates gives,

$$
Q_{T,x,w}(t) = \frac{\delta \dot{W}_{T,x,w}}{\delta q} \quad (3.78)
$$

The generalized aerodynamic load on the tower and foundation, arising from the tower loading, is computed as,

$$
Q_{T,x,wd}^k(t) = \int_0^{H_b} F_{T,wd}(Z, t) \Phi_T^k dZ \quad (3.74)
$$

$$
Q_{T,w}^k(t) = \int_0^{H_b} F_{T,w}(Z, t) \Phi_T^k dZ \quad (3.75)
$$

$$
Q_{T,w}^k(t) = \int_0^{H_b} F_{T,w}(Z, t) \Phi_T^k dZ \quad (3.76)
$$

The total virtual work done by the external wave load on the wind turbine tower is given by,

$$
\delta \dot{W}_{T,x,w} = \int_0^d F_{T,w}(Z, t) \left( \sum_{k=1}^{K} \Phi_T^k \delta q_T^k + \delta q_{F_H,x}^k + Z \delta q_{F_w,x}^k \right) \quad (3.77)
$$

Differentiating the virtual work, $\delta \dot{W}_{T,x,w}$, with respect to the generalized coordinates gives,

$$
Q_{T,x,w}(t) = \frac{\delta \dot{W}_{T,x,w}}{\delta q} \quad (3.78)
$$

The generalized aerodynamic load on the tower and foundation, due to the wave loading, is computed as,

$$
Q_{T,x,w}^k(t) = \int_0^{H_b} F_{T,w}(Z, t) \Phi_T^k dZ \quad (3.79)
$$
3.6 Soil-foundation models

\[ QT_{T,x,w} = \frac{\delta W_{T,x,w}}{\delta q_{F,x}} = \int_{0}^{d} F_{T,w}(Z,t) dZ \]  
(3.80)

\[ QT_{F,y,x,w} = \frac{\delta W_{T,x,w}}{\delta q_{F,y,x}} = \int_{0}^{d} F_{T,w}(Z,t) Z dZ \]  
(3.81)

Thus the total modal load on the tower for the \( k^{th} \) mode is given as,

\[ Q_{T,x}^{k}(t) = Q_{B,T,x} + Q_{T,x,w}^{k} + Q_{T,x,wd}^{k} \]  
(3.82)

\[ Q_{T,y}^{k}(t) = Q_{B,T,y,wd} \]  
(3.83)

The total load on the foundation in the fore-aft is given as,

\[ Q_{F,H,x}(t) = Q_{B,F,H,x} + Q_{T,F,H,x,w} + QT_{F,H,x,wd} \]  
(3.84)

\[ Q_{F,y}(t) = Q_{B,F,y,x} + Q_{T,F,y,x,w} + QT_{F,y,x,wd} \]  
(3.85)

and side-to-side direction as,

\[ Q_{F,H,y}(t) = Q_{B,F,H,y} \]  
(3.86)

\[ Q_{F,y}(t) = Q_{B,F,y} \]  
(3.87)

3.6 Soil-foundation models

Soil has a finite stiffness. Therefore assumptions of a fixed foundation support may be unjustified. In any analysis of a foundation structure, it is therefore important to properly model the actual boundary conditions formed by the supporting soils (DNV/Risø, 2001).

Many modelling methods are available, however most of the developed turbines analysis packages do not allow for the modelling of the soil-foundation interaction through detailed nonlinear and depth-dependent models. Instead, simplified linear foundation models suitable for dynamic analysis are used, such as
3. MODEL FORMULATION FOR A HAWT

- Apparent Fixity (AF) model
- Coupled-Spring (CS) model
- Distributed-Spring (DS) model

The first two are used throughout this thesis and are explained below.

3.6.1 Apparent Fixity model

The AF model is a simple approach to model the clamping effect of the soil by replacing the soil and extending the pile to an effective depth below the seabed. Thus the true foundation and the surrounding soil medium are replaced by a cylinder that is fixed not at the original mudline but at a lower depth derived from a point of apparent fixity for the cantilevered cylinder (Passon, 2006). This model is sometimes used for (preliminary) dynamic analysis in the offshore industry, using tabulated values for the effective fixity length. The values proposed by Barltrop & Adams (1991) are given in Table 3.1, as a function of the soil type and pile diameter $D_p$.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Effective Fixity Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiff clay</td>
<td>$3.5 \times D_p - 4.5 \times D_p$</td>
</tr>
<tr>
<td>Very soft silt</td>
<td>$7 \times D_p - 8 \times D_p$</td>
</tr>
<tr>
<td>General calculations</td>
<td>$6 \times D_p$</td>
</tr>
</tbody>
</table>

Table 3.1: Suggestions for the effective fixity length

The great advantages of this model is its simplicity and the sparse information about the soil properties needed.

A more sophisticated AF model can be achieved by solving equation 3.88 (Bush, 2009). Thus, the apparent fixity length $L_{AF}$ and the flexural rigidity $EI_{AF}$ are derived such that the equivalent cantilever (i.e. the extended section of tower) replacing the true soil-foundation system reproduces the same rotation, $\theta$, and lateral deflection, $w$, at its free end (the original mudline) under the shear force and bending moment applied to the foundation head at the mudline.

\[
\begin{align*}
\begin{bmatrix} w \\ \theta \end{bmatrix} &= \begin{bmatrix} \frac{L_{AF}^3}{3EI_{AF}} & \frac{L_{AF}^2}{2EI_{AF}} \\ \frac{L_{AF}^2}{2EI_{AF}} & \frac{L_{AF}}{EI_{AF}} \end{bmatrix} \begin{bmatrix} H \\ M \end{bmatrix}
\end{align*}
\] (3.88)
3.6 Soil-foundation models

3.6.2 Coupled-Spring model

A stiffness matrix can be used to express the stiffness of the soil-foundation system for dynamic wind turbine analysis (Bush, 2009; DNV/Risø, 2001; Zaaijer, 2006). The stiffness matrix (or CS model) idealizes the foundation compliance as a set of translational and rotational DOFs with coupled springs positioned at the connection between the foundation and support structure i.e. the mudline (Jonkman & Musial, 2010). The relevant DOFs for a laterally loaded foundation are the translation in the plane of interest and the rotation about the horizontal axis perpendicular to this plane. The vertical vibration of embedded footings are insignificant for wind-induced structural responses.

An advantage of the CS model is the condensation of foundation properties in a single matrix at mudline. This provides a significant reduction of complexity and results in acceptable loss of accuracy (Zaaijer, 2002). This simplification facilitates the exchange of information between the Geotechnical Engineer and the Structural Engineer (Zaaijer, 2005). Van der Tempel (2006) states that the full complexity of the non-linear soil system is usually unnecessary, as generally the soil relations remain within the linear elastic region, and that a stiffness matrix model gives excellent agreement with full non-linear models.

In reality, soil behaves in a non-linear manner so foundation springs should too. In practice however, it is common to apply linear spring stiffness, in which case the stiffness matrix must be representative for the loading conditions that will be applied.
3. MODEL FORMULATION FOR A HAWT

A static CS model can be written as,

\[
\begin{bmatrix}
H \\
M
\end{bmatrix} = \begin{bmatrix}
K_{HH} & K_{HM} \\
K_{MH} & K_{MM}
\end{bmatrix} \begin{bmatrix}
w \\
\theta
\end{bmatrix}
\] (3.89)

where \(H\) and \(M\) are the horizontal force and moment applied to the foundation, the terms \(w\) and \(\theta\) are the corresponding translation and rotation of the foundation. The terms \(K_{HH}, K_{MM}\) and \(K_{HM} - K_{MH}\) are the horizontal, rotational and coupling foundation stiffness respectively. Equation 3.89 contains three unknowns in the stiffness matrix, since the off-diagonal elements are assumed to be equal. The method of solving equation 3.89 depends on whether the soil is modelled as linear elastic or non-linear material:

- For linear-elastic soil, the horizontal force and overturning moment can be applied to the foundation separately and a CS model derived. The horizontal force and overturning moment are applied independently to the foundation and the horizontal translation and rotation recorded (for each), then by using the flexibility method (and combining the two load cases) the overall stiffness matrix can be calculated. This is further detailed in section 4.2.2.1.

- For non-linear soil, the response depends on all loads applied to the foundation, thus axial load is important. Hence, the stiffness matrix must be obtained around an operating load. The matrix can be obtained by either the flexibility technique or linearized regression around the operating load.

Both frequency dependent and independent CS models are used in this thesis. The frequency dependent CS models are derived from FEM package Plaxis 3D dynamic and Cone method. These CS models with stiffness and damping properties dependent on frequency are referred to as dynamic stiffness matrix or impedance functions. The impedance functions are connected to the structural superstructure via a substructuring and the EOM (due to the frequency dependence of impedance function) must then be solved in the frequency domain.

For the out-of-plane model, equation 3.89 can be rewritten in the frequency domain (Fourier transform of equation 3.11) as,

\[
\begin{bmatrix}
\hat{H}(\omega) \\
\hat{M}(\omega)
\end{bmatrix} = \left[ S_{i,j}(\omega) \right] \begin{bmatrix}
\hat{q}_{FH}(\omega) \\
\hat{q}_{Fa}(\omega)
\end{bmatrix}
\] (3.90)
3.6 Soil-foundation models

where,

\[
[S_{ij}(\omega)] = \begin{bmatrix}
S_{HH}(\omega) & S_{HM}(\omega) \\
S_{MH}(\omega) & S_{MM}(\omega)
\end{bmatrix}
\]  

(3.91)

The terms \(S_{HH}\) and \(S_{MM}\) are the horizontal translational and rotational impedance functions, \(S_{HM}\) and \(S_{MH}\) are the off diagonal coupling impedance terms, such that the impedance matrix satisfies

\[
[S_{ij}(\omega)] = [S_{ij}]^T(\omega)
\]  

(3.92)

and

\[
[S_{ij}(-\omega)] = [S_{ij}]^*(\omega)
\]  

(3.93)

where the superscripts \(T\) and \(*\) represent the matrix operations transpose and conjugate respectively.

Frequency independent CS models are derived from Plaxis and the DNV/Risø standards. Frequency independent models allow the EOM to be solved in either the time or frequency domain. Expanding equation 3.89, for the coupled in-plane out-of-plane model, gives,

\[
\begin{bmatrix}
H_x(t) \\
M_x(t) \\
H_y(t) \\
M_y(t)
\end{bmatrix} = [\kappa(\bullet)]
\begin{bmatrix}
q_{FH,x}(t) \\
q_{Fg,x}(t) \\
q_{FH,y}(t) \\
q_{Fg,y}(t)
\end{bmatrix}
\]  

(3.94)

the (frequency and time independent) static stiffness term \([\kappa(\bullet)]\) can be expanded to give,

\[
[\kappa(\bullet)] = [K^0] = \begin{bmatrix}
K_{HH,x}^0 & K_{HM,x}^0 & 0 & 0 \\
K_{MH,x}^0 & K_{MM,x}^0 & 0 & 0 \\
0 & 0 & K_{HH,y}^0 & K_{HM,y}^0 \\
0 & 0 & K_{MH,y}^0 & K_{MM,y}^0
\end{bmatrix}
\]  

(3.95)

The term \(K^0\) represent the static stiffness of the soil-foundation system. The stiffness of the foundation in the fore-aft and side-to-side direction are usually taken as equal.
3. MODEL FORMULATION FOR A HAWT

3.6.2.1 UnCoupled-Spring model

The CS model can be simplified to an UnCoupled-Spring (UCS). To obtain the stiffness of the spring elements, a force/moment is simply applied to the foundation, as shown in Figure 3.10, and displacement/rotation recorded. The stiffness may then be recovered as,

\[
\begin{bmatrix}
H \\
M 
\end{bmatrix} = \begin{bmatrix}
K_{HH}^0 & 0 \\
0 & K_{MM}^0 
\end{bmatrix} \begin{bmatrix}
w \\
\theta 
\end{bmatrix}
\]

(3.96)

\[
K_{HH}^0 = \frac{H}{w}, \quad K_{MM}^0 = \frac{M}{\theta}
\]

(3.97)

Figure 3.10: Applied force/moment to obtain uncoupled lateral/rotational springs

3.6.2.2 Regressed CS model

Non-linear soil responds in a path-dependent manner. Therefore, in order to record the foundation’s response accurately (in terms of horizontal translation and rotation), all loads on the foundation must be applied. Thus to solve equation 3.89 at least two separate load analysis, which produce independent results, are required.

The stiffness of the soil-foundation system is dependent on the horizontal force \(H\), moment \(M\), axial load \(AL\), soil stiffness \(ST\), soil layering \(SL\), foundation diameter \(DF\), foundation embedment \(De\), foundation stiffness etc.. The static
3.6 Soil-foundation models

stiffness terms given in equation 3.89 can thus be broken down as functions of their dependants (i.e. anything that if changed would influence the soil-foundation stiffness) and written as,

\[
K_{HH}(H, M, AL, \ldots) = a_{H0} + a_{HH} H + a_{HM} M + a_{HAL} AL + \ldots
\]
\[
K_{MM}(H, M, AL, \ldots) = a_{M0} + a_{MH} H + a_{MM} M + a_{MAL} AL + \ldots
\]
\[
K_C(H, M, AL, \ldots) = a_{C0} + a_{CH} H + a_{CM} M + a_{CAL} AL + \ldots
\]

where the \( a \) terms are regression coefficients. Solving the soil-foundation system, for the translation \( w \) and rotation \( \theta \), for a series of different dependant variables it is possible to carry out a multi-variable linear regression analysis and solve for the regression coefficients \( a = \{a_{H0}, a_{HH}, a_{HM}, \ldots, a_C\} \).

Considering several dependant variables, \( H, M, AL, \ldots \) and several cases (or a spread) for each particular dependant. A multi-variable linear regressed CS model can be produced. Combining equation 3.89 and 3.98 gives,

\[
\{F\} = [Rg_m] \{a\}
\]

which on expansion gives,

\[
\begin{bmatrix}
H_{1,1,\ldots} \\
M_{1,1,\ldots} \\
H_{2,1,\ldots} \\
\vdots \\
M_{k,l,\ldots}
\end{bmatrix} =
\begin{bmatrix}
(q_{1,1,\ldots}) w_{1,1,\ldots} & (q_{1,1,\ldots}) \theta_{1,1,\ldots} & \{0\} & \{0\}
& \{0\} & (q_{1,1,\ldots}) w_{1,1,\ldots} & (q_{1,1,\ldots}) \theta_{1,1,\ldots} & \{0\}
& \vdots & \vdots & \vdots & \vdots
& \{0\} & (q_{k,l,\ldots}) \theta_{k,l,\ldots} & (q_{k,l,\ldots}) \theta_{k,l,\ldots} & \{0\}
\end{bmatrix}
\begin{bmatrix}
a_{H0} \\
a_{HH} \\
a_{M0} \\
\vdots \\
a_{C}
\end{bmatrix}
\]

where,

\[
q_{k,l,\ldots} = 1 + H_{k,l,\ldots} + M_{k,l,\ldots} + AL_{k,l,\ldots} + \ldots
\]

where the subscripts \( k \) and \( l \) represent the total spread of the first two dependant variables.

Note, a linear regressed CS model can be generated for zero dependants in which case the regression coefficients are simply the static stiffness terms (i.e \( a_{H0} = K_{HH}^0 \), \( a_{C0} = K_C^0 \) and \( a_{M0} = K_{MM}^0 \)). Thus a regression model can be used to generate the
true solutions of the static stiffness matrix if the number of model coefficients match the number of equations.

A linear regression analysis can easily be carried out in MATLAB using the `regress` function.

\[
\{ a \} = \text{regress} \left( \{ F \}, [R_{CM}] \right) \quad (3.102)
\]

The regressed CS model (depending on the fit) will be accurate for the regression dependants within the spread of the parameters considered. However for non-linear soil, the spread should be limited as the fit is linear. A regressed CS model was presented by Harte & Basu (2012).

### 3.6.3 Static CS model damping

Frequency dependent CS models inherently contain damping. Static CS models however have none and therefore damping must be specified. The soil-foundation system can be said to have a damping ratio \( \xi_F \), therefore the damping for the horizontal translation and rotational DOFs can be written as,

\[
C_{FH} = 2\xi_{FH} \sqrt{M_t F} K_{HH}
\]

\[
C_{F\theta} = 2\xi_{F\theta} \sqrt{I \cdot F} K_{MM}
\]

where \( \xi_{FH} \) is the translation damping ratio and \( \xi_{F\theta} \) is the rotational damping ratio of the soil-foundation system. Note, no damping is specified for the cross coupling terms (unlike the frequency dependant CS model). The damping ratios in this thesis are estimated by two methods: DNV/Risø standards and Plaxis.

By estimating the strain a wind turbine foundation is likely to experience the DNV/Risø standards give an approximate damping ratio of \( \xi_{F, DNV} = 12.5\% \), as shown in Figure 2.24. This standard damping ratio can be applied to soil-foundation system for each DOF i.e. \( \xi_{F, DNV} = \xi_{FH} = \xi_{F\theta} \).

The damping ratio of the soil-foundation system may also be estimated using a free vibration analyses in Plaxis 3D dynamic. Applying a load (or prescribed displacement) to the soil-foundation model and releasing the load the decaying free vibration response of the system may be captured. From this decay (for an underdamped response \( 0 < \xi < \))
1 and assuming the soil-foundation system can be approximated as a SDOF system, the Logarithmic decrement $\delta$ can be estimated,

$$\delta = \frac{1}{n} \ln \left( \frac{x(t)}{x(t + nT_F)} \right)$$  

(3.105)

where $x(t)$ is the amplitude at time $t$ and $x(t + nT_F)$ is the amplitude of the peak $n$ periods away and $n$ is any integer number of successive positive peaks. The damping ratio, as shown in Figure 3.11, can then be given as,

$$\xi_F = \left( \frac{1}{1 + \left( \frac{2\pi}{\delta} \right)^2} \right) \times 100\%$$  

(3.106)

If the damping ratio is high $\xi > 0.3$ (often the case for soil-foundation models), peak counting maybe inaccurate. A better estimation of the damping ratio may be found by plotting the free vibration of a viscous-underdamped SDOF system, given as,

$$u(t) = Ae^{-\xi\omega_0 t} \left( u_0\cos(\omega_F t) + \frac{v_0 + \xi\omega_0 u_0}{\omega_F} \sin(\omega_F t) \right)$$  

(3.107)

over the actual response of the foundation. The curves may be then matched up by changing the damping ratio. From the foundation response (as recorded from Plaxis),
3. MODEL FORMULATION FOR A HAWT

the initial displacement $u_0$, initial velocity $v_0 = 0$ and time period $T_F$ can be estimated.

$$T_F = \frac{2\pi}{\omega_F}$$  \hspace{1cm} (3.108)

$$\omega_F = \omega_{o,F} \sqrt{1 - \xi_F^2}$$  \hspace{1cm} (3.109)

Therefore the correct damping ratio may be estimated by simply finding the best fit. The damping ratio is established for the horizontal translation and rotational DOF separately. An example of a viscous-undamped SDOF system with various levels of damping is shown in Figure 3.12.

![Figure 3.12: Underdamped SDOF system at different damping ratios](image)

3.6.4 FEM model damping

Soil damping is a combination of radiation and material damping. In FEM models, soil radiation damping is inherently included in the system and depends on the soil, foundation type and boundary conditions. However the amount of material damping must be specified.

Rayleigh damping is a very common and efficient type of damping used in finite element analysis (Ju & Niz, 2007). Rayleigh damping is usually presented in the form:

$$[C] = \alpha_F[M] + \beta_F[K]$$  \hspace{1cm} (3.110)
3.6 Soil-foundation models

where \([C]\), \([M]\) and \([K]\) are the damping, mass and stiffness matrix of the physical system. The terms \(\alpha_F\) and \(\beta_F\) are pre-defined Rayleigh damping constants.

Material damping may be specified in Plaxis using Rayleigh damping. Plaxis helps the user to specify the Rayleigh damping constants. Selecting a target damping ratio at two specific frequencies, Plaxis then estimates the \(\alpha_F\) and \(\beta_F\) Rayleigh damping constants.

Hashash (2004) suggests the two target frequencies be taken as the first natural frequency of the soil column \(f_m\) and the frequency of the input motion \(f_n\), see Figure 3.13.

![Figure 3.13: Material damping formulation](image)

Therefore the \(\alpha_F\) and \(\beta_F\) Rayleigh damping constants may be estimated by specifying the target damping ratio usually 5\% for soil (Wolf & Deeks, 2004b), \(f_m\) (given in equation 4.19) and \(f_n\), for wind turbine analysis this can be taken as 1P (the rotor rotation speed).

3.6.5 Bending moment and Shear forces

The shear force and bending moment including SSI effects in the base of the wind turbine tower can be evaluated by isolating the foundation (Veletsos & Tang, 1990). Solving for the response quantities \(\tilde{q}_{FH}(\omega)\) and \(\tilde{q}_{FB}(\omega)\) in the frequency domain, the Fourier transformed shear force and bending moment in the foundation may be evaluated by solving equation 3.90. Thus the Fourier transformed shear force \(\tilde{H}_F(\omega)\) and
bending moment $\hat{M}_T(\omega)$ in the wind turbine tower (at the base) can be computed as,

$$\hat{H}_T(\omega) = -\omega^2 M_F \hat{q}_{FH}(\omega) + \hat{H}(\omega)$$  \hspace{1cm} (3.111)$$

and

$$\hat{M}_T(\omega) = -\omega^2 I_F \hat{q}_{FH}(\omega) + \hat{M}(\omega)$$ \hspace{1cm} (3.112)$$

Solving and inverting back into the time domain yields the time history response of shear force and bending moment (in wind turbine tower base) including SSI effects.
Chapter 4

Finite Element analysis of the foundation using Plaxis

4.1 Introduction

In the previous chapter a dynamic model of an MDOF HAWT with foundation interaction was formulated. In this chapter, stiffness (both static and frequency dependent) and damping characteristics for several soil-foundation models are computed using the FEM software package Plaxis 3D dynamic. Static and dynamic stiffness coefficients for both surface and embedded foundations are first presented. Then, building upon this, dynamic stiffness coefficients for three wind turbine foundations, both onshore and offshore, are given. Finally several multi-variable linear regression CS model are then presented.

4.2 Impedance functions

The first step in a SSI problem is the evaluation of the dynamic stiffness matrix of the foundation. When the soil is much softer than the foundation, the foundation can be assumed to keep its shape while vibrating. Therefore a generalized massless axisymmetric foundation with a rigid base can be described by six DOFs: one vertical, two horizontal, two rocking and one torsional. The six DOFs and the corresponding forces and moments are shown in Figure 4.1. For a harmonic excitation with a frequency $\omega$,
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

the dynamic stiffness matrix $S$ is related to the vector of forces and moments $F$ and the vector of displacements and rotations $U$ as follows (Liingaard et al., 2007):

$$\{F\} = [S]\{U\}$$  \hfill (4.1)

$$\begin{bmatrix}
H_1 \\
H_2 \\
V \\
M_1 \\
M_1 \\
T
\end{bmatrix} =
\begin{bmatrix}
S_{HH} & 0 & 0 & 0 & -S_{MH} & 0 \\
0 & S_{HH} & 0 & S_{MH} & 0 & 0 \\
0 & 0 & S_{VV} & 0 & 0 & 0 \\
0 & S_{MH} & 0 & S_{MM} & 0 & 0 \\
-S_{MH} & 0 & 0 & S_{MM} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & S_{TT}
\end{bmatrix}
\begin{bmatrix}
w_{H_1} \\
w_{H_2} \\
w_V \\
\theta_{M_1} \\
\theta_{M_2} \\
\theta_T
\end{bmatrix}$$  \hfill (4.2)

The components in $S$ are functions of the harmonic frequency $\omega$ and Poisson’s ratio of the soil $\nu_s$. They describe the dynamic behaviour of the foundation and are called dynamic-stiffness coefficients or impedance functions, and are defined as,

$$S_{ij}(\alpha_0) = K_{ij}^{0}[k_{ij}(\alpha_0) + i\alpha_0c_{ij}(\alpha_0)] \quad (i, j = H, M, T, V)$$  \hfill (4.3)

where $K_{ij}^{0}$ is the static value of the $ij$th stiffness component, $\alpha_0$ is the dimensionless frequency coefficient, $k_{ij}$ and $c_{ij}$ are the dynamic stiffness and damping coefficients, respectively. The dimensionless frequency coefficient is defined as,

$$\alpha_0 = \frac{\omega R_F}{c_s}$$  \hfill (4.4)

Figure 4.1: DOF for a rigid surface footing: (a) displacements and rotations and (b) forces and moments
where \( c_s \) is the shear wave velocity of the soil and \( R_F \) is the radius of the foundation. The real part of equation 4.3 is related to the stiffness and inertia properties of the soil-structure system, whereas the imaginary part describes the damping of the system. The dimensionless spring coefficient governs the force that is in phase with the displacement and the dimensionless damping (or dashpot coefficient) describes the force that is 90° out of phase. For a soil without material dissipation, \( c_{ij} \) reflects the geometric damping, i.e. the radiation of waves into the subsoil. In some situations, it is useful to examine the magnitude and phase angle (of equation 4.3) in addition to the real and imaginary parts of the dynamic stiffness. The magnitude (complex modulus) and the phase angle of \( S_{ij} \) are given as,

\[
|S_{ij}| = K_{ij}^0 \sqrt{(k_{ij})^2 + (a_0 c_{ij})^2} \quad (4.5)
\]

\[
\psi_{ij} = \arctan \left( \frac{a_0 c_{ij}}{k_{ij}} \right) \quad (4.6)
\]

### 4.2.1 Surface footing

The static and dynamic stiffness of a circular rigid massless surface footing founded on a homogeneous linear elastic soil are generated using a FEM based soil-foundation model. The results from Plaxis are compared to known analytical results. For surface footings, coupling between the horizontal and rotational stiffness terms is negligible and ignored. Coupling is however important for embedded foundations. Torsion has also been ignored. Equation 4.2 can be rewritten as,

\[
\begin{pmatrix}
V \\
H \\
M
\end{pmatrix} =
\begin{bmatrix}
S_{VV} & 0 & 0 \\
0 & S_{HH} & 0 \\
0 & 0 & S_{MM}
\end{bmatrix}
\begin{bmatrix}
w_V \\
w_H \\
\theta_M
\end{bmatrix} \quad (4.7)
\]

### 4.2.1.1 Plaxis soil model

Model boundaries should be sufficiently distant to avoid disturbances due to boundary conditions. In dynamic analysis, model boundaries are generally taken further away than in a static analysis.
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

After testing, it was found that truncating the boundaries at a distance of eight times the radius of the foundation (from the centre of the foundation in all directions) gave a model which could be solved in a reasonable time frame and whose boundaries had a negligible impact on the results. Again after testing, the mesh was generated under the setting ‘coarse mesh’ with the plate element (surface footing) refined twice. This gave a model with three to four thousand elements and six to seven thousand nodes. This degree of complexity produced good results while allowing dynamic calculation to be done in a reasonable time frame.

To counteract reflections at the boundaries due to the stress waves caused by the dynamic loading, model boundaries were specified as absorbent boundaries. These special boundary conditions are defined to account for the fact that in reality soil is a semi-infinite medium. Without these absorbent boundaries the waves would be reflected on the model boundaries, returning into the model and disturbing the results. Thus the far filed is replaced with dampers, ensuring that increased stress on the boundary is absorbed without rebounding back into the model thus interfering in the computed results. To reduce calculation time, only half of the soil-foundation system is modelled, using symmetric boundary conditions along the lines of symmetry.

The properties of the massless plate and linear-elastic soil are given in Table 4.1, these were taken from Liingaard et al. (2007). A typical soil bulk unit weight is approximately $\gamma_s = 15 - 23$ KN/m$^3$, hence the value taken, $\gamma_s = 10$ KN/m$^3$, is unrealistic in practice. However, for the purpose of analysis the values are specifically arranged so that the dimensionless frequency $a_0$ (in equation 4.4) equals the frequency $f$ in hertz.

$$a_0 = \frac{\omega R_F}{c_s} = \omega \times \frac{5}{31.62} = \omega \times 0.158 = 2\pi f \times 0.158 \simeq f (Hz)$$

(4.8)

Note, for linear-elastic conditions the soil’s Young modulus, shear modulus and shear wave velocity are all linearly related and functions of soil unit weight and Poisson’s ratio. Therefore any combination of soil parameters could have been taken to test Plaxis against the closed form solutions.

4.2.1.2 Static Stiffness

The static stiffness of the foundation is determined by placing a static load on it. From the displacement response, the static stiffness can be generated, as stiffness is simply
4.2 Impedance functions

Table 4.1: Soil and plate properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>Unit</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material model</td>
<td>-</td>
<td>Linear-Elastic</td>
</tr>
<tr>
<td>Soil unit weight, $\gamma_s$</td>
<td>KN/m$^3$</td>
<td>10</td>
</tr>
<tr>
<td>Young modulus, $E_s$</td>
<td>KN/m$^2$</td>
<td>2660</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_s$</td>
<td>-</td>
<td>0.333</td>
</tr>
<tr>
<td>Shear modulus, $G_s$</td>
<td>MPa</td>
<td>1</td>
</tr>
<tr>
<td>Shear wave velocity, $c_s$</td>
<td>m/s</td>
<td>31.62</td>
</tr>
<tr>
<td>Soil material damping</td>
<td>-</td>
<td>5%</td>
</tr>
<tr>
<td>Analysis type</td>
<td>-</td>
<td>Drained</td>
</tr>
<tr>
<td>Groundwater conditions</td>
<td>-</td>
<td>Dry</td>
</tr>
<tr>
<td>Footing radius, $R_F$</td>
<td>m</td>
<td>5</td>
</tr>
<tr>
<td>Young’s modulus, $E_p$</td>
<td>MPa</td>
<td>210</td>
</tr>
<tr>
<td>Plate density, $\gamma_p$</td>
<td>kg/m$^3$</td>
<td>0</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_p$</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Load divided by displacement. Note, that load control has been used throughout for generating static and dynamic stiffness values.

Results

The static stiffness of the foundation for the vertical, horizontal and rotation DOF have been generated and compared to known analytical formulas. The static stiffness from Plaxis was calculated using equation 4.9 for the horizontal and vertical DOFs and equation 4.10 for the rotational DOF. A screen shot from Plaxis is shown in Figure 4.2 for the vertical DOF.

\[
K^0 = \frac{\text{Force}}{\text{Displacement}} \left( \frac{N}{m} \right) \quad (4.9)
\]

\[
K^0_{MM} = \frac{\text{Moment}}{\text{Rotation}} \left( \frac{Nm}{\text{rad}} \right) \quad (4.10)
\]

Note, in order to generate the required moment, counteracting point loads at opposite
ends of the foundation were used. This gave a moment \( 2R_F P_F \), where \( P_F \) is the point load on the foundation.

The analytical formulas for the vertical, horizontal and rotation stiffness of a surface circular plate are given in equations 4.11, 4.12 and 4.13 respectively. These were given originally by Boussinesq, Borowicka and Mindlin respectively (Kausel, 2010).

\[
K_{VV}^0 = \frac{4G_sR_F}{1 - \nu_s} \quad (4.11)
\]
\[
K_{HH}^0 = \frac{8G_sR_F}{2 - \nu_s} \quad (4.12)
\]
\[
K_{MM}^0 = \frac{8G_sR_F^3}{3(1 - \nu_s)} \quad (4.13)
\]

Results from the analysis are presented in Table 4.2. There is very good agreement between the methods.

<table>
<thead>
<tr>
<th>DOF</th>
<th>Unit</th>
<th>Plaxis</th>
<th>Analytical formulas</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{VV}^0 )</td>
<td>MN/m</td>
<td>30.3</td>
<td>29.99</td>
<td>1%</td>
</tr>
<tr>
<td>( K_{HH}^0 )</td>
<td>MN/m</td>
<td>26.18</td>
<td>23.99</td>
<td>9%</td>
</tr>
<tr>
<td>( K_{MM}^0 )</td>
<td>MNm</td>
<td>507.6</td>
<td>499.8</td>
<td>1.6%</td>
</tr>
</tbody>
</table>

### 4.2.1.3 Dynamic Stiffness

Frequency dependant impedance functions for the surface footing can be generated using a similar method i.e. by relating the applied force to the measured displacement response. For a given excitation frequency \( \omega \), a harmonic load is applied to the foundation. This provides a relation between the force \( F(\omega) \) and the resulting displacements \( U(\omega) \), through the equation,

\[
S(\omega) = F(\omega)/U(\omega) \quad (4.14)
\]

The force can be expanded and given as,

\[
F(\omega, t) = A.sin(\omega t) \quad (4.15)
\]
4.2 Impedance functions

which for a constant frequency changes with time $t$. $A$ is the amplitude of the forcing function. The response of the soil directly under the foundation center is given by,

$$U(\omega, t) = B \sin(\omega t + \psi)$$  \hspace{1cm} (4.16)

where $B$ is amplitude of the displacement response and $\psi$ is the phase difference. Note, $\omega$ is the circular frequency in units rad/s. An example of a harmonic forcing function on the footing and the resulting displacement response of the soil is shown in Figure 4.3.

The phase difference between the two signals can be generated by calculating the time delay between the two signals $T$. The phase of the displacement response is given by,

$$\psi = \omega T$$  \hspace{1cm} (4.17)

The magnitude of the complex stiffness is acquired from the ratio of the force and
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

Figure 4.3: Vertical forcing function (1 KN) operating at a frequency of 2 Hz and the resulting vertical soil displacement response

displacement amplitudes i.e. the stiffness.

\[ |S_{ij}| = K_{ij}^{0} \sqrt{(k_{ij})^2 + (a_0 c_{ij})^2} = \frac{A}{B} \left( \frac{N}{m} \right) \] (4.18)

Results

Following the method provided, for a range of excitation frequencies \( a_0 = [0 - 6] \), impedance functions for the circular surface footing are generated for the vertical, horizontal and rotational DOFs and are given in Figures 4.4 and 4.5. For comparison, the results are plotted against the wave-propagation based Cone method (Wolf & Deeks, 2004b) and known analytical solutions (Veletsos & Tang (1987) and Veletsos & Wei (1971)). The results show good agreement generally.
4.2 Impedance functions

Figure 4.4: Vertical dynamic stiffness of a surface footing, generated using Plaxis, Cone method and analytical formulas

4.2.1.4 Low Frequency Estimates

To validate the results predicted by Plaxis, an estimation of the dynamic stiffness at a very low frequency was carried out. At very low frequencies the static and dynamic results should match exactly.

A sample test was carried out at a frequency of 0.01 Hz. From this test, the maximum displacement returned was 0.003 m, almost 20% bigger than that from the static analysis (0.0026 m), indicating the entire mesh had undergone rigid body displacement. Issues associated with the use of viscous boundaries for axisymmetric problems are outlined in Deeks & Randolph (1994).

The increase in vertical displacement is due to the damping at the viscous boundaries which, for frequencies under the natural frequency of the soil column, are known to give poor results (Kontoe, 2006). Therefore applied excitation frequencies were kept above the natural frequency of the soil domain, where possible. The natural frequency
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

Figure 4.5: Horizontal and rotational dynamic stiffness of a surface footing, generated using Plaxis, Cone method and analytical formulas

of a soil column is given by,

\[ |f_{sc,n}| = \frac{c}{H_{sc}} \left(1 + \frac{2n}{H_{sc}} \right) \]  \hspace{1cm} (4.19)

where \( n = 0 \) for the first natural frequency, \( n = 1 \) for the second natural frequency and so on. The term \( H_{sc} \) is the height of the soil column (\( R_F \times 8 \)) and \( c \) is the wave velocity which depends on the loading, for vertical loading \( c \) is taken as the shear wave velocity \( (c_s = 31.3 \, \text{m/s}) \) of the soil and for horizontal loading \( c \) is taken as the dilation wave velocity \( (c_p = 62.56 \, \text{m/s}) \) of the soil. This implies that for vertical harmonic loading no frequencies lower than 0.2 Hz should be applied, and for horizontal harmonic loading no frequencies lower than 0.39 Hz to this model.

Another test was carried out at a frequency of 0.01 Hz but for a model with a much larger domain (boundaries at \( R_F \times 20 \) in all directions) and fixed boundaries i.e.
zero displacement at the boundaries. The maximum displacement amplitude under the foundation was 0.00256 m, an exact match to the static results. The time history from Plaxis output is shown in Figure 4.6.

![Figure 4.6: Plot from Plaxis 3D output of dynamic time versus soil vertical displacement response (directly under the center of the footing), generated by a 1 KN dynamic vertical force at an excitation frequency of 0.01 Hz](image)

### 4.2.1.5 Surface footing on soil with stiffness linearly increasing with depth

The static and dynamic stiffness of a surface footing founded on linear elastic half-space with Stiffness Linearly Increasing With Depth (SLIWD) is established. The same soil-foundation model is analysed examining the same DOFs. However in this model the Young's modulus of the soil is not constant but instead increases linearly with depth, the stiffness of the soil increases by 10% per meter depth, in Plaxis this is modelled in the advanced settings, \( E_{\text{inc}} = E_s \times 0.1 \text{ KN/m}^2/\text{m} \). The static stiffness results for each DOF are given in Table 4.3 and compared to those derived by the Cone method (with similar soil conditions). The impedance functions for each DOF, for a range of excitation frequencies, \( a_0 = [0 - 6] \), are given in Figures 4.7 and 4.8. The results are again compared to Cone method. The Cone method approximate soil model (soil only increases in stiffness up until a finite depth) was taken as a 46 layered material. The first 45 layers at 1 m thickness, in each subsequent layer the shear modulus is increased by 10%. The last layer was taken as a homogeneous half-space of constant
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

stiffness. Analytical solutions from Veletsos & Tang (1987) and Veletsos & Wei (1971) for a surface footing founded on soil with a constant stiffness are also included as a reference.

Results

The results generally agree. Increased soil stiffness only has a significant effect on the vertical impedance function.

Table 4.3: Static Stiffness for surface footing founded on soil with SSLIWD

<table>
<thead>
<tr>
<th>DOF</th>
<th>Unit</th>
<th>PLAXIS</th>
<th>Cone method</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{VV}^0$</td>
<td>MN/m</td>
<td>34.75</td>
<td>31.54</td>
<td>9.2%</td>
</tr>
<tr>
<td>$K_{HH}^0$</td>
<td>MN/m</td>
<td>45.4</td>
<td>51</td>
<td>11%</td>
</tr>
<tr>
<td>$K_{MM}^0$</td>
<td>MNm</td>
<td>614</td>
<td>598.9</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

Figure 4.7: Vertical dynamic stiffness of a surface footing on soil with SLIWD, generated using Plaxis, Cone method and analytical formulas
4.2 Impedance functions

Figure 4.8: Horizontal and rotational dynamic stiffness of a surface footing on soil with SLIWD, generated using Plaxis, Cone method and analytical formulas

4.2.2 Embedded Foundation

The response of a rigid massless cylindrical foundation embedded in a homogeneous linear elastic soil is now examined, shown in Figure 4.9. The stiffness values for various DOFs are again estimated using Plaxis and compared to Cone method results. For an embedded foundation, the horizontal and rotational stiffness are coupled. Examining the foundation in one plane only equation 4.2 can be rewritten as,

$$
\begin{bmatrix}
V \\
H \\
M
\end{bmatrix} =
\begin{bmatrix}
S_{VV} & 0 & 0 \\
0 & S_{HH} & S_{HM} \\
0 & S_{MH} & S_{MM}
\end{bmatrix}
\begin{bmatrix}
w_V \\
w_H \\
\theta_M
\end{bmatrix}
$$

(4.20)
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

Figure 4.9: Cylindrical embedded foundation

4.2.2.1 Static Stiffness

Static stiffness coefficients are next estimated for an embedded ratio range of $D_e/R_F = 0 - 2$, where $D_e$ is the vertical embedment. This was done using a similar soil-foundation model to that given in the previous section.

A screen shot of a Plaxis model is shown in Figure 4.10 for a foundation with an embedment ratio of $D_e/R_F = 2$. Stiffness coefficients for the vertical DOF can be easily generated using methods previously discussed. However, the results for the horizontal and rotational DOF must be combined to estimate coupling stiffness terms.

To generate a stiffness matrix with coupling on linear-elastic soil, two separate analysis are performed: one with a horizontal load on the base of the foundation and another with a set of opposing vertical forces applied at each side of the foundation in order to create a rocking moment. The first analysis provides a relation between the horizontal force and the resulting displacements and rotations. The second analysis relates the applied moment to the resulting displacements and rotations (Liingaard et al., 2007). This is shown in Figure 4.11 for a monopile foundation (as it is easier to depict than a more complex foundation).

The stiffness matrix can be written as,

$$
\begin{pmatrix}
H \\
M
\end{pmatrix} = \begin{bmatrix}
S_{HH} & -S_{HM} \\
-S_{MH} & S_{MM}
\end{bmatrix}
\begin{pmatrix}
w \\
\theta
\end{pmatrix}
$$

(4.21)
4.2 Impedance functions

Figure 4.10: Screen short from Plaxis showing the soil-foundation model loaded vertically for embedment ratio of $D_e/R_F = 2$

The coupling terms (within the precision of the model) are taken to be equal i.e., $S_{HM} = S_{MH} = S_C$. Equation 4.21 can then be solved simultaneously. Applying a horizontal force $H$ gives the following equation,

$$\begin{pmatrix} H \\ 0 \end{pmatrix} = \begin{bmatrix} S_{HH} & -S_C \\ -S_C & S_{MM} \end{bmatrix} \begin{pmatrix} w_H \\ \theta_H \end{pmatrix}$$

(4.22)

where the terms $w_H$ and $\theta_H$ are the horizontal displacement (directly under the foundation) and the rotation of the foundation respectively, caused by the horizontal force $H$. Isolating the coupling stiffness term,

$$S_C = \left( \frac{S_{MM} \theta_H}{w_H} \right)$$

(4.23)
and substituting back into equation 4.22, the horizontal force can be written as,

\[ H = S_{HH}w_H - \left( \frac{S_{MM} \theta_H}{w_H} \right) \theta_H \]  \hspace{1cm} (4.24)

Similarly for rotation, it can be shown that,

\[ M = - \left( \frac{S_{HH}w_M}{\theta_M} \right) w + S_{MM} \theta_M \]  \hspace{1cm} (4.25)

Combing equations 4.24 and 4.25 gives the following simultaneous equation,

\[
\begin{pmatrix}
S_{HH} \\
S_{MM}
\end{pmatrix} = \frac{1}{w_H^2 \theta_M^2 - w_M^2 \theta_H^2} \begin{pmatrix}
\theta_M^2 & \theta_H^2 \\
w_M^2 & w_H^2
\end{pmatrix} \begin{pmatrix}
Hw_H \\
M \theta_M
\end{pmatrix}  \hspace{1cm} (4.26)
\]

which can be solved for the horizontal and rotational stiffness terms \( S_{HH} \) and \( S_{MM} \). Back substituting, the coupling stiffness terms are then estimated, for \( H = 0 \) and \( M = 0 \), and the average value taken.

**Results**

Figure 4.12 and Table 4.4 shows the static stiffness values for each DOF for the circular footing at different embedments. In Figure 4.12, the results are given as the
ratio of the embedded foundation's static stiffness to that of the surface foundation for
the same DOF. The coupling terms are not included here as there is no equivalent for a
surface footing and are given separately in Figure 4.13. The results are compared to the
Cone model and to empirical formulas given by Pais & Kausel (1988). The empirical
formulas, which are applicable for $D_e/R \leq 2$, are given in equations 4.28 to 4.29.

$$K_{VV} = \frac{4G_s R_F}{1 - \nu_s} \left(1 + 0.54 \frac{D_e}{R_F}\right) \quad (4.27)$$

$$K_{HH} = \frac{8G_s R_F}{2 - \nu_s} \left(1 + \frac{D_e}{R_F}\right) \quad (4.28)$$

$$K_{MM} = \frac{8G_s R_F^3}{3(1 - \nu_s)} \left(1 + 2.3 \frac{D_e}{R_F} + 0.58 \left(\frac{D_e}{R_F}\right)^3\right) \quad (4.29)$$

Figure 4.12: Static stiffness terms for a cylindrical embedded foundation, at several
embedment ratios

4.2.2.2 Dynamic stiffness

Impedance functions for an embedded foundation with an embedment ratio of $D_e/R_F = 2$ are calculated for each DOF. The results are generated for a full and a half soil do-
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

Figure 4.13: Normalised coupling static stiffness ($K_C^0/G_sR_F^2$) for a cylindrical embedded foundation, at several embedment ratios

Table 4.4: Static stiffness values from Plaxis, normalised with regard to shear modulus $G_s$ and the foundation radius $R_F$

<table>
<thead>
<tr>
<th>$D_c/R_F$</th>
<th>$K_{VV}^0/G_sR_F$</th>
<th>$K_{HH}^0/G_sR_F$</th>
<th>$K_{MM}^0/G_sR_F^2$</th>
<th>$K_C^0/G_sR_F^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.11</td>
<td>4.88</td>
<td>4.02</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>7.7</td>
<td>7.5</td>
<td>8.3</td>
<td>-0.82</td>
</tr>
<tr>
<td>1</td>
<td>9.03</td>
<td>9.15</td>
<td>14.14</td>
<td>-2.62</td>
</tr>
<tr>
<td>2</td>
<td>11.4</td>
<td>11.75</td>
<td>41.98</td>
<td>-8.89</td>
</tr>
</tbody>
</table>

main, to investigate the whether a symmetric model in Plaxis can accurately capture the dynamic motion of an embedded foundation, see Figure 4.14. As for axisymmetric problems the a full and half soil domain should in theory give identical solutions. For all the dynamic calculations carried out so far, half of the footing only has been modelled, using symmetric boundary conditions along the lines of symmetry, to save computation time.

The full model had absorbent boundaries specified at each boundary apart for the soil surface. This was also done for the half model apart from at the plain of symmetry where no absorbent boundaries were applied. A surface fixity was also applied to the foundation in the half model. This surface fixity allowed motion in-plane but fixed (or prevented) motion in the out-of-plane direction, as shown in Figure 4.14. The mesh for
4.2 Impedance functions

each model was kept as consistent as possible to ensure comparable results. However, this proved difficult as Plaxis has some issues regarding the meshing of cylindrical structures due to it’s triangular finite elements. The corners of the structure elements peak out because they cannot enclose a perfectly circular shape (Hansen et al., 2012).

Figure 4.14: Screen shot from Plaxis showing the soil-foundation model for the full and symmetric model

**Results**

Impedance functions for the vertical, horizontal, rotation and coupling DOFs are presented in Figures 4.15 and 4.16 for the whole and half Plaxis models. The results have also been plotted against Cone method for comparison. The Cone method soil model was taken as a 21 layered material of constant stiffness. The first 20 layers at 0.5 m thickness \((20 \times 0.5 = 10 \text{ m} = D_c)\) with the last layer being modelled as a homogeneous half-space.

The results agree well indicating that the half models can accurately capture the dynamics of the system, with the proper boundary conditions specified. However, the results should be an exact match and at some dimensionless frequencies the normalised stiffness values varies substantiation between the two models, particularly at \(a_0 = 3.5\). To explore the differences in the normalised stiffness values Table 4.5 was constructed.

\[
\text{Percentage Difference} = \frac{\text{Difference}}{\text{Average}} \times 100
\]  

(4.30)
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

Table 4.5: Sensitivity study

<table>
<thead>
<tr>
<th>Variable</th>
<th>Half model</th>
<th>Full model</th>
<th>Perc. Diff. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_H$ (m)</td>
<td>0.001113</td>
<td>0.001074</td>
<td>-3.62</td>
</tr>
<tr>
<td>$\theta_H$ (deg)</td>
<td>0.000138</td>
<td>0.000149</td>
<td>7.87</td>
</tr>
<tr>
<td>$w_M$ (m)</td>
<td>0.000672</td>
<td>0.000734</td>
<td>8.81</td>
</tr>
<tr>
<td>$\theta_M$ (deg)</td>
<td>0.000174</td>
<td>0.000184</td>
<td>5.21</td>
</tr>
<tr>
<td>$S_{HH}/K_{HH}^0$</td>
<td>4.76</td>
<td>6.78</td>
<td>35.08</td>
</tr>
<tr>
<td>$S_{MM}/K_{MM}^0$</td>
<td>2.13</td>
<td>2.17</td>
<td>1.73</td>
</tr>
<tr>
<td>$S_C/K_C^0$</td>
<td>5.33</td>
<td>8.25</td>
<td>43.03</td>
</tr>
</tbody>
</table>

Examining Table 4.5, the displacements and rotations percentage difference varies between ±10%, this can be explained by the slightly different mesh’s of the two models, which were generated automatically by Plaxis. Therefore the presented method for obtaining the stiffness variables is slightly ill-conditioned.

![Graph](image)

Figure 4.15: Vertical dynamic stiffness of an embedded footing ($D_c/R_F = 2$), generated using Plaxis (using a full and half model) and Cone method.
4.2 Impedance functions

Figure 4.16: Dynamic stiffness of an embedded footing \((D_e/R_F = 2)\), generated using Plaxis (using a full and half model) and Cone method.
4.3 Wind turbine foundations

Three types of wind turbine foundations have been analysed, an onshore Gravity Based Foundation (GBF), an offshore Suction Caisson Foundation (SCF) and an offshore MonoPile Foundation (MPF). The impedance functions are generated assuming the foundations are rigid and embedded in a homogeneous linear-elastic half-space. The analysis has been carried out using the previously discussed methods.

The details of the foundation and soil type are given in Table 4.6. Since the soil is modelled as a linear-elastic material we are only interested in the stiffness properties of the soil. The GBF is taken to be onshore and embedded in a dry material, hence an effective stress (drained) analysis was preformed. The SCF and MPF are located offshore, soil fully saturated, the soil will thus respond in a drained or undrained manner depending on loading conditions and soil type (sands and gravels or silts and clays). Here the soil is taken to respond in an undrained manner (clay material) due to the dynamic excitation and hence the Poisson’s ratio is taken as 0.5 (to avoid numerical instabilities in Plaxis $v_g = 0.495$). Note the the soil stiffness is assumed to remain constant with depth.

<table>
<thead>
<tr>
<th>Properties</th>
<th>GBF</th>
<th>SCF</th>
<th>MPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material model</td>
<td>Linear-Elastic</td>
<td>Linear-Elastic</td>
<td>Linear-Elastic</td>
</tr>
<tr>
<td>Analysis type</td>
<td>Drained</td>
<td>Undrained</td>
<td>Undrained</td>
</tr>
<tr>
<td>Groundwater conditions</td>
<td>Dry</td>
<td>Saturated</td>
<td>Saturated</td>
</tr>
<tr>
<td>Foundation radius, $R_F$</td>
<td>5 m</td>
<td>10 m</td>
<td>3 m</td>
</tr>
<tr>
<td>Embedment depth, $D_e$</td>
<td>3 m</td>
<td>10 m</td>
<td>36 m</td>
</tr>
<tr>
<td>Depth to bedrock, $H_s$</td>
<td>20 m</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Target damping ratio, $\xi_F$</td>
<td>5%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>Poisson’s ratio, $v_g$</td>
<td>0.3</td>
<td>0.495</td>
<td>0.495</td>
</tr>
<tr>
<td>Mass density, $p_s$</td>
<td>17.5 KN/m$^3$</td>
<td>20 KN/m$^3$</td>
<td>20 KN/m$^3$</td>
</tr>
<tr>
<td>Analysis range, $c_s$</td>
<td>$40 - 500$ m/s</td>
<td>$G_s = 1 - 500$ MPa</td>
<td>$G_s = 1 - 500$ MPa</td>
</tr>
</tbody>
</table>

The model boundaries are set at eight time the radius of the foundations from the middle of the foundation in the horizontal direction and from the deepest embedment point in the vertical direction. For the MPF however due to its small radius and large embedment depth, the boundaries were doubled in the horizontal direction. To reduce calculation time, one-half of the soil-foundation system only is modelled, using sym-
metric boundary conditions along the lines of symmetry, this is possible due to the symmetric nature of the foundations modelled. Therefore the dynamic boundary conditions (viscous boundaries) were specified at the \(x_{\text{min}}, x_{\text{max}}, y_{\text{max}}\) and \(z_{\text{min}}\) boundaries (see Figure 4.14). As the model was symmetric with respect to the x-axis, no dynamic boundary was selected at \(y_{\text{min}}\). The boundary at the surface \((z_{\text{max}})\) was also not selected as the object was to model a semi-infinite layer.

As the stiffness of the soil was raised (during testing), the natural frequency of the soil column also raised. Low frequency therefore became an issue for some of the models. As the issue arose, model boundaries were increased to \(20 \times R_F\) and the viscous boundaries switched off.

### 4.3.1 Impedance functions

Impedance functions are presented for the horizontal, rotational and coupling DOFs for each foundation. Vertical vibrations are not critical for HAWT and were not derived.

Impedance functions for the GBF are taken from the base of the footing, however for the SCF and MPF the impedance functions are taken from the top of the foundations. It is important to note where the impedance functions are measured from so that they can be properly included in the EOM and compared with Cone model results. Impedance functions are given in Figures 4.17 - 4.20 for the GBF, in Figures 4.21 - 4.24 for the SCF and Figures 4.25 - 4.28 for the MPF.

The Cone method soil model for the GBF was taken as a 32 layered material of constant stiffness. The first 30 layers at 0.1 m thickness (30\(\times\)0.1 = 3 m = \(D_e\)) with the next layer a homogeneous 17 m layer (bring the soil to a depth of 20 m) and the last layer was modelled as a rigid boundary. The Cone method soil model for the MPF was taken as a 73 layered material of constant stiffness. The first 72 layers at 0.5 m thickness (72\(\times\)0.5 = 36 m = \(D_e\)) with the last layer being modelled as a homogeneous half-space. Similarly the Cone method soil model for the SCF was taken as a 21 layered material of constant stiffness. The first 20 layers at 0.5 m thickness (20\(\times\)0.5 = 10 m = \(D_e\)) with the last layer being modelled as a homogeneous half-space.
Figure 4.17: GBF dynamic stiffness for $c_s = 40$ m/s
4.3 Wind turbine foundations

Figure 4.18: GBF dynamic stiffness for $c_s = 100$ m/s
Figure 4.19: GBF dynamic stiffness for $c_s = 200$ m/s
4.3 Wind turbine foundations

Figure 4.20: GBF dynamic stiffness for $c_s = 500 \text{ m/s}$
Figure 4.21: SCF dynamic stiffness for $G_s = 1$ MPa
4.3 Wind turbine foundations

Figure 4.22: SCF dynamic stiffness for $G_s = 10$ MPa
Figure 4.23: SCF dynamic stiffness for $G_s = 100$ MPa
Figure 4.24: SCF dynamic stiffness for $G_s = 500$ MPa
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

Figure 4.25: MPF dynamic stiffness for $G_s = 1$ MPa
4.3 Wind turbine foundations

Figure 4.26: MPF dynamic stiffness for $G_s = 10$ MPa
Figure 4.27: MPF dynamic stiffness for $G_s = 100$ MPa
Figure 4.28: MPF dynamic stiffness for $G_s = 500$ MPa
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

4.3.2 Discussion

In general, there is good agreement between the impedance functions from Plaxis and those predicted by Cone method. Examining the impedance functions, it is clear that as the stiffness of the soil increases their dependence on frequency decreases, for the frequency range considered. This suggests that for stiffer soils, the foundation stiffness may be treated as frequency independent. However, the radiation damping of the soil-foundation system may still be significant.

The GBF shows the best agreement with Cone method for the DOFs considered. Interestingly there seems to be a slight frequency shift in the dip predicted at a shear wave velocity of \( c_s = 500 \text{ m/s} \).

For the SCF, peaks are present in the impedance functions at a shear modulus of 1 MPa. This phenomenon, due to wave interference in the soil inside the caisson, was also reported by Liingaard (2006). These peaks only shows up in weaker soils for the frequency ranges considered. This phenomenon is not predicted by Cone method. If a longer frequency range was considered for the stiffer soils, similar peaks in the impedance function would be expected. Similar peaks look to present for the MPF at 10 MPa.

4.4 Regressed CS models

There are two options available to obtain the static stiffness matrix for a soil-foundation system founded in non-linear soil: the flexibility technique or linearized regression. A linearized regression model can be obtained around a desired operating point. Here several multi-variable regression CS models are presented for various numbers of dependant variables for a MPF. An exact solution for a CS model is used as a reference. The regression models are linearized around rated operational conditions for the NREL reference offshore 5 MW HAWT.

4.4.1 Soil model

The Plaxis Hardening Soil model with small-strain stiffness (HSsmall) is used to generate all the regression models presented. Brinkgreve et al. (2010) gave empirical formulas for selection of all parameters for the HSsmall model, on the basis of the sands
4.4 Regressed CS models

Table 4.7: Soil parameters for the HSmall model in Plaxis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>25%</th>
<th>50%</th>
<th>80%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>υ_{unsat}</td>
<td>kN/m³</td>
<td>16</td>
<td>17</td>
<td>18.2</td>
<td>19</td>
</tr>
<tr>
<td>υ_{sat}</td>
<td>kN/m³</td>
<td>19.4</td>
<td>19.8</td>
<td>20.3</td>
<td>20.6</td>
</tr>
<tr>
<td>E_{50}^{ref}</td>
<td>kN/m²</td>
<td>15000</td>
<td>30000</td>
<td>48000</td>
<td>60000</td>
</tr>
<tr>
<td>E_{opt}^{ref}</td>
<td>kN/m²</td>
<td>15000</td>
<td>30000</td>
<td>48000</td>
<td>60000</td>
</tr>
<tr>
<td>E_{opt}^{ref}</td>
<td>kN/m²</td>
<td>45000</td>
<td>90000</td>
<td>144000</td>
<td>180000</td>
</tr>
<tr>
<td>G_{opt}^{ref}</td>
<td>kN/m²</td>
<td>77000</td>
<td>94000</td>
<td>114000</td>
<td>128000</td>
</tr>
<tr>
<td>γ_{st,0.7}</td>
<td></td>
<td>1.8 × 10^{-4}</td>
<td>1.5 × 10^{-4}</td>
<td>1.2 × 10^{-4}</td>
<td>1.0 × 10^{-4}</td>
</tr>
<tr>
<td>m</td>
<td></td>
<td>0.622</td>
<td>0.622</td>
<td>0.450</td>
<td>0.388</td>
</tr>
<tr>
<td>ϕ_{s}</td>
<td>deg</td>
<td>31.1</td>
<td>34.3</td>
<td>38.0</td>
<td>40.5</td>
</tr>
<tr>
<td>θ_{s}</td>
<td>deg</td>
<td>1.1</td>
<td>4.3</td>
<td>8.0</td>
<td>10.5</td>
</tr>
<tr>
<td>R_{f}</td>
<td></td>
<td>0.969</td>
<td>0.938</td>
<td>0.900</td>
<td>0.875</td>
</tr>
<tr>
<td>K_{0}</td>
<td></td>
<td>0.4553</td>
<td>0.4262</td>
<td>0.3775</td>
<td>0.3775</td>
</tr>
<tr>
<td>R_{inter}</td>
<td></td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Relative Density (RD). For sands many correlations exist with RD, thus the model parameters are an attempt to stimulate the use of advanced FEM constitutive models for soil on the basis of very limited geotechnical data. The soil parameters were shown to give a reasonable first approximation of the behaviour of sands in geoengineering applications (Brinkgreve et al., 2010) and are given in Table 4.7 for four soil RD. The cohesion $c'$ for sand is usually taken as zero, for modelling purpose in Plaxis a small (negligible) value was specified. The sand is modelled as a drained material.

The foundation self weight will contribute to the stiffness of the soil-foundation system as it will add axial load to the system, the MPF is modelled with a diameter of 6 m, a thickness of 0.6 m and the material is assumed to have a unit weight of 78.85 KN/m³, typical for steel.

4.4.2 Regression models

Four multi-variable regressions CS models, each generated using a different number of dependants, are derived using the method discussed in Section 3.6.2.2. The operating point and spread of dependants chosen for the regression models are given in Table 4.8. The horizontal force and over turning moment are due to wind and wave loading at
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

Table 4.8: Regression dependant variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Operating point</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water level</td>
<td>m</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>Horizontal force</td>
<td>MN</td>
<td>2.5</td>
<td>2.5 ± 20%</td>
</tr>
<tr>
<td>Overturning moment</td>
<td>MNm</td>
<td>140</td>
<td>140 ± 20%</td>
</tr>
<tr>
<td>Axial load</td>
<td>MN</td>
<td>8.6</td>
<td>8.6 ± 20%</td>
</tr>
<tr>
<td>Pile Depth</td>
<td>m</td>
<td>36</td>
<td>30 – 36 – 42</td>
</tr>
<tr>
<td>Sand RD (%)</td>
<td>-</td>
<td>-</td>
<td>25 – 50 – 80 – 100</td>
</tr>
</tbody>
</table>

rated operational conditions and for a moderate sea environment.

The regression coefficients are given in Table 4.9 for the multi-variable regression models with five, four and three dependants and in Table 4.10 for the multi-variable regression models with two dependants at each soil RD.

The size of the matrix $Reg_M$ varies substantially with the spread of the dependants taken (rows) and the number of dependants taken (columns). For the regressed CS model with two dependants $Reg_M = \begin{bmatrix} 18 & \times & 9 \end{bmatrix}$ and for regressed CS model with five dependants $Reg_M = \begin{bmatrix} 648 & \times & 18 \end{bmatrix}$. 
### 4.4 Regressed CS models

#### Table 4.9: Regression coefficients for the multi-variable regression model

<table>
<thead>
<tr>
<th>Reg. coef.</th>
<th>5 dep.</th>
<th>4 dep.</th>
<th>3 dep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{H_D}$</td>
<td>-201310000</td>
<td>-502000000</td>
<td>1203400000</td>
</tr>
<tr>
<td>$a_{H_I}$</td>
<td>74.716</td>
<td>72.41</td>
<td>-232.46</td>
</tr>
<tr>
<td>$a_{H_M}$</td>
<td>0.928</td>
<td>0.57</td>
<td>-0.46</td>
</tr>
<tr>
<td>$a_{H_{RD}}$</td>
<td>-2482500</td>
<td>-2239900</td>
<td>8260300</td>
</tr>
<tr>
<td>$a_{H_{D_e}}$</td>
<td>23380000</td>
<td>9781800</td>
<td>-</td>
</tr>
<tr>
<td>$a_{H_{AL}}$</td>
<td>-11.739</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$a_{C_{0}}$</td>
<td>-423980000</td>
<td>4866900000</td>
<td>-2218200000</td>
</tr>
<tr>
<td>$a_{C_{I}}$</td>
<td>-432.150</td>
<td>-429.24</td>
<td>2870.20</td>
</tr>
<tr>
<td>$a_{C_{M}}$</td>
<td>-36.954</td>
<td>-32.02</td>
<td>-5.14</td>
</tr>
<tr>
<td>$a_{C_{RD}}$</td>
<td>80313000</td>
<td>78929000</td>
<td>5683700</td>
</tr>
<tr>
<td>$a_{C_{D_e}}$</td>
<td>142860000</td>
<td>14028000</td>
<td>-</td>
</tr>
<tr>
<td>$a_{C_{AL}}$</td>
<td>199.490</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$a_{M_{0}}$</td>
<td>-270130000000</td>
<td>-344280000000</td>
<td>573460000000</td>
</tr>
<tr>
<td>$a_{M_{I}}$</td>
<td>-13720.000</td>
<td>-13238</td>
<td>-61148</td>
</tr>
<tr>
<td>$a_{M_{M}}$</td>
<td>608.860</td>
<td>542.39</td>
<td>104.77</td>
</tr>
<tr>
<td>$a_{M_{RD}}$</td>
<td>1409700000</td>
<td>1433900000</td>
<td>727640000</td>
</tr>
<tr>
<td>$a_{M_{D_e}}$</td>
<td>9007300000</td>
<td>1071000000</td>
<td>-</td>
</tr>
<tr>
<td>$a_{M_{AL}}$</td>
<td>-2903.700</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

#### Table 4.10: Regression coefficients for the multi-variable regression model at each soil RD (two dependants)

<table>
<thead>
<tr>
<th>RD</th>
<th>25%</th>
<th>50%</th>
<th>80%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{H_{0}}$</td>
<td>1485500000</td>
<td>2332850000</td>
<td>3180200000</td>
<td>3406400000</td>
</tr>
<tr>
<td>$a_{H_{I}}$</td>
<td>-100.78</td>
<td>-126.195</td>
<td>-151.61</td>
<td>-97.51</td>
</tr>
<tr>
<td>$a_{H_{M}}$</td>
<td>0.20</td>
<td>-1.64781</td>
<td>-3.49</td>
<td>-3.45</td>
</tr>
<tr>
<td>$a_{C_{0}}$</td>
<td>-2154800000</td>
<td>-2748600000</td>
<td>-3342400000</td>
<td>-3192000000</td>
</tr>
<tr>
<td>$a_{C_{I}}$</td>
<td>1437.40</td>
<td>1545.9</td>
<td>1654.40</td>
<td>903.87</td>
</tr>
<tr>
<td>$a_{C_{M}}$</td>
<td>-27.01</td>
<td>-6.1555</td>
<td>14.70</td>
<td>8.90</td>
</tr>
<tr>
<td>$a_{M_{0}}$</td>
<td>57393000000</td>
<td>67740000000</td>
<td>78087000000</td>
<td>76318000000</td>
</tr>
<tr>
<td>$a_{M_{I}}$</td>
<td>-27786</td>
<td>-27088</td>
<td>-26390</td>
<td>-15407</td>
</tr>
<tr>
<td>$a_{M_{M}}$</td>
<td>538.17</td>
<td>174.295</td>
<td>-189.58</td>
<td>-105.89</td>
</tr>
</tbody>
</table>
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS

4.4.3 Analysis

From the regression coefficients presented, the static stiffness matrix can be generated for any combination of conditions within the limits of the dependants. The accuracy of the results will decrease the further one gets from the operating point. For comparison the normalised static stiffness terms at the operating point are generated for each regressed CS model and are given in Table 4.11.

Note the normalised stiffness values may not be an accurate representation of the normalised stiffness of the soil-foundation system compared to linear-elastic soil as the shear modulus values used to obtain the normalised stiffness terms are only reference values, $G_{p}^{ref}$ taken from Table 4.7. The actual shear modulus increases with depth and is strain dependant.

The multi-variable regressed model accuracy increases for decreasing number of dependants (and for tightening the spread of the dependant). This is obviously the case, as with fewer dependants, there is less variance and it is thus easier to fit a linear model. This can be seen from Table 4.11, as the number of dependants drops the models converge on the CS model. The CS model is calculated assuming no dependant variables thus the regression coefficients are simply the static stiffness terms.

The multi-variable regressed CS model with five and four dependants gives significantly different results from the other regressed models (ones with fewer dependants).

The reason for the physically unacceptable results is that a linear fit cannot be found. This is due to the heuristic and empirical nature of the regression fitting process. As a result the linear regression model struggles to find an accurate fit.

In the results presented, as the number of dependants are dropped the regression models converge on the exact solution, thus suggesting that these regression models (with three and two dependants) maybe used to produce static stiffness matrix of a reasonable accuracy.

This is now tested by comparing the stiffness terms predicted by these regression models with new stiffness terms from two alternative CS models for variables within the spread of the models but not specifically included in the model.

The two alternative CS models are categorised for a $H = 2.25$ MN, $M = 126$ MNm, $RD = 60\%$ and a $H = 2.75$ MN, $M = 126$ MNm, $RD = 60\%$. The results from which are plotted in Figure 4.29 and compared to static stiffness terms calculated for the multi-variable regression models with two and three dependants. Using the
### 4.4 Regressed CS models

Table 4.11: Normalised static stiffness terms at the operating point for each regressed CS model at each soil RD

<table>
<thead>
<tr>
<th>RD</th>
<th>$K_{HH}^0/G_s R_F$</th>
<th>$K_{MM}^0/G_s R_F^2$</th>
<th>$K_C^0/G_s R_F^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 dep. Reg. model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>0.15794</td>
<td>55.479</td>
<td>3.1579</td>
</tr>
<tr>
<td>50%</td>
<td>-0.090707</td>
<td>59.331</td>
<td>4.9601</td>
</tr>
<tr>
<td>80%</td>
<td>-0.29256</td>
<td>62.662</td>
<td>6.4382</td>
</tr>
<tr>
<td>100%</td>
<td>-0.38986</td>
<td>63.967</td>
<td>7.1283</td>
</tr>
<tr>
<td>4 dep. Reg. model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>0.24084</td>
<td>57.7</td>
<td>2.5823</td>
</tr>
<tr>
<td>50%</td>
<td>-0.0012924</td>
<td>61.389</td>
<td>4.4477</td>
</tr>
<tr>
<td>80%</td>
<td>-0.19755</td>
<td>64.595</td>
<td>5.9753</td>
</tr>
<tr>
<td>100%</td>
<td>-0.2926</td>
<td>65.828</td>
<td>6.692</td>
</tr>
<tr>
<td>3 dep. Reg. model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>4.5651</td>
<td>254.87</td>
<td>-27.665</td>
</tr>
<tr>
<td>50%</td>
<td>4.4718</td>
<td>215.95</td>
<td>-22.494</td>
</tr>
<tr>
<td>80%</td>
<td>4.4119</td>
<td>185.15</td>
<td>-18.382</td>
</tr>
<tr>
<td>100%</td>
<td>4.3595</td>
<td>169.11</td>
<td>-16.272</td>
</tr>
<tr>
<td>2 dep. Reg. model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>5.4598</td>
<td>278.89</td>
<td>-31.363</td>
</tr>
<tr>
<td>50%</td>
<td>10.985</td>
<td>412.86</td>
<td>-57.274</td>
</tr>
<tr>
<td>80%</td>
<td>6.7604</td>
<td>223.64</td>
<td>-26.541</td>
</tr>
<tr>
<td>100%</td>
<td>6.9778</td>
<td>205.39</td>
<td>-24.665</td>
</tr>
<tr>
<td>CS model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>5.3566</td>
<td>278.24</td>
<td>-31.259</td>
</tr>
<tr>
<td>50%</td>
<td>6.1939</td>
<td>262.8</td>
<td>-30.709</td>
</tr>
<tr>
<td>80%</td>
<td>7.1096</td>
<td>234.04</td>
<td>-28.721</td>
</tr>
<tr>
<td>100%</td>
<td>7.7076</td>
<td>220.8</td>
<td>-28.245</td>
</tr>
</tbody>
</table>

Regression coefficients given in Tables 4.9 and 4.10, for a constant $RD$ and $M$ and for varying $H$ forces.

Examining Figure 4.29 the multi-variable regression model with three dependants produces a stiffness line that is around 20% off the exact stiffness. The multi-variable regression model with two dependants has been calculated for a RD of 50% and 80%, however the predicted stiffness lines surround the exact stiffness terms. This indicates a multi-variable regression model with two dependants ($H$ and $M$) derived at a RD=60% would produce a very accurate fit.
4.5 Conclusions

In this chapter, a method has been presented to estimate impedance functions for surface and embedded foundations using the Plaxis 3D dynamic.

Derived static and dynamic stiffness coefficients for the surface footing were compared to Cone method and analytical formulas. Good agreement was shown for each DOF considered. At excitation frequencies lower than the natural frequency of the soil column results were found to be distorted. This was caused by short comings of the viscous boundaries excited at low frequency. The problem was solved by extending the soil domain and fixing the boundaries.

The flexibility technique was then used to estimate the static and dynamic stiffness coefficients for an embedded foundations. The static stiffness at several embedment depth ratios were estimated and compared to Cone method and analytical formulas. Good agreement was shown for each DOF considered. The dynamic stiffness was es-
timed considering half of a symmetric soil domain model and a full model in Plaxis. The two models were shown to give very similar results justifying the use of half symmetric soil domain models in Plaxis for embedded symmetric foundations.

Impedance functions for three wind turbine foundations were derived using Plaxis and compared to Cone method results. The agreement was in general good, especially for the GBF. For the SCF and MPF, peaks in the impedance function at low soil stiffness were observed, caused by wave interference inside the caisson. The peaks were only predicted by the Plaxis models.

Finally, a multi-variable linear regression CS model analysis was presented for the MPF embedded in non-linear soil medium. The multi-variable models were compared to the exact CS model. It was found that as more dependants were added to the model the results became less reliable and finally unusable.
4. FINITE ELEMENT ANALYSIS OF THE FOUNDATION USING PLAXIS
Chapter 5

Analysis of wind turbines with foundation interaction

5.1 Introduction

A MDOF HAWT model with foundation coupling was formulated in Chapter 3. In Chapter 4, impedance functions for the turbine's soil-foundation system were characterised using several techniques including FEM. In this chapter, the dynamic response of an onshore and an offshore HAWT are formulated for a number of different soil-foundations models.

5.2 Benchmarking the developed model

Before any numerical analyses are carried out, an attempt is made to validate the developed flexible HAWT model. The NREL distributed wind turbine analysis package FAST is used to benchmark the developed model for a simplified 4 DOF Fixed Base (FB) system, using the NREL 5 MW reference offshore wind turbine detailed in Appendix D.

A flexible foundation is then added to the model (via spring support DOFs in the horizontal translation and rotational fore-aft direction). The modal frequencies for the extended 6 DOF model are calculated, considering first a soft soil-foundation system and then an infinitely stiff soil-foundation system. As for an infinitely stiff soil-
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

foundation system, the wind turbines modal frequencies should be unaffected by the foundations inclusion and hence match the FB results.

5.2.1 Fixed base

A comparison is made between the predicted modal frequencies from the developed HAWT model and FAST for a 4 DOF wind turbine model, one blade mode (flapwise) and one tower mode (fore-aft). This is achieved in FAST by disabling all the DOFs apart from \textit{FlapDOFl} and \textit{TwFADOFl}. For the developed wind turbine model, the 4 DOF out-of-plane model is derived using the method given in Section 3.2.1.1 (excluding the foundation). The full EOM for the developed wind turbine model are given in Appendix E.

In FAST modal frequencies for the system are established by carrying out linearization analysis about initial conditions. The analysis generates the system matrix (\(A\) matrix) from which the modal frequencies can be easy calculated using MATLAB. Using a similar method (generating the system matrix \(A\) from the derived EOM), the modal frequencies for the develop modal are calculated, for static conditions \(\Omega_s = 0\) (i.e. no centrifugal stiffening and no aerodynamic damping). Table 5.1 lists the modal frequencies for each DOF.

Table 5.1: Modal frequencies (Hz) for the HAWT for FB conditions given by FAST and the Developed Wind Turbine Model (DWTM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Blade 1</th>
<th>Blade 2</th>
<th>Tower</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAST</td>
<td>0.674</td>
<td>0.677</td>
<td>0.691</td>
</tr>
<tr>
<td>DWTM</td>
<td>0.683</td>
<td>0.683</td>
<td>0.698</td>
</tr>
</tbody>
</table>

Examining Table 5.1, we see that the results agree well. The slight differences between blade frequencies are probably due to the fact that FAST considers the pre-twist of the blades. In FAST the blade does not act in a purely out-of-plane direction and has some small component of in-plane (edgewise) motion.

The agreement between the modal frequencies is expected however, since both codes are developed using a modal formulation and both models are built using the same mode shapes, stiffness and mass distributions for the blade and tower.
5.2 Benchmarking the developed model

5.2.2 Flexible foundation

An additional two DOFs, representing the foundation translation and rotation, are now added to the wind turbine model. A simple surface footing is taken to represent the wind turbine foundation. The foundation's translation and rotation static stiffness are derived using the equations 4.12 and 4.13 given in Section 4.2.1.2. The uncoupled static stiffness terms are derived for an array of soil stiffness conditions ranging from soft soil, \( G_s = 1 \text{ MPa} \), to unrealistically stiff soil, \( G_s = 10,000 \text{ MPa} \), (corresponding to FB conditions).

The modal frequencies for the 6 DOF out-of-plane wind turbine model (the EOM for which are given in Appendix E) are calculated using the same method (i.e. generating the system matrix \( A \)). Note, the surface footing taken is not intended to represent a realistic wind turbine foundation and is taken for illustration purposes only (the stiffness matrix has no cross coupling terms and no damping is considered). Table 5.2 lists the soil and foundation parameters required to calculate the analytical static stiffness terms and the mass and inertia terms required for the EOM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of foundation, ( M_{tF} )</td>
<td>kg</td>
<td>3015929</td>
</tr>
<tr>
<td>Foundation inertia, ( I_F )</td>
<td>kgm²</td>
<td>79419462</td>
</tr>
<tr>
<td>Foundation radius, ( R_F )</td>
<td>m</td>
<td>10</td>
</tr>
<tr>
<td>Foundation depth, ( D_e )</td>
<td>m</td>
<td>4</td>
</tr>
<tr>
<td>Foundation density, ( \rho_F )</td>
<td>kg/m³</td>
<td>2400</td>
</tr>
<tr>
<td>Poisson's ratio, ( \nu_F )</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td>Shear Modulus range, ( G_s )</td>
<td>MPa</td>
<td>1-10,000</td>
</tr>
</tbody>
</table>

Table 5.3 lists the modal frequencies for the 6 DOF wind turbine model at each soil stiffness condition. The modal frequencies are unlabelled and simply arranged in ascending order. Examining Table 5.3 left to right, one notices that, as the stiffness of the soil increases, the modal frequencies converge on the FB results. This validates another aspect of the developed model. Other interesting conclusions can be drawn from Table 5.3: For an unrealistic soft soil-foundation system \((G_s = 1 \text{ MPa})\), the foundation modal frequencies couple with the first tower modal frequency and it becomes impossible to distinguish between them. When the stiffness of the soil-foundation system is raised \((G_s = 10 - 100 \text{ MPa})\) the foundation modal frequencies uncouple from the
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Table 5.3: Modal frequencies (Hz) for the 6 DOF HAWT model calculated for different soil stiffness ($G_s$) conditions

<table>
<thead>
<tr>
<th>Shear Modulus (MPa)</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1,000</th>
<th>10,000</th>
<th>FB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.094</td>
<td>0.220</td>
<td>0.268</td>
<td>0.277</td>
<td>0.277</td>
<td>0.277</td>
<td></td>
</tr>
<tr>
<td>0.431</td>
<td>0.579</td>
<td>0.683</td>
<td>0.683</td>
<td>0.683</td>
<td>0.683</td>
<td></td>
</tr>
<tr>
<td>0.630</td>
<td>0.683</td>
<td>0.683</td>
<td>0.683</td>
<td>0.683</td>
<td>0.683</td>
<td></td>
</tr>
<tr>
<td>0.683</td>
<td>0.683</td>
<td>0.698</td>
<td>0.698</td>
<td>0.698</td>
<td>0.698</td>
<td></td>
</tr>
<tr>
<td>0.683</td>
<td>0.703</td>
<td>1.500</td>
<td>4.635</td>
<td>14.63</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>0.701</td>
<td>1.993</td>
<td>6.302</td>
<td>19.93</td>
<td>63.02</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

tower modal frequency and become distinguishable. The flexible foundation lowers the first tower modal frequency but has very little effect on the blade modal frequencies. Finally, it can be seen as expected, that as the soil stiffness increases (left to right) the foundation modal frequencies increase significantly.

5.3 Onshore HAWT model

Numerical examples are presented to illustrate the effects of SSI on the dynamic response of a 1.5 MW onshore wind turbine (out-of-plane model) attached to a GBF resting on a linear elastic half-space. Transfer functions for displacement of the turbine system are obtained and the modal frequencies of the combined turbine-foundation system estimated. SSI is shown to affect the response of the wind turbine. This is examined in terms of the turbine structural displacement and the base shear and bending moment in the tower and the foundation. The effect of SSI on the rotation of the foundation is also investigated.

5.3.1 Model properties

Model properties for the blades, nacelle and tower were taken from an NREL 1.5 MW 3-bladed HAWT, the structural properties of the turbine are given in Appendix D. Taking the out-of-plane model detailed in Section 3.2.1.1, and considering the first mode of blade and tower vibration only, the EOM are developed for the coupled foundation/nacelleblade wind turbine model. The full EOM for the 6 DOF model are given in Appendix F.
5.3 Onshore HAWT model

The Lagrangian formulation does not account for damping. Modal damping was therefore assumed, the structural damping ratios for the tower and blade are specified in Appendix D. Modal damping ratios due to aerodynamic damping were taken as $\xi_{AD_T} = 4\%$ and $\xi_{AD_b} = 15\%$ for tower and blade bending (first mode) respectively. A high damping ratio for the blade bending mode was assumed due to the known large effects of aerodynamic damping in the Flapwise blade mode (Hansen et al., 2006).

The wind load is generated in accordance with the parameters shown in Table 5.4, assuming a homogeneous isotropic turbulence field. Note, as BEM theory is applied blade geometry and characteristics influence the wind load. The modal wind load on the blade and tower is shown in Figure 5.1. The wind turbine is assumed to be operating under rated conditions for all examples in this section i.e. $\Omega_b = 20$ rpm.

Table 5.4: Wind data for the onshore analysis

<table>
<thead>
<tr>
<th>Properties</th>
<th>Unit</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean wind speed, $\bar{v}(H_h)$</td>
<td>m/s</td>
<td>12</td>
</tr>
<tr>
<td>Air density, $\rho_a$</td>
<td>kg/m$^3$</td>
<td>1.225</td>
</tr>
<tr>
<td>Roughness, $z_o$</td>
<td>-</td>
<td>0.005</td>
</tr>
<tr>
<td>Turbulence intensity, $I_t$</td>
<td>-</td>
<td>10%</td>
</tr>
<tr>
<td>Stability, $\beta$</td>
<td>-</td>
<td>0.143</td>
</tr>
</tbody>
</table>

5.3.2 Soil foundation model

Three soil profiles are modelled for a range of shear wave velocity values. The soil loss factor (soil material damping) is assumed to be constant for all frequencies, i.e. hysteretic damping is assumed. Site A is chosen due to the fact that it can be modelled by analytically derived formulas given by DNV/Risø standards in the form of static springs (stiffness) and damping coefficients, and therefore can be used as a comparison to validate the frequency dependent stiffness and damping coefficient given by Cone method and Plaxis. Site B and Site C are chosen as more realistic soil profiles (with multiple layers) and are modelled solely by Cone method. Site B is taken from (Wolf & Deeks, 2004b) and is based on field conditions found in Western Australia, here the soil stiffness reduces with depth as a results of the cementing action of carbonates on the sand grains near the mudline. Site C is chosen as a more typical soil profile, in that stiffness increases with depth in a layering fashion.
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Details of the foundation and the soil profiles are given in Table 5.5 and Figure 5.2. The GBF was designed assuming a constant and uniform soil density and friction angle.

5.3.2.1 Foundation design

Traditionally geotechnical engineers have relied on lumped Factor Of Safety (FOS) methods, whereby the sum total of resistances is set at a safe multiple of the sum of all actions and forces to prevent failure. A more modern alternative is the limit state approach using specified partial factors, as implemented in the Eurocode 7 (EC7).

An embedded flat circular reinforced concrete foundation slab is designed for the NREL 1.5 MW wind turbine, the design of the foundation is based on EC7, Maunu (2008) and DNV/Riso standards. The embedded foundation is designed for a constant vertical gravity load, 2148.4 KN, and the maximum thrust experienced by the turbine.
5.3 Onshore HAWT model

Figure 5.2: Soil profiles for the GBF

Table 5.5: Foundation and soil properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>Unit</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation radius, $R_F$</td>
<td>m</td>
<td>5</td>
</tr>
<tr>
<td>Foundation embedment Depth, $D_e$</td>
<td>m</td>
<td>3</td>
</tr>
<tr>
<td>Foundation mass, $M_{tf}$</td>
<td>T</td>
<td>664.2</td>
</tr>
<tr>
<td>Foundation moment of inertia Depth, $I_F$</td>
<td>kgm$^2$</td>
<td>345800</td>
</tr>
<tr>
<td>Depth to bedrock, $H_g$</td>
<td>m</td>
<td>20</td>
</tr>
<tr>
<td>Soil loss factor, $n_s$</td>
<td>-</td>
<td>0.05</td>
</tr>
<tr>
<td>Shear wave velocity, $c_s$</td>
<td>m/s</td>
<td>40 – 500</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_s$</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td>Mass density, $\rho_s$</td>
<td>kg/m$^3$</td>
<td>1750</td>
</tr>
<tr>
<td>Angle of friction, $\phi_s'$</td>
<td>deg</td>
<td>35°</td>
</tr>
</tbody>
</table>

during normal operation at near cut-out wind speed, 390 KN. The size of the foundation is based on the bearing capacity of the soil medium.

Figure 5.3 shows the foundation design forces $H_d$ and $V_d$, i.e. the characteristic forces multiplied by their relevant partial load factor. The Load Centre (LC) is the point where the resultant of $H_d$ and $V_d$ intersects the soil foundation interface and implies an eccentricity $e_d$ to the vertical force relative to the centre line of the foundation.

$$e_d = \frac{M_d}{V_d}$$  \hspace{1cm} (5.1)

Where $M_d$ denotes the resulting design overturning moment about the foundation soil interface.

For bearing capacity analysis an effective foundation area $A_{eff}$ is needed. For a circular foundation, with radius $R_F$, an elliptical effective foundation area $A_{eff}$ can be
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Figure 5.3: Forces and soil properties for foundation design

defined as,

\[ A_{eff} = 2 \left[ R_F^2 \cos^{-1} \left( \frac{e_d}{R_F} \right) - e \sqrt{R_F^2 - e_d^2} \right] \]  

(5.2)

the major axes can then be defined as,

\[ b_e = 2(R_F - e_d) \quad \text{and} \quad l_e = 2R_F \sqrt{1 - \left( 1 - \frac{b_e}{2R_F} \right)^2} \]  

(5.3)

The effective area \( A_{eff} \) can be represented by a rectangle with the dimensions \( l_{eff} \) and \( b_{eff} \) given as,

\[ l_{eff} = \sqrt{A_{eff} b_e} \quad \text{and} \quad b_{eff} = \frac{l_{eff}}{l_e} b_e \]  

(5.4)

With the effective dimensions defined, the bearing capacity can be estimated. For fully drained conditions the following general formula can be applied for the bearing capacity of a foundation with a horizontal base and embedded in the soil,

\[ R_d = \frac{1}{2} \gamma_s b_{eff} N_{r_s} s_l + q_b N_q s_q l_q + c' N_c S_c \delta_c \]  

(5.5)
5.3 Onshore HAWT model

where \( R_d \) is the design bearing capacity, \( \gamma_s \) is the unit with of the soil, \( q_b \) is the overburden pressure at the level of the foundation, \( c' \) is the cohesion of the soil, the terms \( N_\gamma, N_q, N_C \) are bearing capacity factors, the terms \( s_\gamma, s_q, s_C \) are shape factors and the terms \( i_\gamma, i_q, i_C \) are the inclination factors (details for the these terms are given in EC7).

The design bearing resistance \( R_d \) of the foundation structure is set at, or below, the design value of the vertical actions \( V_d \) (the vertical load multiplied by a partial factor),

\[
V_d \leq R_d
\]  
(5.6)

<table>
<thead>
<tr>
<th>1.5 MW NREL Wind Turbine Spread Foundation</th>
<th>Reference:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight soil ( \gamma' )</td>
<td>17.5 kN/m³</td>
</tr>
<tr>
<td>Unit weight concrete ( \gamma' )</td>
<td>24.5 kN/m³</td>
</tr>
<tr>
<td>Turbine height (to hub) ( h_t )</td>
<td>82 m</td>
</tr>
<tr>
<td>Turbine tower at ( h_t )</td>
<td>82 m</td>
</tr>
<tr>
<td>Hub height ( h )</td>
<td>82 m</td>
</tr>
<tr>
<td>Foundation depth ( d )</td>
<td>9.68 m</td>
</tr>
<tr>
<td>Foundation diameter ( D )</td>
<td>2148 kN</td>
</tr>
<tr>
<td>Self-weight of foundation ( W_{fg} )</td>
<td>6254 kN</td>
</tr>
</tbody>
</table>

Calculations for ULS:

- Factored vertical loading \( V = q_g \gamma_{WS} W_{fH} = q_{gFS} \)  
  - 12190 9010 12190 9010 kN
- Design horizontal load \( V_h = q_{fg} \)  
  - 580 587 580 587 kN
- Design moment \( M_h = (R_h + 0.6) \)  
  - 4072 41099 4972 41099 kNm
  - Equilibrium (both)  
  - \( \gamma'' = 9.0 \)  
  - 8.0 8.0 8.0 8.0  
  - \( \gamma'' \) \( W_{fS} (0.6) \)  
    - 3.7 3.0 3.7 3.0  
  - Bearing capacity factor \( N_\gamma \)  
    - 33.3 16.9 33.3 16.9  
  - Bearing capacity factor \( N_q \)  
    - 49.2 17.9 49.2 17.9  
  - Bearing capacity factor \( N_C \)  
    - 0 0 0 0  

- Eccentricity of load \( k = M_p V_d \)  
  - 4.09 4.77 4.09 4.77 m
- Load outside modulus for ULS: \( E_{ULS} = \gamma_{WS} W_{fH} \)  
  - 110 110 110 110 m
- Effective breadth \( B = B_e \gamma_{WS} \)  
  - 1.52 0.13 1.52 0.13 m
- Effective area \( A = B_e \gamma_{WS} \)  
  - 14.68 1.26 14.68 1.26 m²
- Effective area \( A_e \)  
  - 5.37 0.14 5.37 0.14
- Minor axis \( a \)  
  - 1.52 0.13 1.52 0.13
- Effective length \( L_e \)  
  - 8.58 9.61 8.58 9.61
- Effective breadth \( B_e \gamma_{WS} \)  
  - 5.60 3.19 5.60 3.19
- Shape factor \( s_\gamma = 1.0 B_e \)  
  - 1.1 1.0 1.1 1.0
- Shape factor \( s_q = 1.0 B_e \)  
  - 0.9 1.0 0.9 1.0
- Shape factor \( s_C \)  
  - 0.9 1.0 0.9 1.0
- Inclination factor \( i_\gamma = 1.0 B_e \gamma_{WS} \)  
  - 0 0 0 0
- Inclination factor \( i_q = 1.0 B_e \gamma_{WS} \)  
  - 0 0 0 0
- Inclination factor \( i_C \)  
  - 0 0 0 0
- Surface load \( q_s \)  
  - 82.8 82.8 82.8 82.8 kN/m²
- Resistance \( R_{RA} \)  
  - 278 901 278 901 kN
- Factored resistance \( R_{\gamma} \)  
  - 12190 12190 12190 12190 kN
- Design vertical loading \( V_d = \)  
  - 92 92 92 92 kN

Figure 5.4: Spread sheet for design of the 1.5 MW turbine foundation
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Using the given method, implemented in EXCEL, a goal seek operation was carried out to find the diameter of the foundation. Figure 5.4 shows that Design Approach 1 Option 1 (DA1.1) is the most efficient. A foundation of diameter 10 m was chosen. This size allows for the recommended 0.1 m extra to deal with the eccentricity of the loading.

The foundation has been sized for the Ultimate Limit State (ULS). However calculations are also required to check for the Serviceability Limit State (SLS). These checks are usually for settlement and rotation under the static loads. Formulas for settlement and rotation are dependent on the Young’s modulus of the soil (i.e. the stiffness). However, in the work that follows, the stiffness of the soil is varied (while the soil strength parameters are held constant) in order to show the effects of SSI on the wind turbines dynamic response. Examining Table 5.5 the shear wave velocity $c_s$ of the soil is varied, for linear elastic soil $c_s$ can related to the shear modulus $G_s$ as,

$$c_s = \sqrt{\frac{G_s}{\rho_s}}$$  \hspace{1cm} (5.7)

which in turn can be related to the Young modulus $E_s$ of the soil as,

$$E_s = 2G_s(1 + \nu_s)$$  \hspace{1cm} (5.8)

thus the soil stiffness properties $c_s$, $G_s$ and $E_s$ vary while the soil strength properties $\nu_s$, $\rho_s$ and $\phi_s$ are held constant. Serviceability checks are therefore not carried out, as the purpose of this design stage was simply to find a realistic size for the foundation, for use in the SSI study.

5.3.2.2 CS models

Three CS model are taken in this study a static CS model taken from the DNV/Risø standards, and two frequency dependant CS models taken from Cone method and Plaxis.

DNV/Risø standards

The standards give formulas for spring stiffness for a circular footing embedded in a stratum overlying bedrock, of a depth $H_s$ i.e. Site A. The horizontal $K_{HH,DNV}^0$ and
rotational $K_{MM,DNV}^0$ static stiffness terms are given as,

$$K_{Hh, DNV}^0 = \left( \frac{8G_s R_F}{2 - \nu_s} \right) \left( 1 + \frac{R_F}{2H_s} \right) \left( 1 + \frac{2D_F}{3R_F} \right) \left( 1 + \frac{5D_F}{4H_s} \right) \quad (5.9)$$

$$K_{MM, DNV}^0 = \left( \frac{8G_s R_F^3}{3(1 - \nu_s)} \right) \left( 1 + \frac{R_F}{6H_s} \right) \left( 1 + \frac{2D_F}{R_F} \right) \left( 1 + \frac{0.7D_F}{H_s} \right) \quad (5.10)$$

Note, no coupling stiffness terms are given and the footing is assumed to be rigid relative to the soil and always in full contact with the soil.

**Cone method**

Impedance function for the gravity based foundation are generated using the executable package CONAN (Wolf & Deeks, 2004b).

To generate impedance functions using CONAN, the following steps can be followed:

1. Prepare a notepad file containing the required information on the foundation and the soil profile
2. Specify the reference point (usually taken at the bottom of the embedded foundation)
3. Specify whether the internal soil (for an embedded foundation) should be excavated or not
4. Following these specifications, choose to solve for the dynamic-stiffness coefficient (option 1)
5. Enter the required DOF (note for embedded foundation choosing $R$ generates the horizontal, rotational and coupling dynamic-stiffness coefficient)
6. Enter the first and last frequency and the frequency step size (in rad/s)
7. Enter whether the output data should be normalised or not (depends on the users requirements)

8. Enter the name of the output file

Figure 5.5: Screen shot of CONAN for Site B with a shear wave velocity of 100 m/s

The frequency range depends on the user’s requirement. For completeness, if the time span $T$ and time step (hence the number of steps $N$) are known the corresponding maximum frequency can be given as,

$$f_{\text{max}} = \frac{N}{2} \frac{2\pi}{T}$$

(5.11)

Using the given method and specifying: a bottom reference point, internal soil excavate, DOF $R$ and a frequency range $f = 0 - 0.1 - 314$ ($T = 200$ at a sampling 0.01). Impedance functions for each Site at each shear wave velocity are generated.

Figure 5.6 shows an sample notepad file for Site B for $c_s = 100$ m/s. The corresponding impedance function are shown in Figure 5.7.
5.3 Onshore HAWT model

Figure 5.6: Example of the input text file needed for CONAN, for Site B with a shear wave velocity of 100 m/s, with the name of each column super-imposed at the bottom of the text file

Plaxis

Impedance function for the GBF were generated using the FEM package Plaxis 3D dynamic. The impedance functions are plotted (against cone model) in Figures 4.17 - 4.20 for each condition of shear wave velocity. A detailed discussion on generating impedance functions with Plaxis was given in section 4.3.1.

5.3.3 Numerical results

Some numerical examples are now presented for to illustrate the effects of SSI on the response of the MDOF onshore 1.5 MW HAWT model. Transfer functions for the system are also obtained and discussed. This type of analysis is essential in understanding the response of the structure due to more complex random excitations. Aerodynamic wind load, developed from BEM theory, including an isotropic homogeneous turbulence field generated using the Kaimal spectrum, is then applied to the structure and the response computed. The analysis is carried out for different soil stiffness conditions, while fully fixed conditions are used as a reference. Soil damping and stiffness
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Figure 5.7: Impedance functions, produced from CONAN implementing Cone method, for Site B with a shear wave velocity of 100 m/s

properties obtained from the DNV/Risø design standards are used as a comparison to validate the dynamic stiffness cone method and Plaxis models. The EOM are solved in the frequency domain following a method outlined in Appendix A.5.

5.3.3.1 Transfer Functions

Transfer functions of different response quantities are determined for a series of soil stiffness conditions. A fully fixed condition is used as a reference. The transfer functions were obtained by considering the output response of the relevant DOF. To obtain the transfer functions with respect to the input at the nacelle, the Fourier spectrum of the
loads applied to the different DOF were normalized by the Fourier spectrum amplitude of a steady load at the nacelle.

Figure 5.8: Structural fundamental modal frequency versus shear wave velocity. Results generated using Cone method, Plaxis and DNV/Risø coefficient to represent the soil-foundation system, Site A

The fundamental modal frequency of the coupled wind turbine system was determined by examination of the displacement transfer function, as resonance of a structure when subjected to a harmonic load can be observed as local peaks in the magnitude of the nodal displacements (Liingaard, 2006). Thus, fundamental modal frequencies as a function of soil shear wave velocity were generated for site A. This was done by modelling the soil foundation system using a static CS model (DNV/Risø) and two frequency dependant CS models (Cone method and Plaxis), the results of which are plotted in Figure 5.8 and given in Table 5.6.

The results from Figure 5.8 and Table 5.6 validates Cone method and Plaxis for a uniform soil profile with regard to DNV/Risø standard formulas. With validation established, Cone model was used to model the more complex layered Site B and Site
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Table 5.6: Tower modal frequencies at each site for the onshore HAWT model

<table>
<thead>
<tr>
<th>$c_s$</th>
<th>Site A</th>
<th>Site B</th>
<th>Site C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CM</td>
<td>Plaxis</td>
<td>DNV/Risø</td>
</tr>
<tr>
<td>40</td>
<td>1.24</td>
<td>1.31</td>
<td>1.29</td>
</tr>
<tr>
<td>70</td>
<td>1.65</td>
<td>-</td>
<td>1.80</td>
</tr>
<tr>
<td>100</td>
<td>2.04</td>
<td>2.10</td>
<td>2.07</td>
</tr>
<tr>
<td>200</td>
<td>2.35</td>
<td>2.36</td>
<td>2.35</td>
</tr>
<tr>
<td>500</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
</tr>
<tr>
<td>FB</td>
<td>2.47</td>
<td>2.47</td>
<td>2.47</td>
</tr>
</tbody>
</table>

Figure 5.9: Nacelle total and relative response

C profiles, modal frequencies for which are presented in Table 5.6.

Transfer functions for Site B are presented in Figure 5.10. Only the low frequency range of the transfer functions for Site B are shown as most of the energy is concentrated in this region. The first peak range is between $1 - 3 \text{ rad/s}$ for the given soil conditions and contains the fundamental modal frequency of the system.

From Figure 5.10, it is evident that as the shear wave velocity of the soil decreases the fundamental modal frequency of the system drops, while for higher values the fundamental modal frequency of the system tends towards a fully fixed response. Therefore, SSI has the effect of lengthening the vibration period of the wind turbine, a fact shown in other SSI studies: Jennings & Bielak (1973); Veletsos & Meek (1974).
5.3 Onshore HAWT model

SSI (generally) has the effect of increasing damping in the system as energy is dissipated by radiation into the soil, and hence SSI should reduce the relative displacement response. The relative displacement of the nacelle response, $NAC_R$, however shows only a slight reduction in peak response as shear wave velocity drops. This can be seen in the transfer function Figure 5.10a. This is because the wind turbine has a relatively large foundation (due to the large over turning moment) compared to its vertical load.

Figure 5.10: Transfer functions: (a) Nacelle relative displacement (b) Nacelle total displacement (c) Foundation horizontal translation (d) Foundation rotation

Total nacelle displacement, $NAC_T$, includes horizontal foundation displacement, lateral displacement due to foundation rotation and relative displacement of the nacelle. Figure 5.10b shows the total displacement of the nacelle increases as shear wave
velocity drops, especially for lower shear wave velocity values. This detrimental effect of SSI has been observed in previous studies for tall slender structures (Luco, 1986; Moghaddasi et al., 2011) and is caused due to the rotation of the foundation, which for tall structures, increases the acceleration of the mass and the associated inertia force. This whipping effect can lead to a corresponding increase in response (Veletsos & Meek, 1974). However total displacements, compared to relative displacements, do not have the same effect on stresses and deformations within the superstructure (Moghaddasi et al., 2011) and therefore are not as critical. Total displacements are still an important design criteria as the motion of the nacelle, for obvious reasons, must be limited.

To avoid resonance, the fundamental modal frequency of the wind turbine system should not coincide with the rotor speed range (1P) and blade passing speed range (3P for three blade turbines). The 1P and 3P areas are plotted in Figure 5.11 to visualize the zones the structural natural frequency should avoid to prevent dynamic interaction (Tong, 2010). In Figure 5.11, \( P_M \) and \( P_R \) represent the rotor speed at cut-in and rated conditions respectively for the variable speed 1.5 MW wind turbine. Overlaid upon this is the range of wind turbine fundamental modal frequencies as a function of shear wave velocity, \( c_s = 40 - 500 \text{ m/s} \). For a soft-stiff design approach the tower’s fundamental modal frequency should lie in between the 1P and 3P range. For stiff soil conditions this is shown to be the case. However for softer soil, the tower’s fundamental modal frequency slips down into the 1P range, this may lead to undesirable dynamic interaction.

5.3.3.2 Time history response

To illustrate the effects of SSI on the wind turbine system, some time histories for the displacement response of the blades, nacelle and foundation are shown in Figures 5.13-5.15. These were generated under a turbulent wind excitation for rated conditions of rotation and wind speed.

Figure 5.13a shows the total displacement response of the nacelle, and as expected there is a large increase in response as the shear wave velocity of the soil drops, caused by the rotation of the foundation increasing the displacement of the nacelle. The nacelle relative displacement, shown in Figure 5.13b and Figure 5.15, illustrates the almost neutral effect of SSI on the relative vibration response.

164
Figure 5.11: Frequency range of 1P and 3P, with structural fundamental modal frequency range due to SSI

The foundation horizontal translation and rotation for various soil stiffness conditions are shown in Figure 5.12. The horizontal translation and rotation response increases with decrease in shear wave velocity of the soil, which is to be expected. For a soil with a shear wave velocity, $c_s = 100$ m/s. An average rotation of around 0.0014 rad is observed, for a 82 m hub height and a foundation depth of 3 m. This corresponds to a horizontal translation of around 119 mm at the nacelle.

Figure 5.12 shows the relative displacement of a blade. The effect of the flexible soil on the blade relative displacement is negligible. This is unsurprising as the interaction with the foundation is dampened out (filtered) through the superstructure.
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Figure 5.12: Blade response time-histories
5.3 Onshore HAWT model

Figure 5.13: Response time-histories: (a) Nacelle total displacement response (b) Nacelle relative displacement response
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

![Response time-histories](image)

Figure 5.14: Response time-histories: (a) Foundation horizontal translation response (b) Foundation rotation response
5.3 Onshore HAWT model

5.3.3.3 Foundation movement

The rotation of a wind turbine foundation must be minimised in order to prevent failure and other serviceability issues concerning wind turbine operations. The maximum rotation yielded from the proposed model for different soil stiffness conditions is shown in Figure 5.16b for Site B (modelled solely by Cone method) and in Figure 5.16a for Site A (where the soil-foundation system was modelled by Cone method, Plaxis and DNV/Risø). DNV/Risø standards specify, for SLS, the maximum allowable tilt of the foundation as 0.0087 rad off the vertical. It can be seen from Figure 5.16b that, at very low soil stiffness, this condition is violated and a maximum tilt of 0.015 rad off the vertical is reached at Site B. Similar rotations are predicted by the different models as shown in Figure 5.16a for Site A. Slight variations are seen to occur for the lowest soil stiffness.

5.3.3.4 Bending moment and shear forces

To examine the effects of SSI on the base shear and bending moment in the wind turbine, as outlined in Section 3.6.5. The model was exposed to a steady wind load for rated conditions of rotational and wind speed. The results are presented in the frequency domain and are shown in Figure 5.17. It is observed that the major peaks occur around $1 - 3$ rad/s and at around $3P$ depending on soil stiffness. Other peaks
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Figure 5.16: Rotation of the foundation versus shear wave velocity: (a) Site A (b) Site B and Site C

occur roughly at multiples of 3P. These peaks are dampened out for soft soil conditions and are only prominent in stiffer soil conditions. The peaks in the range 1 – 3 rad/s are prevalent for softer soils and are affected by SSI. As the shear wave velocity decreases, there is a corresponding drop in frequency at which the peaks occurs.

Sample time histories for base shear and bending moment are presented in Figure 5.18. The time histories show that although the peak shear/moment are very similar regardless of soil stiffness, the frequencies of vibration present are highly dependent on soil stiffness. For a shear wave velocity of 70 m/s compared to 500 m/s the frequency of a peak occurrence is quartered. This will have a significant effect on a fatigue analysis at the tower foundation connection.

5.3.4 Discussion

Fundamental modal frequencies of the wind turbine system were calculated and SSI was shown to lengthen the vibration period of the wind turbine. The static stiffness formulation was shown to give comparable results to the frequency dependant models, under certain conditions.

170
The fundamental modal frequency of the wind turbine system, located on the multi-layered Site B, was shown to dip into the 1P range for softer soil conditions, which should be avoided as it may lead to dangerous dynamic interaction.

SSI is generally considered to have a positive effect on structural vibrations as it adds damping to the system. However, the relative displacement of the nacelle showed only a slight reduction in response for the soil conditions considered, while SSI was found to have a detrimental effect on the total displacement of the nacelle, especially for softer soils.

The rotation of the foundation was shown to increase significantly with decreasing...
soil stiffness and violated prescribed limit state of DNV/Risø standards for lower soil stiffness conditions.

The shear force and bending moment at the base of the tower and in the foundation were computed. No significant difference between the shear and moment in the foundation and tower base was found, as the foundation's inertia was found to be negligible. Peaks in the frequency response were found to occur at multiples of 3P (three times the rotation speed) for stiffer soil conditions. The frequency content in the response time history was significantly affected by SSI, this will have implications from a fatigue point of view.
5.4 Offshore, out-of-plane HAWT model

Numerical examples are presented to illustrate the effects of SSI on the dynamic response of a MDof 5 MW offshore HAWT out-of-plane model, attached to a SCF and MPF resting on a linear elastic half-space. Static and frequency dependant CS models, generated using FEM methods (Plaxis) and the wave prorogation based Cone method for comparison, are used to represent the soil-foundation behaviour at the mudline. Tower displacement response transfer functions are generated using each soil-foundation model at various soil stiffness conditions. The effects of SSI on the displacement transfer functions and the wind turbine dynamics are discussed.

5.4.1 Model properties

Model properties for the turbine are taken from the NREL reference offshore 5 MW 3-bladed HAWT, the structural properties of the turbine are given in Appendix D. Taking the out-of-plane HAWT model detailed in Section 3.2.1.1, the EOM are developed for the coupled foundation/tower/blade wind turbine model considering: the first two blade modes, first three tower modes and the foundation translation and rotational DOF. The full EOM for the 11 DOF model are given in Appendix G.

The structural damping ratio for the tower and blade are given in Appendix D. The modal damping ratios due to aerodynamic damping ratio were taken as $\zeta_{AD_{T}} = 4\%$ and $\zeta_{AD_{B}} = 15\%$ for tower and blade bending respectively.

The wind is assumed to be blowing at rated conditions and two sea states are examined: moderate and extreme. The wind and wave load were generated using the parameters given in Table 5.7. The modal wind load, generated using BEM theory with rotational sampled turbulence, is shown in Figure 5.19. Modal wave load for a moderate and extreme sea environment, generated using Airy linear wave theory, JONSWAP spectrum and Wheeler Stretching, are shown in Figures 5.20 and 5.21 respectively. The spectrum for each wave environment is shown in Figure 5.22.
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Table 5.7: Wind and wave data for offshore out-of-plane HAWT model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean wind speed, ( \bar{v}(H_b) )</td>
<td>m/s</td>
<td>11.4</td>
</tr>
<tr>
<td>Rated rotational speed, ( \Omega_b )</td>
<td>rpm</td>
<td>12.1</td>
</tr>
<tr>
<td>Air density, ( \rho_a )</td>
<td>kg/m(^3)</td>
<td>1.225</td>
</tr>
<tr>
<td>Roughness, ( z_o )</td>
<td></td>
<td>0.005</td>
</tr>
<tr>
<td>Turbulence intensity, ( I_t )</td>
<td></td>
<td>20%</td>
</tr>
<tr>
<td>Stability, ( \beta )</td>
<td></td>
<td>0.11</td>
</tr>
<tr>
<td>Wave height (moderate), ( H_w )</td>
<td>m</td>
<td>6</td>
</tr>
<tr>
<td>Wave height (extreme), ( H_w )</td>
<td>m</td>
<td>9</td>
</tr>
<tr>
<td>Water Depth, ( d )</td>
<td>m</td>
<td>20</td>
</tr>
<tr>
<td>Tower drag coefficient, ( C_{d,T} )</td>
<td></td>
<td>0.7</td>
</tr>
<tr>
<td>Tower inertia coefficient, ( C_{m,T} )</td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 5.19: Wind load: (a) Blade modal load (b) Tower modal load
5.4 Offshore, out-of-plane HAWT model

Figure 5.20: Wave load for moderate sea conditions: (a) Tower modal load (b) Foundation modal load
Figure 5.21: Wave load for extreme sea conditions: (a) Tower modal load (b) Foundation modal load
5.4 Offshore, out-of-plane HAWT model

5.4.2 Soil foundation model

A SCF and MPF embedded in a homogeneous linear-elastic are examined. Two soil-foundation models are taken, a frequency dependant CS model and static CS model. The parameters for the soil and foundations are given in Table 4.6 in Section 4.3.

The soil is modelled for a range of stiffness, or more specifically a range of shear modulus values $G_s = 1 - 10 - 100 - 500$ MPa, as all other soil and foundation parameters are held constant. Therefore the effect of the soil stiffness on the dynamic response of the wind turbine, in terms of modal frequencies and displacement response, can be examined.

**Frequency dependant CS model**

Impedance functions for the two foundations, at each shear modulus level, are generated using Plaxis and Cone method for comparison. The impedance functions are given in Figures 4.21 - 4.28 in Section 4.3.1.
Table 5.8: Normalised static CS stiffness terms with coupling (WC) and without coupling (NC) for the SCF and MPF

<table>
<thead>
<tr>
<th>Foundation</th>
<th>Model</th>
<th>$K^0_{H}/G_sR_F$</th>
<th>$K^0_{M}/G_sR_F^3$</th>
<th>$K^0_{C}/G_sR_F^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCF</td>
<td>WC</td>
<td>15.24</td>
<td>23.94</td>
<td>-9.63</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>11.32</td>
<td>28.57</td>
<td>-</td>
</tr>
<tr>
<td>MPF</td>
<td>WC</td>
<td>51.07</td>
<td>388.70</td>
<td>-377.88</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>10.20</td>
<td>198.59</td>
<td>-</td>
</tr>
</tbody>
</table>

Note, when using Cone method (CONAN) for generating the impedance functions, the internal soil should not be excavated and the reference point changed to the top of the foundation (for both foundations).

**Static CS model**

The static CS models are generated from a static Plaxis analyses. As the soil is linear-elastic, the stiffness terms are generated using the method outlined in Section 3.6.2. CS models with and without coupling are generated to study the effects of the coupling stiffness term on the system. The normalised stiffness terms are given in Table 5.8. The dimension of the SCF were calculated from formulas suggested by Byrne & Houlsby (2003) and the MPF dimensions are taken from the OC3 project (Jonkman & Musial, 2010). As the soil is linear-elastic, and the only parameter changed for each analysis is the shear modulus, the normalised stiffness given in Table 5.8 are applicable to all the soil profiles modelled.

**Damping**

A free vibration study was carried out to determine the damping ratio of the soil-foundation system for the SCF and the MPF. As the soil is linear elastic, the horizontal and rotational damping ratio could be captured separately. To determine the damping ratios, decay curves were matched to the foundations free vibration response, as discussed in Section 3.6.3. Figure 5.23 shows the free vibration response of the foundation for the horizontal translation and rotation DOF. Several SDOF decay curves are plotted over the foundation response and the best fit is taken as the damping ratio. The damping ratio for the SCF are taken as $\xi_{F_H} = 44\%$ and $\xi_{F_\theta} = 45\%$ and for the MPF as $\xi_{F_H} = 41\%$ and $\xi_{F_\theta} = 52\%$. 

178
5.4 Offshore, out-of-plane HAWT model

Figure 5.23: Free vibration response of the SCF for horizontal translation and rotation and SDOF decay curves (for damping ratios between 40-50%)

5.4.3 Numerical results

Some numerical examples are presented in order to illustrate the effects of SSI on the dynamic response of the MDOF offshore HAWT model.

The effects of the soil stiffness and the soil-foundation models on the modal frequencies of the wind turbine for each foundation are discussed. Aerodynamic wind load and wave load is then applied to the structure and the response captured. The analysis is carried out for each soil stiffness condition and each soil-foundation model, a fully fixed condition is used as a reference.

5.4.3.1 Tower modal frequencies with foundation coupling

Modal frequencies for the out-of-plane HAWT model have been calculated for each soil-foundation model at several soil stiffness conditions. Infinitely stiff foundation
conditions were included in all models to ensure the simulation were running correctly. Fixed base conditions were included as a reference.

Tower displacement response transfer functions, with respect to the nacelle loading, for unit loading conditions were generated. To obtain the transfer functions with respect to the input at the nacelle, the Fourier spectrum of the loads applied to the different DOF were normalized by the Fourier spectrum amplitude of the load at the nacelle.

For the frequency dependant CS models, modal frequencies were calculated exclusively by examining the transfer functions for the system. In contrast, for the static CS models, modal frequencies were calculated using eigenvalue analysis in addition to examining the transfer functions of the system.

Using a static CS foundation formulation for the out-of-plane model, the system matrices are time invariant and the foundation impedance is frequency independent. Thus an eigenvalues analysis can easily be carried out. The modal frequencies for the SCF and MPF are presented in Tables 5.9 and 5.10 for the coupled and uncoupled static CS foundation model. They are not labelled and simply arranged in ascending order. Examining the results we can see that the frequencies converge on the FB results as the soil stiffness increases. The blade frequencies are unaffected by the flexible foundation.

The foundation frequencies at lower soil stiffness are of the same order as that of blade and tower modal frequencies and are difficult to distinguish. However, by examining the transfer function for the system in conjunction with the Eigenvalue analysis, the tower modal frequencies can be easily identified. The lower foundation modal frequencies (under 10 Hz) are invisible in the transfer functions as they are highly damped. Transfer functions for the coupled static CS model are shown in Figure 5.24 for the SCF.

For the frequency dependent CS models, modal frequencies were identified as peaks in the transfer functions. The tower modal frequencies are identified and the results given in Tables 5.11 and 5.12. Transfer functions for the frequency depend CS model are shown in Figure 5.25, for the SCF.

The relative displacement transfer function of the tower top (or nacelle) shows a peak reduction for the first modal peak (as shear modulus drops). This indicates that the soil will help to reduce vibration. However, the second and third modal frequencies show a distant peak greater than the FB case. Therefore, depending on the loading and the influence of the second and third modes, the soil may have a positive or negative
5.4 Offshore, out-of-plane HAWT model

Table 5.9: Modal frequencies for the SCF for the static CS foundation models and FB conditions

<table>
<thead>
<tr>
<th></th>
<th>Coupled Static Stiffness</th>
<th>Uncoupled Static Stiffness</th>
<th>FB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>0.169</td>
<td>0.262</td>
<td>0.276</td>
<td>0.277</td>
</tr>
<tr>
<td>0.473</td>
<td>0.764</td>
<td>0.764</td>
<td>0.764</td>
</tr>
<tr>
<td>0.764</td>
<td>0.764</td>
<td>0.764</td>
<td>0.764</td>
</tr>
<tr>
<td>0.764</td>
<td>0.779</td>
<td>0.780</td>
<td>0.780</td>
</tr>
<tr>
<td>0.782</td>
<td>0.916</td>
<td>1.884</td>
<td>2.076</td>
</tr>
<tr>
<td>2.076</td>
<td>2.076</td>
<td>2.076</td>
<td>2.076</td>
</tr>
<tr>
<td>2.076</td>
<td>2.076</td>
<td>2.076</td>
<td>2.102</td>
</tr>
<tr>
<td>2.089</td>
<td>2.102</td>
<td>2.102</td>
<td>2.245</td>
</tr>
<tr>
<td>4.656</td>
<td>6.574</td>
<td>14.914</td>
<td>25.706</td>
</tr>
</tbody>
</table>

Table 5.10: Modal frequencies for the MPF for the static CS foundation models

<table>
<thead>
<tr>
<th></th>
<th>Coupled Static Stiffness</th>
<th>Uncoupled Static Stiffness</th>
<th>Inf</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>0.183</td>
<td>0.265</td>
<td>0.276</td>
<td>0.277</td>
</tr>
<tr>
<td>0.487</td>
<td>0.764</td>
<td>0.764</td>
<td>0.764</td>
</tr>
<tr>
<td>0.764</td>
<td>0.764</td>
<td>0.764</td>
<td>0.764</td>
</tr>
<tr>
<td>0.764</td>
<td>0.779</td>
<td>0.780</td>
<td>0.780</td>
</tr>
<tr>
<td>0.782</td>
<td>1.000</td>
<td>1.981</td>
<td>2.076</td>
</tr>
<tr>
<td>2.076</td>
<td>2.076</td>
<td>2.076</td>
<td>2.076</td>
</tr>
<tr>
<td>2.076</td>
<td>2.076</td>
<td>2.076</td>
<td>2.102</td>
</tr>
<tr>
<td>2.102</td>
<td>2.102</td>
<td>2.102</td>
<td>2.275</td>
</tr>
<tr>
<td>12.920</td>
<td>17.463</td>
<td>49.038</td>
<td>108.750</td>
</tr>
</tbody>
</table>

effect on the vibrational response.

The total displacement transfer function of the tower top show an increases in peak response for all modal frequencies (expect for the first mode at $G_s = 1$ MPa).

Discussion

Some general conclusions can be drawn from the modal frequency analysis. The soil has the effect of lowering the tower’s modal frequencies. The tower’s first mode of
displacement response shows a reduction in frequency with reduction in soil stiffness (shear modulus \( G_s \)). The tower’s second modal frequencies experiences an even greater reduction. A similar finding was reported in by Jonkman & Musial (2010); Zaaijer (2005). The latter stated that this is caused by the relatively small deflections of the foundation for the first mode shape and the relatively high deflections for the second mode shape.

The blade’s first and second modal frequencies, are identifiable (at 0.77 Hz and 2.09 Hz) in the tower displacement transfer functions (and the eigenvalue analysis) and are unaffected by the soil conditions.
### 5.4 Offshore, out-of-plane HAWT model

Table 5.11: Tower modal frequencies for various soil stiffness conditions at rated operational conditions for a SCF modelled by: (a) Impedance function, Plaxis (b) Impedance function, Cone method (c) Static stiffness matrix (d) Static stiffness matrix (No coupling)

<table>
<thead>
<tr>
<th>$G_s$ (MPa)</th>
<th>Model (a) $1^{st}$ (Hz)</th>
<th>Model (b) $1^{st}$ (Hz)</th>
<th>Model (a) $2^{nd}$ (Hz)</th>
<th>Model (b) $2^{nd}$ (Hz)</th>
<th>Model (a) $3^{rd}$ (Hz)</th>
<th>Model (b) $3^{rd}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.154</td>
<td>0.166</td>
<td>0.441</td>
<td>0.453</td>
<td>4.505</td>
<td>4.455</td>
</tr>
<tr>
<td>10</td>
<td>0.262</td>
<td>0.262</td>
<td>0.819</td>
<td>0.874</td>
<td>4.406</td>
<td>4.530</td>
</tr>
<tr>
<td>100</td>
<td>0.275</td>
<td>0.275</td>
<td>1.834</td>
<td>1.879</td>
<td>5.150</td>
<td>5.270</td>
</tr>
<tr>
<td>500</td>
<td>0.277</td>
<td>0.277</td>
<td>2.230</td>
<td>2.240</td>
<td>6.057</td>
<td>6.080</td>
</tr>
<tr>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_s$ (MPa)</td>
<td>Model (c) $1^{st}$ (Hz)</td>
<td>Model (d) $1^{st}$ (Hz)</td>
<td>Model (c) $2^{nd}$ (Hz)</td>
<td>Model (d) $2^{nd}$ (Hz)</td>
<td>Model (c) $3^{rd}$ (Hz)</td>
<td>Model (d) $3^{rd}$ (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>0.169</td>
<td>0.202</td>
<td>0.473</td>
<td>0.524</td>
<td>4.656</td>
<td>4.659</td>
</tr>
<tr>
<td>10</td>
<td>0.265</td>
<td>0.269</td>
<td>0.916</td>
<td>1.140</td>
<td>4.715</td>
<td>4.858</td>
</tr>
<tr>
<td>100</td>
<td>0.276</td>
<td>0.277</td>
<td>1.884</td>
<td>2.061</td>
<td>5.475</td>
<td>5.754</td>
</tr>
<tr>
<td>500</td>
<td>0.277</td>
<td>0.277</td>
<td>2.245</td>
<td>2.296</td>
<td>6.117</td>
<td>6.235</td>
</tr>
<tr>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.12: Tower modal frequencies for various soil stiffness conditions at rated operational conditions for a MPF modelled by: (a) Impedance function, Plaxis (b) Impedance function, Cone method (c) Static stiffness matrix (d) Static stiffness matrix (No coupling)

<table>
<thead>
<tr>
<th>$G_s$ (MPa)</th>
<th>Model (a) $1^{st}$ (Hz)</th>
<th>Model (b) $1^{st}$ (Hz)</th>
<th>Model (a) $2^{nd}$ (Hz)</th>
<th>Model (b) $2^{nd}$ (Hz)</th>
<th>Model (a) $3^{rd}$ (Hz)</th>
<th>Model (b) $3^{rd}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.208</td>
<td>0.178</td>
<td>0.480</td>
<td>0.453</td>
<td>4.38</td>
<td>4.460</td>
</tr>
<tr>
<td>10</td>
<td>0.265</td>
<td>0.265</td>
<td>0.880</td>
<td>0.931</td>
<td>4.65</td>
<td>4.575</td>
</tr>
<tr>
<td>100</td>
<td>0.275</td>
<td>0.275</td>
<td>1.998</td>
<td>1.958</td>
<td>4.98</td>
<td>5.420</td>
</tr>
<tr>
<td>500</td>
<td>0.277</td>
<td>0.277</td>
<td>2.280</td>
<td>2.265</td>
<td>6.250</td>
<td>6.137</td>
</tr>
<tr>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_s$ (MPa)</td>
<td>Model (c) $1^{st}$ (Hz)</td>
<td>Model (d) $1^{st}$ (Hz)</td>
<td>Model (c) $2^{nd}$ (Hz)</td>
<td>Model (d) $2^{nd}$ (Hz)</td>
<td>Model (c) $3^{rd}$ (Hz)</td>
<td>Model (d) $3^{rd}$ (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>0.183</td>
<td>0.106</td>
<td>0.488</td>
<td>0.433</td>
<td>4.857</td>
<td>4.629</td>
</tr>
<tr>
<td>10</td>
<td>0.265</td>
<td>0.233</td>
<td>0.998</td>
<td>0.619</td>
<td>4.713</td>
<td>4.671</td>
</tr>
<tr>
<td>100</td>
<td>0.276</td>
<td>0.272</td>
<td>1.985</td>
<td>1.421</td>
<td>5.589</td>
<td>5.023</td>
</tr>
<tr>
<td>500</td>
<td>0.277</td>
<td>0.277</td>
<td>2.270</td>
<td>2.047</td>
<td>6.183</td>
<td>5.726</td>
</tr>
<tr>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

From the presented transfer function it is clear that SSI effects on the tower response will depend on the frequency of excitation and the contribution of the second and third modes, which will be much greater considering a flexible foundation. Thus SSI can not be said to have a purely positive or purely determinant effect on the tower response in terms of vibration reduction.

At low soil stiffness, for the coupled static stiffness MPF CS model and the frequency dependant SCF CS model, the foundation couples with the third mode and causes it to rise in comparison to the next highest soil stiffness condition. This conclusion has been arrived at by examining the eigenvalue analysis. For the coupled static
stiffness MPF CS model, we can see that the foundation’s first modal frequency, at $G_s = 1$ MPa, is 3.980 Hz. This is very near the third tower modal frequency, thus the modes have coupled and this causes the third tower modal frequency to rise. The same effect caused the rise in the third tower modal frequency at $G_s = 1$ MPa for the frequency dependant SCF CS model. However the foundation modal frequency can not be identified in the transfer function due to high foundation damping.

The SCF static CS model gives results very similar to the frequency dependent CS models. This suggests that a static CS model may be an adequate soil-foundation model for this soil and turbine configuration. The models deviate slightly from each other at softer soil conditions and for higher tower modes. This is because the stiffness is more frequency dependent for softer soils and, in general, stiffness tends to rise with frequency, hence the frequency dependant CS models predict higher tower modal frequencies (for the second and third modes) compared to the static CS model for softer soil conditions. The uncoupled and coupled models give similar results. The results differ more for softer soils and higher modes. The two frequency dependant CS models (given by Plaxis and Cone method) are in very good agreement.

Very similar conclusions can be drawn from examining Table 5.12 for the MPF. The static models agree well with the frequency dependant models for the first tower mode and at higher soil stiffness conditions. Again for higher modes the results deviate especially for the softer soils. For the MPF the coupling terms have a significant impact on the predicted modal frequencies.

### 5.4.3.2 Time history response

To illustrate the effects of SSI on the 11 DOF out-of-plane HAWT model, some time histories for the displacement response of the tower founded on the MPF are now examined. The soil-foundation system, for shear modulus of $G_s = 10$ MPa, is modelled using a frequency dependant CS model (Cone method) and an uncoupled static CS model. The results are compared to FB conditions.

A steady wind load is applied to the turbine model. This analysis was carried out for illustration purpose, as to examine the effects of SSI on the system when no turbulence or wave loading are included. The results are shown in Figure 5.26, (note, no tower aerodynamic damping is included). The relative displacement of the nacelle shows a reduction in response when the foundation coupling is included. This is due to the
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

added soil damping. The total displacement of the tower shows a slight reduction in terms of peak to peak displacement. However the mean is shifted up, relative to the FB response, due to the foundation movement.

![Response time-histories for a steady load, soil-foundation system modelled using a frequency dependant CS model (Cone method); (a) Nacelle relative displacement response (b) Nacelle total displacement response](image)

Figure 5.26: Response time-histories for a steady load, soil-foundation system modelled using a frequency dependant CS model (Cone method): (a) Nacelle relative displacement response (b) Nacelle total displacement response

The HAWT model was then exposed to a turbulent aerodynamic wind excitation generated for rated condition of rotation and wind speed and a moderate sea environment. Figure 5.27a shows the relative displacement response of the nacelle. The beneficial effects of SSI (seen for steady loading conditions) are no longer visible. This might be because the soil lowers the tower’s first modal frequency making the tower
more susceptible to wave loading, thus causing an increase in response compared to the FB results. A similar finding is shown in Figure 5.27b for the total displacement response of the nacelle.

Figure 5.27: Response time-histories, soil-foundation system modelled using a frequency dependant CS model (Cone method): (a) Nacelle relative displacement response (b) Nacelle total displacement response

The soil-foundation system modelled using uncoupled static CS model is now examined. The model is exposed to the same turbulent wind and wave loading conditions. The relative and total displacement response of the nacelle are shown in Figure 5.28. The time histories show a similar trend to those generated using the frequency dependant CS model, however the results are amplified, much greater nacelle motion is
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

predicted (both relative and total). A deviation in results was expected, however, as the modal frequencies predicted using this foundation model did not agree well with the other foundation models considered.

Figure 5.28: Response time histories, soil-foundation system modelled using an uncoupled static CS model: (a) Nacelle relative displacement response (b) Nacelle total displacement response

The effect of the wave loading on the dynamic response of the tower is now examined. The model is exposed to both an extreme and moderate wave environment and results shown in Figure 5.29. Results are also presented for no wave loading as a reference. The nacelle relative displacement is seen to increase as the wave loading is increased. The frequency of the time history response is dependent on the wave loading.
and changes for each analysis. We can also note that the wave loading has a significant effect on the tower displacement response in terms of peak to peak displacement. However the wind load controls the mean displacement.

![Graph showing nacelle relative displacement response for various wave loading conditions](image)

Figure 5.29: Nacelle relative displacement response for various wave loading conditions

A frequency spectrum (Figure 5.30), is generated from the displacement response of the turbine exposed to a steady wind. The results are presented for the relative blade tip and tower top response. From Figure 5.30a we can see that SSI has little impact on the blade response and that the response is dominated by the 1P and first modal blade frequency. From Figure 5.30b, we can see that the soil has the effect of lowering the tower first modal frequency. The blade first modal frequency also appears in the spectrum, which indicates the coupling between the two. It can also be noted that the first mode of vibration dominates the response, as additional modes are not visible in the response spectrum.

5.4.4 Discussion

In this section, the foundation has been represented by three soil-foundation models; a static CS model, with and without cross coupling and a frequency dependent CS model. Tower displacement transfer functions have been presented and peaks for the tower and
blade modes identified. The soil was shown to reduce the tower’s modal frequencies but had no effect on the blade’s modal frequencies.

A static stiffness formulation with coupling was shown to give results similar to those generated from the frequency dependent impedance functions, except for very soft soil conditions. This implies that, for the frequency ranges and turbine configurations considered, a static stiffness formulation with coupling may prove sufficient under certain conditions.

The modal frequency reduction is not linearly dependent on soil parameters, SSI has little impact on the system until a certain minimum soil stiffness is passed, under which the modal frequencies drop off rapidly.

The displacement response of the tower was seen to increase considering the effects of SSI. This was due to the soil interaction softening the tower (in terms of frequency) and making it more susceptible to the wave excitation.

It should be noted that the shear modulus values taken here are design values, as according the DNV/Riso, field values are required to be reduced by 60% for dynamic calculations, see Figure 2.24, $G_s(\gamma_{st}) = 0.4G_0$. 

Figure 5.30: Response spectrum: (a) Blade response (b) Tower response
5.5 Offshore, coupled in-plane out-of-plane HAWT model

Numerical examples are presented to illustrate the effects of SSI on the dynamic response of a MDOF 5MW offshore HAWT coupled out-of-plane in-plane model, attached to a SCF and MPF embedded in non-linear soil. The dynamic response of the turbine model is examined in terms of modal frequencies and displacement response and the effects of SSI on the system are analysed. Time histories for the response of the turbine tower are generated in the fore-aft and side-to-side directions.

5.5.1 Model properties

As in the previous section, model properties for the turbine were taken from the NREL reference 5 MW offshore wind turbine. Taking the coupled in-plane out-of-plane HAWT model detailed in Section 3.2.1.2, the EOM are developed for the coupled foundation/tower/blade wind turbine model considering: the first two blade modes flapwise; the first blade mode edgewise; the roll, tilt and yaw of the nacelle; the drive-train in the nacelle; the first two tower modes (both fore-aft and side-to-side) and the foundation translation and rotational DOF (both fore-aft and side-to-side). The full EOM for the 21 DOF model are given in Appendix H.

The tower mode shapes in this section are calculated using BMODES (Bir, 2012), note MODES was used in the previous sections. BMODES is a higher fidelity model based on finite-elements and is capable of calculating coupled modes for a turbine a tower. BMODES can properly taken into consideration the nacelle-rotor sub assemblies, accounting for its off-center center of mass and its inertia in each direction. Hence the mode shapes are dependent on the direction, either fore-aft or side-to-side. The program allows the user to specify the boundary conditions at the base of the tower. In this work fully fixed (for the FB reference response) and flexible (dependant on the stiffness matrix of the soil-foundation system) boundary conditions are considered.

If there is little direct coupling between the soil and the wind turbine (which is typical for stiff soil), it maybe be adequate to decouple the analysis and calculate the mode shape for the tower assuming fully fixed conditions (as in modes). In reality, however, the dynamic behaviour of the turbine is influenced by considering a flexible foundation system and therefore the tower mode shapes will differ from the ones of a cantilevered beam. This influence, of the flexible soil on the tower mode shapes, is
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

accounted for in BMODES.

5.5.2 Soil-foundation model

Again both a SCF and a MPF are considered. The foundations are founded in non-linear soil, modelled in Plaxis using the HSsmall model. The soil parameters for the HSsmall model are given in Table 4.7. The MPF regressed CS model has been presented in Section 4.4. Using a similar method an exact solution for the CS model for the SCF (around the operating point given in Table 4.8) is created using a regression analysis. The soil profile is modelled for two distinct soil RDs (25% and 100%) and the static stiffness terms for each foundation are given in Table 5.13. As the soil model is non-linear, a full soil domain and foundation is analysed in Plaxis.

| Table 5.13: Static stiffness terms for the SCF and MPF at each soil RD |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | SCF             |                 | MPF             |                 |
| RD= 25%         |                 |                 | RD= 100%        |                 |
| $K_{HH}^0/10^9$ | 7.9624          | 71.754          | 1.1581          | 3.0163          |
| $K_{V}^0/10^{10}$ | -8.9065        | -76.797         | -1.9825         | -3.2888         |
| $K_{MM}^0/10^{11}$ | 11.503         | 90.328          | 5.4277          | 7.6757          |

The SCF is assumed to have a foundation diameter $D_F = 20$ m, an embedment depth ratio $D_e/D_F = 0.5$ and a skirt thickness of $T_F = 0.1$ m. In reality, the lid of the SCF is composed of a hollow cone structure with flange stiffeners connecting the support platform to the lid of the bucket foundation. To account for the additional stiffness and mass provided by the cover and stiffeners, the lid of the suction caisson is modelled unrealistically thick, $T_L = 0.3$ m.

The damping for the SCF and MPF was estimated, by carrying out a free vibrations analysis on the soil-foundation system, for each foundation. The damping ratio was seen to change little regardless of the RD of the soil and therefore an average value for each foundation was taken. The damping ratio for the SCF was taken as $\xi_{FH} = 43\%$ and $\xi_{Fh} = 46\%$ and for the MPF as $\xi_{FH} = 45\%$ and $\xi_{Fh} = 51\%$.
5.5 Offshore, coupled in-plane out-of-plane HAWT model

5.5.3 Modal frequencies

5.5.3.1 Eigenvalue analysis

To solve the eigenvalue problem for the coupled in-plane out-of-plane wind turbine model, we first must consider the governing EOM for the system in free vibration,

\[ [M(t)] \{ \ddot{x} \} + [C(t)] \{ \dot{x} \} + [K(t)] \{ x \} = 0 \] (5.12)

the mass, damping and stiffness matrices are periodic, i.e. time dependent. Insertion of the solution used in a tradition eigenvalue analysis \( y = Y e^{\lambda t} \) yields the eigenvalue problem,

\[ ([A(t)] - \{ \lambda \} [I]) \{ Y \} = 0 \] (5.13)

which can never be fulfilled unless \( Y = 0 \). The so-called snapshot eigenvalue analysis computing periodic azimuth dependent eigenvalues \( \lambda = \lambda(t) \) is nonsense (Hansen & Kallesøe, 2011). Therefore a transformation must be sought to obtain a time-invariant system.

5.5.3.2 Coleman transformation

The time-varying model can be re-formulated as a time-invariant model by introducing the multi-blade coordinate transformation, also referred as a Coleman transformation. The Coleman transformation, a special case of the L-F transformation, is a mathematical tool for three-bladed rotors that allows for mapping the dynamic variables described in local blade coordinates into an non-rotating reference frame. Assuming that the rotor is isotropic, i.e. all blades are identical, identically pitched and symmetrically mounted on the hub. The idea of the transformation is to refer the motions of individual blades in the same coordinate system as the structure supporting the rotor (Hansen, 2003). In this way, the periodic terms in the governing equations are eliminated.

The rotating frame vector \( q(t) \) can be expressed as a function of the non-rotating frame vector \( q_{nr}(t) \) as,

\[ \{ q(t) \} = [B_{cm}(t)] \{ q_{nr}(t) \} \] (5.14)
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

where,

\[
[B_{C_m}(t)] = \begin{bmatrix}
[I_b] & [I_b]\cos(\varphi_1) & [I_b]\sin(\varphi_1) & 0 \\
[I_b] & [I_b]\cos(\varphi_2) & [I_b]\sin(\varphi_2) & 0 \\
[I_b] & [I_b]\cos(\varphi_3) & [I_b]\sin(\varphi_3) & 0 \\
0 & 0 & 0 & [I_{TF}]
\end{bmatrix}
\] (5.15)

and,

\[
[B_{C_m}]^{-1}[\dot{B}_{C_m}] = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & \Omega_b[I_b] & 0 \\
0 & -\Omega_b[I_b] & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \equiv [R]
\] (5.16)

where \(q_{nr}(t)\) denotes the vector of generalized coordinates in the non-rotating frame, \(I_b\) is the identity matrix of size number of blade DOFs and the term \(I_{TF}\) is the identity matrix of size number of tower plus foundation DOFs. Since the motion of nacelle/tower/foundation is described in the ground fixed frame, no transformation is required.

Following work presented by Hansen (2004); Hansen & Kallesøe (2011); Skjoldan & Hansen (2009), the coupled in-plane out-of-plane HAWT model is given in the Coleman domain (i.e. in the non-rotating frame) as,

\[
[M_{C_m}]\ddot{x}_{nr} + (2[M_{C_m}][R] + [C_{C_m}])\dot{x}_{nr} + ([M_{C_m}][R]^2 + [C_{C_m}][R] + [K_{C_m}])x_{nr} = 0
\] (5.17)

where,

\[
[M_{C_m}] \equiv [B]^{-1}[M(t)][B] \\
[C_{C_m}] \equiv [B]^{-1}[C(t)][B] \\
[K_{C_m}] \equiv [B]^{-1}[K(t)][B]
\] (5.18)

Thus the time-invariant system matrix \(A_{C_m}\) can be written as,

\[
[A_{C_m}] = \begin{bmatrix}
0 & [I] \\
-([M_{C_m}]^{-1}([M_{C_m}][R]^2 + [C_{C_m}][R] + [K_{C_m}]) -[M_{C_m}]^{-1}(2[M_C][R] + [C_{C_m}]))
\end{bmatrix}
\] (5.19)
Note, to check that the Coleman transformation is working properly, a simple test can be carried out. The Coleman transformation matrices $M_{cm}$, $C_{cm}$ and $K_{cm}$ should be time-invariant therefore Coleman transformation matrices calculated for different azimuth angles should equate to zero when taken away for each other i.e.,

\[
\begin{align*}
[B]^{-1}[M(\varphi)][B] - [B]^{-1}[M(\varphi + \Theta)][B] &\equiv 0 \\
[B]^{-1}[C(\varphi)][B] - [B]^{-1}[C(\varphi + \Theta)][B] &\equiv 0 \\
[B]^{-1}[K(\varphi)][B] - [B]^{-1}[K(\varphi + \Theta)][B] &\equiv 0
\end{align*}
\]

(5.20)

where $\Theta$ is any angle.

### 5.5.3.3 Campbell diagram

Once the system matrices are time-invariant (and in the ground fixed frame), an eigenvalue analysis can be carried out. As already noted in Section 2.5.1.3, the eigenvalues returned are in complex conjugate pairs and have a real part (damping) and an imaginary part (the modal frequencies). By processing the results of the eigenvalue analysis, the modal frequencies of the system can be obtained.

The HAWT model contains rotating blades, centrifugal stiffening, includes nacelle DOFs that have dependency on the rotational speed $\Omega_b$, a Campbell diagram therefore is a useful visualisation tool.

A Campbell diagram can be obtained by estimating the modal frequencies of the system at different rotational speeds, $\Omega_b = 0 - 15$ rpm. Considering FB conditions, a Campbell diagram has been calculated for the coupled out-of-plane in-plane wind turbine model and is shown in Figure 5.31 for the lower modal frequencies.

Upon the Campbell diagram, the avoidance frequencies of $1P \pm 10\%$ and $3P \pm 10\%$ are plotted along with the zone of operation, i.e. the cut-in speed (6.9 rpm) and rated speed (12.1 rpm). From the literature review presented in Section 2.5.3, we know that in the zone of operation (for a wind turbine) no modal frequencies should pass within $\pm 10\%$ of the rotation frequency $1P$ or of the blade passing frequency $3P$. From Figure 5.31 we can see that this is true for the tower modal frequencies for the FB response, which is expected since the wind turbine has been designed for FB conditions. We can also state the wind turbine is designed as a soft-stiff structure as its first tower modal frequency lies in-between $1P$ and $3P$. However, a blade modal frequency does cross the
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Figure 5.31: Campbell diagram with modal frequencies (Hz) plotted against rotation speed $\Omega_b$ (rpm)

3P line within the zone of operation and this could lead to a resonance issue (for the first flapwise backward whirling mode).

Examining Figure 5.31, we can see that the blade modal frequencies begin to split as the rotor speed is increased. The frequencies split into three separate components for each blade mode, a so-called symmetric component, a Backwards Whirling (BW) component and a Forward Whirling (FW) component (except for the blade edgewise mode which only contains BW and FW components). The frequencies of the BW modes decrease with rotation speed, whereas the frequencies of the FW modes in-
5.5 Offshore, coupled in-plane out-of-plane HAWT model

crease with rotation speed. This splitting of the modal frequencies, of a whirling mode pair (Hansen, 2003), is related to the frame of reference (ground fixed or rotor fixed) of the observer. An observer on the tower top (in the ground fixed frame) will measure the blade frequency $\omega_b$ for all three modes. The observer on a blade will also measure the frequency $\omega_b$ for symmetric blade modes, but for backward and forward whirling modes the same observer will measure the frequencies $\omega_b + \Omega_b$ and $\omega_b - \Omega_b$, respectively (Hansen & Kallesøe, 2011). Thus the frequency of the symmetric component is $\omega_b$ while the frequencies of the whirling components are shifted with frequency $\pm \Omega_b$ and hence increase/decrease as one move from left to right in the Campbell diagram. Therefore FW and BW modes only make sense during operation. The frequency of the symmetric flap mode increases due to centrifugal stiffening of flapwise bending.

The side-to-side tower bending always lies slightly lower than the fore-aft tower bending mode because it contains some tilting of the rotor which has a large inertia (Hansen, 2003). The natural frequencies of the tower bending modes and the shaft torsion (drivetrain) mode are constant with rotation speed.

5.5.3.4 Modal frequencies considering the flexible foundation

In order to carry out the modal analysis, tower mode shapes for the various boundary conditions were first required. A by-product of the BMODES analysis are the tower’s natural frequency, these are presented in Table 5.14. Note, the analysis assumes a fixed rotor nacelle assemble and hence predicts different natural frequency than a complete wind turbine model analysis. From Table 5.14, we can see that, as the soil stiffness (RD) rises, the natural frequencies approach the FB results. Interestingly for the MPF at a RD = 100% higher tower natural frequencies are obtained than for the SCF at a RD = 25%. Examining the static stiffness terms (given in Table 5.13), one would expect the opposite. The difference is not due to the mass or the inertia of the foundation (which has little effect on the natural frequencies) rather it seems to be due to the ratio of the terms in the stiffness matrix.

With the mode shapes in place, an eigenvalues analysis of the system is carried out using the method discussed. The modal frequencies for each foundation at each soil RD is given in Table 5.15. An infinitely stiff soil-foundation system and FB condition are included as a reference and to validate the model.

The tower modal frequencies are then extracted from Table 5.15 and presented in
### Table 5.14: Tower natural frequencies given by Bmodes

<table>
<thead>
<tr>
<th>Mode</th>
<th>SCF</th>
<th>MPF</th>
<th>FB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RD = 25%</td>
<td>RD = 100%</td>
<td>RD = 25%</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; SS</td>
<td>0.25839</td>
<td>0.27068</td>
<td>0.26059</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; FA</td>
<td>0.26026</td>
<td>0.27286</td>
<td>0.26250</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; SS</td>
<td>1.45443</td>
<td>1.56306</td>
<td>1.45397</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; FA</td>
<td>1.64910</td>
<td>1.82473</td>
<td>1.64505</td>
</tr>
</tbody>
</table>

### Table 5.15: Modal frequencies for the two soil profiles (for both the SCF and MPF), an infinitely stiff foundation (Inf) and FB conditions

<table>
<thead>
<tr>
<th>SCF</th>
<th>MPF</th>
<th>Inf</th>
<th>FB</th>
</tr>
</thead>
<tbody>
<tr>
<td>RD = 25%</td>
<td>RD = 100%</td>
<td>RD = 25%</td>
<td>RD = 100%</td>
</tr>
<tr>
<td>0.06703</td>
<td>0.06703</td>
<td>0.06703</td>
<td>0.06703</td>
</tr>
<tr>
<td>0.26006</td>
<td>0.27922</td>
<td>0.26302</td>
<td>0.27506</td>
</tr>
<tr>
<td>0.26206</td>
<td>0.28022</td>
<td>0.26522</td>
<td>0.27625</td>
</tr>
<tr>
<td>0.55782</td>
<td>0.55782</td>
<td>0.55782</td>
<td>0.55782</td>
</tr>
<tr>
<td>0.77714</td>
<td>0.77998</td>
<td>0.77788</td>
<td>0.77971</td>
</tr>
<tr>
<td>0.85255</td>
<td>0.91767</td>
<td>0.91734</td>
<td>0.91755</td>
</tr>
<tr>
<td>0.85322</td>
<td>0.95315</td>
<td>0.92461</td>
<td>0.95315</td>
</tr>
<tr>
<td>0.91767</td>
<td>1.3256</td>
<td>0.92537</td>
<td>1.2604</td>
</tr>
<tr>
<td>0.95315</td>
<td>1.5948</td>
<td>0.95315</td>
<td>1.2699</td>
</tr>
<tr>
<td>1.3262</td>
<td>1.6136</td>
<td>1.3263</td>
<td>1.3261</td>
</tr>
<tr>
<td>1.8437</td>
<td>1.8437</td>
<td>1.8437</td>
<td>1.8437</td>
</tr>
<tr>
<td>2.102</td>
<td>2.1021</td>
<td>2.1021</td>
<td>2.1022</td>
</tr>
<tr>
<td>2.2388</td>
<td>2.2388</td>
<td>2.2388</td>
<td>2.2388</td>
</tr>
<tr>
<td>2.8847</td>
<td>2.8847</td>
<td>2.8847</td>
<td>2.8847</td>
</tr>
<tr>
<td>4.1604</td>
<td>4.4701</td>
<td>4.2566</td>
<td>4.4701</td>
</tr>
<tr>
<td>4.4701</td>
<td>5.0281</td>
<td>4.4701</td>
<td>5.0281</td>
</tr>
<tr>
<td>5.0281</td>
<td>5.0914</td>
<td>5.0281</td>
<td>6.0179</td>
</tr>
<tr>
<td>5.1197</td>
<td>5.3916</td>
<td>5.3227</td>
<td>6.3549</td>
</tr>
<tr>
<td>14.905</td>
<td>45.029</td>
<td>10.469</td>
<td>15.858</td>
</tr>
<tr>
<td>15.084</td>
<td>45.38</td>
<td>10.596</td>
<td>15.902</td>
</tr>
</tbody>
</table>

Table 5.16. The foundation has little impact on the first tower modal frequency, reducing it by around 8% (for the RD = 25% compared to FB conditions). The foundation is seen to have a significant impact on the second tower modal frequency (even for stiff soil conditions). Both the fore-aft and the side-to-side modes are equally affected by the SSI. Similar finding were report by Jonkman & Musial (2010); Zaaijer (2005).
Table 5.16: Tower modal frequencies for the two soil profiles (for both the SCF and MPF), an infinitely stiff foundation (inf) and FB conditions

<table>
<thead>
<tr>
<th>Mode</th>
<th>SCF 25%</th>
<th>SCF 100%</th>
<th>MPF 25%</th>
<th>MPF 100%</th>
<th>inf</th>
<th>FB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st SS</td>
<td>0.260</td>
<td>0.279</td>
<td>0.263</td>
<td>0.275</td>
<td>0.283</td>
<td>0.283</td>
</tr>
<tr>
<td>1st FA</td>
<td>0.262</td>
<td>0.280</td>
<td>0.265</td>
<td>0.276</td>
<td>0.284</td>
<td>0.284</td>
</tr>
<tr>
<td>2nd SS</td>
<td>0.853</td>
<td>1.595</td>
<td>0.925</td>
<td>1.260</td>
<td>2.259</td>
<td>2.273</td>
</tr>
<tr>
<td>2nd FA</td>
<td>0.853</td>
<td>1.614</td>
<td>0.925</td>
<td>1.270</td>
<td>2.303</td>
<td>2.320</td>
</tr>
</tbody>
</table>

The first tower modal frequency in the fore-aft direction for the SCF is plotted against rotation in Figure 5.32. The zone of operation shown in Figure 5.31 is dependent on the generator. The 5 MW turbine could potentially be used to house alternative generators, with may have different cut-in speeds and rated rotations. Examining Figure 5.32, the viable region that a generator could operate in is between 6.5 - 14.3 Hz.
Figure 5.32: Tower first modal frequencies in the fore-aft direction plotted against rotation speed $\Omega_b$ (rpm)
5.5 Offshore, coupled in-plane out-of-plane HAWT model

5.5.4 Time history response

Some time histories are now presented for a HAWT model exposed to rated wind conditions and a moderate sea environment (parameters given in Table 5.7). The time histories are calculated for the turbine founded on the SCF in soil with a RD = 25% and FB conditions as a reference. The blade tip response in the flapwise and edgewise direction is shown in Figure 5.33. Again, the soil is seen to have little impact on the blade relative vibrations.

![Relative blade tip displacement response](image)

Figure 5.33: Relative blade tip displacement response for the turbine founded on SCF (RD = 25%) compared to FB conditions: (a) Flapwise vibrations (b) Edgewise vibrations
Figure 5.34: Tower top relative displacement response for the turbine founded on SCF (RD = 25%) compared to FB conditions: (a) Fore-aft (b) Side-to-side

Figure 5.34 shows the tower top’s (nacelle) relative displacement in the fore-aft and side-to-side direction. Vibrations in the side-to-side direction are clearly damped by SSI effects, thus the soil has a positive effect in terms of vibrations control in this direction. However, in the fore-aft direction the soil interaction clearly has a detrimental effect as the vibrations are significantly increased. Examining the transfer function in the fore-aft direction, shown in Figure 5.36, the soil has two distinct effects on the first tower mode: adding damping and lowering its frequency. For the second tower mode the soil interaction will increase the response. Therefore, depending on the excitation frequency, SSI may have a positive or negative effect on the towers response in terms of vibration suppression. In this example, the second mode has a significant contribution
5.5 Offshore, coupled in-plane out-of-plane HAWT model

to the tower relative response, this may be due to the more accurate mode shapes given from BMODES.

Figure 5.35: Close up of the tower top relative displacement response for the turbine founded on SCF (RD = 25%) compared to FB conditions: (a) Fore-aft (b) Side-to-side

The SCF response in terms of horizontal translation and rotation is shown in Figure 5.37 in the fore-aft and side-to-side directions. As expected, the foundation movement is much greater in the fore-aft direction. The foundation movement is relatively small with a maximum rotation of around 0.00035 rad, well under the SLS provided by the DNV/Riso (0.0087 rad). This was expected as the turbine is only operating under rated wind conditions and a moderate sea environment.
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

![Diagram of Tower Relative Displacement Response Transfer Function](image)

Figure 5.36: Tower relative displacement response transfer function for turbine founded on the SCF (RD = 25%) and FB conditions

5.5.5 Discussion

A coupled in-plane out-plane MDOF HAWT model with foundation coupling was examined and SSI effects on the system analysed.

The EOM contained time varying terms, due to the in-plane motion. Thus before an eigenvalue analysis was carried out the EOM were made time-invariant using the Coleman transformation. A Campbell diagram was presented for a FB response and the blade frequencies were shown to split as the rotational speed increased.

The soil-foundation system, embedded in non-linear soil, was represented by a static CS model. Again, the first modal frequency was shown to be least sensitive and the second modal frequency most sensitive to the SSI affects. The blade modal frequencies and response was shown to be independent of soil conditions.

The foundation was shown to add damping to the system in the side-to-side direction but the effect in the fore-aft direction was cancelled out by the soils effect on the tower’s modal frequency, which made the tower more susceptible to wave excitation.
5.5 Offshore, coupled in-plane out-of-plane HAWT model

Figure 5.37: SCF (RD = 25%) response fore-aft and side-to-side direction: (a) Horizontal translation (b) Rotation
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

5.6 Plaxis based offshore HAWT model

An FEM model is developed to study the effect of unaligned wind and wave loading on the 5 MW offshore HAWT founded on a SCF. The FEM model includes the soil, foundation, support structure and turbine tower. Two soil profiles at a soil relative density of 50% and 100% are modelled and a FB case as a reference. The response of the tower and foundation in the fore-aft and side-to-side directions is examined for a combination of wind-wave alignment events.

5.6.1 Model properties

5.6.1.1 Support structure and turbine tower

Model properties from the NREL reference offshore 5 MW HAWT, detailed in Appendix D, are used to create model in Plaxis.

The turbine tower (unlike the support structure) has a varying wall thickness decreasing with height. Due to limitations of Plaxis plate models, a tapering tower thickness is not possible and instead an average thickness (0.023 m) was chosen. The nacelle and hub were modelled as rigid plate elements resting on the tower top with a position, dimensions and density all specifically chosen to give the correct equivalent mass and center of mass (e.g. the hub was placed a distance of 5 m upwind from the tower center and the nacelle plates were arranged so that the center of mass was 1.9 m downwind of the tower center). The offshore wind turbine is founded on a SCF with a diameter \( D_f = 20 \text{ m} \) and an embedment ratio \( D_e / D_f = 0.5 \). The suction caisson lid is modelled as rigid plate and the skirt as flexible. The Plaxis model is shown in Figure 5.38.

5.6.1.2 Wind and wave loading

Both wind and wave loading are applied to the soil-foundation-tower Plaxis model. The wind load is applied to the hub and the wave load is applied to the support structure at the water depth height (20 m). The position of the wave load is incrementally shifted (by 15°) around the tower, relative to the wind load, until the loads are 90° out of phase with each other, as shown in Figure 5.38. In Plaxis, load time histories can be imported into the model, thus numerically accurate unsteady correlated wind and wave loading are applied. The applied wind load (generated using BEM theory with rotational sam-
5.6 Plaxis based offshore HAWT model

Figure 5.38: Screen shot from Plaxis showing the turbine tower model founded on the suction caisson, with wind and wave loading

pled turbulence) and wave load (generated using Airy linear wave theory, JONSWAP spectrum and Wheeler Stretching) are shown in Figure 5.39.

5.6.1.3 Mesh generation

Dynamic calculations in Plaxis 3D, depending on the model arrangement, can take a relatively long time to run. Run times for the dynamic calculation depend on a variety of parameters: coarseness of the FEM mesh, soil model, soil stiffness, foundation/structure model, model boundaries, run time and time step. Therefore, a Mohr-Coulomb soil model, a fifty second run time at a time step of 0.01 and a coarse mesh setting (with the plate elements refined giving around 4000 elements and 6000 nodes) were chosen. The total real time output for each analysis was 350 s (50 s for each phase at seven phases). The run time of which was approximately 6 days on a In-
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Figure 5.39: Input time histories to Plaxis: (a) Wind load (b) Wave load

tel(R)Core(TM)2Quad 2.5GHz desk top computer.

5.6.1.4 Soil model

The soil is described in Plaxis using a Mohr-Coulomb soil model. Two soil types were modelled corresponding to a sand with a RD of 50% and 100% and a FB case was used as a reference. The soil properties were matched to the parameters given in Brinkgreve et al. (2010) for the HSsmall soil model. This was straightforward for some soil parameters but some effort was required to match the initial stiffness of the soil and the Stiffness Linearly Increasing With Depth (SLIWD) value. The soil parameters for the two soil profiles are given in Table 5.17. As load rotates a full soil domain and tower, support structure and foundation are modelled in Plaxis, see Figure 5.38.
Table 5.17: Mohr-Coulomb soil parameters for the Plaxis wind/wave model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>RD 50%</th>
<th>RD 100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight, $\gamma_{\text{unsat}}$</td>
<td>kN/m$^3$</td>
<td>17</td>
<td>19</td>
</tr>
<tr>
<td>Saturated unit weight, $\gamma_{\text{sat}}$</td>
<td>kN/m$^3$</td>
<td>19.8</td>
<td>20.6</td>
</tr>
<tr>
<td>Young’s modulus, $E_s$</td>
<td>kN/m$^2$</td>
<td>30000</td>
<td>60000</td>
</tr>
<tr>
<td>SLIWD, $E_{\text{inc}}$</td>
<td>kN/m$^2$/m</td>
<td>3000</td>
<td>6000</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_s$</td>
<td>-</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>Cohesion, $c'$</td>
<td>-</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Angle of friction, $\phi'_s$</td>
<td>deg</td>
<td>34.3</td>
<td>40.5</td>
</tr>
<tr>
<td>Dilatancy angle, $\psi'_s$</td>
<td>deg</td>
<td>4.3</td>
<td>10.5</td>
</tr>
<tr>
<td>Shear wave velocity, $G_s$</td>
<td>MPa</td>
<td>1154</td>
<td>23080</td>
</tr>
<tr>
<td>Interface, $R_{\text{inter}}$</td>
<td>-</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Lateral earth pressure $K_0$</td>
<td>-</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

5.6.2 FEM model results

The output response from Plaxis for the turbine tower, support structure and foundation is now presented, for each combination of wind and wave loading.

Figure 5.40 and 5.41 shows the tower top’s and support structure’s (at wave application point) maximum displacement response for each phase. The response is given in the Fore-Aft (FA) and Side-to-Side direction. The magnitude ($= \sqrt{FA^2 + SS^2}$) is also shown. As expected the deflection in the side-to-side direction increases as the wave load is rotated around the structure with respect to the wind load, see Figure 5.38. Note, a phase of zero corresponds to wind and wave loads fully aligned. The magnitude of the displacement decrease as the wave load moves out of phase.

The motion of the foundation in the side-to-side direction, in terms of horizontal translation and rotation, increases as the wave load moves out of phase. The maximum motion in the fore-aft direction shows the opposite effect.

A sample displacement time history, taken in the fore-aft direction for a phase of zero, is shown in Figure 5.44 for the tower top and support structure (at wave application point). From Figure 5.44, we can gauge how the magnitude of the response increases as the soil stiffness drops. We also note there is a change in phase between the three responses. This is can be easily viewed in the frequency domain, a spectrum for the free vibration response of the tower is shown in Figure 5.45. The natural frequency of the tower decreases with decreasing the soil RD. The natural frequency drops by around 7%, comparing the FB response to the soil with a RD = 50%. The resulting
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Figure 5.40: Tower top displacement: (a) Fore-aft (b) Side-to-side (c) Magnitude

Figure 5.41: Displacement at wave application point: (a) Fore-aft (b) Side-to-side (c) Magnitude
natural frequencies values are very similar to those given in the previous section, where a very similar HAWT and soil-foundation system was modelled.

### 5.6.3 Discussion

Drawing on these results some points can be noted. As the RD of the soil decreases, the response of the turbine tower increases (in terms of displacement). This is due to the movement of the foundation in the flexible soil (especially the rotation). The foundation translation and rotation in the side-to-side direction increases as the wave load is rotated around the tower with respect to the wind load. The rotational is significant and may need to be considered in design.

### 5.7 Conclusions

The dynamic response of an onshore and an offshore HAWT including foundation coupling has been analysed for a number of soil-foundation models exposed to aerody-
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Parametric studies were carried out on the developed out-of-plane HAWT model, both onshore and offshore. The stiffness of the soil was altered while holding all other soil, foundation, turbine and loading parameters constant. The soil-foundation system, resting on linear-elastic homogeneous half-space, was modelled using both a static and frequency dependant CS model. In terms of vibration suppression, SSI was found to have little impact on the relative displacement and a detrimental effect on the total displacements of the tower, especially for softer soils. The soil was observed to reduce the tower's modal frequencies but had little effect on the blade's modal frequencies. The static CS models were shown to give comparable results to the frequency dependant CS models, for certain conditions. The cross coupling terms were shown to be important for the SCF and the MPF. However, adequate correlation was found for the GBF without need of these terms.

The dynamic response of the coupled in-plane out-of-plane offshore HAWT model, founded on the SCF and MPF resting in non-linear soil, has been analysed. Again, the tower's modal frequencies were seen to decrease considering the effects of the flexible
5.7 Conclusions

Figure 5.44: Time history of normalised displacement in the fore-aft direction for aligned wind and wave load (phase = 0) for each soil profile: (a) Tower top (b) Wave application point

soil. SSI was observed to add some damping to the system. It was also shown to make the tower more susceptible to wave excitation.

An FEM model of an offshore wind turbine, founded on a SCF, was built in Plaxis to examine the effects of unaligned wind and wave loading on the dynamic response of the turbine tower, support structure and soil-foundation system. The side-to-side displacement of the turbine increased as the wave load was rotated around the turbine with respect to the wave load.

Engineering design can often come down to a compromise between accuracy and
5. ANALYSIS OF WIND TURBINES WITH FOUNDATION INTERACTION

Figure 5.45: Spectrum of the free vibration response of the turbine for each soil profile and FB conditions

computation time. In this chapter, several HAWT models have been analysed: an out-of-plane, a coupled in-plane out-of-plane and a FEM model.

The FEM model is referred to as a “direct method” because the foundation and sub-structure are analysed in a single step. Due to the computation time required to carry out these analyses, they are not particularly amenable for routine design. The substructure approach is therefore recommended instead.

The in-plane model represents a simple (and quick in terms of computation time) method of analysing HAWT, including foundation interaction. The overall softening effects of the foundation on the tower can be swiftly evaluated. However, for a more comprehensive analysis, a coupled in-plane out-of-plane model is recommended. Here foundation effects on the full system’s modal frequencies of the turbine can be scrutinised. Building MDOF HAWT models is a time consuming process therefore the use of software packages is recommended. One such software package is FAST, in the following chapter FAST is used to examine the dynamic response of HAWT including SSI.
Chapter 6

FAST modelling and analysis

6.1 Introduction

In chapter 3, formulation for the dynamic analysis of HAWT has been developed. In chapter 5, soil interaction analysis for the developed wind turbine model, coupled with linear and non-linear soil-foundation systems including wind and wave loading, has been carried out.

In this chapter, the aeroelastic code FAST is used to further investigate the effects of SSI on the dynamic response of wind turbines. FAST is an industrial certified wind turbine simulation software. A key element of FAST is the built-in control strategies available to the user. In FAST the EOM can only be solved in the time domain. However in chapter 5, it was found that for certain conditions static CS models will give comparable results to frequency dependant CS models. Both a SCF and MPF are connected to a HAWT, modelled in FAST, via a static CS model and an Apparent Fixity (AF) model. The soil-foundation system's effects on the natural frequencies and on the shear force and bending moment at the base of the turbine tower are examined. A short examination of SSI effects on the fatigue at the tower foundation connection is also considered. The analyses are carried out for a wind speed just above rated, a moderate sea environment and with the wind turbine's control system switched on and off.
6. FAST MODELLING AND ANALYSIS

6.2 FAST

FAST (Fatigue, Aerodynamics, Structures and Turbulence) is a NREL developed open source aeroelastic design code. Although FAST models the foundation as rigid (or fixed), the code is open source and allows the user to define new subroutines. Alternative soil-foundation system can thus be implemented.

FAST (version, v7.01.00a-bjj, 5.4 MB, 22-February-2012) is used to run all simulations in this chapter. FAST is used for the design and certification of HAWT and can simulate the dynamic response of onshore or offshore wind turbines with two or three blades. It offers both a time-marching analysis and a linearisation analysis (Jonkman & Buhl Jr., 2005); the former was used to obtain the responses time histories and the latter was used to determine natural frequencies of the wind turbine system for the different soil-foundation models.

The program can be used for the design and certification of Horizontal Axis Wind Turbines.

In FAST, numerous types and sizes of wind turbine can be modelled. The direction, upwind or downwind, the number of blades their size and stiffness and that of the tower can all be specified. In this study, a three bladed upwind HAWT is considered.

In FAST, the turbine is comprised of nine rigid bodies (representing the earth, support platform, base plate, nacelle, armature, gears, hub, tail, and structure furling with the rotor, if applicable) and five flexible bodies (the tower, drive shaft, and three blades). The wind turbine model has 24 DOFs. The first six describe rotational and translational platform motions in the three orthogonal directions. These DOFs are defined with respect to a fixed inertia frame of reference. The tower, modelled as a flexible body, rigidly attaches to the platform; it is described by two modes in the fore-aft and side-to-side directions that are cantilevered to the platform. These represent the next four DOFs. The tower is assumed to deflect only in a linear combination of these mode shapes. The blades are also modelled by a linear modal representation. Nine DOFs describe three modes each for the three blades the modes include two flapwise and one edgewise mode per blade. The remaining five DOFS describe nacelle yaw, generator azimuth angle, drivetrain rotational flexibility, rotor-furl, and tail-furl.

The FAST user can enable or disable any of these available DOFs to model their wind turbine as appropriate. In this study the following DOFs are enabled: all the tower bending modes, all the blade bending modes, nacelle yaw, nacelle generator, drivetrain
rotational DOF. For the CS models, the platform DOFs in the surge, sway, pitch and roll direction are enabled.

Since the blades and tower can only deflect as linear combinations of their mode shapes, accuracy of the simulation depends greatly on the accuracy of the user-provided mode shapes. The NREL distributed BMODES is used to generate mode shapes for the blades and tower.

6.2.1 Inflow Turbulence and Aerodynamic Loads

The wind environment is defined in part by a hub-height ten-minute mean wind speed, which is assumed to follow a Rayleigh distribution. This represents the mean wind speed in the longitudinal direction; in the lateral (transverse) and vertical directions, the mean wind speed is zero. TurbSim (an add on to FAST) is used to produce the random stationary processes that are then given to FAST as numerous separate inflow wind time series. TurbSim creates a 3D inflow turbulence field over the entire rotor; thus, wind velocities are simulated on a 2D grid covering the rotor plane. Each grid point on the rotor plane describes the sum of the mean wind velocity and the fluctuating wind field (i.e. the turbulence).

Once a full 3D random wind velocity field has been generated, aerodynamic forces are calculated by the program AeroDyn, which is incorporated into FAST. AeroDyn and FAST source code files are compiled together, forming a single executable. The aerodynamic forces on the blades are computed using the wind fields, airfoil geometries defined for several sections along the length of each blade, rotational speed, and pitch angle.

6.2.2 Hydrodynamic Loads

FAST can also calculate the hydrodynamic loads on the turbine tower for offshore applications. Given the significant wave height and peak spectral period, FAST first calculates the wave spectrum (using the JONSWAP spectrum) and then generates a random, stationary sea surface elevation process composed of irregular, long-crested waves. Linear Airy wave theory is then used to compute water particle kinematics between the mean sea level and the seabed. Wheeler stretching then extends the water particle kinematics to the instantaneous free surface. Once the wave kinematics are
6. FAST MODELLING AND ANALYSIS

calculated along the support structure, horizontal forces on the vertical tower elements is calculated using Morison’s equation.

6.2.3 Control

A major element of the FAST simulation model is the control system. Users can incorporate their own controller algorithms by recompiling FAST with user-defined subroutines. In this work, a generator-torque controller and a full-span rotor-collective blade-pitch controller are implemented based on the control system outlined in Jonkman et al. (2009).

Both the generator-torque and blade-pitch controllers use the generator speed measurement as the sole feedback input. The two control systems are designed to work independently, in the below-rated and above-rated wind-speed range, respectively. The objective of the generator-torque controller is to maximize power capture below the rated operation point. The goal of the blade-pitch controller is to regulate generator speed above the rated operation point (Jonkman et al., 2009).

All simulations in this study are run at above rated conditions. Hence the blade-pitch control system plays a major role in the response. The control system works when the instantaneous wind speed exceeds the rated wind speed; the blade pitches to alleviate aerodynamic loads and maintain constant power production at higher than rated wind speeds. The controller significantly affects the loads experienced by the blade, tower and consequently the foundation.

6.3 Model definition

All analysis in this chapter were carried out on the NREL reference 5 MW HAWT (Jonkman et al., 2009) sited in shallow water, modelled parameters for which are taken from Bush (2009) and detailed in appendix D. Three separate foundation boundary conditions are considered: a SCF, a MPF and FB as a reference. The soil type and MPF dimensions are taken from the IEA Offshore Code Comparison Collaboration (OC3) project (Jonkman & Musial, 2010).
6.3 Model definition

6.3.1 FAST inputs

The offshore turbine is sited in a water depth of 20 m. FAST requires some input parameters to define the wind and wave conditions; these are given in Table 6.1, the foundation properties are given in Table 6.2.

<table>
<thead>
<tr>
<th>Table 6.1: Wind and wave FAST input parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Mean wind speed</td>
</tr>
<tr>
<td>Turb. model</td>
</tr>
<tr>
<td>IEC turb. class</td>
</tr>
<tr>
<td>Wave height, $H_w$</td>
</tr>
<tr>
<td>Wave period, $T_w$</td>
</tr>
</tbody>
</table>

Table 6.2: Parameters for the SCF and the MPF

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>MPF</th>
<th>SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_F$</td>
<td>m</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>$D_e$</td>
<td>m</td>
<td>36</td>
<td>10</td>
</tr>
<tr>
<td>$T_F$</td>
<td>m</td>
<td>0.06</td>
<td>0.1</td>
</tr>
<tr>
<td>$MT_F$</td>
<td>kg</td>
<td>316417</td>
<td>316417</td>
</tr>
<tr>
<td>$I_F$</td>
<td>kgm$^2$</td>
<td>34201222</td>
<td>34201222</td>
</tr>
<tr>
<td>$\xi_{F_h}$</td>
<td></td>
<td>0.42</td>
<td>0.43</td>
</tr>
<tr>
<td>$\xi_{F_d}$</td>
<td></td>
<td>0.44</td>
<td>0.36</td>
</tr>
</tbody>
</table>

6.3.2 Soil

The soil profile, shown in Figure 6.1, was used in the IEA Offshore Code Comparison Collaboration (OC3) project (Jonkman & Musial, 2010).

The soil profile was selected to achieve adequate overall stiffness of the MPF while allowing the SSI to participate in the dynamic response of the structure. The top two layers are composed of a medium-dense sand, and the bottom layer is a dense sand. This soil profile is representative of sites found in the North Sea. Other studies concerned with offshore wind turbine foundations describe a site off the coast of the Netherlands with medium-dense sand overlying dense sand (Zaaijer, 2006).
6. FAST MODELLING AND ANALYSIS

In the OC3 project, the only soil parameters given were the unit weight ($\gamma_a$) and angle of friction ($\phi'_a$). However, to model the sand in Plaxis, continuous soil parameters are needed for the sand layers.

Using the analysis presented by Brinkgreve et al. (2010), the soil properties could be derived. A sand RD for each layer was found (using the provided formulation) with the same angle of friction as that given in the OC3 project. Therefore by simply matching the RD and angle of friction of the sand, soil parameters for the HSsmall model were found. The fitted soil parameters for the three layers are given in Table 6.3.

![Figure 6.1: Soil profile with two HAWT founded on: (a) SCF (b) MPF](image)

6.3.3 Control systems

Both a generator-torque and a collective blade-pitch controller are employed. The control system is implemented as an external Dynamic Link Library (DLL) in the style of Garrad Hassan’s BLADED wind turbine software package. The controller parameters are entered into BladedDLLInterface.f90 as outlined in Jonkman (2012). The controller parameters are given in Table 6.4.
6.4 Foundation models

Table 6.3: Model soil parameters for the three sand layers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Sand 1</th>
<th>Sand 2</th>
<th>Sand 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_s'$</td>
<td>deg</td>
<td>33</td>
<td>35</td>
<td>38.5</td>
</tr>
<tr>
<td>RD</td>
<td>-</td>
<td>40%</td>
<td>56%</td>
<td>84%</td>
</tr>
<tr>
<td>$\gamma_{unsat}$</td>
<td>KN/m$^3$</td>
<td>16.6</td>
<td>17.2</td>
<td>18.4</td>
</tr>
<tr>
<td>$\gamma_{sat}$</td>
<td>KN/m$^3$</td>
<td>19.6</td>
<td>19.9</td>
<td>20.3</td>
</tr>
<tr>
<td>$E_{50}^{ref}$</td>
<td>KN/m$^2$</td>
<td>24000</td>
<td>33600</td>
<td>50400</td>
</tr>
<tr>
<td>$E_{oed}^{ref}$</td>
<td>KN/m$^2$</td>
<td>24000</td>
<td>33600</td>
<td>50400</td>
</tr>
<tr>
<td>$E_{ar}^{ref}$</td>
<td>KN/m$^2$</td>
<td>72000</td>
<td>100800</td>
<td>151200</td>
</tr>
<tr>
<td>$G_0^{ref}$</td>
<td>KN/m$^2$</td>
<td>87200</td>
<td>98080</td>
<td>117120</td>
</tr>
<tr>
<td>$\gamma_{s,0.7}$</td>
<td>-</td>
<td>$1.6 \times 10^{-4}$</td>
<td>$1.44 \times 10^{-4}$</td>
<td>$1.16 \times 10^{-4}$</td>
</tr>
<tr>
<td>$m$</td>
<td>-</td>
<td>0.575</td>
<td>0.525</td>
<td>0.4375</td>
</tr>
<tr>
<td>$c'$</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\psi_s$</td>
<td>deg</td>
<td>3</td>
<td>5</td>
<td>8.5</td>
</tr>
<tr>
<td>$R_f$</td>
<td>-</td>
<td>0.95</td>
<td>0.93</td>
<td>0.895</td>
</tr>
<tr>
<td>$K_0$</td>
<td>-</td>
<td>0.4553</td>
<td>0.4262</td>
<td>0.3775</td>
</tr>
<tr>
<td>$R_{int}$</td>
<td>-</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
</tbody>
</table>

6.4 Foundation models

Two soil-foundation models are used in this study: a CS model and an AF model, a FB model as a reference. The foundation models are derived for both a SCF and MPF system, for both an uncontrolled and controlled turbine. Details of the foundation models are given in Section 3.6.

The foundation system’s horizontal translation and rotation was captured using Plaxis. No vertical springs are included. The foundation is assumed to remain fixed in the vertical direction. This is achieved (in FAST) by disabling the heave and yaw DOFs. Therefore the foundation’s movement is only enabled in the fore-aft and side-to-side directions, see Figure 6.2 and equation 6.1.

FAST solves the system EOM in the time domain, hence only frequency independent soil-foundation models are compatible i.e. only static CS models are analysed in this study.

221
Table 6.4: Baseline control system properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch controller</td>
<td>- collective</td>
<td></td>
</tr>
<tr>
<td>Optimal mode gain</td>
<td>Nm/(rad/s)$^2$</td>
<td>2.33</td>
</tr>
<tr>
<td>Demand power</td>
<td>MW</td>
<td>5.297</td>
</tr>
<tr>
<td>Demanded generator speed</td>
<td>rad/s</td>
<td>91.2</td>
</tr>
<tr>
<td>Optimal mode maximum speed</td>
<td>rad/s</td>
<td>121.68</td>
</tr>
<tr>
<td>Minimum generator speed</td>
<td>rad/s</td>
<td>70.16</td>
</tr>
<tr>
<td>Demanded generator torque</td>
<td>Nm</td>
<td>43093.6</td>
</tr>
<tr>
<td>Maximum pitch angle</td>
<td>deg</td>
<td>90</td>
</tr>
<tr>
<td>Minimum pitch angle</td>
<td>deg</td>
<td>0</td>
</tr>
<tr>
<td>Maximum pitch rate</td>
<td>rad/s</td>
<td>8</td>
</tr>
<tr>
<td>Minimum pitch rate</td>
<td>rad/s</td>
<td>1</td>
</tr>
<tr>
<td>Reference yaw angle</td>
<td>deg</td>
<td>0</td>
</tr>
<tr>
<td>No. of points in torque-speed array</td>
<td>rad/s</td>
<td>7</td>
</tr>
<tr>
<td>Generator speeds array</td>
<td>rad/s</td>
<td>{0, 68.07, 78.54, 86.92, 114.14, 123.6, 146.61}</td>
</tr>
<tr>
<td>Generator torques array</td>
<td>Nm</td>
<td>{0.0, 0.0, 10000, 20000, 30000, 40000, 37000}</td>
</tr>
</tbody>
</table>

### 6.4.1 CS model

The CS model replaces the true soil-foundation system by coupled translational and rotational springs at the mudline, the complete stiffness matrix can be expressed as,

$$
\begin{bmatrix}
H_{\text{surge}} \\
H_{\text{sway}} \\
H_{\text{heave}} \\
M_{\text{roll}} \\
M_{\text{pitch}} \\
M_{\text{yaw}}
\end{bmatrix} =
\begin{bmatrix}
K_{HH}^0 & 0 & 0 & -K_C^0 & 0 \\
0 & K_{HH}^0 & 0 & K_C^0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_C^0 & 0 & K_{MM}^0 \\
-K_C^0 & 0 & 0 & K_{MM}^0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
w_{\text{surge}} \\
w_{\text{sway}} \\
w_{\text{heave}} \\
\theta_{\text{roll}} \\
\theta_{\text{pitch}} \\
\theta_{\text{yaw}}
\end{bmatrix}
\tag{6.1}
$$

Note the stiffness coefficients for heave and yaw are set to zero.

The CS model is implemented in FAST by including the stiffness matrix given in equation 6.1, in a user-defined platform-loading subroutine (UserPtfmLd), thus FAST must be recompiled for each CS model taken. An example of this user-defined subroutine can be found in Appendix J. The platform (of zero height) is positioned at the base of the wind turbine support structure. The stiffness matrix represents the force-
6.4 Foundation models

Figure 6.2: Foundation DOFs and parameters as defined in FAST, shown for a MPF
deformation relationships of the DOFs at the mudline alone. The total mass and inertia
of the foundation are added as a lumped parameters at the mudline in the FAST plat­
form file, example of which can be found in Appendix K.

To determine the stiffness of the soil-foundation system, the loading on the foun­
dation was first required. The mean shear force and bending moment at the mudline
were obtained from the FB model simulations and are given in Table 6.5, for both the
uncontrolled and controlled case. The controller has a significant effect on the results,
more than halving the mean bending moment at the mudline.

Since the shear force and bending moment (at the mudline) in the fore-aft direction
(see Figure 6.2) are much greater than those in the side-to-side direction the stiffness of
the soil foundation system is solved for only in the fore-aft direction. This sub-stiffness
matrix is then applied in both the fore-aft and side-to-side directions.

Models for the soil-foundation system are build in Plaxis for the specified founda­
tions and soil conditions and the response computed. As the soil model used is non­
linear, a range of load cases were needed over which a lineraized model could be solved
(or regressed) for the local translation, rotational and coupling stiffness parameters, as
6. FAST MODELLING AND ANALYSIS

Table 6.5: Mean bending moment and shear force for each DOF at the tower base given by the FB model

<table>
<thead>
<tr>
<th>DOF</th>
<th>Unit</th>
<th>Uncontrolled</th>
<th>Controlled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>KNm</td>
<td>143533</td>
<td>65649</td>
</tr>
<tr>
<td>Roll</td>
<td>KNm</td>
<td>559</td>
<td>4952</td>
</tr>
<tr>
<td>Yaw</td>
<td>KNm</td>
<td>−499</td>
<td>180</td>
</tr>
<tr>
<td>Surge</td>
<td>KN</td>
<td>1286</td>
<td>582</td>
</tr>
<tr>
<td>Sway</td>
<td>KN</td>
<td>−5.4</td>
<td>−7</td>
</tr>
<tr>
<td>Heave</td>
<td>MN</td>
<td>−8.7</td>
<td>−8.7</td>
</tr>
</tbody>
</table>

outlined in Section 3.6.2.

\[
\begin{bmatrix}
H_{\text{surge}} \\
M_{\text{pitch}}
\end{bmatrix} =
\begin{bmatrix}
K^0_{HH} & -K^0_C \\
-K^0_C & K^0_{MM}
\end{bmatrix}
\begin{bmatrix}
w_{\text{surge}} \\
\theta_{\text{pitch}}
\end{bmatrix}
\]

(6.2)

By solving for the response of the foundation at the mean load (in terms of \(H\), \(M\) and axial load) and two additional load cases, taken at ±10% the mean load, the foundation stiffness matrix can be calculated. The ±10% is to ensure the additional load cases yield independent results. A sample regressed CS model for the MPF is shown in Appendix I.

6.4.1.1 Iterative CS model

A FB model will produce different loads at the mudline from those given by a model that accounts for the flexibility of soil. Hence the CS model will produce different forces at the mudline. These forces (from the first CS model) should then be put back onto the soil-foundation system (in Plaxis) and a new CS model produced. This CS model should then be placed back into the FAST and new loads at the mudline calculated. Thus creating an iterative CS model scheme which should be carried out until convergence, as shown in Figure 6.3.

An iterative CS model was carried out until convergence of the loads at the mudline. The convergences study is shown in Table 6.6 for the uncontrolled case. Convergence of the loads at the mudline occurs rapidly. In fact there is almost no difference in shear force and bending moment at the mudline after the first CS model has been solved, indicating that such an iterative model is unnecessary, at least for this particular soil profile and wind turbine arrangement. A similar result was reported by Bush (2009).
6.4 Foundation models

Table 6.6: Fore-aft mean shear force and bending moment for the iterative CS model for each iteration, calculated for the uncontrolled case

<table>
<thead>
<tr>
<th></th>
<th>Unit</th>
<th>FB model</th>
<th>1st iteration</th>
<th>2nd iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPF H</td>
<td>KN</td>
<td>1286</td>
<td>1284</td>
<td>1284</td>
</tr>
<tr>
<td></td>
<td>KNm</td>
<td>143533</td>
<td>144857</td>
<td>144859</td>
</tr>
<tr>
<td>SCF H</td>
<td>KN</td>
<td>1286</td>
<td>1285</td>
<td>1285</td>
</tr>
<tr>
<td></td>
<td>KNm</td>
<td>143533</td>
<td>144293</td>
<td>144293</td>
</tr>
</tbody>
</table>

The static stiffness terms for each DOF are given in Table 6.7. The SCF is significantly stiffer than the MPF in terms of horizontal translation and coupling coefficients. From the results, we can see that the controller has a significant effect on stiffness coefficients. This is because the soil is modelled as a non-linear material. Thus the load and foundation response are not linearly dependant.

Table 6.7: Static stiffness terms for the controlled (C) and uncontrolled (UC) models

<table>
<thead>
<tr>
<th></th>
<th>MPF</th>
<th>SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>UC</td>
<td>C</td>
</tr>
<tr>
<td>$K_{HH}/10^8$</td>
<td>7.39</td>
<td>10.47</td>
</tr>
<tr>
<td>$K_{C}/10^9$</td>
<td>-6.44</td>
<td>-8.16</td>
</tr>
<tr>
<td>$K_{MM}/10^{11}$</td>
<td>1.19</td>
<td>1.35</td>
</tr>
</tbody>
</table>
6. FAST MODELLING AND ANALYSIS

6.4.1.2 Damping

To determine the damping ratio of the soil-foundation system, a free vibration study for each foundation was carried out. The mean load was applied to the foundation and then released and the free vibration response captured. The system is underdamped, therefore a decay curve could be matched to the output, as discussed in Section 3.6.3. A decay curve was matched to free vibration response for both foundations and damping ratio determined in the horizontal translation and rotational DOFs, the results are given in Table 6.2.

6.4.2 AF model

The AF model replaces the soil and the embedded foundation with a cantilever fixed at a point below the mudline. The length of this cantilever $L_{AF}$, referred to as the apparent fixity length, and its flexural rigidity $EI_{AF}$ are derived so the stiffness of this fictitious cantilever represents the stiffness of the true soil-foundation system i.e under a particular shear force and bending moment at the mudline the cantilever must be sized so it produces the same deflection $w$ and rotation $\theta$ at the mudline as that of the true foundation system.

To establish an AF model, three steps can be followed, as shown in Figure 6.4. In the first step, the fore-aft shear force and bending moment values at the mudline, needed to calibrate the stiffness of the cantilever, are obtained from FB model simulations. In
6.4 Foundation models

the second step, the response of the true soil-foundation system for each foundation is found in Plaxis for the appropriate loading conditions (determined in step 1). In the third step, cantilever properties are calculated that produce the same lateral resistance as the true soil-foundation system.

This is done by solving equation 6.3,

\[
\begin{bmatrix}
\frac{L_{AF}^3}{3EI_{AF}} & \frac{L_{AF}^2}{2EI_{AF}} \\
\frac{L_{AF}^3}{2EI_{AF}} & \frac{L_{AF}}{EI_{AF}}
\end{bmatrix}
\begin{bmatrix}
H \\
M
\end{bmatrix}
= \begin{bmatrix}
w \\
\theta
\end{bmatrix}
\]

(6.3)

Following the method presented by Passon (2006.), equation 6.3 can be combined and rearranged to give,

\[
\frac{L_{AF}}{3EI_{AF}} \cdot 2L_{AF}H + 3M - \frac{w}{\theta} = 0
\]

(6.4)

Equation 6.4 can be arranged as a \(2^{nd}\) order polynomial of fixity length \(L_{AF}\). Solving the polynomial leads to two solutions,

\[
L_{AF_{1,2}} = \left[ \frac{1}{4\theta H} (-3\theta M + 3wH + (9\theta^2 M^2 + 30w\theta FM + 9w^2 H^2)^{0.5}) \right]
\]

(6.5)

The positive value, either \(L_{AF_1}\) or \(L_{AF_2}\), is found and the bending stiffness can then be evaluated as either,

\[
EI_{AF_{1,2}} = \left[ \frac{HL_{AF}^3}{3w} + \frac{ML_{AF}^2}{2w} \right]
\]

(6.6)

The mass distribution of the fictitious foundation should result in approximately the same total mass as the real sub-foundation.

6.4.2.1 Implementation

With the apparent length and flexural rigidity established, the AF model can be implemented in FAST. Unlike the CS model, FAST does not have to be recompiled to implement the AF model.

The length of the tower in FAST must be changed (lengthened). This is done by changing the \textit{TwrDraft} height in the platform file, as shown in Appendix L (\textit{TwrDraft}
6. FAST MODELLING AND ANALYSIS

= water depth + $L_{AF}$). The distributed tower properties and mode shapes must be changed in tower file. The distributed tower properties, the mass and flexural rigidity, must be extended to account for the new tower section, as shown in Appendix M.

The computed apparent length and flexural rigidity for the controlled and uncontrolled case are shown in Table 6.8. As expected, the uncontrolled models requires a slightly longer apparent length than the controlled model.

Table 6.8: The translation and rotation of the soil-foundation models and the resulting apparent fixity length and flexural rigidity, for both uncontrolled and controlled cases

<table>
<thead>
<tr>
<th>Model</th>
<th>Foundation</th>
<th>$w$ (m)</th>
<th>$\theta$ (rad)</th>
<th>$L_{AF}$ (m)</th>
<th>$EI_{AF}$ (Nm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td>SCF</td>
<td>0.01079</td>
<td>0.00139</td>
<td>15.169</td>
<td>1.6692e12</td>
</tr>
<tr>
<td></td>
<td>MPF</td>
<td>0.02295</td>
<td>0.00244</td>
<td>18.33</td>
<td>1.1657e12</td>
</tr>
<tr>
<td>Controlled</td>
<td>SCF</td>
<td>0.00387</td>
<td>0.000513</td>
<td>14.799</td>
<td>2.0194e12</td>
</tr>
<tr>
<td></td>
<td>MPF</td>
<td>0.008202</td>
<td>0.000985</td>
<td>16.286</td>
<td>1.1637e12</td>
</tr>
</tbody>
</table>

6.5 Tower mode shapes

The mode shapes for the tower have all being calculated using the NREL distributed BMODES (Bir, 2012). Using BMODE, tower mode shapes can easily be derived for the FB and the AF models. Note for the AF models, the apparent fixity length should be added to the support structure length. For the CS models, the stiffness matrix should be added into BMODES as a boundary conditions, as discussed in Section 5.5.1. A sample BMODES input file is given in Appendix N. The first two tower modes shapes in the fore-aft and side-to-side directions for the FB model and for the CS model MPF (for the uncontrolled case) are given in Figure 6.5. The natural frequencies from BMODES are shown in Table 6.9. The natural frequencies for the AF and CS models agree well.

6.6 Numerical results

6.6.1 Modal frequencies

The loads that an offshore wind turbine experiences are strongly influenced by its modal frequencies. FAST can extract linearised representations of the complete non-linear aeroelastic wind turbine modelled in the code. The linearisation process consists
of two steps: computing a periodic steady state operating point condition for the DOFs and numerically linearising the FAST model about this operating point to form periodic state matrices. From the output state matrices, the full system modes of the HAWT can be obtained through eigenvalue analysis.

Such a linearization and eigenvalue analysis was carried out, about rated conditions for the each soil-foundation model, and the result given in Table 6.10 and 6.11 for the uncontrolled and controlled cases respectively. The modal frequencies are not labelled and simply arranged in ascending order. By examination of the results the tower’s modal frequencies were found and are given in Table 6.12.

The soil-foundation system has the effect of reducing the tower modal frequencies. A reduction of 10-20% and 14-35% in the tower’s first and second modal frequencies is observed considering the flexible foundation. The blade modal frequencies are unaf-
6. FAST MODELLING AND ANALYSIS

Table 6.9: Natural frequencies from BMODES for the different foundation models

<table>
<thead>
<tr>
<th>Model</th>
<th>Mode</th>
<th>Direction</th>
<th>SCF</th>
<th>MPF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>UC</td>
<td>C</td>
</tr>
<tr>
<td>CS</td>
<td>1st</td>
<td>SS</td>
<td>0.2538</td>
<td>0.2577</td>
</tr>
<tr>
<td></td>
<td>1st</td>
<td>FA</td>
<td>0.2556</td>
<td>0.2595</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>SS</td>
<td>1.4379</td>
<td>1.4663</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>FA</td>
<td>1.6285</td>
<td>1.6725</td>
</tr>
<tr>
<td>AF</td>
<td>1st</td>
<td>SS</td>
<td>0.2541</td>
<td>0.2572</td>
</tr>
<tr>
<td></td>
<td>1st</td>
<td>FA</td>
<td>0.2558</td>
<td>0.2590</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>SS</td>
<td>1.4118</td>
<td>1.4312</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>FA</td>
<td>1.5885</td>
<td>1.6174</td>
</tr>
</tbody>
</table>

Table 6.10: Modal frequencies (Hz) for the HAWT for the each soil-foundation models, uncontrolled case

<table>
<thead>
<tr>
<th>FB model</th>
<th>SCF</th>
<th>MPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AF model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>0.000124</td>
<td>0.000124</td>
</tr>
<tr>
<td>0.27895</td>
<td>0.22979</td>
<td>0.25358</td>
</tr>
<tr>
<td>0.28385</td>
<td>0.24087</td>
<td>0.25774</td>
</tr>
<tr>
<td>0.74414</td>
<td>0.74332</td>
<td>0.74323</td>
</tr>
<tr>
<td>0.95032</td>
<td>0.95523</td>
<td>0.94337</td>
</tr>
<tr>
<td>0.95483</td>
<td>0.95975</td>
<td>0.94786</td>
</tr>
<tr>
<td>1.1408</td>
<td>1.1441</td>
<td>1.1366</td>
</tr>
<tr>
<td>1.1502</td>
<td>1.1537</td>
<td>1.1455</td>
</tr>
<tr>
<td>1.6537</td>
<td>1.623</td>
<td>1.5786</td>
</tr>
<tr>
<td>2.0708</td>
<td>1.8813</td>
<td>1.7252</td>
</tr>
<tr>
<td>2.2868</td>
<td>2.083</td>
<td>2.0844</td>
</tr>
<tr>
<td>2.3706</td>
<td>2.1925</td>
<td>2.0913</td>
</tr>
<tr>
<td>2.3749</td>
<td>2.3873</td>
<td>2.349</td>
</tr>
<tr>
<td>2.6736</td>
<td>2.392</td>
<td>2.3527</td>
</tr>
<tr>
<td>3.9406</td>
<td>3.1464</td>
<td>3.8817</td>
</tr>
<tr>
<td>5.8528</td>
<td>3.333</td>
<td>5.8482</td>
</tr>
<tr>
<td>-</td>
<td>3.8913</td>
<td>3.9835</td>
</tr>
<tr>
<td>-</td>
<td>5.8532</td>
<td>5.8538</td>
</tr>
<tr>
<td>-</td>
<td>30.089</td>
<td>9.4685</td>
</tr>
<tr>
<td>-</td>
<td>30.239</td>
<td>9.4961</td>
</tr>
</tbody>
</table>

-these were reported by Jonkman & Musial (2010); Zaaijer (2005).
Table 6.11: Modal frequencies (Hz) for the HAWT for the each soil-foundation models, controlled case

<table>
<thead>
<tr>
<th></th>
<th>SCF</th>
<th></th>
<th>MPF</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FB model</td>
<td>CS model</td>
<td>AF model</td>
<td>CS model</td>
<td>AF model</td>
</tr>
<tr>
<td>0.27895</td>
<td>0.24783</td>
<td>0.25589</td>
<td>0.24836</td>
<td>0.24459</td>
</tr>
<tr>
<td>0.28385</td>
<td>0.28106</td>
<td>0.26012</td>
<td>0.25327</td>
<td>0.2485</td>
</tr>
<tr>
<td>0.74414</td>
<td>0.74346</td>
<td>0.74335</td>
<td>0.74298</td>
<td>0.74294</td>
</tr>
<tr>
<td>0.95032</td>
<td>0.95553</td>
<td>0.94415</td>
<td>0.95325</td>
<td>0.94193</td>
</tr>
<tr>
<td>0.95484</td>
<td>0.96005</td>
<td>0.94864</td>
<td>0.95778</td>
<td>0.94641</td>
</tr>
<tr>
<td>1.1408</td>
<td>1.1442</td>
<td>1.1371</td>
<td>1.1426</td>
<td>1.1358</td>
</tr>
<tr>
<td>1.1502</td>
<td>1.1539</td>
<td>1.146</td>
<td>1.1521</td>
<td>1.1446</td>
</tr>
<tr>
<td>1.6537</td>
<td>1.6315</td>
<td>1.5948</td>
<td>1.5414</td>
<td>1.5325</td>
</tr>
<tr>
<td>2.0708</td>
<td>1.9806</td>
<td>1.7859</td>
<td>1.5629</td>
<td>1.6053</td>
</tr>
<tr>
<td>2.2868</td>
<td>2.0825</td>
<td>2.0855</td>
<td>1.8272</td>
<td>1.9832</td>
</tr>
<tr>
<td>2.3706</td>
<td>2.3578</td>
<td>2.1508</td>
<td>2.0824</td>
<td>2.0843</td>
</tr>
<tr>
<td>2.3749</td>
<td>2.3884</td>
<td>2.3512</td>
<td>2.3806</td>
<td>2.3446</td>
</tr>
<tr>
<td>2.6736</td>
<td>2.3931</td>
<td>2.355</td>
<td>2.3851</td>
<td>2.3483</td>
</tr>
<tr>
<td>3.9406</td>
<td>2.7478</td>
<td>3.8862</td>
<td>3.4069</td>
<td>3.8736</td>
</tr>
<tr>
<td>5.8528</td>
<td>2.9196</td>
<td>5.8484</td>
<td>3.5393</td>
<td>5.8479</td>
</tr>
<tr>
<td>-</td>
<td>3.9012</td>
<td>-</td>
<td>4.0015</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>5.8525</td>
<td>-</td>
<td>5.8548</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>40.93</td>
<td>-</td>
<td>10.431</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>41.119</td>
<td>-</td>
<td>10.447</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.12: Tower modal frequencies (Hz) for each foundation model, for the controlled and uncontrolled cases

<table>
<thead>
<tr>
<th></th>
<th>SCF</th>
<th></th>
<th>MPF</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FB model</td>
<td>CS model</td>
<td>AF model</td>
<td>CS model</td>
<td>AF model</td>
</tr>
<tr>
<td>UC</td>
<td>0.279</td>
<td>0.230</td>
<td>0.254</td>
<td>0.232</td>
</tr>
<tr>
<td></td>
<td>0.284</td>
<td>0.241</td>
<td>0.258</td>
<td>0.245</td>
</tr>
<tr>
<td></td>
<td>2.287</td>
<td>1.881</td>
<td>1.725</td>
<td>1.477</td>
</tr>
<tr>
<td></td>
<td>2.674</td>
<td>2.193</td>
<td>2.091</td>
<td>1.482</td>
</tr>
<tr>
<td>C</td>
<td>0.279</td>
<td>0.248</td>
<td>0.256</td>
<td>0.248</td>
</tr>
<tr>
<td></td>
<td>0.284</td>
<td>0.281</td>
<td>0.260</td>
<td>0.253</td>
</tr>
<tr>
<td></td>
<td>2.287</td>
<td>1.981</td>
<td>1.786</td>
<td>1.541</td>
</tr>
<tr>
<td></td>
<td>2.674</td>
<td>2.358</td>
<td>2.151</td>
<td>1.563</td>
</tr>
</tbody>
</table>
The AF and the CS model predict similar reductions in modal frequencies. However, their agreement is less consistent that shown in Table 6.9. This is due to the coupling between the rotor nacelle assemble (which is fixed in BMODES) and the tower.

Interestingly, the SCF and MPF predict very similar results for the first tower modal frequencies, even though, as shown in Table 6.7, the static stiffness coefficients are much greater for the SCF. This was found to be related to the ratio of the stiffness coefficients in the matrix. However the second tower modal frequencies predicted are significantly lower for the MPF. The modal frequencies predicted for the controlled and uncontrolled cases are different, as a results of the non-linear soil model implied.

Time histories give an insight into the tower bending moment process. Figures 6.6 and 6.7 show time series of the hub-height longitudinal wind speed, the sea surface elevation, and the fore-aft bending moment at the mudline for the different foundation models, uncontrolled and controlled. The mean of each time series has been removed, and the time series has been normalized with respect to its maximum value.

![Figure 6.6: Fore-aft bending moment at the mudline for the various foundation models, for the uncontrolled case](image)

The peak spectral frequency of the waves (shown in Figure 6.8) roughly corre-
6.6 Numerical results

Figure 6.7: Fore-aft bending moment at the mudline for the various foundation models, for the controlled case

sponded to the first modal frequency of the tower. Thus the sea surface elevation and the fore-aft bending moment time series display roughly the same dominant frequency.

6.6.2 Turbine Load Statistics

The load statistics at the mudline are now examined for each foundation model. The minimum, the mean, the maximum and the Standard Derivation (SD) of the shear force and bending moment at the mudline in the fore-aft direction are given in Table 6.13 and Table 6.14, for the uncontrolled and controlled cases respectively.

These statistics were calculated for ten minute simulations with each model exposed to the same wind and wave loading conditions.

Examining the statistics, it can be noted that the inclusion of the foundation, by either the AF or CS model, increases bending moment at the mudline. This is because the foundation interaction reduces the tower's modal frequencies thus making the tower more susceptible to the wave loading. The controller significantly reduces the bending moment at the mudline.
The mean shear force is not influenced by the SSI. The minimum, maximum and as a result the SD of the shear force are all increased considering the foundation interaction. The controller significantly reduces the mean shear force, however it has little effect on the min, max and SD.

The AF and CS models do not agree well in terms of bending moment for any of the cases studied.

Figure 6.9 present wind and wave loading acting on the wind turbine over the ten minute simulation. Figures 6.10 and 6.11 give the response of the MPF head as predicted by the CS model, in terms of horizontal translation and rotation in the fore-aft and side-to-side directions, for the uncontrolled and controlled cases. The shear force and bending moment at the mudline from this model is also shown in Figures 6.12.
Table 6.13: Statistics for shear force (KN) and bending moment (KNm) in the fore-aft direction at mudline, for the uncontrolled case

<table>
<thead>
<tr>
<th>Foundation</th>
<th>Model</th>
<th>min</th>
<th>mean</th>
<th>max</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB</td>
<td>H</td>
<td>-1463</td>
<td>1286</td>
<td>4352</td>
<td>862</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>67750</td>
<td>143534</td>
<td>248900</td>
<td>34590</td>
</tr>
<tr>
<td>CSF</td>
<td>CS</td>
<td>-2011</td>
<td>1285</td>
<td>5275</td>
<td>959</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>63190</td>
<td>144299</td>
<td>265500</td>
<td>35908</td>
</tr>
<tr>
<td>CSF</td>
<td>AF</td>
<td>-1434</td>
<td>1301</td>
<td>5690</td>
<td>952</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>67950</td>
<td>145349</td>
<td>274600</td>
<td>36824</td>
</tr>
<tr>
<td>MPF</td>
<td>CS</td>
<td>-2292</td>
<td>1285</td>
<td>5795</td>
<td>981</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>62070</td>
<td>144859</td>
<td>271000</td>
<td>36969</td>
</tr>
<tr>
<td>MPF</td>
<td>AF</td>
<td>-1612</td>
<td>1307</td>
<td>5336</td>
<td>964</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>63620</td>
<td>146481</td>
<td>271800</td>
<td>37830</td>
</tr>
</tbody>
</table>

Table 6.14: Statistics for shear force (KN) and bending moment (KNm) in the fore-aft direction at mudline, for the controlled case

<table>
<thead>
<tr>
<th>Foundation</th>
<th>Model</th>
<th>min</th>
<th>mean</th>
<th>max</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB</td>
<td>H</td>
<td>-2074</td>
<td>582</td>
<td>3657</td>
<td>811</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>7939</td>
<td>65649</td>
<td>130600</td>
<td>17285</td>
</tr>
<tr>
<td>CSF</td>
<td>CS</td>
<td>-2969</td>
<td>582</td>
<td>4143</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>9335</td>
<td>65982</td>
<td>125100</td>
<td>19179</td>
</tr>
<tr>
<td>CSF</td>
<td>AF</td>
<td>-2021</td>
<td>589</td>
<td>3710</td>
<td>841</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>6760</td>
<td>66493</td>
<td>130100</td>
<td>19933</td>
</tr>
<tr>
<td>MPF</td>
<td>CS</td>
<td>-2986</td>
<td>585</td>
<td>4297</td>
<td>916</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-1484</td>
<td>66363</td>
<td>167800</td>
<td>21758</td>
</tr>
<tr>
<td>MPF</td>
<td>AF</td>
<td>-2037</td>
<td>592</td>
<td>3636</td>
<td>844</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>-313</td>
<td>67303</td>
<td>144400</td>
<td>21849</td>
</tr>
</tbody>
</table>

and 6.13. The shear force and bending moment in the fore-aft direction is seen to dominate the response.

As expected, the control system (due to its effect on the forces at the mudline) has a huge impact on the foundation horizontal translation and rotation, significantly reducing the response.
Figure 6.9: Time history: (a) Wind speed (b) Sea surface elevation

Figure 6.10: Foundation response for the MPF CS model, for the uncontrolled case: (a) Horizontal translation and (b) Rotation
6.6 Numerical results

Figure 6.11: Foundation response for the MPF CS model, for the controlled case: (a) Horizontal translation (b) Rotation

Figure 6.12: Forces at the mudline for the MPF CS model, for the uncontrolled case: (a) Bending moment (b) Shear force
6.6.3 Fatigue

Fatigue is a process in which damage accumulates due to the repetitive application of loads that may be well below the yield point. The process is dangerous because a single application of the load would not produce any ill effects, and a conventional stress analysis might lead to an assumption of safety that does not exist.

One of the most important concept in fatigue design is the S-N diagram. Armed with an applicable stress history for the component and a relevant S-N curve, the fatigue life maybe estimated using Miner law.

A relevant S-N curve for a welded connection in a wind turbine support structure was taken from Van der Tempel (2006). The submerged weld at the mudline considered is a 'transverse loaded butt weld' ground flush to plate, the S-N parameters for which are given in Table 6.15.

From the bending moment at the mudline, the stress $\sigma_{ST}$ on the support structure
6.6 Numerical results

Table 6.15: S-N parameters for transverse loaded butt weld

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endurance limit, $\Delta \sigma_{RC}$</td>
<td>MN/m$^2$</td>
<td>100.375</td>
</tr>
<tr>
<td>Slope, $m_{SN}$</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Slope, $m_{SN}$</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Number of cycle to failure, $N_f$</td>
<td>-</td>
<td>$5 \times 10^6$</td>
</tr>
</tbody>
</table>

(or tower base) can be expressed as,

$$\sigma_{st} = \frac{M}{I_{ST}} R_{ST}$$  \hspace{1cm} (6.7)

where the terms $I_{ST}$ and $R_{ST}$ are the second moment of inertia and the radius of the support structure respectively. The stress can be run through a rainflow counting algorithm and the number of cycles and cycles amplitude recovered. With these parameter established and the S-N parameters given in Table 6.15, the damage $D_{fag}$ may be estimated using Miner’s law,

$$D_{fag} = \sum \left( \left( \frac{n}{N_f} \right) \left( \frac{\sigma}{\Delta \sigma_{RC}} \right)^{m_{SN}} \right)$$  \hspace{1cm} (6.8)

where $n$ is the cycle rate at stress level $\sigma$. The expected time to failure in days can be given as,

$$T_f = \frac{T_0}{D_{fag}}/3600/24/365$$  \hspace{1cm} (6.9)

where $T_0$ is the length of the time history taken (10 minute simulation, $T_0 = 600 \, \text{s}$). This gives the estimated time till complete failure of the component i.e $D_{fag} = 1$. Critical fatigue damage is usually considered to have occurred in wind turbine analysis when a certain amount of damage has occurred, typically for $D_{fag} = 0.1-0.2$. Acceptable overall damage values will depend on the component’s importance.

The fatigue life, taken as the time to reach a damage of 20%, for the CS foundation models is given in Table 6.16. FB results are given as a reference. The fatigue life is normalised with regard to the FB case.

The controller, due to effects on the bending moment at the mudline, significantly increases the fatigue life of the wind turbine support structure. SSI affects decrease the fatigue life of the support structure. This is due to the fact that the flexible soil softens
Table 6.16: Fatigue Life (FL) for each foundation, for both uncontrolled and controlled cases

<table>
<thead>
<tr>
<th>Model</th>
<th>Uncontrolled FL (yrs)</th>
<th>Normalised</th>
<th>Controlled FL (yrs)</th>
<th>Normalised</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB</td>
<td>31.37</td>
<td>1.000</td>
<td>46.97</td>
<td>1.000</td>
</tr>
<tr>
<td>SCF</td>
<td>24.35</td>
<td>0.776</td>
<td>39.45</td>
<td>0.840</td>
</tr>
<tr>
<td>MPF</td>
<td>21.10</td>
<td>0.673</td>
<td>28.36</td>
<td>0.604</td>
</tr>
</tbody>
</table>

the tower making it susceptible to wave excitation. Similar reductions in fatigue life are seen for uncontrolled and controlled cases. The MPF predicts a lower fatigue life compared to the SCF.

To investigate the fatigue life, further wave loading is removed from the models and the analysis recomputed. The load statistics at the mudline with wave loading removed are given in Table 6.17. Interestingly, the mean load (in terms of shear force and bending moment) are almost unchanged by removing the wave loading. The SD is reduced for all cases, indicating that wave loading has more influence on the peak to peak response and that the mean load is dominated by the wind.

As the mean loads are almost unchanged the original CS models, calculated at the two distant operating points (controlled and uncontrolled), can be used again in the next set of analyses. Thus offering a fair comparison. Note that if a higher wind load had been applied the mean loads would also have been relatively unchanged due to the blade pitch controller system.

The fatigue life is calculated and given in Table 6.18. It is shown to increase substantially due to the lower SD in the forces at the mudline. SSI is now shown to have the opposite effect, increasing the fatigue life of the turbine support structure. This is also due to the flexible soil softening the tower’s response. The results are similar for both the uncontrolled and controlled cases.
Table 6.17: Statistics for shear force (KN) and bending moment (KNm) in the fore-aft direction, with wave loading removed for the analyses

<table>
<thead>
<tr>
<th>Foundation</th>
<th>min</th>
<th>mean</th>
<th>max</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC</td>
<td>FB</td>
<td>494</td>
<td>1281</td>
<td>2483</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>86960</td>
<td>143402</td>
<td>238900</td>
</tr>
<tr>
<td>SCF</td>
<td>H</td>
<td>521</td>
<td>1280</td>
<td>2865</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>90010</td>
<td>144124</td>
<td>235700</td>
</tr>
<tr>
<td>MPF</td>
<td>H</td>
<td>423</td>
<td>1279</td>
<td>2846</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>90960</td>
<td>144688</td>
<td>233600</td>
</tr>
<tr>
<td>C</td>
<td>FB</td>
<td>201</td>
<td>577</td>
<td>954</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>30370</td>
<td>65513</td>
<td>97950</td>
</tr>
<tr>
<td>SCF</td>
<td>H</td>
<td>177</td>
<td>577</td>
<td>937</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>30790</td>
<td>65816</td>
<td>96120</td>
</tr>
<tr>
<td>MPF</td>
<td>H</td>
<td>-14</td>
<td>578</td>
<td>1630</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>9522</td>
<td>66174</td>
<td>170300</td>
</tr>
</tbody>
</table>

Table 6.18: Fatigue life for each foundation model, with wave loading removed, for both uncontrolled and controlled cases

<table>
<thead>
<tr>
<th>Model</th>
<th>Uncontrolled (FL yrs)</th>
<th>Normalised</th>
<th>Controlled (FL yrs)</th>
<th>Normalised</th>
</tr>
</thead>
<tbody>
<tr>
<td>FB</td>
<td>81.48</td>
<td>1.000</td>
<td>273.55</td>
<td>1.000</td>
</tr>
<tr>
<td>SCF</td>
<td>93.80</td>
<td>1.251</td>
<td>360.04</td>
<td>1.316</td>
</tr>
<tr>
<td>MPF</td>
<td>134.44</td>
<td>1.650</td>
<td>415.34</td>
<td>1.518</td>
</tr>
</tbody>
</table>

6.7 Conclusion

In this chapter the NREL developed FAST code has been coupled with a AF and CS models derived using the FEM package Plaxis. The effect of the flexible foundation on the modal frequencies, load statistics and fatigue life have been analysed. The analyses were carried out for a wind speed just above rated, a moderate sea state environment and with the wind turbine controller system switch on and off.

The control system was seen to have a huge impact on the shear force and bending moment at the mudline and to increase the fatigue life of the wind turbine significantly. The inclusion of the foundation was shown to lower the first and second tower modal frequency and to increase the bending moment at the mudline.

The stiffness matrix coefficients were found to be dependant on the control strategy.
6. FAST MODELLING AND ANALYSIS

implied, due to the non-linear soil model implied. The predicted modal frequencies were also shown to be affected by the ratio of stiffness coefficients.

The AF and CS models were shown to predict similar modal frequencies, especially for the BMODES analysis. The models however predicted substantially different load statistics at the mudline.

SSI was shown to have a positive effect on the fatigue life of the turbine support structure under aerodynamic wind loading. However when the fatigue life was calculated under a moderate sea environment, SSI had the opposite effect decreasing the fatigue life of the turbine support structure.
Chapter 7

Conclusions

7.1 Introduction

The aim of this thesis was to investigate the dynamic response of a HAWT including soil-structure interaction effects. In order to carry out the analysis MDOF HAWT models were formulated including foundation-interaction. Impedance functions for three HAWT foundations were generated using FEM and other analytical techniques. SSI effects on the dynamic response of HAWT has been analysed using developed models, FEM and FAST.

7.2 Summary and Conclusions

Wind turbine models

MDOF HAWT models including foundation-interaction were formulated for dynamic analysis using an Euler-Lagrangian approach. Two distinct models were developed: an out-of-plane model and a coupled in-plane out-of-plane model.

The out-of-plane model was comprised of a rotor blade system, a nacelle and a flexible tower connected to a foundation system using a substructuring approach. The rotor blade system consisted of three rotating blades and included the effects of centrifugal stiffening due to rotation. The coupled in-plane out-of-plane model was extended to include in-plane motion for the blade, tower and foundation and to include the roll, tilt,
7. CONCLUSIONS

yaw and drivetrain nacelle DOFs.

The models were developed in a generalised fashion (number of blade and tower modes could be altered) capable of describing onshore and offshore HAWT. The foundation could be included using either a static or frequency dependant CS model. Depending on the foundation model taken, the equation of motion could be solved in the frequency or time domain.

Realistic onshore and offshore loads

Aerodynamic loading was generated in accordance with a modified Blade Element Momentum (BEM) theory corrected for Glauert correction and Prandtl's tip loss factor. The Kaimal spectrum was used to create an isotropic and homogeneous turbulence. A more realistic non-homogeneous turbulence field was created using a rotationally sampled spectrum.

Wave loads were simulated in accordance with the Airy linear wave theory, the JONSWAP spectrum and Wheeler Stretching theory.

Soil-foundation models

A major element of this work has been the development of both static and frequency dependant CS models using the geotechnical FEM package Plaxis 3D dynamic. A procedure has been outlined for obtaining impedance functions of both surface and embedded footings using FEM.

Derived static and dynamic stiffness coefficients for the surface footing were compared to results generated using analytical formulas and the wave propagation based Cone method. Good agreement was shown for each DOF considered. The problem of excitation of the viscous boundaries at low frequencies was solved by extending the soil domain and fixing the boundaries appropriately.

The flexibility technique was used to estimate the static and dynamic stiffness coefficients for several embedded foundations. The derived stiffness coefficients were compared to analytical formulas and Cone method. Good agreement was shown for each DOF considered. The dynamic stiffness was estimated considering half of a symmetric soil domain model and a full soil domain model in Plaxis. The two models were shown to give very similar results. Thus justifying the use of half symmetric soil domain models in Plaxis for embedded symmetric foundations, with the proper boundary conditions specified.
7.2 Summary and Conclusions

Building on this work, impedance functions were derived from Plaxis for three HAWT foundations: an onshore Gravity based Foundation (GBF), an offshore Suction Caisson Foundation (SCF) and an offshore MonoPile Foundation (MPF). The results were compared to Cone method and in general there was good agreement, especially for the GBF. For the SCF, peaks in the impedance function at low soil stiffness were observed, caused by wave interference inside the caisson. The peaks were only predicted by the Plaxis models. The impedance functions were estimated for a limited frequency range 0-14 Hz, as most of the controlling modal frequencies for HAWT analysis appear in this range.

A procedure was proposed for developing multi-variable linear regression CS models for foundations embedded in a non-linear soil. Several multi-variable linear regression models, for different number of dependant variables, were estimated for a MPF CS model. It was found that as more dependants were added, the results became less reliable and finally unusable. However upon testing, against two unique CS models generated within the spread of the models, the regression models with three/two dependant variable (such as the horizontal load and moment) were found to give satisfactory results.

Numerical analysis

The dynamic response of an onshore and offshore HAWT was examined for a number of soil-foundation models exposed to aerodynamic wind and wave loading. Two aspects of the developed out-of-plane HAWT model were first validated.

- A comparison between the predicted modal frequencies from the developed HAWT model and NREL distributed aeroelastic code FAST was carried out for a 4 DOF model. The results were shown to agree well.

- The model was expanded to include the foundation (6 DOF). A simple surface footing was taken to represent the HAWT foundation. As the stiffness of the soil was increased, the modal frequencies were seen to converge on the FB results.

The dynamic response of an onshore 1.5 MW 6 DOF out-of-plane HAWT model, founded on a GBF in linear-elastic soil, was analysed. The soil-foundation system was modelled using static and frequency dependant CS models.
7. CONCLUSIONS

SSI effects were observed to decrease the fundamental modal frequency of the system. The static stiffness formulation was shown to give comparable results to the frequency dependant models, under certain conditions.

Flexible soil is usually considered to have a positive effect in terms of structural vibration suppression. The relative displacement of the nacelle however showed only a slight reduction in response for the soil conditions considered, while SSI was found to have a detrimental effect on the total displacement of the nacelle, especially for softer soils.

No significant difference between the shear and moment in the foundation and tower base was found, as the foundation inertia was found to be negligible. Peaks in the frequency response were found to occur at multiples of 3P for stiffer soil conditions. The frequency content in the response time history was significantly affected by SSI. This will have implications from a fatigue point of view.

The dynamic response of an offshore 5 MW 14 DOF out-of-plane HAWT model, founded on a SCF and MPF embedded in linear-elastic soil, was analysed. The soil-foundation was represented by a static CS model (with and without cross coupling) and a frequency dependent CS model.

The soil was shown to reduce the tower’s modal frequencies but had no effect on the blade’s modal frequencies. The second tower modal frequency was more affected by SSI than tower mode one or three.

A static stiffness formulation with coupling was shown to give comparable results to those generated from the frequency dependent models, except for very soft soil conditions. This implies that for the frequency ranges, soil conditions and turbine configurations considered, a static stiffness formulation with coupling may suffice.

The reduction in modal frequency was observed to be linearly independent of soil parameters. Rather it was found that SSI has little impact on the system until a certain minimum soil stiffness is reached under which the modal frequencies drop off rapidly.

The dynamic response of an offshore 5 MW 21 DOF coupled in-plane out-of-plane HAWT model, founded on a SCF and MPF embedded in non-linear soil, was analysed.

A static CS model was used to represent the soil-foundation system. Again, the first tower modal frequency was shown to be least sensitive and the second modal frequency most sensitive to SSI effect. SSI was found to have a noticeable impact on the tower
7.2 Summary and Conclusions

modal frequencies only.

The foundation-interaction was shown to add damping to the system in the side-to-side direction. However in the fore-aft direction and under a moderate wave environment this damping effect was cancelled out by the soil’s softening affect on the tower’s modal frequency causing the tower to become more susceptible to the wave excitation. The second tower mode was seen to have a significant contribution to the system.

FEM models

FEM models were developed to study the effect of unaligned wind and wave loading on the dynamic response of the turbine tower, support structure and soil-foundation system for the 5 MW offshore HAWT founded on a SCF. Two soil profile were modelled with a soil Relative Density (RD) of 100% and 50% and a FB case as a reference.

As the RD of the soil was lowered, the displacement response of the turbine tower was observed to increase. This was due to the movement of the foundation in the flexible soil, especially the rotation. The foundation translation and rotation in the side-to-side direction was observed to increase significantly as the wave load was rotated around the tower with respect to the wind load.

FAST models

The NREL distributed aeroelastic FAST code was coupled with a AF and CS models derived using Plaxis.

The turbine control system was seen to have a large impact on the shear force and bending moment at the mudline. Foundation models were derived for both controlled and uncontrolled load cases. The derived foundation stiffnesses were found to be dependant on the control strategy due to the non-linearity in the soil.

SSI was shown to lower the first and second tower modal frequencies and increase the bending moment experienced at the mudline.

The static stiffness matrix for the SCF was of greater magnitude than the MPF. However it was observed that the two foundations had similar effects on the HAWT in terms of reduction in the tower’s first modal frequencies and bending moment at the mudline. This may be due to the ratio of the stiffness matrix coefficients.

The AF model was unable to accurately capture the dynamic response of the foundation compared to the CS model, in terms of bending moment statistics at the mudline. However there was a general agreement between the two models in terms of reduction
7. CONCLUSIONS

in modal frequencies.

SSI was observed to have a positive effect on the fatigue life of the turbine support structure under aerodynamic wind loading. However, when the fatigue life was calculated with the addition of a moderate sea environment, SSI had the opposite effect of decreasing the fatigue life of the turbine support structure.

Impact of the proposed analysis

HAWT support structures vary in size and dimensions from wind farm to wind farm and depending on the homogeneous of the soil even within the specific wind farm.

An advantage of the CS model is the condensation of foundation properties in a single matrix at mudline, meaning they are less expensive to implement compared to the DS model. The simplification also facilitates the exchange of information between the Geotechnical Engineer and the Structural Engineer. As a result they can be more easily included in existing HAWT analysis packages.

In this thesis, a static CS model has been shown to accurately capture the dynamic foundation interaction compared to a frequency dependant CS model for HAWT analysis for certain conditions. Using FEM methods a procedure has been presented to calculate to the static CS model of a soil-foundation system embedded in non-linear soil for site specific soil and loading conditions with computational ease.

Viewed in this context, the work presented has practical benefits in implementation in addition to the theoretical insights in the analysis.

7.3 Outlook

Wind energy is now an established industry and its market share is only set to increase. To consolidate its position, and fend off competition from other renewable energy sources and traditional energy sectors, it must become cheaper and more reliable. This is only possible through further research.

Pursuant to the work presented in this thesis, the following research would be beneficial.

Due to the level of uncertainty in soil parameters, parametric studies are essential. The CS model lends itself well to these studies. A Monte Carlo simulation could be carried
7.3 Outlook

out using a multi-variable linear regression CS model varying the soil’s relative density and foundation embedment depth.

SSI was observed to have a significant effect on the fatigue life of the support structure. It would be interesting to extend this study and fully analyses SSI effects. Extending the analysis to multiple loads cases: wind loading conditions from cut-in to cut-out, sea environment from calm to extreme and for varying soil stiffness conditions.

The influence of the static stiffness matrix on the system modal frequencies is dependant on the magnitude of the matrix components but also on the ratio of the components. This phenomena has been noted in this work but not fully investigated. Such an investigation could focus on what type of HAWT foundation is most efficient from a SSI view point.

Most monopile foundations are designed employing non-linear p-y curves, a DS model. Although DS and CS models have been compared in Van der Tempel (2006); Zaaijer (2002, 2005) and it has shown that CS model give comparable results. A more comprehensive analysis is still needed.

The foundation stiffness has been shown to be frequency dependent. In this work it has been observed that frequency dependent models, under certain conditions, can be replaced by static models (expect for softer soils). However frequency dependant foundation models may be of particular importance when predicting dynamic response of HAWT to earthquake loading.

These are a few possibilities for further research in the area of dynamic wind turbine analysis including SSI. Due to the complexity of large MW HAWT, the uncertainty surrounding SSI analysis and the huge drive towards renewable energy technologies, it is expected that the area will be a significant research area for years to come.
References

(1922). Electrical World. v. 80, McGraw-Hill. 3


REFERENCES


252
REFERENCES


REFERENCES


HASHASH, Y. (2004). Deepsoil, Pacific Earthquake Engineering Research Center (PEER), Tech. rep., University of Illinois at Urbana-Champaign. 101


254
REFERENCES


IWEA (2010). Jobs and Investment in Irish Wind Energy. 4


255
REFERENCES


REFERENCES


REFERENCES


NAUGHTON, R. (Retrieved 2012). The pioneers: Aviation and airmodelling [online]. 1


258
REFERENCES

veloped wind seas based on the similarity theory of s.a. kitaigorodshii. Journal of
Geophysical Research, 69. 42


resting on layered soil by cone model. Soil Dynamics and Earthquake Engineering,
24, 425 – 434. 49

subjected to wind and earthquake loading. In Proceedings of the 53rd AIAA/AS­
ME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference,
SDM 2012, Honolulu, Hawaii, USA. 66

and concrete wind turbine towers. Engineering Structures, 36, 270–282. 66

RASMUSSEN, F., HANSEN, M.H., THOMSEN, K., LARSEN, T.J., BERTAGNOLIO,
status of aeroelasticity of wind turbines. Wind Energy, 6, 213–228. 83

Foundations. Prentice-Hall. 53

Press. 3

of support structures for offshore wind turbines. In Proceedings of Offshore Wind
Energy Conference., Copenhagen, Denmark. 36

tion Design Utilizing Soil-Foundation-Structure Interaction. In A. Santini &
N. Moraci, ed., 2008 Seismic Engineering Conference Commemorating the 1908
Messina and Reggio Calabria Earthquake, vol. 1020 of American Institute of
Physics Conference Series, 577–584. 52

259
REFERENCES


REFERENCES

Southampton, Boston, USA. 13, 78, 81, 164


National Laboratories, Albuquerque, NM. 83

systems. *Earthquake Engineering & Structural Dynamics, 3*, 121–138. 47, 50, 162,
164

*Journal of the Geotechnical Engineering Division, ASCE, 100*, 479–482. 56

quake Engineering & Structural Dynamics, 15*, 1–21. 53, 110, 114

excited liquid storage tanks. *Earthquake Engineering & Structural Dynamics, 19*,
473–496. 101

quake Engineering & Structural Dynamics, 2*, 87–102. 50

of the Soil Mechanics and Foundations Division, ASCE, 97*, 1227–1248. 53, 110,
114

*Journal of Petroleum Technology, 22*. 44

Co. 2

Inc.,New York, NY. 43, 44, 86

261
REFERENCES


262
Appendix A

Fourier Transforms

A.1 Introduction

Fourier transforms are operations that transform signals from the time domain to the frequency domain and back again. Time/Frequency domain is used to describe the analysis of mathematical function or physical signals, with respect to time/frequency. Typically we have a signal in time domain and a Fourier transform is carried out to give the frequency domain representation of the original function, describing the frequencies present in the original function.

![Figure A.1: Time history](image)

Any signal $x(t)$ in the time domain, as shown in Figure A.1, is the sum of amplitudes of that signal at discrete points in time, and we sum up all points to get the
signal $x(t)$. Another way to represent a signal in the time domain, instead of summing points at each point in time, is to sum up functions that cover all time. Functions that cover all time are essentially waves and can be described using sinusoidal waves with a magnitude and an exponential term. These can also be represented using Euler’s equation, where the real part is cosine and imaginary part is a sine, each with their own amplitudes. Figure A.2 shows a plot of a cosine and sine wave at some frequency $\omega$ with amplitudes $A$ and $B$. Summing up all possible combinations of sinusoidal waves (each at some particular frequency) that have the right amplitudes we can reproduce the function $x(t)$, no matter what it is i.e. any time domain signal can be represented by a sum of sinusoidal waves.

$$e^{j\omega t} = A \cos(\omega t) + iB \sin(\omega t)$$

This is another way to represent a function, not by summing up values at points in time, but by sinusoidal waves that stretch over all time, which have the right amplitude. For Figure A.2 to have an accurate representation of the signal $x(t)$ lots of different sinusoidal waves would need to be plotted, which would be hard to visualise, instead it is easier to represent such signal in the frequency domain. By simply plotting the amplitudes of the cosine and sine parts as a function of $\omega$, the signal can be represented in the frequency domain.
A.2 Fundamentals

To change a time domain signal into the frequency domain, a Discrete Fourier Transform (DFT) can be employed.

\[ \hat{X}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) e^{-i\omega t} \, dt \quad \text{(A.1)} \]

the inverse Fourier transform is,

\[ x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{X}(\omega) e^{i\omega t} \, d\omega \quad \text{(A.2)} \]

using Euler’s rule, we can split the exponential into a real and imaginary part.

\[ \hat{X}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t)(\cos(\omega t) - i \sin(\omega t)) \, dt \quad \text{(A.3)} \]

Which can be simply written as,

\[ \hat{X}(\omega) = \hat{X}_R(\omega) + i \hat{X}_I(\omega) \quad \text{(A.4)} \]

Taking the real part gives,

\[ \hat{X}_R(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) \cos(\omega t) \, dt \quad \text{(A.5)} \]

and the imaginary,

\[ \hat{X}_I(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) \sin(\omega t) \, dt \quad \text{(A.6)} \]

Solving the equation A.5 and A.6 and then taking the modulus we recover the frequency spectrum of the signal. A typical frequency spectrum is shown in Figure A.3.

\[ |\hat{X}(\omega)| = \sqrt{[\hat{X}_R(\omega)]^2 + [\hat{X}_I(\omega)]^2} \quad \text{(A.7)} \]

The frequency domain representation, \( \hat{X}(\omega) \), gives the amplitude of the real and imaginary part of the sinusoid at each frequency \( \omega \).

Performing this integral transformation on real valued time domain signals is computationally expensive therefore a FAST Fourier Transform (FFT) algorithm is usually employed, a clever algorithm that does the same thing as the DFT. To compute a DFT
A. FOURIER TRANSFORMS

![Sample spectra](image)

Figure A.3: Sample spectra

of $N$ points takes $N^2$ arithmetical operations, however an FFT can compute the same result in only $N\cdot log(N)$ operations. The difference in speed can be substantial, especially for long data sets.

A.3 Nyquist criteria

A discrete time signal (unlike a continuous-time signal) is not a function of a continuous argument however one may be obtained by sampling a continuous-time signal. The sampling rate defines the number of samples per second taken from a continuous signal to make a discrete signal.

For example, take a sine wave only two points in time are required to capture this signal. In Figure A.4 the term $T$ is the period of the signal and $\omega$ the circular frequency. The sampling rate is thus given as,

$$\Delta t = \frac{T}{2} \quad (A.8)$$

and the sampling frequency as,

$$f_s = \frac{1}{\Delta t} \quad (A.9)$$

Note the inverse of the sampling frequency is the sampling period which is the time between samples. This is the highest frequency which has to be sampled in order not to lose information about the signal. Consider the signal shown in Figure A.5.
If the blue signal (shown in Figure A.5) is sampled at a rate $\Delta t$ and then reconstructed the signal will be different from the original instead it must be sampled at a rate $\Delta t_1$ to ensure no lose of information. This effect is called aliasing and refers to an effect that causes different signals to become indistinguishable when sampled. To properly analyse time varying data with a maximum frequency of $F$, we need to acquire samples at twice that rate, or $2F$, this is called the Nyquist-Shannon sampling theorem.

In principle a Nyquist frequency just larger than the signal bandwidth is sufficient
A. FOURIER TRANSFORMS

to allow perfect reconstruction of the signal from the samples. However, in practice to avoid aliasing the Nyquist frequency is set strictly greater than maximum frequency component within the signal. Therefore looking at a typical signal in time domain to be transformed using a FFT the sample rate is chosen so that the Nyquist criteria holds, see Figure A.6.

\[ F(t) \]

\[
\begin{align*}
X_0, X_1, X_2, \ldots, X_N
\end{align*}
\]

\[
\omega_0, \omega_1, \omega_2, \ldots, \omega_N
\]

\[
\Delta \omega
\]

\[
\text{Half, } N/2
\]

Figure A.6: Sampling the signal

Examining Figure A.6 gives,

\[
\text{highest frequency, } \quad \omega = \frac{2\pi}{T} \quad (A.10)
\]

\[
\text{sampling, } \quad \Delta t = \frac{T}{2} \quad (A.11)
\]
According to Nyquist criteria,

\[
\text{minimum sampling, } \Delta \text{min} = \frac{T}{2}
\]  

(A.12)

This implies the maximum frequency that can be captured as,

\[
\omega = \frac{2\pi}{T} = \frac{2\pi}{2\Delta t} = \frac{\pi}{\Delta t}
\]  

(A.13)

Where the total duration for large \( N \) is,

\[
T_d = (N - 1)\Delta t \approx N\Delta t
\]  

(A.14)

Therefore the Fourier transforms contains information between 0 and \( f_s \) however we know that the sampling frequency must be of least twice the highest frequency component. Therefore the signal spectrum should be kept entirely below \( f_s/2 \), the Nyquist frequency. Implying the above theory yields a frequency spectrum like the one shown in Figure A.7.

![Figure A.7: Spectra showing the Nyquist Frequency (NF)](image)

Note a real value signal should have a transform magnitude that is symmetrical for positive and negative frequency. So instead of having a spectrum that goes from 0 to \( f_s \) it would be more appropriate to show the spectrum form \(-f_s/2 \) to \(+f_s/2 \).

Since the signal repeats, for some problems, we need only work with half of the data points,

\[
\omega_{N/2} = \frac{\pi}{\Delta t}
\]  

(A.15)
A. FOURIER TRANSFORMS

Figure A.8: Spectra zero centred

\[ [0, \omega_{N/2}] \]  

\[ [0, \frac{\pi}{\Delta t}] \]  

\[ \Delta \omega = \frac{\pi}{\Delta t(N/2)} \]  

Rearranging the above we get frequency step,

\[ \Delta \omega = \frac{2\pi}{N\Delta t} \]  

A.4 Matlab Model

A frequency spectrum of a signal is now computed in MATLAB using the methods presented. First a sample signal is generated. The FFT of the signal is then generated using the build in MATLAB function \textit{fft}. The output is then rearranged by moving the zero-frequency component to the centre of the array using the MATLAB function \textit{ffshift}. The sample signal is sent to another function which computes the so called positive FFT of the signal, it returns the frequency component for each corresponding amplitude for the positive half of the transform only. The syntax for computing the FFT of a signal is presented in listing A.1 and the results shown in Figure A.10.
A.4 Matlab Model

$$|\hat{X}_0(\omega)| = \sqrt{X_{R,0}^2 + X_{I,0}^2}, \quad \omega_0 = 0$$

$$|\hat{X}_1(\omega)| = \sqrt{X_{R,1}^2 + X_{I,1}^2}, \quad \omega_1 = \Delta \omega$$

$$|\hat{X}_2(\omega)| = \sqrt{X_{R,2}^2 + X_{I,2}^2}, \quad \omega_2 = 2\Delta \omega$$

Figure A.9: Half spectra

Listing A.1: Matlab code, Spectra

```
function spec
    fo=4; % freq of wave
    fs = 100; % sampling rate
    Ts = 1/fs; % sampling time interval
    t = 0:Ts:1-Ts; % time interval
    y = 2*sin(2*pi*fo*t); % signal
    N = 2048; % sample points
    X = abs(fft(y,N));
    F = (0 : N - 1)/N;
    figure
    subplot(1,3,1)
    plot(F,X)
    ylabel('Amplitude')
    title('(a)')
    grid on
    X = fftshift(X);
    F = (-N/2:N/2-1)/N;
    subplot(1,3,2)
    plot(F,X)
    ylabel('Amplitude')
    title('(b)')
    grid on
```
A. FOURIER TRANSFORMS

```matlab
%Call positiveFFT which returns
%the frequency range and the positive half of the FFT
[freq, freqRng] = positiveFFT(y,fs);

subplot(1,3,3)
plot(freqRng, abs(freq));
xlabel('Frequency (Hz)')
ylabel('Amplitude')
title('(c)'), grid on

% figure
% plot (freqRng*2, X)
end

%This function get rid of symmetry
function [X,freq] = positiveFFT(y,Fs)
N=length(y);
k = 0:N-1;
T = N/Fs;
freq = k/T; %create freq range
X = fft(y);
cutoff = ceil(N/2); %downsample
X = X(1:cutoff);
freq = freq(1:cutoff);
end
```

Figure A.10: Spectra outputs: (a) Normal spectra (b) Zero centred (c) Half spectra
A.5 Dynamic analyses in the time/frequency domain

The use of Fourier Transforms in solving structural dynamic problems in the frequency domain is now examined. To illustrate this problem a simple SDOF system is solved in time domain and then in frequency domain in the MATLAB environment.

![Figure A.11: SDOF system](image)

The equation of motion for the a SDOF system can be written as,

\[ M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = F(t) \]  \hspace{1cm} (A.20)

Dividing across by the mass \( M \) yields,

\[ \ddot{x} + 2\xi \omega_o \dot{x} + \omega_o^2 x = f \]  \hspace{1cm} (A.21)

where \( \omega_o = \sqrt{\frac{K}{M}}, f = \frac{F(t)}{M} \) and \( \xi = \frac{C}{2\sqrt{MK}} \).

The differential equation (equation A.21) is solved for \( x(t) \) in in the time domain. In the MATLAB this can be done by rearrange equation A.21 into the state-space formulation (equation A.22 ) and using the built-in \textit{ode45} solver. The \textit{ode45} routine uses a variable step, fourth-order, Runge-Kutta method to solve differential equations numerically.

\[
\begin{pmatrix}
\dot{x} \\
\ddot{x}
\end{pmatrix} = \begin{bmatrix}
0 & 1 \\
-\omega_o & -2\xi \omega_o
\end{bmatrix} \begin{pmatrix}
x \\
\dot{x}
\end{pmatrix} + \begin{pmatrix}
0 \\
f
\end{pmatrix} \hspace{1cm} (A.22)
\]

\[ ydot = Ay + F \]  \hspace{1cm} (A.23)
A. FOURIER TRANSFORMS

Equation A.23 is the MATLAB representation of equation A.22. The *ode45* function takes a time span and the initial conditions, *y*, and solves equation A.22 numerous times. The solver varies the step size, increasing its value where possible to increase the speed of computation, and decreasing where needed to maintain accuracy. It then outputs the results to an array, which contains the time, displacement and velocity of the system. The MATLAB code is given in listing A.2 and the solution, in terms of displacement, is given in Figure A.12. Note the initial conditions of displacement and velocity were taken as zero and the forcing function as a simple sine wave.

Listing A.2: Matlab code, Main

```
1 %Solves the SDOF system in time domain using ode45
2 %and then solves the system in the freq domain
3 %and then plots both results against each other
4 function mainsdof
5  
6 global M; %mass
7 global wo; %Nat Freq
8 global I;
9
10 M = 2;
11 K = 10;
12 C = 0.26;
13
14 wo = sqrt(K/M);
15 I = C/(2*M*wo);
16
17 %initial conditions and time span
18 y = [ 0 , 0 ] ;
19 To = linspace(1,100,1000);
20 size (To);
21
22 [ts,ys] = ode45( 'sdof' , To , y );
23
24 figure
25 plot ( ts , ys(:,1),'b')
26 title('time domain')
27 xlabel('Time (s)')
28 ylabel('Displacement (m)')
29 grid on
30
31 [tspanf, disf] = freqsdof;
32
33
```

274
A.5 Dynamic analyses in the time/frequency domain

```matlab
figure
plot ( ts, ys(:,1), 'b', tspanf, disf, 'xr')
xlabel('Time (s)')
ylabel('Displacement (m)')
legend( 'Time Domain', 'Frequency Domain');
grid on
end
```

Listing A.3: Matlab code, SDOF Time domain

```matlab
% Returns the displacement and velocity
function ydot = sdof(t,y)

global M; %mass
global w0;
global I;

\[
F = \begin{bmatrix} 0 & (\sin(t)/M) \end{bmatrix}.
\]

A = \[
\begin{bmatrix} 0 & 1 \\ -w0^2 & -(2*1*w0) \end{bmatrix}.
\]

ydot = A*y + F;
```

The above SDOF system can also be solved in the frequency domain in MATLAB using the in-built functions ifft and ifft (inverse Fast Fourier Transform). The equation of motion is first transformed into the frequency domain,

\[
\begin{align*}
-\omega^2 \tilde{X}(\omega) + i\omega 2\xi \omega_o \tilde{X}(\omega) + \omega_o^2 \tilde{X}(\omega) &= \tilde{F}(\omega) \\
\end{align*}
\]

(A.24)

rearranging gives,

\[
\frac{\tilde{X}(\omega)}{\tilde{F}(\omega)} = \frac{1}{(\omega_o^2 - \omega^2) + i2\omega \xi \omega_o} = \tilde{B}(\omega)
\]

(A.25)

The term \(\tilde{B}\) is called the transfer function. However, we are interested in the amplitude

275
Since the whole solution is needed both the positive and negative sides of the spectrum are required for a complete solution. Due to the fact that for real valued time histories only the first \( N/2 + 1 \) points in the frequency domain are unique, the remaining points are complex conjugates of values in the first half of the record. This implies that the negative range is equal to the complex conjugate of the positive range since,

\[
\hat{B}(-\omega) = \hat{B}^*(\omega)
\]  

(A.27)

the solution can therefore be solved for in two stages up to \( N/2 \),

\[
\hat{B}(\omega_{0-N/2}) = \hat{B}_R(\omega_{0-N/2}) + i\hat{B}_I(\omega_{0-N/2})
\]  

(A.28)
A.5 Dynamic analyses in the time/frequency domain

and beyond,

\[ \hat{B}(\omega_{N/2-N}) = \hat{B}_R(\omega_{N/2-N}) + i\hat{B}_I(\omega_{N/2-N}) \quad (A.29) \]

To solve a dynamic problem in the frequency domain (in MATLAB) the following step should be followed:

Transform the input force time history into the frequency domain,

\[ \hat{F}(\omega) = \text{fft}\{f(t)\} \quad (A.30) \]

Get the transfer function, which for numerical calculations using discrete frequencies is given as,

\[ \hat{B}(\omega_n) = \begin{cases} 
[-\omega_n^2[M] + [K] + i\omega_n[C]]^{-1} & \text{for } n \leq N/2 \\
[\hat{B}^*(\omega_{N-n})] & \text{for } n > N/2 
\end{cases} \quad (A.31) \]

where \( N \) is the number of sample points taken and \( \omega_n (= n\Delta\omega) \) is the discrete circular frequency at each step, noting \( \Delta\omega = 2\pi/T_o \). Note for solving the problem, the time span must be selected so that period \( T_o \) is considerably longer than the duration of excitation (Clough & Penzien, 1995), thus resulting in an interval of zero excitation before and following the excitation load. This is a necessary requirement so that the free vibration response during the intervals of zero excitation will dampen out almost completely. Otherwise, the assumed zero initial conditions at the start of the excitation will not be sufficiently satisfied.

Calculate the response in the frequency domain,

\[ \hat{X}(\omega) = \hat{B}(\omega) \times \hat{F}(\omega) \quad (A.32) \]

Employing an inverse Fourier transform the response of the SDOF system in time domain is recovered,

\[ x(t) = \text{ifft}(\hat{X}(\omega)) \quad (A.33) \]

The corresponding MATLAB code is given in listing A.4. the resulting solution, given in Figure A.13, is seen to match the time domain solution.
A. FOURIER TRANSFORMS

Listing A.4: Matlab code, SDOF Frequency domain

```matlab
1  % frequency domain approach
2  function [timespan,disp] = freqsdof ()
3
4  I=sqrt(-1);
5
6  To=100;
7  K=10;
8  M=2;
9  C = 0.26;
10
11  N=2000;
12  dt=To/N;
13
14  for i=0:N-1
15      f(i+1,1)= sin(i*dt);
16  end
17
18  for i=0:N/2-1
19      B(i+1)=1/(K-M*omega(i,To)^2+I*omega(i,To)*C);
20      B(N-i+1)= conj( B(i+1) ) ;
21  end
22
23  F=fft(f);
24
25  for i=0:N-1
26      X(i+1)=B(i+1)*F(i+1);
27  end
28
29  x=ifft(X);
30
31  for i=0:N-1
32      timespan(i+1)=i*dt;
33  end
34
35  disp = real(x);
36
37  end
38
39  %forcing freq
40  function x=omega(n,To)
41      x=n*(2*pi/To);
42  end
```
A.5 Dynamic analyses in the time/frequency domain

Figure A.13: Displacement response, for the time domain and frequency domain solutions
A. FOURIER TRANSFORMS
Appendix B

Integrated Structural/Plaxis model

Most structures have some direct contact with the ground. Foundations act as buffers, taking loads from the supported structure and transmitting them into the soil. Often structures channel their load into foundation pads through a series of columns i.e. a framed structure. These columns are usually designed assuming the foundation does not deflect i.e. fully fixed conditions. However for softer soils, the foundation will deflect and act as a spring support and thus effect the resultant forces in the columns.

To investigate this static soil-structure interaction problem, an integrated structural/soil model was created, whereby a structural code (capable of solving resultant forces for a framed structures) was linked with a soil-foundation code (Plaxis) and the support stiffness solved for in an iterative manner.

B.1 Method

The framed structure is loaded and the forces resolved for the supporting columns. The resultant column forces are then placed on their corresponding foundation pads in the soil-foundation code. The settlement of each foundation pad is calculated and thus the support stiffness (spring coefficient). The initial assumed fixed base boundary condition in the structural code is replaced with the support stiffness and support forces resolved again. The resultant forces are placed back on their corresponding foundation pads.
B. INTEGRATED STRUCTURAL/PLAXIS MODEL

pads and new support stiffness evaluated. This iterative procedure can be carried out until the forces convergence.

The superstructure is modelled in a structural code developed in MATLAB. The structural code, based on beam elements, is capable of solving any 2-D framed structure. Any structural code could have been used, the only requirement is that the spring stiffness can be feed into the code and the resultant column forces given back in a readable format.

The columns of the framed structure are each connected to spring supports in the vertical and horizontal direction (2-D).

The soil-foundation system is modelled in Plaxis 2-D. Plaxis was used to calculate the deflection of the foundation pads due to the applied force from the column and thus the support stiffness.

B.2 Validation case

This study’s goal was not only to investigate the static soil-structure interaction for a simple structure but also the connection of Plaxis to a structural code. As Plaxis has specialist capability in modelling soil it has however limited capability for analysing and solving structural problems. An example is presented whereby a simple framed structure founded on three foundation pads is solved for in the integrated Struct/Plaxis model, as shown in Figure B.1. The model is then validated by comparison to the same structure solved completely in Plaxis (as Plaxis has the capability to solve simple beam element structures). The example also serves to elaborate the workings of the integrated Struct/Plaxis model.

The beam elements, foundation pads and soil parameters are detailed in Table B.1, B.2 and B.3 respectively.

B.2.1 Integrated Struct/Plaxis model

A control program written in MATLAB (shown in listing B.1) calls the structural code and Plaxis in a loop until convergence. The control program first calls the Structural code, which solves the framed structure (in the x-y direction or horizontal-vertical di-
B.2 Validation case

Table B.1: Framed structure properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EA</td>
<td>KN</td>
<td>2.05e6</td>
</tr>
<tr>
<td>EI</td>
<td>KN.m²</td>
<td>2.05e3</td>
</tr>
<tr>
<td>ρ</td>
<td>kg/m³</td>
<td>0</td>
</tr>
<tr>
<td>P₁</td>
<td>KN</td>
<td>1</td>
</tr>
<tr>
<td>P₂</td>
<td>KN</td>
<td>1</td>
</tr>
</tbody>
</table>

Table B.2: Material properties of the foundation pads

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EA</td>
<td>KN</td>
<td>1.00e9</td>
</tr>
<tr>
<td>EI</td>
<td>KN.m²</td>
<td>1.00e8</td>
</tr>
<tr>
<td>ρ</td>
<td>kg/m³</td>
<td>0</td>
</tr>
<tr>
<td>Depth</td>
<td>m</td>
<td>0.1</td>
</tr>
<tr>
<td>Width</td>
<td>m</td>
<td>1</td>
</tr>
</tbody>
</table>

Table B.3: Soil properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil model</td>
<td>-</td>
<td>Mohr Coulomb</td>
</tr>
<tr>
<td>Analysis type</td>
<td>-</td>
<td>Drained</td>
</tr>
<tr>
<td>Unit weight</td>
<td>KN/m³</td>
<td>18</td>
</tr>
<tr>
<td>Young Modulus</td>
<td>KN/m²</td>
<td>1000</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>-</td>
<td>0.35</td>
</tr>
<tr>
<td>Angle of friction</td>
<td>deg</td>
<td>25</td>
</tr>
</tbody>
</table>

Plaxis is then called (foundcalc.exe). Plaxis is run in batch mode to facilitate the iterative process, and the displacements of the foundation read in. The support stiffness (in the x-y direction) are calculated and placed in a text file (Force.txt) accessible to the structural code. After one iteration, the control file checks for convergence of forces (a 1% tolerance of norm 2) in the x-y direction, in terms of the support forces in this current run compared to the last run. The tolerance (Tor) in each direction is given by,

\[
Tor = \frac{\| F \|}{100} = \frac{\sqrt{\sum_{i=1}^{N_{sup}} F_i^2}}{N_{sup}} / 100
\]  
(B.1)
B. INTEGRATED STRUCTURAL/PLAXIS MODEL

where the term $N_{sup}$ is the number of supports and $F_i$ is the support reaction (in either the x-y direction). After convergences of forces or a max number of iterations, the process is stopped. The foundation displacements, the column forces and support stiffness for each run are recorded. The initial and final displacement and column forces (where shear force $= F_x$ and axial force $= F_y$) are given in Table B.4.
B.2.2 Complete Plaxis Model

The models were combined and ran together in Plaxis to validate the integrated Struct/Plaxis model, as shown in Figures B.3 and B.4. A hinge joint must be applied at the connection between the structure and foundation, as the structural code only considers springs in the x-y direction at support, no moment is transferred.

In order to ensure a fair comparison, the soil domain boundaries were kept the same for both models ($40 \times 15$ m) and the mesh of the soil domain was refined until they corresponded. The results from this analysis are given in Table B.5. The results agree well with the integrated Struct/Plaxis model results validating the procedure.

![Figure B.3: Complete Plaxis model](image)

Table B.4: Integrated Struct/Plaxis model, results

<table>
<thead>
<tr>
<th>Node</th>
<th>Force ($\mathbf{N}$)</th>
<th>Displacement (mm)</th>
<th>$F_x$</th>
<th>$F_y$</th>
<th>$D_x$</th>
<th>$D_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>1</td>
<td>-66.01</td>
<td>-746.02</td>
<td>-0.1107</td>
<td>-1.354</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-35.99</td>
<td>-919.95</td>
<td>-0.041</td>
<td>-1.702</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>102</td>
<td>-334.03</td>
<td>0.264</td>
<td>-0.594</td>
<td></td>
</tr>
<tr>
<td>Final</td>
<td>1</td>
<td>-41.6</td>
<td>-737.8</td>
<td>-0.0698</td>
<td>-1.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-19.9</td>
<td>-924.5</td>
<td>-0.0227</td>
<td>-1.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>61.4</td>
<td>-337.8</td>
<td>0.1570</td>
<td>-0.59</td>
<td></td>
</tr>
</tbody>
</table>
B. INTEGRATED STRUCTURAL/PLAXIS MODEL

![Deformed mesh]

Figure B.4: Complete Plaxis model, deformed mesh

Table B.5: Complete Plaxis Model, results

<table>
<thead>
<tr>
<th>Node</th>
<th>Force (N)</th>
<th>Displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_x$</td>
<td>$F_y$</td>
</tr>
<tr>
<td>1</td>
<td>-41.6</td>
<td>-738</td>
</tr>
<tr>
<td>2</td>
<td>-19.9</td>
<td>-924.4</td>
</tr>
<tr>
<td>3</td>
<td>61.4</td>
<td>-337.8</td>
</tr>
</tbody>
</table>

B.3 Discussion

A method has been presented to connect Plaxis to an outside structural code. The integrated Struct/Plaxis model requires some initial set-up and then runs automatically until convergence of column forces. The method has been validated by comparison to the complete solution solved for in Plaxis. It was observed that the analysis was worth carrying out (for this soil type and framed structure) if the support stiffness was under $K_{sup} = 1$ MN/m.
Listing B.1: Control program

```matlab
%This files call the Struct. code and then Plaxis (in batch mode)
in a loop to refine the spring constants.

% Number of foundation column connections
Num_Sp_supp = 3;
n=1;
% Ensures sum is larger than check
cHECK_x = ones(Num_Sp_supp,1);
cHECK_y = ones(Num_Sp_supp,1);
% Initialise check
r_x=0;
r_y=0;

while sum(check_x)>r_x && sum(check_y) > r_y && n < 20
    if (n==1)
        % Run initial Struct code (Fixed supports)
        Initial_Struct_Static
        % Read in column forces
        Fl(:,:,1) = dlmread('C:\Users......\Force.txt');
        % Calculate converges criteria
        r_x = ( (sqrt(sum(Fl(:,2).^2)) ) /Num_Sp_supp)/100 ;
        r_y = ( (sqrt(sum(Fl(:,1).^2)) ) /Num_Sp_supp)/100;
    else
        % Run initial Struct. code
        Struct_Static
        % Read in column forces
        F2(:,:,n) = dlmread('C:\Users......\Force.txt');
        % Calculate converges criteria
        r_x = ( (sqrt(sum(F2(:,2).^2)) ) /Num_Sp_supp)/100 ;
        r_y = ( (sqrt(sum(F2(:,1).^2)) ) /Num_Sp_supp)/100;
    end

% Run Plaxis
dos('foundcil.exe');
% Read in foundation pad disp.
Dis_Sp(:,,:) = dlmread('C:\Users......\Disp',',',1,0);
% Keep record
Disp_Sp(:,n) = Dis_Sp(:,2:3);

if n>1
    % After first loop get check
    check_y(:,1) = abs( F2(:,2,n) - Fl(:,2) );
```

287
B. INTEGRATED STRUCTURAL/PLAXIS MODEL

```matlab
check_x(:,1) = abs(F2(:,1,n) - F1(:,1));
F1(:,:) = F2(:,n);

%Calculate spring coefficients (S=F/u) for each foundation
Stiff_Spring(:,1,n) = ...
    F2(:,1,n)./Disp_Sp(:,1,n); %in N/m
Stiff_Spring(:,2,n) = F2(:,2,n)./Disp_Sp(:,2,n);

else
    %Calculate spring coefficients for each foundation
    Stiff_Spring(:,1,n) = F1(:,1)./Disp_Sp(:,1,n);
    Stiff_Spring(:,2,n) = F1(:,2)./Disp_Sp(:,2,n);
    %keep record
    F2(:,n) = F1(:,:);
end
end
```
Appendix C

BEM algorithm

This appendix presents an algorithm to calculate the modified BEM method. The algorithm must be carried out at each section of the blade \( r \) and at each time step. Based on work presented by Hansen (2001).

1. Initialize \( a^{(k)}, \dot{a}^{(k)}, a_c, B, \zeta \) (e.g. \( a^{(0)} = 0, \dot{a}^{(0)} = 0, a_c = 0.2, B = 3, \zeta = 0.01 \)), where \( B \) is the number of blades and \( \zeta \) is a tolerance limit.

2. Compute the flow angle, \( \phi_{FA} = \tan^{-1}\left(\frac{(1-a^{(k)})V_o + \ddot{a}}{\Omega r (1+a^{(k)})}\right) \).

3. Compute the angle of attack, \( \alpha_{AOA} = \phi_{FA} - \beta_p - \kappa_t \).

4. Compute Prandtl’s tip loss correction, \( F = \frac{2}{\pi} \cos^{-1}(e^{-f}), \quad f = \frac{B}{2} \frac{R_b - r}{r \sin(\phi_{FA})} \).

5. Retrieve lift and drag coefficients, \( C_{l,b}(\alpha_{AOA}) \) and \( C_{d,b}(\alpha_{AOA}) \), from airfoil data table for the calculated \( \alpha_{AOA} \) and section of the blade \( r \).

6. Compute normal load coefficient, \( C_{N,b} = C_{l,b} \cos(\phi_{FA}) + C_{d,b} \sin(\phi_{FA}) \).

7. Compute tangential load coefficient, \( C_{T,b} = C_{l,b} \sin(\phi_{FA}) - C_{d,b} \cos(\phi_{FA}) \).

8. Compute the solidity, \( \sigma(r) = \frac{cB}{2\pi r} \).

9. if \( \ddot{a} \leq a_c \) then

10. \( a^{(k+1)} = \frac{1}{a + \frac{4f \sin^2(\phi_{FA})}{\sigma C_{N,b}}} \frac{1}{a + \frac{4f \sin^2(\phi_{FA})}{\sigma C_{T,b}}} \).

289
C. BEM ALGORITHM

11. else

12. \[ \tilde{a}^{(k+1)} = \frac{1}{2} \left[ 2 + K(1 - 2a_c) - \sqrt{(K(1 - 2a_c) + 2)^2 + 4(Ka_c^2 - 1)} \right], \quad K = \frac{4F \sin^2(\phi_{FA})}{\sigma C_{N,b}} \]

13. endif

14. \[ \tilde{a}'(k + 1) = \frac{1}{-1 + \frac{4F \sin(\phi_{FA}) \cos(\phi_{FA})}{\sigma C_{T,b}}} \]

15. if \((\tilde{a}^{(k+1)} - \tilde{a}^{(k)}) > \zeta\) & & \((\tilde{a}'^{(k+1)} - \tilde{a}'^{(k)}) > \zeta\)

16. \(k = k + 1\), GO TO \#2

17. else

18. Get the axial and tangential induction factors \(\tilde{a} = \tilde{a}^{(k)}\), \(\tilde{a}' = \tilde{a}'^{(k)}\)

19. endif

20. Compute relative velocity, \(V_{rel} = \sqrt{[V_o(1 - \tilde{a}) + \tilde{w}]^2 + [\Omega_b r(1 + \tilde{a}')]^2} \]

21. Compute local flapwise and edgewise loads, \(p_{N,b} = \frac{1}{2} \rho_o V_{rel}^2 C_{N,b}(\alpha_{AOA})c\), \(p_{T,b} = \frac{1}{2} \rho_o V_{rel}^2 C_{T,b}(\alpha_{AOA})c\)
Appendix D

NREL HAWT models

The Nation Renewable Energy Laboratory (NREL) in the United states of America is a federal laboratory dedicated to the research, development, commercialization and deployment of renewable energy and energy efficiency technologies. Information regarding the properties and operational data for existing wind turbine is generally not publicly available. However, the NREL have made public key parameters for several wind turbines, both onshore and offshore. The 1.5 MW onshore and 5 MW offshore (for shallow waters) HAWT models are used in this work and detailed here.

D.1 NREL 1.5 MW onshore HAWT

This wind turbine is a three-bladed, upwind, variable-speed, collective pitch-controlled machine. It has a 70 m rotor diameter, a maximum rotor speed of 20 rpm, and a rated wind speed of 11.4 m/s. The turbine has a hub height of 83 m, the tower is modelled as a single continuous cylindrical tube with varying diameter and wall thickness.

The blade and tower data for the turbine are given in Table D.1 and Table D.2. In Table D.3 the key properties and some operational data for the 1.5 MW turbine are detailed.

In order to apply modal analysis to the flexible blade and tower components a set of shape functions and natural frequencies must first be derived. This is achieved by
Table D.1: Blade structural properties for the 1.5 MW HAWT

<table>
<thead>
<tr>
<th>Blade Fraction</th>
<th>Structural Twist deg</th>
<th>Blade Mass Density kg/m</th>
<th>Flap Stiffness Nm²</th>
<th>Edge Stiffness Nm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00000</td>
<td>11.100</td>
<td>1447.607</td>
<td>7681.46E+6</td>
<td>7681.46E+6</td>
</tr>
<tr>
<td>0.02105</td>
<td>11.100</td>
<td>180.333</td>
<td>1169.87E+6</td>
<td>1169.87E+6</td>
</tr>
<tr>
<td>0.05263</td>
<td>11.100</td>
<td>183.905</td>
<td>771.88E+6</td>
<td>962.97E+6</td>
</tr>
<tr>
<td>0.10526</td>
<td>11.100</td>
<td>186.138</td>
<td>523.14E+6</td>
<td>833.66E+6</td>
</tr>
<tr>
<td>0.15789</td>
<td>11.100</td>
<td>188.370</td>
<td>274.40E+6</td>
<td>704.35E+6</td>
</tr>
<tr>
<td>0.21053</td>
<td>9.500</td>
<td>178.321</td>
<td>234.57E+6</td>
<td>614.65E+6</td>
</tr>
<tr>
<td>0.26316</td>
<td>7.900</td>
<td>168.271</td>
<td>194.74E+6</td>
<td>524.96E+6</td>
</tr>
<tr>
<td>0.31579</td>
<td>6.300</td>
<td>158.222</td>
<td>154.90E+6</td>
<td>435.26E+6</td>
</tr>
<tr>
<td>0.36842</td>
<td>4.700</td>
<td>148.172</td>
<td>115.07E+6</td>
<td>345.57E+6</td>
</tr>
<tr>
<td>0.42105</td>
<td>3.100</td>
<td>138.123</td>
<td>75.23E+6</td>
<td>255.87E+6</td>
</tr>
<tr>
<td>0.47368</td>
<td>2.600</td>
<td>122.896</td>
<td>62.49E+6</td>
<td>217.87E+6</td>
</tr>
<tr>
<td>0.52632</td>
<td>2.100</td>
<td>107.669</td>
<td>49.75E+6</td>
<td>179.86E+6</td>
</tr>
<tr>
<td>0.57895</td>
<td>1.600</td>
<td>92.442</td>
<td>37.01E+6</td>
<td>141.86E+6</td>
</tr>
<tr>
<td>0.63158</td>
<td>1.100</td>
<td>77.215</td>
<td>24.27E+6</td>
<td>103.85E+6</td>
</tr>
<tr>
<td>0.68421</td>
<td>0.600</td>
<td>61.988</td>
<td>11.53E+6</td>
<td>65.85E+6</td>
</tr>
<tr>
<td>0.73684</td>
<td>0.480</td>
<td>51.861</td>
<td>9.27E+6</td>
<td>54.25E+6</td>
</tr>
<tr>
<td>0.78947</td>
<td>0.360</td>
<td>41.734</td>
<td>7.01E+6</td>
<td>42.66E+6</td>
</tr>
<tr>
<td>0.84211</td>
<td>0.240</td>
<td>31.607</td>
<td>4.75E+6</td>
<td>31.06E+6</td>
</tr>
<tr>
<td>0.89474</td>
<td>0.120</td>
<td>21.480</td>
<td>2.49E+6</td>
<td>19.47E+6</td>
</tr>
<tr>
<td>1.00000</td>
<td>0.000</td>
<td>11.353</td>
<td>0.23E+6</td>
<td>7.87E+6</td>
</tr>
</tbody>
</table>

an eigenvalue analysis of the individual flexible components. This is done using the NREL disturbed MODES (Buhl, 2005). The program takes in the mass and stiffness of the structure as a function of height and gives back the first $n$ modes shapes and natural frequencies. The tower itself acts like a cantilever beam with a heavy mass at the top in the form of the nacelle, the program gives the option of including an end mass. The natural frequencies for the tower and the blade are given in Table D.4. The mode shapes for the blades and tower are shown in Figure D.1 and D.2 respectively.
Figure D.1: First two blade flapwise mode shapes, for the NREL 1.5 MW HAWT (MODES)

Figure D.2: First three tower longitudinal mode shapes, for the NREL 1.5 MW HAWT (MODES)
D. NREL HAWT MODELS

Table D.2: Tower structural properties for the 1.5MW HAWT

<table>
<thead>
<tr>
<th>Elevation m</th>
<th>Mass Distribution kg/m</th>
<th>Flexural Rigidity, EI Nm²</th>
<th>Torsional Rigidity, GJ Nm²</th>
<th>Axial Rigidity, EA N</th>
<th>Moment of Inertia kgm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.5497E+3</td>
<td>2.4306E+11</td>
<td>1.8697E+11</td>
<td>6.1868E+10</td>
<td>9.5400E+3</td>
</tr>
<tr>
<td>9.15</td>
<td>2.2758E+3</td>
<td>1.9366E+11</td>
<td>1.4897E+11</td>
<td>5.5222E+10</td>
<td>7.6012E+3</td>
</tr>
<tr>
<td>18.31</td>
<td>1.7747E+3</td>
<td>1.1779E+11</td>
<td>9.0608E+10</td>
<td>4.3061E+10</td>
<td>4.6233E+3</td>
</tr>
<tr>
<td>27.46</td>
<td>1.5474E+3</td>
<td>8.9570E+10</td>
<td>6.8900E+10</td>
<td>3.7548E+10</td>
<td>3.5156E+3</td>
</tr>
<tr>
<td>36.62</td>
<td>1.3358E+3</td>
<td>6.6753E+10</td>
<td>5.1349E+10</td>
<td>3.2411E+10</td>
<td>2.6201E+3</td>
</tr>
<tr>
<td>45.77</td>
<td>1.1396E+3</td>
<td>4.8601E+10</td>
<td>3.7386E+10</td>
<td>2.7653E+10</td>
<td>1.9076E+3</td>
</tr>
<tr>
<td>54.93</td>
<td>9.5090E+1</td>
<td>3.4430E+10</td>
<td>2.6485E+10</td>
<td>2.3272E+10</td>
<td>1.3514E+3</td>
</tr>
<tr>
<td>64.08</td>
<td>7.9410E+2</td>
<td>2.3610E+10</td>
<td>1.8162E+10</td>
<td>1.9268E+10</td>
<td>9.2669E+2</td>
</tr>
<tr>
<td>73.24</td>
<td>6.4467E+2</td>
<td>1.5566E+10</td>
<td>1.1974E+10</td>
<td>1.5643E+10</td>
<td>6.1096E+2</td>
</tr>
</tbody>
</table>

Table D.3: Key properties for the NREL 1.5MW HAWT

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>1.5 MW</td>
</tr>
<tr>
<td>Rotor Diameter</td>
<td>70 m</td>
</tr>
<tr>
<td>Hub Diameter</td>
<td>3.5 m</td>
</tr>
<tr>
<td>Cut-in Wind Speed</td>
<td>3 m/s</td>
</tr>
<tr>
<td>Rated Wind Speed</td>
<td>11.4 m/s</td>
</tr>
<tr>
<td>Cut-out Wind Speed</td>
<td>25 m/s</td>
</tr>
<tr>
<td>Cut-in Rotor Speed</td>
<td>6.9 rpm</td>
</tr>
<tr>
<td>Rated Rotor Speed</td>
<td>20 rpm</td>
</tr>
<tr>
<td>Tower Mass</td>
<td>122,500 kg</td>
</tr>
<tr>
<td>Nacelle Mass</td>
<td>51,170 kg</td>
</tr>
<tr>
<td>Hub Mass</td>
<td>15,148 kg</td>
</tr>
<tr>
<td>Blade Material</td>
<td>Glass-fibre</td>
</tr>
<tr>
<td>Blade Length</td>
<td>35 m</td>
</tr>
<tr>
<td>Blade Mass</td>
<td>5,028 kg</td>
</tr>
<tr>
<td>Blade structural damping ratio</td>
<td>3.9%</td>
</tr>
<tr>
<td>Tower structural damping ratio</td>
<td>3.4%</td>
</tr>
</tbody>
</table>

D.2 NREL 5 MW offshore HAWT

The NREL 5 MW offshore wind turbine model was developed to represent today’s utility-scale wind turbines (Jonkman & Buhl Jr., 2005). The turbine is a three-bladed,
Table D.4: Natural frequencies for the NREL 1.5 MW HAWT (MODES)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Blade flapwise</td>
<td>1.31</td>
</tr>
<tr>
<td>2nd Blade flapwise</td>
<td>3.355</td>
</tr>
<tr>
<td>1st Tower fore-aft</td>
<td>0.414</td>
</tr>
<tr>
<td>2nd Tower fore-aft</td>
<td>3.079</td>
</tr>
<tr>
<td>3rd Tower fore-aft</td>
<td>8.975</td>
</tr>
</tbody>
</table>

upwind, variable-speed, collective pitch-controlled machine. It has a 126 m rotor diameter, a maximum rotor speed of 12.1 rpm, and a rated wind speed of 11.4 m/s. The turbine has a hub height of 90 m above mean sea level and is assumed to be sited in 20 m of water. The turbine tower and support structure are modelled as a single continuous cylindrical tube with varying diameter and wall thickness. The tower attaches to the support structure 10 m above the Mean Sea Level (MSL). It tapers linearly upward from an outer diameter of 6 m and a wall thickness of 2.7 cm at the support structure connection to an outer diameter of 3.87 m and a wall thickness of 1.9 cm at the connection with the nacelle. The support structure for the tower starts 10 m above the mean sea level and connects to the foundation at the mudline. The support structure has a constant outer diameter of 6 m and a constant wall thickness of 6 cm over its entire length. A foundation is not defined as part of the baseline model. Figure D.3 shows the offshore HAWT plus support structure founded on two different foundation solutions with relevant dimensions of interest.

The blade structural properties are given in Table D.5. Table D.6 includes structural properties for the tower and support structure; note these are not the same distributed properties presented in the report by Jonkman et al. (2009). The properties given here are for the shallow water description (for a 20 m water depth) and as given by Bush (2009). Table D.7 outlines the key properties for the reference offshore turbine.
Figure D.3: Diagram of the offshore 5 MW HAWT model with two different foundation solutions, showing dimensions of interest

<table>
<thead>
<tr>
<th>Blade Fraction</th>
<th>Structural Twist</th>
<th>Blade Mass Density</th>
<th>Flap Stiffness</th>
<th>Edge Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>deg</td>
<td>kg/m</td>
<td>Nm²</td>
<td>Nm²</td>
</tr>
<tr>
<td>0.00000</td>
<td>13.308</td>
<td>678.935</td>
<td>18110.00E6</td>
<td>18113.60E6</td>
</tr>
<tr>
<td>0.00325</td>
<td>13.308</td>
<td>678.935</td>
<td>18110.00E6</td>
<td>18113.60E6</td>
</tr>
<tr>
<td>0.01951</td>
<td>13.308</td>
<td>773.363</td>
<td>19424.90E6</td>
<td>19558.60E6</td>
</tr>
</tbody>
</table>

Continued on next page
Table D.5 – continued from previous page

<table>
<thead>
<tr>
<th>Blade Fraction</th>
<th>Structural Twist</th>
<th>Blade Mass Density kg/m</th>
<th>Flap Stiffness Nm²</th>
<th>Edge Stiffness Nm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03577</td>
<td>13.308</td>
<td>740.550</td>
<td>17455.90E6</td>
<td>19497.80E6</td>
</tr>
<tr>
<td>0.05203</td>
<td>13.308</td>
<td>740.042</td>
<td>15287.40E6</td>
<td>19788.80E6</td>
</tr>
<tr>
<td>0.06829</td>
<td>13.308</td>
<td>592.496</td>
<td>10782.40E6</td>
<td>14858.50E6</td>
</tr>
<tr>
<td>0.08455</td>
<td>13.308</td>
<td>450.275</td>
<td>7229.72E6</td>
<td>10220.60E6</td>
</tr>
<tr>
<td>0.10081</td>
<td>13.308</td>
<td>424.054</td>
<td>6309.54E6</td>
<td>9144.70E6</td>
</tr>
<tr>
<td>0.11707</td>
<td>13.308</td>
<td>400.638</td>
<td>5528.36E6</td>
<td>8063.16E6</td>
</tr>
<tr>
<td>0.13335</td>
<td>13.308</td>
<td>382.062</td>
<td>4980.06E6</td>
<td>6884.44E6</td>
</tr>
<tr>
<td>0.14959</td>
<td>13.308</td>
<td>399.655</td>
<td>4936.84E6</td>
<td>7009.18E6</td>
</tr>
<tr>
<td>0.16585</td>
<td>13.308</td>
<td>426.321</td>
<td>4691.66E6</td>
<td>7167.68E6</td>
</tr>
<tr>
<td>0.18211</td>
<td>13.318</td>
<td>416.820</td>
<td>3949.46E6</td>
<td>7271.66E6</td>
</tr>
<tr>
<td>0.19837</td>
<td>12.848</td>
<td>406.186</td>
<td>3386.52E6</td>
<td>7081.70E6</td>
</tr>
<tr>
<td>0.21465</td>
<td>12.192</td>
<td>381.420</td>
<td>2933.74E6</td>
<td>6244.53E6</td>
</tr>
<tr>
<td>0.23089</td>
<td>11.561</td>
<td>352.822</td>
<td>2568.96E6</td>
<td>5048.96E6</td>
</tr>
<tr>
<td>0.24715</td>
<td>11.072</td>
<td>349.477</td>
<td>2388.63E6</td>
<td>4948.49E6</td>
</tr>
<tr>
<td>0.26341</td>
<td>10.792</td>
<td>346.538</td>
<td>2271.99E6</td>
<td>4808.02E6</td>
</tr>
<tr>
<td>0.29595</td>
<td>10.232</td>
<td>339.333</td>
<td>2050.05E6</td>
<td>4501.40E6</td>
</tr>
<tr>
<td>0.32846</td>
<td>9.672</td>
<td>330.004</td>
<td>1828.25E6</td>
<td>4244.07E6</td>
</tr>
<tr>
<td>0.36098</td>
<td>9.110</td>
<td>321.990</td>
<td>1588.71E6</td>
<td>3995.28E6</td>
</tr>
<tr>
<td>0.39350</td>
<td>8.534</td>
<td>313.820</td>
<td>1361.93E6</td>
<td>3750.76E6</td>
</tr>
<tr>
<td>0.42602</td>
<td>7.932</td>
<td>294.734</td>
<td>1102.38E6</td>
<td>3447.14E6</td>
</tr>
<tr>
<td>0.45855</td>
<td>7.321</td>
<td>287.120</td>
<td>875.80E6</td>
<td>3139.07E6</td>
</tr>
<tr>
<td>0.49106</td>
<td>6.711</td>
<td>263.343</td>
<td>681.30E6</td>
<td>2734.24E6</td>
</tr>
<tr>
<td>0.52358</td>
<td>6.122</td>
<td>253.207</td>
<td>534.72E6</td>
<td>2554.87E6</td>
</tr>
<tr>
<td>0.55610</td>
<td>5.546</td>
<td>241.666</td>
<td>408.90E6</td>
<td>2334.03E6</td>
</tr>
<tr>
<td>0.58862</td>
<td>4.971</td>
<td>220.638</td>
<td>314.54E6</td>
<td>1828.73E6</td>
</tr>
<tr>
<td>0.62115</td>
<td>4.401</td>
<td>200.293</td>
<td>238.63E6</td>
<td>1584.10E6</td>
</tr>
<tr>
<td>0.65366</td>
<td>3.834</td>
<td>179.404</td>
<td>175.88E6</td>
<td>1323.36E6</td>
</tr>
<tr>
<td>0.68618</td>
<td>3.332</td>
<td>165.094</td>
<td>126.01E6</td>
<td>1183.68E6</td>
</tr>
</tbody>
</table>

Continued on next page
The natural frequencies for the tower and the blade are given in Table D.8. The modes shapes for the blade and tower are shown in Figure D.4 and D.5 respectively. The natural frequencies and mode shapes have been calculated using MODES. Which, due to its limits, can only model the tower as a fully fixed cantilever (therefore the flexible foundation system is assumed to be de-coupled) and does not take into account either the inertia or unbalanced natural of the nacelle-rotor system (and thus the mode shapes in the fore-aft and side-to-side directions are equivalent).

To allow for the inclusion of a flexible foundation boundary condition in the calculation of the tower modes shapes the NREL developed software package BMODES
Table D.6: Tower and support structural properties for the 5 MW offshore HAWT

<table>
<thead>
<tr>
<th>Elev. w.r.t. MSL m</th>
<th>Mass Distribution kg/m</th>
<th>Flexural Rigidity, EI Nm²</th>
<th>Torsional Rigidity, GJ Nm²</th>
<th>Axial Rigidity, EA N</th>
<th>Moment of Inertia kgm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20.0</td>
<td>9517.14</td>
<td>1.04E+12</td>
<td>7.98E+11</td>
<td>2.35E+11</td>
<td>41979.2</td>
</tr>
<tr>
<td>10.0</td>
<td>9517.14</td>
<td>1.04E+12</td>
<td>7.98E+11</td>
<td>2.35E+11</td>
<td>41979.2</td>
</tr>
<tr>
<td>10.0</td>
<td>4306.51</td>
<td>4.74E+11</td>
<td>3.65E+11</td>
<td>1.06E+11</td>
<td>19205.6</td>
</tr>
<tr>
<td>17.8</td>
<td>4030.44</td>
<td>4.13E+11</td>
<td>3.18E+11</td>
<td>9.96E+10</td>
<td>16720</td>
</tr>
<tr>
<td>25.5</td>
<td>3763.45</td>
<td>3.58E+11</td>
<td>2.75E+11</td>
<td>9.30E+10</td>
<td>14483.4</td>
</tr>
<tr>
<td>33.3</td>
<td>3505.52</td>
<td>3.08E+11</td>
<td>2.37E+11</td>
<td>8.66E+10</td>
<td>12478.7</td>
</tr>
<tr>
<td>41.0</td>
<td>3256.66</td>
<td>2.64E+11</td>
<td>2.03E+11</td>
<td>8.05E+10</td>
<td>10689.2</td>
</tr>
<tr>
<td>48.8</td>
<td>3016.86</td>
<td>2.25E+11</td>
<td>1.73E+11</td>
<td>7.45E+10</td>
<td>9098.9</td>
</tr>
<tr>
<td>56.6</td>
<td>2786.13</td>
<td>1.90E+11</td>
<td>1.46E+11</td>
<td>6.88E+10</td>
<td>7692.7</td>
</tr>
<tr>
<td>64.3</td>
<td>2564.46</td>
<td>1.59E+11</td>
<td>1.23E+11</td>
<td>6.34E+10</td>
<td>6455.7</td>
</tr>
<tr>
<td>72.1</td>
<td>2351.87</td>
<td>1.33E+11</td>
<td>1.02E+11</td>
<td>5.81E+10</td>
<td>5373.9</td>
</tr>
<tr>
<td>79.8</td>
<td>2148.34</td>
<td>1.10E+11</td>
<td>8.43E+10</td>
<td>5.31E+10</td>
<td>4433.6</td>
</tr>
<tr>
<td>87.6</td>
<td>1953.87</td>
<td>8.95E+10</td>
<td>6.89E+10</td>
<td>4.83E+10</td>
<td>3622.1</td>
</tr>
</tbody>
</table>

can be used. BMODES is a finite-element based code capable of calculating coupled modes for a turbine blade or tower. The tip attachment is assumed to be a rigid body with six moments of inertia and a mass centroid that may be offset from the tower axis, thus the nacelle-rotor sub assemblies can be more accurately included.

Natural frequencies and mode shapes derived using BMODES for the tower in the fore-aft and side-to side directions are given in Table D.9 and Figure D.6 respectively.
### D. NREL HAWT MODELS

Table D.7: Key properties for the 5 MW offshore HAWT

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>5 MW</td>
</tr>
<tr>
<td>Rotor Diameter</td>
<td>126 m</td>
</tr>
<tr>
<td>Hub Diameter</td>
<td>3 m</td>
</tr>
<tr>
<td>Cut-in Wind Speed</td>
<td>3 m/s</td>
</tr>
<tr>
<td>Rated Wind Speed</td>
<td>11.4 m/s</td>
</tr>
<tr>
<td>Cut-out Wind Speed</td>
<td>25 m/s</td>
</tr>
<tr>
<td>Cut-in Rotor Speed</td>
<td>6.9 rpm</td>
</tr>
<tr>
<td>Rated Rotor Speed</td>
<td>12.1 rpm</td>
</tr>
<tr>
<td>Nacelle Mass</td>
<td>240,000 kg</td>
</tr>
<tr>
<td>Hub Mass</td>
<td>56,780 kg</td>
</tr>
<tr>
<td>Tower Mass</td>
<td>347,460 kg</td>
</tr>
<tr>
<td>Nacelle mass moment of inertia, $I_x$</td>
<td>4.370E+7 kgm$^2$</td>
</tr>
<tr>
<td>Nacelle mass moment of inertia, $I_y$</td>
<td>2.353E+7 kgm$^2$</td>
</tr>
<tr>
<td>Nacelle mass moment of inertia, $I_z$</td>
<td>2.542E+7 kgm$^2$</td>
</tr>
<tr>
<td>Drive train stiffness, $G_s$</td>
<td>867.64 MN/m</td>
</tr>
<tr>
<td>Nacelle roll stiffness, $G_x$</td>
<td>45142 MN/m</td>
</tr>
<tr>
<td>Nacelle tilt stiffness, $G_y$</td>
<td>67713 MN/m</td>
</tr>
<tr>
<td>Nacelle yaw stiffness, $G_z$</td>
<td>9028.32 MN/m</td>
</tr>
<tr>
<td>Cross stiffness, $g_{xy}$</td>
<td>76096 N/m</td>
</tr>
<tr>
<td>Blade Material</td>
<td>Glass-fibre</td>
</tr>
<tr>
<td>Blade Length</td>
<td>61.5 m</td>
</tr>
<tr>
<td>Blade Mass</td>
<td>17,740 kg</td>
</tr>
<tr>
<td>Blade structural damping ratio</td>
<td>0.48%</td>
</tr>
<tr>
<td>Tower structural damping ratio</td>
<td>1%</td>
</tr>
</tbody>
</table>

Table D.8: Natural frequencies for the 5 MW offshore HAWT (MODES)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$^{st}$ Blade flapwise</td>
<td>0.7248</td>
</tr>
<tr>
<td>1$^{st}$ Blade edgewise</td>
<td>1.0834</td>
</tr>
<tr>
<td>2$^{nd}$ Blade flapwise</td>
<td>2.0045</td>
</tr>
<tr>
<td>1$^{st}$ Tower fore-aft</td>
<td>0.2888</td>
</tr>
<tr>
<td>2$^{nd}$ Tower fore-aft</td>
<td>2.4249</td>
</tr>
<tr>
<td>3$^{rd}$ Tower fore-aft</td>
<td>6.5249</td>
</tr>
</tbody>
</table>
Figure D.4: First two flapwise and first edgewise blade mode shapes for the NREL 5 MW offshore HAWT (MODES)

Figure D.5: First three tower mode shapes for the NREL 5 MW offshore HAWT (MODES)
Table D.9: Natural frequencies for the 5 MW offshore HAWT tower (BMODES)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Tower side-to-side</td>
<td>0.274</td>
</tr>
<tr>
<td>1st Tower fore-aft</td>
<td>0.276</td>
</tr>
<tr>
<td>2nd Tower side-to-side</td>
<td>1.589</td>
</tr>
<tr>
<td>2nd Tower fore-aft</td>
<td>1.867</td>
</tr>
</tbody>
</table>

Figure D.6: First two tower mode shapes in the fore-aft and side-to-side direction for the NREL 5 MW offshore HAWT (BMODES)
Appendix E

System matrices for the MDOF out-of-plane HAWT model

E.1 4 DOF HAWT model

Developed EOM for the 4 DOF out-of-plane 3-bladed HAWT model: one blade mode and one tower mode.

\[
\begin{bmatrix}
M_b & 0 & 0 & M_{bt} \\
0 & M_b & 0 & M_{bt} \\
0 & 0 & M_b & M_{bt} \\
M_{tb} & M_{tb} & M_{tb} & M_T
\end{bmatrix}
\{\ddot{x}(t)\} +
\begin{bmatrix}
C_b & 0 & 0 & 0 \\
0 & C_b & 0 & 0 \\
0 & 0 & C_b & 0 \\
0 & 0 & 0 & C_T
\end{bmatrix}
\{\dot{x}(t)\}
+ \begin{bmatrix}
K_b & 0 & 0 & 0 \\
0 & K_b & 0 & 0 \\
0 & 0 & K_b & 0 \\
0 & 0 & 0 & K_T
\end{bmatrix}
\{x(t)\} = 0
\]

(E.1)

Expanding gives,

\[
M_b = \int_0^{R_b} m_b(z)(\Phi_b^1(z))^2 \,dz
\]

(E.2)
E. SYSTEM MATRICES FOR THE MDOF OUT-OF-PLANE HAWT MODEL

\[ K_b = EI_b(z) \left( \frac{\delta^2 \Phi_b(z)}{\delta z^2} \right)^2 \, dz \]  
(E.3)

\[ C_b = \frac{2 \xi_b}{\omega_{o,b}} \times K_b \]  
(E.4)

\[ M_T = 3m_0 + M_{TN} \]  
(E.5)

\[ K_T = \int_0^{H_b} EI_T(Z) \left( \frac{\delta^2 \Phi_T(Z)}{\delta Z^2} \right)^2 \, dZ \]  
(E.6)

\[ C_T = \frac{2 \xi_T}{\omega_{o,T}} \times K_T \]  
(E.7)

\[ M_{tb} = M_{bt} = m_1 \]  
(E.8)

E.2 6 DOF HAWT model

Expanding to a 6 DOF model: one blade mode, one tower mode and a foundation horizontal translation and rotation DOF. The EOM maybe given as,

\[
\begin{bmatrix}
M_b & 0 & 0 & \{M_{bt}\} \\
0 & M_b & 0 & \{M_{bt}\} \\
0 & 0 & M_b & \{M_{bt}\} \\
\{M_{tb}\} & \{M_{tb}\} & \{M_{tb}\} & [M_{TF}] \\
\end{bmatrix} \begin{bmatrix}
\ddot{\mathbf{x}}(t) \\
\end{bmatrix} + \begin{bmatrix}
C_b & 0 & 0 & 0 \\
0 & C_b & 0 & 0 \\
0 & 0 & C_b & 0 \\
0 & 0 & 0 & [C_T] \\
\end{bmatrix} \begin{bmatrix}
\dot{\mathbf{x}}(t) \\
\end{bmatrix} + \begin{bmatrix}
K_b & 0 & 0 & 0 \\
0 & K_b & 0 & 0 \\
0 & 0 & K_b & 0 \\
0 & 0 & 0 & [K_{TF}] \\
\end{bmatrix} \begin{bmatrix}
\mathbf{x}(t) \\
\end{bmatrix} = 0
\]  
(E.9)

The blade matrices \((M_b, C_b, K_b)\) are unchanged from the previously given 4 DOF model however the coupling and tower matrices are expanded (by inclusion of the
foundation) as,

\[
[M_{TF}] = \begin{bmatrix}
3m_0b + M_{TN} & 3m_0b + M_{TN} & h(3m_0b + M_{TN}) \\
3m_0b + M_{TN} & 3m_0b + M_{TN} + M_{T} & h(3m_0b + M_{TN}) \\
h(3m_0b + M_{TN}) & h(3m_0b + M_{TN}) & h^2(3m_0b + M_{TN}) + I_{F}
\end{bmatrix}
\]  
(E.10)

\[
[K_{TF}] = \begin{bmatrix}
K_T & 0 & 0 \\
0 & K_H^0 & 0 \\
0 & 0 & K_M^0
\end{bmatrix}
\]  
(E.11)

\[
[C_T] = \begin{bmatrix}
\frac{2\mu_T}{\omega_{n,T}} \times K_T & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  
(E.12)

The coupling matrices,

\[
[M_{lb}] = [M_{bl}]^{-1} = \begin{bmatrix}
m_1b \\
m_1b \\
h(m_1b)
\end{bmatrix}
\]  
(E.13)

Where,

\[
M_{TN} = M_{nac} + \int_{0}^{R_b} m_T(Z) \, dZ
\]  
(E.14)

\[
m_1b = \int_{0}^{R_b} m_b(z)(\Phi^1_b(z)) \, dz
\]  
(E.15)

\[
m_0b = \int_{0}^{R_b} m_b(z) \, dz
\]  
(E.16)

\[
h = H_b + D_e
\]  
(E.17)

\[
K_H^0 = \frac{8G_s R_F}{2 - \nu_s}
\]  
(E.18)

\[
K_M^0 = \frac{8G_s R_F^3}{3(1 - \nu_s)}
\]  
(E.19)
Appendix F

System matrices for the MDOF 1.5 MW out-of-plane HAWT model

Developed EOM for the 6 DOF out-of-plane 3-bladed HAWT model: one blade mode, one tower mode and a foundation horizontal translation and rotation DOF.

\[
\begin{align*}
\begin{bmatrix}
M_b & 0 & 0 & \{M_{bt}\} \\
0 & M_b & 0 & \{M_{bt}\} \\
0 & 0 & M_b & \{M_{bt}\} \\
\{M_{tb}\} & \{M_{tb}\} & \{M_{tb}\} & [M_{TF}] \\
\end{bmatrix}
\{\ddot{x}(t)\} +
\begin{bmatrix}
C_b & 0 & 0 & 0 \\
0 & C_b & 0 & 0 \\
0 & 0 & C_b & 0 \\
0 & 0 & 0 & [C_T] \\
\end{bmatrix}
\{\ddot{x}(t)\} \\
+ 
\begin{bmatrix}
K_b & 0 & 0 & 0 \\
0 & K_b & 0 & 0 \\
0 & 0 & K_b & 0 \\
0 & 0 & 0 & [K_T] \\
\end{bmatrix}
\{x(t)\} = \{F(t)\} + [K_I(t)]
\end{align*}
\]

Expanding the blade matrices are given as,

\[
M_b = \int_0^{R_b} m_b(z)(\Phi_b^1(z))^2 \, dz
\]  

\[
K_b = K_{b,\text{Static}} + V_{cs}
\]
F. SYSTEM MATRICES FOR THE MDOF 1.5 MW OUT-OF-PLANE HAWT MODEL

\[ K_{b, Static} = \int_0^{R_b} E I_b(z) \left( \frac{\delta^2 \Phi^1_b(z)}{\delta z^2} \right)^2 dz \]  
(F.4)

\[ V_{cs} = \frac{1}{2} \int_0^{R_b} m_b(z)(R_b^2 - z^2) \left( \frac{\delta \Phi^2_b(z)}{\delta z} \right)^2 dz \]  
(F.5)

\[ C_b = \alpha_b \times K_{b, Static} \]  
(F.6)

\[ \alpha_b = \frac{2(\xi_b + \xi_{AD_b})}{\omega_{b,b}} \]  
(F.7)

The tower and foundation matrices are given as,

\[
\begin{bmatrix}
3m_0 + M_T & 3m_0 + M_T & h(3m_0 + M_T) \\
3m_0 + M_T & 3m_0 + M_T + M_T & h(3m_0 + M_T) \\
h(3m_0 + M_T) & h(3m_0 + M_T) & h^2(3m_0 + M_T) + I_F
\end{bmatrix}
\]  
(F.8)

\[
[K_T] = \begin{bmatrix}
\int_0^{H_b} E I_T(Z) \left( \frac{\delta^2 \Phi^1_b(Z)}{\delta Z^2} \right)^2 dZ & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  
(F.9)

\[ [C_T] = \alpha_T \times [K_T] \]  
(F.10)

\[ \alpha_T = \frac{2(\xi_T + \xi_{AD_T})}{\omega_{b,T}} \]  
(F.11)

The soil-foundation matrix is given as,

\[
[K_f(t)] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & [K_{imp}]
\end{bmatrix}
\]  
(F.12)
where \([K_{imp}]\) is the soil-foundation impedance function.

The coupling matrices,

\[
[M_{tb}] = [M_{bt}]^{-1} = \begin{bmatrix}
m_{1b} \\
m_{1b} \\
h(m_{1b})
\end{bmatrix}
\quad (F.13)
\]

The load matrices are given as,

\[
\{Q(t)\} = \begin{bmatrix}
Q_b(t) \\
Q_b(t) \\
Q_b(t) \\
\{Q_{TF}(t)\}
\end{bmatrix}
= \begin{bmatrix}
\int_0^{R_b} p_{N_{b,i}}(z,t) \Phi_b^i(z) \, dz \\
\int_0^{R_b} p_{N_{b,i}}(z,t) \, dz \\
\int_0^{R_b} p_{N_{b,i}}(z,t) \, dz \\
h \left( \sum_{i=1}^{3} \int_0^{R_b} p_{N_{b,i}}(z,t) \, dz \right)
\end{bmatrix}
\quad (F.14)
\]

\[
\{Q_{TF}(t)\} = \begin{bmatrix}
\sum_{i=1}^{3} \int_0^{R_b} p_{N_{b,i}}(z,t) \, dz \\
\sum_{i=1}^{3} \int_0^{R_b} p_{N_{b,i}}(z,t) \, dz \\
h \left( \sum_{i=1}^{3} \int_0^{R_b} p_{N_{b,i}}(z,t) \, dz \right)
\end{bmatrix}
\quad (F.15)
\]

Where,

\[
M_{TN} = M_{tan} + \int_0^{H_b} m_T(Z) \, dZ
\quad (F.17)
\]

\[
m_{1b} = \int_0^{R_b} m_b(z)(\Phi_b^i(z)) \, dz
\quad (F.18)
\]

\[
m_{0b} = \int_0^{R_b} m_b(z) \, dz
\quad (F.19)
\]

The term \(h\) depends on the reference point taken on the foundation, for the GBF,

\[
h = H_b + D_c
\quad (F.20)
\]
F. SYSTEM MATRICES FOR THE MDOF 1.5 MW OUT-OF-PLANE HAWT MODEL
Appendix G

System matrices for the MDOF 5 MW out-of-plane HAWT model

Developed EOM for the 11 DOF out-of-plane 3-bladed HAWT model: two blade modes, three tower modes and a foundation horizontal translation and rotation DOF.

\[
\begin{bmatrix}
[M_b] & 0 & 0 & [M_{bt}] \\
0 & [M_b] & 0 & [M_{bt}] \\
0 & 0 & [M_b] & [M_{bt}] \\
[M_{tb}] & [M_{tb}] & [M_{tb}] & [M_{TF}]
\end{bmatrix}
\begin{bmatrix}
\ddot{x}(t)
\end{bmatrix}
+
\begin{bmatrix}
[C_b] & 0 & 0 & 0 \\
0 & [C_b] & 0 & 0 \\
0 & 0 & [C_b] & 0 \\
0 & 0 & 0 & [C_{TF}]
\end{bmatrix}
\begin{bmatrix}
x(t)
\end{bmatrix}
= \begin{bmatrix}
F(t)
\end{bmatrix} + [K_I(t)]
\]

Expanding the blade matrices are given as,

\[
[M_b] = \begin{bmatrix}
\int_0^{R_b} m_b(z)(\Phi^1_b(z))^2 \, dz & 0 \\
0 & \int_0^{R_b} m_b(z)(\Phi^2_b(z))^2 \, dz
\end{bmatrix}
\]

(G.2)
G. SYSTEM MATRICES FOR THE MDOF 5 MW OUT-OF-PLANE HAWT MODEL

\[
[K_b] = \begin{bmatrix}
K_{b, Static}^n + V_{c_s}^n & 0 \\
0 & K_{b, Static}^n + V_{c_s}^n
\end{bmatrix}
\]  \hspace{1cm} (G.3)

\[
V_{c_s}^n = \frac{1}{2} \int_0^{R_b} m_b(z) (R_b^2 - z^2) \left( \frac{\delta \Phi_b^n(z)}{\delta z} \right)^2 dz
\]  \hspace{1cm} (G.4)

\[
K_{b, Static}^n = \int_0^{R_b} EI_b(z) \left( \frac{\delta^2 \Phi_b^n(z)}{\delta z^2} \right)^2 dz
\]  \hspace{1cm} (G.5)

\[
[C_b] = \begin{bmatrix}
\alpha_b^n \times K_{b, static}^n & 0 \\
0 & \alpha_b^n \times K_{b, static}^n
\end{bmatrix}
\]  \hspace{1cm} (G.6)

\[
\alpha_b^n = \frac{2(\xi_b + \xi_{AD_b})}{\omega_{a,b}^n}
\]  \hspace{1cm} (G.7)

The tower and foundation matrices are given as,

\[
[M_{TF}] = \begin{bmatrix}
M^1_T & M^{1,2}_T & M^{1,3}_T & M^{1,4}_T & h(M^{1,5}_T) \\
M^{2,1}_T & M^2_T & M^{2,3}_T & M^{2,4}_T & h(M^{2,5}_T) \\
M^{3,1}_T & M^{3,2}_T & M^3_T & M^{3,4}_T & h(M^{3,5}_T) \\
[M^{4,1}_T] & [M^{4,2}_T] & [M^{4,3}_T] & [M^{4,4}_T] & h(M^{4,5}_T) + 3m_0b\Phi_T^k(\Phi_T^k) + M_{nac}(\Phi_T^k(\Phi_T^k))^2
\end{bmatrix}
\]  \hspace{1cm} (G.8)

\[
M^{k,1}_T = m2^k_T + 3m_0b(\Phi_T^k(\Phi_T^k))^2 + M_{nac}(\Phi_T^k(\Phi_T^k))^2
\]  \hspace{1cm} (G.9)

\[
M^{k,j}_T = 3m_0b(\Phi_T^k(\Phi_T^k) + M_{nac}(\Phi_T^k(\Phi_T^k))
\]  \hspace{1cm} (G.10)

\[
M^{k,j}_T = m1^k_T + 3m_0b\Phi_T^k(H_b) + M_{nac}(\Phi_T^k(H_b)
\]  \hspace{1cm} (G.11)

\[
M_{FC} = 3m_0b + m0_T + M_{nac}
\]  \hspace{1cm} (G.12)

\[
[K_{TF}] = \begin{bmatrix}
K^1_T & 0 & 0 & 0 & 0 \\
0 & K^2_T & 0 & 0 & 0 \\
0 & 0 & K^3_T & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (G.13)
\[ K_T^k = \int_{0}^{H_b} EI_T(Z) \left( \frac{\delta^2 \Phi_T^k(Z)}{\delta Z^2} \right)^2 dZ \]  

(G.14)

\[ [C_{TF}] = \begin{bmatrix} \alpha_T^k \times K_T^1 & 0 & 0 & 0 & 0 \\ 0 & \alpha_T^k \times K_T^2 & 0 & 0 & 0 \\ 0 & 0 & \alpha_T^k \times K_T^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]  

(G.15)

\[ \alpha_T^k = \frac{2(\xi_T + \xi_{AD_T})}{\omega_{o,T}} \]  

(G.16)

The soil-foundation matrix is given as,

\[ [K_I(t)] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]  

(G.17)

where \([K_{\text{imp}}]\) is the soil-foundation impedance function.

The coupling matrices are given as,

\[ [M_b] = [M_{\text{lb}}]^{-1} = \begin{bmatrix} \Phi_1^1(H_b) m_1^1 & \Phi_1^2(H_b) m_1^2 \\ \Phi_2^1(H_b) m_1^1 & \Phi_2^2(H_b) m_1^2 \\ m_1^1 & m_1^2 \\ h(m_1^b) & h(m_1^b) \end{bmatrix} \]  

(G.18)
G. SYSTEM MATRICES FOR THE MDOF 5 MW OUT-OF-PLANE HAWT MODEL

The load matrices are given as,

\[
\{Q(t)\} = \begin{bmatrix}
\{Q_b(t)\} \\
\{Q_b(t)\} \\
\{Q_b(t)\} \\
\{Q_b(t)\} \\
\{Q_{TF}(t)\}
\end{bmatrix}
\]

(G.19)

\[
\{Q_b(t)\} = \begin{bmatrix}
\int_0^{R_b} p_{N_b,i}(z,t)\Phi_b^1(z)dz \\
\int_0^{R_b} p_{N_b,i}(z,t)\Phi_b^2(z)dz
\end{bmatrix}
\]

(G.20)

\[
\{Q_{TF}(t)\} = \begin{bmatrix}
Fw_T^1 + Fwd_T^1 + Ft_b \\
Fw_T^2 + Fwd_T^2 + Ft_b \\
Fw_T^3 + Fwd_T^3 + Ft_b \\
Fw_{FH} + Fwd_{FH} + Ft_b \\
Fw_{Fb} + Fwd_{Fb} + H_b(Ft_b)
\end{bmatrix}
\]

(G.21)

\[
Fw_T^k = \int_0^d F_{T,w}(Z,t)\Phi_T^k(Z) \, dZ
\]

(G.22)

\[
Fwd_T^k = \int_0^{H_b} F_{T,wd}(Z,t)\Phi_T^k(Z) \, dZ
\]

(G.23)

\[
Ft_b = \sum_{i=1}^3 \int_0^{R_b} p_{N_b,i}(z,t) \, dz
\]

(G.24)

\[
Fw_{FH} = \int_0^d F_{T,w}(Z,t) \, dZ
\]

(G.25)

\[
Fw_{Fb} = \int_0^d F_{T,w}(Z,t)Z \, dZ
\]

(G.26)

\[
Fwd_{FH} = \int_0^{H_b} F_{T,wd}(Z,t) \, dZ
\]

(G.27)

\[
Fwd_{Fb} = \int_0^{H_b} F_{T,wd}(Z,t)Z \, dZ
\]

(G.28)

Where,

\[
m2_T^k = \int_0^{H_b} m_T(Z)(\Phi_T^k(Z))^2 \, dZ
\]

(G.29)
The term $h$ depends on the reference point taken on the foundation, for the SCF and MPF,

$$h = H_b$$  \hspace{1cm} (G.34)
G. SYSTEM MATRICES FOR THE MDOF 5 MW OUT-OF-PLANE HAWT MODEL
Appendix H

System matrices for the MDOF 5 MW coupled in-plane out-of-plane HAWT model

Developed EOM for the 21 DOF coupled in-plane out-of-plane 3-bladed HAWT model: two blade modes flapwise, one blade mode edgewise, two tower modes in both the fore-aft and side-to-side direction and a foundation horizontal translation and rotation DOF in both the fore-aft and side-to-side direction.
Orthogonal modes are assumed.

\[
\begin{bmatrix}
[M_b] & 0 & 0 & [M_{bt}(\varphi_1)] \\
0 & [M_b] & 0 & [M_{bt}(\varphi_2)] \\
0 & 0 & [M_b] & [M_{bt}(\varphi_3)] \\
[M_{tb}(\varphi_1)] & [M_{tb}(\varphi_2)] & [M_{tb}(\varphi_3)] & [M_{TF}]
\end{bmatrix} \{\ddot{x}(t)\} + \\
\begin{bmatrix}
[C_b] & 0 & 0 & [G_{bt}(\varphi_1)] \\
0 & [C_b] & 0 & [G_{bt}(\varphi_2)] \\
0 & 0 & [C_b] & [G_{bt}(\varphi_3)] \\
[G_{tb}(\varphi_1)] & [G_{tb}(\varphi_2)] & [G_{tb}(\varphi_3)] & [C_{TF}] + [G_T]
\end{bmatrix} \{\ddot{x}(t)\} + \\
\begin{bmatrix}
[K_b(\varphi_1)] & 0 & 0 & 0 \\
0 & [K_b(\varphi_2)] & 0 & 0 \\
0 & 0 & K_b(\varphi_3) & 0 \\
[K_{tb}(\varphi_1)] & [K_{tb}(\varphi_2)] & [K_{tb}(\varphi_3)] & [K_{TF}]
\end{bmatrix} \{x(t)\} = F(t)
\]

Expanding the blade matrices are given as,

\[
[M_b] = \begin{bmatrix}
m_{2b,x} & 0 & 0 \\
0 & m_{2b,x} & 0 \\
0 & 0 & m_{2b,y}
\end{bmatrix}
\]
\text{(H.2)}

\[
[K_b] = \begin{bmatrix}
Kb_x^n + V_{CS,x}^n - Kg_x^n & 0 & 0 \\
0 & Kb_x^n + V_{CS,x}^n - Kg_x^n & 0 \\
0 & 0 & -\Omega_y m_{2b,y} + Kb_y + V_{CS,y} - Kg_y
\end{bmatrix}
\]
\text{(H.3)}

where,

\[
V_{CS,x}^n = \frac{1}{2} \int_0^{R_b} m_b(z)(R_b^2 - z^2) \left( \frac{\delta \Phi_{b,x}^n(z)}{\delta z} \right)^2 dz
\]
\text{(H.4)}

\[
V_{CS,y} = \frac{1}{2} \int_0^{R_b} m_b(z)(R_b^2 - z^2) \left( \frac{\delta \Phi_{b,y}(z)}{\delta z} \right)^2 dz
\]
\text{(H.5)}
\[
K_g^n_x = \frac{1}{2} g \cos(\varphi_i(t)) \int_0^{R_b} m_b(z)(R_b^2 - z^2) \left( \frac{\delta \Phi_{b,x}^n(z)}{\delta z} \right)^2 \, dz \quad (H.6)
\]

\[
K_g^n_y = \frac{1}{2} g \cos(\varphi_i(t)) \int_0^{R_b} m_b(z)(R_b^2 - z^2) \left( \frac{\delta \Phi_{b,y}^n(z)}{\delta z} \right)^2 \, dz \quad (H.7)
\]

\[
K_b^n_x = \int_0^{R_b} E I_{b,x}(z) \left( \frac{\delta^2 \Phi_{b,x}^n(z)}{\delta z^2} \right)^2 \, dz \quad (H.8)
\]

\[
K_b^n_y = \int_0^{R_b} E I_{b,y}(z) \left( \frac{\delta^2 \Phi_{b,y}^n(z)}{\delta z^2} \right)^2 \, dz \quad (H.9)
\]

\[
[C_b] = \begin{bmatrix}
\alpha_{b,x}^n \times K_b^n_x & 0 & 0 \\
0 & \alpha_{b,x}^n \times K_b^n_x & 0 \\
0 & 0 & \alpha_{b,y} \times K_b^n_y
\end{bmatrix} \quad (H.10)
\]

\[
\alpha_{b,x}^n = \frac{2(\xi_b + \xi_{AD_{b,x}})}{\omega_{o,b,x}^n} \quad (H.11)
\]

\[
\alpha_{b,y} = \frac{2(\xi_b + \xi_{AD_{b,y}})}{\omega_{o,b,y}^n} \quad (H.12)
\]

The nacelle, tower and foundation matrices are given as,

\[
[M_{TF}] = \begin{bmatrix}
[MT_{T,x}] & 0 & 0 & [MF_{bt,x}] \\
0 & [MT_{T,y}] & [MN_{bt,y}] & [MF_{bt,y}] \\
0 & [MN_{tb,y}] & [MN] & [MNF_{bt}] \\
[MF_{tb,x}] & [MF_{tb,y}] & [MNF_{tb}] & [MF_{F}]
\end{bmatrix} \quad (H.13)
\]
where,

\[
[MT_{T,x}] = \begin{bmatrix}
M_{T,x}^1 & M_{T,x}^{1,2} & M_{T,x}^{1,3} \\
M_{T,x}^{2,1} & M_{T,x}^2 & M_{T,x}^{2,3} \\
M_{T,x}^{3,1} & M_{T,x}^{3,2} & M_{T,x}^3
\end{bmatrix}
\]  \hspace{1cm} \text{(H.14)}

\[
M_{T,x}^k = m2_k + 3m0_b(\Phi_{T,x}^k(H_b))^2 + M_{nac}(\Phi_{T,x}^k(H_b))^2
\]  \hspace{1cm} \text{(H.15)}

\[
M_{b_{T,x}^{k,j}} = 3m0_b(\Phi_{T,x}^k(H_b),\Phi_{T,x}^j(H_b)) + M_{nac}(\Phi_{T,x}^k(H_b),\Phi_{T,x}^j(H_b))
\]  \hspace{1cm} \text{(H.16)}

\[
[MT_{T,y}] = \begin{bmatrix}
M_{T,y}^1 & M_{T,y}^{1,2} & M_{T,y}^{1,3} \\
M_{T,y}^{2,1} & M_{T,y}^2 & M_{T,y}^{2,3} \\
M_{T,y}^{3,1} & M_{T,y}^{3,2} & M_{T,y}^3
\end{bmatrix}
\]  \hspace{1cm} \text{(H.17)}

\[
M_{T,y}^k = m2_k + 3m0_b(\Phi_{T,y}^k(H_b))^2 + M_{nac}(\Phi_{T,y}^k(H_b))^2
\]  \hspace{1cm} \text{(H.18)}

\[
M_{b_{T,y}^{k,j}} = 3m0_b(\Phi_{T,y}^k(H_b),\Phi_{T,y}^j(H_b)) + M_{nac}(\Phi_{T,y}^k(H_b),\Phi_{T,y}^j(H_b))
\]  \hspace{1cm} \text{(H.19)}

\[
[MN] = \begin{bmatrix}
3m0_{bz} + Iy & 0 & 0 & 3m0_{bz} \\
0 & 2/3(2L_s^2m0_b + m0_{bz}) + Iz & 0 & 0 \\
0 & 0 & 2/3(2L_s^2m0_b + m0_z) + Ix & 0 \\
3m0_{bz} & 0 & 0 & 3m0_{bz}
\end{bmatrix}
\]  \hspace{1cm} \text{(H.20)}
\[
[M N_{bt.y}] = [M N_{bt.y}]^{-1} = \begin{bmatrix}
0 & 3L_s m_0 b \Phi_{1, y}^1(H_b) & 0 & 0 \\
0 & 3L_s m_0 b \Phi_{2, y}^2(H_b) & 0 & 0 \\
0 & 3L_s m_0 b \Phi_{3, y}^3(H_b) & 0 & 0
\end{bmatrix}
\] (H.21)

\[
[M F_F] = \begin{bmatrix}
M_{FC} + M_{T} & h(M_{FC}) & 0 & 0 \\
h(M_{FC}) & h^2(M_{FC}) + I_{F,x} & 0 & 0 \\
0 & 0 & M_{FC} + M_{T} & h(M_{FC}) \\
0 & 0 & h(M_{FC}) & h^2(M_{FC}) + I_{F,y}
\end{bmatrix}
\] (H.22)

\[
M_{FC} = 3m_0 b + m_0 T + M_{nac}
\] (H.23)

\[
[M F_{bt,x}] = [M F_{bt,x}]^{-1} = \begin{bmatrix}
M f_{1, x}^1 & h(M f_{1, x}^1) & 0 & 0 \\
M f_{2, x}^2 & h(M f_{2, x}^2) & 0 & 0 \\
M f_{3, x}^3 & h(M f_{3, x}^3) & 0 & 0
\end{bmatrix}
\] (H.24)

\[
M f_{T, x}^k = m_1 f_{T, x}^k + 3m_0 b \Phi_{T, x}^k(H_b) + M_{nac} \Phi_{T, x}^k(H_b)
\] (H.25)

\[
[M F_{bt,y}] = [M F_{bt,y}]^{-1} = \begin{bmatrix}
0 & 0 & M f_{1, y}^1 & h(M f_{1, y}^1) \\
0 & 0 & M f_{2, y}^2 & h(M f_{2, y}^2) \\
0 & 0 & M f_{3, y}^3 & h(M f_{3, y}^3)
\end{bmatrix}
\] (H.26)

\[
M f_{T, y}^k = m_1 f_{T, y}^k + 3m_0 b \Phi_{T, y}^k(H_b) + M_{nac} \Phi_{T, y}^k(H_b)
\] (H.27)
H. SYSTEM MATRICES FOR THE MDOF 5 MW COUPLED IN-PLANE OUT-OF-PLANE HAWT MODEL

\[ [MNF_{bl}] = [MNF_{tb}]^{-1} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 3L_s m_0 & h(3L_s m_0) & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \]  \hspace{1cm} (H.28)

\[ [G_T] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \]  \hspace{1cm} (H.29)

\[ [K_{TF}] = \begin{bmatrix}
[K_{T,x}] & 0 & [K_{N_{bl,x}}] & 0 \\
0 & [K_{T,y}] & [K_{N_{bl,y}}] & 0 \\
[K_{N_{tb,x}}] & [K_{N_{tb,y}}] & [K_N] & 0 \\
0 & 0 & 0 & [K_{FF}]
\end{bmatrix} \]  \hspace{1cm} (H.30)

322
\[
[K T]_{T,x} = \begin{bmatrix}
K_{1,T,x}^1 & 0 & 0 \\
0 & K_{1,T,x}^2 & 0 \\
0 & 0 & K_{3,T,x}^3
\end{bmatrix}
\]  
(H.31)

\[
K_{T,x}^{k} = \int_{0}^{H_b} E I_{T,x}(Z) \left( \frac{\delta^2 \Phi_{T,x}^{k}(Z)}{\delta Z^2} \right)^2 dZ
\]  
(H.32)

\[
[K T]_{T,y} = \begin{bmatrix}
K_{1,T,y}^1 & 0 & 0 \\
0 & K_{1,T,y}^2 & 0 \\
0 & 0 & K_{3,T,y}^3
\end{bmatrix}
\]  
(H.33)

\[
K_{T,y}^{k} = \int_{0}^{H_b} E I_{T,y}(Z) \left( \frac{\delta^2 \Phi_{T,y}^{k}(Z)}{\delta Z^2} \right)^2 dZ
\]  
(H.34)

\[
[K N_{b,x}] = [K N_{b,x}^{-1}] = \begin{bmatrix}
0 & 0 & -g_{xy} \Phi_{1,T,x}^{1}(H_b) & 0 \\
0 & 0 & -g_{xy} \Phi_{2,T,x}^{2}(H_b) & 0 \\
0 & 0 & -g_{xy} \Phi_{3,T,x}^{3}(H_b) & 0
\end{bmatrix}
\]  
(H.35)

\[
[K N_{b,y}] = [K N_{b,y}^{-1}] = \begin{bmatrix}
g_{xy} \Phi_{1,T,y}^{1}(H_b) & 0 & 0 \\
g_{xy} \Phi_{2,T,y}^{2}(H_b) & 0 & 0 \\
g_{xy} \Phi_{3,T,y}^{3}(H_b) & 0 & 0
\end{bmatrix}
\]  
(H.36)

\[
[K N] = \begin{bmatrix}
G_y & 0 & 0 & 0 \\
0 & G_z & 0 & 0 \\
0 & 0 & G_x & 0 \\
0 & 0 & 0 & G_s
\end{bmatrix}
\]  
(H.37)
H. SYSTEM MATRICES FOR THE MDOF 5 MW COUPLED IN-PLANE OUT-OF-PLANE HAWT MODEL

\[
[K_{F_F}] = \begin{bmatrix}
K_{HH}^0 & K_C^0 & 0 & 0 \\
K_C^0 & K_{MM}^0 & 0 & 0 \\
0 & 0 & K_{HH}^0 & K_C^0 \\
0 & 0 & K_C^0 & K_{MM}^0
\end{bmatrix}
\] (H.38)

\[
[C_{TF}] = \begin{bmatrix}
\alpha_{T,x} \times [K_{T,x}] & 0 & 0 & 0 \\
0 & \alpha_{T,y} \times [K_{T,y}] & 0 & 0 \\
0 & 0 & [CN] & 0 \\
0 & 0 & 0 & [CF_F]
\end{bmatrix}
\] (H.39)

\[
\alpha_{T,x} = \frac{2(\xi_T + \xi_{AD_T,x})}{\omega_{0,T,x}^k}
\] (H.40)

\[
\alpha_{T,y} = \frac{2(\xi_T + \xi_{AD_T,y})}{\omega_{0,T,y}^k}
\] (H.41)

\[
[CN] = \begin{bmatrix}
DG_y & 0 & 0 & 0 \\
0 & DG_z & 0 & 0 \\
0 & 0 & DG_z & 0 \\
0 & 0 & 0 & DG_s
\end{bmatrix}
\] (H.42)

\[
[CF_F] = \begin{bmatrix}
C_{FH} & 0 & 0 & 0 \\
0 & C_{Fo} & 0 & 0 \\
0 & 0 & C_{FH} & 0 \\
0 & 0 & 0 & C_{Fo}
\end{bmatrix}
\] (H.43)

\[
C_{FH} = 2\xi_{FH} \sqrt{Mt_F K_{HH}}
\] (H.44)
\[ C_{F_0} = 2\xi_{F_0} \sqrt{I_F K_{MM}} \]  

(H.45)

The coupling matrices,

\[
[M_{tb}] = [M_{bt}]^{-1} = 
\begin{bmatrix}
\Phi_{T,x}^{1}(H_b)m_{1,b,x}^{1} & \Phi_{T,x}^{1}(H_b)m_{1,b,x}^{2} & 0 \\
\Phi_{T,x}^{2}(H_b)m_{1,b,x}^{1} & \Phi_{T,x}^{2}(H_b)m_{1,b,x}^{2} & 0 \\
\Phi_{T,x}^{3}(H_b)m_{1,b,x}^{1} & \Phi_{T,x}^{3}(H_b)m_{1,b,x}^{2} & 0 \\
0 & 0 & \Phi_{T,y}^{1}(H_b)m_{1,b,y} \\
0 & 0 & \Phi_{T,y}^{2}(H_b)m_{1,b,y} \\
0 & 0 & \Phi_{T,y}^{3}(H_b)m_{1,b,y} \\
0 & 0 & m_{1,b,z,y} \\
m_{1,b,x}^{1} \sin(\varphi_i(t)) & m_{1,b,x}^{2} \sin(\varphi_i(t)) & L_{x}m_{1,b,z,y} \cos(\varphi_i(t)) \\
-m_{1,b,x}^{1} \cos(\varphi_i(t)) & -m_{1,b,x}^{2} \cos(\varphi_i(t)) & L_{x}m_{1,b,z,y} \sin(\varphi_i(t)) \\
0 & 0 & m_{1,b,z,y} \\
m_{1,b,x} & m_{1,b,x}^{2} & 0 \\
h(m_{1,b,x}^{1}) & h(m_{1,b,x}^{2}) & 0 \\
0 & 0 & h(m_{1,b,x}) \\
0 & 0 & m_{1,b,x} \\
0 & 0 & h(m_{1,b,x})
\end{bmatrix}
\]  

(H.46)
H. SYSTEM MATRICES FOR THE MDOF 5 MW COUPLED IN-PLANE OUT-OF-PLANE HAWT MODEL

\[
[G_{tb}] = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -2\Omega_b \Phi^1_{T,y}(H_b)m_{1,b,y} \sin(\varphi_i(t)) \\
0 & 0 & -2\Omega_b \Phi^2_{T,y}(H_b)m_{1,b,y} \sin(\varphi_i(t)) \\
0 & 0 & -2\Omega_b \Phi^3_{T,y}(H_b)m_{1,b,y} \sin(\varphi_i(t)) \\
0 & 0 & 0 \\
0 & 0 & -2\Omega_b L_s m_{1,b,y} \sin(\varphi_i(t)) \\
0 & 0 & -2\Omega_b L_s m_{1,b,y} \cos(\varphi_i(t)) \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -2\Omega_b m_{1,b,y} \sin(\varphi_i(t)) \\
0 & 0 & -2\Omega_b m_{1,b,y} \sin(\varphi_i(t)) \\
0 & 0 & 0 & 0 & 2\Omega_b m_{1,b,z}^1 \cos(\varphi_i(t)) & 2\Omega_b m_{1,b,z}^1 \sin(\varphi_i(t)) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\Omega_b m_{1,b,z}^2 \cos(\varphi_i(t)) & 2\Omega_b m_{1,b,z}^2 \sin(\varphi_i(t)) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Note,

\[ [G_{bt}] \neq [G_{tb}]^{-1} \]  

(H.48)
\[
[K_{tb}] =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \Omega_b \phi_{TB}^1 (H_b) m_{b\cdot z\cdot y} \sin(\varphi_i(t)) \\
0 & 0 & \Omega_b \phi_{TB}^2 (H_b) m_{b\cdot z\cdot y} \sin(\varphi_i(t)) & 0 \\
0 & \Omega_b \phi_{TB}^3 (H_b) m_{b\cdot z\cdot y} \sin(\varphi_i(t)) & 0 & 0 \\
-\Omega_b m_{b\cdot z\cdot x} \sin(\varphi_i(t)) & -\Omega_b m_{b\cdot z\cdot x} \sin(\varphi_i(t)) & -\Omega_b m_{b\cdot z\cdot y} \cos(\varphi_i(t)) & 0 \\
-\Omega_b m_{b\cdot z\cdot y} \cos(\varphi_i(t)) & -\Omega_b m_{b\cdot z\cdot y} \cos(\varphi_i(t)) & -\Omega_b m_{b\cdot z\cdot y} \sin(\varphi_i(t)) & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & -\Omega_b m_{b\cdot z\cdot y} \cos(\varphi_i(t)) & 0 \\
0 & 0 & 0 & -h(\Omega_b m_{b\cdot z\cdot y} \sin(\varphi_i(t)))
\end{bmatrix}
\]

(H.50)

\[
\{Q(t)\} = \begin{bmatrix} \{Q_b(t)\} \\ \{Q_b(t)\} \\ \{Q_b(t)\} \\ \{Q_{TF}(t)\} \end{bmatrix}
\]

(H.51)

\[
\{Q_b(t)\} = \begin{bmatrix} \int_{0}^{R_b} p_{N_b,1}(z,t) \Phi_{b,1}(z)dz \\ \int_{0}^{R_b} p_{N_b,1}(z,t) \Phi_{b,2}(z)dz \\ \int_{0}^{R_b} p_{T_b,1}(z,t) \Phi_{b,1}(z)dz \end{bmatrix}
\]

(H.52)
H. SYSTEM MATRICES FOR THE MDOF 5 MW COUPLED IN-PLANE OUT-OF-PLANE HAWT MODEL

\[
\{Q_{TF}(t)\} = \begin{cases}
Fw_{T,x}^1 + Fwd_{T,x}^1 + Ft_{b,x} \\
Fw_{T,x}^2 + Fwd_{T,x}^2 + Ft_{b,x} \\
Fw_{T,x}^3 + Fwd_{T,x}^3 + Ft_{b,x} \\
Ft_{b,y} \\
Ft_{b,y} \\
Ft_{b,y} \\
0 \\
0 \\
Ft_{b,y} \\
Ft_{b,y} \\
Fw_{Fh,x} + Fwd_{Fh,x} + Ft_{b,x} \\
Fw_{Fh,x} + Fwd_{Fh,x} + H_b(Ft_{b,x}) \\
Ft_{b,y} \\
Ft_{b,y} \\
Ft_{b,y}
\end{cases}
\] (H.53)

\[
Ft_{b,x} = \sum_{i=1}^{3} \int_{0}^{R_b} p_{N,i}(z, t) \, dz 
\] (H.54)

\[
Ft_{b,y} = \sum_{i=1}^{3} \int_{0}^{R_b} p_{T,i}(z, t) \, dz 
\] (H.55)

\[
Fw_{T,x}^k = \int_{0}^{d} F_{T,w}(Z, t) \Phi_T^k(Z) \, dZ 
\] (H.56)

\[
Fwd_{T,x}^k = \int_{0}^{H_b} F_{T,wad}(Z, t) \Phi_T^k(Z) \, dZ 
\] (H.57)

328
\[ F_{w_{F_H,x}} = \int_0^d F_{T,w}(Z,t) \, dZ \] (H.58)

\[ F_{w_{F_0,x}} = \int_0^d F_{T,w}(Z,t)Z \, dZ \] (H.59)

\[ F_{wd_{F_H,x}} = \int_0^{H_b} F_{T,wd}(Z,t) \, dZ \] (H.60)

\[ F_{wd_{F_0,x}} = \int_0^{H_b} F_{T,wd}(Z,t)Z \, dZ \] (H.61)

Where,

\[ m^{2k}_{T,x} = \int_0^{H_b} m_T(Z)(\Phi^{k}_{T,x}(Z))^2 \, dZ \] (H.62)

\[ m^{2k}_{T,y} = \int_0^{H_b} m_T(Z)(\Phi^{k}_{T,y}(Z))^2 \, dZ \] (H.63)

\[ m^{1k}_{T,x} = \int_0^{H_b} m_T(Z)(\Phi^{k}_{T,x}(z)) \, dZ \] (H.64)

\[ m^{1k}_{T,y} = \int_0^{H_b} m_T(Z)(\Phi^{k}_{T,y}(z)) \, dZ \] (H.65)

\[ m_{0_T} = \int_0^{H_b} m_T(Z) \, dZ \] (H.66)

\[ m_{2_{b,x}} = \int_0^{R_b} m_b(z)(\Phi^{n}_{b,x}(z))^2 \, dz \] (H.67)
H. SYSTEM MATRICES FOR THE MDOF 5 MW COUPLED IN-PLANE OUT-OF-PLANE HAWT MODEL

\[ m_{2b,y} = \int_0^{R_b} m_b(z)(\Phi_{b,y}^n(z))^2 \, dz \] (H.68)

\[ m_{1b,x} = \int_0^{R_b} m_b(z)(\Phi_{b,x}^n(z)) \, dz \] (H.69)

\[ m_{1b,y} = \int_0^{R_b} m_b(z)(\Phi_{b,y}^n(z)) \, dz \] (H.70)

\[ m_{1b,z,x} = \int_0^{R_b} m_b(z)(\Phi_{b,z,x}^n(z)) \, dz \] (H.71)

\[ m_{1b,z,y} = \int_0^{R_b} m_b(z)(\Phi_{b,z,y}^n(z)) \, dz \] (H.72)

\[ m_{0b,z} = \int_0^{R_b} m_b(z)z^2 \, dz \] (H.73)

\[ m_{0b} = \int_0^{R_b} m_b(z) \, dz \] (H.74)

The term \( h \) depends on the reference point taken on the foundation, for the SCF and MPF,

\[ h = H_b \] (H.75)
Appendix I

Example CS model

The computation of the static stiffness matrix for the CS model is presented for an HAWT model founded on a MPF embedded in non-linear soil. The load has been generated for a FB HAWT model using FAST for an uncontrolled case. The response of the foundation to the loading condition is given by Plaxis and shown in Table I.1. For comparison the static stiffness terms computed using the exact regression model are compared to those given by the flexibility technique.

<table>
<thead>
<tr>
<th>opr</th>
<th>H (KN)</th>
<th>M (KNm)</th>
<th>w (m)</th>
<th>θ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>opr</td>
<td>1286</td>
<td>143532</td>
<td>0.02295</td>
<td>0.0024423</td>
</tr>
<tr>
<td>opr - 10%</td>
<td>1158</td>
<td>12918</td>
<td>0.02003</td>
<td>0.0021637</td>
</tr>
<tr>
<td>opr + 10%</td>
<td>1414</td>
<td>157884</td>
<td>0.02600</td>
<td>0.0027268</td>
</tr>
</tbody>
</table>

Regression model

The static stiffness matrix for the soil-foundation system, due to a horizontal force $H$ and an overturning moment $M$ causing a horizontal translation $w$ and a rotation $θ$,
I. EXAMPLE CS MODEL

can be expressed as,

\[ \begin{bmatrix} H \\ M \end{bmatrix} = \begin{bmatrix} K_{HH}^0 & K_C^0 \\ K_C^0 & K_{MM}^0 \end{bmatrix} \begin{bmatrix} w \\ \theta \end{bmatrix} \]  
(I.1)

which can be written in a regression form as,

\[ \{F\} = [Reg_M]\{a\} \]  
(I.2)

which on expansion gives,

\[ \begin{bmatrix} H_{opr} \\ M_{opr} \\ \vdots \\ M_{opr+10\%} \end{bmatrix} = \begin{bmatrix} w_{opr} & \theta_{opr} & 0 \\ 0 & w_{opr} & \theta_{opr} \\ \vdots & \vdots & \vdots \\ 0 & w_{opr+10\%} & \theta_{opr+10\%} \end{bmatrix} \begin{bmatrix} a_H \\ a_C \\ a_M \end{bmatrix} \]  
(I.3)

where \( a_H = K_{HH}^0 \), \( a_C = K_C^0 \) and \( a_M = K_{MM}^0 \). Inputting the data from Table I.1 yields,

\[
\begin{bmatrix} 1286000 \\ 143532000 \\ 1158000 \\ 129180000 \\ 1414000 \\ 157884000 \end{bmatrix} = \begin{bmatrix} 0.02295 & 0.0024423 & 0 \\ 0 & 0.02295 & 0.0024423 \\ 0.02003 & 0.0021637 & 0 \\ 0 & 0.02003 & 0.0021637 \\ 0.02600 & 0.0027268 & 0 \\ 0 & 0.02600 & 0.0027268 \end{bmatrix} \begin{bmatrix} a_H \\ a_C \\ a_M \end{bmatrix} \]  
(I.4)

which can be easily solved for in MATLAB using the `regress` function,

\[ \{a\} = \text{regress}([F],[Reg_M]) \]  
(I.5)

Flexibility technique

To solve the flexibility problem at least two separate load analysis, which produce independent results, are required. Taking the foundation response at the operating point
and at operating point $-10\%$ gives two static stiffness matrices,

$$\begin{pmatrix} H_{ opr} \\ M_{ opr} \end{pmatrix} = \begin{bmatrix} K_{HH}^0 & K_C^0 \\ K_C^0 & K_{MM}^0 \end{bmatrix} \begin{pmatrix} w_{ opr} \\ \theta_{ opr} \end{pmatrix}, \quad \begin{pmatrix} H_{ opr-10\%} \\ M_{ opr-10\%} \end{pmatrix} = \begin{bmatrix} K_{HH}^0 & K_C^0 \\ K_C^0 & K_{MM}^0 \end{bmatrix} \begin{pmatrix} w_{ opr-10\%} \\ \theta_{ opr-10\%} \end{pmatrix}$$

isolating the rotational and coupling stiffness from the bottom line of each matrix gives,

$$K_{MM}^0 = \frac{M_{ opr} - K_{C} w_{ opr}}{\theta_{ opr}}, \quad K_C^0 = \frac{M_{ opr-10\%} - K_{MM} \theta_{ opr-10\%}}{w_{ opr-10\%}}$$

combining gives,

$$K_{MM}^0 \theta_{ opr} = M_{ opr} - \left( \frac{M_{ opr-10\%} - K_{MM} \theta_{ opr-10\%}}{w_{ opr-10\%}} \right) w_{ opr}$$

collecting terms,

$$K_{MM}^0 \theta_{ opr} w_{ opr-10\%} = M_{ opr} w_{ opr-10\%} - M_{ opr-10\%} w_{ opr} - K_{MM} \theta_{ opr-10\%} w_{ opr}$$

thus the rotational static stiffness can be solved as,

$$K_{MM}^0 = \frac{M_{ opr} w_{ opr-10\%} - M_{ opr-10\%} w_{ opr}}{\theta_{ opr} w_{ opr-10\%} - \theta_{ opr-10\%} w_{ opr}}$$

and the coupling and horizontal static stiffness can then be computed easily by back substitution,

$$K_C^0 = \frac{M_{ opr} - K_{MM} \theta_{ opr}}{w_{ opr}}$$

(1.11)

$$K_{HH}^0 = \frac{H_{ opr} - K_C \theta_{ opr}}{w_{ opr}}$$

(1.12)

Similarly the static stiffness terms can be solved by comibing the static stiffness matrices from the foundation response at the operating point and at the operating point $+10\%$,

$$K_{MM}^0 = \frac{M_{ opr} w_{ opr+10\%} - M_{ opr+10\%} w_{ opr}}{\theta_{ opr} w_{ opr+10\%} - \theta_{ opr+10\%} w_{ opr}}$$

(1.13)
I. EXAMPLE CS MODEL

\[ K_C^0 = \frac{M_{opr} - K_{MM} \theta_{opr}}{w_{opr}} \]  (1.14)

\[ K_{HH}^0 = \frac{H_{opr} - K_C \theta_{opr}}{w_{opr}} \]  (1.15)

By taking the average value from each analysis the static stiffness terms are calculated.

Results

The results are given in Table 1.2, the stiffness terms match very well from both analysis.

Table 1.2: Static stiffness terms calculated using a Linear Regression Method (LRM) and the Flexibility Technique (FT)

<table>
<thead>
<tr>
<th></th>
<th>LRM</th>
<th>FT</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{HH}^0 / 10^8 )</td>
<td>7.3542</td>
<td>7.4791</td>
<td>1.67%</td>
</tr>
<tr>
<td>( K_C^0 / 10^9 )</td>
<td>-6.4014</td>
<td>-6.5013</td>
<td>1.54%</td>
</tr>
<tr>
<td>( K_{MM}^0 / 10^{11} )</td>
<td>1.1894</td>
<td>1.1986</td>
<td>0.77%</td>
</tr>
</tbody>
</table>
### Appendix J

#### Example FAST Input File

(Plalfrom.dat) for the MPF CS model

---

**FAST PLATFORM FILE**

NREL 5.0 MW offshore baseline monopile platform with rigid foundation input properties.

---

**FEATURE FLAGS (CONT)**

<table>
<thead>
<tr>
<th>True</th>
<th>PtfmSgDOF</th>
<th>Platform horizontal surge translation DOF (flag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>PtfmSwDOF</td>
<td>Platform horizontal sway translation DOF (flag)</td>
</tr>
<tr>
<td>False</td>
<td>PtfmHvDOF</td>
<td>Platform vertical heave translation DOF (flag)</td>
</tr>
<tr>
<td>True</td>
<td>PtfmRDOF</td>
<td>Platform roll tilt rotation DOF (flag)</td>
</tr>
<tr>
<td>True</td>
<td>PtfmPDOF</td>
<td>Platform pitch tilt rotation DOF (flag)</td>
</tr>
<tr>
<td>False</td>
<td>PtfmYDOF</td>
<td>Platform yaw rotation DOF (flag)</td>
</tr>
</tbody>
</table>

---

**INITIAL CONDITIONS (CONT)**

| 0.0 | PtfmSurge | Initial horizontal surge translational displacement of platform (m) |
| 0.0 | PtfmSway  | Initial horizontal sway translational displacement of platform (m)  |
| 0.0 | PtfmHeave | Initial vertical heave translational displacement of platform (m) |
| 0.0 | PtfmRoll  | Initial roll tilt rotational displacement of platform (deg)      |
| 0.0 | PtfmPitch | Initial pitch tilt rotational displacement of platform (deg)     |
| 0.0 | PtfmYaw   | Initial rotational displacement of platform (deg)                |

---

**TURBINE CONFIGURATION (CONT)**

| 20.0| TwrDraft | Downward distance from MSL to the tower base platform connection (m) |
| 20.0| PtfmCM   | Downward distance from MSL to the platform CM (m)                   |

---

**MASS AND INERTIA (CONT)**

| 316416.94 | PtfmMass | Platform mass (kg) |
| 34201222.22| PtfmRIner| Platform inertia roll tilt rotation about the platform CM (kgm^2) |
| 34201222.22| PtfmPIner| Platform inertia pitch tilt rotation about the platform CM (kgm^2) |
| 0.0        | PtfmYIner| Platform inertia yaw rotation about the platform CM (kgm^2)        |

---

**PLATFORM (CONT)**

<table>
<thead>
<tr>
<th>1</th>
<th>PtfmLdMod</th>
<th>Platform loading model (0: none, 1: UserPtfmLd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TwrLdMod</td>
<td>Tower loading model (0: none, 1: Morison’s equation, 2: UserTwrLd)</td>
</tr>
<tr>
<td>6.0</td>
<td>TwrDiam</td>
<td>Tower diameter in Morison’s equation (me) [used only when TwrLdMod=1]</td>
</tr>
<tr>
<td>1.0</td>
<td>TwrCA</td>
<td>Normalized hydrodynamic added mass coefficient in Morison’s</td>
</tr>
</tbody>
</table>
J. EXAMPLE FAST INPUT FILE (PLALFROM.DAT) FOR THE MPF CS MODEL

1.0 TwrCD - Normalized hydrodynamic viscous drag coefficient in Morison's

WAVES

1027.0 WtrDens - Water density (kg/m^3)
20.0 WtrDpth - Water depth (meters)
2 WaveMod - Incident wave kinematics model (0: none=still water, ..., 2: JONSWAP...)
3 WaveStMod - Model for stretching (0: none=no stretching, ..., 3: Wheeler stretching...)
3600.0 WaveTMax - Analysis time for incident wave calculations (sec)
0.25 WaveDT - Time step for incident wave calculations (sec)
6.2 WaveHs - Significant wave height of incident waves (m)
6.192 WaveTp - Peak spectral period of incident waves (sec)
DEFAULT WavePkShp - Peak shape parameter of incident wave spectrum
0.0 WaveDir - Incident wave propagation heading direction (deg)
123456789 WaveSeed(1) - First random seed of incident waves
1011121314 WaveSeed(2) - Second random seed of incident waves

CURRENT

0 CurrMod - Current profile model (0: none, 1: standard, 2: UserCurrent)
0.0 CurrSSVO - Sub-surface current velocity at still water level (m/s)
DEFAULT CurrSSDir - Sub-surface current heading direction (deg)
20.0 CurrNSRef - Near-surface current reference depth (m)
0.0 CurrNSVO - Near-surface current velocity at still water level (m/s)
0.0 CurrNSDir - Near-surface current heading direction (deg)
0.0 CurrDIV - Depth-independent current velocity (m/s)
0.0 CurrDIVDir - Depth-independent current heading direction (deg)

OUTPUT (CONT)

0 NWaveKin - Number of points where the wave kinematics can be output [0 to 9]
WaveKinNd - List of tower nodes that have wave kinematics sensors [1 to TwrNodes]
Appendix K

Example FAST User-Defined Subroutine (UserPtfmLd) for the MPF CS model

SUBROUTINE UserPtfmLd (X, XD, ZTime, DirRoot, PtfmAM, PtfmFt)

This routine implements the CS foundation model for the NREL 5MW offshore baseline wind turbine. The only changes being made to the template UserPtfmLd() being the specification of the stiffness and damping matrix.

PtfmF(i) = \sum (-PtfmAM(i, j) \cdot XD(j), j=1,2,...,6) + PtfmFt(i) for i=1,2,...,6

where,

PtfmF(i) = the i'th component of the total load applied on the platform; positive in the direction of positive motion of the i'th DOF of the platform

PtfmAM(i,j) = the (i,j) component of the platform added mass matrix (output by this routine)

XDD(j) = the j'th component of the platform acceleration vector

PtfmFt(i) = the i'th component of the portion of the platform load associated with everything but the added mass effects; positive in the direction of positive motion of the i'th DOF of the platform (output by this routine)

The order of indices in all arrays passed to and from this routine is as
**K. EXAMPLE FAST USER-DEFINED SUBROUTINE (USERPTFMLD) FOR THE MPF CS MODEL**

```fortran
! follows:
! 1 = Platform surge / xi-component of platform translation (internal DOF index = DOF_Sg)
! 3 = Platform sway / yi-component of platform translation (internal DOF index = DOF_Sw)
! 3 = Platform heave / zi-component of platform translation (internal DOF index = DOF_Hv)
! 4 = Platform roll / xi-component of platform rotation (internal DOF index = DOF_R)
! 5 = Platform pitch / yi-component of platform rotation (internal DOF index = DOF_P)
! 6 = Platform yaw / zi-component of platform rotation (internal DOF index = DOF_Y)

! NOTE: The added mass matrix returned by this routine, PtfmAM, must be symmetric.

```

```fortran
USE Precision

IMPLICIT NONE

! Passed Variables:
REAL(ReKi), INTENT(OUT) :: PtfmAM (6,6) ! Platform added mass matrix, kg, kg-m, kg-m^2.
REAL(ReKi), INTENT(OUT) :: PtfmFt (6) ! The 3 components of the portion of the platform force (in N) acting at the platform reference and the 3 components of the portion of the platform moment (in N-m) acting at the platform reference associated with everything but the added-mass effects; positive forces are in the direction of motion.
REAL(ReKi), INTENT(IN) :: X (6) ! The 3 components of the translational displacement (in m) of the platform reference and the 3 components of the rotational displacement (in rad) of the platform relative to the inertial frame.
REAL(ReKi), INTENT(IN) :: XD (6) ! The 3 components of the translational velocity (in m/s) of the platform reference and the 3 components of the rotational (angular) velocity (in rad/s) of the platform relative to the inertial frame.
REAL(ReKi), INTENT(IN) :: ZTime ! Current simulation time, sec.

CHARACTER(1024), INTENT(IN) :: DirRoot ! The name of the root file including the full path to the current working directory. This may be useful if you want this routine to write a permanent record of what it does to be stored with the simulation results: the results should be stored in a file whose name (including path) is generated by appending any suitable extension to DirRoot.

```

```fortran
! Local Variables:
REAL(ReKi) :: Damp (6,6) ! Damping matrix.
REAL(ReKi) :: Stiff (6,6) ! Stiffness/restoring matrix.
INTEGER(4) :: I ! Generic index.
INTEGER(4) :: J ! Generic index.
```
Damp (1,1) = (13759072.12, 0.0, 0.0, 0.0, 0.0, 0.0)  
Damp (2,1) = (0.0, 13759072.12, 0.0, 0.0, 0.0, 0.0)  
Damp (3,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  
Damp (4,1) = (0.0, 0.0, 0.0, 2060457353.95, 0.0, 0.0)  
Damp (5,1) = (0.0, 0.0, 0.0, 0.0, 2060457353.95, 0.0)  
Damp (6,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  

Stff (1,1) = (738641235.5, 0.0, 0.0, 0.0, -6440794804.3, 0.0)  
Stff (2,1) = (0.0, 738641235.5, 0.0, 6440794804.3, 0.0, 0.0)  
Stff (3,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  
Stff (4,1) = (0.0, 6440794804.3, 0.0, 119312319997., 0.0, 0.0)  
Stff (5,1) = (-6440794804.3, 0.0, 0.0, 0.0, 119312319997., 0.0)  
Stff (6,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  

Pt fmAM(1,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  
Pt fmAM(2,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  
Pt fmAM(3,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  
Pt fmAM(4,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  
Pt fmAM(5,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  
Pt fmAM(6,1) = (0.0, 0.0, 0.0, 0.0, 0.0, 0.0)  

Pt fmFt(1) = 0.0  
Pt fmFt(2) = 0.0  
Pt fmFt(3) = 0.0  
Pt fmFt(4) = 0.0  
Pt fmFt(5) = 0.0  
Pt fmFt(6) = 0.0  

DO J = 1, 6  
    DO I = 1, 6  
        Pt fmFt(I) = Pt fmFt(I) - Damp(I,J)*XD(J) - Stff(I,J)*X(J)  
    ENDDO  
ENDDO  
RETURN  
END SUBROUTINE UserPt FMld  

!=========================================================================================

339
K. EXAMPLE FAST USER-DEFINED SUBROUTINE (USERPTFMLD) FOR
THE MPF CS MODEL
Appendix L

Example FAST Input File

(Plalfrom.dat) for the MPF AF model

--- FAST PLATFORM FILE ---
NREL 5.0 MW offshore baseline monopile platform with rigid foundation input properties.

--- FEATURE FLAGS (CONT) ---
- False PtfmSgDOF  - Platform horizontal surge translation DOF (flag)
- False PtfmSwDOF  - Platform horizontal sway translation DOF (flag)
- False PtfmHVDOF  - Platform vertical heave translation DOF (flag)
- False PtfmPDOF   - Platform pitch tilt rotation DOF (flag)
- False PtfmYDOF   - Platform yaw rotation DOF (flag)

--- INITIAL CONDITIONS (CONT) ---
- 0.0 PtfmSurge  - Initial horizontal surge translational displacement of platform (m)
- 0.0 PtfmSway   - Initial horizontal sway translational displacement of platform (m)
- 0.0 PtfmHeave  - Initial vertical heave translational displacement of platform (m)
- 0.0 PtfmRoll   - Initial roll tilt rotational displacement of platform (deg)
- 0.0 PtfmPitch  - Initial pitch tilt rotational displacement of platform (deg)
- 0.0 PtfmYaw    - Initial rotational displacement of platform (deg)

--- TOWER CONFIGURATION (CONT) ---
- 38.33 TwrDraft - Downward distance from MSL to the tower base platform connection (m)
- 38.33 PtfmCM   - Downward distance from MSL to the platform CM (m)
- 38.33 PtfmRef  - Downward distance from MSL to the platform reference point (m)

--- MASS AND INERTIA (CONT) ---
- 0.0 PtfmMass    - Platform mass (kg)
- 0.0 PtfmRIner   - Platform inertia roll tilt rotation about the platform CM (kgm^2)
- 0.0 PtfmPIner   - Platform inertia pitch tilt rotation about the platform CM (kgm^2)
- 0.0 PtfmYIner   - Platform inertia yaw rotation about the platform CM (kgm^2)

--- PLATFORM (CONT) ---
- 1 PtfmLdMod    - Platform loading model (0: none, 1: UserPtfmLd)

--- TOWER (CONT) ---
- 1 TwrLdMod     - Tower loading model (0: none, 1: Morison's equation, 2: UserTwr1d)
- 6.0 TwrDiam     - Tower diameter in Morison's equation (m) [used only when TwrLdMod=1]
- 1.0 TwrCA       - Normalized hydrodynamic added mass coefficient in Morison's
### Example Fast Input File (PLALFROM.DAT) for the MPF AF Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TwrCD</td>
<td>Normalized hydrodynamic viscous drag coefficient in Morison's</td>
</tr>
<tr>
<td>WtrDens</td>
<td>Water density (kg/m³)</td>
</tr>
<tr>
<td>WtrDpth</td>
<td>Water depth (meters)</td>
</tr>
<tr>
<td>WaveMod</td>
<td>Incident wave kinematics model {0: none=still water, ..., 2: JONSWAP...}</td>
</tr>
<tr>
<td>WaveStMod</td>
<td>Model for stretching {0: none=no stretching, ..., 3: Wheeler stretching...}</td>
</tr>
<tr>
<td>WaveTMax</td>
<td>Analysis time for incident wave calculations (sec)</td>
</tr>
<tr>
<td>WaveDT</td>
<td>Time step for incident wave calculations (sec)</td>
</tr>
<tr>
<td>WaveHs</td>
<td>Significant wave height of incident waves (m)</td>
</tr>
<tr>
<td>WaveTp</td>
<td>Peak spectral period of incident waves (sec)</td>
</tr>
<tr>
<td>WavePkShp</td>
<td>Peak shape parameter of incident wave spectrum</td>
</tr>
<tr>
<td>WaveDir</td>
<td>Incident wave propagation heading direction (deg)</td>
</tr>
<tr>
<td>WaveSeed(1)</td>
<td>First random seed of incident waves</td>
</tr>
<tr>
<td>WaveSeed(2)</td>
<td>Second random seed of incident waves</td>
</tr>
<tr>
<td>GHWvFile</td>
<td>Root name of GH Bladed files containing wave data</td>
</tr>
<tr>
<td>CurrMod</td>
<td>Current profile model {0: none, 1: standard, 2: UserCurrent}</td>
</tr>
<tr>
<td>CurrSSVO</td>
<td>Sub-surface current velocity at still water level (m/s)</td>
</tr>
<tr>
<td>CurrSSDir</td>
<td>Sub-surface current heading direction (deg)</td>
</tr>
<tr>
<td>CurrNSRef</td>
<td>Near-surface current reference depth (m)</td>
</tr>
<tr>
<td>CurrNSVO</td>
<td>Near-surface current velocity at still water level (m/s)</td>
</tr>
<tr>
<td>CurrNSDir</td>
<td>Near-surface current heading direction (deg)</td>
</tr>
<tr>
<td>CurrDIV</td>
<td>Depth-independent current velocity (m/s)</td>
</tr>
<tr>
<td>CurrDIDir</td>
<td>Depth-independent current heading direction (deg)</td>
</tr>
<tr>
<td>NWaveKin</td>
<td>Number of points where the wave kinematics can be output [0 to 9]</td>
</tr>
<tr>
<td>WaveKinNd</td>
<td>List of tower nodes that have wave kinematics sensors [1 to TwrNodes]</td>
</tr>
</tbody>
</table>

342
# Appendix M

**Example FAST Input File (Tower.dat)**

for the MPF AF model

---

### FAST TOWER FILE

<table>
<thead>
<tr>
<th>NREL 5.0 MW offshore baseline monopile tower with rigid foundation input properties.</th>
</tr>
</thead>
</table>

---

### TOWER PARAMETERS

<table>
<thead>
<tr>
<th>Number of input stations to specify tower geometry</th>
<th>Number of input stations to specify tower geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>NTwInpSt</td>
</tr>
</tbody>
</table>

---

### TOWER PARAMETERS

<table>
<thead>
<tr>
<th>- Tower 1st fore-aft mode structural damping ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>- Tower 1st side-to-side mode structural damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

---

### TOWER ADJUSTMENT FACTORS

<table>
<thead>
<tr>
<th>- Factor to adjust tower mass density (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

---

### DISTRIBUTED TOWER PROPERTIES

<table>
<thead>
<tr>
<th></th>
<th>HtFract</th>
<th>TMassDen</th>
<th>TwFAStif</th>
<th>TwSSStif</th>
<th>TwGJStif</th>
<th>TwEAStif</th>
<th>TwFAIner</th>
<th>TwSSIner</th>
<th>TwFAcgOf</th>
<th>TwSScgOf</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>(kg/m)</td>
<td>(Nm^2)</td>
<td>(Nm^2)</td>
<td>(N)</td>
<td>(kg)</td>
<td>(kg m)</td>
<td>(kg m)</td>
<td>(m)</td>
<td>(m)</td>
<td></td>
</tr>
<tr>
<td>0.0000</td>
<td>17262.3</td>
<td>1.1456E12</td>
<td>1.1656E12</td>
<td>8.97018E11</td>
<td>2.64272E11</td>
<td>76142.6</td>
<td>76142.6</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.1456</td>
<td>9517.14</td>
<td>1.0371E12</td>
<td>1.0371E12</td>
<td>7.98098E11</td>
<td>2.35129E11</td>
<td>41979.2</td>
<td>41979.2</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.3830</td>
<td>4306.51</td>
<td>4.7449E11</td>
<td>4.7449E11</td>
<td>3.65133E11</td>
<td>1.06396E11</td>
<td>19205.6</td>
<td>19205.6</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.4457</td>
<td>4030.44</td>
<td>4.1308E11</td>
<td>4.1308E11</td>
<td>3.1787E11</td>
<td>99576000000</td>
<td>16720</td>
<td>16720</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.5069</td>
<td>3763.45</td>
<td>3.5783E11</td>
<td>3.5783E11</td>
<td>2.75356E11</td>
<td>92979000000</td>
<td>14483.4</td>
<td>14483.4</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.5688</td>
<td>3505.52</td>
<td>3.083E11</td>
<td>3.083E11</td>
<td>2.37242E11</td>
<td>86607000000</td>
<td>12478.7</td>
<td>12478.7</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.6300</td>
<td>3256.66</td>
<td>2.6408E11</td>
<td>2.6408E11</td>
<td>2.0322E11</td>
<td>80459000000</td>
<td>10689.2</td>
<td>10689.2</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.6919</td>
<td>3016.86</td>
<td>2.248E11</td>
<td>2.248E11</td>
<td>1.72987E11</td>
<td>74534000000</td>
<td>9098.9</td>
<td>9098.9</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.7538</td>
<td>2786.13</td>
<td>1.9006E11</td>
<td>1.9006E11</td>
<td>1.46252E11</td>
<td>68834000000</td>
<td>7692.7</td>
<td>7692.7</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>0.8150</td>
<td>2564.46</td>
<td>1.5949E11</td>
<td>1.5949E11</td>
<td>1.22735E11</td>
<td>63357000000</td>
<td>6455.7</td>
<td>6455.7</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>
M. EXAMPLE FAST INPUT FILE (TOWER.DAT) FOR THE MPF AF MODEL

0.8769 2351.87 1.3277E11 1.3277E11 1.02167E11 58105000000 5373.9 5373.9
0.9381 2148.34 1.0954E11 1.0954E11 84291000000 53077000000 4433.6 4433.6
1.0000 1953.87 89490000000 89490000000 68863000000 48272000000 3622.1 3622.1

-- TOWER FORE-AFT MODE SHAPES --

1.1015 TwFAM1Sh(2) - Mode 1, coefficient of $x^2$ term
-1.7033 TwFAM1Sh(3) - , coefficient of $x^3$ term
3.4301 TwFAM1Sh(4) - , coefficient of $x^4$ term
-2.2015 TwFAM1Sh(5) - , coefficient of $x^5$ term
0.3732 TwFAM1Sh(6) - , coefficient of $x^6$ term

-- TOWER SIDE-TO-SIDE MODE SHAPES --

1.0949 TwSSM1Sh(2) - Mode 1, coefficient of $x^2$ term
-1.6966 TwSSM1Sh(3) - , coefficient of $x^3$ term
3.4227 TwSSM1Sh(4) - , coefficient of $x^4$ term
-2.1982 TwSSM1Sh(5) - , coefficient of $x^5$ term
0.3773 TwSSM1Sh(6) - , coefficient of $x^6$ term

-- TOWER FORE-AFT MODE SHAPE S --

1.1015 TwFAM1Sh(2) - Mode 2, coefficient of $x^2$ term
152.4561 TwFAM2Sh(3) - , coefficient of $x^3$ term
-200.1367 TwFAM2Sh(4) - , coefficient of $x^4$ term
284.7567 TwFAM2Sh(5) - , coefficient of $x^5$ term
-119.6579 TwFAM2Sh(6) - , coefficient of $x^6$ term

-- TOWER SIDE-TO-SIDE MODE SHAPE S --

1.0949 TwSSM1Sh(2) - Mode 2, coefficient of $x^2$ term
-423.5649 TwSSM2Sh(3) - , coefficient of $x^3$ term
548.411 TwSSM2Sh(4) - , coefficient of $x^4$ term
-740.1529 TwSSM2Sh(5) - , coefficient of $x^5$ term
966.4363 TwSSM2Sh(6) - , coefficient of $x^6$ term
-350.1294 TwSSM2Sh(6) - , coefficient of $x^6$ term
Appendix N

Example BMODES Input File for the MPF CS model

================================ BModes v3.00 Main Input File ==================================

NREL 5MW Tower

-------- General parameters --------

true Echo Echo input file contents to *.echo file if true.
2 beam_type 1: blade, 2: tower (-)
0. romg: rotor speed, automatically set to zero for tower modal analysis (rpm)
1. romg_multiplier: rotor speed multiplicative factor (-)
87.6 radius: tower height above ground level [onshore] or MSL [offshore] (m)
0. hub_rad: hub radius measured along coned blade axis OR tower rigid-base height (m)
0. precone: built-in precone angle, automatically set to zero for a tower (deg)
0. bl_thp: blade pitch setting, automatically set to zero for a tower (deg)
3 hub_conn: tower-base boundary [1: cantilevered; 2: free-free; 3: axial and torsion]
0. modepr: number of modes to be printed (-)
1 TabDelim (true: tab-delimited output tables; false: space-delimited tables)

-------- Blade-tip or tower-top mass properties --------

3.500003109E+005 tip_mass blade-tip or tower-top mass (kg)
-0.4137754432 cm_loc tip-mass c.m. from the tower axis measured along x-tower axis (m)
1.9669893542 cm_axial tip-mass c.m. tower tip measures axially along the z axis (m)
4.370E7 ixx_tip blade lag mass moment of inertia (kg-m^2)
2.353E7 iyy_tip blade flap mass moment of inertia (kg-m^2)
2.542E7 izz_tip torsion mass moment of inertia (kg-m^2)
0. ixy_tip cross product of inertia about x and y reference axes (kg-m^2)
1.169E6 izx_tip cross product of inertia about z and x reference axes (kg-m^2)
0. iyz_tip cross product of inertia about y and z reference axes (kg-m^2)

-------- Distributed-property identifiers --------

1 id_mat: material_type [1: isotropic; non-isotropic composites]
'CS_monopile_tower_sec.dat' : sec_props_file name of beam section properties file (-)

Property scaling factors

1.0 sec_mass_multiplier: mass density multiplier (-)
1.0 flp_iner_multiplier: blade flap or tower f-a inertia multiplier (-)

345
N. EXAMPLE BMODES INPUT FILE FOR THE MPF CS MODEL

```
1.0 lag_iner_mult: blade lag or tower s-s inertia multiplier (-)
1.0 flp_stff_mult: blade flap or tower f-a bending stiffness multiplier (-)
1.0 edge_stff_mult: blade lag or tower s-s bending stiffness multiplier (-)
1.0 tor_stff_mult: torsion stiffness multiplier (-)
1.0 axial_stff_mult: axial stiffness multiplier (-)
1.0 cg_offst_mult: eg offset multiplier (-)
1.0 sc_offst_mult: shear center multiplier (-)
1.0 tc_offst_mult: tension center multiplier (-)

--- Finite element discretization ---------------------------------------------
61 nselt:  no of blade or tower elements ( - )
Distance of element boundary nodes from flexible-tower root (normalized wrt tower length), el_loc():
0  0.003481894 0.010445682 0.017409471 0.024373259 0.031337047 0.038300836
0.045264624 0.052228412 0.059192201 0.066155989 0.073119777 0.080083565 0.08704735
0.094011142 0.10097493 0.107938719 0.114902507 0.121866295 0.128830084 0.135793872
0.13990 0.149721448 0.156685237 0.163649025 0.170612813 0.177576602 0.18454039
0.191504178 0.198467967 0.205431755 0.212395543 0.219359331 0.226323120.233286908
0.240250696 0.247214485 0.250696379 0.320334262 0.37971 0.424791072 0.45961
0.486635 0.51366 0.54068 0.5677 0.594715 0.62173 0.64875 0.67577
0.70279 0.72981 0.75683 0.78385 0.81087 0.83789 0.864905 0.89192 0.91894 0.94596 0.97298
1.0

--------- Properties of tower support subsystem (read only if beam_type is 2)  -------------
1  tow_support:  aditional tower support [0: no additional support; 1: monopile ]  ( - )
20.0 draft :  depth of tower base from the ground or the MSL (mean sea level) ( m )
20.0 cm_pform :  distance of platform c.m. below the MSL ( m )
316417.0 mass_pform :  platform mass (kg)
Platform mass inertia 3X3 matrix (i_matrix_pform):
34201222.0 0. 0. 0. 34201222.0 0. 0. 0. 34201222.0
20.0 ref_msl :  distance of platform reference point below the MSL ( m )
Platform-reference-point-referred hydrodynamic 6X6 matrix (hydro_M);
Platform-reference-point-referred hydrodynamic 6X6 stiffness matrix (hydro_K):

Mooring-system 6X6 stiffness matrix (mooring_K):
738641235.5 0.0 0.0 0.0 -6440794804.3 0.0 0.0
0.0 738641235.5 0.0 6440794804.3 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 6440794804.3 0.0 119312319997. 0.0 0.0 0.0
-6440794804.3 0.0 0.0 0.0 119312319997. 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0

Distributed (hydrodynamic) added-mass per unit length along a flexible tower length:
0  n_secs_m_distr: number of sections added mass per unit length is specified (-)
0. 0  : z_distr_m [row array of size n_added_m_pts; section the flexible tower base] (m)
0.0  : distr_m [row array of size n_added_m_pts; added distributed masses] (kg/m)
```

END of Main Input File Data

346