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Lattice Input on the Inclusive τ Decay V_{us} Puzzle

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Recent analyses of flavor-breaking hadronic- τ -decay-based sum rules produce values of $|V_{us}| \sim 3\sigma$ low compared to 3-family unitarity expectations. An unresolved systematic issue is the significant variation in $|V_{us}|$ produced by different prescriptions for treating the slowly converging $D = 2$ OPE series. We investigate the reliability of these prescriptions using lattice data for various flavor-breaking correlators and show the fixed-scale prescription is clearly preferred. Preliminary updates of the conventional τ -based, and related mixed τ -electroproduction-data-based, sum rule analyses incorporating B-factory results for low-multiplicity strange τ decay mode distributions are then performed. Use of the preferred FOPT $D = 2$ OPE prescription is shown to significantly reduce the discrepancy between 3-family unitarity expectations and the sum rule results.

The conventional inclusive hadronic τ decay determination of $|V_{us}|$ ¹ is obtained by applying the finite energy sum rule (FESR) relation, involving polynomial weight $w(s)$ and kinematic-singularity-free correlator $\Pi(s)$ with spectral function $\rho(s)$,

$$\int_0^{s_0} w(s)\rho(s) ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s)\Pi(s) ds, \quad (1)$$

to the flavor-breaking (FB) difference $\Delta\Pi_\tau \equiv \left[\Pi_{V+A;ud}^{(0+1)} - \Pi_{V+A;us}^{(0+1)} \right]$, where

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$\Pi_{V/A;ij}^{(J)}(s)$ are the spin $J = 0, 1$ components of the flavor ij , vector (V) or axial vector (A) current-current 2-point functions. The spectral functions, $\rho_{V/A;ij}^{(0+1)}$, hence also $\Delta\rho_\tau$, are related to the normalized differential decay distributions, $dR_{V/A;ij}/ds$, of flavor ij V- or A-current-induced τ decay widths, $R_{V/A;ij} \equiv \Gamma[\tau^- \rightarrow \nu_\tau \text{ hadrons}_{V/A;ij}(\gamma)]/\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]$, by

$$dR_{V/A;ij}/ds = 12\pi^2 |V_{ij}|^2 S_{EW} \left[w_\tau(y_\tau) \rho_{V/A;ij}^{(0+1)}(s) - w_L(y_\tau) \rho^{(0)}(s) \right] / m_\tau^2, \quad (2)$$

with $y_\tau = s/m_\tau^2$, V_{ij} the ij CKM matrix element, $w_\tau(y) = (1-y)^2(1+2y)$, $w_L(y) = y(1-y)^2$, and S_{EW} a short-distance electroweak correction factor. The $J = 0$ (longitudinal) contributions in (2) are well known phenomenologically and, due to problems with the corresponding $D = 2$ OPE series, usually subtracted from dR/ds ^{1,2}. The subtracted result, $dR_{V/A;ij}^{(0+1)}/ds$, allows the construction of $J = 0 + 1$ reweighted analogues, $R_{V+A;ij}^w(s_0) = \int_0^{s_0} ds [w(s)/w_\tau(y_\tau)] dR_{V+A;ij}^{(0+1)}(s)/ds$, for any $w(s)$ and $s_0 < m_\tau^2$. Defining $\delta R_{V+A}^w(s_0) = \left[R_{V+A;ud}^w(s_0)/|V_{ud}|^2 \right] - \left[R_{V+A;us}^w(s_0)/|V_{us}|^2 \right]$, one has, for s_0 large enough to allow use of the OPE on the RHS of (1), ¹

$$|V_{us}| = \sqrt{R_{V+A;us}^w(s_0) / \left[\frac{R_{V+A;ud}^w(s_0)}{|V_{ud}|^2} - \delta R_{V+A}^{w,OPE}(s_0) \right]}. \quad (3)$$

This relation has usually been employed in un-reweighted form, with $w = w_\tau$, and the single value $s_0 = m_\tau^2$ ¹. This has the advantage that $R_{V+A;ud,us}^{w_\tau}(m_\tau^2)$ is determinable from branching fraction information alone, but the disadvantage of precluding tests of the s_0 - and $w(s)$ -independence of the analysis, which could otherwise be used to investigate potential systematic uncertainties (in particular, those associated with the treatment of OPE contributions). Such self-consistency tests were carried out in Refs. 2, 3, 4, and non-trivial $w(s)$ - and s_0 -dependences observed, suggesting shortcomings in the experimental data and/or OPE representation.

The most obvious potential OPE problem lies in the rather slow convergence of the $D = 2$ OPE series. In terms of the running \overline{MS} quantities $m_s(Q^2)$ and $\bar{a} \equiv \alpha_s(Q^2)/\pi$, the $D = 2$ series, which is known to 4-loops, is given by

$$[\Delta\Pi_\tau(Q^2)]_{D=2}^{OPE} = \frac{3}{2\pi^2} \frac{m_s^2(Q^2)}{Q^2} \sum_{k=0} c_k^\tau \bar{a}^k \quad (4)$$

with $c_k^\tau = 1, 7/3, 19.93, 208.75$ for $k = 0 \dots 3$ ⁵. Since $\bar{a}(m_\tau^2) \simeq 0.10$, $c_3^\tau \bar{a}^3 > c_2^\tau \bar{a}^2$ at the spacelike point on the contour for all $s_0 \leq m_\tau^2$. The problematic convergence complicates the assessment of $D = 2$ truncation errors, and manifests itself, e.g., in the ~ 0.0020 difference in $|V_{us}|$ values obtained using two alternate (CIPT or FOPT) versions of the 4-loop-truncated, w_τ -weighted series.

An alternate determination employs the FB combination $\Delta\Pi_{\tau-EM} \equiv 9\Pi_{EM} - 5\Pi_{ud;V}^{(0+1)} + \Pi_{ud;A}^{(0+1)} - \Pi_{us;V+A}^{(0+1)}$ in place of $\Delta\Pi_\tau$ ⁶. Inclusive electroproduction cross-sections fix the electromagnetic (EM) spectral function. By construction, the $\Delta\Pi_{\tau-EM}$ $D = 2$ series is strongly suppressed, having the form (4), with $c_k^\tau \rightarrow$

$c_k^{\tau-EM} = 0, -1/3, -4.384, -44.943$ for $k = 0 \dots 3$. The $D = 4$ series is also strongly suppressed. OPE contributions to $\Delta\Pi_{\tau-EM}$ FESRs, hence also estimated OPE errors, are thus very small⁶, and the resulting $|V_{us}|$ errors essentially entirely experimental. A check of this predicted suppression is thus of interest.

We investigate the relative merits of the fixed-scale (FOPT-like) and local-scale ($\mu^2 = Q^2$, i.e., CIPT-like) treatments of the $\Delta\Pi_{\tau}$ $D = 2$ series, and the level of $\Delta\Pi_{\tau-EM}$ suppression, by comparing OPE expectations and lattice data for the two correlator combinations over a range of Euclidean Q^2 . Five RBC/UKQCD domain wall fermion ensembles are employed, three, with $m_{\pi} = 293, 349, 399$ MeV, having $1/a = 2.31$ GeV⁸, and two, with $m_{\pi} = 171, 248$ MeV, having $1/a = 1.37$ GeV⁷. For technical reasons, conserved-local versions of the flavor us 2-point functions are numerically challenging and hence, for $\Delta\Pi_{\tau}$, local-local versions are used. To check that this does not produce residual lattice artifacts which would impact our conclusions, we have also performed the OPE-lattice comparison, using conserved-local data, for the alternate flavor-diagonal FB combination $\Delta\Pi_{diag} \equiv \Pi_{V;\ell\ell} - \Pi_{V;ss}$, whose $D = 2$ series is very similar to that of $\Delta\Pi_{\tau}$ ($c_k^{\tau} \rightarrow c_k^{diag} = 1, 8/3, 24.32, 253.69$ for $k = 0 \dots 3$ in (4)). The results confirm those of the local-local study.

Representative OPE-lattice data comparisons for $\Delta\Pi_{\tau}$ are shown, for the $1/a = 2.13$ GeV, $m_{\pi} = 293$ MeV ensemble, in Fig. 1. The left (right) panel comparison employs the fixed-scale (local-scale) prescription for the $D = 2$ OPE series. The fixed-scale versions match much better the Q^2 dependence of the lattice results, with the 3-loop-truncated version thereof best matching the overall normalization.

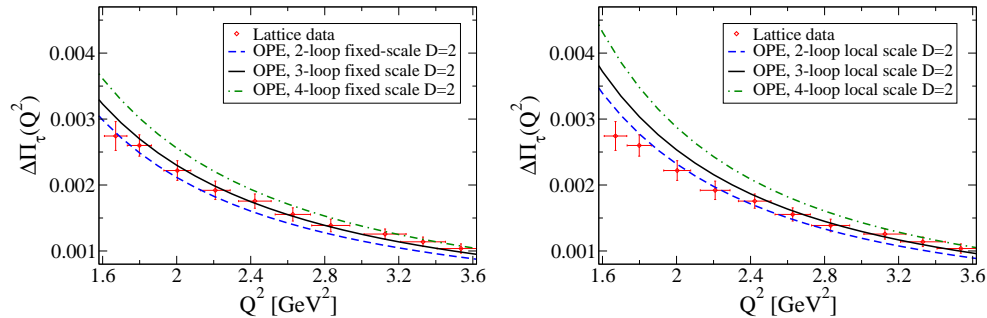
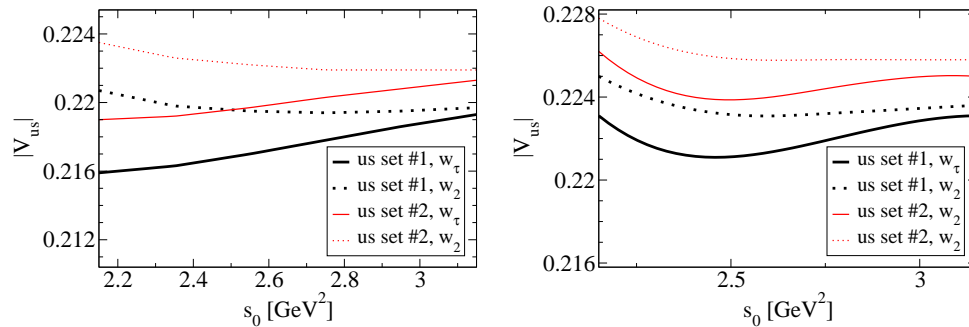


Fig. 1. OPE and lattice $\Delta\Pi_{\tau}$ data, $1/a = 2.31$ GeV, $m_{\pi} = 293$ MeV ensemble, $O(\bar{a}^{1,2,3})$ $D = 2$ OPE truncation, fixed-scale (left panel) or local-scale (right panel) $D = 2$ prescription

The comparison of lattice data for $\Delta\Pi_{\tau}$ and $\Delta\Pi_{\tau-EM}$ confirms the very strong suppression of $\Delta\Pi_{\tau-EM}$ ⁴ (see Ref. 4 for the relevant figure).

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 Fig. 2. $|V_{us}|$ from preliminary updates of the FB $\Delta\Pi_\tau$ and $\Delta\Pi_{\tau-EM}$ FESRs

We turn to preliminary updates of the $|V_{us}|$ analyses. For the $D = 2$ OPE series, we employ the 3-loop-truncated FOPT prescription favored by lattice data, and for the ud spectral integrals, OPAL data⁹, as updated in Ref. 10. For the us spectral integrals, recent B-factory results are used for the $K\pi$ ¹¹, $K^-\pi^-\pi^+$ ¹² and $K_s\pi^-\pi^0$ ¹³ exclusive mode distributions, and ALEPH results¹⁴, updated for current branching fractions (BFs), for all other modes. Contributions from the latter lie higher in the spectrum, and have much larger errors. The B-factory distributions are unit normalized, and also require current BFs for their overall scales. We work with BFs obtained in a $\pi_{\mu 2}$, $K_{\mu 2}$ -constrained HFAG fit, supplemented by the update to $B[\tau^- \rightarrow K_s^0\pi^-\pi^0\nu_\tau]$ produced by the recent Belle result¹³. Other non-trivial shifts in the us BFs also remain possible. To illustrate the changes to $|V_{us}|$ that could result, we consider also an alternate set of us BFs with the recent larger, but not yet finalized, BaBar results¹⁵ for $B[\tau^- \rightarrow K^-n\pi^0\nu_\tau]$, $n \leq 3$, used in place of those of the HFAG fit. The first set of us BFs is labelled “ us BF set #1” below, the second, alternate set “ us BF set #2”. Changes to the us BFs alter the inclusive us spectral distribution, and hence can affect both the magnitude of $|V_{us}|$ and the s_0 -dependence of the results. The significantly larger preliminary BaBar $K^-\pi^0$ BF is particularly relevant for the FB FESRs considered here, which weight more strongly the low- s part of the spectrum. We consider FESRs employing the weights w_τ and $w_2(y) = (1-y)^2$. w_2 weights less strongly the higher- s , large-error region of the us spectral distribution. Differences between results obtained using the two different weights can thus point to issues with the us spectral distribution.

$|V_{us}|$ results obtained from the w_τ and w_2 versions of the $\Delta\Pi_\tau$ FESR are shown, as a function of s_0 , and also the choice of the input us BF set, in the left panel of Fig. 2. Similar results for the $\Delta\Pi_{\tau-EM}$ FESR are shown in the right panel. w_2 results, which are less sensitive to the large-error high- s region, show better s_0 -stability in both cases. For w_τ , s_0 -stability is also better for the $\Delta\Pi_{\tau-EM}$ case,

where OPE contributions are suppressed. The convergence of w_τ results to the more stable w_2 ones as $s_0 \rightarrow m_\tau^2$, seen for both the $\Delta\Pi_\tau$ and $\Delta\Pi_{\tau-EM}$ FESRs, suggests the possibility of residual OPE problems in the w_τ case, where cancellations on the contour play a larger role. Finally we note that results obtained using the FOPT prescription preferred by the lattice data agree better with 3-family unitarity expectations than do those (not shown here) obtained using CIPT, as do those obtained using us BF set #2 in place of us BF set #1. More details of these analyses will be presented elsewhere.

We close by stressing the preference for FOPT over CIPT for the $D = 2$ OPE series. The prescription which underlies CIPT (of summing logarithmic terms to all orders while truncating the series of non-logarithmic terms), though plausible, is motivated by heuristic arguments not generally valid for divergent series¹⁶, and performs poorly when tested against lattice data for the FB correlators.

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