Testing Distributional Models for the Irish Equity Market

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Abstract: This study outlines distributional properties and tests the applicability of different models for the ISEQ index and its major constituents. A Stability under Additions procedure is applied to raw, filtered and rescaled returns. The filtering process uses GARCH(1, 1) and GARCH-M(1, 1) specifications, removing the influence of temporal anomalies and non-synchronous trading effects. Rescaling involves the standardising of returns using the time varying models estimates of conditional variance. Results support the ordinary stable and mixtures of stables distributions. The lack of normality is affected by first and second moment dependence.

I INTRODUCTION

The Irish equity market has attracted greater investor attention in recent years with an almost sevenfold increase in turnover since 1990. Much of the focus is due to the popularity of equity markets in general, but is also a response to improvements in trading conditions, and the general structure of the Dublin Stock Exchange. Of paramount importance to investors and researchers is that they correctly assess the risk return relationship that the market offers. Most of the research examining this relationship applies pricing models and statistical tests assuming normality, and thus there is a constant variance. However, if normal characteristics are lacking, it may lead to empirical problems. For instance, findings for any procedure that requires a constant

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variance, for example, regression analysis, can be questioned if a time varying second moment is documented.

Many alternative distributional models have been suggested for their applicability in correctly describing characteristics of equity returns and these can be classified under two headings. First the stable paretian family including the mixtures of stables (Ghose and Kroner, 1995), and second the ARCH family including the GARCH-M (Poon and Taylor, 1992) specifications. These models have both recognised and have similar ability in removing the causes of non normality such as conditional heteroskedasticity (Mills, 1996). In particular, while the kurtosis findings show an improvement for the adjusted data in contrast to raw returns, it still persists (Taylor, 1986).

Literature on changing characteristics has indicated two particular distributional types: mixtures of stables and mixtures of normals. The mixtures of stable paretian model involves either a changing of the scale or skewness parameters. In contrast, the mixtures of normals model involves price series with the same mean and changing variance. Causes of the changing moments have focused on market anomalies due to trading and temporal conditions, as well as nonsynchronous data. These features have been documented for the Irish equity market with time periods involving this paper's realm of analysis, with evidence of seasonalities and thin trading (Cotter, 1997; and Murray, 1995).

Models incorporating conditional dependence have been developed and applied to equity market prices with varying success. Conditional heteroscedastic based approaches have received voluminous empirical attention in finance, and a comprehensive review can be found in Pagan (1996). The basis of these approaches is that the source of leptokurtosis is due to heteroscedasticity in the variance of a distribution. As many of the ARCH related models may be applicable to equity returns, it is necessary to have a justified rationale for examining any particular specification. Gallant et al. (1992) find that temporal and trading factors have an influence on both the conditional mean and variance of US data. Incorporating both the first and second moment influences together suggests a risk return relationship. Given this relationship, a GARCH-M model introduced by McCurdy and Morgan (1988) can be applied. The application of this model to speculative data has been extended to deal with autoregressive effects.

Previous analysis of time series properties of the ISEQ include measuring autocorrelation and determining day of the week effects (Lucey, 1994). This paper readdresses this issue by testing the applicability of different distributional models for the ISEQ and FTSE indexes using a Stability under Additions procedure. As the Irish market is heavily influenced by a number of relatively

1. Risk return relationships are often formally inherent in the make-up of asset pricing models, for example, the Capital Asset Pricing Model (CAPM).
large stocks, an analysis of different sized equities provides a more detailed picture of the market's operation. The data is examined under three forms: raw, filtered and rescaled returns. A filtering process removes trading and temporal anomalies influencing the raw returns through GARCH(1, 1) and GARCH-M(1, 1) models. Rescaling involves standardising the filtered returns by using the estimates of the conditional variance, obtained from the time varying frameworks. These approaches determine how influential the anomalies are in determining distributional conclusions. Causes of non-normal properties and the robustness of our GARCH models are also detailed.

The outline of the paper is as follows: initially, a description of the data is given, showing summary statistics. This is followed by a methodology section where a description of the filtering and rescaling procedures, as well as the stability under additions test, is given. Next, results are presented and discussed in the empirical findings section. Finally, conclusions are detailed.

II DATA

There has been considerable progress in the structure of Dublin's equity market which bears well for its future. Whereas the Irish stock exchange is small by international standards, there has been a substantial improvement in its capitalisation value in recent times. Two important structural factors positively influencing the size of the index have been the introduction of relatively large companies from the State owned sector, for example Greencore and Irish Life, and food co-operatives which went public, such as Waterfood Foods. In an international context, there is considerable room for more increases in size, as FTSE's value represents over 100 per cent of UK GDP. In comparison, ISEQ represents under 50 per cent of Irish GDP.

Trading conditions on the Dublin market have improved considerably (with the exception of thin trading) in recent times. Thin trading is largely associated with small companies as large companies account for most of the active volume. For instance, the largest company in the index is Allied Irish Bank, representing over 15 per cent of ISEQ's total value, and the top five account for over 55 per cent of the total market. Otherwise, the Irish market offers very attractive trading options to investors. This can be seen by examining recent changes in trade levels. In 1997, a relatively busy year on the Dublin market, the total turnover was IR£23,017m in comparison to IR£3,346m in 1990. Increases in the level of trade have largely resulted from a reduction in transaction costs and innovative usage of technology. Another recent development has been the reorganisation of the classification of small companies. In the past, a number of the smaller sized Irish equities have been quoted on the Unlisted Securities Market, the Small Commodities Market and the Exploration Securities Market.
With very little interest in the first two markets, these were replaced in 1996 with the Developing Companies Market.

Summary statistics for daily returns between January 1987 and December 1997 are shown in Table 1, where details of the first four moments plus the Jarque-Bera normality test are presented. Each of the moments is calculated assuming the other variables are constant. The more commonly used compounded returns are generated using the first difference of the natural logarithms of all closing daily prices, known hereafter as raw returns:

$$\log(P_t) - \log(P_{t-1})$$

The FTSE100 index is chosen as a matter of comparison, which is also the case for major constituents of the Irish market, namely, Allied Irish Bank, Bank of Ireland and Jefferson Smurfit. The other equities included cover a representative size range in market capitalisation terms, Silvermines, (with a market capitalisation at the end of 1997) of under IR£75m, and Grafton of between IR£75 and IR£500m.

Table 1: Summary Statistics of Equity Returns

<table>
<thead>
<tr>
<th>Series</th>
<th>Mean (%D)</th>
<th>Max (%)</th>
<th>Min (%)</th>
<th>Standard Deviation (%)</th>
<th>Skewnessa</th>
<th>Kurtosisa</th>
<th>Jarque-Bera Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISEQ</td>
<td>4.41x10-2</td>
<td>8.54</td>
<td>-12.77</td>
<td>9.72x10-1</td>
<td>-1.58*</td>
<td>23.61*</td>
<td>67923.9*</td>
</tr>
<tr>
<td>FTSE100</td>
<td>3.73x10-2</td>
<td>7.59</td>
<td>-13.02</td>
<td>9.27x10-1</td>
<td>-1.67*</td>
<td>25.89*</td>
<td>80273.4*</td>
</tr>
<tr>
<td>Allied Irish Bank</td>
<td>5.97x10-2</td>
<td>13.11</td>
<td>-15.68</td>
<td>1.60</td>
<td>-0.58*</td>
<td>11.59*</td>
<td>16274.1*</td>
</tr>
<tr>
<td>Bank of Ireland</td>
<td>7.96x10-2</td>
<td>12.10</td>
<td>-13.70</td>
<td>1.56</td>
<td>-0.08</td>
<td>8.78*</td>
<td>9256.2*</td>
</tr>
<tr>
<td>Jefferson Smurfit</td>
<td>2.82x10-2</td>
<td>24.06</td>
<td>-24.95</td>
<td>1.97</td>
<td>-0.67*</td>
<td>25.52*</td>
<td>78103.5*</td>
</tr>
<tr>
<td>Grafton</td>
<td>1.11x10-1</td>
<td>34.83</td>
<td>-29.48</td>
<td>2.16</td>
<td>-0.28*</td>
<td>71.61*</td>
<td>613742.0*</td>
</tr>
<tr>
<td>Silvermines</td>
<td>-3.37x10-3</td>
<td>35.67</td>
<td>-38.57</td>
<td>2.77</td>
<td>-0.20*</td>
<td>38.31*</td>
<td>172866.0*</td>
</tr>
</tbody>
</table>

* represents significant at 5 per cent level.
(a) A normal distribution has a skewness and kurtosis coefficient of zero and three respectfully.

ISEQ exhibited positive returns over the full period of analysis. However, for sub-periods the index returns varied considerably, being disproportionately affected by certain factors. Down side factors include the problems in financial markets caused by the October 1987 crash, the Gulf War in 1990, and the currency crises with its dampening of investors expectations in 1992. On the upside, ISEQ was positively influenced by the currency devaluation of 1993, and the performance of world markets in general, such as the historic high for the US market in 1997. Returns from the ISEQ compared favourably with the London market and there was a large similarity in their pattern. The Dublin market also contains many individual equities offering a wide variation in
returns, with a high proportion of companies of similar size to Grafton and Silvermines having returns following distinct courses. This represents a good choice for investors seeking holdings with a wide range of return possibilities.

A second important performance indicator for investors is equity risk, conventionally measured by the standard deviation of returns. Volatility for the ISEQ is less than 1 per cent per day, and very similar to FTSE's risk levels. There is also a high degree of similarity in the risk pattern of both indices through time with, for example, 1996 exhibiting low volatility and 1987 high volatility. Investors attracted to firm specific, rather than market risk, have a diverse range to choose from on the Dublin market as the volatility of the representative equities differ significantly from the market index. Also, firm specific risk varies inversely with company size. Investors looking for highly risky options should note that as firm size decreased (for instance Allied Irish Bank to Silvermines), the corresponding volatility measure increased.

The issue of normality has often been addressed by examining both the skewness and kurtosis coefficients (Badrinath and Chatterjee, 1991). While there is no clear consensus regarding the sign of the skewness coefficient, generally the magnitude is small and close to zero for speculative prices, thereby suggesting a symmetrical distribution. The skewness coefficient, \( \beta \), examines the concentration of returns to the right or left of the mean value. Our findings contradict the hypothesis that the data is symmetric, based on the test statistic \( t = \beta/(SE \beta) \). Negative skewness is prominent for our data in Table 1, with both indexes exhibiting similar values. Extreme negative factors such as the worldwide effects of the Asian market crises manifest themselves as outliers, leading to a conclusion of asymmetry. Skewness findings are incorporated in the stability under additions procedure when testing different distributional hypotheses.

The degree of kurtosis, \( \alpha \), refers to the extent to which the distribution of returns are located around average values. The general finding for stock market returns is that they are more peaked than a normal distribution, which is bell shaped. This is because an excess of returns are bunched too closely to the mean, as well as far away from it, in comparison to the normal distribution. The test statistic \( t = \alpha/(SE \alpha) \) examines whether the kurtosis value is in line with a normal distribution or not. The general conclusion of leptokurtosis is accepted as significant kurtosis is shown for all data in Table 1 with similar values for the stock indexes. The issue of normality is formally tested using the Jarque-Bera statistic. Normality is rejected at conventional levels and is in line with previous studies on the distributional properties of equity returns. This conclusion can be clearly seen in Figure 1, indicating significant leptokurtosis, and to a lesser extent, negative skewness, in comparison to the bell shaped normal distribution.
Figure 1: Histogram of ISEQ with Normal Curve

III METHODOLOGICAL ISSUES

A stability under additions test is applied to examine the distributional properties of raw equity returns. This focuses on the kurtosis coefficient and allows for interdependency with other distributional moments. Two other distinct sets of ISEQ's returns are tested using the stability under additions test. First, residuals from a filtering technique based on GARCH specifications are examined. Second, the filtered returns are rescaled using the time varying models' estimates of conditional variance. This section discusses the stability under additions test, the filtering, and rescaling processes.

3.1 Stability Under Additions Test

The stability under additions test relies on the stable paretian model defined by its characteristic function:

$$\log \phi(t) = i \delta t - \gamma |t| \alpha (1 + \beta (t/|t|) \tan (\pi \alpha / 2))$$

for $0 < \alpha \leq 2$  \hspace{1cm} (2)

where $t$ is any real number, $\delta$ is the location parameter, $\gamma$ is the scale parameter, $\beta$ is the skewness parameter, and $\alpha$ is the kurtosis parameter and is also the characteristic exponent.
The characteristic exponent describes the shape of the areas around the mean and the tails of the distribution. Specific types of stable distributions realise different values. The special case of the normal distribution is described when \( \alpha = 2 \). Speculative data from spot markets have a kurtosis value between 1 and 2 (Hall et al., 1989). In this case, the distribution has a finite mean, but the variance is infinite. In practice this infinite variance implies that erratic behaviour is shown by returns for large samples and this is indicated by larger standard deviations than for Gaussian processes. This conclusion falls neatly into the volatility clustering arguments that support time varying models such as GARCH where periods of high volatility can occur consecutively, as demonstrated by fat tails. The second distributional moment generally aligned with non-normality is the skewness coefficient, for example, see Table 1. The stable distribution recognises skewness by allowing the parameter take on values in the interval \(-1 \leq \beta \leq 1\).

Distributional characteristics are tested for using the stability under additions test, developed by McCulloch (1986). This approach uses a fractal approach similar to Fama and Roll's (1971) test, but has the additional property of allowing the skewness parameter to take on a range of values \((-1 \leq \beta \leq 1\)). As non-trivial skewness is found for ISEQ's returns, it is best to apply a procedure that incorporates varying skewness in the calculation of the kurtosis coefficient. Also, this newer method has an advantage of removing a small asymptotic bias in Fama and Roll's estimates of \( \alpha \). Five sample quantiles, and linear interpolations of tabulated index numbers are the basis of this technique. Indexes for both \( \alpha \) and \( \beta \) coefficients are measured using the following:

\[
V_\alpha = \frac{V_{0.95} - X_{0.05}}{X_{0.75} - X_{0.25}}
\]

and

\[
V_\beta = \frac{X_{0.95} + X_{0.05} - 2X_{0.5}}{X_{0.75} - X_{0.25}}
\]

where \( V_\alpha \) and \( V_\beta \) are sample estimates of corresponding population values. These indexes are dependent on each other. In contrast, summary statistics assume that each coefficient is calculated with other parameters being constant. Interdependent values for \( \alpha \) and \( \beta \) are tabulated by McCulloch (1986).

The procedure operates by comparing the characteristic exponent across different sums of observations, based on a property of stable distributions that they are invariant under addition, and for a particular stable distribution, the kurtosis coefficient will remain relatively unchanged across different sums of
observations, that is $\alpha_1 = \alpha_2 = \ldots = \alpha_n$. However, when approximating sample values, $\alpha$ estimates can be affected by sampling error, and the extent of this could be compounded with the stability under additions test when you move to larger sums of observations, as the sample size decreases. To control for possible sampling error, the number of overlapping sums of observations is limited to 20.

3.2 Filtering Technique

The purpose of the filtering technique is to remove documented anomalies from the data, thereby generating smooth returns series'. By using (5) as the relationship between the sequences $\{Y_t\}$ and $\{|Z_t|\}$, the estimation of the variance parameter relies on a linear GARCH ($p, q$).

$$Y_t = bZ_t + \epsilon_t$$  \hspace{1cm} (5)

Extending (5) to include an estimate of the conditional variance gives a GARCH-M model, outlined in (6).

$$Y_t = bZ_t + c\epsilon_t + \epsilon_t$$  \hspace{1cm} (6)

In this case, the time varying model involves a joint estimation of the mean involving the disturbance term plus the hetrasdeskastic process that estimates the second moments.

The specific GARCH-M filtering process adopted here is similar in its objective to that of Engle and Ng (1993). It involves a joint estimation of our mean equation incorporating the conditional variance, as well as the heteroscedastic process that estimates the second moments and is given in (7). The model removes temporal anomalies, such as the day of the week effects (see Gallant et al., 1992 for a discussion); autocorrelation effects which result in nonsynchronous data (for example, thin trading); and the influence of conditional variance. A GARCH(1, 1) specification similar to (7) without the measure of the conditional variance is included, due to mixed evidence for a GARCH-M application with the FTSE100 (Poon and Taylor, 1992).

$$Y_t = a_0 + \sum A_s X_{t-s} + A_{11}D_M + A_{12}D_T + A_{13}D_W + A_{14}D_{Th} + A_{15}D_F + A_{16}D_H + A_{17}h_t + \epsilon_t$$  \hspace{1cm} (7)

Where $Y_t$ is the return series to be filtered; $X_{t-s}$ is the lagged raw returns series; possible temporal anomalies are given by dummy variables so that $D_M = 1$ if Monday, $D_T = 1$ if Tuesday, $D_W = 1$ if Wednesday, $D_{Th} = 1$ if Thursday, $D_F = 1$ if
Friday and \( D_{n} = 1 \) if a holiday. Otherwise they are 0. The measure of the conditional variance is denoted by \( h_{t} \). The residuals, \( \varepsilon_{t} \), are the filtered returns series.

The risk measure is allowed to vary over time and is assumed to follow a GARCH (1,1) process given by Equation (8) where adjustments are made for day of the week and holiday effects, and a trend.

\[
h_{t} = b_{0} \varepsilon_{t-1}^{2} + \beta h_{t-1} + b_{1} D_{M} + b_{2} D_{T} + b_{3} D_{W} + b_{4} D_{Th} + b_{5} D_{F} + b_{6} D_{H} - b_{7} T_{t} + b_{8} T_{t}^{2} E_{t} | V_{t-1} \sim N(O, h_{t})
\]

(8)

Linear and quadratic trend variables, \( T_{t} \) and \( T_{t}^{2} \) are included in the variance specification, but not directly in the mean equation (Gallant et al., 1992).

3.3 Rescaled Returns

A related set of returns to those generated by the filtering process are the rescaled returns, and these are obtained as follows:

\[
Z_{t} = \frac{\varepsilon_{t} - \varepsilon}{\sqrt{h_{t}}}
\]

(9)

where \( h_{t} \) is an estimate of the actual conditional variance.

It is common to focus in on a simplified variation of (9), where the filtered returns series is divided by the conditional standard deviation, as the results are similar. Both Taylor (1986) and Bollerslev (1988) suggest that much of the serial dependence exhibited in the second moments of an asset's return can be removed if one examines the standardised returns after applying an ARCH related approach.

Bollerslev (1988) suggests a similar diagnostic to determine whether the conditional heteroskedastic model is able to capture the serial dependence in the data. The test is based on examining the autocorrelation function of the standardised filtered returns, \( \varepsilon_{t} / h_{t} \). Also, Ding and Granger (1996) suggest that an examination of autocorrelation values for the squared returns series indicates whether ARCH effects are present. Thus, a comparison of dependency for raw and rescaled returns indicates whether the influence of serial dependence in the second moments have been removed after filtering. To determine whether the Irish equity market exhibits dependency in the first and second moments, sample autocorrelations are calculated with 30 lags. Also, for lags at an aggregate level, the Ljung-Box Portmanteau Q statistics is used to formally assess whether all calculated autocorrelations are zero.
IV EMPIRICAL FINDINGS

The empirical findings for the applicability of various distributional models for the Irish equity market are now discussed. The stability under additions test is applied to returns obtained from different filtering procedures based on related time varying ARCH models. These models are estimated by maximum likelihood methods using the algorithm developed by Berndt et al. (1974). Possible causes of the distributional findings are identified by measuring autocorrelation values for the equity data. Also, the ability of the filtering technique to remove ARCH effects is analysed using the autocorrelation findings.

The values for the characteristic exponent for the raw compounded returns are shown in Table 2. The classifications are 1, 2, 5, 10, 15 and 20 loosely corresponding to daily, bidental, weekly, biweekly, triweekly and monthly continuously compounded returns. A null hypothesis of the ISEQ index belonging to a normal distribution is rejected conclusively using the stability under additions test as none of the kurtosis coefficients have values equal to 2. The FTSE100 and major constituents of the Irish market are also not normally distributed. The characteristic exponent findings suggest that the underlying distribution characterising the ISEQ is a mixtures of stables distribution as the \( \alpha \) coefficient values increase across the sums of observations, and reach 2 for monthly returns. Individual Irish equities have similar kurtosis coefficients to the ISEQ for daily returns as one would expect from assets which are so prominent in the makeup of the market portfolio. In contrast, the characteristic exponent for the FTSE100 is relatively higher. There is no clear evidence to support the ordinary stable distribution for the data analysed.

Table 2: Estimates of the Characteristic Exponent for Raw Equity Returns

<table>
<thead>
<tr>
<th>Series</th>
<th>( n = 1 )</th>
<th>( n = 2 )</th>
<th>( n = 5 )</th>
<th>( n = 10 )</th>
<th>( n = 15 )</th>
<th>( n = 20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISEQ</td>
<td>1.48</td>
<td>1.49</td>
<td>1.56</td>
<td>1.59</td>
<td>1.81</td>
<td>2.00</td>
</tr>
<tr>
<td>FTSE100</td>
<td>1.87</td>
<td>1.72</td>
<td>1.99</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Allied Irish Bank</td>
<td>1.49</td>
<td>1.54</td>
<td>1.77</td>
<td>2.00</td>
<td>1.70</td>
<td>1.76</td>
</tr>
<tr>
<td>Bank of Ireland</td>
<td>1.39</td>
<td>1.56</td>
<td>1.62</td>
<td>1.72</td>
<td>1.70</td>
<td>1.76</td>
</tr>
<tr>
<td>Jefferson Smurfit</td>
<td>1.43</td>
<td>1.48</td>
<td>1.57</td>
<td>1.64</td>
<td>1.66</td>
<td>1.54</td>
</tr>
</tbody>
</table>

\( n \) is the different sums of observations.

In order to determine whether the stable paretian family, and in particular, the mixtures of stables is not only the most applicable classification, but also, and of more importance, whether Irish equity returns actually follow this

2. Convergence is not achieved for Silvermines and Grafton using the GARCH and GARCH-M specifications so these are excluded from further analysis.
distribution, the issue is investigated further. The stability under additions is reapplied to the equity data after filtering using GARCH and GARCH-M specifications. Trading and seasonality effects (and conditional volatility influences in the case of a GARCH-M) are thereby removed from the raw compounded returns. These filtered returns are then rescaled using the measures of conditional variance from the GARCH models, and the findings are shown in Table 3.\(^3\) In general, both GARCH filters generate similar conclusions, but the kurtosis values for the ISEQ index are higher for the GARCH(1, 1) specification. Normality is again rejected for the equity returns.

### Table 3: Estimates of the Characteristic Exponent for Rescaled Returns

<table>
<thead>
<tr>
<th>Series</th>
<th>(n = 1)</th>
<th>(n = 2)</th>
<th>(n = 5)</th>
<th>(n = 10)</th>
<th>(n = 15)</th>
<th>(n = 20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISEQ GARCH(1, 1)</td>
<td>1.51</td>
<td>1.56</td>
<td>1.65</td>
<td>1.45</td>
<td>1.71</td>
<td>1.64</td>
</tr>
<tr>
<td>FTSE100 GARCH(1, 1)</td>
<td>1.92</td>
<td>1.73</td>
<td>1.82</td>
<td>1.97</td>
<td>1.86</td>
<td>1.87</td>
</tr>
<tr>
<td>Allied Irish Bank</td>
<td>1.56</td>
<td>1.63</td>
<td>1.70</td>
<td>1.87</td>
<td>1.68</td>
<td>1.63</td>
</tr>
<tr>
<td>Bank of Ireland</td>
<td>1.45</td>
<td>1.59</td>
<td>1.60</td>
<td>1.55</td>
<td>1.55</td>
<td>1.48</td>
</tr>
<tr>
<td>Jefferson Smurfit</td>
<td>1.43</td>
<td>1.48</td>
<td>1.56</td>
<td>1.57</td>
<td>1.70</td>
<td>1.59</td>
</tr>
<tr>
<td>ISEQ GARCH-M(1, 1)</td>
<td>1.48</td>
<td>1.47</td>
<td>1.54</td>
<td>1.44</td>
<td>1.31</td>
<td>1.38</td>
</tr>
<tr>
<td>FTSE100 GARCH-M(1, 1)</td>
<td>1.85</td>
<td>1.78</td>
<td>1.79</td>
<td>1.77</td>
<td>1.70</td>
<td>1.74</td>
</tr>
<tr>
<td>Allied Irish Bank</td>
<td>1.48</td>
<td>1.54</td>
<td>1.59</td>
<td>1.55</td>
<td>1.59</td>
<td>1.64</td>
</tr>
<tr>
<td>Bank of Ireland</td>
<td>1.44</td>
<td>1.59</td>
<td>1.77</td>
<td>1.77</td>
<td>1.58</td>
<td>1.83</td>
</tr>
<tr>
<td>Jefferson Smurfit</td>
<td>1.41</td>
<td>1.47</td>
<td>1.57</td>
<td>1.63</td>
<td>1.56</td>
<td>1.41</td>
</tr>
</tbody>
</table>

\(n\) is the different sums of observations.

Kurtosis values for daily returns remain relatively unchanged from raw to rescaled returns, but the patterns across the sums of observations changes. As the characteristic exponent values for the ISEQ index remain reasonably constant, this supports the hypothesis that the data is from an ordinary stable distribution, in contrast to the mixtures of stables hypothesis. Thus the ISEQ index indicates an infinite variance, as demonstrated by the characteristic of volatility clustering. Similar conclusions can be made for the individual equities and the FTSE100 index. Obtaining these differing conclusions from raw to rescaled returns indicates a benefit from applying the GARCH filters as it shows that the influence of trading anomalies is important in determining the distributional characteristics of equity data.

3. Filtered returns are not included for analysis as the kurtosis values for rescaled returns are very similar.
A summary of the extent to which first and second moment correlation occurs in the Irish equity market is shown in Table 4. The ISEQ index is not strict white noise as significant dependence exists in the levels for 11 of 30 lags. Similar conclusions can be made for the other series analysed. Strong ARCH effects are indicated for the ISEQ index and these are stronger than the first order dependence, corresponding to a general finding for equity markets (Taylor, 1986). As the other series' indicate similar findings, it is no surprise to see the application in finance of a vast number of models that are inherently setup to deal with such a characteristic.

Table 4: Summary of Autocorrelation Values for Equity Returns

<table>
<thead>
<tr>
<th>Nos. of Significant Lags</th>
<th>Level Returns</th>
<th>GARCH(1,1)</th>
<th>GARCH-M(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISEQ</td>
<td>11 (196.0)</td>
<td>2 (31.2)</td>
<td>0 (7.4)</td>
</tr>
<tr>
<td>FTSE100</td>
<td>5 (61.6)</td>
<td>1 (23.1)</td>
<td>1 (20.6)</td>
</tr>
<tr>
<td>Allied Irish Bank</td>
<td>4 (51.7)</td>
<td>2 (28.8)</td>
<td>1 (90.7)</td>
</tr>
<tr>
<td>Bank of Ireland</td>
<td>3 (60.8)</td>
<td>1 (33.8)</td>
<td>2 (30.6)</td>
</tr>
<tr>
<td>Jefferson Smurfit</td>
<td>3 (47.6)</td>
<td>0 (21.3)</td>
<td>2 (31.7)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nos. of Significant Lags</th>
<th>Squared Returns</th>
<th>GARCH(1,1)</th>
<th>GARCH-M(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISEQ</td>
<td>21 (1.63x10³)</td>
<td>17 (662.0)</td>
<td>0 (1.87x10⁻²)</td>
</tr>
<tr>
<td>FTSE100</td>
<td>12 (1.55x10³)</td>
<td>1 (36.5)</td>
<td>6 (552.0)</td>
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<tr>
<td>Allied Irish Bank</td>
<td>14 (498.0)</td>
<td>2 (39.6)</td>
<td>0 (2.5)</td>
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<tr>
<td>Bank of Ireland</td>
<td>15 (375.0)</td>
<td>1 (29.2)</td>
<td>11 (210.0)</td>
</tr>
<tr>
<td>Jefferson Smurfit</td>
<td>6 (176.0)</td>
<td>6 (95.4)</td>
<td>0 (0.6)</td>
</tr>
</tbody>
</table>

Notes: (a) Ljung Box statistics are in parentheses.
(b) The series |Rt| refers to the unfiltered returns whereas the others are the respective GARCH(1,1) and GARCH-M(1,1) filtered returns.

The two filtering techniques are able to remove much, but not all of the first order correlation for the series analysed. In particular, the null hypothesis of no autocorrelation is accepted after ISEQ's returns are filtered with the GARCH-M specification. Both conditional heteroskedastic models are able to remove much of the serial correlation in the variance parameter, as is expected from these types of models (Bollerslev, 1988). This may be a result of these models explicitly dealing with the influence of thin trading on the Irish equity market. However for the ISEQ index, the GARCH-M specification clearly dominates GARCH(1, 1) in removing second order dependence. The FTSE100 series does not concur with this finding, and confirms the suggestion that a
GARCH-M model may not be fully appropriate in describing the behaviour of these returns (Poon and Taylor, 1992).

V CONCLUSIONS

This study examines distinct distributional hypotheses for the Irish equity market, by focusing on the ISEQ index. The major constituents, as well as the FTSE100 are included for comparative purposes. The paper uses a stability under additions test to determine kurtosis values, allowing the moments of a distribution to be interdependent on each other. The key to the procedure is calculating the characteristic exponent of a stable model, which can take on distinct sets of values, thereby determining the underlying distribution of the data analysed. ISEQ's returns are tested in three forms after applying a filtering process to remove data anomalies, as well as giving estimates of the conditional variance. These are filtered, rescaled (obtained from standardising the filtered returns) and raw compounded returns. Two filtering processes, namely, GARCH(1, 1) and GARCH-M(1, 1) specifications are utilised. The distributional findings are analysed as to determine the influence of dependency in both the mean and variance parameters.

Excess kurtosis and skewness are found in Irish equity returns. Support is offered for the ISEQ belonging to an ordinary stable distribution after removal of temporal and nonsynchronous trading effects. This distributional finding allows for varying parameters such as scale and skewness, and more importantly, the existence of volatility clustering. As ISEQ's raw compounded returns indicate mixtures of stables distribution, it demonstrates the benefits of filtering. Both first and second moment dependence are documented for the raw returns. The filters remove much of the dependence, with the GARCH-M(1, 1) dominating the GARCH(1, 1) model for the ISEQ index.

Given the attention that the Irish equity market attracts, it is important that its practical applications work effectively. A main focus has been on pricing models and inferences based on related statistical procedures. Key assumptions including normality and independence are incorporated in these procedures. This paper’s motivation is to examine whether these characteristics hold, and it finds a lack of normality and independence. As a consequence, investors and researchers should be aware of these characteristics, and in future, should incorporate these findings into applied pricing procedures and their related testing methodologies. Alternative procedures including ARCH related frameworks and processes that cater for non normal properties should be applied.
REFERENCES


