Productivity Trends in Ireland: A Rejoinder

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Introduction

In his reply to our earlier note (Sapsford and Kelly, 1980) Katsiaouni (1980) puts forward a number of justifications for the particular choice of sub-periods in his analysis of trend growth rates in output per man-hour in manufacturing between 1953 and 1973. However, despite these arguments the fact still remains that as a consequence of the incorrect selection of sub-periods, Katsiaouni's parameter estimates are unsatisfactory. In particular, as will be shown in the following section, Katsiaouni's estimated growth rate for his first chosen sub-period, is an upward biased estimate of the true growth rate during the "early part" of the study period.

Statistical Consequences of the Mis-specification of Sub-Periods

Consider Katsiaouni's estimate for his first sub-period, namely, 1953 to 1964, and recall that the results reported in our original note suggested the occurrence in 1957 of the structural switch. Under such circumstances it is easy to show that Katsiaouni's estimate, being derived from the "hybrid" period 1953-1964 (which begins to the left of, and ends to the right of, the switching point) is an upward biased estimate of the true growth rate during the first, structurally stable, sub-period, namely, 1953-1957. In addition, we also demonstrate that application of the regression method to this "hybrid" data period likewise yields an upward biased estimate.

To demonstrate this result, write the switching model in terms of specification (1) of our original note and assume, in the usual way, that $E(u_1) = E(u_2) = 0$.

Thus we can write

$$\ln P_t = \alpha_1 + \beta_1 t + u_{1t}, \quad \text{for } 1 \leq t \leq n_0$$

and

$$\ln P_t = \alpha_2 + \beta_2 t + u_{2t}, \quad \text{for } n_0 \leq t \leq n$$

(1)
Now, if one were to mistakenly evaluate an end points estimate (say $\hat{\beta}$) for the period beginning $j$ ($>0$) observations before and ending $k$ ($>0$) observations after switching point $n_0$, it is easy to show, using (1) and (2) that

$$E(\hat{\beta}) = \frac{\beta_1 j + \beta_2 k}{(j + k)}$$

(3)

from which we see that $\hat{\beta}$ is biased as a measure of both of the true growth rates $\beta_1$ and $\beta_2$.

The results reported in our original note imply that $n_0 = 1957$ and that $\beta_2 > \beta_1$ and using (3) we see that the bias in $\hat{\beta}$ as an estimate of $\beta_1$ is

$$\text{Bias } \hat{\beta} = E(\hat{\beta}) - \beta_1 = (\beta_2 - \beta_1) \cdot \frac{k}{(j + k)} > 0$$

(4)

The corresponding expressions for the regression estimate (say $\hat{\beta}$) computed for the same period, are as follows

$$E(\hat{\beta}) = \frac{\beta_1 \sum_{i=n_0-1}^{n_0} (t_i - \bar{t})^2 + \beta_2 \sum_{i=n_0+1}^{n_0+k} (t_i - \bar{t})^2}{\sum_{i=n_0-j}^{n_0+k} (t_i - \bar{t})^2}$$

(5)

where $\bar{t} = \frac{1}{n_0+k} \sum_{i=n_0-j}^{n_0+k} t_i$

from which

$$\text{Bias } \hat{\beta} = E(\hat{\beta}) - \beta_1 = (\beta_2 - \beta_1) \cdot \frac{\sum_{i=n_0+1}^{n_0+k} (t_i - \bar{t})^2}{\sum_{i=n_0-j}^{n_0+k} (t_i - \bar{t})^2} > 0$$

Conclusion

As was made clear in our original note, we have considered only one aspect of Katsiaouni's work. However, despite Katsiaouni's defence of his selected sub-periods, important statistical problems do remain. The possibility that his subsequent disaggregated estimates of the growth rates of productivity and other variables are likewise biased raises some nagging doubts about the validity of the use of these estimates as data for estimation of Verdoorn type regression equations and in particular, about potential problems of the errors in variables sort.

REFERENCES
