

ORIENTATION ESTIMATION USING KALMAN FILTERS WITH MULTIPLE SENSORS

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ABSTRACT

The ability to repeatedly and accurately position robotic end-effectors is a key requirement for modern industrial automation. Parallel developments in domestic and service robotics are opening up exciting possibilities for future manufacturing scenarios that would see robots and humans work far more closely together. These robots will typically be lighter, more mobile, less task specific and far more interactive than typical manufacturing robots of today. Reliable and precise detection of position and orientation in conjunction with the ability to plan and control complex kinematic motions will be vital. Currently, a large variety of sensors exist to measure orientation. The reliability and other attributes of these sensor technologies vary greatly. By using multiple sensors more accurate orientation estimates can be achieved. This paper presents the use of Kalman filters to realise sensor fusion of accelerometer, gyroscope and incremental encoder data. Four Kalman filters are implemented and tuned based on data from a system incorporating the relevant sensors. The filters use either gyroscope or encoder data for the predict step and accelerometer or encoder orientation estimates, or a combination of the two, for the measurement step. There are distinct differences between the responses of these implementations when the system is in operation.

KEYWORDS: Kalman filter, orientation estimate, sensor fusion

1. INTRODUCTION AND BACKGROUND

Measurement of orientation is important in a variety of fields. In robotics, orientation and position tracking of all parts of the robot is vital for the system to operate effectively. The capacity to accurately measure the orientation of each link in a robotic system, particularly under dynamic conditions, increases the accuracy and speed with which robotic devices can operate. In a manufacturing context this can reduce lead time, improve tolerances and increase reliability.

A number of sensors exist with which to establish an orientation estimate. Two common means include incremental encoders and inertial measurement units (IMU), which consist of three accelerometers and three gyroscopes orientated orthogonally.

1.1 Sensors

Incremental encoders can be used to measure rotation about a joint. As the joint rotates, two out of phase square signals are generated. By analysing when these signals change state, the rotation of the joint is measured [1], [2]. The angle between the parent and child link can then be inferred provided the original

orientation is known. Some encoders provide an additional index digital output which gives a high signal at only one point in the rotation, thus giving a reference for measurement of absolute position. Because orientation is measured relative to the previous joint, errors in the orientation of each link in a system are cumulative. Additionally it is possible that transitions from high to low on the digital lines may be missed by the controller. An encoder is required for each degree of freedom of the joint and must be positioned on the joint itself which increases complexity.

Accelerometers can be used to discern orientation by identifying the direction in which gravity is acting [3]. This method estimates the orientation relative to the global reference frame. However, when the sensor is in motion, it experiences accelerations in addition to gravity. This results in incorrect estimates of orientation, though these are corrected once the sensor comes to rest.

Gyroscopes measure rotational velocity. By integrating this measurement, the rotation of the sensor can be established. This method is however prone to drift as each successive measurement is dependent on all previous measurements and so error is cumulative [4]. For this reason gyroscopes are rarely used in isolation for the determination of orientation.

Figure 1 shows a comparison of the orientation estimates possible from the sensors detailed above. The accelerometer estimate varies widely from the other estimates especially when the sensors orientation changes rapidly. However it will return to a stable, accurate measurement of orientation once the sensor comes to rest. The gyroscope estimate initially agrees with that of the accelerometer but by the end of the test the estimate has drifted by several degrees. The encoder estimate differs by several degrees at start up, as may be the case if the start position is not known precisely. By the end of the test, the offset from the accelerometer estimate is still the same, showing that changes in the orientation were successfully measured. Also shown in figure 1 is a Kalman filter estimate of orientation. This type of filter is commonly used with IMU data and is further explained in the following section.

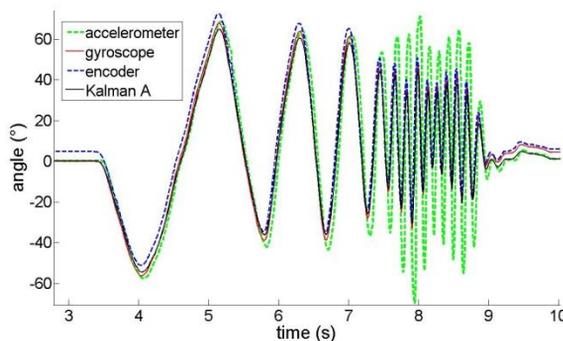


Figure 1: Orientation estimation using established methods. “Kalman A” is a filter commonly used with IMU sensors.

1.2 The Kalman Filter

First proposed by R.E. Kalman in 1960 [5], the Kalman filter provides a recursive solution to the discrete-data linear filtering problem [6]. It can predict the state of a linear system using information from a number of sources of

information regardless of the accuracy of each source, as well as information about the dynamics of the system and past states. The filter has several advantages including the ability to predict future as well as the current state of the system and the relatively small amount of information which must be stored between successive iterations. The Kalman filter successively minimises the mean square of the error. As a recursive data processing algorithm, it has been said that “the Kalman filter is optimal with respect to virtually any criterion which makes sense”[7].

The Kalman filter consists of two distinct process - predict and measure. In the predict step, the current state of the system is updated based on the previous orientation estimate and any control inputs. In the measurement step this estimate is corrected based on the measured value with regard for the accuracy with which the measurement is taken and the trust placed in the predicted estimate. If the state of the system is required between the times when measurements are taken it is possible to calculate a predicted state without the necessity of taking a measurement.

Applications of the Kalman filter are wide ranging from economics[8] to improving the accuracy of a measured quantity such as voltage [6], but in particular it has been used in the area of autonomous navigation. The following description of the discrete Kalman filter commonly implemented for IMU data is based on [6], [9], [10].

The predicted estimate of the state of the system is known as the priori state. The rotational velocity from the gyroscope is typically included as a control input here. The priori state is calculated as in equation 1.

$$\hat{x}_{k|k-1} = F \hat{x}_{k-1|k-1} + B \dot{\theta}_k \quad (1)$$

$\hat{x}_{k|k-1}$ is an estimate of the state (indicated by the “hat”), as opposed to the actual state at the measurement k , based on the measurements up to and including measurement $k-1$. In the case of a Kalman filter which uses a gyroscope reading in the predict step for orientation estimation, the state includes the angle and also the bias in the gyroscope reading (how much it has drifted). An estimate of the state is found using equation 2.

$$\hat{x}_k = \begin{bmatrix} \theta \\ \dot{\theta}_b \end{bmatrix} = \begin{bmatrix} \text{angle} \\ \text{gyro bias} \end{bmatrix} \quad (2)$$

$\hat{x}_{k-1|k-1}$ is the previous estimate of the state of the system from the last time the Kalman filter updated. F is the state transition model, which is applied to the previous state to adjust it before it is used to estimate the next.

$$F = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \quad (3)$$

This is the identity matrix with an extra term to account for an adjustment based on the gyroscope bias and the time step (the time since the state was last updated, Δt). $\dot{\theta}_k$ is the new gyroscope measurement adjusted to units of degrees per second. B is called the control input model and describes how the new

measurement should modify the previous state. In this case, as it is applied to the gyroscope reading.

$$B = \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \quad (4)$$

B takes the form shown in equation 4 as the angular velocity times the time step is the change in angle and there is no new information about the bias from the new gyroscope reading.

Next the priori state convergence matrix (P) is calculated. This quantifies the level of trust placed on the priori state calculated above. The smaller the values, the more the state is trusted. The priori state convergence matrix is given by equation 5.

$$P_{k|k-1} = FP_{k-1|k-1}F^T + Q_k \quad (5)$$

Q_k is the process noise covariance matrix. It is a measure of covariance in angle estimate and gyroscope bias. If these quantities are considered independent, then this reduces to the variance in each.

$$Q_k = \begin{bmatrix} Q_\theta & 0 \\ 0 & Q_{\dot{\theta}_b} \end{bmatrix} \Delta t \quad (6)$$

The values in this matrix may be dynamic but in the case of the filters featured here they are assumed static and tuned to improve the response of the filters as detailed in table 1. The dependency on Δt recognises that if the time between measurements is greater, so too is the associated error.

The first stage in the measurement part of the filter is called the innovation. It is calculated using equation 7.

$$\tilde{y}_k = z_k - H\hat{x}_{k|k-1} \quad (7)$$

z_k is the measured quantity, in this case the angle estimate found using the accelerometer readings. H is a transfer matrix. It maps the priori state into the observed space.

$$H = [1 \ 0] \quad (8)$$

Given the form of H in equation 8, it simply selects the angle measurement from the priori state. It is evident that the innovation will therefore not be a matrix but a single number in this case. A covariance matrix is also required for the innovation. This is calculated as shown in equation 9.

$$S_k = HP_{k|k-1}H^T + R \quad (9)$$

This matrix indicates how much confidence is placed in the measurement. The effect of H and its transpose is to pick out P_{00} from the priori matrix. R is the measurement covariance matrix. In this case, it is the measurement noise in the accelerometer reading (the variance). This can be modelled or tuned. It can be

seen then that as with \tilde{y}_k , for this case, S_k is a number rather than a matrix. A quantity known as the Kalman gain is used to quantify the trust placed in the innovation. This is defined in equation 10.

$$K_k = P_{k|k-1} H^T S_k^{-1} \quad (10)$$

The final estimate for the angle can now be calculated as described in equation 11.

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k \quad (11)$$

The current estimate of the state, is updated by adding the difference between this and the accelerometer value, adjusted by the confidence in that difference. The new state is known as the posteriori. Lastly the priori convergence matrix is updated to the posteriori convergence matrix for the next iteration. This will then describe the confidence in the estimated state of the system. Equation 12 shows this, where I is the identity matrix.

$$P_{k|k} = (I - K_k H) P_{k|k-1} \quad (12)$$

In the next iteration this matrix will be $P_{k-1|k-1}$. Here the model is self-correcting based on how much the estimate of the angle had to be adjusted.

While this process is quite complicated, it is reasonably efficient in code as only a few variables must be saved between iterations rather than many previous measurements as is required in some models[7]. This filter was modified in order to accept different combinations of measurements from the different sensors as detailed below.

2 KALMAN FILTER IMPLEMENTATIONS

Four Kalman filters were implemented, as summarised in table 1. “Kalman A” is implemented as described above. The other Kalman filters are similar, with differences as described below.

“Kalman B” replaces the accelerometer measurement with the angle inferred from the encoder, knowing the initial orientation. This angle measurement is more stable than that from the accelerometer and so the orientation should be more consistent during dynamic conditions. “Kalman C” uses the rotation recorded by the encoder in the predict step (the number of increments counted since the previous predict computation). This allows the filter to maintain absolute orientation measurement provided by the accelerometer while benefiting from the stability of the encoder measurements. In this case matrix B must be adjusted in order to account for the different type of data being used. The Δt term in equation 4 is replaced by the number of degrees per increment of the encoder.

“Kalman D” uses both the accelerometer and encoder measurements in the measurement step while the gyroscope updates the predict step. In accordance with the principles of the Kalman filter, the addition of more measurements leads to a better estimate of the system state. The predict step is implemented as detailed in the previous section. In the measurement step, the measurement

vector, z_k , now has two values instead of one. This means that the matrix H now takes the form of equation 13.

$$H = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \quad (13)$$

This means that the innovation matrix, \tilde{y}_k is now a vector. The elements of equation 9 are also modified. R , the measurement covariance matrix is now a 2X2 matrix. The diagonals are the variance in the measurements from each sensor. The other elements are the covariance between the sensors, here assumed to be zero. The Kalman gain, posteriori state and posteriori convergence matrix can then be calculated as before.

For ease of development, only rotation around a single axis is considered but this could be expanded to a three dimensional orientation estimate. Once data was collected as described in the following section, the filters were manually tuned. Table 1 shows the tuned values of process and measurement noise used with the filters described above.

	Predict	Measure	Process noise angle (Q_θ) (degrees ²)	Process noise bias ($Q_{\hat{\theta}_b}$) ((degrees/s) ²)	Measurement noise (R) (degrees ²)
Kalman A	Gyroscope	Accelerometer	0.001	0.002	0.035
Kalman B	Gyroscope	Encoder	0.001	0.0001	0.04
Kalman C	Encoder	Accelerometer	0.03	0.000001	0.035
Kalman D	Gyroscope	Accelerometer & Encoder	0.0001	0.002	0.035 0.04

Table 1: Summary of the Kalman filters implemented

3 DATA GENERATION

In order to generate the required sensor data a single degree of freedom system was constructed which incorporated an IMU and encoder on a common shaft supported by bearings. The Invensense MPU6050[11] was chosen as the IMU. As well as providing raw accelerometer and gyroscope readings, the IMU itself computes an orientation estimate using a built in Digital Motion Processor (DMP). This can be used to sample the accelerometer and gyroscope readings at a high frequency, process them, and send the output to the master at a user defined frequency. The encoder used was an E4P OEM [12] which is capable of providing 360 pulses per revolution on each digital line, therefore providing 1440 updates of position per revolution.

The testing apparatus consists of a crank rocker mechanism operated by a geared electric motor which is powered by a variable DC power supply. The pivot of the rocker is the shaft on which the encoder is attached and the IMU is also positioned on this shaft as close to the centre of rotation as possible. This mechanism ensures that the movement of the rocker is consistent and predictable at any chosen speed. The furthest deviations of the rocker from the vertical can be measured under static conditions. These easily achieved measurements can then be compared with those found under dynamic conditions using each of the methods detailed above. During testing care was taken to position the rocker vertically before testing began.

Prior to testing the accelerometers and gyroscopes were calibrated. The IMU was placed on a flat horizontal surface so as gravity acts exclusively in one axis. The relevant offsets were calculated so as to ensure the readings reflected this. As the sensor was stationary, the gyroscopes should detect no rotation. Again the relevant offsets were calculated.

Testing sequences of 25-30 second duration were conducted during which time the electric motor turned the crank at nine different speeds as detailed in the results. Two ATmega328 microcontrollers[13] were used to connect to the sensors. One of these monitored the encoder and the other the IMU. Readings from these sensors were sent to a computer via a serial connection for logging and post-processing. Only raw encoder and IMU readings as well as orientation estimates from the DMP were recorded. All analysis using the Kalman filters was conducted after testing so as to minimize the computational task on the microcontrollers.

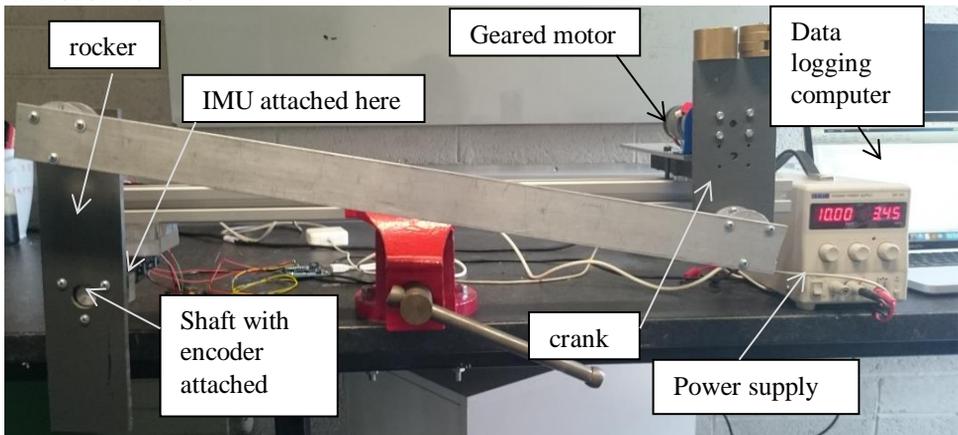


Figure 2: Testing apparatus

4 RESULTS

A sample output when the system is operating at 0.36 Hz is shown in figure 3. The results were produced after the filters were tuned manually. As it is difficult to see differences in the angle estimates produced when the full graph is shown, magnified graphs at the point when the rocker changes direction are shown in figure 4.. Static measurements show that the actual angle of the system has maximum and minimum values of 41.75° and -45.5° respectively. In the case of figure 4, the maximum of the angle curve should be 41.75 .

It is apparent that when the shaft is rotated at a relatively slow speed there is little difference between the estimates. At higher speeds there are more distinct differences. The DMP angle estimate tends to underestimate the orientation angle. It was observed that when the rocker moved at a frequency greater than 1.22Hz, the orientation gathered from the DMP was no longer stable and so this is not included in the analysis.

The encoder measurements are quite accurate particularly at low velocities. As expected the curve produced is not smooth as changes are made in 0.25 degree increments. Higher precision encoders would produce a smoother output.

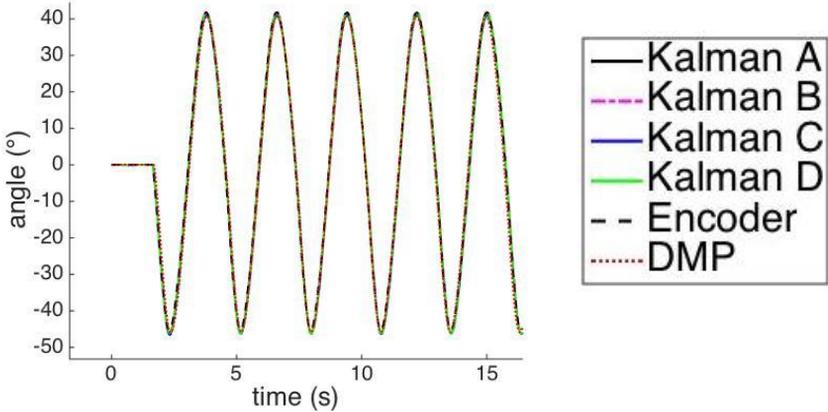


Figure 3: The output of the Kalman filters, encoder and digital motion processor when the system moves at 0.36 Hz

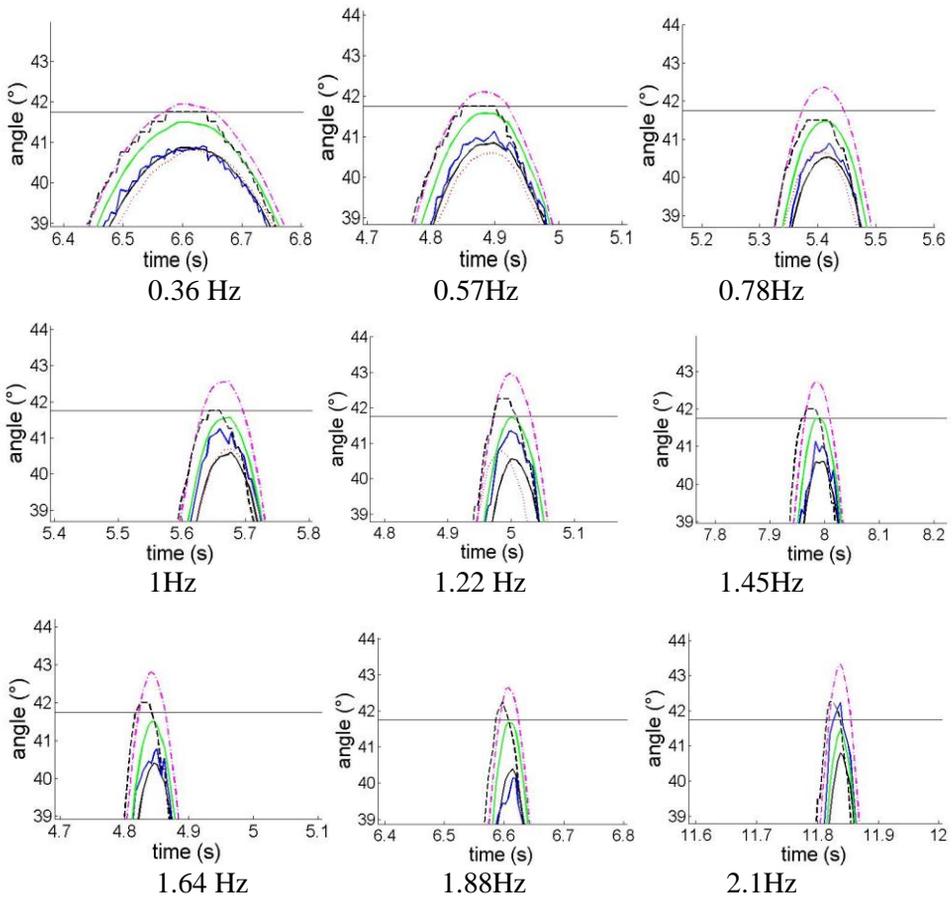


Figure 4: A close examination of the angle estimates as the rocker changes direction. The horizontal line shows the statically measured angle which was the maximum in this direction (41.75°).

The “Kalman A” filter agrees closely with the DMP estimate as both use the same combination of sensors. The curve produced is relatively smooth. “Kalman B” overshoots in all of the tests. The curve produced however is quite smooth. “Kalman C” shows improved performance when compared with “Kalman A”, however the curve produced contains more noise as may be expected with the more abrupt changes which are synonymous with the encoder as opposed to the gyroscope. Finally, “Kalman D” produced an estimate which shows improved accuracy when compared to the encoder or “Kalman A”.

As there will be some variability in how each angle estimate performs during testing, the average difference between each angle estimate and the known orientation at the limits of the rocker were calculated, and are graphed in figure 5. This shows that the encoder is most accurate at low rotational velocities. As the velocity increases, the error is comparable with that of “Kalman A”. “Kalman C” and “Kalman D” show improved accuracy when there are large rotational velocities.

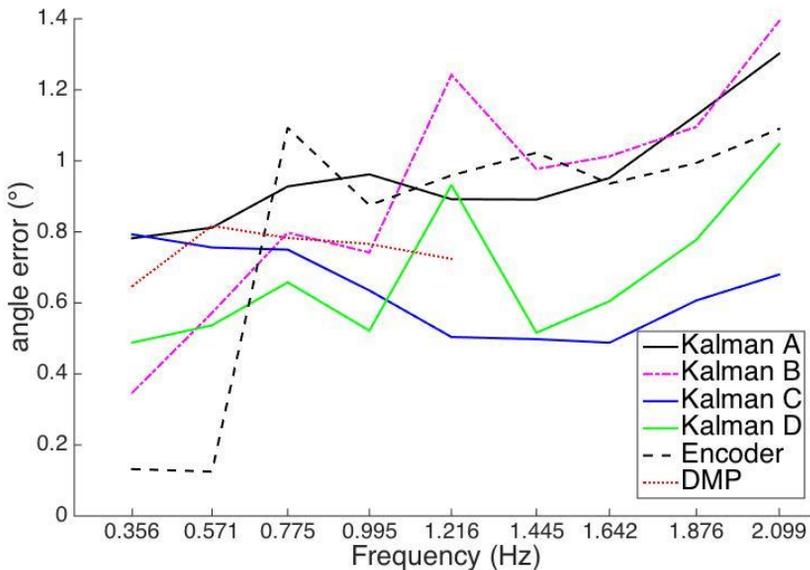


Figure 5: The angle estimate using the four Kalman filters, the DMP and the encoder as the shaft is rotated at different speeds (approximate frequencies shown)

5 CONCLUSION

Orientation estimation can be enhanced by considering inputs from additional sensors. The Kalman filter has been shown to be a useful tool with which to incorporate each of these measurements into a single estimate. The validity of the Kalman filters implemented here will require further testing to verify their response to a wide variety of operating conditions such as when the type of rotation changes during testing. Additionally further tuning of each of the filters has the capability to further improve performance. Ultimately the feasibility of attaching particular sensors and the required accuracy will play a critical role in the choice of sensor employed. The approach outlined here can then be used to produce an optimised orientation estimate based on the available data.

6 REFERENCES

- [1] L. Krisztián and A. G. Kocsis, “Implementation of speed measurement for electrical drives equipped with quadrature encoder in LabView FPGA,” *Acta Tech. Corviniensis - Bull. Eng.*, vol. 6, pp. 123–126, 2013.
- [2] I. I. Incze, A. Negrea, M. Imecs, and C. Szabó, “Incremental Encoder Based Position and Speed Identification : Modeling and Simulation,” *Acta Univ. Sapientiae, Electr. Mech. Eng.*, vol. 2, pp. 27–39, 2010.
- [3] P. Cheng and O. Bengt, “Joint-Angle Measurement Using Accelerometers and Gyroscopes—A Survey,” *IEEE Trans. Instrum. Meas.*, vol. 59, no. 2, pp. 404–414, 2010.
- [4] Y. S. Suh, “Attitude Estimation Using Low Cost Accelerometer and Gyroscope,” in *Korea-Russia International Symposium*, 2003.
- [5] R. E. Kalman, “A New Approach to Linear Filtering and Prediction Problems 1,” *ASME–Journal Basic Eng.*, vol. 82, no. 82, Series D, pp. 35–45, 1960.
- [6] G. Welch and G. Bishop, “An Introduction to the Kalman Filter,” Citeseer, 2006.
- [7] P. S. Maybeck, *Stochastic models , estimation , and control*, vol. 1. Academic Press, 1979.
- [8] G. K. Pasricha, “Kalman Filter and its Economic Applications,” *Munich Pers. RePEc Arch.*, no. 22734, 2006.
- [9] K. Lauszus, “A practical approach to Kalman filter and how to implement it,” 2012. [Online]. Available: <http://blog.tkjelectronics.dk/2012/09/a-practical-approach-to-kalman-filter-and-how-to-implement-it/#comment-57783>. [Accessed: 19-Mar-2015].
- [10] J. L. C. Miranda, “Application of Kalman Filtering and PID Control for Direct Inverted Pendulum Control,” California State University, 2009.
- [11] InvenSense, “MPU-6000 and MPU-6050 Product Specification,” 2013.
- [12] US Digital, “OEM Miniature Optical Kit Encoder.”
- [13] Atmel, “ATmega48PA/88PA/168PA/328P,” 2008.