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I.M. 46/66

DEPARTMENT OF TRANSPORT AND POWER

METEOROLOGICAL SERVICE

INTERNAL MEMORANDUM

THE FLUCTUATIONS OF DUBLIN SUMMERS

- by -

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Received 25/1/66.

Issued 28/4/66.

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R.M. Poulter (1962) introduced a useful index for expressing the quality of summer weather. W.A. Morgan (1965) has applied a similar technique to summarise Dublin's summers from 1880 to 1964. These are reproduced in Table 1, and are plotted in Figure 1

TABLE 1. Summer Weather Index for Dublin (Phoenix Park)

Decade Commencing	LAST DIGIT OF YEAR 0	1	2	3	4	5	6	7	8	9
1880	339	316	332	312	351	348	355	435	302	346
1890	318	338	325	393	311	323	361	357	370	394
1900	325	362	328	306	374	346	387	313	351	335
1910	325	392	258	355	365	319	342	353	345	353
1920	314	376	322	336	303	382	375	312	333	332
1930	334	311	353	401	388	388	337	348	305	344
1940	379	344	348	360	338	342	304	382	327	390
1950	359	356	352	353	315	364	321	354	279	403
1960	346	337	329	332	339					

Mean value 345

At first sight, Figure 1 suggests typically random fluctuations about a mean, but a simple consideration reveals that randomness is unlikely. It is well known that in a random sequence the number of peaks, and the number of troughs, is each one third of the total number of data.* In this case the plot has 85 points and the expected number of peaks in a random distribution is 28. The actual number is 33, and although the excess is not necessarily significant, it is suggestive, and encourages further investigation. What it suggests is a periodicity of length less than 3 years, and supporting evidence is soon forthcoming.

* The proof of this is simple, but, since it is omitted from such textbooks as I have inspected, it is set out here as Appendix A.

Figure 2 summarises the results of auto-correlation, i.e. the values of the simple correlation coefficient between the sequence of index values and the same sequence displaced by 1,2,3 ..... years. As with Figure 1, the dominant feature is the presence of frequent oscillations on either side of the mean in short intervals. The value for n = 2 is not appreciable, so that if there is a period of length less than 3 years it is probably a submultiple of 12,15, 24,34 or 36, the values for which r is largest.

Table 2 and Figure 3 present the results of harmonic analysis of some trial periods. In Figure 3 the linear scale for the ordinates is of (amplitude)^2 to emphasise the peaks. For the abcissa what was actually plotted were the reciprocals of the chosen period lengths, to open out the region of special interest.

It must be mentioned that there are ambiguities with this type of periodogram analysis. Where the data are for integral intervals (in this case one year intervals) it is not possible to distinguish between real periods a little less and a little greater than 2 in length. Thus if the data fit to

$$I = A\cos\left(\frac{2\pi n}{p} - \phi\right)$$

they fit equally well to

$$A\cos\left\{\frac{2\pi(p-1)n}{p} + \phi\right\}$$

What we believe to be a period of length 2.42 years may really be 1.70 years. The only way of determining which alternative is preferable is to use shorter intervals, e.g. seasons instead of years. It is not easy to divide the data in this case, and it has not been attempted.

TABLE 2. Periodogram Data

Period (Years)	Amplitude	Period (Years)	Amplitude	Period (Years)	Amplitude	Period (Years)	Amplitude
1 7/8	12.0	2 8/13	8.5	4 2/7	2.8	11 1/3	7.1
1 15/17	11.2	2 8/11	2.1	4 1/2	6.5	12	5.8
1 8/9	10.8	2 3/4	2.3	4 6/7	5.7	13	4.9
1 17/19	8.3	2 10/13	1.7	5	4.2	14	2.9
1 14/15	8.0	2 5/6	5.9	5 1/7	5.0	14 1/2	3.5
2	9.0	2 6/7	5.6	5 1/2	4.9	15	6.8
2 1/14	8.0	2 8/9	7.3	5 2/3	2.3	15 1/2	6.6
2 1/8	10.8	2 9/10	6.0	6	3.6	15 2/3	7.1
2 2/15	11.2	2 10/11	6.8	6 1/2	1.2	16	7.6
2 1/7	12.0	2 11/12	7.2	6 3/5	9.3	17	7.2
2 2/13	11.2	2 11/13	7.8	6 4/5	9.4	17 1/2	8.2
2 1/6	5.8	2 14/15	8.3	7	8.6	18	6.3
2 1/5	8.5	2 26/27	8.2	7 1/5	7.5	19	3.5
2 3/13	6.1	3	9.6	7 1/2	6.3	22	1.0
2 1/4	2.4	3 1/12	5.8	7 3/4	6.6	24	2.8
2 2/7	5.3	3 1/11	6.3	8	9.4	26	3.8
2 4/13	1.4	3 3/13	8.1	8 1/2	6.8	28	1.2
2 5/14	8.5	3 1/4	5.5	8 2/3	7.1	29	2.3
2 3/8	9.4	3 1/3	5.7	8 3/4	10.0	30	1.6
2 5/13	13.8	3 2/5	4.7	9	11.3	31	1.4
2 2/5	16.2	3 5/9	5.0	9 1/4	9.3	32	5.4
2 5/12	14.5	3 3/5	5.5	9 1/2	9.2	34	5.1
2 3/7	10.0	3 2/3	7.8	9 2/3	6.1	35	5.2
2 6/13	8.9	3 3/4	2.7	10	4.6	36	5.8
2 1/2	5.2	4	3.2	11	3.7	37	4.4

For the integral periods (2,3,4 ... 37 years) the mean of the amplitudes is 5.0 with standard deviation 2.59. One cannot therefore expect to find significant any amplitude less than about 5 + 3 x 2.59, i.e. 12.8, and only one period satisfies this test. Its length is a little less than $2\frac{1}{2}$ years. To determine it more precisely recourse was had to the method of harmonic dials, by computing the phase and amplitude for successive sections of the entire sequence of years. This was done, not only for the neighbourhood of $2\frac{1}{2}$ years but also for values near the less conspicuous peaks of the periodogram.

The 85 years were divided into five groups of 17 each, and the results plotted in Figure 4 are the coefficients a and b in $I = a\cos\frac{2\pi n}{p} + b\sin\frac{2\pi n}{p}$, expressible also as $A\cos\left(\frac{2\pi n}{p} - \phi\right)$ where p is the length of the trial period and $\phi = 0$ would indicate a period with maximum in the middle of the first year, 1880.

If a trial period were exactly correct, and had persisted without change of phase, and without perturbing influences, the five lines in the diagram would coincide. If the true period differs only slightly from the value tried, there will be a regular drift from I to V: the amount of the drift can be used to evaluate the adjustment necessary. Inspecting Figure 4 we find:

Periods $1\frac{8}{9}$ and $2\frac{1}{7}$ years. No persistence or uniform trend. Periods near these values may exist, but are liable to frequent changes of phase.

Period $2\frac{5}{13}$ years. Except for III the trend is uniform from $\phi = -82^\circ$ in 1880-96 to $\phi = +71^\circ$ in 1948-64. This is consistent with the true period being longer by .036 years, i.e. 2.421 years.

Period $2\frac{2}{5}$ years. The similar trend here leads to an estimate of 2.426 years.

Period $2\frac{5}{12}$ years. The scatter is less than in any other case examined, supporting the belief that there is a true period near this length, 2.417 years.

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Period 3 years. The phases could be interpreted as decreasing from $+109^{\circ}$ to -282°, which leads to an estimate of 2.856 years, but no supporting evidence has been found.

Period 9 years. Although the phase is consistent for the last four of the five groups of years the amplitude is erratic.

SOLAR CYCLE.

Poulter's evaluation (1962) of a similar index for Kew led him to identify an 11-year periodicity as the only one of consequence. This is not apparent in the Dublin data. Nor is there appreciable evidence for any linkage with the solar cycle. Table 3 summarises the data for years near sunspot maximum and minimum.

Table 3.

	Years from Sunspot Maximum					Years from Sunspot Minimum				
	-2	-1	0	+1	+2	-2	-1	0	+1	+2
Mean Index	343	331	353	330	351	367	326	349	344	337
Number of good summers	0	0	1	1	1	2	0	2	1	2
Number of bad summers	1	1	1	3	0	1	2	0	1	2

The mean values do not suggest any difference between the summers of many sunspots and those of very few. The "good" and "bad" summers referred to by the lower lines of Table 3 are the best and worst of the 85, with index values above 386 and below 313 respectively. There is obviously no significant connection with the sunspots.

STRATOSPHERIC OSCILLATION.

In recent years several important papers have appeared on the subject of an approximately 2-year oscillation found to be very well marked in the equatorial stratosphere. (Reed 1960 etc). It is clear

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that such an oscillation must have world-wide consequences and should be detectable in climatic indices. Present opinions differ as to the nature of the oscillation. Reed himself (1965) postulates a period fluctuating in length between 21 and 30 months, with a mean value of 26 months. Probert-Jones (1964) on the other hand believes there to be two interacting periods, of length $22\frac{1}{2}$ and 27 months. He points out that these could be one sixth and one fifth respectively of an 11.2 year solar cycle. As the solar cycle is itself liable to perturbation one cannot expect its harmonics to recur regularly, and there is thus considerable doubt as to what period or periods can be established as "real". Although earlier observations confirm that the stratospheric oscillation is not a new phenomenon, the data suitable for rigorous analysis began only in the 1950s and many years must elapse before any reliable assessment can be accomplished.

The present investigation has yielded unreliable periods of $1\frac{8}{9}$ and $2\frac{1}{7}$ years ($22\frac{2}{5}$ and $25\frac{5}{7}$ months), plus a more persistent one of 2.42 years (29 months). Even the latter, however, is of doubtful prognostic value.

PROGNOSTIC VALUE OF 29-MONTHS PERIOD.

The broken line in Figure 1 is the plot of the 29-month periodicity with amplitude 20 (arbitrarily increased from the value obtained in the periodogram). The fluctuations match well for most of the time, the year-to-year change having the right sign in 63 out of the 84 cases. Most of the good and bad years are fitted as to sign of the departure from normal, but not otherwise and there were notable exceptions, such as 1892-93 and 1934.

The most disappointing feature, however, is that one of the times of worst fit is since 1960. It is therefore necessary to wait for a few years before discovering whether the 29-months period has been re-established.

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Appendix A

PEAKS AND TROUGHS IN A RANDOM DISTRIBUTION.

A simple and useful test for randomness (and hence also a test for the presence of systematic influences) is to count the peaks and troughs in a plot of the figures. For random data the peaks and troughs each number one third of the total number of points.

PROOF. Suppose that the variable x ranges between a and b and that the probability of a value between x and x + dx is p(x)dx. Then the probability that the adjacent points on either side are lower is in each case $\int_a^x p(x)dx$. As x may take any value from a to b the total probability that the point is a peak is

$$\int_a^b \left\{ \int_a^x p(x)dx \right\}^2 p(x)dx = \frac{1}{3} \left\{ \int_a^b p(x)dx \right\}^3 = \frac{1}{3}$$

since the total probability $\int_a^b p(x)dx$ is unity.

Obviously the same result is obtainable for troughs. This ignores the possibility that adjoining points are equal, but that is trivial if the range of values is large compared with the unit employed.

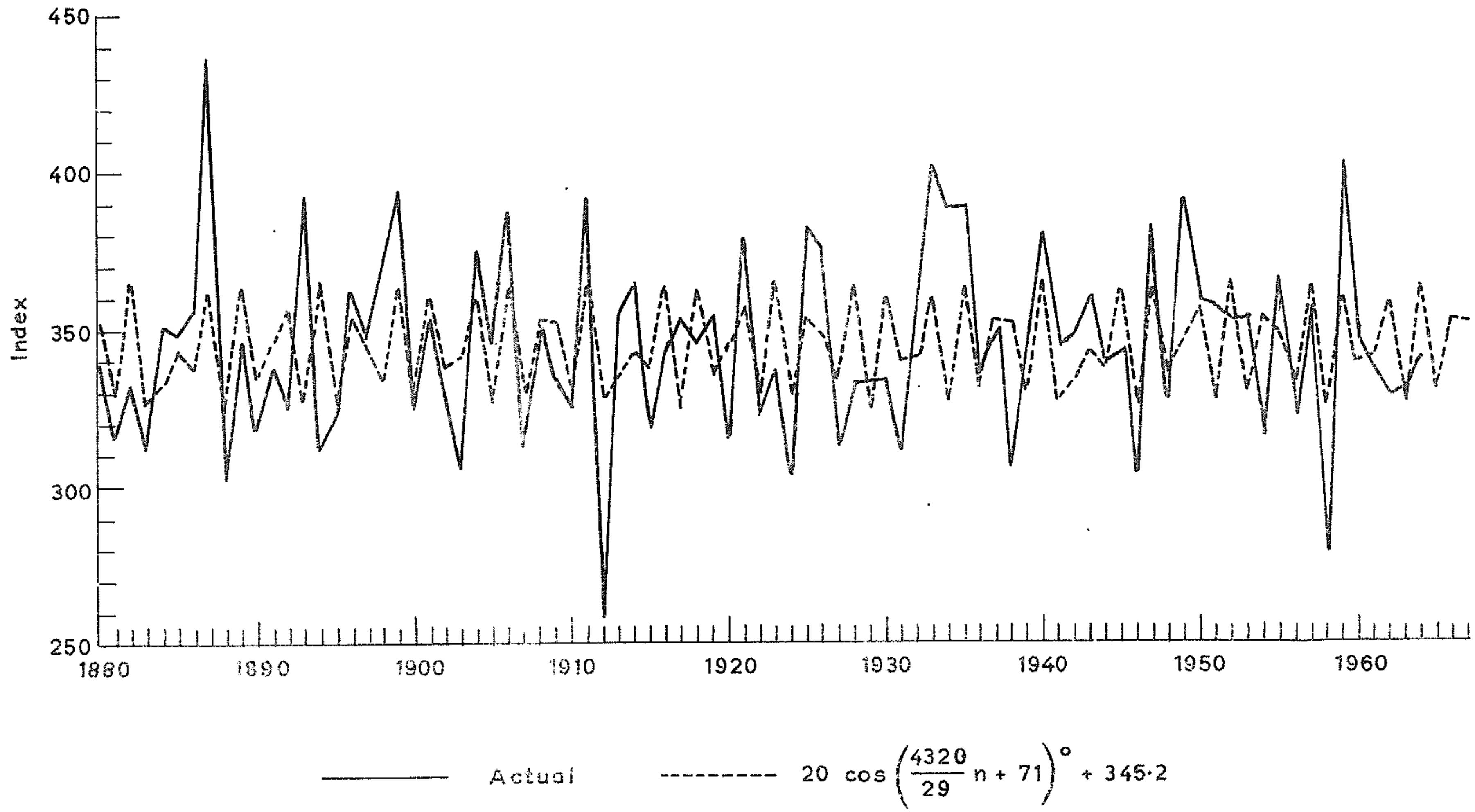

Fig. 1 Dublin Summers 1880 –1964

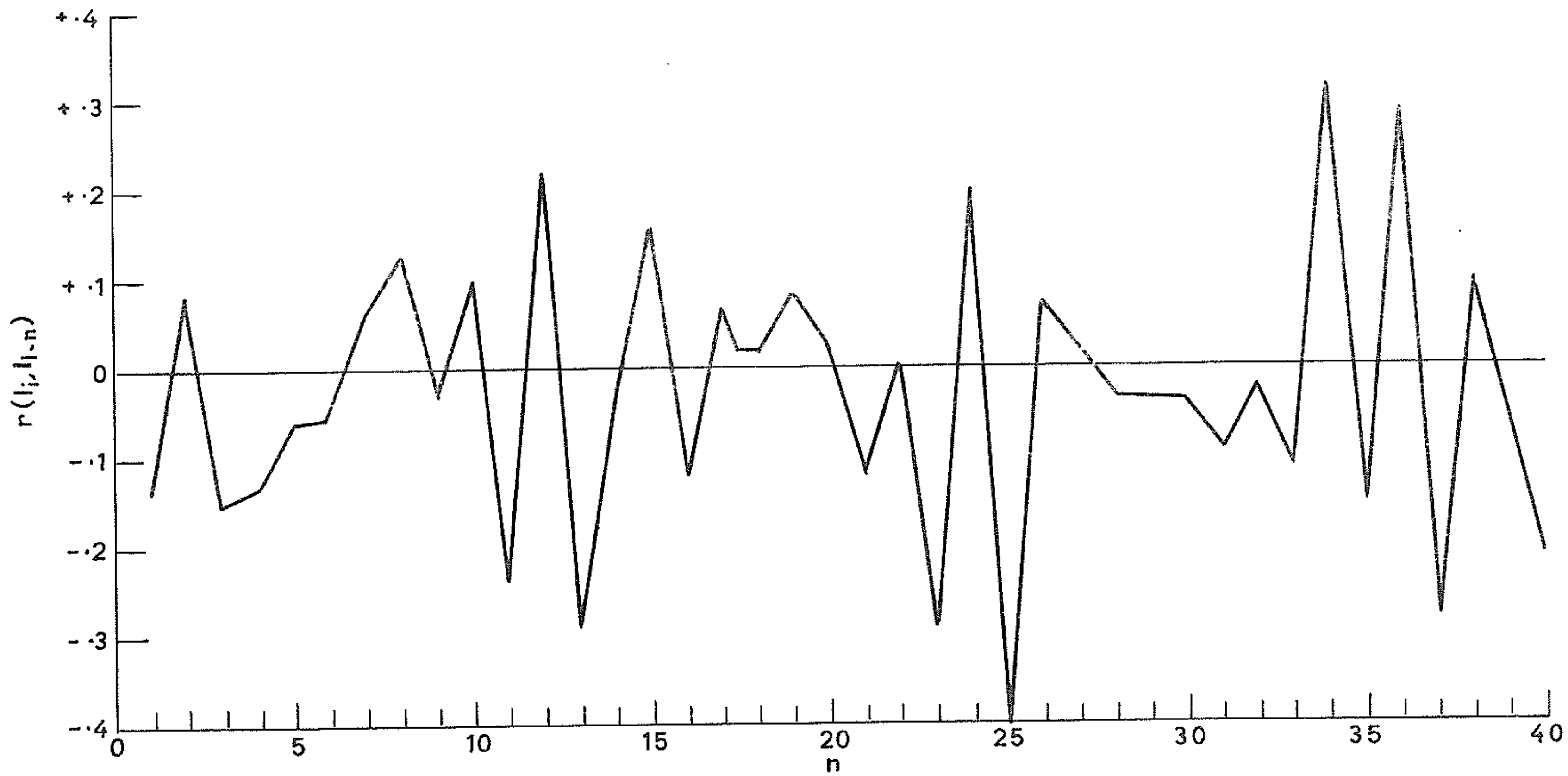

Fig. 2 Auto-correlation coefficients, Dublin Summers 1880-1964.

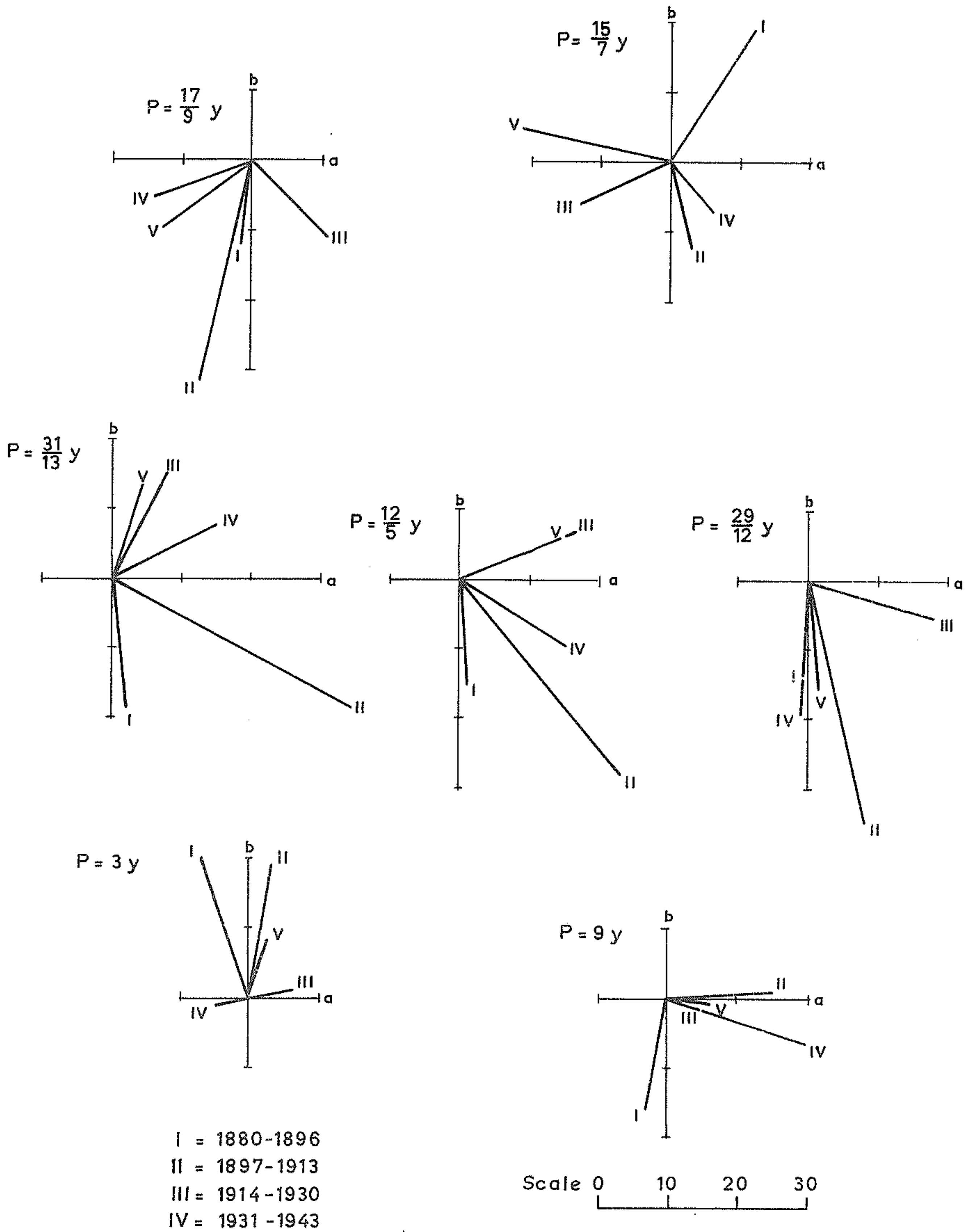

Fig. 4 Harmonic Dials. Values of a, b in $\left(a \cos \frac{2\pi n}{P} + b \sin \frac{2\pi n}{P}\right)$