SECTION 1: INTRODUCTION

The purpose of this paper is to show what can be learned about the reaction of Irish consumers to price and income changes by using the published time series data and the currently-accepted models of consumer behaviour.

Two difficulties present themselves immediately. First, the data are relatively scarce. At the time of writing, there are some twenty annual observations of consumer expenditure and, while there is a fairly fine division in the tables of current price expenditure (which provide 15 categories), the tables of constant price expenditure reduce this to eight. Secondly, there is at the moment no generally accepted "best" model of consumer behaviour. Many are available, and each has its strengths and weaknesses. It is true that some can be rejected because their assumptions are unacceptable and some because they cannot be estimated; but having done this one cannot discover any one model which is clearly better than all others.

The first difficulty cannot be overcome. With some effort, one might increase the number of consumption categories, but this would increase the number of parameters to be estimated in most models very considerably and it is doubtful if the length of the time series is sufficiently great to make it possible to estimate such an expanded system.

The strategy adopted here in regard to the second difficulty is to estimate all the most plausible models. Any aspects of consumer behaviour which are consistent in all the results can be regarded as reasonably reliable, while the remainder will at least indicate areas of doubt. Further, it will be possible to discover how well each model fits the Irish data, and that may provide some further information about the nature of consumer response to price and income changes. The remainder of the paper is organised as follows:

Section 2 sets out certain basic results of consumption theory which are relevant to all the models;
Section 3 describes the models used;  
Section 4 describes the estimating methods;  
Section 5 comments on the data;  
Section 6 describes the results, and draws conclusions from them;  
Section 7 is a summary.

SECTION 2: THEORETICAL BACKGROUND

The standard theory of consumer behaviour imposes a considerable number of restrictions on the parameters of any demand system. Since frequent references will be made to these, it is convenient to state them briefly here. Their derivation is not given as it is available in many places, for example, in Brown & Deaton (1). Strictly speaking, the restrictions apply only to the behaviour of a single utility-maximising consumer, and it is at least debatable whether they are relevant to a group of consumers even if each is a utility-maximiser. However, these constraints are of considerable importance in regard to the models presented below. There are four principal restrictions, namely, aggregation, homogeneity, symmetry, and negativity.

(1) Aggregation: This implies that total expenditure must continue to exhaust the budget after all income and price changes. There are two sub-cases:-

(1A) Engel aggregation. This refers to the reaction to income changes.

Define

\[ e = \text{the column vector of expenditure elasticities.} \]
\[ E = \text{the matrix of price elasticities.} \]
\[ W = \text{the column vector of average budget shares.} \]
\[ i = \text{a column vector of units.} \]

The Engel aggregation may be written as

\[ W'e = 1 \]  
(2.1)

(1B) Cournot aggregation. This refers to the reaction to price changes.

It may be written

\[ W'E + W' = 0 \]  
(2.2)

(2) Homogeneity: The homogeneity restrictions reflect the well-known fact that a rational consumer will not change his purchase-plan in the face of an equal proportionate change in income and all prices.

They may be written

\[ E_i + e = 0 \]  
(2.3)

or alternatively, if we further define

\[ Q_p = \text{the matrix of the derivatives of the quantities with regard to the prices.} \]
\[ q_y = \text{the column vector of the derivatives of the quantities with regard to total expenditure.} \]
\[ p = \text{the column vector of prices} \]
\[ q = \text{the column vector of quantities.} \]

\[ [Q_p + q_y q'] p = 0 \]  
(2.4)

(3) Symmetry: Consistency of behaviour requires that, provided the consumer is compensated for the change in income caused by price variations, the change in the quantity of
good i caused by a variation of one unit in the price of good j must be equal to the change in the quantity of j caused by a unit variation in the price of i.

This may be written

\[ S = S' \] .................. (2.5)

where S is the matrix defined in (2.4) above.

(4) Negativity The most obvious aspect of this set of conditions is that the compensated demand curve for each good must slope downward; provided the consumer is compensated for the income change, a rise in the price of any good must lead to a fall in quantity and vice versa. However, much more than this is involved. In the words of Brown & Deaton "one way of interpreting the matter is to say that the compensated demand curve for any fixed-proportion bundle of goods slopes downward".

Negativity imposes the condition that the S-matrix referred to above be negative semi-definite. In particular, the elements of main diagonal of S must all be negative.

SECTION 3: THE MODELS

(i) THE LINEAR EXPENDITURE SYSTEM (LES):

The demand functions of this system were developed by Klein and Rubin (2), before the utility function from which they might be derived was known. Later, Samuelson (3) and Geary (4) showed that if a consumer maximises a utility function of the form

\[ u(q) = \sum b_i \log (q_i - c_i) \] .................. (3.1)

(where b and c are constants, and q is quantity) subject to the income constraint and to the imposition of symmetry on the S-matrix, the resulting demand functions are

\[ q_i = c_i + b_i \frac{Y - \sum_{j} p_j c_j}{p_j} \] .................. (3.2)

The \( c_i \) may be any function which is homogeneous of degree zero in the prices, but in the most widely-used form the \( c_i \) are constants.

The elasticities may be found to be:

Income \[ E_i = \frac{b_i}{W_i} \] (where \( W_i \) is the average budget share)

Own Price: \[ E_{ii} = -1 + (1 - b_i) \frac{c_i}{q_i} \]

Cross Price: \[ E_{ij} = -b_i \frac{p_j c_j}{p_i q_i} \]

Substitution of these in equations 2.1, 2.2, 2.3 will show that, provided \( \sum b_i = 1 \), the equations satisfy the conditions of Cournot and Engel aggregation and of homogeneity. Symmetry has, of course, been imposed in formulating the system. Negativity is not imposed, but it will be satisfied if the \( b_i \) are found to be all positive, and if \( (Y - \sum p_j c_j) \) is also positive.
The LES has many attractive qualities. In its basic form, it satisfies almost all the restrictions of economic theory, yet it requires the estimation of relatively few parameters. The parameters have a simple interpretation, the $c_i$ being required quantities and the $b_i$ the fractions of supernumerary income (the amount by which actual income exceeds the sum needed to buy the required bundle of goods) which is devoted to additional purchases of commodity $i$. The form of the model makes it easy to incorporate as part of a bigger model. Finally, the system has been estimated many times, and a good deal is known about its characteristics.

There are, however, some flaws. It is possible for the model to predict negative quantities for sets of values of the parameters which are otherwise quite reasonable - but since this usually happens only at very low income values it is probably not very important. As long as the $b_i$ are all positive, none of the commodities can be an inferior good, a characteristic which is likely to become more important as one increases the number of categories of expenditure used. While own price elasticities greater than unity in absolute value can be accommodated (if $c_i$ is negative) the system does not really measure price responses. It concentrates on income effects, and the price response measured might be described as the income aspect of the price change. Finally, the method of computation is tedious even by comparison with other complete systems of demand equations. A maximum likelihood estimator has become available only very recently.

(ii) THE ROTTERDAM SYSTEM

This is primarily derived from the work of Theil (5). It begins from the general set of demand functions

$$ q = f(y,p) $$

where, as before $q, y$ and $p$ are interpreted as vectors. One may write the differentials as

$$ dq = q_y dy + Q_p dp $$

where $q_y$ is the vector of quantity derivatives with regard to money income and $Q_p$ in the matrix of derivatives of quantity with regard to price. Using the matrix of compensated price derivatives defined above

$$ S = Q_p + q_y q $$

one may by substitution write

$$ dq = q_y (dy - q dp) + S dp $$

The system may be multiplied by $y^{-1} \hat{p}$, where $\hat{p}$ is a matrix with the prices on the main diagonal and zero elsewhere, and using the fact that $dq_i = q_i d\log q_i$ etc., it becomes

$$ y^{-1} \hat{p} q_y d\log q = y^{-1} \hat{p} q_y (d\log y - y^{-1} pq d\log p) + y^{-1} \hat{p} S \hat{p} d\log P $$

$$ W d\log q = b(d\log Y - W' d\log P) + C d\log P $$

where the elements of $W$ are the average budget shares, those of $b$ the marginal propensities to consume and $C$ is defined in (3.3)

Elasticities:

$$ e_i = \frac{b_i}{W_i} ; \quad E_{ii} = -b_i + \frac{c_{ii}}{W_i} ; \quad E_{ij} = -b_j \frac{W_j}{W_i} + \frac{c_{ij}}{W_i} $$
The principal advantages of the Rotterdam model are that the theoretical constraints can be expressed as linear combinations of the elements of b and C and that they can be applied to the model one at a time. Thus, it is the only system in use at the moment which enables one to test the applicability of the constraints to a particular body of data.

One may substitute equations (2.1) to (2.5) in (3.4) and show that (using i to stand for a column vector of units of appropriate length):

(i) Engel aggregation requires that \( i'b = 1 \) (that is, the sum of the elements of the b vector should be unity).

(ii) Cournot aggregation requires \( i'C = 0 \) (the sum of each of the columns of the C matrix should be zero).

(iii) Homogeneity imposes the condition that \( Ci = 0 \) (the sum of each row of the C matrix should also be zero).

(iv) Symmetry implies \( C = C' \) (the C matrix should be symmetric).

Finally, if the condition of negativity is to be satisfied, then the C matrix should be negative semi-definite.

The usual procedure is to begin by estimating the system with both forms of aggregation imposed, since these are logically necessary. Then homogeneity is imposed, and the probability of its being a realistic restriction is assessed. If it is found acceptable, symmetry is imposed and tested in a similar way. A practical method of imposing negativity is not yet known, since the restrictions are in the form of inequalities, and require programming methods.

The Rotterdam system thus has the advantage of working from the general form of the demand function rather than imposing a particular formulation. It also has the very considerable merit of enabling one to impose and test virtually all the theoretical restrictions in a reasonably simple manner. These are very substantial advantages. Unfortunately, there is a theoretical fault. The model is expressed in terms of differentials, and these must be such that they satisfy Young’s theorem of the equality of cross-partial derivatives, specifically

\[
\frac{\partial Q_p}{\partial y} = \frac{\partial q_y}{\partial p}
\]

However, if these are satisfied, it can then be shown that the model is based on the Bergson demand functions which imply constant budget shares. Such functions are, of course, a very bad basis for a demand system.

Various arguments have been advanced to defend the Rotterdam system against this flaw. In essence, they amount to the statement that all demand systems are an approximation to the truth and the differentials used in the Rotterdam system may provide a good approximation even if the basic equations are inappropriate. Support for the system is increased by the fact that, to quote Brown & Deaton (1) “it has been applied over a wide range of countries and time periods, producing results which are plausible, conform to other evidence, and show in some respects considerable uniformity between studies”. In a comparative study of demand systems Parks (6) found that the Rotterdam model fitted a long Swedish series better than four other models. It seems reasonable to conclude that it represents a model which is, at least, worthy of examination.

(iii) THE INDIRECT ADDILOG SYSTEM

The term ‘indirect’ refers to the fact that in this system utility is written as a function of income and prices. Since the quantities of the various goods consumed are functions of income and prices, utility is thus an indirect function of these variables. The form which has received most attention and which is used here is
\[ U = \sum_{i} a_i Y^{b_i} \]

It is largely due to the work of Houthakker (7) and may be shown to lead to the following demand functions

\[ q_i = q_i Y^{b_i} \left( b_i + 1 \right) / \sum_{j} a_j Y^{b_j} \]

The elasticities may be calculated as

\[ \epsilon_i = (1 + b_i) - \sum_j b_j w_j \]

The model is very flexible in the sense that it imposes few restrictions on the values of the income and price elasticities. The only theoretical restriction on the \( b_i \) is that they must be greater than -1, and so income elasticities less than zero are possible as well as values in the range 0 to +1, corresponding to inferior, normal and luxury goods. Similarly, price elasticities may be less than or greater than -1. Finally, the cross elasticities may be positive or negative and this allows goods to be complements or substitutes. Unfortunately, this wealth of possibilities depends on very few parameters. The equations (3.7) show that all the elasticities are determined by the \( b_i \) only, and the form of the equations is such that these are likely to be determined by income behaviour. Thus as in the linear expenditure system, the price responses measured may really be merely the income aspect of price changes. This of course, is no drawback if the real world corresponds to the system.

(iv) THE DOUBLE-LOG SYSTEM

This consists of a series of eight regressions in each of which the dependent variables is the logarithm of ‘quantity’ (actually constant price expenditure) of one commodity group and the independent variables are the logarithms of total money expenditure, the implied consumption price index (found by dividing total money expenditure by total constant price expenditure) and the actual price of the commodity group. Since the variables are in logarithmic form, the coefficients are, of course, elasticities. Because the original variables produced results with a high degree of autocorrelation, their first differences were used instead, the equations being estimated without a constant.

No attempt was made to impose any constraints on the coefficients. The system has nothing to recommend it from the theoretical point of view. Indeed, as has often been pointed out, the assumption of constant income elasticities is not realistic. Yet it is often used, frequently with good results. It is included here to make it possible to see if more elaborate models are really preferable to a simple approach.
THE DIRECT ADDILOG SYSTEM

This system was also estimated. However, the results were so obviously unacceptable on both a theoretical and statistical basis, that no attempt is made to present them.

SECTION 4 : ESTIMATING METHODS

THE LINEAR EXPENDITURE SYSTEM

This was estimated using the LINEX programme, developed by Carlevaro and Rossier (8). The programme provides maximum likelihood estimators, using an iterative method of calculation.

Because of the budget constraint, the errors in any observation must sum to zero, so that (if there are N expenditure categories) there are only N-1 independent disturbances. Therefore, the likelihood function must be condensed by dropping one of the equations of the system; the choice of the equation to be dropped makes no difference to the final result. The programme then starts with an initial set of values for the parameters, say $d_0$, and calculates $M_Q$ (the matrix of second derivatives with regard to the parameters) and $G_Q$ the vector of gradients of the likelihood function with regard to the parameters. The succeeding values of the parameters are then calculated according to

$$d_{i+1} = d_i + 1_i \left( -M_i \right)^{-1} G_i \tag{4.1}$$

Where $1_i$ is a scalar chosen to maximise the likelihood function in the gradient direction.

It is known that procedures of this type may fail to converge if the initial value of the parameters $d_0$ are far from the maximum likelihood values. To overcome this, the programme uses least squares to calculate approximate values for $d$, which are then used as $d_0$. A typical run uses 12 least squares, and 10 maximum likelihood iterations. In most cases, the least squares results are found to be quite close to the final maximum likelihood output.

In the estimation procedure, the $b_i$ are constrained to be positive and to sum to unity. No constraints are imposed on either the value or the sign of the $c_i$.

THE ROTTERDAM SYSTEM

The problem of singularity arises here just as it does in the linear expenditure system. Barten (9) has shown how this may be overcome by writing the likelihood function in the form

$$L = \frac{1}{2} T \ln N - \frac{1}{2} T (N - 1) (1 + \ln 2\pi) - \frac{1}{2} T \ln A \tag{4.2}$$

where $N$ is the number of commodity-groups, $T$ is the number of observations and $A$ is defined by

$$A = \frac{1}{T} \sum_t V_t V_t^T + i_i$$

where $V_t$ is the column vector of disturbances in observation $t$ and $i$ is a column vector of units of the same length. If we define $V$ as the matrix of disturbances over the entire set of observations, $Y$ as the matrix of dependent variables $W_i \ d \log q_i$, $X$ as the matrix of independent variables

$$(\Sigma_i Y_i \ d \log P_1, \ d \log P_2, \ldots, \ldots) \quad (\text{all with } T \text{ rows})$$

and $D$ as the matrix of coefficients $(b,C)$ then we may write
\[ V = Y - XD' \]  
(4.3)

Substituting this, and the definition of \( A \) into (4.2), differentiating with regard to \( D \) and solving, gives

\[ D' = (X'X)^{-1} X'Y \]  
(4.4)

the ordinary least squares estimators. These estimates may be shown to satisfy both Engel and Cournot aggregation.

The imposition of homogeneity requires that the rows of \( C \) should sum to zero. This simply amounts to the imposition of a single linear restriction on each of the regressions in (4.4). The method of doing this is well known, and may be shown to lead to

\[ D' = (X'X)^{-1} - k(X'X)^{-1} RR' (X'X)^{-1} X'Y \]  
(4.5)

where \( R \) is the vector of the form necessary to impose the restriction namely, \((0,1,1,...)\) and \( k = 1 - (R'X^2R) \)

Symmetry may be imposed by forming the matrix of parameters into a vector, and imposing a further set of linear restrictions on them. However, the matter is not relevant here, since tests showed that homogeneity was barely acceptable at the 95 per cent level of probability and symmetry was rejected with a very high probability.

No programme was available to carry out the estimation. However, the procedures are relatively simple and do not require iteration, so a programme was written to carry out the necessary transformations of the variables, estimate the parameters, and calculate the various test-statistics. The result given below refer to the model on which homogeneity (but not symmetry) has been imposed.

THE INDIRECT ADDILOG SYSTEM

The system was estimated, using the GAD programme developed by Carlevaro and Sadoulet (10). This is a very valuable programme, as it is capable of estimating not only the Indirect Addilog, but the Direct Addilog, and General Addilog systems too. The programme is a rather lengthy one, and a full description of it is given in (10). The basic procedure only is described here.

Define

\[ d_t = \text{the vector of expenditure in time } t. \]
\[ x_t = \text{the vector of independent variables, namely total expenditure and prices.} \]
\[ u_t = \text{the vector of disturbances.} \]
\[ b = \text{the vector of coefficients to be estimated.} \]

Let

\[ d_t = f(x_t, b) + u_t \]

\[ F(b|d,x) = -\frac{1}{2} (d - f)' (d - f) \]

\[ H = I_T \sum (d_t - f(x_t, b)) (d_t - f(x_t, b))' \]

\( \Theta \) is Kronecker product

72
Since the problem of singularity arises here too, the system is first concentrated by dropping the $n^{th}$ equation. Least squares methods are then used to obtain initial estimates of $b$. The process then continues in an iterative manner, the typical step being described by

\[
\begin{align*}
    b^{k+1} & = b^k + z^{k+1} \left( \frac{\delta f}{\delta b} \right)^T H^{-1} \left( \frac{\delta f}{\delta b} \right) \\
    & = b^k + z^{k+1} \Delta b^{k+1}
\end{align*}
\]

The two matrices in square brackets are each calculated from the $b^k$, and $z^{k+1}$ is a constant calculated to maximise

\[F(b^k + z^{k+1})\]

The process is stopped when the $b_i$ converge to some predetermined limit. In the present case, 50 iterations were required.

The equation system does not determine the $a_i$ fully, but only up to a scale factor. They are therefore normalised by setting

\[a_i = 1\]

The constraint

\[a_i > 0\]

is also imposed.

SECTION 5 : DATA

The data are taken from the Central Statistics Office’s “National Income and Expenditure” booklets (11). Two sets of figures are shown for each year (Tables A.10 + B.10, and A.11 + B.11 in the 1972 booklet) giving expenditure on various commodity groups at current and constant prices. The latter has the smaller number of groups; nine are given, namely

- Food and non-alcoholic beverages
- Alcoholic beverages and tobacco
- Clothing, footwear and personal equipment
- Fuel and power
- Durable household goods
- Transport equipment
- Other goods
- Other expenditure
- Expenditure by non-residents

Since the two tables provide a measure of total expenditure and quantity of each commodity group, a price index for any group can be calculated by division.

The following are the commodity groups used in the calculations:

1. Food and non-alcoholic beverages
2. Alcoholic beverages
3. Clothing, footwear and personal equipment
4. Fuel and power
5. Durable household goods
6. Transport equipment
7. Other goods, including tobacco
8. Other expenditure.

The main change is the transfer of tobacco from group 2 to 7. It was not felt that alcohol and tobacco necessarily formed a homogeneous group, so an index of tobacco price was obtained by enquiry from the CSO and this was applied to the current-price series for tobacco to obtain constant-price figures. The latter were then subtracted from the 'Alcoholic beverages and tobacco' group to give the 'Alcoholic beverages' figure. Initially, tobacco was included as a separate commodity-group. However, the quantity consumed per head (the dependent variable) showed very little variation throughout the period and the results were generally statistically insignificant. Further, the inclusion of a series which was virtually a vector of constants tended to interfere with the results in the other commodity-groups. Therefore, it was decided to include tobacco in the 'Other goods' category, and thus restore the number of groups to eight.

'Expenditure by non-residents' is shown as a separate category in the CSO tables. However, it is impossible to divide it into commodity-groups so this item is ignored. The data for each commodity group thus includes expenditure by residents and non-residents. Experiments were made using a series:

'Other home expenditure' = 'Other expenditure' less 'Expenditure by non-residents'.

This was justified by the belief that expenditure by non-residents would be largely devoted to services which are the main item in the 'Other expenditure' category. However, this change made no important difference to any of the results.

Little adjustment of the data were necessary. The constant-price series for 1953-1964 had to be reworked to base 1968=100, instead of 1958=100. There were some slight discrepancies between the figures taken from earlier sources, and those in the 1971 tables for the years 1958-1960. These discrepancies appeared in the years where the two series overlapped. When this happened, the following procedure was adopted:

1. The price index was calculated from the latest figures available for constant and current-price expenditure, and then recalculated to base 1968=100.
2. The constant-price percentage variations in the earlier figures were calculated, and these variations were applied to the 1958 constant price figure, giving a constant-price series back to 1953.
3. The constant-price figures were multiplied by the price index calculated in step 1 above, to give current price figures for 1953-1957.

In all cases, the quantity series are divided by population to give quantity per head. The calculations use all the data available, namely - the observations from 1953-1972.

SECTION 6 : THE RESULTS

(i) INCOME ELASTICITIES

The main results are presented in Table 1. Since the estimated coefficients are virtually impossible to compare, the table shows the elasticities which they imply. In all cases except that of the regression, the elasticities are not constant, but are dependent on price and quantity. The values

74
TABLE 1
ELASTICITIES 1972

<table>
<thead>
<tr>
<th>Expenditure</th>
<th>Food</th>
<th>Alcohol</th>
<th>Clothing</th>
<th>Fuel &amp; Power</th>
<th>Household Durables</th>
<th>Transport Equipment</th>
<th>Other Goods</th>
<th>Other Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Expenditure</td>
<td>0.5712</td>
<td>1.3089</td>
<td>1.2673</td>
<td>0.9822</td>
<td>1.5773</td>
<td>1.9073</td>
<td>0.8784</td>
<td>1.0052</td>
</tr>
<tr>
<td>Rotterdam</td>
<td>0.5805</td>
<td>1.1535</td>
<td>1.7456</td>
<td>1.6049</td>
<td>1.6673</td>
<td>2.1234</td>
<td>0.9244</td>
<td>0.6963</td>
</tr>
<tr>
<td>Indirect Addilog</td>
<td>0.5052</td>
<td>1.6417</td>
<td>1.1690</td>
<td>0.9348</td>
<td>1.6760</td>
<td>2.5483</td>
<td>0.7113</td>
<td>0.9608</td>
</tr>
<tr>
<td>Regression</td>
<td>0.5539</td>
<td>1.2486</td>
<td>1.5567</td>
<td>1.3642</td>
<td>2.0175</td>
<td>4.4400</td>
<td>0.8526</td>
<td>0.5153</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Own Price</th>
<th>Food</th>
<th>Alcohol</th>
<th>Clothing</th>
<th>Fuel &amp; Power</th>
<th>Household Durables</th>
<th>Transport Equipment</th>
<th>Other Goods</th>
<th>Other Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Expenditure</td>
<td>-0.3880</td>
<td>-0.7464</td>
<td>-0.6855</td>
<td>-0.5868</td>
<td>-0.8342</td>
<td>-0.9827</td>
<td>-0.5095</td>
<td>-0.6156</td>
</tr>
<tr>
<td>Rotterdam</td>
<td>-0.4272</td>
<td>-0.4840</td>
<td>-1.0148</td>
<td>+0.1105</td>
<td>-0.4757</td>
<td>-1.5890</td>
<td>-0.7585</td>
<td>+0.4256</td>
</tr>
<tr>
<td>Indirect Addilog</td>
<td>-0.1450</td>
<td>-0.9716</td>
<td>-0.5471</td>
<td>-0.2920</td>
<td>-1.0025</td>
<td>-1.8404</td>
<td>-0.1584</td>
<td>-0.4753</td>
</tr>
<tr>
<td>Regression</td>
<td>-0.0721</td>
<td>-0.0678</td>
<td>-0.3105</td>
<td>+0.0157</td>
<td>-0.3995</td>
<td>-1.5057</td>
<td>-0.4343</td>
<td>-0.3157</td>
</tr>
</tbody>
</table>
shown in the table are those calculated at the end-point of the series, namely 1972. This is done because the author feels that the final values are of more practical significance than the more conventional mean values. No confidence intervals are given because the GAD programme does not calculate them for the indirect addilog model so a uniform comparison would not be possible.

The upper part of the table shows income elasticities. In general there is a good deal of uniformity in these, not only in the pattern between the various commodity groups, but even in the actual numerical results. This uniformity is a little surprising when one takes into account the differences in the models used. In particular, the linear expenditure system and the indirect addilog models work from specific utility functions which are very different, yet the results from these two are quite close.

It would appear reasonable to believe that the set of values estimated by the various systems for any elasticity provides an estimate of the range in which the true value might lie. To facilitate comparison, the ranges are presented in Table 2.

<table>
<thead>
<tr>
<th>Commodity Group</th>
<th>Range for Elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.51 to 0.58</td>
</tr>
<tr>
<td>Alcohol</td>
<td>1.15 to 1.64</td>
</tr>
<tr>
<td>Clothing</td>
<td>1.17 to 1.75</td>
</tr>
<tr>
<td>Fuel and Power</td>
<td>0.93 to 1.60</td>
</tr>
<tr>
<td>Household Durables</td>
<td>1.57 to 2.02</td>
</tr>
<tr>
<td>Transport Equipment</td>
<td>1.91 to 4.44</td>
</tr>
<tr>
<td>Other Goods</td>
<td>0.71 to 0.92</td>
</tr>
<tr>
<td>Other Expenditure</td>
<td>0.52 to 1.01</td>
</tr>
</tbody>
</table>

Using the customary definition of inferior, normal and luxury goods (income elasticity less than zero, between zero and unity and greater than unity respectively) one would appear to be justified in classifying the various commodity groups as follows:

(i) No inferior commodity groups
(ii) Income elasticity of food very close to 0.55
(iii) “High normal” range: other goods, other expenditure
(iv) “Low luxury” range: Alcohol, Clothing, Fuel.
(v) “High luxury” range: Household Durables, Transport Equipment.

This classification corresponds well with common-sense and other evidence. A possible exception is Other Expenditure; since this is composed very largely of expenditure on services, one might have expected to find it in the “high-luxury” range.

Table 3 compares the results from the time series analysis with elasticities for similar categories calculated by Pratschke from the 1966 Household Budget Inquiry (12). The time-series figures have been recalculated to show the elasticities at the 1966 levels of expenditure and prices.
No category in the cross-section data corresponds, even approximately to the "Other Goods" category used in this paper, so this group is ignored.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.49 to 0.55</td>
<td>0.51</td>
</tr>
<tr>
<td>Alcohol</td>
<td>1.25 to 2.07</td>
<td>1.79</td>
</tr>
<tr>
<td>Clothing</td>
<td>1.16 to 1.88</td>
<td>1.14</td>
</tr>
<tr>
<td>Fuel &amp; Power</td>
<td>0.91 to 1.77</td>
<td>0.32</td>
</tr>
<tr>
<td>Household Durables</td>
<td>1.65 to 2.02</td>
<td>1.20</td>
</tr>
<tr>
<td>Transport Equipment</td>
<td>2.44 to 4.44</td>
<td>3.74 (Motor Vehicles)</td>
</tr>
<tr>
<td>Other Goods</td>
<td>0.85 to 0.92</td>
<td>--</td>
</tr>
<tr>
<td>Other Expenditure</td>
<td>0.52 to 0.98</td>
<td>1.52</td>
</tr>
</tbody>
</table>

Of the cross-section figures, only the elasticities for food, alcohol and transport equipment fall within the time series range. There does not seem to be a pattern in the differences shown by the others. It seems reasonable to conclude that the cross-section elasticities provide only a poor guide to the figures which are appropriate to a situation where income and prices change over time.

(ii) PRICE ELASTICITIES

The estimates of the price elasticities are presented in the bottom half of Table 1. The outstanding fact is that there is very little agreement between the various estimates and the ranges involved are too great to be of any practical value. It is true that in the case of food, the elasticity seems to be low, certainly less (in absolute value) than 0.5. One might also gather with a reasonable amount of confidence that the elasticity for transport equipment is substantially above unity, and probably about 1.5. It is true that in this case, the linear expenditure estimate is slightly below unity. However, as inspection of the elasticities derived from (3.2) will show, since the $b_i$ are constrained to be positive, the system can only generate a price elasticity greater (in absolute value) than unity if the c-value is negative. However, the c-value is determined mainly by income behaviour, so it is quite possible for a high price elasticity to be smothered by the much stronger income information.

The only other 'hard' fact which seems to emerge is that the price elasticities tend on the whole to be less than unity. This, at least, is consistent with most empirical evidence, for example, Parks and Bartens recent cross-country estimate (13). Apart from this, while the ranges shown in Table 1 may be of some practical value, they do not give us any confidence that we are estimating the true price elasticities. If there were some theoretical basis for selecting one system as being the most reliable, it would be possible to make some advance, but as Parks (6) has pointed out, no such basis is available. The situation seems to be, that if we use the demand systems which are currently fashionable, we can obtain from each of them estimates of the various price
elasticities; but there are substantial differences between these estimates, and we have no way of knowing which is the most realistic.

(iii) OVERALL GOODNESS OF FIT

If it could be shown that one of the demand systems fits the data very much better than all the others that might provide grounds for regarding the elasticity estimates which it provides as the most dependable. For this reason, two measures of goodness of fit are shown in Table 4. The upper half of the table shows the residual variance ratio, calculated from each system for each of the commodity groups. This is defined as $\frac{\sum e^2_{it}}{\sum (q_{it} - \bar{q}_{it})^2}$, where $e_i$ is the vector of residuals. In the case of the Rotterdam and regression estimates, (both of which are originally calculated in first differences) the estimated first differences are used to form estimated quantities, and the goodness of fit is calculated from the latter. The residual variance ratio is used rather than the more usual coefficient of determination because in some cases (for example, the constrained Rotterdam) the mean of the residuals is not zero so the variance of the dependent variable is less than the sum of ‘explained’ and ‘residual’ variance. Thus, if $R^2 = \frac{‘explained’}{total\ variance}$ is used, it will give an over-estimate of the goodness of fit.

The results (Table 4) are inconclusive. It is obvious that the least squares regression is very much worse than all the others. The lowest (best) value in each category is marked with an asterisk. On this criterion, the linear expenditure system leads with four best, and the Rotterdam system comes next with three. However, the differences are not big enough to give a clear superiority to any one system.

The comparison of systems equation by equation is somewhat unsatisfactory, and it is desirable to find a means of comparing the entire systems. This is provided by the concept of average information inaccuracy proposed by Theil (5). The information inaccuracy for any observation is defined as

$$ I = \sum_i w_i \log \frac{w_i}{w_i^*} $$

where $w_i$ is the average budget share observed and $w_i^*$ is the average budget share calculated from the demand system under investigation. Budget shares are used because they sum to unity, and thus may be tested as probabilities. The values for $I$ may then be averaged over the observations. This method satisfies the postulates of information theory, and provides a statistic which combines the performance of all the commodity groups, each one being weighted according to its importance in the consumers’ budget. It can be shown (5) that the statistic is always non-negative and of course the smaller it is, the better the system fits the observed data. The results are given in the lower part of Table 4. The values are very small, but this is to be expected because it can be shown that if $e_{it}$ is defined as $(w_{it} - w_{it}^*)$, the value of $I$ is approximately equal to $\sum_i e_{it}^2$. Since the $w_i$ and $w_i^*$ are all between 0 and 1 and the differences are quite small, $e_{it}^2$ is likely to be very small indeed (5).

The values serve to confirm the impression given by the upper part of the Table. The regression is by far the worst. The others are of the same order of magnitude, and while the Rotterdam system is the best, the difference is not sufficiently great to give us confidence in the clear superiority of the Rotterdam estimates over the others. Since tests of significance for the $I$ values are not known, this conclusion must be tentative. It is interesting to note that the Rotterdam system was also found to be superior in a study done by Parks (6), using a long series of Swedish data.
<table>
<thead>
<tr>
<th></th>
<th>Food</th>
<th>Alcohol</th>
<th>Clothing</th>
<th>Fuel</th>
<th>Household Durables</th>
<th>Transport Equipment</th>
<th>Other Goods</th>
<th>Other Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotterdam</td>
<td>.040</td>
<td>.026*</td>
<td>.043</td>
<td>.068*</td>
<td>.0092</td>
<td>.034</td>
<td>.036</td>
<td>.002*</td>
</tr>
<tr>
<td>Linear Expenditure</td>
<td>.023</td>
<td>.070</td>
<td>.027*</td>
<td>.078</td>
<td>.0085*</td>
<td>.028*</td>
<td>.026*</td>
<td>.033</td>
</tr>
<tr>
<td>Indirect Addilog</td>
<td>.014*</td>
<td>.031</td>
<td>.033</td>
<td>.080</td>
<td>.011</td>
<td>.030</td>
<td>.035</td>
<td>.034</td>
</tr>
<tr>
<td>Regression</td>
<td>.017</td>
<td>.090</td>
<td>.059</td>
<td>.492</td>
<td>.019</td>
<td>.538</td>
<td>.036</td>
<td>.765</td>
</tr>
</tbody>
</table>

**AVERAGE INFORMATION INACCURACY COEFFICIENTS**

<table>
<thead>
<tr>
<th></th>
<th>Rotterdam</th>
<th>Linear Expenditure</th>
<th>Indirect Addilog</th>
<th>Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.000456</td>
<td>.000708</td>
<td>.00594</td>
<td>.0218</td>
</tr>
<tr>
<td>Year</td>
<td>Food</td>
<td>Alcohol</td>
<td>Clothing</td>
<td>Fuel</td>
</tr>
<tr>
<td>------</td>
<td>------</td>
<td>---------</td>
<td>----------</td>
<td>------</td>
</tr>
<tr>
<td>1953</td>
<td>49.0</td>
<td>10.7</td>
<td>16.4</td>
<td>6.9</td>
</tr>
<tr>
<td>1954</td>
<td>50.2</td>
<td>11.1</td>
<td>16.0</td>
<td>7.1</td>
</tr>
<tr>
<td>1955</td>
<td>54.3</td>
<td>12.0</td>
<td>16.6</td>
<td>8.6</td>
</tr>
<tr>
<td>1956</td>
<td>54.0</td>
<td>12.5</td>
<td>17.1</td>
<td>8.7</td>
</tr>
<tr>
<td>1957</td>
<td>56.2</td>
<td>12.6</td>
<td>16.0</td>
<td>8.8</td>
</tr>
<tr>
<td>1958</td>
<td>60.5</td>
<td>12.8</td>
<td>16.6</td>
<td>8.2</td>
</tr>
<tr>
<td>1959</td>
<td>61.8</td>
<td>13.3</td>
<td>16.5</td>
<td>8.3</td>
</tr>
<tr>
<td>1960</td>
<td>64.0</td>
<td>14.4</td>
<td>18.4</td>
<td>8.5</td>
</tr>
<tr>
<td>1961</td>
<td>65.9</td>
<td>15.7</td>
<td>20.2</td>
<td>9.2</td>
</tr>
<tr>
<td>1962</td>
<td>68.8</td>
<td>18.1</td>
<td>20.8</td>
<td>10.0</td>
</tr>
<tr>
<td>1963</td>
<td>70.5</td>
<td>19.4</td>
<td>22.2</td>
<td>10.2</td>
</tr>
<tr>
<td>1964</td>
<td>78.2</td>
<td>22.0</td>
<td>24.5</td>
<td>10.9</td>
</tr>
<tr>
<td>1965</td>
<td>82.4</td>
<td>23.9</td>
<td>26.6</td>
<td>11.1</td>
</tr>
<tr>
<td>1966</td>
<td>84.5</td>
<td>25.4</td>
<td>26.0</td>
<td>11.5</td>
</tr>
<tr>
<td>1967</td>
<td>86.3</td>
<td>27.7</td>
<td>29.9</td>
<td>12.1</td>
</tr>
<tr>
<td>1968</td>
<td>96.7</td>
<td>31.5</td>
<td>33.7</td>
<td>13.3</td>
</tr>
<tr>
<td>1969</td>
<td>103.3</td>
<td>37.4</td>
<td>38.6</td>
<td>14.8</td>
</tr>
<tr>
<td>1970</td>
<td>112.0</td>
<td>42.5</td>
<td>42.2</td>
<td>17.1</td>
</tr>
<tr>
<td>1971</td>
<td>120.3</td>
<td>48.4</td>
<td>46.7</td>
<td>20.0</td>
</tr>
<tr>
<td>1972</td>
<td>133.8</td>
<td>54.4</td>
<td>51.7</td>
<td>23.4</td>
</tr>
</tbody>
</table>
## Table 6

### Price Indices

<table>
<thead>
<tr>
<th>Year</th>
<th>Food</th>
<th>Alcohol</th>
<th>Clothing</th>
<th>Fuel</th>
<th>Durables</th>
<th>Transport Equipment</th>
<th>Other Goods</th>
<th>Other Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953</td>
<td>.656</td>
<td>.517</td>
<td>.785</td>
<td>.771</td>
<td>.747</td>
<td>.724</td>
<td>.505</td>
<td>.571</td>
</tr>
<tr>
<td>1954</td>
<td>.656</td>
<td>.515</td>
<td>.785</td>
<td>.770</td>
<td>.742</td>
<td>.706</td>
<td>.510</td>
<td>.576</td>
</tr>
<tr>
<td>1955</td>
<td>.684</td>
<td>.518</td>
<td>.788</td>
<td>.793</td>
<td>.740</td>
<td>.722</td>
<td>.519</td>
<td>.590</td>
</tr>
<tr>
<td>1956</td>
<td>.685</td>
<td>.539</td>
<td>.801</td>
<td>.878</td>
<td>.776</td>
<td>.753</td>
<td>.583</td>
<td>.619</td>
</tr>
<tr>
<td>1957</td>
<td>.708</td>
<td>.563</td>
<td>.807</td>
<td>.934</td>
<td>.798</td>
<td>.787</td>
<td>.621</td>
<td>.652</td>
</tr>
<tr>
<td>1958</td>
<td>.770</td>
<td>.588</td>
<td>.812</td>
<td>.921</td>
<td>.814</td>
<td>.786</td>
<td>.644</td>
<td>.677</td>
</tr>
<tr>
<td>1959</td>
<td>.763</td>
<td>.612</td>
<td>.817</td>
<td>.894</td>
<td>.823</td>
<td>.795</td>
<td>.647</td>
<td>.685</td>
</tr>
<tr>
<td>1960</td>
<td>.759</td>
<td>.625</td>
<td>.831</td>
<td>.870</td>
<td>.826</td>
<td>.801</td>
<td>.661</td>
<td>.702</td>
</tr>
<tr>
<td>1962</td>
<td>.795</td>
<td>.712</td>
<td>.862</td>
<td>.907</td>
<td>.854</td>
<td>.815</td>
<td>.722</td>
<td>.750</td>
</tr>
<tr>
<td>1963</td>
<td>.805</td>
<td>.741</td>
<td>.882</td>
<td>.933</td>
<td>.863</td>
<td>.833</td>
<td>.745</td>
<td>.777</td>
</tr>
<tr>
<td>1964</td>
<td>.862</td>
<td>.818</td>
<td>.932</td>
<td>.954</td>
<td>.899</td>
<td>.849</td>
<td>.807</td>
<td>.828</td>
</tr>
<tr>
<td>1965</td>
<td>.912</td>
<td>.847</td>
<td>.955</td>
<td>.947</td>
<td>.922</td>
<td>.872</td>
<td>.855</td>
<td>.867</td>
</tr>
<tr>
<td>1966</td>
<td>.920</td>
<td>.912</td>
<td>.983</td>
<td>.957</td>
<td>.940</td>
<td>.907</td>
<td>.927</td>
<td>.913</td>
</tr>
<tr>
<td>1967</td>
<td>.936</td>
<td>.963</td>
<td>.983</td>
<td>.986</td>
<td>.976</td>
<td>.970</td>
<td>.953</td>
<td>.944</td>
</tr>
<tr>
<td>1968</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>1969</td>
<td>1.059</td>
<td>1.152</td>
<td>1.034</td>
<td>1.036</td>
<td>1.070</td>
<td>1.117</td>
<td>1.092</td>
<td>1.079</td>
</tr>
<tr>
<td>1970</td>
<td>1.138</td>
<td>1.237</td>
<td>1.122</td>
<td>1.166</td>
<td>1.151</td>
<td>1.244</td>
<td>1.185</td>
<td>1.172</td>
</tr>
<tr>
<td>1972</td>
<td>1.341</td>
<td>1.402</td>
<td>1.348</td>
<td>1.400</td>
<td>1.323</td>
<td>1.525</td>
<td>1.325</td>
<td>1.409</td>
</tr>
</tbody>
</table>
Irish consumer expenditure is divided into eight commodity groups, using annual observations for the period 1953 - 1972.

The data are used in combination with the following four demand models to obtain estimates of income and own-price elasticities of demand:

1. The linear expenditure system.
2. The Rotterdam system (with homogeneity imposed).
3. The indirect addilog system.
4. A double-log single equation system (in first differences).

The estimated income elasticities show a reasonable degree of similarity for all commodity-groups, and are probably sufficiently reliable to be useful for many policy purposes. The central values are: Food, 0.55; Alcohol, 1.3; Clothing, 1.5; Fuel and Power, 1.2; Household Durables, 1.7; Transport Equipment, 2.5; Other Goods, 0.85; Other Expenditure, 0.7. However, these estimates are substantially different from published cross-section elasticities.

In the case of the own-price elasticities, only two commodity groups show even approximate similarity. These are Food (between -0.1 and -0.4) and Transport Equipment (about -1.5). It is not possible to find a theoretical or empirical basis for accepting one set of estimates rather than another. The evidence suggests that overall, the Rotterdam model is best, but the margin of superiority is small. It would appear that reliable estimates of price elasticities cannot be obtained until more is known about the estimating characteristics of the demand models.

REFERENCES

DISCUSSION

Dr. J.L. Pratschke   It gives me very great pleasure indeed to propose a vote of thanks to Dr. O'Riordan for his important and stimulating paper. I have a great respect for Dr. O'Riordan's mastery of this area, to which he continues to make a significant contribution. I feel sure that we all, in digesting this latest instalment, look forward keenly to the next.

It seems to me not surprising that O'Riordan finds it difficult to choose between the alternative approaches he adopted. Even in single equation regression approaches, the problem of choosing between alternative function forms remains a difficult one. The various adjusted coefficients of determination suggested by Leser, Mahajan and myself, amongst others, do not seem adequate; nor indeed have the tests for uniformity of fit - the cross sectional parallel to serial correlation of errors - being particularly helpful in discriminating between alternatives. In the single-equation case, it seems probable that the approach of Box and Cox, is the most hopeful, but even here one must be careful in using standard statistical procedures for evaluation of significance tests.

Because of these problems in the relatively simple case of single equation estimation, Theil's information theory approach does not seem particularly helpful, even in principle; its similarity to Leser's adjusted $R^2$ forces one to be rather sceptical of its power to discriminate between alternatives.

Given that, I find it difficult to suggest any basis on which one might select from the four methods presented by O'Riordan. I am less willing than the author to express - more or less - confidence in the elasticity estimates he presents than those presented by other researchers in the past. Not only is it difficult to match O'Riordan's commodity groups with those from Household Budget Inquiry data, of the type that Leser and I used, but it is also difficult to match those with the studies of, for example, Casey, Hart or O'Riordan's earlier studies. Casey's paper came nearest to presenting an Irish comparison of LES estimates with Engel functions for a limited range of good commodities and certainly discovered notable differences. I would prefer, however, to wait a little longer, before deciding which approach will in time prove the more flexible in providing useful forecasts. My personal suspicion - or perhaps it is a prejudice - is that even when the $C$ vector (of equation (3.2)) is treated as a function rather than a constant, the LES approach is still not as flexible as the much simpler, but in my view more fertile Engel function approach as far as income elasticities are concerned. The attractions of the large body of data on which Engel formulations could be based are very great for those who would wish to build in demographic, social structure or other less commodity-oriented variables into their demand structure.

Without any doubt, however, Dr. O'Riordan is to be congratulated not only on a most useful research project well done, but also for the concise way in which he has sketched the essential attributes of the alternative systems reviewed. Not only other researchers, but also teachers and students of applied demand analysis in Ireland are deeply in his debt.
It is a most pleasing duty to propose a vote of thanks to him.

References

(1) Leser, C.E.V., “Demand Relationships for Ireland” \(\textit{The Economic and Social Research Institute, Paper No. 4, Dublin April 1962.}\)


Mr Noel O’Gorman: seconded the vote of thanks, expressed his appreciation of Dr O’Riordan’s contribution to research into the behaviour of Irish consumers and offered some remarks on the usefulness of this knowledge to the public sector.

One can identify at least four areas of public policy formulation where this knowledge would be a valuable asset, the best-known area being that of taxation. Information on the relevant price and income responses would assist policy-makers in determining both the degree and the appropriate mix of direct and indirect taxation adjustments which would ensure the harmony of revenue targets and demand management objectives. Dr O’Riordan has in recent years directed much attention to the implications for taxation policies of price elasticity. He has, indeed, suggested that certain taxes - in particular the duties on petrol and tobacco - may be approaching their limit as a revenue source. In view of this, I think some reference to the role of price elasticity is justified.

Revenue elasticities should not be confused with price elasticities. A positive revenue elasticity implies that, even in the absence of increase in consumption induced by rising incomes, an increase in the tax per unit will produce extra revenue from the levies on the commodity concerned; this could come about even with a fall in the total value of sales.

84
On the simplifying assumption that the tax-exclusive price is exogenous, it can be shown that the revenue elasticity \( r_x \) for a taxed commodity is \( 1 + \frac{1}{x} e_x \) (\( x \) being the tax per unit and \( e_x \) being its own-price elasticity - the latter is generally negative). For each and every commodity with an own-price elasticity greater than minus one, it is clear that \( r_x \) is necessarily positive i.e. an increase in the rate of tax produces additional revenue. Indeed, only for commodities whose retail price includes a large proportion of taxation, or the demand for which is extremely price-sensitive, is there a real possibility of a negative revenue elasticity. Thus, even for heavily-taxed commodities, this possibility is rather remote. For the most heavily-taxed commodity, tobacco, to have a negative revenue elasticity its own-price-elasticity would need to be of the order of -1\( \frac{1}{2} \); for petrol and alcoholic drink the relevant value would be in the region of -2 while for motor vehicles it would be between -4 and -5. Dr O'Riordan's price-elasticity estimates indicate that in no case is the tax-revenue elasticity negative. However, we cannot entirely rule out the possibility that a less aggregative approach might yield such results.

Since it appears that few, if any, commodities, can be ruled out of tax increases solely because of their revenue elasticity, attention must focus on the use of price and income responses in selecting the optimal taxation package in the light of general economic objectives and revenue requirement. If I might take a simple example, the Government may wish to provide a stimulus to aggregate demand by cutting direct taxation but, because of constraints on borrowing, revenue must also be increased. From the income elasticities or other measures of the income response, the distribution across the various consumption categories of the addition to demand could be estimated. The consequential increase in revenue from existing indirect taxes can then be deduced, so that the net reduction in aggregate tax revenue is known. By selectively increasing indirect taxes, concentrating on commodities with relatively low price-elasticities - and on those which are import-intensive - it should be possible to maintain total tax revenue with minimal dilution of the initial stimulus.

Of course, this exercise in fine-tuning would require elasticity estimates which were both extremely accurate and very extensive. Dr O'Riordan has however, shown us in his paper that the current state of economic theory and the quality of the available data preclude this. Were such estimates available, I am confident that they would be put to good use by the "tax-gathers".

A second area where price and income elasticities could be put to work by the public sector is in deciding the optimal allocation of current resources, resources which are as a rule scarce. In this context we must divide the services provided by the public sector into two broad classes. The first covers the social services, such as health and education, which are so heavily subsidised that the charges for them, where any exist, are only a fraction of the cost of providing them; this approximates to what the price would be in a free-market situation. The second covers those services - we may describe them as economic services - the charges for which are close to the economic price; among these are electricity supply, post office services and public transport. While price and income elasticities of demand must exist for the former class of services, there seems little possibility of estimating them. In any case it is likely that at current income levels the aggregate demand for them, given the full economic price, would be at a level that was socially (and economically) undesirable. However, in the case of the economic services, the quality of decision-making should benefit from a knowledge of the relevant price responses.

To illustrate this point, public transport is a useful example: prior knowledge of consumer response to fare adjustments would be of immense value. With this information to hand the optimal level of changes, on the strictly economic criterion of profit-maximisation, or possibly loss-minimisation, could then be determined. The merits of subsidisation below this level would largely depend on social considerations. It would of course be necessary to take account of the logical consequences for other areas of public authorities' expenditure such as on road-building and maintenance.
In so far as Dr O'Riordan's results are concerned, there is one which may be of particular relevance to the example I have taken. The demand for "transport equipment", which reflects the demand for private means of transport, appears to be particularly sensitive both to the income level and to its relative price. The effects of the energy crisis on economic growth and on the cost of private motoring may well prove to be the silver lining on public transport's cloud.

Turning briefly to the third area I have in mind, better information on time-series income elasticities should help to improve the quality of long-term projections of public capital requirements. The development of these economic services entails a substantial volume of investment by the public sector, broadly-defined. To the extent that the quality of long-term forecasts of the demand for these services could be improved, the magnitude of future capital requirements could be more accurately assessed in relation to the availability of resources.

Better information on consumers' response to price and income changes should also be of value in the formulation of policy measures to act on balance of payments problems, although elasticities of a different kind might be necessary for this purpose. There has been some discussion of late on the possible benefits of imposing import restrictions with a view to protecting domestic employment. I must leave aside, for the purpose of this exercise, the question of our obligations as a member of the European Community.

The value of a tariff imposition cannot be adequately assessed without a reliable estimate of the relevant price elasticities. If, for example, the demand for the imported good in question were totally insensitive to a change in its relative price, the effect would simply be to aggravate domestic price rises while leaving the import cost unaffected. As regards the benefits of a deflationary policy stance in an adverse payments situation, this would obviously depend on the sensitivity of imports with respect to the level of domestic demand, this being an income elasticity concept.

In concluding Mr O'Gorman thanked Dr O'Riordan for a most interesting and thorough paper and expressed his interest in the results of Dr O'Riordan's continuing research. Research into consumer behaviour fulfilled an important function if it helped to improve the quality of decision-making whether at national level or at the level of the individual firm.

Dr. R.C. Geary: This interesting paper is not for everyone and there is no reason it should be so, in this learned society. It will be fully understood only by those who are up-to-date in the elasticity industry, of whom the speaker is not one, nowadays.

The author's strategy is to show in the first part of the paper that, according to each of his four approaches, the structural form of the relations is determined. I found this part of the paper very useful, noting particularly that the Rotterdam System required no utility function. What would its utility function be? The second part consisted in using the maximum likelihood method to estimate the coefficients, given the functional forms of the four methods. It is interesting to know that the computer has programs for solving the N not necessarily linear equations in N parameters. This was the bugbear of the ML approach heretofore. Is this interpretation correct?

I confess to being completely baffled by Section 4, to the point of respectfully suggesting its omission; later I ask for extensions in other directions. If the computer can achieve these miracles of estimation - Praise be to God! There is no need to say how; barely mention continued approximation. I would ask the author anyway to look at formula (4.2). It has a normal theory look but why all those constants which vanish on setting up the standard equations for estimating the coefficients.

I do not understand the formula for the indvar matrix X, on page 72. Would the author please explain?
I was somewhat unhappy about the transition from theory to statistical practice. How, for instance, did the author statisticize $d \log q$, $d \log v$, $1 \log p$ in the Rotterdam System approach? Were the first difference approximations used? Might I suggest that in the definitive version of the paper far more of the statistical workings be given. Why should not the actual and computed values of the $q_i$ for each system be given, with graphs perhaps? It would make for better understanding, conviction and appreciation of a valuable paper.

I have now some encouraging words for the lecturer who is despondent about the inconsistent showing of his various estimates of own-price elasticities. If these are inconsistent, may not the fault be with the concept of elasticity, this single parameter which tries to describe a multifarious economic system, a sheer impossibility of course. Are estimates of elasticities in other countries, to which he refers in his paper, any better than his? Even if they were I would still be unconvinced.

The concept of demand elasticity in economics was a useful one for class-room exposition. It predated by long term modern econometrics. Multivariate analysis is designed to describe and to forecast the economic system. It is simply impossible to think that one can say "one per cent rise in income will lead to x per cent rise in quantity demand for commodity A" and the own price and cross-price elasticities are even more incredible. For one thing, even if they do not look too bad (e.g. that famous test "the sign is right"), the ranges (statistical confidence limits of single systems, or from several systems) are too large to be of any use. Even if the concept of elasticity has any validity, it must surely be in regard to very specific commodities (say milk) in very specific consuming groups (say households with income £3,000 - £4,000). What can one make elasticity-wise of a group like Food? It would be quite otherwise if, as the author should, set out his results as macro-economic relationships, not only in structural but in reduced form.

It is really in the lower part of Table 4 that the paper comes into its own. The contrast between the AII coefficients between the regression method (which in this context must be regarded as a naïve approach) and any of the other three is remarkable and encouraging. To repeat, why not supply actual-calculated graphs for each method and each commodity group (or whatever function - log, delta etc - was used)?

In appraising adequacy of representation conventional measures of goodness of fit (s-squared, R-squared or any the author uses) are not enough. He should also try the Durbin-Watson d or our own tau on disturbances. These, applied collectively, may indicate a clear preference for one of the three systems. If so we would be a long step on the way towards that philosopher's stone we all seek - how this part of the economic system works in Ireland.

Mr. C. McCarthy: It is a pleasure to echo the appreciative remarks of earlier speakers on Dr. O'Riordan's paper. I would like to raise just two points.

First of all, the difficulty Dr. O'Riordan had encountered in discriminating between the different models he has estimated and his general agnosticism, if I may call it that, about the prospects of getting good estimates of price responses are by no means unusual and many investigations on these lines conclude with similar observations. I feel this is principally a data problem however, and not a problem with either the model specifications or with estimation methods.

The specification of systems of demand equations is quite sophisticated and their estimation requires the data to yield up rather subtle information about behaviour. We would expect that a sample of high quality would be necessary in order to distinguish cross-price effects, for example.

In practice, most investigations utilise aggregate time series data and the deficiencies of this kind of data for the purpose at hand are considerable. The principal weaknesses appear to be:
(i) Sample sizes are usually quite small - perhaps no more than twenty annual observations.

(ii) The assumption that consumers' tastes are given must be violated in samples of this kind.

(iii) Annual data involve a high degree of time aggregation, a problem that is too seldom referred to.

(iv) There is a high degree of commodity group aggregation whose legitimacy is itself an empirical question that must be very much open.

(v) The aggregation over individuals involved may conceal important composition effects, for example demographic shifts, and does nothing to cater for shifts in the income distribution, which cross-section evidence suggests are potentially disruptive.

(vi) Changes in the quality of goods imply that hedonic price corrections are necessary. Uncorrected price indices must overstate the relative prices of certain durable goods.

This is by no means an exhaustive list but it does suggest that the sample Dr. O'Riordan has had to use is far from ideal for his purpose. Data collected at more regular intervals, say quarterly, and for a wider classification of commodities would meet some of these objections, but panel data are probably the only really satisfactory alternative.

My second point concerns the income elasticities. We know that for certain durable categories there must be limits to per capita ownership levels such that demand must ultimately decline to a pure replacement demand and the (per capita) income elasticity to zero. The incorporation of prior information of this kind might help in the estimation of income elasticities, although it may be objected that durables may not really belong in systems of this kind at all.

Both in the paper he has read this evening and in his earlier work, Dr O'Riordan has made a substantial contribution to our knowledge of consumers' behaviour in Ireland and it is a pleasure to congratulate him on his efforts.

Mr. Peter Sloane: I would like to begin by congratulating Dr. O'Riordan on a very fine paper. In commenting on the paper I will begin by referring to Dr. Pratschke's call for greater commodity disaggregation. There can be little doubt that it is highly desirable to work on as fine a division of goods as possible. However, it must be remembered that most complete systems of demand equations imply some kind of separability. Such assumptions may be fairly acceptable when using a broad commodity division as Dr. O'Riordan does but the same can hardly be said if commodity groups are highly disaggregated. In the case of the LES, for example, it would not be too implausible to argue that the marginal utility of 'transport equipment' is independent of the quantity of 'food and non-alcoholic beverages'. However, it could hardly be claimed that the marginal utility of (say) mutton is independent of the quantity of beef!

Assumptions of additivity also have important implications for the estimation of price elasticities. In the LES, additivity implies that the ratio of the income and own-price elasticities will be approximately constant whereas in the Indirect Addilog system the sum of the elasticities will tend to be constant. These constraints are different and yet they are both 'forced' on the data in the process of estimation. Given that changes in income are normally much larger than changes
in relative prices it is likely that the income elasticities will be estimated fairly well but the brunt of the constraints will fall on the estimates of the price elasticities. It is not surprising, therefore, that there will be significant differences in the price-elasticity estimates generated by the various models.

There is one further minor point I would like to make. It would be useful to examine the effect of imposing the homogeneity condition on the Double-Log estimates. It is clear that this method of estimation performs very poorly, but the relatively high value of the Information Inaccuracy coefficient may be largely attributable to the impossibility of imposing the adding-up constraints. It would be interesting to see if the imposition of homogeneity led to any improvement.