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INTRODUCTION OF A THERMODYNAMICALLY HYPERELASTIC MODEL FOR PEAT

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1. Introduction

Peat deposits consist of the remains of dead plant vegetation, at various levels of decomposition, which have accumulated under waterlogged conditions. Such deposits are considered problematic due to low shear strength and very high compressibility.

At present, predictions of the geotechnical behaviour of peat for design practice are mostly based on constitutive theories developed for fine-grained mineral soil. Some specific models have been developed for soft clay and peat; e.g. isotache models (den Haan 1996) for simulating the 1D creep of amorphous peat and the two-level micro-structure rheological model (Berry & Poskitt 1972) for 1D consolidation of fibrous peat. In the early studies, only 1D consolidation (compression) conditions were considered. Many conventional models (e.g. C_α/C_c after Mesri & Castro (1987)) for constitutive modelling of peat do not consider the its micro-mechanism assumptions of consolidation/compression. Constitutive laws developed from routine curve-fit modelling are not in the differential form. Hence it is not convenient to directly extend such relations from 1D to 3D. Such constitutive relations may also violate physical laws. For instance, the stored plastic work in the modified Cam-clay model is ignored in the energy equation, which is not consistent with the yield function and flow rule. Artificial separation of hydraulic and mechanical behaviours is also not realistic for soil.

This paper introduces a hyperelastic model (after Nagel & Kelly (2012)) for peat within a thermodynamical framework. The thermodynamically consistent approach has been used previously for sand (Zhao 2011) and clay (Collins & Kelly 2002) but not for peat or other highly organic soils. This phenomenological model is a coupled mechanical and hydraulic model developed in 3D form, obeying the thermodynamic laws.

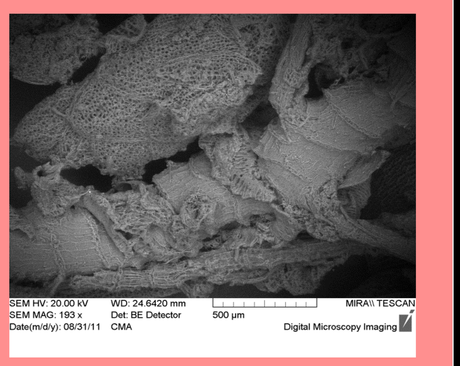

Figure 1. SEM image of *Sphagnum* peat fibres.

2. Volume Fraction Concept

The peat is assumed a biphasic material composed of a porous, organic solid structure (solid matrix) fully saturated with pore water. In the model, the solid matrix and pore water are intrinsically immiscible and incompressible. The hyperelastic model is proposed for amorphous peat. From initial work, the basic isotropic material model is introduced upon which more complex behaviour (structural anisotropy, creep etc.) can be added at a later stage.

To utilize the constitutive laws of continuum mechanics, the porous structure of peat is considered as 'smeared' over the solid and fluid phases. The considered domain is defined by the solid constituent. The solid and fluid constituents simultaneously present in every point. The volume fraction concept (de Boer 2005) is employed and assumes the porous solid always models a control space of B_s, with the boundary ∂B_s in the actual placement. Only the fluid contained in the pores can leave the control space. Using elements of the theory of mixtures restricted by the volume fraction concept, the porous media is modelled with a macroscopic body, where neither a geometrical interpretation of the poro-structure nor the exact location of individual components of the body is considered.

The porous structure of the saturated material consists of constituents α, with real volumes v^α: where α denotes the solid (S) and pore water (F) constituents. The boundary ∂B_s is a material surface for the solid phase and non-material surface for the pore water phase. The concept of volume fractions can be formulated as

$$\varphi^\alpha(x,t) = \frac{dv^\alpha}{dv} \qquad (1)$$

where: x, the position vector of the actual placement; t, corresponding time. The volume fractions φ^α satisfy

$$\varphi^S + \varphi^F = 1 \qquad (2)$$

where φ^F is porosity. Based on the volume fraction concept, all geometric and physical quantities (e.g. motion, deformation and stress) are defined in the entire control space. Hence they can be interpreted as the statistical average of the real quantities.

REFERENCES

Berry PL & TJ Poskitt (1972) The consolidation of peat. *Géotechnique 22(1)*, 27–52.

Collins IF & PA Kelly (2002) A thermomechanical analysis of a family of soil models. *Géotechnique 52(7)*, 507–518.

De Boer R (2005). *Trends in Continuum Mechanics of Porous Media*. Dordrecht, The Netherlands: Springer.

Den Haan EJ (1996) A compression model for non-brittle soft clays and peat. *Géotechnique 46(1)*, 1–16.

Houlsby GT & AM Puzrin (2006) *Principles of Hyperplasticity*. London, UK: Springer.

Mesri G & A Castro (1987) Time– and stress–compressibility interrelationship. *J. Geotech. Eng., ASCE 113(3)*, 230–247.

Nagel T & D Kelly (2012) Apparent behaviour of charged and neutral materials with ellipsoidal fibre distributions and cross-validation of finite element implementations. *J. Mech. Behav. Biomed. 9*, 122–129.

Zhao Q (2011) *A Thermomechanical Approach to Constitutive Modeling of Geomaterials*. Master's thesis, University of Hong Kong, China.

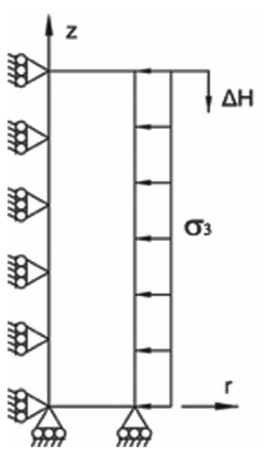

Figure 2. Triaxial compression model

3. Principles of Thermodynamics

The constitutive framework obeys the First and Second Laws of Thermodynamics. The First Law states that energy can neither be created nor destroyed during a process; it can only be transformed from one state to another. The Second Law clarifies the direction of energy transfer and makes a statement on the reversibility of processes. From the laws of thermodynamics, the balance principles (balance of mass, momentum and moment of momentum and balance of energy) have to be satisfied for each constituent α, in consideration of all interactions with external agencies.

According to Houlsby & Puzrin (2006), constitutive models violating thermo-dynamics cannot be used with any confidence in describing material behaviour, unless under some rather particular well-defined conditions. The main advantage of embedding constitutive models in a thermodynamical framework is that they cannot produce thermodynamically unreasonable results. The framework makes considerable use of potential functions which are closely related to variational and extremum principles (Houlsby & Puzrin 2006). The thermodynamical approach also allows a number of competing models to be cast within a single framework, enabling them to be more readily compared.

4. Isotropic Hyperelastic Model

From the principle of effective stress and considering the volume fractions φ^α and φ^α, the total stress of constituent α can be decomposed as

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_S + \boldsymbol{\sigma}_F = \boldsymbol{\sigma}_S^E - p\varphi^S \boldsymbol{I} - p\varphi^F \boldsymbol{I} = \boldsymbol{\sigma}_S^E - p\boldsymbol{I} \qquad (3)$$

where: p, pore water pressure; $\boldsymbol{I}$, the second-order unity tensor; $\boldsymbol{\sigma}_S^E$, the effective stress acting in the soil skeleton for which we provide a hyperelastic model.

In hyperelasticity, Cauchy stress can be derived from a Helmholtz free-energy function $\psi(\boldsymbol{C})$ by

$$\boldsymbol{\sigma} = \frac{2}{J}\boldsymbol{F}\frac{\partial\psi(\boldsymbol{C})}{\partial\boldsymbol{C}}\boldsymbol{F}^T \qquad (4)$$

where the deformation gradient $\boldsymbol{F}$ has been used to define the right Cauchy-Green tensor $\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F}$

As the Cauchy stress is not work conjugate to any convenient deformation measure, the 2nd Piola-Kirchhoff stress $\boldsymbol{T}$ is adopted as the stress measure. From the relation $\boldsymbol{T} = \boldsymbol{F}^{-1}J\boldsymbol{\sigma}\boldsymbol{F}^{-T}$ the 2nd Piola-Kirchhoff stress can be derived from the Helmholtz free-energy function as

$$\boldsymbol{T} = 2\frac{\partial\psi(\boldsymbol{C})}{\partial\boldsymbol{C}} \qquad (5)$$

Any isotropic tensor function can be expressed in terms of the principal invariants of its argument. The free Helmholtz energy density function can therefore be written as

$$\psi = \psi(I_1(\boldsymbol{C}), I_2(\boldsymbol{C}), I_3(\boldsymbol{C})) \qquad (6)$$

To reduce the number of material parameters and limit model complexity, a formulation independent of I_2 was chosen. Thus, the coupled hydraulic and mechanical strain function can be expressed as

$$\psi_{iso} = C_{10}(I_1 - lnI_3 - 3) + D_2(lnI_3)^2 \qquad (7)$$

where C_{10} and D_2 are material parameters. At small strains, these parameters are related to the linear-elastic Young's modulus E and Poisson's ratio ν

$$C_{10} = \frac{E}{4(1+\nu)}; \qquad D_2 = \frac{C_{10}\nu}{2(1-2\nu)} \qquad (8)$$

5. Numerical Implementation

The material model has been implemented using Abaqus, coding the coupled hydraulic and mechanical model as a UMAT user subroutine in Fortran. A total Lagrangian description for a mixed $u-p$ formulation has been implemented.

At the time of writing this paper, the coded hyperelastic model is being validated against triaxial compression data for amorphous peat. An axisymmetric model is used in simulating the test conditions (Figure 2), with an applied all-around confining pressure σ_3 simulating the effective confining pressure. The specimen's top surface is loaded axially via a rigid platen having zero contact friction. The deviator stress is obtained from the reaction force generated between the platen and the top of the specimen.

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6. Conclusions

As a starting point towards a new approach for modelling peat, a thermodynamically consistent hyperelastic model adopted from the biomechanical field has been presented, drawing analogies between the properties/behaviour of peat and some bio-tissues. Due to the complexity of the nature of peat, further features should be added to the isotropic hyperelastic model in order to adequately capture the material's significant creep and structural anisotropy features.

The application of the hyperelastic model is strictly only valid within the material's 'elastic' region. For peat, it is particularly difficult to separate the viscoelastic and elastic responses in compression and to define the extent of its elastic region. This can lead to under/over-estimations of elastic parameters values determined from experimental data for performing numerical simulations. Since the hyperelastic model does not consider soil hardening during compression, deformation paths towards failure cannot be approached. In order to simulate the stress paths towards failure, (more) realistic formulations of yield functions, flow rules and hardening rules should be developed specifically for peat.