Verdoorn’s Law: A Retrospective View

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Abstract: We examine the origins of the Verdoorn law and the key role it plays in Kaldor’s growth analysis. We suggest that two empirical studies of the Irish economy (Kennedy, 1971 and Kennedy and Dowling, 1975) can be interpreted within a Kaldorian framework. We contrast this framework with the alternative methodology based on the neo-classical theory of the firm and suggest that the latter may have provided a better picture of how economies hit by the supply shocks of the 1970s reacted to those shocks. Finally, we illustrate how Verdoorn “type” laws can be derived using the neo-classical approach.

I INTRODUCTION

In this paper we examine some aspects of Verdoorn’s law and its applications to the study of the supply side of the Irish economy, and compare this approach with a more orthodox mainstream neo-classical approach. An “orthodox” approach to the study of supply issues is by means of the neo-classical apparatus of production functions and theories of the firm, where factor inputs are treated as derived demands in an optimising framework. These factor demand equations can, of course, be rewritten as factor productivity relationships and, in the case of labour inputs, a relationship between output and labour productivity is implicit in them. Such a relationship has been separately studied under the name of the “Verdoorn law” and there has been much controversy surrounding the interpretation of this law and its role in economic theories of growth.¹ Two major studies of economic growth in Ireland over the period 1947 to 1972 specifically invoked the Verdoorn law and certain elements of Kaldor’s growth analysis (Kennedy, 1971 and Kennedy and Dowling, 1975). A more recent review of growth and transformation of developed capitalist economies, in addition to advocating a growth theory along broadly Kaldorian lines, expressed explicit hostility to applications of neo-classical methodology to growth analysis (Cornwall, 1977).

¹Boulier (1984) is the most recent example to hand.
Of course these particular aspects are merely elements of a very extensive methodological controversy. We have a very limited objective, namely to attempt to arrive at an understanding, much assisted by hindsight, of some of the main features of the growth analysis contained in Kennedy (1971) and Kennedy and Dowling (1975). We feel this to be of some importance since it appears to us that the work of Kennedy and Dowling, in so far as it can be classified, is more in the Kaldorian than the neo-classical mould. An attempt to make such methodological assumptions explicit may be of interest in the light of the neo-classical research programme which has dominated empirical work in Ireland and elsewhere since the publication of the above works.

In Section II we review Verdoorn's original statement of his "law" and examine his reasons for working with a simple relationship between productivity growth and output growth. We also briefly examine some of the uses Kaldor and others have made of the Verdoorn law and the growth models that have developed round it. Section III is devoted to a re-examination of two studies of Irish economic growth (Kennedy, 1971 and Kennedy and Dowling, 1975). Finally, in Section IV we draw some general conclusions about the relative value of different approaches and attitudes to the study of the supply side of the economy and how the major economic shocks suffered by the Irish economy during the 1970s may have rendered the use of Verdoorn's law a less appropriate guide to the complex issues involved.

II VERDOORN'S LAW AND KALDOR'S ELABORATIONS

The relationship between productivity and output growth was first noted in the early paper by Verdoorn (Verdoorn, 1949). Using data for the volume of industrial production and labour productivity for a range of countries and for different time periods, Verdoorn suggested the existence of a "fairly constant relationship over a long period between the growth of labour productivity and the volume of industrial production". Letting \( q \) and \( Q \) represent labour productivity and output, this relationship (subsequently referred to as "Verdoorn's law") can be written as

\[
q_t = a_0 + a_1 Q_t \tag{2.1}
\]

where \( a_1 \) measures the Verdoorn elasticity and \( a_0 \) measures "autonomous" changes in labour productivity. Since \( q = Q/L \) (\( L \) measuring labour inputs), we may rewrite (2.1) also in the form

\[
L = -a_0 + (1 - a_1) Q \tag{2.2}
\]

Verdoorn's original empirical analysis (carried out using data for varying

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2. Originally published in Italian, a translation by Dr. Finola Kennedy, was very kindly made available to us.
periods between 1841 and 1938) suggested a value for \( a_1 \) of 0.45 with tight lower and upper bounds of 0.41 and 0.57. These early results were the motivating force behind a large subsequent empirical analysis by other researchers, continuing up to the present day.\(^3\) Much of this work has been carried out in a context where the neo-classical theory of the firm (i.e., production functions, profit maximisation, etc.) is explicitly rejected and where the Verdoorn law is used as an alternative way of handling "technology" considerations. Before turning to these aspects, it is interesting to examine Verdoorn's original motivations for working with the productivity-output relationship, particularly in the light of his very recent contribution to the continuing "Verdoorn" controversy (Verdoorn, 1980).

Verdoorn was very much concerned with the early plans for economic revival and recovery in post-war Europe. Two aspects of planning evaluation were of importance:

(i) If a plan were couched in terms of projected output and projected labour productivity, could it be checked for technical feasibility and plausibility?

(ii) If the plan objective was to absorb a given availability of labour, what output growth would be required?

To answer these questions, Verdoorn dismissed the use of production analysis from the very start.\(^4\) It is not very difficult to suggest reasons why the production function approach was not explored. The available techniques were largely restricted to Cobb-Douglas two factor functions, with neutral technical change. Data availability (particularly for capital stock and investment) was poor. Statistical estimation was a cumbersome business for all but simple bivariate models. However, one reason which cannot be advanced to explain Verdoorn's lack of enthusiasm for production functions is any rejection by him of the neo-classical theory of the firm since, in his 1949 paper, he provided a justification of the constancy of the Verdoorn elasticity using a simple neo-classical growth model. His 1980 paper made minor corrections to this derivation and essentially uses what has become known as the Solow growth model (Solow, 1956). This model assumes a single commodity produced by labour and capital under standard neo-classical conditions with a constant savings ratio and an exponentially growing labour force.

From such illustrative derivations of Verdoorn's law, it emerges that the Verdoorn elasticity is only constant asymptotically, i.e., in the very long run. Verdoorn concluded, rather ruefully, that "the 'law' that has been given my name appears, therefore, to be much less generally valid than I was led to believe in 1949" (Verdoorn, 1980, p. 385).

3. McCombie (1982) provides the most recent comprehensive empirical survey of the area.
4. "Since one cannot count either on the assumption of a constant annual rate of growth of productivity, nor on the use of the production function, one needs to adopt a third method of approach".
An interesting aspect of his 1980 paper is that it shows that Verdoorn always thought of his "law" as essentially interpretable within a neo-classical framework of analysis. Indeed in his 1949 paper he stressed the fact that his "law" appeared to lead directly to a research programme of further investigation.5

Verdoorn's 1949 paper remained rather neglected until Kaldor's famous inaugural lecture (Kaldor, 1966) in which he used the Verdoorn law to examine the slow growth rate of the post-war UK economy. He couched Verdoorn's insights within a more general framework, suggesting that neo-classical economic models could not explain the widely differing growth rates in similarly developed countries which occurred in the 1953 to 1965 period. A recent issue of the *Journal of Post-Keynesian Economics* contained the results of a symposium on Kaldor's growth laws (Thirlwall, 1983) published to honour Kaldor's seventy-fifth birthday. In that issue, Thirlwall summarised very succinctly the key elements of Kaldor's growth analysis.

(i) Kaldor suggests that fast rates of economic growth are associated with rapid growth in the manufacturing sector — in other words the manufacturing sector is the "engine" of growth which induces productivity growth in other sectors. This viewpoint was used in his inaugural lecture to explain the poor UK growth record in terms of the UK's "premature maturity", i.e., having reached the stage of roughly equal productivity in all sectors of the economy before attaining a high level in manufacturing industry.

(ii) The Verdoorn law (often referred to as Kaldor's second law) describes the manner in which rapid output growth in manufacturing leads to productivity growth owing to static and dynamic returns to scale. Furthermore, as a result of the increase in activity in manufacturing, labour is transferred into that sector from other sectors where there are either diminishing returns or no relationship between output and productivity. Hence, overall productivity growth is positively related to output and employment growth in manufacturing and negatively associated with growth of employment outside manufacturing. As opportunities for the transfer of factors begin to dry up, the driving role of manufacturing industry becomes weaker and the overall growth rate diminishes.

(iii) The initiation of a process of growth is determined by demand from the agriculture sector at first, but by export demand at a later stage. Under certain conditions a rapid growth of exports can set up a cumulative process (or virtuous circle) through the link between output growth and productivity growth given certain price and wage responses.

5. "In the cases where (elasticity) differences are very remarkable, I would suggest treatment of a more general nature. Taking account of other variables (-----) one can try to find less rigid rules between the requirements of capital and those of labour. With this aim in view, the method put forward, while not directly applicable for practical purposes, serves to establish points of departure for research in this direction".
Although Kaldor's analysis of this process is rich in insights, his writings on the subject have tended to present his ideas verbally and in a manner which makes it difficult to check internal consistency and to use the approach for policy analysis. Subsequent writers have brought the Kaldorian model into a more analytic form and have clarified its structure (Dixon and Thirlwall, 1975; Thirlwall, 1980; Cornwall, 1977). In all these works the central role played by the Verdoorn law is emphasised. However, more recently Thirlwall (1983) seems to back away from the importance of Verdoorn's law. Such an eclectic attitude may pose problems for further analytic work within the Kaldor framework since the hypotheses underlying it have become very difficult to test empirically using economic data.

III VERDOORN'S LAW AND THE ANALYSIS OF IRISH GROWTH

Kaldor's approach and Verdoorn's law appeared to hold out the prospect of providing explanations, at once comprehensive and analytically simple, of the process of economic growth and development. Hence, it is not surprising that the methodology was widely applied in theoretical and empirical studies. In particular, the Irish economy of the period 1947 to 1972, with its rapid economic growth after 1960, its dualistic nature and the export-led characteristics of growth, was such as to invite the application of Kaldorian ideas.

The first such study, Kennedy (1971) was concerned with two main issues: the causes of productivity growth differences among Irish manufacturing industries and the existence and explanation of the association between long-term changes in productivity and in output, both in broad industrial groupings and for individual industries, i.e., the Verdoorn law. The existence and stability of Verdoorn's law was established both for broad industrial groups (in particular, the standard ten-industry groupings of the Irish Census of Industrial Production (CIP)), and for a wide selection of the individual CIP industries. Other explanatory variables in the productivity-output relationship were investigated, the most important being the capital stock, \( K \). It was found that high capital-labour (\( K/L \)) ratios tended to be associated with high productivity for industry groupings in cross-sections. However, the importance of this finding was questioned on the grounds that the above relationship involved circular reasoning. In order to avoid the above circularity, a measure of the change in combined factor inputs

6. "Moreover, Verdoorn's law is not (contrary to the popular view) an indispensable element of the complete Kaldor growth model. ... Moreover, a breakdown of the Verdoorn relation, if it has broken down, does not undermine the model either (Thirlwall, 1983, pp. 354 and 357).
7. Thirlwall (1984) is the most recent example.
8. The capital measure being used was derived from insurance estimates and thus reflected the expected future returns to capital. However, the remainder of net output in a given year provides an approximation to the industries future profit. Hence, the regression is largely tautological since the explanatory variable is so largely defined by reference to the "explained" variable.
was calculated, weighting each input by the relative return to the factor in a base year, thereby deriving a measure of combined factor productivity. It was found that the inclusion of capital did not contribute much to explaining variations in productivity between industries and that any capital effect is dominated by the output effect. No reasonable variation in weights changed this conclusion. Hence, any role of the capital stock in "explaining" the Verdoorn law was effectively dismissed by Kennedy (1971).

The influence of earnings, unit costs, and prices on the Verdoorn law was then examined. The empirical findings tended to reject any role for these variables in the Verdoorn law. For example, labour productivity growth was only very weakly correlated with growth in annual earnings, which, moreover, showed little inter-industry variation. As a consequence, productivity was strongly negatively correlated with unit labour costs. Changes in materials costs likewise could explain little of the variation in productivity, nor also could gross output prices.

From the perspective of the neo-classical theory of the firm these results are difficult to rationalise. The first criticism that can be levelled against the methodology being employed is that it lacks any consistent and articulated model of firm's behaviour which is, in some formal sense, being tested by data. Secondly, and with the benefit of hindsight, it is clear that since relative prices were rather stable during the 1950s and 1960s, it would be quite difficult to find statistically significant relative factor price and factor substitution effects anyway. Thirdly, the Verdoorn technique, although a close approximation to a neo-classical analysis when relative prices are stable, is likely to provide little guidance during a period of highly unstable relative prices such as that which followed the first oil and commodity price shocks of the early 1970s.

In seeking a "theory" to explain the Verdoorn law, a range of other possible elements was examined in Kennedy (1971). Two major elements were finally isolated: technical change and economies of scale. The issues of endogenous technical change and economies of scale were held to be closely interrelated and the most plausible explanation of Verdoorn's law, consistent with observed movements in relative costs and prices, was one that allowed for such inter-relationships. Such an explanation placed its main emphasis on stimuli provided by the growth in output. In the Irish case, the role of technological progress was seen in terms of applying new techniques developed elsewhere. Unfortunately, no analytical model or empirical examination of these interesting theories was provided so they must remain plausible but untested hypotheses underlying the Verdoorn law. In fact, a reading of later work on the Verdoorn law in the "post-Keynesian" tradition (Cornwall, 1977) leads one to suspect that "post-

9. However, Boyle and Sloan (1982) using CIP data for the period 1953-1973, and a time-series approach, found highly significant factor substitution effects.
Keynesians regard the law as an analytic model and test of these theories and not independent from them. This would also explain the relative lack of interest by "post-Keynesians" in any attempt to base the Verdoorn relationship on the neo-classical theory of the firm.

Finally, given the initial claims by Verdoorn and Kaldor in respect of the stability of the Verdoorn law, Kennedy's conclusions on the matter were considerably less sanguine than the notion of a "universal constant" permeating much early work. In fact, such conclusions could be said to invalidate partially Verdoorn's original reasons for rejecting the production function approach, which he held to be less stable than the simpler productivity-output relationship to which his name has been given.

While Kennedy (1971) dealt with the manufacturing sector alone and was confined to, what one might classify as, issues of industrial technology and growth, the second work, Kennedy and Dowling (1975) (subsequently referred to as KD) attempted the more ambitious task of investigating the origins of the more rapid Irish economic growth and development of the 1960s compared with the relative stagnation of the 1950s. In that study it was stated that no attempt was to be made to apply rigorous tests to various "theoretical" models since the objective was to analyse Irish experience rather than verify theories of growth (KD, p. xv). However, if one were obliged to characterise the spirit of the study, it could be held to be very much in a Kaldorian mould, with a strong emphasis on the export-led growth element of Kaldor's later work. One could go as far as saying that elements of the Kaldorian framework are not so much tested as used as an interpretive maintained hypothesis in the study. Such a characterisation might be considered unduly restrictive of a work which explores many alternative hypotheses and does not reject any in any formal sense. However, the following observations could be advanced in support of our interpretation. First, the Verdoorn law is assigned a crucial (if somewhat understated) role in completing the virtuous circle of growth. Second, the treatment of consumption is in the tradition of Kaldor's savings function. Although no distinction is made between the MPCs out of profits and wage income, (largely, we suspect, due to data problems), the lower MPC out of farmers' income is highlighted when compared with non-farm income. In addition, many "developmental" explanatory

10. "The elasticity of productivity with respect to output may vary quite considerably from one time period to another or from one country to another. Our basic claim is more limited, namely, that output growth, through its influence on technological progress and in other ways, generally plays a large role in determining differences in productivity growth among manufacturing industries over the longer run. ... It may additionally be possible to define limits within which the elasticity will generally lie, but these limits may be wider than has sometimes been supposed" (Kennedy, 1971, p. 240).
11. "If it were established that exports generate a faster rate of growth of capacity and output, it would still be necessary, in order to complete the virtuous circle, to show that faster output growth leads to faster productivity growth and a greater decline (or lower increase) in unit costs and prices, thereby maintaining the country's competitive advantage" (KD, p. 72).
variables are included in the estimated consumption function. Third, investment is assigned a rather minor role in KD. In fact, no behavioural theory of investment is advanced, but the incremental capital-output ratio (ICOR) is used in an *ex-post* type of analysis. Since the ICOR is defined as

\[
\text{ICOR} = \frac{\text{Investment}}{\text{Change in Output}} = \frac{I}{\Delta Q_t}
\]

if ICOR is fixed and known, investment \((I_t)\) is simply

\[
I_t = \text{ICOR} \cdot \Delta Q_t
\]

If, on the other hand, ICOR is variable, then its determinants must be understood before one has a theory of investment. Finally, and most important, there is a very explicit absence of any detailed analysis of the determination of wages and prices and of their role in resource allocation within the economy.\(^{12}\)

There is also a tendency to short-circuit the price system in discussions of policy effects.\(^{13}\)

Taken together, the above two works have an interesting interrelationship. The first (Kennedy, 1971), interprets Irish industrial developments of the 1950s and 1960s by means of Verdoorn’s law in the absence of any formal neo-classical theory of the firm. The second proceeds to interpret Irish economic development largely in the light of some elements of the Kaldorian approach. Recent attempts have been made to formalise these Kaldorian ideas within a complete macromodel and to relate long-term growth to short-term balance of payments constraints (Dixon and Thirlwall, 1975; Thirlwall, 1980). This was an aspect treated empirically by KD but in a manner where the long-term growth aims and the short-term constraints were difficult to relate to each other within any analytical framework. In our final section we attempt to evaluate how suitable this approach is in exploring the nature of the supply side of the Irish economy in the 1970s and 1980s.

\**IV EVALUATION**

Although Verdoorn attempted to explain the relationship which now bears his name within the neo-classical theory of the firm, subsequent use and interpretations have been carried out against a background where the neo-classical paradigm was explicitly rejected (Cornwall, 1977; Thirlwall, 1983). Hence, in attempting to evaluate the Irish applications of Verdoorn’s law, one should

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12. It should be noted that Geary (1976) in his extended review of KD, criticised its lack of an open economy perspective with respect to price determination.

13. "If Government grants for investment in plant and training of workers tend to be devoted to export-oriented industries ... the result may be a movement towards greater efficiency and a better allocation of resources" (KD, p. 69).
consider whether it was being used because of its simplicity (the full neo-classical treatment being deemed too complicated or too demanding on data availability and statistical estimation), or whether it was the expression of a full or partial rejection of the neo-classical approach. No clear-cut answer is available. Undoubtedly both reasons were at work in Irish applications of Verdoorn’s law. What is clear, however, is that the empirical work on the supply side of Irish economy which followed Kennedy, 1971 and KD, carried out during the 1970s to date, has been motivated mainly by neo-classical ideas and concepts. Such work can be characterised by three main guiding principles;

(i) Firms are assumed to be motivated by objectives such as profit maximisation or cost-minimisation.

(ii) Firms are assumed to operate in fairly simple market structures, often perfectly competitive.

(iii) The production process is describable by a production or cost function with some degree of factor substitutability.

An extensive research programme follows from the above and is well represented by Irish empirical work. It includes such topics as factor demand systems, factor substitution, the cost of capital, the nature of technological change, and leads ultimately to improved policy analysis either by itself or within formal economy-wide or sector-wide macroeconometric models.14

Verdoorn’s law, by its very nature, does not link employment (and/or productivity) to relative or real factor prices. In Section III, we noted how Kennedy (1971) carried out a number of simple econometric tests where factor prices were included in a productivity relationship and proved to be statistically insignificant. Yet this is an interesting finding in itself with a properly constructed neo-classical model and one which should caution one against the rejection of such a framework. In terms of the empirical importance of the omission of prices from Verdoorn’s law, it is not surprising that these factors should prove insignificant for a period when there were few major supply side shocks serving to distort relative factor costs. However, the use of Verdoorn’s law may be inappropriate during a period such as the mid-1970s when notable distortions to energy prices vis-à-vis the general price level increased the running cost of capital relative to labour.15 It is when confronted with such price distortions that the dichotomy between the Verdoorn and the neo-classical approaches becomes pronounced. At the very least, the estimated Verdoorn coefficients will be subject to specification bias due to missing variables (Katz, 1968). An interesting topic of research would be to attempt to encompass Verdoorn’s law within the neo-classical theory of the firm, in particular to

15. Even for a data period terminating in 1973, Boyle and Sloane (1982) found highly significant relative factor price terms in explaining factor proportions. See also Higgins (1981) for the more recent data.
capture the key Kaldorian theme of increasing returns to scale, i.e., to obtain solid micro-foundations to Verdoorn's law. Analytically one could proceed in two possible ways:

(i) postulating perfectly competitive factor and output markets, non-increasing returns to scale at the firm level and increasing returns to scale at the level of the industry (i.e., Marshallian economies of scale).

or,

(ii) Postulating increasing returns to scale at the level of the firm, together with the assumption of demand constrained firms operating in non-competitive market structures.

The work of Katz (1968 and 1969) attempted to develop approach (i) but violated the returns to scale restrictions (Katz, 1969, p. 79). In the case of the second approach, for simplicity of exposition let us assume a Cobb-Douglas technology of the form

\[ Q = A \exp (yt)K^aL^\beta \] (4.1)

where \( Q \) represents output, \( K \) the capital stock, \( L \) the labour input, \( y \) the (constant) rate of technical progress and \( A \), \( a \) and \( \beta \) fixed parameters. If the firm is demand constrained and attempts to minimise the cost of producing a fixed output \( Q \), given exogenous wage rates \( (w) \) and cost of capital \( (r) \), the derived demand for labour and capital may be written as

\[ \log \left( \frac{Q}{L} \right) = \text{constant} + \left( \frac{a}{a + \beta} \right) \log \left( \frac{w}{r} \right) + \left( \frac{a + \beta - 1}{a + \beta} \right) \log Q + \left( \frac{\gamma}{a + \beta} \right) t \] (4.2a)

\[ \log \left( \frac{K}{L} \right) = \text{constant} - \left( \frac{a}{a + \beta} \right) \log \left( \frac{w}{r} \right) + \left( \frac{a + \beta - 1}{a + \beta} \right) \log Q + \left( \frac{\gamma}{a + \beta} \right) \] (4.2b)

Taking logarithmic derivations (denoted by "'"') yields

\[ \dot{q}_L = \left( \frac{\gamma}{a + \beta} \right) + \left( \frac{a}{a + \beta} \right) \left( \frac{\dot{w}}{r} \right) + \left( \frac{a + \beta - 1}{a + \beta} \right) \dot{Q} \] (4.3a)

\[ \dot{q}_K = \left( \frac{\gamma}{a + \beta} \right) - \left( \frac{\beta}{a + \beta} \right) \left( \frac{\dot{w}}{r} \right) + \left( \frac{a + \beta - 1}{a + \beta} \right) \dot{Q} \] (4.3b)

where \( q_L \) denotes labour productivity. Clearly Equation (4.3a) is a "Verdoorn style" relationship and, in a period of stable relative factor prices (i.e., \((\dot{w}/r)=0\)) will reduce to the simple relationship (2.1) above.

The way would now be open to examine (4.3a,b) in greater detail, by choosing flexible functional form production functions, non-neutral technical change, introducing other intermediate inputs (energy, raw materials), etc. It also leads one naturally to look at related issues, such as the process of wage bargaining, the
nature of labour and capital markets, the malleability of the capital stock, topics which can be explored in isolation, or within the context of a fully articulated macroeconomic model.

In conclusion, if Verdoorn's law is used as an alternative to a complete neo-classical factor demand system on grounds of simplicity or necessity, an interpretation of the "law" from the perspective of the neo-classical theory of the firm should alert one to possible specification errors in times of supply-side shocks. On the other hand, if use of the "law" is for reasons of rejection of the neo-classical theory of the firm, this rejection should be made quite explicit, and the consequences in terms of elaborating an alternative paradigm recognised.

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