INTRODUCTION OF A THERMODYNAMICALLY HYPERELASTIC MODEL FOR PEAT

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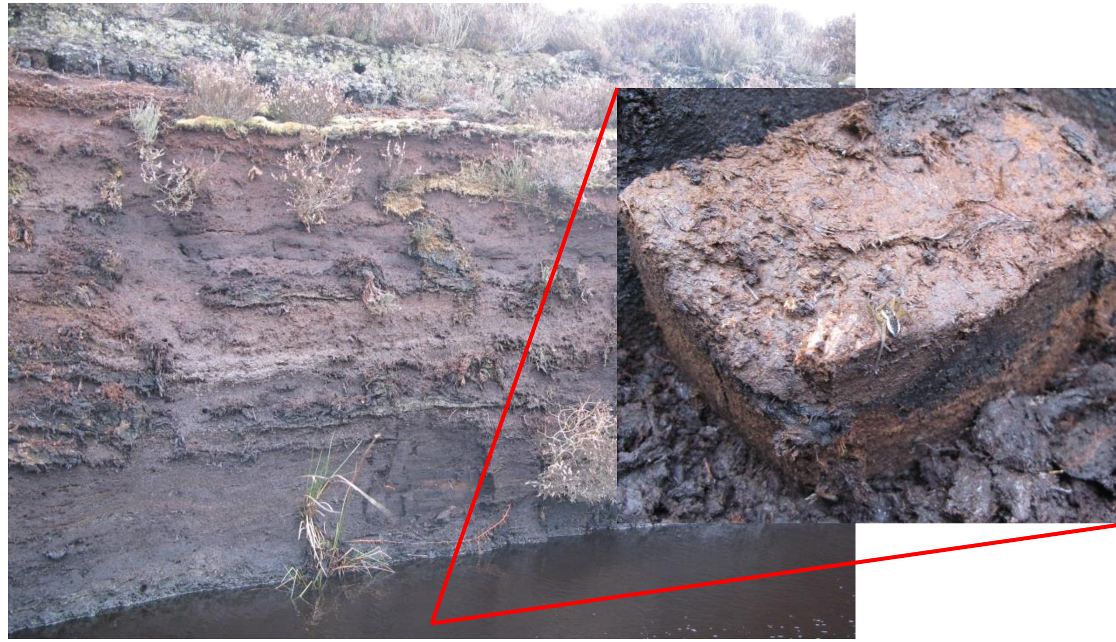
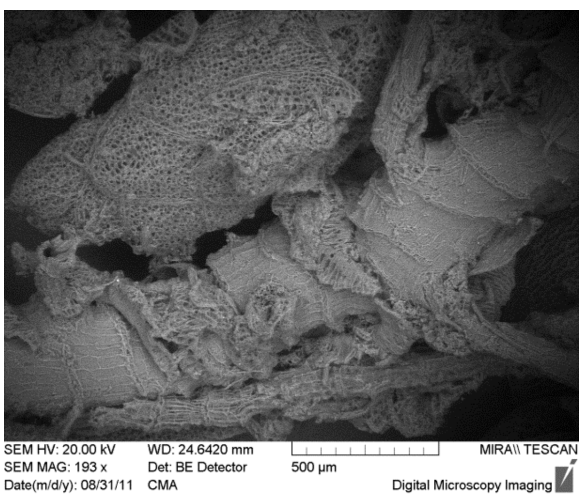
Introduction

What is peat?

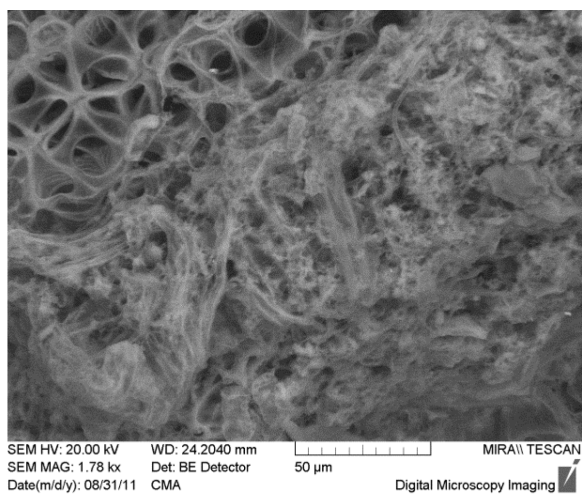


Undisturbed peat block taken from vertical face bank

Below: SEM and optical images



Zhang and O'Kelly (2014)



O'Kelly and Pichan (2013)

Introduction

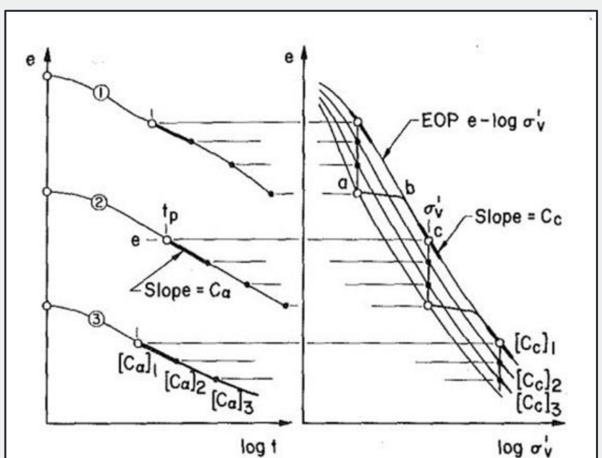
How does peat behave under loading?

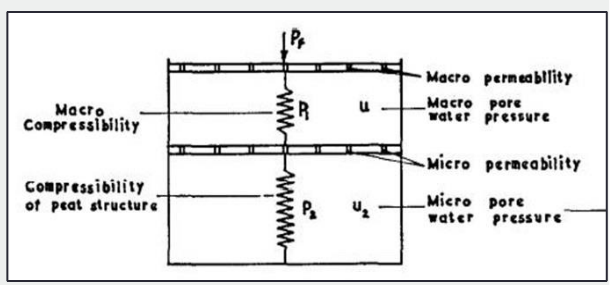
- Compared with mineral soils at similar water contents, peat has remarkably high strength;
- Extreme compressibility; e.g. specimens tested in drained triaxial compression may undergo up to 50% axial strain without shear failure occurring/reaching peak deviatoric stress (Adams, 1961);
- Significant secondary compression (creep);
- Fibrous nature introduces (strong) structural anisotropy;
- Fibrous peat has very high initial permeability which decreases dramatically under loading.

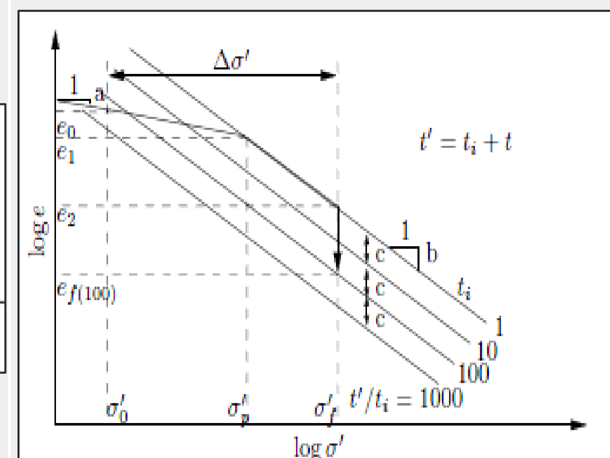
Constitutive Models Implemented for Peat

The main focus of constitutive models implemented for peat has been on its time-dependent behaviour under 1D loading.

Empirical Models	Rheological Models	$\sigma - \epsilon - \dot{\sigma} - \dot{\epsilon}$ Models
C_α / C_c concept: Mesri & co-workers (1977; 1979; 1985; 1987)	Barden (1968); Berry & Poskitt (1972); Edil & co-workers (1984; 1992; 1994)	Yin & Graham's (1989) EVP model (clay); den Haan (1996) *abc* model





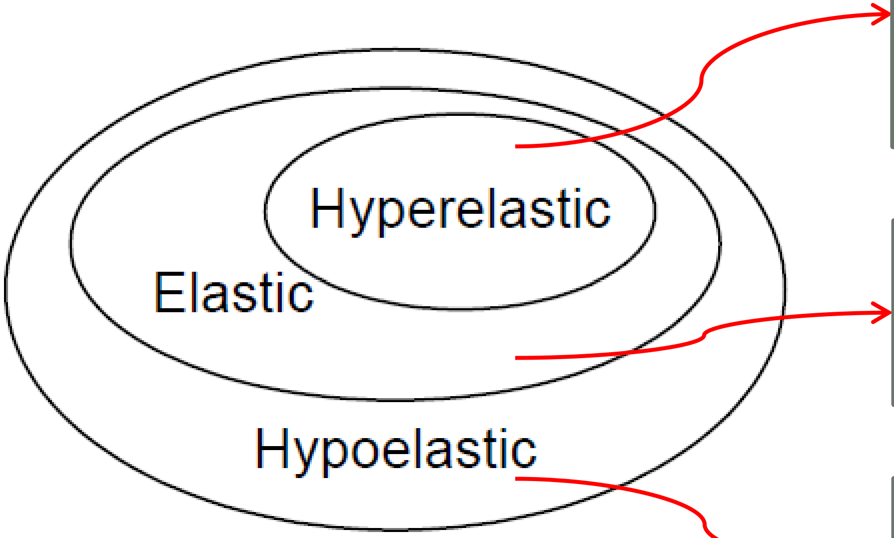


Why thermodynamically consistent approach?

- Constitutive models that violate thermodynamics cannot be used with any confidence in describing material behaviour, unless some rather particular and well-defined conditions apply (Houlsby & Puzrin, 2006).
- Main advantage of embedding constitutive models in a thermodynamical framework is that they cannot produce thermodynamically unreasonable results.
- The framework makes considerable use of potential functions that are closely related to variational and extremum principles.
- Within this single framework, a number of competing models can be more readily compared.

Why hyperelastic model?

- Analogue of the mechanical behaviour of peat with rubber and some bio-tissues.
- Houlsby & Puzrin (2006) claim that a hyperelastic model is preferred in modelling the elastic behaviour of soils.



$$\sigma_{ij} = \frac{\partial f(\varepsilon_{ij})}{\partial \varepsilon_{ij}} \qquad \dot{\sigma}_{ij} = \frac{\partial^2 f(\varepsilon_{ij})}{\partial \varepsilon_{ij}\,\partial \varepsilon_{kl}}\dot{\varepsilon}_{kl}$$

$$\sigma_{ij} = f_{ij}(\varepsilon_{ij}) \qquad \dot{\sigma}_{ij} = \frac{\partial f_{ij}(\varepsilon_{ij})}{\partial \varepsilon_{kl}}\dot{\varepsilon}_{kl}$$

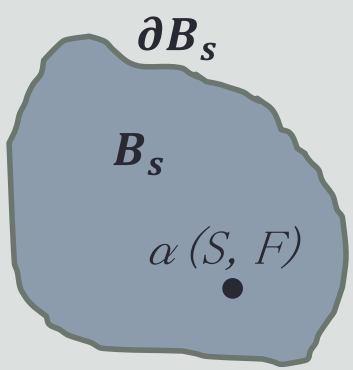
$$\dot{\sigma}_{ij} = f_{ijkl}(\varepsilon_{ij}, \sigma_{ij})\dot{\varepsilon}_{kl}$$

Volume Fraction Concept

Assumptions

- Biphasic material (solid matrix saturated with porewater)
- Solid matrix and porewater (fluid) are considered to be intrinsically immiscible and incompressible.
- The porous solid models the control space. Only the pore water contained in the pores can leave control space.
- The solid and porewater constituents simultaneously present in every point.

Formulation



$$\varphi^{\alpha}(x,t) = \frac{dv^{\alpha}}{dv}$$

$$\varphi^{S} + \varphi^{F} = 1$$

$$\rho^{\alpha}(x,t) = \varphi^{\alpha}(x,t)\rho^{\alpha R}(x,t)$$

Thermodynamically consistent approach

Balance Principles:

- Balance of mass $$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega} \rho(\boldsymbol{x}, t)\,\mathrm{d}\Omega = 0$$
- Balance of momentum $$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega} \rho\boldsymbol{v}\,\mathrm{d}\Omega = \int_{\partial\Omega} \boldsymbol{t}\,\mathrm{d}\Gamma + \int_{\Omega} \rho\boldsymbol{b}\,\mathrm{d}\Omega$$
- Balance of energy

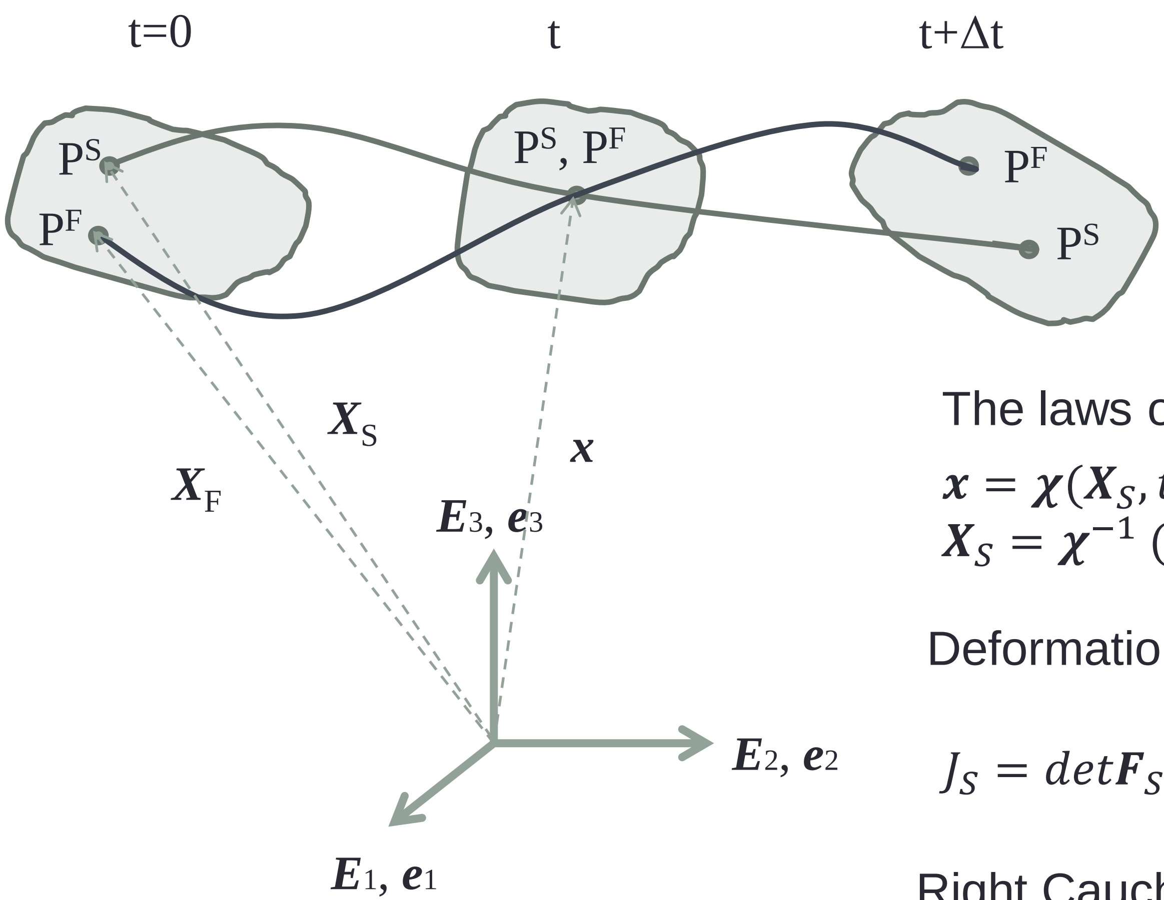
$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega} \frac{1}{2}\rho\boldsymbol{v}^2\,\mathrm{d}\Omega}_{\frac{\mathrm{d}}{\mathrm{d}t}E_{\mathrm{kin}}} + \underbrace{\int_{\Omega} \boldsymbol{\sigma} : \boldsymbol{d}\,\mathrm{d}\Omega}_{P_{\mathrm{int}}} = \underbrace{\int_{\partial\Omega} \boldsymbol{t}\cdot\boldsymbol{v}\,\mathrm{d}\Gamma + \int_{\Omega} \rho\boldsymbol{b}\cdot\boldsymbol{v}\,\mathrm{d}\Omega}_{P_{\mathrm{ext}}}$$

- Entropy inequality (2nd law of thermodynamics)

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega} \rho s\,\mathrm{d}\Omega - \int_{\Omega} \rho\frac{r}{\theta}\,\mathrm{d}\Omega + \int_{\partial\Omega} \frac{\boldsymbol{q}}{\theta}\cdot\boldsymbol{n}\,\mathrm{d}\Gamma \geq 0$$

Isotropic Hyperelastic Model

Strain measure



The laws of motion:

$$\boldsymbol{x} = \chi(\boldsymbol{X}_S, t) \quad \text{and} \quad \boldsymbol{x} = \chi(\boldsymbol{X}_F, t)$$
$$\boldsymbol{X}_S = \chi^{-1}(\boldsymbol{x}, t) \quad \text{and} \quad \boldsymbol{X}_F = \chi^{-1}(\boldsymbol{x}, t)$$

Deformation gradient: $\boldsymbol{F}_S = Grad\ _S\boldsymbol{x}$

$$J_S = det\boldsymbol{F}_S = \frac{d\Omega}{d\Omega_0}$$

Right Cauchy-Green tensor: $\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F}$

Isotropic Hyperelastic Model

<u>Stress measure</u>

- Effective stresses for solid and porewater (fluid) phases in spatial configuration:

$$\boldsymbol{\sigma}_S = -\phi_S p \boldsymbol{I} + \boldsymbol{\sigma}_S^E$$

$$\boldsymbol{\sigma}_F = -\phi_F p \boldsymbol{I} + \boldsymbol{\sigma}_F^E$$

- Assume dissipative stresses induced by the fluid viscosity are negligible, thus $\boldsymbol{\sigma}_F^E = 0$.

- Pulling back these stress relations into a material description yields the 2nd Piola-Kirchhoff stresses:

$$\boldsymbol{T}_S = -J_S \phi_S p \boldsymbol{C}_S^{-1} + \boldsymbol{T}_S^E$$

$$\boldsymbol{T}_F = -J_S \phi_F p \boldsymbol{C}_S^{-1}$$

- Total stress: $$\boldsymbol{T} = \boldsymbol{T}_S^E - p J_S \boldsymbol{C}_S^{-1}$$

Isotropic Hyperelastic Model

- Free Helmholtz energy density function:

$$\psi = \psi(I_1(\boldsymbol{C}), I_2(\boldsymbol{C}), I_3(\boldsymbol{C}))$$

$$I_1(\boldsymbol{C}) = \boldsymbol{C} : \boldsymbol{I}$$

$$I_2(\boldsymbol{C}) = \frac{1}{2}\left[(\boldsymbol{C} : \boldsymbol{I})^2 - \boldsymbol{C}\boldsymbol{C} : \boldsymbol{I}\right]$$

$$I_3(\boldsymbol{C}) = \det\boldsymbol{C} = J^2$$

- Coupled hydraulic and mechanical strain energy function obtained by adapting the Neo-Hookean law:

$$\psi_{iso} = C_{10}(I_1 - lnI_3 - 3) + D_2(lnI_3)^2$$

- At small strains, C_{10} and D_2 related to linear elastic Young's modulus E and Poisson's ratio ν

$$C_{10} = \frac{E}{4(1+\nu)}; \qquad D_2 = \frac{C_{10}\nu}{2(1-2\nu)}$$

Numerical implementation

- The material model has been implemented using ABAQUS, with the coupled hydraulic and mechanical material model coded as a UMAT user-subroutine in Fortran.
- A total Lagrangian description for a mixed $u - p$ formulation (where u is the solid matrix deformation and p the porewater pressure) has been implemented.
- At the time of writing the paper, the coded hyperelastic model is being validated against triaxial compression data for amorphous peat.

Conclusions & Recommendations

- In this research, a thermodynamically consistent hyperelastic model adopted from the biomechanical field has been presented as a starting point towards a new approach for modelling peat. This model draws analogies between the properties/behaviour of peat and some bio-tissues.

- Due to the complexity of the nature of peat, further features (time-dependency, structural anisotropy, plasticity when extending to larger strains, etc.) will be added to the isotropic hyperelastic model presented.

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Thank You!

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