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ANALYTICAL APPROACHES TO EVALUATING SUSTAINABLE TRANSPORTATION STRATEGIES

XIAOQING JABER

A dissertation submitted to the University of Dublin in the partial fulfilment of the requirements for the Degree of Doctor of Philosophy

Department of Civil, Structural & Environmental Engineering
Trinity College Dublin

2009
DECLARATION

I hereby declare that this dissertation, in whole or in part has not been submitted as an exercise for a degree at this or any other University. I further declare except where reference is given in the text, the work is entirely my own. I also give the library permission to lend or copy the thesis, upon request, for academic purposes.

April 2009

Xiaoqing Jaber
Dedicated to my husband Wafa, my parents Yunian and Huai, and my brother Qian
SUMMARY

Sustainable transportation strategies and policies not only generate an efficient and equitable transportation system but also balance other economic, social and environmental objectives. The objective of this thesis is to carry out a comprehensive equilibrium analysis of most widely applied transportation policies and strategies from the viewpoint of sustainability, mainly concentrating on their impact on vehicular emissions. This study adopts the two best known and most widely used macroscopic emission estimation models belonging to the category of average-speed models. These are Mobile Source Emissions Factor Model or MOBILE and TRANSYT-7F model. The evaluated transportation strategies and policies include road pricing, road network design, traveller information provision services, and speed limit control. The traffic assignment is simulated by employing different traffic assignment techniques, which consist of deterministic user equilibrium (DUE), stochastic user equilibrium (SUE), system optimum (SO), and system equity (SE). The impact of those strategies on travellers' route choice behaviour under different equilibriums is studied. The consequent impact on vehicular emissions is analysed.

In this thesis, transportation problems are formulated into various mathematical programs subject to different scenarios. The model proposed in chapter 3 to study the impact of road pricing is a single-level minimisation model, although it is bi-level in nature. The basic models proposed in chapter 4 to analyse the impact of various road network design schemes are also formulated as bi-level programs. Additionally, in chapters 4 and 5, multi-objective optimisation models are developed to assist policy makers or transportation planners in their multi-objective decision making. In chapters 6 and 7, the models to study the impact of traveller information provision services are formulated as a nonlinear complementarity problem (NCP). In chapters 6 and 7, models proposed to derive the marginal cost pricing policy are formulated as convex programming, although the convexity of the emissions cost function in chapter 7 is conditional. The goal of formulating these analytical modes is to seek a sustainable solution. These models, which are a valid representation of the performance of the transportation system under different transportation strategies, are solved by the Generalised Reduced Gradient (GRG) algorithm.
The obtained results are briefly summarised as follows. In terms of analysing the impact of road pricing on vehicular emissions, the results show that the tighter the vehicular emission standards, the higher the toll charges required. It is also shown that vehicular emission standards have a direct impact on the following: overall vehicular emissions; operational strategies and profit of public transit; the mode and route choices of travellers; residential and employment distributions; profits of land owners; and rents. The government should consider these impacts when determining the vehicular emission standard of each road. With respect to analysing different road network design schemes whilst considering the land-use transport interaction over time, the results show that it is difficult to comment on which scheme was the best in general after considering each party's perspective. Tradeoffs are also seen to exist between the objectives of all related parties. This implies that the government has to consider tradeoffs between parties' objectives carefully. Regarding evaluating the traveller information provision services, it is found that the service charges, information quality, travellers' own perception of the trade-offs among their multiple criteria and their value of time (VOT) levels have a strong implication on the system performance. An analytical methodology is also proposed to study the impacts of strategic interactions among traffic managers, toll road operators as well as information providers using the proposed NCP formulation. A sensitivity analysis is carried out to demonstrate the analytical methodology and illustrate the impact of strategic interactions among speed limit control, traveller information provision services, and tolling on both congestion and vehicular emissions.

In summary, this thesis presents analytical models, approaches, and analyses incorporating economic, social, and environmental aspects, which allow transportation policy makers and transportation planners to evaluate sustainability of various transportation strategies at a strategic level. The proposed models and the derived marginal tolls provide a useful methodology which enables transportation policy makers to balance the tradeoffs amongst different parties from economic, social, and environmental perspectives. The analyses carried out in this thesis demonstrate policy implications of various transportation strategies. The conclusions drawn from the results could help policy makers to identify optimal sustainable transportation strategies and policies, which can simultaneously reduce negative impacts of transportation and avoid conflicting objectives.
SUMMARY

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## TABLE OF CONTENTS

SUMMARY ............................................................................................................................................... iv  
ACKNOWLEDGEMENTS .................................................................................................................. vi  
TABLE OF CONTENTS ..................................................................................................................... vii  
LIST OF FIGURES ............................................................................................................................... xii  
LIST OF TABLES ................................................................................................................................ xiv  

### CHAPTER 1: INTRODUCTION

1.1 Background ..................................................................................................................................... 1  
1.2 The Objectives of the Thesis ....................................................................................................... 3  
1.3 Outline of the Thesis ................................................................................................................... 5  

### CHAPTER 2: LITERATURE REVIEW

2.1 Traffic Assignment Techniques .................................................................................................. 9  
   2.1.1 Deterministic User Equilibrium (DUE) ........................................................................... 9  
   2.1.2 Stochastic User Equilibrium (SUE) ............................................................................... 12  
   2.1.3 System Optimum (SO) ................................................................................................... 14  
   2.1.4 System Equity (SE) ........................................................................................................ 16  
2.2 Land Use-Transport Interaction ............................................................................................... 18  
2.3 Emission Estimation Approaches ............................................................................................ 24  
   2.3.1 MOBILE Model ............................................................................................................. 24  
   2.3.2 TRANSYT-7F Model .................................................................................................... 25  
2.4 Literature Review of Transportation Policies and Strategies .................................................... 27  
   2.4.1 Road Pricing Problems ................................................................................................. 27  
   2.4.2 Road Network Design Problems (NDP) ..................................................................... 37  
   2.4.3 Traveller Information Provision Services ..................................................................... 39  
   2.4.4 Speed Limit Control ...................................................................................................... 40
CHAPTER 3: INCORPORATING LAND USE, TRANSPORT AND ENVIRONMENTAL CONSIDERATIONS INTO TIME-DEPENDENT TOLLING STRATEGIES

3.1 Introduction................................................................................................................................42
3.2 Model Formulation..................................................................................................................43
   3.2.1 Lower Level Time-Dependent Land-use Transport Problem .......................43
   3.2.2 Upper Level Problem .........................................................................................51
   3.2.3 The Proposed Bi-level Model ..............................................................................53
3.3 Numerical Studies ....................................................................................................................54
   3.3.1 The Impacts of Time-Dependent Tolls ..............................................................54
   3.3.2 The Impact of Maximum Allowable Link Emissions on Link Tolls ...............60
3.4 Summary ....................................................................................................................................62

CHAPTER 4: TIME-DEPENDENT ROAD NETWORK DESIGN FRAMEWORKS WITH LAND USE AND ENVIRONMENTAL CONSIDERATIONS: POLICY IMPLICATIONS

4.1 Introduction............................................................................................................................64
4.2 Formulation............................................................................................................................66
   4.2.1 Time-Dependent Lowry-Type Constraints and Modal-Split/Assignment Constraints ..............................................................66
   4.2.2 Road Network Design Constraints .................................................................67
   4.2.3 Financial Constraints ......................................................................................67
   4.2.4 Objective Functions .........................................................................................70
   4.2.5 Considerations in Road Network Improvement Projects ..........................73
   4.2.6 Three Models Derived from the Proposed Single-Objective Framework ..................................................................................76
   4.2.7 Model Extension: Multi-Objective Optimisation ........................................78
4.3 Numerical Studies ....................................................................................................................80
   4.3.1 Scenario Setting ..............................................................................................81
   4.3.2 Performance of Each Scheme ........................................................................83
   4.3.3 Performance of the Cost Recovery Design under Landowner Equity Consideration ...........................................................................88
CHAPTER 5: SIMULTANEOUS OCCURRENCE OF BRAESS' AND EMISSION PARADOXES

5.1 Introduction ......................................................................................................................... 94
5.2 When Does Nagurney’s Emission Paradox Occur in Braess’ Network? ......................... 95  
  5.2.1 Description of Braess’ Network and Emission Estimation Model ................. 96
  5.2.2 User Equilibrium Assignment Conditions with Emissions ....................... 97
  5.2.3 Occurrence Conditions of the Emission Paradox ............................................. 99
5.3 When does Braess’ Paradox Occur? .............................................................................. 103
5.4 Internalising Congestion and Emission Externalities Simultaneously ................. 105  
  5.4.1 The Traditional Marginal Cost Congestion Pricing ........................................... 105
  5.4.2 The Marginal Cost Congestion and Emission Pricings ..................................... 107
5.5 Discussion of Paradoxes with Corresponding Pricing Schemes ....................... 116
5.6 Summary .............................................................................................................................. 119

CHAPTER 6: MIXED SUE BEHAVIOUR UNDER THE TRAVELLER INFORMATION PROVISION SERVICES WITH HETEROGENEOUS MULTI-CLASS MULTI-CRITERIA DECISION MAKING

6.1 Introduction .......................................................................................................................... 121
6.2 NCP Model formulation and performance measures ...................................................... 123  
  6.2.1 NCP Formulation of the Heterogeneous Multiclass Mixed SUE Assignment Problem with an Environmental Criterion ........................................... 123
  6.2.2 Performance Measures of the Total System Travel Time (TSTT) and Vehicular Emissions ................................................................. 128
6.3 Existence of Paradoxes in Traveller Information Provision Services ....................... 129
6.4 A System Optimised Pricing Policy in a SUE Assignment Problem ....................... 132  
  6.4.1 Traveller Information Services with Net Economic Benefit Maximisation .................. 132
  6.4.2 The Equivalent Logit-Based SUE Assignment Conditions and a System Optimised Toll ............................................................. 135
  6.4.3 Karush-Kuhn-Tucker (K-K-T) Optimality Conditions ..................................... 138
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.4.4</td>
<td>Weights in a Bicriteria Model: Relation to the Optimal Toll</td>
<td>139</td>
</tr>
<tr>
<td>6.4.5</td>
<td>Illustrative Examples</td>
<td>140</td>
</tr>
<tr>
<td>6.5</td>
<td>Sensitivity Analysis</td>
<td>144</td>
</tr>
<tr>
<td>6.5.1</td>
<td>Information Quality and Service Charges: Impact on Traveller Information Provision Services with Endogenous Market Penetration</td>
<td>145</td>
</tr>
<tr>
<td>6.5.2</td>
<td>Heterogeneous Multiclass Multicriteria Decision Making: Implications on Traveller Information Provision Services with Endogenous Market Penetration</td>
<td>148</td>
</tr>
<tr>
<td>6.6</td>
<td>Summary</td>
<td>158</td>
</tr>
</tbody>
</table>

**CHAPTER 7: A METHODOLOGY FOR EVALUATING THE JOINT IMPLEMENTATION OF TRAVELLER INFORMATION PROVISION SERVICES, SPEED LIMIT CONTROL, AND TOLLING**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>162</td>
</tr>
<tr>
<td>7.2</td>
<td>Methodology</td>
<td>163</td>
</tr>
<tr>
<td>7.2.1</td>
<td>Emission Estimation Model</td>
<td>163</td>
</tr>
<tr>
<td>7.2.2</td>
<td>NCP Formulation</td>
<td>164</td>
</tr>
<tr>
<td>7.3</td>
<td>Sensitivity Analysis</td>
<td>166</td>
</tr>
<tr>
<td>7.3.1</td>
<td>Incorporating the Speed Limit Control into Traveller Information Provision Services</td>
<td>166</td>
</tr>
<tr>
<td>7.3.2</td>
<td>Implementation of Tolling, Speed Limit Control, and Traveller Information Provision Services</td>
<td>174</td>
</tr>
<tr>
<td>7.4</td>
<td>Internalisation of Speed-Varying Emission Externality in a SUE Traffic Assignment Problem</td>
<td>176</td>
</tr>
<tr>
<td>7.5</td>
<td>Summary</td>
<td>181</td>
</tr>
</tbody>
</table>

**CHAPTER 8: CONCLUSIONS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>Research Summary</td>
<td>184</td>
</tr>
<tr>
<td>8.2</td>
<td>Research Findings and Contributions</td>
<td>185</td>
</tr>
<tr>
<td>8.3</td>
<td>Critical Assessment</td>
<td>189</td>
</tr>
<tr>
<td>8.4</td>
<td>Recommendations for Future Research</td>
<td>190</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 2.1 Schematic representation of an integrated urban land use-transport model. 23
Figure 3.1 The land use transport and environment relationship over time. 44
Figure 3.2 Interaction between land use, transport, and environment at a time instant. 44
Figure 3.3 The transport network for Scenario 1. 55
Figure 3.4 The overall vehicular emissions and the maximum allowable vehicular emissions over time. 56
Figure 3.5 Number of passengers between OD pair E1-R3 on the public transit system and the total transit revenue. 56
Figure 3.6 Total employments in zones 1 and 2 in each design year. 57
Figure 3.7 Number of residents in zones 3, 4 and 5 over time. 58
Figure 3.8 Total travel cost as well as emissions on link E1-R5. 59
Figure 3.9 The transport network for Scenario 2. 60
Figure 3.10 The impact of the maximum allowable link emissions of link E1-R3 on its link toll. 61
Figure 4.1 The scenario network. 81
Figure 4.2. Travel time and travel cost over time. 84
Figure 4.3. Vehicular emissions over time. 85
Figure 4.4 Changes in landowner profits against toll on link 2. 90
Figure 4.5 Change in social surplus against toll on link 2. 90
Figure 4.6 Change in consumer surplus against toll on link 2. 91
Figure 5.1 Braess' networks. 96
Figure 5.2 The occurrence of two paradoxes in Braess' network, where $\beta_1 - \beta_2 = 10 - 1 > 0$ and $\alpha_1 - \alpha_2 = 50 - 10 > 0$. 104
Figure 5.3 The relationship between path flow and the demand under the marginal cost congestion pricing. 106
Figure 6.1 The example network. 130
Figure 6.2 The impact of service charges and information quality on TSTT. 147
Figure 6.3 The impact of service charges and information quality on overall vehicular emissions. 147
Figure 6.4 A plot of market penetration (MP) against service charges, when $\theta$ is 2, 4, 6 and 8 $\epsilon^{-1}$ respectively. 148
Figure 6.5 A plot of TSTT and market penetration (MP) against weight for emissions on link 1. ................................................................. 150
Figure 6.6 Total emissions and market penetration (MP) against weight for emissions on link 1. ................................................................. 150
Figure 6.7 The difference of systematic utilities against weight for emissions on link 1. .................................................................... 151
Figure 6.8 Average travel time versus weight for emissions on link 1. ............... 152
Figure 6.9 Vehicular emissions versus weight for emissions on link 1. ............... 152
Figure 6.10 Total emissions and market penetration (MP) against weight for emissions on link 2. ................................................................. 153
Figure 6.11 The difference of systematic utilities against weight for emissions on link 2. .................................................................... 154
Figure 6.12 Weights for vehicular emissions versus VOT of class 2 users............. 156
Figure 6.13 TSTT against VOT of class 2 users..................................................... 156
Figure 6.14 Total vehicular emissions against VOT of class 2 users.................... 157
Figure 6.15 Vehicular emissions against VOT of class 2 users............................ 157
Figure 6.16 Total MP and MP of two classes against VOT of class 2 users......... 158
Figure 7.1 The example network ........................................................................ 167
Figure 7.2 The impact of service charges and the speed limit of link 1 on overall vehicular emissions....................................................... 169
Figure 7.3 The impact of service charges and the speed limit of link 1 on TSTT...... 169
Figure 7.4 The impact of information quality on overall vehicular emissions when speed limits on link 1 are 50 and 110 km/hr respectively................................................................. 170
Figure 7.5 The impact of information quality on TSTT when speed limits on link 1 are 50 and 110 km/hr respectively.............................. 170
Figure 7.6 The impact of information quality on MP when speed limits on link 1 are 50 and 110 km/hr respectively.............................. 171
Figure 7.7 MP versus speed limit on link 1 with various service charges............. 172
Figure 7.8 Vehicular emissions on two links against speed limit on link 1. ......... 172
Figure 7.9 Total emissions against speed limit on link 1.................................... 173
Figure 7.10 Total emissions against toll............................................................. 175
Figure 7.11 TSTT against toll........................................................................... 175
LIST OF TABLES

Table 4.1 A summary of three network design schemes.........................................................80
Table 4.2 The objective measure of each party under three schemes.................................85
Table 4.3 A comparison of the objective measure of each party with and without
landowner equity consideration..........................................................................................89
Table 4.4 Vehicular emissions and its relative reduction with the implementation of
three schemes..................................................................................................................92
Table 5.1 Summary of the occurrence of the emission paradox under each flow pattern. .....101
Table 5.2 Summary of demand conditions under which three flow patterns occur
according to different relationships of parameters in link performance functions........101
Table 5.3 Demand conditions under which braess’ and emission paradoxes occur.... 103
Table 6.1 Changes in TSTT, overall vehicular emissions, and flows on links 1 and 2
before and after the information provision......................................................................132
Table 6.2 The system performance before and after the marginal cost pricing under SUE. ........................................................................................................141
Table 6.3 The optimal tolls under different emission weights with the same total
emission constraint..........................................................................................................142
Table 6.4 The system performance before and after the marginal cost pricing under SUE. ........................................................................................................144
Table 6.5 The optimal tolls under different emission constraints with various emission
weights.............................................................................................................................144
Table 6.6 The weights corresponding to various VOTs.......................................................155
CHAPTER 1
INTRODUCTION

1.1 BACKGROUND

Motor vehicles are responsible for a large proportion of harmful greenhouse gases in the environment. With increasing demands on transportation networks worldwide, vehicular emissions are growing rapidly. Dings (2008) indicated that the transport sector today is not moving in a sustainable direction: CO\textsubscript{2} emissions and oil consumption have risen 32 percent since 1990, cancelling out the progress made in other sectors. This urgent situation requires policy makers and transportation planners to integrate sustainability into the transportation planning process so as to meet Kyoto Protocol targets.

There is no universally accepted definition of sustainability, sustainable development or sustainable transport (Beatley, 1995). Generally, sustainability is a planning perspective that considers economic, social and environmental goals. Transportation facilities and activities have significant negative impacts on sustainability. Some examples are traffic congestion, fatalities and injuries, noise, air and water pollution, greenhouse gas emissions, diminishing energy resources and biological and ecosystem damage. The challenge of developing and maintaining a sustainable transportation system lies in minimising these negative impacts whilst offering strong transportation benefits (TRB, 2005).

To quantify these complex phenomena and to help explain how they change over time, sustainable transportation indicators can be broadly classified into economic, social and environmental aspects. According to Litman and Burwell (2006), Litman (2007), and TRB (2008), indicators in the economic aspect include social welfare, consumer surplus, transportation facility costs, land use, and travel costs of road users amongst others. Indicators in the social aspect include travel time, equity, human health, auto crash and injuries. Examples of indicators in the environmental aspect include climate change,
vehicular emissions, noise, and water pollution. Some of these indicators may overlap. For example, air pollution from vehicular emissions is an environmental concern, but it is also a social concern because it is related to human health. Transportation strategies and policies that increase system efficiency and simultaneously reduce these negative impacts are considered to be sustainable. Thus, sustainable transportation policies and strategies help to achieve multiple goals of reducing negative impacts and increasing transportation efficiency.

There are some sustainability principles which provide guidelines in the analysis of sustainability of transportation policies and strategies and also help to identify such policies and strategies which make progress toward sustainability objectives. According to principles proposed by the Victoria Transport Policy Institute (2007), sustainable planning solutions must reflect integrated analysis. Some of the proposed principles are as follows:

- Integrated and strategic planning;
- Comprehensive analysis;
- Focusing on goals, performance and outcomes;
- Consideration of the equity issue and the time concept.

Based on these principles, analysing sustainability of transportation policies requires planning that accounts for economic, social and environmental impacts. This requires a comprehensive analysis and holistic evaluation of transportation policies and strategies. The analysis and evaluation can help decision-makers to understand the effects of their decisions. Analysing sustainability of transportation policies and strategies also requires that planning should be based on goals and outcomes, such as increased social welfare and consumer surplus, reduced vehicular emissions and total system travel time/cost, etc. The goals can be any economic, social or environmental indicator or a combination thereof. Equity issues should also be incorporated into the analysis of sustainability of transportation policies, as transportation policies can have significant equity impacts on society and future generations. The time concept should also be integrated into the analysis, which means focusing on long-term planning.

Since transportation networks are large-scale in nature, it is required to develop appropriate analytical models and approaches incorporating economic, social, and
environmental aspects, which allow policy makers and transportation planners to handle the dimensionality of the problem and help to avoid those that solve one transportation problem but exacerbate other economic, social or environmental problems.

1.2 THE OBJECTIVES OF THE THESIS

The main objective of this thesis is to provide a comprehensive evaluation of most applied transportation strategies and policies from the viewpoint of sustainability using traffic assignment techniques. The focus is on the environmental aspect, specifically vehicular emissions, whilst considering other economic and social aspects. In this thesis, traffic assignment techniques considered are deterministic user equilibrium (DUE), stochastic user equilibrium (SUE), system optimum (SO), and system equity (SE). Transportation strategies and policies evaluated include road pricing, road network design, traveller information provision services, and speed limit control. By carrying out the equilibrium analysis of these strategies and policies, the potentials of different policy options in reducing transport emissions are investigated. The implications of their implementation on vehicular emissions are discussed. Although there are various sustainable indicators mentioned above, in this thesis, the analysis of the impact of transportation strategies is only carried out on some economic and social aspects, such as social welfare, consumer surplus, travel time/cost of road users, and total system travel time/cost. The land use-transport interaction is taken into account to analyse the impact of some transportation strategies on land use. The goal of this study is to show conflict among some sustainable indicators and to propose analytical tools to evaluate and solve problems. The objective is to identify sustainable transportation strategies and policies which can provide a balanced and sustainable solution and simultaneously help to reduce negative impacts of transportation and avoid conflicting objectives.

Therefore, the objectives of this research are as follows:

- To conduct a literature review of
  - traffic assignment techniques and the land use-transport interaction.
o transportation problems with most applied transportation strategies and policies, which include road pricing, road network design, traveller information provision services, and speed limit control.

o derived optimal pricings, which are used to control congestion and vehicular emissions simultaneously.

- To develop optimisation models to determine time-dependent optimal tolls while considering the dynamic relationships between land use, transport, and environment. The novelty is to study the effect of tightening link vehicular emission standards on link tolls.

- To propose time-dependent road network design frameworks with land use and environmental considerations so that the land use transport interaction can be dealt with when determining optimal network design schemes. Impacts of different network design schemes on related parties will be evaluated. These parties include land owners in terms of their profits and the government in terms of vehicular emissions.

- To analytically examine the simultaneous occurrence of Braess' and emission paradoxes and to derive congestion and emission pricing policies and compare them with existing marginal cost of congestion and emission pricings. The occurrence of Braess' and emission paradoxes and their corresponding pricing schemes will also be discussed.

- To propose a normalised multi-objective weighting method for computing optimal toll charges in such a way that allows decision makers to trade-off between two conflicting objectives, alleviating congestion versus reducing vehicular emissions. A multi-objective analysis will be carried out by adjusting the associated weighting coefficients associated with different objectives.

- To propose a multi-class, multi-criteria mixed SUE assignment model under the traveller information provision services with heterogeneous road travellers and elastic market penetration. The impacts of information quality and service charges of the traveller information provision on congestion and vehicular emissions will be evaluated. The implication of heterogeneous multi-class multi-criteria decision making on traveller information provision services with endogenous market penetration will be analysed.

- To derive a system optimised pricing policy in a multi-class, multi-criteria mixed SUE assignment problem. The relationship between derived optimal tolls
and weights in a bi-criteria model on controlling vehicular emissions will be studied.

- To propose an analytical methodology for evaluating the impact of the integration of the traveller information provision services, speed limit control, and tolling on vehicular emissions and congestion. A sensitivity analysis of strategic interactions among speed limit control, traveller information provision services and tolling will be carried out. The speed-varying marginal cost pricing in a mixed SUE problem will be derived.

The ultimate goal is to identify and seek optimal transportation strategies and policies which can maintain a sustainable transportation network with respect to controlling congestion and vehicular emissions.

1.3 OUTLINE OF THE THESIS

The remainder of this thesis is organised as follows:

Chapter 2 introduces the concepts and formulations of the conventional traffic assignment techniques, the land use-transport interaction, and emission estimation approaches used in this thesis. Chapter 2 subsequently presents a literature review of most applied transportation strategies and policies, which include road pricing, road network design, traveller information provision services, and speed limit control.

Chapter 3 develops a single-level minimisation model to determine time-dependent optimal tolls whilst considering the dynamic relationships between land use, transport, and environment as well as the maximum allowable link emissions from a system equity (SE) viewpoint. The traffic is assigned according to deterministic user equilibrium (DUE). Numerical studies are set up to illustrate the importance of considering land use, transport and environment over time in determining optimal time-dependent tolls. Some policy implications of implementing tolls are discussed in this chapter.
Optimisation frameworks for road network design are proposed in Chapter 4 considering the land use-transport interaction over time. Unlike existing models, the optimisation frameworks can determine the optimal designs automatically without trial-and-error once the objective(s) is/are clearly defined. Moreover, these frameworks allow the evaluation of the impacts of the optimal designs on the related parties including government, landowners, toll road operators, transit operators, and road users. This information can be used to help network planners and profit-makers with decision-making by the elimination of many alternative designs. Numerical studies are carried out to illustrate the models and demonstrate the impact of road network improvements on related parties. In particular, the impact on landowners in terms of their profits and the impact on the government in terms of vehicular emissions are shown. These are demonstrated under different network design schemes, through a simple example. However, these models can be applied to general networks.

In Chapter 5, Braess’ network is employed as a special case of road network design to analytically examine the occurrence of the emission paradox and the simultaneous occurrence of Braess’ and emission paradoxes. The objective of this chapter is to answer the following questions: 1. Does the emission paradox always occur? 2. Do Braess’ paradox and the emission paradox occur at the same time? 3. If not, under which condition(s) does the emission paradox occur (In Braess’ network example, some conditions under which Braess’ paradox occurs have been discussed by Pas and Principio (1997))? 4. If they both occur, does this simultaneous occurrence affect pricings? 5. Can traditional marginal cost congestion pricing remove Braess’ and emission paradoxes at the same time? In addition, this chapter proposes some marginal cost congestion and emission pricing policies and compare them with existing marginal cost of congestion and emission pricing proposed by Nagurney (2000a). Different toll charges derived from these pricing policies are compared. The goal is to seek a solution which can maintain a sustainable network regarding two conflicting objectives, alleviating congestion versus reducing vehicular emissions.

Chapter 6 studies travellers’ mixed SUE behaviour under the traveller information provision services with heterogeneous multi-class multi-criteria decision making. A multi-class, multi-criteria mixed SUE assignment model under the traveller information provision services is proposed. The model considers heterogeneous road travellers and
elastic market penetration. The impact of traveller information provision services and travellers’ multi-class multi-criteria route choice decision making on vehicular emissions and congestion is studied. The route choice behaviour of equipped and unequipped travellers is formulated as an optimisation program, in which the net economic benefit is maximised and the total generated emissions are constrained. The model has an interpretation from both economic and behavioural viewpoints. The solution of this program satisfies the logit-based SUE assignment. By deriving KKT optimality conditions from this program, the marginal cost pricing policy is obtained. This pricing is not only class-dependent but also link-dependent. It is demonstrated that marginal cost pricing is applicable in the network under the logit-based mixed SUE. At equilibrium this pricing can guarantee that the flow pattern is system optimised, but travellers still behave in a manner following the mixed SUE. The effectiveness of this marginal cost pricing on reducing vehicular emissions and congestion is investigated. The stochastic nature of the proposed mixed equilibrium model and how it results in a paradoxical phenomenon are discussed. Namely that this marginal cost pricing cannot simultaneously decrease vehicular emissions and total system travel time (TSTT). It is argued that this phenomenon is not a paradox but a pseudo paradox, as in the proposed mixed stochastic network loading model, travellers’ perceived travel times are the ones to be minimised not the measured TSTT. More discussion and investigation is carried out in this chapter.

Chapter 7 presents an analytical methodology for evaluating the impact of the integration of the traveller information provision services, speed limit control and tolling on vehicular emissions and congestion. A sensitivity analysis of strategic interactions among these is provided. The optimal speed-varying marginal tolls are derived in a transportation system where travellers follow a mixed SUE traffic assignment so that uncertainty in travellers’ behaviour is captured. The derived marginal tolls are dependent on the average speed, the speed limit, the congestion level and capacity of the link. Although an optimal toll with similar interpretation has been previously studied by Yin and Lawphongpanich (2006), it is argued that the differences between these two derived tolls lie in three aspects, which are the nature of the model, the equilibrium principle of traffic assignment, and the goal to limit total emissions. Finally, the convexity of the proposed model is discussed and the critical speed which dictates the convexity of the emission cost function is derived. It is pointed out that the
purpose of formulating the proposed model is to seek a balanced and sustainable solution with an optimal toll pattern which supports a mixed SUE assignment and simultaneously guarantees that the maximum allowable vehicular emissions are not violated.

The final chapter, Chapter 8, provides a general conclusion of this thesis and discusses the shortcomings of the research and some possible research directions for further study.
CHAPTER 2

LITERATURE REVIEW

2.1 TRAFFIC ASSIGNMENT TECHNIQUES

The purpose of this section is to provide a general review of the traffic assignment techniques utilised in this thesis. These techniques include Deterministic User Equilibrium (DUE), Stochastic User Equilibrium (SUE), System Optimum (SO), and System Equity (SE). This section introduces the concepts and the formulations of DUE, SUE, SO, and SE, which are the main principles characterising the network users’ route choice behaviour based on different assumptions. Detailed discussions on classical DUE, SUE and SO can be found in Sheffi (1985) and Patriksson (1994). In addition to traditional DUE and SO (referred to as Wardrop’s first and second principles), this chapter also reviews the extended DUE and SO proposed by Rilett and Benedek (1994), in which the generalised cost is a function only of a negative product rather than of travel time. This negative product may be air pollution, noise pollution, or some such consequence of vehicular traffic.

2.1.1 Deterministic User Equilibrium (DUE)

2.1.1.1 DUE proposed by Wardrop (1952)

The flow pattern is known as Deterministic User Equilibrium (DUE) and is used if all users in the network have complete and accurate information about the network and if the flow pattern is stable over time on a strongly connected network with strictly increasing congestion effects of all links. DUE is characterised by Wardrop’s first principle (Wardrop, 1952; Patriksson, 1994):

*The journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route.*
The traffic flows that satisfy this criterion are usually referred to as the DUE flows. In other words, at equilibrium, no one can reduce his travel time by unilaterally choosing another route.

In the past, several mathematical models have been proposed in order to precisely characterise the DUE principle. They can be broadly categorised into the following four types: the standard DUE model, the elastic demand DUE model, the queuing DUE model and the DUE trip distribution/assignment model. This section only provides a review on the formulations of the first model, as it is the focus of this study.

It is assumed that the OD demand $q^\alpha$ is fixed for any OD pair $rs$, and travel time of each link $t_a$ is a positive, separable, strictly increasing and continuously differentiable function of its own traffic flow $v_a$. The link flow pattern of DUE is a solution of the strictly convex optimisation model (Dafermos and Sparrow, 1969) as follows:

$$\min \sum_{a \in A} \int_0^{v_a} t_a(\omega)d\omega$$

subject to

$$v_a = \sum_{rs} \sum_p f_p^\alpha \delta_p^a, \forall a,$$  \hspace{1cm}  (2.1)

$$q^\alpha = \sum_p f_p^\alpha, \forall rs,$$  \hspace{1cm}  (2.2)

$$f_p^\alpha \geq 0, \forall p, rs,$$  \hspace{1cm}  (2.3)

where $f_p^\alpha$ denotes the traffic flow on path $p$ between OD pair $rs$. $\delta_p^a = 1$ if path $p$ using link $a$, and $\delta_p^a = 0$ otherwise. The first order optimal conditions of the above strictly convex optimisation problem (2.1)-(2.4) are expressed as follows:

$$f_p^\alpha (c_p^\alpha - \mu^\alpha) = 0, \forall p, rs,$$  \hspace{1cm}  (2.5)

$$c_p^\alpha - \mu^\alpha \geq 0, \forall p, rs,$$  \hspace{1cm}  (2.6)
where 
\[ c_p^\gamma = \sum_a t_a(v_a) \delta \mu^a, \forall p, rs, \] (2.7)

and \( \mu^\gamma \) is the Lagrange multiplier associated with equation (2.3). The first order optimality conditions (2.5) and (2.6) imply Wardrop’s first principle as follows:

\[ f_p^\gamma > 0 \Rightarrow c_p^\gamma = \mu^\gamma, \forall p, rs, \] (2.8)

\[ f_p^\gamma = 0 \Rightarrow c_p^\gamma \geq \mu^\gamma, \forall p, rs. \] (2.9)

Equations (2.8) and (2.9) state that: when the path carries flow, travel time on this path is equal to the lowest path travel time; when the path does not carry any flow, travel time on this path is greater or equal to the lowest path travel time.

The objective function (2.1) is a strictly convex function with respect to link flows, therefore the DUE link flows are unique. However, Sheffi (1985) indicated that the objective function is not strictly convex with respect to path flows, so there could be many solutions in terms of path flows.

2.1.1.2 DUE proposed by Rilett and Benedek (1994)

Rilett and Benedek (1994) proposed a new DUE assignment principle. They assumed that there is a negative product \( X \), where \( X \) may be air pollution, noise pollution, or a similarly undesirable consequence of vehicular traffic, the amount of which could be controlled through the use of a centralised Route Guidance System (RGS). If the network users choose their routes in order to minimise the amount of \( X \) they produce on their trip, the traffic assignment can be modelled by using a standard DUE traffic assignment model with \( X \) being the sole parameter in the generalised cost function instead of travel time.

By replacing the travel time of each link \( t_a \) in equation (2.1) with a cost function of \( X \), \( X_a(v_a) \), the newly proposed DUE conditions can be obtained as follows:
where $X_p^n$ is the measured amount of $X$ produced on path $p$ between OD pair $rs$; $\mu^n$ is the Lagrange multiplier associated with equation (2.3). Equations (2.10) and (2.11) state that: when the path carries flow, the amount of $X$ produced on this path is equal to the lowest amount of $X$ produced on this path, otherwise the path does not carry any flow. Rilett and Benedek (1994) indicated that although routing traffic based solely on individual drivers' minimising the amount of $X$ they produce is unrealistic, the solution provides a base condition that represents the situation where drivers care only about the pollution they produce.

### 2.1.2 Stochastic User Equilibrium (SUE)

In the DUE problem, it is assumed that each network user has accurate information about travel costs and that all network users are uniform and rational in their decision-making. However, in reality, the network users choose their routes based on perceived travel times rather than the actual travel time. The perceived travel times can be considered as random variables distributed across the population of users (Sheffi, 1985). Equilibrium will be reached when no road user can improve his travel time by unilaterally changing routes. This definition characterises the Stochastic User Equilibrium (SUE) condition.

The SUE is a more general statement of equilibrium than the DUE. If the perceived travel times are assumed to be entirely accurate, road users would perceive the same travel time and the SUE would be identical to the DUE.

According to Sheffi (1985), for given OD trip rates $q^n$, the SUE conditions can be characterised as follows:

$$f_p^n = q^n P_p^n, \forall p, rs,$$  

(2.12)
where \( v_a = \sum_p \sum_r f_p^a \delta_p^a \); \( P_p^a \) is the probability that route \( p \) between \( r \) and \( s \) is chosen given a set of measured travel times. It means, 

\[
P_p^a = P_p^a(t) = \Pr(C_p^a < C_q^a, \forall l \neq p | t),
\]

where \( C_p^a \) denotes perceived route travel time expressed as follows:

\[
C_p^a = c_p^a + \xi_p^a, \forall p, rs, \tag{2.15}
\]

where \( c_p^a \) is the measured travel time on path \( p \) between OD pair \( rs \); \( \xi_p^a \) is an additive random "error term". Equation \( (2.12) \) characterises the SUE condition, which means, at SUE, the travel time will be such that equation \( (2.12) \) is satisfied for the equilibrium path flows. These path flows, in turn, will be associated with link flows that satisfy equations \( (2.13) \) and \( (2.14) \) for the equilibrium travel times.

The distribution of the SUE flows depends on the probability distribution of the random variable, as well as on the characteristics of the utility functions (Sheffi, 1985). The logit-based SUE and probit-based SUE models are two specific discrete choice models arising from two different specifications of the error term. The study in this thesis employs and focuses on the logit-based SUE problem, therefore only a review on the logit-based SUE model is given as follows.

The logit-based SUE is derived from the assumption that the random variable \( \xi_p^a \) in the equation \( (2.15) \) is as follows:

\[
\xi_p^a = -\frac{1}{\theta} \xi_p^a, \forall p, \tag{2.16}
\]

where \( \xi^a \) are the random components assumed to be identically and independently distributed (i.i.d.) Gumbel variates, and the positive parameter is a constant used to scale the perceived travel time and calibrate the variance in the travel time perception.
The path choice probability in the logit-based SUE can be expressed as follows:

$$P^*_p = \frac{\exp(-\theta c^*_{p,rs})}{\sum_k \exp(-\theta c^*_k)}, \forall p,rs.$$  \hspace{1cm} (2.17)

Equation (2.17) can be further derived into:

$$P^*_p = \frac{1}{1 + \sum_{k \neq p} \exp[ \theta (c^*_{p,rs} - c^*_k)]}, \forall p,rs,$$  \hspace{1cm} (2.18)

Equation (2.18) means that the choice probability of a given path can be expressed as a function of the difference between the measured travel times of that and all other alternatives. The logit-based SUE path flow pattern can be obtained by substituting equation (2.17) or (2.18) into the SUE condition (2.12).

### 2.1.3 System Optimum (SO)

#### 2.1.3.1 SO proposed by Wardrop (1952)

Wardrop's second principle states: At equilibrium the average journey time is a minimum (Wardrop, 1952). This means that each network user behaves cooperatively in choosing his route to ensure the total system travel time is minimised. The traffic flow pattern satisfying Wardrop's second principle is generally considered as a system optimal (SO) one. Economists argue this SO flow pattern can be achieved with marginal cost pricing.

The SO assignment can also be formulated as a mathematical program. The feasible set of the SO flow pattern is the same as the one in the DUE assignment expressed in equations (2.2)-(2.4), but the objective function is to minimise total system travel time as follows:

$$\min \sum_{v \in a \alpha} t_v(v_v) v_v.$$  \hspace{1cm} (2.19)
The first order optimality conditions of the SO problem (2.19) and (2.2)-(2.4) are expressed as follows:

\[ f_p''(c_p' - \mu_p') = 0, \forall p, rs, \tag{2.20} \]

\[ c_p'' - \mu_p' \geq 0, \forall p, rs, \tag{2.21} \]

where \( \mu_p' \) is the Lagrange multiplier associated with equation (2.3), which is the lowest OD travel time at SO; \( c_p'' \) is the path travel time expressed as:

\[ c_p'' = \sum_a \bar{t}_a(v_a) \delta_a, \forall p, rs, \tag{2.22} \]

where

\[ \bar{t}_a(v_a) = t_a(v_a) + v_a \frac{dt_a(v_a)}{dv_a}, \forall a. \tag{2.23} \]

The first order optimality conditions (2.20) and (2.21) are similar to those in the DUE case. The difference lies in the link time function in equation (2.23) used to calculate the path travel time. Equation (2.23) represents the marginal contribution of an additional network user on link \( a \) to the total travel time on this link. It is composed of \( t_a(v_a) \) and \( v_a \frac{dt_a(v_a)}{dv_a} \), which are respectively the travel time that additional user experienced when link flow is \( v_a \), and additional travel time burden that this user imposes on all travellers on this link (Sheffi, 1985). It is noted that the SO assignment solution can be obtained by directly solving the DUE assignment model (2.1)-(2.4) if the link travel time function \( t_a(v_a) \) is replaced by equation (2.23).

### 2.1.3.2 SO proposed by Rilett and Benedek (1994)

Rilett and Benedek (1994) proposed a new SO assignment concept. If the objective of the RGS is to minimise the total amount of \( X \) produced, the objective could be achieved by giving explicit routes to the individual vehicles through the centralised RGS. The
traffic assignment can be modelled by using the classical SO concept in which the generalised cost is a function only of $X$ rather than of travel time.

By replacing the travel time of each link $t_a$ in equation (2.19) with a cost function of $X$, $X_a(v_a)$, the optimality conditions of the newly proposed SO can be obtained as follows:

$$ f_p^n (\bar{X}_p - \bar{\mu}^n) = 0, \forall p, rs, \quad (2.24) $$

$$ \bar{X}_p - \bar{\mu}^n \geq 0, \forall p, rs, \quad (2.25) $$

where $\bar{\mu}^n$ is the Lagrange multiplier associated with equation (2.3), which is the lowest amount of $X$ produced between OD pair $rs$ at SO; $\bar{X}_p^n$ is the amount of $X$ on path $p$ between OD pair $rs$ expressed as:

$$ \bar{X}_p^n = \sum_a \bar{X}_a(v_a) \delta_p^a, \forall p, rs, \quad (2.26) $$

where

$$ \bar{X}_a(v_a) = X_a(v_a) + v_a \frac{dX_a(v_a)}{dv_a}, \forall a. \quad (2.27) $$

The first order optimality conditions (2.24) and (2.25) are similar to equations (2.10) and (2.11) in the DUE case proposed by Rilett and Benedek (1994). The difference lies in the link cost function of $X$ in equation (2.27) used to calculate the amount of $X$ produced on the path $p$. Equation (2.27) represents the marginal contribution of an additional network user on link $a$ to the total amount of $X$ on this link. It is composed of $X_a(v_a)$ and $v_a \frac{dX_a(v_a)}{dv_a}$, which are respectively the amount of $X$ that additional user experienced when link flow is $v_a$, and additional burden of amount of $X$ that this user imposes on all travellers on this link.

### 2.1.4 System Equity (SE)
Rilett and Benedek (1994) proposed two system equity (SE) concepts in which the assignments of vehicles is to route traffic through a network based on the objectives of the residents of a neighbourhood rather than those of the system operators or drivers. The first SE concept corresponds to traditional assignment techniques that have an explicit link capacity constraint in which the link capacity is not a function of the number of vehicles on the link but rather the cumulative amount of pollutant \( X \) that the vehicles produce on the link. The second SE concept states that the vehicles may be assigned to a street network in such a way as to ensure the amount of \( X \) released on all streets (or a subset of streets) is the same. In the following, the mathematical interpretations of these two SE concepts are provided.

### 2.1.4.1 SE1 - Constraining the Negative Effects

The first SE concept implies that people living near major roadway corridors may wish that traffic is routed through a given network by the system operators in such a way that noise levels or vehicular emissions on the adjacent streets do not exceed some maximum safety standard (i.e. for health reasons). This can be expressed as follows:

\[
Q_{a,r}^X \leq \hat{Q}_{a,r}^X, \forall a,r,
\]

(2.28)

where \( Q_{a,r}^X \) and \( \hat{Q}_{a,r}^X \) are the amount of \( X \) produced and the maximum allowable amount of \( X \) on link \( a \) in period \( r \) respectively. Constraint (2.28) is a link constraint on the amount of \( X \), which is to ensure that the amount of \( X \) produced on link \( a \) in period \( r \) is less than or equal to the maximum allowable level on the link in that period.

By adding constraint (2.28) to the classical DUE, SUE and SO assignments, the optimal flow pattern not only satisfies the corresponding equilibrium conditions but also ensures that the amount of \( X \) produced on link \( a \) in period \( r \) does not exceed the maximum allowable level. It is noted that if the maximum allowable level is too low, there may be no solution to satisfy equilibrium conditions as well as ensure system equity (SE).
2.1.4.2 SE2 - Distributing the Negative Effects Evenly

In the second SE concept, it is assumed that vehicles are routed through the network (on the basis of the routes directly broadcast to the vehicles) such that no one group of people living near the traffic network is affected more than any other group of people. It means that as long as traffic will be travelling through a given area, the negative effects should be distributed evenly between corridors. Rilett and Benedek (1994) indicated that this may at first seem to be an extreme case, but there are currently a number of scenarios in which traffic control devices are operated such that the negative externalities of traffic (i.e. noise) are distributed as evenly as possible among competing routes. The mathematical expression regarding the second SE can be shown as follows:

\[
Q^X_{a,\tau} = Q^X_{b,\tau}, \forall a, b, \tau
\]  

(2.29)

where \(Q^X_{a,\tau}\) and \(Q^X_{b,\tau}\) are the amount of \(X\) produced on links \(a\) and \(b\) in period \(\tau\). Constraint (2.29) is to ensure that the amount of \(X\) produced on link \(a\) in period \(\tau\) is the same as the one on link \(b\) in period \(\tau\). At equilibrium, the traffic is assigned to ensure the amount of \(X\) is distributed evenly over the network. It is noted that there may be no solution if the travel demands on two competing or adjacent links \(a\) and \(b\) are considerably different.

2.2 LAND USE-TRANSPORT INTERACTION

Firstly, this section introduces the concept of land use-transport interaction as well as linked and integrated land use-transport models proposed in the last few decades. Subsequently, a general description of a typical integrated model which reflects land use-transport equilibrium is given.

The concept of land use-transport interaction adds a new aspect to the relationship between supply and demand (Cabanellas et al., 2004). Land use and transport interact until supply-demand equilibrium is reached. Martinez (2002) indicated that total benefits generated by a transport project can be estimated correctly only if the travel-
demand model properly forecasts the combined land use and transport system equilibrium, i.e. the travel-demand model incorporates all technological and access effects, as well as land use-transport feedback.

Webster et al. (1988) pointed out that the intricate connection between land use and transport hints that any policy involving the land use and transport development will inevitably affect the other one. There are four relationships:

1. The way transport changes affect travel characteristics;
2. The way changes in land use factors affect other land use factors;
3. The way land use changes affect travel characteristics, and;
4. The way transport changes affect land use factors.

Since changes in the land use pattern lead to changes in travel behaviour, the land use and transport systems interact through the mechanism of feedback between the two systems.

Simmonds and Banister (2007) proposed that “land use” is related to “transport” for at least three reasons as summarised below:

1. Activities and the interactions between them generate the demands for transport;
2. Those activities and interactions are to a greater or lesser extent influenced by the availability of transport;
3. The linkages between transport and activities may be important to the appraisal of transport strategies, especially when trying to consider whether the transport system is providing the kinds of accessibilities that activities (i.e. people and businesses) require, rather than simply providing mobility.

They also pointed out that it is important to recognise that the “land use” system is never static, and that “transport” is only one of the factors that influence how it changes.

The Bureau of Transport Economics (1998) reviewed four broad types of urban transport models. These are conventional four-step transport models, behavioural travel-demand models, linked land use-transport models, and integrated urban land use-transport models. It is indicated that the evolution of these models from the traditional
four-step approach to linked, then to the integrated approach, represents an increasing understanding of the urban transport system and its interaction with the land use system. As the focus is on land use-transport interaction, this section only gives a brief summary of their review on linked and integrated land use-transport models. The discussions below (Sections 2.2.1 & 2.2.2) are extracted from the Bureau of Transport Economics (1998).

2.2.1 Linked Land Use-Transport Models

Much attempt was made in the 1970s to link transport models with land use models, which results in the development of so-called “linked land use-transport models”. Examples of such models include the SELNEC model (Wilson et al. 1969; SELNEC, 1971, 1972) and the works of Putman (1973, 1975a, b, c) and Echenique et al. (1973). The main characteristic of these models is that the transport model no longer treats land use variables as exogenous and the feedback from the transport system to the land use pattern is explicitly recognised. De la Barra (1989) refers to these types of models as linked land use-transport models. These models incorporate the effect of transport costs on the location decisions of households and firms. However, they fail to link, as in the case of the traditional four-step approach, total travel demand directly to travel costs.

From the generalised cost calculations, two main feedbacks are incorporated into the model structure: one goes back up to the trip distribution stage in an instantaneous way, and the other back to the location of activities in a lagged fashion through the notion of accessibility. Inclusion of feedback mechanisms in the linked system is an important improvement over the conventional four-step models which assume a uni-directional causal relationship.

Nevertheless, the linked land use and transport model suffers from a key weakness. The model still treats travel demand at the trip generation stage as being inelastic (unresponsive to) with respect to travel cost. While this may be true for journey-to-work trips, the assumption cannot be applied to other types of trips such as shopping and recreational journeys. The demand for these trips is likely to be cost-sensitive. De la Barra (1989) discusses other problems associated with the linked approach. One is the redundancy of the trip distribution model in the transport system. Another problem is
the use of a simple average method in the calculation of generalised composite (mean) costs.

2.2.2 Integrated Land Use-Transport Models

Integrated land use-transport models began to gain popularity in the 1980s. Their development represents increasing recognition of intricate connections between land use and transport systems, and of the corresponding need to model these systems in a fully integrated way. A key feature of the integrated approach is that travel behaviour is modelled as a response to price signals, namely transport costs. The integration is achieved by explicit recognition of the two-way interaction between land use and transport systems, as well as by incorporating a wide range of theories (such as microeconomic, entropy or information, random utility, time geography, economic and welfare economics theories) and modelling techniques (such as spatial interaction, random utility and input-output models, and mathematical programming). Webster et al. (1988) provided a summary of the various theories and techniques. The integrated approach currently constitutes the state of best practice in urban transport modelling and has been extensively used in transport planning and policy analysis, although scope exists for further improvement.

Some integrated and empirically applied land use-transport models developed over the past 15 years or so were reported by the International Study Group on Land Use-Transport Interaction (ISGLUTI) (Webster et al., 1988). The ISGLUTI study, coordinated through the British Transport and Road Research Laboratory, carried out comparisons of nine different land use-transport models, representing an important milestone in research on integrated models.

Integrated land use-transport models can be classified into two fairly distinct groups: predictive models and 'optimising' (or normative) models. Predictive models are based on a set of behavioural relationships. They are concerned with explaining the changing patterns of the land use and transport systems, and with predicting or assessing the impacts of a change in exogenous variables or in policies imposed on these systems. Such models include BOYCE et al., CATLAS/NYSIM/METROSIM, ITLUP, MASTER, and MEPLAN, etc.
Optimising models aim to map out those land use configurations which will optimise some community objective or a set of objectives. They are designed to evaluate a particular policy or a set of policies in terms of its effect on an objective function (Webster et al., 1988). Such models include TRANSLOC and TOPAZ, although the distinction is less clear in practical applications because of the incorporation of some behavioural elements in the optimising models. For a more detailed and systematic review of integrated land use-transport models, one can refer to Webster et al. (1988) and Southworth (1995).

Figure 2.1 provides a schematic representation of an integrated urban land use-transport model. The model consists of two sub-models: one for the land use system and the other for transport. The representation of the land use system is rather simplified, focusing largely on components that interact directly with the transport system. Land use sub-models are intended to explain how spatial choices are made for residential and employment locations. These are basically stipulated as a function of, among other things, locational accessibilities, which in turn depend on zonal attractiveness and travel costs. The spatial distributions of residents and firms are assumed to create major demand for travel, which drives the development of the transport system. The interplay of demand with supply through transport costs forms the nucleus of interconnected causes and effects within the transport system. The land use and transport systems are integrated through a mechanism of feedbacks between the two systems. The land use system supplies the transport system with estimates of the location and volume of travel generators. The transport system affects the land use system through the notion of accessibility, often in a temporally lagged manner. As an integral part of such accessibility, changes in travel costs become part of the mechanism used to relocate labour, residence, and other urban economic activities.

Integrated land use and transport models such as the one illustrated in Figure 2.1 have wide applications in planning and policy analysis, which can be used to predict future land use patterns and transport demand by introducing changes in exogenously determined variables such as population and basic employment. Common policy applications can be classified into three broad categories: regulatory, pricing and investment policies (Webster et al., 1988). Regulatory policies include those that regulate the use of space or time. Pricing policies are those which directly affect the
price of land, buildings or transport. Investment policies are those which directly affect the capacity of the transport network, and hence travel speeds.

**Figure 2.1 Schematic representation of an integrated urban land use-transport model.**

Source Adapted from Southworth (1995).

The general framework described in Figure 2.1 can be modified or extended to evaluate policies for particular objectives, such as those to limit peripheral urban development, to encourage use of public transport and reduce car dependence, and to conserve resources such as time or energy. The model could also be coupled with a detailed economic module to assess the welfare implications of various policies.
2.3 EMISSION ESTIMATION APPROACHES

In the last three decades, different models have been developed to estimate emissions for given traffic levels and characteristics. The most broadly applied model employs information on the average speed as a proxy for driving cycle information. At the opposite end of the spectrum are instantaneous models, which simulate emissions on a second-by-second basis using precise driving-cycle information. In the middle ground are a number of models that integrate some queuing information (Anderson et al., 1996). The study in this thesis adopts the two best known and most widely used models of the category of average-speed models, which are Mobile Source Emissions Factor Model or MOBILE and TRANSYT-7F model.

2.3.1 MOBILE Model

The model, known as Mobile Source Emissions Factor Model or MOBILE, was originally developed by the U.S. Environmental Protection Agency (USEPA) in 1978 to estimate the amounts of emissions from motor vehicles. It is the primary emissions factor tool employed by national, state, and local agencies to estimate on-road mobile-source emissions, make policy decisions, assess associated environmental impacts and generate national, regional, and urban emissions inventories (TRB, 2000). The model estimates carbon monoxide (CO), hydrocarbons (HC), and nitric oxide (NO) emissions as a function of a number of factors, with vehicle speed and vehicle-miles-travelled (VMT) as the only traffic measures.

The MOBILE model was developed based on laboratory-based dynamometer test cycles (USEPA, 1999). Anderson et al. (1996) stated the test as follows: For each category of vehicle, the model estimates generated emissions over a trip of a given length. The estimate depends upon the average speed over the trip and a set of factors that might affect emissions, including air temperature and vehicle-fleet characteristics. MOBILE can account for the variations in the rates of emissions of most pollutants over three periods of operation: the period following a cold start, the period following a hot start, and the period of hot stabilised operation. It also accounts for hot-soak emissions.
following a trip, and, in the case of HC, for crankcase as well as tail-pipe emissions. They also pointed out that while the method of calculating emissions in MOBILE is trip based, its outputs are emissions factors, which are defined in terms of emissions in grams per unit of distance travelled. This is done by assuming a systemwide distribution of trip lengths and a percentage of trips beginning with cold starts (Anderson et al., 1996).

According to Hallmark (2004), mobile source emissions (typically in grams) can be estimated by multiplying vehicle activity by emission rates which can be exclusively provided by MOBILE model. The expression is as follows:

\[ Q^t_a = h^t_a(\bar{s}_a)v_a, \forall a, k, \]  

(2.30)

where \( Q^t_a \) is total emissions per unit of time of pollutant \( k \) from traffic travelling along link \( a \); \( \bar{s}_a \) is the average speed of travel on link; \( h^t_a(\bar{s}_a) \) is emission factor of link \( a \) generated by MOBILE model as a function of \( \bar{s}_a \); \( v_a \) is the traffic volume on link \( a \) per unit of time. Once the link characteristics and average driving speed are identified, the link emissions factor is given. The aggregate emissions for the entire area can be calculated by summing across links as follows:

\[ Q^k = \sum_a h^t_a(\bar{s}_a)v_a, \forall k. \]  

(2.31)

Equation (2.31) calculates the total emissions per unit of time of pollutant \( k \) over the entire network.

2.3.2 TRANSYT-7F Model

TRANSYT (TRAffic Network StudY Tool) is a macroscopic optimisation and simulation tool which was originally developed by the United Kingdom Transport and Road Research Laboratory (TRRL) in the late 1960s and has undergone several revisions since then (Robertson, 1969; Chowdhury and Sadek, 2003). TRANSYT-7F is
a U.S. version of TRANSYT developed by the University of Florida for the Federal Highway Administration (FHWA) in the late 1970s and early 1980s.

TRANSYT-7F has incorporated its own emission estimation models and is capable of network-level analysis like many traffic simulation and optimisation models, such as Synchro/SimTraffic, INTEGRATION, and Paramics which, however, belong to the domain of microscopic models (Federal Highway Administration, 2008). The TRANSYT model has two main features: the simulation of traffic flow and the optimisation of traffic signal plans. In terms of simulation, TRANSYT considers platoon dispersion for its computations. It models platoons of vehicles rather than individual vehicles. This function allows TRANSYT to very realistically model traffic flow on arterials and in road networks and to estimate a number of important traffic flow performance measures which include delay, stops and queuing, fuel consumption-pollutant emissions, and travel time (Chowdhury and Sadek, 2003). Among these measures, the emission rate of pollutant \( k \) is estimated using a macroscopic relationship and is expressed as follows:

\[
ROP_{a,j} = \frac{A^{i}e^{B^{i}v_{a,j}}}{C^{i}\bar{s}_{a,j}}, \forall a, j, k,
\]

(2.32)

where \( ROP_{a,j} \) is the production rate of vehicular emissions of pollutant \( k \) (grams/veh ft) on link \( a \) for vehicle type \( j \); \( A^{i} \), \( B^{i} \) and \( C^{i} \) are constants for measuring the emissions of pollutant \( k \). \( \bar{s}_{a,j} \) is the average speed of vehicle type \( j \) on link \( a \) (ft/sec), which is obtained by dividing the link distance \( l_{a} \) with the average travel time of vehicle type \( j \) on link \( a \).

The generated emissions of pollutant \( k \) for each vehicle of type \( j \) traveling on link \( a \) can be expressed as follows:

\[
Q^{i}_{a,j} = ROP_{a,j}l_{a} = \frac{A^{i}e^{B^{i}v_{a,j}}l_{a}}{C^{i}\bar{s}_{a,j}}, \forall a, j, k,
\]

(2.33)

where \( l_{a} \) is the link length in feet. The overall vehicular emissions can be calculated as follows:

\[
Q = \sum_{j} \sum_{a} \sum_{k} Q^{i}_{a,j}v_{a}.
\]

(2.34)
The overall emissions $Q$ can be obtained as in equation (2.34) by summing the emissions of all the pollutants for all vehicle types across the network.

2.4 LITERATURE REVIEW OF TRANSPORTATION POLICIES AND STRATEGIES

This section presents a literature review of most applied transportation strategies and policies, which include road pricing, road network design, traveller information provision services, and speed limit control. These strategies and policies are implemented to mitigate traffic congestion and assist traffic management in order to increase transport system efficiency and achieve specific planning objectives. However, these strategies and policies may result in unexpected outcomes, which may have considerable impact on transportation, land use as well as environment. The review provided in this section is, therefore, focused on evaluating these strategies and policies pertaining to these issues.

2.4.1 Road Pricing Problems

The concept of road pricing needs no introduction. It is supported by a long history of economic and transportation analyses (e.g. Pigou, 1920; Knight, 1924; Walters, 1961; Vickrey, 1963; Newbery, 1990; Button and Verhoef, 1998; Verhoef, 2000; Hau, 2005a and Hau, 2005b), pointing to its potential benefits of travel time minimisation, economic benefit maximisation, market efficiency related to externality pricing, cost recovery, and effective management of transportation demand amongst others. Some of these perspectives are summarised in Lo and Hickman (1997). Some surveys of road pricing include the research of Morrison (1986), Hau (1992), Lewis (1993), Arnott et al. (1994), Johansson and Mattsson (1995), and Verhoef (1996).

Previous road pricing studies can be broadly classified into three types: Empirical, theoretical, and simulation studies. Empirical studies (e.g. Newbery, 1988; Phang, 1997; O’Mahony, 1999; O’Mahony et al., 2001; Lee, 2002; Dickson, 2006; Eliasson and
Mattsson, 2006; De Palma et al., 2006; Bureau and Glachant, 2008; Kunchornrat et al., 2008) focus on the lessons learnt from toll policy implementations and its impacts to society, economy and environment. Theoretical studies focus on the economic theory of the first best pricing problem (e.g. Vickrey, 1969), the cordon-based network congestion pricing (e.g. Zhang and Yang, 2004; Ho et al., 2005), road pricing under the dynamic traffic assignment framework (e.g. Yang and Huang, 1997; Lo and Szeto, 2005), developing efficient solution algorithms for toll design problems (Meng et al., 2001; Sumalee, 2005), and the impact of road pricing on the transport network reliability (e.g., Chan and Lam, 2005). Simulation studies (Miyamoto et al., 1996) can simulate the probable results of a given road pricing strategy while covering a lot of aspects like land use considerations, emissions, and very detailed land-use transport interaction, but do not prescribe what the strategy ought to be. Some recent research focusing on the analysing the impact of road pricing on land use is broadly reviewed as follows.

Lam et al. (1996) indicated that the conventional approach aimed to produce comprehensive, long-term plans for land use and transport in considerable detail, but tended to ignore the role of road pricing policy, thus ending up with solutions that might not be efficient or economical. They presented a computer analysis system (or model) which enables the analysis of coordinated tunnel toll pricing policies by optimising an 'objective function' while satisfying the associated and other constraints. The possibility of integrating the optimal road pricing policies in the land use and transport planning were discussed. A case study based on Hong Kong data demonstrated the efficiency of optimising tolls on two of the three harbour crossing tunnels in Hong Kong.

De Palma et al. (2006) provided a comprehensive study on modelling of urban road pricing and its implementation. The study is focused on the design of urban road pricing schemes, and their spatial and temporal impacts, using quantitative transport (and land use) models. The policy implications of road pricing, including welfare and equity aspects, were studied for Paris, Brussels and Oslo using state of the art planning models. They also gave a review on the future prospects of road pricing.

Gupta et al. (2006) examined the traffic, land use and welfare impacts of road pricing in the Austin (Texas, USA) region, including the introduction of planned toll roads, bridge tolls, and a downtown cordon toll. Different tolling strategies were also studied,
including fixed versus variable toll rates. Austin-calibrated DRAM-EMPAL models were used to predict future household and job distributions. Results included traffic redistribution over space and time, long-term location choice changes, and traveller welfare implications. In addition, the bridge tolls were expected to successfully redistribute traffic, while the downtown area appeared highly sensitive to cordon tolls.

Martin (2006) suggested that road pricing is closely related with several other important areas of public policy including land use and development, climate change, the links between local traffic conditions, local communities, and other transport policies, modes and funding. The impacts on other forms of transport from the road user charging (RUC) schemes are of great importance because a well-planned and implemented scheme can generate increased demand for public transport, walking and cycling, and an overall reduction in demand for road use.

McDonald (2008) presented a model of a monocentric city with traffic congestion to study the impact of road pricing on land use. He showed that policies, such as charging time-of-day tolls to shift the rush hour to other times, can reduce congestion costs and the amount of population redistribution needed to achieve an efficient population distribution.

To date, little attention was focused on incorporating environmental and land use considerations into analytical road pricing models. In fact, toll charging can reduce the traffic level and hence emissions. In the long run, tolls will alter the travellers' residential locations. Recently, May et al. (2005) and Vold (2005) carried out studies on defining optimal land use transport strategies whilst considering its impacts on vehicular emissions and their environmental costs. However, they did not examine the impact of controlling maximum allowable link emissions on toll charges. Even if the overall vehicular emissions are under control, emissions on some links may exceed their acceptable levels, which are harmful to human health. In addition, the population is changing over time. It is important to consider the time dimension in analytical toll design models in additional to land use and the environment.
Recently, much research has been carried out to analyse the implication of road pricing on vehicular emissions. Some were focused on empirical studies; others were focused on theoretical analysis. A review regarding these two aspects is provided as follows.

**Empirical Studies**

Beamon (1996) developed a model to quantify the reductions in congestion and automobile emissions resulting from two types of road pricing implementations: a Five-Dollar Cordon Scheme and a One-Dollar-Per-Mile Area Link Charging Scheme. The policy issues and possibilities for implementation of road pricing in a democratic society were also investigated. The results showed that both schemes were also found to reduce the amount of automobile emissions.

Mayeres *et al.* (1996) estimated the marginal external costs of urban transportation. These included the marginal external cost of congestion, accidents, air pollution and noise. The costs were computed for cars, buses, trams, metro, and trucks. They addressed three important points. Firstly, marginal external costs are always computed for a given economic equilibrium. Due to the implementation of social cost pricing, the marginal external costs change if the economic equilibrium changes. What is needed is, therefore, a marginal external cost function, rather than a point estimate of the external cost in the present equilibrium. Secondly, simply charging consumers the ‘equilibrium’ social cost of car and truck use per km is not necessarily the best pricing principle. A different equilibrium will imply a different level of external costs, hence the importance of external cost functions, rather than point estimates. This calls for external cost information expressed as a function of the externality problems themselves (e.g. per gram of a pollutant) rather than per vehicle km. In addition, it needs to be emphasized is that external costs are, by definition, costs borne by others. They applied the methodology to the urban area of Brussels for the year 2005. They pointed out that it is very important to realise that the marginal external cost estimates in their paper are only valid for the particular transport situation in Brussels in the year 2005 under unchanged policy conditions.

Such measures are important for policy makers when deciding about public investments and policy instruments in order to regulate environmental impacts, e.g. from road transportation and industry. They found out that the mean willingness to pay (WTP) for a 50% reduction of harmful substances where the respondents live and work was about 2000 SEK/year, which is of the same order of magnitude as earlier stated preference studies in Nordic countries. Most parameters in the econometric analysis had the expected sign. WTP was increasing in income, wealth and education; it was larger for men, members of environmental organisations, people living in big cities (which are on average more polluted), and people who own their house or apartment. It was lower for retired people. However, the additional WTP for people in big cities, although significantly higher than for other people, was lower than expected, indicating a possible insensitivity-to-scope effect.

Nash et al. (2001) examined the implications of the valuation of air pollution and other externalities of transport infrastructure use in the context of the European Commission’s proposals for pricing based on marginal social costs. The aim was to examine the implications for transport prices, and hence transport demand and air pollution from the transport sector, by comparing existing variable taxes and charges with forecasts of marginal costs for different passenger and freight modes in 2010. The main finding was that economically efficient prices will only have a positive impact on transport emissions in a limited range of transport contexts.

Proost and Van Dender (2001) computed four different marginal external costs in the present equilibrium: air pollution, accidents, noise, and congestion. The gap between marginal social costs and prices showed that congestion and unpaid parking were the dominant sources of inefficiencies. Air pollution costs were significant as well. The effects of a typical air quality policy (regulation of car emission technology) and two typical fuel-based policies (minimum fuel efficiency policy and fuel taxes) were compared with the effects of three alternative transport policies (full external cost pricing, cordon pricing, parking charges). Regulation of emission technology and of fuel efficiency beyond the 2005 levels did not lead to welfare gains, whereas transport pricing policies yielded substantial gains for the urban area under study.
Proost et al. (2002) analysed the gap between present transport prices and efficient transport prices. Efficient transport prices are those prices that maximise economic welfare, including external costs (congestion, air pollution, accidents). The methodology was applied to six urban and interregional case studies using one common optimal pricing model. The case studies covered passenger as well as freight transport and covered all modes. They found that prices need to be raised most for peak urban passenger car transport and to a lesser extent for interregional road transport. Optimal pricing results for public transport were more mixed. They also showed that current external costs on congested roads are a bad guide for optimal taxes and tolls: the optimal toll that takes into account the reaction of demand is often less than one third of the present marginal external cost.

Ubbels et al. (2002) discussed the potential environmental effects of a kilometre charge for car traffic in the Netherlands. They pointed out that this kilometre charge would replace the existing taxes on new cars and on car ownership. It would lead to a substantial increase in the variable costs of car use. It may lead to a doubling of these costs while at the same time the average costs of car use would not increase because the fixed taxes are strongly reduced. Four alternatives for the kilometre charge were formulated. These were estimated to lead to substantial reductions of energy and certain emissions. This study carried out for the Dutch situation showed that the introduction of a differentiated kilometre charge can lead to a reduction of environmental pollution caused by car traffic (depending on the kind) with 20% till 70%. In addition, a significant decrease of congestion on roads can be expected.

Mayeres et al. (2004) tested two archetypes of pricing rules: marginal social cost (MSC) pricing and average cost (AC) pricing and illustrated the direct effects of MSC pricing and compared them with the direct effects of AC pricing. They also claimed that a full evaluation of the efficiency and equity impacts of transport policies requires a general equilibrium rather than a partial equilibrium approach. They then presented results obtained with a computable general equilibrium (CGE) model for Belgium. However, they noted that the model only considers the time costs of congestion. The effects of congestion on the emission factors or the accident risks are not yet incorporated.
Begg and Gray (2004) and Chatterjee and Gordon (2006) each demonstrated research studies of analysing the impact of road pricing in UK. Begg and Gray (2004) indicated that the introduction of a national road charging scheme would reduce CO₂ and further reduce local air pollution. Chatterjee and Gordon (2006) found out that congestion and carbon dioxide emissions are most effectively limited with congestion-based road charging.

Parry and Small (2005) derived the second-best optimal gasoline tax, disaggregating it into components that reflect external costs of congestion, accidents, and air pollution (local and global), as well as a “Ramsey tax” component that reflects the appropriate balance between excise taxes and labour taxes in financing the government’s budget. They applied the formula to the US and UK based on a detailed assessment of evidence on underlying parameter values, thereby illustrating why, and to what extent, the optimal tax may differ across countries, and under what circumstances, if any, current rates might be justified.

Beevers and Carslaw (2005) presented a comprehensive analysis of the impact of the London congestion charging scheme (CCS) using detailed traffic data, combined with the Environmental Research Group’s road traffic emissions model. Several important results are apparent from the analysis of the effect of the London congestion charging scheme. They found out that the reduction in congestion led to a significant effect associated with increases in vehicle speed and that the changes at slower speeds disproportionately affected vehicle emissions. Additionally, there was a reduction in emissions of CO₂, providing evidence that a scheme of this kind would assist in attaining government targets relating to climate change.

Mitchell et al. (2005) applied traffic assignment, pollutant emission, and dispersion models to a major UK city so as to assess the air quality impacts of five road pricing schemes. Schemes were evaluated with reference to: exceedence of air quality standards for six pollutants; greenhouse gas emission; redistribution of pollution, an environmental justice concern; and road network performance as traffic speed and trip distance. They compared results to alternatives of do nothing, network development and clean fuel promotion. The air quality benefits of a modest distance-based charge were highlighted. However, whilst road pricing showed potential as an air quality
management tool, its value and suitability were strongly sensitive to prior air quality and emission source apportionment in the application city.

De Palma and Lindsey (2006) have studied the impacts of road pricing in Paris. The morning peak is studied using METROPOLIS. Time-independent tolls on selected links, time-varying cordon tolls, and a network-wide toll proportional to travel time are considered. They found out that welfare gains from the link and cordon tolls are relatively small. The comprehensive travel-time-based toll yields much higher benefits. In all cases, benefits to users amount to a large fraction of toll revenues and exceed the monetised value of reductions in noise, accidents and vehicle emissions.

Theoretical Analysis

Johansson (1997) derived optimal speed-dependent charges with respect to congestion, emissions, and the corresponding excessive fuel consumption and emissions due to the congestion. The optimal road-charge was derived in a standard cost-benefit framework, based on the Hicks/Kaldor efficiency criteria, in which they explicitly maximised the net benefit per time unit. The importance of the derived general tax charge is that it is expressed as a function of the speed, and not, as in most studies, as a function of the flow, which simplifies the interpretation. It was shown that a road-user should pay a charge corresponding not only to its own emissions, but also to the increased emission and fuel consumption of other road-users. It was demonstrated in a numerical example that these 'system effects' may be significant. Nevertheless, he pointed out that only the static deterministic equilibrium solution for a given infrastructure was considered. All the obviously important dynamics and uncertainties of the system were neglected.

Johansson and Sterner (1998) discussed the scope for environmental road pricing in the future and focused on a number of important factors and trends which individually point in various directions. They pointed out that the interest in pricing traffic efficiently has been spreading to other areas such as health effects, regional environmental effects, global warming, noise, barrier effects, road damage and accidents (e.g. De Borger et al., 1996; Kageson, 1993; Maddison et al., 1996; Mayeres, 1993; Newbery, 1988; Rothengatter, 1994; Verhoef, 1994). This is because of a general increased environmental awareness and the fact that modern information technology has made
various road-pricing systems realistic possibilities, at least in the near future. Still, each
element in the optimal externality correcting tax has typically been treated separately.
They also pointed out that, in order to obtain a first-best solution, it is necessary to
differentiate the charge perfectly with respect to the variables which determine all the
other external costs, such as congestion, noise, external accident costs, barrier effects,
road wear and tear, and dirting.

Calthrop and Proost (1998) addressed three important issues. Firstly, empirical work is
still necessary to better identify marginal external costs, including congestion, accident
and environmental costs. Secondly, any assessment of policy options should treat
externalities simultaneously. The use of pricing instruments and emissions standards
were discussed within their framework. Thirdly, they emphasised the role of
government. Designing the optimal road-pricing institutions requires consideration of
horizontal and vertical tax competition, while double-dividend arguments are central to
the question of securing public support.

Verhoef (2000) discussed a number of issues that will become increasingly important
now that the concept of marginal external cost pricing becomes more likely to be
implemented as a policy strategy in transport in reality. One of these important issues is
that factors affecting the shape and position of the marginal inter-sectoral external cost
curve are those factors that determine the emissions of pollutants or noise per vehicle
kilometre. To demonstrate the long-run optimality of marginal external cost pricing,
three simple models were considered. One of them is with factors behind the marginal
environmental cost curve. He recalled that transport externalities include a large variety
of effects – congestion, emissions, noise annoyance, and accidents – optimal individual
charges should therefore vary at least according to the following dimensions: the used
vehicle (technology), the actual state of this vehicle, the kilometrage, the time of driving,
the place of driving, the actual route chosen, and the driving style.

Nagurney (2000a) presented models, accompanied by analyses, and numerical
illustrative examples which, through a variety of policy instruments, guarantee that the
transportation network in question will be sustainable. It means that given the behaviour
of the travellers on the networks, the environmental goals, specifically maximum
vehicular emissions standards, can be achieved. The vehicular emissions are calculated
using the emission estimation approach in MOBILE model. She derived optimal tolls and proposed emission pricing policies for transportation networks to be sustainable, in which travellers behave in user-optimising and system-optimising fashions. She pointed out the work is the first to develop an integrated theory of sustainable transportation networks and establishes the foundation for this important area of research and practice.

Calthrop and Proost (2002) examined environmental pricing, defined broadly to include emission taxes, product taxes and subsidies and compared it to alternative approaches. They used a simple formal model to demonstrate the relative efficiency properties of pricing solutions, before highlighting some of the associated implementation problems. They also extended the scope and realism of the analysis by examining the choice between regulatory instruments in the presence of several market distortions. The basic case for environmental taxes, set at the correct level, was shown to remain. Although it is impossible for them to cover all modes and all real world case studies of pricing. They illustrated the central point in the context of air pollution from car use and concluded that the essential insight from the model applies to a whole range of environmental problems in the transport sector. They also mentioned that a general introduction to the choice of regulatory instruments to tackle environmental damage can be found in Kolstad (1999).

Mayeres (2002) described that Fullerton and West (2002) investigated the extent to which the optimal Pigouvian pollution tax can be mimicked by a tax on fuel and on car characteristics such as engine size, vintage, or the absence of pollution control equipment. They find that 71% of the welfare gain under the Pigouvian tax can be realised with a combined tax on size, fuel and vintage; 62% is obtainable via a fuel tax alone.

Johansson (2006) used a stylised model to derive a first-best road charge with respect to different possible externality components. Although the model captures nothing of the large heterogeneity over time, in space and between people that are essential to any road pricing system in practice, the main conclusions did not depend on these simplifying assumptions. It was shown that not only should road users pay for the direct time and environmental costs that they impose on other road users and other people; they should also pay a charge corresponding to the increase in others' fuel costs and wear-and-tear
costs. Moreover, they should also pay for the increase in others’ environmental charges, since other cars will be more polluting, and their damage per emission unit will be higher due to increased population density, when the traffic increases. He pointed out that this derived optimal first-best road charge has never been shown in a utility-theoretic model before. One can of course argue that practical road-pricing systems must reflect trade-offs between allocative efficiency and simplicity, and can hence not take into account all theoretical subtleties. It is nevertheless valuable to know the theoretical benchmark solution before making all necessary simplifications.

Yin and Lawphongpanich (2006) derived optimal tolls which can be used to internalise congestion and emissions externalities simultaneously. They formulated a model, in which the objective of the problem was to minimise a weighted combination of congestion and traffic emissions. Emissions were estimated using TRANSYT-7F model. The derived optimal toll is reminiscent of the expression in Johansson (1997) and suggests that a road user should pay a charge corresponding not only to the traffic emissions he generates, but also to the increased emissions and travel time of other users. It should be noted that, under the derived optimal pricing, the travel flow pattern is system-optimised. In addition, uncertainty of travel behaviour was not captured.

2.4.2 Road Network Design Problems (NDP)

Transportation infrastructure is in an active phase of planning and development in many parts of the world, especially in some major cities in Asia and Europe. Transportation projects are expensive. Due to constrained government expenditures, they must be carefully scrutinised for cost-effectiveness (Szeto and Lo, 2006). Traditionally, the analysis involved belongs to the discipline of road network design. In the past, much research (e.g. LeBlanc, 1975; Boyce and Janson, 1980; Marcotte, 1986; Chen and Alfa, 1991; Friesz et al., 1993; Davis, 1994; Meng et al., 2001; Chen and Yang, 2004; Chiou, 2004) has been performed on the static approach to the discipline.

Yang and Bell (1998) provides a comprehensive review on the static approach to this discipline. Recently, researchers have considered the time dimension of transport network design. Three time scales are typically considered in the literature: seconds,
days, and years. The smallest time scale (e.g. Heydecker, 2002) is used to capture the within-day dynamics such as queuing phenomena, the fluctuation of demand within a day, and the departure choice of travellers. The medium scale (e.g. Friesz and Shah, 2001) is used to capture the route adjustment behaviour of travellers from day to day. The largest scale (e.g. Szeto and Lo, 2006) is used to capture the changing demand, gradual network upgrades, and cost and benefit over a long period of time. This is to maintain a similar social equity level over years, and to determine the optimal infrastructure improvement timetable, and its associated financial arrangement and tolling scheme.

All the previous efforts on transport network design, however, were focused on the transport system alone and tend to ignore the interaction between land use and transport. Although Shan and Gao (2000) proposed a combined bilevel model of urban land use and transportation network design, their model is static without considering the land use-transport interaction over time. In reality, the transport system interacts with the land use system. When a new road is built or an existing road is widened, the travel costs between some zones decrease, and hence the accessibilities for those zones increase. Increases in the accessibilities lead to changes in population and employment distributions, and in turn a new travel demand pattern. The new travel demand pattern leads to a new traffic pattern and new congestion locations, which may require further improvements in the future. Ignoring the interaction may result in wrong allocations of budgets on (road) network improvements or starting the improvements at wrong locations or at suboptimal time. In addition, the impact of road network improvement policies to the land use system and the benefit of landowners cannot be evaluated without considering the interaction.

In addition, very few studies (e.g. Rosqvist, 2003; Zhang et al., 2006) have attempted to analyse the impact of road network design on vehicular emissions. Little research (e.g. Frank et al., 2000) has been conducted to link land use with vehicle emissions. However, the new travel demand pattern after implementing the road network design project will indirectly affect vehicular emissions as well as land use. Actually it may cause many negative environmental externalities in the neighbourhood. For example, building a new road may result in more noise pollution, more air pollutants and higher risk of accidents.
among things. Therefore, the impact of road network design on land use and environment should be examined simultaneously.

2.4.3 Traveller Information Provision Services

Over the last two decades, there has been a considerable interest in the transportation area in analysing the effects of providing traveller information. Much research work has been undertaken to evaluate the impact of traveller information provision in terms of welfare economic considerations (e.g. Arnott et al., 1991; Verhoef et al., 1995; Emmerink et al., 1996a; Verhoef et al., 1996; Emmerink et al., 1998), potential travel time savings (e.g. Abdel-Aty and Abdalla, 2004; Adler and Michael, 1994; Adler, 2001; Adler et al., 1999; Adler et al., 2005; Jou, 2001; Jou et al., 2004; Jou et al., 2005; Mahmassani and Liu, 1999; Srinivasan and Krishnamurthy, 2004), driver behaviour (e.g. Bonsall, 1992; Emmerink et al., 1996b; Iida et al., 1992; Kobayashi, 1994; Lotan and Koutsopoulos, 1993; Mahmassani and Herman, 1990; Mannering et al., 1994; Van Berkum and Van Der Mede, 1993; Yang et al., 1993), safety implications (e.g. Al-Deek et al., 1993; Srinivasan et al., 1995; Al-Deek et al., 1998; Moriarty, 2003), and the efficiency of road usage (e.g. Al-Deek and Kanafani, 1993; Emmerink et al., 1995a,b; Mahmassani and Jayakrishnan, 1991; Mahmassani and Peeta, 1993), and so on.

Only very few studies were focused on studying environmental impacts of traveller information provision, particularly from increased vehicular emissions. For example, Al-Deek et al. (1995) proposed an analytical method for evaluating the impact of Advanced Traveller Information System (ATIS) on air quality in a simple network where traffic experienced incident congestion. Kanninen (1996) discussed congestion relief and environmental impacts expected of Intelligent Transportation Systems (ITS). It was mentioned that ITS might induce latent travel demand which would probably increase vehicular emissions.

The impact of traveller information on vehicular emissions requires more evaluation, because any underestimation or overestimation of environmental impacts of traveller information provision could lead to an unexpected outcome. Conventionally, it is intended that providing travellers with the dissemination of traffic information would
allow equipped vehicles to spread from the congested to the less congested areas. Hence the travel times of both equipped drivers and the total system travel time (TSTT) would be reduced. However, diverted traffic may impose extra environmental externalities, such as air pollution, noise pollution, and accidents, on existing drivers and residents living in the neighbourhood. The increased pollution is hazardous to human health, so in this case, the aim of implementing traveller information provision is not only to enhance the mobility of a transportation system but also to maintain the system which is sustainable in an environmentally-friendly way.

Al-Deek et al. (1995) pointed out that under different market penetration levels, ATIS may have different impacts on vehicular emissions. Actually the market penetration level, the proportion of the number of drivers who buy the service and the total number of drivers in the system, is determined by the cost of purchasing the information system, the value of time of drivers as well as the level of information quality provided to drivers. Therefore, the key to determining the impacts of traveller information provision on vehicular emissions lies in the investigation of these parameters and the way in which they determine the level of market penetration (and consequently their impact on vehicular emissions). Similarly, these parameters are also determinants of the system performance in terms of TSTT due to the changes in market penetration. However, the provision of traveller information services may not be able to reduce TSTT and the total vehicular emissions simultaneously. Therefore, more analysis is required to analyse the impact of traveller information provision services on these two issues at the same time.

2.4.4 Speed Limit Control

It is the well-recognised fact that proper control of traffic speed can contribute to both a reduction in accidents and improved efficiency of highway operations (Kang et al., 2004). In the past, great attention has been given to study the effects of speed limit control on congestion (e.g. Shi, 2002; Hegyi, 2005; Kerner, 2007; Ghods et al., 2007), safety (e.g. Wevers and Lu, 2007; Allaby et al., 2007; Awadallah, 2007), and vehicular emissions (Joumard, 1986; Taylor, 2000; Rietveld and Shefer, 1998; Fergusson, 1994; Panis et al., 2006; Keller et al., 2008).
With the implementation of traveller information provision services in many urban cities, travellers with better information about a transport network may change their route choice behaviour and this may affect the impacts of the speed limit control on vehicular emissions as well as congestion. Intuitively speaking, it seems that a higher speed limit on the transport network would result in a lower Total System Travel Time (TSTT). However, this higher speed limit would lead to higher vehicular emissions. Therefore, when travellers are guided to travel on a route with the higher speed limit to save their travel times, this may lead to an increase in the vehicular emissions.

To date, no research has been carried out on the impact of the joint implementation of traveller information provision services and speed limit control on vehicular emissions and congestion. The joint implementation of these two strategies may produce a significant impact on these.
CHAPTER 3

INCORPORATING LAND USE, TRANSPORT AND ENVIRONMENTAL CONSIDERATIONS INTO TIME-DEPENDENT TOLLING STRATEGIES

3.1 INTRODUCTION

To date, little attention was focused on incorporating environmental and land use considerations into analytical road pricing models. In fact, toll charging can reduce the traffic level and hence emissions. In the long run, tolls will alter the travellers' residential locations. Until recently, May et al. (2005) and Vold (2005) have carried out some studies on defining optimal land use transport strategies while considering their impacts on vehicular emissions and their environmental costs. However, they did not examine the impact of controlling maximum allowable link emissions on toll charges. Even if the overall vehicular emissions are under control, emissions on some links may exceed their acceptable levels, which are harmful to human health. In addition, the population is changing over time. It is important to consider the time dimension in analytical toll design models in addition to land use and the environment.

With these considerations, this chapter presents a single-level minimisation model to determine time-dependent optimal tolls while considering the dynamic relationships between land use, transport, and the environment as well as the maximum allowable link emissions from a system equity (SE) viewpoint. This model is in fact bi-level in nature. The upper level formulates the decision maker's problem whose aims at selecting time-dependent tolls to minimise the total travel cost while ensuring emissions on each link is lower than its maximum allowable emissions. The lower level formulates a time-dependent land-use transport problem. This lower level is expressed as two sets of constraints, one for the transport system and the other for the land use system. These constraints are encapsulated into the upper level, resulting in a single level model while considering the land-use transport interaction over time. The resulting
model can then be solved by many existing efficient optimisation methods. In the following, section 2 describes the formulation of the proposed model. Section 3 covers the numerical studies. Finally, section 4 gives a summary.

3.2 MODEL FORMULATION

It is considered that there is a strongly connected multi-modal transportation network with multiple Origin-Destination (OD) flows over the planning horizon \([0, T]\). The planning horizon is divided into \(N\) equal design periods. The network is further divided into \(M\) subnetworks, one for each mode, to account for the unique travelling speed of each mode. The mode here can be an individual mode or a combined mode. The flows of trucks and articulated lorries are treated as background flows and are given. They represent the movements of goods and are converted into Passenger Car Units (PCU). Also the following assumptions are made to simplify the analysis: 1.) Basic employment, basic employment growth rate, and the zonal attractiveness are known; 2.) Each zone has only one characteristic, and the residential zone and the employment zone are not mixed; 3.) Traffic assignment follows the user-equilibrium principle, and; 4.) The link cost and travel demand functions are separable. These assumptions are not restrictive from a modelling perspective and can be relaxed easily in further studies. With these considerations, the time-dependent toll design problem can be formulated as a bi-level problem. The lower level problem is the time-dependent land-use transport problem and the upper level is the decision maker's problem.

3.2.1 Lower Level Time-Dependent Land-use Transport Problem

3.2.1.1 Lower Level Problem Structure

Due to the complexity of the lower level problem, the structure is shown by two figures: Figures 3.1 and 3.2. Figure 3.1 illustrates the time dimension in the lower level problem together with the land-use-transport-environment interaction. Figure 3.2 shows in details how land-use, transport and the environment interact at a time instant. Decision variables are highlighted in Figure 3.2. Based on this structure, the lower level problem
can then be described by two sets of mathematical constraints: time-dependent Lowry-based land-use constraints and time-dependent traffic assignment constraints.

\[ T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow \ldots \rightarrow T_n \]

**Figure 3.1** The land use, transport and the environment relationship over time.

**Figure 3.2** Interaction between land use, transport, and the environment at a time instant.

### 3.2.1.2 Time-dependent Lowry-based Land-use Constraints
Time-dependent Lowry-based land-use constraints are developed based on Lowry’s (1964) model, which classifies the land-use into three categories: basic sector, household sector and non-basic sector. The basic sector includes industries, businesses and administrative establishments whose goods and services are exported outside the urban area. It generates a centripetal flow of capital into the city generating growth and surpluses. It is generally assumed that this sector is less constrained by urban location problems since the local market is not the main concern. This consideration is an exogenous element of the Lowry model and must be given. The non-basic sector includes businesses, administrative establishments and other retailing services that deal with providing goods and services for local residential population. Since this sector strictly serves the local / regional demand, the location choice is oriented to the household sector. Employment levels are also assumed to be linked with the local population. The household sector consists of residential population. The number of residents is related to the number of basic and non-basic jobs available. Their residential locations are also closely linked to the place of work. Since the residential and non-basic sector location choices depend on each other, the household and non-basic sectors finally distribute themselves to achieve equilibrium.

The time-dependent Lowry-based land-use constraints extend the Lowry-type equilibrium to a dynamic framework. In each design period, Lowry-type equilibrium is assumed to be held. The equilibrium is depicted by a number of constraints. The first one describes how to allocate residents who work in employment zone $i$ to residential zone $j$ using the gravity-type model:

$$ R_{ij,r} = E_{r,i} B_{r,i} W_{r,i} \exp(-\beta' c_{ij,r}), \forall i, j, r, $$

where

$$ B_{r,i} = 1 / \sum_j W_{r,j} \exp(-\beta' c_{ij,r}), \forall i, r. $$

$R_{ij,r}$ is the number of residents travelling between OD pair $ij$ in period $r$ or the number of work-to-home trips (or the number of total employment trips) between OD pair $ij$ in period $r$. This is the number of residents in zone $j$ that work in zone $i$; $E_{r,i}$ is the total employment in zone $i$ in period $r$; $W_{r,i}$ is the attractiveness of zone $j$ in period $r$, which can be represented by the availability of floor space for residential use. The attractiveness of each zone is assumed to follow the following function:
\[ W^{*}_{i,t+1} = W^{*}_{i,t} \left( 1 + \tilde{h}_{i,t} \right), \forall i, \tau, \]  

where \( \tilde{h}_{i,t} \) is the growth rate of attractiveness of zone \( i \) over time. \( \beta' \) is the parameter to regulate the effect of transport cost on distribution of residents. A high value of \( \beta' \) will result in the residents being allocated close to their place of work; if \( \beta' \) tends to infinity, all residents will live and work in the same zone. On the other hand, if \( \beta' \) tends to zero, the residents whose work in zone \( i \) will locate to all residential zones equally. \( c_{ij,t} \) is the composite travel cost between OD pair \( ij \) in period \( t \), representing the inter-zonal impedance and will be defined later. The term \( B_{i,t} \) is to ensure a correct allocation of residents to zone \( j \) in period \( t \) so that \( \sum R_{j,t} \exp(-\beta' c_{ij,t}) = 1 \) and \( \sum_{j} R_{j,t} = E_{i,t} \). The latter means that the total number of work-to-home trips from employment zone \( i \) in period \( t \) must be equal to the number of jobs available in that zone in period \( t \), \( E_{i,t} \) (i.e. the number of people working in employment zone \( i \) in period \( t \) must be equal to the number of jobs available in that zone in the same period).

The total employment in zone \( i \) in period \( t \), \( E_{i,t} \), in (3.1), is the sum of the basic employment \( E_{i,t}^B \) and the service employment or non-basic employment, \( E_{i,t}^S \), in zone \( i \) in period \( t \):

\[ E_{i,t} = E_{i,t}^B + E_{i,t}^S, \forall i, \tau. \]  

(3.4)

The basic employment in the employment zone is supposed to grow linearly over time:

\[ E_{i,t+1}^B = E_{i,t}^B \left( 1 + \tilde{h}_{E,t} \right), \forall i, \tau, \] 

(3.5)

where \( \tilde{h}_{E,t} \) is the growth rate of basic employment. The service employment in zone \( i \) in period \( t \), \( E_{i,t}^S \), in (3.4) is equal to the number of service employment trips starting from zone \( i \) in period \( t \) or simply the number of service employees working there in that period:

\[ E_{i,t}^S = \sum_{j} E_{j,t}^S, \forall i, j, \tau. \] 

(3.6)
where $E_{i,j}^S$ is the number of service employees who work in zone $i$ living in zone $j$ (i.e. the number of service employment trips between OD pair $ij$ in period $r$). The number of service employment trips $E_{i,j}^S$ is obtained by:

$$E_{i,j}^S = sR_{i,j}A_{i,j}W_i^α \exp(-\beta'c_{i,j}), \forall i, j, r,$$

where $A_{i,j} = 1/\sum_i W_i^α \exp(-\beta'c_{i,j}), \forall j, r;$

$s$ is a service employment-to-population ratio; $W_i^α$ is the attractiveness in zone $i$ in period $r$, which can be the availability of floor space for commercial use; $\beta'$ is the parameter to regulate the effect of transport cost on distribution of service employees, and its function is similar to $\beta'$ in (3.1). A high value of $\beta'$ will result in service employment being allocated close to the residential location, and a small value will result in service employment being allocated to all residential locations equally. The term $A_{i,j}$ is to ensure a correct allocation of service employment to zone $i$, which has a similar function to the term $B_{i,r}$ in (3.2). The term $sR_{j,r}$ in (3.7) is the total number of employees in zone $j$ in period $r$. The total number of residents in zone $j$ in period $r$, $R_{j,r}$, in (3.7) is defined as:

$$R_{j,r} = \mu \sum_i R_{i,j}, \forall j, r;$$

where $R_{i,j}$ is the number of employees who work in zone $i$ and live in zone $j$ in period $r$, and $\mu$ is a population-to-employment ratio. According to (3.9), in each period, the total number of employees in zone $j$, $\sum_i R_{i,j}$ multiplied by the population-to-employment $\mu$ gives the total number of residents in that zone $R_{j,r}$.

### 3.2.1.3 Time-dependent Traffic Assignment Constraints

The time-dependent traffic assignment constraints represent the transport model in this framework and describe the route and mode choices over time. These constraints are made up of Wardrop’s conditions, travel cost constraints, as well as the modal split, flow conservation, and non-negativity conditions.
Equilibrium Conditions of Wardrop’s First Principle

These conditions are supposed to be held in each design period for each mode. They require that for each mode $k$ and for each period $\tau$, route $p$ between OD pair $ij$ will not be used if its travel cost is higher than the lowest travel cost between OD pair $ij$. Conversely, any used route $p$ must have its travel cost equal to the lowest travel cost between OD pair $rs$. Mathematically, these conditions can be stated as:

$$f_{p,i,j}^{k,\tau} \left[ c_{p,i,j}^{k,\tau} - \pi_{i,j}^{k,\tau} \right] = 0, \forall p,i,j,k,\tau,$$  \hspace{1cm} (3.10)

$$c_{p,i,j}^{k,\tau} - \pi_{i,j}^{k,\tau} \geq 0, \forall p,i,j,k,\tau,$$  \hspace{1cm} (3.11)

where $f_{p,i,j}^{k,\tau}$ is the representative hourly flow of mode $k$ on route $p$ between OD pair $ij$ in period $\tau$; $c_{p,i,j}^{k,\tau}$ is the travel cost on route $p$ between OD pair $ij$ by mode $k$ in period $\tau$; $\pi_{i,j}^{k,\tau}$ is the lowest travel cost between OD pair $ij$ by mode $k$ in period $\tau$:

$$\pi_{i,j}^{k,\tau} = \min \left[ c_{p,i,j}^{k,\tau}, \forall p \right].$$  \hspace{1cm} (3.12)

Conditions (3.10) and (3.11) constitute the nonlinear complementarity conditions for the route choice assignment principle for each mode $k$ in each period $\tau$. According to (3.10), if route $p$ carries a positive mode $k$’s flow in period $\tau$, (i.e., $f_{p,i,j}^{k,\tau} > 0$), then its associated route cost $c_{p,i,j}^{k,\tau}$ must be equal to the lowest cost $\pi_{i,j}^{k,\tau}$ through the condition $c_{p,i,j}^{k,\tau} - \pi_{i,j}^{k,\tau} = 0$. If no mode $k$’s flow is on path $p$ in period $\tau$, the term $\left[ c_{p,i,j}^{k,\tau} - \pi_{i,j}^{k,\tau} \right]$ in (3.10) is unrestricted and the travel cost $c_{p,i,j}^{k,\tau}$ can be greater than or equal to $\pi_{i,j}^{k,\tau}$ according to (3.11). Actually, condition (3.11) ensures $\pi_{i,j}^{k,\tau}$ to be the lowest cost among all the possible routes between OD pair $ij$ by mode $k$ in period $\tau$.

Travel Cost Constraints

Travel costs depend on flows, network characteristics such as free flow travel times and capacities of links, and out-of-pocket costs such as tolls and fares. Route costs depend on route flows, in which the latter depends on link flows through:

$$v_{a,k}^{\tau} = \sum_{y} \sum_{p} f_{p,y,a}^{k,\tau} \delta_{p,a}^{\tau}, \forall a,k,\tau,$$  \hspace{1cm} (3.13)
where \( v_a^k \), is the hourly flow of mode \( k \) on link \( a \) in period \( \tau \), and \( \delta_a^{p,k} \) is a link-path incidence indicator for mode \( k \), which equals one if link \( a \) is on route \( p \), and zero otherwise. Equation (3.13) states that for each mode \( k \), the link flow in each period is obtained by adding all route flows on that link in that period together. The link time \( t_{a,\tau}^k \) (such as travel time, waiting time, or walking time) relates link flows through the link performance function:

\[
t_{a,\tau}^k = t_{a,\tau}^k(v_{a,\tau}),
\]

where \( v_{a,\tau} = [v_a^k] \) is the link flow vector in period \( \tau \). This link performance function is non-separable as the link time on link \( a \) depends on the flows on other links. The link performance function adopted in road traffic assignment in this chapter is:

\[
t_{a,\tau}^m = t_{a,\tau}^{0,m} + \alpha_0 + \alpha_1 \left( \frac{v_{a,\tau}^m}{c_{a,\tau}} \right)^{\alpha_2}, \forall a, m, \tau,
\]

where the superscript \( m \) stands for the mode that travels in the road network only (which is different from the superscript \( k \) that is used for representing any mode considered in this chapter); \( t_{a,\tau}^{0,m} \) and \( c_{a,\tau} \) are the free flow travel time for mode \( m \) and the capacity of link \( a \); \( \alpha_0 = 1 \), \( \alpha_1 = 0.15 \), and \( \alpha_2 = 4 \) are parameters of the link performance function of link \( a \) for mode \( m \). Equation (3.15) is the typical Bureau of Public Roads (BPR) function, which describes the monotonic relationship between the link travel time \( t_{a,\tau}^m \) and the link flow \( v_{a,\tau}^m \).

The route cost \( c_{p,i,j}^k \) is the sum of the link-wise additive costs \( g_{p,i,j}^k \) and the route specific costs \( \theta_{p,i,j}^k \):

\[
c_{p,i,j}^k = g_{p,i,j}^k + \theta_{p,i,j}^k, \forall p, i, j, k, \tau.
\]

The link-wise additive costs \( g_{p,i,j}^k \) are defined by summing up link attributes, which include link tolls \( \rho_a^k \), and congestion-dependent attributes; for instance, travel time (and other costs such as fuel consumption) spending on road networks, or walking, on-board and boarding/boarding time spending on transit networks. The link-wise additive cost \( g_{p,i,j}^k \) can be written as:
\[ g_{p,i,j,k,r}^k = \sum_a (\psi \delta_{a,r}^k + \rho_{a,r}^k), \forall p,i,j,k,r, \]  

(3.17)

where \( \psi \) is the cost of unit (travel) time, and therefore \( \psi \delta_{a,r}^k \) is the (travel) time cost on link \( a \) by mode \( k \) in period \( r \); \( \rho_{a,r}^k \) is the toll for mode \( k \) using link \( a \) in period \( r \). The route specific costs \( \theta_{p,i,j,k,r}^k \) are non-linear and/or nonadditive over links; for instance, some types of tolls on road networks (e.g. non-linearly proportional to distance), or waiting time and some fare structures for transit networks (e.g. zone-wise prices).

The composite travel cost between OD pair \( ij \) in period \( r \), \( c_{i,j,r} \) is defined as:

\[ c_{i,j,r} = -\ln \left[ \sum_k \left( \exp \left( -\bar{\beta} \left( \pi_{i,j,r}^k + \theta^k \right) \right) \right) \right], \forall i,j,r, \]  

(3.18)

where \( \bar{\beta} \) is the parameter in the logit model to regulate the effect of the mode travel cost \( \pi_{i,j,r}^k + \theta^k \); \( \pi_{i,j,r}^k \) is the lowest travel cost between OD pair \( ij \) by mode \( k \) in period \( r \) defined in (3.12); \( \theta^k \) is the mode-specific cost. The composite cost is obtained by aggregating the mode travel cost \( \pi_{i,j,r}^k + \theta^k \) over all modes. The derivation of this composite cost can be found in Ben Akiva and Lerman (1987).

The Modal Split, Flow Conservation, and Non-negativity Conditions

Modal split can be obtained by the logit model:

\[ q_{i,j,k,r}^k = R_{i,j} \left\{ \frac{\exp \left( -\bar{\beta} \left( \pi_{i,j,r}^k + \theta^k \right) \right)}{\sum_k \exp \left( -\bar{\beta} \left( \pi_{i,j,r}^k + \theta^k \right) \right)} \right\}, \forall i,j,k,r, \]  

(3.19)

where \( q_{i,j,k,r}^k \) is the demand for mode \( k \) between OD pair \( ij \) in period \( r \), and \( R_{i,j} \) is the number of residents who work in zone \( i \) and live in zone \( j \) defined in (3.1). The demand for mode \( k \) between OD pair \( ij \) in period \( r \), \( q_{i,j,k,r}^k \), in (3.19) is equal to the sum of the route flows of that mode between the OD pair in the same period so that route flows are conserved in each mode between each OD pair in each period:

\[ q_{i,j,k,r}^k = \sum_p f_{p,i,j,k,r}^k, \forall i,j,k,r. \]  

(3.20)

Moreover, route flows in (3.20) must be non-negative:
3.2.1.4 Lower Level Optimisation Model

Given the time-dependent tolls, the lower level model is formulated as follows:

**Lower Level Model**

\[ \text{min} \ G, \]

subject to

- time-dependent Lowry-based land-use model constraints (3.1)-(3.9), and;
- time-dependent traffic assignment constraints (3.10)-(3.21).

where \( f, E, R \) represent, respectively, the vectors of path flows, service employment trips, and residential trips. Since \( f_{p,j,t}^i \geq 0 \) and \( c_{p,j,t}^i - \pi_{y,j,t}^i \geq 0 \) are ensured by the time-dependent traffic assignment constraints (3.11) and (3.21), path flows, service employment trips, and residential trips will satisfy (3.1)-(3.21) when the gap \( G \) is zero.

Many gap functions can serve this purpose. This chapter adopts the following:

\[ G = \sum_p \sum_y \sum_t f_{p,j,t}^i \left[ c_{p,j,t}^i - \pi_{y,j,t}^i \right]. \]

(3.22)

This gap function must have non-negative values if \( f_{p,j,t}^i \geq 0 \) and \( c_{p,j,t}^i - \pi_{y,j,t}^i \geq 0 \) because the sum of non-negative numbers must be non-negative. One property of this gap function is that if \( f_{p,j,t}^i \geq 0 \), \( c_{p,j,t}^i - \pi_{y,j,t}^i \geq 0 \) and the gap function attains its minimum value of zero, the time-dependent Wardrop’s condition (3.10) is satisfied. This property will be used in developing a lower level optimisation model.

3.2.2 Upper Level Problem

The upper level is the decision maker’s problem. This study assumes that the decision maker (or the government) is primarily concerned with the total travel cost (TTC), and secondly with vehicular emissions and design (e.g. toll) or regulation (e.g. link emission) constraints when making decisions.
3.2.2.1 Total Travel Cost (TTC)

TTC is mathematically formulated as:

\[ TTC = \sum_{\tau} \sum_{k} \sum_{\eta} \sum_{p} f_{p,\eta,\tau} c_{k,\eta,\tau}. \] (3.23)

Equation (3.23) states that TTC is the sum of the products of route flows and their corresponding travel costs over all routes, modes, OD pairs, and time. This TTC function forms the objective function of the bi-level model.

3.2.2.2 Estimation of Vehicular Emissions Based on MOBILE Model

There are two types of vehicular emissions: link and network (or overall). The link vehicular emissions are defined through the link emission factor approach:

\[ Q_{m,\tau} = \sum_{a} Q_{m,a,\tau} = \sum_{m} h_{m,a,\tau} v_{m,a,\tau}, \forall a, \tau, \] (3.24)

where \( Q_{m,a,\tau} \) is the vehicular emissions for traffic mode \( m \) on link \( a \) in period \( \tau \); \( v_{m,a,\tau} \) represents the hourly traffic flow for mode \( m \) on link \( a \) in period \( \tau \); \( h_{m,a,\tau} \) is the emission factor for mode \( m \) on link \( a \) in period \( \tau \), which is assumed to be given for all links. The factors affecting the value of \( h_{m,a,\tau} \) are discussed in Nagurney (2000a). This link emission factor approach according to MOBILE model has been adopted by Nagurney et al. (1998) and others. According to (3.24), the vehicular emissions for mode \( m \) on a particular link is the product of the link flows of mode \( m \) and the corresponding emission factor, and the total vehicular emissions on this link are the sum of vehicular emissions for all modes travelling on this link. The overall vehicular emissions are the sum of the vehicular emissions on each link:

\[ Q_{\tau} = \sum_{a} Q_{a,\tau}, \forall \tau. \] (3.25)

3.2.2.3 Link Emission Constraint

With the definition of link vehicular emissions, a link emission constraint can be defined to ensure that link emissions are smaller than or equal to the maximum allowable emissions on link \( a \) in period \( \tau \):
\[ Q_{a,r} \leq \hat{Q}_{a,r}, \forall a, r, \]  \hspace{1cm} (3.26)

where \( \hat{Q}_{a,r} \) are the maximum allowable vehicular emissions on link \( a \) in period \( r \).

### 3.2.2.4 Toll Constraints

For political and other reasons it is not always feasible to set the toll charge too high while in practice only non-negative tolls are possible and charged at certain links. Mathematically, these conditions are expressed as:

\[ \rho_{a,r}^k \geq 0, \forall a,k,r, \] \hspace{1cm} (3.27)
\[ \rho_{a,r}^k \leq \tilde{\rho}_{a,r}^k, \forall a,k,r. \] \hspace{1cm} (3.28)

Condition (3.27) ensures that all tolls are greater than or equal to zero. Condition (3.28) requires the toll charged on link \( a \) in period \( r \) to be less than the maximum allowable toll \( \tilde{\rho}_{a,r}^k \) for that link. If the link is not allowable to charge any toll, \( \tilde{\rho}_{a,r}^k \) is set to zero.

### 3.2.3 The Proposed Bi-level Model

The proposed bi-level model is formulated as follows:

\[ \text{Bi-level Model} \]
\[ \min_{\mathbf{E, R, \rho}} \mathbf{TTC}, \]
subject to
- time-dependent Lowry-based land-use constraints (3.1)-(3.9);
- time-dependent traffic assignment constraints (3.10)-(3.21);
- link emission constraint (3.26), and;
- toll constraints (3.27)-(3.28).

The proposed bi-level model is formulated as a single level minimisation program where the lower level problem is expressed as constraints (3.1)-(3.21) instead of the optimisation model, Lower Level Model, described in Section 3.2.1.4. The advantage of formulating the bi-level problem as a single level problem is that the existing single optimisation algorithms and packages can be used to solve for solutions. In this study, Bi-level Model is solved using the Generalised Reduced Gradient (GRG) method (Abadie and Carpentier, 1969).
3.3 NUMERICAL STUDIES

Two scenarios are set up in this section. The first one is to illustrate the effects of tolls on the overall vehicular emissions, link emissions, total travel cost, transit revenue, residents’ working and living locations, and the importance of considering land use, transport and the environment (in terms of link emissions) over time in determining optimal time-dependent tolls. The second one is to study the impact of maximum allowable link emissions on link tolls.

3.3.1 The Impacts of Time-Dependent Tolls

The scenario network is shown in Figure 3.3 with five nodes, six (road) links and six origin-destination (OD) pairs. The six OD pairs are E1-R3, E1-R4, E1-R5, E2-R3, E2-R4, and E2-R5. E1 and E2 represent employment zones 1 and 2 respectively. R3, R4, and R5 correspondingly represent residential zones 3, 4, and 5. There is a public transit operated between each OD pair (but the transit is not shown here). Link E1-R3 is the sole toll link. No truck flows are considered here for the sake of simplicity. The parameters adopted in this scenario are shown below.

**Transport model parameters:** Value of time: $\psi = \€ 10$/hour; free flow travel times:

- Car-specific constant: $\theta_{\text{car}} = 0.8$; free flow travel times: $t_{13}^{\text{car}} = t_{24}^{\text{car}} = 20$ mins, $t_{15}^{\text{car}} = t_{25}^{\text{car}} = t_{23}^{\text{car}} = t_{54}^{\text{car}} = 15$ mins;
- Car-specific constant: $\theta_{\text{car}} = 0.8$; free flow travel times: $t_{13}^{\text{car}} = t_{24}^{\text{car}} = 12$ mins, $t_{15}^{\text{car}} = t_{25}^{\text{car}} = 10$ mins; transit fares: $t_{13}^{\text{transit}} = t_{24}^{\text{transit}} = t_{23}^{\text{transit}} = \€ 2$
- Transit-specific constants: $\theta_{13}^{\text{transit}} = \theta_{24}^{\text{transit}} = \theta_{23}^{\text{transit}} = 1.8$
- Transit-specific constants: $\theta_{13}^{\text{transit}} = \theta_{24}^{\text{transit}} = \theta_{23}^{\text{transit}} = 1$

**Land use model parameters:** Basic employment: $E_{1,1}^B = 1000$ jobs, $E_{2,1}^B = 800$ jobs;
- Population to employment ratio: $\mu = 5$; Service employment to population ratio: $s = 0.1$;
- Accessibility parameters: $\beta' = 0.8$, $\beta'' = 0.6$; parameter for the composite cost: $\bar{\beta} = 1$;
Attractiveness: $W^a_{1,1} = W^a_{2,1} = 1000$ jobs; $W^a_{3,3} = W^a_{4,1} = W^a_{5,1} = 1000$ houses; growth rates of basic employment and attractiveness: $\tilde{h}_{E,j} = \tilde{h}_{W,j} = 0.04$.

Environment parameters: Emission factors: $h^a_{13,7} = 0.7$ litre/veh, $h^a_{24,7} = 0.6$ litre/veh, $h^a_{33,7} = 0.4$ litre/veh, $h^a_{43,7} = 0.3$ litre/veh, $h^a_{53,7} = 0.5$ litre/veh; overall maximum allowable vehicular emissions: $\bar{Q}_r = 1680$ litres/hour. Planning parameter: Planning horizon: 3 years, length of each period: 1 year.

In this scenario, the Lower Level Model is employed to study the impact of tolls on transport, land use and environmental systems and is solved by the GRG method. For illustrative purposes, the tolls are constant over time. The toll levels on link 1 are varied from €0 to €5. The overall vehicular emissions on the entire network are plotted in Figure 3.4. From this figure, it is observed that the tolls can be implemented as a measure to control the overall vehicular emissions. However, the decision maker has to select the toll level carefully, because not all tolls can achieve the overall vehicular emissions to be less than the maximum allowable vehicular emissions. For example, when the toll is greater than €2.4, the overall vehicular emissions are under the maximum allowable vehicular emissions of $1.68E3$ litres/hour as shown by the horizontal line in the figure. In particular, when the toll is greater than €3, the overall vehicular emissions remain at the same level. This raises an issue for the government on how to select tolls to regulate the overall vehicular emissions. An analytical model is needed for this purpose.
Not only can the implementation of tolls on links lower the overall traffic emissions but it also has a significant impact on the public transit system and the land use pattern. Figure 3.5 depicts the number of passengers on OD pair E1-R3 on the public transit system and the total transit revenue over the entire planning horizon. It is demonstrated from this figure that the toll implementation on link E1-R3 causes travellers between OD pair E1-R3 to change their mode choice and travel by public transit instead of private cars. When the toll level rises, more passengers are attracted to the public transit system in order to decrease their travel costs. Beyond the toll level of €3, the number of passengers on transit between OD pair E1-R3 goes stable. Figure 3.5 also reveals that the implementation of tolls can generate more transit revenue. When there is no toll on
the transport network, the total transit revenue is only €1.35E7. However, after charging a toll of €3 on link E1-R3, the revenue is increased by 21.5% due to the increase in passengers. It is concluded that the implementation of tolls can divert some travellers to take public transit and raise its revenue. This has important implications for transit operators on their operational strategies and profits.

Figure 3.6 shows the rearrangement of employment due to the changes in tolls. This is because their travel costs are altered after the toll implementation and people are sensitive to travel costs in selecting where they work. Figure 3.7 indicates the change in the number of residents in residential zones 3, 4 and 5 over time. From Figure 3.7, it is revealed that there are dramatic changes in the numbers of residents in zones 3 and 5 over three years when the tolls are ranged from €0-€3. This is again due to the travel costs. People will decide their residential locations based on their travel costs to their work location. However, not all tolls can affect the land use pattern in terms of residential and employment distributions. As shown in the above two figures, the land use pattern remains stable after the toll value of €3.

To sum up, these two figures clearly demonstrate that tolls can alter the choices of working and living locations of people, which has a strong implication for land owners’ profits, the rents of residents and a living environment. This is because the rents, the profits, and the living environment depend on the population size or the number of residents. More residents will result in higher rents and hence higher land owners’
profits. More residents will also result in few spaces for recreational uses and higher competition for resources and facilities, and hence a poor living environment. Tolls must be carefully selected to deal with the concerns of land owners and residents.

The total travel cost before and after tolling are revealed in Figure 3.8a. It is observed that the total travel cost is increased substantially (more than 13\%) when charging tolls on link E1-R3. This illustrates that the implementation of tolls can have a negative impact on the total travel cost. In particular, comparing Figure 3.8a with Figure 3.4, it is observed that there is a trade-off between the overall vehicular emissions and the total travel cost. The higher the toll, the fewer the emissions but the higher the total travel cost (although the increase in total travel cost is diminishing quickly). The total travel

Figure 3.7 Number of residents in zones 3, 4 and 5 over time.

58
cost should be taken into account in a modelling stage in addition to the overall vehicular emissions.

Figure 3.8b shows that the link emissions on link E1-R5 as a result of tolls on link E1-R3. This figure shows that the higher the tolls, the higher the vehicular emissions on link E1-R5, because more traffic will be diverted to link E1-R5. In the extreme case, the vehicular emissions on link E1-R5 can rise more than 15%. In addition, their relationship is not linear and two kinks are observed in this figure due to the sudden changes in the overall network flow pattern. One more key observation is that although the overall vehicular emissions are acceptable when the tolls are greater than €2.8 (see Figure 3.4), the link emissions exceed the allowable vehicular emissions if the allowable emissions on link E1-R5 are 6500 litres. Therefore the government should regulate not only the overall vehicular emissions but also link vehicular emissions since link emission standards can be violated even though the overall emission standard is satisfied. In this case, the government may need to control the traffic volume on some links so that the link vehicular emission standards on these links are not violated. To conclude, the total travel cost and link emissions should be considered in determining optimal time-dependent tolls.

![Figure 3.8 Total travel cost as well as emissions on link E1-R5.](image)
3.3.2 The Impact of Maximum Allowable Link Emissions on Link Tolls

With all the considerations in the first scenario, another scenario is set up to study the effects of the maximum allowable link emissions on its link toll. The proposed Bi-level Model is employed for this purpose and solved by the GRG method. A simple network is shown in Figure 3.9, which has three nodes, two OD pairs, two road network links and one transit link. The two OD pairs are OD pairs E1-R2 and E1-R3. The three nodes are E1, R2 and R3 representing employment zone 1, residential zones 2 and 3 respectively. The two road links are link E1-R2 and link E1-R3. Link E1-R3 has a higher emission factor due to its longer distance compared with link E1-R2. The transit link connects OD pair E1-R3. The government tries to lower the vehicular emissions on link E1-R3 by charging tolls on this link over time, and also regulates the vehicular emissions on link E1-R2 so that the emissions are smaller or equal to its maximum allowable emissions.

The following parameters are adopted:

**Transport model parameters:** Value of time: \( \psi = €10/\text{hour} \); free flow travel times: \( t_{12}^{\text{car}} = 18 \text{ mins} \), \( t_{13}^{\text{car}} = 30 \text{ mins} \); Car-specific constant: \( \theta^{\text{car}} = 0.8 \); free flow travel times: \( t_{13}^{\text{transit}} = 10 \text{ mins} \); transit fares: \( \theta_{13}^{\text{transit}} = €3 \); Transit-specific constants: \( \theta_{13}^{\text{transit}} = 1.8 \); maximum allowable toll: \( \bar{\rho}_{u} = €10 \).
**Land use model parameters:** Basic employment: $E_{E_{1,1}}^b = 1000$ jobs; Population to employment ratio: $\mu = 5$; Service employment to population ratio: $s = 0.15$; Accessibility parameters: $\beta' = 0.8$, $\beta'' = 0.6$; parameter for the composite cost: $\bar{\beta} = 1$; Attractiveness: $W_{1,1}^c = 1000$ jobs; $W_{2,1}^c = W_{3,1}^c = 1000$ houses; growth rates of basic employment and attractiveness: $\tilde{h}_{E,ij} = \tilde{h}_{w,ij} = 0.04$.

**Environment parameters:** Emission factors: $h_{l,12,12}^{cap} = 0.5$ litre/veh, $h_{l,13,13}^{cap} = 1.1$ litres/veh; maximum allowable vehicular emissions: $\hat{Q}_{l,12,r} = 4000$ litres/hour. Planning parameter: Planning horizon: 2 years; length of each period: 1 year.

![Figure 3.10](image)

**Figure 3.10 The impact of the maximum allowable link emissions of link E1-R3 on its link toll.**

Figure 3.10 illustrates the effect of the maximum allowable link emissions of link E1-R3 on its link toll. As you can see, higher maximum allowable link emissions result in a lower link toll. It is reasonable as a tighter link emission standard only allows few vehicles on that link, and few vehicles on that link can be achieved by setting a higher toll on it. The implication is that the maximum allowable link emissions can have direct impacts on the transport system (in terms of the number of transit passengers, transit revenue, the total travel cost), the land use system (in terms of population and employment distributions) and the environmental system (in terms of link and overall emissions).
emissions). It is because the maximum allowable link emissions determine link tolls, and the link tolls can have impacts on the transport, land use and environmental systems over time as shown in the previous scenario. The government should consider the impacts on land use and transport systems in addition to the environmental system when determining the maximum allowable link emissions.

3.4 SUMMARY

In this chapter, the bi-level model is proposed to determine the optimal tolls over time to control vehicular emissions while capturing the land-use transport interaction over time. To illustrate the importance of considering land use, transport and the environment over time in determining optimal time-dependent tolls, a numerical study was set up. The results show that:

1. The implementation of tolls can have a negative impact on the total travel cost although the tolls can lower the overall vehicular emissions. There is a trade-off between the overall vehicular emissions and the total travel cost. The total travel cost should be taken into account in a modelling stage in addition to the overall vehicular emissions.

2. Link emission standards can be violated even though the overall emission standard is satisfied. Link vehicular emissions should also be considered in determining optimal time-dependent tolls.

3. The implementation of tolls can generate more transit revenue and more transit passengers. This has important implications for transit operators on their operational strategies and profits.

4. Tolls can alter traveller's choices in terms of their working and living locations, which has strong implications for land owners' profits, the rents of residents and the general living environment. Tolls must be carefully selected so as to consider the concerns of land owners and residents in addition to the concern of transit operators.
Another study was also performed to analyse the effect of maximum allowable link emissions (or link emission standard) on link tolls, which has not been studied in the transportation literature to date. The results show that higher maximum allowable link emissions lead to a lower toll on that link. This observation together with the previous four findings imply that the maximum allowable link emissions can have direct impacts on the road and transit systems, the land use system and the environmental system. The government should consider the impacts on land use, transport and environmental systems when determining the maximum allowable link emissions.
CHAPTER 4

TIME-DEPENDENT ROAD NETWORK DESIGN
FRAMEWORKS WITH LAND USE AND ENVIRONMENTAL CONSIDERATIONS: POLICY IMPLICATIONS

4.1 INTRODUCTION

Nowadays, many road network improvement projects are still ongoing, especially in some major cities in Asia and Europe. These projects are expensive. With respect to constrained government expenditure, especially for road network improvements, the government should carefully select cost-effective improvement projects to be implemented. Traditionally, the analysis involved belonged to the discipline of road network design. In the past, road network design was normally focused on the effect of designs on the transport system alone but not on land use, and land use-transport interaction tended to be ignored over time. This may result in obtaining suboptimal designs. Additionally, the impacts of road network improvement policies on the land use system, especially the benefit of landowners cannot be evaluated without considering the interaction. Furthermore, road network design policies considering the interaction between land use and transport systems would have a substantial impact on the environment, specifically on vehicular emissions.

With the above considerations, a general time-dependent road network design framework is developed to encapsulate the Lowry-type land use consideration so that the land use-transport interaction can be dealt with when determining optimal designs. More importantly, unlike existing models, the optimal designs can be determined automatically through the optimisation procedure without trial-and-error once the objective is clearly defined. In addition, the effects of road network improvement policies on the land use side such as subsidising road network improvements using public fund or transit revenue, cost recovery, and build-operate-transfer (BOT) on the
profits of landowners and their profit distribution as well as population and employment changes can be studied using the proposed framework. The time scale considered in the framework is in years as in Szeto and Lo (2006), since the pace of the adjustment process inside the land use system is slow compared with those occurring inside the transport system like the day-to-day route adjustment process or the second-to-second traffic dynamics. Nonetheless, a second smaller time dimension can be easily added to the proposed models to cope with the dynamics inside the transport system without conceptual difficulty and is left to future study. Since the largest time scale is used here, the inherent advantages of the model proposed by Szeto and Lo (2006) can be found in this framework. The framework is formulated as a single-level single-objective optimisation program that can be solved by many existing optimisation software. However, it is in fact bi-level in nature. The upper level formulates the decision maker’s problem. The lower level formulates the time-dependent land-use transport problem. This lower level is expressed as two sets of constraints, one for the transport system and the other for the land use system. These constraints are encapsulated into the upper level, resulting in a single level model while considering the land-use transport interaction over time.

To incorporate the considerations of various parties involved in road network improvement projects, a multi-objective optimisation framework is then developed through the hybrid approach. A numerical study using a small network is also set up to clearly illustrate the frameworks, the effect of the implementation of road network improvement projects on the related parties, and the trade-offs between various objectives of the related parties. The proposed frameworks can help network planners and profit makers with decision-making by the elimination of many alternative designs. Three improvement schemes are considered: exact cost recovery, build-operate-transfer (BOT), and to use the increase in transit profit to subsidise road improvement projects. The scenario under each scheme is formulated individually using the proposed frameworks, and the corresponding optimal design is obtained by employing the generalised reduced gradient method (Abadie and Carpentier, 1969) to solve the models.
4.2 FORMULATION

It is considered that there is a strongly connected multi-modal transportation network with multiple Origin-Destination (OD) flows over the planning horizon $[0, T]$. The planning horizon is divided into $N$ equal design periods. The network is further divided into $M$ subnetworks, one for each mode, to account for the unique travelling speed of each mode. The mode here can be an individual mode or a combined mode. With this consideration, the proposed framework is formulated as a single-level, single-objective constrained optimisation program as follows:

$$\max y(x), \quad (4.1)$$

subject to

- time-dependent Lowry-type constraints and modal-split/assignment constraints;
- road network design constraints;
- financial constraints, and;

where $y(x)$ is the objective function and $x$ is the vector of decision variables including tolls and capacity enhancements. In the following, the framework is discussed.

4.2.1 Time-Dependent Lowry-Type Constraints and Modal-Split/Assignment Constraints

The time-dependent Lowry-type constraints are developed based on the Lowry-type land use model (Lowry, 1964) and describe the interaction between employment and population over time according to the travel costs obtained from the transport model. The transport model is represented by the time-dependent modal-split assignment constraints that depict the modal choice and Wardropian (Wardrop, 1952) route choice behaviour in each design period based on the travel demand obtained from the Lowry model. The details of the time-dependent constraints can be found in chapter 3 or in Li et al. (2007). Note that one of the differences between the proposed single objective framework and the one in chapter 3 or in Li et al. (2007) is that the latter only considers time-dependent tolls while the former considers both time-dependent tolls and capacity.
enhancement.

4.2.2 Road Network Design Constraints

They include link improvement constraints and toll constraints. Link improvement constraints are included to address the fact that a link (in road networks) cannot be built or expanded beyond an upper limit due to space limitation, and that the improvement must be non-negative:

\[ \bar{c}_{a,m}^{w} \leq u_{a,m}^{w}, \forall a, m, r, \]  \hspace{1cm} (4.2) \\
\[ y_{b,r} \geq 0, \forall b, r, \] \hspace{1cm} (4.3)

where \( u_{a,m}^{w} \) is the maximum allowable capacity of link \( a \) for mode \( m \) that uses the highways or roads. Equation (4.2) is the maximum allowable capacity constraint, which is to limit the total capacity of each link after road expansion or highway construction in period \( r \), \( \bar{c}_{a,m}^{w} \), to be less than its maximum allowable capacity \( u_{a,m}^{w} \); Equation (4.3) is the non-negativity condition of capacity improvements. Toll constraints cater for scenarios that due to political reasons, the toll cannot be collected on certain links or set too high. These constraints can also be found in chapter 3 or in Li et al. (2007).

4.2.3 Financial Constraints

They depict the relationship between the improvement costs, toll revenues, and subsidy. These constraints include cost and revenue functions and the cost recovery constraint.

Cost and revenue functions: The toll revenue \( T_{r} \), and the improvement and maintenance cost \( K_{r} \) in period \( r \) can be expressed in terms of the equilibrium link flow \( v_{a,m}^{w} \), the toll \( \rho_{a,m}^{w} \) and the improvement \( y_{b,r} \) as follows:

\[ T_{r} = \sum_{m} \sum_{a} m v_{a,m}^{w} \rho_{a,m}^{w} \forall r, \]  \hspace{1cm} (4.4)
where $h_{b, r}$ and $w_{b, r}$ are the improvement (or construction) and maintenance cost functions of link $b$ in period $r$ respectively; $\bar{b}_{b, 0}$, $\bar{b}_{b, 1}$, $\beta_{b, 0}$, $\beta_{b, 1}$, $\beta_{b, 2}$ are parameters of these cost functions; $n$ converts link flows from an hourly basis to a period basis; $\bar{j}$ is the inflation rate; $l_b$ is the length of link $b$. Equation (4.4) calculates the toll revenue in period $r$, which is the sum of the product of the link flow and toll in that period. Equation (4.5) computes the improvement and maintenance cost in period $r$ by adding the improvement and maintenance cost of all links; Equation (4.6) is the time-dependent improvement cost function; The term $(1 + \bar{j})^{-1}$ represents the inflation factor: for the same capacity enhancement, the improvement cost increases by $\bar{j}$ 100% each period; The term $\bar{b}_{b, 0}/y_{b, 1}$ models the improvement cost of link $b$ in period 1 (i.e., the base period); Equation (4.6) depicts the general relationship that the improvement cost of a link is proportional to the extent of the widening (and hence capacity gain) and its length. This function is adopted for illustration and simplicity; other functional forms can be adopted in this framework without difficulty. Equation (4.7) is the time-dependent maintenance cost function, which is set to be: $\beta_{b, 0} + \beta_{b, 1} \left( \sum_m (nv_{b, r}^m) \right)^{\beta_{b, 2}}$, in the base period, consisting of the fixed cost $\beta_{b, 0}$ and the variable cost $\beta_{b, 1} \left( \sum_m (nv_{b, r}^m) \right)^{\beta_{b, 2}}$, in which $\sum_m (nv_{b, r}^m)$ is the link flow on link $b$ in period $r$. Again, the maintenance cost depends on the inflation factor $(1 + \bar{j})^{-1}$.

Cost recovery constraints: Cost recovery can be classified into three types: partial, exact, and profitable (Lo and Szeto, 2008). Partial (exact) cost recovery occurs when the cost in a design period is partially (exactly) recovered by the revenue, adjusted to
present value terms. Profitable cost recovery occurs when, in present value terms, the revenue more than covers the cost, with a surplus or profit at the end of the planning horizon. These three cost recovery schemes can be mathematically formulated using one equation:

$$\sum_{\tau} \frac{T_{\tau}}{(1+i)^{\tau-1}} + \sum_{\tau} \frac{S_{\tau}}{(1+i)^{\tau-1}} - \sum_{\tau} \frac{K_{\tau}}{(1+i)^{\tau-1}} = TOP,$$

where \( T_{\tau} \), \( K_{\tau} \), and \( S_{\tau} \) are, respectively, toll revenue, improvement and maintenance cost, and subsidy or contribution for network improvements in period \( \tau \); \( TOP \) is the profit or surplus of the toll road operator; \( i \) is the discount rate.

The first term on the left hand side (LHS) of (4.8) is the total discounted toll revenue for the entire planning horizon. Similarly, the second (third) term is the total discounted government subsidy (the total discounted improvement and maintenance cost). The cost recovery equation (4.8) requires that, in present value terms, the total toll revenue plus the total subsidy minus the total improvement and maintenance cost equals the surplus or profit. Depending on the values of \( S_{\tau} \) and \( TOP \), equation (4.8) reduces to a) the partial cost recovery equation if \( S_{\tau} \) is positive and \( TOP \) is zero; to b) the exact cost recovery equation if all \( S_{\tau} \) and \( TOP \) are zero; or to c) the profitable cost recovery equation if all \( S_{\tau} \) are zero and \( TOP \) is positive.

In this case the subsidy \( S_{\tau} \) is obtained from the increase in transit profit (which is numerically the same as the increase in transit revenue when the operation and maintenance cost is fixed). \( \Delta U_t^{k'} \), the subsidy can be calculated by:

$$S_{\tau} = \Delta U_t^{k'} = U_{t,\text{after}}^{k'} - U_{t,\text{before}}^{k'},$$

where

$$U_{t,\tau} = \sum_{\rho} \sum_{\varphi} p_{\rho,\varphi,\tau} n_{\rho,\varphi,\tau}, \forall \tau, k',$$

where \( p_{\rho,\varphi,\tau} \) and \( U_{t,\tau} \) are respectively the fare and revenue of transit mode \( k' \) on route \( p \) between OD pair \( ij \) in period \( \tau \) whose profit is used to subsidise road network improvements. Equation (4.10) states that the subsidy due to the revenue of transit mode
$k'$ is the sum of the product of the fare, $p_{p,i}^{x^c}$, and the corresponding passenger flow $n_{p,i}^{x^c}$ in the period considered.

4.2.4 Objective Functions

The objective thus depends on who is the decision maker. In cases where the improvement projects involve the private sector (i.e. The builder and operator are the private sector) like build-operate-transfer projects, the objective is usually profit-maximising, and the objective function is $TOP$ defined by (4.8).

In the case where the funding is wholly from the government who is in charge of a road network design, the decision maker is the government who usually considers a number of objectives from the viewpoint of society. The main one is societal benefit, or equivalently the change in societal benefit after implementing a transport policy like implementing a road construction project (because the societal benefit before the implementation is a constant that does not affect finding the optimal design during optimisation). This can be measured by the change in social surplus (SS).

The change in social surplus: The change in social surplus, $\Delta SS$, is the difference between the SS after and before the implementation, and is equal to the sum of the change in consumer surplus (CS), $\Delta CS$, the change in land owner profit, $\Delta LOP$, the change in toll revenue, $\Delta T$, the change in transit revenue, $\Delta U$, minus the change in net tax revenue, $\Delta R$, the change in improvement and maintenance cost for the toll road, $\Delta K$, and the change in operation and maintenance cost of transit modes, $\Delta Y$:

$$\Delta SS = \Delta CS + \Delta LOP + \Delta T + \Delta U - \Delta R - \Delta K - \Delta Y.$$  \hspace{1cm} (4.11)

The change in consumer surplus, $\Delta CS$, in equation (4.11) measures the difference between what consumers would be willing to pay for travel and what they actually pay. It internalises the effect of network congestion and the public’s propensity to travel. For the same network and demand characteristics, a higher CS (positive change in CS)
implies a better performing system. Here, an approximation to this change, in present value terms, is employed (Williams, 1976), which can be expressed as follows:

$$\Delta CS = \sum_{\tau} \sum_{i} \sum_{k} \frac{\Delta CS_{i,k}^{r}}{(1+i)^{r-1}},$$

$$\Delta CS_{i,k}^{r} = (1/2)(\pi_{i,k}^{r,\text{before}} + \pi_{i,k}^{r,\text{after}})(\pi_{i,k}^{r,\text{before}} - \pi_{i,k}^{r,\text{after}}), \forall i, j, k, r,$$  

where the superscripts 'before' and 'after' denote before and after improvement project implementations, respectively; $\bar{r}$ is the interest rate; $\frac{1}{(1+r)^{r-1}}$ is the discount factor for period $r$. According to (4.12), the change in CS is the sum of the change in CS for all modes and for all OD pairs over time, discounted to present value terms. Equation (4.13) is the rule-of-half definition for CS.

The change in landowner profit $\Delta LOP$ is the sum of the change of each individual discounted land owner profit $\Delta LOP_{j,r}$ over time:

$$\Delta LOP = \sum_{\tau} \sum_{j} \frac{\Delta LOP_{j,r}}{(1+i)^{r-1}}.$$  

The difference of a land owner's profit before and after the network improvement project implementations can be written as follows:

$$\Delta LOP_{j,r} = LOP_{j,r}^{\text{after}} - LOP_{j,r}^{\text{before}}, \forall j, r,$$  

where $LOP_{j,r}^{\text{before}}$ and $LOP_{j,r}^{\text{after}}$ represent the profits of land owner $j$ before and after the network improvement project implementation in period $r$; The Land Owner Profit (LOP) in residential zone $j$ in period $r$ can be expressed as follows:

$$LOP_{j,r} = R_{j,r}r_{j,r} - M_{j,r}^{H}, \forall j, r,$$  

where $R_{j,r}$ is the total number of residents in zone $j$ in period $r$ as in equation (3.9); $r_{j,r}$ is house rent per resident in residential zone $j$ in period $r$; the maintenance cost, $M_{j,r}^{H}$, on houses can be formulated as a linear function as follows:

$$M_{j,r}^{H} = M_{j,r} + m^{H}R_{j,r}, \forall j, r,$$
where $M_{j,r}$ is the fixed maintenance cost on houses in residential zone $j$ in period $r$; $m^h$ is a parameter. The rent $r_{j,r}$ is assumed to increase over time due to inflation:

$$r_{j,r+1} = r_{j,r} (1 + 	ilde{j}),$$

(4.18)

where $\tilde{j}$ is the inflation rate.

The change in toll revenue $\Delta T$ can be similarly calculated by:

$$\Delta T = \sum_r \frac{(T_{r, after} - T_{r, before})}{(1 + \tilde{i})^{r-1}}.$$  

(4.19)

The term in the numerator is the difference of toll revenue after and before the implementation of network improvement projects. This term is discounted by $\frac{1}{(1 + \tilde{i})^{r-1}}$ to form the discounted change in toll revenue in period $r$. The sum of the discounted change in toll revenue in all periods is the change in toll revenue according to (4.19).

The change in transit revenue can be defined in a way similar to the change in toll revenue:

$$\Delta U = \sum_r \sum_s \left( U_{r,s}^{k, after} - U_{r,s}^{k, before} \right),$$

(4.20)

where $U_{r,s}^{k}$ follows the definition in (4.10).

The change in tax revenue, $\Delta R$, is equal to the total discounted subsidy from the government, $\sum_r \frac{S_r}{(1 + \tilde{i})^{r-1}}$:

$$\Delta R = \sum_r \frac{S_r}{(1 + \tilde{i})^{r-1}}.$$  

(4.21)

There is only one term in the numerator as the subsidy before the implementation is zero.
Similarly, the change in improvement and maintenance cost for toll roads, $\Delta K$, can be defined like the change in tax revenue:

$$\Delta K = \sum_{r} \frac{K_r}{(1+i)^{r-1}},$$

Again, the improvement and maintenance cost is zero before the implementation, so there is only one term in the numerator.

The change in operation and maintenance cost of transit $\Delta Y$ is actually zero if it is assumed that this cost is fixed and independent of the number of passengers.

**4.2.5 Considerations in Road Network Improvement Projects**

Developing a specific model requires taking into account the parties involved in the implementation of road network projects. In general, the implementation of road network projects involve many parties, including road users, private landowners, private transit operators, private toll road operators, and the government. Each of these parties has distinctive objectives as discussed below.

*Road users: the shortest travel time and the lowest travel cost.* Travellers are concerned with their actual travel times and travel costs. The actual travel time is the shortest travel time between an OD pair:

$$\tilde{g}_{a,r}^{k} = \min \left[ \hat{g}_{p,a,r}^{k} \right], \forall i, j, k, r,$$

where

$$\hat{g}_{p,a,r}^{k} = \sum_{p} t_{p,a}^{k}, \forall p, i, j, k, r;$$

$\tilde{g}_{a,r}^{k}$ is the shortest travel time for mode $k$ between OD pair $ij$ in period $r$; $\hat{g}_{p,a,r}^{k}$ is the travel time of mode $k$ on path $p$ between OD pair $ij$ in period $r$. The travel cost is the lowest travel cost, $\pi_{a,r}^{k}$.
Private landowners: discounted profit or the change in landowner’s (discounted) profit. In the case of private landowners, they are concerned with their own total discounted profit, which is the sum of the discounted landowner profit in each year. This can be formulated as:

\[ LOP_j = \sum \frac{LOP_{j,r}}{(1+i)^{r+1}}, \forall j, \]  

(4.25)

where \( LOP_j \) is the discounted profit of landowner \( j \), and \( LOP_{j,r} \) follows the definition in (4.16)-(4.17).

After the project implementation, there must be a change in landowner profit due to redistribution of residents. This change in landowner’s profit can be used as an alternative to formulate the objective of the landowner, since the profit before the implementation is fixed. This change can be written as:

\[ \Delta LOP_j = \sum \frac{\Delta LOP_{j,r}}{(1+i)^{r+1}}, \forall j, \]  

(4.26)

where \( \Delta LOP_{j,r} \) is defined in (4.15)-(4.17). A positive value of \( \Delta LOP_j \) means the project implementation is beneficial to the landowner, and vice versa.

Private transit operators: profit. Like private landowners, the objectives of transit operators are profit-driven. The profit of the private transit operator can be written as:

\[ U_k' = \sum \frac{U_{r}^k - Y_{r}^k}{(1+i)^{r+1}}, \forall k', \]  

(4.27)

where \( U_{r}^k \) is the revenue of transit operator \( k' \) in period \( r \) defined in (4.10); \( Y_{r}^k \) is the operation and maintenance cost of transit operator \( k' \) in period \( r \).

Private toll road operators: profit. The objective of each private toll road operator is to maximise his total discounted profit \( TOP_k' \), which is the difference between the total discounted revenue \( T_k \) and the total discounted cost \( K_k' \):
\[ TOP_b = T_b - K_b, \forall b, \]  
\[ (4.28) \]

where
\[ T_b = \sum_t \sum_m \frac{m \nu_{a,\tau}^m \rho_{a,\tau}^m}{(1 + i)^{r-\tau}}, \forall b; \]  
\[ (4.29) \]
\[ K_b = \sum_t \frac{h_{b,\tau}}{(1 + i)^{r-\tau}}, \forall b; \]  
\[ (4.30) \]

The subscript \( b \) represents the toll road operator; \( m \nu_{a,\tau}^m \rho_{a,\tau}^m \) is the toll revenue in period \( \tau \) from mode \( m \) on road networks, and \( h_{b,\tau} \) is the improvement cost following the earlier definition.

**Government:** vehicular emissions, average network travel time, and equity among landowners. The government has a lot of concerns, including the whole societal benefit, the congestion problem, the environmental issue, the equity issues among travellers, among private toll road operators, and among landowners, and so on. Here, only three measures are discussed and formulated, which are vehicular emissions, average network travel time, and the equity constraints among landowners.

**Vehicular emissions and average network travel time (ANTT).** Vehicular emissions can be calculated as in equations (3.24) and (3.25) in chapter 3. For ease of reference, the formulation is provided again in this section. There are two types of vehicular emissions: link and network (or overall). The link vehicular emissions are defined through the link emission factor approach as follows:

\[ Q_{a,\tau} = \sum_m Q_{a,\tau}^m = \sum_m h_{a,\tau}^m \nu_{a,\tau}^m, \forall a, \tau, \]  
\[ (4.31) \]

where \( Q_{a,\tau}^m \) is the vehicular emissions for traffic mode \( m \) on link \( a \) in period \( \tau \); \( \nu_{a,\tau}^m \) represents the hourly traffic flow for mode \( m \) on link \( a \) in period \( \tau \); \( h_{a,\tau}^m \) is the emission factor for mode \( m \) on link \( a \) in period \( \tau \), which is assumed to be given for all links. According to (4.31), the vehicular emissions for mode \( m \) on a particular link is the product of the link flows of mode \( m \) and the corresponding emission factor, and the total vehicular emissions on this link are the sum of vehicular emissions for all modes.
travelling on this link. The overall vehicular emissions are the sum of the vehicular emissions on each link:

$$Q_t = \sum_u Q_{u,t} \forall \tau.$$  \hfill (4.32)

Average network travel time is defined as follows:

$$ANTT = \frac{\sum_k \sum_a \sum_m \sum_r v_{a,r}^m t_{a,r}^m}{\sum_r \sum_m \sum_a v_{a,r}^m}.$$  \hfill (4.33)

The numerator is the total travel time of all modes using road networks over the planning horizon, whereas the denominator is the total traffic over the planning horizon. This measure only considers the average speed of all modes on a road network and hence the congestion level of the network, which is different from consumer surplus that considers both the effect of network congestion and the public's propensity to travel.

Non-negative changes in all landowner profits. In general, the implementation of road improvement projects may result in different changes in landowner profits. Some changes can be greater than the other, and some changes can be even negative. This raises the issue of equity among landowners. Here the simplest case of equity is considered that all changes must be nonnegative. If all changes are nonnegative, it means that inequity does not exist.

4.2.6 Three Models Derived from the Proposed Single-Objective Framework

Based on the considerations above as well as different combinations of objective functions and constraints discussed above, many specific single-objective optimisation models are developed. In this section, three specific models are provided, which will be used in the numerical study. They are the profit maximisation model, the social surplus
maximisation model under exact cost recovery, and the social surplus maximisation model under cross-subsidisation:

**Profit maximisation model (PM model)**

The profit maximisation model can be obtained by setting \( y(x) \) in (4.1) to be \( \text{TOP} \) defined by (4.8):

\[
\max_{E,R,f,y,p} \text{TOP}
\]

subject to
time-dependent Lowry-type constraints and modal-split/assignment constraints;
road network design constraints, and;
financial constraints (4.4)-(4.6),

where \( E, R, f, y, p \) represent, respectively, the vectors of the number of service employment trips, the number of work-to-home trips, path flows, capacity improvement, and tolls. Note that the cost recovery condition (4.8) is included in the objective function rather than in financial constraints. This model is suitable to aid decision-making in the build-operate-transfer projects.

**Cost recovery model (CR model)**

This can be formulated as follows:

\[
\max_{E,R,f,y,p} \Delta \text{SS}
\]

subject to
the same constraints as in the PM model, and;
the cost recovery condition (4.8) with \( \text{TOP} = 0 \) and \( S_r = 0, \forall \tau \),

where \( \Delta \text{SS} \) is defined by (4.11)-(4.21). This model formulates the problem from the government’s perspective, assuming the toll revenue generated to be able to recover the improvement and maintenance cost. In the case when the improvement and maintenance cost is very expensive and the toll revenue generated is not able to recover the cost, the model gives no improvement, zero toll charges and no change in SS.
Cross-subsidisation model (CS model)

This can be formulated as follows:

\[
\max_{f, b, G, p} \Delta S
\]

subject to the same constraints as in the CR model, except \( S_r = 0, \forall r \); the cross-subsidisation condition (4.9)-(4.10),

where \( p \) is the vector of transit fares. This model also formulates the problem from the perspective of the government, assuming that there is a transit profit and the increase in profit is enough to build the toll road. In reality, the change in transit profit can be negative but the transit can still have a profit. In this case, the profit can still be used to subsidise the toll road construction and its maintenance but the cross-subsidisation condition requires modifications.

4.2.7 Model Extension: Multi-Objective Optimisation

Multi-optimisation framework

The above single-objective optimisation model may not be able to give a design that makes every party happy, as will be shown in the numerical study. If this happens, a compromised design can be available using the following multi-objective optimisation framework extended from the proposed framework discussed before:

\[
\max \sum_i w_i y_i(x)
\]

subject to the same as the single-objective framework;

\[
\sum_i w_i = 1
\]

\[
w_i \geq 0
\]

\[
\tilde{y}_i(x) \geq \epsilon
\]

where \( y_i(x) \) is the \( i \)-th (normalised) objective function; \( x \) is the vector of decision variables; \( w_i \) is the (normalised) weight for \( i \)-th objective function; \( \epsilon \) is the aspiration
level or the satisfactory objective value, and $\tilde{y}_j(x)$ is the $j$-th objective function that does not appear in the weighted objective function in (4.34).

In the above framework, the objective function (4.34) is formed by summing all the weighted objective functions. Condition (4.35) is the weight constraint, which requires the sum of all weights to be one to normalise all the weights. Condition (4.36) is the nonnegativity condition of the weights. Condition (4.37) is the performance constraint (or $\varepsilon$-constraint), which considers the objective that does not include in (4.34). The objective function is set to be greater than the desirable or satisfactory objective value to ensure that, at optimal, the $j$-th objective value is at least equal to the satisfactory value.

Two points are worth mentioning. First, the weights are input, and their relative magnitudes represent the relative importance of those objectives. Second, the larger the value of $\varepsilon$, the tighter the constraint, the closer to the optimal $j$-th objective value at optimal. The following is an example of the performance constraint.

Equity constraints: The landowner equity constraints can be expressed as:

$$\Delta LOP_j \geq 0, \forall j,$$

which ensures that all the changes in landowner profits are non-negative. However, incorporating these constraints in the single-objective optimisation discussed before can reduce the optimal objective value, which will be seen in the numerical study.

**Cost recovery model under equity consideration (CR-equity model)**

This multi-objective optimisation model will be used in the numerical study and is formulated as follows:

$$\max_{\varepsilon, R, \omega, \rho} \Delta SS$$

subject to the same constraints as in the CR model, and;

the landowner equity constraint (4.38)
where $\Delta SS$ is defined by (4.11)-(4.21). The key difference between this model and the CR model is that this model has the landowner equity constraints, avoiding reduction in landowner profit due to the implementation of network improvement projects.

### 4.3 NUMERICAL STUDIES

This study is set up to compare the three schemes of road network design, namely build-operate-transfer, cost recovery, and cross subsidisation, illustrate the impacts of the implementation of road network improvements on the related parties, especially on landowners, and show the trade-off between various objectives. The three schemes are summarised in Table 4.1. The build-operate-transfer (BOT) scheme allows a private company to build a toll road and collect tolls to recover the construction and maintenance cost within a franchised period; and after the franchised period is over, all these toll roads are transferred back to the government. This scheme is very common now in Asia and Europe. The exact cost recovery scheme uses toll revenue to exactly recover the construction and maintenance cost. The tolling and improvement strategy is to maximise the change in SS, rather than to maximise the profit as in the BOT scheme. Since the objective of this scheme is to maximise the change in SS, the private sector is not willing to be involved. The builder and operator is thus the government. This scheme can be found in India. The cross subsidisation scheme is similar to the exact cost recovery scheme except that the increase in transit profit is used to subsidise the construction and maintenance cost of the toll road. This scheme is not common and only applicable to the place like Ireland when the transit system is government-owned and can generate a huge profit.

### Table 4.1 A summary of three network design schemes.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Build-operate-transfer (BOT)</th>
<th>Exact cost recovery</th>
<th>Cross-subsidisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective</td>
<td>Maximise Profit</td>
<td>Maximise $\Delta SS$</td>
<td>Maximise $\Delta SS$</td>
</tr>
</tbody>
</table>
**Operation and management method**

| Builder and operator | Use toll revenue to recover the construction and maintenance cost | Use toll revenue to *exactly* recover the construction and maintenance cost | Use the increase in transit profit to recover the construction and maintenance cost |

4.3.1 Scenario Setting

The network adopted is shown in Figure 4.1. There are 3 links in this network: link 1, link 2, and link 3. Links 1 and 2 are links whose travel time, \( t_{a,t} \), is given by the BPR function as in equation (3.15). Link 3 is a separate transit link, as represented by a dash line in the figure. There are 3 zones too: E1, R2, and R3, in which ‘E’ stands for an employment zone whereas ‘R’ stands for a residential zone. The attractiveness of each zone is assumed to follow the following function:

\[
W_{i,t}^{\alpha} = W_{i,t}^{\alpha} \left(1 + \tilde{h}_{i,t}^Z\right),
\]

where \( \tilde{h}_{i,t}^Z \) is the growth rate of attractiveness of zone \( i \) over time. The basic employment in the employment zone is supposed to grow linearly over time:

\[
E_{i,t+1}^B = E_{i,t}^B \left(1 + \tilde{h}_{i,t}^E\right),
\]

where \( \tilde{h}_{i,t}^E \) is the growth rate of basic employment. The three zones form two OD pairs: E1-R2 and E1-R3. Both OD pairs are connected by highways but only OD pair E1-R2
has a separate transit connection. In other words, there are two modes for OD pair E1-R2 but there is only one mode for OD pair E1-R3.

The parameters in this study include:

a) Land use parameters:
\[ E_{i,1}^h = 5000 \text{ jobs}; \quad W_{i,1}^r = 3000 \text{ jobs}; \quad W_{2,1}^r = W_{3,1}^r = 3000 \text{ houses}; \quad \mu = 5; \quad s = 0.1; \]
\[ \beta' = 0.04 \text{ €}^{-1}; \quad \beta'' = 0.03 \text{ €}^{-1}; \quad M_{i,r} = \text{€100}; \quad m = \text{€0.01} / \text{household}; \]
\[ \hat{h}_{i,1} = \hat{h}_{i,2} = \hat{h}_{i,3} = 0.05; \quad \hat{h}_{i,2} = 0.04, \quad r_{2,0} = r_{3,0} = 12 \times \text{€1000} \times 10 = \text{€120000} \]

b) Environment parameters: Emission factors: \( h_i = 0.8 \text{ litre/veh}, \quad h_j = 1.2 \text{ litre/veh} \)

c) Transport network parameters:
\[ c_i^0 = c_j^0 = 3000 \text{ vph}; \quad u_i = u_j = 10000 \text{ vph}; \quad t_i^0 = t_j^0 = 5 \text{ hours}; \quad t_j^0 = 4 \text{ hours}; \]

d) Transit’s operation and maintenance cost: \( Y_r = \text{€1000000} \)

e) Parameters of improvement cost functions: \( b_{1,1} = b_{2,3} = 1, \quad b_{1,0} = b_{2,0} = \text{€2000} \)

f) Parameters of maintenance cost functions: \( \beta_{2,0} = \text{€1200}, \quad \beta_{2,3} = \text{€0.001}, \quad \beta_{2,2} = 1 \)

g) Parameters in travel cost functions: \( \psi = \text{€15/h}; \quad \theta^{\text{work}} = 16; \quad \theta^{\text{misc}} = 30; \]
\[ \beta = 0.05 \text{ €}^{-1} \]

h) Interest and inflation rates: \( \bar{i} = 0.03; \quad \bar{j} = 0.01 \)

i) Converting factor: \( n = 365 \text{days} \times 24 \text{hours} \times 10 \text{years} = 87600 \text{hours/period} \)

j) Length of each period: 10 years

k) Planning horizon and franchised period: \([0,50]\)

l) Specific parameters for each scheme:
   a. BOT: the transit fare on link 3, \( p_{3,12,r}^t = \text{€40}; \) the toll on link 1, \( p_{1,r}^l = \text{€0}; \)
      Maximum toll: \( \rho_{\text{max}}^l = \text{€ 5}; \)
   b. Cost recovery: the transit fare on link 3, \( p_{3,12,r}^t = \text{€40}; \) the toll on link 1, \( p_{1,r}^l = \text{€0}; \)
   c. Cross-subsidisation: the tolls on both links 1 and 2, \( p_{1,r}^l = p_{2,r}^l = \text{€0}. \)

These values are chosen for illustrative purposes.
4.3.2 Performance of Each Scheme

The optimal designs under the three schemes are obtained by solving the PM model, the CR model, and the CS model using the generalised reduced gradient method (Abadie and Carpentier, 1969). The corresponding performance measures are shown in Figures 4.2 and 4.3, and Table 4.2. In general, they show that road network improvements have different impacts to related parties, including road users, private landowners, transit operators, private toll road operators, and the government.

![Figure 4.2a. Travel time between OD pair E1-R2.](image)

![Figure 4.2b. Travel time between OD pair E1-R3.](image)
Figure 4.2c. Travel cost between OD pair E1-R2.

Figure 4.2d. Travel cost between OD pair E1-R3.

Figure 4.2. Travel time and travel cost over time.

Figure 4.3a. Vehicular emissions between OD pair E1-R2.
Fig. 4.3b. Vehicular emissions between OD pair E1-R3.

**Figure 4.3.** Vehicular emissions over time.

**Table 4.2** The objective measure of each party under three schemes

<table>
<thead>
<tr>
<th>Party</th>
<th>Objective measure</th>
<th>Build-operate-transfer (BOT)</th>
<th>Exact cost recovery</th>
<th>Cross-subsidisation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Landowners</strong></td>
<td>Change in profit of landowner 2 (€)</td>
<td>-9260000000</td>
<td>-13500000000</td>
<td>-19400000000</td>
</tr>
<tr>
<td></td>
<td>Change in profit of landowner 3 (€)</td>
<td>9260000000</td>
<td>13500000000</td>
<td>1940000000</td>
</tr>
<tr>
<td><strong>Toll road operator</strong></td>
<td>Profit of toll road operator</td>
<td>97700000000</td>
<td>0</td>
<td>-25500000000</td>
</tr>
<tr>
<td></td>
<td>Construction and maintenance cost (€)</td>
<td>17700000000</td>
<td>57900000000</td>
<td>255000000000</td>
</tr>
<tr>
<td></td>
<td>Toll revenue (€)</td>
<td>115000000000</td>
<td>57900000000</td>
<td>0</td>
</tr>
<tr>
<td><strong>Transit operator</strong></td>
<td>Change in profit (€)</td>
<td>243000000000</td>
<td>162000000000</td>
<td>255000000000</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td>$\Delta SS$ (€)</td>
<td>945000000000</td>
<td>1120000000000</td>
<td>1610000000000</td>
</tr>
<tr>
<td></td>
<td>$\Delta CS$ (€)</td>
<td>604000000000</td>
<td>957000000000</td>
<td>1610000000000</td>
</tr>
<tr>
<td>Average network travel time (min)</td>
<td>3.58E+02</td>
<td>3.26E+02</td>
<td>3.16E+02</td>
<td></td>
</tr>
<tr>
<td>----------------------------------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>Emissions on link 1 (litres)</td>
<td>1.44E+04</td>
<td>1.34E+04</td>
<td>1.11E+04</td>
<td></td>
</tr>
<tr>
<td>Emissions on link 2 (litres)</td>
<td>3.36E+04</td>
<td>3.80E+04</td>
<td>2.60E+04</td>
<td></td>
</tr>
<tr>
<td>Overall emissions (litres)</td>
<td>4.80E+04</td>
<td>5.14E+04</td>
<td>3.71E+04</td>
<td></td>
</tr>
</tbody>
</table>

Road users. They are concerned with their travel times and travel costs. According to Figure 4.2, the travel times and travel costs increase over time due to the increase in population over time and increase in travel demand. However, after the implementation of any road improvement projects, travel time and travel cost are less than those before.

Private Landowners. Private landowners are concerned with their own profit. As shown in Table 4.2, the profit landowner 2 will be reduced but that of landowner 3 will be increased if anyone of the schemes is implemented. Landowner 2 will object to any implementation unless the government provides him a subsidy to raise the profit back to the original level.

Toll road operators. Private toll road operators are concerned with the profit from the project. The BOT scheme will result in generating a profit, but this profit may not be too attractive as the rate of return (i.e. toll revenue/construction and maintenance cost) is about 6%, which is less than the usual norm of 10-15%. From the private toll road operator viewpoint, the project is not attractive if no subsidy is further given from the government. However, if the government gives the operator a subsidy to raise the rate of return to the minimum of 10%, the SS will decrease further.

It is worthwhile to point out that in the cost recovery scheme, the builder and operator is the government, whose objective is to maximise the change in SS subject to cost
recovery. So you can see in Table 4.2 that the profit is zero and the construction cost and maintenance cost is equal to the toll revenue.

Transit operators. When the transit operator is private and profit-driven, the operator will welcome the implementation of the BOT and cost recovery scheme because both schemes will raise the transit profit as reflected by the positive change in transit profit. In particular the operator will prefer the BOT scheme more as the change is larger.

When the transit operator is the government, the positive change in transit profit means the implementation is good to society, as the change in transit profit (or the change in transit revenue minus the change in transit’s operation and maintenance cost) is part of the change in SS. Note that when under cross-subsidisation, the change in transit profit is equal to the construction and maintenance cost of toll roads.

Government. From the government’s perspectives, the three schemes are beneficial to the society, as the change in social surplus ($\Delta SS$) is positive. In addition, from the congestion or road network performance point of view, the three schemes do improve the situation, since the change in consumer surplus ($\Delta CS$) is positive and the average network travel time is lower than 437 min which is the average network travel time without the implementation of any scheme. However, the government needs to consider the unequal change in profit between the landowners when any one of these schemes is implemented and may require subsidising the private toll road operator when the BOT scheme is implemented. According to Figure 4.3, vehicular emissions on both links increase over time with the increase in population and demand. However, the implementation of all the schemes has negative impacts on vehicular emissions on link 2, although all the schemes lower vehicular emissions on link 1. Comparatively, the cross subsidisation scheme has the least negative impact on vehicular emission on link 2 and the most positive impact on vehicular emissions on link 1. Additionally, Table 4.2 shows that the implementation of this scheme leads to a lowest overall vehicular emissions compared with other two schemes. This scheme also results in a reduction in overall vehicular emissions, which were $3.96E+04$ litres before the implementation of any scheme.
It is difficult to comment which scheme is the best in general after considering the perspectives of all the above parties. In this particular example, the cross subsidisation gives the highest ΔSS and ΔCS and the lowest overall vehicular emissions and average network travel time. This scheme is the best from the government point of view. Especially when the government is primarily concerned about overall vehicular emissions, the implementation of this scheme can lower the level of overall vehicular emissions. However, the implementation of another two schemes cause overall vehicular emissions to rise significantly. The BOT scheme gives the largest profit from toll and transit revenue, and is the best from the private transit and toll road operator point of view. When the transit operator is private, cross-subsidisation is not possible and exact cost recovery gives a better performance in terms of ΔSS, ΔCS, and the average network travel time. Then, cost recovery is the second best. However, all these observations and conclusions are based on this specific case, and cannot be generalised to other study. Nevertheless, a general observation can be made. *All schemes can lead to an unequal change in landowner profit, and some landowner’s profit can be reduced.* Landowners will object the implementation of the scheme if this happens. To avoid this to happen, their consideration has to be taken into account when designing road improvement projects.

### 4.3.3 Performance of the Cost Recovery Design under Landowner Equity Consideration

To deal with the consideration of landowners, equity constraints can be added to the three models, ensuring that the changes in landowner profits are nonnegative. For illustrative purposes, only equity constraints are added to the CR model to form the CR-equity model. This CR-equity model is solved by the generalised reduced gradient method and gives the performance measures associated with the optimal design in Table 4.3 and Figure 4.3. For the ease of comparison, the performance measures with and without landowner equity considerations are also provided in Table 4.3 and Figure 4.3.
Table 4.3 A comparison of the objective measure of each party with and without landowner equity consideration.

<table>
<thead>
<tr>
<th>Party</th>
<th>Objective measure</th>
<th>Without equity</th>
<th>With equity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Landowners</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in profit of landowner 2 (€)</td>
<td>-1350000000 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Change in profit of landowner 3 (€)</td>
<td>1350000000 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><strong>Toll road operator</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit of toll road operator</td>
<td>0 0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Construction and maintenance cost (€)</td>
<td>5790000000 58000000000</td>
<td>58000000000 58000000000</td>
<td></td>
</tr>
<tr>
<td>Toll revenue (€)</td>
<td>5790000000 58000000000</td>
<td>58000000000 58000000000</td>
<td></td>
</tr>
<tr>
<td><strong>Transit operator</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in profit or revenue (€)</td>
<td>16200000000 45300000000</td>
<td>45300000000 45300000000</td>
<td></td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔSS (€)</td>
<td>112000000000 45200000000</td>
<td>45200000000 45200000000</td>
<td></td>
</tr>
<tr>
<td>ΔCS (€)</td>
<td>95700000000 -78700000</td>
<td>95700000000 -78700000</td>
<td></td>
</tr>
<tr>
<td>Average network travel time (min)</td>
<td>3.26E+02 3.65E+02</td>
<td>3.26E+02 3.65E+02</td>
<td></td>
</tr>
<tr>
<td>Emissions on link 1 (litres)</td>
<td>1.34E+04 1.57E+04</td>
<td>1.34E+04 1.57E+04</td>
<td></td>
</tr>
<tr>
<td>Emissions on link 2 (litres)</td>
<td>3.80E+04 2.39E+04</td>
<td>3.80E+04 2.39E+04</td>
<td></td>
</tr>
<tr>
<td>Overall emissions (litres)</td>
<td>5.14E+04 3.96E+04</td>
<td>5.14E+04 3.96E+04</td>
<td></td>
</tr>
</tbody>
</table>

These results clearly show trade-offs between the perspectives of each of the parties. The equity scheme is worse than the original cost recovery design in terms of ΔSS, ΔCS and the average network travel time. ΔSS and ΔCS are smaller and the average network travel time is higher, meaning that the society receives less benefit and the road network is more congested when landowner equity is ensured. In particular, ΔCS is negative, which is highly unacceptable. Road travellers face higher travel time and cost compared with the situation without considering equity. However, the private transit operator will favour the equity scheme as the change in profit is larger. Nevertheless, the equity scheme is better regarding the overall vehicular emissions, as the overall vehicular

89
emissions are smaller in the equity scheme than the one in the cost recovery scheme without considering the equity issue among land owners.

Figure 4.4 Changes in landowner profits against toll on link 2.

Figure 4.5 Change in social surplus against toll on link 2.
Figure 4.6 Change in consumer surplus against toll on link 2.

To illustrate the trade-offs further, Figures 4.4-4.6 are provided, which are obtained by solving the CR model while setting tolls to be constant during the planning horizon. From these figures, you can see that if the government tries to increase the toll level from the optimal value of €2 in terms of SS, or to reduce the decrease of profit of landowner 2, or to minimise the difference between the two changes in landowner profits, the changes in SS and CS have to be reduced. Moreover, in the extreme case, if the landowner equity is ensured, the toll charge is €35, which may be too high and may be objected by road travellers.

Table 4.4 shows vehicular emissions and its relative reduction with the implementation of three schemes. The relative reduction in vehicular emissions (RRVE) is calculated as follows:

$$RRVE = \frac{Q^b - Q^a}{Q^b} \times 100\%,$$

where $Q^b$ and $Q^a$ represent vehicular emissions before and after the implementation of a road network design scheme. As shown in Table 4.4, cross subsidisation is the only scheme which brings about the positive relative reduction in overall vehicular emissions. In terms of the relative reduction in link emissions, the implementation of cross subsidisation scheme also leads to a highest relative reduction in vehicular emission on link 1, which is 29.19%. Although, under this scheme, the relative reduction in vehicular emissions on link 2 is negative, it is relatively smaller than the negative
reductions obtained in BOT and the cost recovery scheme without equity, which are -40.36% and -58.93% respectively. Comparatively speaking, BOT is better than the cost recovery scheme without equity in terms of the relative reduction in overall vehicular emissions as well as emissions on link 2. However, the cost recovery scheme without equity generates higher relative reduction in emissions on link 1. It is noteworthy that the cost recovery scheme with equity gives rise to approximately the same overall and link vehicular emissions as without improvement. In this case, the government may favour this scheme under the considerations of the land owner equity together with the environmental concern, because the implementation of this road network design scheme will do no harm to the land owners’ profits and the environment in terms of vehicular emissions.

Table 4.4 Vehicular emissions and its relative reduction with the implementation of three schemes.

<table>
<thead>
<tr>
<th>Vehicular emissions (litres)</th>
<th>Without improvement</th>
<th>Build-operate-transfer (BOT)</th>
<th>Cost recovery without equity</th>
<th>Cross subsidisation</th>
<th>Cost recovery with equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions on link 1</td>
<td>15683.3</td>
<td>14363.6</td>
<td>13366.6</td>
<td>11106.1</td>
<td>15686.9</td>
</tr>
<tr>
<td>Emissions on link 2</td>
<td>23932.3</td>
<td>33591.8</td>
<td>38035.6</td>
<td>25971.3</td>
<td>23944.7</td>
</tr>
<tr>
<td>Overall emissions</td>
<td>39615.6</td>
<td>47955.5</td>
<td>51402.2</td>
<td>37077.5</td>
<td>39631.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relative reduction</th>
<th>Emissions on link 1</th>
<th>Emissions on link 2</th>
<th>Overall emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions on link 1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Relative reduction</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Overall emissions</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

92
4.4 SUMMARY

In this chapter, a single-objective road network design framework is proposed to consider the land-use transport interaction over time. This framework allows the evaluation of the impact of the design on related parties including landowners and contrasts to existing models that cannot be used for such purpose as the land use transport interaction over time is not captured. This flexibility helps network planners and private firms with decision-making. This framework is formulated as a single-level maximisation program, and can be solved by many existing optimisation methods. Through the hybrid approach, the framework is also extended to consider multi-objectives. This multi-objective framework can aid the government making decisions considering the objective of each related party and eliminating a large number of alternative designs without trial-and-error.

This chapter also illustrates the models and the impacts of road network improvements on related parties, especially landowners in terms of their profits and the government in terms of vehicular emissions, under different network design schemes through a simple example, although the models can be applied to general networks. The results show that it is difficult to comment which scheme is the best in general after considering each party's perspective and that trade-offs exist between the objectives of all related parties. Moreover, all schemes lead to unequal changes in landowner profits. This raises the issue of landowner equity. If the purpose is to ensure that their profits must not be reduced, other considerations such as societal benefit and the road network performance may get worse. If vehicular emissions are the primary concern, cross subsidisation scheme is a better choice, because this scheme is the only one which gives rise to a positive relative reduction in overall vehicular emissions. In comparison, cost recovery scheme with equity is also a better choice from the government point of view, since the implementation of this scheme is not harmful to the land owners' profits and the environment in terms of vehicular emissions. With various concerns and possibilities, the government has to carefully select the road network design scheme while considering the trade-offs.
CHAPTER 5

SIMULTANEOUS OCCURRENCE OF BRAESS’ AND EMISSION PARADOXES

5.1 INTRODUCTION

Braess (1968) presented a remarkable example and demonstrated a counterintuitive phenomenon that adding a new link can increase total system travel cost (TSTC) for all travellers. However, its impact in relation to vehicular emissions caused by network expansion is less often discussed. The impact of ignoring an emission paradox can be significant. The road network improvement may be able to mitigate congestion but may increase vehicular emissions such as nitrogen oxide, carbon monoxide, nitrogen dioxide, sulphur dioxide, ozone and particulates. These emissions are harmful to human health. Up until now, only Nagurney (2000a, 2000b) has raised this great interest recently in discussing the emission paradox. Through specific numerical examples, Nagurney demonstrated that three distinctive paradoxical phenomena can occur as regards the total generated emissions. One of them revealed that, in the well-known Braess’ network, the total emissions also increase after adding the new link to the network, which is analogous to the classical Braess’ paradox. Nagurney pointed out that so-called “improvements” to the transportation network may actually induce increases in the total emissions. This further calls attention to a careful network expansion. Nevertheless, road network expansion is an expensive investment. Due to the limited budget, transport road designers should design carefully and take into account some unexpected outcomes possibly arising from road expansion, such as the increase in total system travel cost (TSTC) and overall vehicular emissions. Particularly, when the increase in emissions is ignored, road network expansion can worsen the existing environment. Therefore, it is pivotal to consider and study the simultaneous occurrence of both Braess’ and emission paradoxes so that transport road designers can avoid unnecessary waste of the governments’ budget and make sure that the design brings about an efficient and environmentally-sustained transport system.
This, thus, arises following questions to be discussed further: 1. Does the emission paradox always occur? 2. Do Braess' paradox and the emission paradox occur at the same time? 3. If not, under which condition(s) does the emission paradox occur (in Braess' network example, some conditions under which Braess' paradox occurs have been discussed by Pas and Principio (1997))? 4. If they both occur, does this simultaneous occurrence affect pricings? 5. Can traditional marginal cost congestion pricing remove Braess' and emission paradoxes at the same time? The objective of this chapter is to answer the above questions using the classical Braess' network. In addition, some marginal cost congestion and emission pricing policies are proposed and compared with existing marginal cost of congestion and emission pricing proposed by Nagurney (2000a). Different toll charges derived from these pricing policies are compared. Moreover, more demand conditions of the occurrence of Braess' paradox which are not considered by Pas and Principio (1997) will be examined in this study. The remainder of this chapter is organised as follows. The next section analyses the conditions under which the emission paradox occurs in Braess' network. The third section examines the conditions under which Braess' paradox occurs. The fourth section compares and discusses some marginal cost pricing schemes derived from the proposed models and the other existing model. The fifth section discusses the occurrence of Braess' and emission paradoxes and corresponding pricing schemes. Finally, the last section gives concluding remarks.

5.2 WHEN DOES NAGURNEY'S EMISSION PARADOX OCCUR IN BRAESS' NETWORK?

Nagurney (2000a) demonstrated that, in the well-known Braess' network, total emissions also increase after adding a new link to the network, which is analogous to the classical Braess' paradox. This section analyses the conditions under which Nagurney's emission paradox occurs in the classical Braess' network that have not been studied before. The result will be useful in analysing the simultaneous occurrence of both Braess' and emission paradoxes.
5.2.1 Description of Braess’ Network and Emission Estimation Model

Braess’ network is shown in Figure 5.1. Before a new link is added (Figure 5.1a), the network consists of four nodes, and four links, and one OD pair $rs = (1, 4)$. The four-link network has two paths between this OD pair, which are respectively path 1 (1-2-4) and path 2 (1-3-4). After a new link is added to connect nodes 2 and 3 (Figure 5.1b), this network has one more path, path 3 (1-2-3-4). Before and after adding the new link, the total travel demand is the same and fixed. The following notations are adopted: $c_a$ is the travel cost on link $a$; $\eta_p$ is the total cost on path $p$; $\eta'_p$ is the marginal of total cost on path $p$; $\tilde{\eta}'_p$ is the generalised marginal of the total cost on path $p$; $\alpha_i$ and $\alpha_s$ are the free flow travel costs, and $\alpha_1, \alpha_2 > 0$; $\beta_i$ and $\beta_s$ are the delay parameters in which a lower delay parameter means a higher capacity on a link, and $\beta_i, \beta_s > 0$; $v_a$ is the flow on link $a$; $f_p$ is the path flow on path $p$; $h_a$ is the emission factor on link $a$; $D$ is the total travel demand, and $D^*$ is the travel demand between OD pair $rs$; $Q$ is the total emissions; TSTC is Total System Travel Cost.

\[
\begin{align*}
c_{12} &= \beta_1 v_{12} \\
c_{13} &= \alpha_1 + \beta_2 v_{13} \\
c_{24} &= \alpha_1 + \beta_2 v_{24} \\
c_{34} &= \beta_1 v_{34} \\
c_{23} &= \alpha_2 + \beta_3 v_{23}
\end{align*}
\]

(a) Four-link network  
(b) Five-link network

Figure 5.1 Braess’ networks.
According to the emission models developed by the United States Environmental Protection Agency (USEPA), the volume of emissions can be calculated by multiplying the emissions factor and the vehicle activity (or link load), which is the key in the estimation of vehicular emissions (Allen, 1995; Anderson et al., 1996; DeCorla-Souza et al., 1994). This relationship in calculating vehicular emissions is also utilised in this chapter described as follows:

\[ Q = \sum_a h_a v_a, \]

where \( Q \) is the total emissions, \( h_a \) is the emission factor on link \( a \), which is assumed to be given for all links, and \( v_a \) is the flow on link \( a \). According to equation (5.1), link flows are required to calculate the total emissions generated in the network. With the total emissions, the conditions when Nagurney’s emission paradox occurs can be examined. In the following, flow patterns and the total emissions before and after adding the new link are obtained.

### 5.2.2 User Equilibrium Assignment Conditions with Emissions

Before the addition of link 2-3, two paths’ costs are equal and the flow on each path is the same when User Equilibrium condition is reached. One can obtain:

\[ \eta_i = \eta_2 = \frac{D(\beta_i + \beta_3)}{2} + \alpha_i, \quad \text{and} \]
\[ f_i = f_2 = D/2. \]

The total emissions in the network are then calculated as follows:

\[ Q_{\text{four links}} = h_{12} v_{12} + h_{24} v_{24} + h_{31} v_{31} + h_{34} v_{34}, \quad \text{or} \]
\[ Q_{\text{four links}} = D(h_2 + h_3 + h_4)/2. \]

After link 2-3 is added, there are three possible flow patterns, which are respectively: 1. There are flows on all three paths. (i.e. \( f_i, f_2, f_3 > 0 \)); 2. Only the new path, path 3, is used. (i.e. \( f_1 = f_2 = 0, f_3 > 0 \)); 3. Only the new path is not used. (i.e. \( f_1 = f_2 > 0, f_3 = 0 \)), which is the same as the case that new link is not added in the four-link network. As it will be seen later, the occurrence of each flow pattern depends on the network configuration (or the parameters in the performance function of each link).
It is noted that due to the symmetrical characteristic of Braess’ network, the flows on paths 1 and 2 are always the same, i.e. \( f_1 = f_2 \). In the following, the three flow patterns and the corresponding total emissions are described.

1. **All three paths are used** (i.e. \( f_1, f_2, f_3 > 0 \)).

In this case, travel costs on three paths are equal, and the User Equilibrium flow pattern must satisfy the following conditions:

\[
\begin{align*}
    f_1 &= f_2 = \frac{\alpha_3 - \alpha_1 + D(\beta_1 + \beta_3)}{\beta_1 + 3\beta_2}, \\
    f_3 &= D - 2f_1, \\
    \eta_1 &= \eta_2 = \eta_3 = \alpha_1 + D\beta_1 + (\beta_2 - \beta_1) \left[ \frac{\alpha_3 - \alpha_1 + D(\beta_1 + \beta_3)}{\beta_1 + 3\beta_2} \right],
\end{align*}
\]

and the total emissions in the network are:

\[
Q_{\text{flows}} = f_1(h_{34} + h_{31} - h_{12} - h_{34} - 2h_{32}) + D(h_{12} + h_{34} + h_{23}).
\]

2. **Only new path (path 3) is used** (i.e. \( f_1 = f_2 = 0, f_3 > 0 \)).

In this case, paths 1 and 2 carry no flow and only new path (path 3) carries flow. The User Equilibrium flow pattern must satisfy:

\[
\begin{align*}
    f_1 &= f_2 = 0, \\
    f_3 &= D, \\
    \eta_1 &= \eta_2 = \alpha_1 + \beta_1D \geq \eta_3 = D(2\beta_1 + \beta_2) + \alpha_3.
\end{align*}
\]

One can then obtain:

\[
D \leq \frac{\alpha_1 - \alpha_3}{\beta_1 + \beta_2}.
\]

The total emissions, in this case, are as follows:

\[
Q_{\text{flows}} = f_3(h_{12} + h_{23} + h_{34}) = D(h_{12} + h_{23} + h_{34}).
\]

3. **Only the new path (path 3) is not used** (i.e. \( f_1 = f_2 > 0, f_3 = 0 \)).
In this case, path 3 carries no flow and the total emissions are the same as described in equation (5.4). The User Equilibrium flow pattern must follow:

\[
f_i = f_2 = D/2, \quad f_3 = 0, \quad \text{and} \quad \eta_1 = \eta_2 = \alpha_1 + \beta_1 D/2 + \beta_2 D/2 \leq \eta_3 = \alpha_2 + \beta_1 D/2 + \beta_2 D/2. \tag{5.11}
\]

From equation (5.11), one has:

\[
(\beta_1 - \beta_2)D \geq 2(\alpha_1 - \alpha_2). \tag{5.12}
\]

According to the above three types of flow patterns, the changes in total emissions can be expressed as follows

1. All three paths are used (i.e., \( f_1, f_2, f_3 > 0 \)):

\[
Q_{\text{five links}} - Q_{\text{four links}} = \left( D - D/2 \right) \left( h_{24} + h_{34} - h_{14} - 2h_{23} \right) = (f_i - D/2)A. \tag{5.13}
\]

where \( f_i \) must satisfy equation (5.5) and \( A = h_{24} + h_{34} - h_{14} - 2h_{23} \).

2. Only new path (path 3) is used (i.e., \( f_1 = f_2 = 0, f_3 > 0 \)):

\[
Q_{\text{five links}} - Q_{\text{four links}} = \left( D/2 \right) \left( h_{14} + 2h_{23} + h_{34} - h_{14} - h_{23} \right) = (-D/2)A. \tag{5.14}
\]

3. Only new path (path 3) is not used (i.e., \( f_1 = f_2 > 0, f_3 = 0 \)):

\[
Q_{\text{five links}} - Q_{\text{four links}} = 0. \tag{5.15}
\]

### 5.2.3 Occurrence Conditions of the Emission Paradox

Based on the above changes in total emissions, whether Nagurney’s emission paradox occurs in Braess’ network can be determined. The emission paradox occurs if and only if the change in total emission is positive (i.e., \( Q_{\text{five links}} - Q_{\text{four links}} > 0 \)). According to equation (5.13), \( f_i - D/2 = f_1 - \frac{f_1 + f_2 + f_3}{2} = -f_3/2 < 0\). This implies that the emission paradox occurs if \( A < 0 \). Similarly, according to (5.14), \(-D/2 < 0\). This implies that the emission paradox occurs if \( A < 0 \). According to (5.15), the emission paradox must not occur. These implications are summarised in Table 5.1; the emission paradox occurs if \( A < 0 \) and the new path carries flow (the old paths may or may not carry flows). When \( A < 0 \), it is assumed that emission factors only depend on distance travelled, or the emission factor of each link is proportional to the travel distance on that link.
According to the triangle formula, \((h_3 - h_{12} - h_{13}) < 0\) and \((h_{43} - h_{34} - h_{23}) < 0\). One then has 
\[ A = (h_3 - h_{12} - h_{13}) + (h_{43} - h_{34} - h_{23}) < 0. \]
However, if emission factors depend on not only distance travelled but also some other factors, which have been pointed out by DeCorla-Souza et al. (1994), then \(A\) can be any real values. From the above expressions, it is observed that whether Nagurney’s emission paradox occurs or not depends on the flow pattern on the new network as well as the emission factor of each link. The result is summarised in Table 5.1.

Therefore, it is essential to know the demand conditions under which the new path will carry flows and under which the new path will never carry flows when examining the occurrence of the emission paradox. According to different relationships of parameters in link performance functions, immediately in Table 5.2 below the demand conditions under which three flow patterns occur based on equations (5.10) and (5.12) are summarised. For example, in condition 1, all three flow patterns occur but under different demand conditions. When \(\frac{\alpha_i - \alpha_j}{\beta_i + \beta_j} < D < \frac{2(\alpha_i - \alpha_j)}{\beta_i + \beta_j}\), there are flows on all three paths; when \(D \leq \frac{\alpha_i - \alpha_j}{\beta_i + \beta_j}\), only path 3 carries flows; however, when \(D \geq \frac{2(\alpha_i - \alpha_j)}{\beta_i + \beta_j}\), path 3 carries no flow. In condition 2, there are only two flow patterns occurred. When \(D > \frac{2(\alpha_i - \alpha_j)}{\beta_i + \beta_j}\), all three paths carry flows. When \(D \leq \frac{2(\alpha_i - \alpha_j)}{\beta_i + \beta_j}\), the new path will not be used. Parameters’ conditions 3 and 4 have the same demand conditions under which flow patterns 1 and 2 occur, and under any demand value flow pattern 3 does not occur. When parameters in link performance functions fall into condition 5, under any demand value, only flow pattern 1 happens, in which all three paths are utilised. It is noticeable that the same flow pattern occurs in conditions 6, 7, 8 and 9. Under any demand value only flow pattern 3 occurs and flow patterns 1 and 2 do not occur. This implies, in these cases, the new link does not play any role in the network after construction.
Table 5.1 Summary of the occurrence of the emission paradox under each flow pattern.

<table>
<thead>
<tr>
<th>Flow patterns</th>
<th>Conditions of parameters</th>
<th>1. ( f_1, f_2, f_3 &gt; 0 )</th>
<th>2. ( f_1, f_2 = 0, f_3 &gt; 0 )</th>
<th>3. ( f_1, f_2 &gt; 0, f_3 = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A &lt; 0 )</td>
<td>The emission paradox must happen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A \geq 0 )</td>
<td>The emission paradox never happens</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2 Summary of demand conditions under which three flow patterns occur according to different relationships of parameters in link performance functions.

<table>
<thead>
<tr>
<th>Flow patterns</th>
<th>Conditions of Parameters</th>
<th>1. ( f_1, f_2, f_3 &gt; 0 )</th>
<th>2. ( f_1, f_2 = 0, f_3 &gt; 0 )</th>
<th>3. ( f_1, f_2 &gt; 0, f_3 = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 - \beta_2 &gt; 0; \alpha_1 - \alpha_2 &gt; 0 )</td>
<td>( \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} &lt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} )</td>
<td>( D \leq \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} )</td>
<td>( D \geq \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} )</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 - \beta_2 &lt; 0; \alpha_1 - \alpha_2 &lt; 0 )</td>
<td>( D &gt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} )</td>
<td>Never happen under any demand value</td>
<td>( D \leq \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} )</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 - \beta_2 &lt; 0; \alpha_1 - \alpha_2 &lt; 0 )</td>
<td>( D \geq \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} )</td>
<td>Never happen under any demand value</td>
<td>( D \leq \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} )</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 - \beta_2 = 0; \alpha_1 - \alpha_2 &gt; 0 )</td>
<td>Must happen under any demand value</td>
<td>Never happen under any demand value</td>
<td>Never happen under any demand value</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 - \beta_2 &lt; 0; \alpha_1 - \alpha_2 &lt; 0 )</td>
<td>Never happen under any demand value</td>
<td>Must happen under any demand value</td>
<td>Never happen under any demand value</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 - \beta_2 &lt; 0; \alpha_1 - \alpha_2 &lt; 0 )</td>
<td>Never happen under any demand value</td>
<td>Must happen under any demand value</td>
<td>Never happen under any demand value</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 - \beta_2 = 0; \alpha_1 - \alpha_2 = 0 )</td>
<td>Never happen under any demand value</td>
<td>Must happen under any demand value</td>
<td>Never happen under any demand value</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 - \beta_2 = 0; \alpha_1 - \alpha_2 = 0 )</td>
<td>Never happen under any demand value</td>
<td>Must happen under any demand value</td>
<td>Never happen under any demand value</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 - \beta_2 = 0; \alpha_1 - \alpha_2 = 0 )</td>
<td>Never happen under any demand value</td>
<td>Must happen under any demand value</td>
<td>Never happen under any demand value</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 - \beta_2 = 0; \alpha_1 - \alpha_2 = 0 )</td>
<td>Never happen under any demand value</td>
<td>Must happen under any demand value</td>
<td>Never happen under any demand value</td>
<td></td>
</tr>
</tbody>
</table>
If the new link does not play any role in the network, the emission paradox will not occur. According to Table 5.2 conditions under which the emission paradox occurs or not can be summarised. When \( A < 0 \), it is obvious that the emission paradox is more likely to occur when parameters in link performance functions satisfy conditions 1 and 2. In conditions 3, 4 and 5, the emission paradox must occur if \( A < 0 \), because the new link is utilised under any demand value. However, in condition 6, 7, 8 and 9, the emission paradox does not occur, as the new link does not play any role under any demand value after construction. After examining the demand conditions under which different flow patterns occur, it is shown that under conditions 6, 7, 8 and 9, there is no point in building the new link, as in any circumstance it does not play any role in the network. In terms of cost and benefit, the road designer should examine the network configuration carefully to avoid such redundant construction.

According to parameters in Nagurney’s example the parameters in Braess’ network are \( \alpha_i = 50, \alpha_j = 10, \beta_1 = 10, \beta_2 = 1, D = 6, h_{12} = h_{13} = h_{24} = h_{34} = h_{23} = 0.1 \), one can easily evaluate under which condition Nagurney’s emission paradox occurs. As \( A = h_{14} + h_{1} - h_{12} - h_{13} - 2h_{23} = -0.2 < 0 \), also \( \beta_1 - \beta_2 > 0, \alpha_1 - \alpha_2 > 0 \) falls into condition 1, then there is \( D = 6 < \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} = 8.89 \). In this situation, the emission paradox occurs as the new path carries some flows.

It is demonstrated that the occurrence of the emission paradox largely depends on parameters of link performance functions, the total demand as well as link emission factors. Pas and Principio (1997) have demonstrated some demand conditions under which Braess’ paradox occurs, which also depends on parameters of link performance functions and total demand. Therefore, it is interesting to discuss if there is simultaneous occurrence of Braess’ and emission paradoxes. In the following, Pas and Principio (1997)’s work is firstly extended to further examine all other conditions under which Braess’ paradox occurs, which they did not discuss. And then the simultaneous occurrence of Braess’ and emission paradoxes is discussed.
5.3 WHEN DOES BRAESS' PARADOX OCCUR?

Pas and Principio (1997) have examined some demand conditions under which Braess' paradox occurs when \( \beta_1 - \beta_2 > 0, \alpha_1 - \alpha_2 > 0 \), which is condition 1 in this study. Here, all other demand conditions under which Braess' paradox occurs are further examined. From Table 5.2, it is shown that in conditions 6, 7, 8 and 9 the new path must not carry any flow, so Braess' paradox will not occur. Therefore, only conditions 2, 3, 4, and 5 are examined. Both demand conditions for the occurrence of Braess' and emission paradoxes are summarised in Table 5.3 below. It indicates that, in condition 1, Braess' and emission paradoxes occur when demand falls into the range described in Table 5.3. In condition 2, Braess' paradox never happens, but the emission paradox may happen under certain demand ranges. In conditions 3, 4 and 5, the emission paradox must occur, however Braess' paradox never happens. In conditions 6, 7, 8 and 9, both Braess' and emission paradoxes never take place.

Table 5.3 Demand conditions under which Braess' and emission paradoxes occur.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Braess' Paradox</th>
<th>Emission Paradox (( A &lt; 0 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \beta_1 - \beta_2 &gt; 0, \alpha_1 - \alpha_2 &gt; 0 );</td>
<td>When ( \frac{2(\alpha_1 - \alpha_2)}{3\beta_1 + \beta_2} &lt; D &lt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} ), Braess' paradox occurs</td>
<td>When ( D &lt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} ), the emission paradox occurs</td>
</tr>
<tr>
<td>2. ( \beta_1 - \beta_2 &lt; 0, \alpha_1 - \alpha_2 &lt; 0 );</td>
<td>Braess' paradox never occurs under any demand value</td>
<td>When ( D &gt; \frac{2(\alpha_1 - \alpha_2)}{(\beta_1 - \beta_2)} ), the emission paradox occurs</td>
</tr>
<tr>
<td>3. ( \beta_1 - \beta_2 &lt; 0, \alpha_1 - \alpha_2 &gt; 0 );</td>
<td></td>
<td>The emission paradox must occur under any demand value</td>
</tr>
<tr>
<td>4. ( \beta_1 - \beta_2 = 0, \alpha_1 - \alpha_2 &gt; 0 );</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. ( \beta_1 - \beta_2 &lt; 0, \alpha_1 - \alpha_2 = 0 );</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6, 7, 8 and 9.</td>
<td>Both paradoxes never occur under any demand value</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.2 shows the occurrence of two paradoxes in the classical Braess’ network. As shown, under Braess’ network parameters with Nagurney’s emission factors, there are three regions:

1. Both paradoxes do not occur (when demand is larger than or equal to 8.89 units).
2. Both paradoxes occur at the same time (when demand is between 2.58 and 8.89 units).
3. The emission paradox occurs but Braess’ paradox does not (when demand is less than 2.58 units).

The existence of the last region implies that road network design for mitigating congestion alone may not be able to avoid the increase in vehicular emissions.

Moreover, it is observed that the range of demand for the occurrence of the emission paradox includes the range for the occurrence of Braess’ paradox, which implies that the emission paradox is more likely to occur. However, it is hard to know whether this is always true for any network. This is left for future studies.

Figure 5.2 The occurrence of two paradoxes in Braess’ network, where
\[ \beta_1 - \beta_2 = 10 - 1 > 0 \text{ and } \alpha_1 - \alpha_2 = 50 - 10 > 0. \]
5.4 INTERNALISING CONGESTION AND EMISSION EXTERNALITIES SIMULTANEOUSLY

In this section, different marginal cost pricings are discussed and compared so as to seek a best pricing scheme to tackle the occurrence of Braess’ and the emission paradoxes. Firstly it begins with traditional marginal cost pricing that is marginal cost congestion pricing. And then a model is proposed to minimise the total emissions subject to constraining TSTC not to be higher than the required one. Marginal cost pricing is derived from a proposed model. Then another model based on Yin and Lawphongpanich (2006) is proposed and marginal cost pricing is also derived based on this proposed model. Marginal cost pricings are compared with existing pricing proposed by Nagurney (2000a) which has a similar objective to control congestion and vehicular emissions. Different pricing schemes derived according to different optimality conditions from these models are discussed and compared below.

5.4.1 Traditional Marginal Cost Congestion Pricing

Traditional road pricing such as first best congestion pricing has been considered as one of the most reliable tools to relieve congestion. However, its net impact on controlling vehicular emissions has been a conjecture with some argument in the transport arena. It is often assumed solving congestion problem by charging road users would result in decreased vehicular emissions (Jakobsson et al., 2000; May et al., 2002; Parry and Bento, 2002; Steininger et al., 2007; Yang and Bell, 1997). However, other researchers have noticed that this conventional wisdom that improved traffic flow or reduced vehicular miles travelled means less congestion and better air quality has been challenged (Hall, 1995, described by Nagurney, 2000a). Therefore, this controversial issue of the impact of traditional congestion pricing on controlling vehicular emissions awaits more investigation. Herein, the effectiveness of traditional marginal cost congestion pricing on controlling vehicular emissions is examined.

Pas and Principio (1997) have pointed out that when demand satisfies the following inequality, total system travel time is reduced after the new link is added:
However, from the conclusion drawn above, when the new link is utilised, the emission paradox must occur if \( A < 0 \). Nagurney (2000a) discovered the emission paradox occurrence in the classical Braess’ network, where the emission factors satisfy \( A < 0 \). It is expected that marginal cost congestion pricing would lead to decreased vehicular emissions as it relieves congestion. However, it is not true in Braess’ network. After marginal cost congestion pricing, the emission paradox still occurs when demand satisfies inequality (5.16). As under this condition, the new link carries some flows. For example, when demand is 4 units, which is smaller than \( \frac{\alpha_1 - \alpha_2}{\beta_1 - \beta_2} = 4.44 \). After adding the new link, all three paths carry flows. By equalising marginal cost on each path, one then has \( f_1 = f_2 = 1.85, f_3 = 0.3 \). The resulting vehicular emissions are 0.83. However, before adding the new link, the resulting vehicular emissions are 0.8. It shows that marginal cost congestion pricing does not lead to decreased vehicular emissions, even the congestion is mitigated. Figure 5.3 below demonstrates the occurrence of the emission paradox under marginal cost congestion pricing in the classical Braess’ network.

The emission paradox still occurs under the marginal cost congestion pricing as the new path carries flows.

The emission paradox does not occur under the marginal cost congestion pricing as the new path carries no flow.

**Figure 5.3** The relationship between the path flow and demand under marginal cost congestion pricing.
When demand is smaller than 4.44, the emission paradox still occurs even under marginal cost congestion pricing. When demand is greater than 4.44, the emission paradox does not occur as the new link has no flow under marginal cost congestion pricing.

5.4.2 Marginal Cost Congestion and Emission Pricings

As shown above, marginal cost congestion pricing alone cannot deal with the occurrence of the emission paradox. Two models are then proposed to tackle this problem. Moreover, the marginal cost of congestion and emission pricing policy proposed by Nagurney (2000a) is reviewed. These pricings are also further compared in this section.

Model 5.1: Minimising the Total Emissions Subject to TSTC Constraint

As the main concern is to minimise the total emissions, a model is proposed to minimise the total emissions subject to constraining TSTC not to be higher than the previous one in the four-link network. The model can be described as follows:

\[
\text{Minimise } Q = \sum_{a} h_{a} \sum_{p} f_{p} \delta_{a}^{p} \quad (5.17)
\]

subject to:

\[
\sum_{p} f_{p} = D^{n}, \forall rs, \quad (5.18)
\]

\[
\sum_{p} f_{p} \eta_{p} \leq \overline{\text{TSTC}}, \quad (5.19)
\]

\[
f_{p} \geq 0, \forall p. \quad (5.20)
\]

Where condition (5.17) is the objective function, which is to minimise the total emissions. Conditions (5.18)-(5.20) are the constraints, which are respectively to guarantee that the path flow pattern satisfies the total travel demand, total system travel cost from the flow pattern does not exceed the previous TSTC, and to ensure the path flows are nonnegative.
It is noted that since the objective function is convex as it is linear, travel cost functions in constraint (5.19) are increasing functions, which means constraint (5.19) is also convex, and other constraints are convex as they are linear. Hence, according to equations (5.17)-(5.20), the KKT optimisation conditions can be derived and described as follows:

\[
H_p(f^*_p, \gamma^*) = \sum_a h_a \delta^*_a + \gamma^* \eta'_p(f^*_p) \begin{cases} 
\phi^*, & \text{if } f^*_p > 0 \\
\geq \phi^*, & \text{if } f^*_p = 0,
\end{cases}
\]  
(5.21)

\[
TSTC = \sum_p f_p \eta_p \begin{cases} 
= 0, & \text{if } \gamma^* > 0 \\
\geq 0, & \text{if } \gamma^* = 0.
\end{cases}
\]  
(5.22)

where \(H_p\) denotes the generalised marginal of emissions on path \(p\); \(f^*_p\) and \(\gamma^*\) are an equilibrium path flow pattern and the Lagrange Multiplier associated with TSTC constraint (5.19); \(h_a\) is the emission factor on link \(a\); \(\delta^*_a = 1\), if link \(a\) is contained on path \(p\), and 0, otherwise; \(\sum_a h_a \delta^*_a\) is the sum of the emission factors on link \(a\) which comprise path \(p\); \(\eta'_p\) represents the marginal of total cost on path \(p\); \(\gamma^* \eta'_p(f^*_p)\) denotes the marginal of emissions on path \(p\) due to marginal travel costs generated by travellers on path \(p\); when the equilibrium is reached, the marginal of total cost on path \(p\) is higher, the marginal of emissions on path \(p\) is higher, and when the marginal of total cost on path \(p\) is lower, the marginal of emissions on path \(p\) is lower; \(\phi^*\) is the lowest generalised marginal of emissions between OD pair \(rs\); \(\eta_p\) represents the total cost on path \(p\); \(\sum_p f_p \eta_p\) denotes total system travel cost over all paths. Equation (5.21) means all used paths have the equal and minimal generalised marginal of emissions, and all unused paths have the greater generalised marginal of emissions. Equation (5.22) describes when the Lagrange Multiplier is greater than zero, \(\gamma^* > 0\), total system travel cost is the same as the required TSTC, if \(\gamma^* = 0\), total system travel cost is smaller than the required TSTC.

When \(\gamma^* > 0\), total system travel cost is the same as the required TSTC, the optimality condition (5.21) can be modified as follows:
\[ \tilde{\eta}_p'(f_p', \gamma^*) = \eta_p'(f_p') + \sum_a \frac{h_a \delta_a^p}{\gamma^*} \begin{cases} = \mu^\alpha, & \text{if } f_p' > 0 \\ \geq \mu^\alpha, & \text{if } f_p' = 0 \end{cases} \quad (5.23) \]

and when \( \gamma^* = 0 \), total system travel cost is smaller than and equal to the required TSTC, the optimality condition (5.21) can then be reduced as follows:

\[ H_p(f_p', \gamma^*) = \sum_a h_a \delta_a^p \begin{cases} = \varphi^\alpha, & \text{if } f_p' > 0 \\ \geq \varphi^\alpha, & \text{if } f_p' = 0 \end{cases} \quad (5.24) \]

In equation (5.23), \( \tilde{\eta}_p' \) denotes the generalised marginal cost on path \( p \); \( \eta_p' \) represents the marginal of total cost on path \( p \). Condition (5.23) is to calculate the generalised marginal travel cost, which is the sum of the marginal of total cost on path \( p \) and the marginal of total cost due to emissions generated on the same path. Equation (5.23) also implies that the generalised marginal travel costs on used paths for each OD pair are equal and minimal, and unused paths have greater or equal generalised marginal travel costs. In this case, a pricing policy can be implemented to charge each traveller a toll amount as follows:

\[ \rho_p = \eta_p' - \eta_p + \frac{\sum_a h_a \delta_a^p}{\gamma^*} \quad (5.25) \]

where \( \rho_p \) is the toll on path \( p \); \( \eta_p' - \eta_p \) represents congestion pricing; and \( \frac{\sum_a h_a \delta_a^p}{\gamma^*} \) denotes emission pricing. This combined emission and congestion toll can guarantee that both the total emissions are minimised as well as that the total system travel cost does not exceed the required level. When the Lagrange Multiplier is larger, the emission toll is smaller. When it is smaller, the corresponding emission toll is larger. Also it is noteworthy that conditions (5.21) and (5.23) are equivalent when \( \gamma^* > 0 \). This indicates when used paths with minimum and equal generalised marginal of emissions will have minimum and equal generalised marginal of total cost. After the imposition of congestion and emission tolls, the traffic flow pattern is system optimised, but travellers behave in a user-optimised manner.
Equation (5.24) indicates that, when TSTC is smaller than its limit, the traffic is routed through the network such that the generalised marginal of emissions on all routes used between the same OD pair is equal and minimum, and all unused routes have equal or greater generalised marginal of emissions. In this case, no pricing policy is required. A similar System Optimised traffic assignment regarding to emission minimisation (SO-emissions) has been studied by Rilett and Benedek (1994) and Benedek and Rilett (1998) respectively. Rilett and Benedek (1994) discovered that the reduction in total system travel time and the reduction in environmental pollution, specifically vehicular emissions, may actually conflict. In the proposed model, this problem is tackled. The TSTC constraint is to ensure TSTC does not exceed its limit. Therefore, in the proposed model, not only system optimality regarding the total emissions minimisation can be achieved, also TSTC will not exceed its limit. When total system travel cost is smaller than the required TSTC, \( \gamma' = 0 \), the SO-emissions assignment is the same as the macroscopic traffic assignment used in Benedek and Rilett (1998). This traffic assignment can be implemented with the advent of Intelligent Transportation System, i.e. in-vehicle route guidance navigation system, in which network information is provided to travellers so that the total emissions are minimised. The SO-emissions flow pattern obtained in this model is different from the SO-TSTC flow pattern obtained under the conventional System Optimised equilibrium condition, in which total system travel cost is the main concern to be minimised.

Now Nagurney's example is solved by using model (5.17)-(5.20), but demand is 4 units, because which is under the range marginal cost congestion pricing cannot remove the emission paradox. It is calculated that TSTC without adding the new link is 288 units. Assuming the new network's TSTC cannot be greater than the previous one, then 288 units is the required TSTC. The following optimal solution is then obtained:

\[
 f_1^* = f_2^* = 2, f_3^* = 0, TSTC = 288, Q = 0.8, \gamma' = 0. \tag{5.26}
\]

There are no Braess' and emission paradoxes in this case if the traffic is assigned according to the SO-emissions equilibrium. Therefore, no pricing policy is needed under such a traffic assignment methodology.

*Model 5.2: Minimising Both TSTC and the Total Emissions*
Yin and Lawphongpanich (2006) proposed a multi-objective method for computing optimal toll charges in such a way that allows decision makers to trade-off between these two conflicting objectives. They associated each objective function with a weighting coefficient and minimised the sum of the objectives so that multiple objective functions were transformed into a single objective function. However, their objective function was not normalised or scaled. In this case, the role of weighting coefficients may be greatly misleading. The effectiveness of their computed tolls from this multi-objective method is therefore questionable. As Miettinen (1999) has emphasised that normalising objective functions is to ensure their objective values are approximately of the same magnitude. In this way, one can manoeuvre this weighting method to produce solutions of a desired nature in proportion to the range of the objective functions. Therefore, a normalised multi-objective weighting method is proposed based on their work, which combines both congestion and emission minimisation into its objective function. The model can be described as follows:

\[
\text{Minimise} \left[ e \left( \frac{TSTC}{TSTC} \right) + \lambda \left( \frac{Q}{Q} \right) \right] = \left[ \sum_{p} \frac{f_{p} \eta_{p}}{TSTC} + \lambda \left( \frac{\sum_{a} \sum_{p} f_{p} \delta_{p}^a}{Q} \right) \right] \quad (5.27)
\]

subject to:

\[
\sum_{p} f_{p} = D^{re}, \forall rs,
\]

\[
f_{p} \geq 0, \forall p.
\]

Where equations (5.28) and (5.29) are the same as equations (5.18) and (5.20), the objective function in equation (5.27) is a combined objective function in which both TSTC and the total emissions are minimised. In equation (5.27), where \( e \) and \( \lambda \), weights for TSTC and the total emissions, are nonnegative. They have a relationship as follows:

\[
e = 1 - \lambda. \quad (5.30)
\]

\[
\sum_{p} \frac{f_{p} \eta_{p}}{TSTC} \quad \text{and} \quad \sum_{a} \sum_{p} \frac{f_{p} \delta_{p}^a}{Q}
\]

are to normalise TSTC and the total emissions by dividing its required objective values so that, in the objective function (5.27), both TSTC and the total emissions are of approximately the same magnitude (Osyczka, 1984; Osyczka, 1992; described by Miettinen, 1999). The model (5.27)-(5.29) provides a multi-
objective analysis in handling problems of congestion and emissions. According to each party's concern, by adjusting weights of $\varepsilon$ and $\lambda$ between TSTC and the total emissions, one can strike a balance between these two.

According to conditions (5.27)-(5.29), the KKT optimisation conditions can be derived and described as follows:

$$\tilde{\eta}_p(f_p) = \eta'_p(f_p) + \frac{\lambda \cdot TSTC \sum_a h_a \delta_a^p}{\varepsilon \tilde{Q}} \begin{cases} = \mu^\alpha, & \text{if } f_p^* > 0 \\ \geq \mu^\gamma, & \text{if } f_p^* = 0, \end{cases} \quad (5.31)$$

where $\tilde{\eta}_p$ denotes the generalised marginal cost on path $p$; $\eta'_p$ represents the marginal of total cost on path $p$; $h_a$ is the emission factor on link $a$; $\delta_a^p = 1$, if link $a$ is contained on path $p$, and 0, otherwise; $f_p^*$ is an equilibrium path flow pattern and $\frac{\lambda \cdot TSTC \sum_a h_a \delta_a^p}{\varepsilon \tilde{Q}}$ denotes the marginal of total cost on path $p$ due to the emissions generated by travellers on path $p$. When the required $TSTC$ and $\tilde{Q}$ are predetermined, this cost on path $p$ is actually dependent on the ratio of weighting between two objectives, $\frac{\lambda}{\varepsilon}$, and also dependent on the path emission factors, $\sum_a h_a \delta_a^p$. When one has more emphasis on TSTC rather than the total emissions, the weight associated with TSTC can be greater than the one associated with the total emissions, i.e. $\varepsilon > \lambda$, the weight is biased to TSTC, and the marginal cost regarding to emissions, $\frac{\lambda \cdot TSTC \sum_a h_a \delta_a^p}{\varepsilon \tilde{Q}}$, is smaller. When one has more concern on the total emissions, the weight associated with the total emissions can be greater than the one associated with TSTC, i.e. $\varepsilon < \lambda$, the weight is biased to the total emissions, and the marginal cost regarding to emissions, $\frac{\lambda \cdot TSTC \sum_a h_a \delta_a^p}{\varepsilon \tilde{Q}}$, is greater. Equation (5.31) is to calculate the generalised marginal travel cost, which is the sum of the marginal of total cost on path $p$ and the marginal of total cost due to emissions generated on this path. $\mu^\gamma$ equals the minimum generalised marginal of travel cost between OD pair $rs$. Therefore, equation (5.31) also implies that the generalised marginal travel costs on used paths for each OD
pair are equal and minimal, and unused paths have greater generalised marginal travel costs or equal minimum travel costs.

According to condition (5.31), one can also write up the equation for calculating an optimal toll as follows:

\[
\rho_p = \eta' - \eta_p + \frac{\lambda \widetilde{TSTC} \sum h^\mu}{\varepsilon Q}, \quad \forall \neq 0,
\]

where \( \rho_p \) is the toll on path \( p \); \( \eta' - \eta_p \) represents congestion pricing; and \( \frac{\lambda \widetilde{TSTC} \sum h^\mu}{\varepsilon Q} \) denotes emission pricing. As shown, when the weight is biased to the total emissions, this emission pricing is larger. When the weight is biased to TSTC, this emission pricing is smaller. For example, again when demand is 4 units, the required TSTC and the total emissions are respectively 288 and 0.8 units, which are the same when there is no new link added, the following optimal solutions are then obtained. Here, only solutions from two combinations of the weighting coefficients are demonstrated for the illustrative purpose, which are respectively when \( \varepsilon = 0.9 > \lambda = 0.1 \) and \( \varepsilon = 0.1 < \lambda = 0.9 \):

- \( \varepsilon = 0.9 > \lambda = 0.1, f_1^* = f_2^* = 2, f_3^* = 0, TSTC = 288, Q = 0.8, \rho_1 = 30, \rho_2 = 52; \)
- \( \varepsilon = 0.1 < \lambda = 0.9, f_1^* = f_2^* = 2, f_3^* = 0, TSTC = 288, Q = 0.8, \rho_1 = 670, \rho_2 = 1012 \).

It is obvious that when more weight is biased to TSTC, the total toll on each path is lower, as emission pricing is lower. When all of the weight is biased to TSTC, there is no emission pricing at all, only congestion pricing. When more weight is biased to the total emissions, the total toll on each path is higher, as emission pricing is higher.

**Model 5.3: Minimising TSTC Subject to the Total Emissions' Constraint**

Nagurney (2000a) proposed a pricing policy to charge travellers emission/congestion tolls for maintaining environmentally sustainable system-optimised transportation networks. This policy serves a twofold purpose to mitigate congestion as well as to
guarantee the environmental quality standard is satisfied. Nagurney formulated this emission problem in a transportation system into a mathematical programming, in which the total system travel cost is minimised subject to constraining the total emissions not to violate maximum allowable vehicular emissions.

After deriving KKT optimality conditions, the resulting congestion and emission tolls can be described as follows:

\[ \rho_p = \eta'_p - \eta_p + r^* \sum_a h_a \delta_a^p \] (5.33)

where \( \rho_p \) is the toll on path \( p \); \( \eta'_p \) represents the marginal of total cost on path \( p \); \( h_a \) is the emission factor on link \( a \); \( \delta_a^p = 1 \), if link \( a \) is contained on path \( p \), and 0, otherwise; \( r^* \) is the equilibrium marginal cost of emission abatement; \( r^* \sum_a h_a \delta_a^p \) denotes the marginal of total cost on path \( p \) due to the emissions generated by travellers on path \( p \), it also represents emission "pricing" on path \( p \); \( \eta'_p - \eta_p \) represents congestion pricing.

When demand is 4 units, the total emissions before adding the new link is 0.8, then it is expected that the required total emissions after adding the new link is also 0.8. The following optimal solution is then obtained:

\[ f^*_1 = f^*_2 = 2, f^*_3 = 0, TSTC = 288, Q = 0.8, \tau^* = 632.4. \] (5.34)

In this way, there are no Braess' and the emission paradoxes as the new link does not play any role in the network. And the following tolls are charged in the network:

\[ \rho_1 = \rho_2 = 148.5, \rho_3 = 229.7 \] (5.35)

These tolls on three paths are actually congestion/emission pricing, under which travellers are assigned to the network according to SO-TSTC assignment. Unlike in Model 5.1, there is no need to charge travellers if they are assigned to the network according to the SO-emissions assignment through the in-vehicle route guidance navigation system.

To compare solutions obtained in the previous three models, it is found that the flow patterns as well as the resulting TSTC and the total emissions are exactly the same.
However, toll charges are quite different. In Model 5.1, as travellers follow the SO-emissions traffic assignment methodology in which travellers are routed so as to minimise the total emissions, there is no pricing policy involved. In Model 5.2, when the weight is biased to the total emissions, the weighting coefficient associated with the total emissions is larger, and hence tolls are larger, which is higher than the ones obtained in Nagurney’s model (Model 5.3). However, when the weight is biased to TSTC, the weighting coefficient associated with TSTC is larger, and thus tolls are smaller. Therefore, the tolls obtained in Model 5.2 are greatly dependent on the weighting coefficients which are the reflection of the decision maker’s preference in terms of TSTC and the total emissions. Hence, the benefit of Model 5.2 is that it proposes a multi-objective method for computing optimal toll charges in such a way that allows decision makers to trade-off between these two conflicting objectives, alleviating congestion versus reducing vehicular emissions by adjusting the associated weighting coefficients (Yin and Lawphongpanich, 2006).

After comparing the optimal solution from these three models, it is found that to remove the emission paradox completely requires that the new link is not used. However, this is just contradiction of the purpose to build this new link. If one has to seek a solution which can maintain a sustainable network regarding TSTC and emissions, one has to allow some flows on the new link, therefore resulting in a small fraction of emissions to increase. In this case, Nagurney’s model (Model 5.3) can be employed, and a numerical example is demonstrated to show how marginal cost of congestion and emission pricing removes Braess’ paradox and ensures the emissions generated are under its maximum allowable value by relying on Nagurney’s optimality conditions. When Braess’ network is employed, all congestion function parameters and emission factors remain at the same, but the total demand is 4 units. Before the new link is added, as travellers choose paths in a user-optimised fashion, path flows are equal on both paths as: \( f_1 = f_2 = 2 \). The generated emissions can be easily calculated:

\[
\sum_{i} h_i (y_i) = h_{13} y_{13} + h_{24} y_{24} = 0.8.
\]

After the new link is added, the following user-optimised flow pattern can be obtained: \( f_1 = f_2 = 0.31 \), and \( f_3 = 3.38 \). Hence, the new generated emissions are \( h_{12}(3D - 2f_1) = 0.1(3\times4 - 2\times0.31) = 1.14 \), which is greater than the total generated emissions in the original network. And the increase is nearly 30% after adding the new link. Assuming if only a 3% increase in
emissions is allowed in the new network, then the maximum allowable vehicular emissions are 0.824 correspondingly. As the new network emission level violates maximum allowable emissions, a new SO-TSTC flow pattern has to be determined with the imposition of marginal cost of congestion and emission pricing. Thus, the system-optimised flow pattern which just meets maximum allowable vehicular emissions of 0.824 is as follows:

\[ f_1^* = f_2^* = 1.88, \quad f_3^* = 0.24, \]

and the equilibrium marginal cost of emission abatement is as: \( r' = 8.8 \), and the generalised marginals of the total costs on three paths get equalised to:

\[ \tilde{\eta}_1^* = \tilde{\eta}_2^* = \tilde{\eta}_3^* = 97.92. \]

After the imposition of marginal costs of congestion and emission pricing, the generated total emissions are precisely equal to the maximum allowable vehicular emissions, 0.824. Also the total cost of the new network without marginal cost of congestion and emission pricing is 348.92. However, the one with marginal cost of congestion and emission pricing is 287.41. These indicate that Braess’ paradox is removed and the maximum vehicular emissions requirement is met under marginal costs of congestion and emission pricings. It is noteworthy that there are multiple solutions to satisfy the given maximum vehicular emissions. Any resulting vehicular emissions which are between 0.8 and 0.824 will generate one set of feasible solution. For example, one of the feasible solutions satisfying the given maximum allowable vehicular emissions of 0.824 is as follows: \( f_1^* = f_2^* = 1.885, \quad f_3^* = 0.23, \quad \tilde{\eta}_1^* = \tilde{\eta}_2^* = \tilde{\eta}_3^* = 98.09, \quad r' = 10.1, \quad \text{TSTC}=287.42, \quad Q=0.823. \)

5.5 DISCUSSION OF PARADOXES WITH CORRESPONDING PRICING SCHEMES

There are different occurrence conditions of two paradoxes according to the parameters of link congestion functions. As discussed earlier on, under parameter conditions 6, 7, 8 and 9, there is no Braess’ and the emission paradoxes under any demand value, as the new path is not utilised. Therefore, only conditions 1-5 are discussed. According to
different situations, various pricing schemes are employed to mitigate problems. Five
discussions are presented as follows for parameter conditions 1-5:

1. **Condition 1:**

Pas and Principio (1997) summarised four situations depending on the total demand
which may occur in a given network when \( \beta_1 - \beta_2 > 0 \) and \( \alpha_1 - \alpha_2 > 0 \), which is
condition 1 in this study. Here, their work was extended to consider the occurrence of
the emission paradox and corresponding pricing schemes in Braess' paradox network
when there is \( A < 0 \).

1) Under the following relationship,

\[
D \leq \frac{2(\alpha_1 - \alpha_2)}{3\beta_1 + \beta_2},
\]

(5.36)

Braess' paradox does not occur as demand is low. However, Nagurney's emission
paradox must occur even when demand is low. In this case, marginal cost congestion
and emission pricing can be employed to ensure that vehicular emissions will not
violate the previous one.

After adding the new link, the flow on path 1 is shown in equation (5.5), if demand
follows the above inequality, then equation (5.5) can be written as follows:

\[
f_1 = f_2 = \frac{\alpha_1 - \alpha_2 + D(\beta_1 + \beta_2)}{\beta_1 + 3\beta_2} \leq \frac{\alpha_1 - \alpha_2 + \frac{2(\alpha_1 - \alpha_2)}{3\beta_1 + \beta_2} (\beta_1 + \beta_2)}{\beta_1 + 3\beta_2}.
\]

After some manipulation, one can obtain \( f_1 = f_2 \leq \frac{(\alpha_2 - \alpha_1)(\beta_1 - \beta_2)}{(\beta_1 + 3\beta_2)(3\beta_1 + \beta_2)} \), as there is
\( (\alpha_2 - \alpha_1)(\beta_1 - \beta_2) < 0 \), therefore, \( f_1 = f_2 \leq 0 \). In this case, path 3 carries the total
demand. The total emissions before and after adding the new links are shown as follows:

\[
Q^{\text{four links}} = (h_{12} + h_{24})D/2 + (h_{13} + h_{34})D/2 = 2Dh_{12},
\]

\[
Q^{\text{five links}} = D(h_{12} + h_{23} + h_{34}) = 3Dh_{12}.
\]

It is apparent that Nagurney's emission paradox always occurs when demand falls into
condition (5.36).
2) Both Braess’ paradox and Nagurney’s emission paradox occur, when demand is under the following range:

\[
\frac{2(\alpha_i - \alpha_z)}{3\beta_1 + \beta_z} < D < \frac{\alpha_i - \alpha_z}{\beta_1 - \beta_z}.
\]

Marginal cost of congestion pricing can remove Braess’ paradox, but cannot remove the emission paradox. In this case, marginal cost congestion and emission pricing will do for both purposes.

3) Both paradoxes occur, when demand is under the following range:

\[
\frac{\alpha_i - \alpha_z}{\beta_1 - \beta_z} < D < \frac{2(\alpha_i - \alpha_z)}{\beta_1 - \beta_z}.
\]

If marginal cost congestion pricing is employed, there is no emission paradox, as the new link does not carry any flow. Therefore, under this demand range, marginal cost congestion pricing alone can do both purposes.

4) Both paradoxes do not occur, when demand is under the following range:

\[
D \geq \frac{2(\alpha_i - \alpha_z)}{\beta_1 - \beta_z}.
\]

There is no flow on the new path, the flow pattern is exactly the same as the pattern without network expansion, and therefore both TSTC and total emission remain at the same.

2. **Condition 2:**

1) Both paradoxes do not occur, as demand is low, new added link does not play any role in the network, the total system travel costs and the total emissions remain at the same:

\[
D \leq \frac{2(\alpha_i - \alpha_z)}{(\beta_1 - \beta_z)}.
\]

2) The emission paradox occurs, as the new path carries some flow. Marginal cost of congestion and emission pricing can be employed:

\[
D > \frac{2(\alpha_i - \alpha_z)}{(\beta_1 - \beta_z)}.
\]
3. **Conditions 3, 4 and 5:**

In this case, only the emission paradox occurs under any demand value. Marginal cost of emission and congestion pricing can be employed to solve the problem.

4. **Conditions 6, 7, 8 and 9:**

In this case both Braess’ and emission paradoxes will never take place, because even if the new link is built, there is always no flow on the new path. If one tries to build the new link, this means this new link will not be used under any circumstances. So it is redundant to build it. Because the new link never carries any flow, the total emissions will always remain at the same level, so does TSTC. Therefore, no emission and Braess’ paradoxes occur.

### 5.6 SUMMARY

In this chapter, the occurrence of the emission paradox and the simultaneous occurrence of Braess’ and emission paradoxes are analytically examined in the classical Braess’ network. It is found that the emission paradox does not always occur. The occurrence of the emission paradox depends on demand for travel, the parameters of link performance functions as well as link emission factors. Moreover, Braess’ and emission paradoxes do not always occur at the same time. More importantly, it is found that under some conditions of parameters in link performance functions, the emission paradox does occur but Braess’ paradox does not. This implies that road network design for mitigating congestion alone may not be able to avoid the increase in vehicular emissions. A more comprehensive view of road network design considering both congestion and emissions simultaneously is necessary to avoid the occurrence of the emission paradox.

Apart from these, another traffic assignment technique in which the objective function is to minimise the total emissions rather than total system travel cost (TSTC) in the conventional traffic assignment technique is demonstrated. A normalised multi-objective weighting method is proposed based on Yin and Lawphongpanich (2006), in which both congestion and emission minimisation are combined into the objective function. The proposed models are further compared with the existing model proposed
by Nagurney (2000a), which is also designed to tackle congestion and emission problems. These different pricing schemes are compared and discussed. It is found that if the traffic is assigned under the SO-emissions assignment technique, no toll charges are needed, and Braess’ and emission paradoxes will not occur. If the traffic is assigned under the SO-TSTC assignment, marginal cost congestion and emission pricing can indeed remove both paradoxes. Nagurney’s marginal cost of emission and congestion pricing policy is better than the one derived from the proposed multi-objective method. Because the normalisation of the multi-objective function generates very high toll value, even the larger weight coefficient associated with TSTC is taken. However, the normalised multi-objective Model 5.2 has its own advantage as it proposes a multi-objective method for computing optimal toll charges in such a way that allows decision makers to trade-off between two conflicting objectives, alleviating congestion versus reducing vehicular emissions, by adjusting the weighting coefficients (Yin and Lawphongpanich, 2006).

In addition, it is found that traditional marginal cost congestion pricing cannot remove the emission paradox. This means the conventional concept of reduced congestion implying improved vehicular emissions is not always right. Moreover, it is shown that under any pricing schemes, the new link should not be used so that the total emissions do not violate the previous level. However this is just the contradiction of the purpose to build the new link. With this consideration, if policy makers have to seek a solution which can sustain, it allows a small fraction of emissions to increase. Subject to maximum vehicular emissions, it is found that there are uncountable optimal solutions which can ensure TSTC and total vehicular emissions to be smaller than their maximum levels after adding the new link. Finally, the occurrence of both paradoxes with corresponding pricing schemes is discussed. It is concluded that marginal cost congestion pricing cannot remove the emission paradox, marginal cost congestion and emission pricing can do for both purposes to relieve congestion and control vehicular emissions simultaneously. This implies that the policy makers and the transport designers need to examine the network configuration very carefully to choose the optimal pricing scheme, as an improper pricing scheme may lead to a suboptimal design.
CHAPTER 6

MIXED SUE BEHAVIOUR UNDER THE TRAVELLER INFORMATION PROVISION SERVICES WITH HETEROGENEOUS MULTI-CLASS MULTI-CRITERIA DECISION MAKING

6.1 INTRODUCTION

In this chapter, a multi-class multi-criteria mixed SUE assignment model is proposed under the traveller information provision services with heterogeneous road travellers and elastic market penetration. Traveller heterogeneity is considered by assuming a discrete set of VOTs for several traveller classes. The travellers of each class having the same VOT are further divided into different groups with different travel cost perception variations. It is assumed that equipped travellers have lower perception variation due to provided traveller information and unequipped travellers have higher perception variation due to the lack of current traffic information. The route choice behaviour of the equipped and unequipped travellers can then be modelled to follow a mixed-Stochastic User Equilibrium (SUE). Elastic market penetration is also considered to capture the elastic behaviour of demands within each class for the services. The model includes a proposed multinomial logit-based market penetration model, in which demand of each group of drivers depends on the negative average generalised travel costs of all groups of drivers within this class. Elastic market penetration of each traveller class, which is the proportion of the number of equipped travellers and the total number of travellers within this class, will determine total elastic market penetration, which is the proportion of the total number of equipped travellers and the total number of travellers for all the classes.

Traditionally, the travellers consider two criteria when making route choices, which are travel times and travel costs. For those who choose to use information provision services, they make their decisions based on the obtained information of travel times and costs on a network. For those who do not use the services, they make route choices
based on their experience or their knowledge of the network. Nagurney et al. (2002) pointed out that it is not unreasonable to assume that certain classes of travellers factor an environmental criterion into their decision making process with the increasing concern of the degradation of the environment. Also with the growing interests in studying Intelligent Transportation Systems, it is expected that the emissions can be broadcast to the travellers as well as travel times and costs (Nagurney et al., 2002). The consideration of the environmental criterion regarding emissions may cause travellers to change their previous choices which they would have made when emissions were not taken into account. Additionally, the weights of travellers' multiple criteria of travel times, costs and emissions will have direct impacts on their route choices as well, and consequently total system travel time and emissions. Therefore, it would be very meaningful to consider incorporating the environmental criterion into the travellers' decision making process when analysing the impact of traveller information provision services. However, no one has attempted this in the transportation literature to date.

In this chapter, a multi-class multi-criteria mixed SUE assignment model is developed under traveller information provision services based on Szeto (2007), where user heterogeneity and multi-criteria decision making are not considered. The travellers' multi-criteria decision making process is incorporated into the proposed model in which there is an explicit environmental criterion. The multi-class multi-criteria mixed SUE assignment model is different from those proposed in Yang and Zhang (2002) and Huang and Li (2007), as the latter models do not consider the environmental criterion. Furthermore, market penetration is exogenous in Huang and Li (2007), but it is elastic and endogenous in the proposed model. This multi-class multi-criteria mixed SUE assignment problem with driver heterogeneity and demand elasticity is modelled as a nonlinear complementary problem (NCP). The model then can be solved by any existing optimisation program. In this study, the model is solved using the Generalised Reduced Gradient (GRG) method (Abadie and Carpentier, 1969).
6.2 NCP MODEL FORMULATION AND PERFORMANCE MEASURES

6.2.1 NCP Formulation of the Heterogeneous Multi-class Mixed SUE Assignment Problem with an Environmental Criterion

The traveller information provision service is a navigational measure to provide drivers with the information of road network situations. It assists travellers in making decisions regarding their route choices in order to alleviate traffic congestion. Lo and Szeto (2002) pointed out that the assumption has been made in many previous studies (e.g., Yang, 1998; Lo et al., 1999) to consider equipped travellers to have perfect information and unequipped travellers to have imperfect information. The level of information provided can be reflected by the parameter of travel cost perception variation. In this chapter, the multi-class route choice behaviour of equipped and unequipped travellers is modelled to follow the principle of Stochastic User Equilibrium (SUE) by only varying travel cost perception variation, which implies that all the equipped travellers have a lower travel cost perception variation and all the unequipped travellers have a higher travel cost perception variation. It is assumed there are \( N \) information service providers (ISP) who provide traffic information service over the whole road network for the corresponding \( N \) equipped driver groups, who only pay for the information from one service provider. There is an unequipped driver group who do not pay for any information service. Therefore, this problem has \( N+1 \) driver groups in each class having their own VOT. Each driver is assumed to be a multi-criteria decision maker, who considers travel times, travel costs and generated emissions when making route choices. Let \( M \) denote the number of driver classes in the network and let \( m = 1, 2, ..., M \). The problem has \( M \) classes of drivers. For the drivers in any class \( m \), they all follow \( N+1 \) sets of SUE conditions with all groups of equipped drivers having lower perception variations than those of unequipped drivers. The SUE conditions are formulated using the logit model:

\[
\begin{align*}
&f^m_{p,i,m} - w^m_{p,i,m} \cdot q^m_{i,m} = 0, \forall rs, p, i, m, \quad (6.1) \\
&w^m_{p,i,m} = \frac{\exp(-\theta \cdot \bar{\eta}^m_{p,i,m})}{\sum_i \exp(-\theta \cdot \bar{\eta}^m_{i,m})}, \forall rs, p, i, m, \quad (6.2)
\end{align*}
\]
where $f_{p,m}^n$, $w_{p,m}^n$, and $\eta_{p,m}^n$ are respectively the route flows, the proportion, and the generalised travel cost or the disutility of group $i$ drivers in class $m$ on route $p$ between origin-destination (OD) pair $rs$; $\theta_i$ is the parameter representing the travel cost perception variation of group $i$ drivers, interpreted as the information quality available to them or interpreted as the accuracy of traffic information provided by an information service provider (ISP) to equipped drivers; $q_{p,m}^n$ is the demand of group $i$ drivers in class $m$ between OD pair $rs$.

Equation (6.1) means that the demand of each group in class $m$ is loaded into each route according to the proportion defined by the logit model (6.2). As shown in (6.2), the difference between all groups in class $m$ is on the travel cost perception variation, represented by $\theta_i$. A higher $\theta_i$ means a smaller travel cost perception variation and therefore better information quality. In general, it is expected that the equipped vehicles have a higher $\theta_i$ than the unequipped ones. The value of $\theta_{N+1}$, on the other hand, represents people’s familiarity with the network conditions without traveller information services. All of these parameters can be calibrated to the quality of information available by the methodology proposed in Lo and Szeto (2002). The generalised route travel cost or the disutility $\eta_{p,m}^n$ in (6.2) is a weighted average of three criteria: travel times, travel costs and emission costs, which in turn are functions of the link flow $v_a$. The link flow $v_a$ can be determined by summing up all the route flows on that link:

$$v_a = \sum_{m \in M} \sum_{p \in P^m} \sum_{r \in N+1} f_{p,m}^n \cdot \delta_a^p, \forall a,$$

where $\delta_a^p$ is the link-route incidence indicator — $\delta_a^p = 1$ if link $a$ is on route $p$; $\delta_a^p = 0$ otherwise, and $P^m$ is the set of paths between OD pair $rs$.

The link travel time $t_a$ in this study is obtained from the link flow $v_a$ based on the Bureau of Public Roads (BPR) type performance function:

$$t_a = t_a^0 \left[ 1 + \alpha \left( \frac{v_a}{c_a^0} \right)^\beta \right], \forall a,$$
where $t^o_a$ is the link's free flow travel time; $c^o_a$ is its capacity; $\alpha, \beta$ are parameters of the BPR function, whose typical values are $\alpha = 0.15, \beta = 4$. This monotonic and continuous link travel time function is adopted for illustrative purposes. Other continuous link travel time functions can be used in this problem.

Once link travel time is known, the route travel cost can be computed as follows:

$$\eta_{p,i,m}^a = C_{ISP,i} + \sum_a \left( B_m t_a + \rho_a \right) \cdot \delta_a^p, \forall rs, p,i,m, \quad (6.5)$$

where $\rho_a$ is the toll on link $a$, which equals zero for a toll-free link. $B_m$ is the value of time of class $m$ drivers. $C_{ISP,i}$ is the information service charge for group $i$ drivers. It is the average service charge per trip derived from the monthly charge, and is zero when $i$ is $N+1$ (i.e., when the drivers are unequipped). The term in the brackets is the travel cost on link $a$, which is the sum of the travel time cost $B_m t_a$ and its toll $\rho_a$. The route travel cost is the service charge and the sum of the travel costs of the links on that route.

According to Nagurney et al. (2002), the environmental costs associated with travelling on link $a$ can be expressed as follows:

$$e_a = e_a(v_a), \forall a. \quad (6.6)$$

If a single pollutant is simply considered, $e_a(v_a)$ can be assumed to be the average emissions of this single pollutant generated by the travellers on link $a$, for example, carbon monoxide.

Once the travel time, travel cost and emission costs are clearly defined, one can obtain the following generalised route cost:

$$\eta_{p,i,m}^a = \sum_a t_a \omega_{i,a,m} \delta_a^p + \eta_{p,i,m}^a \sum_a (\omega_{c,a,m} \delta_a^p) + \sum_a e_a \omega_{e,a,m} \delta_a^p, \forall rs, p,i,m, \quad (6.7)$$

where $\eta_{p,i,m}^a$ is the generalised travel cost or the disutility of group $i$ drivers in class $m$ on route $p$ between origin-destination (OD) pair $rs$; $\omega_{i,a,m}$, $\omega_{c,a,m}$ and $\omega_{e,a,m}$ denote the non-negative weights associated with traveller's travel time, cost and emissions on link $a$ respectively. The weights $\omega_{i,a,m}$, $\omega_{c,a,m}$ and $\omega_{e,a,m}$ are not only class-dependent but also link-dependent. Equation (6.7) means that each group $i$ drivers in class $m$ has
their own perception of the trade-offs among travel time, travel cost, and emissions generated when they travel on route $p$ between origin-destination (OD) pair $rs$. The trade-offs are represented by these weights associated with each criterion. The link-dependent weights allow us to incorporate some link-dependent factors as safety, comfort, view, sociability factors, as well as sensitivity to pollution (Nagurney et al., 2002).

The demand $q_{i,m}^n$ in (6.1) is modelled through the newly proposed multinomial logit market penetration model:

$$q_{i,m}^n = \tilde{q}_m^n g_{i,m}^n, \forall rs, i, m, \quad (6.8)$$

$$g_{i,m}^n = \frac{\exp(\rho \phi_{i,m}^n)}{\sum_{k=1}^{N+1} \exp(\rho \phi_{k,m}^n)}, \forall rs, i, m, \quad (6.9)$$

$$\sum_{m} \tilde{q}_m^n = \tilde{q}^n, \forall rs, m, \quad (6.10)$$

where $\tilde{q}_m^n$ and $\tilde{q}^n$ are the total travel demand of class $m$ drivers and the total travel demand of all classes drivers between OD pair $rs$, which are fixed; and $g_{i,m}^n$ is the proportion of group $i$ drivers in class $m$ between OD pair $rs$; $\rho$ is the scale parameter; $\phi_{i,m}^n$ is the systematic utility received by group $i$ drivers in class $m$ between OD pair $rs$, which is assumed to be equal to the driver-group-specific constant $\omega_i$ minus average generalised route travel cost of those drivers:

$$\phi_{i,m}^n = \omega_i - \left( \sum_{p} w_{p,i,m} \cdot \eta_{p,i,m}^n \right), \forall rs, i, m. \quad (6.11)$$

The driver-group-specific constant $\omega_i$ is a parameter capturing benefits other than travel time saving such as convenience of having the device. The market penetration model (6.8)-(6.9) is the generalisation of the special case in Lo and Szeto (2002). This model is reduced to the special case when there are only one class of drivers and one information service provider and $\rho = 1$. Under this situation, the systematic utility received by group $i$ drivers is:

$$\phi_{i}^n = -C_{bps} + \omega_i - B \left\{ \sum_{p} \left[ w_{p,i} \left( \sum_{a} \left( t_{a} \cdot \delta_{a}^p \right) \right) \right] \right\}, i = 1, 2, \forall rs. \quad (6.12)$$
The term in the parentheses in (6.12) is the travel time on route \( p \). The term in braces is the average route travel time of group \( i \) drivers between OD pair \( rs \). The first and second terms in equation (6.12) are zero for unequipped drivers. From (6.12), the difference of systematic utilities can be determined as follows:

\[
\phi_2^n - \phi_1^n = C_{ISP,i} - \omega - B \left\{ \sum_p \left[ w_{p,2} \left( \sum_u (t_u \cdot \delta_u^p) \right) \right] - \sum_p \left[ w_{p,1} \left( \sum_u (t_u \cdot \delta_u^p) \right) \right] \right\}, \forall rs \tag{6.13}
\]

The term in the braces in (6.13) is the travel time saving defined in Lo and Szeto (2002). Since there is only one information provider and the demand for the service can be expressed as a function of the difference of systematic utilities:

\[
q_i^n = \tilde{q}_i^n g_i^n = \tilde{q}_i^n \frac{\exp(\phi_i^n)}{\exp(\phi_1^n)+\exp(\phi_2^n)} = \tilde{q}_i^n \frac{1}{1+\exp(\phi_2^n-\phi_1^n)}, \forall rs \tag{6.14}
\]

By substituting (6.13) into (6.14), the elastic market penetration model proposed in Lo and Szeto (2002) can be obtained. Hence, the market penetration model (6.8)-(6.9) is the generalisation of the one there.

To obtain the NCP formulation, the corresponding route flows is multiplied by the SUE conditions (6.1), substitute (6.8) into the resultant expression, and add the non-negativity conditions to obtain the following:

\[
\begin{align*}
f_{p,i,m} & \left( f_{p,i,m} - w_{p,i,m} \tilde{q}_m^n g_{i,m}^n \right) = 0 \\
f_{p,i,m} & \geq 0, \forall rs, p, i, m. \\
f_{p,i,m} - w_{p,i,m} \tilde{q}_m^n g_{i,m}^n & \geq 0
\end{align*} \tag{6.15}
\]

According to (6.15), if \( f_{p,i,m} > 0 \), the term \( f_{p,i,m} - w_{p,i,m} \tilde{q}_m^n g_{i,m}^n \) must be zero; (6.1) must be satisfied, and \( f_{p,i,m} \) is apportioned according to the logit split expressions (6.2) and (6.9). If \( f_{p,i,m} = 0 \), the term \( f_{p,i,m} - w_{p,i,m} \tilde{q}_m^n g_{i,m}^n \) can take any value. However, this will never happen because theoretically, the SUE assignment assigns positive flows to each route. Finally, the constraint \( f_{p,i,m} - w_{p,i,m} \tilde{q}_m^n g_{i,m}^n \geq 0 \) is added for mathematical completeness. At the SUE solution, as contended above, the term \( f_{p,i,m} - w_{p,i,m} \tilde{q}_m^n g_{i,m}^n \)
becomes zero, and hence satisfies this last constraint automatically. Put everything together and let:

\[
y = \begin{bmatrix} \theta_i, i = 1, \ldots, N \\ C_{R_1}, i = 1, \ldots, N \end{bmatrix},
\]

(6.16)

\[
x(y) = \begin{bmatrix} f''_{p,m} \forall rs, p, i, m \end{bmatrix}, \text{ and}
\]

(6.17)

\[
F(x) = \begin{bmatrix} f''_{p,m} - w''_{p,m} \tilde{q}_m g''_{m} \forall rs, p, i, m \end{bmatrix},
\]

(6.18)

the NCP (6.15) can then be expressed as finding \( x^* \geq 0 \) such that:

\[
F(x^*) \geq 0, x^*(y)^T . F(x^*) = 0,
\]

(6.19)

where \( w''_{p,m} \) and \( g''_{m} \) are defined by (6.2)-(6.11).

In this study, it is assumed that a congestion problem is a primary concern of decision makers (governments). Traveller information services are provided to guide equipped travellers from congested areas to the less congested ones. It is anticipated that a provision of traveller information services could possibly relieve the congestion problem. In addition, governments also consider the impact of the traveller information provision on vehicular emissions. With the above NCP formulation, route flow patterns of equipped travellers and unequipped travellers can be solved, and hence the impacts of the traveller information provision on traffic incurred congestion and emissions can be evaluated. In the following, the functions of system performance regarding congestion and vehicular emissions are defined.

### 6.2.2 Performance Measures of the Total System Travel Time (TSTT) and Vehicular Emissions

The total system travel time (TSTT) is calculated to measure system performance regarding congestion and it is the sum of the travel times of all drivers on all links, expressed as:

\[
\text{TSTT} = \sum_a v_a t_a.
\]

(6.20)

According to (6.20), TSTT is a function of link flows, and hence is a function of route flows based on (6.3).
In terms of vehicular emissions, there are two types of vehicular emissions: link and network (or overall). The link vehicular emission is defined through the link emission factor approach. The key of estimating vehicular emissions is the relationship that volume of emissions is equal to the product of an emission factor and link load (DeCorla-Souza et al., 1995). This emission factor obtained using MOBILE model proposed by the Environmental Protection Agency (EPA) is based on the federal test procedure (FTP), typical driving conditions for an urban vehicle trip (DeCorla-Souza et al., 1995). This link emission factor approach adopted by Nagurney et al. (1998) and others is formulated as in equations (3.24) and (3.25) in chapter 3, but vehicle types and time dimensions are not considered in this study.

6.3 EXISTENCE OF PARADOXES IN TRAVELLER INFORMATION PROVISION SERVICES

Scenarios 1 and 2 are set up to evaluate the effectiveness of traveller information provision services on TSTT and vehicular emissions. In these two scenarios, it is assumed that all the travellers are homogeneous and have identical VOT. They are only concerned with the route travel cost in their route choice decision making. These assumptions are made to remove possibilities that some other factors may affect the efficiency of the services, such as travellers' VOT and perception of multi-criteria decision making. The multi-class multi-criteria mixed SUE model is then simplified to study a single class single criterion route choice behaviour of all the travellers.

In scenario 1, two situations are compared, which are respectively before and after the provision of traveller information services. In the situation with the services in place, the travellers are all equipped, which means 100% market penetration. In the situation without these services, the travellers are all unequipped. By comparing the performance measures of these two situations, the effectiveness of the services can be evaluated. In these two situations, only travel time perception variation is varied, which implied that before the information provision all the unequipped travellers had a higher travel time perception variation, and after the information provision, all the equipped travellers had
a lower travel time perception variation. The scenario example used here is broadly based upon an existing section of the road network in Ireland between North of Balbriggan and Dundalk in Co. Louth, which consists of two links, two nodes and one OD pair shown in Figure 6.1 below.

Figure 6.1 The example network.

In scenario I, the parameter values adopted are as follows: a) demand parameters: $q^{13} = 11000$ vph; b) route choice parameters: value of time $B = € 15/ hr$; travel cost perception variation parameter: unequipped drivers $\theta_1 = 0.05 \text{ €}^{-1}$; equipped drivers $\theta_2 = 15 \text{ €}^{-1}$; c) toll operation parameter: toll $\rho_1 = \rho_2 = 0 \text{ €}$; d) information service parameter: service charge $C_{ISP} = 0.5 \text{ €}$; e) vehicular emission parameter: link emission factors $h_1 = 1.3 \text{ litre/veh}$; $h_2 = 0.8 \text{ litre/veh}$; f) Network parameters: $c^0_1 = 12000 \text{ vph}$; $c^0_2 = 2000 \text{ vph}$; $\alpha_0 = 1$; $\alpha_1 = 0.15$; $\alpha_2 = 4$; $t^0_1 = 21 \text{ mins}$; $t^0_2 = 37 \text{ mins}$. They are assumed to be fixed and selected for illustrative purposes. In general, they should correspond to network characteristics and a real situation.

In this scenario, TSTT and total vehicular emissions are calculated in two situations. One is where all drivers do not have information provision services equipped in their vehicles (lower $\theta$ value) and another is where all drivers are equipped with traveller information provision services (higher $\theta$ value). Table 6.1 below shows the changes in TSTT and overall vehicular emissions before and after the traveller information provision. It is obvious that TSTT is decreased after all the drivers are equipped with traveller information services. However, the overall vehicular emissions are increased after the information provision. This means that in this case the traveller information provision service worsens the overall vehicular emissions on the network. These changes are caused by the better information provided to all the travellers after they use the information provision services. This indicates that the traveller information provision cannot reduce TSTT and total vehicular emissions simultaneously. Table 6.1
demonstrates the changes in flows on links 1 and 2 before and after the information provision. As shown, before the information provision, that is when $\theta$ is equal to 0.5, there are more vehicles travelling on link 1 than the ones on link 2. However, after the information provision, all drivers are guided to travel link 1. Because link 1 is a high emission factor link with less free flow travel time and larger capacity to accommodate more vehicles, when all drivers are guided onto link 1, overall system travel time can be reduced but there is an increase in overall vehicular emissions. In this case, providing traveller with better information can lower TSTT but raise overall vehicular emissions.

In scenario 2, the parameter values adopted are the same as scenario 1 with the exception of the following: e) vehicular emission parameter: link emission factors $h_1 = 0.6$ litre/veh; $h_2 = 1.3$ litre/veh; f) Network parameters: $c_1^o = 5000$ vph; $c_2^o = 12000$ vph. Like in scenario 1, these parameters are assumed to be fixed and selected for illustrative purposes. In general, they should correspond to the network characteristics and the real situation.

In this scenario, it is found that there is a paradoxical phenomenon that providing travellers with better information cannot always decrease TSTT. It is demonstrated in Table 6.1 that after the information provision, TSTT increases. On the contrary, providing traveller information leads to a fall in overall vehicular emissions. This again indicates that the traveller information provision cannot reduce TSTT and total vehicular emissions simultaneously, even it cannot always lower TSTT as anticipated. Table 6.1 further reveals changes in flows on links 1 and 2. It is shown that there are more travellers on link 1 after the information provision. As the total travel demand is fixed, there are fewer travellers on link 2. Compared with link 2, link 1 is a lower emission factor link with less free flow travel time but less link capacity. Thus, when more drivers switch their routes to travel on link 1, TSTT increases because link 1 cannot accommodate more vehicles with a smaller link capacity. Nonetheless, overall vehicular emissions fall because more drivers travel link 1, which is the lower emission factor link.
Table 6.1 Changes in TSTT, overall vehicular emissions, and flows on links 1 and 2 before and after the information provision.

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theta (€⁻¹)</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>TSTT (min)</td>
<td>453824</td>
<td>349366</td>
</tr>
<tr>
<td></td>
<td>255465</td>
<td>406120</td>
</tr>
<tr>
<td>Overall Vehicular Emis (litre)</td>
<td>12586</td>
<td>10201</td>
</tr>
<tr>
<td></td>
<td>14300</td>
<td>9058</td>
</tr>
<tr>
<td>Flow on Link 1 (Veh)</td>
<td>7572</td>
<td>5855</td>
</tr>
<tr>
<td></td>
<td>11000</td>
<td>7489</td>
</tr>
<tr>
<td>Flow on Link 2 (Veh)</td>
<td>3428</td>
<td>5145</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3511</td>
</tr>
</tbody>
</table>

6.4 A SYSTEM OPTIMISED PRICING POLICY IN A SUE ASSIGNMENT PROBLEM

In this section an optimisation model is formulated, the solution of which satisfying the logit-based SUE assignment not only maximises the net economic benefit of all the travellers but also ensures that total emissions does not violate the maximum level. This means, at equilibrium, the flow pattern is system optimised, but travellers still behave in a manner following the mixed SUE. A system optimised pricing policy, which incorporates the congestion and emission externalities, is derived and proposed to tackle existence of the paradoxes in traveller information provision services.

6.4.1 Traveller Information Services with Net Economic Benefit Maximisation

Based on the classical consumer theory, in this section, route choice behaviour of equipped and unequipped travellers is formulated as an optimisation program, in which the net economic benefit is maximised. So the model has an interpretation from both economic and behavioural viewpoints. The solution of this program satisfies the logit-based SUE assignment.
According to the trip consumer approach in Oppenheim (1995), an individual traveller considered as a consumer of urban trips maximises direct utility through an optimal choice of the aggregate demand. The direct utility of a representative group \( i \) traveller \( f_{p,n}^{r_i,m} \) with a level of travel cost perception variation \( \theta_i \) in class \( m \) corresponding to the aggregate demand \( q_{i,m}^r \) can be defined as follows:

\[
U_{i,m} = -\frac{1}{\theta_i} \sum_{n \in P} f_{p,n}^{r_i,m} \ln f_{p,n}^{r_i,m} + \frac{1}{\theta_i} \sum_{n \in P} q_{i,m}^r \ln q_{i,m}^r. \tag{6.21}
\]

Then the gross direct utility of representative consumers in class \( m \) with various levels of travel cost perception variations (various \( \theta_i \) ) can be expressed as follows:

\[
\sum_i U_{i,m} = -\sum_i \left( \frac{1}{\theta_i} \sum_{n \in P} f_{p,n}^{r_i,m} \ln f_{p,n}^{r_i,m} \right) + \sum_i \left( \frac{1}{\theta_i} \sum_{n \in P} q_{i,m}^r \ln q_{i,m}^r \right). \tag{6.22}
\]

The net economic benefit of class \( m \) consumers is the difference between the gross direct utility of travellers in class \( m \), \( \sum_i U_{i,m} \), and the total travel cost in class \( m \),

\[
\sum_{n \in P} \sum_{p \in P^r} f_{p,n}^{r_i,m} \eta_{p,n}^{r_i,m},
\]

and can be expressed as follows:

\[
NEB_m = \sum_i \left( \frac{1}{\theta_i} \sum_{n \in P} f_{p,n}^{r_i,m} \ln f_{p,n}^{r_i,m} \right) + \sum_i \left( \frac{1}{\theta_i} \sum_{n \in P} q_{i,m}^r \ln q_{i,m}^r \right) - \sum_{n \in P} \sum_{p \in P^r} f_{p,n}^{r_i,m} \eta_{p,n}^{r_i,m}. \tag{6.23}
\]

One then can obtain the net economic benefit of all consumers from all classes as follows:

\[
NEB = \sum_m \left[ \sum_i \left( \frac{1}{\theta_i} \sum_{n \in P} f_{p,n}^{r_i,m} \ln f_{p,n}^{r_i,m} \right) + \sum_i \left( \frac{1}{\theta_i} \sum_{n \in P} q_{i,m}^r \ln q_{i,m}^r \right) - \sum_{n \in P} \sum_{p \in P^r} f_{p,n}^{r_i,m} \eta_{p,n}^{r_i,m} \right]. \tag{6.24}
\]

The maximisation of the net economic benefit is equivalent to the minimisation of the negative net economic benefit. In this model, the negative net economic benefit is minimised so that a convex minimisation program can be formulated and later the strict convexity of \( F(f) \) in the following Model 6.1 with respect to flow variable, \( f_{p,n}^{r_i,m} \) over the feasible region can be proved.
Model 6.1:

\[
\begin{align*}
\min \ F(f) = & \sum_{m} \left( \sum_{i} \frac{1}{\theta_i} \sum_{p \in \mathcal{P}_m} f_{p,i,m} \ln f_{p,i,m} - \sum_{i} \frac{1}{\theta_i} \sum_{n \in \mathcal{N}_m} q_{i,n,m} \ln q_{i,n,m} \right) \\
+ & \sum_{i} \sum_{n \in \mathcal{N}_m} f_{p,i,m} n_{p,i,m} \right) \\
\end{align*}
\]  \hspace{1cm} (6.25)

subject to

\[
\sum_{i} \sum_{p \in \mathcal{P}_m} f_{p,i,m} = \sum_{n \in \mathcal{N}_m} q_{i,n,m} = q^* \quad \forall rs, p, i, m, \hspace{1cm} (6.26)
\]

\[
\sum_{a \in \mathcal{A}_m} \sum_{i} \sum_{p \in \mathcal{P}_m} f_{p,i,m} \delta_{a} \leq Q \quad \forall a, rs, p, i, m \hspace{1cm} (6.27)
\]

\[
f_{p,i,m} \geq 0 \quad \forall rs, p, i, m \hspace{1cm} (6.28)
\]

where \( \theta_i \) is the parameter representing the travel cost perception variation of group \( i \) drivers, interpreted as the information quality available to them or interpreted as the accuracy of traffic information provided by the information service provider (ISP) to equipped drivers; \( f_{p,i,m} \) and \( n_{p,i,m} \) are respectively the route flows and the route travel cost of group \( i \) drivers in class \( m \) on route \( p \) between origin-destination (OD) pair \( rs \); \( q_{i,n,m} \) is the demand of group \( i \) drivers in class \( m \) between OD pair \( rs \); \( q^* \) is the total travel demand between OD pair \( rs \); \( h_a \) is the emission factor on link \( a \), \( Q \) is the maximum allowable vehicular emissions in the whole transportation system. \( q^*_m \), \( q^* \) and \( Q \) are exogenously decided. Equation (6.26) guarantees that the path flow pattern satisfies the travel demands. Constraint (6.27) guarantees that the path flow pattern does not exceed the maximum allowable vehicular emissions. Constraint (6.28) is to ensure route flows are non-negative. It is observed that conditions (6.26)-(6.28) correspond to Linear System 4.1 in Nagurney (2000a), the existence of a solution to this linear system of equations and inequalities guarantees that the demand associated with OD pairs can be satisfied by a path flow pattern, which also satisfies the maximum allowable vehicular emissions simultaneously. This means that the solution guarantees viability of a transportation system with given OD pairs and travel demands. Nagurney (2000a) has pointed out that there may be more than one such flow pattern to Linear System 4.1. It will be proved later that the solution \( f_{p,i,m}^* \) to the minimisation problem is a strict local minimum of \( F(f) \) and then that is the unique minimum of \( F(f) \) over the feasible region defined by conditions (6.26)-(6.28).
6.4.2 The Equivalent Logit-Based SUE Assignment Conditions and a System Optimised Toll

Now the solution of Model 6.1 can be derived to have the form of the logit-based Stochastic User Equilibrium assignment conditions. This is demonstrated by proving that the first-order conditions for Model 6.1 are equivalent to the logit-based SUE assignment conditions. The Lagrangian for the minimisation Model 6.1 can be formulated as:

$$L(f, \lambda, \mu) = F(f) + \sum_{\nu} \lambda_{\nu} \left( \sum_{m} \sum_{i} q_{i,m} - \sum_{m} \sum_{\nu} f_{\nu,m} \right) + \mu \left( \sum_{a} h_{a} \sum_{\nu} \sum_{p \in p^{\nu}} f_{\nu,m}^{\nu} \delta_{a}^{\nu} - \bar{Q} \right), \tag{6.29}$$

where $\lambda_{\nu}$ and $\mu$ are Lagrange multipliers associated with constraints (6.26) and (6.27) respectively. Equating the partial derivatives of $L(f, \lambda, \mu)$ with respect to the flow variables $f_{\nu,m}^{\nu}$ to zero will lead to the conditions that a stationary point of $F(f)$ subject to constraints (6.26)-(6.28) must satisfy. It is important to note that $\frac{\partial L}{\partial f_{\nu,m}^{\nu}}$ does not exist when $f_{\nu,m}^{\nu} = 0$, because of the singularity at $f_{\nu,m}^{\nu} = 0$ in the logarithmic function in the objective function $F(f)$, a solution to the stationary point conditions will only be valid if all components of $f_{\nu,m}^{\nu}$ are strictly positive. The first-order conditions obtained by calculating the partial derivatives of $L(f, \lambda, \mu)$ with respect to the flow variables $f_{\nu,m}^{\nu}$ are expressed as follows:

$$\frac{\partial}{\partial f_{\nu,m}^{\nu}} L(f, \lambda, \mu) = \frac{\partial}{\partial f_{\nu,m}^{\nu}} F(f) - \lambda_{\nu} + \mu \sum_{a} h_{a} \delta_{a}^{\nu} = 0, \forall rs, p, i, m, \tag{6.30}$$

where $\frac{\partial}{\partial f_{\nu,m}^{\nu}} F(f) = \frac{1}{\theta_{i}} \left( \ln f_{\nu,m}^{\nu} + 1 \right) + \eta_{\nu,m}^{\nu} + \sum_{m} \sum_{\nu} f_{\nu,m}^{\nu} \frac{d\eta_{\nu,m}^{\nu}}{df_{p,m}^{\nu}}, \forall rs, p, i, m$.

Therefore

$$\frac{\partial}{\partial f_{\nu,m}^{\nu}} L(f, \lambda, \mu) = \frac{1}{\theta_{i}} \left( \ln f_{\nu,m}^{\nu} + 1 \right) + \eta_{\nu,m}^{\nu} + \sum_{m} \sum_{\nu} f_{\nu,m}^{\nu} \frac{d\eta_{\nu,m}^{\nu}}{df_{p,m}^{\nu}} - \lambda_{\nu} + \mu \sum_{a} h_{a} \delta_{a}^{\nu} = 0, \forall rs, p, i, m. \tag{6.31}$$
Equation (6.31) can be modified as follows:

\[
\frac{1}{\theta_i} \left( \ln f_{p,i,m}^* + 1 \right) + \eta_{p,i,m}^r + \sum_m \sum_i f_{p,i,m}^* \frac{d\eta_{p,i,m}^m}{df_{p,i,m}^m} + \mu \sum_a h_a \delta_a^p = \lambda^r, \forall rs, p,i,m. \quad (6.32)
\]

It is observed that the left-hand side of equation (6.32) consists of the sum of four items. The first of which is the derivative of the first item in the objective function \( F(f) \), which has the form of an entropy function defined on the path flow. The second is route travel cost, which is defined in equation (6.5). The third is congestion externality, which is the additional travel time burden that an additional traveller inflicts on each one of the travellers already using path \( p \) (Sheffi, 1985). The fourth is the marginal contribution of the total cost on path \( p \) due to the vehicular emissions generated by travelling on path \( p \) (Nagurney, 2000a). The Lagrange multiplier \( \mu \) associated with vehicular emission constraint (6.27) is interpreted as being the marginal cost of emission abatement. The sum of the second, third and fourth items on the left-hand side of equation (6.32) can be interpreted as the marginal cost of group \( i \) drivers in class \( m \) travelling on path \( p \) between OD pair \( rs \), \( \tilde{\eta}_{p,i,m}^r \), associated with the congestion and emission externalities that the additional traveller imposes on others:

\[
\tilde{\eta}_{p,i,m}^r = \eta_{p,i,m}^r + \sum_m \sum_i f_{p,i,m}^* \frac{d\eta_{p,i,m}^m}{df_{p,i,m}^m} + \mu \sum_a h_a \delta_a^p, \forall rs, p,i,m. \quad (6.33)
\]

Then equation (6.32) becomes:

\[
\frac{1}{\theta_i} \left( \ln f_{p,i,m}^* + 1 \right) + \tilde{\eta}_{p,i,m}^r = \lambda^r, \forall rs, p,i,m. \quad (6.34)
\]

Huang (1995) assumed that there was a set of “efficient” paths (i.e. \( f_{p,i,m}^* > 0, p \in E^r \)) between OD pair \( rs \), which can be represented by \( E^r \). This assumption is adopted with little modification. It is assumed that \( E_{i,m}^r \) is a set of “efficient” paths for group \( i \) drivers in class \( m \) (i.e., \( f_{p,i,m}^* > 0, p \in E_{i,m}^r \)) between OD pair \( rs \). For any path \( k \in E_{i,m}^r \), equation (6.34) also holds as follows:

\[
\frac{1}{\theta_i} \left( \ln f_{k,i,m}^* + 1 \right) + \tilde{\eta}_{k,i,m}^r = \lambda^r, \forall rs, k,i,m. \quad (6.35)
\]

Observing equations (6.34) and (6.35) are equal, there is:

\[
\frac{1}{\theta_i} \left( \ln f_{p,i,m}^* + 1 \right) + \tilde{\eta}_{p,i,m}^r = \frac{1}{\theta_i} \left( \ln f_{k,i,m}^* + 1 \right) + \tilde{\eta}_{k,i,m}^r.
\]
After some manipulations, there is:

$$\exp(\ln f_{p,i,m}^n) \exp(\theta \tilde{\eta}_{p,i,m}^n) \exp(-\theta \tilde{\eta}_{k,j,m}^n) = \exp(\ln f_{k,j,m}^n),$$

then

$$f_{p,i,m}^n \exp(\theta \tilde{\eta}_{p,i,m}^n) \exp(-\theta \tilde{\eta}_{k,j,m}^n) = f_{k,j,m}^n.$$  \hspace{1cm} (6.36)

Substituting equation (6.36) into flow conservative condition (6.26), there is:

$$\sum_m \sum_i f_{p,i,m}^n \exp(\theta \tilde{\eta}_{p,i,m}^n) \sum_{k \in E_{i,m}} \exp(-\theta \tilde{\eta}_{k,j,m}^n) = \sum_m \sum_i q_{i,m}^n = q^n.$$  \hspace{1cm} (6.37)

Rearranging the above, there is:

$$f_{p,i,m}^n = q_{i,m}^n \exp(-\theta \tilde{\eta}_{p,i,m}^n) \left/ \sum_{k \in E_{i,m}} \exp(-\theta \tilde{\eta}_{k,j,m}^n) \right., p \in E_{i,m}.'$$

Observing that equation (6.37) obtained from the first-order optimality conditions of Model 6.1 has the form of the logit-based SUE assignment conditions. The differences between the above SUE condition and the ones derived in Sheffi (1985) and Yang (1999) are the usage of the route cost function. Here, the route cost function is equation (6.33), where $\tilde{\eta}_{p,i,m}^n$ is the marginal cost of group $i$ drivers in class $m$ travelling on path $p$ between OD pair $rs$ associated with the congestion and emission externalities. However, the one in Sheffi (1985) uses the usual travel cost function without considering the congestion and emission externalities. The one derived in Yang (1999) only incorporates the congestion externality in the travel cost function not considering the emission externality. The solution of Model 6.1 satisfying the logit-based SUE assignment not only maximises the net economic benefit of all the travellers but also ensures that total emissions does not violate its constraint. This means, at equilibrium, the flow pattern is system optimised, but travellers still behave in a manner following the mixed SUE. To achieve such an optimal equilibrium point in any network where equipped and unequipped travellers follow the mixed SUE assignment, it is required that the congestion and emission externalities should be simultaneously incorporated into the travel cost function. This can be obtained by charging travellers a toll equal to the congestion and emission externalities, which is the difference between the marginal travel cost and generalised travel cost as follows:

$$\rho_{p,i,m}^n = \tilde{\eta}_{p,i,m}^n - \eta_{p,i,m}^n = \sum_m \sum_i f_{p,i,m}^n \frac{d\eta_{p,i,m}^n}{df_{p,i,m}^n} + \mu \sum_a h_a \delta_a^p, \forall rs, p, i, m, \hspace{1cm} (6.38)$$
where $\sum_{m} \sum_{i} f_{p,j,m}^{\alpha} \frac{dn_{p,j,m}^{\alpha}}{df_{p,j,m}}$ and $\mu \sum_{a} h_{a} \delta_{a}^p$ are respectively the congestion and emission externalities.

6.4.3 Karush-Kuhn-Tucker (K-K-T) Optimality Conditions

Since the objective function (6.25) is to maximise the net economic benefit, Karush-Kuhn-Tucker (K-K-T) optimality conditions can be derived for the system-optimised problem given by (6.25)-(6.28). K-K-T conditions for Model 6.1 can be stated as follows:

$$\frac{1}{\theta} (\ln f_{p,j,m}^{\alpha} + 1) + \eta_{p,j,m}^{\alpha} + \sum_{m} \sum_{i} f_{p,j,m}^{\alpha} \frac{dn_{p,j,m}^{\alpha}}{df_{p,j,m}}$$

$$-\lambda^{\alpha} + \mu^{\alpha} \sum_{a} h_{a} \delta_{a}^p \geq 0, \forall rs, p, i, m,$$

$$\left[ \frac{1}{\theta} (\ln f_{p,j,m}^{\alpha} + 1) + \eta_{p,j,m}^{\alpha} + \sum_{m} \sum_{i} f_{p,j,m}^{\alpha} \frac{dn_{p,j,m}^{\alpha}}{df_{p,j,m}} \right]^{\top} f_{p,j,m}^{\alpha} = 0, \forall rs, p, i, m,$$

$$\mu^{\alpha} \left( \sum_{a} h_{a} \sum_{m} \sum_{i} \sum_{p e p^r} f_{p,j,m}^{\alpha} \delta_{a}^p - \bar{Q} \right) = 0, \forall rs, p, i, m,$$

$$\mu^{\alpha} \geq 0,$$

$$\sum_{m} \sum_{i} \sum_{p e p^r} f_{p,j,m}^{\alpha} - \sum_{m} \sum_{i} q_{p,j,m}^{\alpha} = 0, \forall rs, p, i, m,$$

$$f_{p,j,m}^{\alpha} \geq 0.$$

where $\lambda^{\alpha}$ and $\mu^{\alpha}$ are Lagrange multipliers associated with constraints (6.26) and (6.27) respectively. Because of the singularity at $f_{p,j,m}^{\alpha} = 0$ in the logarithmic function in the objective function $F(f)$, it is required that all $f_{p,j,m}^{\alpha}$ are strictly positive. Therefore, the equation (6.39) must hold as equality, which has been proved to be the conditions equivalent to the logit-based SUE conditions. This implies that at equilibrium the system optimised flow pattern follows the logit-based SUE. In conditions (6.41) and (6.42), the positive marginal cost of emission abatement $\mu^{\alpha}$ is to ensure the vehicular
emissions not to exceed the maximum allowable vehicular emissions. When the vehicular emissions are smaller than its maximum allowable value, the marginal cost of emission abatement is zero. Conditions (6.43) and (6.44) are respectively the flow conservation and non-negativity constraints. The above conditions (6.39)-(6.44) are only necessary conditions and hold at an optimal point which minimises the objective function. In order to show that the SUE equivalent minimisation Model 6.1 has only one solution, it is sufficient to prove that the objective function (6.25) is strictly convex in the vicinity of $f'$ (and convex elsewhere) and the feasible region defined by constraints (6.26)-(6.28) is convex. The uniqueness requirements for the sufficient conditions have been proved in appendix A.

### 6.4.4 Weights in a Bicriteria Model: Relation to the Optimal Toll

If it is assumed that the travellers in the network are environmentally conscious and consider travel costs and generated emissions in their route choice decision making. There is the following generalised route cost function, which incorporates two criteria that travellers consider:

$$
\bar{\eta}_p^{\alpha} = \eta_p^{\alpha} + \omega_{e,p,m} \sum_a e_a \delta_a^{\alpha}, \forall r, p, i, m, \tag{6.45}
$$

where $\omega_{e,p,m}$ is the weight associated with emissions generated on path $p$ for class $m$ drivers. Obviously, the weight associated with travel cost is assumed to be 1. It is assumed as in Nagurney et al. (2002) that the emissions function $e_a$ on link $a$ is equal to the emission factor on the link, which represents emissions generated by a single traveller travelling on that link. Then the above generalised route cost function can be rewritten as follows:

$$
\bar{\eta}_p^{\alpha} = \eta_p^{\alpha} + \omega_{e,p,m} \sum_a h_a \delta_a^{\alpha}, \forall r, p, i, m. \tag{6.46}
$$

To achieve the optimal equilibrium solution in Model 6.1, a toll should be charged and equal to:
The above equation represents the optimal toll which guarantees a SUE flow pattern is system optimised. When the weight associated with generated emissions is equal to lagrange multiplier $\mu$, traveller only needs to pay a toll equal to the congestion externality. When the traveller is not aware of the generated emissions, she or he needs to pay a toll equal to the congestion and emission externalities. When the traveller has very high awareness of generated emissions (i.e., $\omega_{e,p,m} > \mu$), he pays a toll less than the congestion externality. This implies that the weight that the traveller place on emissions is directly related to the charged optimal toll. The higher the awareness of generated emissions is, the lower the optimal toll is.

### 6.4.5 Illustrative Examples

Here, two examples are given for an illustrative purpose. It is assumed that there are two classes of travellers with different VOTs. Each class is further divided into two groups, equipped and unequipped travellers. The example network in Figure 6.1 is employed again and the parameters adopted in the first example are as follows: a) demand parameters: $q_{1,1}^{13} = 150$ vph; $q_{2,1}^{13} = 700$ vph; $q_{1,2}^{13} = 800$ vph; $q_{2,2}^{13} = 2600$ vph; b) route choice parameters: value of time $B_1 = € 15/ hr$; $B_2 = € 10/ hr$; travel cost perception variation parameter: unequipped drivers $\theta_1 = 0.15 €/1$; equipped drivers $\theta_2 = 2 €/1$; c) toll operation parameter: toll $\rho_1 = \rho_2 = 0 €$; d) information service parameter: service charge $C_{ISP} = 0.5 €$; e) vehicular emission parameter: link emission factors $h_1 = 0.9$ litre/veh; $h_2 = 1.3$ litre/veh; maximum allowable vehicular emissions: $Q = 4310$ litre; f) Network parameters: $c_0 = 2500$ vph; $c_0 = 2000$vph; $\ell_1^0 = 21$ mins; $\ell_2^0 = 29$ mins.

Table 6.2 demonstrates the system performance before and after the implementation of the marginal cost pricing. Before setting a maximum allowable vehicular emission
constraint in the network, no toll is needed. When travellers are only concerned with travel costs in their decision making process, the calculated TSTT and total emissions are respectively 118363.66 minutes and 4351.86 litres. After setting the emission constraint to be 4310 litres, the marginal cost pricing reduces the total emissions to be 4310 litres but increases TSTT to 120513.73 minutes whether or not travellers are concerned with generated emissions in their decision making. In this case, the marginal cost pricing under the logit-based SUE does not necessarily diminish TSTT. This point has been indicated in Yang (1999), although Yang’s marginal cost pricing only considers congestion externality.

Table 6.3 shows the optimal tolls under various emission weights when the same maximum allowable vehicular emissions are required. It is revealed that congestion tolls are class-dependent. Class 1 drivers face higher congestion tolls than class 2 drivers when they traverse the same link. It is observed that emission tolls are not class-dependent like congestion tolls, but are link-dependent. Emission tolls are related to the emission factors on links. The higher the emission factor, the higher the emission toll, and vice versa. When travellers are more environmentally conscious (higher $\omega_{e}$), the marginal cost of emission abatement is less. Therefore, emission tolls are less. However, congestion tolls remain. It is observed that the difference between optimal marginal costs of abatement is 5, which is exactly the same as the weight travellers place on generated emissions in their decision making. This coincides with the relationship derived in equation (6.47).

Table 6.2 The system performance before and after the marginal cost pricing under SUE.

<table>
<thead>
<tr>
<th></th>
<th>TSTT (min)</th>
<th>Total Emissions (litre)</th>
<th>Maximum Allowable Vehicular Emissions (litre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Marginal Cost Pricing $\omega_{e}=1, \omega_{z}=0$</td>
<td>118363.66</td>
<td>4351.86</td>
<td>None</td>
</tr>
<tr>
<td>After Marginal Cost Pricing $\omega_{e}=1, \omega_{z}=5$</td>
<td>120513.73</td>
<td>4310.00</td>
<td>4310</td>
</tr>
<tr>
<td></td>
<td>120513.73</td>
<td>4310.00</td>
<td>4310</td>
</tr>
</tbody>
</table>
Table 6.3 The optimal tolls under different emission weights with the same total emission constraint.

<table>
<thead>
<tr>
<th>Class 1 Drivers (B₁=15 €/hr)</th>
<th>Class 2 Drivers (B₂=10 €/hr)</th>
<th>the Marginal Cost of Emission Abatement μ</th>
<th>Maximum Allowable Vehicular Emissions (litre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link 1</td>
<td>Link 2</td>
<td>Link 1</td>
<td>Link 2</td>
</tr>
<tr>
<td>Congestion Toll (€)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ωᵩ = 1, ωᵝ = 0</td>
<td>6.86</td>
<td>0.59</td>
<td>4.58</td>
</tr>
<tr>
<td>Emission Toll (€)</td>
<td>11.54</td>
<td>16.66</td>
<td>11.54</td>
</tr>
<tr>
<td>ωᵩ = 1, ωᵝ = 5</td>
<td>6.86</td>
<td>0.59</td>
<td>4.58</td>
</tr>
<tr>
<td>Emission Toll (€)</td>
<td>7.04</td>
<td>10.16</td>
<td>7.04</td>
</tr>
</tbody>
</table>

In the second example, the same network and parameters are adopted as in the first example except the followings: link emission factors $h_1 = 1.3$ litre/veh; $h_2 = 0.9$ litre/veh; maximum allowable vehicular emissions: $\bar{Q} = 4950$ litre and 4840 litre respectively. Table 6.4 illustrates how the system performance changes before and after implementing the marginal cost pricing with various levels of total emission constraints. It shows that the marginal cost pricing decreases both TSTT and total emissions, which are 118363.66 minutes and 4998.14 litres originally. To lower total emissions such that they are less than 4950 litres, the marginal cost pricing used in the network is where the traveller has no environmental consciousness. Surprisingly, total emissions, which are 4851.37 litres, are less than the maximum permitted level, which is 4950 litres.

It is shown in Table 6.5 that with this emission constraint there are only congestion tolls. If the maximum allowable vehicular emissions are further lowered to be 4840 litres, it can be seen in Table 6.4 that total emissions are further reduced to the maximum allowable level. However, in this case, congestion tolls alone are not enough. As shown in Table 6.5, there are emission tolls on all links, which is 0.96 euro on link 1 and 0.67 euro on link 2. As discussed in the last example, emission tolls are link-dependent. Link 1 has a higher emission toll since link 1 has a higher emission factor. If the travellers are
allowed to be environmentally conscious ($\omega_r = 0.3$) in the same network with the same maximum allowable emissions, which are 4840 litres, emission tolls are reduced but congestion tolls remain the same. Observing that, before and after travellers place some weights on emissions, the difference between optimal marginal costs of emission abatement is 0.3, which is exactly equal to the weight the traveller puts on generated emissions. This implies that there is a strong relationship between weights associated with emissions and the marginal cost pricing, more specifically the marginal cost of emission pricing. It is found that an increased weight associated with emissions results in less optimal tolls.

It is also observed in Table 6.4 that TSTT is pushed up from 115374.94 minutes to 115437.87 minutes when a tighter emission constraint is adopted. This means that there is a trade-off between TSTT and total emissions. When one tries to lower total emissions, TSTT is worsened. As appeared in the last example, the marginal cost pricing cannot lower TSTT and total emission simultaneously. This does not mean that the system is not optimised. The phenomenon that the marginal cost pricing under SUE cannot necessarily decrease TSTT is not a paradox but a pseudo paradox. This is rooted in the stochastic nature of the mixed equilibrium model, which is the randomness of the perceived travel times (Sheffi, 1985). With traveller information provision services equipped travellers still have perception variation of travel times, which is only less than that of unequipped travellers. Therefore, in the mixed stochastic network loading model, travellers’ perceived travel times are minimised. An improved system does not necessarily mean a reduction in TSTT. An optimised system here is referred to as an optimal situation with a given emission bound. Whether the marginal cost pricing under the logit-based SUE can reduce TSTT and total emissions simultaneously or not depends on the topology, characteristics and the emission bound of a network.
Table 6.4 The system performance before and after the marginal cost pricing under SUE.

<table>
<thead>
<tr>
<th></th>
<th>TSTT (min)</th>
<th>Total Emissions (litre)</th>
<th>Maximum Allowable Vehicular Emissions (litre)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Marginal Cost Pricing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_c = 1, \omega_e = 0$</td>
<td>118363.66</td>
<td>4998.14</td>
<td>None</td>
</tr>
<tr>
<td>After Marginal Cost Pricing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_c = 1, \omega_e = 0$</td>
<td>115374.94</td>
<td>4851.37</td>
<td>4950</td>
</tr>
<tr>
<td>$\omega_c = 1, \omega_e = 0$</td>
<td>115437.87</td>
<td>4840.00</td>
<td>4840</td>
</tr>
<tr>
<td>$\omega_c = 1, \omega_e = 0.3$</td>
<td>115437.87</td>
<td>4840.00</td>
<td>4840</td>
</tr>
</tbody>
</table>

Table 6.5 The optimal tolls under different emission constraints with various emission weights.

<table>
<thead>
<tr>
<th></th>
<th>Class 1 drivers (B₁=15 €/hr)</th>
<th>Class 2 drivers (B₂=10 €/hr)</th>
<th>the Marginal Cost of Emission Abatement $\mu$</th>
<th>Maximum Allowable Vehicular Emissions (litre)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Link 1</td>
<td>Link 2</td>
<td>Link 1</td>
<td>Link 2</td>
</tr>
<tr>
<td>$\omega_c = 1, \omega_e = 0$</td>
<td></td>
<td></td>
<td>3.50</td>
<td>2.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\omega_c = 1, \omega_e = 0$</td>
<td></td>
<td></td>
<td>3.34</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.96</td>
<td>0.67</td>
</tr>
<tr>
<td>$\omega_c = 1, \omega_e = 0.3$</td>
<td></td>
<td></td>
<td>3.34</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.57</td>
<td>0.40</td>
</tr>
</tbody>
</table>

6.5 SENSITIVITY ANALYSIS
6.5.1 Information Quality and Service Charges: Impact on Traveller Information Provision Services with Endogenous Market Penetration

It is demonstrated in section 6.3 that there are paradoxical phenomena present when implementing traveller information provision services and the traveller information provision cannot reduce TSTT and total vehicular emissions simultaneously. With this consideration, the changes in TSTT and overall vehicular emissions are investigated in a mixed SUE assignment problem where one group of drivers is equipped drivers who can obtain pre-trip traveller information, and another group of drivers is not equipped. These two groups of drivers follow the mixed SUE assignment, in which they have different perception variations. Market penetration of traveller information provision services is elastic here, because travellers choose whether or not to buy services based on the information quality and charges for the services. The example network in Figure 6.1 is employed again and the parameters adopted are the same as in scenario 1 with the exception of the following: a) demand parameters: \( \bar{q}^{13} = 4500 \) vph; b) route choice parameters: travel cost perception variation parameter: unequipped drivers \( \theta_1 = 0.15 \) €\(^{-1}\); equipped drivers \( \theta_2 = 2, 4, 6, 8 \) €\(^{-1}\); convenience of having the device: \( \omega = 1.5 \) €; e) vehicular emission parameter: link emission factors \( h_1 = 1.3 \) litre/veh; \( h_2 = 0.9 \) litre/veh; f) Network parameters: \( c_1^0 = 2500 \) vph; \( c_2^0 = 2000 \) vph; \( t_1^0 = 21 \) mins; \( t_2^0 = 29 \) mins.

Figures 6.2 and 6.3 respectively plot changes in TSTT and overall vehicular emissions against service charges for the provision of information under varied values of travel time perception variation parameter \( \theta \). It is observed that when the service charge is smaller than 3 euros, the higher the \( \theta \) value is, the higher TSTT and overall vehicular emissions are. A higher \( \theta \) means that drivers have a smaller travel time perception variation implying they have better traffic information, and vice versa. Therefore, results depict that providing better information quality to drivers is likely to lead to a rise in both TSTT and overall vehicular emissions. However, when the service charge is greater than 3.5 euros, the level of information quality provided to drivers has a minor effect on both TSTT and overall vehicular emissions. That is because market penetration is low, which means there are few travellers using the services. The better
information quality provided to few equipped drivers will not affect TSTT and vehicular emissions of a whole system to a large extent. Figures 6.2 and 6.3 also demonstrate how TSTT and overall vehicular emissions change against various levels of service charges. For example, Figure 6.2 shows that, regardless of the level of information quality (theta), TSTT decreases when the service charge is raised, until the service charge is around 4 euros. TSTT then increases slowly with further increase in service charge before becoming stable. In Figure 6.3, overall vehicular emissions continuously fall when the service charge rises.

Figure 6.4 shows how market penetration changes when the service charge increases. It is obvious that market penetration decreases when the service charge is increased. When the service charge is at around 9.2 euros, market penetration is zero. At this level, no driver buys the information provision service due to its high charge. If this situation is considered as an original case where no one buys information provision services, once the service charge starts falling, more travellers are willing to buy the service and consequently market penetration rises. As shown in Figure 6.2, until the service charge is around 3 euros in a case when $\theta$ is at value of 2, where market penetration is more than 20%, TSTT is over its original level as represented by the horizontal line. This indicates the level where no driver uses the service. In other words, when the service charge is smaller than 3 euros, where market penetration is higher than 20%, TSTT is worse off. Nonetheless, when the service charge is ranged from 3 euros to 9.2 euros, TSTT is not greater than its previous level. This implies that, under this range of the service charge, the paradoxical phenomenon in terms of TSTT does not occur. Nevertheless, overall vehicular emissions are always worse in this case. Additionally, in Figure 6.4, it is observed that the effect of information quality on the level of market penetration. Although the market penetration rises with the decrease of the service charge, it falls with better information quality (higher theta) when the service charges are at a very low level.
The original level of TSTT when market penetration is zero

Figure 6.2 The impact of service charges and information quality on TSTT.

The original level of overall vehicular emissions when market penetration is zero

Figure 6.3 The impact of service charges and information quality on overall vehicular emissions.
Figure 6.4 A plot of market penetration (MP) against service charges, when $\theta$ is 2, 4, 6 and 8 €\(^{-1}\) respectively.

The results are case specific, but it implies in general that the levels of information quality and service charges have substantial impacts on TSTT and overall vehicular emissions. This indicates that the utilisation of information provision services may not always play a good role in terms of relieving the congestion problem in the network and it may worsen the overall vehicular emissions simultaneously. With the above implications, the government has to control the service charges and information quality so as to avoid the increases in both TSTT and total vehicular emissions after the provision of traveller information services.

6.5.2 Heterogeneous Multi-class Multi-criteria Decision Making: Implications on Traveller Information Provision Services with Endogenous Market Penetration

6.5.2.1 Non-Cooperative Multi-criteria Decision Making
In the last section, the effectiveness of traveller information provision services has been analysed in the simplified single class single criterion model where drivers have identical VOT and only consider route travel costs when making route choices. It is found that the service charges and information quality are key factors affecting the system performance substantially. These factors are controlled by the information provider. In this section, it is assumed that travellers are multi-criteria decision makers who perceive their travel disutility or generalised travel cost on a route as a weighting of travel time, travel cost and emissions generated. This permits each class of travellers to perceive their travel times, costs and generated emissions in an individual manner. Numerical experiments are carried out to analyse how travellers’ own perception of the trade-offs among their multiple criteria affects the system performance with the traveller information provision services. These trade-offs are represented by the non-negative weights associated with each criterion, which are respectively travel time, travel cost and generated emissions. Here, a weighting scheme proposed by Dafermos (1981) is adopted, in which the generalised cost is a weighted average of the three criteria, but the weights in this study are link-dependent. This weighting scheme is expressed as:

\[\omega_{t,a,m} + \omega_{c,a,m} + \omega_{e,a,m} = 1, \forall a,m,\]  

(6.48)

where \(\omega_{t,a,m}, \omega_{c,a,m}\) and \(\omega_{e,a,m}\) stand for the non-negative weights associated with traveller’s travel time, cost and emissions on link \(a\) respectively.

The network shown in Figure 6.1 is employed again and the same parameters used in section 6.5.1 are adopted but with travel cost perception variation parameter for equipped drivers \(\theta_2 = 2\, \text{€}^{-1}\). The weights are assumed to be as follows: \(\omega_{t,1} = \omega_{t,2} = 0\); \(\omega_{c,2} = 0.5\); \(\omega_{t,1} + \omega_{t,2} = 1\). It is assumed as in Nagurney et al. (2002) that the emissions function \(e_i(v_i)\) on link \(a\) is equal to the emission factor on the link, which represents emissions generated by a single traveller travelling on that link. Then there is: \(e_1 = h_1 = 1.3\,\text{litre/veh}; e_2 = h_2 = 0.9\,\text{litre/veh}\).

Figures 6.5 and 6.6 plot the changes in TSTT, total emissions and market penetration versus weights associated with emissions on link 1. It is shown that total emissions and market penetration increase when weights associated with emissions on link 1 rise.
However, TSTT falls at the beginning but ascends when the weight associated with emissions on link 1 keeps rising.

Figure 6.5 A plot of TSTT and market penetration (MP) against weight for emissions on link 1.

Figure 6.6 Total emissions and market penetration (MP) against weight for emissions on link 1.
Figure 6.7 shows how weights associated with emissions on link 1 influence the difference of systemic utilities between unequipped and equipped travellers, which is the determinant factor of the demand for the services. The larger difference of systematic utilities means the larger difference of generalised travel costs between unequipped and equipped travellers. As mentioned earlier, the adopted weighting scheme has an interpretation of a weighted average. That means when the traveller places more weights on one criterion, the weights on other criteria are reduced. If travellers are only concerned with travel costs and emissions, when weights associated with emissions on one link increase, weights associated with travel costs on this link descend. In this case, changes in weights associated with emissions indirectly affect the difference of generalised travel costs between unequipped and equipped travellers, and consequently market penetration. Market penetration ascends with the increase of the difference of systematic utilities between unequipped and equipped travellers. Figure 6.8 further demonstrates that how weights associated with emissions on link 1 influence the average travel times of unequipped and equipped travellers. When weights are between 0.3 and 0.6, the average travel time of equipped travellers is less than that of unequipped travellers.

![Figure 6.7 The difference of systematic utilities against weight for emissions on link 1.](image)
Figure 6.8 Average travel time versus weight for emissions on link 1.

Figure 6.9 shows changes in vehicular emissions on both links against weights for emissions on link 1. Surprisingly, the increase in weights associated with emissions on link 1 leads to a fall in emissions on link 2 but a rise on link 1. More interestingly, when travellers place more weights on the environmental criterion on link 1, the link with a higher emission factor, total emissions increase contrary to one’s expectation.

Figure 6.9 Vehicular emissions versus weight for emissions on link 1.
In the next example, the same experiment will be carried out as the last one but studying the changes in the weights associated with emissions on link 2. The weights are assumed to be as follows: $\omega_{t,1} = \omega_{c,2} = 0$; $\omega_{c,1} = \omega_{e,1} = 0.5$; $\omega_{e,2} + \omega_{e,2} = 1$. Figures 6.10 and 6.11 plot the changes in total emissions, market penetration and the difference of system utilities versus weights associated with emissions on link 2. In this case, total emissions decline when weights associated with emission on link 2 increase. However, market penetration firstly falls and then rises sharply due to the changes in the systematic utilities. This reflection of the changes in the difference of system utilities is shown in Figure 6.11.

These results indicate that travellers' own perception of the trade-offs among their multiple criteria has an enormous effect on the system performance with the traveller information provision services. This is because travellers are allowed to perceive the trade-offs among their multiple criteria in an individual conduct. They do not take into account the perception of the trade-offs that other groups of travellers have when making decisions. Their multi-criteria decision makings are non-cooperative.

![Figure 6.10 Total emissions and market penetration (MP) against weight for emissions on link 2.](image-url)
Weights for emissions on link 1

<table>
<thead>
<tr>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1.2</td>
<td>-1.4</td>
<td>-1.6</td>
<td>-1.8</td>
<td>-2</td>
</tr>
<tr>
<td>-2</td>
<td>-2.2</td>
<td>-2.4</td>
<td>-2.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The difference of systematic utilities (euros)

Figure 6.11 The difference of systematic utilities against weight for emissions on link 2.

6.5.2.2 Mixed Equilibrium Market Penetration with User Heterogeneity

In this section, some numerical experiments are carried out to study travellers’ route choice behaviour in response to travel information provision services in the network with a discrete set of VOTs for several user classes. Travellers make the trade-offs among their multiple criteria of travel times, travel costs, and generated emissions depending on their value of times (VOTs). To explain the heterogeneity of preferences among travellers, VOTs are incorporated in travellers’ multi-criteria decision making. It is assumed that there is a special case where travellers decide the weights associated with travel costs directly depending on their VOTs, which can be expressed as follows:

$$\omega_{c,m} = \frac{60}{\kappa B_m}, \forall a, m, $$

(6.49)

where $B_m$ is value of time of class $m$ travellers; $\kappa$ is a scale parameter; 60 is to convert the value of time $B_m$ from an hour unit to a minute unit. The weight associated with each criterion satisfies the aforementioned weighting scheme in equation (6.48).

The example network in Figure 6.1 is employed again and the parameters adopted are the same as in last examples in the last section except the followings: a) demand
parameters: $\tilde{q}_1^{13} = 20\%$ $\tilde{q}_2^{13} = 850$ vph; $\tilde{q}_2^{13} = 80\%$ $\tilde{q}_2^{13} = 3400$ vph; b) route choice parameters: value of time $B_1 = \text{€} 20$/ hr; $B_2 = \text{€} 10$/ hr; $\kappa = \text{€} 8$/ min/ €; c) the weights: $\omega_{1,1,1} = \omega_{2,2,1} = 0.1$; $\omega_{1,1,2} = \omega_{1,2,2} = 0$. According to (6.49), $\omega_{1,1,1} = \omega_{1,2,2} = 0.375$. Then, $\omega_{2,1,1} = \omega_{2,2,1} = 0.525$. The weights corresponding to various levels of VOTs for class 2 drivers are listed in Table 6.6 below.

Table 6.6 The weights corresponding to various VOTs.

<table>
<thead>
<tr>
<th>VOT (€)</th>
<th>Time</th>
<th>Cost</th>
<th>Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>0.750</td>
<td>0.250</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0.577</td>
<td>0.423</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>0.469</td>
<td>0.531</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>0.375</td>
<td>0.625</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>0.313</td>
<td>0.688</td>
</tr>
<tr>
<td>32</td>
<td>0</td>
<td>0.234</td>
<td>0.766</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>0.188</td>
<td>0.813</td>
</tr>
<tr>
<td>48</td>
<td>0</td>
<td>0.156</td>
<td>0.844</td>
</tr>
</tbody>
</table>

Figure 6.12 shows the changes in weights associated with emissions versus VOTs of class 2 users. It illustrates that when users have higher VOTs they place more weight on emissions. This is because they put less weight on travel costs they face, which is also revealed in Table 6.6. Figures 6.13, 6.14 and 6.15 display the changes in TSTT, total emissions and vehicular emissions on two links against VOTs of class 2 users. It shows the changes in VOTs of class 2 users greatly affect TSTT, total emissions, and vehicular emissions on two links. The increase in VOTs of class 2 users results in an increase in emissions on link 2 but a decrease in TSTT, total emissions and emissions on link 1. Figure 6.16 depicts the effects of VOTs of class 2 users on MP of the total system and MP of two classes respectively. As shown, total MP and MP of two classes ascend with the increase in VOTs of class 2 users. It shows that the changes in VOTs of one class of users influence not only their own market penetration for the services also market
penetration of other class of users for the services, and consequently market penetration of the total system for the services. Hence, it is essential to incorporate user heterogeneity in route choice when evaluating the network performance under traveller information provision services.

Figure 6.12 Weights for vehicular emissions versus VOT of class 2 users.

Figure 6.13 TSTT against VOT of class 2 users.
Figure 6.14 Total vehicular emissions against VOT of class 2 users.

Figure 6.15 Vehicular emissions against VOT of class 2 users.
6.6 SUMMARY

In this chapter, a multi-class multi-criteria mixed SUE assignment model is presented under the traveller information provision services incorporating elasticity of the demand for the services and traveller heterogeneity. The term “multi-class” refers to heterogeneous travellers with a discrete set of VOTs. The travellers of each class are further divided into different groups with various levels of travel cost perception variation. Equipped travellers are assumed to have lower perception variation and unequipped travellers are assumed to have higher perception variation. The route choice behaviour of the equipped and unequipped travellers is modelled to follow a mixed-Stochastic User Equilibrium (SUE). The elasticity of the demand for the services is measured by market penetration, which is the proportion of the total number of equipped travellers and the total number of travellers. The elasticity of market penetration is to reflect travellers’ choices in response to information quality and charges of the services. In this study, the travellers are assumed to be multi-criteria decision makers, who have their own perception of trade-offs among travel time, travel cost, and generated emissions. These trade-offs are represented by nonnegative class-dependent and link-dependent weights associated with three criteria. It is noteworthy
that the multi-class multi-criteria mixed SUE assignment model is the first to incorporate an environmental criterion into a policy analysis regarding traveller information provision services.

Through some numerical experiments, it is revealed that there are paradoxical phenomena when implementing traveller information provision services. TSTT and overall vehicular emissions cannot be reduced simultaneously. More importantly, it is found that better information quality does not lead to a better system performance in terms of TSTT and overall vehicular emissions. On the contrary, both TSTT and overall vehicular emissions can be worsened.

Next, route choice behaviour of equipped and unequipped travellers is formulated as an optimisation program, in which the net economic benefit is maximised. So the model has an interpretation from both economic and behavioural viewpoints. The solution of this program satisfies the logit-based SUE assignment. It is demonstrated that the marginal cost pricing is applicable in the network under the logit-based mixed SUE. At equilibrium this pricing can guarantee that the flow pattern is system optimised, but travellers still behave in a manner following the mixed SUE. It is also found that there is the relationship between the marginal cost pricing and the weight associated with emissions in the bicriteria model. It shows that a more weight associated with emissions leads to a lower optimal toll, more specifically the marginal cost of emission toll. It is also found that there is the phenomenon that the marginal cost pricing under mixed SUE cannot necessarily decrease TSTT. It is pointed out that this phenomenon is not a paradox but a pseudo paradox because of the stochastic nature of the mixed equilibrium model. In the mixed stochastic network loading model, travellers’ perceived travel times are the ones to be minimised, not the measured TSTT. An improved system is not referred to as a decreased TSTT. An optimised system here is referred to as an optimal situation with a given emission bound. Whether the marginal cost pricing under the logit-based SUE can reduce TSTT and total emissions simultaneously or not depends on the topology, characteristics and the emission bound of a network.

The sensitivity analysis is also carried out to identify factors which affect the effectiveness of traveller information provision services. The effectiveness of traveller information provision services is investigated in the simplified single class single
criterion model where drivers have identical VOT and only consider route travel costs when making route choices. It is found that the service charges and information quality are key factors affecting the system performance substantially. Results also demonstrate that there is a range of the service charge under which the paradoxical phenomenon regarding TSTT is not likely to occur. This implies that the government has to control the service charges and information quality so as to avoid the increases in both TSTT and total vehicular emissions.

It is further assumed that travellers are multi-criteria decision makers who perceive their travel disutility or generalised travel cost on a route as a weighting of travel time, travel cost and emissions generated. In this case, changes in weights associated with emissions indirectly affect the difference of systematic utilities between unequipped and equipped travellers, and consequently market penetration, TSTT and total emissions. Surprisingly, the increase in weights associated with emissions on one link leads to a fall in emissions on another link but a rise on itself. More interestingly, when travellers place more weight on the environmental criterion on a link with a higher emission factor, total emissions increase contrary to the expectation. These results indicate that travellers' own perception of the trade-offs among their multiple criteria has a strong impact on the system performance with the traveller information provision services. As it is assumed that the travellers perceive the trade-offs among their multiple criteria in an individual manner, in their decision making process they do not consider the trade-offs that other groups of travellers perceive. Their multi-criteria decision makings are therefore non-cooperative.

To explain the heterogeneity of preferences among travellers, VOTs are incorporated in travellers' multi-criteria decision making. It is assumed that there is a special case where travellers decide the weights associated with travel costs directly depending on their VOTs. Numerical studies show that the changes in VOTs of one class of users affect not only their own market penetration for the services also market penetration of other class of users for the services, and consequently market penetration of the total system for the services, TSTT as well as total emissions. Hence, it is essential to incorporate user heterogeneity in route choice when evaluating the network performance under traveller information provision services.
From the above analysis, it is concluded that there are strong implications for traveller information provision services when travellers of a network are heterogeneous multi-class multi-criteria decision makers. It is, thus, considerably important to incorporate travellers' heterogeneity and multi-criteria decision making into route choice when evaluating the effectiveness of traveller information provision services.
CHAPTER 7

A METHODOLOGY FOR EVALUATING THE JOINT IMPLEMENTATION OF TRAVELLER INFORMATION PROVISION SERVICES, SPEED LIMIT CONTROL, AND TOLLING

7.1 INTRODUCTION

This chapter presents an analytical methodology for evaluating the impact of the integration of the traveller information provision services, speed limit control, and tolling on vehicular emissions and congestion. The idea of the integration is straightforward. The vehicular emissions depend on the driving speed of a vehicle (Den Tonkelaar, 1994). The speed limit on each route dictates the free flow travel time of this route in a network and the average travel time on the route, dependent on the free flow travel time, affects the average travel speed once a travel distance is fixed. This means that the speed limit indirectly affects the average driving speed on the route and hence the vehicular emissions. To integrate speed limit control into the analysis of traveller information provision services, the multi-class multi-criteria mixed SUE assignment model proposed in chapter 6 is employed and link free flow travel time is relaxed from a fixed parameter into a variable, which depends on the link speed limit. This relaxation allows one to incorporate the concept of speed limit control into the study in chapter 6 and to study the impacts of the strategic interaction between traveller information provision services and speed limit control. Further the macroscopic emissions estimation relationship used in TRANSYT-7F model is adopted to replace the one in the Mobile model in chapter 6. Because the model used in TRANSYT-7F belongs to the domain of average speed models, the speed limit indirectly dictates the average speed which is dominant to the volume of emissions in this case. The replacement of the emission estimation model allows us to assess the impact of the joint implementation of traveller information provision services and speed limit control on vehicular emissions. It is noted that TRANSYT-7F model can be applied to estimate fuel consumption, carbon monoxide (CO) emissions, hydrocarbon emissions, and nitrogen oxide emissions.
It was decided that in this study, only CO emission rates would be calculated. The reasons for this are as follows. Firstly, as indicated in Rilett and Benedek (1994), the assignment results obtained on the basis of all the pollutants would be similar due to the similarity in the form of the production functions. Secondly, Alexopoulos et al. (1993) pointed out that according to PERPA Annual Report (1990) CO may be considered as the best tracer to determine the traffic contribution to the overall atmospheric pollution of the area, since CO is almost solely emitted by vehicles. Thirdly, CO is dangerous to human health, particularly to those with heart disease, because it reduces the blood's ability to carry oxygen. So the scope of the study in this chapter is to analyse the combined impact of traveller information provision services and speed limit control on the vehicular emissions, specifically CO emissions, and congestion in terms of total system travel time (TSTT). The rest of this chapter is organised as follows: Section two depicts the emission estimation model in TRANSYT-7F and the modified NCP formulation of the multi-class multi-criteria mixed SUE assignment model. Section three provides a sensitivity analysis of strategic interactions among speed limit control, traveller information provision services and tolling. Section four derives the speed-varying marginal cost pricing and finally, section five summarises this chapter.

7.2 METHODOLOGY

7.2.1 Emission Estimation Model

In chapter 6, MOBILE model is utilised to estimate vehicular emissions under traveller information provision services. In this chapter, to incorporate speed limit control into the analysis, the macroscopic emissions estimation relationship used in TRANSYT-7F model is adopted, which can be applied to estimate fuel consumption, carbon monoxide (CO) emissions, hydrocarbon emissions, and nitrogen oxide emissions. This emission estimation approach, belonging to the domain of average speed models, has been used in previous research (e.g. Rilett and Benedek, 1994; Benedek and Rilett, 1998; Yin and Lawphongpanich, 2006) to calculate CO emissions. The estimation of vehicular emissions is defined as follows:
\[ ROP_{a,j} = \frac{A^{co} e^{B^{co} \bar{v}_a}}{C^{co} \bar{x}_{a,j}}, \forall a, j \]  

(7.1)

where \( ROP_{a,j} \) is the production rate of vehicular emissions (grams/veh-ft) on link \( a \) for vehicle type \( j \); \( A^{co}, B^{co} \) and \( C^{co} \) are constants. For measuring CO emissions, these are 3.3963, 0.01456 and 1000 respectively. \( \bar{x}_{a,j} \) is the average speed of vehicle type \( j \) on link \( a \) (ft/sec), which is obtained by dividing the link length \( l_a \) with the average travel time of vehicle type \( j \) on link \( a \). In this study, the unit of speed is converted into km/hr so that the production rate of CO emissions is in grams/veh-km. The converted function of CO emissions is not expressed here. It is assumed that that all types of vehicles have the same average speed when they travel on the same link, generated CO emissions for each vehicle travelling on link \( a \) can be expressed as follows:

\[ CO_a = ROP_a l_a = 3.3963 \frac{e^{0.01456 \bar{x}_a}}{1000}, \forall a, \]  

(7.2)

where \( l_a \) is the link length in feet. Substituting \( \bar{x}_a = \frac{l_a}{t_a(v_a)} \) into the above, then there is:

\[ CO_a(v_a) = 3.3963 \cdot t_a(v_a) \cdot \frac{e^{0.01456 \bar{x}_a}}{1000}, \forall a, \]  

(7.3)

where \( t_a(v_a) \) is travel time on link \( a \) in seconds.

The overall vehicular emissions can be calculated as follows:

\[ Q = \sum_a Q_a = \sum_a CO_a(v_a) \cdot v_a, \]  

(7.4)

where \( Q_a \) is CO emissions on link \( a \); \( v_a \) represents the hourly traffic flow on link \( a \); \( Q_a \) is the production of the CO emissions of one vehicle on this link and the link load. The overall CO emissions \( Q \) can be obtained as in equation (7.4) by summing CO emissions on each link.

### 7.2.2 NCP Formulation

NCP formulation of the heterogeneous multi-class multi-criteria mixed SUE assignment problem in chapter 6 is modified by relaxing link free flow travel time as follows:
where \( t_{a,j}^0 \) and \( s_{a,j} \) are the free flow travel time and the speed limit on link \( a \) for vehicle type \( j \); \( l_a \) is the length of the link \( a \). Vehicle types are incorporated here for generalisation, as in reality speed limits may be different for various vehicle types. As it raises the issue of driving safety, it is not the concern of this study. The same speed limit is adopted for all types of vehicles travelling on the same link as follows:

\[
t_{a}^0 = \frac{l_a}{s_{a}}, \forall a.
\]  

(7.6)

Once the length and speed limit of the link are given, the link free flow travel time can be calculated as in equation (7.6). The calculated free flow travel time on link \( a \) is then substituted into equation (6.4) in chapter 6.

The NCP formulation in this chapter can be developed by finding \( x^* \geq 0 \) such that:

\[
F(x^*) \geq 0, \quad x^*(y)^T \cdot F(x^*) = 0,
\]  

(7.7)

\[
F(x) = \left[ f_{p,i,m}^{n} - w_{p,i,m}^{n} \tilde{q}_{m}^{n} g_{i,m}^{n}, \forall rs, p, i, m \right], \text{ and}
\]  

(7.8)

\[
x(y) = \left[ f_{p,i,m}^{n}, \forall rs, p, i, m \right],
\]  

(7.9)

where \( f_{p,i,m}^{n}, w_{p,i,m}^{n} \) and \( g_{i,m}^{n} \) are defined by (6.1)-(6.11) and (7.6). In this study, it is assumed that there are one traffic manager, \( A \) toll road operators, and \( N \) traveller information service providers. The traffic manager is responsible for regulating the speed limits on all the links and it is assumed that there are \( a \) links. Each toll road operator is in charge of tolling on only one link. It is assumed that their primary goal by limiting speed, tolling and providing traveller information is to relieve traffic congestion. The individual toll road operators are concerned with the congestion levels on the links they are in charge of without considering the impact of their decisions on other links or routes. Traveller information providers, however, are concerned with the congestion level in the whole network. \( y \) in equation (7.9) can now be expressed as follows:
Equation (7.10) represents the strategic interactions among the traffic manager, \( A \) toll road operators and \( N \) traveller information service providers, who are respectively in charge of speed limit \( s_a \), toll \( \rho_a \), as well as information quality \( \theta_i \) and service charge \( C_{ISP,i} \).

The above NCP formulation allows one to evaluate the joint implementation of traveller information provision services, speed limit control, and tolling strategies. The route flow pattern of equipped travellers and unequipped travellers under the joint implementation of these three strategies can be solved. A change in any of these three strategies would lead to a new flow pattern, which determines corresponding total system travel time (TSTT) and vehicular emissions. Therefore the proposed NCP formulation, utilised as the analytical methodology, allows one to study the impact of the strategic interactions among the traffic manager, toll road operators as well as information providers on a transportation system. In this study, the focus is on vehicular emissions and TSTT.

### 7.3 Sensitivity Analysis

#### 7.3.1 Incorporating the Speed Limit Control into Traveller Information Provision Services

In this section, firstly the impact of traveller information provision services and the speed limit control on TSTT and overall vehicular emissions is studied in a mixed SUE assignment problem, in which the equipped and unequipped drivers have different perception variations. Market penetration of traveller information provision services is elastic here, because travellers decide whether or not to buy services based on quality and service charges of traveller information. It is assumed that all drivers are
homogeneous, and they each have the same value of time. The example network in chapter 6 is employed and shown in Figure 7.1, which consists of two links, two nodes and one OD pair, where link 1 is a highway link and link 2 is a road link. It is assumed that the speed limit on each route is uniform and the compliance rate of the speed limits is 100%. Also the speeds of vehicles are constant along each link with 0% grade on all roads. As Benedek and Rilett (1998) have indicated, speed and grade variations are essential considerations in estimating vehicular emissions but are not modelled in macroscopic traffic assignment models. The parameters adopted are as follows: 

a) demand parameters: \( q_{1} = 4250 \) vph; 
b) route choice parameters: value of time \( B = € 15/\) hr; 
c) toll operation parameter: toll \( \rho_1 = \rho_2 = € 0 \); 
d) information service parameter: service charge \( C_{ISP} = € 0 - 8 \); 
e) Network parameters: \( c_i^o = 2500 \) vph; \( c_i^a = 2000 \) vph; \( \alpha_0 = 1 \); 
\( \alpha_1 = 0.15 \), \( \alpha_2 = 4 \); 
\( s_1 = 50-110 \text{ km/hr} \); 
\( s_2 = 60 \text{ km/hr} \); 
\( l_1 = 55 \text{ km} \); 
\( l_2 = 30 \text{ km} \). They are assumed to be fixed and are selected for illustrative purposes. In general, they should correspond to network characteristics and a real situation.

![Figure 7.1 The example network.](image)

Figures 7.2 and 7.3 below respectively plot changes in overall vehicular emissions and TSTT against service charges of information provision under varied values of speed limits on link 1. Results indicate that both higher and lower speed limits, such as 110 and 50 km/hr, do not lead to lower vehicular emissions. It is largely dependent on the service charges of traveller information services. For example, when the speed limit on link 1 is 50 km/hr, the level of vehicular emissions varies when the service charge changes. When the service charge is low, resulting in high market penetration, the vehicular emissions are at a high value. As service charge increases the vehicular emissions decrease. When the service charge is smaller than 2.2 euros, the speed limit
of 50 km/hr brings about the highest vehicular emissions as compared with any other given speed limit. However, when the service charge is greater than 2.2 euros, the speed limit of 110 km/hr leads to the highest value of vehicular emissions.

It is observed in Figure 7.2 that intermediate speed limits, such as 70 and 80 km/hr, maintain a lower level of emissions. When the service charge is below 2.2 euros, the lowest vehicular emissions occur when the speed limit is at 80 km/hr. Nevertheless, this does not always occur after the service charge is over 2.2 euros, where the lowest vehicular emissions are produced when the speed limit is at 70 km/hr. These findings imply that neither higher or lower speed limits lead to a lower level of vehicular emissions. In this case, it depends on the value of service charges, which directly affects the market penetration.

In Figure 7.3, it is found that the higher the speed limit on link 1, the lower the TSTT. However, when the speed limit is low at 50 km/hr, TSTT decreases when the service charge rises. As known, a lower level of service charge results in high market penetration. This means that when the speed limit on link 1 is low at 50 km/hr, higher market penetration would lead to the increase in total system travel time. This is because more traffic is diverted to link 2 which has a higher travel speed limit, hence a shorter free flow travel time. But with the increase in the traffic volume on link 2, travel time on this link rises as well. In this case, when more users utilise traveller information services, it leads to an increased TSTT. But when the higher speed limit is implemented on link 1, the changes in TSTT are seen to be marginal when the service charges vary. Therefore, it is concluded that governments have to control the speed limit and service charges with caution when traveller information provision services are implemented so as to balance the conflict between vehicular emissions and TSTT.
Figure 7.2 The impact of service charges and the speed limit of link 1 on overall vehicular emissions.

Figure 7.3 The impact of service charges and the speed limit of link 1 on TSTT.
Figures 7.4 and 7.5 illustrate the impacts of information quality on overall vehicular emissions and TSTT when speed limits are 50 and 110 km/hr respectively. In this case, the sensitivity analysis is carried out to change travel time perception variation, $\theta$. A higher value of $\theta$ means a lower perception variation and better information quality. It was found that the information quality has different effects on vehicular emissions and TSTT.
congestion with varied speed limits. For instance, when the speed limit is 50 km/hr, the information quality has major impacts on both vehicular emissions and TSTT. When the information quality is better, both vehicular emissions and TSTT increase. However, when the speed limit is at 110 km/hr, only the vehicular emissions rise slightly with the increase in $\theta$. There is almost no effect on TSTT when $\theta$ increases. Figure 7.6 illustrates the impact of information quality on market penetration when speed limits on link 1 are 50 and 110 km/hr respectively. It is shown that when speed limit is at 50 km/hr, market penetration is more sensitive to the information quality. However, when the speed limit is at 110 km/hr, market penetration remains almost at the same level as information quality changes.

![Figure 7.6 The impact of information quality on MP when speed limits on link 1 are 50 and 110 km/hr respectively.](image)

Figure 7.6 The impact of information quality on MP when speed limits on link 1 are 50 and 110 km/hr respectively.

Figure 7.7 depicts market penetration versus speed limit on link 1 as service charges vary. It can be seen that the changes in service charges and speed limits have a strong impact on the level of market penetration. For a given speed limit, higher service charge results in lower market penetration. If the service charge remains at the same, market penetration firstly declines with the increase in speed limit and then climbs up slightly. These findings imply that it is important to consider the speed limit control while evaluating the impact of traveller information services. This is because the speed limit, which determines free flow travel time, affects the drivers’ average travel time, which may have a strong influence on the demand for the information provision services, and hence on the system performance in terms of vehicular emissions and TSTT.
Figure 7.7 MP versus speed limit on link 1 with various service charges.

Figure 7.8 Vehicular emissions on two links against speed limit on link 1.
Figures 7.8 and 7.9 illustrate the changes in link and total vehicular emissions against speed limit on link 1 when service charge and information quality are respectively 0.5 euro and 2 \( \text{€}^{-1} \), as well as 3.5 euros and 14 \( \text{€}^{-1} \). It shows that speed limit control has a considerable impact on link and total vehicular emissions. Speed limit on one link considerably affects vehicular emissions not only on this link but also on adjacent links between OD pair. This is because a change in speed limit on one link results in a change in free flow travel time on this link, which influences travellers' route choices between the same OD pair, and hence leads to changes in link total emissions. In this case, when speed limit on link 1 increases, emissions on link 2 fall but emissions on link 1 rise. This raises an equity issue regarding emissions on both links. As observed in Figure 7.8, when service charge and information quality are 0.5 euro and 2 \( \text{€}^{-1} \), system equity (SE) regarding emissions on both links is achieved at a speed limit of 77 km/hr. However, when service charge and information quality increase to 3.5 euros and 14 \( \text{€}^{-1} \), emissions on two links are equal when the speed limit on link 1 is 73 km/hr. It is shown in Figure 7.9 that total emissions fall rapidly when the speed limit increases from 50 km/hr to 75 km/hr and then suddenly increase when the speed limit rises continuously. This implies that the strategic interaction between speed limit control and traveller information provision services has a significant effect on overall vehicular emissions as well as link emissions, more importantly causing emission inequity among links between OD pair.
In this case, the traffic manager needs to implement speed limit control with great caution and should take into account the strategic interaction with traveller information provision services.

7.3.2 Joint Implementation of Tolling, Speed Limit Control, and Traveller Information Provision Services

In this analysis, to demonstration the impact of joint implementation of tolling, speed limit control and traveller information services on TSTT and vehicular emissions, toll road operators are incorporated into the previous study. Two links are charged with tolls separately. Figures 7.10 and 7.11 demonstrate changes in total emissions and TSTT versus toll when speed limit and information quality are at different levels. As shown in Figures 7.10 and 7.11, changes in total emissions and TSTT are completely different if a toll is charged at a different location. For example, the first and third lines on the top of Figures 7.10 and 7.11 represent changes in total emissions and TSTT when a toll is charged on link 1 and link 2 respectively with speed limit and theta at 50 km/hr and 6 €\(^{-1}\) separately. Both total emissions and TSTT increase when toll is charged on link 1. However, they both decrease when a toll is charged on link 2.

The same also happens to total emissions and TSTT when speed limit and theta are respectively 50 km/hr and 2 €\(^{-1}\). Nevertheless, when speed limit is 80 km/hr, the trends in total emissions and TSTT vary. When a toll is charged on link 1, regardless of whether theta is 2 €\(^{-1}\) or 6 €\(^{-1}\), total emissions decrease slightly until the toll reaches 1 euro, and then rise steadily as the toll keeps increasing. If a toll is charged on link 2 when theta is chosen to be 2 €\(^{-1}\), total emissions remain almost the same and when theta is 6 €\(^{-1}\), total emissions rise slowly. It is observed that, when speed limit is 80 km/hr, charging a toll on link 1 leads to lower total emissions as shown in Figure 7.10. However, charging a toll on link 2 results in lower TSTT as shown in Figure 7.11.
Figure 7.10 Total emissions against toll.

Figure 7.11 TSTT against toll.
The above observation indicates that tolling location affects the system performance regarding total emissions and TSTT. It also reveals the strategic interactions among tolling, speed limit control and traveller information provision services. This implies that a holistic analysis of the costs and benefits of the joint implementation of these strategies is required and should be carried out before any individual scheme is adopted.

7.4 INTERNALISATION OF SPEED-VARYING EMISSION EXTERNALITY IN A SUE TRAFFIC ASSIGNMENT PROBLEM

Johansson (1997 and 2006) derived optimal speed-dependant charges due to congestion and emissions in a deterministic traffic assignment problem without considering uncertainties. In contrast, in this section, optimal speed-varying charges are derived in a more realistic situation where travellers follow a mixed SUE traffic assignment to capture uncertainty in travellers’ behaviour. Here, Model 6.1 in chapter 6 is employed but $h_a$ in equation (6.27) is substituted with $CO_a$ in equation (7.2) or (7.3). This means that emissions on a link are not simply dictated by an emission factor on this link but depend on the average driving speed of vehicles on the link.

For ease of reference, the modified Model 6.1 is demonstrated as follows:

Model 7.1:

$$\min_f F(f) = \sum_m \left( \sum_i \frac{1}{\theta_i} \sum_{p \in P_i} f_{p,m}^{\alpha} \ln f_{p,m}^{\alpha} - \sum_i \frac{1}{\theta_i} \sum_{p \in P_i} q_{i,m}^\alpha \ln q_{i,m}^\alpha \right)$$

$$+ \sum_p \sum_{i,m} \sum_{p \in P_i} f_{p,m}^{\alpha} \eta_{p,i,m}$$

subject to

$$\sum_p \sum_{i,m} f_{p,m}^{\alpha} = \sum_m q_{i,m}^\alpha = q^\alpha, \forall rs, p, i, m$$

$$\sum_a CO_a(v_a) \sum_i \sum_{p \in P_i} \sum_{m} f_{p,m}^{\alpha} \beta_a^p \leq \bar{Q}, \forall a, rs, p, i, m$$

$$f_{p,m}^{\alpha} \geq 0, \forall rs, p, i, m$$

where $f_{p,m}^{\alpha}$ and $\eta_{p,i,m}$ are defined by (6.1)-(6.11) and (7.6). $q_{i,m}^\alpha$, $q^\alpha$ and $\bar{Q}$ are exogenously decided.
Analogous to the derivation carried out in section 6.4.2 in chapter 6, the following is obtained:

\[ f_{p,i,m} = q_{i,m}^* \exp(-\theta \tilde{\eta}_{p,i,m}^*) / \sum_{k \in \Omega_{p,m}} \exp(-\theta \tilde{\eta}_{k,i,m}^*), p \in E_{i,m}. \]  

(7.15)

where

\[ \tilde{\eta}_{p,i,m}^* = \eta_{p,i,m}^* + \sum_m \sum_i f_{p,i,m} \frac{d\tilde{\eta}_{p,i,m}^*}{df_{p,i,m}} \]

\[ + \mu(\sum_a CO_a(v_a)\delta_a^p) + \mu(\sum_m \sum_i f_{p,i,m} \frac{d\sum CO_a(v_a)\delta_a^p}{df_{p,i,m}}), \forall rs, p, i, m. \]

(7.16)

Equation (7.15) means that, at equilibrium, Model 7.1 has the form of the logit-based SUE assignment conditions, in which \( \tilde{\eta}_{p,i,m}^* \) represents the marginal cost of group \( i \) drivers in class \( m \) travelling on path \( p \) between OD pair \( rs \). In equation (7.16), the first item on the right hand side is route travel cost, which is defined in equation (6.5). The second is congestion externality, which is the additional travel time burden that an additional traveller inflicts on each one of the travellers already using path \( p \) (Sheffi, 1985). The Lagrange multiplier \( \mu \) associated with emission constraint (7.13) is interpreted as being the marginal cost of emission abatement. The third is the marginal contribution of the total cost on path \( p \) due to the generated vehicular emissions on path \( p \) (Nagurney, 2000a). The fourth is emission externality that is the additional emission cost that the additional traveller imposes on each of the travellers already using path \( p \).

Equation (7.16) indicates that a traveller should pay a charge equal to the sum of his own emission costs and the congestion and emission externalities that he imposes on others. And this charge can be expressed as follows:

\[ \rho_{p,i,m} = \tilde{\eta}_{p,i,m}^* - \eta_{p,i,m}^* \]

\[ = \sum_m \sum_i f_{p,i,m} \frac{d\tilde{\eta}_{p,i,m}^*}{df_{p,i,m}} \]

\[ + \mu(\sum_a CO_a(v_a)\delta_a^p) + \mu(\sum_m \sum_i f_{p,i,m} \frac{d\sum CO_a(v_a)\delta_a^p}{df_{p,i,m}}), \forall rs, p, i, m. \]

(7.17)
The above optimal toll is analogous to the one derived in Yin and Lawphongpanich (2006). However, they are different in nature on three aspects as follows:

1) The nature of the model;
2) The equilibrium principle of traffic assignment; and
3) The goal to limit total emissions.

Firstly, the model in Yin and Lawphongpanich (2006) is a multi-objective model to minimise a weighted combination of congestion and total emissions. The proposed model in this study formulates route choice behaviour of equipped and unequipped travellers as an optimisation program, in which the net economic benefit is maximised. So this model has an interpretation from both economic and behavioural viewpoints.

Secondly, in Yin and Lawphongpanich (2006), the optimal flow pattern follows a system optimum (SO) traffic assignment. However, the optimal flow pattern in this model satisfies the logit-based SUE assignment, which reflects uncertainty in route choice behaviour. Thirdly, the goal in Yin and Lawphongpanich (2006) is to determine charges that induce a flow pattern with less congestion and emissions. Nagurney (2000a) indicated that in an optimisation problem whose objective function minimises both total travel cost and total emissions, the environmental quality standard (maximum allowable vehicular emissions) may not be met by the solution of the problem. This means the solution of the model in Yin and Lawphongpanich (2006) may not guarantee a sustainable network in terms of vehicular emissions. The goal of this model, however, is to control total emissions, keeping it lower than its maximum allowable value. So the derived optimal toll in this model can guarantee a sustainable network in which the flow pattern follows the SUE assignment and maximum allowable vehicular emissions are not violated.

Equation (7.17) can be further derived as follows:

\[ \rho_{p,i,m}^{\pi} = \sum_{m} f_{p,i,m}^{\pi} \frac{d\eta_{p,i,m}^{\pi}}{df_{p,i,m}} + \mu \left( \sum_{u} \frac{A_{e}}{C_{e}} t_{u}(v_{u}) e^{\delta_{u}} \right) \]

\[ + \mu \left[ \sum_{m} f_{p,i,m}^{\pi} \frac{A_{e}}{C_{e}} \sum_{u} e^{\delta_{u}} (1 - B_{e}) \frac{d_{s}(v_{u})}{df_{p,i,m}} \right], \forall r, s, p, i, m. \tag{7.18} \]

where \( A_{e}, B_{e} \) and \( C_{e} \) are related constants for measuring CO emissions, and they are 3.3963, 0.01456 and 1000 respectively. Equation (7.18) shows that the optimal toll
derived from Model 7.1 is composed of a congestion charge and an emission charge. The emission charge includes the traveller's own emission cost and additional emission cost he imposes on all travellers. As observed, the emission charge is speed-varying, which means the emission charge that the traveller pays depends on his own average driving speed.

Observing the third item on the right hand side of equation (7.18), all the components are non-negative except \(1-B^{\infty \alpha s_0}\). Note that \(\frac{dt_a(v_a)}{df_{p,j,m}} > 0\), because the link performance function is assumed to be positive and increasing as in Sheffi (1985). If a path is assumed to be composed of only one link, when \(1-B^{\infty \alpha s_0} > 0\), the traveller imposes a positive additional emission cost on all travellers. Substituting \(B^\alpha = 0.01456\) into the above, one can obtain \(s_0 < 68.7 \text{ ft/sec (75.4 km/hr)}\), which coincides with the calculated critical speed in Benedek and Rilett (1998). This means that when the average driving speed is lower than 75.4 km/hr, the traveller needs to pay positive emission externality he inflicts on others in addition to his own emission cost. Therefore, emission externality may not always be positive. It largely depends on average driving speed.

Substitute \(t_a(v_a)\) in equation (6.4) in chapter 6 into \(s_0 = \frac{l_a}{t_a(v_a)}\), then there is:

\[
\bar{s}_a = \frac{s_a}{1 + \alpha \left(\frac{v_a}{c_\alpha^0}\right)^\beta}, \forall \alpha, \quad (7.19)
\]

where \(s_a\), \(v_a\) and \(c_\alpha^0\) are respectively speed limit, flow and capacity on link \(\alpha\).

Substituting equation (7.19) into (7.18), the optimal toll is expressed as follows:

\[
\rho_{p,j,m}^{a} = \sum_m \sum_l f_{p,j,m}^{n \ast} \frac{\partial T_{p,j,m}^{n \ast}}{\partial f_{p,j,m}^{n \ast}} + \mu (\sum_a \frac{A_{C_{\alpha}}^{\infty \alpha} t_a(v_a) e^{\frac{B^{\infty \alpha s_0}}{1+\alpha \left(\frac{v_a}{c_\alpha^0}\right)^\beta}} \delta_{\alpha}^\rho})
\]

\[
+ \mu \left[\sum_m \sum_l f_{p,j,m}^{n \ast} \frac{A_{C_{\alpha}}^{\infty \alpha} t_a(v_a) e^{\frac{B^{\infty \alpha s_0}}{1+\alpha \left(\frac{v_a}{c_\alpha^0}\right)^\beta}} \delta_{\alpha}^\rho} \right] \left(1 - \frac{B^{\infty \alpha s_0}}{1 + \alpha \left(\frac{v_a}{c_\alpha^0}\right)^\beta} \frac{dt_a(v_a)}{df_{p,j,m}^{n \ast}} \delta_{\alpha}^\rho \right). \quad (7.20)
\]
Equation (7.20) further shows that the optimal toll is dependent on the speed limit, the congestion level and the capacity.

In appendix A it is demonstrated that Model 6.1 has a unique solution by proving that the objective function (6.25) is strictly convex in the vicinity of \( f^{*} \) (and convex elsewhere) and the feasible region defined by constraints (6.26)-(6.28) is convex. Here, the uniqueness of the solution in Model 7.1 is investigated. It is observed that the difference between Model 6.1 and Model 7.1 lies in constraints (6.27) and (7.13). In constraint (6.27) the emission function is linear, which guarantees the convexity of constraint (6.27) according to Theorem 3.1 in Simmons (1975). However, the emission function (7.3) is non-linear in Model 7.1.

According to Benedek and Rilett (1998), when the average speed is less than the critical speed, the emission function is convex. To calculate the critical speed, there is:

\[
\frac{dCO_{a}(v_{a})}{dv_{a}} = 0, \forall a.
\]  
(7.21)

Then there is:

\[
\frac{A^{\infty}}{C^{\infty}} e^{\tau v_{a}} (1 - B^{\infty} \cdot \hat{s}_{a}) \frac{dt_{a}(v_{a})}{dv_{a}} = 0, \forall a.
\]  
(7.22)

where \( \hat{s}_{a} \) is the critical speed on link \( a \). As \( \frac{A^{\infty}}{C^{\infty}}, e^{\tau v_{a}} \) and \( \frac{dt_{a}(v_{a})}{dv_{a}} \) are positive, the critical speed can then be expressed as follows:

\[
\hat{s}_{a} = \frac{1}{B^{\infty}}, \forall a.
\]  
(7.23)

If speed limit is greater than critical speed and average speed on a link as \( \bar{s}_{a} \leq \hat{s}_{a} \leq s_{a} \), when the average speed is expressed as follows, the emission function is convex, and increases with traffic volume \( v_{a} \) monotonously:

\[
\bar{s}_{a} \leq \hat{s}_{a} \leq s_{a} = \frac{1}{B^{\infty}}, \forall a.
\]  
(7.24)

Equation (7.24) indicates the range of speed limit (average speed) which can guarantee a unique solution of Model 7.1. As only CO emissions are estimated, then there is:
\[ \bar{s}_a \leq \hat{s}_a \leq s_a = \frac{1}{B_{av}} = 68.7 \text{ ft/sec} = 75.4 \text{ km/hr}. \]  

(7.25)

This means if the speed limit \( s_a \leq 68.7 \text{ ft/sec} = 75.4 \text{ km/hr} \), the feasible region defined by constraints (7.12)-(7.14) is convex. The objective function (7.11) has been proved in appendix A to be strictly convex, this strict convexity in the vicinity of the minimum point can guarantee that there is a unique solution to the SUE equivalent minimisation optimisation programme (7.11)-(7.14).

It should be noted that when the speed limit is not within the range shown in equation (7.24), the emission function is non-convex. However, using a non-convex function does not mean the solution is not optimal, it simply indicates that there is no guarantee that the solution is optimal (Benedek and Rilett, 1998). In this study, the purpose of formulating the proposed model is to seek an optimal toll pattern which supports a SUE assignment and guarantees that the maximum allowable vehicular emissions are not violated. Therefore Model 7.1 and the derived speed-varying tolls are still meaningful.

### 7.5 SUMMARY

In the literature to date, no one has attempted to incorporate the speed limit control when studying the impacts of traveller information provision services and tolling on vehicular emissions and TSTT. However, the speed of a vehicle plays an important role in estimating the volume of vehicular emissions. Therefore, in this chapter, an analytical methodology is proposed to evaluate the impact of the integration of the traveller information provision services, speed limit control, and tolling particularly on vehicular emissions. To address this important issue, the macroscopic emission estimation relationship used in TRANSYT-7F model is adopted, which belongs to the domain of average speed models, to consider the effect of the speed limit control while analysing the impacts of traveller information provision services and tolling on TSTT and overall vehicular emissions. The route choice behaviour of the equipped and unequipped travellers is modelled to follow a mixed Stochastic User Equilibrium (SUE) where elastic market penetration is captured. This mixed SUE problem is modelled through the Nonlinear Complementarity Problem (NCP) approach and solved by Generalised Reduced Gradient (GRG) method.
The proposed NCP formulation, utilised as the analytical methodology, allows one to study the impact of the strategic interactions among speed limit control, traveller information provision services and tolling on a transportation system. Through the sensitivity analysis, it is found that it is essential to consider speed limit control whilst evaluating the impact of traveller information services. This is the case because the free flow travel time (which depends on the speed limit) determines the driver’s average travel time. This in turn significantly affects the demand for information provision services, and hence the system performance in terms of vehicular emissions and TSTT. More importantly, the speed limit control may cause emission inequity among links between an OD pair. In this case, the traffic manager needs to implement speed limit control with great caution and should take into account the strategic interaction with traveller information provision services. In addition, results show strong strategic interactions among tolling, speed limit control and traveller information provision services. Also, the tolling location affects the system performance regarding total emissions and TSTT. This implies that, what is required is to carry out a holistic analysis of the costs and benefits of the joint implementation of these strategies before any individual scheme is implemented. The proposed methodology can be used for this purpose to help policy makers balance the conflict between vehicular emissions and TSTT.

Finally, optimal speed-varying marginal tolls are derived in a transportation system where travellers follow a mixed SUE traffic assignment so that uncertainty in travellers’ behaviour is captured. The derived analytical expression indicates that a traveller should pay a charge equal to the sum of his own emission costs and the congestion and emission externalities that he imposes on others. Although an optimal toll with similar interpretation has been previously derived and studied by Yin and Lawphongpanich (2006), the difference between their toll and marginal tolls derived in this study has been discussed from three aspects, which are the nature of the model, the equilibrium principle of traffic assignment, and the goal to limit total emissions. Their toll was derived in a multi-objective model to minimise a weighted combination of congestion and total emissions, and optimal flow pattern follows a system optimum (SO) traffic assignment. The goal of their model was to determine optimal charges that induce a flow pattern with less congestion and emissions. Nevertheless, the proposed model in this chapter formulates route choice behaviour of equipped and unequipped travellers.
following a mixed SUE traffic assignment and the net economic benefit is maximised. So the model has an interpretation from both economic and behavioural viewpoints. Also the goal of the model is clear: to guarantee that maximum allowable emissions are not violated. As such, the derived optimal toll is more meaningful, guaranteeing a sustainable network in which the flow pattern follows a mixed SUE assignment. Additionally, it is demonstrated that the derived emission toll is dependent on the average speed, the speed limit, the congestion level and capacity of the link.

Furthermore, the convexity of the proposed model is discussed and the critical speed which dictates the convexity of the emission cost function is derived. Due to the restriction of convexity of the emission cost function, it is discussed that the proposed model cannot always guarantee uniqueness of the solution. However, it does not limit the meaning of the proposed model and derived speed-varying marginal tolls, as the purpose of formulating the model is to seek an optimal toll pattern which supports a mixed SUE assignment and simultaneously guarantees that the maximum allowable vehicular emissions are not violated.
CHAPTER 8

CONCLUSIONS

8.1 RESEARCH SUMMARY

Sustainable transportation strategies and policies should be able to provide multiple economic, social and environmental benefits. This means that they not only generate an efficient and equitable transportation system but that they also balance other economic, social and environmental objectives. However, current transportation strategies can only solve one or two problems but exacerbate others (Litman, 2008). For example, adding roadway capacity reduces congestion but tends to increase total vehicle traffic, which creates increased accidents and air pollution. The fact is that emissions from vehicles are growing faster than in any other sector as any improvement in technology is directly outweighed by the sheer increase in traffic (UITP, 2007). Unsustainable transportation strategies and policies will make it difficult for local authorities to meet Kyoto emission reduction targets.

With this consideration, the objective of this thesis is to carry out a comprehensive equilibrium analysis of most widely applied transportation polices and strategies from the viewpoint of sustainability, mainly concentrating on their impact on vehicular emissions. This study adopted the two best known and most widely used macroscopic emission estimation models belonging to the category of average-speed models. These are Mobile Source Emissions Factor Model or MOBILE and TRANSYT-7F model. The evaluated transportation strategies and policies include road pricing, road network design, traveller information provision services, and speed limit control. The traffic assignment is simulated by employing different traffic assignment techniques, which consist of deterministic user equilibrium (DUE), stochastic user equilibrium (SUE), system optimum (SO), and system equity (SE). The impact of those strategies on travellers’ route choice behaviour under different equilibriums is studied. The consequent impact on vehicular emissions is analysed.
In this thesis, transportation problems were formulated into various mathematical programs subject to different scenarios. The model proposed in chapter 3 to study the impact of road pricing was a single-level minimisation model, although it was bi-level in nature. The basic models proposed in chapter 4 to analyse the impact of various road network design schemes were also formulated as bi-level programs. Additionally, in chapters 4 and 5, multi-objective optimisation models were developed to assist policy makers or transportation planners in their multi-objective decision making. In chapters 6 and 7, the models to study the impact of traveller information provision services were formulated as a nonlinear complementarity problem (NCP). In chapters 6 and 7, models proposed to derive the marginal cost pricing policy were formulated as convex programming, although the convexity of the emissions cost function in chapter 7 was conditional. The goal of formulating these analytical models is to seek a sustainable solution to handle some problems caused by some current transportation strategies. These models, which are a valid representation of the performance of the transportation system under different transportation strategies, were solved by the Generalised Reduced Gradient (GRG) algorithm.

8.2 RESEARCH FINDINGS AND CONTRIBUTIONS

This section presents the important research findings and contributions of this thesis, which are summarised from chapters 2 to 7 as follows:

Chapter 2 introduced the concepts and formulation of the conventional traffic assignment techniques, the land use-transport interaction, and most widely used macroscopic emission estimation approaches. It also gave a literature review of most applied transportation strategies and policies so as to emphasise the importance of this research to the field.

The contribution of Chapter 3 is the examination of the impact of controlling maximum allowable link emissions on toll charges, which has never been studied in the literature, although some researchers have carried out studies on defining optimal land use
transport strategies whilst considering its impact on overall vehicular emissions. The results showed that the tighter the vehicular emission standards, the higher the toll charges required. It was also shown that vehicular emission standards have a direct impact on the following: overall vehicular emissions; operational strategies and profit of public transit; the mode and route choices of travellers; residential and employment distributions; profits of land owners; and rents. The government should consider these impacts when determining the vehicular emission standard of each road.

Chapter 4 proposed optimisation frameworks for road network design considering the land use-transport interaction over time. Unlike existing models, the optimisation frameworks can determine the optimal designs automatically without trial-and-error once the objective(s) is/are clearly defined. Moreover, these frameworks allow the evaluation of the impacts of the optimal designs on the related parties including government, landowners, toll road operators, transit operators, and road users. They also aid network planners and profit-makers with decision-making by the elimination of many alternative designs. A numerical study was set up to examine the effect of road network design on these related parties, in particular landowners in terms of their profits and the government in terms of vehicular emissions. Three road network improvement schemes were compared, which are exact cost recovery, build-operate-transfer, and cross-subsidisation (using the increase in transit profit to subsidise road improvement projects). The results showed that it is difficult to comment on which scheme was the best in general after considering each party's perspective. Trade-offs were also seen to exist between the objectives of all related parties. This implies that the government has to consider trade-offs between objectives of parties carefully.

Braess’ paradox is well-known and has already been examined extensively in the literature. However, the emission paradox has not. If the emission paradox is not considered, the road network improvement that mitigates congestion may increase harmful vehicular emissions. Thus the contribution of Chapter 5 was to analytically examine the occurrence of the emission paradox and the simultaneous occurrence of Braess’ and emission paradoxes in the classical Braess’ network. It was found that the occurrence of the emission paradox depends on the demand for travel, the parameters of link performance functions, as well as link emission factors. It was also found that Braess’ and emission paradoxes do not always occur at the same time, and that the
emission paradox is more likely to occur than the Braess paradox in some networks. More importantly, it was discovered that under certain conditions of parameters in link performance functions, the emission paradox does occur but Braess’ paradox does not. This implies that road network design for mitigating congestion alone may not be able to curb the increase in vehicular emissions. A more comprehensive view of road network design considering both congestion and emissions simultaneously is necessary to avoid the occurrence of the emission paradox.

In addition, some marginal cost congestion and emission pricing policies were proposed and compared with the existing marginal cost of congestion and emission pricing. Different toll charges derived from these pricing policies were compared. The occurrence of both paradoxes with corresponding pricing schemes was discussed. It was concluded that the marginal cost congestion pricing cannot remove the emission paradox but that the marginal cost congestion and emission pricing can remove both of the paradoxes. This implies that policy makers and transport designers need to examine the network configuration very carefully before any optimal pricing scheme is chosen so as to avoid exacerbating other problems.

Chapter 6 proposed a multi-class, multi-criteria mixed SUE assignment model under the traveller information provision services with heterogeneous road travellers and elastic market penetration. The novelty of this study is to incorporate the travellers’ multi-criteria decision making process into the proposed mixed SUE model in which there is an explicit environmental criterion, which has not been attempted in the transportation literature to date. Some numerical experiments were carried out in order to evaluate the effectiveness of traveller information provision services. It was found that the service charges, information quality, travellers’ own perception of the trade-offs among their multiple criteria and their VOT levels have a strong implication on the system performance with traveller information provision services. The proposed model has an interpretation from both economic and behavioural viewpoints. The solution of this program satisfies the logit-based SUE assignment. The marginal cost pricing was then derived from the proposed model. This marginal cost pricing is not only class-dependent but also link-dependent. However, it was found that it cannot lower TSTT and vehicular emissions at the same time. It was pointed out that this is due to the stochastic nature of the proposed mixed equilibrium model. An optimised system here is
referred to as an optimal flow pattern which satisfies a given emission bound. In this chapter, it was also proved that the objective function in the proposed SUE equivalent minimisation Model 6.1 is strictly convex in the vicinity of \( f^* \) (and convex elsewhere) and the feasible region is convex, which guarantee the uniqueness of the solution.

Chapter 7 adopted the macroscopic emission estimation relationship used in TRANSYT-7F model to consider the effect of the speed limit control whilst analysing the impacts of traveller information provision services and tolling on TSTT and overall vehicular emissions. The contribution of this chapter lies in the proposed analytical methodology, which allows one to study the impacts of strategic interactions among a traffic manager, toll road operators as well as information providers using the proposed NCP formulation. A sensitivity analysis was carried out to demonstrate the analytical methodology and illustrate the impact of strategic interactions among speed limit control, traveller information provision services, and tolling on both congestion and vehicular emissions. Although only CO emissions were considered in the analysis, it is noted that the principle applies to estimate all the pollutants.

To capture uncertainty in travellers’ behaviour, optimal speed-varying marginal tolls were further derived in a transportation system where travellers follow a mixed SUE traffic assignment. The derived marginal tolls are dependent on the average speed, the speed limit, the congestion level and capacity of the link. The difference between the marginal cost speed-dependent tolls in this study and others were discussed from three aspects. These were the nature of the model, the equilibrium principle of traffic assignment, and the goal to limit total emissions. After comparison, it was concluded that the speed-varying marginal tolls in this study are more meaningful. Finally, the critical speed which dictates the convexity of the emission cost function was derived. It was also pointed out that there is the limitation of the proposed model, which is the restriction of convexity of the emission cost function. This means that the uniqueness of the solution cannot be guaranteed like the one in chapter 6. However, this does not affect the meaning of formulating the proposed model, which is to seek an optimal toll pattern in a mixed SUE assignment and to guarantee the generated emissions do not exceed the maximum allowable levels.
In summary, this thesis presented analytical models, approaches, and analyses incorporating economic, social, and environmental aspects, which allow transportation policy makers and transportation planners to evaluate sustainability of various transportation strategies at a strategic level. The proposed models provided a useful methodology which enables transportation policy makers to balance the trade-offs amongst different parties from economic, social, and environmental perspectives. The analyses carried out in this thesis demonstrated policy implications of various transportation strategies. The conclusions drawn from the results could help policy makers to identify optimal sustainable transportation strategies and policies, which can simultaneously reduce negative impacts of transportation and avoid conflicting objectives.

8.3 CRITICAL ASSESSMENT

Although this thesis has covered a wide horizon in the research area of the analysis of the impact of transportation strategies in terms of sustainability, there are a number of limitations to the research from theoretical, practical, and computational aspects, which have been summarised broadly as follows.

- All the proposed models are static by assuming that the traffic condition remains at a steady state under recurrent congestion. These models do not allow the study under incident conditions or non-recurrent congestion. It would be meaningful to capture en-route traveller behaviour and network uncertainty in the proposed models in a dynamic traffic assignment framework in the future research.

- Although the proposed models are theoretically sound, there are limitations from a practical point of view. For example. It is assumed that a complete path set is given. In reality, there are a large number of paths and most of them may not even be used. It is impossible to make path enumeration, especially for large networks. Developing a link-based formulation for all the models would be another important future direction.

- The proposed models provide a useful tool for evaluating the effects of various transportation strategies at a strategic level. However, the models were only
applied to small networks for illustrative purposes. It would be a significant study by applying the proposed models to the real networks in Ireland. This requires some real local data to verify the corresponding models and efficiency of the relevant algorithms.

- In this thesis, all the models were solved by the generalised reduced gradient (GRG) algorithm, which can be a good solution method to solve problems in small networks. Considering solving large networks with a huge number of paths, it would be difficult and time-consuming to find a global solution. In some cases, there may be more than one optimal solution in path-based formulations. Therefore, it should be more desirable to find a global optimum by applying or designing more efficient solution algorithms and computational methods.

Nevertheless, these limitations offer opportunities for further research. A detailed discussion for further research is provided in the next section.

8.4 RECOMMENDATIONS FOR FUTURE RESEARCH

The study in this thesis raises a lot of future research directions, which are discussed as follows:

In terms of the study presented in Chapter 3, it is meaningful to carry out the following research. Firstly, the movements of goods are considered to be given in this study. Relaxing this assumption is one of future research directions. Secondly, from the equity point of view, it is not fair to charge the same toll to travellers who have different values of time, therefore it would be vitally important to consider optimal tolls in multi-class, multi-criteria transportation networks in which travellers consider multi-criteria of travel cost, travel time and incurred vehicular pollution. Thirdly, one can extend this study to consider the impact of road pricing on society such as social welfare and the land use owner's profits. It would also be interesting to model the transport network under the stochastic user equilibrium assumption in order to study the performance of the transport system effectively, as the model with the deterministic user equilibrium
assumption does not represent the real world scenario. Finally, extending this framework to consider capacity expansion, road construction and the gradually upgraded transit network is another possible direction, as many parts of Asia and Europe are in an active phase of a massive transport development.

In Chapter 4, the study opens up a number of research directions. First, the proposed frameworks only consider single class drivers with one trip purpose and the Wardrop’s travel principle. A possible direction is to extend them to consider multi-class drivers in which each class of drivers has its own value of time, routing strategy, and trip purpose. Second, the network and the demand here are assumed to be deterministic. In reality, they are not. Capturing uncertainty in demand and supply in the proposed frameworks can be another direction. Third, BOT projects are in fact competing with others. It is thus worthwhile to extend the proposed framework to model the competitive situation between BOT projects. Finally, the proposed models are path-based, and thus unsuitable for large networks involving many paths. Moreover, the problem is highly non-convex, which is difficult to solve for global solutions efficiently. Developing a link-based formulation and an efficient global optimisation technique for this problem represents another worthy research direction.

In Chapter 6, the total travel demand is assumed to be fixed. Kanninen (1996) pointed out that ITS may induce potential travel demand. Ignoring latent travel demand may overestimate the benefits of information provision services. It is thus interesting to study the effect of information provision in a network with an elastic travel demand. As indicated, the proposed models are static. It would be meaningful to model mixed SUE behaviour under endogenous market penetration of traveller information services in a dynamic traffic assignment framework. This will allow the study of the impact of traveller information provision services to capture the changes in departure times and the impact of traveller information provision services under non-recurrent network congestion. Besides, the derived marginal tolls are static. It would be very important to derive time-varying marginal tolls under the SUE assignment in a dynamic traffic assignment problem, as Carey and Srinivasan (1993) pointed out that dynamic social cost externality depends not only on the level of congestion, but also on the rate of increase or decrease of congestion. Also, it would be very meaningful to calibrate the
parameters of perception variations $\theta_i$, which represent travel uncertainty, once the survey data is available. The method proposed by Huang (1995) can be employed for this purpose. In addition, extending the existing model to incorporate travel time, cost and network uncertainties would be another challengeable research direction.

In Chapter 7, there are the following extensions raised in this research. Firstly, although the model is a multi-class multi-criteria framework, the analysis is carried out with the assumption that all drivers have the same VOT. However, people with a higher value of time will travel faster than other people (see, for example, Rietveld and Shefer, 1998; Rienstra and Rietveld, 1996). Additionally, it is assumed in this study that a uniform speed limit is applied on each route. Nevertheless, Rietveld and Shefer (1998) have pointed out that with a uniform speed limit, it is impossible to consider variety among drivers. Without considerable difficulty, the model can be extended to consider various speed limits on the same route in the transport system where drivers have different VOTs. Secondly, it was assumed that the compliance rate of the speed limit is 100%, but in reality drivers are not completely obedient to the speed limit (see, for example, Thornton and Lyles, 1996; Elliott et al., 2005). It is, thus, rather important to capture this driver behaviour when studying the impact of the speed limit control, traveller information provision services, and tolling on TSTT and vehicular emissions. Besides, extending the existing model to incorporate travel time, cost and network uncertainties would be another essential but challengeable research direction.
REFERENCES


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APPENDIX A: PROOF OF UNIQUENESS CONDITIONS

The usefulness of the logit-based SUE equivalent minimisation model (6.25)-(6.28) would be limited if it does not have unique minimum (Sheffi, 1985). In order to show that the SUE equivalent minimisation Model 6.1 has only one solution, it is sufficient to prove that the objective function (6.25) is strictly convex in the vicinity of \( \mathbf{f}^* \) (and convex elsewhere) and the feasible region defined by constraints (6.26)-(6.28) is convex. The strict convexity in the vicinity of the minimum point is to guarantee that it is a local minimum. The convexity of feasible region is to ensure there are no other local minima.

Strict Convexity of the Objective Function and Convexity of the Feasible Region

It is well known that one way to determine whether a function is strictly convex or not is to evaluate whether the Hessian of the function is positive definite or not. If the Hessian matrix of second-order partial derivatives is positive definite, then the function is strictly convex. In addition, the sum of two or more convex functions is convex (Simmons, 1975). Observing that the objective function (6.25) consists of three items, it is assumed:

\[
\sum_i \frac{1}{\theta_i} \sum_{rs} f_{r,m}^\alpha \ln f_{p,i,m}^\alpha = F_i(f),
\]

\[
- \sum_i \frac{1}{\theta_i} \sum_{rs} q_{r,m}^\alpha \ln q_{i,m}^\alpha = F_z,
\]

\[
\sum_i \sum_{rs} f_{p,i,m}^\alpha T_{p,i,m}^\alpha = F_b(f).
\]

Since travel demand between OD pair \( rs \), \( q_{i,m}^\alpha \), and perception variation \( \theta_i \) are given, \( F_z \) is a constant. It only needs to prove that \( F_i(f) \) and \( F_b(f) \) are convex functions. According to Theorem 1 in Evans (1973), \( F_i(f) \) is strictly convex. Now one can demonstrate that \( F_b(f) \) is also strictly convex by proving its Hessian matrix is positive definite. The first-order partial derivative of \( F_b(f) \) is expressed as follows:
\[ \frac{\partial}{\partial f_{p,i,m}} F_3(f) = \eta_{p,i,m}^n + \sum_i f_{p,i,m}^n \frac{d\eta_{p,i,m}^n}{df_{p,i,m}^n} = \eta_{p,i,m}^n + \sum_i f_{p,i,m}^n \frac{d(\sum B_{a} \delta_{a}^p)}{df_{p,i,m}^n}, \forall rs, p, i, m, \]

and the second-order partial derivative is as follows:

\[ \frac{\partial^2 F_3(f)}{\partial f_{p,i,m}^n \partial f_{k,i,m}^n} = \begin{cases} \frac{d(\sum B_{a} \delta_{a}^p)}{df_{p,i,m}^n} + \sum_i f_{p,i,m}^n \frac{d^2(\sum B_{a} \delta_{a}^p)}{df_{p,i,m}^n} \frac{d(f_{p,i,m}^n)^2}{d(f_{p,i,m}^n)^2}, & \text{for } p = k, \\ 0, & \text{otherwise} \end{cases} \]

thus the Hessian matrix of \( F_3(f) \) is as follows:

\[ H = \nabla^2 F_3(f) = \begin{bmatrix} \frac{\partial^2 F_3(f)}{\partial f_{p,i,m}^n \partial f_{p,i,m}^n} & 0 & 0 & \cdots \\ 0 & \frac{\partial^2 F_3(f)}{\partial f_{p,i,m}^n \partial f_{i,m}^n} & 0 & \cdots \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & & \frac{\partial^2 F_3(f)}{\partial f_{p,i,m}^n \partial f_{k,i,m}^n} \end{bmatrix}. \]

Because the link performance functions \( t_a \) are assumed to be positive and increasing, the Hessian matrix is a diagonal matrix with strictly positive elements on the diagonal. Hence, the Hessian is positive definite. The function \( F_3(f) \) is thus strictly convex. It is now concluded that the objective function is strictly convex.

According to Theorem 3.1 in Simmons (1975), any linear function is both convex and concave over all of \( E^n \). Therefore, the linear constraints are convex. The feasible region defined by constraints (6.26)-(6.28) is a convex set. The strict convexity of the objective function and convexity of the feasible region guarantee the solution \( f^* \) to be a global minimum. In other words, there is a unique optimal solution for the convex program (6.25)-(6.28).
APPENDIX B: OUTLINE OF GENERALISED REDUCED GRADIENT ALGORITHM

The following gives the detailed algorithmic steps of the Generalised Reduced Gradient method:

Step 1: **Initialisation**

Choose stopping tolerance $\varepsilon > 0$ and any starting feasible solution $x^0$. Partition vector $x$ into basic variables $x_b$ and non-basic variables $x_n$. Therefore, the initial point $x^0$ can be represented by $\begin{bmatrix} x_b^0 \\ x_n^0 \end{bmatrix}$. Construct the basis matrix $B = \frac{\partial h}{\partial x_b}(x^0)$ and the non-basis matrix $N = \frac{\partial h}{\partial x_n}(x^0)$. Set $k = 0$.

Step 2: **Direction finding**

Compute the generalised reduced gradients $r^k_n$ at $x^k$ by:

$$ r^k_n = \frac{\partial \text{TCS}(x^k)}{\partial x_b} - \frac{\partial \text{TCS}(x^k)}{\partial x_n} B^{-1} N. $$

Determine the displacement direction $d^k = \begin{bmatrix} d^k_n \\ d^k_n \end{bmatrix}$ by:

$$ d^k_n = -B^{-1} N d^k_n, $$

$$ d^k_n = \begin{bmatrix} d^k_n \end{bmatrix}, $$

where $d^k_n = \begin{cases} r^k_n & \text{if } r^k_n > 0 \text{ or } x^k_n > 0 \\ 0 & \text{otherwise} \end{cases}$, $r^k_n(x^k_n)$ is the element of $r^k_n(x^k_n)$.

Step 3: **Convergence test**
If $\|d^k\| \leq \varepsilon$, stop.

**Step 4: Feasibility limit**

Compute feasibility limiting step $\lambda^k_{\text{max}}$ by:

$$
\lambda^k_{\text{max}} = \begin{cases} 
\infty & \text{if } d > 0 \\
\min \left\{ \frac{x^k_j}{-d^k_j}, \forall j \in \{n, b\} | d^k_j < 0 \right\} & \text{otherwise}
\end{cases}
$$

in which $d^k_s$ is the element of $d^k$.

**Step 5: Line search**

Perform a one-dimensional search to determine the step size $\lambda^k$ solving:

$$
\max TCS \left( x^k + \lambda^k d^k \right)
$$

subject to $0 \leq \lambda^k \leq \lambda^k_{\text{max}}$.

**Step 6: Move**

Set $x^k = x^k + \lambda^k d^k$.

**Step 7: Correction step:**

Step 7.1 Set $l = 0$ and $z^0 = x^k$.

Step 7.2 Compute $z^{l+1}$ by:

$$
z^{l+1} = z^l - B^l h \left( z^l, x^k \right).
$$

Step 7.3 If $z^{l+1} \geq 0$ and $\|h(z^{l+1}, x^k)\| \leq \varepsilon$, Set $x^{k+l} = z^{l+1}$, $x^{k+1} = x^k$, and $k = k + 1$. Go to Step 1.

Step 7.4 If $z^{l+1} \geq 0$, set $l = l + 1$ and go to Step 6.2.
Step 7.5 Substitute the basic variable that becomes negative by a non-basic variable. Form a new basis $B = \frac{\partial h}{\partial x_b}(x^k)$ and

$$N = \frac{\partial h}{\partial x_n}(x^k).$$

Go to Step 1.
APPENDIX C: A GLOSSARY OF TERMS

Accessibility

A measure of the ability or ease of all people to travel among various origins and destinations.

Capacity

Capacity refers to a rate of vehicular or person flow that can be expected to traverse a point or uniform section of a lane or roadway during a specific period, which is most often a peak 15-minute period, and which is not the maximum volume that can be accommodated during an hour, under prevailing roadway, traffic, and control conditions.

Charge/Fare

Payment in the form of coins, bills, tickets and tokens collected for provided services.

Congestion Pricing

The policy of charging drivers a fee that varies with the level of traffic on a congested roadway. Congestion pricing is designed to allocate roadway space in a more efficient manner. Congestion pricing is also known as relief tolling, variable pricing, and road pricing.

Emissions

A substance released into the air, often with the burning of fossil fuels such as oil, gas and coal.

Emission Charges

Vehicle charges that vary based on emission rates.

Market Penetration

The proportion of the number of equipped travellers and the total number of travellers within this class.
Modal Split
A term which describes how many people use alternative forms of transportation. Frequently used to describe the percentage of people using private automobiles as opposed to the percentage using public transportation.

Mode
A particular form of travel (e.g., bus commuter tail, train, bicycle, walking or automobile.

Model
An analytical tool (often mathematical) used by transportation planners to assist in making forecasts of land use, economic activity, travel activity and their effects on the quality of resources such as land, air and water.

Multimodal
Refers to the availability of multiple transportation options, especially within a system or corridor.

Net Economic Benefit
Refers to social welfare, which is the difference between the total network user benefit and the total social cost.

Network
The configuration of streets or transit routes and stops that constitutes the total system.

Operating Cost
The total costs to operate and maintain a transit system including labor, fuel, maintenance, wages and salaries, employee benefits, taxes, etc.

Public Transportation
Transportation by bus, rail, or other conveyance, either publicly or privately owned, which provides to the public general or special service on a regular and
continuing basis. Also known as "mass transportation," "mass transit" and "transit".

Private Transit Operator
An employee of a private-owned transit system who spends his or her working day in the operation of the transit system.

Revenue
Receipts derived from or for the operation of transit service, revenue from other commercial sources, and operating assistance from governments. Revenue from a transit system includes all fares, transfer charges, and zone charges paid by transit passengers.

Road Pricing
The general term for any charge for use of a roadway. It is sometimes limited to direct charges, such as tolls, or may include other vehicle fees, including fuel taxes, license fees and parking charges.

Route
A specified path taken by a transit vehicle usually designated by a number or a name, along which passengers are picked up or discharged.

Subsidy
Funds granted by federal, state or local government.

Sustainable Development
The process of meeting the development needs of all without compromising or jeopardising the ability of future generations to meet their essential needs.

Toll Road
A section of road where motorists are charged a direct user fee (or toll).

Transfer Passenger
A passenger who transfers to a line after paying a fare on another line.
**Transportation Strategy**

A general approach to solving a transportation problem. Examples of transportation strategies for roadways include: "do nothing" or "no-build," add a travel lane, convert a travel lane to an HOV lane, reconstruct or widen a roadway, or implement Transportation Demand Management and Transportation Systems Management.

**Transit Fare**

The price charged to one adult for one transit ride; excludes transfer charges, zone charges, express service charges, peak period surcharges and reduced fares.

**Transit System**

An organisation (public or private) providing local or regional multi-occupancy-vehicle passenger service. Organisations that provide service under contract to another agency are generally not counted as separate systems.

**United States Environmental Protection Agency (USEPA)**

A Federal agency charged with protecting the natural resources on the nation.
APPENDIX D: PUBLISHED RESEARCH

1. **Refereed Journal Paper**


2. **Papers in Refereed Conference Proceedings**


