Factor Demand and Factor Substitution in Selected Sectors of the Irish Food Industry

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Précis: In this paper the degree of substitution between capital services, labour, material and fuel was estimated for the meat and milk processing sectors of the Irish food industry. The responsiveness of these inputs to price changes was also estimated. Capital services and labour were strong substitutes in production in all sectors. The price elasticity of demand for labour services was -0.74 for the bacon sector, -1.2 for the other meat processing sector and -0.63 for the dairy processing sector. Predictions of the demand for labour based on fixed coefficient type relationship between labour and output will be misleading when the capital-labour-price ratio also changes.

I INTRODUCTION

Output in any industry is produced by combining labour, capital and materials. Economists have had a long interest in the technology of production as is manifested by the large volume of published work in this area. A number of such studies have also been carried out with Irish aggregate manufacturing data (Farley, 1971; Henry, 1972; Geary and McDonnell, 1980). This paper focuses on three sectors of the Irish food industry, as classified in the Census of Industrial Production.

These are:-

- (a) Bacon factories.
- (b) Slaughtering, preparation and preserving of meat other than by bacon factories. (I will refer to this group as "Other Meat Processing".)

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(c) Creamery butter, cheese, condensed milk, chocolate crumb, ice cream and other edible milk products. (I will refer to this group as "The Dairy Processing Sector".)

These three industrial groupings are important because they process the output of the dairy, cattle, sheep and pig sectors which make up the bulk of agricultural output in Ireland. In 1978 they employed 17,500 persons, which was 65 per cent of employment in primary food and drink manufacturing.¹

It is sometimes argued that Ireland has great scope for expanding production in grass based products and that this expansion could lead to considerable expansion in manufacturing employment (e.g., NESC, 1977). But change in the demand for labour in any industry depends on more than changes in output. It also depends on changes in the relative prices of inputs, the nature of substitution possibilities between inputs and the nature of technical change. It is important to have some indication of the nature of the underlying technology of production before attempting to make predictions about employment in any sector of manufacturing.

This paper investigates the underlying technology of production of the above mentioned sectors of the Irish food industry. Specifically it gives estimates of the partial elasticities of substitution between capital, labour, materials and fuel, of the related price elasticities of demand for these inputs and of the bias in technical change.

Theoretical Framework

For each sector it is assumed that the relationship between gross output and inputs is represented by a twice differentiable production function:²

$$Q = Q(K, L, M, F)$$
(1)

Where Q = gross output, K = capital services, L = labour services, M = material inputs and F = fuel. It is also assumed that these production functions exhibit constant returns to scale. Assuming cost minimising behaviour on the part of producers, the theory of duality between production and costs³ implies that the technology of production can also be represented by a cost function:

$$\mathbf{C} = \mathbf{C}(\mathbf{P}_{\mathbf{k}}, \mathbf{P}_{1}, \mathbf{P}_{m}, \mathbf{P}_{f}, \mathbf{Q})$$
(2)

1. The primary sector also includes flour milling and animal feed preparations, canning and freezing of vegetables and fruit, sugar refining and malting.

2. For notational convenience subscripts for the sectors are left out.

3. See Diewert (1974 or 1978) or Varian (1978) for good discussion on this topic.

Where C = total costs, P_k = price of capital, P_1 = price of labour services, P_m = price of materials and P_f = price of fuel.

In order to obtain measures of the technology of production (i.e. partial elasticities of substitution and related measures) from the parameters of a cost function it is necessary to specify a function which is linearly homogeneous, concave and non-decreasing in factor prices (at least locally). The translog cost function (Christensen *et al.*, 1973), satisfies these conditions. Assuming constant returns to scale and including time (t) as a measure of technical change⁴ the translog unit cost function is:

$$\ln C = b_o + \sum_i b_i \ln P_i + \frac{1}{2} \sum_i \sum_j b_{ij} \ln P_i P_j + b_t \ln t + \frac{1}{2} b_{tt} (\ln t)^2$$

$$+ \sum_i b_{ti} \ln t \ln P_i \qquad i, j = k, l, m, f.$$
(3)

Linear homogeneity in input prices implies that:

$$\sum_{i} b_{ij} = 0, \sum_{j} b_{ij} = 0, \sum_{i} b_{i} = 0 \text{ and } \sum_{i} b_{ti} = 0. \qquad i, j = k, l, m, f.$$

For a twice continuously differentiable cost function, the matrix of second partial derivatives, with respect to p, evaluated at the point of minimum cost is symmetric.⁵ This symmetry condition implies that $b_{ij} = b_{ji}$ for all i and j.

The partial derivatives of the translog cost function with respect to the logs of input prices give the demand functions for inputs expressed in terms of shares of the inputs in total output.⁶

$$\frac{\partial \ln C}{\partial \ln P_i} = \frac{\partial C}{\partial P_i} \frac{P_i}{C} = S_i = b_i + \sum_j b_{ij} \ln P_j + b_{ti} \ln t \qquad (4)$$
$$i, j = k, l, m, f.$$

Where S_i is the share of the ith input.

The partial elasticities of substitution (σ_{ij}) and input demand elasticities (η_{ii}) can be obtained from the parameters of the cost function (and hence

6. This follows from Shephard's Lemma - See Varian, 1978, p. 32 for a proof.

^{4.} Binswanger (1974), Kako (1978) and Willis (1979) have included a time trend variable to account for technical change. Underlying this procedure is the assumption that technical change has occurred at a constant rate over time.

^{5.} This condition is referred to as Young's Theorem (Allen, 1938, pp. 301-302).

of the share equations) by the following expressions.⁷

$$\sigma_{ij} = \frac{b_{ij} + 1}{S_i S_i}, \qquad \eta_{ii} = \frac{b_{ii} + S_i - 1}{S_i}, \qquad \eta_{ij} = S_j \sigma_{ij} \qquad (5)$$

The coefficients of the time trend variable in the share equations purports to measure the nature of bias in technical change. If it is negative for a particular factor share equation, this implies that technical change was saving in that factor.

Empirical Estimation, Data and Results

The parameters of the cost function which are required to measure partial elasticities of substitution and bias in technical change are estimated from the factor share equations. Four factors, capital services (k), labour services (l), materials (m) and fuel (f) are included for each of the three sectors. The assumptions of linear homogeneity in factor prices and the symmetry restrictions mean that if we have estimates of the parameters of any three equations the parameters of the remaining equation will be automatically determined. The fuel share equation is being excluded. It is assumed that firms make errors in their cost minimising behaviour, so an additive error term (e_i) is added to each share equation. Imposing linear homogeneity in factor prices, the three equations being estimated take the form:

$$S_{i} = b_{i} + \sum_{j} b_{ij} \ln \left[\frac{P_{j}}{P_{f}} \right] + b_{it} \ln t + e_{i}$$

$$i, j = k, l, m.$$
(6)

The symmetry restrictions can be imposed by combining these three equations into a single stacked equation. The disturbance terms in the three equations are likely to be correlated because deviations from cost minimisation should affect all factor shares. Because of this the three equations are treated as a set of seemingly unrelated regression equations. In order to make the values of the estimated parameters independent of the equation left out, Zellner's Iterative seemingly unrelated regression estimation method is used.

7. See Binswanger (1974) for these derivations.

II DATA

The data used are mainly from the Census of Industrial Production annual and quarterly inquiries for Irish manufacturing. Expenditures on materials, wages and salaries and fuel are expressed as proportions of gross output to get the shares of these factors. The share of capital is taken as the residual share (remainder of net output in the published figures). The price of labour services is obtained by dividing the wage and salaries cost by total hours worked. Total hours worked is the weighted average hours worked in each quarter by production workers, multiplied by the 12 monthly average of the total number of persons engaged in the industry. For material inputs a Divisia price index is developed. Detailed information on the generation of price indices is given in the appendix. For fuel a general energy price index given by Scott (1980) is used. For capital services an index developed by Geary and McDonnell (1979) is used⁸. All prices are expressed in index form with 1958 equal to 1. The parameters of the input demand model are estimated separately for the bacon industry, other meat processing and the dairy processing industry. The time period used is 1953 to 1973; this was dictated by data availability.

III RESULTS

Since the translog cost function is a second order approximation to the true cost function it will not be well behaved globally; it is not concave in input prices at all price levels and its input demand functions are not positive at all input price levels (monotonicity). The estimated translog cost function must be checked for concavity and monotonicity for the input price levels in the data used for estimation. Monotonicity will be satisfied if the predicted cost shares are positive. Concavity will be satisfied if the matrix of partial elasticities of substitution is negative semi-definite.

Monotonicity is satisfied for all years in each of the three industrial groupings. Concavity is not satisfied for all observations for any of the three industries. For the bacon industry and other meat processing industries it is satisfied in 17 out of 21 years. For the dairy processing industry it is not satisfied at any of the observation points. Since the principal minors of the matrix of partial elasticities of substitution are random variables, negative semi-definiteness should be tested for statistically; this would prove difficult because the standard errors of the principal minors are difficult to estimate.

^{8.} This cost of capital services employs as the price of investment goods, part of the Gross Domestic Capital Formation deflator derived from the National Accounts. Its use for these three sectors may be inappropriate. I am indebted to the referee for pointing this out.

Finding a matrix of estimated partial elasticities of substitution which is not negative semi-definite is not then conclusive proof that the cost function is not concave in input prices. But it does raise serious questions as to whether the observed data are consistent with the hypothesis of cost minimisation subject to a translog cost function and hence of the use of the estimated parameters of the translog cost function to represent the underlying production technology. Lack of concavity has also been found by Geary and McDonnell (1980) for some years for aggregate manufacturing in Ireland. Table 1 presents estimates of the parameters of the cost function with symmetry and homogeneity restrictions imposed.

	Bacon factories	Other meat processing	Dairy processing
b _m	0.8636**	0.8404**	0.8216**
ь b ₁	0.0574**	0.0519**	0.0666**
b _k	0.0682**	0.098**	0.0824**
b _f	0.0108**	0.0097**	0.0294**
b _{mm}	0.0943**	0.0469	0.1653**
b _{ml}	-0.0296**	-0.0108	-0.063**
b _{mk}	-0.0563**	-0.0311	-0.0802**
b _{mf}	-0.0084*	-0.0050	-0.02215**
bu	0.0154*	-0.0103	0.0183**
b _{lk}	0.0204*	0.0263*	0.0351**
b _{lf}	-0.0062**	-0.0053**	0.00966*
b _{kk}	0.0279	-0.0009	0.0689*
b _{kf}	0.0080**	0.0056**	-0.0238**
b _{ff}	0.0066*	0.0046	0.0363**
b _{mt}	-0.00074	0.01298*	0.01894**
b _{lt}	0.0043*	0.00094	-0.0057*
b _{kt}	-0.004	-0.0135**	-0.0105
system R ²	0.99	0.92	0.96

Table 1: Estimates of the parameters of the translog cost functions

Notes:

(i) Standard errors of the estimated parameters are given in Appendix Table 1.

(ii) **denotes 1 per cent level of significance and * denotes 5 per cent level of significance.

(iii) Key: M = Materials, L = Labour, K = Capital, F = Fuel and t = Time (proxy for technical change).

The nature of the bias in technical change is given by the coefficients of the time trend variable. If these coefficients are not significantly different from zero for all share equations, then technical change is "Hick's neutral", that is, it causes isoquants to shift homothetically. The results indicate that technical change has been "capital saving" in "other meat processing". It has been material using in the other meat and dairy processing sectors. With regard to labour, technical change has been neutral for the other meat processing sector, labour using for the bacon sector and labour saving for the dairy processing sector. Without detailed information on changes in the composition of output and changes in industrial processes in each of these sectors, little useful comment can be made on the reasons for the results obtained here. The method used for measuring the effect and technical change is crude and the time trend variable is probably picking up the effect of changes in other variables not included in the model.

It is of interest to see if the commonly used Cobb-Douglas function technology is valid for these industries. For a Cobb-Douglas technology to be valid, all the coefficients of the price variables in the cost function should equal zero (i.e., b_{mm} to b_{ff} equal to zero). It is clear from Table 1 that this is not true for any of the three sectors.

The main purposes of this study is to measure the nature and degree of substitution between inputs and responsiveness of input demand to input price changes. These are measured by the partial elasticities of substitution and the related price elasticities of input demand. Partial elasticities of substitution (σ_{ij}) and price elasticities of demand inputs (η_{ij}) vary from year to year in the translog cost formulation. As was shown earlier they can be calculated directly from the estimated parameters of the cost function. These elasticities are given for three years, 1953, 1963 and 1973, in Tables 2 and 3, for the bacon and other meat processing sectors, and in Table 4 for the dairy processing sector.

From Table 2 it can be seen that the partial elasticities of substitution for the bacon industry and the "other meat processing" are similar in signs and fairly similar in magnitude. This is what one would expect since the bacon factories process a large amount of beef and sheepmeat as well as pigmeat and hence produce a similar output to the other meat processing plants (these process beef and sheepmeat only). Capital and labour are strong substitutes in production (partial elasticity of substitution is positive) as is capital and fuel. Labour and fuel are complements in production. The partial elasticities of substitution between material and capital and material and fuel are not significantly different from zero. Labour and materials are weak substitutes. They were expected to be complements. A possible explanation of these results is that an increase in the price of materials (mainly cattle, sheep and pigs) in these industries relative to the price of labour could result in more intensive processing of the materials which in turn would result in an increase in the labour input for a given level of output. A multiple output specification of the technology of production would be necessary to investigate this possibility. Humphrey and Moroney (1975) in a study of US manufacturing industries found that natural resources, which would to some extent correspond to materials in this study, were substitutes for labour in the food and beverage industries.

		Bacon f	Bacon factories		Other meat processing		
	1953	1963	1973	1953	1963	1973	
σ _{mm}	-0.010	-0.041	-0.104	-0.083*	-0.109**	-0.093**	
σ_{ml}	(0.026) 0.286 (0.181)	(0.028) 0.542** (0.116)	(0.032) 0.592** (0.104)	(0.034) 0.721 (0.236)	(0.036) 0.767** (0.197)	(0.034) 0.729** (0.229)	
σ_{mk}	(0.181) -0.044 (0.384)	-0.106	(0.104) 0.30 (0.257)	0.520	(0.157) 0.573 (0.361)	(0.223) 0.572 (0.362)	
σ_{mf}	-0.221	(0.107) (0.031) (0.266)	(0.237) -0.08 (0.297)	0.261 (0.268)	0.260	(0.302) -0.227 (0.445)	
σ_{1}	-13.339** (2.830)	-9.536** (1.086)	-8.160** (0.755)	-26.90** (4.136)	-20.925^{**}	-25.630** (3.823)	
$\sigma_{ m lk}$	8.146* (3.574)	5.531*	3.236**	9.035**	6.674** (2.186)	7.823**	
$\sigma_{ m lf}$	-16.05** (3.060)	-7.096** (1.453)	-6.036** (1.263)	-14.541** (4.401)	-11.351** (3.498)	-23.553** (6.953)	
σ_{kk}	-7.901 (6.809)	-7.930 (7.135)	-6.181* (2.497)	-12.635* (5.46)	-10.803* (4.104)	-11.025* (4.258)	
$\sigma_{ m kf}$	17.928 [*] * (4.042)	14.282** (3.172)	9.188 ^{***} (1.955)	10.884** (3.568)	9.363 ^{**} (3.019)	15.342** (5.177)	
$\sigma_{ m ff}$	18.743 (52.91)	-33.604 (31.08)	-32.257 (33.76)	-51.08 (52.47)	-51.662 (50.0)	-2.196 (141.7)	

Table 2: Estimates of partial elasticities of substitution for 1953, 1963 and 1973 in thebacon and other meat processing industries

**denotes 1 per cent level of significance.

* denotes 5 per cent level of significance.

Standard errors in parentheses.

The own price elasticities of demand for inputs in these two industries are given in Table 3. The own price elasticity of demand for material is very low. This means that a change in the price of materials has little effect on the amount of materials used at a given level of output. The price elasticity of the demand for labour is inelastic in the bacon industry and elastic in the meat processing industry (but not significantly different from 1 in this industry). The amount of labour demanded is fairly responsive to *ceteris paribus* changes in the price of labour services. The own price elasticity of demand for capital is also inelastic in the case of the bacon industry and very close to 1 in the other meat processing sectors.

	Bacon factories			Other meat processing		
	1953	1963	1973	1953	1963	1973
$\eta_{\rm mm}$	-0.009	-0.035	-0.083*	-0.072*	-0.093**	-0.08*
<i>n</i> .,	(0.023) -0.625**	(0.024) -0.721**	(0.026) -0.740**	(0.029) -1.188**	(0.030) -1.135**	-1.178**
.01	(0.132)	(0.082)	(0.068)	(0.183)	(0.149)	(0.176)
$\eta_{\mu\nu}$	-0.482	-0.472	-0.622*	-0.937*	-0.924*	-0.926*
. ""	(0.415)	(0.425)	(0.251)	(0.405)	(0.351)	(0.358)
$\eta_{\rm ff}$	-0.146	-0.341	-0.341	-0.393	-0.407	-0.01
-11	(0.412)	(0.316)	(0.329)	(0.404)	(0.394)	(0.663)

 Table 3: Estimates of the price elasticities of demand for inputs for 1953, 1963 and 1973

 in the bacon and other meat processing industries

**denotes 1 per cent level of significance.

* denotes 5 per cent level of significance.

Standard errors are in parentheses.

The own price elasticity of demand for fuel is not significantly different from zero for either industry.

The results for the dairy processing sector are presented in Table 4. The lack of concavity in the cost function (referred to earlier) is reflected in the positive values of own price elasticities for some inputs; this is in conflict with the neo-classical theory of the firm. It is only in the case of materials that the own price elasticity (and elasticity of substitution) is positive and significantly different from zero. The dairy processing sector is dominated by firms which are operated as co-operatives for the milk suppliers. In this situation we would expect that the objectives of the co-operatives would be to minimise the cost of production subject to paying the maximum price possible for its main raw material - milk. It is not surprising then that the price elasticity of demand for milk is positive because the co-operative, unlike other firms will not try to substitute other inputs for milk when its price increases. A more complete model for this sector should take into account the inputs used in the farm sector for the production of the main raw material (milk) for the dairy sector, since dairy processing in a farmers' cooperative is to a large extent an extension of the farming operation. In the dairy sector, capital and labour are strong substitutes and the demand for labour is price inelastic.

Partial elasticities of substitution			Input demand elasticities			
1953	1963	1973		1953	1963	1973
0.067*	0.063*	0.043	$\eta_{\rm mm}$	0.058*	0.054*	0.036
(0.025)	(0.026)	(0.027)		(0.022)	(0.022)	(0.023)
-0.736**	-0.344*	-0.262	η_{it}	-0.521**	-0.609**	-0.633**
(0.191)	(0.145)	(0.138)	- 11	(0.153)	(0.118)	(0.107)
-0.311	-0.490	-0.111	$\eta_{ m kk}$	0.049	0.167	-0.112
(0.352)	(0.401)	(0.298)	AR	(0.415)	(0.469)	(0.340)
-0.287	-0.228	-0.731	$\eta_{ m ff}$	0.851	0.757	1.396
(0.360)	(0.344)	(0.485)	•11	(0.579)	(0.549)	(0.754)
-12.455**	-11.207**	-20.641**		· · ·	· · ·	
(3.66)	(2.166)	(1.808)				
12.894**	11.333***	7.850**				
(3.717)	(3.23)	(2.141)				
12.648*	9.498*	11.646*				
(5.127)	(3.741)	(4.686)				
0.690	2.677	-1.308				
(5.89)	(7.514)	(3.956)				
-16.047*	-17.257*	-17.164*				
(6.218)	(6.659)	(6.625)				
42 914	36 226	91 565				
(29.223)	(26.28)	(49.411)				
	Partial elas 1953 0.067* (0.025) -0.736** (0.191) -0.311 (0.352) -0.287 (0.360) -12.455** (3.66) 12.894** (3.717) 12.648* (5.127) 0.690 (5.89) -16.047* (6.218) 42.914 (29.223)	Partial elasticities of su19531963 0.067^* 0.063^* (0.025) (0.026) -0.736^{**} -0.344^* (0.191) (0.145) -0.311 -0.490 (0.352) (0.401) -0.287 -0.228 (0.360) (0.344) -12.455^{**} -11.207^{**} (3.66) (2.166) 12.894^{**} 11.333^{**} (3.717) (3.23) 12.648^* 9.498^* (5.127) (3.741) 0.690 2.677 (5.89) (7.514) -16.047^* -17.257^* (6.218) (6.659) 42.914 36.226 (29.223) (26.28)	Partial elasticities of substitution195319631973 $0.067*$ $0.063*$ 0.043 (0.025) (0.026) (0.027) $-0.736**$ $-0.344*$ -0.262 (0.191) (0.145) (0.138) -0.311 -0.490 -0.111 (0.352) (0.401) (0.298) -0.287 -0.228 -0.731 (0.360) (0.344) (0.485) $-12.455**$ $-11.207**$ $-20.641**$ (3.66) (2.166) (1.808) $12.894**$ $11.333**$ $7.850**$ (3.717) (3.23) (2.141) $12.648*$ $9.498*$ $11.646*$ (5.127) (3.741) (4.686) 0.690 2.677 -1.308 (5.89) (7.514) (3.956) $-16.047*$ $-17.257*$ $-17.164*$ (6.218) (6.659) (6.625) 42.914 36.226 91.565 (29.223) (26.28) (49.411)	Partial elasticities of substitution1953196319730.067*0.063*0.043 η_{mm} (0.025)(0.026)(0.027)-0.736**-0.344*-0.262 η_{l1} (0.191)(0.145)(0.138)-0.311-0.490-0.111 η_{kk} (0.352)(0.401)(0.298)-0.287-0.228-0.731 η_{ff} (0.360)(0.344)(0.485)-12.455**-11.207**-20.641**(3.66)(2.166)(1.808)12.894**11.333**7.850**(3.717)(3.23)(2.141)12.648*9.498*11.646*(5.127)(3.741)(4.686)0.6902.677-1.308(5.89)(7.514)(3.956)-16.047*-17.257*-17.164*(6.218)(6.659)(6.625)42.91436.22691.565(29.223)(26.28)(49.411)	InputInput19531963197319530.067*0.063*0.043 η_{mm} 0.058*(0.025)(0.026)(0.027)(0.022)-0.736**-0.344*-0.262 η_{ll} -0.521**(0.191)(0.145)(0.138)(0.153)-0.311-0.490-0.111 η_{kk} 0.049(0.352)(0.401)(0.298)(0.415)-0.287-0.228-0.731 η_{ff} 0.851(0.360)(0.344)(0.485)(0.579)-12.455**-11.207**-20.641**(3.66)(2.166)(1.808)(1.808)12.894**11.333**7.850**(3.717)(3.23)(2.141)12.648*9.498*11.646*(5.127)(3.741)(4.686)0.6902.677-1.308(5.89)(7.514)(3.956)-16.047*-17.257*-17.164*(6.218)(6.659)(6.625)42.91436.22691.565(29.223)(26.28)(49.411)	Partial elasticities of substitutionInput demand elasticities of substitution195319631973195319630.067*0.063*0.043 η_{mm} 0.058*0.054*(0.025)(0.026)(0.027)(0.022)(0.022)-0.736**-0.344*-0.262 η_{II} -0.521**-0.609**(0.191)(0.145)(0.138)(0.153)(0.118)-0.311-0.490-0.111 η_{kk} 0.0490.167(0.352)(0.401)(0.298)(0.415)(0.469)-0.287-0.228-0.731 η_{ff} 0.8510.757(0.360)(0.344)(0.485)(0.579)(0.549)-12.455**-11.207**-20.641**(0.579)(0.549)(3.66)(2.166)(1.808)(0.579)(0.549)12.894**11.333**7.850**(3.717)(3.23)(2.141)12.648*9.498*11.646*(5.127)(3.741)(4.686)0.6902.677-1.308(5.89)(7.514)(3.956)-16.047*-17.257*-17.164*(6.218)(6.659)(6.625)42.91436.22691.565(29.223)(26.28)(49.411)

 Table 4: Estimates of partial elasticities of substitution and price elasticities of demand for inputs for the dairy processing sector, 1953, 1963 and 1973

**denotes 1 per cent level of significance.

* denotes 5 per cent level of significance.

Standard errors in parentheses.

IV CONCLUSIONS

By using the well established duality relationship between production functions and cost functions, it is possible to estimate the technology of production (i.e., the partial elasticities of substitution between inputs) from the cost function of the firm. In this paper, an attempt is made to estimate partial elasticities of substitution from the parameters of a translog cost function for three sectors of the Irish food industry. The translog cost function is used because it is a second order approximation to any arbitrary cost function and it imposes no *a priori* restrictions on the values of the partial elasticities of substitution.

In general it can be concluded that in none of the three sectors is the tech-

nology of production consistent with the commonly used Cobb-Douglas or CES production functions. Capital and labour are strong substitutes in all three industrial groups and the price elasticity of demand for labour is relatively large. This means that if the wage rate increases relative to the price of capital, capital will be substituted for labour and the quantity of labour used will decline (*ceteris paribus*). Predictions of the demand for labour based on fixed coefficient type relationship between labour input and output are likely to be seriously misleading if the relative price of capital to labour changes. The effect of technical change on labour demand varies across the three sectors; it is labour using in the bacon industry, neutral in the other meat processing sector and labour saving in the dairy processing sector. The differences in production technologies between sectors of the food industries, in particular the differences between dairy processing, and meat processing highlight the desirability of analysing the food industry (or indeed any other industry) at a disaggregated level.

It must be mentioned that the translog cost function and translog production function are not self-dual. This is so because they are approximations to the true functions. Because of this, different values of the partial elasticities of substitution could be obtained if they were estimated from the translog production function, but the differences should be small if the translog cost function is a good approximation for the true function. Burgess (1975) and more recently Geary and McDonnell (1980) have found substantial divergences between estimates of elasticities based on translog production and cost functions.

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APPENDIX

Data: The sources of the data for this paper were the Census of Industrial Production quarterly and annual inquiries, published in the Irish Statistical Bulletin; in addition, figures for annual employment were obtained from the Irish Statistical Abstracts, figures for the price of capital services were obtained from Geary and McDonnell (1979) and figures for the price of fuel were obtained from Scott (1980).

The method of constructing the various variables used is described briefly in this appendix.

Share of Materials: Total cost of all materials (except fuel) as a proportion of gross output.

Share of Labour: Total wages and salaries as a proportion of gross output.

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Share of Fuel: Cost of fuel as a proportion of gross output.

Share of Capital: This was taken as the residual proportion of gross output.

Price of Material: For the bacon industries, the price per ton of pigmeat, cattlemeat and sheepmeat was obtained from the "Materials Used" tables of the Census of Industrial Production. Since no quantity data were given for other meat industries, the price per ton of sheepmeat and cattlemeat obtained for the bacon sector was also used for this sector. The price of other materials (excluding fuel) was taken as the wholesale price index for "crudely transformed products". These "other materials" were a small proportion of total materials cost.

A Divisia price index for material inputs was calculated as follows:

$$Pm_{t} = 0.5 \Sigma \left| \frac{x_{i}^{t+1}}{C^{t+1}} - \frac{x_{i}^{t}}{C^{t}} \right| (\ln Px_{i}^{t+1} - \ln Px_{i}^{t})$$

Where $x_i = \text{cost}$ of input i (where i = pigmeat, cattlemeat, sheepmeat and other materials).

C = total cost of these materials.

 $Px_i = price of input i (pigmeat, etc.).$

t = time index.

A Divisia index for material used in the dairy industry was also generated. Separate prices were got for whole milk, separated milk, cream, sugar and other material; these were combined into a Divisia price index.

Price of Labour: The total cost of labour was taken as wages and salaries. The total amount of labour used was obtained as follows: The hours worked per wage worker per week was obtained from the *Quarterly Industrial Inquiry* for each quarter. A weighted average hours worked for all workers was obtained by weighting the average hours worked per week in each quarter by the average number of persons engaged (wages and salaries) in the quarter. This average hours worked per person per week was multiplied by the 12 monthly average of number of persons engaged (*Statistical Abstracts of Ireland*) and multiplied by 52 to convert to annual total hours worked. This annual total hours worked was divided into total wages and salaries to give a measure of labour cost per hour and converted into an index.

Cost of Capital: Geary and McDonnell (1979) estimates of the cost of capital were used. The index used here was one which allowed for Industrial Development Authority investment grants, for tax relief on investment and which assumed a geometric depreciation rate on investment.

Cost of Fuel: For this Scott's (1980) general energy price index was used. This is an index of the price of major fuels; oil, electricity, coal, gas and turf. All indices were based at 1958 = 1.

	Bacon factories	Other meat processing	Milk processing
)	0.009	0.01	0.011
	0.0035	0.0038	0.004
ւ Տե	0.0089	0.01	0.0127
bmm	0.0204	0.0259	0.019
b _{m1}	0.0075	0.0091	0.007
b	0.0207	0.0263	0.0215
b _{mf}	0.0023	0.018	0.0062
b ₁₁	0.0062	0.0081	0.0064
նե	0.0102	0.0101	0.011
blf	0.0011	0.0015	0.0043
b _{kk}	0.0253	0.03	0.029
bkf	0.0019	0.002	0.0087
b _{ff}	0.0032	0.003	0.0115
b _{mt}	0.005	0.0052	0.0054
ын b ₁₊	0.0019	0.0024	0.002
b	0.004	0.005	0.006

Appendix Table 1: Standard errors of the estimates of the parameters of the translog cost functions