

The Irish Aggregate Import Demand Equation: the 'Optimal' Functional Form

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Précis: A more general approach than that of "best fit" for choosing empirically the appropriate formulation of the aggregate import demand function for Ireland is presented. This approach leads to the choice of a particular form, from a class of forms, for a given specification of the aggregate import demand function. It is found that the log linear form is preferred to the linear form where gross national income and the domestic to foreign price ratio are the explanatory variables. It is also found that the inclusion of a partial adjustment mechanism does not significantly improve the model specification.

I INTRODUCTION

In studies of aggregate import demand (Kreinin, 1967; Houthakker and Magee, 1969; Khan, 1974; and Magee, 1975), two functional forms have principally been used:

- (i) a linear formulation in which imports are assumed to be a linear function of the explanatory variables selected, and
- (ii) a log linear formulation in which the logarithm of imports demanded is assumed to be a linear function of the logarithms of the explanatory variables.

A recurring problem encountered in the literature is the choice of the appropriate functional form from within the restricted class of linear and logarithmic functions. The choice is made difficult and ultimately quite

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arbitrary for two reasons. First, at the theoretical level, there are no *a priori* economic criteria to indicate that one functional form is superior to the other. Secondly, at the empirical level, discrimination between functional forms based on certain "goodness of fit" criteria, principally the coefficient of determination (R^2), can be quite arbitrary. There are both economic and statistical implications of using one form of the equation rather than another. The use of a linear functional form, for instance, implies a decreasing price elasticity of import demand and an income elasticity tending toward unity. While the use of a logarithmic formulation implies constant elasticities with respect to price and income, this may be considered theoretically too restrictive in the case of an import function. Statistically, mis-specification of the functional form results in the violation of the classical properties for the error term. This results in the estimates being biased and inconsistent (Kmenta, 1971), and thus weakens its predictive power.

Previous studies of Irish import functions by Leser (1967), Baker and Durkan (1969), and McAleese (1970a, 1970b) have all used linear or log linear or lagged variants of either of these two forms for estimation purposes. McAleese (1970b, p. 399) identifies the problem of choosing between the two functional forms and the inability of researchers to select the appropriate functional form because of the absence of appropriate selective criteria.

The purpose of this paper is to present a more general approach than that of "best fit" for choosing empirically the appropriate formulation of the aggregate import demand function for Ireland. The procedure is based on the work of Box and Cox (1964). A generalised functional form of the import demand function is specified. This form includes as special cases both the linear and the log linear forms and also includes a whole range of other possible forms. The Box-Cox procedure consists essentially of determining which member of this class is optimal in a certain sense (to be defined) and also in determining whether or not a specific form within the class (e.g., linear) is acceptable in relation to the given observations.

The layout of the paper is as follows. In Section II the aggregate import demand equation in linear and log linear form is specified. Both these equations are modified to include a partial adjustment mechanism. The generalised functional form of the import demand equation is then defined along with the maximum likelihood method of estimating its parameters. The empirical results are presented in Section III and the summary and conclusion in Section IV.

II THE GENERALISED FORM OF THE AGGREGATE IMPORT DEMAND FUNCTION

The simplest specification of an aggregate import demand equation relates the quantity of imports to the ratio of the price of imports to domestic

prices and to the level of domestic real income.¹ This gives

$$M^d = F(P, Y) \quad (1)$$

where

M^d = the quantity of imports demanded,

P = the ratio of the price of imports to the domestic price level, and

Y = the real Gross National Product.

The sign of the partial derivative, $\delta M^d / \delta P$, is expected to be negative, while the sign of $\delta M^d / \delta Y$ is generally expected to be positive.² The linear formulation of the aggregate import equation for time t is

$$M_t^d = a_0 + a_1 P_t + a_2 Y_t + e_t \quad (2)$$

where e is assumed to be a random error term. If a logarithmic relationship is considered preferable, then the aggregate demand for the imports for time t is

$$\log M_t^d = \beta_0 + \beta_1 \log P_t + \beta_2 \log Y_t + e_t \quad (3)$$

Equations (2) and (3), as formulated, are *ex ante* relationships and the replacement of import demand, M^d , by actual imports implies instantaneous adjustment to changes in relative prices and real income. This, however, is regarded as an excessively restrictive assumption and can be relaxed by specifying a partial adjustment mechanism for imports in which the change in imports is related to the difference between the *ex ante* demand for imports in period t and the actual level of imports in the previous period. For the linear form, this model reduces to

$$M_t = a_0^* + a_1^* P_t + a_2^* Y_t + a_3^* M_{t-1} + e_t \quad (4)$$

Similarly, for the logarithmic form, the partial adjustment mechanism yields

$$\log M_t = \beta_0^* + \beta_1^* \log P_t + \beta_2^* \log Y_t + \beta_3 \log M_{t-1} + e_t \quad (5)$$

In this paper, equations specified as in (2) and (3) are termed Model I, and equations specified as in (4) and (5), which contain the partial adjustment mechanism, are termed Model II. Applying a power transformation to each of the variables and writing $M_t^d = M_t$ for notational convenience,

1. See Leamer and Stern (1970, ch. 1).

2. For a theoretical elaboration of this point, see Magee (1975). The ambiguity arises to the extent that imports can be viewed as the difference between domestic consumption and domestic production of importables less exports. If domestic income rises, domestic consumption may rise faster (slower) than domestic production. The partial derivative ($\delta M^d / \delta Y$) could then be positive (negative).

the generalised functional form in the case of equilibrium import demand (Model I) is

$$\frac{M_t^\lambda - 1}{\lambda} = \gamma_0 + \gamma_1 \frac{P_t^\lambda - 1}{\lambda} + \gamma_2 \frac{Y_t^\lambda - 1}{\lambda} + e_t \tag{6}$$

while for dynamic import demand (Model II) the form is

$$\frac{M_t^\lambda - 1}{\lambda} = \delta_0 + \delta_1 \frac{P_t^\lambda - 1}{\lambda} + \delta_2 \frac{Y_t^\lambda - 1}{\lambda} + \delta_3 \frac{M_{t-1}^\lambda - 1}{\lambda} + e_t \tag{7}$$

For $\lambda = 1$, equations (6) and (7) become identical to the linear specifications (2) and (4). For $\lambda = 0$, equations (6) and (7) reduce to the log linear equations (3) and (5). It should be noted that for $\lambda = 0$, the expressions involving λ appear to become indeterminate. However, if we expand, say, the transformed dependant variable, we obtain

$$\begin{aligned} \frac{M_t^\lambda - 1}{\lambda} &= \frac{e^{\log M_t^\lambda} - 1}{\lambda} = \frac{e^{\lambda \log M_t} - 1}{\lambda} \\ &= \frac{1}{\lambda} \{1 + \lambda \log M_t + \frac{1}{2}(\lambda \log M_t)^2 + \dots - 1\} \\ &= \log M_t + \frac{\lambda}{2} (\log M_t)^2 + \dots \end{aligned}$$

For $\lambda = 0$, $\frac{M_t^\lambda - 1}{\lambda} = \log M_t$ and similarly for the other variables.

The Box-Cox procedure as applied to Model I is presented here. Application of the procedure to Model II involves, in effect, merely the inclusion of an additional explanatory variable and does not materially affect the method. For notational convenience equation (6) is rewritten as

$$M_t(\lambda) = \gamma_0 + \gamma_1 P_t(\lambda) + \gamma_2 Y_t(\lambda) + e_t = \mu_t(\lambda) + e_t \tag{8}$$

The probability density of the untransformed observations, M_t , and hence the likelihood *in relation to these original observations* is obtained by multiplying this normal density by the Jacobian $J(\lambda; \underline{M})$ of the transformation. In our case

$$J(\lambda; \underline{M}) = \prod_t M_t^{\lambda - 1} \tag{9}$$

and, hence, the likelihood given the original observations is

$$\begin{aligned} L(\gamma_0, \gamma_1, \gamma_2, \sigma^2, \lambda/\underline{M}) &= \frac{1}{(2\pi)^{1/2n}} \cdot \frac{1}{\sigma^n} \cdot \exp \frac{-1}{2\sigma^2} \sum (M_t(\lambda) \\ &\quad - \mu_t(\lambda))^2 \cdot \prod_t M_t^{\lambda \cdot 1} \end{aligned} \tag{10}$$

The maximum likelihood estimation of the unknown parameters $(\gamma_0, \gamma_1, \gamma_2, \sigma^2, \lambda)$ is a two-stage procedure. First, for given λ , equation (10) gives the likelihood for a standard least squares problem apart from a constant factor. Hence, the maximum likelihood estimates of the γ 's are the least squares estimates for the regression problem with dependent variable $M_t^{(\lambda)}$ and explanatory variables $P_t^{(\lambda)}$ and $Y_t^{(\lambda)}$, and the maximum likelihood estimate of σ^2 , denoted for fixed λ by $\hat{\sigma}^2(\lambda)$, is

$$\hat{\sigma}^2(\lambda) = \frac{S^2(\lambda)}{n} \quad (11)$$

where $S^2(\lambda)$ is the residual sum of squares in the analysis of variance of $M_t^{(\lambda)}$. Hence for fixed λ , the maximised log likelihood is, except for a constant factor,

$$L_{\max}(\lambda) = -\frac{1}{2}n \log \hat{\sigma}^2(\lambda) + (\lambda - 1) \sum_t \log M_t \quad (12)$$

By performing the above analysis on the transformed observations, $M_t(\lambda)$, for a trial series of values of λ , it is possible to plot $L_{\max}(\lambda)$ against λ and from this plot to read off the estimated value of λ for which $L_{\max}(\lambda)$ is itself maximised with respect to λ . Based on the orthodox large sample theory of maximised likelihood estimation, it can be shown that a 100 $(1 - \alpha)$ per cent confidence interval for λ is given by values of λ such that

$$L_{\max}(\hat{\lambda}) - L_{\max}(\lambda) < \frac{1}{2} \chi_1^2(\alpha) \quad (13)$$

Finally, this confidence interval enables us to examine the acceptability or otherwise of any hypothesised value of λ (e.g., in particular $\lambda = 0$ (log linear) or $\lambda = 1$ (linear)).

The Box-Cox procedure as applied here consists of considering the generalised functional forms of equations (6) and (7) for a series of values of λ , obtaining the maximum likelihood estimates of the parameters of the transformed model for each such λ , finding the value of λ for which the log likelihood in relation to the original observations is maximised, and finally examining in particular the position of the linear and log linear models within this class of functions.

III EMPIRICAL RESULTS

Annual data for the period 1953 to 1975 inclusive are used in this study. Real imports in year t , M_t , are the value of imported goods and services at constant 1970 prices. Real Gross National Product (GNP) in year t , Y_t , is GNP at constant 1970 prices. The price variable for year t , P_t , is the ratio

Table 1: Parameter estimates, t -values^a, R^2 s and the d statistics for the linear and log linear forms of Models I and II

Model	Dependent Variable	Constant	P_t	$\text{Log } P_t$	Y_t	$\text{Log } Y_t$	M_{t-1}	$\text{Log } M_{t-1}$	R^2	d
I	M_t	-423.62 (- 11.19)	- 83.96 (- 1.05)		0.69 (29.47)				.9878	2.15
	$\text{Log } M_t$	- 6.75 (- 15.69)		-0.52 (-3.37)		1.79 (30.76)			.9910	2.02
II	M_t	-390.88 (- 2.61)	-115.10 (- 0.92)		0.65 (3.42)		0.06 (0.24)		.9876	0.26
	$\text{Log } M_t$	- 4.99 (- 3.09)		-0.60 (-3.60)		1.37 (3.62)		0.22 (1.15)	.9913	1.02

a t -values in parentheses.

of the import price index (base 1970) to the Wholesale Price Index (base 1970), both in year t . The Wholesale Price Index series is taken from the Irish Statistical Bulletin (various issues). All other series are taken from the Data Bank of Annual Economic Time Series, 1977 of the Central Bank of Ireland's Research Department.

First, certain statistics associated with equations (2) and (3) (the linear and log linear forms of Model I) and equations (4) and (5) (the linear and log linear forms of Model II) are estimated. These include the parameter estimates and their respective t -values, the coefficient of determination (R^2) and the Durban-Watson statistic for autocorrelation (d). These statistics are presented in Table 1.

The results from Model I show that the parameter estimates have the signs expected from theory in both the linear and log linear case. The parameter estimate for income has about the same degree of significance for both the linear and log linear form. At the 5 per cent significance level the parameter estimate for the price variable is not significant for the linear form, but is significant for the log linear form. Both the linear and log linear form give a very high R^2 and a d statistic which allows acceptance of the hypothesis that there is no autocorrelation. Hence, by reference to these criteria it is difficult to discriminate between these two forms of specification.

Similarly, in the case of Model II the signs of the parameter estimates for both forms of the equation are as expected. Again, as in Model I, the income parameter estimate has approximately the same level of significance under both forms while the price variable parameter estimate is not significant under the linear form, but is significant under the log linear form. The parameter estimate for the lagged variable is not significant under either form. Again, the model seems to be well specified under either form, given the magnitude of the R^2 . Whereas the hypothesis of non-autocorrelation must be rejected under the linear form, under the log linear form the d statistic lies in the inconclusive region. However, recognising that the d statistic is inappropriate in the case of lagged models, an alternative test (Durbin, 1970) was carried out which allowed acceptance of the hypothesis that there is no autocorrelation in either the linear or log linear form of Model II. On the basis of these results, one thus cannot justify the choice of one form rather than the other in the case of either Model I or II.

$L_{\max}(\lambda)$ is calculated for Models I and II for values of λ from -1.5 to $+1.5$ at intervals of 0.1 . This series of values includes $\lambda = 0$ and $\lambda = 1$. A 95 per cent confidence interval for λ is derived as explained in Section II. $L_{\max}(\lambda)$ plotted against λ and the 95 per cent confidence interval for λ is shown in Figure 1. It may be observed that $L_{\max}(\lambda)$ in the range examined is a uni-model function having a unique maximum value in the case of both Models I and II.

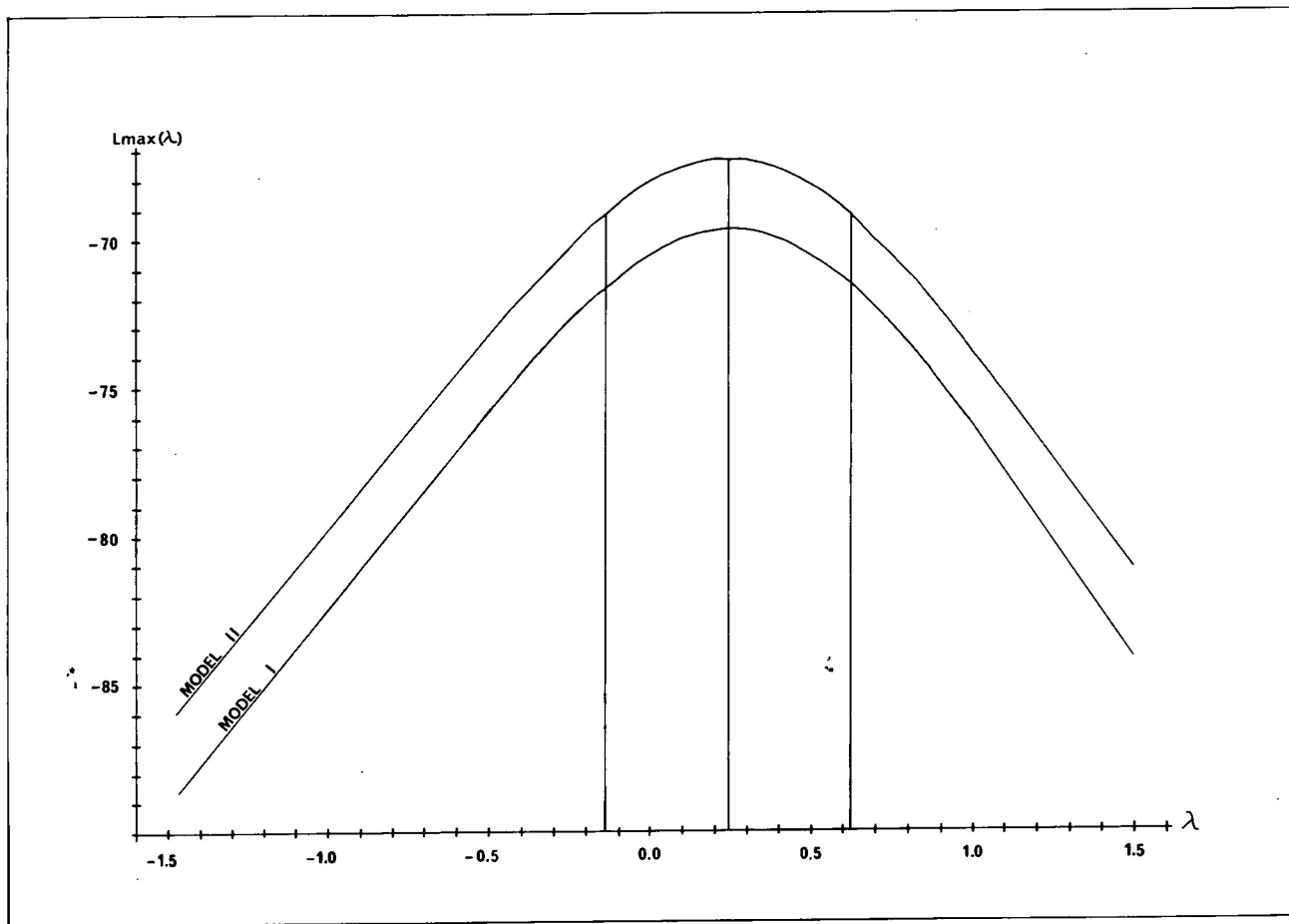


Figure 1. $L_{\max}(\lambda)$, for Models I and II, for the range $\lambda = -1.5$ to 1.5.

The maximum value of $L_{\max}(\lambda)$ for Model I occurs at $\lambda = 0.24$. The 95 per cent confidence interval for λ is $(-0.14, 0.62)$. This includes $\lambda = 0$ and does not include $\lambda = 1$. The hypothesis is that $\lambda = 0$ cannot be rejected whereas the hypothesis is that $\lambda = 1$ can be rejected. It may, therefore, be concluded that the log linear formulation of the import demand function as specified in Model I is superior to a linear formulation. The maximum value of $L_{\max}(\lambda)$ for Model II occurs also at $\lambda = 0.24$ and the 95 per cent confidence interval for λ is $(-0.15, 0.62)$. As in the case of Model I, this includes $\lambda = 0$ and excludes $\lambda = 1$. Thus, in the case of Model II also, the log linear formulation is superior to the linear formulation.

The import demand elasticity with respect to price and income is important for longer-term forecasting. It is of interest to compare the elasticities derived in this study with those of previous studies. The elasticities derived from the log linear form in this study are the more appropriate to compare with elasticities from other studies, since the log linear form was found to be acceptably close to the optimal functional form. It might be noted that the income and price elasticities derived from the optimal functional form are both very close to those derived from the log linear form in both Models I and II.

The price elasticity derived from Models I and II are below the lowest value in McAleese's (1970a) range and considerably below Leser's (1967) value (Table 2). Similarly, the income elasticities derived from Models I and II are below the lowest value of McAleese's while that of Model I is greater, and that of Model II less, than the value derived by Leser.

Table 2: *The price and income elasticities of import demand*

Elasticity	Log Linear		McAleese	Leser
	Model I	Model II		
Price	-0.56	-0.68*	-0.89 - -1.53	-1.38
Income	1.79	1.56	1.87 - 2.15	1.61

*Long-run elasticities

IV CONCLUSIONS

This paper is concerned with finding the appropriate form for an aggregate import demand function for Ireland and, in particular, discriminating between the linear and log linear formulations of a standard specification. The absence of theoretical grounds for choosing a particular formulation necessitated an empirical approach.

The Box-Cox procedure led to the choice of the log linear formulation from a whole class of formulations and, in particular, over the linear form-

ulation in both a static and a partial adjustment model. The partial adjustment model added nothing to that of the static model. The log linear formulation of the static model thus seems the more appropriate form and specification of the aggregate import demand function for Ireland.

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