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T. Trevor West 1938–2012

Timothy Trevor West, universally known as Trevor West, was a mathematician of considerable accomplishment, a politician who employed his political skills in many ways, a sportsman and a writer on aspects of Irish history and heritage that related to his political and sporting interests and to his family background.

Trevor was at Trinity College Dublin (TCD) as an undergraduate 1956–1960 and joined the staff in Mathematics in 1966 following a doctorate \[2\] completed under the supervision of Frank Smithies† at St. John’s College, Cambridge, a Research Fellowship in Glasgow (1963–1965) and a year at UCLA. He remained on the staff in Mathematics at TCD until he retired in 2004 but continued up to 2009 as Chairman of the Dublin University Central Athletic Club (DUCAC), a position he held from 1976 until then. As his exceptionally long tenure in the busy DUCAC role indicates, Trevor was very much interested in sports (of almost all kinds but including cricket, rugby and football). It is significant that he stayed on in the job until a big building project, incorporating a major sports facility on campus, was completed. His political skills and influence in College were in constant use behind the scenes on behalf of sport, but between 1970 and 1982 he was more publicly involved in politics as a member of the upper house of the Irish Parliament (the Senate), representing the constituency for graduates of Dublin University.

A number of obituaries of Trevor have appeared in the press or online, and they have dealt quite extensively with the very many non-mathematical aspects of his life, but our aim here is to discuss his publications and the work that flowed from them.
1. Olagunju

When Trevor arrived in Cambridge, he had a strong academic record and was joined there by his TCD classmate Robin Harte (who studied under J. H. Williamson), both admitted largely on the recommendation of one of their lecturers Arthur C. Allen.†

It seems that the first task given to Trevor on arrival was to straighten out some papers of P. A. Olagunju, another student from TCD who had gone to Cambridge a few years earlier. Unfortunately, Olagunju died in a motor accident in early 1960 (see [1]) and so Trevor was asked to attend to the referee’s comments on the papers. This project clearly had a significant influence on Trevor’s thesis work, as the thesis contains several references to Olagunju.

In addition, however, Cambridge was a very active place for functional analysis and spectral theory at that time. John Ringrose had graduated earlier, but was a lecturer and associated with St. John’s during 1961–1963; Graham Allan,† Harry Dowson, Ben Garling, Barry Johnson† and John Pym were among the postgraduate students who were more or less contemporaries of Trevor (and Robin Harte). It occurs to me to note that Functional Analysis was in an exceptionally active phase at the time. The classics (7, 17, 6, 12) would have provided a new accessibility to the field, and influenced Trevor, though of course some of the theory was established and (13) had been available for a while. Frank Bonsall† was appointed to a chair at Newcastle in 1959 and his influence on Functional Analysis in the United Kingdom was already considerable by the early 1960s.

The main result of Olagunju [1] is a characterization of operators $K: X \to X$ with spectrum $\sigma(K) \subset [0, \infty)$, where $X$ is a finite-dimensional (complex) vector space. The condition is $\text{trace}(Kp(K)\overline{p}(K)) \geq 0$ for all (complex) polynomials $p$, where $\overline{p}(z) = \overline{p}(\overline{z})$. More generally, when $X$ is a locally convex space and there are restrictions on $K$, the condition is $\text{trace}(K^3p(K)\overline{p}(K)) \geq 0$. The final section, added by Trevor, considers $\sigma(K) \subset \mathbb{R}$ and a similar condition with $K^3$ replaced by an even power. In his thesis [2] and in [3] (where he restricts to the Hilbert space case), Trevor considered conditions involving traces for operators $K$ to have spectra that lie in a circle.

The other paper (10) of Olagunju dealt with the non-existence of a continuous (positive definite) inner product on $\ell^\infty(\Lambda)$ for uncountable sets $\Lambda$ and in fact Trevor returned to this matter in a joint paper with his TCD colleague Timothy Murphy [12] where they relate the existence of a continuous inner product on $C(\Lambda)$ (for $\Lambda$ a compact Hausdorff space) to the non-existence of an uncountable family of disjoint non-empty open subsets of $\Lambda$, and motivate their work in part via considerations of Lax (8). They deduce for a Banach space $X$ that if the closed unit ball $X^*_1$ of the dual space satisfies this non-existence condition in the weak*-topology, then $X$ admits a continuous inner product, and further that if $X$ is uniformly convex, then the converse statement holds.

Not directly related to [1] or to (10), but a little related to ideas in (8), Trevor considered in his thesis [2] and in [4] the question of whether the spectrum $\sigma(T)$ of a bounded operator $T$ on an incomplete normed space $X$ would be different from $\sigma(\overline{T})$, with $\overline{T}$ being the extension of $T$ to the completion of $X$. In general, $\sigma(T) \neq \sigma(\overline{T})$ but the contrary was stated in an exercise in (6).

2. The West decomposition

The most well-known result from Trevor’s thesis [2] (published in [6]) is referred to as the ‘West decomposition’. It applies to Riesz operators $T$ on a (complex) Hilbert space $H$ and asserts that $T$ can be written as $T = Q + C$ with $Q$ quasinilpotent (so $\sigma(Q) = \{0\}$) and $C$ compact.

General considerations of Riesz operators on Banach spaces were expounded by Trevor [5], but the notion originates from Ruston [14] who defined them as operators $T$ having the same spectral properties as compact operators (the origin is the only possible accumulation point of $\sigma(T)$ and the non-zero points $\lambda \in \sigma(T) \setminus \{0\}$ are associated with finite-dimensional generalized eigenspaces). Ruston also characterized them as operators which become quasinilpotent on passing to the Calkin algebra (quotient of the bounded operators modulo the compacts).

In [6], Trevor discussed the case of operators on Banach spaces also and noted that if $T$ is a Riesz operator with $\sigma(T) \setminus \{0\}$ finite, then it is straightforward that $T = Q + C$ with $QC = QC = 0$, but several questions were unresolved then, and led to further work by Trevor and many others. With Alastair Gillespie [7] (in the first of several joint papers) he showed that one could not get commutativity $CQ = QC$ in general, but it remains open even now whether the West decomposition holds on general Banach spaces. The ‘red book’ [28] is a good source for some of the generalizations of the original theorem (obviously up to 1982), but I think a generalization by Stampfli [15] deserves explicit mention. By Stampfli [15, Theorem 4], if $T$ is an operator on a Hilbert space, then there is a compact $C_0$ with $\sigma(T - C_0) \subseteq \sigma(T - C)$ for all compacts $C$. This result, known as the Stampfli decomposition, quickly implies the West decomposition when $T$ is assumed to be a Riesz operator.

Perhaps the most significant effort to generalize the West decomposition to general Banach spaces $X$ was by Davidson and Herrero [5] who required that $X$ satisfy an unconditional decomposition into finite-dimensional subspaces $X_n$ with an $\ell^p$ norm on the direct sum of the $X_n$ (where $1 \leq p \leq \infty$). In fact, [5] also established that the Stampfli decomposition holds for operators on the same class of Banach spaces and rather recently in [16] it was shown that the classes of Banach spaces for which two related decomposition results hold coincide.

3. Collaborations

During his year in UCLA (1965–1966), Trevor began to collaborate extensively with Rien Kaashoek (also at UCLA that year), who had recently been a student of A. C. Zaanen in Leiden. Their research concentrated on semigroups, or in fact semi-algebras, which are essentially multiplicatively closed cones in Banach algebras over $\mathbb{R}$. This topic was initiated by Bonsall (see for instance [1, 2, 4]) and it is interesting to note from (2) that one of Bonsall’s aims was to capture the Perron–Frobenius theorem, the spectral theoretic result for $n \times n$ matrices with non-negative entries. Thus, in addition to the sequence of articles [9, 8, 11] with Kaashoek, and their monograph [17], Trevor’s more recent interest in the Perron–Frobenius theorem (for instance [36]) was in part due to this theme. The monograph [17] concentrates on semi-algebras which are sufficiently like finite-dimensional ones as to be locally compact (at the origin).

Another long-standing collaboration was with Alastair Gillespie, beginning with the work [7] already mentioned (published while Alastair was a PhD student of Bonsall) and their other joint works [14, 15, 16, 18] are related to the same general theme (semigroups rather than semi-algebras), as are Trevor’s [10, 13] and the single joint work [20] with Martha Bertman of Clarkson College, Potsdam, New York. The focus in these works was more concretely on properties of operators (on Banach or Hilbert spaces) and, for instance, reflexivity of operators and $G$-operators are discussed in several of these papers. Connections with spectral theory are rarely far away. Chapter I of [17] contains some results on compact semigroups (with jointly or separately continuous multiplication).

Trevor’s collaboration with Tom Laffey of University College Dublin resulted in just two publications [29, 34], with the more substantial being a decomposition theorem that $T = V + F$ holds for Fredholm operators $T$ on a Banach space, where $F$ is finite rank and $V$ is
left- and/or right-invertible according as the index of $T$ is $\geq 0$ or $\leq 0$ and $V, F$ come close to commuting in the sense that $[V, F]^2 = 0$.

4. Students and younger collaborators

Trevor had two PhD students in the formal sense, Roger Smyth and Ed Bach, but he and Rien Kaashoek jointly helped supervise Lothrop Mittenthal at UCLA, who was in the US army and more senior in age than they were. While Gerard Murphy\(^1\) was not Trevor’s student and Micheál Ó Searcoid was in fact a student of Robin Harte’s (in Cork), Trevor took a keen interest in the mathematical welfare of Roger, Gerard and Micheál especially, and collaborated with them over a considerable period.

The first paper [19] with Roger was on a question that I find particularly attractive, that of lifting elements from a quotient $A/I$ of a Banach algebra $A$ by an ideal $I$ to $A$ while preserving (or preserving within any $\varepsilon > 0$) the spectral radius. They gave some examples and counterexamples and finished with a conjecture that such lifting should be possible (at least within $\varepsilon$) when $A$ is a $C^*$-algebra. That conjecture was quickly proved by Gert Pedersen\(^\dagger\) in (11), but a very elegant formula was established by Gerard and Trevor [23] for the spectral radius $\rho(a)$ of an element $a$ of a $C^*$-algebra $A$, namely $\rho(a) = \inf_{h = h^* \in A} \| e^{h} a e^{-h} \|$, which implies the Pedersen result. Note that the West decomposition is the case of lifting quasinilpotents from the Calkin algebra (with $\varepsilon = 0$, so that the lift is quasinilpotent), but it does not seem appropriate to delve further into the background of the problem here. A recent work [9] generalizes the formula from [23] and may be consulted for the background.

Trevor wrote slightly more joint papers with Gerard Murphy than with any other of his collaborators, but the aforementioned ‘red book’ [28] deserves another mention. Perhaps the main innovation there is the systematic use of the socle for Banach algebras, an ideal that turns out to be the finite ranks when the algebra is $B(X)$, the bounded operators on a Banach space. It is still a good reference for algebraic aspects of Banach algebra theory.

Roger Smyth recalls that Trevor was the driving force behind [28], negotiating a deal with the publisher, taking charge of the copy editing, and using his political skill when different authors suggested incompatible approaches to the material.

5. Epilogue

Trevor continued to publish until he retired, returning in his final mathematical paper [37] to the semi-algebra theory he had considered with Kaashoek in the context of the Perron–Frobenius theorem. Perhaps it is of interest that Bonsall revisited semi-algebras in his last paper (3).

Unfortunately, Trevor felt unable to continue with mathematical research after 2004, but there remain some interesting aspects of his work that could prove fruitful in the future. Not all of his works have been mentioned here, but the impressive extent of it should be clear.

Trevor will be remembered also for his other activities, many of which are not covered in detail here (politics, sport, for example) and by the many students in TCD with whom he engaged in a unique way over his years in the College. He had a considerable influence on Mathematics in Ireland, starting rather soon after his appointment to TCD. Operating with the support of other newly appointed academics, he persuaded the Royal Irish Academy (RIA)

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to run symposia on mathematical topics and to set up a ‘Symposium sub-committee of the Irish National Committee for Mathematics’ with himself as its secretary. The first symposium organized under the auspices of the RIA was a broadly defined one on Group Representations and Quantum Mechanics, which was a resounding success both academically and financially, and a springboard for a succession of further symposia in the 1970s. Two of these were organized primarily by Trevor, one on Spectral Theory in March 1974 during which B. Sz.-Nagy† gave an address to the Academy and was admitted as an honorary member, and a second on Harmonic Analysis in December 1975. The latter was billed as being in honour of Gavin Brown† and Bill Moran on the occasion of their departure from Liverpool for positions in Australia and was retrospectively retitled Harmonic Analysis and Topological Algebras in a proceedings volume [21] published by the Academy. At that time, the RIA adhered to its traditional practice of publishing articles (for its Proceedings, Section A, Mathematical & Physical Sciences) as separate pamphlets and the papers arising from these two symposia were the first times the RIA agreed to publish in a journal-like format. Indeed Trevor worked over a long time to persuade the Academy to switch to a journal format and achieved that aim in 1980.

In the meantime, in December 1976, the Irish Mathematical Society (IMS) was formed to give a voice and a focus for mathematical activities in Ireland outside the constraints of the RIA, a development in which Trevor was crucially involved. Finbarr Holland of Cork, with whom Trevor cooperated on symposia and other matters, was the first President of the IMS. A further landmark event organized by Trevor was the first joint meeting of the LMS and IMS, held in Trinity College Dublin on 21–22 March 1986. The topic was $C^*$-Algebras and Operator Theory and the speakers were E. C. Lance (Leeds), R. G. Douglas (Stonybrook), W. B. Arveson† (Berkeley) and A. Connes (Paris). The LMS Bulletin (18) records the social arrangements for that meeting as including a reception at the RIA followed by a dinner at the Kildare Street and University Club. It was a part of Trevor’s political and social skill that he looked after visitors to Dublin well, and maintained contacts with many of the mathematicians he had met, including those he had met at Cambridge, Glasgow and UCLA. His network extended also to Indiana, Berkeley, Tübingen and Warsaw along with the places where his collaborators were based.

It was fitting that many of his collaborators and associates came to a meeting (dubbed the ‘WestFest’) organized by Donal O’Donovan in honour of Trevor held in TCD on 19 and 20 December 2005. Although Trevor did not organize conferences himself after 1986, he very much encouraged Gerard Murphy to organize a series of very successful international meetings in Cork. Trevor attended them, contributing noticeably to the good atmosphere at them, and moreover turned his family mansion in Midleton into a dormitory for some of us attending the earlier meetings.

That mansion came from his mother’s side of the family, more specifically his mother’s step-father, and Trevor’s last book [O5] was in part a tribute to the business activities of his step-grandfather. Trevor’s father was for a long time headmaster of Midleton College, which Trevor attended 1946–1954 and later served many years on its Board of Governors. His book [O3] relates to his dedication to that school.

The malting business involved farmers who grew the barley and is in part why Trevor admired Horace Plunkett‡ (who set up the Irish co-operative movement in 1889, was elected as an MP at Westminster representing South Dublin in 1892, knighted and elected FRS 1903, was deeply involved in agricultural reform in Ireland and in the debates over Irish Home Rule, and was a member of the Irish Senate in 1922–23). Trevor modelled his political aspirations in some way on the example of Plunkett and his most ambitious non-mathematical book [O1] seeks to

‡Sir H. C. Plunkett (1834–1932).
recognize the achievements of that man, competing in a way with several other accounts, some by eminent historians, of the co-operative movement and the man.

Unlike Horace Plunkett, who never married, Trevor met Maura Lee during his involvement with the quatercentenary of TCD in 1992 and they married soon after that. They were well suited and we extend our sympathies to her and to Trevor’s brothers John and Brian.

We can view Trevor as a man who believed in tradition, in Trinity, in sport, in Midleton, in Irish affairs, but his strong view of the importance of traditions also included an appreciation of the necessity for innovations, many of which he initiated in Mathematics and in the other aspects of his life. His cheerful presence will be missed by many.

Acknowledgement. Special thanks are due to Robin Harte and Finbarr Holland for their extensive help in preparing this account of Trevor.

References


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Other writing of T. T. West

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