Analysing Ireland's Algebra Problem

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In recent years, there has been concern regarding the performance of Irish second level students in mathematics and in particular algebra. In response, the Irish Government have introduced 'Project Maths' which is a major reform of second level mathematics. It was introduced on a phased basis in 2008 and involves changes to what students learn, how they learn it and how it is assessed. One of the main changes is the promotion of a functions based approach to teaching algebra. This is much different from the transformational rule based approach which had dominated Irish classrooms. The new algebra strand and functions based approach was introduced in second level schools in September 2011. This paper aims to investigate the immediate effect, if any, of such an approach on student's transformational algebraic ability. This will be done by analysing the mathematics diagnostic scores of incoming third level students at an Irish University.

Keywords: Algebra, curriculum change, teaching approaches.

THE STUDY

This paper aims to investigate the immediate effect (if any) of the functions based approach to teaching algebra that has been implemented in Irish schools instead of the transformational (rule and procedure) based approach which was previously used. The paper will do this through analysing the results of a diagnostic test taken by incoming first year students at an Irish University. There are eight algebra based questions on the diagnostic test and all of them are transformational based in nature. Hence this paper will compare student's algebraic ability from a technical perspective in aspects such as manipulating terms and solving equations.

BACKGROUND TO THE STUDY

Irelands 'Algebra Problem'

Algebra has long been identified as an area of difficulty in the teaching and learning of mathematics. In the introductory note to his monumental *Arithmetica*, written ca. 250 AC, Diophantus of Alexandria mentions the discouragement that students usually feel when learning what we now term 'algebraic techniques' to solve word-problems. Fast forwarding to 1982, Cockcroft identified algebra as a source of considerable confusion and negative attitudes among students. This was followed by Herscovics and Linchevski (1994, p. 62) who reported that many students consider algebra an unpleasant, even alienating experience and find it difficult to understand."*How can we multiply by x when we don't know what x is?"* (12 year old student). Furthermore Artigue and Assude (2000) posit that many students see algebra as the area where mathematics abruptly becomes a non-understandable world.

Evidence of this confusion is common in Irish classrooms. Chief Examiners' Reports have identified algebra as an area of weakness over the past number of years. According to these reports, Irish student performance in algebra has shown little or no progress in the last fifteen years. In the 1999 Junior Certificate (JC) Higher Level paper, there were two questions based primarily on algebra, while other parts of questions also involved algebra. The long questions on algebra were both low scoring and unpopular choices. Question 3, yielding an average mark of 24.3 out of 50, was the lowest scoring on the paper thus reflecting the extent of candidates' difficulties with algebra. Furthermore, candidates often ignored parts of other questions which involved the topic. In the 2003 JC Higher Level paper, the Chief Examiner Report (2003) concluded that while there was some improvement in relation to algebraic skills, further improvement was still needed. Questions 3 and 4 relating to algebra demonstrated that the algebraic

skills of candidates need to be enhanced so that they can handle with ease topics such as manipulation of formulae, quadratic equations, solving inequalities and setting up equations. Again the lowest scoring question on the paper (Question 4, yielding an average mark of 29.1 out of 50), was based on algebra (Chief Examiners Report, 2003). This was also the case in the most recent Chief Examiners Report (2006). On this occasion Question 4 yielded an average mark of 27.1 out of 50. This report noted that improvements were required in areas such as simplifying and removing brackets from algebraic expressions, particularly expressions containing minuses and also simplifying algebraic fractions. Hence on the evidence of these reports it is clear that although algebra has long enjoyed a place of distinction in the mathematics curriculum, many students have difficulty in understanding and applying even its most basic concepts. *"Algebra means hours of instruction that you don't even come close to understanding"* (seventh-grade student as cited in House, 1988, p. 1).

Reasons for such poor performance in algebra

In Ireland, research has suggested that there has been an over reliance on traditional methods when teaching algebra. Transformational based activities have dominated lessons. Algebra was a paper and pencil activity involving the following of rules and procedures. A minimalist approach to algebraic sense making took place. Each day of instruction was textbook led and focused on a particular type of manipulation. For example, the textbook started by introducing the concept of a variable, followed by the notion of algebraic expressions and then equations (Kieran, 1992). This structure ensured that algebra was considered as a series of skills to be mastered (Chazan, 1996). Success in the subject was determined by the ability to memorise procedures by rote, nothing else (Bracey, 1992).

Research has found that many mathematics teachers felt there was no alternative to teaching mathematics through the traditional chalk and talk or the common method of following sections through a textbook (Lyons et al., 2003). Using such a method with algebra forced students to memorise procedures and solve artificial problems that had no meaning to their lives. They were drilled on the possession of mathematical rules and manipulations and they were graded not on their understanding of the mathematical concepts, but on producing the right symbol series. As a result what students learned was a collection of procedures and skills to be performed, having no logical coherence, very little connection with previously learned arithmetic, and no applications in other school subjects or in the outside world (MacGregor, 2004). Although such procedures and skills are important outcomes of learning algebra, what students need even more is a sound understanding of algebraic concepts and the ability to use knowledge in new and often unexpected ways. Students need to be given the opportunity to construct their own mathematical knowledge along with understanding its importance and usefulness in every day applications.

The 'Solution'

As a result of Irish students' poor performance in mathematics and indeed algebra, the Irish Government have introduced 'Project Maths' [1]. This is an ambitious reform of Irish second level education [2] which has an overall aim to teach mathematics in a way which leads to real understanding (Department of Education and Skills (DES), 2010). It involves changes to what students learn in mathematics, how they learn it and how they will be assessed. The initiative is designed to ensure an appropriate balance between understanding mathematical theory and concepts and developing practical applications skills. There is a much greater emphasis placed on student understanding of mathematical concepts, with increased use of contexts and applications. The focus is on students understanding the concepts involved, building from the concrete to the abstract, from the informal to the formal and learning to apply their knowledge in familiar and unfamiliar contexts (DES, 2010). Changes have being phased in over a number of years covering 5 strands of mathematics (Number, Algebra, Statistics and Probability, Geometry and Trigonometry, Functions), with assessment in the examinations being adapted as each strand of mathematics comes on stream. The assessment reflects the different emphasis on problem solving and applications in the teaching and learning of the subject.

Changes to the teaching, learning and assessment of algebra

Although the initiative first began in 24 pilot schools in 2008, it was not rolled out nationally until 2010 and the new algebra strand was not introduced to all schools until September 2011. In a major shift from the transformational based approach which had dominated Irish classrooms, the new strand advocates a functions based approach to teaching algebra. The

functions based approached envisages that students may be able to work with variables and the rules of arithmetic and learn to use algebraic notation and techniques themselves. There is an opportunity for students to see algebraic notation arising as a natural and useful consequence of expressing generality (Pegg & Redden, 1990).

Through Project Maths, the new approach reflects inquiry methods through which students take responsibility when dealing with new problems rather than rehearsing known procedures. Students examine functions derived from some kind of context, e.g., familiar everyday situations, imaginary contexts or arrangements of tiles or blocks. They express generalisations mathematically using algebraic symbolism, interpret expressions as rules for functions and use the Cartesian plane as a space to display and consider a variety of meanings of the results (Chazan & Yerushalmy, 2003). Therefore students represent the problem using words, numbers, symbols, tables and graphs. This approach builds on the learner's prior knowledge and allows them to see different representations which should enable a deeper understanding of the topic.

The change in teaching approach to algebra has also led to a change in the assessment approach for the strand. The Project Maths assessment reflects the changes in emphasis of a functions based approach in which students are required to take an everyday problem, solve it mathematically using tables, functions and graphs and then interpret their results in the context of the problem.

Success of Project Maths and the Functions Based Approach?

The phased implementation of Project Maths means that 2014 was the first year in which all strands of the revised syllabi were examined. Indeed it will be 2017 before a first cohort of students who have experienced all 5 strands of Project Maths from 1st to 6th year [3] will be examined. Thus it is very early to make any conclusions regarding the successes / failures of the initiative. However, an interim report commissioned by the NCCA and conducted by the National Foundation for Educational Research (NFER, UK) has been published and includes findings on students' attitudes and achievements. Overall this report found that there is emerging evidence of the positive impacts of Project Maths on students' experiences of, and attitudes towards, mathematics (Jeffes et al., 2013). Furthermore, students' are achieving more at individual strand level, and in some instances students appear to be successfully drawing together their knowledge across different mathematics topics (Jeffes et al., 2013). This suggests that students are beginning to acquire a deeper understanding of mathematics and how it can be applied (Jeffes et al., 2013).

With a specific reference to algebra, the revised strand was first examined at LC level in June 2012. The NFER report found that out of the five strands, algebra is in the lowest two strands, both in terms of student confidence and student achievement (Jeffes et al., 2013). Furthermore although students' performance in other strands is similar to international students who participated in TIMSS 2007, students appear to find algebra especially difficult when compared to international standards (Jeffes et al., 2013). However on a positive note for the functions based approach, the report found that students feel more motivated to learn algebra when it is taught in a way that makes it seem more relevant to everyday life and when they can see that it interlinks with other mathematics topics.

METHODOLOGY

The methodology of this study involves comparing student's results on a number of transformational based algebra questions from a university diagnostic test. The results will be compared between two cohorts, 2011 in which students were taught using traditional methods, and 2013 in which students were taught using a functions based approach.

The instrument

The diagnostic test from which the data is compared was designed and implemented in the University of Limerick (UL) in 1997. It was designed to help identify students who may be at risk of failing service mathematics examinations (O'Donoghue, 1999; Gill, 2006). Within the design process, a team of experienced service mathematics lecturers analysed and adjusted an initial list of 70 questions, reducing this to the final 40 question version. Thirty four of these questions are set at a LC Ordinary Level standard or below, with the other six questions set at a LC Higher Level standard. To ensure the validity of this test, it was then piloted in Irish second level schools and compared with the SEFI Core Level Zero syllabus for engineers, the Irish

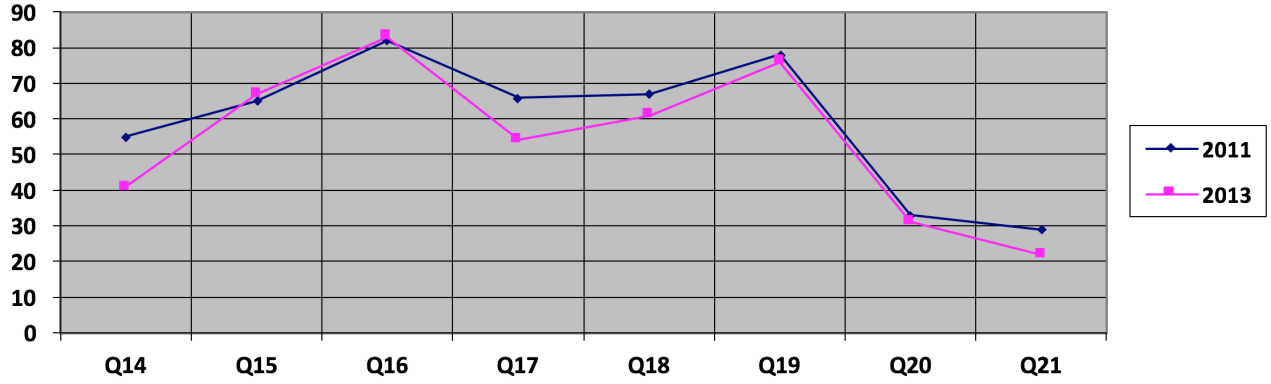

Figure 1: Student results in eight algebra questions

JC mathematics syllabus, the Irish LC mathematics syllabus and further diagnostic tests (Gill et al., 2010).

The test was intended specifically to suit the mathematical level of the students in each of the targeted service mathematics modules in UL, namely Science Mathematics 1 and Technological Mathematics 1. Since 1998 students from these cohorts have been assigned the 40 question diagnostic test which assesses basic skills and procedures in various mathematical topics. These topics include arithmetic (13 questions), algebra (8 questions), geometry (4 questions), trigonometry (3 questions), co-ordinate geometry (4 questions), complex numbers (2 questions), differentiation (3 questions), integration (2 questions), and modelling (1 question). The test is presented to students in their first mathematics lecture of each year since and they are not informed beforehand that they are required to take it.

The layout of the test and each of the 40 questions have remained unchanged over the years to ensure reliability. The UL database which contains data from these diagnostic tests dating from 1998 to 2013 currently holds information on over 10,100 students.

THE STUDY

This paper will focus on the results of students from the 8 algebra questions (see appendix 1) in the year 2011 in comparison to the results from 2013. It is important to note that the algebra questions included in the test are transformational based in nature and hence do not reflect a student's overall algebraic ability. The years 2011 and 2013 were selected as the algebra strand of Project Maths was first phased into Irish schools in September 2011. Hence the students who took the 2011 UL diagnostic test would have been taught algebra using the transformational based approach. The students who took the 2013 UL diagnostics test would have been taught algebra using the functions based approach. 685 students took the diagnostic test in 2011 and 645 took it in 2013. Analysis of this data will aid the authors is answering the following research question:

Is there a difference in the transformational algebraic ability of incoming third level students who have been taught using different approaches?

RESULTS

Descriptive analysis of the data found that there was a statistically significant difference (t=4.463, p =.000) between the mean algebra scores of the students in 2011 (M: 59.01; SD: 24.07) compared to the mean scores in 2013 (M: 53.16; SD: 23.74). In order to get a more in-depth analysis of this finding, the results of each of the eight algebra questions from both years were compared (see Figure 1).

Students who had been taught using the transformational based approach scored higher in six out of the eight algebra questions. There was a statistically significant difference in the mean scores of students in four of the eight algebra questions (Q14, 17, 18, 21) in 2011 when compared to 2013 (see Table 1).

A closer inspection of the four questions (See Appendix 1 Q14, 17, 18, 21) in which there were statistically significant differences between the cohorts does not reveal any correlation between the questions. Question 14 involves the rearrangement of formula, Question 17 and 18 involve solving equations (quadratic and simultaneous respectively) and Question 21 concerns the subtraction of algebraic fractions. Similarly, no correlation appears to exist between

Question	Mean and SD 2011	Mean and SD 2013	Independent T-test
14	.54 (.499)	.40 (.490)	t=5.390, p =.000
15	.65 (.478)	.66 (.475)	t=-3.52, p =.725
16	.82 (.383)	.82 (.386)	t=0.156, p =.876
17	.66 (.476)	.52 (.500)	t=5.087, p =.000
18	.66 (.474)	.59 (.492)	t=2.551, p =.011
19	.77 (.420)	.75 (.436)	t=1.131, p =.258
20	.33 (.471)	.30 (.460)	t=1.138, p =.255
21	.29 (.453)	.22 (.414)	t=2.895, p =.004

Table 1: Mean Score, Standard Deviation and Independent T-tests

the questions in which there were no statistically significant differences (See Appendix 1 – Q15, 16, 19, 20). Question 15 involves substitution, Question 16 involves solving a linear equation, Question 19 is the expansion of brackets while Question 20 concerns solving inequalities. Each of the eight questions are JC Higher Level / LC Ordinary Level standard and would have been on the traditional syllabus, in addition to being on the current Project maths syllabus.

DISCUSSION AND CONLCUSION

Despite the perceived success of Project Maths and the functions based approach to teaching algebra (Jeffes et al., 2013), the analysis of the UL diagnostic test results shows that there is statistically significant differences in the 2013 mean algebra scores of students when compared to 2011. Analysis of the LC grades shows that both cohorts had very similar grades in mathematics upon completing second level. The mean number of LC points achieved by these students through mathematics in 2011 was 54.27 while, in 2013 this cohort produced a mean number of 53.64 points. However the performance of the 2013 cohort in the algebra section of the diagnostic test is significantly lower than the 2011 cohort. This would suggest that the introduction of the new algebra strand in Project Maths and the change in teaching approach has had a detrimental effect on student's transformational algebraic ability.

The findings of the study are hardly surprising given that diagnostic test used to gather the data focuses solely on transformational activities and the emphasis on such activities has been diminished under the new curriculum. However the eight algebra questions contained in the test (See Appendix 1) are of a very basic standard and all Irish students would have encountered such concepts at Junior Cycle / Senior Cycle level in both syllabuses. Thus the results of this study, in this regard are surprising. Whatever the approach to teaching algebra at second level, it is expected that students entering third level on degree programmes with a mathematics component, should be competent in rearranging basic formula, expanding brackets, substitution and solving basic equations. The authors argue that such skills should be acquired by second level students regardless of whether they are being taught algebra using a transformational, a functions based or indeed any other approach.

Thus the challenge for Irish mathematics educators on the basis of this study is to ensure that the transition to the functions based approach does not result in a neglect of teaching the technique and rule-bound aspects of the algebraic language (Prendergast & O'Donoghue, 2014). While the transition to the functions based activities of Project Maths is a welcome move, it is also important that the algebraic purpose behind such activities is not lost. This results with students not knowing how to move from their informal constructions to a formal and algebraic relationship (Stacey & MacGregor, 1997).This was evidenced in the UK where the search for meaning and the consequent suppression of symbolism led to a situation in the early 1990s where students were doing hardly any symbol manipulation (Sutherland, 1997). Problem solving by whatever means had all but replaced algebra (Kieran, 2004). The hope was that, in focusing on algebraic understanding, the techniques would take care of themselves. However, a study carried out by Artigue in France in the mid 1990's on the use of DERIVE in French classrooms found that the techniques did not take care of themselves (Kieran, 2004). As anticipated, the researchers found that the teachers were emphasising the conceptual elements while neglecting the role of the procedural work in algebra learning. However, this emphasis on conceptual work was producing neither a clear understanding of the proce-

dural aspects, nor a definite enhancement of students' conceptual understanding, "easier calculation did not automatically enhance students reflections and understanding" (Lagrange, 2003, as cited in Kieran, 2004, p. 28). Thus traditional exposition and practice must be retained alongside more opportunities for practical work, problem solving, investigations and discussion and providing purpose to the activities (Sutherland, 1997). The work of 'Project Maths' in Ireland must facilitate the curriculum and teachers in making such an evolution. While the new curriculum is having many positive benefits it is important that techniques and conceptual understanding are taught together rather than in opposition.

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ENDNOTES

1. 'Project Maths' is the name under which the reform of the Irish mathematics curriculum has been implemented.

2. There are three levels to the Irish education system – primary level, second level and third level. Second level students are typically aged 12–18 years.

3. 1st to 6th year – Second level education is typically of 6 years duration.

APPENDIX 1

Algebra Q.14 - Q.21

14. Solve for $h : V = \pi r^2 h$

Ans _____ Don't know ☐

15. Evaluate $ab + 2bc - 3ac$ when $a = 3$, $b = -2$ and $c = 4$

Ans _____ Don't know ☐

16. Solve the equation: $3(x + 2) - 24 = 0$

Ans _____ Don't know ☐

17. Solve for x: $x^2 + x - 6 = 0$

Ans _____ Don't know ☐

18. Solve the set of equations:

$$\begin{aligned} 2x + y &= 7 \\ x + 2y &= 5 \end{aligned}$$

Ans Don't know ☐

19. Write out $(x + 3y)(a - 2b)$ in an equivalent form without brackets

Ans _____ Don't know ☐

20. Solve for x: $3 - 6x < 21$

Ans _____ Don't know ☐

21. Simplify $\dfrac{1}{x-1} - \dfrac{2}{x+1}$

Ans _____ Don't know ☐