The Economic and Social Review, Vol. 27, No. 4, July, 1996, pp. 389-396

NOTES and COMMENTS

Geary's Count of Sign Changes as a Test of Unit Roots: Some Further Evidence

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I INTRODUCTION

In a recent issue of this journal, Honohan (1996) examined the performance of Geary's (1970) count of sign changes or τ test as a test of unit roots. The test is based on counting the number of times the plot of a series crosses the trend line joining the first and last observations. The null hypothesis of a unit root is rejected when the number of crossings is "large". Honohan argues that the sign change test is a useful informal test mainly because of its simplicity — it can be calculated by hand and the 95 per cent critical values may be approximated by twice the square root of the sample size. In the abstract to his paper, Honohan suggests that, despite its simplicity, the test has surprisingly good power.

In this note I present some Monte Carlo evidence which looks at the relative performance of the Geary τ test of a unit root. I compare its performance with the commonly used Dickey-Fuller test (Dickey and Fuller, 1979; 1981 and Fuller, 1976). It is well known that the Dickey-Fuller test lacks power so it is interesting to see if the Geary τ test is more powerful than the Dickey-Fuller test. I also examine the performance of the Geary τ statistic when there is non-zero drift. Honohan only examined the power of the Geary

τ test when the data generation process or DGP is a first order autoregressive or AR(1) process with zero drift. A priori, the critical values of the Geary τ test should not depend on the amount of drift present.

II THE MONTE CARLO EXPERIMENTS

In the Monte Carlo experiments reported here the same AR(1) DGP as Honohan's (1996) is used:

$$x_t = \alpha + \rho x_{t-1} + u_t$$

except that the drift α was not constrained to be zero. Values of 0, 0.1, 0.25, 0.5, 0.75 and 1 for α were used. Dickey and Fuller (1981) used a similar range of values for α. The random error terms u_t were generated as n.i.d.(0,1). Honohan also considered the case where u_t had a uniform distribution but this case is less interesting. The empirical size and power of the Geary τ and Dickey-Fuller test statistics were examined for a range of sample sizes (T = 50, 100, 250 and 500) and values of the autoregressive parameter (ρ = 1, 0.99, 0.975, 0.95, 0.9 and 0.8). The null hypothesis is, of course, $\rho = 1$. The critical values for the Dickey-Fuller test are taken from MacKinnon (1994) who calculated approximate asymptotic distribution functions. When there is no drift ($\alpha = 0$), the Dickey-Fuller test statistic is τ_μ, the t statistic on x_{t-1} when Δx_t is regressed on a constant and x_{t-1}. When there is drift ($\alpha \neq 0$), the Dickey-Fuller test statistic is $\tau_{u\tau}$, the t statistic on x_{t-1} when Δx_t is regressed on a constant, x_{t-1} and a time trend. Finally, the Monte Carlo results are all based on 5,000 replications which may be too few. Honohan used 1,000 replications.

III THE MONTE CARLO RESULTS

Table 1 shows the 95 per cent and 99 per cent critical values of Geary's count of sign changes test for unit roots based on 5,000 replications. Honohan presented non-integer critical values which are, in effect, interpolations based on randomisation. No interpolation is done since the actual test statistic is always a count. It is clear that the rule of thumb "twice the square root of the sample size" only provides a rough approximation to the actual 95 per cent critical values. As expected the critical values do not appear to depend on the value of the drift term α.

Tables 2 to 5 show the empirical size and power of the Geary and Dickey-Fuller tests for various sample sizes and various values of α and ρ. The empirical size of the Geary τ test may differ from its nominal size since

Table 1: *95 Per Cent and 99 Per Cent Critical Values of Geary's Count of Sign Changes Test for Unit Roots*

Monte Carlo Results Based on 5,000 Replications

	Sample Size			
Drift	*T = 50*	*T = 100*	*T = 250*	*T = 500*
$\alpha = 0.00$	14 (16)	19 (24)	30 (38)	44 (54)
$\alpha = 0.10$	14 (16)	18 (22)	30 (37)	43 (52)
$\alpha = 0.25$	14 (16)	18 (24)	30 (36)	42 (54)
$\alpha = 0.50$	14 (16)	18 (24)	30 (38)	42 (54)
$\alpha = 0.75$	14 (16)	18 (24)	30 (36)	42 (54)
$\alpha = 1.00$	14 (16)	18 (24)	30 (38)	43 (54)

integer critical values are used. The empirical size of the Dickey-Fuller test will differ from its nominal size since asymptotic critical values are used. The results show that the empirical size of the Dickey-Fuller test is generally closer to the nominal 5 per cent or 1 per cent level than the Geary τ test. However, the differences in the empirical size of the two tests are very small.

The interesting question is whether the Geary τ test is more powerful than the Dickey-Fuller test. The results in Tables 2 to 5 suggest that this is not the case. The Geary τ test is more powerful than the Dickey-Fuller test in less than 10 per cent of the 480 entries in Tables 2 to 5. Moreover, when the Geary τ test is more powerful, the difference in power tends to be small. However, the Dickey-Fuller test is significantly more powerful than the Geary τ test in many cases — particularly for smaller values of ρ and/or larger values of T.

IV CONCLUSIONS

The Geary τ test is a simple informal test of a unit root. However, it is not a substitute for a more formal test. The rule of thumb "twice the square root of the sample size" only provides a rough approximation to the actual 95 per cent critical values of the test. The Monte Carlo results also suggest that the Geary τ test is less powerful than the commonly used Dickey-Fuller test. The test is far less powerful particularly for smaller values of ρ and/or larger values of T. Of course, the Geary τ test may be more powerful than the Dickey-Fuller test when there are structural breaks or moving average errors but this is not very likely a priori.

Table 2: *Empirical Size and Power of Geary and Dickey-Fuller Tests for Sample Sizes of 50 at Nominal 5 Per Cent and 1 Per Cent Significance Levels*

Monte Carlo Results Based on 5,000 Replications

Drift	*Test*	*Empirical Size*	*Empirical Power*				
		$\rho = 1.000$	$\rho = 0.990$	$\rho = 0.975$	$\rho = 0.950$	$\rho = 0.900$	$\rho = 0.800$
$\alpha = 0.00$	Geary τ	0.0538 (0.0198)	0.0486 (0.0206)	0.0618 (0.0240)	0.0664 (0.0254)	0.0836 (0.0288)	0.1736 (0.0808)
	Dickey-Fuller τ_{μ}	0.0628 (0.0182)	0.0576 (0.0136)	0.0662 (0.0164)	0.0910 (0.0240)	0.1410 (0.0374)	0.3640 (0.1256)
$\alpha = 0.10$	Geary τ	0.0520 (0.0192)	0.0504 (0.0184)	0.0568 (0.0196)	0.0664 (0.0254)	0.0952 (0.0392)	0.1792 (0.0820)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0668 (0.0182)	0.0564 (0.0154)	0.0654 (0.0162)	0.0714 (0.0216)	0.1046 (0.0304)	0.2328 (0.0746)
$\alpha = 0.25$	Geary τ	0.0516 (0.0190)	0.0518 (0.0202)	0.0510 (0.0176)	0.0622 (0.0244)	0.0812 (0.0326)	0.1766 (0.0756)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0624 (0.0176)	0.0586 (0.0154)	0.0634 (0.0156)	0.0768 (0.0204)	0.1062 (0.0276)	0.2306 (0.0756)
$\alpha = 0.50$	Geary τ	0.0538 (0.0180)	0.0484 (0.0172)	0.0550 (0.0208)	0.0546 (0.0226)	0.0828 (0.0298)	0.1840 (0.0834)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0672 (0.0192)	0.0598 (0.0178)	0.0644 (0.0174)	0.0762 (0.0244)	0.0944 (0.0268)	0.2288 (0.0762)
$\alpha = 0.75$	Geary τ	0.0550 (0.0232)	0.0446 (0.0162)	0.0350 (0.0122)	0.0450 (0.0168)	0.0814 (0.0320)	0.1744 (0.0764)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0622 (0.0158)	0.0556 (0.0168)	0.0560 (0.0154)	0.0690 (0.0170)	0.0994 (0.0294)	0.2180 (0.0666)
$\alpha = 1.00$	Geary τ	0.0520 (0.0188)	0.0374 (0.0130)	0.0328 (0.0104)	0.0262 (0.0096)	0.0698 (0.0248)	0.1790 (0.0816)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0560 (0.0132)	0.0494 (0.0120)	0.0548 (0.0136)	0.0624 (0.0182)	0.1058 (0.0290)	0.2322 (0.0748)

Table 3: *Empirical Size and Power of Geary and Dickey-Fuller Tests for Sample Sizes of 100 at Nominal 5 Per Cent and 1 Per Cent Significance Levels*

Monte Carlo Results Based on 5,000 Replications

Drift	*Test*	*Empirical Size*	*Empirical Power*				
		$\rho = 1.000$	$\rho = 0.990$	$\rho = 0.975$	$\rho = 0.950$	$\rho = 0.900$	$\rho = 0.800$
$\alpha = 0.00$	Geary τ	0.0506 (0.0126)	0.0522 (0.0124)	0.0630 (0.0148)	0.0894 (0.0212)	0.1708 (0.0494)	0.3964 (0.1692)
	Dickey-Fuller τ_{μ}	0.0568 (0.0108)	0.0600 (0.0134)	0.0786 (0.0164)	0.1302 (0.0290)	0.3356 (0.0956)	0.8822 (0.5632)
$\alpha = 0.10$	Geary τ	0.0810 (0.0178)	0.0870 (0.0232)	0.1034 (0.0272)	0.1404 (0.0434)	0.2584 (0.0978)	0.5336 (0.2738)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0578 (0.0134)	0.0556 (0.0142)	0.0680 (0.0146)	0.0950 (0.0230)	0.2010 (0.0650)	0.6958 (0.3546)
$\alpha = 0.25$	Geary τ	0.0874 (0.0142)	0.0836 (0.0092)	0.0802 (0.0104)	0.1470 (0.0226)	0.2470 (0.0498)	0.5262 (0.1808)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0612 (0.0140)	0.0544 (0.0142)	0.0668 (0.0148)	0.1018 (0.0246)	0.2056 (0.0586)	0.6942 (0.3276)
$\alpha = 0.50$	Geary τ	0.0832 (0.0114)	0.0630 (0.0086)	0.0684 (0.0064)	0.0954 (0.0146)	0.2392 (0.0390)	0.5150 (0.1806)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0526 (0.0106)	0.0508 (0.0104)	0.0624 (0.0136)	0.0952 (0.0252)	0.2078 (0.0590)	0.6936 (0.3460)
$\alpha = 0.75$	Geary τ	0.0800 (0.0122)	0.0456 (0.0068)	0.0322 (0.0046)	0.0570 (0.0060)	0.2024 (0.0400)	0.5136 (0.1730)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0524 (0.0126)	0.0502 (0.0111)	0.0582 (0.0130)	0.1032 (0.0264)	0.2222 (0.0650)	0.6752 (0.3246)
$\alpha = 1.00$	Geary τ	0.0876 (0.0118)	0.0210 (0.0026)	0.0118 (0.0014)	0.0310 (0.0048)	0.1736 (0.0244)	0.5140 (0.1806)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0598 (0.0120)	0.0402 (0.0092)	0.0580 (0.0132)	0.1024 (0.0266)	0.2218 (0.0666)	0.6726 (0.3304)

Table 4: *Empirical Size and Power of Geary and Dickey-Fuller Tests for Sample Sizes of 250 at Nominal 5 Per Cent and 1 Per Cent Significance Levels*

Monte Carlo Results Based on 5,000 Replications

Drift	*Test*	*Empirical Size*	*Empirical Power*				
		$\rho = 1.000$	$\rho = 0.990$	$\rho = 0.975$	$\rho = 0.950$	$\rho = 0.900$	$\rho = 0.800$
$\alpha = 0.00$	Geary τ	0.0634 (0.0102)	0.0772 (0.0150)	0.1274 (0.0258)	0.2752 (0.0760)	0.5672 (0.2656)	0.8382 (0.6670)
	Dickey-Fuller τ_{μ}	0.0578 (0.0136)	0.0700 (0.0144)	0.1602 (0.0364)	0.4762 (0.1640)	0.9704 (0.7612)	1.0000 (1.0000)
$\alpha = 0.10$	Geary τ	0.0642 (0.0106)	0.0762 (0.0136)	0.1258 (0.0284)	0.2674 (0.0742)	0.5558 (0.2648)	0.8342 (0.6606)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0582 (0.0118)	0.0636 (0.0118)	0.1108 (0.0246)	0.2752 (0.0776)	0.8448 (0.5062)	1.0000 (0.9980)
$\alpha = 0.25$	Geary τ	0.0586 (0.0152)	0.0510 (0.0138)	0.0878 (0.0264)	0.2234 (0.0862)	0.5400 (0.3168)	0.8302 (0.7056)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0536 (0.0098)	0.0564 (0.0116)	0.1122 (0.0278)	0.2916 (0.0852)	0.8518 (0.5106)	1.0000 (0.9986)
$\alpha = 0.50$	Geary τ	0.0666 (0.0102)	0.0136 (0.0014)	0.0244 (0.0042)	0.1264 (0.0280)	0.4884 (0.2108)	0.8414 (0.6576)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0602 (0.0128)	0.0536 (0.0118)	0.1324 (0.0300)	0.3088 (0.0928)	0.8470 (0.5164)	1.0000 (0.9984)
$\alpha = 0.75$	Geary τ	0.0642 (0.0170)	0.0018 (0.0012)	0.0016 (0.0004)	0.0482 (0.0132)	0.4192 (0.2304)	0.8258 (0.6978)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0508 (0.0104)	0.0592 (0.0138)	0.1860 (0.0478)	0.3604 (0.1220)	0.8602 (0.5302)	1.0000 (0.9990)
$\alpha = 1.00$	Geary τ	0.0654 (0.0106)	0.0000 (0.0000)	0.0002 (0.0000)	0.0166 (0.0022)	0.3356 (0.1240)	0.8246 (0.6388)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0510 (0.0114)	0.0608 (0.0140)	0.2516 (0.0810)	0.4352 (0.1642)	0.8706 (0.5558)	1.0000 (0.9988)

Table 5: *Empirical Size and Power of Geary and Dickey-Fuller Tests for Sample Sizes of 500 at Nominal 5 Per Cent and 1 Per Cent Significance Levels*

Monte Carlo Results Based on 5,000 Replications

Drift	*Test*	*Empirical Size*	*Empirical Power*				
		$\rho = 1.000$	$\rho = 0.990$	$\rho = 0.975$	$\rho = 0.950$	$\rho = 0.900$	$\rho = 0.800$
$\alpha = 0.00$	Geary τ	0.0548 (0.0112)	0.0966 (0.0224)	0.2474 (0.0820)	0.5366 (0.2660)	0.8270 (0.6576)	0.9306 (0.8806)
	Dickey-Fuller τ_{μ}	0.0484 (0.0096)	0.1214 (0.0254)	0.4654 (0.1440)	0.9720 (0.7500)	1.0000 (0.9998)	1.0000 (1.0000)
$\alpha = 0.10$	Geary τ	0.0504 (0.0138)	0.0822 (0.0254)	0.2150 (0.0834)	0.5188 (0.2896)	0.8206 (0.6926)	0.9302 (0.8928)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0472 (0.0096)	0.0854 (0.0190)	0.2830 (0.0830)	0.8392 (0.4836)	1.0000 (0.9980)	1.0000 (1.0000)
$\alpha = 0.25$	Geary τ	0.0634 (0.0108)	0.0356 (0.0042)	0.1414 (0.0290)	0.4920 (0.2042)	0.8274 (0.6306)	0.9418 (0.8804)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0498 (0.0096)	0.0968 (0.0222)	0.3036 (0.0912)	0.8348 (0.4902)	1.0000 (0.9994)	1.0000 (1.0000)
$\alpha = 0.50$	Geary τ	0.0682 (0.0104)	0.0016 (0.0000)	0.0248 (0.0034)	0.2684 (0.0848)	0.7776 (0.5698)	0.9432 (0.8810)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0540 (0.0110)	0.1424 (0.0356)	0.4170 (0.1342)	0.8696 (0.5374)	0.8470 (0.9988)	1.0000 (1.0000)
$\alpha = 0.75$	Geary τ	0.0656 (0.0104)	0.0000 (0.0000)	0.0018 (0.0002)	0.1058 (0.0208)	0.6996 (0.4846)	0.9376 (0.8710)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0478 (0.0104)	0.2586 (0.0896)	0.5682 (0.2384)	0.9132 (0.6130)	1.0000 (0.9978)	1.0000 (1.0000)
$\alpha = 1.00$	Geary τ	0.0504 (0.0110)	0.0000 (0.0000)	0.0000 (0.0000)	0.0192 (0.0036)	0.5550 (0.3672)	0.9314 (0.8750)
	Dickey-Fuller $\tau_{\mu\tau}$	0.0514 (0.0114)	0.4526 (0.2086)	0.7770 (0.4506)	0.9364 (0.6828)	1.0000 (0.9982)	1.0000 (1.0000)

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