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DYNAMIC RESPONSE OF WIND TURBINE TOWER ASSEMBLIES

by

Paul J. Murtagh

Thesis submitted to the University of Dublin, Trinity College

for the Degree of Doctor of Philosophy

May 2005
DECLARATION

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Paul J. Murtagh

May 2005
Cogito ergo sum

René Descartes (1596-1650)
DEDICATION

I would like to dedicate this thesis, not the collection of theory and print it is, but rather the sacrifice and personal endeavour it represents, to my parents Aiden and Mona. I solemnly thank you both for your support and belief over the years, and especially of late. I love you both dearly.
ACKNOWLEDGEMENTS

I would like to thank my family, the Murtaghs and the Finns, for all their support, both financial and spiritual, during my tenure at Trinity College. I am especially grateful to Aiden for his generosity in sponsoring my numerous ‘keep me sane’ holidays to the States and to Clara for our regular feasts of Mexican food.

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I must also thank Dr. Brian Broderick and Dr. Biswajit Basu for their supervision, encouragement and financial support. Of the latter, I feel compelled to say the following: I feel privileged to have worked with the brilliant Dr. Basu, who showed atypical interest in me and my work, and showed uncharacteristic generosity with his time, patience and encouragement. I know that if perchance, I ever become the teacher, I will endeavour to emulate his example. I now understand the true meaning of the term philosophy.

I would like to thank the technical staff in the department, Chris O’Donovan, Gerard McGranaghan and particularly George Jones for aiding me with the construction and testing of my model. I would like to thank Dr. Kevin McNamara and Xin Wang for allowing me to use the wind tunnel in National University of Ireland, Galway. I would also like to acknowledge the generosity of RPS – MCOS Consulting Engineers for their sponsorship of me during my second year.
ABSTRACT

As wind turbine towers are composed of several flexible inter-connected components, it is important that the engineer has the ability to understand the dynamic behaviour of the system, including that of its components, in order to ensure the serviceability and survivability of the structure. The object of this thesis is to investigate, both theoretically and experimentally, the free and forced vibrations of wind turbine tower assemblies, using model order reduction techniques. This approach reduces the number of degrees-of-freedom of the system rendering the dynamic analysis more computationally efficient.

First, discrete, distributed parameter and finite element models are proposed in order to obtain the natural frequencies and mode shapes of the individual components of the wind turbine tower, as well as a coupled tower and blades assembly. Next, the forced vibration response of all individual and coupled assemblies is modelled in the time domain with both classical and non-classical damping. A gust response factor design methodology is also proposed using a two degree-of-freedom coupled tower and blades assembly. A physical reduced scale wind turbine tower model was then constructed, and its transfer function obtained experimentally. This model was then immersed within a turbulent wind flow created in a wind tunnel where its response was recorded. The measured response was validated using a theoretical model based on random vibration theory, using the previously measured transfer function.

The main contributions to knowledge that stem from this thesis include the development of reduced order models to calculate dynamic response of assemblies including centrifugal stiffening of blades, rotationally sampled turbulence and blade-tower interaction, as well as the development of a two degree-of-freedom gust response factor for wind turbine towers for use in equivalent static wind loading design. The principal findings advocate that blade-tower interaction has a large effect on the dynamic response of wind turbine tower and magnitude of gust response factor.

Keywords: Wind engineering, wind turbine towers, model order reduction, centrifugal stiffening, gust response factor, system identification
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PRINCIPAL NOTATION

\( \{ a_j(t) \} \)  modal acceleration time-history of mode ‘j’
\( a_k \)  randomly generated Fourier coefficient
\( A_i \)  area of node ‘i’
\( \hat{A}_{M1}(\omega) \)  fundamental modal acceleration response spectrum
\( \hat{A}_{M2}(\omega) \)  second modal acceleration response spectrum
\( A_T \)  total surface area of structure
\( b \)  width of tower
\( b_k \)  randomly generated Fourier coefficient
\( B \)  Background component of response
\( c_a(z) \)  aerodynamic damping force at elevation ‘z’
\( [C_a] \)  aerodynamic damping matrix
\( [C_B] \)  damping matrix of blade
\( \overline{C}_{B,j} \)  modal damping of blade for mode ‘j’
\( [C_{CS}] \)  coupled system damping matrix
\( \overline{C}_{CS,j} \)  coupled system modal damping of mode ‘j’
\( C_D \)  coefficient of drag
\( \text{Coh}(k,l:f) \)  coherence between nodes ‘k’ and ‘l’
\( [C_s] \)  structural damping matrix
\( [C_T] \)  damping matrix of tower and nacelle
\( CT_i \)  tensile centrifugal axial force at node ‘i’
\( \overline{C}_{T,j} \)  modal damping of tower for mode ‘j’
\( [C_{T,T}] \)  total damping matrix of tower and nacelle
\( C_{UU}(\tau) \)  autocovariance function of variable ‘U’
\( \text{D-D} \)  Displacement-to-displacement
\( \hat{D}_{M1}(\omega) \)  fundamental modal fluctuating displacement response spectrum
\( \hat{D}_{M2}(\omega) \)  second modal fluctuating displacement response spectrum
\( D_N \)  diameter of spherical mass of nacelle
\( E_I \)  flexural rigidity
\( f \)  frequency
\( f_a, f_b, f_c \)  flexibility at nodes a, b, and c
\( f(x) \)  probability density function
\( f_{CS,1} \)  fundamental frequency of coupled system
\( f_{CS,2} \)  second natural frequency of coupled system
\( f_{DI} \)  mean drag force at node ‘i’
\( f_{Dj}^*(t) \)  fluctuating drag force at node ‘i’
\( f_{D_Mj} \)  mean modal drag force for mode ‘j’
\( f_{D_Mj}^*(t) \)  fluctuating modal drag force of mode ‘j’
\( f_{T1} \)  fundamental frequency of tower and nacelle
\( f_{T1} \)  fundamental frequency of tower and nacelle
\( f_{T_M1}(\omega) \)  fundamental modal fluctuating drag force spectrum
\( \hat{f}_{D,M1}(\omega) \)  second modal fluctuating drag force spectrum
\( F \)  Equivalent Spring force
F(X,0), F(X,1), F(X,2), F(X,3)  Solution functions of power series
F_{CS,1} fundamental modal force of coupled system
F_{CS,2} second modal force of coupled system
F-D  Force-to-displacement
F_{D}(t) total nodal drag force
F_{D,i}(t) total drag force at node ‘i’
\{F_{D,j}(t)\} total modal drag force for mode ‘j’
F_j(t) modal force for mode ‘j’
[F] flexibility matrix of tower and nacelle
G gust response factor
G_i axial force due to gravity at node ‘i’
H height at top of tower
|H(f)|, |H(ω)| modulus of complex frequency response function
H_{11}(f) fundamental modal transfer function ordinate at node ‘1’
H_{12}(f) second modal transfer function ordinate at node ‘1’
[l] influence vector
I(z) turbulence intensity at elevation ‘z’
J_{NAC} rotary mass moment of inertia of nacelle
k Von-kârmâns constant
\bar{K}_{B,j} modal stiffness of blade for mode ‘j’
[K_B] stiffness matrix of blade
[K_{CS}] coupled system stiffness matrix
\bar{K}_{CS,j} coupled system modal stiffness of mode ‘j’
[K_{GB}] geometric stiffness matrix of blade
K_S equivalent spring stiffness
[K_T] stiffness matrix of tower and nacelle
\bar{K}_{T,j} modal stiffness of tower and nacelle for mode ‘j’
L_B length of rotating blade
L_S length scale
L_T length of tower
L^x \_u integral length scale along global x-axis due to longitudinal turbulence (u)
m_B(x) mass of blade as a function of distance along blade ‘x’
m_i lumped mass at node ‘i’
\bar{m}_B mass per unit length of blade
\bar{m}_T mass per unit length of tower
M'(t) fluctuating bending moment
\{M'_b(t)\} fluctuating bending moment at base-point
M_B total mass of rotating blade
[M_B] mass matrix of blade
\bar{M}_{B,j} modal mass of blade for mode ‘j’
[M_{CS}] coupled system mass matrix
\bar{M}_{CS,j} coupled system modal mass of mode ‘j’
\{M_m'(t)\} fluctuating bending moment at mid-point
M_{NAC} mass of nacelle at free end of tower
M_S equivalent mass of spring
[M_T] mass matrix of tower and nacelle
\( \bar{M}_{Tj} \) modal mass of tower and nacelle for mode ‘j’

\( M_{T}(L) \) bending moment on tower at distance ‘L’

\( M_{3B} \) mass of three rotating blades

\( n \) Monin co-ordinate

\( N_i \) total axial force at node ‘i’

\( N_i \) expected ratio of zero-crossing

\( \bar{p}_i \) mean force at node ‘i’

\( p_i'(t) \) fluctuating force at node ‘i’

\( P_i(t) \) total force at node i

\( \{q_i(t)\} \) base acceleration

\( \{Q(\omega)\} \) Fourier domain tower base physical co-ordinate

\( r_1, r_2 \) frequency ratios for modes ‘1’ and ‘2’

\( R \) resonant component of response

\( R_h \) radius of hub

\( s_1, s_2 \) points ‘1’ an ‘2’

\( s_{T,j} \) complex eigenvalue of tower and nacelle for mode ‘j’

\( S_{FF}(f) \) drag force PSDF

\( S_{FF}(f_i) \) drag force PSDF ordinate at the fundamental frequency

\( S_{MF,MF}(f) \) fluctuating modal drag force PSDF for mode ‘j’

\( S_{Vv}(z,f) \) fluctuating wind velocity PSDF at elevation ‘z’

\( S_{VVV}(f) \) fluctuating wind velocity cross PSDF

\( S_{V_{Vv}}(f),S_{Vv}(f) \) fluctuating wind velocity auto PSDF

\( S_{XX}(f) \) displacement response PSDF

\( t \) time

\( T \) time duration of response

\( u \) longitudinal velocity component of turbulence

\( u(L_B,t) \) relative displacement between blade tip and base

\( u_N \) amplitude of mode shape at node ‘n’

\( \{u(t)\} \) nodal displacement vector of blade

\( \{\dot{u}(t)\} \) nodal velocity vector of blade

\( \{\ddot{u}(t)\} \) nodal acceleration vector of blade

\( v \) transverse velocity component of turbulence

\( v_* \) friction velocity

\( \bar{v}(H) \) mean wind velocity at elevation ‘H’

\( \bar{v}_i \) mean wind velocity at node ‘i’

\( v_i'(t) \) fluctuating wind velocity at node ‘i’

\( \bar{v}_k \) mean wind velocity at node ‘k’

\( \bar{v}_j \) mean wind velocity at node ‘j’

\( \bar{v}(z) \) mean wind velocity at elevation ‘z’

\( \bar{v}(z_i) \) mean wind velocity at elevation ‘z_i’

\( \bar{v}(z_2) \) mean wind velocity at elevation ‘z_2’

\( V \) amplitude of sinusoidal variation of mean wind velocity

\( V_{BB} \) blade base shear due to flapping motion

\( V_{BLL} \) blade base shear due to lead/lag motion

\( \{V_B(t)\} \) total base shear of blade
\[
\{V_B'(t)\} \quad \text{effective base shear force of blade}
\]
\[
\{V_{Mj}(t)\} \quad \text{modal blade base shear force for mode 'j'}
\]
\[
V(t) \quad \text{total wind velocity}
\]
\[
V_T(L) \quad \text{shear force on tower at distance 'L'}
\]
\[
w \quad \text{vertical velocity component of turbulence}
\]
\[
w(x,t) \quad \text{transverse displacement response at distance x}
\]
\[
x \quad \text{variable x}
\]
\[
\bar{x} \quad \text{mean displacement of tower}
\]
\[
x'(t) \quad \text{fluctuating displacement of tower}
\]
\[
\{x(t)\} \quad \text{nodal displacement of tower and nacelle}
\]
\[
\{\dot{x}(t)\} \quad \text{nodal velocity of tower and nacelle}
\]
\[
\{\ddot{x}(t)\} \quad \text{nodal acceleration of tower and nacelle}
\]
\[
X(\omega) \quad \text{tower displacement response spectrum}
\]
\[
X(t) \quad \text{total displacement of tower}
\]
\[
y \quad \text{distance from strain point to neutral axis}
\]
\[
Y_j(x) \quad \text{normalised mode shape of blade for mode 'j'}
\]
\[
z_1, z_2 \quad \text{elevation of points '1' and '2'}
\]
\[
z_0 \quad \text{roughness length}
\]
\[
\{z(t)\} \quad \text{nodal displacement vector of coupled system}
\]
\[
\{\dot{z}(t)\} \quad \text{nodal velocity vector of coupled system}
\]
\[
\{\ddot{z}(t)\} \quad \text{nodal acceleration vector of coupled system}
\]
\[
\{Z(\omega)\} \quad \text{Fourier domain coupled system physical co-ordinate}
\]
\[
\alpha \quad \text{mean wind velocity gradient exponent}
\]
\[
\varepsilon'(t) \quad \text{fluctuating strain}
\]
\[
\{\eta_{Bj}(t)\} \quad \text{blade displacement in modal co-ordinates for mode 'j'}
\]
\[
\{\dot{\eta}_{Bj}(t)\} \quad \text{blade velocity in modal co-ordinates for mode 'j'}
\]
\[
\{\ddot{\eta}_{Bj}(t)\} \quad \text{blade acceleration in modal co-ordinates for mode 'j'}
\]
\[
\{\eta_{CSj}(t)\} \quad \text{coupled system displacement in modal co-ordinates for mode 'j'}
\]
\[
\{\dot{\eta}_{CSj}(t)\} \quad \text{coupled system velocity in modal co-ordinates for mode 'j'}
\]
\[
\{\ddot{\eta}_{CSj}(t)\} \quad \text{coupled system acceleration in modal co-ordinates for mode 'j'}
\]
\[
\eta_{CSj}(\omega) \quad \text{Fourier domain coupled system modal co-ordinate for mode 'j'}
\]
\[
\{\eta_{Tj}(t)\} \quad \text{tower displacement in modal co-ordinates for mode 'j'}
\]
\[
\{\dot{\eta}_{Tj}(t)\} \quad \text{tower velocity in modal co-ordinates for mode 'j'}
\]
\[
\{\ddot{\eta}_{Tj}(t)\} \quad \text{tower acceleration in modal co-ordinates for mode 'j'}
\]
\[
\theta_1, \theta_2, \theta_3 \quad \text{angle blades 1, 2 and 3 make with horizontal global axis}
\]
\[
\mu \quad \text{mean of variable}
\]
\[
\xi_{Tj} \quad \text{modal damping ratio of tower for mode 'j'}
\]
\[
\xi_{Bj} \quad \text{modal damping ratio of tower for mode 'j'}
\]
\[
\xi_{CSj} \quad \text{modal damping ratio of coupled system for mode 'j'}
\]
\[
\rho \quad \text{density of air}
\]
\[
\sigma_u \quad \text{standard deviation of fluctuating velocity due to longitudinal turbulence}
\]
\[
\sigma_x \quad \text{standard deviation of variable 'x'}
\]
\[
\sigma_x \quad \text{standard deviation of displacement response}
\( \tau \)  
dummy temporal variable

\( \phi_{T,j}(k) \)  
mode shape component of tower at node ‘k’ for mode ‘j’

\( \phi_{I,j}(l) \)  
mode shape component of tower at node ‘l’ for mode ‘j’

\( \phi_{T,j} \)  
tower top node component of mode shape ‘j’

\( \chi \)  
aerodynamic admittance function

\( \chi_1 \)  
aerodynamic admittance function ordinate at the fundamental frequency

\( \omega \)  
circular frequency

\( \omega_{B,j} \)  
circular natural frequency of blade for mode ‘j’

\( \omega_{BF,1} \)  
fundamental flapping frequency of blade

\( \omega_{BLL,1} \)  
fundamental lead/lag frequency of blade

\( \omega_{CS,j} \)  
circular natural frequency of coupled system for mode ‘j’

\( \omega_k \)  
k\(^{th}\) discretized circular frequency

\( \omega_{j,D} \)  
damped circular natural frequency of mode ‘j’

\( \omega_{T,j} \)  
circular natural frequency of tower and nacelle for mode ‘j’

\( \omega_{TD,j} \)  
damped circular natural frequency of tower and nacelle for mode ‘j’

\( \Delta \)  
scalar constant

\( \Phi_T(0), \Phi_T(L) \)  
tower displacement at distances ‘0’ and ‘L’

\( \Phi_T'(0), \Phi_T'(L) \)  
tower slope at distances ‘0’ and ‘L’

\( \Phi_{T,j} \)  
mode shape of tower and nacelle for mode ‘j’

\( \Phi_{T,j} \)  
complex eigenvector of tower and nacelle for mode ‘j’

\( \Phi_{B,j} \)  
mode shape of blade for mode ‘j’

\( [\Phi_{CS}] \)  
mode shape matrix of coupled system

\( \Psi \)  
peak factor

\( \Omega \)  
rotational frequency of blade
CHAPTER 1 - INTRODUCTION

1.1 GENERAL INTRODUCTION

Wind turbine towers are the means with which the kinetic energy contained within wind may be harnessed and transmuted into electrical energy, directly available to the consumer. It is only in the last decade that wind turbines have become an economically viable option for widespread electrical energy production. As more wind turbine towers are placed in different wind environments in different areas around the world, it is necessary that the engineer has a comprehensive understanding of the behaviour of the structure under dynamic loading. The motivation behind this thesis is to develop a comprehensive structural behavioural understanding and subsequent design methodology for wind turbine towers, through the use of reduced order models, aimed at ensuring survivability and requisite performance during their design lives.

There is much debate in society of late regarding the causes of global warming, though few can deny it is actually occurring. The prevailing theory is that this phenomenon is generally man made through the release of greenhouse gases, in part from fossil fuel combustion. At present, electricity is primarily generated worldwide by burning fossil fuels such as oil and coal, which releases the aforementioned greenhouse gases into the atmosphere. However, there exist abundant alternative sources of clean renewable fuel to generate electricity, and one of these is in the form of wind energy. Wind turbines, therefore, represent a clean and affordable means of providing electrical power.

The neo-popularity of wind turbines stems from the fact that units have now become financially viable mainly due to mass production, and from an increase in environmental awareness due to Earth summits such as those in Rio de Janeiro and Kyoto. The seminal summit held in Kyoto, Japan in 1997, resulted in the formation of the Kyoto Protocol. This treaty demanded that global production of greenhouse gases produced by developed countries be reduced by approximately 5% below 1990 levels by 2008-2012. In the context of Irish governmental obligation, the administration pledged to limit the increase of Irish greenhouse gas emissions to 13% above 1990 levels by 2008-2012. The Irish Government acknowledges that in order to meet this target, it must foster renewable sources of energy to meet the country’s electricity needs, and wind energy is abundantly available for this purpose. As of January 2004, 257 MW of wind electricity generating
capacity was installed on the island of Ireland, with another 300 MW of wind power planned for the near future.

Governments and the general public have long understood that our development is linked to our understanding of wind, and are slowly coming to the realisation that our ability to harness wind energy is vital in order to deliver a sustainable future. As wind is an integral component of this thesis, it is proposed to quickly explore it’s effects on human civilisation along five directions, anthropology, the arts, academics, engineering and power generation.

Anthropology
Wind has undoubtedly interested man from earliest time. The human mind has evolved into a mechanism that reacts to stimuli by first interpreting them in three dimensional space and in conjunction with our senses, discerns whether or not the stimulus represents a threat. Wind, therefore, perplexes our species as one cannot see, hear or taste it. Wind is in fact the only one of the platonic elements which is noticeable by a single sense alone, the sense of touch. Wind may have also acted as an evolutionary stimulus, inspiring mammals to evolve both physically and mentally out of the necessity to survive in harsh wind environments. Even today, wind renders one incapable of surviving in certain climates without specialised clothing, due to the pseudo cooling of the body, known as wind chill. In ancient times, wind even influenced the layout of cities, such as Kahun in ancient Egypt, around the year 2000 B.C. Buildings occupied by the city’s social elite were oriented to face the cooler north winds.

The Arts
Wind has also had a measurable influence on our conceptual expression, especially in theology and the arts. In mythology, wind was usually feared as a destructive force and ascribed godlike powers. In ancient Greece, wind, as with other meteorological phenomena, was an integral part of their polytheistic theology, being personified in no less than eight Gods. In fact, a white marble tower known as The Tower of the Winds was constructed in about 50 B.C. by the architect and astronomer Andronikos of Kyrrhos, and stands to this day in the agora in Athens. It is octagonal in shape, with each side facing the direction of each of the eight wind gods. This theory of eight characteristic winds was later endorsed by the well known Roman architect Marcus Vitruvius. The Grecians also believed that the winds, the venti, were kept in a cave and overseen by the god Aeolus. In fact, all major polytheistic theologies have wind gods or wind goddesses, including, to
name but a few: Aquilo (Roman), Njord (Norse), Chi Po (Chinese), Minlil (Babylonian), Varuna (Hindu) and Quetzalcoatl (Aztec).

In literary circles, the wind has provided inspiration to many of history's greatest poets and dramatists. In 1599, the most celebrated English language poet and dramatist, William Shakespeare (1564-1616), wrote the aptly named song Blow, Blow, Thou Winter Wind contained in his play As You Like It. Though open to interpretation, this song is probably a simile comparing the harshness of wind with the harshness of man. The Irish Noble Poet Laureate William Butler Yeats (1865-1939) used an inference of wind action often, as seen in his poem To A Child Dancing In The Wind (1916) and in his anthology The Wind Among The Reeds compiled in 1899. The often morose American poet Emily Dickinson (1830-1886) was inspired by the wind on numerous occasions, writing The Wind Trapped Like A Tired Man, A Wind That Rose and There Came A Wind Like A Bugle.

Academics

Some of the greatest minds in the history of civilisation have set about explaining the formation and composition of wind. The first recorded theories of wind and wind generation came from the one time student of Plato and subsequent tutor of Alexander the Great, Aristotle. He was born in 384 B.C., at Stagirus, near Macedonia and in 350 B.C. wrote the classic Meteorologica, in which he attempted to explain various meteorological phenomena. While his ideas on water vapour and precipitation are accurate by contemporary standards, he failed to fundamentally understand the nature of wind. He was aware that air was indeed a fluid, as he compared the flow of air to a flowing river, but was unable to state why this flow of air did not dry up, as a river does. He did however, acknowledge that the sun played a part in the formation of wind, though he erroneously attributed wind formation to rising water vapour from solar heating.

The enigmatic Leonardo Da Vinci (1452-1519) is credited with the first tangible contributions to the field of aerodynamics. Da Vinci was reportedly fascinated with the concept of flight, and carried out studies on the size and shape of bird's wings during his investigations into drag and lift forces. A design for a glider capable of supporting a single pilot was found among the multitude of his sketches. Alas, Da Vinci never reportedly took to the sky. He also derived the concept of the wind tunnel principle. This stated that the effects of wind blowing on a stationary object at a given velocity are equivalent if the body
is moving with the same velocity through stationary air. This principle would support the construction of wind tunnels two hundred years later.

The imaginative ideas proposed by Aristotle would go unchallenged for centuries until the advent of the barometer by Evangelista Torricelli (1608-1647) and the thermometer by Galileo Galilei (1564-1642) allowing the renaissance philosopher Sir Francis Bacon (1561-1626) to publish *Historia Naturalis et Experimentalis* in 1622. This was a collection of his observations on naturally occurring phenomena and contained a section entitled *Historia Ventorum*, on the origin of history of winds.

A mathematical quantification of the forces experienced by a body in motion was not conceived until Sir Issac Newton (1642-1727) developed the ideas of mechanics which he published in his 1687 treatise on motion entitled *Philosophiae Naturalis Principia Mathematica*. Contained in this work was his three famous laws of motion. The second law, which is of primary interest to the dynamics of structures, states that the force experienced by a body is directly proportional to the body's rate of change of momentum. This law was the origin of the inertial forces experienced by all structures undergoing acceleration/retardation. Newton also correctly postulated in Book II, Section VIII, entitled *Of motion propagated through fluids* that the resistive force experienced by a body within a fluid is proportional to the square of the velocity of the fluid passing the body.

Newton is also credited with being one of the fathers of calculus which subsequently allowed mathematicians and fluid dynamists such as Daniel Bernoulli (1700-1782), Claude Navier (1785-1836), Irish born George Stokes (1819-1903), and Irish born Osbourne Reynolds (1842-1912) to develop the laws of fluid dynamics, all of which are still in use today. Stokes helped derive the well known Navier-Stokes equation that describes the flow of fluids. Reynolds developed the very useful dimensionless measure of the onset of turbulence, known as the Reynolds number.

One of the first published works on the topic of mechanical dynamics came from the Nobel Laureate John William Strutt Raleigh (1842-1919). He published his truly seminal *Theory of Sound, Volume 1* (1877) which set out the fundamentals of the vibration of various types of entities and included the necessary mathematics to describe their dynamic motions. Thus, the necessary advancements in the mathematics of motion and fluid dynamics would
allow engineers to later develop the contemporary theories of wind engineering in use today.

**Engineering**

The term wind engineering was first suggested by Professor Jack Cermak, Emeritus Professor from Colorado State University, in the early 1970s and is generally accepted as concerning the effects of wind action on structures. Prior to the advent of this phrase, wind effects on structures were broadly classified under the term industrial aerodynamics. The birth of industrial aerodynamics may be attributed to the industrial revolution of the 18th century, with the increased construction of large-scale bridge structures brought about by transportation needs. The British engineer John Smeaton (1724-1794) developed some of the first empirical design expressions to quantify static wind loading. In 1759, he published a paper entitled *An Experimental Enquiry Concerning the Natural Powers of Water and Wind to Turn Mills and Other Machines Depending on Circular Motion* in which he addressed the relationship between wind pressure and velocity for moving objects. Subsequent study by other researchers found a constant of proportionality between pressure and the square of wind velocity, equal to 0.005, which became known as the so-called Smeaton coefficient.

The construction of bridges continued unabated throughout the 19th century, but with several high profile bridge failures due to wind action, engineers conceded that the understanding of wind action on structures was still wholly inadequate. Bridges such as the Menai Straits bridge in Wales (1850), the Wheeling bridge in Ohio (1864) and the Tay bridge in Scotland (1879) all failed due to wind action. The value of the Smeaton coefficient used in the design of bridges was later found to be erroneous, and after the Tay bridge disaster, prompted the National Physical Laboratory to conduct further wind tests, which resulted in the amendment of Smeaton’s coefficient to 0.003.

The American skyscraper boom of the 1930s provided the next thrust for engineering studies of wind effects on structures. The first tentative coding was subsequently developed specifically to design structures to withstand wind, though only static loading was considered. Experiments were carried out on the famous Empire State Building in New York City in which the engineer J.C. Rathbun observed that the building vibrated ‘like the tines of a tunes fork’. The dramatic bridge failure in 1940 of the Tacoma Narrows bridge, along with the near failure of the newly built Golden Gate Bridge in San Francisco...
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reinforced the need to understand the dynamic effects of wind loading. Subsequent experiments on the latter showed that the bridge contained flexural and torsional modes at 0.13 Hz and 0.145 Hz respectively, and it was torsional vibrations to which bridges at the time were vulnerable.

Engineers soon learned that the behaviour of structures immersed within turbulent fluid such as a wind flow could also be studied within a closed volume of that fluid, and the wind tunnel was ideal for this task. The English mathematical Benjamin Robins (1707-1751) is credited with the first instigation of the aforementioned Da Vinci wind tunnel principle. Robins constructed a whirling arm, 4 feet long (1220 mm) which rotated about a centrifuge and achieved arm tip velocities of a few feet per second. Sir George Cayley (1773-1857) is generally regarded as the father of aerodynamics and carried out many experiments measuring lift and drag using a whirling arm 5 feet (1524 mm) in length which was capable of arm tip velocities of between 10 and 20 feet per second. In 1804 he used the data derived from his wind testing to fly an unmanned glider, believed to have been the first flight of a heavier-than-air body in history. Although the whirling arm provided much of the relevant experimental data needed to achieve sustained flight, one of its major flaws was that models ultimately flew in their own wakes. The presence of turbulence thus made it difficult to accurately measure the relative velocity of the model and the air. This drawback prompted the genesis of the wind tunnel.

Frank H. Wenham (1824-1908) is recognized with the design and operation of the first wind tunnel in 1871. He was a member of the Aeronautical Society of Great Britain and successfully petitioned that body for funding to build his wind tunnel. It was 12 feet (3660 mm) in length with a cross sectional area of 18 inches squared (0.209 m²). A steam driven fan blower propelled air down the tube against the model. Wind tunnels were also instrumental in the first successful powered manned flight by the Wright brothers on the 17th of December 1903 in Kitty Hawk, North Carolina. With Orville at the controls, their longest flight that day lasted 59 seconds and covered 862 feet (287 m). In the course of their research, they built two wind tunnels allowing them to optimise the lift-to-drag ratio by experimenting with different wing configurations.

Jack Cermak was responsible for the erection of the first boundary layer wind tunnel in 1954, located in Colorado State University. The first major scale model experiment into wind effects on structures was carried out in 1964. Cermak was aided by Alan Davenport
with the motivation of measuring the response of a scale model of the now ill-fated World Trade Centre towers in New York City. It was not until about 1960 that the emphasis really shifted from static wind design to dynamic wind design, when Davenport, later connected to the University of Western Ontario, used random vibration theory to define wind loading in terms of statistical moments and turbulence spectra. The idea of a statistical theory of turbulence had been around for about twenty years, originally conceived and derived by the Russian mathematician Andrey Nikolaevich Kolmogorov (1903-1987), though English Physicist Geoffrey Ingram Taylor (1886-1975) was independently working on this idea around the same time. This idea was further advanced by the work of Theodore Von-Kármán (1881-1963), who had previously identified vortex shedding in wakes, allowing him to publish a mathematical expression quantifying the energy contained within a turbulent wind flow, as a function of the average size of turbulent eddies. Davenport went on to develop the idea of the gust loading/response factor in 1967. The basis of this concept allowed for a realistic portrayal of the inherit randomness contained within the wind, and included intrinsic mechanical dynamic phenomena such as damping and dynamic amplification. The gust loading factor would go on to become the basis of most global contemporary wind codification.

Power Generation
The first historic use of wind power can be traced to the ancient construction of boats and the realisation that wind may be used to propel vessels. The ancient Egyptians used papyrus boats with primitive sails to navigate along the Nile, and in the time of King Sahure (2500 B.C.) large sea-going cedar constructed ships traded with their neighbours using wind power for propulsion. The late period (~500 B.C.) of the Egyptian civilisation saw them become more dependent on sailing technology imported from Greece and Rome. The Trireme, in use about 400 B.C., was a sophisticated war ship used by both the ancient Greeks and Romans and was powered by oar and sail. The vessel had a square canvas sail and carried around 130 oarsmen and 30 other persons, including marines, carpenters and other trades. The vessel acted without sails in stormy conditions and when entering into combat.

The use of wind energy for something other than travel is said to have originated in the Asian civilisations of China, Tibet, India, Afghanistan, and Persia. The first historical evidence of the use of a wind collecting machine comes from the Hero of Alexandria, who described the use of a primitive horizontal axis wind turbine in the second to third century
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B.C. However, scholars are unsure as to the purpose of this machine and suspect it as being a toy. Stronger evidence exists detailing the use of windmills in Persia from about 700 B.C. to about 900 A.D. They used a vertical axis design known as a Panemone, which was not particularly efficient as it turned on the principle of drag only. They were used primarily for grinding grain and pumping water. In fact, the English word mill is derived from the Latin word molina, a term ascribed to a machine that grinds grain. Some scholars proclaim China as being the birthplace of the windmill, though the earliest documentation of the use of windmills in China comes from the Chinese statesman Yehlu Chhu-Tshai in 1219 A.D. Its purpose was also to grind grain and to pump water.

It is thought that the use of wind power spread to Europe around the 11th century A.D. with accounts of windmills being found in England from this period. Also, German crusaders are thought to have brought back the skill of windmill construction from Syria around the 12th century A.D. The European windmill then evolved from the existing vertical axis Persian windmill to a horizontal axis version. The Europeans understood that the horizontal axis design is much more efficient in the conversion of kinetic to mechanical energy. Their design, though simple, contained a gearing system coupled with the horizontal water wheel assembly designed by Marcus Vitruvius. The Dutch then set about refining existing horizontal designs in the late 14th century, and furthered the concept of the windmill to include a masonry structure, supporting the rotating windmill arms, which included the accommodation of the windmill operator and storage space for large quantities of grain.

Over the next five hundred years, the European windmill saw further refinement and designers even started using sails that worked on the principle of both aerodynamic drag and lift. The advent of the steam turbine during the industrial revolution caused a subsequent decline in the use of windmills in Europe. However, in the United States, the popularity of the windmill was growing, especially in rural settings, being used to pump water. The American windmill originally comprised of wooden paddles with a tail used for orientation in the wind, but by 1870, wooden blades had been replaced by steel blades, which were lighter and shape malleable. Between 1850 and 1970, over six million small windmills were installed in the United States.

In the late 19th century, the first windmill ever to generate electricity was built by Charles F. Brush (1849-1929), in Cleveland, Ohio in 1888. It contained a picket fence rotor system.
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17 m in diameter and even contained a step-up gearbox with a ratio of 1:50. Though it's electrical output was minimal, it demonstrated that windmills could indeed be used to generate electricity relatively easily with low levels of technology. The so-called ‘Danish Edison’ Poul La Cour (1846-1908), developed his own electricity generating windmill which incorporated many of the aerodynamic principles in use today. The development of windmills continued slowly, but events such as the two world wars and the great American depression arrested progress.

The first use of large scale electricity generation connected to an electricity grid was demonstrated in the Ukraine, in 1931, and a decade later a 1.25 MW unit was constructed in Vermont in the United States. In 1945 however, operation of this unit was suspended after the stainless steel blade broke off near the hub, apparently due to metal fatigue. The post World War II advanced designs by devotees such as Professor Ulrich Hütter, and Johannes Juul paved the way for what would subsequently be called the first generation of modern wind turbines.

The impetus towards the development of large scale first generation wind turbines draws it’s origins from the so-called energy crisis in 1973, when the availability and cost of fossil fuels became untenable, due to the actions of OPEC coupled with geo-instability, particularly in the middle east. The term turbine originates from the Latin turbo meaning spinning top and whirlwind, and would now be used instead of the antiquated windmill. The development of the first large scale wind turbines came about towards the end of the 1970s when in United States, the National Aeronautics and Space Administration (NASA) designed the MOD-0 and MOD-1 wind turbines. These were quickly followed by the MOD-2 which was rated at 2.5 MW. In Europe, governmental funding had yielded the construction of the Nibe wind turbine in Denmark. In 1982, the Growian wind turbine was built near Marne in Germany and was until recently the biggest wind turbine ever built, having a rating of 3 MW. All the units described above, with the exception of the Nibe, had two blade rotor systems.

During the mid-eighties the European Commission (EC) began to actively fund research into the development of high energy yielding wind turbines, and established its WEGA project. The initial phase of this programme, termed WEGA I, lasted from 1984 to 1986 and developed several new models of wind turbines, all with three blades. Although the operation of the units installed under this programme was not successful, the experience
gathered by researchers would prove valuable in the design stage for the second EC funded programme, WEGA II. The WEGA II prototypes were erected between 1994 and 1996 and represent the second generation of modern wind turbines. Along with wind turbines built under the European Union’s (EU) THERMIE research programme, the design of these turbines form the basis of the wind turbines in use today. WEGA II wind turbines generally have three rotor blades, though under THERMIE experimental single blade units have also been designed and tested.

1.2 ORGANISATION OF THE THESIS
This thesis is concerned with the vibration of wind turbine tower assemblies subjected to stationary wind loading, including the separate vibration of tower and blades and the coupled vibration of both due to tower/blade interaction. The body of the work is segmented into nine chapters.

Chapter 2 presents most of the concepts the reader will meet in the course of reading this thesis, and provides conceptual and/or mathematical explanations of those concepts. Topics discussed include the current understanding of wind formation, including turbulence, the statistical separation of wind force time-histories, autocorrelation, cross-correlation, and power spectral densities. Also included is the difference between free and forced vibrations, and the methods of solving equations of motion in the time domain. The parts of the wind turbine tower, the mechanism with which the blades rotate and associated dynamics of tower and blades is also included.

Chapter 3 presents a thorough review of pertinent literature describing similar published work. Texts of interest to students of structural dynamics, and reports published on the design of many aspects of wind turbine towers are initially reviewed. Literature of the classification of wind turbulence via spectra, studies of free vibrations of structures using both discrete and continuous models are included, as are forced vibration topics of interest including the simulation of time-histories, spatial correlation of forces and rotationally sampled velocity spectra. The use of wind tunnel testing is also addressed, with the scaling laws that govern the use of wind tunnels and scale models initially presented. Various published studies on the use of section and scale models and the response phenomena investigated are revealed. Finally, the topic of experimental system identification tells of what approaches researchers have used in order to obtain the dynamic properties such as natural frequency and damping from full scale and scaled models of structures.
Chapter 4 illustrates the analytical approaches proposed in order to obtain the free vibration properties of the towers, rotating blades, and coupled tower and blade assemblies. Both lattice and tubular towers are considered, as well as prismatic and tapered blades. The free vibration properties of the prismatic blades contain behavioural effects inherent to the operation of wind turbine blades, such as centrifugal stiffening and axial self-weight loading due to rotation. The mathematical approaches utilised include discrete system mechanics, the finite element technique and continuous system mechanics. Obtaining the free vibration properties of all assemblies is the first step in any dynamic analysis, and these properties may now be used to carry out a forced vibration analysis.

The forced vibration analyses of all assemblies is presented in Chapter 5. These assemblies include the tower and nacelle, the rotating blades, and a tower connected to three rotating blades. A forced vibration analysis of the tower and nacelle is carried out by first simulating random nodal wind loading to act on the nodes of the assembly. These nodal wind loads are spatially correlated over the length of the structure. The response of the structure is first obtained, with classical damping, using the mode acceleration technique and a numerically integrated superposition technique. The response of the same structure subjected to the same loading, but with non-classical damping due to the presence of aerodynamic damping, is then obtained. The response of classically damped prismatic wind turbine blades is next estimated, and the solution contains the effects of rotationally sampled turbulence on the response. Lastly, the response of a tower coupled to three rotating blades is obtained, including blade-tower interaction.

As a long and detailed forced vibration analysis is generally not viable in a design environment, a gust response factor design methodology for wind turbine towers is presented in Chapter 6. This approach is an extension of the traditional gust response factor methodology, as it includes the presence of a second mode of vibration. The gust response factor of a coupled two degree-of-freedom wind turbine model may be obtained using a closed form solution derived for this purpose.

Chapter 7 describes the first of the experimental chapters detailing a scale model wind turbine tower that was custom built in the Civil Engineering Laboratory, in Trinity College. The model consists of a plastic tubular tower, a nacelle containing an electric motor, and a rotor system containing an aluminium hub with three flexible plastic blades. A mechanical actuator was employed in order to identify the model’s dynamic properties,
such as natural frequencies of vibration and modal damping ratios. The ultimate goal of the testing was to experimentally obtain the transfer function of the coupled tower-blades model for use in a stochastic analysis.

Chapter 8 is the second experimental chapter and details a series of wind tunnel tests carried out using the model introduced in Chapter 7. The testing was carried out at the wind tunnel in the National University of Ireland, Galway. The model was immersed within a turbulent wind flow and it's response, in the form of acceleration and strains, was recorded. The mean components of the response parameters was extracted leaving the fluctuating components that may be represented using random vibration theory. An analytical model was proposed in order to validate the random vibration of the model in the wind tunnel, using the wind properties measured in the wind tunnel and the transfer functions obtained in Chapter 7.

The last chapter presents a summary of the work executed, and includes the conclusions that may be drawn from the analytical and experimental results obtained throughout the thesis. Some recommendations for future research are also offered, in order to better understand the nature of wind loading on a wind turbine tower, and it's associated structural response.
CHAPTER 2 - THEORETICAL CONSIDERATIONS

2.1 INTRODUCTION
This chapter introduces and explains the relevant theory and mathematical expressions underpinning the concepts addressed in later chapters. The aspects focused on include wind creation, wind loading classification, mathematical procedures to quantify structural response to dynamic loading, wind effects on structures, and a brief description of the components of contemporary wind turbine towers, as well as their associated dynamics.

2.2 ATMOSPHERIC PHYSICS
Wind may be defined as the movement of an air mass relative to the surface of the earth. It occurs due a pressure difference between points in space at equal elevations. This phenomenon is fundamentally caused by the differential heating of the Earth’s atmosphere by solar radiation. Although the sun’s energy instigates this phenomenon, it is the earth itself that facilitates the creation of wind. The earth’s atmosphere is largely transparent to solar radiation, and radiation with a wavelength greater that 3.0 μm is mostly transmitted through the atmosphere without attenuation (Irish Meteorological Service). Thus, the portion of solar radiation not absorbed by the atmosphere or scattered back into space is absorbed by the earth’s surface. The earth, upon receiving this energy, heats up and emits its own energy in the form of terrestrial radiation, typically with wavelengths of the order of 10.0 μm (Simiu and Scanlan, 1996). The atmosphere is however, capable of absorbing terrestrial radiation, and does so, re-emitting some of this energy back towards the earth’s surface.

While the resulting temperature differences in the atmosphere bring about the creation of wind, several other atmospheric processes also contribute (Simiu and Scanlan, 1996). These include atmospheric radiation between layers of air, the expansion and contraction of air, molecular and eddy conduction, and the contraction and evaporation of water vapour. Next, it is necessary to consider the mechanisms that affect the motion of wind, focusing on the wind closest to the earth’s surface, which is mainly of interest to engineers. Three mechanisms are responsible for the direction of wind flows (Simiu and Scanlan, 1996). The first is known as the horizontal pressure gradient force, and this force or pressure initiates the horizontal movement of an air mass. Air will simply move from an area of high pressure to an area of low pressure. The second process is due to the earth’s rotation. When a particle of air moves in the atmosphere without being subjected to any
external force, it will in theory continue in a straight line. However, as the earth spins on an axis 66° 30' (Simiu and Scanlan, 1996) to its elliptical orbit around the sun, the air particle will experience an apparent force, known as the Coriolis force, which will cause it to deviate from the straight line. Due to the earth’s rotation, in the northern hemisphere, a wind flow travelling north will deflect to the east, and a south bound wind flow will deflect to the west. The reverse is true to an observer in the southern hemisphere. The last process that impacts the motion of wind is friction. When any fluid passes over a surface, a boundary layer is formed due to friction between the fluid and the surface. The magnitude of the boundary layer usually depends on the roughness of the surface and the associated frictional forces are a function of the fluid velocity and viscosity (Hamill, 1995).

The flow of air over the earth’s surface is also subject to the laws of fluid mechanics, and the boundary layer created is known as the atmospheric boundary layer. It is characterised by a zero velocity flow condition at the earth’s surface with increasing wind velocities with height above the ground. The wind velocity is greatest at the top of the atmospheric boundary layer, which is known as the gradient speed. The depth of this layer can range from between a few hundred meters to several kilometres, depending on the terrain type and wind intensity (Simiu and Scanlan, 1996). For normal winds, it will extend over a height of between 1-2 kilometres (Jeary, 1997). The reduction in wind velocity at the earth’s surface is due to frictional drag between the air flow and any elements that protrude into the wind flow, such as structures, trees and mountains. These frictional drag forces are transmitted through the layers of air by Reynolds stresses and by exchange of momentum due to the vertical movement of air (ESDU, 1974), and subsequently arrest the velocity of the wind. At the top of the atmospheric boundary layer, these Reynolds stresses have effectively dissipated and discontinue to affect the flow.

It is within the atmospheric boundary layer that turbulence originates. Turbulent flow is characterised by fast moving chaotic motion in which the particles move randomly (Hamill, 1995). In wind engineering, turbulence is typified by the presence of swirling vortices of rotating air of various sizes. The aforementioned exchange of momentum due to the vertical movement of air is attributed to the formation and decay of turbulent eddies (ESDU, 1974). These eddies mix together in three dimensional space, and the resulting fluctuations in wind velocities are termed gusts. Gusts vary in both space and time.
The problem of developing a coherent and complete understanding of turbulence in fluids has notoriously been difficult, even inspiring the British mathematician Horace Lamb (1849 – 1934) in 1932 to comment as “When I die and go to heaven, there are two matters on which I hope enlightenment. One is quantum electrodynamics and the other is turbulence of fluids. About the former, I am really optimistic”. However, some insight was provided in 1941 by the Russian mathematician Andrey Nikolaevich Kolmogorov (1903 - 1987). His ideas, including his first (Kolmogorov, 1941a) and second (Kolmogorov, 1941b), hypotheses helped yield what is known as the Kolmogorov cascade. The basis of this concept is that large turbulent eddies keep interacting with each without losing energy. They invariably make smaller eddies due to this interaction, and once the eddies reach a small enough size, their energy may be dissipated through viscosity. Thus, there exists an inertial sub range, comprised of eddies of intermediate scales, above which eddies pulsating at certain frequencies may dissipate their energy through viscosity. He also postulated that the energy contained within the inertial sub-range follows the ‘−5/3’ law. This law states that in a logarithmic plot of turbulent spectral energy against frequency or wave number, the slope of the plot in the inertial sub range equals −5/3.

This work by Kolmogorov allowed Von-kàrmàn (1948), and subsequently others, to experimentally quantify the energy contained within turbulent wind flow, including that in the inertial sub range, and propose mathematical expressions which one day would form the basis of contemporary stochastic wind engineering.

2.3 RANDOM VIBRATION THEORY APPLIED TO WIND LOADING

Because wind turbulence varies in both space and time, it is convenient to express the occurrence of turbulence in terms of random vibration theory. A time-variant wind velocity time-history is illustrated in figure 2.1.

Applying random vibration theory to this time-history facilitates the segregation of the time-history into a mean or average component, and a time-varying fluctuating component. This concept may be extended to all aspects of random-theory based wind engineering, such as drag force, and even response quantities such as displacement, base shear and base bending moment of a structure.
MEAN COMPONENT OF WIND VELOCITY

The mean component of wind velocity is time-invariant and pseudo-static in nature, and as mentioned in the section on the atmospheric boundary layer, varies with height above the earth’s surface. Two models have been proposed to quantify the variation of mean wind velocity with height. The first model was proposed by Hellman (1916) and is known as the power law. It is expressed as

\[ \bar{v}(z_1) = \bar{v}(z_2) \left( \frac{z_1}{z_2} \right)^\alpha \]  

(2.1)

where \( \bar{v}(z_1) \) and \( \bar{v}(z_2) \) denote the mean wind velocity at elevations \( z_1 \) and \( z_2 \) respectively, and \( \alpha \) is an exponent. However, this model is subject to several inaccurate assumptions. A second model is regarded by meteorologists as being superior to the power law model, known as the logarithmic law. This model, which stems from work by Millikan (1938), Schubauer and Tchen (1961) and Csanady (1967), is expressed as

\[ \bar{v}(z) = \left( \frac{1}{k} \right) \nu \ln \left( \frac{z}{z_0} \right) \]  

(2.2)
where $\bar{v}(z)$ is the mean wind velocity at height $z$, $k$ is known as Von-kármán's constant (usually about 0.4), $v_*$ is the friction velocity and $z_0$ is the roughness length. The friction velocity is equal to the square root of the ratio of the shear force at the surface of the boundary layer, $\gamma$, to the density of air, $\rho$, as

$$v_* = \sqrt{\frac{\gamma}{\rho}} \quad (2.3)$$

The expressions given by equations 2.1 and 2.2 are both used in this thesis.

**FLUCTUATING COMPONENT OF WIND VELOCITY**

Applying a statistical analysis of a measured wind velocity time-history shows that the distribution of data is approximately Gaussian or Normally distributed. When the mean component is removed from a wind velocity time-history, a fluctuating component with zero mean is left remaining. A plot of the probability distribution for the fluctuating component of the velocity time-history is illustrated in figure 2.2.

![Figure 2.2 Probability density function for Normal/Gaussian distribution](image)

The probability density function, $f(x)$, which describes the distribution of variable $x$ about a zero mean is given by the expression (Nigam and Narayanan, 1994)

$$f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp \left[ -\left( \frac{x}{\sigma_x} \right)^2 \right] \quad (2.4)$$
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where \( \sigma_x \) is the standard deviation of the variable \( x \). The first moment of area about the \( x = 0 \) axis is termed the mean, \( \mu \), and is given by the expression

\[
\mu = \int_{-\infty}^{\infty} xf(x)dx = 0
\]  
(2.5)

The second moment of area about the \( x = 0 \) axis is termed the variance, and is equal to

\[
\sigma_x^2 = \int_{-\infty}^{\infty} x^2 f(x)dx
\]  
(2.6)

The standard deviation is a measure of how the data is spread out from the mean, and is found as the square root of the variance.

Next, it is convenient to investigate the relationship between the two values of a fluctuating wind velocity time-history acting at the same point in space, but separated by a time lag \( \tau \). This relationship provides information about the size of eddies or gusts contained within the air flow, and is known as covariance. There are nine covariance functions in total, corresponding to the three orthogonal axes of Euclidean space (xyz), but the most important are the autocovariance functions. These act along the same axis only. They are obtained as the mean of the product of the value of a fluctuating velocity component occurring at the same point, but at different times \( t \) and \( t + \tau \), as

\[
C_{uu}(\tau) = \overline{U(t)U(t+\tau)} = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} U(t)U(t+\tau)dt
\]  
(2.7)

in which \( U \) denotes the \( u \) (longitudinal), \( v \) (transverse) or \( w \) (vertical) fluctuating velocity components and \( T \) is the averaging time period. Figure 2.3 illustrates the autocovariance function as a function of \( \tau \). When the autocovariance functions are normalised by their respective variances, they become what it known as autocorrelation functions. A Fourier Transform of an autocorrelation function yields a very useful representation of the magnitude of turbulence known as a power spectral density function (PSDF).
The space and time dependent power spectral density, $S_{yy}(z,f)$ of a fluctuating wind time-history is a measure of the energy contained within turbulent wind as a function of frequency.

It can be useful to conceptualise wind flow as being a series eddies or vortices of varying size and speed being carried along by the mean wind component, which eventually strike some structure. Depending on their size, these eddies impart their kinetic energy into the structure at certain frequencies. The power spectral density function relates the size of kinetic energy to the size of gust. Considerable research has been carried out to fit a mathematical functional to this distribution of energy with frequency. The velocity spectrum primarily considered in this thesis was first proposed by Kaimal et al (1972), and is represented by

$$\frac{fS_{yy}(z,f)}{\sigma_i^2} = \frac{200n}{(1+50n)^{5/3}}$$

(2.8)

where $f$ denotes frequency (Hz), and $n$ is known as the Monin coordinate, obtained from the expression

$$n = \frac{fz}{\bar{v}(z)}$$

(2.9)
Equation 2.8 clearly represents the energy distribution in the inertial sub range as following the '-5/3 law' (Simiu and Scanlan, 1996). Other expressions for the fluctuating velocity PSDF have been provided by von Kármán (1948), Davenport (1961b), and Harris (1971).

![Comparison of wind velocity PSDFs used in wind engineering](image)

Figure 2.4 Comparison of wind velocity PSDFs used in wind engineering

The expressions proposed by the latter authors are compared within that of Kaimal et al (1972) in figure 2.4.

The wind velocity spectrum experienced by rotating blades however, is somewhat different to that represented by equation 2.8. Due to the rotation of the blades, energy is shifted from the lower frequency range to multiples of the rotational frequency of the blades, giving rise to what is known as a rotationally sampled spectrum. This phenomenon is illustrated in figure 2.5, taken from Harrison et al (2000). The model spectrum is the typical fluctuating velocity spectrum like that of equation 2.8. The rotational spectrum is that as experienced by the blades, with the redistribution of energy being clearly visible. In effect, eddies of specific sizes are created due to the rotation of the blades and subsequently impart their energy into the blades.

The relationship between the fluctuating velocity components at two points separated by a given distance is also of interest, and is known as spatial or cross-covariance. This
relationship may or may not include a time lag, $\tau$. The cross covariance function for two points $s_1$ and $s_2$ is,

$$C_{UU}(s_1, s_2; \tau) = \frac{U(s_1, t)U(s_2, t + \tau)}{\langle U(s_1, t)U(s_2, t) \rangle} = \lim_{T \to \infty} \frac{1}{T} \int_0^T U(s_1, t)U(s_2, t + \tau) dt \quad (2.10)$$
where $U$ denotes the $u$, $v$, or $w$ component of the fluctuating velocity. Figure 2.6, taken from ESDU (1974) illustrates the longitudinal cross-covariance function as a function of $\tau$. A negative value of cross-covariance may be obtained as either of the fluctuating components of velocity may be negative, so the product and mean product may become negative.

An average length of eddies along an appropriate direction may be obtained, and is known as an integral length scale of turbulence. It is obtained by the integration of the spatial or cross-covariance function over the total range of the appropriate variable, divided by the variance (mean square) of the variable. There are nine integral length scales, for three gust components along the $u$, $w$ and $v$ axis, along the three orthogonal dimensions of Euclidean space, $x$, $y$, and $z$. The three integral length scales associated with the longitudinal component of turbulent, $u$, are shown in figure 2.7.

\[ L_u = - \frac{\sigma_u^2}{C_{uu}(x; \tau)} dx \]  

(2.11)

Figure 2.7 Integral length scales for the longitudinal component of turbulence ‘$u$’
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where \( \sigma_u \) is the standard deviation of the longitudinal component of the wind velocity. It is also equal to the integration of the autocorrelation function. Counihan (1975) gives the following expression for \( L_u^* \), for a height range of 10-240m,

\[
L_u^* = Cz^m
\]  

(2.12)

where \( C \) and \( m \) are obtained from an empirical derived plot. According to Counihan (1975), the integral length scales, \( L_u^* \) and \( L_u^z \) are approximately equal to one third and one half of \( L_u^* \), given by equation 2.12. Duchène-Marullaz (1980) offer an estimate of \( L_u^* \) as equal to a fifth of \( L_u^* \) and Clough and Penzien (1993) suggest an expression for \( L_u^z \) with \( z \) being the elevation in metres, as

\[
L_u^z \equiv 6z^{0.5}
\]  

(2.13)

2.4 NECESSARY ASPECTS OF STRUCTURAL DYNAMICS

Structures may be subjected to two possible kinds of vibrations, free and forced vibrations. In carrying out a dynamic analysis, it is usual for an engineer to conveniently discretize a structure into ‘\( m \)’ degrees of freedom. The structure, thus, becomes a multi degree-of-freedom (MDOF) entity, allowing the structure’s stiffness, mass, damping and loading characteristics to be formulated into a series of matrices and the equations for free and forced vibrations easily derived, ready for the solution of all unknowns.

Free vibrations occur when the structure is subjected to initial conditions, such as an initial displacement or velocity, and set in motion. Free vibrations do not result from any external loading or forcing, and if the structural damping is low, it may be omitted and later incorporated in the forced vibration phase of the dynamic analysis. Thus, for undamped free vibration motion, the motion is harmonic and does not decay. For a MDOF system, the equation of motion which represents free vibrational motion is

\[
[M][\dddot{u}(t)] + [K][u(t)] = \{0\}
\]  

(2.14)

where \([M]\) and \([K]\) are the ‘\( n \times n \)’ mass and stiffness matrices, with \( n \) being the number of nodes created in the discretization process, \( \{\dddot{u}(t)\} \) is the time-variant nodal acceleration.
vector and \( \{u(t)\} \) is the nodal displacement vector. With some mathematical manipulation, equation 2.14 can be changed into a characteristic equation, where the eigenvalues are the system’s natural frequencies, and the eigenvectors are the system’s mode shapes. It is possible to have damped free vibrations, but the motion will decay with time. Also, if the structural damping is low, the damped natural frequencies are approximately equal to the undamped natural frequencies.

When the MDOF system is subjected to external loading, be it steady-state and/or transient, the vibratory motion that ensues is classed as a forced vibration. The equation of motion that describes a MDOF system in forced vibrational motion is

\[
\begin{bmatrix} M \end{bmatrix} \{\ddot{u}(t)\} + \begin{bmatrix} C \end{bmatrix} \{\dot{u}(t)\} + \begin{bmatrix} K \end{bmatrix} \{u(t)\} = \{P(t)\} \tag{2.15}
\]

where \([C]\) is an ‘n x n’ damping matrix, \([P(t)]\) is the loading vector and \(\{\dot{u}(t)\}\) is the nodal velocity vector. Both equations 2.14 and 2.15 are cast in the time-domain, meaning that they use Newtonian mathematics to describe the motion of the system as a function of time. The solution of equation 2.15, when structural behaviour is linear only, and with classical damping, is obtained by virtue of the fact that the nodal displacement, may be expressed as the weighted sum of the product of two independent spatial and temporal variables, as

\[
\{u(t)\} = \sum_{j=1}^{m} \Phi_j \{\eta_j(t)\} \tag{2.16}
\]

where \(\Phi_j\) is known as a shape function or mode shape of mode ‘j’, \(\eta_j(t)\) is known as the generalised/modal/principal coordinate of mode ‘j’ and ‘m’ is the number of modes being used to represent the response. Substituting equation 2.16 into equation 2.15 and pre-multiplying by the inverse of the shape function, renders the forced vibration equation of motion uncoupled as

\[
\{\ddot{\eta}_j(t)\} + 2\xi_j\omega_j \{\dot{\eta}_j(t)\} + \omega_j^2 \{\eta_j(t)\} = \frac{\{F_j(t)\}}{M_j} \tag{2.17}
\]

where \(\xi_j\) denotes the modal damping, \(\omega_j\) denotes the circular natural frequency, \(\{F_j(t)\}\) denotes the time-varying modal force, \(M_j\) denotes the modal mass, subscript ‘j’ denotes
the \( j^{th} \) mode of vibration and an over-dot represents a temporal differentiation. The quantity of the right hand side of equation 2.17 is known as the modal participation factor.

Equation 2.16 is known as the mode displacement technique as the modal displacement coordinate is only used to evaluate the response. Another technique, termed the mode acceleration technique, is also available. According to Craig (1981), it is superior to the mode displacement technique in equation 2.16, as fewer modes of vibration are needed to converge the solution to a desired level of accuracy. Rewriting equation 2.15 in terms of the nodal displacement yields

\[
\{u(t)\} = [K]^{-1}\{P(t)\} - [C]\{\dot{u}(t)\} - [M]\{\ddot{u}(t)\} \tag{2.18}
\]

Inserting the modal displacement solution of equation 2.16 into equation 2.18 gives

\[
\{u(t)\} = [K]^{-1}\{P(t)\} - [K][C]\sum_{j=1}^{m}\Phi_j\{\ddot{\eta}_j(t)\} - [K][M]\sum_{j=1}^{m}\Phi_j\{\dddot{\eta}_j(t)\} \tag{2.19}
\]

Equation 2.19 can be simplified to give

\[
\{u(t)\} = [K]^{-1}\{P(t)\} - \sum_{i=1}^{m}\left(\frac{2\zeta_i}{\omega_i}\right)\Phi_i\{\dddot{\eta}_i(t)\} - \sum_{i=1}^{m}\left(\frac{1}{\omega_i^2}\right)\Phi_i\{\dddot{\eta}_i(t)\} \tag{2.20}
\]

The solution of the modal coordinate and its derivatives needed in equations 2.17 and 2.20 are obtained in this thesis using the approach developed by Nigam and Jennings (1968).

The solution of equation 2.15, when structural behaviour is non-linear, and with arbitrary damping, may be obtained through use of a step-by-step integration method, such as the average acceleration method or the linear acceleration method, as proposed by Newmark (Clough and Penzien, 1993).

### 2.5 Wind Effects on Structures

As well as the obvious amplification effect of dynamic wind loading on flexible structures, the spatial correlation of wind forces over the surface of a body, and especially along the
height of a tall body, is also important. Spatial correlation relates the similarity of pressures or forces over the surface of a body. It is a function of frequency, or the size of eddies enveloping the structure. Since turbulent wind may be visualised as an array of eddies being carried along by a mean wind, if the size of the structure is small compared to the dominant eddy size, then the aerodynamic force will be the same (fully correlated) over the entire structure. However, if the size of the structure is much larger than the dominant eddy size, then the aerodynamic forces will only be partially correlated on the structure. On this understanding, the aerodynamic admittance factor was introduced, to represent partially correlated forces on a body. An expression for this factor was obtained empirically, and given by Nigam and Narayanan (1994) as

$$\chi = \frac{1}{1 + \left(\frac{2fb}{\bar{v}} \right)^5}$$  \hspace{1cm} (2.21)

where $\chi$ represents the aerodynamic admittance function. This function is dependant on three parameters, frequency, $f$ (size of gust eddy), width, $b$, and nodal mean wind velocity, $\bar{v}$. It is used to adjust a velocity or force spectrum, to allow for the possibility of force/pressure not being fully correlated.

Tall structures are faced with a similar problem, in that forces or pressures may not be fully correlated over the entire height of the structure. In effect, all forces along the height of the structure will be correlated to some degree, and the strength of this correlation is termed coherence. Coherence is a measure of how well two signals (fluctuating wind velocities/forces) are related to each other when separated by some distance. It is linked to the idea of cross-covariance introduced in Section 2.3, but is a Fourier transform representation. The coherence function proposed by Davenport (1968) is

$$\text{coh}(s_1, s_2; f) = \exp \left( -\frac{|s_1 - s_2|}{L_s} \right)$$  \hspace{1cm} (2.22)

where $|s_1 - s_2|$ is the spatial separation between points $s_1$ and $s_2$, and $L_s$ is a length scale given by

$$L_s = \frac{\hat{v}}{fD}$$  \hspace{1cm} (2.23)
where
\[ \tilde{\nu} = 0.5\left[\bar{v}(s_1) + \bar{v}(s_2)\right] \] (2.24)

in which \( D \) is a decay constant, and \( \bar{v}(s_1) \) and \( \bar{v}(s_2) \) are the mean wind velocities at points \( s_1 \) and \( s_2 \) respectively. Davenport’s expression for coherence is used in this thesis.

### 2.6 Wind Turbine Towers

The horizontal axis wind turbine tower (HAWT) is by far the most common type of wind turbine assembly in use worldwide. It consists of three main modules, the tower, the nacelle and the rotor system and is illustrated in figure 2.8.

![Figure 2.8 Horizontal axis wind turbine](NORDEX GmbH)

#### 2.6.1 The Tower

The function of the tower is to support the nacelle and rotor system and transfer all net loads to the foundation. The tower may be constructed from steel reinforced concrete, pre-
stressed concrete, tapered tubular steel, or steel members arranged to form a lattice frame. Concrete towers have been used in Denmark, but were very heavy in comparison with steel tubular or lattice towers. The lattice tower, as shown in figure 2.9, consists of steel sections joined together, and these are easily transportable and built up on site.

Figure 2.9 Lattice tower supporting nacelle and rotors
(NORDEX GmbH)

Figure 2.10 Tubular tower supporting nacelle and rotors
(NORDEX GmbH)

The availability of concrete suppliers in the locality of the tower erection site has a direct economic bearing on the choice of concrete as viable material, and this has contributed to the decline of concrete towers. The lattice option enjoyed initial popularity, but a sharp decline in its usage has become apparent of late, as it is deemed not aesthetically pleasing, though this is clearly debateable. The lattice tower will use approximately half the material of the tubular tower. Be that as it may, the tapered tubular steel section is by far the most commonly used worldwide. The steel tubular tower, as in figure 2.10, is constructed from curved steel plates which are welded together to form a conical section. The erected tower may comprise two or three of these sections lifted one on top of the other by crane. A ladder runs the length of the tower, allowing maintenance personnel protected access to the nacelle.
2.6.2 The Nacelle

The nacelle is a protective casing that houses the electrical and mechanical equipment needed to generate electricity. It is usually manufactured from steel. It contains the various shafts which connect the rotor blades, via the gearbox, to the electrical generator, as is illustrated in figure 2.11. For a typical 1 mega-watt (MW) three blade assembly, the rotors will turn relatively slowly, usually in the order of twenty to thirty revolutions per minute (Danish Wind Engineering Association). The rotational energy created by this, is transferred from the rotor blades into the gearbox via the low speed driveshaft.

![Diagram of nacelle containing shafts, gearbox and electrical generator](NORDEX GmbH)

The gearbox accepts this rotation and steps up the frequency by a factor of approximately fifty (Danish Wind Engineering Association). This rotary energy is then transmitted to the electrical generator via the high speed shaft. The electrical generator, usually an induction or asynchronous generator, is now capable of an electrical output of the magnitude of 600 to 3000 Kilowatts.

2.6.3 The Rotor System

The rotor system consists of two parts, the rotor hub and the rotor blades. Both components are usually made from glass fibre reinforced plastic (GRP). The hub is usually conical and is connected to the nacelle. The blades are the long members which stretch out radially from the hub, as viewed in figure 2.12. The function of the blades is to collect kinetic energy imparted by the wind and transfer this energy into the electrical generator housed in
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the nacelle. The contemporary wind turbine concept, sometimes dubbed the 'Danish concept' is to have three blades in the rotor system. Wind turbines with two or even one blade(s) have been built though are still considered experimental. While a reduction in the number of blades has an obvious economic advantage, one or two blade units need to run at a higher speed of rotation which usually increases acoustic pollution in the vicinity of the tower. The main reason for three blades, or at least an odd number of blades, is in the stability of the wind turbine. A rotor with an odd number of blades (at least three) may be considered to be a disc when calculating the dynamic properties of the system. When all three blades are undergoing vibration, the bending moment at the hub is theoretically assumed to equal zero.

![Figure 2.12 A three blade rotor and hub system (NORDEX GmbH)](image)

When one blade enters the wind shade of the tower, the aerodynamic loading on it decreases and hence the blade root bending moment decreases, creating a net bending moment at the hub. If there are still two blades undergoing full aerodynamic loading, this net bending moment is minimised. This phenomenon is of greater interest if only two or one blade(s) are present.

The outermost part of the blade resembles an aircraft wing, a so-called aerofoil, and when immersed in a fluid flow behaves similarly, being subject to the same aerodynamic forces, namely drag and lift. The shape of the wind turbine blade is of interest, as it determines the efficiency of the conversion of kinetic energy to mechanical energy. Figure 2.13 illustrates
a typical blade section and demonstrates the generation of the lift force that is instrumental in moving the blade. The wind flow (shown in green) separates on contact with the blade. Due to the geometric shape of the blade, the flow moving over the top of the blade moves faster than that on the underside of the blade. There is a consequent reduction in pressure on the top of the blade, bringing about a suction force (displayed in orange) called lift, compelling the blade to move upwards. Thus, the blades are set in rotary motion by a combination of drag and lift forces.

2.6.4 Tower and blade dynamics
This section explains the types of dynamic loading and response to which the wind turbine towers and blades are subjected throughout their operational life. Both tower and rotating blades constitute flexible continuous entities, each subjected to wind loading. Some intricate dynamic loadings must be considered due to blade rotation and blade/tower dynamic interaction.

TOWER DYNAMICS
The tower is primarily subjected to aerodynamic loading i.e. fluctuating wind drag forces over its height, which cause it to vibrate. It is also subjected to dynamic tower/blade interaction forces, in the form of a shear force imparted at the top of the tower. This is caused by unbalanced aerodynamic loading on the blades due to their motion in and out of the towers wind shade and due to a reduction and sudden increase in tower wind drag loading due to this same blade movement in and out of the wind shade. As the tower is continuous, it will in theory have an infinite number of natural frequencies and mode
shapes to characterise its free vibration behaviour. In practice, only the lower modes of vibration (usually the fundamental alone) will be of interest to the engineer. These frequencies and shape functions will depend on the geometry and material properties of the tower, namely the length, mass per unit length, elastic modulus and mass moment of inertia. The mass and rotary inertia of the nacelle sitting on top of the tower will also affect the dynamic characteristics of the tower. Harrison et al (2000) state that the most common stiffness approach for modern three blade units is to have the first tower bending frequency in the region of $1.5 \Omega$, where $\Omega$ is the blade rotational frequency in radians per second. Towers of this stiffness are deemed as ‘soft’.

**BLADE DYNAMICS**

The rotating blades are subjected to three main types of loading (Hansen, 2000), gravity loading, inertial loading, and aerodynamic loading. Gravity loading is derived from the effects of self-weight on the rotating blades. This will cause a dynamic axial force along the length of the blade, and a dynamic bending moment and stress reversal at the blade root. Inertial loading will be experienced by the blade, when it accelerates or decelerates, with the force created being in the direction of rotation. A centrifugal force will also be experienced by the blades due to their rotation, if the blades are sloped back towards the tower. This phenomenon is known as coning and is used to reduce the bending moments at blade root. The centrifugal force, which is acting vertically, will have two components, the first acting along the direction of the blade’s longitudinal axis, and the second acting at right angles. Aerodynamic loading comes from the action of the dynamic wind force against the blade. Imbalance effects also contribute to rotor loading; if the centre of gravity of one of the blades is not at its centre of rotation, extra dynamic gravitational and centrifugal bending moments are created (Harrison et al, 2000).

![Figure 2.14 Tapering geometry of wind turbine blade.](image-url)
If the blades do not have the same mass distribution, an extra net centrifugal force is created due to rotation which will be transmitted into the tower. Rotor blades will primarily vibrate in flexure along two orthogonal planes, causing natural frequencies and mode shapes in the flapping direction and in the lead/lag direction. The degrees- The flapping natural frequency will be the lower of the two due to the lower stiffness of the blade with respect to this deformation. Figure 2.14 illustrates the tapering nature of wind turbine blades, and suggests the distribution of material available for flapping and lead/lag motions. For what is termed as a 'stiff' blade, Harrison et al (2000) give the flapping frequency as being usually between 3 \( \Omega \) to 4 \( \Omega \), and the lead/lag frequency between 5 \( \Omega \) to 6 \( \Omega \). Blades may also be subject to dynamic axial loadings requiring knowledge of the axial modes, and twisting motions, requiring an analysis of the torsional modes. Figure 2.15 illustrates the degrees-of-freedom typically associated with the dynamic response of a wind turbine blade.

Figure 2.15 The degrees-of-freedom associated with the dynamic response of a wind turbine blade
CHAPTER 3 - REVIEW OF LITERATURE

3.1 INTRODUCTION
This chapter presents a brief overview of past research relevant to the themes contained within this thesis. The scope of this thesis is an investigation into wind effects on flexible structures, and in particular the wind turbine tower. This chapter reviews all published work consulted during the tenure of this thesis, under the broad heading of wind effects on structures. This topic may be subdivided into several constituent parts, starting with the free vibrations of structures, certain elements of atmospheric physics and the classification of wind loading through mathematics, and finishing with forced vibrations of structures. Included in the latter is a review of published experimental research on the system identification of vibrating systems, and details of wind tunnel testing, along with published results on the responses of structures, and evaluation of aerodynamic damping.

3.2 WIND EFFECTS ON STRUCTURES AND WIND TURBINE TOWERS
As mentioned in Chapter 1, modern wind engineering is perhaps only forty to fifty years old. Hence, there has only been a relatively small number of practitioners who would have reached expert status. Two of which, Emil Simiu and Robert Scanlan have compiled a definitive and comprehensive text entitled ‘Wind Effects on Structures’ (Simiu and Scanlan, 1996), the latest being the third edition, which truly represents a holistic guide to current wind engineering theory. This text contains detailed explanations of the formation of wind, mathematical along-wind and across-wind structural response estimation, advice on wind tunnel testing and applications to design scenarios. The text is mathematically rigorous, especially in dealing with random vibration theory. Nigam and Narayanan (1994), in an extremely inclusive presentation of the applications of random vibrations, take a chapter to review current wind engineering practice, focusing mainly on structural response to stochastic wind loading. This includes an introduction to the gust response factor methodology, taking into account improvements such as aerodynamic admittance and acceptance.

Text books on the design considerations of modern wind turbine towers are only lately starting to make their way onto the market, written mainly by experts from Northern Europe. Harrison et al (2000) provide a brief introduction into the modern history of wind turbine towers in Europe and comment lightly on the dynamics of the components of the system. The treatment of the dynamic intricacies undoubtedly experienced by all towers
and blades is not particularly rigorous. However, the authors presented a thorough discussion on the economics of wind turbine technology instead. Their handling of design issues such as nacelle layout and mechanical configuration, component material and weight and control of rotors in unfavourable wind climates is quite in-depth. Hansen (2000) tackles the design problem more from the perspective of fluid dynamics, giving a comprehensive explanation into two and three dimensional aerodynamic theory applied to wind turbines. Hansen also deals with necessary issues such as the control of the rotor system, and provides an inclusive review of all sources of loading experienced by the rotor system. The text also addresses, though vaguely, the important topic of blade fatigue due to stress reversal.

Government sponsored research institutions have also contributed greatly to the design of wind turbine towers and rotor systems, providing easily obtainable reports aimed at helping manufacturers develop comprehensive design methodologies. In the United States, the National Renewable Energy Laboratory (NREL) based in Colorado has issued a series of detailed reports termed ‘Design Guidelines’, providing a review of the current design regulations necessary for the certification of wind turbine towers in the United States. Topics covered include design of yaw and pitch systems (NREL, 2000a), gearbox design (NREL, 2000b), the origin of loading (NREL, 2003), structural strength and fatigue requirements (NREL, 2004a), and control and protection systems (NREL, 2004b).

3.3 ATMOSPHERIC PHYSICS

Although the problem of turbulence is, to quote the Nobel Physics Laureate Richard Feynman, “the last great unsolved problem in classical physics”, the Russian mathematician Andrey Nikolaevich Kolmogorov (1903-1987) contributed significantly to the current understanding of turbulence in wind flow. He published the first of his famous hypotheses (Kolmogorov, 1941a) which stated that “For the locally isotropic turbulence the [velocity fluctuation] distributions are uniquely determined by the kinetic viscosity, and the rate of average dispersion of energy per unit mass [energy flux]”. In the same volume of the journal ‘Comptes Rendus (Doklady) de l’Académie des Sciences de l’URSS’, he published his second hypothesis (Kolmogorov, 1941b) stating that “For pulsations [velocity fluctuations] of intermediate orders where the length scale is large compared to the scale of the finest pulsations, whose energy is directly dispersed into heat due to
viscosity, the distribution laws are uniquely determined by energy flux and do not depend on viscosity."

These two concepts allowed Kolmogorov to establish the so called ‘Kolmogorov cascade’ explained in Chapter 2, and allowed the distribution of energy within a turbulent flow to be quantified as a function of eddy size or frequency. On the back of this work, Theodore Von-kàrmàn (Von-kàrmån, 1948) was the first to propose a mathematical expression for the distribution of energy within turbulent wind flow, yielding a velocity spectrum at a height related to the size of gusts at that height (integral length scales). However, this expression was found to be inconsistent with the Kolmogorov cascade idea, regarding the size of eddies in the inertial sub range, and is not used in circumstances, where the higher frequency components of the spectrum are expected to be of importance. Davenport (Davenport, 1961b) subsequently proposed another expression for the velocity spectrum, but this expression was also subject to inherent flaws. The spectrum was independent of height, the expression being obtained as an average of measurements made at several different heights. The expression was also seen to overestimate the energy in the higher frequency range of the spectrum by as much as 100% to 400% (Simiu and Scanlan, 1996) and implied that the spectrum has a zero value at zero frequency, which according to Lumley and Panofsky (1967) is erroneous.

Harris (1971) was next to propose an expression for the velocity spectrum, which was also independent of height. It did, however, guarantee a non-zero integral length scale of turbulence. The Davenport and Harris spectra both used a mean wind velocity at a reference height of 10 m. Harris (1990) subsequently provided an expression based on a modified version of the Von-kàrmàn spectrum to include the variation of spectral energy with height. Kaimal et al (1972) developed the first expression for this variation of spectral energy with height. Their expression correlated quite closely with the energy contained within the inertial sub range, but still implies zero spectral energy and zero frequency. However, the Kaimal spectrum represents the best available quantification of the spectral energy contained within the wind.

Connell (1980) reported that a rotating blade is subjected to an atypical fluctuating wind velocity spectrum, known as a rotationally sampled spectrum. Due to the rotation of the blades, the spectral energy distribution is altered, with variance shifting from the lower frequencies to peaks located at integer multiples of the rotational frequency. Kristensen
and Frandsen (1982), following on from earlier work carried out by Rosenbrock (1955), developed a simple model to predict the power spectrum associated with a rotating blade, which was significantly different to a spectrum where the rotation is not considered. Kristensen and Frandsen (1982) used this simple model for the rotationally sampled spectrum and compared it to experimentally obtained results, showing reasonable agreement. Madsen and Frandsen (1984) used the rotationally sampled spectrum to obtain the structural response of rotating blades. The phenomenon of rotational sampling has also been validated experimentally by Verholek (1978). Verholek used eight anemometers equally spaced along the circumference of a circle having a diameter of 24.4 m, with a centre of similar distance above the ground. A similar experiment was carried out by Hardesty et al (1981) who used a CO₂ laser anemometer. Sørensen et al (2002) investigated fluctuations in power generated in wind farms, including the effects of rotationally sampled turbulence on rotor blades. However, it appears that a workable mathematical expression to quantify this rotationally sampled spectrum as a function of height and blade rotational speed has not yet been developed.

3.4 WIND INDUCED STRUCTURAL VIBRATIONS

This section provides a review, mainly based on published journal articles, of the necessary steps the engineer will undertake to investigate the response of flexible structures to dynamic wind loading. Usually, the first step carried out is to obtain the structure’s free vibration properties, which then leads to the forced vibration phase of the dynamic analysis.

3.4.1 Free vibrations

The free vibrations of single-degree-of-freedom (SDOF), multi-degree-of-freedom (MDOF) and continuous systems are covered in most elementary texts on structural dynamics, including Clough and Penzien (1993) and Craig (1981). Both texts deal with free vibration issues such as the derivation of the partial and ordinary differential equations of motion for continuous and discrete systems respectively, forming the necessary mass and stiffness matrices for SDOF and MDOF systems, solving for natural frequencies and mode shapes for continuous and discrete systems, and applying energy methods such as the Rayleigh-Ritz technique. Craig (1981) also addresses the subject of finite element modelling of systems and even introduces the concept of component mode synthesis for systems composed of multi-bodies.
This thesis is especially concerned with the free vibrations of systems including strategically placed lumped masses. Konstantakopoulos (1999) investigated the dynamic properties of towers subjected to additional masses, taking particular interest in the rotary inertia of the masses. Konstantakopoulos investigated the effects of localised lumped masses at certain positions along the length of the tower, on the tower’s eigen-properties. The presence of the lumped masses, along with their rotary inertias, rendered the dynamics of the system strongly non-linear. Konstantakopoulos also estimated the forced vibration response of the system to seismic and wind loading. Clough and Penzien (1993) presented the problem of a continuous cantilever beam with a rigid mass at the free end and derived the boundary conditions. Murtagh et al (2002) solved this equation of motion for free vibrations, obtaining an expression in closed form that yielded the eigenvalues and eigenvectors of the cantilever beam with a rigid mass at its free end. This model is of particular interest to this thesis and its solution is presented in Chapter 4. Similar studies involving beams with concentrated mass have been carried out by Rossi et al (1993), who investigated the free vibrations of Timoshenko beams carrying concentrated masses, and Posiadala (1997) who carried out a similar study. Auciello (1996) investigated the transverse vibration of a linear tapered cantilever beam with a tip mass and Uscilowska and Kolodziej (1998) carried out a free vibration analysis of a column immersed in a fluid while carrying a tip mass. Tomski et al (1994) considered a model of a cantilever beam under a prestressing force along with a concentrated mass and investigated the effects of the prestressing force and the concentrated mass of the beam’s natural frequency.

Recent authors using the finite element technique for free vibration analyses of structures in wind engineering include Bazeos et al (2002), who used the software code NASTRAN in a dynamic analysis of a prototype wind turbine steel tower. The first few eigenmodes of the tower were obtained using this finite element based software for use in a forced vibration analysis due to seismic excitation. Dutta et al (2002) used the finite element method to obtain the free vibration characteristics of structures which were subsequently subjected to tornado loading. Murtagh et al (2002) presented a finite element analysis of lattice towers supporting utility masses, using the software code ANSYS. The free vibration results obtained matched closely with those obtained in closed-form using an equivalent continuous model.
Literature on the calculation of the free vibrations properties of wind turbine blades has not been widely disseminated within structural engineering journals. These properties are usually difficult to obtain due to the complex geometry of the blades and the effects of blade rotation. Baumgart (2002) proposed an analytical model of a wind turbine blade using a combination of finite elements and virtual work, which allowed for the complex geometry of the blade. The modal parameters of the blade obtained from the model showed good correlation with equivalent experimental results. The act of rotation has a considerable effect on the free vibration properties of the blade. This phenomenon is of interest to many industrial fields, such as the previously mentioned wind turbine tower blades, as well as aircraft rotor blades and turbine rotor blades. As a result, considerable research has been conducted towards gaining a comprehensive understanding of the rotordynamics of beams, and in particular, the centrifugal stiffening effects associated with that rotation. This rotation causes an apparent increase in stiffness along the blade.

Naguleswaran (1994) presented an approach to determine the free vibration characteristics of a spanwise rotating beam subjected to centrifugal stiffening. In this publication, the mode shape equation was solved using a power series. Nagelswaran (1994) and Banerjee (2000) both used the Frobenius method to obtain the natural frequencies of spanwise rotating uniform beams for several cases of boundary conditions. Chung and Yoo (2002) used the finite element method to obtain the dynamic properties of a rotating cantilever whereas Lee et al (2001) carried out experimental studies on the same. Interested parties may also refer to subsequent work by Murphy (1998), Fung and Yau (1999, 2001) and Chen and Chen (2002). All studies showed that the natural frequencies rise as the rotational frequency of the blade increases.

As the wind turbine is in fact a multi-body system, its free vibration characteristics must be obtained using a technique such as the substructure synthesis or component mode synthesis method. These approaches decompose the system into several sub-entities whose individual dynamic properties may be evaluated through analytical or experimental methods. The mass and stiffness matrices of the coupled system may then be assembled by envoking compatibility at structural interfaces. A numerical technique such as the assumed modes method, Galerkin method, or finite element method may then be employed to obtain the dynamic characteristics of the multi-body system. Jang et al (2002) used the finite element method in conjunction with substructure synthesis to estimate the free vibration properties of a spinning flexible disk-spindle system, and obtained good agreement with

The literature on the free vibrations of wind turbine towers is not extensive. Lee et al (2002) recently provided a comprehensive mathematical framework which yielded the natural frequencies and mode shapes of a horizontal wind axis turbine system including tower, nacelle and blade interaction. Equations representing the dynamic behaviour of each of these components were first derived separately and then unified, yielding a coupled mathematical system. The eigenvalues and eigenvectors of this system could be extracted using Floquet theory.

3.4.2 Forced vibrations

In carrying out forced vibration analyses, the lumped parameter technique is very popular among researchers due to its relative ease of use. Usually, the only difficulty is in obtaining the stiffness matrix for use in free vibration analysis. Several methods may be employed in this regard, such as using any of the various static analysis software programs to obtain the flexibility matrix or employing a structural analysis technique such as the moment area method. Aksogun et al (2003) recently used the lumped mass technique to represent coupled shear walls in a study of response to time-varying loading. Shrimali and Jangid (2002) modelled continuous liquid contained within a storage tank as a series of convective, impulsive and rigid lumped masses and estimated the seismic response of the tank. Kanda et al (1997) investigated the seismic safety of several lumped mass models and developed a probabilistic procedure to demonstrate this safety. Chang and Tung (1990) modelled the forced vibration of an offshore structure as a lumped parameter system subjected to wave action. In wind engineering, the lumped parameter technique was used by Wu et al (1998) in a study on the control of transmission towers under stochastic wind loading. Lavassas et al (2003) also used this technique to assess the accuracy and reliability of a more computationally expensive finite element analysis of a wind turbine tower.

A wind-induced forced vibration analysis makes use of the two topics discussed previously, the free vibration properties of the structure and the mathematical characterization of the wind loading. Random vibration theory-based forced vibration

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analysis falls into two categories, time domain and frequency domain, and both approaches are employed in the research described in this thesis.

The pioneering work carried out by Davenport during the 1960s shifted the emphasis in wind effects on structures from static-based to dynamic-based modelling. In two seminal publications, Davenport (1961a) redefined wind loading in terms of statistically-based parameters and expressed the dynamic energy contained in the wind in terms of turbulence spectra (1961b). This work allowed for the creation of the frequency domain based systems approach for the response of flexible structure to wind action. This research ultimately led to the publication of his further seminal paper (Davenport, 1967) in which the idea of the gust loading/response factor (GRF) was first introduced. The basis of this concept allowed for an accurate portrayal of the inherent randomness contained in the wind, facilitating the simplification of wind-induced dynamic loading into equivalent static wind loading. The gust loading factor is defined as the ratio of the maximum response parameter to the mean of that response parameter, and although it is just a scalar number, it is derived from the effects of dynamic loading, such as mechanical and aerodynamic admittance. This scalar number, may then be multiplied by the mean wind loading, allowing a flexible structure to be designed with adequate stiffness and strength. The structure was assumed to behave as a single degree-of-freedom entity with a linear mode shape. Velozzi and Cohen (1968) also contributed to early work on the GRF, including the effects of pressure correlation between the windward and leeward faces of the structure. Vickery (1970) slightly augmented the Davenport type methodology to provide increased sensitivity to meteorological parameters, and also investigated non-linear mode shapes. However, the initial GRF methodology was subject to erroneous assumptions, such as turbulence being independent of height.

Subsequent improvements to the GRF approach came from Simiu (1980) and Solari (1982), who both addressed the issue of height dependent turbulence. The appropriateness of the originally assumed linear mode shape has been addressed by several authors, including Tamura et al (1996), and Zhou et al, (2000, 2002b). The GRF was usually obtained using a ratio of displacements, and while this yielded accurate expected maxima for displacements, it was found to fall short in providing estimates of other response maxima, such as bending moment and shear force. In this regard, new models of the GRF have emerged, such as those suggested by Zhou and Kareem (2001), and Holmes (1994), the former being based on base bending moment, rather than displacement. The GRF approach has been applied to lattice towers by Holmes (1994, 1996) and to guyed
structures by Davenport and Sparling (1998). Harikrishna et al (1999) carried out a analytical study on the magnitude of GRFs for a 52 m lattice tower which included experimental validation.

The GRF methodology has become the basis of most modern design codification worldwide (Zhou et al, 2002a), such as those used by the United States (ASCE, 1998), Canada (NRCC, 1996), Japan (Architectural Institute of Japan, 1996), Australia (Australian standards, 1989) and Europe (CEN, 2004).

As wind pressures may not be fully correlated over the surface of a body, the aerodynamic admittance function may be used to relate the size of eddy imparting energy into a structure to the size of the structure. Liepmann (1952) appears to be one of the first authors to introduce the concept of aerodynamic admittance allowing Davenport (1962) to use it in a study of the buffeting of suspension bridges. Bearman (1971) investigated this phenomenon experimentally using a square plate immersed within a turbulent flow. He obtained the well known characteristic shape of the function, to which Nigam and Narayanan (1994) presented a fitted mathematical expression.

Spatial correlation, or coherence relates the similarity of signals measured over a spatial distance within a random field. This is particularly relevant to long structures where the relationship between force components at different points is needed to accurately quantify the magnitude of all loading. Earthquake engineers have studied the relationship between ground accelerations at different points on the earth’s surface; publications in this regard include Hao et al (1989) and Harichandran and Vanmarcke (1986). Coherence is also of great importance to the wind engineer, especially if gust eddies are smaller than the height of a structure. Some of the earliest investigations into the spatial correlation of wind forces were carried out in the 1960s by Panofsky and Singer (1965) and Davenport (1968) who both studied the vertical structure of turbulence. This work was later augmented by Vickery (1970) and Brook (1975). The latter studied the spatial correlation of turbulence within an urban environment.

Recent publications involving coherence in wind engineering include Højstrup (1999), who experimentally measured the degree of horizontal and vertical correlation in observed power spectra of two wind turbines separated by a specific distance. This work included ambient equilibrium turbulence due to boundary layer turbulence as well as non-
equilibrium turbulence due to eddies formed by blade rotation. The results obtained compared well with the established analytical expression for coherence prediction, though mainly at low frequencies. Sørensen et al (2002) investigated the fluctuations in power output experienced by wind turbine towers within a wind farm. These fluctuations are due to the dynamic interaction of wind turbines with each other, and Sørensen et al employed the 'Davenport type' coherence function within the study. Minh et al (1999) studied the buffeting of scale model and full scale long-span bridges in Japan. They also used the 'Davenport type' coherence function to account for the spatial correlation of wind turbulence along the length of the bridge.

The transfer function, or complex frequency response function, is a necessary component of any input-output approach-based forced vibration analysis. In order to obtain a transfer function, the differential equation that describes the structures motion must be converted into algebraic form, by means of a mathematical transform. To this end, Jeary (1997) uses the Laplace Transform technique and Clough and Penzien (1993) use the Fourier Transform technique.

Proving that the input-output approach is widely applicable in wind engineering, Saunderson et al (1999) developed transfer functions in a novel study on the dynamic response of a spruce tree under stochastic high wind action. The transfer function was also used in a wind induced transport study by Baker (1993) who investigated the displacement of a road vehicle to strong winds. Much use of the transfer function concept is made in the study of the control of structures. In this regard, Kareem and Kline (1995) recently developed transfer functions which incorporated the characteristics of multiple and tuned mass dampers to estimate structural response due to wind and seismic excitations. Suhardjo et al (1992) investigated the active control of wind excited structures, using the transfer function to minimise the energy output by the system. Most dynamic analysis software codes have dedicated facilities to obtain a discretized model's transfer function. Trowbridge et al (1991) used the transfer function capability of the software code NASTRAN to predict the transient structural deformation and force within rod and plate structures. Murtagh et al (2004) derived the transfer functions of a series of lattice towers using the software code ANSYS.

Forced vibration analysis may also be executed in the time domain, and in some instances this approach is preferable to the frequency domain based approach as at it allows for the
inclusion of behavioural non-linearity and response coupling. As introduced in Chapter 2, the mode acceleration method is employed within this thesis to obtain the response time-history of a discrete system. Williams (1945) is credited with the first implementation of the mode acceleration method. Singh (1986) presented a method of obtaining the seismic response of a non-classically damped system, based on the spectral response mode-acceleration technique. This approach has been found to be more computationally efficient than the alternative mode displacement method. Akgun (1993) presented an augmented algorithm based on the mode acceleration method which improves convergence when computing stresses in large models.

Short of possessing actual input time-histories as measured in the field, the engineer is tasked with the artificial generation of relevant time-histories using widely published spectral density functions. The means by which this is possible can be divided into three categories, the first based on a Fast Fourier Transform (FFT) algorithm, the second based on Wavelet Theory and the third based on the Auto-Regressive Moving Average (ARMA) method. Suresh Kumar and Stathopoulos (1997) simulated wind pressure time histories with both Gaussian and non-Gaussian distributions, on low building roofs, by using a FFT based algorithm, that made use of a previously measured pressure spectrum. Kitagawa and Nomura (2003) used wavelet theory to generate wind velocity time-histories by assuming that eddies of varying scale and strength may be represented on the time axis by wavelets of corresponding scales. Time-histories were obtained by employing the inverse Wavelet transform, and the artificially created wind characteristics were compared to those of natural wind, showing good agreement. Minh et al (1999) investigated the time-domain buffeting of long-span bridges by using the digital filtering ARMA method to numerically generate time histories of wind turbulence. This method allowed for the simultaneous generation of two wind velocity component time-histories, while allowing for the inclusion of spatial correlation.

When a mode superposition technique, such as the mode acceleration or mode displacement method, is used to obtain the response of an ‘n’ dimensional discrete system, the equation of motion is uncoupled using the eigenvectors (mode shapes) of the system. This is only possible if the damping matrix is proportional to the mass and/or stiffness matrix, in which case is said to be classically damped. Systems in which the damping matrix is not proportional to the mass and/or stiffness matrix are deemed to be non-classically damped systems. The equation of motion may not be decoupled as before, but
must be cast in state-space having a dimension of ‘2n’. Eigenvectors may be obtained for
this ‘2n’ dimensional system, with the bottom half of the eigenvector being the actual
mode shape or shape function. This phenomena has traditionally been the interest of the
earthquake engineers. Singh (1980) obtained the root mean square (RMS) response of
structures by the square root of sum of squares (SRSS) method for non-proportionally
systems in the frequency domain using the modal decomposition method to obtain spectral
moments. Chang and Mohraz (1990) presented a formulation which used a recursive
procedure based on the exact solution of the differential equation of motion for non-linear
classically and non-classically damped systems in order to obtain modal responses.

Literature on the effects of aerodynamic damping on systems in wind engineering is
relatively scarce. Jeary (1997) explains that the phenomenon of aerodynamic damping will
exist wherever there is a coupling between the motion of the structure and an on-coming
wind flow. However, Simiu and Scanlan (1996), while acknowledging the potential of
aerodynamic damping to reduce wind induced resonant responses, advised against the
consideration of aerodynamic damping due to it’s very uncertain determination. This
temporary reticence may have been alleviated by Holmes (1996) who studied the
aerodynamic damping of lattice towers. Although his work concentrated solely on the
fundamental mode of vibration, Holmes demonstrated in a numerical example that the
fundamental critical aerodynamic damping ratio was nearly three times that of the
conventionally-assumed critical structural damping ratio. The presence of aerodynamic
damping will render any MDOF system’s damping non-classical.

Various commercial software codes have been developed by engineers for the dynamic
analysis of the various components of a wind turbine tower. Buhl (1994) presented
guidelines for the use of the software code ADAMS to determine the free and forced
vibrations of wind turbine towers. Cambanis and Stapountzis (2001) investigated
blade/tower interaction by considering the fluid interaction between rotating blades and
tower using the computational fluid dynamics software FLEUNT. Savory et al (2001)
recently used the finite element software code ABAQUS in a study on tornado effects on
structures. The author’s used the direct integration capability of the code to solve the
forced vibration equation of motion, yielding the response of lattice transmission towers to
non-stationary wind loading.
3.5 WIND TUNNEL TESTING

In order to study the wide range of effects that ambient wind flow may have on structures, the wind tunnel provides a means of testing structural response to a prescribed flow, without the need to build an expensive and impractical full scale model. There are two types of wind tunnels (Bain et al, 1971), the 'open circuit' tunnel and the 'closed circuit' tunnel. The open circuit tunnel draws air in from the atmosphere through an intake and expels the air back out into the atmosphere through a diffuser. The closed circuit tunnel consists of a test section along with a wind return section, and the air remains in the tunnel system, being circulated by a fan. The test section may be constructed with a square, rectangular, octagonal, circular or elliptical cross-section.

Two strategies are available to the wind tunnel modeller; a linear scale model (reduced scale compared to the full model) may be built and tested or a full scale section of the full model may be utilised. When the former is being tested, a complex scaling convention must be adopted when any scale other than 1:1 is being investigated. This convention may be identified as similitude, and is discussed in this section in detail.

Similitude is the concept that governs the legitimacy of wind tunnel model scaling, by ensuring that there is an equivalency between the scale model and the full scale model. In wind tunnel testing, this may be separated into physical similitude and flow similitude. The crux of this modelling strategy is that the similarities between the scale model and the full model are expressed as a series of dimensionless numbers belonging to either physical or flow similitude. The three parameters that control similitude between a scale model and a full scale model are mass, length and time (Jeary, 1997). It is not possible to have a common scaling factor applicable to all three, accept at a scale of 1:1. An example of this occurs when the lengths of the scale model are scaled down by a factor of 'x' from the full model. The mass of the scale model must now be scaled down by a factor of 'x^3', reflecting the change in volume.

Dimensional analysis may be employed to obtain the dimensionless numbers necessary to maintain the rules of similitude. If a force of F is assumed to act on a body immersed in a fluid flow, it may be shown that the force is proportional to six parameters, namely the density ρ, the mean wind velocity in turbulent flow $\bar{v}$, some typical dimension D, some frequency f, fluid viscosity μ and acceleration due to gravity g.
3.5.1 Physical Similitude

In order for a scale model and a full scale model to have physical similitude, they must have the same aerodynamic shape, have the same mass and stiffness distribution and have the same dimensionless similitude quantities. The first dimensionless quantity is known as the dimensionless density number, as instructed by Bain et al (1971), Walshe, (1972) and Simiu and Scanlan (1996). This number may be expressed as a ratio of model material density to fluid density, as

\[
\left( \frac{\rho_m}{\rho_f} \right)_{SM} = \left( \frac{\rho_m}{\rho_f} \right)_{FSM}
\]  

where \(\rho_m\) is the density of the model’s material, \(\rho_f\) is the density of fluid, and subscripts SM and FSM denote scale model and full scale model respectively. The next dimensionless similitude number, which is probably the most important, is known as the dimensionless reduced frequency number (Bain et al (1971), Walshe, (1972), Simiu and Scanlan (1996) and Jeary (1997)). This number may be expressed as

\[
\left( \frac{D f_j}{v} \right)_{SM} = \left( \frac{D f_j}{v} \right)_{FSM}
\]

where \(D\) is a typical dimension of the model, \(f_j\) is the \(j^{th}\) natural frequency (Hz) and \(v\) is the mean wind velocity in a turbulent flow past the structure. The next dimensionless number is termed the dimensionless gravitational number (Bain et al (1971), Walshe, (1972), Simiu and Scanlan (1996) and Jeary (1997)). This number is also known as the Froude number, and is a ratio of inertia forces to gravitational forces, expressed as

\[
\left( \frac{v^2}{D g} \right)_{SM} = \left( \frac{v^2}{D g} \right)_{FSM}
\]

where \(g\) is acceleration due to gravity. This similitude may, however be ignored in most cases, except for situations in which the gravitational restoring force is significant, such as experienced by suspension bridges. The last similitude rule being considered regards the structural damping (Bain et al (1971), Walshe, (1972), Simiu and Scanlan (1996)). Damping ratios must remain constant between scaled model and full scale model, in order
to ensure proper proportionality of oscillatory deflections between the two. Thus, mode shapes must remain consistent between the scale model and the full model. Natural frequency ratios must also remain constant between scale model and full scale model.

3.5.2 Fluid Similitude

A set of rules also exists for the flow conditions observed in open air and in the wind tunnel, which will affect the full scale and scaled models respectively. These rules comprise a set of dimensionless flow related numbers, the first of which is called the dimensionless Reynolds number (Bain et al (1971), Walshe, (1972), Simiu and Scanlan (1996) and Jeary (1997)). The Reynolds number is equal to the ratio of the inertia forces to the viscous forces and may be represented in dimensionless form as

$$\left( \frac{D\bar{v}}{v} \right)_{SM} = \left( \frac{D\bar{v}}{v} \right)_{FSM}$$

where $v$ is the kinematic viscosity of the fluid. It is usually not possible to achieve Reynolds number similitude as it is impractical to test the scale model at the required higher wind velocities or to lower the fluid viscosity. Normal wind tunnel testing under standard gravity and atmospheric conditions thus operates with scale violations for Reynolds number similitude.

The second dimensionless flow similitude number is given by Walshe (1972) as the dimensionless turbulence intensity number. Turbulence intensity is a quantitative measure of turbulence at a specific point in space, and is equal to the ratio of the RMS of fluctuating wind velocity to the mean component of wind velocity expressed as

$$\left( \frac{\sigma_x}{\bar{v}}, \frac{\sigma_y}{\bar{v}}, \frac{\sigma_z}{\bar{v}} \right)_{SM} = \left( \frac{\sigma_x}{\bar{v}}, \frac{\sigma_y}{\bar{v}}, \frac{\sigma_z}{\bar{v}} \right)_{FSM}$$

where $\sigma_{x,y,z}$ are the root mean square values of fluctuating wind velocity along a three dimensional xyz orthogonal axis system of reference. The final flow similitude rule concerns the characteristic size of eddies within the turbulent flow and is called the dimensionless scales of turbulence number (Walshe, (1972) and Jeary (1997)). This rule is especially important as it concerns the spatial correlation of pressures along the height of the model. It is expressed as
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\[
\begin{pmatrix}
L'_u \\
D
\end{pmatrix}_{SM} = \begin{pmatrix}
L'_u \\
D
\end{pmatrix}_{FSM}
\quad (3.6)
\]

where \( L'_u \), \( L'_y \) and \( L'_z \) are the average size of eddies along the x, y, or z axis respectively, due to the longitudinal component \((u)\) of the fluctuating wind velocity.

The wind tunnel testing reported in the literature is usually motivated by either full scale testing of sections of a structure, scaled modelling of a structure to evaluate wind-induced phenomena such as response coupling or vortex shedding, or estimation of the efficiency of vibration control approaches. Gioffrè et al (2004) tested a full scale section of a mobile phone mast to evaluate the drag and lift forces generated within a specific flow regime. Once these forces were recorded, a reliable quantification of wind loading on the structure could be determined. Carril et al (2003) tested three section models of a rectangular sectioned lattice tower in the Boundary Layer Wind Tunnel Laboratory at the University of Western Ontario in Canada, and compared the measured RMS values for drag and cross-wind forces to those predicted in all the major design codes in use worldwide.

Thepmongkorn and Kwok (2002) carried out a wind tunnel test on the along-wind, across-wind and twisting response of a model having an eccentric centre of mass and stiffness. Yan-li at al (2003) built a 2 m high 1:100 scale model of a guyed mast and measured the model’s acceleration response to a user generated wind input. The measured acceleration RMS of the guyed model matched closely with that of an equivalent analytical model. Cho et al (1998) used a frame model of a five-story building measuring 1.17 m high, which consisted of brass members welded together, and included several viscoelastic dampers positioned throughout the body of the model. The effectiveness of the viscoelastic dampers in vibration suppression were tested under wind action.

Wu and Pan (2002) constructed a 1:300 scale model of a high rise building which included an active mass driver to facilitate reduction in response parameters, and obtained the model’s response to realistically generated wind loading. Kwon et al (2000) investigated the use of a passive aerodynamic control method to decrease the magnitude of flutter experienced by a bridge under wind action. Flutter in bridges is primarily caused by interaction between the deck and the surrounding air flow. Mechanical plates were strategically positioned on a section model and the premise of the control method was that
orientation of the plates could be altered, disrupting the air flow and directly affecting the conditions necessary for flutter.

It is also possible to measure the aerodynamic damping that occurs in a wind tunnel test. Marukawa et al (1996) investigated the aerodynamic damping of tall buildings by using a 1:500 scale stick model immersed in a wind flow. The random decrement technique (RDT) was used to evaluate the aerodynamic damping ratios from the wind induced response. The structural damping ratio was first evaluated by free vibration, and subtracted from the total damping to yield the aerodynamic damping ratio. The RDT is a popular system identification tool available to estimate damping in wind-excited responses. Tamura and Suganuma (1996) used the RDT to estimate the damping ratios of full scale towers during strong winds. Kijewski and Kareem (2000) discussed the reliability of the RDT and introduced a mathematical theory termed ‘Bootstrap Theory’ which may be used to estimate the variance of random decrement signatures.

3.6 EXPERIMENTAL SYSTEM IDENTIFICATION

Obtaining the inherent dynamic properties of a system, often known as system identification (SI), is of obvious interest to engineers. Though a structure may be initially designed using analytical response predictions, its true dynamic characteristics may only be accurately found by measuring its behaviour when subjected to either damped free or forced vibration. Jayakumar (1987) used the then newly developed pseudo-dynamic testing procedure to investigate the system parameters of a full scale six story steel structure subject to earthquake motions. Of interest in this work were the elastic and inelastic responses of the structure caused by small and large amplitude oscillations respectively, and the linear and non-linear analytical models developed for the purpose of comparison. Fukuwa et al (1996) carried out a series of free and forced vibration tests on a steel framed building in a study investigating the range of natural frequency and damping values obtained using several different SI approaches. Using measured response time-histories, the SI approaches adopted included the derivation of the Fourier amplitude response spectrum and logarithmic decay method, the random decrement method, sweep excitation testing, and experimentally-derived transfer functions. While all approaches yielded a similar value for fundamental frequency, the damping ratio was largely dependent on the type of test carried out. Maneetes and Linzell (2003) investigated the free vibration response of a steel bridge initially excited by a mechanical shaker. The acceleration free
vibration time-history was recorded and transferred to the frequency domain via FFT, where the fundamental frequency could be observed.

Shaking table testing is primarily used to simulate a seismic input into the base of a structure. Filiatrault and Tremblay (1998) used a shake table to estimate the lower modes of vibration and modal structural damping of a model frame in an investigation into the dynamic behaviour of tension-only braced frames. They used a dedicated software code developed by a commercial company to obtain the system parameters. Peckan et al (2000) published a study on the dynamic behaviour of a 1:4 scale six story braced frame under the action of three realistic earthquakes. Dynamic properties were obtained from the story level transfer functions and estimation of viscous damping was carried out through the use of the half power method (Clough and Penzien, 1993).

System parameter estimation though the use of wavelet theory is proving an increasingly popular approach. Goggins (2004) estimated the fundamental frequency and equivalent viscous damping ratio of a braced frame under seismic action simulated from a shake table, using wavelet theory. Yin et al (2004) proposed an approach based on the wavelet transform for frequency response functions for linear systems, to yield the dynamic properties such as natural frequency and damping. Le and Argoul (2004) presented a wavelet based methodology for obtaining the natural frequency, viscous damping ratio, and mode shape for any linear mechanical system. These parameters may be obtained either from the phase or the modulus of the wavelet transform. Le and Argoul also discuss some of the difficulties associated with the use of the continuous wavelet transform for the purpose of systems identification. Hans et al (2000) used their wavelet theory basis approach to quantify the amount of damping contained within a building, when it was excited by both harmonic and shock loading.

The use of scale models in order to better understand the dynamics of a systems is not an unusual practice. Proving the practitioner has a solid understanding of the associated scaling laws, this approach is both practical and inexpensive. Mettler et al (2000) investigated certain dynamic rotor-flight phenomena experienced by helicopters in hover and cruise conditions. They used frequency domain-based SI techniques developed by the U.S. Army, and compared the necessary experimentally measured response characteristics with those of an equivalent analytical model.
CHAPTER 4 - FREE VIBRATIONS OF WIND TURBINE TOWER ASSEMBLIES

4.1 INTRODUCTION

This chapter presents the analytical models, in both discrete and continuous form, derived in order to obtain the free vibration properties of all the flexible components of a wind turbine tower. These include the rotating blades, the tower carrying the nacelle, and the tower/nacelle coupled to three rotating blades. Obtaining the free vibration properties of any flexible system is the first step in carrying out a dynamic analysis. These properties, including natural frequencies of vibration and associated mode shapes, are brought about when the system is set in motion by an initial condition (for example, displacement and/or velocity), and does not include any external sources of loading. Generally, if the system is lightly damped, damping may be excluded and incorporated into the forced vibration portion of the dynamic analysis. Thus, the parameters that affect the free vibration properties of the components of a wind turbine tower are mass and stiffness. As free vibrational motion is assumed to be undamped, the motion once started, will continue indefinitely, and the system will theoretically vibrate at certain frequencies of oscillation, the natural frequencies, and will assume specific shapes at those frequencies.

This chapter is split into four subsequent sections. The first, Section 4.2, demonstrates the approaches adopted to obtain the free vibration characteristics of the tower, carrying a concentrated mass at its top representing the nacelle. As towers are used worldwide in both the tubular and lattice configurations, methods used to obtain the free vibration properties of each are included. Three such computational methods are presented in this section. The first method, described in Section 4.2.1, uses a discrete model order reduction technique to simplify the motion of the tower/nacelle into a system of several degrees-of-freedom, termed a multi degree-of-freedom (MDOF) system. This method may be used for both the tubular and lattice towers. The second computational method, presented in Section 4.2.2, uses the finite element technique to obtain the free vibration properties of both lattice and tubular towers. A commercially available software code is used in this regard. The model is input into the software by selecting the geometry, including the mass and stiffness characteristics, and the program uses mathematical solvers in order to extract the natural frequencies and mode shapes of the model.

The third method is applicable to the tubular tower only, and uses continuous system mechanics. The tower is represented by a system with an infinite number of degrees-of-
freedom, and carries a lumped mass at its top representing the nacelle. The equation of motion for this system is solved for free vibrations, and the translational and rotary inertia of the nacelle are included in the solution.

The free vibration properties of the rotating blades are considered in the next section, Section 4.3. Obtaining the free vibration properties of the blades differs conceptually from the approaches used for the tower, as the blades are affected by certain phenomena due to their rotation. When the blades rotate, they experience an increase in stiffness due to the centrifugal forces which act on them. This action is known as centrifugal stiffening. The blades will also experience self-weight or gravity effects, depending on their position. This gravity effect will act to stiffen the blades, having the effect of raising values of natural frequency. The blades are modelled as both discrete and continuous systems, allowing the three computational methods described in the previous paragraph to be employed in conjunction with the rotating blades. The discrete parameter method is introduced in Section 4.3.1, and may account for both gravity and centrifugal stiffening effects due to rotation on the blade’s dynamic characteristics. The use of the finite element method is demonstrated in Section 4.3.2, and though this method may include realistic blade geometry, the specific software code used cannot take gravity or centrifugal stiffening effects into account. The blades are lastly modelled as continuous systems in Section 4.3.3, where the motion of the blades are represented by a system of infinite degrees-of-freedom. While this method includes centrifugal stiffening effects due to rotation, it does not include the aforementioned gravity effects.

Once the free vibration properties of both the tower and rotating blades are found separately, the two systems may be mathematically coupled together, as demonstrated in Section 4.4. A discrete model of a wind turbine tower where the motion of the rotating blades is coupled to the motion of a tower is presented. The rotating blades and tower are each modelled as single degree-of-freedom systems, and by way of compatibility, are coupled together to form a discrete two degree-of-freedom system. This model thus includes phenomena inherent to the behaviour of wind turbine towers, including blade tower interaction and centrifugal stiffening and gravity effects on the rotating blades. This method can be used to couple as many blades as desired, with the tower.

A series of numerical examples are illustrated in Section 4.5, showing the applicability of the analytical methods proposed to obtain the free vibration properties of each of the
Section 4.5.1 obtains the free vibration properties of typical lattice and tubular towers using each of the three methods detailed in Section 4.2. Numerical examples are also presented for each of the three methods used to estimate the free vibration properties of the rotating blades, as illustrated in Sections 4.5.2. The models considered in this section include prismatic cantilever blades that include the effects due to rotation, as well as tapering blades. The free vibration properties of two coupled models of a tower and three rotating blades are illustrated in Section 4.5.3, including the effects of blade rotation of the dynamics of the coupled system. Section 4.6 lastly provides a discussion on the numerical examples for the towers, rotating blades, and coupled tower-blade systems.

### 4.2 FREE VIBRATIONS OF TOWERS

This section deals with the analytical methods used to obtain the free vibration properties of both continuous and lattice wind turbine towers. Both types of towers carry concentrated mass at the top, representing the mass of the nacelle and rotor blades. Three such analytical methods were employed, a discrete lumped parameter method, a finite element software based method, and a continuous distributed parameter method.

#### 4.2.1 Discrete modelling

Although a continuous or lattice tower in reality represent systems with infinite degrees of freedom (DOF), the number of DOF needed to accurately represent the free vibration motion of the system may be reduced, simplifying the mathematical representation of the system. The simplified system is now a reduced order system, and is referred to as a MDOF system. The tower may be modelled by lumping the structural parameters, i.e. mass and stiffness at certain nodes of interest. The free vibration equation of motion for an undamped MDOF system is

\[ [M_T] \{\ddot{x}(t)\} + [K_T] \{x(t)\} = 0 \]  

(4.1)

where \([M_T]\) is the mass matrix, and \([K_T]\) is the stiffness matrix, \(\{x(t)\}\) denotes nodal the displacement vector, \(\{\ddot{x}(t)\}\)denotes the nodal acceleration vector, and subscript T denotes tower. Equation 4.1 represents an eigenvalue or characteristic value problem. The eigenvalues or natural frequencies, \(\omega_{T,j}\), may be obtained by solving the equation

\[ \text{DET}\left[[K_T] - \omega_{T,j}^2[M_T]\right] = 0 \]  

(4.2)
where ‘j’ denotes the jth mode of vibration, and DET denotes the determinant of the resulting matrix. The mass of the tower is discretized into nodal values and cast within a diagonal mass matrix as

\[
[M_T] = \begin{bmatrix}
m_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & 0 & m_N
\end{bmatrix}
\]  

where ‘N’ is the number of nodes chosen. The stiffness matrix for both the tubular and lattice tower models is obtained by first evaluating the flexibility matrix. The flexibility matrix is a measure of nodal displacement caused by a unit force. For the tubular tower, which is assumed to be prismatic, the flexibility matrix may be determined using the approach illustrated in figure 4.1.

The displacement (flexibility matrix coefficients) at nodes ‘a’, ‘b’ and ‘c’, due to a unit load placed at node ‘a’ are given by Ghali and Neville (1997) as

\[f_a = \frac{l^3}{3EI}\]  

\[f_b = f_a + \frac{dl^2}{2EI}\]  

Figure 4.1 Flexibility matrix coefficients for a prismatic cantilever with a unit load
where $E_l$ is the stiffness of the tower. The flexibility coefficients for the lattice tower may be obtained using a static analysis program as found in all finite element software code. A unit force is placed at a particular node, and the displacements of that node and all other nodes are noted. Continuing this process for unit loads at all other nodes yields the flexibility matrix, where $\delta_{ij}$ is the displacement of node $i$ due to a unit load applied at node $j$. Assuming that the nodes are numbered from the top of the bottom, this matrix which will be of the form

$$[F_T] = \begin{bmatrix}
\delta_{11} & \delta_{12} & \cdots & \delta_{1(N-1)} & \delta_{1N} \\
\delta_{21} & \delta_{22} & \cdots & \delta_{2(N-1)} & \delta_{2N} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\delta_{(N-1)1} & \delta_{(N-1)2} & \cdots & \delta_{(N-1)(N-1)} & \delta_{(N-1)N} \\
\delta_{N1} & \delta_{N2} & \cdots & \delta_{N(N-1)} & \delta_{NN}
\end{bmatrix}$$ (4.7)

The stiffness matrix is found as the matrix inversion of $[F_T]$ as

$$[K_T] = [F_T]^{-1}$$ (4.8)

Values of natural frequency may be obtained by employing equation 4.2, a trial value of $\omega_{T,j}$ is adopted and changed until the sign of the matrix determinant changes, indicating a root of the characteristic equation. To obtain the mode shapes, it is assumed that the tower is in free-vibration simple harmonic motion, and the equation of motion for the system, equation 4.1, may ultimately be expressed in matrix form as

$$\begin{bmatrix}
e_{11} & e_{12} & e_{13} & \cdots & e_{1N} \\
- & - & - & \cdots & - \\
e_{21} & e_{22} & e_{23} & \cdots & e_{2N} \\
e_{31} & e_{32} & e_{33} & \cdots & e_{3N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
e_{N1} & e_{N2} & e_{N3} & \cdots & e_{NN}
\end{bmatrix}\begin{bmatrix}
u_1 \\
u_2 \\
u_3 \\
\vdots \\
u_N
\end{bmatrix} = \begin{bmatrix}0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}$$ (4.9)
where, $e_{ij} = K_{T,j} - \omega_{T,j}^2 M_{T,j}$ and $u_N$ is the amplitude of the nodal displacement. In equation 4.9, for the fundamental mode of vibration, the top nodal amplitude is normalised to unity. It is, therefore, more convenient to express equation 4.9 as

$$\begin{bmatrix} e_{11} & [E_{10}] \\ [E_{01}] & [E_{00}] \end{bmatrix} \begin{bmatrix} 1 \\ \{u_0\} \end{bmatrix} = \begin{bmatrix} 0 \\ \{0\} \end{bmatrix}$$

(4.10)

Using equation 4.10, it is evident that all nodal displacement amplitudes may be calculated as

$$\{u_0\} = -[E_{00}]^{-1}[E_{01}]$$

(4.11)

from which the fundamental mode shape is obtained as

$$\Phi_{T,1} = \begin{bmatrix} 1 \\ \{u_0\} \end{bmatrix}$$

(4.12)

This approach may be repeated for any arbitrary mode of vibration, though it must be specified which node is to be normalised to unity.

4.2.2 Finite Element modelling

A finite element (FE) software code ANSYS v.5.6.1 was also employed to obtain the natural frequencies and mode shapes of the lattice and tubular wind turbine towers. Figures 4.2 and 4.3 illustrate the lattice tower and tubular tower models, as created in ANSYS. The program has an inbuilt modal analysis capability which extracts and stores the relevant natural eigen-properties requested by the user. The approach used to determine the natural frequencies and mode shapes of any system, as with any finite element software code, follows a hierarchical order of events, such as

- Building the model
- Defining boundary conditions
- Extracting the eigenvalues and eigenvectors
- Reviewing the eigenvalues and eigenvectors obtained.
BUILDING THE MODEL

First, the user must specify which finite element will most closely represent the dynamic behaviour of the structure under consideration. The model of the lattice tower, as illustrated in figure 4.2, was built using a three dimensional truss element, termed 'LINK8' in order to simulate a pin-jointed structure. This element has three displacement DOF at each node and does not have any bending or shear stiffness. Information regarding this element may be found in Appendix A.1. The element used to construct the tubular model, as illustrated in figure 4.3, was a three-dimensional pipe element, termed 'PIPE16' which has six DOF at each node. Information of this element can be found in Appendix A.2. After the element type has been chosen, the user must assign real constants to the element, such as cross-sectional area and moment of inertia. Then the user must prescribe material properties to the element, such as modulus of elasticity and density. The model may be built up using either key-points or nodes. If the former is chosen, the user must create lines between the key-points, and then mesh the lines in order to create nodes and elements. If the user directly creates nodes, elements must be created in between the nodes.
DEFINING BOUNDARY CONDITIONS
Once the model has been built and discretized, boundary conditions must be specified. For a cantilever type structure, the boundary conditions include the restraining of all DOF at the fixed end. For the lattice tower model, all DOF of the nodes at the base of the structure were restrained, and similarly for the tubular tower.

EXTRACTING THE EIGENVALUES AND EIGENVECTORS
The next step involves extracting the eigenvalues and eigenvectors from the now discretized and restrained model. ANSYS provides several extraction solvers available for this task, depending on the nature of the model and the number of DOF present. These include a reduced method (ANSYS, 1999a), which condenses the system’s equation of motion in terms of master DOFs and uses the Householder-Bisection-Inverse iteration extraction technique to extract the eigen-properties. A subspace iteration method (ANSYS, 1999b), as described by Bathe (1982), is also available in which trial eigen-properties are suggested and converge to the actual eigen-properties through an iterative process. ANSYS also contains damped eigen-property solvers (ANSYS, 1999c, 1999d) in which the complex eigen-properties of a system can be extracted if that system is non-classically damped. The numerical examples presented in Section 4.5.1 were obtained using the subspace iteration method.

The user must specify the number of modes to extract, the frequency range that will contain the modes, and whether the user wants the modes normalised to the mass matrix or to unity.

REVIEWING THE EIGENVALUES AND EIGENVECTORS OBTAINED
The program contains a post-processor that allows the user to view the determined natural frequencies of vibration, along with their corresponding normalised mode shapes. The mode shapes can be output in either pictorial or numerical form, and usually contain three components along the three-dimensional global axis. The pictorial form of the mode shapes may also be output to a graphics file.

4.2.3 Continuum modelling
The tubular tower carrying a concentrated nacelle and rotor system mass is lastly modelled using continuous system mechanics as a cantilever beam of uniform cross-section with a spherical mass at the free end. The continuous beam represents the tubular tower and the
spherical mass represents the nacelle and rotor system, and is illustrated in figure 4.4. The tower is of height $L_T$, and the spherical mass has the same density as the cantilever beam and its radius ($D_N/2$) is derived from the corresponding user defined mass of the nacelle and rotor system.

For the case of beam flexure of a continuous system, the partial differential equation of motion that represents free-vibration motion is

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \bar{m}_T \frac{\partial^4 w(x,t)}{\partial t^2} = 0 \quad (4.13)$$

In equation 4.13, $EI$ is the stiffness of the beam, $\bar{m}_T$ is the mass per unit length of the tower, $w(x,t)$ denotes the transverse displacement response at a distance $x$ from the ground at an instant of time $t$ and $\partial^n$ denotes a partial differential to the $n^{th}$ degree.

![Figure 4.4 Cantilever beam (tower) and spherical mass (nacelle)](image)

After separation of variables, it can be seen that equation 4.13 yields two ordinary differential equations,

$$\ddot{Y}(t) + \omega_{T,j}^2 Y(t) = 0 \quad (4.14)$$

and
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\[ \Phi_{T,j}'''(x) - a^4 \Phi_{T,j}(x) = 0 \]  \hspace{1cm} (4.15)

where, \( Y(t) \) denotes time dependent amplitude, \( \Phi_{T,j}(x) \) represents mode shape of mode ‘j’, prime denotes a differentiation with respect to distance, the overdot denotes a differentiation with respect to time, and ‘a’ is a constant which depends on the eigenvalue. Equation 4.15 may be solved by introducing a solution in the form

\[ \Phi_{T,j}(x) = G \exp(st) \]  \hspace{1cm} (4.16)

which leads to

\[ \Phi_{T,j}(x) = G_1 \exp(iax) + G_2 \exp(-iax) + G_3 \exp(ax) + G_4 \exp(-ax) \]  \hspace{1cm} (4.17)

In equation 4.17, \( G_1, G_2, G_3 \) and \( G_4 \) are complex constants. Setting the imaginary parts of equation 4.17 to zero and expressing the exponential in terms of its trigonometric and hyperbolic equivalents, an equation representing the modal amplitude for a continuous beam is

\[ \Phi_{T,j}(x) = A_1 \cos(ax) + A_2 \sin(ax) + A_3 \cosh(ax) + A_4 \sinh(ax) \]  \hspace{1cm} (4.18)

where \( A_1, A_2, A_3 \) and \( A_4 \) are numerical constants, \( a^4 = \frac{\omega_{T,j}^2 \bar{m}_T}{EI} \) where \( \omega_{T,j} \) is the \( j \)th natural frequency of the tower/nacelle. The numerical constants must satisfy the boundary conditions of the beam, namely the displacement, slope, shear and moment at the beam ends. The boundary conditions for the tubular tower/spherical nacelle model may be defined as the displacement, \( \Phi_{T,j}(0) \), and slope, \( \Phi_{T,j}'(0) \), at the fixed end, and the moment, \( M_T(L) \), and shear, \( V_T(L) \), at the free end, where the rigid mass is located. The four boundary conditions to be satisfied are

\[ \Phi_{T,j}(0) = 0 \]  \hspace{1cm} \[ \Phi_{T,j}'(0) = 0 \]  \hspace{1cm} (4.19)

\[ M_T(L) = \frac{\omega_{T,j}^2 \Phi_{T,j}(L) J_{NAC}}{EI} \]  \hspace{1cm} \[ V_T(L) = \frac{\omega_{T,j}^2 \Phi_{T,j}'(L) M_{NAC}}{EI} \]  \hspace{1cm} (4.20)

where \( M_{NAC} \) is the mass at the free end and \( J_{NAC} \) is the corresponding rotary mass moment of inertia. Incorporating the boundary conditions into equation 4.18, one obtains four
equations in terms of the constants $A_1$, $A_2$, $A_3$ and $A_4$. Arranging in matrix form and evaluating the determinant of the resulting $4 \times 4$ matrix to zero, one obtains the equation as

$$\begin{align*}
\left[ a^4 - \frac{\omega_{t,j}^4 J_{NAC}^3 M_{NAC}}{(EI)^2} \right] \cosh(aL)\cos(aL) - \left[ \frac{\omega_{t,j}^2 M_{NAC}^3 + \omega_{t,j}^2 J_{NAC}^3 a}{EI} \right] \cosh(aL)\sin(aL) \\
+ \left[ \frac{\omega_{t,j}^2 J_{NAC}^3 - \omega_{t,j}^2 J_{NAC}^3 a^3}{EI} \right] \cos(aL)\sinh(aL) = -a^4 - \frac{\omega_{t,j}^4 J_{NAC}^3 M_{NAC}}{(EI)^2}
\end{align*}$$

that yields the eigenvalues of the system. The required eigenvalue may be found using an iterative technique such as the Newton-Raphson method or Goalseek in MS EXCEL. The mode shape may be obtained by expressing one of the fixed end constants in terms of the other (in this case $A_1$ in terms of $A_2$) and substituting into equation 4.18 to yield

$$\Phi_{t,j}(x) = A_2 \left[ \sin(ax) - \sinh(ax) + \frac{\sin(aL) + \sinh(aL)}{\cos(aL) + \cosh(aL)} \right] (-\cos(ax) + \cosh(ax))$$

4.3 FREE VIBRATIONS OF BLADES

The analytical approaches used to obtain the free vibration properties of rotating wind turbine blades are presented in this section. The blades are modelled as discrete and continuous systems, and also may have either prismatic or tapering geometries. As described in Section 4.2, three analytical methods may be employed to obtain the free vibration properties. The first is a discrete lumped parameter method, the second is a computer software based finite element code, and the third is a continuous distributed parameter method.

The geometry of the actual wind turbine blades is usually very complex, with both mass and stiffness varying along all three global axes. The software based finite element method is best equipped to deal with the most complex tapering geometry, though this option represents the most computationally expensive option. The other two approaches are for blade geometries that are assumed to be prismatic. Two types of free vibration blade motion is of interest, termed flapping motion and lead/lag motion. The former may be visualised as motion in the direction of the wind flow, and lead/lag motion occurs in a plane perpendicular to the wind flow. The flapping stiffness will generally be considerably less than the lead/lag stiffness.
Although the free vibration characteristics of any system are solely based on mass and stiffness distributions, the free vibration properties of rotating blade’s will also depend on their rate of rotation. As the blades rotate, they experience an outward axial force, known as centrifugal stiffening, which will have the effect of increasing the blades stiffness. Also, as the blade’s go from the horizontal to the vertical position, a dynamic axial self-weight force will also affect the blades stiffness. Both of these phenomena are included in two of the three following sections.

4.3.1 Discrete modelling

Rotating wind turbine blades in reality represent systems with infinite numbers of degrees-of-freedom. However, the vibration motion of these blades may be simplified by representing the motion of the blades by just several degrees-of-freedom, creating a MDOF system. The structural parameters of the blades may be discretized and lumped at specific nodes of interest. This approach can include the blade rotation effects such as centrifugal stiffening and blade self-weight loading. The blade is discretized into a lumped parameter system comprising of ‘m’ degrees of freedom. The eigenvalues of the blade for either flapping or lead/lag motion may be obtained from the equation

\[
\text{DET}\left([K_B^j] - \omega_{B,j}^2 [M_B]\right) = 0
\] (4.23)

where, \([K_B^j] = [K_B + K_{Gb}]\) represents the modified stiffness matrix due to the geometric stiffness matrix accounting for the effect of axial load (centrifugal stiffening and blade self-weight), \(\omega_{B,j}\) is the natural frequency of mode ‘\(j\)’, \([M_B]\) is the mass matrix, and subscript B denotes blade. The flexural stiffness matrix \([K_B]\), may be obtained using the approach for prismatic members presented in Section 4.2.1, or using the static analysis capability of a finite element software code to generate the flexibility matrix.

The geometric stiffness matrix, \([K_{GB}]\), contains force contributions due to blade rotation which are always tensile, and contributions from the self weight of the blade, which may be both tensile and compressive, depending on blade position. The geometric stiffness matrix is obtainable as
\begin{equation}
[K_{GB}] = \begin{bmatrix}
\frac{N_1}{l_1} & -\frac{N_1}{l_1} & \cdots & 0 \\
\frac{-N_1}{l_1} & \frac{N_1 + N_2}{l_1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots \\
0 & 0 & -\frac{N_{N-1}}{l_{N-1}} & \frac{N_{N-1} + N_N}{l_{N-1}} \\
\end{bmatrix}
\end{equation}

where $N_i$ is the axial force at node ‘i’, $l_i$ is the length of beam segment between the nodes ‘i’ and ‘i+1’, and ‘N’ is the total number of nodes. The magnitude of the tensile centrifugal axial force, $C_{T_i}$, may be obtained in discrete form (from the continuous form by Naguleswaran (1994)), as

$$C_{T_i} = 0.5\Omega^2 (M_B L_B - m_i x_i)$$

where $\Omega$ is the blade rotational frequency, $M_B$ is the total mass of the blade, $L_B$ is the total length of the blade, $m_i$ is the cumulative mass at node ‘i’ and $x$ is the distance of node ‘i’ from the centre of rotation. The nodal axial force due to gravity (self weight), $G_i$, may be obtained from geometry and depends on the angle of the blade to the horizontal $\theta$, see figure 4.7a. Values of $N_i$ are obtained from the expression

$$N_i = C_{T_i} \pm G_i$$

with the sign convention that tensile forces are positive and compressive forces are negative.

4.3.2 Finite element modelling

The finite element software code ANSYS v.5.6.1 was employed to build a finite element model of a blade, with tapering geometry, as illustrated in figure 4.5. The model was created using a beam element, termed ‘BEAM44’. Specific information regarding this element may be found in Appendix A.3. This element has six degrees-of-freedom at each node, and the user may specify different nodal values of cross-sectional area and moment of inertia. The modal analysis capability of ANSYS provides a means to obtain the natural frequencies and mode shapes of the model.
However, ANSYS cannot account for rotational effects such as centrifugal stiffening and axial self-weight loading, so the free vibration properties of the blade can only be obtained under static equilibrium (i.e. at a rotational frequency of 0 rads\(^{-1}\)).

### 4.3.3 Continuum modelling

The last method used to model the rotating blade is as a distributed-parameter system using continuum mechanics. The blade is modelled using Euler-Bernoulli beam theory and consists of a prismatic beam of length \(L_B\) connected to a circular hub of radius \(R_H\), as illustrated in figure 4.6. The beam is rigidly attached to the hub and radially free at the other end. The beam is modelled as a continuous entity, having an infinite number of degrees-of-freedom.

Naguleswaran (1994) provided the basis of the following strategy to evaluate the natural frequency of a span-wise (out-of-plane relative to figure 4.6) rotating beam. First, the incremental forces acting on an elemental section of the beam are considered. These include shear force, bending moment and centrifugal tension forces due to rotation.
Figure 4.6 Continuous blade and hub with applied forces on an incremental element

Summation of these forces along with normalisation of length and displacement yields the equation for motion for the beam vibrating in its dimensionless mode shape in the ‘xy’ plane as

\[ D^4Y(X) - 0.5v(1+2\rho_0)D^2Y(X) + v\rho_0D[XDY(X)] + 0.5vD(X^2DY(X)) - \mu Y(X) = 0 \quad (4.27) \]

where \( D = d/dX \), \( X = x/L_B \), \( Y(X) = y(x)/L_B \), \( \rho_0 = R_H/L_B \), \( v = \eta^2 = \frac{\bar{m}_B \Omega^2 L_B^4}{EI_{xy}} \), \( \mu = \bar{m}_B \omega_B^2 L_B^4 / EI_{xy} \), \( y(x) \) denotes displacement of the blade as a function of \( x \), the distance along the blade from the hub, \( \bar{m}_B \) is the mass per unit length of the blade, \( \Omega \) is the rotational speed in the ‘zx’ plane, \( EI_{xy} \) is the flexural stiffness for bending in the ‘xy’ plane and \( \omega_B \) is the natural frequency of the blade. \( \rho_0 \), \( \eta \) and \( \mu \) are known as the offset, rotational and frequency parameters respectively.

Equation 4.27 is solved by employing the Frobenius method, which involves the use of a power series such as

\[ F(X, c) = \sum_{n=0}^\infty a_{n+1}(c)X^{c+n} \quad (4.28) \]

to represent the dimensionless displacement, \( Y(X) \), where \( a_{n+1}(c) \) is a numerical coefficient which is a function of the integer \( n \), and \( c \) is an undetermined exponential constant. In employing the Frobenius method, equation 4.28 is inserted into equation 4.27 and the solution of equation 4.27 ultimately contains four solution functions, \( F(X,0) \), \( F(X,1) \), \( F(X,2) \), \( F(X,3) \), each expressed as...
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\[ F(X,0) = 1 + 0.5v(1 + 2\rho_0)X^2 / 2 + \sum_{n=0}^{\infty} a_{n+5}(0)X^{n+4} \]  

\[ F(X,1) = X + 0.5v(1 + 2\rho_0)X^3 / 6 - \nu\rho_0X^4 / 24 + \sum_{n=0}^{\infty} a_{n+5}(1)X^{n+5} \]  

\[ F(X,2) = X^2 + 0.5v(1 + 2\rho_0)X^4 / 12 - \nu\rho_0X^5 / 30 + \sum_{n=0}^{\infty} a_{n+5}(2)X^{n+6} \]  

\[ F(X,3) = X^3 + 0.5v(1 + 2\rho_0)X^5 / 20 - \nu\rho_0X^6 / 40 + \sum_{n=0}^{\infty} a_{n+5}(3)X^{n+7} \]  

The general solution of equation 4.27 is

\[ Y(X) = C_1F(X,0) + C_2F(X,1) + C_3F(X,2) + C_4F(X,3) \]  

where constants \( C_1, C_2, C_3, \) and \( C_4 \), are evaluated from the boundary conditions. The boundary conditions for a beam fixed at the hub are

\[ Y(0) = 0 \]  

\[ DY(0) = 0 \]

resulting in \( C_1 \) and \( C_2 \) being equal to 0. Equation 4.28 may be rewritten as

\[ F(X,c) = \sum_{n=0}^{\infty} A_{n+1}(X,c) \]  

in which the ascending terms, \( A_{n+1} \) must be evaluated. Expressions for these terms are given as

\[ A_1(X,c) = X^c, \]  

\[ A_2(X,c) = 0, \]  

\[ A_3(X,c) = \frac{0.5v(1 + 2\rho_0)X^{c+2}}{[(c + 2)(c + 1)]} \]
and the remainder of the terms may be obtained using the expression

$$A_{n+5}(X, c) = \frac{X^2 B_{n+5}(X, c)}{[(c + n + 4)(c + n + 3)(c + n + 2)(c + n + 1)]}$$  \hspace{1cm} (4.41)$$

in which

$$B_{n+5}(X, c) = 0.5v(1+2\rho_0)(c + n + 2)(c + n + 1)A_{n+3}(X, c) - \nu \rho_0 (c + n + 1)^2 X A_{n+2}(X, c) -$$

$$[0.5v(c + n)(c + n + 1) - \mu]X^2 A_{n+1}(X, c)$$ \hspace{1cm} (4.42)$$

The derivatives of equations 4.37 to 4.41 are also of interest. The first derivatives of each term is given by

$$D A_1(X, c) = cX^{c-1}$$ \hspace{1cm} (4.43)$$

$$D^2 A_1(X, c) = c(c - 1)X^{c-2}$$ \hspace{1cm} (4.44)$$

$$D^3 A_1(X, c) = c(c - 1)(c - 2)X^{c-3}$$ \hspace{1cm} (4.45)$$

$$D A_2(X, c) = 0 = D^2 A_2(X, c) = D^3 A_2(X, c)$$ \hspace{1cm} (4.46)$$

$$D A_3(X, c) = \frac{0.5v(1+2\rho_0)X^{c+1}}{(c + 1)}$$ \hspace{1cm} (4.47)$$

$$D^2 A_3(X, c) = 0.5v(1+2\rho_0)X^c$$ \hspace{1cm} (4.48)$$

$$D^3 A_3(X, c) = c0.5v(1+2\rho_0)X^{c+1}$$ \hspace{1cm} (4.49)$$
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\[ D_A(x, c) = \frac{-v \rho_0 c X^{c+2}}{(c+2)(c+1)} \] (4.50)

\[ D^2 A(x, c) = \frac{-v \rho_0 c X^{c+1}}{(c+1)} \] (4.51)

\[ D^3 A(x, c) = -v \rho_0 c X^c \] (4.52)

\[ D_{n+5} A_n(x, c) = \frac{X B_{n+5}(x, c)}{((c + n + 3)(c + n + 2)(c + n + 1))} \] (4.53)

\[ D^2 A_{n+5}(x, c) = \frac{B_{n+5}(x, c)}{((c + n + 2)(c + n + 1))} \] (4.54)

\[ D^3 A_{n+5}(x, c) = 0.5v(1+2\rho_0)D_{A_{n+3}}(x, c) - v \rho_0(c + n + 1)A_{n+2}(x, c) - [0.5v(c + n)(c + n + 1)-\mu]X_{A_{n+1}}(x, c)/(c + n + 1) \] (4.55)

The frequency equation of a fixed-free beam is given by

\[ f(\mu) = D^2 F(1,2)D^3 F(1,3) - D^3 F(1,2)D^2 F(1,3) = 0 \] (4.56)

Equation 4.56 contains four separate terms which must be evaluated, requiring the second and third derivatives of the solution functions to be obtained. A trial value of \( \mu \) is suggested and entered into equation 4.56. This trial value is altered in steps until the resultant of equation 4.56 changes sign, indicating a root or natural frequency. Thus, the fundamental natural frequency of the blade, for either flapping or lead/lag motion can be obtained.

The normalized mode shape equation for a clamped-free beam in its \( j \)th mode shape is

\[ Y_j(x) = \frac{[D^2 F(1,3)F(X,2)-D^2 F(1,2)F(X,3)]}{[D^2 F(1,3)F(1,2)-D^2 F(1,2)F(1,3)]} \] (4.57)

Again, four fixed terms must be evaluated along with the two terms which are functions of \( X \).
4.4 FREE VIBRATIONS OF TOWER/NACELLE COUPLED WITH BLADES

As a three blade wind turbine tower in reality is a multi-body dynamic system, the free vibration properties of the combined tower/nacelle and rotating blades assembly are of obvious importance. The motion of the blades will directly affect the motion of the tower, and vice versa. Once the free vibration properties of the tower/nacelle and rotating blades are each obtained independently, they can be coupled together mathematically. In this section, three rotating blades, vibrating in their flapping and lead/lag mode shapes are coupled to a tower and nacelle.

4.4.1 Discrete modelling

In order to facilitate the coupling of the tower/nacelle to three rotating blades, a number of assumptions are made. It is first assumed that vibration of the tower/nacelle and rotating blades occurs in their fundamental mode shapes only, so each system may be generalised into a single degree-of-freedom (SDOF) system. This is a reasonable assumption as it is generally believed that the higher modes contribute negligibly to the dynamic response. Then the two SDOF systems each representing the tower/nacelle and three rotating blades are coupled together to form a discrete two DOF model, as illustrated in figures 4.7a and 4.7b. A three blade wind turbine tower system, as in figure 4.7a is therefore represented by the simple model shown in figure 4.7b, comprising two masses and two springs. The bottom mass and spring represent the tower and nacelle, and the top mass and spring represent the three rotating blades.

![Figure 4.7a Tower-blade system](image)

![Figure 4.7b Equivalent two DOF system](image)
The modal mass of the tower/nacelle, $\bar{M}_{T,1}$, associated with the displaced shape of the tower, $\Phi_{T,1}$, may be obtained as

$$\bar{M}_{T,1} = \int_0^{L_T} \Phi_{T,1}(x)^2 m_T(x)dx + \Phi_{T,1}(L)^2 M_{NAC}$$

(4.58)

where $L_T$ is the height of the tower, and $m_T(x)$ is the variation of mass along the tower. The tower is assumed to be prismatic, so equation 4.58 can be re-written as

$$\bar{M}_{T,1} = \bar{m}_T \int_0^{L_T} \Phi_{T,1}(x)^2 dx + \Phi_{T,1}(L)^2 M_{NAC}$$

(4.59)

where $\bar{m}_T$ is the mass per unit length of the tower. The above integral may be evaluated by use of numerical integration. The tower modal stiffness $K_{T,1}$, may be found as

$$K_{T,1} = \omega_{T,1}^2 \bar{M}_{T,1}$$

(4.60)

When the rotating blades of a wind turbine vibrate, they impart a shear force, maximum at the base of the blade, into the tower. It is this force transmission that strongly couples the motion of the blades to the tower, especially when the blades are vibrating in the flapping direction (deemed out-of-plane motion relative to figure 4.7a). When the blades are vibrating in the flapping mode, the shear force transmitted into the tower will always be horizontal, regardless of the position of the blade. When the blades are vibrating in a lead/lag mode (deemed in-plane motion relative to figure 4.7a), the shear force transmitted into the tower will have a horizontal and vertical component, the magnitude of each changes with the position the blade. However, the horizontal component of the latter is only of interest for blade/tower coupling.

The blade base shear, $V_{BF}$, imparted by a blade undergoing flapping motion, into the tower, may be obtained by integrating the elastic forces in the blade over its length as

$$V_{BF} = \int_0^{L_B} f_s(x,t)dx = \int_0^{L_B} \omega_{BF,1}^2 m_B(x)u(x,t)dx$$

(4.61)
where $L_B$ is the length of the blade, $\omega_{BF,i}$ is the flapping blade natural frequency and $f_s(x,t)$ and $u(x,t)$ are the elastic force and displacement of the beam, respectively, as functions of space and time. The blades are assumed to have prismatic geometries, so equation 4.61 may be rewritten as

$$V_{BF} = \int_0^{L_B} f_s(x,t) dx = \overline{m}_B \omega_{BF,i}^2 \int_0^{L_B} u(x,t) dx \quad (4.62)$$

The blade base shear, $V_{BLL}$, imparted by a blade undergoing lead/lag motion, into the tower, may be obtained by integrating the elastic forces in the blade over its length and considering the horizontal component of the force only as

$$V_{BLL} = \sin(\theta) \int_0^{L_B} f_s(x,t) dx = \sin(\theta) \int_0^{L_B} \omega_{BLL,i}^2 m_B(x) u(x,t) dx \quad (4.63)$$

where $\theta$ is the angle the blade makes with the horizontal global axes $X_g$, as in figure 4.7a. and $\omega_{BLL,i}$ is the lead/lag blade natural frequency. For a prismatic blade, the blade base shear for lead/lag motion is given by

$$V_{BLL} = \sin(\theta) \int_0^{L_B} f_s(x,t) dx = \sin(\theta) \overline{m}_B \int_0^{L_B} \omega_{BLL,i}^2 u(x,t) dx \quad (4.64)$$

For a SDOF system, the displacement of that system may be decomposed into two components, a single mode shape (for either flapping or lead/lag motion), $\Phi_{B,i}(x)$, and a time varying harmonic function $\eta_{B,i}(t)$, as expressed by

$$u(x,t) = \Phi_{B,i}(x) \eta_{B,i}(t) \quad (4.65)$$

Substituting equation 4.65 into equations 4.62 and 4.64 gives an expression for the base shear for flapping and lead/lag motion as

$$V_{BF} = \overline{m}_B \omega_{BF,i}^2 \int_0^{L_B} \Phi_{B,i}(x) dx \quad (4.66)$$

and
FREE VIBRATIONS OF WIND TURBINE TOWER ASSEMBLIES

\[ V_{\text{BLL}} = \sin(\theta) \bar{m}_B \omega_{\text{BLL},1}^2 \eta_{\text{B},1}(t) \int_0^{L_B} \Phi_{\text{B},1}(x) \, dx \]  

(4.67)

In equations 4.66 and 4.67, \( \eta_{\text{B},1}(t) \) represents the generalised coordinate, representing the relative displacement at the top of the blade. The base shear for three blades, \( V_{3\text{BF}} \), vibrating in their flapping modes, when all three blades are vibrating in-phase, is given by

\[ V_{3\text{BF}} = 3V_B = 3\bar{m}_B \omega_{\text{BF},1}^2 \int_0^{L_B} \Phi_{\text{B},1}(x) \, dx \]  

(4.68)

In the lead/lag case, the total resultant shear due to the three blades must be obtained. In finding this force, it assumed that the vibration of one of the blades is out-of-phase with that of the other two. If all three blades are vibrating in-phase, the resultant base shear is zero. Thus, the total base shear imparted by the three blades (\( V_{2\text{BLL}} \)) may be found as

\[ V_{2\text{BLL}} = V_{\text{B}1}[\sin\theta_1] + V_{\text{B}2}[\sin\theta_2] + V_{\text{B}3}[\sin\theta_3] \]  

(4.69)

where blades 1, 2 and 3 are inclined at angles \( \theta_1 \), \( \theta_2 \) and \( \theta_3 \) measured anti-clockwise to the horizontal, as in figure 4.7a, and \( V_{\text{B}1,2,3} \) correspond to the base shear exerted by blades 1, 2 and 3 respectively. In equation 4.69, an absolute value of all trigonometric functions is taken to ensure the correct directionality of force vectors in 360°.

The three rotating blades are replaced by a spring of equivalent stiffness to study the vibrational characteristics of the blade and its interaction with the tower. The force (F) exerted by the equivalent spring is the product of it’s stiffness (\( K_s \)) and the relative displacement at the tip of a blade, \( u(L_B, t) \) as

\[ F = K_s u(L_B, t) \]  

(4.70)

\( K_s \) is obtained as the ratio of the blade base shear, \( F ( = V_{3\text{BF}}, \text{or } V_{2\text{BLL}}) \) to \( u(L_B, t) \), where \( u(L_B, t) ( = \eta_{\text{B},1}(t)) \) is assumed to be the relative displacement at the tip of the blade. The equivalent mass of this SDOF system representing the three rotating blades, \( M_s \) may be obtained as
The tower and three rotating blades systems are readily coupled in matrix form as

\[
\begin{bmatrix}
M_s & 0 \\
0 & M_{T,1}
\end{bmatrix} - \omega_{cs}^2 \begin{bmatrix}
K_s & -K_s \\
-K_s & K_s + K_{T,3}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  

(4.72)

The eigenvalues of the coupled system, \( \omega_{cs} \), may be obtained by equating the determinant of the left hand side of equation 4.72 to zero. Since the flapping or lead/lag vibration of the rotor blades causes a base shear to be transmitted to the top of the tower, the coupled system will either vibrate in-plane or out-of-plane relative to figure 4.7a. Thus, the in-plane deformation of the blades will only affect the in-plane free vibration motion of the tower. This phenomenon also holds for out-of-plane motion of the blades.

4.5 NUMERICAL EXAMPLES

A series of numerical examples are presented to demonstrate the applicability of each of the analytical approaches previously described. Numerical examples for the three methods used to obtain the free vibration properties of both the lattice and tubular tower and nacelle are first provided. Next, the free vibration properties of a rotating blade is obtained, using the three analytical approaches contained in Section 4.3. Lastly, two numerical examples of a tower and nacelle system coupled to three rotating blades are illustrated. The geometry of the models assumed in this section is representative of that used in practise.

4.5.1 Free vibration properties of towers

Two models are considered in this section, a lattice tower and a tubular tower, both are carrying concentrated masses at the top, representing the mass of the nacelle. The lattice tower is a three dimensional structure of height 60 m, base width 6 m, top width 3 m, constructed of steel angular sections, as shown in figure 4.2. The near vertical members measure 200x200L24, and the oblique and horizontal members measure 100x100L12. The physical properties of all angled sections are sourced from The Steel Construction Institute (1997). The density of steel is taken as 7850 kgm\(^{-3}\), and the total mass of tower is 27222.76 kg. A range of concentrated masses are placed at the top the tower, and the ratio of these masses to the total mass of the tower is termed the mass ratio (mr). Mass ratios placed on top of the tower are 10 % or 2722 kg, 20 % or 5444 kg, and 30 % or 8166 kg.
The tubular tower is modelled as a three dimensional uniform steel tower of circular hollow cross-section, as illustrated in figures 4.3 and 4.4. Tower height of 60 m is considered, with a width of 3 m, and a thickness of 0.015 m. The density of steel is taken as 7850 kgm\(^{-3}\). The total mass of tower is 66253 kg. The mass ratios placed on top of the tower are 10 % or 6625 kg, 20 % or 13250 kg, or 30 % or 19876 kg. For the continuum model, the spherical mass has a diameter of 6.4 m, thickness of 0.02 m, with rotary inertia values of 0 kgm\(^{2}\), 846 kgm\(^{2}\), 1691 kgm\(^{2}\), 2536 kgm\(^{2}\), corresponding to mass ratios of 0 %, 10 %, 20 % and 30 % respectively.

**DISCRETE MODEL**

**Lattice Tower**

Table 4.1 presents the first three natural frequencies, \(f_{r,1}\), \(f_{r,2}\) and \(f_{r,3}\), for the lattice tower with mass ratios of 0 %, 10 %, 20 % and 30 %, obtained using the discrete model analytical formulation.

<table>
<thead>
<tr>
<th>(mr) (%)</th>
<th>(f_{r,1}) (Hz)</th>
<th>(f_{r,2}) (Hz)</th>
<th>(f_{r,3}) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.671</td>
<td>5.555</td>
<td>10.027</td>
</tr>
<tr>
<td>10</td>
<td>1.432</td>
<td>5.220</td>
<td>9.772</td>
</tr>
<tr>
<td>20</td>
<td>1.257</td>
<td>5.045</td>
<td>9.645</td>
</tr>
<tr>
<td>30</td>
<td>1.132</td>
<td>4.905</td>
<td>9.565</td>
</tr>
</tbody>
</table>

Table 4.1 Natural frequencies of lattice tower obtained using the discrete model

![Figure 4.8 1st mode shape with 'mr' of 0 %](image1)

![Figure 4.9 2nd mode shape with 'mr' of 0 %](image2)

![Figure 4.10 3rd mode shape with 'mr' of 0 %](image3)
Figures 4.8 to 4.19 show the mode shapes obtained using the discrete model. Figures 4.8 to 4.10 illustrate the first three mode shapes for the lattice tower with a mass ratio of 0 %. Figures 4.11 to 4.13 show the first three mode shapes for the lattice tower with a mass ratio of 10 %, Figures 4.14 to 4.16 show the first three mode shapes for the lattice tower with a mass ratio of 20 %, and Figures 4.17 to 4.19 show the first three mode shapes for the lattice tower with a mass ratio of 30 %.
of 10 %. The mode shapes for a mass ratio of 20 % are shown in figures 4.14 through 4.16 and figures 4.17 to 4.19 show the first three mode shapes for a mass ratio of 30 %.

**Tubular tower**

Table 4.2 presents the first three natural frequencies of the tubular tower, $f_{T,1}$, $f_{T,2}$ and $f_{T,3}$, for mass ratios of 0 %, 10 %, 20 % and 30 % using the discrete parameter analytical method.

<table>
<thead>
<tr>
<th>mr (%)</th>
<th>$f_{T,1}$ (Hz)</th>
<th>$f_{T,2}$ (Hz)</th>
<th>$f_{T,3}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.840</td>
<td>4.918</td>
<td>12.732</td>
</tr>
<tr>
<td>10</td>
<td>0.716</td>
<td>4.440</td>
<td>12.048</td>
</tr>
<tr>
<td>20</td>
<td>0.629</td>
<td>4.234</td>
<td>11.825</td>
</tr>
<tr>
<td>30</td>
<td>0.563</td>
<td>4.106</td>
<td>11.693</td>
</tr>
</tbody>
</table>

Table 4.2 Natural frequencies of tubular tower obtained using the discrete model

Figures 4.20 to 4.31 illustrate the mode shapes for the tubular tower with varying mass ratios. Figures 4.20 to 4.22 demonstrate the first three mode shapes for a mass ratio of 0 %. Figures 4.23 to 4.25 show the first three mode shapes obtained using a mass ratio of 10 %. A mass ratio of 20 % was used to obtain the mode shapes presented in figures 4.26 to 4.28 and figures 4.29 to 4.31 illustrate the first three mode shapes generated with a mass ratio of 30 %.
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Figure 4.23 1\textsuperscript{st} mode shape with 'mr' of 10%

Figure 4.24 2\textsuperscript{nd} mode shape with 'mr' of 10%

Figure 4.25 3\textsuperscript{rd} mode shape with 'mr' of 10%

Figure 4.26 1\textsuperscript{st} mode shape with 'mr' of 20%

Figure 4.27 2\textsuperscript{nd} mode shape with 'mr' of 20%

Figure 4.28 3\textsuperscript{rd} mode shape with 'mr' of 20%

Figure 4.29 1\textsuperscript{st} mode shape with 'mr' of 30%

Figure 4.30 2\textsuperscript{nd} mode shape with 'mr' of 30%

Figure 4.31 3\textsuperscript{rd} mode shape with 'mr' of 30%
FINITE ELEMENT ANALYSIS

Lattice tower

Table 4.3 presents the natural frequencies of the lattice tower, $f_{T,1}$, $f_{T,2}$ and $f_{T,3}$, obtained using the finite element software code ANSYS for mass ratios of 0 %, 10 %, 20 % and 30 %.

<table>
<thead>
<tr>
<th>MR (%)</th>
<th>$f_{T,1}$ (Hz)</th>
<th>$f_{T,2}$ (Hz)</th>
<th>$f_{T,3}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.679</td>
<td>6.439</td>
<td>13.313</td>
</tr>
<tr>
<td>10</td>
<td>1.440</td>
<td>5.771</td>
<td>12.030</td>
</tr>
<tr>
<td>20</td>
<td>1.266</td>
<td>5.433</td>
<td>11.576</td>
</tr>
<tr>
<td>30</td>
<td>1.141</td>
<td>5.246</td>
<td>11.365</td>
</tr>
</tbody>
</table>

Table 4.3 Natural frequencies of lattice tower obtained using the finite element model

Figures 4.32 to 4.43 illustrate the first three mode shapes of the lattice tower obtained using various mass ratios. Figures 4.32 to 4.34 demonstrate the lattice tower mode shapes with a mass ratio of 0 %. A mass ratio of 10 % was used to generate the mode shapes presented in figures 4.35 to 4.37. Figures 4.38 to 4.40 show the first three mode shapes for a mass ratio of 20 % and figures 4.41 to 4.43 illustrate the mode shapes obtained with a mass ratio of 30 %.

Figure 4.32 1\textsuperscript{st} mode shape with ‘mr’ of 0 %

Figure 4.33 2\textsuperscript{nd} mode shape with ‘mr’ of 0 %

Figure 4.34 3\textsuperscript{rd} mode shape with ‘mr’ of 0 %
Tubular Tower

Table 4.4 presents the first three mode shapes of the tubular tower, $f_{r,1}$, $f_{r,2}$ and $f_{r,3}$, obtained using the finite element software code ANSYS for varying mass ratios.

<table>
<thead>
<tr>
<th>mr (%)</th>
<th>$f_{r,1}$ (Hz)</th>
<th>$f_{r,2}$ (Hz)</th>
<th>$f_{r,3}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.845</td>
<td>5.161</td>
<td>13.937</td>
</tr>
<tr>
<td>10</td>
<td>0.712</td>
<td>4.528</td>
<td>12.529</td>
</tr>
<tr>
<td>20</td>
<td>0.627</td>
<td>4.257</td>
<td>12.075</td>
</tr>
<tr>
<td>30</td>
<td>0.566</td>
<td>4.107</td>
<td>11.854</td>
</tr>
</tbody>
</table>

Table 4.4 Natural frequencies of tubular tower obtained using the finite element model

Figures 4.44 to 4.55 show the first three mode shapes of the tubular tower obtained using the finite element software code ANSYS. Figures 4.44 to 4.46 show the mode shapes for a mass ratio of 0 %, and figures 4.47 to 4.49 illustrate the same three mode shapes but with a mass ratio of 10 %. A mass ratio of 20 % was used to generate figures 4.50 to 4.52 and a mass ratio of 30 % was used to create the first three mode shapes of the tubular tower, as pictured in figures 4.53 to 4.55.

Figure 4.44 1\textsuperscript{st} mode shape with 'mr' of 0 %

Figure 4.45 2\textsuperscript{nd} mode shape with 'mr' of 0 %

Figure 4.46 3\textsuperscript{rd} mode shape with 'mr' of 0 %
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Figure 4.47 1st mode shape with 'mr' of 10%

Figure 4.48 2nd mode shape with 'mr' of 10%

Figure 4.49 3rd mode shape with 'mr' of 10%

Figure 4.50 1st mode shape with 'mr' of 20%

Figure 4.51 2nd mode shape with 'mr' of 20%

Figure 4.52 3rd mode shape with 'mr' of 20%

Figure 4.53 1st mode shape with 'mr' of 30%

Figure 4.54 2nd mode shape with 'mr' of 30%

Figure 4.55 3rd mode shape with 'mr' of 30%
CONTINUUM MODEL

Tubular tower
Table 4.5 presents the first three natural frequencies obtained for the tubular tower, $f_{T,1}$, $f_{T,2}$ and $f_{T,3}$, for varying mass ratios using the continuum model.

<table>
<thead>
<tr>
<th>mr (%)</th>
<th>$f_{T,1}$ (Hz)</th>
<th>$f_{T,2}$ (Hz)</th>
<th>$f_{T,3}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.847</td>
<td>5.317</td>
<td>14.889</td>
</tr>
<tr>
<td>10</td>
<td>0.716</td>
<td>4.668</td>
<td>13.377</td>
</tr>
<tr>
<td>20</td>
<td>0.630</td>
<td>4.389</td>
<td>12.891</td>
</tr>
<tr>
<td>30</td>
<td>0.570</td>
<td>4.236</td>
<td>12.652</td>
</tr>
</tbody>
</table>

Table 4.5 Natural frequencies of tubular tower obtained using the continuum model

Figures 4.56 to 4.67 illustrate the first three mode shapes obtained for the tubular tower using the continuum model. Figures 4.56 to 4.58 show the first three mode shapes for a mass ratio of 0 %, while figures 4.59 to 4.61 present mode shapes for a mass ratio of 10 %. A mass ratio of 20 % was used to obtain the mode shapes in figures 4.62 to 4.64 and figures 4.65 to 4.67 show the tubular tower mode shapes with a mass ratio of 30 %.

Figure 4.56 1st mode shape with ‘mr’ of 0 %
Figure 4.57 2nd mode shape with ‘mr’ of 0 %
Figure 4.58 3rd mode shape with ‘mr’ of 0 %
Figure 4.59 1\textsuperscript{st} mode shape with ‘mr’ of 10 %

Figure 4.60 2\textsuperscript{nd} mode shape with ‘mr’ of 10 %

Figure 4.61 3\textsuperscript{rd} mode shape with ‘mr’ of 10 %

Figure 4.62 1\textsuperscript{st} mode shape with ‘mr’ of 20 %

Figure 4.63 2\textsuperscript{nd} mode shape with ‘mr’ of 20 %

Figure 4.64 3\textsuperscript{rd} mode shape with ‘mr’ of 20 %

Figure 4.65 1\textsuperscript{st} mode shape with ‘mr’ of 30 %

Figure 4.66 2\textsuperscript{nd} mode shape with ‘mr’ of 30 %

Figure 4.67 3\textsuperscript{rd} mode shape with ‘mr’ of 30 %
4.5.2 Free vibration properties of blades

Several numerical examples regarding the free vibration properties of rotating wind turbine blades are illustrated in this section. In particular, two separate models of a blade are presented. The first model is a uniform cantilever beam used with the discrete and continuum analytical models only, as seen in figure 4.6, and the second is a tapered beam used with the finite element model, as illustrated in figure 4.5. The prismatic cantilever blade is of rectangular hollow cross-section. The blade is 30 m in length with a width of 2.4 m, a depth of 0.4 m and a thickness of 0.01 m. The elastic modulus of the blade is taken as $6.5 \times 10^{10}$ Nm$^{-2}$. The mass moment of inertia in the flapping and lead/lag directions is $1.92 \times 10^3$ m$^4$ and $3.39 \times 10^2$ m$^4$ respectively. The density of the material used to construct the blade was taken as 2100 kgm$^{-3}$, giving each blade a mass per unit length of 116.76 kgm$^{-1}$ and a total mass of 3502.80 kg. The tapered blade model built using the finite element code ANSYS, as illustrated in figure 4.5, has a length of 30 m, a width of 1.1 m at both ends and a width of 3.0 m at its widest intermediate point. The modulus of elasticity of the tapered blade was taken as $6.5 \times 10^{10}$ Nm$^{-2}$ and the density of the blade’s material was assumed to be 2100 kgm$^{-3}$.

**DISCRETE MODEL**

Table 4.6a presents the first two flapping and lead/lag natural frequencies, $f_{B,1}$ and $f_{B,2}$, obtained for the prismatic blade modelled as a discrete system. The blade is stationary and is shown at three positions, $\theta = 0^\circ$, $90^\circ$, and $210^\circ$, where $\theta$ (or $\theta_1$ in figure 4.7a) is the angle the blade’s longitudinal axis makes with the horizontal measured anticlockwise.

<table>
<thead>
<tr>
<th>$\theta$ (Degrees)</th>
<th>Flapping</th>
<th>Lead/lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{B,1}$ (Hz)</td>
<td>$f_{B,2}$ (Hz)</td>
</tr>
<tr>
<td>0</td>
<td>0.637</td>
<td>3.919</td>
</tr>
<tr>
<td>90</td>
<td>0.630</td>
<td>3.913</td>
</tr>
<tr>
<td>210</td>
<td>0.638</td>
<td>3.920</td>
</tr>
</tbody>
</table>

Table 4.6a Flapping and lead/lag natural frequencies for the discrete model with $\Omega = 0$ rads$^{-1}$

Figures 4.68 and 4.69 show the first and second flapping and lead/lag mode shapes of the blade not undergoing rotation, as a function of blade position $\theta$. Table 4.6b showcases the first and second flapping and lead/lag natural frequencies, $f_{B,1}$ and $f_{B,2}$, for the blade.
modelled as a discrete system. The blade is rotating with a rotational frequency $\Omega$ of 0.785 radians$^{-1}$ (one complete revolution every eight seconds) and the blade is shown at three different positions, corresponding to $\theta = 0^\circ$, $90^\circ$, and $210^\circ$.

$$\Omega = 0.785 \text{ rads}^{-1}$$

<table>
<thead>
<tr>
<th>$\theta$ (Degrees)</th>
<th>Flapping</th>
<th>Lead/lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{r,1}$ (Hz)</td>
<td>$f_{r,2}$ (Hz)</td>
</tr>
<tr>
<td>0</td>
<td>0.651</td>
<td>3.931</td>
</tr>
<tr>
<td>90</td>
<td>0.643</td>
<td>3.931</td>
</tr>
<tr>
<td>210</td>
<td>0.652</td>
<td>3.932</td>
</tr>
</tbody>
</table>

Table 4.6b Flapping and lead/lag natural frequencies for the discrete model with $\Omega = 0.785$ rads$^{-1}$

Figures 4.70 and 4.71 illustrate the first and second flapping and lead/lag mode shapes for the discrete blade, as a function of blade position $\theta$. Table 4.6c shows the first and second flapping and lead/lag natural frequencies for the discrete blade rotating at a rotational frequency $\Omega$ of 1.57 rads$^{-1}$ (one complete revolution every four seconds) and the blade is shown at three different positions, corresponding to $\theta = 0^\circ$, $90^\circ$, and $210^\circ$. Figures 4.72 and 4.73 illustrate the first and second flapping and lead/lag mode shapes of the discrete blade system, as a function of the three blade positions denoted by $\theta$. Table 4.6d presents the first and second natural frequencies for the blade modelled using the discrete parameter method for flapping and lead/lag motion.

$$\Omega = 1.57 \text{ rads}^{-1}$$

<table>
<thead>
<tr>
<th>$\theta$ (Degrees)</th>
<th>Flapping</th>
<th>Lead/lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{r,1}$ (Hz)</td>
<td>$f_{r,2}$ (Hz)</td>
</tr>
<tr>
<td>0</td>
<td>0.690</td>
<td>3.961</td>
</tr>
<tr>
<td>90</td>
<td>0.683</td>
<td>3.955</td>
</tr>
<tr>
<td>210</td>
<td>0.691</td>
<td>3.962</td>
</tr>
</tbody>
</table>

Table 4.6c Flapping and lead/lag natural frequencies for the discrete model with $\Omega = 1.57$ rads$^{-1}$

The blade is rotating with a rotational frequency of $\Omega = 3.14$ rads$^{-1}$ and is oriented in three positions corresponding to $\theta = 0^\circ$, $90^\circ$, and $210^\circ$. 

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Table 4.6d Flapping and lead/lag natural frequencies for the discrete model with $\Omega = 3.14 \text{ rads}^{-1}$

<table>
<thead>
<tr>
<th>$\theta$ (Degrees)</th>
<th>$f_{T,1}$ (Hz)</th>
<th>$f_{T,2}$ (Hz)</th>
<th>$f_{T,1}$ (Hz)</th>
<th>$f_{T,2}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.826</td>
<td>4.080</td>
<td>2.734</td>
<td>16.526</td>
</tr>
<tr>
<td>90</td>
<td>0.819</td>
<td>4.074</td>
<td>2.732</td>
<td>16.525</td>
</tr>
<tr>
<td>210</td>
<td>0.826</td>
<td>4.080</td>
<td>2.734</td>
<td>16.527</td>
</tr>
</tbody>
</table>

Figures 4.74 and 4.75 illustrate the first and second mode shapes for flapping and lead/lag motion for the blade rotating at $\Omega = 3.14 \text{ rads}^{-1}$, for the three blade positions denoted by $\theta$.

Figure 4.68 1\textsuperscript{st} flapping and lead/lag mode shapes with varying position, $\Omega = 0 \text{ rads}^{-1}$

Figure 4.69 2\textsuperscript{nd} flapping and lead/lag mode shapes with varying position, $\Omega = 0 \text{ rads}^{-1}$

Figure 4.70 1\textsuperscript{st} flapping and lead/lag mode shapes with varying position, $\Omega = 0.785 \text{ rads}^{-1}$

Figure 4.71 2\textsuperscript{nd} flapping and lead/lag mode shapes with varying position, $\Omega = 0.785 \text{ rads}^{-1}$
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Figure 4.72 1st flapping and lead/lag mode shapes with varying position, $\Omega = 1.57 \text{ rads}^{-1}$

Figure 4.73 2nd flapping and lead/lag mode shapes with varying position, $\Omega = 1.57 \text{ rads}^{-1}$

Figure 4.74 1st flapping and lead/lag mode shapes with varying position, $\Omega = 3.14 \text{ rads}^{-1}$

Figure 4.75 2nd flapping and lead/lag mode shapes with varying position, $\Omega = 3.14 \text{ rads}^{-1}$

**FINITE ELEMENT MODEL**

Table 4.7 lists the first and second flapping and lead/lag natural frequencies, $f_{3,1}$ and $f_{3,2}$, obtained from the modal analysis capability of the finite element software code ANSYS.

<table>
<thead>
<tr>
<th>$\Omega$ (rads$^{-1}$)</th>
<th>Flapping</th>
<th>Lead/lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1$ (Hz)</td>
<td>$f_2$ (Hz)</td>
</tr>
<tr>
<td>0</td>
<td>0.697</td>
<td>3.998</td>
</tr>
</tbody>
</table>

Table 4.7 Flapping and lead/lag natural frequencies for the finite element model
The code can not take rotary or self weight effects into account, so a blade rotational frequency of $\Omega = 0$ rads$^{-1}$ is designated. Figures 4.76 and 4.77 illustrate the first and second flapping mode shapes as output from ANSYS. Figures 4.78 and 4.79 show the corresponding first and second lead/lag mode shapes.

### Flapping modes

![3D Dynamic Analysis of a Wind Turbine Blade](image1)

Figure 4.76 1st flapping mode shape of tapering wind turbine blade

![3D Dynamic Analysis of a Wind Turbine Blade](image2)

Figure 4.77 2nd flapping mode shape of tapering wind turbine blade
Lead lag modes

Figure 4.78 1\textsuperscript{st} lead/lag mode shape of tapered wind turbine blade

Figure 4.79 2\textsuperscript{nd} lead/lag mode shape of tapered wind turbine blade
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CONTINUUM MODEL
Table 4.8 presents the first and second flapping and lead/lag natural frequencies, $f_{b,1}$ and $f_{b,2}$, for the rotating blade modelled as a continuum. The blade is rotating with four different rotational frequencies, $\Omega$. The continuum model does not take the axial self weight effect of blade rotation into account, so the blade cannot be attributed a spatial position.

<table>
<thead>
<tr>
<th>$\Omega$ (rads$^{-1}$)</th>
<th>Flapping</th>
<th>Lead/lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{b,1}$ (Hz)</td>
<td>$f_{b,2}$ (Hz)</td>
</tr>
<tr>
<td>0</td>
<td>0.642</td>
<td>4.025</td>
</tr>
<tr>
<td>0.785</td>
<td>0.659</td>
<td>4.040</td>
</tr>
<tr>
<td>1.570</td>
<td>0.705</td>
<td>4.020</td>
</tr>
<tr>
<td>3.140</td>
<td>0.865</td>
<td>4.247</td>
</tr>
</tbody>
</table>

Table 4.8 Flapping and lead/lag natural frequencies obtained using continuum model

Figures 4.80 to 4.87 illustrate the first and second flapping and lead/lag mode shapes of the continuum blade. Figures 4.80 and 4.81 denote a stationary blade, figures 4.82 and 4.83 a blade with a rotational frequency of 0.785 rads$^{-1}$, figures 4.84 and 4.85 a rotational frequency of 1.57 rads$^{-1}$, and figures 4.86 and 4.87 a rotational frequency of 3.14 rads$^{-1}$.

Figure 4.80 1st flapping and lead/lag mode shapes for $\Omega = 0$ rads$^{-1}$

Figure 4.81 2nd flapping and lead/lag mode shapes for $\Omega = 0$ rads$^{-1}$
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Figure 4.82 1st flapping and lead/lag mode shapes for $\Omega = 0.785$ rad$^{-1}$

Figure 4.83 2nd flapping and lead/lag mode shapes for $\Omega = 0.785$ rad$^{-1}$

Figure 4.84 1st flapping and lead/lag mode shapes for $\Omega = 1.57$ rad$^{-1}$

Figure 4.85 2nd flapping and lead/lag mode shapes for $\Omega = 1.57$ rad$^{-1}$

Figure 4.86 1st flapping and lead/lag mode shapes for $\Omega = 3.14$ rad$^{-1}$

Figure 4.87 2nd flapping and lead/lag mode shapes for $\Omega = 3.14$ rad$^{-1}$
4.5.3 Free vibration properties of tower/nacelle coupled with blades

Two numerical examples of tower and blade coupling are presented. The first example, termed Assembly 1, is chosen to investigate the magnitudes of the natural frequencies of the coupled model due to both flapping and lead/lag coupling, when the fundamental frequency of the blades is not close to the fundamental frequency of the tower and nacelle. A second numerical example, termed Assembly 2, is presented, in which the fundamental frequency of the rotating blades are approximately equal to the fundamental frequency of the tower and nacelle. Flapping and lead/lag coupling is also investigated for this example.

**ASSEMBLY 1**

A uniform tubular tower of circular hollow cross-section height of 60 m, width 3 m, and thickness 0.015 m is considered. The density of steel was taken as 7850 kgm$^{-3}$. The total mass of the tower is 66253 kg. A mass ratio of 30% or 19876 kg was placed at the top of the tower representing the nacelle. This consisted of a spherical mass with a diameter of 6.4 m, thickness of 0.02 m, and rotary inertia of 2536 kgm$^2$. The fundamental frequency of tower and nacelle is 0.570 Hz.

The rotor system comprises three rotating prismatic blades, each assumed to be geometrically similar and of the same material. The blades are 30 m in length, with one of the blades assumed to have an azimuth angle $\theta = 90^\circ$ and the other two are at $\theta = 210^\circ$ and $330^\circ$. Thus, the method outlined in Section 4.3.1 is used to find the free vibration properties of the blades. Tables 4.9 and 4.10 present the first and second natural frequencies, $f_{CS,1}$ and $f_{CS,2}$, of the coupled system for flapping and lead/lag motion respectively.

<table>
<thead>
<tr>
<th>$\Omega$ (rads$^{-1}$)</th>
<th>$f_{CS,1}$ (Hz)</th>
<th>$f_{CS,2}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.52451</td>
<td>1.35914</td>
</tr>
<tr>
<td>0.785</td>
<td>0.52459</td>
<td>1.36629</td>
</tr>
<tr>
<td>1.57</td>
<td>0.52484</td>
<td>1.38781</td>
</tr>
<tr>
<td>3.14</td>
<td>0.52567</td>
<td>1.47031</td>
</tr>
</tbody>
</table>

Table 4.9 Flapping natural frequencies of coupled system

<table>
<thead>
<tr>
<th>$\Omega$ (rads$^{-1}$)</th>
<th>$f_{CS,1}$ (Hz)</th>
<th>$f_{CS,2}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.542725</td>
<td>3.48374</td>
</tr>
<tr>
<td>0.785</td>
<td>0.542726</td>
<td>3.48624</td>
</tr>
<tr>
<td>1.57</td>
<td>0.542729</td>
<td>3.49441</td>
</tr>
<tr>
<td>3.14</td>
<td>0.542738</td>
<td>3.52674</td>
</tr>
</tbody>
</table>

Table 4.10 Lead/lag natural frequencies of coupled system
Figures 4.88 to 4.103 illustrate the first and second flapping and lead/lag mode shapes of the two DOF coupled blades/tower Assembly 1. Figures 4.88 and 4.89 represent the first and second flapping mode shapes when the blades are stationary, and figures 4.90 and 4.91 represent the same mode shapes but for the lead/lag case of coupling. Figures 4.92 and 4.93 illustrate the first and second flapping mode shapes when the blades are rotating at 0.785 rads$^{-1}$ and figures 4.94 and 4.95 show the corresponding two mode shapes for lead/lag motion.

The first and second flapping modes for blade rotation of 1.57 rads$^{-1}$ are shown in figures 4.96 and 4.97 and the lead/lag mode shapes for the same rotational frequency are illustrated in figures 4.98 and 4.99. Figures 4.100 and 4.101 show the first and second flapping modes for blade rotation of 3.14 rads$^{-1}$, and the lead/lag coupling mode shapes for this rotational frequency are shown in figures 4.102 and 4.103.
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**ASSEMBLY 1**

The second model consists of a uniform tubular tower of circular hollow cross-section of height 50 m, with a width of 2.7 m, and a thickness of 0.015 m. The mass moment of inertia of the tower is $1.14 \times 10^{11}$ m$^4$. The tower is constructed from steel with an elastic modulus of $2.1 \times 10^{11}$ Nm$^{-2}$, and density of steel 7850 kgm$^{-3}$. The total mass of tower is 49662.1 kg. A nacelle of mass 19864.84 kg sits on top of the tower, with a diameter of 6.4 m, and rotary inertia of 2534.75 kgm$^2$, representing a mass ratio of 40%. The fundamental frequency of tower and nacelle system is 0.678 Hz.

The rotor system being coupled to the tower consists of three identical blades, each of length 30 m, with a hub radius of 3 m, a width of 2.4 m, a depth of 0.4 m, and a thickness 0.01 m. The density of blade material is taken as being 2100 kgm$^{-3}$, giving each blade a total mass of 3502.80 kg. The blades have a flapping moment of inertia of $1.92 \times 10^{-3}$ m$^4$,
and a lead/lag moment of inertia of $3.39 \times 10^{-2} \text{ m}^4$. One of the blades is assumed to have an azimuth angle $\theta = 90^\circ$ and the other two are at $\theta = 210^\circ$ and $330^\circ$.

<table>
<thead>
<tr>
<th>$\Omega$ (rads$^{-1}$)</th>
<th>$f_{CS,1}$ (Hz)</th>
<th>$f_{CS,2}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.55022</td>
<td>0.78236</td>
</tr>
<tr>
<td>0.785</td>
<td>0.55587</td>
<td>0.79077</td>
</tr>
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<th>$\Omega$ (rads$^{-1}$)</th>
<th>$f_{CS,1}$ (Hz)</th>
<th>$f_{CS,2}$ (Hz)</th>
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Table 4.11 Flapping natural frequencies of coupled system

Table 4.12 Lead/lag natural frequencies of coupled system

The method outlined in Section 4.3.1 is used to find the free vibration properties of the blades. Tables 4.11 and 4.12 present the first and second natural frequencies, $f_{CS,1}$ and $f_{CS,2}$, of the coupled system for flapping and lead/lag motion respectively. Figures 4.104 to 4.119 illustrate the first and second flapping and lead/lag mode shapes for the two DOF coupled blades/tower Assembly 2. Figures 4.104 and 4.105 show the first and second flapping mode shape for stationary blades, while the respective lead/lag mode shapes are shown in figures 4.106 and 4.107. Figures 4.108 and 4.109 show the first and second mode shapes for blade rotation of 0.785 rads$^{-1}$, with the corresponding lead/lag mode shapes presented in figures 4.110 and 4.111. A blade rotational frequency of 1.57 rads$^{-1}$ was used to generate the first and second flapping modes in figures 4.112 and 4.113, with the corresponding lead/lag mode shapes presented in figures 4.114 and 4.115. Figures 4.116, 4.117 show the flapping mode shapes of a rotational frequency of 3.14 rads$^{-1}$, with the corresponding lead/lag mode shapes shown in figures 4.118 and 4.119.
NORMALISED DISPLACEMENT

Figure 4.108 1st flapping mode shape
$\Omega = 0.785 \text{ rads}^{-1}$

Figure 4.109 2nd flapping mode shape
$\Omega = 0.785 \text{ rads}^{-1}$

Figure 4.110 1st lead/lag mode shape
$\Omega = 0.785 \text{ rads}^{-1}$

Figure 4.111 2nd lead/lag mode shape
$\Omega = 0.785 \text{ rads}^{-1}$

Figure 4.112 1st flapping mode shape
$\Omega = 1.57 \text{ rads}^{-1}$

Figure 4.113 2nd flapping mode shape
$\Omega = 1.57 \text{ rads}^{-1}$

Figure 4.114 1st lead/lag mode shape
$\Omega = 1.57 \text{ rads}^{-1}$

Figure 4.115 2nd lead/lag mode shape
$\Omega = 1.57 \text{ rads}^{-1}$

Figure 4.116 1st flapping mode shape
$\Omega = 3.14 \text{ rads}^{-1}$

Figure 4.117 2nd flapping mode shape
$\Omega = 3.14 \text{ rads}^{-1}$

Figure 4.118 1st lead/lag mode shape
$\Omega = 3.14 \text{ rads}^{-1}$

Figure 4.119 2nd lead/lag mode shape
$\Omega = 3.14 \text{ rads}^{-1}$
4.6 DISCUSSION ON NUMERICAL EXAMPLES

This section will briefly discuss the numerical examples presented in Section 4.4, involving the free vibrations of tower and nacelle, rotating blades and coupled tower/nacelle and rotating blades. Three methods were presented to obtain the free vibration properties of the tower and nacelle systems, where the tower can be either of lattice or tubular type, and each type carried mass ratios of 0 %, 10 %, 20 % and 30 %. For the lattice tower case, a discrete and finite element model were presented, where the latter had much more DOF than the former. The discrete model had 5 DOFs and the finite element model had 20. The fundamental natural frequencies obtained using both approaches showed close correlation, with percentage differences observed at mass ratios of 0 %, 10 %, 20 % and 30 % of 0.5 %, 0.6 %, 0.7 % and 0.8 % respectively. The second natural frequency showed a difference of 14 %, 10 %, 7 % and 7 % at mass ratios of 0 %, 10 %, 20 % and 30 %, and the third natural frequency showed a discrepancy of 25%, 19%, 17% and 16% at mass ratios of 0 %, 10 %, 20 % and 30 % respectively. The natural frequencies obtained using the finite element method proved to be the most accurate, as this approach contained the greatest number of DOFs due to a more complex system discretisation.

For the tubular tower case, all three analytical methods were used to analyse the structure, the continuum model contained the most number of DOF, whilst the discrete model contained the least number of DOF, at 5. However, all three analytical methods agreed closely regarding the fundamental frequency of vibration, with the continuum model yielding the greatest accuracy due to it having the greatest number of DOF. Relative to the continuum model, the fundamental frequency observed using the discrete model and finite element model (which had 20 DOFs) showed a percentage difference of 0.8 % and 0.2 % for a mass ratio of 0 %, 0 % and 0.6 % for a mass ratio of 10 %, 0.2 % and 0.5 % for a mass ratio of 20 % and 1.2 % and 0.7 % for a mass ratio of 30 %. The second natural frequency found using the discrete and finite element methods showed a discrepancy of 7.5 % and 3 % respectively, for a mass ratio of 0%, 5 % and 3 % respectively, for a mass ratio of 10 %, 4 % and 3 % respectively, for a mass ratio of 20 %, and 3 % and 3 % respectively, for a mass ratio of 30 %. The third natural frequency found using the discrete and finite element methods showed a discrepancy of 14 % and 6 % respectively, for a mass ratio of 0%, 10 % and 6 % respectively, for a mass ratio of 10 %, 8 % and 6 % respectively, for a mass ratio of 20 %, and also 8 % and 6 % respectively, for a mass ratio of 30 %.
A uniform blade was analysed using both the discrete and continuum analytical methods with each method having its own distinct merits. The discrete approach is the only model presented of the three that can consider centrifugal stiffening and axial self-weight effects, but it generally contains the lowest number of DOF, at 8. However, when comparing the fundamental flapping and lead/lag natural frequencies obtained using the discrete and continuum models, a percentage difference of approximately 2 % for the flapping natural frequency and 0.7 % for the lead/lag natural frequency was observed, for a rotational frequency of 0 rads⁻¹. These values changed to 2 % and 0.8 %, for the flapping and lead/lag fundamental natural frequency for a rotational frequency of 0.785 rads⁻¹, 3 % and 0.8 % for the flapping and lead/lag fundamental natural frequency with a rotational frequency of 1.57 rads⁻¹, and 5 % and 1 % for the flapping and lead/lag fundamental natural frequency for a rotational frequency of 3.14 rads⁻¹. The second flapping and lead/lag natural frequencies show discrepancies of 3 % and 3 %, 3 % and 2.6 %, 2 % and 2.6 % and 4 % and 2.6 % for the flapping and lead/lag second natural frequency for rotational frequencies of 0 rads⁻¹, 0.785 rads⁻¹, 1.57 rads⁻¹, and 3.14 rads⁻¹ respectively. The free vibration properties obtained using the finite element ANSYS method were included to show that a more realistic blade geometry may be considered, though this method does have a drawback as centrifugal stiffening effects may not be included using this version of ANSYS. However, for slow blade rotational frequencies, this phenomenon does not appear to alter the free vibration parameters of the blade greatly.

Lastly, two different models were presented to show the effect of the proximity of blade and tower/nacelle fundamental frequencies on the natural frequencies of the coupled system, and to show the effect of blade rotational frequency on the coupled system. In Assembly 1, the fundamental flapping frequency of the blade at rest was approximately twice the fundamental frequency of the tower and nacelle, where in Assembly 2, the fundamental flapping frequency of the blade at rest was a little less than that of the tower and nacelle. In Assembly 1, the first natural frequency of the coupled system due to blade flapping, when the blades are at rest, was 7.981 % less than the fundamental frequency of the tower and nacelle, where the second natural frequency of the coupled system due to blade flapping was approximately 138.494 % greater than the fundamental frequency of the tower/nacelle. These values changed to 7.966 % and 139.700 % respectively, for a rotational frequency of 0.785 rads⁻¹, 7.923 % and 143.475 % respectively, for 1.57 rads⁻¹, and 7.778 % and 157.965 % respectively for 3.14 rads⁻¹.
The first natural frequency of the coupled system due to blade lead/lag motion, but when the blades are stationary, was 4.785 % less than the fundamental frequency of the tower and nacelle. The second natural frequency was 511.183 % greater than the fundamental frequency of the tower and nacelle. The corresponding percentage differences for rotational velocities of 0.785 rads⁻¹, 1.57 rads⁻¹ and 3.14 rads⁻¹ were 4.785 % and 511.622 %, 4.784 % and 513.054 % and 4.783 % and 518.727 %.

In Assembly 2, the first natural frequency of the coupled system due to blade flapping, when the blades are at rest, was approximately 18.846 % less the fundamental frequency of the tower and nacelle, where the second natural frequency of the coupled system due to blade flapping was approximately 15.392 % greater than the fundamental frequency of the tower/nacelle. These values changed to 18.013 % and 16.632 % respectively, for a rotational frequency of 0.785 rads⁻¹, 15.941 % and 20.537 % respectively, for 1.57 rads⁻¹, and 11.533 % and 37.220 % respectively, for 3.14 rads⁻¹. The first natural frequency of the coupled system due to blade lead/lag motion, but when the blades are stationary, was 6.147 % less than the fundamental frequency of the tower and nacelle. The second natural frequency was 321.003 % greater than the fundamental frequency of the tower and nacelle. The corresponding percentage differences for rotational velocities of 0.785 rads⁻¹, 1.57 rads⁻¹ and 3.14 rads⁻¹ were 6.146 % and 321.508 % respectively, 6.144 % and 322.990 % respectively, and 6.136 % and 328.935 % respectively. It may be interesting to observe that in Assembly 2, the coupling of the two modes may cause a significant influence of the effect of the second mode on the fundamental mode of response of the tower-blade system.
CHAPTER 5 - FORCED VIBRATIONS OF WIND TURBINE TOWER ASSEMBLIES

5.1 INTRODUCTION

In this chapter, the analytical approaches used to predict the forced vibration behaviour of a wind turbine tower are presented. This approach represents the second component of a dynamic analysis, and is intrinsically linked to the availability of a systems free vibration properties, as explained in Chapter 4. Several models are created using the discrete parameter approach, and using mathematical techniques, the response of various component of a wind turbine system can be obtained. These include a wind turbine tower and nacelle, a rotating blade, and a wind turbine tower with three rotating blades coupled together. The first section; Section 5.2, presents the analytical approaches adopted to estimate the response of the wind turbine tower and nacelle only, subject to random wind loading. The analytical approach proposed to obtain the response of a rotating blade is discussed in Section 5.3. The next section; Section 5.4, describes the analytical method derived in order to obtain the response of a wind turbine tower coupled together with three rotating blades. The following section; Section 5.5, provides a series of typical models and uses the analytical approaches discussed in Sections 5.2, 5.3 and 5.4 to generate response time-histories of the models under dynamic wind loading. The last section in this chapter, Section 5.6 outlines the conclusions that may be drawn from the numerical examples and previously discussed theoretical approaches.

Section 5.2 is subdivided into two sub-sections, in which two analytical approaches are proposed in order to obtain the response of a wind turbine tower and nacelle with classical and non-classical damping. The first subsection deals with the estimation of displacement response of the tower/nacelle, using the mode acceleration method. This method is a modal truncation approach which requires only the first few modes of vibration to be included, and accounts for the rest of the modes by using the pseudo-static component of the response. This method is only applicable for classically damped systems. The following subsection provides an approach based on the numerical integration of the solution of tower response, and is applicable to both classically and non-classically damped systems. The latter system is brought about by the presence of aerodynamic damping, which has the effect of decreasing structural response.
Section 5.3 presents the estimation of the response of discrete prismatic rotating wind turbine blades subjected to random wind loading, and contains a sub-section, considering classically damped systems only. Rotating wind turbine blades are subjected to a special kind of wind turbulence, termed rotationally sampled turbulence, which is due to ambient turbulence induced by frictional elements on the surrounding terrain such as trees and hills, and turbulence induced due to the rotational motion of the blades. Both types of turbulence were addressed in a stochastic sense using a rotationally sampled wind turbulence spectrum. The response of the wind turbine blades, with classical damping, was thus obtained including inherent wind turbine blade phenomena, such as centrifugal stiffening, axial self weight effects, and the aforementioned rotationally sampled spectra. Response parameters simulated included displacement and shear forces at the base of the blades.

Section 5.4 explains the approach adopted in order to mathematically couple three rotating and vibrating wind turbine blades to a hub, which is connected to the nacelle, at the top of a wind turbine tower. This section has one sub-section, in which the classically damped discrete wind turbine blades, analysed by Section 5.3, are coupled primarily in the Fourier domain, to the discrete tower and nacelle system, introduced in Section 5.2. The equations of motion of both sub-systems are first cast and an expression for the shear force transmitted by the blades into the tower is obtained. Using this expression, the equations of motion of both sub-systems can be coupled easily in the Fourier domain, and a time-domain representation of combined response subject to random wind loading is facilitated by use of inverse Fourier transform. Thus, this section provides an approach to obtain the response of a coupled wind turbine tower and rotor system, including the effects of blade-tower dynamic interaction.

A section presenting numerical examples of all the assemblies previously mentioned is illustrated in Section 5.5. This section is sub-divided into four subsequent sub-sections. The first, Section 5.5.1, presents the simulated nodal drag force time-histories and forced vibration response of a wind turbine tower and nacelle with classical damping. The results in this sub-section were obtained using the mode acceleration technique. The next sub-section, Section 5.5.2, illustrates the displacement response of the same wind turbine tower and nodal drag force loading as of Section 5.5.1, obtained using a numerical integration super-position method for systems with classical and non-classical damping. The response due to non-classical damping contains the effects of aerodynamic damping on the magnitude of the displacement response. The next section, Section 5.5.3, illustrates the
displacement response time-histories simulated for rotating wind turbine blades. The effects of blade position and blade rotational frequency is investigated, and the displacement response at blade tip and shear force at blade base are obtained and presented. The investigations into blade-tower coupling are presented in Section 5.5.4, where three coupled assemblies are documented. The first assembly does not include blade-tower dynamic interaction and the latter two do. The coupled response of all three assemblies are formulated and compared for a tower and blade system of arbitrary dimensions. All numerical results obtained are discussed and compared where possible in the Section 5.6.

5.2 FORCED VIBRATIONS OF TOWERS
This section presents the analytical approaches derived to estimate the forced vibration responses of wind turbine towers when they are acted on by random wind loading. The towers in this section are assumed to be discrete lumped parameter MDOF systems, and this approach is both mathematically assessable, and accurate in rendering the free vibration characteristics. Two cases are considered, the first assumes that the system is classically damped, or that the damping matrix of the system is proportional to the mass matrix. This occurs when only structural damping is considered. The second case assumes that the system is non-classically damped, or that the damping matrix of the system is not proportional to the mass matrix. This phenomena occurs with the presence of both structural and aerodynamic damping.

5.2.1. Tower and nacelle with classical damping
The tower and the nacelle are modelled together as a discrete parameter system, whose structural parameters are cast in matrix form. Prior to any forced vibration analysis, the free vibration properties of the system first be obtained. To this end, the approach presented in Chapter 4, Section 4.2.1 may be employed, which will yield the natural frequencies and mode shapes of the system. The forced vibration differential equations of motion for a discrete tower and nacelle system, is given by

\[
[M_T]\{\ddot{x}(t)\} + [C_T]\{\dot{x}(t)\} + [K_T]\{x(t)\} = \{P(t)\} \tag{5.1}
\]

where \([M_T]\), \([C_T]\) and \([K_T]\) denote the mass, structural damping, and stiffness matrices of the tower respectively, where subscript T denotes tower. The notation \(\{x(t)\}\), \(\{\dot{x}(t)\}\) and
\( \{\dot{x}(t)\} \) denote the displacement, velocity and acceleration vectors respectively, and \( \{P(t)\} \) is a time-varying load vector.

If the structural damping matrix is proportional to either the mass and/or stiffness matrix, then equation 5.1 may be transformed from physical coordinates to generalised or modal coordinates by use of modal orthogonality. The tower displacement in physical coordinates may be represented as a linear combination of two sub variables as

\[
\{x(t)\} = \sum_{j=1}^{m} [\Phi_{T,j}] \{\eta_{T,j}(t)\}
\]  

where \( [\Phi_{T,j}] \) and \( \{\eta_{T,j}(t)\} \) are the mode shape and modal coordinate of the \( j^{th} \) mode of the tower, and ‘\( m \)’ is the number of DOF (number of nodes used to discretize the tower). Inserting equation 5.2 into equation 5.1, and pre-multiplying by the transpose of the modal matrix, \( [\Phi_{T}]^T \) gives

\[
[\Phi_{T}]^T [M_{T}][\Phi_{T}] \{\dot{\eta}_{T}(t)\} + [\Phi_{T}]^T [C_{T}][\Phi_{T}] \{\ddot{\eta}_{T}(t)\} + [\Phi_{T}]^T [K_{T}][\Phi_{T}] \{\eta_{T}(t)\} = [\Phi_{T}]^T \{P(t)\}
\]

Equation 5.3 can be further expressed in individual modal form as

\[
\tilde{M}_{T,j} \ddot{\eta}_{T,j}(t) + \tilde{C}_{T,j} \dot{\eta}_{T,j}(t) + \tilde{K}_{T,j} \eta_{T,j}(t) = \tilde{F}_{j}(t) \quad ; \quad j = 1, \ldots, m
\]

where \( \tilde{M}_{T,j} \) is the modal mass, \( \tilde{C}_{T,j} \) is the modal damping, \( \tilde{K}_{T,j} \) is the modal stiffness of the tower and \( \tilde{F}_{j}(t) \) is the modal force. \( \tilde{C}_{T,j} \) is usually expressed as

\[
\tilde{C}_{T,j} = 2\xi_{T,j} \omega_{T,j} \tilde{M}_{T,j}
\]

where \( \xi_{T,j} \) is the modal damping ratio as a percentage of critical, and \( \omega_{T,j} \) is the \( j^{th} \) circular natural frequency. Each modal force, \( \tilde{F}_{j}(t) \) is made up of contributions for ‘\( N \)’ nodal force time-histories, as
where \( P_1(t) \) to \( P_N(t) \) are the nodal force time-histories. These must first be simulated using a mathematical technique. However, in this section, the modal force time-histories of each mode will be simulated directly. Using random vibration theory, the nodal force time-history at node ‘i’ may be decomposed into a mean component, \( \bar{P}_i \), and a fluctuating component, \( p_i'(t) \) as

\[
P_i(t) = \bar{P}_i + p_i'(t)
\]

The nodal force in wind engineering is represented by drag force. Equation 5.7 becomes

\[
F_{D,i}(t) = \bar{f}_{D,i} + f_{D,i}'(t)
\]

where \( F_{D,i}(t) \) is the total drag force time-history at node ‘i’, \( \bar{f}_{D,i} \) is the mean nodal drag force and \( f_{D,i}'(t) \) is the fluctuating drag force time-history at node ‘i’. The mean drag force is given by

\[
\bar{f}_{D,i} = 0.5C_D A_i \rho \bar{V}_i^2
\]

where \( C_D \) is the drag coefficient, \( A_i \) is the area associated with node ‘i’, and \( \rho \) is the air density. Thus, the modal mean drag force for mode ‘j’, \( \bar{f}_{D,Mj} \) is obtained as

\[
\bar{f}_{D,Mj} = [\Phi]_{TF} \begin{bmatrix} \bar{f}_{D,1}(t) \\ \bar{f}_{D,2}(t) \\ \vdots \\ \bar{f}_{D,N}(t) \end{bmatrix}
\]

Now, the modal fluctuating drag force time-histories must be simulated. However, as wind turbine towers are generally tall structures, it cannot be assumed that the pressure on every point on the structure will be the same at all times. In other words, as small eddies (relative
to the height of the structure) engulf the tower, pressure will not be fully correlated over the entire height of the structure. This phenomenon is termed as spatial correlation or coherence. It is possible to simulate modal fluctuating drag force time-histories that include this incomplete pressure correlation as a function of eddies frequency (size).

Nigam and Narayanan (1994) presented an expression for the modal fluctuating drag force power spectrum, \( S_{MFjMFj}(f) \), which includes spatial correlation information, for a continuous line-like structure. When this continuous structure is discretized into a MDOF system, the expression reads as

\[
S_{MFjMFj}(f) = (C_D A_T \rho)^2 \sum_{k=1}^{N} \sum_{l=1}^{N} S_{vkvli}(f) \bar{v}_k \bar{v}_l \phi_{T,j}(k) \phi_{T,j}(l)
\]  

(5.11)

where \( A_T \) is the total area of the structure, ‘k’ and ‘l’ are spatial nodes, \( S_{vkvli}(f) \) is the velocity auto PSDF when \( k = l \) and the cross PSD function when \( k \neq l \), \( \bar{v}_k \) and \( \bar{v}_l \) are the mean wind velocities at nodes \( k \) and \( l \) respectively, and \( \phi_{T,j}(k) \) and \( \phi_{T,j}(l) \) are the node \( k \) and \( l \) components of the ‘\( j \)'th mode shape. The auto and cross PSD terms may be evaluated as

\[
S_{vkvli}(f) = \sqrt{S_{vkvk}(f)S_{vivi}(f)\text{coh}(k,l,f)}
\]  

(5.12)

with \( S_{vkvk}(f) \) and \( S_{vivi}(f) \) being the velocity auto PSD at nodes \( k \) and \( l \) respectively and \( \text{coh}(k,l,f) \) is the spatial coherence function between nodes \( k \) and \( l \). \( S_{vkvk}(f) \) and \( S_{vivi}(f) \) may be found from the expression by Kaimal et al (1972) expressed as

\[
\frac{f S_{vv}(z,f)}{v_z^2} = \frac{200n}{(1 + 50n)^{5/3}}
\]  

(5.13)

In equation 5.13, \( z \) is the elevation, \( f \) is frequency (Hz), \( S_{vv}(z,f) \) is the power spectral density function of the fluctuating wind velocity as a function of elevation and frequency, \( v_z \) is the friction velocity (explained in Chapter 2), and \( n \) is the known as the Monin coordinate. The latter two terms may be obtained from the expressions

\[
\bar{v}(z) = \frac{1}{k} v_z \ln \frac{z}{z_0}
\]  

(5.14)
FORCED VIBRATIONS OF WIND TURBINE TOWER ASSEMBLIES

and

$$n = \frac{f_z}{\bar{v}(z)}$$  \hspace{1cm} (5.15)

with $\bar{v}(z)$ being the mean wind velocity at an elevation of $z$, $k$ is Von-kàrmàn’s constant (typically around 0.4), and $z_0$ is the roughness length.

The coherence function is typically a measure of the relationship between two time-histories separated by a specific distance and the function suggested by Davenport (1968), $\text{coh}(k, l; f)$, is used in this section. This function, which relates the frequency dependent spatial correlation between nodes $k$ and $l$, is given by

$$\text{coh}(k, l; f) = \exp\left(-\frac{|k-l|}{L_s}\right)$$  \hspace{1cm} (5.16)

where $|k-l|$ is the spatial separation and $L_s$ is a length scale given by

$$L_s = \frac{\hat{v}}{fD}$$  \hspace{1cm} (5.17)

where

$$\hat{v} = 0.5(\bar{v}_k + \bar{v}_l)$$  \hspace{1cm} (5.18)

and $D$ is a decay constant. The fluctuating modal drag force time-histories, may now be simulated. This may be facilitated by virtue of the fact that any arbitrary fluctuating modal drag force time-history, $f_{D,M_j}(t)$, with zero mean, may be represented by a discrete Fourier transform (DFT) with a discretized version of a continuous frequency content, as

$$f_{D,M_j}(t) = \sum_{k=1}^{\infty} a_k \cos(\omega_k t) + \sum_{k=1}^{\infty} b_k \sin(\omega_k t)$$  \hspace{1cm} (5.19)

where $a_k$ and $b_k$ are the Fourier coefficients, $\omega_k$ is the $k^{th}$ discretized circular frequency ($\omega = 2\pi f$, $f$ is frequency in Hz) and $t$ is the time instant. This velocity signal is generated in conjunction with the fluctuating modal drag force PSD in equation 5.11. The PSDF is
conceptually divided into infinitesimal frequency bands of size \( df \). The area under the PSD function between the limits of \( f_i \) and \( f_i + df \) is equal to the variance of the signal at the discrete frequency \( f_i \). The Fourier coefficients in equation 5.19 are obtained as normally distributed random numbers, generated with zero mean and standard deviation \( \sigma_i \). The fluctuating modal drag force time-history is hence composed of a number of contributions from a discretized form of a continuous frequency band. The modal mean and fluctuating drag force components may be added together to form the total modal drag force, \( F_{D,Mj}(t) \).

In order to obtain the response of the wind turbine tower in physical coordinates, the mode acceleration technique is employed, as derived in Chapter 2, Section 2.3. The equation that describes the response of the structure using the mode acceleration method is

\[
\{x(t)\} = [K_T]^{-1}\{F_d(t)\} - \sum_{j=1}^{G} \left( \frac{2\xi}{\omega_{T,j}} \right) \{\Phi_{T,j}\} \{\hat{\eta}_{T,j}(t)\} - \sum_{j=1}^{G} \left( \frac{1}{\omega_{T,j}^2} \right) \{\Phi_{T,j}\} \{\ddot{\eta}_{T,j}(t)\} \quad (5.20)
\]

which may be derived from manipulating equation 5.1. In equation 5.20, \( \{F_d(t)\} \) is the nodal drag force vector, \( \{\dot{\eta}_{T,j}(t)\} \) and \( \{\ddot{\eta}_{T,j}(t)\} \) are the first and second temporal derivatives of the modal co-ordinate of the tower, and \( G \) is the number of modes being considered in the response. The first term of equation 5.20 represents the pseudo-static response, while the second and third terms represent the dynamic response.

A linear MDOF system ultimately comprises of ‘m’ single degree-of-freedom (SDOF) systems, allowing the total response of the MDOF entity to be obtained as the linear combination of the response of these SDOF entities. Hence, the response expressed by equation 5.20 may be obtained by considering a series of SDOF systems of natural frequency \( \omega_{T,j} \) and damping \( \xi_{T,j} \), and obtaining solutions for \( \{\dot{\eta}_{T,j}(t)\} \) and \( \{\ddot{\eta}_{T,j}(t)\} \). The modal coordinate \( \{\eta_{T,j}(t)\} \), along with its first and second temporal derivatives may then be used to evaluate \( \{x(t)\} \).

Nigam and Jennings (1968) presented an algorithm to obtain the modal co-ordinate and its first derivative, for a viscously damped SDOF oscillator subjected to an excitation acceleration process. They assumed that the input acceleration time-history could be approximated by a piecewise linear function, thus yielding a semi-analytical process.
allowing the displacement and velocity response time-histories to be computed. The fundamental equation based on this algorithm is expressed as

\[
\begin{pmatrix}
\{\eta_{i+1}\} \\
\{\dot{\eta}_{i+1}\}
\end{pmatrix}
= \begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{bmatrix}
\begin{pmatrix}
\{\eta_i\} \\
\{\dot{\eta}_i\}
\end{pmatrix} + \begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix}
\begin{pmatrix}
a_{i+1}\n
\end{pmatrix}
\]

(5.21)

where the subscript ‘i’ denotes the ‘i’th time instant, \(\{\eta(t)\}\) is the generalised coordinate, \(a_i\) is the excitation acceleration value at time ‘i’ and \(c_{11}, c_{12}, c_{21}, c_{22}, b_{11}, b_{12}, b_{21}, b_{22}\) are constants, given by

\[
c_{11} = \exp(-\omega_j \xi_j t_{\text{inc}}) \left\{ \cos(\omega_{j,D} t_{\text{inc}}) + \frac{\omega_j \xi_j}{\omega_{j,D}} \sin(\omega_{j,D} t_{\text{inc}}) \right\}
\]

(5.22)

\[
c_{12} = \frac{\exp(-\omega_j \xi_j t_{\text{inc}})}{\omega_{j,D}} \left\{ \sin(\omega_{j,D} t_{\text{inc}}) \right\}
\]

(5.23)

\[
c_{21} = \frac{-\omega_j^2 \exp(-\omega_j \xi_j t_{\text{inc}})}{\omega_{j,D}} \left\{ \sin(\omega_{j,D} t_{\text{inc}}) \right\}
\]

(5.24)

\[
c_{22} = \exp(-\omega_j \xi_j t_{\text{inc}}) \left\{ \cos(\omega_{j,D} t_{\text{inc}}) - \frac{\omega_j \xi_j}{\omega_{j,D}} \sin(\omega_{j,D} t_{\text{inc}}) \right\}
\]

(5.25)

\[
b_{11} = \frac{1}{\omega_j^2 \omega_{j,D} t_{\text{inc}}} \left[ \exp(-\omega_j \xi_j t_{\text{inc}}) \left\{ \left( -\frac{\omega_j^2 + \omega_{j,D}^2}{\omega_j} \right) \sin(\omega_{j,D} t_{\text{inc}}) \right\} + \frac{2 \omega_j \xi_j \omega_{j,D}}{\omega_j^2} \right]
\]

(5.26)

\[
b_{12} = \frac{1}{\omega_j^2 \omega_{j,D} t_{\text{inc}}} \left[ \exp(-\omega_j \xi_j t_{\text{inc}}) \left\{ \left( -\frac{\omega_j^2 + \omega_{j,D}^2}{\omega_j^2} \right) \sin(\omega_{j,D} t_{\text{inc}}) \right\} + \frac{2 \omega_j \xi_j \omega_{j,D}}{\omega_j^2} \right]
\]

(5.27)
\[
\begin{align*}
  b_{21} &= \frac{1}{\omega_j^2 \omega_{j,D} t_{inc}} \left[ \exp(-\omega_j \xi_j t_{inc}) \left\{ (\omega_j \xi_j + \omega_j^2 t_{inc}) \sin(\omega_j t_{inc}) + (\omega_{j,D} \cos(\omega_{j,D} t_{inc})) \right\} - \omega_{j,D} \right] \\
  b_{22} &= \frac{1}{\omega_j^2 \omega_{j,D} t_{inc}} \left[ \exp(-\omega_j \xi_j t_{inc}) \left\{ \xi_j \omega_j \sin(\omega_j t_{inc}) + \omega_{j,D} \cos(\omega_{j,D} t_{inc}) \right\} + \omega_{j,D} \right]
\end{align*}
\] (5.28) (5.29)

in which \( \omega_{j,D} \) is the damped natural frequency of mode ‘j’ given by

\[
\omega_{j,D} = \omega_j \sqrt{1 - \xi_j^2}
\] (5.30)

where \( \xi_j \) is the j\(^{th}\) modal damping ratio; \( t_{inc} \) is a uniform time increment expressed as

\[
t_{inc} = t_{i+1} - t_i
\] (5.31)

and \( t_i \) is the ‘i\(^{th}\)’ time instant. The modal excitation acceleration time-history \( \{a_j(t)\} \), which is thus made up of individual acceleration values of \( a_i \), may easily be obtained for each SDOF entity as

\[
\{a_j(t)\} = \{F_{D,M_j}(t)\} / M_{T,j}
\] (5.32)

The second derivative of the generalised coordinate is given by

\[
\{\ddot{\eta}_{T,j}(t)\} = \left\{ \frac{F_{D,M_j}(t)}{M_{T,j}} \right\} - 2\xi_{T,j} \omega_{T,j} \{\dot{\eta}_{T,j}(t)\} - \omega_{T,j}^2 \{\eta_{T,j}(t)\}
\] (5.33)

The first and second temporal derivatives of the modal coordinates can be inserted into equation 5.20 to find the dynamic components. The pseudo-static component must be obtained by use of inverse modal orthogonality involving the modal drag force vector, \( \{F_{D,M}(t)\} \) and the mode shape matrix, as

\[
\{F_D(t)\} = \left[ [\Phi_T]^T \right]^{-1} \{F_{D,M}(t)\}
\] (5.34)
5.2.2 Tower and nacelle with non-classical damping

The response of a wind turbine tower with non-classical damping will now be derived. When aerodynamic damping is included in the system, it will render the system's damping matrix non-proportional to the system's mass and/or stiffness matrices. Thus, the modes cannot be decoupled using the approaches used in Chapter 4. The wind turbine tower and nacelle is again being modelled as a discrete lumped parameter system and this approach makes it a MDOF system. The differential equations of motion that describes the forced motion of the system is slightly different to equation 5.1, and is given by

\[
[M_T]\{\ddot{x}(t)\} + [C_{T,T}][\dot{x}(t)] + [K_T]\{x(t)\} = \{P(t)\} \tag{5.35}
\]

where \([C_{T,T}]\) represents the total damping matrix of the tower. This is made up of two matrix components, one due to the inherent damping properties of the structure, \([C_s]\) and one due to aerodynamic damping, \([C_a]\) brought about by the relative motion of the structure to the wind. This relationship is expressed as

\[
[C_{T,T}] = [C_s] + [C_a] \tag{5.36}
\]

The structural damping matrix, \([C_s]\), associated with an MDOF classically damped system may be taken a scalar factor times the mass matrix as

\[
[C_s] = \Delta[M_T] \tag{5.37}
\]

where \(\Delta\) is a scalar constant. This constant can be chosen so as to create specific magnitudes of modal damping using the undamped mode shapes and the modal orthogonality condition. Thus, the structural damping matrix is a diagonal matrix directly proportional to the mass matrix.

The aerodynamic damping matrix must be built up from first principles. The aerodynamic damping force as a function of height, \(c_a(z)\), may be found from Nigam and Narayanan (1994) as

\[
c_a(z) = \rho C_D \bar{v}(z) A \tag{5.38}
\]
where \( \rho \) is the density of air, \( C_D \) is the coefficient of drag, \( \bar{v}(z) \) is the mean wind velocity which is function of height \( z \), and \( A \) is the incremental area at height \( z \). As the tower is discretized into several lumped parameters, the damping force associated with a node is given by

\[
c_{a,i} = \rho C_D \int_{h_i}^{h_{i+1}} \bar{v}(z)A_i dz
\]

(5.39)

where \( c_{a,i} \) is the \( i^{th} \) damper connected between tower heights of \( h_i \) and \( h_{i+1} \). The mean wind velocity profile is taken to be a power law, as

\[
\bar{v}(z) = \bar{v}(H) \left( \frac{z}{H} \right)^\alpha
\]

(5.40)

where \( \bar{v}(H) \) is the mean wind velocity at the top of the tower at height \( H \), and \( \alpha \) is termed the mean wind velocity profile exponent. The aerodynamic damping matrix will have the form

\[
[C_a] = \begin{bmatrix}
c_{a,1} & -c_{a,1} & \cdots & 0 \\
-c_{a,1} & c_{a,1} + c_{a,2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & -c_{a,N-1} & c_{a,N-1} + c_{a,N}
\end{bmatrix}
\]

(5.41)

where ‘N’ is the maximum number of nodes used to discretise the tower. The total damping matrix will be non-proportional to either the mass and/or stiffness matrices, so equation 5.35 cannot be decoupled into its current form. Instead, equation 5.35, with a dimension of ‘n’ must be re-cast in state space, with dimensionality of ‘2n’. This new ‘2n’ equation may now be decoupled using a traditional eigen-analysis. Equation 5.35 is now recast as

\[
[A]\{\ddot{y}\} + [B]\{y\} = [0]
\]

(5.42)

with the forcing term \( P \), purposely omitted. The matrices in equation 5.42 are expressed as

\[
[A] = \begin{bmatrix}
0 & \begin{bmatrix} M_T \end{bmatrix} \\
\begin{bmatrix} M_T \end{bmatrix} & \begin{bmatrix} C_{T,T} \end{bmatrix}
\end{bmatrix}
\]

(5.43)
and

\[ [B] = \begin{bmatrix} -[M_t] & [0] \\ [0] & [K_t] \end{bmatrix} \]  \hspace{1cm} (5.44)

and

\[ \{y\} = \begin{bmatrix} \{\dot{x}(t)\} \\ \{x(t)\} \end{bmatrix} \]  \hspace{1cm} (5.45)

The related eigen-problem form of equation 5.42 may be expressed as

\[ [B][\Phi_{T,j}] = -s_{T,j}[A][\Phi_{T,j}] \]  \hspace{1cm} (5.46)

where \( s_{T,j} \) represents the \( j^{th} \) eigenvalue and \( \{\Phi_{T,j}\} \) represents the \( j^{th} \) eigenvector. Due to the symmetry of matrices \([A]\) and \([B]\), the eigen-solutions \( \{s_{T,j}, \Phi_{T,j}\} \) occur in conjugate pairs (Igusa, 1984). It has been stated in the literature that the complex eigenvalues \( s_{T,j} \) will have negative real parts for a well supported structural system (Singh, 1980). The eigenvectors, \( \{\Phi_{T,j}\} \), are composed of the first ‘n’ rows being the product of the complex eigenvalue by the damped mode shape \( \{\Phi_{T,j}\} \), the last ‘n’ rows being the damped mode shape, as

\[ \{\Phi_{T,j}\} = \begin{bmatrix} s_{T,j}\{\Phi_{T,j}\} \\ \{\Phi_{T,j}\} \end{bmatrix} \]  \hspace{1cm} (5.47)

For the sake of comparing the eigen-properties of the non-classically damped system to those of a classically damped system, the eigenvalue is composed of a modal damping ratio, \( \xi_{T,j} \), an undamped circular frequency, \( \omega_{T,j} \), and a damped circular frequency, \( \omega_{TD,j} \), as

\[ s_{T,j} = -\xi_{T,j}\omega_{T,j} \pm i\omega_{TD,j} \hspace{0.5cm} ; \hspace{0.5cm} j = 1, \ldots, m \]  \hspace{1cm} (5.48)

Note that the damped circular frequency is imaginary. The undamped circular natural frequency is equal to the modulus of the eigenvalue,
The modal damping ratio is equal to the ratio of the negative real component of the eigenvalue to the undamped natural circular frequency, as

$$\xi_{T,j} = -\frac{\text{Re}(s_{T,j})}{|s_{T,j}|} \quad (5.50)$$

The damped circular natural frequency is given by the expression

$$\omega_{TD,j} = \omega_{T,j}\sqrt{1 - \frac{\xi_{T,j}^2}{}} \quad (5.51)$$

The ‘2n’ physical response coordinate \(\{y\}\), may be transformed into modal coordinates using the transformation of

$$\{y\} = \{\hat{\Phi}_T\}\{z(t)\} \quad (5.52)$$

where \(\{z(t)\}\) is the modal coordinate. Inserting equation 5.52 into equation 5.42 and including the forcing term on the right hand side gives

$$[A]\{\hat{\Phi}_T\}\{\dot{z}\} + [B]\{\hat{\Phi}_T\}\{z\} = [P] \quad (5.53)$$

where

$$[P] = \left\{ \begin{array}{c} 0 \\ \{F_D(t)\} \end{array} \right\} \quad (5.54)$$

with \(\{F_D(t)\}\) being the nodal drag force vector of dimension ‘n’. Pre-multiplying equation 5.53 by \(\{\hat{\Phi}_T\}^T\) gives

$$\{\Phi_T\}^T [A]\{\hat{\Phi}_T\}\{\dot{z}\} + [B]\{\hat{\Phi}_T\}\{z\} = [P] \quad (5.55)$$

Equation 5.55 may be simplified to give
\[
\begin{align*}
\begin{bmatrix} \tilde{A}_j \end{bmatrix} \begin{bmatrix} \tilde{z}_j \end{bmatrix} + \begin{bmatrix} \tilde{B}_j \end{bmatrix} \begin{bmatrix} z_j \end{bmatrix} &= \begin{bmatrix} \tilde{\Phi}_{T,j} \end{bmatrix}^T [P] \quad (5.56) \\
\begin{bmatrix} \tilde{A}_j \end{bmatrix} &= \begin{bmatrix} \tilde{\Phi}_{T,j} \end{bmatrix}^T [A] \begin{bmatrix} \tilde{\Phi}_{T,j} \end{bmatrix} = -s_{T,j} \begin{bmatrix} \tilde{B}_j \end{bmatrix} \quad (5.57) \\
\text{and} \\
\begin{bmatrix} \tilde{B}_j \end{bmatrix} &= \begin{bmatrix} \tilde{\Phi}_{T,j} \end{bmatrix}^T [B] \begin{bmatrix} \tilde{\Phi}_{T,j} \end{bmatrix} \quad (5.58)
\end{align*}
\]

Dividing equation 5.56 across by \( \begin{bmatrix} \tilde{A}_j \end{bmatrix} \) yields
\[
\begin{align*}
\{ \tilde{z}_j \} - s_{T,j} \begin{bmatrix} z_j \end{bmatrix} &= \begin{bmatrix} \tilde{\Phi}_{T,j} \end{bmatrix}^T [P] \begin{bmatrix} \tilde{A}_j \end{bmatrix} = Q_j(t) \quad (5.59)
\end{align*}
\]

The solution of equation 5.59 is given by
\[
\begin{align*}
z_j(t) &= \int_0^t Q_j(t) e^{s_{T,j}(t-\tau)} d\tau \\
&= \frac{1}{\begin{bmatrix} \tilde{A}_j \end{bmatrix} } \begin{bmatrix} \tilde{\Phi}_{T,j} \end{bmatrix}^T [P] \begin{bmatrix} \tilde{A}_j \end{bmatrix} = Q_j(t) \quad (5.60)
\end{align*}
\]

where \( t \) is the total time of the response, and \( \tau \) is a dummy variable. Using the mode superposition technique, the total displacement response of the system is given by
\[
\begin{align*}
\{x(t)\} &= \sum_{j=1}^{2n} \begin{bmatrix} \Phi_{T,j} \end{bmatrix} \{z_j(t)\} = \sum_{j=1}^{2n} \int_0^t b_j(\tau) e^{s_{T,j}(t-\tau)} d\tau \quad (5.61)
\end{align*}
\]

where
\[
b_j(\tau) = \begin{bmatrix} \Phi_{T,j} \end{bmatrix} \{Q_j(t)\} \quad (5.62)
\]

Using the fact that \( b_j(\tau) \) and \( s_{T,j} \) occur in conjugate pairs, the solution of the total displacement response \( \{x(t)\} \), as given in equation 5.61 is expanded as
\[
\begin{align*}
\{x(t)\} &= \sum_{j=1}^{\frac{n}{2}} \left\{ \int_0^t b_j(\tau) e^{-\xi_j \omega_{T,j} \text{sgn}(\omega_{T,j}) \chi(t-\tau)} d\tau + \int_0^t b_j(\tau) e^{+\xi_j \omega_{T,j} \text{sgn}(\omega_{T,j}) \chi(t-\tau)} d\tau \right\} \quad (5.63)
\end{align*}
\]

where an overbar denotes a complex conjugate.
Grouping the real and imaginary parts together, equation 5.63 is now expressed as

\[
\{x(t)\} = 2 \sum_{j=1}^{n} \left\{ \int_{0}^{t} -\text{Imag}(b_j(\tau)) e^{(\xi_j,\omega_j,\tau,\omega_{TD,\tau})(t-\tau)} \sin \omega_{TD,j}(t-\tau) d\tau + \int_{0}^{t} \text{Real}(b_j(\tau)) e^{(\xi_j,\omega_j,\tau,\omega_{TD,\tau})(t-\tau)} \cos \omega_{TD,j}(t-\tau) d\tau \right\} (5.64)
\]

The first part of equation 5.64 may be expressed as

\[
F = 2 \sum_{j=1}^{n} \sin \omega_{TD,j} \left[ \int_{0}^{t} -\text{Imag}(b_j(\tau)) e^{(\xi_j,\omega_j,\tau,\omega_{TD,\tau})(t-\tau)} \cos \omega_{TD,j} \tau d\tau \right] - \cos \omega_{TD,j} \left[ \int_{0}^{t} -\text{Imag}(b_j(\tau)) e^{(\xi_j,\omega_j,\tau,\omega_{TD,\tau})(t-\tau)} \sin \omega_{TD,j} \tau d\tau \right] (5.65)
\]

and the second part of equation 5.64 is

\[
S = 2 \sum_{j=1}^{n} \cos \omega_{TD,j} \left[ \int_{0}^{t} \text{Real}(b_j(\tau)) e^{(\xi_j,\omega_j,\tau,\omega_{TD,\tau})(t-\tau)} \cos \omega_{TD,j} \tau d\tau \right] + \sin \omega_{TD,j} \left[ \int_{0}^{t} \text{Real}(b_j(\tau)) e^{(\xi_j,\omega_j,\tau,\omega_{TD,\tau})(t-\tau)} \sin \omega_{TD,j} \tau d\tau \right] (5.66)
\]

Thus, the response components in equations 5.65 and 5.66 may be numerically integrated in a computer algorithm using some appropriate time step.

### 5.3 FORCED VIBRATIONS OF BLADES

This section demonstrates the mathematical techniques used to estimate the forced vibration response of a rotating wind turbine blade, when it is subjected to rotationally sampled random wind turbulence. The blades are modelled as MDOF discrete parameter systems allowing rotational effects such as centrifugal stiffening and axial self weight loading to be incorporated. The blades are assumed to be classically damped, with aerodynamic damping not being considered.
5.3.1 Rotating blades with classical damping

The rotating wind turbine blades are modelled as discrete lumped parameter systems consisting of uniform cantilever beams of rectangular hollow cross section. The differential equations of motion characterising the dynamic behaviour of a discretized MDOF blade is represented by

\[
[M_B]\{\ddot{u}(t)\} + [C_B]\{\dot{u}(t)\} + [K_B]\{u(t)\} = \{P(t)\}
\]  

(5.67)

where \([M_B]\), \([C_B]\) and \([K_B]\) are the mass, structural damping and stiffness matrices (including structural and geometric stiffness) of the blade respectively, where subscript \(B\) denotes blade. The notation of \(\{u(t)\}, \{\dot{u}(t)\}\) and \(\{\ddot{u}(t)\}\) denote the displacement, velocity and acceleration vectors respectively, and \(\{P(t)\}\) is a time-varying load vector. As the structural damping matrix is assumed to be proportional to the mass and/or stiffness matrices, the undamped free vibration properties of the system may be obtained using the approach presented in Chapter 4, Section 4.3.1. The blade displacement in physical coordinates may be represented as the weighted sum of a linear combination of two sub variables as

\[
\{u(t)\} = \sum_{j=1}^{m}[\Phi_{B,j}]\{\eta_{B,j}(t)\}
\]  

(5.68)

where \([\Phi_{B,j}]\) and \(\{\eta_{B,j}(t)\}\) are the mode shape and modal coordinate of the \(j^{th}\) mode. Inserting equation 5.68 into equation 5.67, and pre-multiplying by the transpose of the blade modal matrix, \([\Phi_B]^T\) gives

\[
[\Phi_B]^T[M_B][\Phi_B]\{\ddot{\eta}_B(t)\} + [\Phi_B]^T[C_B][\Phi_B]\{\dot{\eta}_B(t)\} + [\Phi_B]^T[K_B][\Phi_B]\{\eta_B(t)\} = [\Phi_B]^T\{P(t)\}
\]  

(5.69)

which in modal form reads

\[
\bar{M}_{B,j}\ddot{\eta}_{B,j}(t) + \bar{C}_{B,j}\dot{\eta}_{B,j}(t) + \bar{K}_{B,j}\eta_{B,j}(t) = \bar{F}_j(t)
\]  

(5.70)
where $\bar{M}_{B,j}$ is the modal mass, $\bar{C}_{B,j}$ is the modal damping, $\bar{K}_{B,j}$ is the modal stiffness of the blade, and $\bar{F}_j(t)$ is the modal force. $\bar{C}_{B,j}$ for a classically damped system may be expressed as

$$\bar{C}_{B,j} = 2\xi_{B,j} \omega_{B,j} \bar{M}_{B,j}$$  \hspace{1cm} (5.71)

where $\xi_{B,j}$ is the modal damping ratio as a percentage of critical, and $\omega_{B,j}$ is the $j^{th}$ circular natural frequency of the blade. Similar to the classically damped tower and nacelle, the response of the blade will also be obtained using the mode acceleration method, the equation of which is expressed as

$$\{\ddot{u}(t)\} = [K_{B}]^{-1} \{P(t)\} - \sum_{j=1}^{G} \left( \frac{2\xi_{B,j}}{\omega_{B,j}} \Phi_{B,j} \right) \{\dot{\eta}_{B,j}(t)\} - \sum_{j=1}^{G} \left( \frac{1}{\omega_{B,j}^2} \right) \Phi_{B,j} \{\ddot{\eta}_{B,j}(t)\}$$  \hspace{1cm} (5.72)

where $\{P(t)\}$ represents the nodal loading vector, $\{\dot{\eta}_{B,j}(t)\}$ and $\{\ddot{\eta}_{B,j}(t)\}$ are the first and second temporal derivatives of the modal co-ordinate and $G$ is the number of modes being considered in the response. The nodal loading on the blade, $\{P(t)\}$, similar to the tower, is from the drag force brought about by the wind and may be replaced by the notation $\{F_D(t)\}$. A total nodal drag force time history $F_{D,i}(t)$, at node ‘$i$’ is expressed as

$$F_{D,i}(t) = 0.5C_D A_i \rho \left[ \bar{v}_i + v_i'(t) \right]^2$$  \hspace{1cm} (5.73)

where $C_D$ is the drag coefficient, $A_i$ is the nodal area, $\rho$ is the air density, $\bar{v}_i$ is the mean wind velocity at node ‘$i$’ and $v_i'(t)$ is the fluctuating velocity component at node ‘$i$’. The mean wind velocity will follow a sinusoidal variation with time due to change in blade position as it rotates, and this variation is given by

$$\bar{v}_i = V \cos(\Omega t)$$  \hspace{1cm} (5.74)

where $V$ is the required amplitude necessary to represent the mean wind velocity as a function of nodal position or height above the ground, $\Omega$ is the rotational frequency of the blade and $t$ is the time. The fluctuating component will be simulated using the DFT
technique discussed in Section 5.2.1, and a fluctuating wind velocity PSDF. The fluctuating velocity time-history at node ‘i’, \( v_i'(t) \), with zero mean, may be represented by a DFT with a discretized version of a continuous frequency content, as

\[
v_i'(t) = \sum_{k=1}^{\infty} a_k \cos(\omega_k t) + \sum_{k=1}^{\infty} b_k \sin(\omega_k t)
\]  

(5.75)

where \( a_k \) and \( b_k \) are the Fourier coefficients, \( \omega_k \) is the \( k^{th} \) discretized circular frequency and \( t \) is the time instant. Time-histories were simulated using equation 5.75 and a modified version of the fluctuating wind velocity spectrum offered by Kaimal et al (1972) expressed as

\[
\frac{fS_{vv}(H,f)}{v_r^2} = \frac{200n}{(1 + 50n)^{5/3}}
\]

(5.76)

In equation 5.76, \( H \) is the hub elevation, \( f \) is frequency (Hz), \( S_{vv}(H,f) \) is the PSDF of the fluctuating wind velocity as a function of hub elevation and frequency, \( v_r \) is the friction velocity (ms\(^{-1}\)), and \( n \) is the Monin coordinate. The latter two terms may be obtained from the expressions

\[
\bar{v}(H) = \frac{1}{k} v_r \ln \frac{H}{z_0}
\]

(5.77)

and

\[
n = \frac{fH}{\bar{v}(H)}
\]

(5.78)

with \( \bar{v}(H) \) being the mean wind velocity at hub height, \( k \) is Von-kàrmàn’s constant and \( z_0 \) is the roughness length.

This spectrum, while applicable to stationary bodies, does not accurately reflect the energy within a turbulent flow around rotating bodies, such as turbine blades. A modified spectrum, known as a rotationally sampled spectrum, must be used instead. Rotationally sampled spectra are used to quantify kinetic energy as a function of frequency for rotor blades within a turbulent wind flow. In order to represent this redistribution of spectral
energy, the following procedure was adopted. The required redistribution of spectral energy was facilitated by identifying the specific frequencies $\omega_k$, in equation 5.75, equalling $1\Omega$, $2\Omega$, $3\Omega$, and $4\Omega$, and then deriving their Fourier coefficients $a_k$ and $b_k$ according to specific standard deviation values. These values are obtained by virtue of a number of assumptions. First, the variance of wind velocity spectral energy at height $H$ was calculated numerically using equation 5.76, and based on this value, a value of variance was allocated to each node along the length of the blade.

Madsen and Frandsen (1984) observed that the peaks of redistributed spectral energy in a rotationally sampled spectrum tend to become more pronounced as distance increases along the beam, away from the hub. It is, therefore, assumed that the variance values increase by an arbitrary value of 10 %, for each successive blade node radiating out from the hub. It is also assumed that 30 % of total variance values at each node are localised into peaks at $1\Omega$, $2\Omega$, $3\Omega$, and $4\Omega$. This 30 % energy is subsequently divided into four peaks, that is 15 %, 7.5 %, 4.5 % and 3 % per peak, with the maximum energy being localised at a frequency of $1\Omega$, descending to a minimum value at a frequency of $4\Omega$. This procedure is illustrated in figure 5.1. Thus, nodal fluctuating velocity time-histories, $v_i'(t)$, with specific energy-frequency relationships were obtained using equation 5.75. The total nodal drag force time-history at any arbitrary node may now be obtained. Due to the magnitude of turbulence created by the rotation of the blades, as well as the ambient wind turbulence,

![Figure 5.1 Energy distribution within rotationally sampled spectrum at arbitrary node](image-url)
FORCED VIBRATIONS OF WIND TURBINE TOWER ASSEMBLIES

pressures were assumed to be uncorrelated along the entire length of the blades. Thus, spatial coherence information incorporated in the response of the tower in Section 5.2.1 in neglected in this section. The total modal drag force time-history, \( F_{D,Mj}(t) \) may be obtained by virtue of modal orthogonality as

\[
\begin{bmatrix}
F_{D,1}(t) \\
F_{D,2}(t) \\
\vdots \\
F_{D,N}(t)
\end{bmatrix} = \Phi_B \begin{bmatrix}
F_{D,M1}(t) \\
F_{D,M2}(t) \\
\vdots \\
F_{D,MN}(t)
\end{bmatrix}
\]

(5.79)

As the system is linear, it is composed of ‘m’ SDOF systems, allowing the total response of the MDOF system to be obtained as the linear combination of the response of these SDOF entities. In order to solve equation 5.72 for the total blade response, the modal coordinates \( \{ \eta_{B,j}(t) \} \), along with its first and second temporal derivates must be obtained.

The approach reported by Nigam and Jennings (1968) will again be used to obtain these vectors. This technique obtains the modal co-ordinate and its first derivative, for a viscously damped SDOF oscillator subjected to an excitation acceleration. The fundamental equation based on this algorithm is given by equation 5.21, and the frequency and damping dependant constants, given in equations 5.22 to 5.31. The modal excitation acceleration time-history \( \{ a_j(t) \} \), which is thus made up of individual acceleration values of \( a_i \), may easily be obtained for each SDOF entity as

\[
\{ a_j(t) \} = \frac{\{ F_{D,Mj}(t) \}}{M_{B,j}}
\]

(5.80)

where \( \{ F_{D,j}(t) \} \) is the modal force time history mode ‘j’. The second derivative of the modal coordinate is given by

\[
\{ \ddot{\eta}_{B,j}(t) \} = \frac{F_{D,Mj}(t)}{M_{B,j}} - 2\xi_{B,j}\omega_{B,j} \{ \dot{\eta}_{B,j}(t) \} - \omega_{B,j}^2 \{ \eta_{B,j}(t) \}
\]

(5.81)

The total base shear, \( \{ V_B(t) \} \), exerted by a rotating blade is equal to the summation of the inertia forces over the entire length of the blade as
\[ \{V_B(t)\} = \overline{m}_B \int_0^{L_B} \{\ddot{u}(t)\} \, dx \quad (5.82) \]

where \( \overline{m}_B \) is the mass per unit length of the blade and \( L_B \) is the length of the blade. The acceleration in physical coordinates, \( \{\ddot{u}(t)\} \) may be represented as before as a linear combination of mode shapes and second temporal derivatives of the modal coordinates. The latter having been calculated using equation 5.81.

### 5.4 FORCED VIBRATIONS OF TOWER COUPLED TO ROTOR SYSTEM

This section presents the analytical derivation used to couple three rotating wind turbine blades to a tower and nacelle. This model then obtains the response at the top of the tower due to dynamic wind drag forces along the length of the tower, and due to blade/tower dynamic interaction. In order to couple the tower and rotating blades, it is necessary to define the equation of motion for the tower including the forces transmitted by the vibrating blades. This equation of motion reads

\[
[M_T]\{\ddot{x}(t)\} + [C_T]\{\dot{x}(t)\} + [K_T]\{x(t)\} = \{F_D(t)\} + \{V_B'(t)\} \quad (5.83)
\]

where \( \{V_B'(t)\} \) is the effective base shear force vector from the three rotating blades, acting at the top of the tower. The tower displacement in physical coordinates is represented as before, as the summation of a linear combination of two sub variables as

\[
\{x(t)\} = \sum_{j=1}^{m} [\Phi_{T,j}] \{\eta_{T,j}(t)\} \quad (5.84)
\]

Inserting equation 5.84 into equation 5.83 and pre-multiplying by the transpose of the modal matrix, \([\Phi_T]^T\) gives

\[
[\Phi_T]^T[M_T][\Phi_T]\{\ddot{\eta}_T(t)\} + [\Phi_T]^T[C_T][\Phi_T]\{\dot{\eta}_T(t)\} + [\Phi_T]^T[K_T][\Phi_T]\{\eta_T(t)\} = [\Phi_T]^T\{F_D(t)\} + [\Phi_T]^T\{V_B'(t)\} \quad (5.85)
\]

The effective shear force transmitted into the tower due to blade vibration is given by
\[
V_B'(t) = [\ddot{u}_1(t) + \ddot{x}(t)]m_1 + \ldots + [\ddot{u}_q(t) + \ddot{x}(t)]m_q
\]  

(5.86)

where \(\ddot{u}_q(t)\) is the blade nodal acceleration at node ‘q’ and \(\ddot{x}(t)\) is the acceleration at the top of the tower/base of the blades. Equation 5.86 may be simplified to

\[
V_B'(t) = V_B(t) + \ddot{x}(t)M_{3B}
\]  

(5.87)

with \(V_B(t)\) being the absolute base shear, given by equation 5.82, and \(M_{3B}\) is the total mass of the three blades. Equation 5.85 is simplified to give

\[
[M_{T,j}]{\ddot{\eta}_{T,j}(t)} + [C_{T,j}]{\ddot{\eta}_{T,j}(t)} + [K_{T,j}]{\eta}_{T,j}(t) = \{F_{D,M_j}(t)\} + \{V_{M_j}(t)\} \quad j = 1, \ldots, m
\]  

(5.88)

with the vector \(\{V_{M_j}(t)\}\) representing the modal blade base shear force time-history. This time-history vector may be formed as

\[
\{V_{M_j}(t)\} = [\Phi_T]^T \{V_B'(t)\} = \begin{bmatrix}
\phi_{T,11} & \cdots & \phi_{T,1m} \\
\vdots & \ddots & \vdots \\
\phi_{T,N1} & \cdots & \phi_{T,Nm}
\end{bmatrix}^T \begin{bmatrix}
V_B(t) + M_{3B}\ddot{x}(t) \\
\vdots \\
0
\end{bmatrix}
\]  

(5.89)

where ‘N’ is the total number of nodes and ‘m’ is number of modes being included in the response. Equation 5.89 can be simplified to give the following expression for the modal base shear as

\[
V_{M_j}(t) = \phi_{T,jj} \{V_B(t) + M_{3B}\ddot{x}(t)\}
\]  

(5.90)

Equation 5.84 may be inserted into equation 5.90 to give

\[
V_{M_j}(t) = \phi_{T,jj} \left\{ V_B(t) + M_{3B} \sum_{i=1}^q \phi_{T,ii} \ddot{\eta}_{T,i}(t) \right\}
\]  

(5.91)

Equation 5.91 is expressed in its canonical form as
\[ \{ \ddot{\eta}_{T,j}(t) \} + 2\xi_{T,j}\omega_{T,j}\{ \ddot{\eta}_{T,j}(t) \} + \omega_{T,j}^2 \{ \eta_{T,j}(t) \} = \left\{ \frac{F_{D,Mj}(t)}{M_{T,j}} \right\} \]

\[ + \left\{ \phi_{T,j}\{ V_B(t) \} + \phi_{T,j}M_{3B}n \sum_{i=1}^{n} \phi_{T,i}\{ \ddot{\eta}_{T,i}(t) \} \right\} \frac{1}{M_{T,j}} \]

(5.92)

Taking a Fourier Transform of equation 5.92 gives

\[ -\omega^2 \{ \eta_{T,j}(\omega) \} + 2\xi_{T,j}\omega_{T,j}\omega \{ \eta_{T,j}(\omega) \} + \omega_{T,j}^2 \{ \eta_{T,j}(\omega) \} = \left\{ \frac{F_{D,Mj}(\omega)}{M_{T,j}} \right\} + \left\{ \frac{\phi_{T,j}\{ V_B(\omega) \}}{M_{T,j}} \right\} \]

\[ + \omega^2 M_{3B} \phi_{T,j}n \sum_{i=1}^{n} \phi_{T,i}\{ \eta_{T,i}(\omega) \} \]

(5.93)

Assuming that the inclusion of the first ‘k’ modes will yield a satisfactory level of response accuracy and arranging the common terms together yields an expression for the frequency domain modal co-ordinate as

\[ \eta_{T,j}(\omega) = \frac{\phi_{T,j}V_B(\omega) - \phi_{T,j}M_{3B}\omega^2 \sum_{i=1}^{k} \phi_{T,i}\eta_{T,i}(\omega)}{-\omega^2 + 2\xi_{T,j}\omega_{T,j}\omega + \omega_{T,j}^2 + \frac{\phi_{T,j}^2M_{3B}\omega^2}{M_{T,j}}} \]

(5.94)

Equation 5.94 leads to a set of ‘k’ simultaneous equations, as

\[ \left[ -\bar{M}_{T,j}\omega^2 + \bar{M}_{T,j}2\xi_{T,j}\omega_{T,j}\omega + \bar{M}_{T,j}\omega_{T,j}^2 + \phi_{T,j}\bar{M}_{3B}\omega^2\phi_{T,j} \right] \eta_{T,j}(\omega) \]

\[ + \sum_{i=1}^{k} \left[ \phi_{T,i}M_{3B}\omega^2\phi_{T,ii} \right] \eta_{T,i}(\omega) = \frac{F_{D,Mj}(\omega)}{M_{T,j}} + \phi_{T,j}V_B(\omega) \]

; \ j = 1,\ldots, k \]

(5.95)

For the case when k = 3 (first three modes considered), it is convenient to adopt the simplifying notation of

\[ \left[ -\bar{M}_{T,1,2,3}\omega^2 + \bar{M}_{T,1,2,3}2\xi_{T,1,2,3}\omega_{T,1,2,3}\omega + \bar{M}_{T,1,2,3}\omega_{T,1,2,3}^2 + \phi_{T,1,2,3}\bar{M}_{3B}\omega^2\phi_{T,1,2,3} \right] \eta_{T,1,2,3}(\omega) = A_i \]

; \ i = 1, 2, 3 \]

(5.96)
The solutions of equation 5.95 expanded out for the first three modes yields expressions for the modal co-ordinates as follows:

\[ \eta_1(\omega) = \frac{\lambda \delta - \tau \beta}{\delta \alpha - \gamma \beta} \quad (5.104) \]

\[ \eta_2(\omega) = \frac{\lambda \gamma - \tau \alpha}{\gamma \beta - \delta \alpha} \quad (5.105) \]

\[ \eta_3(\omega) = \frac{1}{A_3} \left[ B_3 - C_{31} \left( \frac{\lambda \delta - \tau \beta}{\delta \alpha - \gamma \beta} \right) - C_{32} \left( \frac{\lambda \gamma - \tau \alpha}{\gamma \beta - \delta \alpha} \right) \right] \quad (5.106) \]

where

\[ \lambda = -B_3 C_{13} + A_3 B_1 \quad (5.107) \]

\[ \delta = A_3 A_2 - C_{23} C_{32} \quad (5.108) \]

\[ \tau = -B_3 C_{23} + A_3 B_2 \quad (5.109) \]
The total displacement at the top of the tower, \( \{X(\omega)\} \), is hence

\[
\{X(\omega)\} = \left[\Phi_{11}\right]\{\eta_1(\omega)\} + \left[\Phi_{12}\right]\{\eta_2(\omega)\} + \left[\Phi_{13}\right]\{\eta_3(\omega)\} \tag{5.113}
\]

An inverse Fast Fourier Transform (IFFT) of \( \{X(\omega)\} \) yields the displacement response time-history at the top of the tower including blade vibration effects.

5.5 NUMERICAL EXAMPLES

A series of numerical examples are first presented in order to illustrate the responses of the various assemblies described in Section 5.2. The displacement response of the tower and nacelle, with classical damping, subjected to random wind loading is first obtained using the mode acceleration technique. The response of the same system is obtained using an integral superpositioning technique for classical and non-classical damping. Next, the displacement response of rotating blades is obtained, along with the shear force at its base. The response of an arbitrary dimensioned coupled tower, nacelle and blades model is lastly computed, with rotating and vibrating blades, and random wind loading along the length of the tower.

5.5.1 Tower and nacelle with classical damping

A wind turbine tower, consisting of a uniform cantilever of circular hollow cross-section is considered in this section. It has a height of 60 m, a width of 3 m, and thickness of 0.015 m. The tower is constructed from steel with a density of 7850 kgm\(^{-3}\), giving the tower a total mass of 66253 kg. The elastic modulus of steel was taken as 2.1 \( \times 10^{11} \) Nm\(^{-2}\). The tower carries a nacelle mass of 19876 kg. The discrete parameter approach, presented in Section 4.2.1 was used to obtain the free vibration properties of the system. The mean wind velocity at top of tower was taken as being 30 m s\(^{-1}\) and the mean wind velocity profile exponent was 0.16 (Simiu and Scanlan, 1996). The density of air was taken as 1.25 kgm\(^{-3}\).
and the coefficient of drag was 1.2. The system was discretized into eight nodes (with the nodes numbered from top to bottom), giving the total number of degrees-of-freedom as eight. The first modal damping ratio was 0.7 %, the second was 0.095 % and the third 0.032 %. These values were obtained using a specific scalar constant, as used in equation 5.37. Eight spatially correlated total nodal drag force time-histories were simulated, as presented in figures 5.2 to 5.9.

Figure 5.2 Drag force at node 1

Figure 5.3 Drag force at node 2

Figure 5.4 Drag force at node 3

Figure 5.5 Drag force at node 4

Figure 5.6 Drag force at node 5

Figure 5.7 Drag force at node 6
The decay constant associated with the coherence was taken as being equal to 9. The response of the tower and nacelle at the top of the tower is presented in figure 5.10, and shows a maximum displacement of approximately 1.1 m.

Figure 5.8 Drag force at node 7

Figure 5.9 Drag force at node 8

Figure 5.10 Tip response of wind turbine tower and nacelle subject to random wind loading
5.5.2 Tower and nacelle with classical and non-classical damping

The response of the tower and nacelle system obtained using the integral superposition technique of Section 5.2.2, for cases of classical and non-classical damping, is presented in this section. The classically damped case occurs due to structural damping only, and the response of this case is illustrated in figure 5.11. The first three modes of vibration were included in the response, with modal damping ratios of 0.7 %, 0.095 % and 0.032 % respectively. The maximum observed response, for a mean wind velocity of 30 ms\(^{-1}\) at the top of the tower, and a mean wind velocity profile exponent of 0.16, was approximately 1.1 m.

![Figure 5.11 Tip response of wind turbine tower and nacelle excluding aerodynamic damping](image)

For the case of non-classical damping, aerodynamic damping was included in the response of the structure and the response of the structure subjected to random wind loading was estimated. The wind loading applied to the structure was similar to that used in the case of classical damping. The first three structural modal damping ratios were also the same, but with the inclusion of aerodynamic damping, the effective total modal damping ratios, given by equation 5.50, were 0.99 %, 0.4 % and 0.3 %. This suggests that at high wind velocities, aerodynamic damping is considerable (Holmes, 1996). Figure 5.12 presents the simulated
displacement response time-history at the top of the tower, including aerodynamic damping. The maximum observed displacement response at the top of the tower was approximately 1.0 m.

![Graph showing tip response of wind turbine tower and nacelle including aerodynamic damping.](image)

Figure 5.12 Tip response of wind turbine tower and nacelle including aerodynamic damping

5.5.3 Rotating blades with classical damping

The flapping response of a rotating blade in two spatial positions undergoing various rotational frequencies is presented in this section. The blades are modelled as discretized prismatic cantilever beams of rectangular hollow cross-section connected to a circular hub of radius 3 m, 30 m in length, 2.8 m wide and 0.8 m deep, with a thickness of 0.01 m. The density of blade material was taken as 2100 kgm$^{-3}$, giving each blade a total mass of 4510.80 kg. Each blade has a flapping moment of inertia equal to 9.53 x 10$^{-3}$ m$^4$. Two blade positions are investigated, $\theta = 90^\circ$ and $210^\circ$ (or $330^\circ$), each rotating at 1.57 rads$^{-1}$ and 3.14 rads$^{-1}$. The blades were discretized into eight nodes, giving eight degrees-of-freedom and the first three modal damping ratios were taken as being 1% of critical. A mean wind velocity of 20 ms$^{-1}$ was assumed at the base of the blades. Due to the level of turbulence experienced by rotating blades, the simulated nodal drag force time-histories were assumed not to experience any spatial correlation. The free vibration properties of the blades were obtained using the approach outlined in Section 4.3.1.
\[ \Omega = 1.57 \text{ radians}^{-1} \]

Figure 5.13 displays the blade response time-history simulated for a blade initially at an azimuth angle \( \theta = 90^\circ \) and rotating with a rotational frequency of 1.57 radians \(^{-1}\). The maximum observed displacement appears to be approximately 0.75 m.

![Blade Response Time-History](image)

Figure 5.13 Displacement time-history of blade rotating at \( \Omega = 1.57 \text{ radians}^{-1} \) when \( \theta = 90^\circ \)

![Fourier Transform](image)

Figure 5.14 Fourier transform of fluctuating wind velocity at blade tip with \( \Omega = 1.57 \text{ radians}^{-1} \) and \( \theta = 90^\circ \)
Figure 5.15 Displacement time-history of blade rotating at \( \Omega = 1.57 \text{ rad s}^{-1} \) when \( \theta = 210^\circ \)

Figure 5.16 Fourier transform of fluctuating wind velocity at blade tip with \( \Omega = 1.57 \text{ rad s}^{-1} \) and \( \theta = 210^\circ \)
Figure 5.17 Total base shear time-history from three blades rotating at $\Omega = 1.57 \text{ rads}^{-1}$

A Fourier transform of the simulated fluctuating wind velocity time-history at the tip of the blade is presented in figure 5.14. From this figure, it is possible to observe the peaks of elevated energy due to the phenomenon of rotational sampling. The displacement response at the tip of the blade at a spatial position of $\theta = 210^\circ$ is shown in figure 5.15. The maximum displacement appears to be approximately 0.81 m. A Fourier transform of the simulated fluctuating wind velocity time-history at the tip of the blade is presented in figure 5.15. In this figure, the energy peaks at $1.57 \text{ rads}^{-1}$, $3.14 \text{ rads}^{-1}$, $4.71 \text{ rads}^{-1}$ and $6.28 \text{ rads}^{-1}$ are clearly visible. Blades located at $\theta = 210^\circ$ and $330^\circ$ are dynamically equivalent, so when the base shear from each of those blades is added to the base shear to the blade at $\theta = 90^\circ$, the total base shear that would be transferred into the hub is obtained, as is illustrated in figure 5.17. A maximum shear force of approximately 150 kN is observed.

$\Omega = 3.14 \text{ RADS}^{-1}$

The tip displacement response time-history for a blade at its azimuth position $\theta = 90^\circ$ and subsequently rotating at a rotational frequency of $3.14 \text{ rads}^{-1}$, is presented in figure 5.18. The maximum observed displacement is approximately 0.73 m. A Fourier transform of the simulated fluctuating wind velocity acting at the tip of the blade is presented in figure 5.20.
On inspection of this figure, it is possible to see the peaks in energy at integer multiples of the rotational frequency.

Figure 5.18 Displacement time-history of blade rotating at $\Omega = 3.14 \text{ rads}^{-1}$ when $\theta = 90^\circ$

Figure 5.19 Fourier transform of fluctuating wind velocity at blade tip with $\Omega = 3.14 \text{ rads}^{-1}$ and $\theta = 90^\circ$
Figure 5.20 Displacement time-history of blade rotating at $\Omega = 3.14$ rad$^{-1}$ when $\theta = 210^\circ$

Figure 5.21 Fourier transform of fluctuating wind velocity at blade tip with $\Omega = 3.14$ rad$^{-1}$ and $\theta = 210^\circ$
Another identical blade, rotating at the same rotational frequency but originally at the position of $\theta = 210^\circ$ (or $330^\circ$) was analysed and its displacement response time-history is demonstrated in figure 5.20. A maximum displacement of approximately 0.71 m was observed. A Fourier transform of fluctuating wind velocity acting at the top blade node is presented in figure 5.21, and clearly shows the elevated energy levels at frequencies of 3.14 rads$^{-1}$, 6.28 rads$^{-1}$, 9.42 rads$^{-1}$ and 12.56 rads$^{-1}$. Again, the blades at $\theta = 210^\circ$ and $330^\circ$ are dynamically similar so the total base shear being transmitted into the hub is the sum of the base shear from the blades at $\theta = 90^\circ$, $210^\circ$ and $330^\circ$ and is illustrated in figure 5.22. The maximum value appears to be about 130 kN.

5.5.4 Tower coupled to rotor system

This section illustrates the effects of blade-tower interaction on the magnitude of the response of the system. A wind turbine tower, modelled as a discrete prismatic cantilever of circular hollow cross-section carries a nacelle, to which three rotating blades are attached. The geometric and material properties of the wind turbine tower and nacelle are identical to those specified in Section 5.5.1 and the three rotating blades are similar to those described in Section 5.5.3.
FORCED VIBRATIONS OF WIND TURBINE TOWER ASSEMBLIES

The mean wind velocity at top of tower was taken as being 20 ms$^{-1}$ and the mean wind velocity profile exponent was 0.16. The density of air was taken as 1.25 kgm$^{-3}$ and the coefficient of drag was 1.2. The system was again discretized into eight nodes, and the first three modes were included in the response. The first modal damping ratio was 0.7 % of critical, the second was 0.095 % and the third was 0.032 %.

**BLADE-TOWER INTERACTION NOT INCLUDED**

Figure 5.23 represents the response at the top of the tower excluding blade-tower interaction. In this instance, the mass of the three rotating blades is added to the mass of the nacelle, but the shear force created due to blade vibration is not transferred into the top of the tower. A maximum displacement value of 0.111 m was observed.

**BLADE-TOWER INTERACTION INCLUDED**

The response of the system, including blade-tower interaction will now be presented. The base shear time-histories simulated due to the vibration of the rotating blades, presented in figures 5.17 and 5.22, are transferred into the top of the tower and the response at the top of the tower is estimated.

![Figure 5.23 Displacement response time-history at the top of the tower excluding blade-tower interaction](image-url)
Figure 5.24 Displacement response time-history at the top of the tower including blade-
tower interaction with $\Omega = 1.57 \text{ rads}^{-1}$

Figure 5.25 Displacement response time-history at the top of the tower including blade-
tower interaction with $\Omega = 3.14 \text{ rads}^{-1}$
Two blade rotational frequencies are being considered, 1.57 \text{ rads}^{-1} and 3.14 \text{ rads}^{-1}. Figure 5.24 illustrates the displacement response time-history simulated for the blade rotational frequency of 1.57 \text{ rads}^{-1}, and shows a maximum displacement of approximately 0.39 m. The displacement response time-history estimated when the blades are rotating at 3.14 \text{ rads}^{-1} is presented in figure 5.25. This figure shows a maximum displacement of 0.23 m.

5.6 DISCUSSION ON NUMERICAL EXAMPLES

The magnitudes of the displacement response results obtained using the numerical examples will now be discussed. Two analytical methods were presented in order to obtain the displacement response of a linear classically damped wind turbine tower carrying a nacelle at the top, which was subject to random wind loading along its length. The first method uses the mode acceleration method in order to estimate the response of the system, and may only use the first few modes of vibration, due to the presence of the pseudo static response component. Random wind drag loading was simulated in the time-domain, containing information regarding spatial correlation of pressure, which acted along the length of the tower. The response of the same system subject to identical wind loading was subsequently obtained by numerically solving the response of the system using a superposition technique. The response time-history profiles for each solution method are practically identical, with both solutions providing the same maximum displacement response value. Both solutions contain initial transient components, which take time to decay out of the response. The response of the same system was found using the numerical integrating super-position technique, including aerodynamic damping, and the maximum displacement response was found to decreases by approximately 9 \%, due to the presence of the aerodynamic damping.

The response of classically damped rotating wind turbine blades was also obtained using the mode acceleration technique. Response time-histories, in the form of displacement and base shear forces, were obtained for two different rotational frequencies of 1.57 \text{ rads}^{-1} and 3.14 \text{ rads}^{-1}. The difference between the simulated displacement time-histories observed at \theta = 90^\circ and \theta = 210^\circ for both rotational frequencies appears inconclusive. Thus, the axial self weight of the blades appears not to have a substantial effect on the response of the blades. However, the centrifugal axial force due to rotation does appear to affect the magnitude of the response of the blades. The displacement response for the 1.57 \text{ rads}^{-1} case at \theta = 90^\circ, was generally higher than the 3.14 \text{ rads}^{-1} case at \theta = 90^\circ, as evident from a comparison of figures 5.13 and 5.18. This phenomenon is however less evident comparing
each rotational frequencies at $\theta = 210^\circ$. The effect of rotation on the blades is clearly visible when the total base shear time-histories for the 1.57 rads$^{-1}$ case, figure 5.17 is compared to the 3.14 rads$^{-1}$ case in figure 5.22. Because the blade in the latter case is rotating faster, it has more centrifugal stiffness which makes it displace less. Because this blade is stiffer, it will experience less inertia forces which will have the effect of reducing the shear force at the base of the blade.

The effect of whether or not blade-tower dynamic interaction is included in the response of a coupled model is clearly evident from the three numerical examples presented. In the first example, blade-tower interaction is not considered, the mass of the three rotating blades are simply lumped together with the mass of the nacelle. The approach yielded the lowest response magnitude, and represents the least dynamically realistic. When blade-tower interaction is considered, the response of the system increases significantly. As the blade rotation case of 1.57 rads$^{-1}$ created the largest base shear force time-history, when these three blades are coupled to the tower the largest coupled response is observed. This response is over three times greater than the case where blade-tower interaction is not considered. The blade rotating case of 3.14 rads$^{-1}$ created slightly less base shear force, so the coupled response of the tower and these three blades was approximately 40% less than the 1.57 rads$^{-1}$ coupled model. However, the maximum response from the 3.14 rads$^{-1}$ coupling case was over twice that of the case where blade-tower interaction was not considered.

It is evident that some of the maximum displacement response values obtained in Section 5.5 are too large to be realistic, where responses of these magnitudes would rarely been observed in practise. However, these maximum response values are purely a function of two parameters, the first being the geometry of the assemblies, which was at best representative of those used in practise. The second parameter was the wind environment, which was assumed to be aggressive. The mean wind velocities used in Section 5.5 were typically two to three times higher than those that an average wind turbine tower would typically operate optimally.
6.1 INTRODUCTION

This chapter presents the forced vibration models created to derive the gust response factors (GRFs) for wind turbine towers. At the design stage, it is not feasible to carry out long and detailed forced vibration analyses of the kind presented in Chapter 5, so a simplified theoretical approach such as the GRF is ideally suited in this regard. It allows practising engineers to design wind turbine towers under the action of dynamic wind loading. The use of such a design approach for wind turbine towers, as proposed in this chapter, appears entirely novel, mainly due to the unavailability of a simplified model of the system, which includes blade/tower dynamic interaction, for use with the gust factor design methodology.

The GRF is fundamentally a scalar number, equal to the ratio of the maximum magnitude of structural response to the mean or average magnitude of that response. This factor takes into account the mechanical admittance properties of the structure, as well as the turbulence properties of the wind loading. The GRF is very useful because using it has the effect of converting a system under dynamic loading to a system under equivalent static loading. A maximum design criterion used in ultimate limit state design may be obtained as the product of the GRF and the mean static value of that criterion. This philosophy is popular in wind engineering because it allows for the safe design of structures subject to wind loading, without the engineer requiring to have a comprehensive knowledge of wind effects on structures, and negating the necessity for a detailed wind-induced dynamic analysis. Also, the method of obtaining the GRF outlined in most design codes is relatively simple, as expressions for obtaining the GRF are usually offered in closed form.

Two types of models are derived in this chapter: a wind turbine tower represented by a single degree-of-freedom (DOF) model, and the same tower represented by two degrees-of-freedom. For the case of the former, the mass of the tower, nacelle and rotor system is lumped at the top of the tower, allowing motion of the entire wind turbine tower to be represented by the single degree-of-freedom. This scenario represents the most common approach especially with the GRF, as all GRFs are derived by considering one degree-of-freedom only. The second model uses two degrees-of-freedom to represent the motion of the wind turbine tower, and includes phenomena inherent to wind turbine behaviour, such as blade/tower interaction, and centrifugal stiffening of the rotor blades due to rotation.
This philosophy of adding a second degree-of-freedom is uncommon, but provides a more realistic basis for the development of accurate GRFs for use in the design of wind turbine towers.

Section 6.2 presents the derivation and validation of the GRF for the wind turbine tower represented by the single degree-of-freedom. Section 6.2.1 derives the GRF based on a frequency domain approach, and Section 6.2.2 describes a method to derive the GRF in the time domain. This method is based on a number of forced vibration analyses which yield the GRF from their average maximum and mean response values.

Section 6.3 describes the proposed approach to develop a two degree-of-freedom model for the GRFs for wind turbine towers. Section 6.3.1 outlines the method used to estimate the two degree-of-freedom GRF in the frequency domain, and this section is followed by Section 6.3.2., that obtains the two-degree-of-freedom GFR in the time domain.

Section 6.4 illustrates several numerical examples used to obtain GRFs for hypothetical towers and contains two sub sections. The first, Section 6.4.1, presents the GRFs obtained using the single degree-of-freedom model in the time and frequency domains, and includes several figures of GRFs obtained by applying the frequency domain method to two wind turbine assemblies with varying mean wind velocities. The next section, Section 6.4.2, demonstrates the GRFs obtained using the two degree-of-freedom model. GRFs were simulated in both the time and frequency domains for four different orientations of the two wind turbine assemblies, corresponding to varying blade rotational frequencies. Included in this section are several figures of GRFs obtained using the frequency domain method for the two assemblies with varying blade rotational frequencies for varying mean wind velocities.

Section 6.5 discusses all the results obtained, by firstly comparing the GRFs obtained for the single degree-of-freedom model in both the time and frequency domains. The GRFs found in the time and frequency domains using the two degree-of-freedom model are also compared. Finally, the GRFs obtained from the single degree-of-freedom and two degree-of-freedom models are compared.
6.2 GUST RESPONSE FACTOR FOR SINGLE DOF MODEL

This section details the derivation of the GRF for a wind turbine tower modelled as a single DOF system. The structural parameters of the continuous tower, nacelle and rotor system are discretized and motion of the system is characterized by a single DOF, illustrated by figures 6.1a and 6.1b. Methods to derived GRFs for the wind turbine tower are presented in both the frequency and time domains.

6.2.1 GRF in the frequency domain

The traditional GRF as derived by Davenport (1967) and used in most design codification worldwide is based on the application of random vibration theory to wind loading on a single DOF structure. Since the GRF uses random vibration theory, it is derived from a stochastic analysis, being equal to the ratio of a maximum response parameter of an ensemble average of responses, to the mean response of that parameter, (based on an ensemble average).

The gust response factor is obtained from a linear input-output relationship between two variables, namely wind loading and structural displacement. This relationship is characterised by the power spectral density function (PSDF) of the output, the
displacement, being equal to the product of the square of the amplitude of complex frequency response function and the PSDF of the input, the wind drag force, and is defined by the equation

\[ S_{xx}(f) = |H(f)|^2 S_{ff}(f) \quad (6.1) \]

where \( S_{xx}(f) \) is the displacement response PSDF, \( |H(f)| \) is the modulus of the complex frequency response function at the top of the tower and \( S_{ff}(f) \) is the PSDF of the wind drag force. All three parameters are functions of frequency, \( f \). When equation 6.1 is evaluated, the integral of \( S_{xx}(f) \) between the limits of a defined frequency band (i.e. the area under the curve) will yield the variance (mean square value) of the displacement response. The square root of the variance yields the standard deviation (root mean square) of the displacement response, which is further used to obtain the GRF.

The wind velocity flowing past a body induces a drag force on the body acting along the direction of the wind velocity. Since the velocity of a wind flow is random in both space and time, it is quantified using a statistical approach. Thus, statistical moments such as the mean, standard deviation and variance may be used. The total wind velocity, \( V(t) \), can be conceptually divided into a time-invariant mean component, \( \bar{v} \) and a zero mean fluctuating component, \( v'(t) \), given by the equation

\[ V(t) = \bar{v} + v'(t) \quad (6.2) \]

The fluctuating component is usually modelled statistically using a PSDF, like those suggested by figure 2.4 in Chapter 2. In a similar manner, the total wind drag, \( F_D(t) \), may be decomposed into two elements, a mean drag force, \( \bar{f}_D \), and a fluctuating drag force, \( f_D'(t) \), as given by the equation

\[ F_D(t) = \bar{f}_D + f_D'(t) \quad (6.3) \]

The fluctuating drag force may also be modelled statistically by a PSDF, and this constitutes the function \( S_{ff}(f) \) in equation 6.1. This fluctuating drag force PSDF, acting on a point-like body is given by the following equation, and is related to the fluctuating wind velocity PSDF, \( S_{vv}(f) \) as
GUST RESPONSE FACTOR

\[ S_{FF}(f) = \frac{4\tilde{r}_D^2}{(\bar{v})^2}S_{vv}(f) \]  \hspace{1cm} (6.4)

The mean component of the drag force \( \tilde{F}_D \), given by

\[ \tilde{F}_D = 0.5C_D A \rho \bar{v}^2 \]  \hspace{1cm} (6.5)

where \( C_D \) denotes the drag coefficient, \( A \) denotes the surface area of the tower and three rotating blades, and \( \rho \) denotes the density of air. The fluctuating wind velocity spectrum considered in this section is the expression offered by Kaimal et al (1972) as

\[ \frac{fS_{vv}(H,f)}{v_*^2} = \frac{200n}{(1 + 50n)^{5/3}} \]  \hspace{1cm} (6.6)

where \( H \) is the height of the tower, \( v_* \) is the friction velocity (\( m/s \)), and \( c \) is the Monin coordinate. The latter two terms may be obtained from the expressions

\[ \bar{v}(H) = \left( \frac{1}{k} \right) v_* \log_e \left( \frac{H}{z_0} \right) \]  \hspace{1cm} (6.7)

\[ n = \frac{fH}{\bar{v}(H)} \]  \hspace{1cm} (6.8)

with \( \bar{v}(H) \) being the mean wind velocity at height \( H \), \( k \) is Von-kàrmàn’s constant (typically around 0.4), and \( z_0 \) is the roughness length.

Equation 6.4 is valid for point-like bodies only, where the drag forces are assumed to be fully correlated over that body. If the size of the body is large compared to the smallest gust eddy size of interest, then the drag forces will not be fully correlated and equation 6.4 will not hold true. Equation 6.4 may be augmented to include the effects of incomplete pressure correlation, known as aerodynamic admittance, to yield
\[
S_{V_{T}}(f) = \frac{4(\overline{v}_{0})^{2}}{v^{2}} \chi^{2} \left( \frac{fb}{v} \right) S_{V_{V}}(f)
\] (6.9)

where \( \chi \) represents the aerodynamic admittance function. This function is dependant on three parameters, frequency (size of gust eddy), width \( b \), and mean wind velocity, and is represented by the empirically derived expression given by Nigam and Narayanan (1994) as

\[
\chi = \frac{1}{1 + \left[ \frac{2fb}{v} \right]^{3}}
\] (6.10)

The complex frequency response function is fundamentally a measure of how the structure will respond to a unit harmonic load and for a single DOF system is given by

\[
H(f) = \left( \frac{1}{4\pi^{3}f_{T,1}^{2}\overline{M}_{T,1}(1-\xi_{T,1}^{2}+2i\xi_{T,1}r_{1})} \right)
\] (6.11)

where \( f_{T,1} \) is the fundamental frequency, \( \overline{M}_{T,1} \) is the fundamental modal mass, \( \xi_{T,1} \) is the fundamental modal damping ratio, \( i \) denotes the complex operator equal to \( \sqrt{-1} \), and \( r_{1} \) is the non-dimensional frequency ratio given by the equation

\[
r_{1} = \frac{f}{f_{T,1}}
\] (6.12)

In the above equations, superscript \( T \) denotes tower. Analogous to wind velocity and wind drag force, the total displacement response, \( X(t) \) may be separated into a mean part, \( \bar{x} \), and fluctuating part, \( x'(t) \), as

\[
X(t) = \bar{x} + x'(t)
\] (6.13)

The GRF for displacement is equal to the ratio of the maximum displacement response to the mean displacement response. The maximum displacement response, \( X_{\text{max}}(t) \) may be obtained as
where $\Psi$ is the peak factor and $\sigma_x$ is the root mean square (RMS) value of displacement response, obtained from the evaluation of equation 6.1. The mean displacement component is obtained as the ratio of the mean drag force to the fundamental model stiffness of the tower, $K_{T,1}$ as

$$\bar{x} = \frac{\bar{f}_D}{K_{T,1}} \quad (6.15)$$

The peak factor may be obtained from the expression given by Crandall (1970) as

$$\Psi = \left(2 \log_e N_i T\right)^{0.5} + \frac{0.5772}{\left(2 \log_e N_i T\right)^{0.5}} \quad (6.16)$$

where $T$ is the time duration of the response and $N_i$ is the expected rate of zero-crossing by the fluctuating component of the response, which for a single DOF system is approximately equal to the natural frequency, $f_1$, for a narrow band process. As the gust response factor, $G$, is the ratio of the maximum response to the mean response, dividing equation 6.14 by $\bar{x}$ gives an expression for $G$ as

$$G = 1 + \Psi \frac{\sigma_x}{\bar{x}} \quad (6.17)$$

The RMS of the displacement response, $\sigma_x$, required by equation 6.17, was obtained by numerically integrating $S_{XX}(f)$ using the trapezoidal rule. Alternatively, the gust response factor can be derived in closed form. As equation 6.1 represents the input/output relationship for the system, it is necessary to obtain a closed form solution for the variance of the displacement response power spectral density function, $S_{XX}(f)$. To this end, a method of decomposition is employed, in which it is assumed that the displacement response power spectral density may be separated into two components: a background component and a resonant component. The background component may be thought of as the response component resulting from the energy contained within the wind being spread over a wide band of frequencies in the vicinity of the fundamental mode of vibration, but not at the fundamental modal frequency itself.
The resonant component of the response results from the energy contained in the wind acting directly at the fundamental mode of vibration. Figure 6.2 illustrates the decomposition of the total response into background and resonant components, with the stiffness term removed from the transfer function, for illustrative purposes. The blue curve is the displacement response power spectral density function. The square root of the area under this curve is the RMS value of the displacement response needed to formulate the GRF. The black curve is the fluctuating wind drag force power spectral density function, and the red curve is the product of the transfer function squared, and the wind velocity drag force power spectral density function ordinate at a frequency corresponding to the fundamental frequency.

The assumption made by the decomposition process states that the area under the blue curve is approximately equal to the area under the red curve plus the area under the black curve. The concept behind figure 6.2 is reiterated in the equation
\[
\sigma_x^2 = \int_0^\infty |H(f)|^2 S_{ff}(f) df = \int_0^\infty S_{ff}(f) df + S_{ff}(f_{r,1}) \int_0^\infty |H(f)|^2 df \tag{6.18}
\]

where the first term of the right hand side of equation 6.18 represents the background component, and the second term represent the resonant component. The background component, \(B\), was originally given by Davenport (1977) as

\[
B = \frac{4(\bar{f}_D)^2}{(4\pi^2 f_{T,1}^2 \bar{M}_{T,1})^2 (\bar{v})^2} \int_0^\infty \left(\frac{fb}{v}\right) S_{vv}(f) df \tag{6.19}
\]

The resonant component, \(R\), is given by

\[
R = \frac{(\bar{f}_D)^2 \chi_1^2 S_{vv}(f_{r,1}) \pi f_{r,1}}{16\pi^4 f_{T,1}^4 \bar{M}_{T,1}^2 (\bar{v})^2 \zeta_{T,1}} \tag{6.20}
\]

in which

\[
\chi_1 = \chi(f_{r,1} b/\bar{v}) \tag{6.21}
\]

The total energy output in the response is approximately equal to the summation of the background and resonant energies, the RMS of which is obtained as

\[
\sigma_x = \sqrt{B + R} \tag{6.22}
\]

The RMS of the displacement response may now be input into equation 6.17 to obtain the gust response factor.

6.2.2 GRF in the time domain

The GRF for the wind turbine tower, with its motion represented by a single DOF, will now be derived in the time domain. As the GRF derived previously in the frequency domain is actually a statistical quantity, a number of sample time-histories of wind drag forces are used to carry out time domain analyses. The GRF obtained from each analysis is averaged over the ensemble, in order to compare the time-domain GRF with the frequency domain GRF.
Although the fluctuating wind drag force component is modelled by a wind velocity power spectral density function, it is possible to transfer the information contained within this PSDF, which is in the frequency domain, into information which is in the time-domain. In other words, it is possible to simulate a fluctuating wind drag force time-history that has the same random characteristics and total energy as that of the wind drag force PSDF. This is facilitated by the fact that any arbitrary fluctuating drag force time-history, with zero mean, may be represented by a Discrete Fourier Transform (DFT) with a discretized version of a continuous frequency content, as

\[ f_{D}(t) = \sum_{k=1}^{\infty} a_k \cos(\omega_k t) + \sum_{k=1}^{\infty} b_k \sin(\omega_k t) \]  

(6.23)

where \( a_k \) and \( b_k \) are the Fourier coefficients, \( \omega_k \) is the \( k^{th} \) discretized circular frequency (\( \omega = 2\pi f \), \( f \) is frequency in Hz) and \( t \) is the time instant. This drag force time-history is generated from the drag force power spectral density function given by equation 6.9, which may be conceptually divided into infinitesimal frequency bands of size \( df \). The area under the PSDF between the limits of \( f_i \) and \( f_i + df \) is equal to the variance of the signal \( \sigma_i^2 \), at the discrete frequency \( f_i \). The Fourier coefficients in equation 6.23 are obtained as normally distributed random numbers, generated with zero mean and standard deviation \( \sigma_i \). The fluctuating drag force time-history is hence composed of a number of contributions from a discretized form of a continuous frequency band. Once the fluctuating drag force is simulated, it may be added to the mean component to give the total drag force time-histories in equation 6.3.

The availability of these drag force time-histories allows the use of the mode displacement technique to obtain the structural response of the wind turbine tower due to the wind loading. Using this concept, the total displacement response of the structure at the top of the tower \( \{X(t)\} \), is found as the product of the mode shape and the modal co-ordinate, as

\[ \{X(t)\} = [\Phi_{T,1}] \{\eta_{T,1}(t)\} \]  

(6.24)

where \([\Phi_{T,1}]\) is the fundamental mode shape (normalised to unity) and \( \{\eta_{T,1}(t)\} \) is the fundamental modal co-ordinate. Nigam and Jennings (1968) presented an algorithm to obtain the modal co-ordinate and its first derivative, of a viscously damped single DOF
oscillator subjected to an excitation. They assumed that the input acceleration time-history could be approximated by a piecewise linear function, thus yielding a semi-analytical process allowing the displacement and velocity response time-histories to be computed. The fundamental equation based on this algorithm is expressed as

\[
\begin{bmatrix}
\eta_{i+1} \\
\dot{\eta}_{i+1}
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
\eta_i \\
\dot{\eta}_i
\end{bmatrix} +
\begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix}
\begin{bmatrix}
a_i \\
a_{i+1}
\end{bmatrix}
\]  

(6.25)

where \( \eta_i(t) \) is the modal coordinate at time instant ‘i’, \( a_i \) is the excitation acceleration value at time ‘i’ and \( c_{11}, c_{12}, c_{21}, c_{22}, b_{11}, b_{12}, b_{21}, b_{22} \), are frequency and damping dependant constants given by equations 5.22 to 5.31 in Section 5.2.1 of Chapter 5.

The acceleration time-history for the fundamental mode of vibration is made up of individual acceleration values of \( a_i \), which may be obtained as

\[
\{a_i(t)\} = \frac{\{F_D(t)\}}{M_{T,i}}
\]  

(6.26)

where \( F_D(t) \) is the total drag force simulated using the approach presented in equation 6.23. Substituting the acceleration time-history obtained in equation 6.26 into equation 6.25, and in conjunction with equations 5.22 to 5.31, the fundamental modal co-ordinate and it’s first temporal derivative may be obtained. Thus, using equation 6.24, the displacement time-history at the top of the wind turbine tower can be estimated. The ratio of the maximum and mean values in this total displacement response represent the required gust factor, \( G \). This process must be repeated a number of times and an average value of \( G \) obtained, which may be compared with the frequency-domain based value given by equation 6.17.
6.3 GUST RESPONSE FACTOR FOR TWO DOF MODEL

This section presents the derivation of the GRF for a wind turbine tower modelled as a two DOF system. The structural parameters of the continuous tower and nacelle, and rotor system are discretized and the motion of each system is first represented by a single degree-of-freedom. Both assemblies are then coupled together to form a two degree-of-freedom system, using the philosophy outlined in Section 4.4 of Chapter 4, and illustrated in figures 6.3a and 6.3b. This two degree-of-freedom model considers phenomena intrinsic to the behaviour of a coupled wind turbine tower, such as blade/tower interaction and centrifugal stiffening of the rotor blades due to rotation. Two methods to derive the GRFs for the two degree-of-freedom model are included in this section. The first method obtains the GRF through the frequency domain and the second obtains the GRF through the time domain.

6.3.1 GRF in the frequency domain

Similar to the derivation used for the single DOF model, the GRF obtained in this section will be obtained through the frequency domain as a ratio of the maximum response to the mean of that response, in a statistical sense. The input-output relationship that relates the random wind loading to the displacement response at the top of the tower, denoted by node 2, is given by the equation
in which $S_{XX}(f)$ is the displacement response power spectral density function, $|H_2(f)|$ is the complex frequency response function at node 2 and $S_{FF}(f)$ is the wind drag force power spectral density function, given by equation 6.9. The free vibration characteristics of the two degree-of-freedom model, namely the natural frequencies and mode shapes, are needed and are obtained using the modelling approach proposed in Section 4.4 of Chapter 4. The coupled model will have a mode shape matrix, $[\Phi_{CS}]$, as

$$
[\Phi_{CS}] = \begin{bmatrix}
\phi_{11} & \phi_{12} \\
\phi_{21} & \phi_{22}
\end{bmatrix}
$$

(6.28)

where $\phi_{11}$ and $\phi_{12}$, $\phi_{21}$ and $\phi_{22}$ are the first and second mode shape components at nodes 1 and 2 respectively. The complex frequency response function at node 2 due to a unit force at node 2 may be expressed as

$$
H_2(f) = \phi_{21} \eta_{CS,1}(f) + \phi_{22} \eta_{CS,2}(f)
$$

(6.29)

where $\eta_{CS,1}(f)$ and $\eta_{CS,2}(f)$ are the first and second modal co-ordinates as a function of frequency, where the subscript CS denotes coupled system. Equation 6.29 is further expressed as

$$
H_2(f) = \phi_{21} \left( \frac{F_{CS,1}}{4 \pi^2 f_{CS,1}^2 \bar{M}_{CS,1} (1 - r_1^2 + 2 \xi_{CS,1} r_1 f)} \right) + \phi_{22} \left( \frac{F_{CS,2}}{4 \pi^2 f_{CS,2}^2 \bar{M}_{CS,2} (1 - r_2^2 + 2 \xi_{CS,2} r_2 f)} \right)
$$

(6.30)

where $F_{CS,1}$ and $F_{CS,2}$ are the first and second unit modal forces, $f_{CS,1}$ and $f_{CS,2}$ are the first and second natural frequencies, $\bar{M}_{CS,1}$ and $\bar{M}_{CS,2}$ are the first and second modal masses, $\xi_{CS,1}$ and $\xi_{CS,2}$ are the first and second modal damping ratios, and

$$
r_i = \frac{f}{f_{CS,i}}
$$

(6.31)
are the dimensionless frequency ratios. The modal forces are obtained from the expressions

\[
[F] = [\Phi_{CS}]^T [P] = \begin{bmatrix}
\phi_{11} & \phi_{21} \\
\phi_{12} & \phi_{22}
\end{bmatrix} \begin{bmatrix}
0 \\
1
\end{bmatrix}
\] (6.33)

by applying a unit load at node 2. In equation 6.33 a unit force is applied to node 2 only as, ultimately, the response at the top of the tower is required, because the tower will be designed to resist the total load applied at this point (as with the GRF for the SDOF case).

The total displacement response, \( X(t) \), as before, may be separated into a mean part, \( \bar{x} \), and fluctuating part \( x'(t) \), and the mean component has contributions from the two modes of vibration as

\[
\bar{x} = \frac{\phi_{12} \bar{f}_{D,1}}{K_{CS,1}} + \frac{\phi_{12} \bar{f}_{D,2}}{K_{CS,2}}
\] (6.34)

with \( K_{CS,1} \) and \( K_{CS,2} \) being the first and second modal stiffnesses, and \( \bar{f}_{D,1} \) and \( \bar{f}_{D,2} \) are the first and second mean modal drag forces respectively. The mean modal drag forces are obtained from the equation

\[
[\bar{f}_{D,M}] = [\Phi_{CS}]^T [\bar{f}_D] = \begin{bmatrix}
\phi_{11} & \phi_{21} \\
\phi_{12} & \phi_{22}
\end{bmatrix} \begin{bmatrix}
\bar{f}_1 \\
\bar{f}_2
\end{bmatrix}
\] (6.35)

The GRF may be obtained by inserting equation 6.34 and equation 6.16 into equation 6.17. The RMS of the displacement response, \( \sigma_x \), may be obtained by numerically integrating the displacement response PSDF, \( S_{XX}(f) \), into equation 6.27. However, the GRF can be also derived in closed form based on a closed form solution for the RMS of \( S_{XX}(f) \). Similar to the single DOF gust response factor model, a decomposition strategy is adopted, in which the displacement response PSDF is separated into two components: a background component and a resonant component. The latter term will contain contributions from two
Figure 6.4 Pictorial explanation of two DOF gust response factor modes of vibration. The second mode will also be a factor in the magnitude of the background component. This decomposition strategy is illustrated in figure 6.4.

Figure 6.4 illustrates the decomposition of the total response into background and resonant components. In figure 6.4, the modal stiffness terms have been purposely omitted to aid the visual explanation the decomposition strategy. The blue curve represents the displacement response power spectral density function. The square root of the area under this curve is the RMS of displacement response needed to formulated the GRF. The black curve represents the fluctuating wind drag force power spectral density function. The green curve represents the product of the first modal transfer function squared, and the wind drag force power spectral density function evaluated at the fundamental frequency of the coupled system. The red curve represents the product of the second modal transfer function squared, and the wind velocity drag force power spectral density function evaluated at the second modal frequency of the coupled system. The premise behind the decomposition process is that the area underneath the blue curve is approximately equal to the sum of the areas underneath the black, red and green curves. This decomposition strategy is presented mathematically as
\[ \sigma_x^2 = \int_0^\infty |H(f)|^2 S_{yy}(f)df = \int_0^\infty S_{yy}(f)df + S_{FF}(f_1) \int_0^\infty |H(f)|^2 df + S_{FF}(f_2) \int_0^\infty |H(f)|^2 df \quad (6.36) \]

or

\[ \sigma_x^2 = B + R_1 + R_2 \quad (6.37) \]

where the first term of the right hand side of equation 6.36 represents the background component (B), and the second and third terms represent the resonant component (R_1 and R_2). The background component was derived to be equal to

\[ B = \frac{4(f_D)^2}{(v)^2} \int_0^\infty S_{VV}(f) \mathcal{X}^2 \left( \frac{fb}{v} \right) \left[ \frac{\phi_{21} f_{CS,1}}{4\pi f_{CS,1}^2 M_{CS,1}} + \frac{\phi_{22} f_{CS,2}}{4\pi f_{CS,2}^2 M_{CS,2}} \right] df \quad (6.38) \]

The resonant component is comprised of two terms representing the first and second modes of vibration. The first term, R_1 may be evaluated from

\[ R_1 = \frac{\phi_{21} f_{CS,1}}{16\pi f_{CS,1}^4 M_{CS,1}} \frac{2 f_{CS,1}^2 S_{VV}(f_{CS,1}) \pi f_{CS,1}}{(v)^2 4 \xi_{CS,1}} \quad (6.39) \]

in which

\[ \mathcal{X}_1 = \mathcal{X}(f_{CS,1} b / v) \quad (6.40) \]

The second term associated with the second mode, R_2 is

\[ R_2 = \frac{\phi_{21} f_{CS,2}}{16\pi f_{CS,2}^4 M_{CS,2}} \frac{2 f_{CS,2}^2 S_{VV}(f_{CS,2}) \pi f_{CS,2}}{(v)^2 4 \xi_{CS,2}} \quad (6.41) \]

in which

\[ \mathcal{X}_2 = \mathcal{X}(f_{CS,2} b / v) \quad (6.42) \]

The total energy output to the response is approximately equal to the summation of the background and resonant energies, the RMS of which is obtained as

\[ \sigma_x = \sqrt{B + R_1 + R_2} \quad (6.63) \]
The RMS of the response may now be input into equation 6.56 to obtain the gust response factor.

6.3.2 GRF in the time domain

The GRF for the two degree-of-freedom wind turbine tower will now be derived in the time domain. As the frequency domain GRF derived in the previous section is the ensemble average of GRFs, a number of time domain analyses must be executed, and the GRF obtained from each sample needs to be averaged, in order to compare the time-domain GRF with the frequency domain GRF. It is convenient to express this total drag force in terms of the first and second total modal drag forces, \( F_{D,M1}(t) \) and \( F_{D,M2}(t) \) respectively, as

\[
\begin{bmatrix}
F_{D,M1}(t) \\
F_{D,M2}(t)
\end{bmatrix}
= 
\begin{bmatrix}
\Phi_{CS}^T \\
\Phi_{CS}^T
\end{bmatrix}
\begin{bmatrix}
0 \\
F_d(t)
\end{bmatrix}
= 
\begin{bmatrix}
\Phi_{CS}^T \\
\Phi_{CS}^T
\end{bmatrix}
\begin{bmatrix}
0 \\
\bar{F}_D
\end{bmatrix}
+ 
\begin{bmatrix}
\Phi_{CS}^T \\
\Phi_{CS}^T
\end{bmatrix}
\begin{bmatrix}
0 \\
\bar{F}'_D(t)
\end{bmatrix}
\]  

(6.44)

where superscript \( T \) denotes matrix transpose. The first and second mean modal drag forces, \( \bar{F}_{D,M1} \) and \( \bar{F}_{D,M2} \) are obtainable from the equation

\[
\begin{bmatrix}
\bar{F}_{D,M1} \\
\bar{F}_{D,M2}
\end{bmatrix}
= 
\begin{bmatrix}
\phi_{11} & \phi_{12} \\
\phi_{21} & \phi_{22}
\end{bmatrix}
\begin{bmatrix}
0 \\
\bar{F}_D
\end{bmatrix}
\]  

(6.45)

and correspondingly the first and second fluctuating modal drag forces, \( f_{D,M1}'(t) \) and \( f_{D,M2}'(t) \) are found from the equation

\[
\begin{bmatrix}
f_{D,M1}'(t) \\
f_{D,M2}'(t)
\end{bmatrix}
= 
\begin{bmatrix}
\phi_{11} & \phi_{12} \\
\phi_{21} & \phi_{22}
\end{bmatrix}
\begin{bmatrix}
0 \\
f_D'(t)
\end{bmatrix}
\]  

(6.46)

The fluctuating modal drag forces may be simulated as time-histories from knowledge of the first and second modal fluctuating drag force PSDs, \( S_{MF1MF1}(f) \) and \( S_{MF2MF2}(f) \). Expressions for these PSDs are given by

\[
S_{MF1MF1}(f) = S_{FF}(f)\phi_{21}^2
\]  

(6.47)
These fluctuating modal drag force time-histories may be simulated using the DFT technique presented in Section 6.2.2. The mode displacement technique is then used to obtain the structural response of the wind turbine tower due to the wind loading. Using this concept, the total displacement response at the top of the tower (node 2), \( \{X_2(t)\} \), is found as a linear combination of the weighted contributions of the two modes of vibration as

\[
\{X_2(t)\} = \phi_{21} \{\eta_{CS,1}(t)\} + \phi_{22} \{\eta_{CS,2}(t)\}
\]  

(6.49)

It is hence necessary to obtain the modal co-ordinate time-histories, \( \eta_{CS,1}(t) \) and \( \eta_{CS,2}(t) \). The algorithm presented by Nigam and Jennings (1986), as used in Section 6.2.2, is again used to obtain the modal co-ordinate and its first derivative. The equation that governs this algorithm is

\[
\begin{bmatrix}
\eta_{i+1} \\
\dot{\eta}_{i+1}
\end{bmatrix} = \begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{bmatrix} \begin{bmatrix}
\eta_i \\
\dot{\eta}_i
\end{bmatrix} + \begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix} \begin{bmatrix}
a_i \\
\dot{a}_i
\end{bmatrix}
\]  

(6.50)

where \( \eta_i(t) \) is the modal coordinate at time instant ‘i’, \( a_i \) is the acceleration value at time ‘i’ and \( c_{11}, c_{12}, c_{21}, c_{22}, b_{11}, b_{12}, b_{21}, b_{22} \) are frequency and damping dependant constants. These constants may be found from equations 5.22 to 5.31 in Section 5.2.1 in Chapter 5.

The modal acceleration time-history for mode ‘j’, \( \{a_j(t)\} \), which is thus made up of individual acceleration values of \( a_i \), may easily be obtained from the equation

\[
\{a_j(t)\} = \frac{\{F_{D,Mj}(t)\}}{M_{CS,j}}
\]  

(6.51)

where \( M_{CS,j} \) is the ‘j’th modal mass of the coupled system and \( F_{D,Mj}(t) \) is the total modal drag force for mode ‘j’ simulated through the DFT process. Solving equation 6.50 for each mode in turn yields the modal co-ordinates, which used in conjunction with equation 6.49 give the total displacement response. The ratio of the maximum and mean values in this total displacement response represent the required gust response factor, G. This process
must be repeated a number of times and an average value of \( G \) obtained, which may be compared with the stochastic frequency-domain based value of \( \text{GRF} \).

6.4 NUMERICAL EXAMPLES

A series of numerical examples are presented to show the difference in the \( \text{GRFs} \) obtained using the single degree-of-freedom approach presented in Section 6.2 and the two degree-of-freedom approach presented in Section 6.3. Two separate wind turbine tower assemblies are presented, henceforth referred to as Assembly 1 and Assembly 2, and for the purpose of comparison, each assembly will in turn be modelled by the single DOF and two DOF approaches. Each assembly contains the same wind turbine tower, created with typical but arbitrary dimensions. The tower is modelled as a uniform hollow tubular steel tower, of height 50 m, a width of 2.7 m, and a shell thickness of 0.015 m. This tower height was specifically chosen so the fundamental frequency of the tower/nacelle would be close to that of the rotating blades. The density of the steel was taken as 7580 kgm\(^{-3}\), giving the tower a total mass of 49662.1 kg. The elastic modulus of the steel was taken as 2.1 \( \times 10^{11} \) Nm\(^{-2}\), giving the tower a second moment of area of \( 1.14 \times 10^{-1} \) m\(^4\). The tower carried a nacelle mass of 20076.69 kg.

The primary difference between the two assemblies is the rotor system. The rotor system of Assembly 1 consists of three geometrically identical blades of rectangular hollow cross section, see figure 4.6 in Chapter 4 for reference, with a length of 30 m, a width of 2.8 m, a depth of 0.8 m, and a shell thickness of 0.01 m. Each blade is connected at its fixed end to a hub of radius 2.5 m. The blades are hypothetically fabricated from a glass fibre reinforce epoxy of density 2100 kgm\(^{-3}\), giving each blade a mass of 4511 kg. The elastic modulus of the material was taken as being 6.5 \( \times 10^{10} \) Nm\(^{-2}\) giving the blades a flapping second moment of area of \( 9.53 \times 10^{-3} \) m\(^4\). The coefficient of drag for the tower was taken as 1.2, and the density of air was 1.225 kgm\(^{-3}\).

The rotor system of Assembly 2 also contains three geometrically identical blades of rectangular hollow cross section, but with a length of 30 m, a width of 2.4 m, a depth of 0.4 m, and a shell thickness of 0.01 m. The hub radius was again taken as 2.5 m. The material and aerodynamic properites of the blades of Assembly 2 are identical to those of Assembly 1, giving the blades a flapping second moment of area of \( 1.92 \times 10^{-3} \) m\(^4\).
6.4.1 Single DOF gust response factor

Assemblies 1 and 2 were both initially modelled using as the single DOF wind turbine tower approach as presented in Section 4.2.3 of Chapter 4, where the mass of the three blades is lumped together with the mass of the nacelle. In Assembly 1, the fundamental frequency of tower, nacelle and three blades was calculated to be 3.557 rads$^{-1}$. The corresponding modal mass is 45492.84 kg, found using equation 4.59 in Chapter 4. The fundamental frequency of the tower, nacelle and three rotating blades used in Assembly 2 was 3.681 rads$^{-1}$, with a corresponding modal mass of 42481.72 kg, and an assumed fundamental modal damping ratio of 1% of critical was assumed for both assemblies.

GRFs were obtained for the two assemblies located in an onshore wind environment. The factor which dictates the wind environment is the roughness lengths $z_0$, as used in equation 6.7. Davenport (1977) gives the range of onshore (open grassland) roughness lengths as being from 0.01 m to 0.1 m, from which a roughness length of 0.08 m was chosen. Figures 6.5 and 6.6 present GRFs obtained using the frequency domain numerically integrated and closed form solution methods for Assemblies 1 and 2 respectively, using a range of mean wind velocities from 5 ms$^{-1}$ to 20 ms$^{-1}$. The numerically integrated and closed form solutions for single degree-of-freedom GRFs for the two assemblies were almost identical.

![Figure 6.5 GRFs for the onshore single DOF wind turbine tower of Assembly 1](image-url)
In order to validate these frequency domain values of GRF, ten response time-histories were simulated using the time domain approach of Section 6.2.2, for both Assemblies 1 and 2. Both single DOF wind turbines were located onshore using the roughness length of 0.08 m and were subjected to a mean wind velocity of 20 ms$^{-1}$ at the top of the tower. The average GRF of the ten simulated analyses for Assembly 1 was calculated at 2.674. The corresponding frequency domain GRFs for the same system were calculated as 2.925 using the numerically integrated method, and 2.921 using the closed form method. The average GRF of the ten simulated analyses for Assembly 2 was calculated at 2.782, compared to a numerically integrated frequency domain GRF value of 2.907, and a closed form value of 2.911.

6.4.2 Two DOF gust response factor
Assemblies 1 and 2 are modelled using the two DOF approach presented in Section 4.4 of Chapter 4. This approach allows for the dynamic interaction between the rotating blades vibrating in the flapping direction, and the tower and nacelle. The effects of centrifugal stiffening on the free vibration properties of the blades are also included. For Assemblies 1 and 2, four models are considered, corresponding to four different rotational frequencies of the rotor system. The first rotational frequency considered is 0 rads$^{-1}$, or when the blades are stationary.
This occurs when the blades are purposely stalled at high wind velocities. The other blade velocities considered are 0.785 rad s\(^{-1}\) (one completed revolution every eight seconds), 1.57 rad s\(^{-1}\) (one completed revolution every four seconds), and 3.14 rad s\(^{-1}\) (one completed revolution every two seconds). The free vibration properties of the four models of Assembly 1 are illustrated in Table 6.1, with the corresponding properties of Assembly 2 shown in Table 6.2. GRFs were obtained for all four orientations of both assemblies, assumed to be located onshore, i.e. employing a roughness length of 0.08 m, as the single DOF case.

### Table 6.1 Properties of the two DOF models of Assembly 1

<table>
<thead>
<tr>
<th>(\Omega = 0) rad s(^{-1})</th>
<th>MODE 1</th>
<th>MODE 2</th>
<th>(\Omega = 0.785) rad s(^{-1})</th>
<th>MODE 1</th>
<th>MODE 2</th>
<th>(\Omega = 1.57) rad s(^{-1})</th>
<th>MODE 1</th>
<th>MODE 2</th>
<th>(\Omega = 3.14) rad s(^{-1})</th>
<th>MODE 1</th>
<th>MODE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_{CS}) (rad s(^{-1}))</td>
<td>3.296</td>
<td>8.540</td>
<td>(\omega_{CS}) (rad s(^{-1}))</td>
<td>3.296</td>
<td>8.585</td>
<td>(\omega_{CS}) (rad s(^{-1}))</td>
<td>3.303</td>
<td>9.239</td>
<td>(\omega_{CS}) (rad s(^{-1}))</td>
<td>3.487</td>
<td>4.956</td>
</tr>
<tr>
<td>(\Phi_{CS})</td>
<td>1.213</td>
<td>-5.595</td>
<td>(\Phi_{CS})</td>
<td>1.210</td>
<td>-5.607</td>
<td>(\Phi_{CS})</td>
<td>1.177</td>
<td>-5.762</td>
<td>(\Phi_{CS})</td>
<td>1.177</td>
<td>-5.762</td>
</tr>
<tr>
<td>(\xi_{CS})</td>
<td>0.01</td>
<td>0.01</td>
<td>(\xi_{CS})</td>
<td>0.01</td>
<td>0.01</td>
<td>(\xi_{CS})</td>
<td>0.01</td>
<td>0.01</td>
<td>(\xi_{CS})</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

### Table 6.2 Properties of the two DOF models of Assembly 2
Figures 6.7 to 6.10 present the GRFs obtained by numerical integration and using the closed form solution, for the four wind turbine tower models of Assembly 1, with varying rotational frequencies. Figures 6.11 to 6.14 illustrate the numerically integrated and closed form GRFs for the four blade rotational frequency models of Assembly 2. From comparison of tables 6.1 and 6.2, it is evident that the coupled natural frequencies of Assembly 1 are relatively far apart, whereas those of Assembly 2 are close together. Hence, the two assemblies offer a good basis on which to gauge the accuracy of the closed form solution of the two degree-of-freedom GRF, compared to the corresponding numerically-integrated solution.

Ten response time-histories for each of the four models of Assemblies 1 and 2 were simulated in the time domain using the approach of Section 6.3.2. Each two degree-of-freedom wind turbine was placed onshore using the roughness length of 0.08 m and subjected to mean wind velocities of 20 ms$^{-1}$ at the top of the tower. For the stationary blades case ($\Omega = 0$ rads$^{-1}$), the average GRF of the ten simulated analyses was calculated to be 2.738 for Assembly 1 and 2.365 for Assembly 2.

Figure 6.7 GRFs obtained by numerical integration and closed form for the two DOF model with $\Omega = 0$ rads$^{-1}$, of Assembly 1
Figure 6.8 GRFs obtained by numerical integration and closed form for the two DOF model with $\Omega = 0.785 \text{ rad s}^{-1}$, of Assembly 1

Figure 6.9 GRFs obtained by numerical integration and closed form for the two DOF model with $\Omega = 1.57 \text{ rad s}^{-1}$, of Assembly 1
Figure 6.10 GRFs obtained by numerical integration and closed form for the two DOF model with $\Omega = 3.14 \text{ rad}^{-1}$, of Assembly 1.

Figure 6.11 GRFs obtained by numerical integration and closed form for the two DOF model with $\Omega = 0 \text{ rad}^{-1}$, of Assembly 2.
Figure 6.12 GRFs obtained by numerical integration and closed form for the two DOF model with $\Omega = 0.785$ rads$^{-1}$, of Assembly 2

Figure 6.13 GRFs obtained by numerical integration and closed form for the two DOF model with $\Omega = 1.57$ rads$^{-1}$, of Assembly 2
Figure 6.14 GRFs obtained by numerical integration and closed form for the two DOF model with $\Omega = 3.14 \text{ rads}^{-1}$, of Assembly 2

For the case of the blades rotating at 0.785 rads$^{-1}$, the averaged GRFs from the ten simulated analyses were calculated as 2.660 for Assembly 1 and 2.414 for Assembly 2. The case with a blade rotational frequency of 1.57 rads$^{-1}$ found an average time-domain GRF of 2.662 for Assembly 1 and 2.501 for Assembly 2, for the ten simulated time-histories. The averaged time-domain GRFs for the blade rotation case of 3.14 rads$^{-1}$ were calculated to be 2.662 for Assembly 1 and 2.600 for Assembly 2.

6.5 DISCUSSION ON NUMERICAL EXAMPLES

Two sets of GRFs are plotted in figures 6.5 and 6.6, obtained with a single DOF wind turbine tower. In both figures, the set in orange was obtained by use of numerical integration, and the second set in green was obtained using the derived closed form solution. Both sets show excellent agreement, validating the decomposition strategy adopted for the single DOF approach. A GRF of 2.674 was obtained for Assembly 1 using the time domain method with a mean wind velocity of 20 ms$^{-1}$ at the top of the tower. This value differed from the values obtained using the frequency domain method by approximately 9%. A GRF of 2.782 was obtained for Assembly 2 using the time domain method with a mean wind velocity of 20 ms$^{-1}$ at the top of the tower. This value differed
from the frequency domain GRFs by approximately 4%. These discrepancies can be attributed to the relatively small sample size of responses generated in the time domain.

For the two DOF approach, the GRFs for all orientations of Assembly 1 are plotted in figures 6.7 to 6.10, in which the sets in orange denote the GRFs obtained using numerical integration, and the sets in green represent the GRFs derived in closed form. The differences between these two solution methods appear negligible. Thus, for the coupled Assembly 1, the GRFs obtained using the closed form solution prove very accurate. This is because the modes are not too close together. The GRFs calculated for Assembly 2 are shown in figures 6.11 to 6.14. The difference between the numerically integrated GRF and the GRFs obtained in closed form are clearly visible. For a mean wind velocity of 20 ms\(^{-1}\) with stationary blades, a percentage difference of approximately 10% is observed. This difference decreases as blade velocity increases, with a disparity of approximately 4% for the blades rotating at 3.14 rads\(^{-1}\). The differences between GRFs obtained using the two methods is brought about due to the proximity of the two modes. As they are so close together, they are not fully decoupled. As the modes move further apart, as happens when the blade rotational frequencies increases or as in Assembly 1, the modes become more decoupled, and the closed form GRF converges towards the numerically integrated value.

<table>
<thead>
<tr>
<th>Assembly</th>
<th>(\Omega) (rads(^{-1}))</th>
<th>(G_{num})</th>
<th>(G_{avg})</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2.916</td>
<td>2.738</td>
<td>6.104</td>
</tr>
<tr>
<td></td>
<td>0.785</td>
<td>2.907</td>
<td>2.660</td>
<td>8.496</td>
</tr>
<tr>
<td></td>
<td>1.57</td>
<td>2.909</td>
<td>2.662</td>
<td>8.491</td>
</tr>
<tr>
<td></td>
<td>3.14</td>
<td>2.914</td>
<td>2.662</td>
<td>8.648</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2.457</td>
<td>2.365</td>
<td>3.744</td>
</tr>
<tr>
<td></td>
<td>0.785</td>
<td>2.464</td>
<td>2.414</td>
<td>2.029</td>
</tr>
<tr>
<td></td>
<td>1.57</td>
<td>2.497</td>
<td>2.501</td>
<td>0.160</td>
</tr>
<tr>
<td></td>
<td>3.14</td>
<td>2.650</td>
<td>2.600</td>
<td>1.887</td>
</tr>
</tbody>
</table>

Table 6.3 Comparison between frequency domain and time domain GRFs using two DOF approach

The GRFs obtained using the time domain method show good agreement with those estimated using numerical integration within the frequency domain method, and for Assemblies 1 and 2, the percentage differences are presented in table 6.3. For all four
systems with varying rotational frequencies subjected to a mean wind velocity of 20 ms\(^{-1}\) at the top of the tower, the maximum observed difference between the time and frequency domain GRFs was approximately 9\%.

It is of interest to compare the GRFs obtained using the single and two degree-of-freedom models. An arbitrary mean wind velocity of 20 ms\(^{-1}\) is adopted for the purpose of this comparison. For Assembly 1, the GRF obtained from the single DOF approach is about 2.9 in magnitude, with the GRFs estimated using the two DOF approach also around 2.9, depending on the rotational frequency of the rotor system. This suggests that an accurate measure of the GRF of a wind turbine tower whose first and second modes are relatively far apart, does not depend on the choice of model, i.e. the lumped single DOF model or the two coupled two DOF model. However, for the wind turbine tower represented by Assembly 2, the choice of modelling approach does have a bearing on the magnitude of the GRF. For a mean wind velocity of 20 ms\(^{-1}\), the GRF obtained using the single DOF approach was approximately 2.9, but the GRFs estimated using the two DOF approach were between 2.4 and 2.6, depending on the rotational frequency of the rotor blades. Thus, for a wind turbine tower whose first and second modes are close to each other, the most accurate portrayal of the GRF is obtained using the coupled two DOF approach. As the GRF philosophy is primarily used in design, it is recommended that the coupled two DOF model be used whenever closely-spaced modes are present. If the single DOF model is used in an equivalent static wind loading design of a wind turbine tower (excluding fatigue considerations), the magnitudes of the design variables, such as dynamic bending moments and shear forces, may be overestimated, leading to a conservative design.
CHAPTER 7 - EXPERIMENTAL TESTING FOR SYSTEM IDENTIFICATION

7.1 INTRODUCTION

This chapter presents the experimental testing carried out for system identification in the Vibration and Dynamic Testing Laboratory, at the Department of Civil, Structural and Environmental Engineering, Trinity College Dublin. Chapter 4 presented a series of approaches used to compute the dynamic properties of an arbitrary theoretical system. This chapter has a similar basis in that it describes a series of experiments to measure the dynamic properties of a real scale model. The free vibration properties of the assemblies presented in Chapter 4 were required to subsequently carry out the forced vibration analyses presented in Chapter 5. The experimental results presented in this chapter are used to provide the necessary inputs for the validation of the forced vibration analysis of the response of the scale model in wind tunnel tests, described in Chapter 8.

This chapter is divided into six sections, with Section 7.2 describing the experimental set-up used for system identification, including the data acquisition equipment and instrumentation. The model examined in the system identification tests is presented in Section 7.3, and is illustrated in detail. The next section, Section 7.4, describes the experimental results obtained, including free and forced vibration analyses. Section 7.5 details the mathematical models derived in order to validate the frequency response functions experimentally obtained. This section also compares the experimental and analytical results obtained. Section 7.6 deals with fitting mathematical expressions of multi-modal transfer functions to those obtained experimentally, for later use in Chapter 8.

The experimental set-up is explained in detail in Section 7.2. There are four components in the experimental set-up. The first component is the wave generator, which is discussed in Section 7.2.1. This facility allows the user to create an arbitrary broadband wave, and when interfaced with a shaking table, results in the creation of a realistic random motion. However, the experiments conducted for this project were limited to narrowband harmonic motions only. The wave generator was connected to a mechanical actuator, as described in Section 7.2.2, which creates the motion prescribed by the wave generator. The actuator was connected to a custom built moveable bed mounted on a set of ball bearings, on which the model was placed. Recording instruments were used to track the motion of the structure under the action of the actuator, in particular an accelerometer and a linear variable differential transducer (LVDT). The analogue signals output by the instruments
are sent to a signal conditioner, as described in Section 7.2.4, to be converted from analogue form to digital form. Once in digital form, the signals could be converted into metric units and output to a personal computer. A dedicated software package, as illustrated in Appendix B, is tasked with this conversion, and has been used to view the response of the model on a computer screen, and output results to a ASCII file.

The model used throughout the testing was a custom built scale model wind turbine tower assembly as presented in detail in Section 7.3. The model comprised of a flexible tower, a nacelle, which houses a motor and gearing system, and a rotor system consisting of three flexible blades. The tower is presented in Section 7.3.1, the nacelle, is illustrated in Section 7.3.2, and the rotor system, is detailed in Section 7.3.3.

The motivation behind this testing was two fold. The first motivation was to use system identification techniques to estimate the fundamental frequency of vibration and damping of the chosen assemblies, and the results of these tests are presented in Section 7.4.1. Free vibration time-histories for the blade only, the tower and nacelle only, and for the coupled tower/blades system are presented. For the coupled case, free vibration time-histories are presented for non-rotating blades with three blade orientations, as well as rotating blades with blade rotations of 1.57 and 3.14 radians per second. Frequency domain representations of all the coupled time-histories are included, in order to clearly illustrate the modal contributions. The second testing objective was to use the mechanical actuator and moveable bed to perform a forced vibration analysis on all assemblies. Using a range of user defined harmonic motions, displacement to displacement (D-D) transfer functions were determined. These are presented in Section 7.4.2 as the maximum displacement of the tip of the tower relative to that of the moving base, divided by the maximum displacement of the moving base. Displacement response plots for all assemblies were also be obtained, as the maximum tip response relative to the moving base.

In Section 7.5, two analytical derivations are presented. The first is concerned with the blade, and the other is concerned with the tower and nacelle. Both derivations act to theoretically estimate the displacement response plot for the blade and tower and nacelle, and use the previously recorded sinusoidal base motions, and estimated fundamental modal damping ratios of Section 7.4.1. These two displacement frequency response plots are then compared to the experimentally observed displacement response plots of Section 7.4.2, to validate the experimental results and the associated conceptual approach.
Section 7.6 deals with obtaining the force to displacement (F-D) transfer functions for the coupled model including rotating and non-rotating blades. As the transfer functions obtained experimentally are D-D transfer functions, they must be converted to F-D transfer functions to be of use in a wind induced stochastic analysis, as in Chapter 8. In this regard, a multi-modal expression capable of expressing the dynamic information is derived, and fitted to the experimentally obtained D-D transfer function to obtain the necessary values of modal mass. Then, a multi-modal expression is proposed for the F-D transfer function, and using the values of modal mass obtained from the D-D transfer function, the F-D transfer functions are obtained. The fitted and experimentally observed D-D transfer functions are compared in a series of figures.

7.2 EXPERIMENTAL SET-UP
A scale model, comprising tower, nacelle and rotor system was built, primarily to investigate the coupling effects between a tower and rotating blades undergoing harmonic vibration, with the ultimate aim of experimentally obtaining the transfer function of the system for use in a stochastic analysis of the model. The model was initially placed on a custom made moveable bed suspended on ball bearings, which in turn was connected to a mechanical actuator. The actuator was capable of moving the moveable bed with a harmonic motion of known frequency and amplitude.

An accelerometer was fixed at the top of the tower in order to measure the time-variant accelerations of the model when it was in motion. An LVDT was connected to the moveable bed which measured the displacement at the base of the model, in real time. The analogue signals output from the accelerometer and LVDT were passed through a signal conditioner to convert them from analogue form to a digital signal. These digital signals were then received by a personal computer, where a dedicated software package could interpret them in terms of metric units, and output the acceleration and displacement response of the model. Figure 7.1 presents a schematic diagram of the experimental set-up.
7.2.1 Wave generator

The wave generator is a digital to analogue conversion device that reads a specific type of wave input by the user, and converts that information into an analogue signal output by the device. The wave generator was manufactured by Thurlby Thandar Instruments, based in the UK, model designation TGA1241. The user is capable of creating any arbitrary wave, either narrow or broadband, in order to model a specific transient load time-history. The wave generator has a maximum frequency of 40 MHz, and allows the frequency to be specified up to the fifth decimal place. The signal generator was used in this section to create a series of harmonic waves of predefined frequency and amplitude.

7.2.2 Mechanical actuator

A mechanical actuator is a mechanical device that imposes a user specified motion on a test specimen or scale model. The actuator employed in the tests presented in this chapter was an Electrodynamic shaker VP 4, manufactured by Derritron Vibration Products. A steel test frame (in yellow) was built to specifically house the actuator, as illustrated in figure 7.2. A loading arm extends out from the actuator and is connected to a moveable bed, which is resting on a bed of ball bearings.
The moveable bed is restrained so as to provide fore and aft motion in the direction of the actuator thrust only, and the ball bearings act to reduce friction, so as not to add to the damping of the system. The principle on which the actuator works is based on the electromagnetic. At the core of the actuator is a strong permanent magnet, surrounded by an intricately wound electrical coil. The coil, also referred to as the armature, is mounted on flexible copper springs. An alternating current (ac) is supplied to the coil which produces a magnetic field, in-effect magnetising the coil. A force is produced perpendicular to the orientation of the coil, due to the presence of the magnetic field and the strong magnet. The ac current changes the polarity of the magnetic field having the effect of forcing the actuator fore and aft, at a user specified frequency and amplitude.

7.2.3 Instrumentation
Two distinct types of instruments were used to record the response of the model. An accelerometer was placed at the top of the tower and measured the magnitude of accelerations and retardations experienced by the model. The absolute displacement response at this point could be obtained by converting the experimentally measured
acceleration time-history. A LVDT was connected to the base of the model and measured the displacement of the base at a user specified frequency.

**ACCELEROMETER**
The Departmental laboratory possesses several accelerometers of the EGCS-D1S type, manufactured by Entran Devices Incorporation, from the USA. The accelerometers measure uniaxial acceleration, and have a range of ± 5 g, with a sensitivity of 28.83 mV/g. It works on the principle of piezo-resistance. The accelerometer contains an internal spring element, made of a crystal, metal or ceramic material. Several strain-sensitive gauges are connected to the body of this spring element, and the gauges are in turn connected in a Wheatstone bridge. When the spring element deflects while undergoing acceleration, the gauges are deformed, and produce a measurable change in electrical output. This analogue output may be calibrated to give an absolute acceleration in terms of the acceleration due to gravity. The accelerometer was connected to the top of the tower, as illustrated in figure 7.3, and the in-plane acceleration (relative to figure 7.3) was measured.

![Accelerometer connected to the underside of the nacelle plate](image)

**LVDT**
An LVDT is an analogue device used to measure linear displacement, and is pictured in figure 7.5. The testing laboratory possesses several LVDTs manufactured by the RDP electronics group, based in the UK. The LVDT used in these tests was a type ACT 2000, serial 30336. It is primarily composed of two components, the first is the inner component,
EXPERIMENTAL TESTING FOR SYSTEM IDENTIFICATION

Figure 7.4 Configuration of LVDT

consisting of a non-magnetic push rod with a magnetic armature at the end. The second component is the outer casing into which the inner component is inserted. The operation principle is illustrated in figure 7.4. The transformer has a primary coil, fed by an ac energy supply, and two secondary coils to the left and right of the primary coil. When the armature moves, an ac current is induced in the first and second secondary coils due to their coupling with the primary coil. When the armature is in its central position, as shown in figure 7.4, the voltage induced in secondary coil 1, $V_1$, is equal and opposite to the voltage induced in secondary coil 2, $V_2$, and the total output voltage, $V_0$, is zero. As the armature moves out of its central position, the ac voltage induced in secondary coil 1 increases or decreases relative to the ac voltage induced in secondary coil 2, and the resultant output voltage is no longer zero. This voltage can be linearly related to displacement using a calibration coefficient.

The LVDT is relatively low end technology and possesses several drawbacks. If the LVDT is connected to the tip of a cantilever type structure undergoing large displacement, the displacement profile of the tip will be in an arc. Thus, the inner component of the LVDT will move up and down and will be in contact with the outer component, inducing friction. This friction will have the effect of adding to the stiffness and damping of the test system, which is unwelcome especially during system identification testing. Also, this friction may affect the accuracy of free vibration tests, particularly if the model has low mass.
7.2.4 Data acquisition system

The data acquisition system consists of a signal conditioner and an analogue to digital (A/D) converter. The data acquisition system used was the System 6000, model 6100, manufactured by the Vishay Measurements Group, based in the USA. It contains sixteen cards, with each prescribed it’s own channel: two for LVDTs, two high level channels for accelerometers, and twelve channels for strain gauges. The System 6000 has a maximum scanning rate of 10,000 Hz. It receives analogue signals in the form of voltage differentials, and converts the signals into a digital equivalent. The digital signal may be sent to a software code operating on a personal computer with a digital operating system.

A dedicated personal computer (PC) has been used to record the experimental data. It is a Dell Optiplex GX 240, with a Pentium 4 processor with a clock speed of 2.0 GHz., with 512 MB of random access memory, a hard disk of 80 GB, and is serviced by the Microsoft Windows 2000 operating system. The PC is interfaced with the System 6000 signal conditioner using a PCMA card, and contains the software code Strainsmart used to analyse the digital signals received by the PC from the system 6000. Further details on the Strainsmart software may be found in Appendix B.
7.3 EXPERIMENTAL SCALE MODEL

The model was custom built at the Department laboratory and comprises flexible tower, nacelle, and rotor system. The rotor system is capable of rotating at a user specified rotational frequency as it is connected to a motor fixed onto the nacelle plate. Figure 7.6 shows the entire model resting on the flat bed prior to the start of the experiments.

![Figure 7.6 Scale model wind turbine tower](image)

Due to the difficulty in fabricating a model with very specific geometry, the model was built to no particular scale, although the overall height was limited to under one metre due to the size limitations at the wind tunnel at National University of Ireland, Galway (see Chapter 8), where the same model was also tested. However, a tower to blade length ratio of approximately 1:0.6, was adopted, which is similar to what is used in practice.
7.3.1 The tower
The tower was constructed from a tubular section of unpolymerised polyvinyl chloride (uPVC), as typically found in sanitation works. The tower is approximately 500 mm in length, though it has an effective length of 487 mm due to the steel nut that is used to fix the bottom of the tower to the flat plate. Using a micrometer, the tower was found to have an average diameter of 21.29 mm and an average thickness of 1.088 mm. A series of load displacement tests yielded an average elastic modulus value for the pipe, obtained indirectly from the flexural rigidity, of $3.45 \times 10^9$ Nm$^{-2}$. Figure 7.7 illustrates the size of the tower.

![PVC tower of circular hollow cross section](image)

Figure 7.7 PVC tower of circular hollow cross section

7.3.2 The nacelle
To simulate a nacelle, a 6 mm thick rectangular steel plate, of width 80 mm, length 60 mm, and mass 0.280 kg, was fixed at the top of the tower. This was achieved by attaching a cylindrical component of similar diameter to the tower, to the base of the plate and subsequently inserting it into the tower. A nut is then placed over the tower at the joining position and tightened, providing a robust fixity between steel plate and tower.
An accelerometer was attached to the underside of the plate by means of an angle bracket. The purpose of the steel plate was initially to provide mass loading on the tower, but it also was tasked with supporting the motor which turns the rotor blades. A gearing system was built up to considerably step down the turning speed of the motor, which unhindered would typically turn at a rate of excess of 10,000 revolutions per minute. The motor and gearing system were asymmetrically bolted to the base of the nacelle plate to ensure that the orientation of the drive shaft was along the centre line of the nacelle plate, as illustrated in figure 7.8. A counter mass was positioned on the opposite side of the nacelle plate, to ensure an even distribution of mass in order to counteract any possible yawing forces brought about due to mass imbalances. The total mass of the nacelle, including motor, nacelle plate and accelerometer was 0.538 kg.

![Motor and gearing system fixed to nacelle plate](image)

**Figure 7.8 Motor and gearing system fixed to nacelle plate**

### 7.3.3 The rotor system

The rotor system comprises of three flexible blades, masses at the free end of each blade, and a hub that grips the ends of the blades, connecting them to the drive shaft of the motor.
THE BLADES
The blades were made from a phenolic material cut to size by machine. The original sheet had a mean thickness of 2.3 mm, and blade lengths of 320 mm and widths of 25 mm were chosen. When the blades are fixed into the hub, they become cantilevers with effective lengths of 300 mm. Although, the phenolic material is very flexible, with an elastic modulus value of $7.5 \times 10^9$ Nm$^{-2}$, the relatively short length of the blades ensured that their fundamental frequency was much higher than that of the tower and nacelle. Thus, it was decided to attach a steel mass to the free end of the blades in order to increase their flexibility, and reduced their fundamental frequencies. Thus, in order to study the coupling effects between the tower/nacelle and the rotor blades, masses of 0.064 kg were attached to the blades so that the fundamental frequency of each blade was close to that of the tower and nacelle. Rectangular masses of length 18 mm, height 13 mm and width 32 mm, were attached approximately 15 mm from the free end edge of the blades. Figure 7.9 presents a blade, painted white for clarity, along with the free end mass.

![Figure 7.9 Longitudinal section of blade including end mass](image)

THE HUB
The hub is the clamping device used to restrain each of the flexible blades, and is itself rigidly connected to the drive shaft of the motor. When the motor turns the drive shaft, the hub also turns, and this rotary motion is transferred to each of the blades. The hub consists of two circular aluminium disks each milled from solid blocks. The first circular disk measures 60 mm in diameter with an average thickness of 4 mm, and has a 4 mm diameter hole at its centre. Three holes of diameter 3 mm were drilled into the face of the disk for
use in fixing it to the second disk, each separated by an angle of 120°. The second circular

disk is geometrically similar to the first disk with two additional features. First, three

rebates approximately 1.5 mm deep were milled into the face of the disk, each measuring

25 mm by 20 mm, and separated by an angle of 120°. Second, a cylindrical housing

located at the back of disk is used to connect the hub to the drive shaft of the motor.

The three blades were positioned into the rebates on the second disk, and the first disk was

placed over the three blades. The two disks were bolted together using the three holes,

rendering the three blades as cantilevers firmly fixed into the hub. The rotor system was

then connected to the motor by threading the drive shaft along the internal space within the

hub. A grub screw was tightened at the back of the hub, ensuring tight contact between the
drive shaft and hub. Figure 7.10 illustrates the assembled rotor system connected to the

drive shaft of the motor. The total mass of the three blades, end masses and hub was 0.351

kg. The combined mass of the nacelle and rotor system was 0.890 kg, which in relation to
the total mass of the tower, 0.0611 kg, represents a mass ratio of nearly 1500 %. This mass
ratio does not reflect conditions in full scale, where it typically lies between 40 % - 50 %,
but was chosen to ensure that the fundamental frequency of the scale model would be close
to those of full scale wind turbines, namely between 1 to 3 Hz.

Figure 7.10 Rotor system connected to the drive shaft of the motor
7.4 EXPERIMENTAL RESULTS
The objectives of the experiments presented in this chapter were two fold. The first was to determine the fundamental frequencies of vibration and corresponding modal damping ratios of all considered assemblies. This was to be achieved using free vibration tests. The second was to estimate the D-D transfer functions for all assemblies using forced vibration tests. These D-D transfer functions may be converted to F-D transfer functions for use in a stochastic analysis at a later stage (see Chapter 8).

7.4.1 System identification from free vibration analyses
A series of free vibration tests were carried out on the blade only, the tower and nacelle only, and the tower coupled to three blades. For the last case, free vibration tests were carried out for both rotating and non-rotating blades. Using the free vibration time-histories, the fundamental frequency and associated modal damping ratio can be obtained. The fundamental frequency may be estimated as the reciprocal of the time duration taken for one complete oscillation, assuming damping to be low. The modal damping ratio is obtained by employing the logarithmic decrement technique (Clough and Penzien, 1993). This approach assumes that the free vibration motion of the system will decrease exponentially due to the presence of damping, and uses information from two peaks on the acceleration time-history, as

\[
\ln \frac{a_n}{a_{n+m}} = \frac{2m\pi \xi}{\sqrt{1-\xi^2}}
\]

where \(a_n\) is the acceleration at the \(n^{th}\) peak, and \(a_{n+m}\) is the acceleration at the \((n+m)^{th}\) peak, and \(\xi\) is the damping ratio as a percentage of critical. The greatest accuracy is obtained when ‘n’ and ‘m’ are furthest apart.

**BLADES**
Five free vibration tests were carried out using the blade and its end mass only. These were carried out by rigidly attaching one end of the blade to the moveable bed, and setting the free end in motion by applying an initial displacement. The accelerometer was attached at this free end. Figure 7.11 presents the acceleration response obtained in one such test, and it is evident that that free vibration decay of the blade is indeed exponential. An average damped fundamental frequency of 3.10 Hz was identified using equation 7.1, and an average fundamental modal damping ratio of 1.68 % of critical was calculated.
Five free vibration tests were also carried out on the tower and nacelle model, one of which is presented in figure 7.12. The free vibration decay again appears exponential. An average damped fundamental frequency of 3.69 Hz was measured, along with an average fundamental modal damping ratio of 2.30 % of critical, found using equation 7.1.
TOWER, NACELLE AND NON-ROTATING BLADES
For the coupled tower/blades model with non-rotating blades, three blade orientations were considered. Position A, as shown in figure 7.13, is characterised by one of the blades at its azimuth. Position B, as in figure 7.14, has one blade perfectly horizontal and Position C, illustrated in figure 7.15, in which one of the blades is in the downward vertical position.

Free vibration tests were recorded for the coupled tower/blades model with non-rotating blades in position A, and from the free vibration time-history illustrated in figure 7.16, it is evident that two distinct vibration frequencies are presented.

Figure 7.13 Position A
Figure 7.14 Position B
Figure 7.15 Position C

Position A
Free vibration tests were recorded for the coupled tower/blades model with non-rotating blades in position A, and from the free vibration time-history illustrated in figure 7.16, it is evident that two distinct vibration frequencies are presented.
The Fourier transform of the acceleration time-history, presented in figure 7.17, confirms the multi-modal character of the response. Both the first and second modes of vibration are contributing to the response of the structure. The damped first and second natural frequencies appear to be approximately 2.25 Hz and 4.98 Hz respectively. As the free vibration decay appears exponential, equation 7.1 was employed to calculate an average fundamental modal damping ratio of 2.7% of critical, for five free vibration tests.

![Figure 7.17 Fourier transform of acceleration time-history at blade position A](image)

**Position B**

Several free vibration tests for the coupled tower/blades model with non-rotating blades in position B were carried out, and an acceleration time-history at the top of the tower is presented in figure 7.18. It is evident that the response again contains contributions from more than one mode of vibration, and this is substantiated by the Fourier transform of the acceleration response presented in figure 7.19. The first two modes mainly contribute to the response, and they appear at the frequencies of approximately 2.25 Hz and 5.00 Hz respectively. The response in figure 7.18 appears to decay exponentially allowing a fundamental modal damping ratio of approximately 2.9% of critical to be obtained using equation 7.1, from an average of five tests.
EXPERIMENTAL TESTING FOR SYSTEM IDENTIFICATION

Figure 7.18 Free vibration time-history of coupled system at blade position B

Figure 7.19 Fourier transform of acceleration time-history at blade position B
Position C

Five free vibration tests were carried out for the coupled model with its blades oriented at position C, and a free vibration acceleration time-history at the top of the tower is presented in figure 7.20.

Figure 7.20 Free vibration time-history of coupled system at blade position C

Figure 7.21 Fourier transform of acceleration time-history at blade position C
On inspection of this figure, it is clear that more than one mode of vibration is participating in the acceleration response of the model. Taking a Fourier transform of this time-history confirms that the second mode is contributing to the response, as illustrated in figure 7.21. The damped fundamental frequency appears to be at 2.24 Hz and the second natural frequency is 5.02 Hz. The free vibration decay is exponential, allowing an average fundamental modal damping ratio of 3.0% of critical be estimated using equation 7.1.

**TOWER, NACELLE AND ROTATING BLADES**

Free vibration acceleration time-histories were recorded with the coupled model for two cases of blade rotational frequency, 1.57 rads⁻¹ and 3.14 rads⁻¹.

**Blade rotation of 1.57 rads⁻¹**

A series of five free vibration tests were performed for the coupled model undergoing blade rotation at 1.57 rads⁻¹. A free vibration acceleration time-history recorded is illustrated in figure 7.22. Similar to the non-rotating cases, the response appears to be composed of more than one mode of vibration. A Fourier transform of the free vibration time-history indeed shows that the second mode is participating in the response, as shown in figure 7.23. From this plot, the first and second damped natural frequencies are obtained as 2.26 Hz and 5.03 Hz respectively.

![Figure 7.22 Free vibration time-history of coupled system with blade rotation of 1.57 rads⁻¹](image)
The free vibration acceleration response characterised by figure 7.22 seems to decay exponentially, enabling an average fundamental modal damping ratio of 2.2 % to be estimated using equation 7.1.

**Blade rotation of 3.14 rads**

Five free vibration acceleration time-histories were also recorded for the coupled model undergoing blade rotation at 3.14 rads. One of these time-histories is pictured in figure 7.24. It is evident that compared to all the other coupled free vibration time-histories, the fast motion of the blades has rendered the free vibration response more broad banded. It is not possible to discern by visual inspection what modes are contributing to the acceleration response, so a Fourier transform of the response was obtained, and is presented in figure 7.25. A higher response than before is observed at frequencies greater than 15 Hz, but overall, the response appears to primarily have contributions from the first and second modes of vibration. From this figure, the first and second damped natural frequencies are 2.25 Hz and 5.02 Hz, respectively. The free vibration decay in figure 7.24 still appears to be exponential, allowing an average fundamental modal damping ratio of 2.3 % to be estimated using equation 7.1.
Figure 7.24 Free vibration time-history of coupled system with blade rotation of $3.14 \text{ rads}^{-1}$

Figure 7.25 Fourier transform of acceleration time-history with blade rotation of $3.14 \text{ rads}^{-1}$
Table 7.1 shows the first and second damped natural frequencies of the coupled system, $f_{CS,1}$ and $f_{CS,2}$, and the fundamental modal damping ratios, $\xi_{CS,1}$, for all the rotating and non-rotating coupled assemblies. In this table, the values of natural frequency do not seem to be dependent of blade position or blade rotation frequency, and the fundamental modal damping ratio appears between 2% to 3% of critical.

<table>
<thead>
<tr>
<th>CASE</th>
<th>$f_{CS,1}$ (Hz)</th>
<th>$f_{CS,2}$ (Hz)</th>
<th>$\xi_{CS,1}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0$ rad$^{-1}$, Position A</td>
<td>2.25</td>
<td>4.98</td>
<td>2.7</td>
</tr>
<tr>
<td>$\theta = 0$ rad$^{-1}$, Position B</td>
<td>2.25</td>
<td>5.00</td>
<td>2.9</td>
</tr>
<tr>
<td>$\theta = 0$ rad$^{-1}$, Position C</td>
<td>2.25</td>
<td>5.02</td>
<td>3.0</td>
</tr>
<tr>
<td>$\theta = 1.57$ rad$^{-1}$</td>
<td>2.26</td>
<td>5.03</td>
<td>2.2</td>
</tr>
<tr>
<td>$\theta = 3.14$ rad$^{-1}$</td>
<td>2.25</td>
<td>5.02</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Table 7.1 Dynamic properties of all assemblies found from free vibration tests

### 7.4.2 Transfer function and displacement response

Forced vibration analyses were carried out for several types and orientations of assemblies using the wave generator and mechanical actuator described in Section 7.2. These assemblies included the blade only, the tower and nacelle only, and the coupled tower and blades assembly, in both rotating blade and non-rotating blade forms. The wave generator created a harmonic wave of known frequency and via the mechanical actuator, harmonic motion was induced at the base of the assemblies, using the moveable bed.

The accelerometer measured the acceleration at the top of the assembly, and the LVDT measured the displacement at the base of the assembly. The recorded acceleration could be represented by a sinusoidal expression, and thus converted to a displacement by dividing the acceleration by the square of the frequency of the wave, in radians per second. The harmonic displacement time-history at the base of the tower may be subtracted from the harmonic displacement time-history at the top of the tower to obtain the relative displacement response at the top of the tower. The amplitude of this relative displacement time-history constitutes the displacement response at the top of the tower, as a function of the frequency of the input harmonic base motion.

The D-D transfer function, which is ultimately a function of the frequency of input harmonic base motion, was obtained as the ratio of the amplitude of the relative
displacement time-history at the top of the tower, to the amplitude of the displacement time-history at the base of the tower.

**BLADES**

Figure 7.26 presents the experimentally obtained forced vibration displacement response for a blade with a rigid mass at its free end. The displacement response slowly increases with frequency until it quickly peaks at approximately 125 mm, at a frequency of about 3.1 Hz. This represents the fundamental frequency of the system and is identical to the value of 3.1 Hz determined from the free vibration test in section 7.4.1. A negligible response is observed after a frequency of 4 Hz. The displacement response also appears to converge to the static displacement value as the frequency tends towards zero.

A plot of the observed D-D transfer function is presented in figure 7.27. A maximum (dimensionless) value of the D-D transfer function of just under 30 is observable. The D-D transfer factor appears to be converging to zero as the frequency tends to zero.
A forced vibration analysis was subsequently carried out for the tower with a lumped mass at its free end, and the resultant displacement response is demonstrated in figure 7.28. From viewing this figure, it is evident that two response peaks occur in close proximity to each other: the first and largest, at a frequency of about 3.65 Hz, and the second and smaller peak occurring at a frequency of approximately 3.95 Hz. The first peak occurs due to the excitation of the fundamental flexural mode, observed at 3.69 Hz from the free vibration test in section 7.4.1 and the second peak is brought about by the excitation of the first torsional mode of the system, clearly visible during the experiment. The displacement response observed due to the resonant excitation of the fundamental flexural mode is about 75 mm and nearly 55 mm for that of the torsional mode. The displacement response is considerably less either side of these peaks, at approximately 10 mm. The D-D transfer function of the system was also obtained, and is presented in figure 7.29. The dimensionless transfer function value at the fundamental flexural mode was close to 20. The presence of the torsional mode does not appear to be very distinct however, though it is observable, having a magnitude of approximately 15. The value of the transfer function appears to approach zero at the lower end of the frequency range.
Figure 7.28 Displacement response observed at the top of the tower

Figure 7.29 D-D transfer function observed at the top of the tower
TOWER, NACELLE AND NON-ROTATING BLADES

The displacement response and D-D transfer function plots obtained from testing the coupled tower, nacelle and non-rotating blades model, for the three blade positions illustrated by figures 7.12 to 7.14, are presented in this section.

Position A

Figure 7.30 presents the experimentally-recorded displacement response for the coupled system with non-rotating blades in position A. This displacement response plot shows three peaks, the first and largest, at nearly 70 mm, corresponds to the fundamental flexural frequency of the system at about 2.35 Hz, which is close to the value of 2.25 Hz estimated from the free vibration test in section 7.4.1. The second peak, at nearly 40 mm, is caused by the excitation of the blades when they are excited by the vibration of the tower, and occurs at a frequency of 3.2 Hz. This represents a local mode of the coupled system. The third peak, also close to 30 mm, is the second flexural mode of the system, at nearly 4.75 Hz. This value differs from the value of 4.98 Hz estimated from the free vibration tests.

The plot of the D-D transfer function at the top of the tower, as presented in figure 7.31, shows a similar trend of three peaks, but the dimensionless magnitude of the first and second flexural modes, at about 17 and 16 respectively, are approximately twice that of the intermediate peak at nearly 8.

![Figure 7.30 Displacement response observed at the top of the tower for blade position A](image)
Position B

The experimentally-obtained displacement response for the coupled system with non-rotating blades in position B is presented in figure 7.32. This figure shows three peaks in structural response, at frequencies of about 2.35 Hz, 3.27 Hz and 4.85 Hz. The first peak represents the greatest response magnitude, at over 90 mm, and is caused by the excitation of the fundamental flexural mode of vibration. This occurs at a frequency close to the value of 2.25 Hz observed from the free vibration test in section 4.7.1. The second peak shows a much smaller response magnitude of just over 20 mm, and occurs due to the excitation of the blades due to the vibration of the tower. This peak is again a local mode of the coupled system. The third response peak represents the second flexural natural frequency of the coupled system, showing a response magnitude of nearly 55 mm. This occurs at a frequency of 4.85 Hz, close to the value of 5.00 Hz observed from the free vibration tests.

The corresponding D-D transfer function for this blade orientation is shown in figure 7.33. It possesses the characteristic three peaks due to the first and second flexural modes and the local blade vibrating mode, with each having dimensionless magnitudes of 23, 18 and 5 respectively. Thus, the intermediate mode would not be expected to contribute greatly in the total response of the coupled model. The magnitude of the transfer function is observed to drop sharply at the lower end of the frequency range.

Figure 7.31 D-D transfer function observed at the top of the tower for blade position A
Figure 7.32 Displacement response observed at the top of the tower for blade position B

Figure 7.33 D-D transfer function observed at the top of the tower for blade position B
Position C

Figure 7.34 presents the displacement response of the coupled assembly with the non-rotating blades in position C. The displacement response function exhibits the presence of three modes of vibration within the frequency range. The first response peak, which is the largest of the three, being close to 80 mm, was observed at a frequency of 2.35 Hz, and corresponds to the first flexural mode of vibration. This value is close to the value of 2.24 Hz observed during the free vibration tests. The second peak, at over 50 mm, occurs at a frequency of nearly 3.5 Hz, and is caused by the excitation of two of the blades due to the motion of the tower. This peak represents a local mode of the coupled system. It occurs at a higher frequency than the local modes in positions A and B because the blades have a varying fundamental frequency depending on their positions, primarily due to the presence of the lumped mass at their free ends. The third peak, occurring at a frequency of 4.85 Hz, causes a response of about 45 mm due to the damped resonant vibration of the second flexural mode at that frequency. This mode occurs at a frequency close to that of 5.02 Hz, as estimated from the free vibration test. The corresponding D-D transfer function is shown in figure 7.35. The presence of the three modes is again evident, only this time the local mode figures very prominently with a magnitude of about 23.
Figure 7.35 D-D transfer function observed at the top of the tower for blade position C

The first and second flexural modes have transfer function magnitudes close to 20 and 18 respectively. It is evident that the position of the blades had the effect of greatly magnifying the response of the local mode, compared to positions A and B.

**TOWER, NACELLE AND ROTATING BLADES**

Both the displacement response and the D-D transfer functions were obtained for the coupled model of tower, nacelle and three rotating blades, at two blade rotational frequencies of 1.57 rads⁻¹ and 3.14 rads⁻¹.

**Blade rotation of 1.57 rads⁻¹**

For the case of blade rotational frequency of 1.57 rads⁻¹, the experimentally-obtained displacement response is presented in figure 7.36. From this plot, it is evident that two distinct modes of vibration are present. The first peak occurs at a frequency of 2.28 Hz, with a response magnitude of 67 mm. This frequency corresponds to the fundamental flexural frequency of the coupled system. The value of this frequency closely correlates with the value of 2.25 Hz observed from the free vibration tests. The second peak of nearly 40 mm occurs at a frequency of about 4.85 Hz and is due to the excitation of the second flexural mode. This value is close to that of 5.03 Hz observed in the free vibration tests.
Figure 7.36 Displacement response observed at the top of the tower for blade rotation at 1.57 rads$^{-1}$

Figure 7.37 D-D transfer function observed at the top of the tower for blade rotation at 1.57 rads$^{-1}$
The presence of an intermediate local mode, as observed previously, is unclear, as the blades are changing position due to rotation. Figure 7.37 illustrates the D-D transfer function for this case of rotation, with a factor of over 20 observed for the first flexural mode, and 17 for the second flexural mode.

**Blade rotation of 3.14 rads\(^{-1}\)**

Figure 7.38 presents the experimentally-observed displacement response function of the coupled system when the blades are rotating at 3.14 rads\(^{-1}\). Three response peaks are definable, the first peak with by far the largest magnitude of 120 mm, occurs at a frequency of 2.22 Hz, and represents the damped resonant response of the first flexural mode. This value closely correlates with the value of 2.25 Hz estimated from the free vibration tests. The second peak, located close to 2.7 Hz, represents a local mode of the system, and exhibited a response of about 30 mm. This occurs because the rotating blades were being excited at a frequency of 2.7 Hz because of the motion of the tower. The third peak occurs at the second flexural mode of vibration, at a frequency of close to 5 Hz, causing a structural response of more than 30 mm. This value correlates closely with the corresponding value observed from the free vibration tests. The corresponding D-D transfer function is shown in figure 7.39.

![Figure 7.38 Displacement response observed at the top of the tower for blade rotation at 3.14 rads\(^{-1}\)](image-url)
Figure 7.39 D-D transfer function observed at the top of the tower for blade rotation at 3.14 rads\(^{-1}\)

It confirms the participation of the two flexural modes of the system, and the local mode of the blades. The first and second flexural modes experience a transfer function factor of 26 and 17 respectively, and the local blade mode has a factor of 11.

7.5 ANALYTICAL VERIFICATION OF FORCED VIBRATION ANALYSES

In order to validate the experimental displacement responses presented in Section 7.4.2, two analytical models were derived. The first model, predicts the displacement response of the blade and it's free end mass, using the fundamental frequency and damping ratio and the recorded base displacements measured with the LVDT. The second model, predicts the displacement response of the tower and the nacelle at the top of the tower in a similar fashion.

7.5.1 Displacement responses of blade, and tower and nacelle

**BLADE**

The blades are modelled as discrete lumped parameter systems, whose free vibration properties of fundamental frequency, mode shape and fundamental modal mass may be
obtained following the approach outlined in Section 4.3.1 of Chapter 4. The fundamental modal damping of the system is presented in Section 7.4.1. Once the free vibration properties have been found, the displacement response function may be simulated. The equation of motion of a blade subject to harmonic base acceleration is

\[
[M_B]\{\ddot{u}(t)\} + [C_B]\{\dot{u}(t)\} + [K_B]\{u(t)\} = -[M_B]\{I\}\{\ddot{q}(t)\} \tag{7.2}
\]

where \([M_B]\), \([C_B]\) and \([K_B]\) are the mass, damping and stiffness matrices of the blade, \([I]\) is an influence vector of ones, \([u(t)\) is the nodal displacement of the blade, \([q(t)\) is the displacement at the base of the blade, and an overdot denotes a temporal differentiation. As the linear displacement co-ordinate may be separated into a sum of the product of it’s spatial and temporal components, \([u(t)\) is expressed as

\[
\{u(t)\} = \sum_{j=1}^{m} [\Phi_{Bj}^T]\{\eta_{Bj}(t)\} \tag{7.3}
\]

where ‘j’ refers to the ‘j’th mode, ‘m’ is the number of degrees-of-freedom (DOF) characterising the system, and subscript B denotes blade. Inserting equation 7.3 into equation 7.2 and pre-multiplying by the transpose of the blade’s mode shape matrix \([\Phi_B]\) gives

\[
[\Phi_B]^T[M_B][\Phi_B]\{\ddot{\eta}_{Bj}(t)\} + [\Phi_B]^T[C_B][\Phi_B]\{\dot{\eta}_{Bj}(t)\} + [\Phi_B]^T[K_B][\Phi_B]\{\eta_{Bj}(t)\} = -[\Phi_B]^T[M_B]\{I\}\{\ddot{q}(t)\} \tag{7.4}
\]

Equation 7.4 may be simplified to

\[
[M_{Bj}]\{\ddot{\eta}_{Bj}(t)\} + [C_{Bj}]\{\dot{\eta}_{Bj}(t)\} + [K_{Bj}]\{\eta_{Bj}(t)\} = -[\Phi_{Bj}]^T[M_B]\{I\}\{\ddot{q}(t)\} \tag{7.5}
\]

where \([M_{Bj}], [C_{Bj}]\) and \([K_{Bj}]\) are the blade modal mass, damping and stiffness respectively, for the j’th mode of vibration. Dividing across by the modal mass of the blade gives

\[
\{\ddot{\eta}_{Bj}(t)\} + 2\xi_{Bj}\omega_{Bj}\{\dot{\eta}_{Bj}(t)\} + \omega_{Bj}^2\{\eta_{Bj}(t)\} = \frac{-[\Phi_{Bj}]^T[M_B]\{I\}\{\ddot{q}(t)\}}{M_{Bj}} \tag{7.6}
\]
Taking a Fourier Transform of equation 7.6 yields

\[-\omega^2 \{\eta_{B,j}(\omega)\} + 2i\xi_{B,j}\omega \{\eta_{B,j}(\omega)\} + \omega^2 \{\eta_{B,j}(\omega)\}\]

which on simplification yields

\[\{\eta_{B,j}(\omega)\} = \frac{\omega^2 [\Phi_{B,j}]^T \{M_B\} \{Q(\omega)\}}{M_{B,j}(-\omega^2 + 2i\xi_{B,j} \omega + \omega_{B,j}^2)} (7.8)\]

An inverse Fourier transform of equation 7.8 will yield the \(j^{th}\) modal co-ordinate in the time domain. Assuming that inclusion of the fundamental mode alone will yield accurate estimates of the response, the total response at of the blade, \(\{u(t)\}\) is given by

\[\{u(t)\} = [\Phi_{B,j}] \{\eta_{B,j}(t)\} (7.9)\]

The displacement response function that depicts the model’s response as a function of frequency must now be built up using the different harmonic base acceleration inputs. These harmonic base acceleration inputs have measured indirectly by the LVDT during the experimentation stage.

**TOWER AND NACELLE**

The tower and nacelle is also modelled using a discrete mathematical formulation, and the free vibration properties of this system may be obtained using the approach presented in Section 4.2.1 of Chapter 4. The fundamental modal damping of the system is presented in Section 7.4.1. Once the free vibration characteristics of the system have been estimated, the frequency response function may be simulated using the approach presented below.

The equation of motion of the tower and nacelle subject to harmonic base inertial loading is

\[[M_T]\{\ddot{x}(t)\} + [C_T]\{\dot{x}(t)\} + [K_T]\{x(t)\} = -[M_T]\{I\}\{\ddot{q}(t)\} (7.10)\]
where \([M_T], [C_T] \) and \([K_T]\) are the mass, damping and stiffness matrices of the tower, \([I]\) is an influence vector of ones, \([x(t)]\) is the nodal displacement of the tower, \([q(t)]\) is the displacement at the base of tower, an overdot denotes a temporal differentiation. Owing to the fact that any linear displacement coordinate may be separated into a sum of the product of its spatial and temporal components, \([x(t)]\) is expressed as

\[
\{x(t)\} = \sum_{j=1}^{m} \left[ \Phi_{T,j} \right] \{\eta_{T,j}(t)\}
\]

(7.11)

where 'j' denotes the jth mode and subscript 'T' denotes tower. Inserting equation 7.11 into equation 7.10 and pre-multiplying by the transpose of the tower’s mode shape matrix gives

\[
\left[ \Phi_T \right]^T [M_T] [\Phi_T] \{\eta_T(t)\} + \left[ \Phi_T \right]^T [C_T] [\Phi_T] \{\eta_T(t)\} + \left[ \Phi_T \right]^T [K_T] [\Phi_T] \{\eta_T(t)\} = -\left[ \Phi_T \right]^T \{[M_T] [I] \{\ddot{q}(t)\}\}
\]

(7.12)

Equation 7.12 may be simplified to

\[
[M_{T,j}] \{\ddot{\eta}_{T,j}(t)\} + [C_{T,j}] \{\ddot{\eta}_{T,j}(t)\} + [K_{T,j}] \{\eta_{T,j}(t)\} = -\left[ \Phi_{T,j} \right]^T \{[M_T] [I] \{\ddot{q}(t)\}\}
\]

(7.13)

where \(M_{T,j}, C_{T,j}\) and \(K_{T,j}\) are the tower’s modal mass, damping and stiffness respectively, for the jth mode of vibration. Dividing across by the modal mass of the tower gives

\[
\{\ddot{\eta}_{T,j}(t)\} + 2\xi_{T,j}\omega_{T,j} \{\dot{\eta}_{T,j}(t)\} + \omega_{T,j}^2 \{\eta_{T,j}(t)\} = \frac{-\left[ \Phi_{T,j} \right]^T \{[M_T] [I] \{\ddot{q}(t)\}\}}{M_{T,j}}
\]

(7.14)

Taking a Fourier Transform of equation 7.14 yields

\[
-\omega^2 \{\eta_{T,j}(\omega)\} + 2\xi_{T,j}\omega_{T,j}\omega \{\eta_{T,j}(\omega)\} + \omega_{T,j}^2 \{\eta_{T,j}(\omega)\} = \frac{\omega^3 [\Phi_{T,j}]^T \{[M_T] [I] \{Q(\omega)\}\}}{M_{T,j}}
\]

(7.15)

which on simplification yields

\[
\{\eta_{T,j}(\omega)\} = \left[ \frac{\omega^3 [\Phi_{T,j}]^T \{[M_T] [I] \{Q(\omega)\}\}}{M_{T,j}(-\omega^2 + 2\xi_{T,j}\omega_{T,j}\omega + \omega_{T,j}^2)} \right]
\]

(7.16)
An inverse Fourier transform of equation 7.16 will yield the $j^{th}$ modal co-ordinate in the time domain. Assuming that inclusion of the fundamental mode alone will yield accurate estimates of the response, the total response of the tower, $\{x(t)\}$ is given by

$$\{x(t)\} = [\Phi_{T,j}] \{\eta_{T,j}(t)\}$$ (7.17)

The displacement response function that depicts the response of the model as a function of frequency must now be built up using the different harmonic base acceleration inputs. These harmonic base acceleration inputs have been measured indirectly by the LVDT during the experiments.

7.5.2 Comparison of experimental and analytical displacement responses

The experimentally-obtained displacement response functions for the blade and tower/nacelle assemblies are compared to the requisite functions obtained using the analytical approaches described in Section 7.5.1. Figure 7.40 compares the displacement responses obtained experimentally and theoretically at the tip of a blade, including its end mass. Similar base acceleration inputs were applied for both response plots, and a fundamental modal damping ratio of 1.68 % was included in the theoretical analysis. The theoretical and experimental fundamental frequency of the blade and end mass were almost identical, and judging from the close agreement observed in figure 7.40, the theoretical fundamental mode shape and corresponding modal mass were also accurate.

Figure 7.41 compares the experimental and theoretical displacement response functions for the tower and nacelle, and these two plots also show close agreement. A fundamental modal damping ratio of the tower/nacelle system of 2.3 % was used in the theoretical analysis. The fundamental frequency obtained using the theoretical and experimental methods was almost identical, and on account of the close agreement observed in figure 7.51, the fundamental mode shape and modal mass may also be deemed accurate.

The close agreement between the experimental and theoretical displacement response plots for the blade and tower/nacelle assemblies verifies the experimental approach undertaken, and the analytical approaches proposed. In figures 7.40 and 7.41, poor agreement between theoretical and experimental displacement response is observed at the lowest frequencies. It is thought that this poor agreement was caused by the quality of accelerometer used in the experiment, and the estimation of structural damping, described in Section 7.4.1.
Figure 7.40 Experimental and theoretical displacement response at top of blade

Figure 7.41 Experimental and theoretical displacement response at top of tower and nacelle
7.6 IDENTIFICATION OF TRANSFER FUNCTIONS

The transfer functions measured in Section 7.4 relate displacement to displacement. For many applications, it is necessary to convert these types of transfer functions to force to displacement transfer functions. To achieve this, an expression for the D-D transfer function must first be derived, which is applicable to the coupled assembly under consideration. The coupled assembly may be modelled as a discrete two node system. The system thus contains two degrees-of-freedom, horizontal displacement at both nodes. The differential equation of motion of each assembly when subject to base acceleration is

\[
[M_{cs}]{\ddot{z}(t)} + [C_{cs}]{\dot{z}(t)} + [K_{cs}]{z(t)} = -[M_{cs}]{1}{\ddot{q}(t)} \tag{7.18}
\]

where \([M_{cs}], [C_{cs}], \) and \([K_{cs}]\) are the mass, damping and stiffness matrices of the two DOF coupled model, \({1}\) is an influence vector of ones, \({z(t)}\) is the nodal displacement of the coupled system, \({q(t)}\) is the displacement at the base of the coupled system, and an overdot denotes a temporal differentiation. The displacement coordinate \(z(t)\), may be separated into a sum of the product of its spatial and temporal components, as

\[
{z(t)} = \sum_{j=1}^{2} [\Phi_{cs,j}]{\eta_{cs,j}(t)} \tag{7.19}
\]

where subscript CS denotes coupled system. Inserting equation 7.19 into equation 7.18 and pre-multiplying by the transpose of the system’s mode shape matrix gives

\[
[\Phi_{cs}]{\Phi_{cs}^T}[M_{cs}]{\ddot{\eta}_{cs}(t)} + [\Phi_{cs}]{\Phi_{cs}^T}[C_{cs}]{\dot{\eta}_{cs}(t)} + [\Phi_{cs}]{\Phi_{cs}^T}[K_{cs}]{\eta_{cs}(t)}
\]

\[
= -[\Phi_{cs}]{\Phi_{cs}^T}[M_{cs}]{1}{\ddot{q}(t)} \tag{7.20}
\]

Equation 7.20 may be simplified to

\[
[M_{cs,j}]{\ddot{\eta}_{cs,j}(t)} + [C_{cs,j}]{\dot{\eta}_{cs,j}(t)} + [K_{cs,j}]{\eta_{cs,j}(t)} = -[\Phi_{cs,j}]{\Phi_{cs,j}^T}[M_{cs}]{1}{\ddot{q}(t)} \tag{7.21}
\]

where \(M_{cs,j}, C_{cs,j}\) and \(K_{cs,j}\) are the coupled system’s modal mass, damping and stiffness respectively. Dividing across by the modal mass of the coupled system gives
\[
\{\ddot{\eta}_{CS,j}(t)\} + 2\xi_{CS,j}\omega_{CS,j}\{\dot{\eta}_{CS,j}(t)\} + \omega_{CS,j}^2 \{\eta_{CS,j}(t)\} = \frac{-[\Phi_{CS,j}]^T ([M_{CS}]) \{\ddot{q}(t)\}}{M_{CS,j}} \tag{7.22}
\]

Taking a Fourier Transform of equation 7.22 yields

\[
-\omega^2 \{\eta_{CS,j}(\omega)\} + 2\xi_{CS,j}\omega_{CS,j}\{\eta_{CS,j}(\omega)\} + \omega_{CS,j}^2 \{\eta_{CS,j}(\omega)\} = \frac{\omega^2[\Phi_{CS,j}]^T ([M_{CS}]) \{Q(\omega)\}}{M_{CS,j}} \tag{7.23}
\]

which on simplification yields

\[
\{\eta_{CS,j}(\omega)\} = \left( \frac{\omega^2[\Phi_{CS,j}]^T ([M_{CS}]) \{Q(\omega)\}}{M_{CS,j}(-\omega^2 + 2\xi_{CS,j}\omega_{CS,j}\omega + \omega_{CS,j}^2)} \right) \tag{7.24}
\]

As the total response of the structure is composed of contributions from two modes, inserting equation 7.24 into equation 7.19, and dividing the resulting equation by \{Q(\omega)\} gives

\[
\frac{\{Z(\omega)\}}{\{Q(\omega)\}} = \phi_{11} \left( \frac{\omega^2[\Phi_{CS,j}]^T ([M_{CS}]) \{I\}}{M_{CS,j}(-\omega^2 + 2\xi_{CS,j}\omega_{CS,j}\omega + \omega_{CS,j}^2)} \right) + \phi_{12} \left( \frac{\omega^2[\Phi_{CS,2}]^T ([M_{CS}]) \{I\}}{M_{CS,2}(-\omega^2 + 2\xi_{CS,2}\omega_{CS,2}\omega + \omega_{CS,2}^2)} \right) \tag{7.25}
\]

which is the mathematical representation of the D-D transfer function of the 2 DOF model. In equation 7.25, \(\phi_{11}\) and \(\phi_{12}\) are the first and second mode shape components at the top node of the coupled model. The mode shapes of the system are both unknown, so theoretical mode shapes must be provided. Then, as the only remaining unknowns in equation 7.25 are the modal mass values, equation 7.25 may be fitted to the D-D transfer functions obtained experimentally, to yield the exact values of modal mass to suit the theoretical mode shapes.

For the wind tunnel testing described in Chapter 8, it was necessary to replace the D-D transfer function with a F-D transfer function, \(H(f)\), expressed for a two DOF system as
where $F_{CS,1}$ and $F_{CS,2}$ are the first and second modal forces, $f_{CS,1}$ and $f_{CS,2}$ are the first and second natural frequencies, $M_{CS,1}$ and $M_{CS,2}$ are the first and second modal masses, $\xi_{CS,1}$ and $\xi_{CS,2}$ are the first and second modal damping ratios, $i$ denotes the complex operator iota, $\sqrt{-1}$, and

$$r_1 = \frac{f}{f_{CS,1}}$$

(7.27)

and

$$r_2 = \frac{f}{f_{CS,2}}$$

(7.28)

The unit modal forces are obtained from the expressions

$$\begin{bmatrix} F_{CS,1} \\ F_{CS,2} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(7.29)

Using equation 7.25, D-D transfer functions were fitted to the experimentally observed D-D transfer functions for the coupled tower and blades model with blade positions A, B and C, and for the blade rotation cases of 1.57 rads$^{-1}$ and 3.14 rads$^{-1}$. The first and second natural frequencies of each system were identified from the transfer function plots of the relevant cases. It was not possible to experimentally measure the mode shapes of each assembly, so their theoretical mode shapes were derived using the approach outlined in Section 4.5.3 of Chapter 4. These mode shapes represent the approximate mode shapes of the coupled assemblies and hence, the modal masses obtained using these mode shapes will not yield an accurate portrayal of the actual modal masses. Thus, the mode shapes were scaled by a constant factor in relation to equation 7.25, in order to find the necessary values of first and second modal mass to fit the D-D transfer function given by equation 7.25 to the experimentally obtained D-D transfer function. A first and second modal damping ratio of 2.2 % of critical was also considered. This value was chosen as it was the lowest observed modal damping ratio of all rotating and non-rotating cases. The fitted modal parameters of all coupled assemblies are presented in table 7.2.
Table 7.2 Assumed modal parameters of all coupled model orientations

For the non-rotating blades assemblies with positions of A, B and C, figures 7.42, 7.43, and 7.44 compare the fitted D-D transfer functions with the experimentally observed D-D transfer functions. From these three plots, it is evident that the intermediate local mode is not included in the mathematical expression given by equation 7.25. Figures 7.45 and 7.46 present the fitted and experimentally observed D-D transfer functions for the blade rotational cases of 1.57 rads$^{-1}$ and 3.14 rads$^{-1}$ respectively, with the intermediate local modes omitted from the fitted expression.

The F-D transfer functions may be obtained from equation 7.26, which makes use of the dynamic parameters presented in table 7.2. Figure 7.7 illustrates the F-D transfer functions for all rotating and non-rotating blade-tower assemblies, and these will be used in Section 8.5 of Chapter 8.

In figures 7.42 to 7.46, it is evident that the fitted D-D transfer functions do not include the intermediate local modes between the first and second flexural modes. These local modes are a product of the geometry and material properties of the scale model, and occur primarily due to the lumped masses attached to the ends of the rotating blades. This mass distribution does not occur in full scale wind turbine blades, therefore, the intermediate local modes were excluded from the transfer function formulations.
Figure 7.42 Experimental and fitted D-D transfer functions for the coupled model with non-rotating blades at position A.

Figure 7.43 Experimental and fitted D-D transfer functions for the coupled model with non-rotating blades at position B.
Figure 7.44 Experimental and fitted D-D transfer functions for the coupled model with non-rotating blades at position C.

Figure 7.45 Experimental and fitted D-D transfer functions for the coupled model with blades rotating at 1.57 rads$^{-1}$. 
Figure 7.46 Experimental and fitted D-D transfer functions for the coupled model with blades rotating at 3.14 rad\(^{-1}\).

Figure 7.47 F-D transfer functions for all orientations of coupled assembly.
CHAPTER 8 – WIND TUNNEL EXPERIMENTATION AND VALIDATION

8.1 INTRODUCTION

This chapter describes the forced vibration testing carried out at the wind tunnel testing facility, in the Department of Civil Engineering, National University of Ireland, Galway (NUIG). The motivation behind this work was to experimentally carry out a controlled wind induced forced vibration analysis of a wind turbine tower and, in conjunction with the transfer functions obtained in Chapter 7, validate theoretical predictions of the vibration response of the assemblies using a random theory approach. This chapter is divided into five subsequent sections. The first section, presents the experimental set-up used in the wind tunnel, including details about the operation of the wind tunnel and the hardware used to measure the wind properties and the response of the wind turbine model. Section 8.3 presents the wind properties measured in the wind tunnel, while Section 8.4 presents the observed behaviour of the model when it was placed within the generated wind flow. In Section 8.5, the experimental results obtained in Section 8.4 are compared with predictions obtained using a series of mathematical models.

In each test, the scale model wind turbine tower, described in detail in Chapter 7, was placed in the wind tunnel and the response of the model was recorded under the action of specifically generated wind turbulence. Section 8.2.1 presents the most pertinent information regarding the wind tunnel, including its history, type and physical layout. This section also contains a schematic of the wind tunnel layout, which includes some of the more important dimensions. Also discussed in this section are the operations and limitations of the facility, including the methods employed to generate user defined mean wind velocity profiles and magnitudes of turbulence, during wind tunnel operations. Section 8.2.2 briefly presents the model used in the course of experimentation, and in particular, the instruments and their placement in order to requisitely capture the response of the model. The principles behind the use of strain gauges placed on the model is explained in this section. Next, Section 8.2.3 deals with the hardware and software used in obtaining the data pertaining to the wind properties, and the response of the model. The instruments used for capturing the wind properties include the manometer, to estimate the mean wind velocity in the wind tunnel, and the hot film anemometer, which is used to measure the power spectral density, turbulence intensity and integral length scales of turbulence.
Section 8.3 presents the wind properties measured in the wind tunnel using, in particular, the hot film anemometer. Section 8.3.1 illustrates the mean wind velocity profile observed in the wind tunnel, and Section 8.3.2 presents the turbulence properties observed within the wind flow, power spectral densities at specific heights, and include integral length scales and turbulence intensities.

Section 8.4 presents the experimental response of the model to the turbulent wind flow. Responses for the cases of rotating and non-rotating blades are included, and of the latter, the three blade orientations of Positions A, B, and C, are of interest. These three blade orientations are illustrated in figures 7.13, 7.14 and 7.15 respectively, in Chapter 7. The experimental results illustrated in this section include the strain and acceleration time-histories obtained from the non-rotating blades assembly with blade positions of A, B and C, and the rotating blade assembly with blade rotations of 1.57 rads$^{-1}$ and 3.14 rads$^{-1}$.

Section 8.5 presents the theoretical studies undertaken to validate the observed experimental response of the model. A mathematical representation of the model, based on random theory, is presented in Section 8.5.1, along with necessary derivations for acceleration response at the top of the tower and the fluctuating component of the strain response at the mid and base points of the tower. Section 8.5.2 presents the simulated acceleration and fluctuating strain time-histories obtained using the theoretical model. The three non-rotating blades cases with positions A, B and C, are considered, as well as the rotating cases of 1.57 rads$^{-1}$ and 3.14 rads$^{-1}$.

Section 8.6 compares the observed experimental and simulated theoretical response values of the fluctuating strain and acceleration results, including peak values and the second statistical moment of the time histories, the root mean square (RMS). All cases of blade rotation and non-rotation are considered.
8.2 EXPERIMENTAL SET-UP

This section presents the entire experimental set-up used in carrying out the forced vibration wind tunnel testing. Included in this section is the physical layout of the wind tunnel, the method of operation, and the practices used to generate turbulence within the wind flow. Also discussed in this section is the equipment used to measure the wind properties observed within the flow and the subsequent response of the model.

8.2.1 The wind tunnel

HISTORY, TYPE AND PHYSICAL LAYOUT

The wind tunnel at the Department of Civil Engineering, NUIG, was purchased from the Department of Engineering, at the University of Leicester in July, 1982. It is an open circuit wind tunnel, in so far as the air is initially drawn into the wind tunnel by a suction force created by an impeller situated at the rear of the wind tunnel, as illustrated in figure 8.1. The air is fed through an intake screen, shown in figure 8.2, which has the effect of ensuring that the air enters the wind tunnel uniformly, and also acts as a filter to counter the ingestion of large objects into the wind tunnel during operation, which might damage the impeller. Once past the intake filter, the air enters the test section of the wind tunnel. This section measures approximately 9.99 m in length, with a width of 2.4 m and a height of 1.98 m. The wind tunnel is housed in a building located next to the Civil Engineering Department at NUIG, which protects the facility from environment damage and also helps to reduce the amount of dust suspended in the air entering the test section. Figure 8.3 illustrates a schematic of the layout of the test section.

Figure 8.1 Impeller used to draw in air
Figure 8.2 Air intake filter of wind tunnel

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Figure 8.3 Schematic of test section including turbulence aids
The exterior surface of the intake and the interior surface of test section is fabricated from a polished wood, to minimise friction during operation. However, when the wind tunnel runs at a mean wind velocity of 5 m s\(^{-1}\), a boundary layer with a height of 0.15 m above the test section floor is unavoidably formed. Figure 8.4 illustrates the wind tunnel, and in particular the test section, (in white) as elevated above the ground. This elevation is necessary for two reasons. First, it isolates the facility from external sources of vibration that may be transmitted into the super-structure from passing vehicles or impact testing from the adjoining laboratory. These sources of noise may affect the sensitivity of instruments used to measure the wind flow properties, and in particular affect the estimation of the dynamic response of a flexible model. The second reason for elevation stems from the fact that the motor used to rotate the impeller needs to be housed under the impeller.

Access into the test section is possible through two doors (one is open in figure 8.4), both of which must be closed during the operation of the wind tunnel. This is to ensure safety and also, to ensure that the pressure inside the test section is only allowed to dissipate out of the back of the wind tunnel.
OPERATION AND LIMITATIONS

The wind tunnel makes use of a hydraulic system to power the motor of the impeller. A house underneath the rear of the wind tunnel houses a backup pump, hydraulic oil tank and main hydraulic pump. Figure 8.5 shows the main hydraulic pump in the left foreground, with the oil sump behind it, along with a wall mounted control panel. The backup pump must first be run for approximately ten minutes prior to operation of the impeller motor. The main hydraulic pump then draws oil from the sump and forces it through pipes upward into the impeller motor at high pressure. This high pressure fluid then forces rotating elements within the motor to move, ultimately converting hydraulic energy into mechanical (torque) energy, turning the impeller. The hydraulic fluid is then recycled to the sump where it is re-pressurised, allowing the cycle to repeat itself. This system is prone to leakages and over heating, especially if run for long periods of time. For this reason, the wind tunnel cannot safely create a mean wind speed at an elevation of 1.0 m greater than 5 ms⁻¹, to some extent limiting the scope of testing.
GENERATION OF TURBULENCE

In order to mimic realistic turbulent wind environments in the wind tunnel, compared to those observed in the open air, two procedures are used in practice. The two most important phenomena that must be created in the wind tunnel are a mean wind velocity profile that increases with height above the ground, and a power spectral density of wind energy that contains energy across a wide band of frequencies (many different sizes of eddies). The former is accomplished by using a grid or roughness elements and the latter is brought about using several spires.

Roughness elements

Roughness elements are commonly used to create a user desired mean wind velocity profile. They consist of objects stuck to the floor of the test section. The principle on which they work is based on that observed in full scale open air tests. The objects cause large amounts of friction with the oncoming wind, having the effect of retarding the flow close to the floor of the wind tunnel, producing a slower wind velocity. This friction induces Reynolds stresses which are subsequently transferred up through the layers of air, also slowing the oncoming wind velocity.

Figure 8.6 Roughness elements used to create mean wind velocity profile
As height above these objects increases, the retardation of flow reduces. The wind tunnel at NUIG uses wooden cubes of 0.04 m dimension, with square based spacing of 0.2 m, as observed in figure 8.6. Approximately 6 m of roughness elements are present, with another 0.85 m of smooth carpet in front of the model.

**Spires**

Spires represent a very cost effective or low cost technology method of conveniently creating a turbulent wind flow within the wind tunnel. The spires used at NUIG are made from wood, standing 1.15 m high, with a base width of 0.2 m and a tip width of 0.02m, as pictured in figure 8.7. The spires are painted yellow to increase visibility. Several spires are usually placed upwind of the test section, against the air intake, and depending on their height and spacing, have the effect of creating turbulence. The spires have the effect of cutting up the oncoming flow into much smaller eddies. The eddies created close to the bottom, where the spacing is smallest, are usually the smallest in size or highest in frequency. The eddies created at higher elevation, where the spacing is greater are usually bigger, and are at lower frequencies. These eddies flow downwind and continue to interact with each other, subsequently creating eddies of varying size.

![Figure 8.7 Spire used in the wind tunnel to generate turbulence](image)
Figure 8.6 illustrates four spires being used upwind of the roughness elements. The spires had a base spacing of approximately 0.6 m from each other, at a distance of approximately 0.3 m from the sides of the test section.

8.2.2 The model and instrumentation

The model used throughout the range of tests in the wind tunnel is the same as that extensively described in Chapter 7. Figure 8.8 illustrates the model in position in the wind tunnel, downwind of the air intake. To avoid unnecessary repetition, the makeup of the model will not be repeated in this chapter. However, two points regarding the model not already discussed in Chapter 7 will be addressed in this section. The first point is the tower fixed end connection with the floor of the wind tunnel. A circular hole of diameter 0.17 m exists approximately 6.85 m from the air intake in the test section of the wind tunnel. A wooden plate of similar diameter and 0.02 m depth was fabricated at the Department of Civil Engineering workshop in Trinity College. This plate had a hole of 0.01 m diameter at its centre point, along with two 0.002 m deep and 0.025 m wide rebates running parallel along its top. Two aluminium members were fitted into these rebates and fixed in place, which would hold the plate in place by resting on the wind tunnel floor.

Figure 8.8 The model placed within the wind tunnel
The circular plate, and aluminium members were next attached with a bolt to the inner thread embedded in the base of the tower, thus rigidly fixing the model to the circular wooden plate. The plate and model could then be placed into the hole in the wind tunnel floor, ensuring a rigid connection of both systems. Figure 8.9 presents a close up view of the circular plate attached to the base of the model, and placed within the wind tunnel floor.

In order to record the response of the model when subjected to a turbulent wind flow, an accelerometer was placed at the top of the tower, along with strain gauges at the windward and leeward mid-point and base-point of the tower. The principle behind the use of the accelerometer is explained in Chapter 7, Section 7.2.3 and is not repeated here. The model possessed relatively high stiffness, so the strain gauges were also used as sensors on the tower to augment the accelerometer in the event of structural acceleration being minimal, as even with low accelerations, strains would be measurable. Figure 8.9 illustrates the position of the accelerometer at the top of the tower and the strain gauges along the length of the tower.

**STRAIN GAUGES**

The strain gauges used were manufactured by the Vishay Micromeasurements Group in the United States of America (USA). They act uniaxially, or in one direction only, and are placed vertically on the surface of the tower. There are two parts to the strain gauge, the strain gauge itself which has two metallic legs attached to it, and the connector pad. Both legs must first be bonded onto the surface of the tower, with superglue, in close proximity to each other. Then the two legs are soldered onto the connector pad along with two wires which will carry the input/output signals to an analogue to digital (A/D) converter.

The strain gauges work on the principle of electrical resistance and have a fundamental resistance of 350 ohms. An input voltage of 2 to 3 volts is supplied to the strain gauges by the A/D converter. When the tower bends, the metallic legs of the strain gauges either increase or decrease, causing a change in resistance in the strain gauges. The strain gauge and the A/D converter constitute a Wheatstone bridge, which converts this change in resistance to an absolute voltage. This voltage is the output received by the A/D converter, and is usually measured in millivolts. These millivolts may then be linearly related to strain using what is known as a gauge factor, which in the testing was equal to 2.1. Strain gauges will usually have small amounts of residual strain due to the bonding process and due to
thermal expansion effects. Thus, it is necessary to zero the strain gauges before each set of tests.

![Figure 8.9 Position of accelerometer (top) and strain gauges (mid-point and base-point)](image)

8.2.3 Data acquisition for wind and structural response properties
This section discusses the hardware and software required to produce an output to measure and record the wind properties generated within the wind flow, along with the corresponding response of the model. Data is produced by all the measuring instruments in analogue form, usually voltage, and must be passed through an A/D converter to yield results in digital metric form.
**WIND PROPERTIES**

When the wind tunnel is operating in conjunction with the spires and roughness elements, a specific wind environment will be created in the wind tunnel. The wind properties mainly of interest include the real-time mean wind velocity, the mean wind velocity profile with elevation above the wind tunnel floor, and the fluctuating wind velocity time-history. The latter will yield a quantification of the turbulence present in the wind flow, including power spectral density, turbulence intensity and integral length scales of turbulence. Figure 8.10 illustrates the data acquisition equipment owned and operated by the Department of Civil Engineering at NUIG. The instrument in the foreground (light blue) is the hot film anemometer which is connected to a hot film probe which is immersed in the wind flow. The instrument directly behind the hot film anemometer (grey/green cover) in figure 8.10 is the manometer. Both the manometer and hot film anemometer are subsequently connected to a A/D card at the rear of the personal computer, also visible in figure 8.10.

**Manometer and pitot tube**

The electric manometer and pitot tube may be used to obtain the velocity of a fluid in motion and is used to estimate the instantaneous wind velocity in the wind tunnel at an elevation of approximately 1.0 m. This system works on the principle of converting the

![Figure 8.10 Data acquisition equipment used to measure wind parameters](image)
kinetic energy of the flow into potential energy. The system used at NUIG is the model FCO14 obtained from Furness Control Limited, in Sussex, England. The pitot tube is immersed in the flow and is connected to the manometer, as is conceptually shown in figure 8.11. The conversion from kinetic to potential energy occurs at the stagnation point, at the mouth of the pitot tube. A pressure known as the 'static' pressure results from this energy conversion. The manometer measures the free stream or dynamic pressure and obtains the difference between these static and dynamic pressures.

Figure 8.11 Pitot tube and manometer (Schematic diagram)

Converting the resulting difference in static and dynamic pressure into a fluid velocity depends on the flow regime encountered, like whether the flow is considered incompressible, subsonic compressible or supersonic compressible. The manometer outputs an analogue signal to an A/D converter allowing the mean wind velocity to be obtained. Figure 8.12 presents the pitot tube elevated by a mechanical arm above the test section.
The hot film anemometer is used to record fluctuating wind velocities at a high frequency, something that the pitot tube and manometer are incapable of achieving. It works on the principle of obtaining the wind velocity of a fluid by measuring the amount of heat convected away by the fluid. The apparatus consists of two components, the hot film probe and the hot film anemometer, both interconnected. The probe usually consists of a thin film typically made of platinum or tungsten with a diameter of 4–10 μm, and length 0.01 m, as illustrated in figure 8.13.

The hot film anemometer used in NUIG is of the constant temperature type, manufactured by TSI Incorporated (USA) and was purchased from BIRAL in Bristol, England. It operates at a temperature of approximately 450 °C. The mechanism works by feeding the film an adjustable current in order to maintain its constant temperature. The fluid velocity is fundamentally a function of the input current and flow temperature. However, the latter can be measured during the testing, leaving the current as the only variable of interest.
As the heat from the film is lost to the surroundings, the increase in current needed to maintain constant temperature is recorded and, by Ohm's Law, the dynamic voltage is obtained and output by the anemometer in analogue form to the A/D card. The system is very useful as it may be used to obtain the along-wind power spectral density, turbulence intensity and integral length scales of turbulence.

**A/D converter and accompanying software**

All analogue signals created using the pitot tube manometer and hot film anemometer are received by an A/D converter card embedded at the back of the personal computer. The card used for this A/D conversion is the DASH-16 multi-function expansion board manufactured by the Keithley MetraByte Corporation (USA) and contains sixteen single ended channels and eight differential channels. Data from the digital channels may be converted into metric measurements by employing a known gauge factor and output to a computer screen using a computer software package or ASCII file. The former will now be discussed. A dedicated software package was written in visual basic by the researchers in NUIG and has been used for the purpose of this testing. The software entitled ‘Wind Tunnel Console’ was capable of taking the digital signals from the instruments and converting them into metric measurements output to the screen. The program has six separate algorithms available to the user, categorised as:
- Motor for mechanical arm
- Wind velocity measurement
- Hot-wire calibration
- Turbulence analyser
- Mean wind velocity profile with elevation
- Pressure measurements

The algorithm controlling the motor of the mechanical arm was used to move the mechanical arm holding the pitot tube or hot wire probe to a user defined three-dimensional spatial point. The desired co-ordinates and the speed of the motor are input by the user, and the algorithm moves the arm accordingly. The wind velocity algorithm displayed the calibrated digital output from the pitot tube and manometer. The user selected a scanning frequency (one or two seconds) and the channel being read on the A/D card. Figure 8.14 presents a screen shot taken during a test showing a mean wind velocity of approximately 5 ms$^{-1}$.

![Screen shot of manometer recording mean wind velocity](image)

Figure 8.14 Screen shot of manometer recording mean wind velocity

The Wind Tunnel Console program also has an inbuilt algorithm to calibrate the hot film anemometer, which must be done often especially if there is a lot of dust present in the wind tunnel. The calibration process is based on King's Law, which is given by the equation

$$E^2 = A + Bu^n$$  \hspace{1cm} (8.1)
where $E$ is the voltage across the hot film, $A$ and $B$ are constants and $n$ is a further constant of value 0.45 and $u$ is the fluid velocity normal to the wire. Constant $A$ is obtainable by measuring the voltage across the wire in zero flow conditions ($u = 0 \text{ m s}^{-1}$) and $B$ may then be found for a known wind velocity.

Once the hot film anemometer has been calibrated, the turbulence analyser algorithm may be used. The sampling frequency and time duration are selected and a wind velocity profile is measured which, having been converted to a calibrated digital form is stored by the program. The variance (mean square) of the wind velocity time-history is obtained, along with the mean wind velocity. Thus, the turbulence intensity is available. As the mean wind velocity is known, it may be extracted from the measured wind velocity, leaving the fluctuating component. A Fourier transform of this fluctuating component transfers it into the frequency domain and yields the smoothed power spectral density. The program fits the Von-kármán spectrum to the observed spectrum by best fit, and as the turbulence intensity is known, the integral length scale of turbulence is obtainable.

The program also possess the capability of storing a series of hot film measured wind velocity time-histories which can be used to obtain a mean wind velocity profile with elevation. The user can take a number of time-histories at increasing elevations, and the program will fit a best fit power law to the mean wind velocities, and output the power law exponent.

The last option in the program is concerned with measuring the pressure induced by the wind flow in the test section. A pressure transducer interfaced with the A/D card may be sequentially connected to up to forty-eight pressure taps placed at strategic points within the wind flow, scanning the resultant pressures at a user defined frequency and outputting the pressures for use in deriving pressure coefficients.

**STRUCTURAL PROPERTIES**

The data acquisition system used to record the structural response of the model was brought to the wind tunnel from the Department of Civil Engineering at Trinity College Dublin. This data acquisition system was the same as that used in Chapter 7. The hardware consisted of the System 6000 A/D converter, manufactured by the Vishay Measurements Group (USA) and accompanying a personal computer, and the complete set-up is shown in
figure 8.15. The System 6000 has up to twelve channel dedicated to strain gauges, four of which were used in the wind tunnel experiments. It also has two high level channels available to accelerometers, one of which was utilised for the experiments. The System 6000 has a scanning rate of up to 10,000 Hz. The Strainsmart software also created by the Vishay Measurements Group was used to log the structural response data, and output it in ASCII form for later analysis.

Figure 8.15 Data acquisition equipment for response of model

8.3 RECORDED WIND PROPERTIES
As both the spires and the roughness elements were used in the test section, they created specific wind conditions that must be measured. The mean wind velocities at specific elevations above the ground along with the corresponding power spectral densities, turbulence intensities and integral length scales of turbulence were obtained.

8.3.1 Mean wind velocity profile
The mean wind velocity profile was obtained in the wind tunnel using the hot film anemometer using a scanning frequency of 5000 Hz for a duration of 50 seconds. Three tests were carried out at elevations of 0.04 m, 0.08 m, 0.1 m, 0.15 m, 0.2 m, 0.25 m, 0.3 m,

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0.4 m and 0.5 m above the test section floor, and the average taken. A power law was then used to fit this profile, as is given the equation

\[
\bar{v}(z) = \bar{v}(z_{0.5}) \left( \frac{z}{z_{0.5}} \right)^{\alpha}
\]  

(8.2)

where \( \bar{v} \) denotes mean wind velocity, \( z \) denotes elevation above the ground, subscript of 0.5 denotes an elevation of 0.5 m, and \( \alpha \) denotes the power law exponent. The mean wind velocity profile exponent was obtained by fitting equation 8.2 to the observed data using the least squares method, and was found to be 0.253. Figure 8.16 shows the wind velocity power law profile in red.

![Figure 8.16 Mean wind velocity profile recorded in the wind tunnel](image)

8.3.2 Turbulence parameters

The hot film anemometer was used to obtain the properties associated with the fluctuating component of a wind flow, namely the turbulence intensity, power spectral densities and longitudinal integral length scales of turbulence. First, the turbulence intensity as a function of elevation, \( I(z) \), was found as the ratio of the standard deviation of the longitudinal component (u direction) of the wind velocity, \( \sigma_u(z) \), to the mean wind velocity, \( \bar{v}(z) \), and is given by the equation
This is the simplest method used to measure the amount of turbulence present at a specific elevation. Figure 8.17 illustrates the distribution of turbulence intensity values as a function of elevation. The values obtained were the average of three tests, run at a sampling frequency of 5000 Hz for a time duration of 50 seconds. From figure 8.17, it is evident that greater turbulence exists closer to the wind tunnel floor, particularly due to the prevalence of larger Reynolds stresses in this region, caused by the friction from the roughness elements and the smaller eddies shed from the turbulence spires.

\[ I(z) = \frac{\sigma_v(z)}{\bar{v}(z)} \]  

(8.3)

Once the mean component has been separated from the measured wind velocity time-history, a Fourier transform of the zero mean fluctuating component gives the Fourier Amplitude of the energy contained within the fluctuating component as a function of frequency. As the ensemble of auto-correlation functions contains only one entry, spectral smoothing is employed to fit the power spectral density function to the Fourier Amplitude curve. The Von-kàrman wind velocity spectrum was then fitted to the observed spectrum using a non-linear curve fitting approach. The Von-kàrman wind velocity spectrum, \( S_\nu(f) \), where \( L_\nu \) is the longitudinal integral length scale of turbulence, is given by the equation.

---

Figure 8.17 Turbulence intensity profile with elevation

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\[ S_{vv}(f) = \frac{4I(z)^2L^x_z
abla(z)}{\left[1 + 70.8 \left(\frac{fL^x_z}{\nabla(z)}\right)^2\right]^{5.6}} \]  

(8.4)

It is evident from equation 8.4, that the wind velocity spectrum is a function of three main variables. Both the turbulence intensity and mean wind velocity are known, leaving the integral length scale of turbulence as the only remaining unknown. The latter can be extracted from equation 8.4, once the Von-kármán spectrum has been fitted to the observed spectrum. Using this approach, table 8.1 was created, showing the obtained integral length scales of turbulence for spectra at different elevations. These integral length scales of turbulence represent the average physical size of eddies observed during the testing.

<table>
<thead>
<tr>
<th>Elevation (m)</th>
<th>Integral length scale of turbulence (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.279</td>
</tr>
<tr>
<td>0.4</td>
<td>0.348</td>
</tr>
<tr>
<td>0.3</td>
<td>0.328</td>
</tr>
<tr>
<td>0.25</td>
<td>0.32</td>
</tr>
<tr>
<td>0.2</td>
<td>0.272</td>
</tr>
<tr>
<td>0.15</td>
<td>0.325</td>
</tr>
<tr>
<td>0.1</td>
<td>0.331</td>
</tr>
<tr>
<td>0.08</td>
<td>0.334</td>
</tr>
<tr>
<td>0.04</td>
<td>0.228</td>
</tr>
</tbody>
</table>

Table 8.1 Integral length scales of turbulence at elevation

The fitted power spectral densities at each elevation are presented in figures 8.18 to 8.26.

Figure 8.18 Von-kármán fluctuating wind velocity spectrum at \( z = 0.5 \) m

Figure 8.19 Von-kármán fluctuating wind velocity spectrum at \( z = 0.4 \) m
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Figure 8.20 Von-kármán fluctuating wind velocity spectrum at \( z = 0.3 \) m

Figure 8.21 Von-kármán fluctuating wind velocity spectrum at \( z = 0.25 \) m

Figure 8.22 Von-kármán fluctuating wind velocity spectrum at \( z = 0.2 \) m

Figure 8.23 Von-kármán fluctuating wind velocity spectrum at \( z = 0.15 \) m

Figure 8.24 Von-kármán fluctuating wind velocity spectrum at \( z = 0.10 \) m

Figure 8.25 Von-kármán fluctuating wind velocity spectrum at \( z = 0.08 \) m

Figure 8.26 Von-kármán fluctuating wind velocity spectrum at \( z = 0.04 \) m
8.4 EXPERIMENTAL RESPONSE OF THE MODEL

Five forced vibrations tests were carried out in the wind tunnel, the first three having stationary blades but with different orientations, and the other two having blades rotating at 1.57 rads\(^{-1}\) and 3.14 rads\(^{-1}\). The non-rotating blade cases are termed as positions, A, B and C, having blade orientations illustrated in figures 7.13, 7.14 and 7.15. For all tests, the mean components of the recorded strain time-histories were removed, leaving the fluctuating component, as represented by random theory.

TOWER, NACELLE AND NON-ROTATING BLADES

**Position A**

The blade orientation of position A is illustrated by figure 7.13 and is characterised by one of the blades in the upward vertical position. Figures 8.27 to 8.36 illustrate the magnitudes of the fluctuating component of the strains and accelerations measured in the test over a duration of ten seconds. The maximum fluctuating strains observed by the strain gauges located at the mid-points were 85 micro-strain (\(\mu\varepsilon\)) at the windward face, as in figure 8.27, and 118 \(\mu\varepsilon\) at the leeward face, as presented in figure 8.29. Figures 8.28 and 8.30 represent the frequency domain representations of these strain time-histories. In both figures, the first and second modes of vibration are observed to contribute to the strain response. Figures 8.31 and 8.33 illustrate the fluctuating strain time-histories recorded during the same ten second interval at the windward and leeward gauges positioned at the base of the tower. The maximum windward fluctuating strain observed was 120 \(\mu\varepsilon\), as illustrated in figure 8.31, along with the corresponding maximum leeward fluctuating strain of 219 \(\mu\varepsilon\),

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure8.27.png}
\caption{Fluctuating strain time-history observed at windward mid-point}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{figure8.28.png}
\caption{Fourier transform of strain at windward mid-point}
\end{figure}
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Figure 8.29 Fluctuating strain time-history observed at leeward mid-point

Figure 8.30 Fourier transform of strain at leeward mid-point

Figure 8.31 Fluctuating strain time-history observed at windward base-point

Figure 8.32 Fourier transform of strain at windward base-point

Figure 8.33 Fluctuating strain time-history observed at leeward base-point

Figure 8.34 Fourier transform of strain at leeward base-point
as in figure 8.33. Figures 8.32 and 8.34 contain the frequency domain information for the fluctuating strains at the windward and leeward base-points, showing contributions from the first two modes of vibration. Figure 8.35 presents the acceleration response time-history at the top of the tower over the ten second period, yielding a maximum acceleration of 0.0824 g, or 0.808 ms\(^2\). Figure 8.36 is a Fourier transform of this acceleration response at the top of the tower. Energy is primarily concentrated around the first two modes of vibration, with a greater contribution from the fundamental mode. In the Fourier transform plots of the strain and acceleration responses, the first and second modes of vibration were found at frequencies close to those obtained from the free vibration tests presented in Section 7.4.1 of Chapter 7.

**Position B**

Position B is denoted by a blade orientation illustrated by figure 7.14, and is characterised by one of the blades being fully horizontal. The forced vibration results obtained are again presented in both the time and frequency domains. Figures 8.37 to 8.46 illustrate the magnitude of the fluctuating strains and accelerations obtained from the test. Figures 8.37 and 8.39 illustrate the windward and leeward tower mid-point strain time-histories observed over ten seconds, showing a maximum windward fluctuating strain of 91 \(\mu\)e at the windward face and 131 \(\mu\)e on the leeward side. Figures 8.38 and 8.40 illustrate the energy distribution with frequency for the strain time-histories at the windward and leeward faces respectively. Both figures show a similar distribution of energy, in particular around the first and second modes of vibration.
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Figure 8.37 Fluctuating strain time-history observed at windward mid-point

Figure 8.38 Fourier transform of strain at windward mid-point

Figure 8.39 Fluctuating strain time-history observed at leeward mid-point

Figure 8.40 Fourier transform of strain at leeward mid-point

Figure 8.41 Fluctuating strain time-history observed at windward base-point

Figure 8.42 Fourier transform of strain at windward base-point
Figures 8.41 and 8.43 present ten seconds of the fluctuating strain time-histories obtained at the windward and leeward faces at the base of the tower. The maximum windward fluctuating strain observed was 134 με, with a corresponding maximum leeward fluctuating strain of 225 με. Frequency domain representations of these figures can be viewed in figures 8.42 and 8.44, both showing contributions to the strain response from the first two modes of vibration. Figure 8.45 illustrates ten seconds of the acceleration time-history obtained at the top of the tower. A maximum acceleration value of 0.082 g was observed, which translates to an acceleration of 0.80 ms⁻¹. A Fourier transform of this time-history was taken and is presented in figure 8.46. It shows that the energy is concentrated in the first two modes of vibration, and in particular the energy concentrated around the fundamental mode is higher than that around the second mode. From the
Fourier transform plots presented in this section, the first and second modal frequencies appear identical to those obtained from a free vibration test of this assembly, detailed in Section 7.4.1 of Chapter 7.

**Position C**

Non-rotating blade position C is characterised by a blade orientation illustrated in figure 7.24, and possesses one blade in the vertical downward position. Time and frequency domain representations of all forced vibration responses are presented. Figures 8.47 to 8.56 illustrate the magnitude of the strains and accelerations obtained from this test. Figures 8.47 and 8.49 demonstrate the fluctuating strain time-histories obtained over a ten second period at the windward and leeward mid-point on the tower respectively. A maximum fluctuating strain of $77 \, \mu e$ was observed at the windward face, with a maximum leeward fluctuating strain of $113 \, \mu e$.

Figures 8.51 and 8.53 demonstrate ten seconds of the base-point fluctuating strain time-histories observed at the windward and leeward faces of the tower, respectively. A maximum windward fluctuating strain of $118 \, \mu e$ was reordered, along with a maximum leeward fluctuating strain of $131 \, \mu e$. Corresponding frequency domain representations for the windward and leeward cases at the mid point and base point are found in figures 8.48, 8.50, 8.52 and 8.54. All figures show that the first two modes of vibration are primarily contributing to the overall strain response.
Figure 8.49 Fluctuating strain time-history observed at leeward mid-point

Figure 8.50 Fourier transform of strain at leeward mid-point

Figure 8.51 Fluctuating strain time-history observed at windward base-point

Figure 8.52 Fourier transform of strain at windward base-point

Figure 8.53 Fluctuating strain time-history observed at leeward base-point

Figure 8.54 Fourier transform of strain at leeward base-point
Figure 8.55 presents ten seconds of the observed acceleration time-history at the top of the tower. A maximum value of 0.0897 g, or 0.879 ms\(^{-1}\) was recorded. A Fourier transform of figure 8.55 is presented in figure 8.56, showing the energy concentration of the tower response as a function of frequency of oscillation, with concentration around the first two modes. However, in figure 8.56, the second peak, corresponding to the second mode, appears to be comparable to the first peak due to the fundamental mode. This is physically not consistent with the linear behaviour of the tower previously observed. Nevertheless, the participation of the first two modes of vibration of the tower-blade assembly is still emphasised through the observation. The first and second modal frequencies observed in the Fourier transform plots presented in this section seem to be very close to those identified in the free vibration tests described in Section 7.4.1 in Chapter 7.

**TOWER, NACELLE AND ROTATING BLADES**

**Blade rotation of 1.57 rads\(^{-1}\)**

Forced vibration tests were carried out with the three blades rotating at approximately 1.57 rads\(^{-1}\), or 15 complete revolutions of each blade per minute. A power regulator was connected to the motor and the voltage input to the motor was altered until the desired rotational frequency was obtained. Figures 8.57 to 8.66 illustrate the strain and acceleration results obtained, presented in both the time and frequency domains. Figures 8.57 and 8.59 demonstrate ten seconds of the fluctuating strain time-history recorded at the windward and leeward faces of the mid-point of the tower. A maximum windward fluctuating strain of 85 με and a maximum leeward fluctuating strain of 103 με were observed. Figures 8.61
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Figure 8.57 Fluctuating strain time-history observed at windward mid-point

Figure 8.58 Fourier transform of strain at windward mid-point

Figure 8.59 Fluctuating strain time-history observed at leeward mid-point

Figure 8.60 Fourier transform of strain at leeward mid-point

Figure 8.61 Fluctuating strain time-history observed at windward base-point

Figure 8.62 Fourier transform of strain at windward base-point
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Figure 8.63 Fluctuating strain time-history observed at leeward base-point

Figure 8.64 Fourier transform of strain at leeward base-point

Figure 8.65 Acceleration time-history observed at the top of the tower

Figure 8.66 Fourier transform of acceleration at the top of the tower

and 8.63 demonstrate the time-history of fluctuating strain at the windward and leeward base-points over a ten second period, respectively. The maximum windward fluctuating strain at the base was 130 µε, while the corresponding maximum leeward fluctuating strain was 190 µε. Fourier domain representations of the windward and leeward mid and base-points are included as figures 8.58, 8.60, 8.62 and 8.64, and all show that the strain response is predominantly composed of contributions from the first two modes of vibration, as also observed earlier. Figure 8.65 represents ten seconds of the recorded acceleration time-history at the top of the tower. A maximum acceleration value of 0.123 g or 1.207 ms\(^{-1}\) was observed. Figure 8.66 is the frequency domain representation of figure 8.66, showing contributions of the first two modes to the acceleration response. The peak corresponding to the second mode is slightly higher than that corresponding to the first
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mode, which seems to be at odds with the linear behaviour of the tower-blades model assumed in this study. The first and second modal frequencies observed in the Fourier transform plots in this section show close correlation with those observed during the free vibration testing, described in Section 7.4.1 in Chapter 7.

**Blade rotation of 3.14 rads\(^{-1}\)**

Forced vibration tests were also carried out with the three blades rotating at approximately 3.14 rads\(^{-1}\), (30 complete revolutions of each blade per minute). The voltage input to the motor from the power regulator was increased from the previous case, having the effect of increasing the rotational frequency of the motor. Figures 8.67 to 8.76 illustrate the strain and acceleration results obtained, including time and frequency domain representations. Figures 8.67 and 8.69 present fluctuating strain time-histories observed at the windward and leeward faces at the mid-point of the tower, over a ten second duration, respectively.
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Figure 8.71 Fluctuating strain time-history observed at windward base-point

Figure 8.72 Fourier transform of strain at windward base-point

Figure 8.73 Fluctuating strain time-history observed at leeward base-point

Figure 8.74 Fourier transform of strain at leeward base-point

Figure 8.75 Acceleration time-history observed at the top of the tower

Figure 8.76 Fourier transform of acceleration at the top of the tower

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A maximum windward fluctuating strain of 80 $\mu$ε was observed along with a maximum leeward strain of 113 $\mu$ε. Frequency domain representations of the windward and leeward mid-point strains are illustrated in figures 8.68 and 8.70. Both plots show that the first two modes of vibration are primarily contributing to the strain response. Figures 8.71 and 8.73 demonstrate the fluctuating strain time-history obtained for the ten second interval at the windward and leeward faces at the base of the tower. A maximum windward strain of 131 $\mu$ε was recorded at the base, with a corresponding maximum leeward strain recorded of 188 $\mu$ε. Figures 8.72 and 8.74 represent the energy/frequency distributions for fluctuating strain at the windward and leeward base-points respectively. From both figures, it is evident that the first two modes of vibration show the most participation in the strain response.

Figure 8.75 shows ten seconds of the recorded acceleration time-history response at the top of the tower. From this plot, the maximum observed acceleration was 0.29 g, or 2.87 ms$^{-1}$. A Fourier transform of the data presented in figure 8.75 is illustrated in figure 8.76. In this figure, the first two modes are seen to contribute to the response, thought the second peak corresponding to the second mode appears to be greater than the first peak, as observed earlier and apparently inconsistent with linear behaviour of the model. The first and second modal frequencies observable from the Fourier transform plots of the measured responses appear very close to those identified during the free vibration tests, detailed in Section 7.4.1 of Chapter 7.

**DISCUSSION ON FORCED VIBRATION RESULTS**

The magnitudes of the forced vibration results presented in the previous sections are first compared for the non-rotating blades in positions A, B, and C. It is evident that the orientation of the three rotating blades relative to the tower has a bearing on the magnitude of both the acceleration response and strain response of the tower. The case that brought about the greatest observed acceleration response was position C, in which one of the blades was in a downward vertical position, though the differences between the three cases were marginal. The case which caused the biggest observed fluctuating strains was position B. At the windward faces of the mid-point and the base-point of the tower, position B recorded fluctuating strains of up to 11 % greater than positions A and C, while the maximum leeward fluctuating strain of position B was marginally higher at the mid and base-point compared to position A and C.
When the three blades are set in motion, the fluctuating strains at both faces and at both positions correlate closely with those observed in the non-rotating cases, indicating that the rotating action of the blades does not greatly affect the magnitude of the fluctuating strains created. However, the measurements recorded by the accelerometer increased dramatically, by up to 50 % for rotation at 1.57 rads⁻¹ compared to position B, and up to 142 % for rotation at 3.14 rads⁻¹ compared to position B. As this increase is considerable relative to the previous four cases of acceleration, and is not reflected in the strain response readings, it is thought to be the result of experimental error.

8.5 THEORETICAL RESPONSE OF THE MODEL

8.5.1 Mathematical derivation of response

The acceleration and fluctuating strain responses observed experimentally are compared with those estimated using a mathematical model of the wind turbine tower. The tower is modelled as a two node, two degree-of-freedom system, as illustrated in figure 8.77. An approach based on the input-output relationship for a linear system will be used to estimate the responses of the structure. Thus, the input (wind loading) and transfer function are needed. The transfer functions obtained from the system identification testing, as presented in Chapter 7, Section 7.6, will be used in this regard.

![Figure 8.77](image)

Figure 8.77 Two node representation of wind turbine tower model tested in the wind tunnel

The coupled model (CS) has a mode shape matrix, \([\Phi_{CS}]\), as
[\Phi_{cS}] = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \quad (8.5)

where \( \phi_{11} \) and \( \phi_{12} \) are the first, and \( \phi_{21} \) and \( \phi_{22} \) are the second mode shape components, at nodes 1 and 2 respectively. The transfer function ordinates for the displacement of the tip of the tower due to the first and second modes of vibration, termed \( H_{11}(f) \) and \( H_{12}(f) \), are expressed as

\[ H_{11}(f) = \phi_{11} \eta_{cS,1}(f) \quad (8.6) \]

and

\[ H_{12}(f) = \phi_{12} \eta_{cS,2}(f) \quad (8.7) \]

where \( \eta_{cS,1}(f) \) and \( \eta_{cS,2}(f) \) are the first and second modal co-ordinates. These are given by

\[ \eta_{cS,1}(f) = \left( \frac{F_{cS,1}}{4\pi^2 \bar{M}_{cS,1}(1 - r_1^2 + 2\xi_{cS,1}r_1)} \right) \quad (8.8) \]

and

\[ \eta_{cS,2}(f) = \left( \frac{F_{cS,2}}{4\pi^2 \bar{M}_{cS,2}(1 - r_2^2 + 2\xi_{cS,2}r_2)} \right) \quad (8.9) \]

where \( F_{cS,1} \) and \( F_{cS,2} \) are the first and second unit modal forces, \( f_{cS,1} \) and \( f_{cS,2} \) are the first and second natural frequencies, \( \bar{M}_{cS,1} \) and \( \bar{M}_{cS,2} \) are the first and second modal masses of this coupled system, \( \xi_{cS,1} \) and \( \xi_{cS,2} \) are the first and second modal damping ratios, and

\[ r_1 = \frac{f}{f_{cS,1}} \quad (8.10) \]

and

\[ r_2 = \frac{f}{f_{cS,2}} \quad (8.11) \]

The unit modal forces are obtained from the expressions.
The mode shapes of each assembly used with equation 8.12 are given in Table 7.2. The total nodal drag force, \( F_D(t) \), experienced by the model immersed within the wind flow is made up of a mean component, \( \bar{F}_D \), and a fluctuating component, \( f_D'(t) \), as

\[
F_D(t) = \bar{F}_D + f_D'(t)
\]

(8.13)

However, as the aim of this section is to theoretically validate the random theory representation of the experimental response of the assemblies, the fluctuating component of the total drag force is only of interest. The modal drag force PSDF, associated with the first mode of vibration, \( S_{MF1MF1} \), is expressed as

\[
S_{MF1MF1}(f) = (C_D A_T \rho)^2 \sum_{k=1}^{2} \sum_{l=1}^{2} \sqrt{S_{VIVI}(f)S_{VkVk}(f)\bar{v}_k \bar{v}_l \phi_1(k)\phi_1(l)}
\]

(8.14)

where \( C_D \) denotes the drag coefficient, \( A_T \) is the total area of the structure, \( k \) and \( l \) are either nodes 1 or 2, \( S_{VIVI} \) is the auto PSDF at node 1, \( S_{VkVk} \) is the auto PSDF at node \( k \), \( \bar{v}_k \) and \( \bar{v}_l \) are the mean wind velocities at nodes \( k \) and 1, and \( \phi_1(k) \) and \( \phi_1(l) \) are the first mode shape components at nodes \( k \) and 1. The second modal drag force PSDF, \( S_{MF2MF2} \), may be expressed in a similar fashion as

\[
S_{MF2MF2}(f) = (C_D A_T \rho)^2 \sum_{k=1}^{2} \sum_{l=1}^{2} \sqrt{S_{VIVI}(f)S_{VkVk}(f)\bar{v}_k \bar{v}_l \phi_2(k)\phi_2(l)}
\]

(8.15)

where \( \phi_2(k) \) and \( \phi_2(l) \) are the second mode shape components at nodes \( k \) and 1.

Fluctuating modal drag force time histories, for the first and second modes were simulated by the discrete Fourier transform technique discussed in Chapter 5, and these are termed \( f_{D,M1}'(t) \) and \( f_{D,M2}'(t) \) respectively. The fluctuating wind velocity PSDFs are modelled using the Von-kàrman spectrum given in equation 8.4. A Fourier transform of the simulated first and second fluctuating modal drag force time-histories is taken, giving \( \hat{f}_{D,M1}() \) and \( \hat{f}_{D,M2}() \). The first and second modal fluctuating displacement response
Fourier spectra, $\hat{D}_{m1}(\omega)$ and $\hat{D}_{m2}(\omega)$ are obtained as the product of the modal force Fourier spectra and the relevant transfer functions, as

$$\hat{D}_{m1}(\omega) = H_{11}(\omega)\hat{f}_{D,M1}(\omega) \quad (8.16)$$

and

$$\hat{D}_{m2}(\omega) = H_{12}(\omega)\hat{f}_{D,M2}(\omega) \quad (8.17)$$

where $\omega$ is circular frequency. The fluctuating first and second modal acceleration Fourier Spectra, $\hat{A}_{m1}(\omega)$ and $\hat{A}_{m2}(\omega)$ may be found as

$$\hat{A}_{m1}(\omega) = -\omega^2\hat{D}_{m1}(\omega) \quad (8.18)$$

and

$$\hat{A}_{m2}(\omega) = -\omega^2\hat{D}_{m2}(\omega) \quad (8.19)$$

An inverse Fourier transform of the first and second modal acceleration Fourier spectra gives the first and second modal fluctuating acceleration time-histories, and adding the products of these and their mode shape components yields the total fluctuating acceleration time-history at the top of the tower, $\{\ddot{a}_1(t)\}$. The fluctuating acceleration time-history at node 2, $\{\ddot{a}_2(t)\}$, may be found using equations 8.18 and 8.19 and their corresponding mode shape components. The fluctuating dynamic strains at the windward mid-point and base-points may be obtained from the moment curvature relationship. The fluctuating strain, $\varepsilon'(t)$, is obtained from the expression

$$\varepsilon'(t) = \frac{M'(t)y}{EI} \quad (8.20)$$

where $M'(t)$ is the fluctuating bending moment, $y$ is the distance from the strain point to the neutral axis, $E$ is the elastic modulus of the material and $I$ is the second moment of area of the tower. The fluctuating bending moment at the mid point of the tower is equal to the product of the fluctuating acceleration time-history at node 1, $\{\ddot{a}_1(t)\}$, by the mass at node 1, $m_1$, and the distance between node 1 and the midpoint, $(H/2)$, as
The fluctuating bending moment at the base of the tower, $M_b'(t)$, is obtained as the product of the mass at nodes 1 and 2, by their respective nodal acceleration time-histories, and their corresponding lever arms, as

$$\{M_b'(t)\} = m_1 \{\ddot{a}_1(t)\} (H/2) \quad (8.21)$$

where $m_1$ is the mass at node 1 and $\{\ddot{a}_1(t)\}$ is the fluctuating acceleration time-history at node 1.

$8.5.2$ Simulated theoretical results

This section presents the simulated acceleration and fluctuating strain responses of the scale model wind turbine tower, using the mathematical approach described in section 8.5.1. For the three non-rotating and the two rotating blade cases, the acceleration time-histories at the top of the tower, and the fluctuating strain-histories at the windward mid and base points are derived. The drag coefficient assumed for the tower was 1.2.

$TOWER, NACELLE AND NON-ROTATING BLADES$

Position A

Figures 8.78 to 8.83 illustrates the response time-histories and Fourier spectra simulated for the model in non-rotating blade position A. Figure 8.78 illustrates the windward fluctuating strain time-history simulated over a ten second period, showing a maximum of 72 $\mu$e. Figure 8.79 represents the Fourier transform of the fluctuating strain time-history, and shows that two predominant frequencies contribute to the strain response. These peaks correspond to the first and second natural frequencies of vibration. Figure 8.80 shows the windward base-point fluctuating strain time-history simulated over a ten second period, and shows a maximum fluctuating strain value of 141 $\mu$e. Figure 8.81 shows the fluctuating base strain time-history in a frequency domain representation, showing that the fundamental mode of vibration primarily contributes to the strain response, with a much smaller contribution for the second mode. The simulated acceleration response time-history at the top of the tower is shown in figure 8.82, and shows a maximum acceleration value of 0.098 g, or 0.96 ms$^{-1}$. A Fourier transform of this time-history, illustrated in figure
WIND TUNNEL TESTING AND VALIDATION

Figure 8.78 Fluctuating strain time-history simulated at windward mid-point

Figure 8.79 Fourier transform of fluctuating strain at windward mid-point

Figure 8.80 Fluctuating strain time-history simulated at windward base-point

Figure 8.81 Fourier transform of fluctuating strain at windward base-point

Figure 8.82 Acceleration time-history simulated at top of the tower

Figure 8.83 Fourier transform of acceleration at the top of the tower
8.83, shows peaks close to the first and second natural frequencies.

**Position B**

Figures 8.84 to 8.89 show the response time-histories and Fourier spectra simulated for non-rotating blades position B. Figure 8.84 illustrates the fluctuating strain time-history simulated at the windward mid-point of the tower. A maximum fluctuating strain value of 74 $\mu$e was calculated. The Fourier transform of this time-history illustrated in figure 8.85 shows two peaks in the output energy, corresponding to the first and second modes of vibration. Figure 8.86 illustrates the simulated base-point fluctuating strain time-history at the windward face, yielding a maximum strain value of 147 $\mu$e. Figure 8.87 presents the Fourier transform of this strain time-history, showing that the majority of the response energy is output through the first mode.
The simulated acceleration response time-history over a ten second period is presented in figure 8.88, and shows a maximum value of 0.101 g, or 0.99 ms\(^{-1}\). Figure 8.89 represents a Fourier transform of this acceleration time-history, and shows that two predominant frequencies contribute to the acceleration response, corresponding to the first and second natural frequencies.

**Position C**

Figures 8.90 to 8.95 illustrate the response time-histories and Fourier spectra obtained from the mathematical model for non-rotating blades position C. Figure 8.90 shows the simulated fluctuating strain at the windward midpoint of the tower, with a peak strain value of 62 με. A Fourier transform of this strain time-history is shown in figure 8.91, and shows two principal frequencies contributing to the fluctuating strain response, the first and second natural frequencies. Figure 8.92 shows the fluctuating strain time-history simulated at the windward base-point, showing a maximum value of 109 με. Figure 8.93 is a Fourier transform plot of the fluctuating strain at the base, and shows concentration of energy about the fundamental frequency of the system.

Figure 8.94 illustrates the acceleration time-history simulated at the top of the tower over a ten second period. A maximum acceleration value of 0.085 g, or 0.834 ms\(^{-1}\) is evident. A Fourier transform of this time-history was taken and is presented in figure 8.95. The distribution of energy appears to be maximum about the first and second natural frequencies.
WIND TUNNEL TESTING AND VALIDATION

Figure 8.90 Fluctuating strain time-history simulated at windward mid-point

Figure 8.91 Fourier transform of strain at windward mid-point

Figure 8.92 Fluctuating strain time-history simulated at windward base-point

Figure 8.93 Fourier transform of strain at windward base-point

Figure 8.94 Acceleration time-history simulated at top of tower

Figure 8.95 Fourier transform of acceleration at top of tower
Blades rotation of 1.57 rads⁻¹

Figures 8.96 to 8.101 illustrate the response time-histories and associated Fourier spectra for the model with blade rotation of 1.57 rads⁻¹. Figure 8.96 shows the estimated fluctuating strain time-history at the windward mid-point of the tower, showing a maximum strain of 70 με. A Fourier Transform of this time-history is presented in figure 8.97 and shows the energy to be concentrated around two peaks, corresponding to the first and second natural frequencies. The simulated base fluctuating strain at the windward side is presented in figure 8.98, and shows a peak of 129 με. Figure 8.99 is a Fourier amplitude plot of this time-history, and shows the model to be vibrating predominantly at the first natural frequency. Figure 8.100 shows the simulated acceleration time-history at the top of the tower, and shows a peak acceleration value of 0.096 g or 0.942 ms⁻¹.

Figure 8.96 Fluctuating strain time-history simulated at windward mid-point

Figure 8.97 Fourier transform of strain at windward mid-point

Figure 8.98 Fluctuating strain time-history simulated at windward base-point

Figure 8.99 Fourier transform of strain at windward base-point
A Fourier spectrum of this response is illustrated in figure 8.101, which shows a contribution from two specific frequencies to the acceleration response, corresponding to the first and second natural frequencies.

**Blade rotation of 3.14 rads⁻¹**

Figures 8.102 to 8.107 represent the simulated response time-histories and associated Fourier spectra for the blade rotation case of 3.14 rads⁻¹. Figure 8.102 shows the predicted fluctuating strain time-history at the windward mid-point of the tower, showing an observed maximum of 73 με. A corresponding Fourier transform of this plot is presented in figure 8.103, and shows a distinct contribution from the first two modes to the overall fluctuating strain response.
The simulated base-point windward fluctuating strain time-history may be viewed in figure 8.104, with a maximum of 131 με being evident. Figure 8.105 represents the Fourier spectrum of this time-history and shows the first mode to be predominantly contributing to the response. Figure 8.106 represents the simulated acceleration response at the top of the tower, and shows a maximum observable value of 0.077 g, or 0.755 ms⁻¹. A Fourier amplitude representation of this time-history is illustrated in figure 8.107, and shows that the first and second natural frequencies are mainly contributing to the acceleration response at the top of the tower.
8.6 COMPARISON OF EXPERIMENTAL AND THEORETICAL RESPONSES

The experimental and theoretical windward strain responses at the mid-point and the base-point, as well as the accelerations at the top of the tower, are now compared using their statistical moments and peak values. All five cases are included, the non-rotating blade positions of A, B and C, and the two blade rotation cases of 1.57 rads⁻¹ and 3.14 rads⁻¹. Table 8.2 presents the experimental and theoretical peak and root mean square (RMS) values for fluctuating strains and accelerations of the model in Position A.

<table>
<thead>
<tr>
<th>Position A</th>
<th>Experimental</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>RMS</td>
</tr>
<tr>
<td>Strain at mid-point</td>
<td>84.92</td>
<td>28.08</td>
</tr>
<tr>
<td>Strain at base-point</td>
<td>120.37</td>
<td>40.71</td>
</tr>
<tr>
<td>Acceleration at top (g)</td>
<td>0.082</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Table 8.2 Comparison between experimental and theoretical responses for Position A

The theoretical fluctuating peak strain at the mid-point is approximately 16 % lower than that observed experimentally. The corresponding theoretical RMS value of mid-point fluctuating strain was also underestimated, by a factor of 24 %. The theoretical model overestimated the peak fluctuating base strain by 15 %. The experimental and theoretical strain RMS values differ by a reduced factor of 11 %. The experimental and theoretical peak accelerations differ by approximately 16 %, with the theoretical model overestimating the value. The theoretical acceleration RMS was also overestimated by the model, by almost 24 %. Table 8.3 presents the experimental and theoretical mean and RMS values for the strains and acceleration of the non-rotating model in position B. In this instance, the theoretical and experimental fluctuating peak strains at mid-point differ by a factor of 18 %, along with the corresponding RMS values that disagree by approximately 32 %.

<table>
<thead>
<tr>
<th>Position B</th>
<th>Experimental</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>RMS</td>
</tr>
<tr>
<td>Strain at mid-point</td>
<td>90.70</td>
<td>38.23</td>
</tr>
<tr>
<td>Strain at base-point</td>
<td>133.06</td>
<td>54.64</td>
</tr>
<tr>
<td>Acceleration at top (g)</td>
<td>0.082</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Table 8.3 Comparison between experimental and theoretical responses for Position B
The theoretical model overestimated the peak fluctuating strain at the base-point, by 10\%, as well as the corresponding RMS value by a factor of 8\%. The theoretical model then overestimated the acceleration at the top of the tower by a factor of 33\%. The theoretical RMS values were also overestimated by a factor of 19\%.

Table 8.4 shows the statistical moment and peak value comparison between experimental and theoretical strain and acceleration response for the non-rotating case at Position C. The theoretical fluctuating peak strain at the mid-point underestimates the experimental value by 20\%. The corresponding RMS values however appear close, differing by 17\%. The values presented in table 8.4 show a peak base-point experimental fluctuating strain 8\% greater than the theoretically predicted value. The theoretical model overestimated the base-point fluctuating strain RMS by 14\%. Contrary to Positions A and B, the theoretical model predicted peak and RMS acceleration values that were very close to those observed experimentally, differing by 4\% and 7\% respectively.

<table>
<thead>
<tr>
<th>Position C</th>
<th>Experimental</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain at mid-point</td>
<td>77.49</td>
<td>62.17</td>
</tr>
<tr>
<td>Strain at base-point</td>
<td>118.32</td>
<td>108.74</td>
</tr>
<tr>
<td>Acceleration at top (g)</td>
<td>0.089</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Table 8.4 Comparison between experimental and theoretical responses for Position C

Comparison results for the first of the rotating cases is presented in table 8.5. For a blade rotation of 1.57 rads$^{-1}$, the theoretical model underestimated the peak fluctuating strain at the mid-point by 17\%, as was the RMS value by a factor of 16\%.

<table>
<thead>
<tr>
<th>$\Omega = 1.57$ rads$^{-1}$</th>
<th>Experimental</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain at mid-point</td>
<td>84.66</td>
<td>70.45</td>
</tr>
<tr>
<td>Strain at base-point</td>
<td>129.66</td>
<td>128.95</td>
</tr>
<tr>
<td>Acceleration at top (g)</td>
<td>0.124</td>
<td>0.096</td>
</tr>
</tbody>
</table>

Table 8.5 Comparison between experimental and theoretical responses for blade rotation of 1.57 rads$^{-1}$
The theoretical model very accurately predicted the peak fluctuating base-point strain but overestimated the corresponding RMS value by 18%. The theoretical model also underestimated the peak acceleration at the top of the tower, by 23%, and the RMS value by a factor of 22%.

Table 8.6 compares the statistical moments for the experimental and theoretical models, with a blade rotation of 3.14 rads⁻¹. First, the theoretical model underestimates the peak fluctuating strain at the mid-point by 7%, and underestimates the RMS value by a factor of 11%. The peak values of the base-point fluctuating strain observed experimentally and predicted theoretically were almost identical. The corresponding RMS values showed less correlation, differing by approximately 17%. The experimental peak and RMS acceleration values observed were not indicative of the previous rotation and non-rotation cases, as they both appear over three times bigger. The large magnitude of these parameters was most probably the result of experimental error, as the accelerometer appears to have recorded erroneous values of acceleration.

<table>
<thead>
<tr>
<th>$\Omega = 3.14 \text{ rads}^{-1}$</th>
<th>Experimental</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Peak</td>
<td>RMS</td>
</tr>
<tr>
<td>Strain at mid-point</td>
<td>77.99</td>
<td>23.06</td>
</tr>
<tr>
<td>Strain at base-point</td>
<td>130.82</td>
<td>36.76</td>
</tr>
<tr>
<td>Acceleration at top (g)</td>
<td>0.294</td>
<td>0.117</td>
</tr>
</tbody>
</table>

Table 8.6 Comparison between experimental and theoretical strains for blade rotation of 3.14 rads⁻¹

The percentages differences between experimentally observed and theoretically predicted response parameters, as documented in this section, are mainly due to a number of necessary assumptions in derivation of the theoretical model, and due to the difficulty in accurately measuring certain parameters in the wind tunnel. In the theoretical model, it was assumed that the mean wind velocity acting on the blades was similar to that at the top of the tower, regardless of blade rotation. Also, a drag coefficient was adopted from the literature for use in the theoretical model, and because of the very low Reynolds number (~5000) observed in the wind tunnel, this drag coefficient had to be a lower end estimation of the literature values.
CHAPTER 9 – EXECUTIVE SUMMARY AND CONCLUSIONS

9.1 EXECUTIVE SUMMARY

The study examines both theoretically and experimentally, the free and forced vibrations of wind turbine tower assemblies. As these assemblies are invariably flexible, contain low levels of damping, and are subject to random dynamic wind loading, a dynamic analysis must be carried out in either the time or frequency domain to ultimately estimate the responses of the individual components of the wind turbine tower, as well as the tower and nacelle coupled to rotating blades, in order to ensure the serviceability and survivability of the structure during it’s design life. As a wind turbine tower is a flexible multi-body system composed of a number of continuous sub-systems, a dynamic analysis using a software based finite element model of a wind turbine tower would contain a huge number of degrees-of-freedom, making a forced vibration analysis computationally expensive. The object of the thesis was to create a series of reduced order models of the tower, the rotating blades, and the rotating blades coupled to tower, and using these models, investigate the free and forced vibration of each individual component, and of the coupled tower and blades structure. These reduced order models will allow for the investigation of multi-body behavioural phenomena such as modal coupling, loading phenomena such as spatial correlation of force, and atypical spectral energy distribution in air turbulence, and be more computationally efficient than a finite element based solution.

First, an analytical approach was proposed to obtain the free vibration parameters, i.e. natural frequencies and mode shapes, of a tower carrying a nacelle, rotating wind turbine blades, and three wind turbine blades connected to a tower. Two types of towers were investigated, a three-dimensional lattice tower, constructed from steel angle sections, and a hollow prismatic tubular tower. Three approaches were proposed to obtain the free vibration properties of the tubular tower, a discrete lumped parameter model, a distributed parameter continuous model and a finite element model. Each approach contained varying degrees of complexity, yielding approximately similar results. The lattice tower was accurately analysed using the discrete parameter and finite element approaches. The three approaches were also used to obtain the free vibration properties of the rotating blades. The discrete parameter and distributed parameter approaches were used in conjunction with prismatic rotating blades, each model having the capability of including centrifugal stiffening of the blade due to its rotation. The discrete parameter model also considered the effect of self weight on the overall stiffness of the blade. The finite element model was
used to analyse a tapering blade, with mass and stiffness varying along the length of the blade. However, this method could not include centrifugal stiffening or axial self weight effects due to rotation. A reduced order model investigating the free vibration behaviour of a tower coupled to three rotating blades was also proposed. This model initially investigated the free vibration properties of each of the sub-systems individually, and then generalised the motion of the tower and blades into a single degree-of-freedom each. Thus, the two degrees-of-freedom were joined together by way of compatibility, and the natural frequencies and mode shapes of the new two degree-of-freedom system could be extracted. Using this two degree-of-freedom approach, several behavioural effects were studied, including the case when the fundamental frequency of the blades are close to that of the tower and nacelle, and the effect of blade rotational frequency of the natural frequencies on the coupled system.

Next, the forced vibration response of all individual and coupled assemblies was investigated. This was carried out primarily in the time domain, and included systems with classical and non-classical damping. The latter is brought about with the inclusion of the aerodynamic damping that occurs due to the relative motion between the wind and the structure. For the classically damped individual systems, such as the rotating blades and tower and nacelle, the mode acceleration method is used to estimate the structural response due to random wind loading. The random wind loading acting on the tower and blades is simulated in the time domain using a discrete Fourier transform technique in conjunction with a velocity or force power spectral density function. The simulated loading acting along the length of the tower includes the possibility of incomplete correlation of forces due to the presence of turbulent eddies of high frequency or low spatial size. The loading simulated to act on the blades contains elevated energy levels at frequencies corresponding to integer products of the rotational frequency, known as rotational sampling. The response of the classically damped tower was also obtained using a numerically integrated superposition method, and when subject to the same loading, yielded an identical response to the mode acceleration method. The numerically integrated superposition method was subsequently used to obtain the response of the same tower subject to the same loading as before, but including aerodynamic damping in the response. The response at the top of a wind turbine tower, carrying a nacelle with three flexible rotating blades was also obtained in the time domain. The blades were subject to random wind loading and through their subsequent vibration, imparted a shear force from their bases into the top of the tower. This had the effect of amplifying the response of the tower, with itself being acted upon by
spatially correlated wind drag force. Thus, the response of the tower due to blade-tower dynamic interaction was compared with the conventional approach of lumping the entire mass of the rotor system with the mass of the nacelle, excluding blade-tower interaction.

The coupled two degree-of-freedom model was used to develop an extension of the gust response factor methodology for the design of flexibly structures in wind engineering. The traditional gust response factor approach assumes that a structure may be accurately modelled as a single degree-of-freedom system, but when a second mode is close to the fundamental mode of the system, as predicted by the coupled two degree-of-freedom model, it will contribute to the overall forced response of the system and affect the magnitude of the gust response factor. The approach proposed in this thesis includes this second mode in the formulation of the gust response factor, and suggests a closed form solution to find the two degree-of-freedom gust response factor.

In order to experimentally validate the use of random vibration theory in the analytical portion of the thesis, experimental testing was carried out in the vibrations laboratory in Trinity College Dublin and the wind tunnel in National University of Ireland, Galway. A scale model of a wind turbine tower was constructed, consisting of a plastic tubular tower, carrying a steel plate housing a motor and gearing system, and an aluminium hub with three flexible phenolic blades, in turn connected to the motor. The motor was capable of rotating the blades at an arbitrary rotational frequency. The model was fixed onto a custom built moveable bed mounted on ball bearings, which was connected to a mechanical actuator. A LVDT was attached to the moveable bed, measuring the displacement of the bed in real time. An accelerator was fixed to the top of the tower allowing the acceleration response of the model to be obtained accurately. A series of harmonic base motions of known frequency and amplitude were imparted into the moveable base by the actuator, which when used with the instruments, allowed the displacement response and transfer function of the scale model to be obtained experimentally.

The model was immersed within an artificially generated turbulent wind flow created in the wind tunnel, and the wind properties, such as the mean wind velocity profile with height, turbulence intensities, fluctuating wind velocity power spectral densities, and integral length scales of turbulence, were measured. Then the acceleration response at the top of the tower and the strain at the mid-point and base-point of the tower was recorded. The random vibration theory validation was accomplished by theoretically simulating the
acceleration response and the fluctuating strain responses using a numerical model. This model took into account the experimentally obtained transfer functions, and the wind parameters measured in the wind tunnel.

9.2 CONCLUSIONS

This section presents the conclusions that may be drawn based on the numerical modelling and experimental studies presented throughout this thesis. The free vibration properties of an arbitrary angle-sectioned lattice tower carrying a concentrated nacelle mass at the top may be evaluated accurately using either a discrete parameter formulation, or a finite element formulation. The discrete model contained a fraction of the degrees-of-freedom of the finite element model, but it very accurately estimated the fundamental frequency of the system. Good accuracy was observed for the second mode, and acceptable accuracy was observed for the third mode. Thus, to obtain the free vibration properties of a lattice tower, a discrete model of reduced order may be employed instead of a more complicated finite element software model with a modal analysis capability, as it is less computationally expensive and more cost effective.

For the prismatic tubular tower with nacelle mass, a discrete parameter approach, a finite element approach and a distributed parameter approach may all be used to accurately obtain the natural frequencies and mode shapes of the system. All three models showed close results for the fundamental frequency. For the second natural frequency, the finite element method appeared closest to the continuum model, with the discrete model also close. The finite element model also appeared closest to the continuum model for the third mode, while the discrete approach still showed good accuracy. Thus, the discrete parameter model may be used instead of the other two models as it is less computationally expensive, while still yielding accurate information about the first three modes.

The free vibration properties of prismatic rotating blades, rotating at any arbitrary rotational frequency, may be obtained using a discrete parameter approach or a distributed parameter approach. The latter can take centrifugal stiffening into account but does not however take the effect of the self weight of the blade into consideration as the blade changes position. The discrete approach includes both centrifugal stiffening and self weight effects, but the latter is not particularly significant for blades of low mass. Both approaches showed close agreement for both the first and second natural frequencies. The discrete method appears superior to the continuum method as it includes the effect of axial
self-weight, and represents a significantly simpler formulation. A software based finite element approach may be used to obtain the modal properties of a blade with tapering geometry. The proposed model used a beam element which allowed for an unequal distribution of mass and stiffness to be prescribed. There was a drawback in using this approach in that conventional finite element software codes do not have the ability to include centrifugal stiffening and axial self weight effects, so the blades may only be analysed when they are considered to be at rest. This approach, however, was considerably more efficient than creating a tapering blade using a conventional solid modelling approach.

It was evident that the first two natural frequencies of the coupled tower/nacelle and rotor blades model were coupled together when the fundamental frequencies of both the tower and nacelle and rotating blades were similar. When the fundamental frequencies of the tower/nacelle and rotating blades were further apart, the natural frequencies of the coupled system tended also to be far apart, with less influence on each other. The natural frequencies of the coupled system also began to move further apart as blade rotational frequency increased. This is because the bending stiffness of the rotor blades component of the coupled system increased as blade rotational frequency increased.

The response of any linear elastic wind turbine tower, with classical damping, modelled as a discrete lumped parameter system carrying a nacelle mass, subject to random wind loading may be obtained in the time domain using the mode acceleration technique. This technique allowed the response to be accurately estimated using only a small number of the initial modes of vibration, due to the presence of a pseudo-static component in the response. This approach, when used in conjunction with a discrete Fourier transform technique to simulate time-varying loading, considered spatial correlation of the wind forces along the length of the tower. The response of the linear wind turbine tower, with either classical or non-classical damping may also be evaluated using the numerical integration super-position technique, though this approach is more computationally expensive than the mode acceleration approach. In this technique, a solution of the forced vibration equation of motion was obtained through numerical integration and the response obtained for the classical damping case was identical to the response obtained using the mode acceleration case. When the response of the wind turbine tower and nacelle was estimated using the numerical integration technique, including aerodynamic damping, the response of the system decreased due to the presence of the aerodynamic damping.
The response of any linear rotating prismatic wind turbine blade with classical damping may be obtained using the mode acceleration technique. This response was obtained as a function of the rotational frequency of the blade, which had the effect of stiffening the blades as they rotated, and as a function of the rotationally sampled turbulence experienced by the blades undergoing rotating. The spatial position of the blades did not have a large bearing on their response, but the rotational frequency at which they rotated at did have a considerable effect. As the blade's rotational frequency increased, the blades became stiffer and exhibited a decreased displacement response when undergoing loading. The shear force at the base of a blade was also observed to reduce at higher rotational speeds.

When investigating the response of a coupled tower and blades system, the numerical examples showed that it is necessary to consider blade-tower interaction, or transmission of shear force from the blades to the tower, in order to accurately portray the true dynamic nature of the coupled entity. When blade-tower interaction was not considered, as when the mass of the blades was simply lumped together with the mass of the nacelle, the response of the system was largely underestimated. When the shear force being transmitted from the vibrating blades into the tower was greatest, as in the case of a slowly rotating rotor system, the response of the system was greatest. As the blade rotational frequency increased, the stiffness of the blades increased, causing a decrease in the overall response of the system.

In calculation of the gust response factor, the single degree-of-freedom models gave almost identical gust response factors using the numerically integrated and the closed form methods, in the frequency domain, thus validating the accuracy and applicability of the tradition single degree-of-freedom gust response factor. The gust response factors obtained using a time domain averaging approach show good agreement with those obtained using the numerical integration method of frequency domain, provided that the sample size used in the time domain method was not less than ten. For the two degree-of-freedom model, the numerically integrated and closed form estimates of the GRP obtained in the frequency domain showed excellent agreement if the modes of the coupled model were relatively far apart. If this is the case, the gust response factors obtained for the two degree-of-freedom model, which included blade-tower interaction were almost identical with those of the single degree-of-freedom model that does not consider blade-tower interaction. Thus, the use of the single degree-of-freedom lumped parameter approach was satisfactory in estimating the gust response factor for the wind turbine tower, providing that the
fundamental frequencies of the rotor blades were far from that of the tower/nacelle. When the modes of the two degree-of-freedom model were very close together, occurring because the fundamental frequencies of the blades were close to that of the tower/nacelle, the disagreement was observed to be a function of blade rotational frequency. The agreement was least when the blades were stationary, and most when the blades were rotating at 3.14 rads$^{-1}$. This difference in GRF values occurred because the two modes were not fully decoupled due to their close proximity, and hence, the closed form values of GRF were underestimated. For the two degree-of-freedom model, the gust response factors obtained in the time-domain agreed well for all assemblies with those obtained using the numerically integrated method in the frequency domain, provided the sample size used in the time domain was not less than ten.

During the system identification testing, the free vibration time-histories of the blade only, and tower/nacelle only, showed exponential decays at one predominant frequency, allowing the fundamental frequency and fundamental modal damping ratios to be obtained. The free vibration time-histories of coupled tower and blade models, with various blade orientations, also showed exponential decay, but the responses were broadbanded. The damped natural frequencies of these assemblies were obtainable from Fourier transform plots of the free vibration time-histories, indicating that the response contained contributions from the first and second modes of vibration. Using the experimental set-up, displacement response functions for the blade only, tower and nacelle only, and coupled tower and blades assembly were obtained, and the blade and tower/nacelle displacement responses were validated using an analytical formulation. The confidence thus afforded to the experimental set-up allowed displacement-to-displacement transfer functions to be obtained experimentally for the coupled tower and blades assemblies. For the non-rotating blades cases of the coupled model, the position of the blades had a bearing on the magnitude of the transfer functions obtained due to the excitation of specific modes of vibration. The highest transfer function values were observed for the blade orientation with one of the blades being fully horizontal. The action of blade rotation appeared to have only a minor effect of the magnitude of transfer functions obtained, with higher factors observed for the higher blade rotational frequencies. The experimentally obtained displacement-to-displacement transfer functions were converted to force-to-displacement transfer functions by first fitting a multi-modal expression to the displacement-to-displacement transfer function and obtaining the values of modal mass of each of the two modes. These values of
modal mass may then be used to estimate the force-to-displacement transfer functions, which were used in the validation of the wind tunnel tests.

The use of a wind tunnel in carrying out a forced vibration analysis of a flexible reduced scale multi-body structure provides insight into the dynamic interaction between structure and the wind flow, and the dynamic interaction between the various flexible components of the structure. A turbulent wind flow was easily created by employing roughness elements and spires, which represented a cost effective and efficient method of generating a user defined mean wind velocity profile, and spectral energy distributions. The most obvious limitation regarding the operation of the wind tunnel was its inability to generate a mean wind velocity of above 5 ms\(^{-1}\) at a one metre elevation. However, this limitation did not adversely affect the operation of the experiments, as the instruments were sufficiently able to measure the induced structural response. The position of the blades relative to the tower in the non-rotating blade cases seemed to have a slight effect on the magnitudes of fluctuating strains induced. The case that seemed to induce the most fluctuating strain at the mid and base-points was when one of the blades was fully horizontal. This was closely followed by the rotating cases of 3.14 rads\(^{-1}\) and 1.57 rads\(^{-1}\). However, the action of blade rotation did not particularly affect the response of the model. Theoretically predicted peak and root mean square fluctuating strains showed good agreement with those observed experimentally. Theoretical peak accelerations were generally predicted higher than those measured, as are the root mean square values. Overall, the proposed analytical model based on random vibration theory gave an accurate prediction of the fluctuating component of the response of the scale model wind turbine tower.

9.3 CRITICAL ASSESSMENT OF THE APPROACH AND THE MAIN ASSUMPTIONS

The main approach adopted in this thesis was to model the along-wind response of a wind turbine tower complete with rotating blades using a series of reduced order models. This approach considered specific degrees-of-freedom only, in particular unidirectional displacement degrees-of-freedom, and did not include an orthogonal displacement degree-of-freedom, which would model an across-wind response, or a torsional degree-of-freedom, which would allow the yawing response to be calculated. However, non-inclusion of these degrees-of-freedom would not adversely affect the along-wind dynamic response of the wind turbine tower.
In adopting this approach, a number of assumptions were made, and the adequacy of these assumptions will now be discussed. The geometry of the tubular tower was simplified into being of uniform hollow cross-section, whereas in practice the width of the tower is tapered from the base to the tip. Also, the torsional response of the tower was neglected, which will be of interest when one investigates yawing. The geometry of the rotating blades was considerably simplified into being of uniform rectangular hollow cross-section, whereas the geometry in practice is of an aerofoil cross-section, tapering along both principle axes. Non-linearity of the blade response was not considered, which occurs in practice. The free vibration behaviour of the coupled tower/blades model was also considerably simplified. It was assumed that the flapping and lead/lag modes of the blades could be completely uncoupled, whereas in reality modal coupled between the two modes occurs. Thus, it is a simplification to assume that the flapping/lead/lag modes of the blades will only coupled to the flapping/lead/lag motion of the tower.

The geometry of reduced scale model used in the experimental studies was considerably simplified in comparison with that in actuality, primarily to aid fabrication. The tower did not have a tapering of cross-section over its height, and the blades had a solid rectangular cross-section. During the wind tunnel testing, the mean wind velocity constraint discussed in Chapter 8 resulted in a Reynolds number which was low, making it difficult to obtain a drag coefficient from the literature. Thus, a drag coefficient based on a lower end literature estimation was assumed.

9.4 RECOMMENDATIONS FOR FURTHER STUDY

Although an attempt was made, by way of this research thesis, to develop a series of analytical models of reduced order, capable of predicting the response of the individual and coupled components of a wind turbine tower, further research is still required to obtain a realistic design-orientated analytical representation of the response of wind turbine tower assemblies. The areas that would benefit from additional theoretical and experimental study include:

*The rotating blades* – develop a reduced order technique (with relatively few degrees-of-freedom) that predicts the natural frequencies and mode shapes of a wind turbine blade with realistic geometry, including an aerofoil cross section with varying mass and stiffness distributions. This model may be based on the finite element method incorporating a model
order reduction technique. The model should be able to include the stiffening of the blade due to centrifugal and self-weight effects. This model should be validated by a series of full scale experimental system identification tests, covering the cases of when the blade is stationary, and when it is rotating under the action of wind loading. A mathematical expression that relates the transfer of energy from the lower frequency range to multiplies of the blade rotational frequency, as observed during rotational sampling, should also be developed for use in response calculations of rotating wind turbine blades. This expression may be similar to the fluctuating wind velocity spectra commonly used in wind engineering, but should include several other important variables, such as the elevation above the ground, the rotational frequency of the blades, and the distance along the blade from blade tip. The correlation of forces along the length of the blades, and the correlation of drag forces on each of the blades relative to each other should also be addressed. To account for the rotationally sampled turbulence acting on the rotating blades, a series of full scale experimental tests should be carried out to identify expressions that define the correlation of force at points along the length of a blade and at points on different blades, as a function of wind velocity, frequency (or size or eddies) and, probably, some numerical decay constant.

Coupled tower and rotor system – Provide experimental validation of the analytical approach proposed in this thesis for the forced vibration response of a wind turbine tower including blade-tower interaction. This should also include an experimental validation of the two degree-of-freedom gust response factor. With this experimental validation, the analytical approach proposed in this thesis may be used to optimise the design process for a wind turbine tower. The reduced order models developed in this thesis should be augmented to include dynamic wave and current loading for offshore wind turbine towers, and include soil-structure interaction for wind turbines placed onshore in unfavourable soil-conditions. As the analytical models involving blade and tower coupling investigated the along-wind wind response only, the across-wind response of the coupled model should also be investigated. This would include identification of specific model geometries that may induce vortex shedding.

Experimental work - Develop an analytical formulation to evaluate force-to-displacement transfer functions for any scale model wind turbine tower, using the geometric and material properties of the model. This analytical formulation could then be used to accurately estimate the transfer function of a full scale wind turbine tower, provided it has been
validated experimentally. The occurrence of rotationally sampled fluctuating wind velocity spectra may also be investigated in a wind tunnel. This would involve measuring the fluctuating wind velocity acting on rotating bodies immersed within the generated wind flow. This approach may also be used to develop an expression for the rotationally sampled spectrum, instead of full scale testing. The two degree-of-freedom gust response factor may also be experimentally validated in the wind tunnel, if full scale tests are impractical. The wind tunnel may also be used to continue the research into the evaluation of aerodynamic damping, and in particular the aerodynamic damping of wind turbine towers.
REFERENCES


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REFERENCES


A.1 TRUSS ELEMENT FOR LATTICE TOWER

‘LINK8’ is a spar which may be used in a variety of engineering applications. This element can be used to model trusses, sagging cables, links, springs, etc. The three-dimensional spar element is a uniaxial tension-compression element with three degrees of freedom at each node: translations in the nodal x, y, and z directions. As in a pin-jointed structure, no bending of the element is considered.

INPUT DATA

The geometry, node locations, and the coordinate system for this element are shown in . The element is defined by two nodes, the cross-sectional area, an initial strain, and the material properties. The element x-axis is oriented along the length of the element from node I toward node J.
A.2 PIPE ELEMENT FOR TUBULAR TOWER

‘PIPE16’ is a uniaxial element with tension-compression, torsion, and bending capabilities. The element has six degrees of freedom at two nodes: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes.

Figure A.2 Elastic straight pipe ‘PIPE16’ as used in ANSYS

INPUT DATA

The geometry, node locations, and the coordinate system for this element are shown in figure A.1. The element input data include two or three nodes, the pipe outer diameter and wall thickness, stress intensification and flexibility factors, internal fluid density, exterior insulation density, and thickness, corrosion thickness allowance, insulation surface area, pipe wall mass, axial pipe stiffness, rotodynamic spin, and the isotropic material properties.

The element X-axis is oriented from node I toward node J. For the two-node option, the element Y-axis is automatically calculated to be parallel to the global X-Y plane. Several orientations are shown in figure A.2. For the case where the element is parallel to the global Z-axis (or within a 0.01 percent slope of it), the element Y-axis is oriented parallel to the global Y-axis (as shown). For user control of the element orientation about the element X-axis, the third node option is used. The third node (K), if used, defines a plane (with I and J) containing the element X and Z axes (as shown in figure A.2). Input and output locations around the pipe circumference identified as being at 0° are located along the element Y-axis, and similarly 90° is along the element Z-axis.
A.3 BEAM ELEMENT FOR TAPERED BLADE

BEAM 44 is a uniaxial element with tension, compression, torsion, and bending capabilities. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. This element allows a different unsymmetrical geometry at each end and permits the end nodes to be offset from the centroidal axis of the beam. The effect of shear deformation is available as an option. Another option is available for printing the forces acting on the element in the element coordinate directions. Stress stiffening and large deflection capabilities are also included.

![Diagram of BEAM 44 element](image)

**Figure A.3 3-D Tapered unsymmetric beam 'BEAM44' as used in ANSYS**

**INPUT DATA**

The geometry, node locations, and coordinate system for this element are shown in figure A.3. The element is located by a reference coordinate system (x',y',z') and offsets. The reference system is defined by nodes I, J, and K, or an orientation angle, as shown in figure A.3. The principal axes of the beam are in the element coordinate system (x,y,z) with x along the cross-section centroid (C.G.). The element x-axis is oriented from node I (end 1) toward node J (end 2). For the two-node option, the default ($\theta = 0^\circ$) orientation of the element y-axis is automatically calculated to be parallel to the global X-Y plane. Several orientations are shown in figure A.3. For the case where the element is parallel to the global Z-axis (or within a 0.01 percent slope of it), the element y-axis is oriented parallel to the global Y-axis (as shown). For user control of the element orientation about the element x-axis, use the $\theta$ angle or the third node option. If both are defined, the third node option takes precedence. The third node (K), if used, defines a plane (with I and J) containing the element x and z-axes (as shown). If this element is used in a large deflection analysis, it should be noted that the location of the third node (K), or the angle $\theta$, is used only to initially orient the element.

The element real constants describe the beam in terms of the cross-sectional area, the area moments of inertia, the extreme fiber distances from the centroid, the centroid offset, and the shear constants. The moments of inertia (IZ and IY) are about the lateral principal axes of the beam. The torsional moment of inertia at end 1 (IX1), if not specified, is assumed equal to the polar moment of inertia at end 1 (IZ1 + IY1). The moment of inertia values at
end 2 (IX2, IY2, and IZ2), if blank, default to the corresponding end 1 values. The element torsional stiffness decreases with decreasing values of IX.

The offset constants (DX, DY, DZ) define the centroid location of the section relative to the node location. Offset distances are measured positive from the node in the positive element coordinate directions. All real constants (except the centroidal offset constants DX, DY, and DZ) for end 2 of the beam, default to the corresponding end 1 values, if zero. The "top" thicknesses at end 1, TKZT1 and TKYT1, default to the "bottom" thicknesses at end 1, TKZB1 and TKYB1, respectively. Also the "top" thicknesses at end 2, TKZT2 and TKYT2, default to the "top" thicknesses at end 1, TKZT1 and TKYT1, respectively. The thicknesses are measured from the centroid of the section.
APPENDIX B – STRAINSMART SOFTWARE

The software used to obtain the digital response of the model was Strainsmart, version 3.10, written by the Vishay Measurements Group from the USA. There are several steps in setting up a project using the Strainsmart software. They are summarised as follows:

**General Information**
The name of the project is first entered, as in figure B.1. The program creates a system file with this name which will subsequently contain all the sensor information and recorded data. The user can specify that the project file is to be compressed, if envisaged as being large to save hard disk space. The user may also assign a password to the project, disabling other uses from altering the program.

**Sensor Information**
The user must then specify what sensors are to be used in the course of the testing. Details regarding a sensor may be input directly, such as a A/D calibration coefficient, or an electrical resistance value for use in a whetstone bridge. The program also contains a database, into which the user may input sensor data and save it for future use. Figure B.2 shows an accelerometer and a LVDT retrieved from the master database by the user.

**Material Information**
Material properties may be input into the program, such as the elastic modulus of the material being testing. This may then be used at a later stage to calculate stresses from strain measurements.

**Channel Information**
The user must scan the System 6000 to see how many of the sixteen channels are available, and then is tasked with assigning the proper channels to the relevant sensor. Figure B.3 shows the high level (accelerometer) card is assigned to channel 14 and the LVDT is assigned to channel 15. Figure B.4 confirms this assignment, awaiting instruction from the user to proceed.

**Data acquisition**
Once the sensors have been installed and assigned their relevant channels, the user is now in a position to record some sensor readings. A scan session is first created, figure B.5...
shows several of these, and a scanning frequency is chosen, as illustrated in figure B.6. Before the recording starts, all sensors must be zeroed, so as to create a benchmark for measurement, as presented in figure B.7. The program then provides a brief summary of user specified recording details, as in figure B.8. And then commences recording, as demonstrated in figure B.9.

**Review of recorded data**

Lastly, the user can view the recorded data in terms of A/D counts in graphical format, as illustrated in figure B.10. It is then necessary to reduce this data into ASCII format, in which the raw data in A/D counts will be recalibrated into metric measurement units, specifically millimetres for all LVDT readings and acceleration due to gravity for all acceleration readings.