
Brendan C. O’Kelly
Associate Professor, Department of Civil, Structural and Environmental Engineering, Museum Building, Trinity College Dublin, Dublin 2, Ireland.
e-mail: bokelly@tcd.ie Tel. +353 1896 2387 Fax. +353 1677 3072

First submission: 21st December 2011
Resubmitted: May 2012

Discussion:

The authors have reported a laboratory study with two main themes, namely: (i) an assessment of various waste products as substitutes for lime in amended biosolids (sewage sludge); (ii) the correspondence between undrained shear strength \( s_u \) deduced by shear-vane and triaxial compression (TC) tests. This discussion concerns itself with the second theme in which the authors, having studied strain rate \( (\dot{\varepsilon}) \) effects for a very soft site-specific material \( (s_u = 3–5 \text{ kPa}) \) in TC tests, concluded that \( s_{u\text{vane}} > s_{u\text{triaxial}} \) by a factor \( (F) \) of 1.8 and 1.6 for biosolids and 20% lime-amended biosolids, respectively. Specifically, these \( F \) values were deduced by the authors from a time to failure \( (t_f) \) of \( \sim 5 \text{ sec} \) observed for \( s_{u\text{vane}} = 6–10 \text{ kPa} \) using a Geonor handheld shear-vane apparatus, compared with \( t_f = 60 \text{ min} \) for 75-mm diameter by 150-mm long specimens sheared at \( \dot{\varepsilon} = \sim 0.33\%/\text{min} \) in TC under a confining pressure \( (\sigma_3) \) of 100 kPa.
However, it is important to note that the factor $F$ is not exclusively governed by relative differences in $\dot{\varepsilon}$ between shear-vane and $TC$ tests. The different apparatus approach the estimation of strength in different ways, with torsional-shear failure occurring around a cylindrical surface in vane-shear, compared with general ductile bulging in $TC$. The deduced strength value is material specific and depends, among other factors, on its consistency (water content), degree of saturation (gas voids content), specimen boundary conditions and applied confinement pressure, as well as scale effects concerning the specimen size in $TC$ to the size and aspect ratio of the cruciform vane. For example, the vane specimens prepared and tested in PVC tubes by the authors would have only experienced lateral confinement (i.e., applied vertical stress was 0 kPa). Hence, the mean confining pressure applied in the authors’ vane tests was ~0 kPa, compared with ~100 kPa for the $s_{\text{triaxial}}$ data used in deducing the values of $F$ reported in Eqs. 5 and 6 of the Kayser et al. (2011) study. These confinement pressures would correspond to overburden depths in a biosolids monofil of 0 and ~9.1 m, based on the reported in-situ bulk unit weight of 1.1 t/m$^3$.

Furthermore, ageing effects on test-specimens that are allowed to stand and cure (undisturbed and at constant composition) over an extended period before strength testing cause a strength gain (thixotropy). In the case of unamended biosolids (i.e., for pH < 11), internal reactions also occur, including: chemical changes of the solids and pore fluid; biodegradation of organics and accumulation of biogas produced by ongoing microbial activity. The slow but steady rate of biogas accumulation over the course of long-duration tests progressively reduces the degree of specimen saturation and also the effective confining pressure, with the latter calculated as the applied confining pressure minus the pore fluid pressure (O’Kelly 2006). This is accentuated under the undrained condition in the reported $TC$ tests, for which the test-specimen was fully enclosed by a rubber membrane. Consequently, the pore fluid pressure increases with elapsed time, between setting up the specimen in the test apparatus and the moment at which shear failure occurs under $TC$, thereby reducing the mobilized strength (O’Kelly 2005a). Another consideration is that the shear-vane test might not be performed under a truly undrained condition for test-material of slurry or very soft consistency on account of some flow of material occurring outward from between the blades and around the cruciform vane (Landva 1980), rather than the development of a purely cylindrical failure surface enclosing the vane, with $s_{\text{vane}}$ calculations based on the latter scenario.

Below, the discusser presents some data of remolded undrained shear strength ($s_{uw}$) for freshly-prepared unamended biosolids, having a degree of saturation of 94.1–97.4%, which was tested in both vane-shear and $TC$. This biosolids material was comprised of ~70% volatile solids by dry mass, a specific gravity of 1.55 (similar to values reported by the authors) and Atterberg liquid limit (LL) and plastic limit (PL) values of 314% and 53% respectively. The physio-mechanical and chemical properties of this biosolids material have been reported in full elsewhere (O’Kelly 2005a; 2005b; 2006; 2008). The drive motor of the laboratory vane apparatus rotated the vane-head at an angular rotation of 9°/min, thereby transmitting a known torque via a calibrated spring to the 12.7 mm by 12.7 mm cruciform vane which was embedded in the test-specimen. 38-mm diameter by 76-mm long specimens were sheared undrained in single- and multi-stage $TC$ tests (British Standards Institution 1990) using $\dot{\varepsilon} = 1.6%/\text{min}$, with standard corrections applied to the mobilized deviator stress for the reinforcing effect of the specimen membrane. Note that the water content (w) has been expressed in terms of liquidity index ($IL$, Eq. 1), with IL values of unity and zero corresponding to the LL and PL conditions respectively.
\[ IL = \frac{w - PL}{LL - PL} \] (1)

Referring to Figs. 1 to 3, the following observations are made in relation to data presented by the discusser, although the same general trends may be surmised for the authors’ study, particularly for unamended biosolids:

1. In TC tests, \( t_f \) was set at 9.5 min, given that these tests had been performed at an \( \dot{\varepsilon} \) of 1.6%/min, with specimen failure under general ductile bulging deemed to have occurred by 20% axial strain. However, in vane tests, \( t_f \) is a function of the torsional stiffness of the spring, speed of the drive motor/vane-head, and also the \( s_{ur} \) of the material itself (Fig. 1(a)). The vane \( t_f \) is approximately proportional to \( s_{ur} \) for a given torsion spring but importantly, the vane \( t_f \) increases with increasing sensitivity of the torsion spring fitted in the vane apparatus. The rotation of the cruciform vane at shear failure would also appear to be dependent on these factors (Fig. 1(b)). The angular rotation of the authors handheld vane at shear failure was \( \sim 30^\circ \) (\( s_u = 6–10 \) kPa), based on the reported \( \sim 1 \) revolution/min and \( t_f = 5 \) sec. If the factor \( F \) were solely \( \dot{\varepsilon} \) dependent, its value should decrease with increasing \( s_{ur} \) on account of the relative increase in vane \( t_f \). Hence, there are other significant reasons at play.

2. The authors have highlighted significant increases in \( s_{u,\text{triaxial}} \) brought about by an increase in \( \dot{\varepsilon} \) (decreasing \( t_f \)). However, the vane data in Fig. 2 for sets of two or three specimens prepared at the same water content and dry density but sheared at different rates suggest that the \( \dot{\varepsilon} \) dependence of \( s_{ur} \) is also a function of \( IL \) (water content), with proportionally greater increases in \( s_{ur} \) occurring for material at higher \( IL \) values. This may explain the strong \( \dot{\varepsilon} \) dependency of \( s_{u,\text{triaxial}} \) observed for very soft materials (\( s_u \leq 8 \) kPa) in Fig. 4 from Kayser et al. (2011), in which the \( \sim 30\% \) gain in shear strength per tenfold increase in \( \dot{\varepsilon} \) is approximately three times that experienced by mineral fine-grained soil in TC (Koumoto and Houlsby 2001). Also, the \( \dot{\varepsilon} \) dependency of lime-amended biosolids was slightly less than that of biosolids alone (30% and 34% respectively), conceivably since the former had a lower water content (higher \( s_u \)). Furthermore, given the very large increases in \( s_{ur,vane} \) mobilized for very modest
reductions in $t_f$ (Fig. 2(b)), it is possibly that the semi-logarithmic relationship of $s_{ur\text{triaxial}}$ against $\dot{\varepsilon}$ reported in literature for undrained saturated mineral soils, and which has been applied by the authors, may not be appropriate for biosolids under vane-shear conditions.

3. Since biosolids material is partially saturated on account of occluded biogas bubbles generated internally, an increase in confining pressure produces an increase in $s_{ur}$ (see data for stiff and very stiff biosolids shown in Fig. 3). In common with other unsaturated soils, the Mohr-Coulomb failure envelop for biosolids has a concave curvature, becoming less steep for higher confining pressures. Similar behavior was reported by the authors for very soft biosolids in Figs. 3(c) and 3(d) of Kayser et al. (2011). As observed in these figures, a large difference in mobilized deviator stress occurred between $\sigma_3 = 0$ and 100 kPa, as compared with the difference in response under $\sigma_3 = 100, 200$ and 400 kPa. Hence, the relative difference in deduced $s_{ur\text{triaxial}}$ values increases with reducing confinement. Importantly, the factor $F$ is sensitive to differences in the mean confining pressure applied in different test apparatus, particularly as has been shown, when comparisons are being made relative to the unconfined condition. Ideally, the confining pressure applied in $TC$ tests should simulate typical field-overburden conditions, with unconfined compression ($\sigma_3 = 0$) providing a lower bound $s_{ur\text{triaxial}}$ value for a given $\dot{\varepsilon}$.

4. The strengths mobilized in vane-shear were peak values whereas a different failure criterion was applied by the authors and discusser for the $TC$ tests. The value of $s_{ur\text{triaxial}}$ was taken as the shear resistance (i.e., half of the mobilized deviator stress) at 20% axial strain, generally without reaching peak (e.g. see Figs. 3(a) and 3(c) in Kayser et al. (2011)). The deviator stress invariably continued to monotonically increase with marginally, although in some cases significantly, higher values of $s_{ur\text{triaxial}}$ mobilized for larger strains. This was particularly true for unamended biosolids under higher confinement (see Fig. 3(c) in Kayser et al. (2011)).
5. The thixotropic strength ratio of cured strength to $s_{ur}$ increases with increasing $IL$. The authors’ TC tests reported in Figs. 3 and 4 of Kayser et al. (2011) were performed on very soft materials and for $t_f = 1$–600 min. Hence, some varying degree of thixotropic hardening is likely to have occurred over the course of some of these tests, and would have been more significant for the longer duration test of $t_f = 10$ h on the freshly-prepared biosolids. Another consideration is that the shear resistance developed over the pre-defined cylindrical failure surface in vane shear is proportional to the applied torque, which is controlled either by the constant speed of the drive motor or alternatively by manual rotation at a uniform rate for laboratory and handheld vanes respectively. Hence, the applied loading may be viewed as similar to stress-controlled in vane shear, compared with the strain-controlled loading conditions in TC. The compacted specimens tested by the authors and discussers presumably had some degree of cross anisotropy arising from the specimen preparation method. This difference in loading conditions could be significant in some instances, depending on the degree of material anisotropy, in which case the geo-mechanical behavior, and hence the deduced $s_{ur}$ value, would also be stress-path dependent.

The $w : s_{ur}$ relationship given by Eq. 2 has been confirmed theoretically (Koumoto and Houlsby 2001) using critical state theory and also experimentally over the full plastic range, for both fine-grained mineral (Sharma and Bora 2003) and organic (Zentar et al. 2009) soils.

$$w = a s_{ur}^{-b}$$

where: coefficient $a$ (%) is the water content value for $s_{ur} = 1$ kPa; coefficient $b$ is the gradient of the linear function relating water content (as %) and $s_{ur}$ (in kPa) on a bi-logarithmic plot.

Figure 4 shows shear-vane and TC data presented in this manner for the biosolids material reported by the discussers, and is a particularly useful way of comparing the different strength approaches, with the results of regression analysis of the log $w$: log $s_{ur}$ correlations reported in Table 1. The regression coefficient $r$ values were very close to unity, indicating very strong correlations. Note that the range of $s_{u,vane}$ is similar to that reported by the authors in Fig. 5 from Kayser et al. (2011). Also, the range of $s_{u,triaxial}$ values mobilized under different $\dot{e}$ for the authors biosolids material ($w = 330\%$) are reasonably consistent with the TC correlation line in Fig. 4, but significantly greater than that mobilized in vane shear.

Clearly, the factor $F$ relating vane and triaxial strengths is not a constant, even considering a single, homogeneous material (biosolids alone in this case) tested under specific conditions in vane shear and TC. In particular, $s_{u,vane} = s_{u,triaxial}$ for $w = 125\%$ and 105\% ($IL = 0.28$ and 0.20) under TC with $\sigma_3 = 100$ and 300 kPa, respectively. These values apply specifically for
the biosolids material and test-conditions described by the discusser. Furthermore, $s_{ur \text{ triaxial}} > s_{ur \text{ vane}}$ for water contents greater than these transition points, and vice versa.

**INSERT**

Figure 4. Water content against $s_{ur}$ for unamended biosolids.

Table 1. Regression analysis of log $w$: log $s_{ur}$ data. Note: $n$, number of data points; $r$, regression coefficient.

Shearing of physically-identical specimens of relatively homogeneous and isotropic material using different test apparatus should mobilize similar strength values provided that the different approaches to strength measurement apply similar confining pressures, rates of shear strain, and also that the failure criterion has been consistently applied.

The following observations are made regarding the data presented in Fig. 4 of Kayser et al. (2011) for very soft specimens sheared in TC under $\sigma_3 = 100$ kPa, but at different $\dot{\varepsilon}$ values:

- Ageing effects are evident, including an increase in $s_u$ due to chemical reactions and thixotropic hardening occurring over the one-week curing period for the 20% lime-amended biosolids specimens. In the case of unamended biosolids, ongoing biodegradation (with reported pH of 8.1–8.6) is likely to have produced the deviation of $s_{u \text{ triaxial}}$ for $t_f = 600$ min below the undrained strength regression line.

The data in Fig. 4 of Kayser et al. (2011) are reassessed in Fig. 5 of the present study by the discusser, as follows:

- Data points A and C of unconfined $s_{u \text{ triaxial}} = 2.4$ and 4.5 kPa under $\dot{\varepsilon} = 0.33\%$/min ($t_f = 60$ min) for biosolids and 20% lime-amended biosolids, respectively, correspond to the peak values of deviator stress reported in Figs. 3(c) and 3(d) of Kayser et al. (2011).

- Data points B and D are deduced $s_{u \text{ triaxial}} = 4.9$ and 8.4 kPa for $t_f = 0.08$ min (i.e., 5 sec) determined by extrapolation back from points A and C, parallel to the regression lines for $\sigma_3 = 100$ kPa. These unconfined $s_{u \text{ triaxial}}$ values are in reasonable agreement with the nearest comparable vane data of $s_{u \text{ vane}} = 5.1$ and 8.3 kPa reported for 3-week aged biosolids and 10% lime-amended biosolids tested, respectively, in Table 2 of the original paper. Given some additional thixotropic hardening/chemical reactions had occurred for these materials prior to strength testing, this would suggest values of $s_{ur}$...
measured in unconfined triaxial compression are reasonably consistent with \( s_{u,vane} \) for very soft biosolids materials tested by the authors.

**INSERT**

Figure 5. Reasonable agreement between unconfined TC and shear-vane for very soft biosolids materials.

Hence, in conclusion, the field (Kayser et al. (2011)) and laboratory (O’Kelly 2005a; 2006) shear-vane apparatus appear to provide an expedient and reasonably reliable means of assessing the shear strength value of very soft biosolids materials. Furthermore, shear-vane tests are easier to perform compared with more onerous TC tests, which also necessitate specialist sampling and specimen preparation techniques.

**References**


Table 1. Regression analysis of $\log w$: $\log s_{ur}$ data. Note: $n$, number of data points; $r$, regression coefficient.

<table>
<thead>
<tr>
<th>Test method</th>
<th>$n$</th>
<th>$w$ (%)</th>
<th>$IL$</th>
<th>$a$</th>
<th>$b$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vane</td>
<td>20</td>
<td>104–186</td>
<td>0.19</td>
<td>0.51</td>
<td>257.7</td>
<td>0.229</td>
</tr>
<tr>
<td>TC (100 kPa)</td>
<td>17</td>
<td>56–203</td>
<td>0.01</td>
<td>0.57</td>
<td>429.3</td>
<td>0.397</td>
</tr>
<tr>
<td>TC (300 kPa)</td>
<td>4</td>
<td>106–214</td>
<td>0.20</td>
<td>0.62</td>
<td>450.0</td>
<td>0.382</td>
</tr>
</tbody>
</table>

Figure 1. Shear failure in vane tests for different torsion springs and material consistency (stiffness of springs in order 1 to 3 was 1.0, 1.9 and 3.3 kN/m$^2$).

(a) Time to failure

(b) Angular rotation of vane
Figure 2. Vane $s_{ur}$ mobilized for different times to failure.

(a) Against time.

(b) Against logarithm of time.
Figure 3. Shear resistance against axial strain for biosolids specimens in multi-stage undrained TC tests under $\sigma_3 = 100, 200$ and $300$ kPa, with $\dot{\varepsilon} \approx 1.6/%/\text{min.}$

Figure 4. Water content against $s_{ur}$ for unamended biosolids.
Figure 5. Reasonable agreement between unconfined $TC$ and shear vane for very soft biosolids materials.