1. INTRODUCTION

Conventional foundation design is achieved using traditional bearing capacity approaches to calculate the ultimate bearing capacity ($q_{ult}$) of the foundation:

$$q_{ult} = 0.5 \gamma B N_1 S_1 + c N_c + \gamma D N_q S_q$$  \hspace{1cm} (Eq 1)

Where:
- $\gamma$ = Unit Weight of Soil
- $B$ = Foundation Width
- $c$ = cohesive strength
- $D$ = foundation depth
- $S_1 S_q$ are shape factors
- $N_1 , N_c , N_q$ are bearing capacity factors which depend on the friction angle ($\phi'$) of the soil.

In un cemented sands, the second part of Equation 1 reduces to zero, and the non geometrical variables are the bearing capacity factors. As noted by Randolph et. al. (2003), the accuracy with which bearing capacity factors for a given soil can be determined has increased substantially in recent years (See Martin 2003), and the main uncertainty in the implementation of equation 1, lies in the choice of an appropriate friction angle. The primary complication being the difficulty in sampling cohesionless soils to allow strength testing on representative samples, and the stress-dependent effects on choice of friction angle.

To overcome some of these problems greater reliance is being placed on in-situ tests such as the Cone Penetration Test (CPT) to derive engineering parameters such as the friction angle, or through the use of empirical formulae to estimate the bearing capacity of the soil. Randolph et al. (2003) summarise the results of tests performed on shallow foundations, and buried piles in the laboratory and field, and reported ratios of the bearing pressure ($q$) at displacements of 5 and 10% of the foundation diameter, normalized by the CPT end resistance ($q_e$). The data suggest $q/q_e$ ratios of 0.09–0.16 at displacements of 5% of the foundation diameter, increasing to 0.13-0.21 at 10%.

Due to the relatively large ultimate bearing resistance of sand, actual foundations are usually remote from failure and the designer is primarily concerned with estimating the settlement of the footing under working stress conditions. This is typically achieved by assuming that in this stress range, the foundation response can be assumed to be linear-elastic. Once again the key difficulty for the designer is the choice...
of an appropriate youngs modulus (E) and a myriad of recommendations are available in the literature to assist in the choice of parameters.

With such uncertainties in mind a field experiment which examined the load-settlement response of a model footing was performed at the University College Dublin (UCD) dense sand test bed site.

2. FIELD WORK

2.1. Ground conditions

The fieldwork was carried out at the base of a sand quarry on the outskirts of Blessington, a small village located 25km to the southwest of Dublin. The material is a horizontally bedded uniformly graded sand deposited at the bed of a glacial lake. The particle grading between beds varies from silty sand to coarser sand depending on the lake level at the time of deposition. The material can be classified as very silty sand (with fines content ≈ 30%), to slightly silty Sand, with the mean particles size (D₅₀) decreasing from 0.32mm in the coarse sand to 0.1mm in the silty sand.

Periodic minor advances of the ice sheet, varying water table levels in the lake and the recent removal by quarrying of the upper 15m of sand/gravel have resulted in over-consolidation of the underlying material, with oedometer test results suggesting a past maximum vertical effective stress of between 700 to 800 kPa. A total of nine Cone Penetration Tests (CPT) were performed in the area, with one being performed adjacent to the foundation experiment. All CPT end resistance (qₑ) values were remarkably consistent below a depth of 0.5m below ground level (bgl). The variability in near ground conditions is unsurprising due to the tracking of heavy quarrying equipment. Typical CPT profiles are shown in Figure 1 and qₑ values typically in the range 12 – 20 MPa.

The water table level in the area was about 5m below the test level, however, the natural water content of samples above the water table level were 11-12%. Sand replacement tests, in addition to maximum and minimum void ratio determinations, indicated that the in-situ sand is at a relative density of close to 100%, has a bulk unit weight of 20.3 kN/m³ and a degree of saturation of 71%. Direct shear box tests reveal peak friction angles (𝜙ₑ') varying from 36-39°.

![CPT qₑ profiles at test site](image)

Figure 1  CPT qₑ profiles at test site

2.2. Experimental Procedure

The test foundation was a 0.25m square, steel plate which was founded at a depth of 0.5 bgl to avoid the variable upper layer of sand. The footing was loaded using a 20 tonne cone truck as reaction, and displacements were measured by two Linear Variable Displacement Transducers (LVDTs) located at each side of the foundation which were supported on a 4m long reference beam. The applied load was measured using a 200kN load cell, which was attached (along with the LVDTs) to a datalogger which recorded load and displacement at 1sec interval throughout the test. Load was applied in increments of approximately 25 kN, and unload-reload loops were performed at load levels of 85, 125 and 180 kN.
3. EXPERIMENTAL RESULTS

3.1. Ultimate Bearing Resistance

The mobilization of the bearing pressure resistance ($q$) under the footing, with settlement ($s$) is shown in Fig. 2. The footing tilted slightly from the initial stages of the test, and the settlement shown is the mean value of the two LVDT readings. After an initial bedding error of approximately 1mm, due mainly to the densification of the loose sand bed placed beneath the footing, it appears that the load-settlement response was approximately linear-elastic for bearing pressures of up to 1000kPa, with creep settlements recorded between the load increments becoming increasingly significant at higher stress levels. A permanent plastic deformation of approximately 3mm had occurred at the end of the first unload stage from a bearing pressure of 1400kPa. The reload response of the footing was approximately linear-elastic for bearing pressures up to the new pre-consolidation pressure of 1400kPa. Significant yielding occurred at a settlement of approximately 10mm, and the ultimate bearing capacity ($q_{ult}$) of 2950kPa was reached at approximately 21mm settlement.

Figure 3, shows the average pressure normalized by the CPT $q_c$ value against normalized displacement ($s/D$). Ratios of $q/q_c$ at 5 and 10\% of the plate diameter are 0.175 and 0.2 respectively. Whilst the value at $s/D = 10\%$ is in keeping with the lab and field test data reported by Randolph et al. (2003), the value at $s/D=5\%$ is higher than those reported. The graph suggests that the stiffness response at this strain level may have been affected by the unloading-reloading sequences. These would not affect the ratio at the higher ratio of $s/D$ as ultimate conditions were achieved at $s/D = 8\%$. The figure confirms that $q/q_c=0.2$ provided a good estimate of ultimate resistance at this site.
3.2. Load-Displacement Response

By far the greatest uncertainty associated with the behaviour of vertically loaded shallow foundations is to attempt to describe their load displacement response. Sophisticated models to attempt to predict this response have been developed, however, these typically require a large number of input parameters and require the designer to make subjective assumptions regarding yield surface shape etc. (See Lehane and Fahey 2002). The Elastic Modulus $E'$ is routinely used to estimate settlement of a footing using equation 2:

$$s = \frac{\pi q D (1-\nu^2)}{4E'} \quad \text{(Eqn 2)}$$

where $q$ is the applied pressure, $s$ is the footing settlement, $\nu$ (poisons ratio) is typically 0.2 in a drained material and $E'$ is the youngs modulus. This expression was used to derive the $E'$ response of the footing during the test using the measured pressure and settlement. The response shown in Figure 4 (the unload-reload loops are removed for clarity), shows $E'$ reducing rapidly with increasing strain level from a maximum measured (given the resolution of the test) of 54MPa at $s/D = 1.7\%$ to 18MPa at $s/D=12\%$. The choice of an appropriate $E'$ value for design is a matter of great debate. Lunne & Christoferson (1985) suggest that during the initial loading of shallow footings the footing response can be modeled using a constant $E'$ value given by:

$$E' = \alpha q_c \quad \text{(Eqn 3)}$$

Where $\alpha = 5$ for over-consolidated soil with $q_c < 50,000$ kPa. This results in an $E'_{\text{design}}$ value of 75 MPa. This modulus is assumed (Tomlinson 2001) to correspond to $E_{25}$, i.e. the modulus at a mobilised stress ratio of 25%, where the mobilised stress ratio is defined as (mobilised stress/ultimate stress). From Figure 2, the displacement at a mobilised stress ratio of 25% (3000/4=750 kPa) was approximately 1.2mm or 0.5% strain. The $E'_{\text{design}}$ value of 75 MPa is seen from Figure 4 to be reasonable. However, it is also clear that the modulus drops below this value at relatively low strain levels.

Hyperbolic models have been widely used to describe the degradation of stiffness with strain level in soils. These models retain the simplicity of linear-elastic models, whilst attempting to describe the materials response to loading up to yield. Mayne (2000) suggests the settlement of a shallow footing can be estimated using the approach shown in Equation 4, which involves the estimation of two unknowns, $Q_{\text{ult}}$ and $E_0$.

$$\delta = \frac{Q \cdot I_{\text{inv}}}{D E_0 [1-(Q/Q_{\text{ult}})0.3]} \quad \text{(Eqn 4)}$$

Where $Q_{\text{ult}}$ is the ultimate load which the footing can support ($= q_{\text{ult}} \cdot$ Footing area), $I_{\text{inv}}$ is a displacement influence factor from elastic theory (See Poulos and Davis 1974), and $E_0 = 2G_0(1+\nu)$. 

Figure 4 Degradation of $E'$ with increasing strain level during loading

As shown in section 2.1, the $q_{\text{ult}}$ value could be estimated with reasonable accuracy using the $q_c$ approach. Therefore $G_0$ needs to be estimated or measured. Sampling induced strains make the estimation of $G_0$ from laboratory tests difficult, this leaves two possibilities (i) Correlation with in-situ test measurements of (ii) In-situ $G_0$ measurement using geophysical methods.

Schnaid et al (2004) present a comprehensive review of CPT based methods to estimate $G_0$ including work by Robertson (1997) who suggests:
\[ G_0 = \beta \sqrt[3]{q_c \sigma'_v p_a} \]  
(Eqn 5)

Where \( \beta \) varies from 140 for unaged sand to 280 for aged sand, \( \sigma'_v \) is the in-situ vertical effective stress and \( p_a \) is atmospheric pressure.

Shear wave velocity profiles were obtained in the field using the Multi Channel Analysis of Surface Waves (MASW) method (Park et al. 1999, Donohue et al. 2003) at the test site. Two MASW survey lines were performed to test the repeatability of the survey. The lines were perpendicular to each other and crossed at the centre geophone.

The field set up for each of the survey lines consisted of 24 receivers (10 Hz geophones) at 1m intervals collinear with the source location. A number of different source locations were chosen for each profile, at source receiver offsets of 1, 6, 13 and 25m. This was necessary to determine the optimum acquisition parameters thereby minimising the influence of near field and far offset effects (Park et al., 2002). The software Surfseis developed by the Kansas Geological Survey was used to perform the inversion procedure to produce shear wave velocity – depth profiles which may then be converted into corresponding \( G_{\text{max}} \) profiles using equation 6 below:

\[ G_{\text{max}} = \rho V_s^2 \]  
(Eqn 6)

Where \( V_s \) = shear wave velocity (m/s) and \( \rho \) = density (kg/m\(^3\)).

Predictions made using equation 5 (with \( \beta = 280 \)) are compared in Figure 5 to profiles of \( G_0 \) measured in MASW tests performed at the test site. It is clear that equation 5 produces a good approximation of the in-situ \( G_0 \) profile.

3.3. Comparison of load-displacement response estimated by linear-elastic and hyperbolic methods

Linear elastic methods remain popular in design practice because of their simplicity. It is clear from Figure 4 that it is difficult to choose a representative \( E' \) value for the footing response. Implementing the constant \( E'_{\text{design}} \) value of 75MPa in Equation 2 to estimate the footing response results in the prediction of the initial footing behaviour shown in Figure 6. The method over-predicts bearing pressure mobilisation for the initial loading phase, with the over-prediction worsening as displacement increases (allowing for the 1mm bedding in of the footing). In contrast the reload stiffness is significantly underestimated.

The two MASW and CPT derived were shown to provide consistent estimate of \( G_0 \) and an average value at 0.5m bgl of 82 Mpa was determined for use in Equation 4. \( q_{\text{ult}} \) was assumed to be equal 0.2\( q_c \). The predicted footing response shown in Figure 6 is more encouraging. Equation 4 allows degradation of the stiffness as the bearing resistance is mobilised and settlement occurs. Although the method over-predicts the total settlement required to mobilise \( q_{\text{ult}} \), this is at least in part due to the un-load re-load loops performed, as one of the drawbacks of the hyperbolic model is that it cannot account for changes in the stress path direction. The model can be seen to provide good estimates of settlement under working stresses. Assuming a factor of safety of 3, the allowable stress on the footing is \( \approx \)1000 kPa, with the predicted settlement of 4mm at the stress level being very close to the measured value.
4. CONCLUSION

CPT based design methods were shown to provide good estimates of both the ultimate capacity and displacement under working load of a shallow footing on sand. It should be noted however, that the load displacement response was shown to be strongly stress path dependent and therefore the accuracy of the settlement prediction may have been somewhat fortuitous.

Independent measurements of $G_0$ using two standard, relatively inexpensive and quick in-situ test techniques (MASW & CPT) produced compatible profiles of $G_0$. High quality independent measurements such as these provide the designer with useful insights of the stress history of the deposit and highlight the relevance of these techniques in similar materials.

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6. REFERENCES


