The Economic and Social Review, Vol. 31, No. 3, July, 2000, pp. 187-214

Poverty in Ireland, 1987-1994: A Stochastic Dominance Approach*

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Abstract: Poverty dominance analysis uses stochastic dominance to provide rankings of distributions in terms of poverty which are not sensitive to the choice of poverty line. This analysis is carried out for Ireland using Household Budget Survey data for 1987 and 1994 including tests for the statistical significance of the results. We find that for a wide range of absolute poverty lines, poverty in Ireland fell over the 1987-1994 period. When relative poverty lines are used, second-order dominance for 1987 over 1994 is found for the case of expenditure and third-order dominance for 1994 over 1987 for the case of income.

I INTRODUCTION

Results from poverty studies are frequently sensitive to the choice of poverty line (the means of identifying the poor) and poverty measure (the measure obtained when aggregating the incomes/expenditures of households below the poverty line). Since these choices are typically at the discretion of the analyst, this can give rise to the suggestion that the results obtained are not robust. Potentially different results could be obtained by the choice of a different poverty line/measure. Few conclusions can be drawn if poverty trends differ substantially

*This paper draws substantially upon *The Measurement and Analysis of Poverty in Ireland: Some New Perspectives*, an unpublished MA thesis by Fiona Smith. We gratefully acknowledge the helpful comments of Anthony Murphy and two anonymous referees. We remain responsible for any remaining errors.

when different poverty measures are applied or the position of the poverty line is changed. Analysts usually overcome this problem by employing a number of poverty lines/measures but this only partially overcomes the problem since it may still be possible to obtain different results by the choice of another poverty line or measure.

Thus what is ideally needed is an approach which is robust to the choice of poverty line. Poverty dominance analysis, which is an application of stochastic dominance, provides such a method. Stochastic dominance, in relation to poverty, relates to the ranking of income/expenditure distributions, i.e. it examines whether one distribution has unambiguously more or less poverty than another over a range of potential poverty lines. In this paper we apply such poverty dominance analysis to the Irish Household Budget Surveys of 1987 and 1994 using both income and expenditure data. We also test the statistical significance of our results. This is a potentially attractive approach since as Deaton (1997) puts it "… the reduction of a distribution to a single number is perhaps a lot more aggregation than we want, and more fundamental insights into the levels of living can often be obtained from graphical representations of either the whole distribution or some part of it" (Deaton, 1997, p. 158).

The remainder of the paper is as follows: in Section II we briefly explain the poverty dominance approach and show its links with other social welfare measures. In Section III we apply this approach to the Irish Household Budget Survey of 1987 and 1994 and present results for income and expenditure, while in Section IV we repeat the analysis this time using purely relative poverty lines. In all cases we test for the statistical significance of our results. In Section V we provide some concluding comments.

II STOCHASTIC DOMINANCE AND POVERTY DOMINANCE ANALYSIS

Studies of poverty typically present results for a variety of poverty measures. There is a wide range of such measures but it seems fair to suggest that the following three measures are among the most popular: (a) the headcount ratio (b) the income gap measure and (c) the Foster-Greer-Thorbeck (FGT) P_α measure where $\alpha = 2$ (see Foster *et al.*, 1984). These are now briefly explained. Suppose we have n households with income/expenditure[1] $x_1, x_2, \ldots, x_n$ and the poverty line is z. Then if q households have incomes below z, the headcount ratio is simply H = q/n. However, the headcount ratio has a number of deficiencies as a

1. The choice of income or expenditure is not a trivial one and we discuss it in more detail below. For the purposes of this section of the paper we use "income" but in Section III we will present results for both income and expenditure.

poverty measure, the most important of which are the fact that it takes no account of the depth of poverty and that it does not satisfy the principle of transfers (i.e. it is unchanged following a transfer of income from poor to a less poor household when both households are below the poverty line). The first of these deficiencies can be remedied by the choice of an income gap measure as our poverty measure. The income gap measure sums all the proportionate shortfalls below the poverty line for poor households: $\sum_{i=1}^{q} \frac{z - x_i}{z}$. This measure is then normalised by dividing by the total number of households, n. This can also be expressed as H.I where $I = \frac{z - \bar{x}_p}{z}$ and $\bar{x}_p$ is average income for poor households. This measure takes account of the depth of poverty but does not satisfy the principle of transfers.

To take account of the principle of transfers we use the FGT P_α measure when $\alpha = 2$. This measure weights the income gaps by the gaps themselves thus awarding a higher weight to poorer households. Thus we have $P_2 = \frac{1}{n} \sum_{x_1<z}^{q} \left(\frac{z - x_i}{z} \right)^2$. When $\alpha = 0$ this measure equals H, and when $\alpha = 1$ the measure equals H.I.

This approach suffers from two drawbacks however. First, the rankings of different income distributions in terms of poverty may be sensitive to where the poverty line, z, is drawn. Second, the ranking may be sensitive to the particular poverty measure chosen. Poverty dominance addresses both of these issues.

Poverty dominance analysis is an application of stochastic dominance to distributions of households' income. Until recently, probably the main application of stochastic dominance in economics was in relation to assets with monetary payoffs where it is used to rank the payoff distributions of assets in terms of their level of return and the dispersion of the return, i.e. the level of risk attached to the asset. However, it is also extremely useful in income distribution and poverty analysis.

Suppose we have two distributions with cumulative density functions (CDF) F(x) and G(x) respectively. Then CDF F(x) first-order stochastically dominates G(x) if and only if, for all monotone non-decreasing functions $\alpha(x)$:

$$\int \alpha(x)dF(x) \geq \int \alpha(x)dG(x)$$

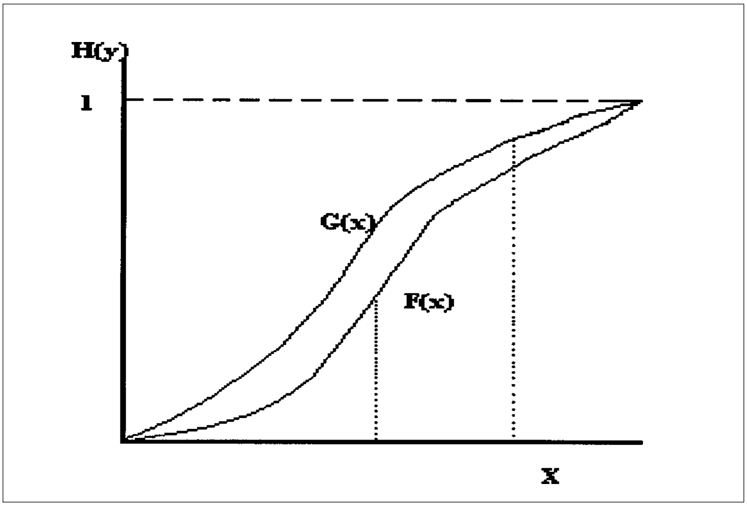
where the integral is taken over the whole range of x. Thus the average value of α is at least as large in distribution F as it is in distribution G, as long as the valuation function is such that more is better, i.e. it is monotone non-decreasing. In this sense distribution F stochastically dominates distribution G. An equivalent way of expressing this is to say that for all x,

$$G(x) \geq F(x)$$

so that the CDF of distribution G is always at least as large as that of distribution F, i.e. distribution G always has more mass in the lower part of the distribution.

In terms of a diagram, the cumulative distribution points H(y), on the vertical axis, are proportional to the area under the curves and to the left of x. As we can see from Figure 1, distribution G(x) is everywhere above distribution F(x) and so the probability of getting at least x is higher under F(x) than G(x), thus F(x) first-order stochastically dominates G(x).

Figure 1: *First-Order Stochastic Dominance*



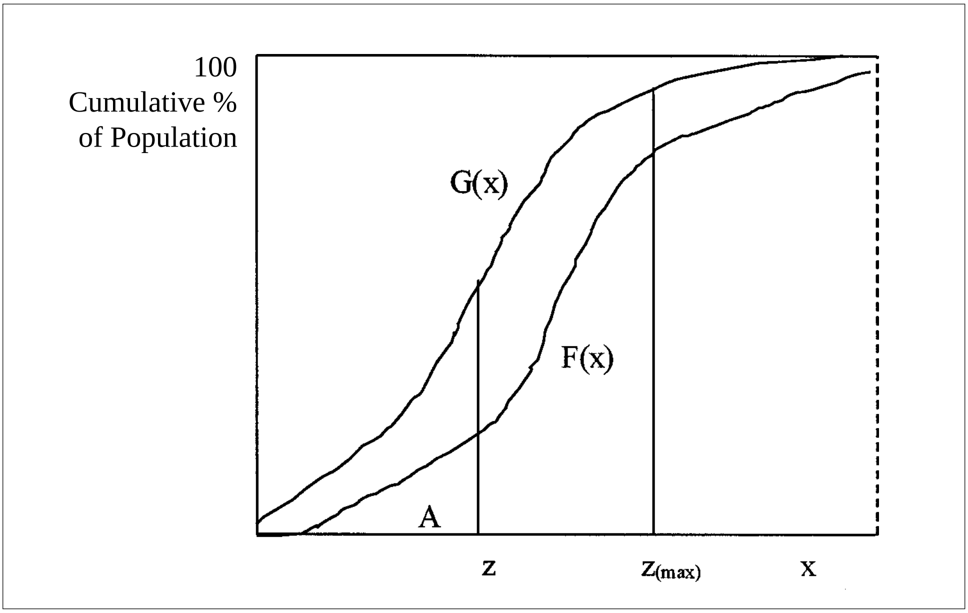
So how is this related to poverty analysis? Suppose we decide upon a poverty line and denote it as z. If we have n households in total and if q households have income below z, then as outlined above, the headcount ratio, is $P_0 = q/n$. In this case the CDFs are referred to as Poverty Incidence Curves and each point on the graph gives the proportion of the population consuming less than or equal to the amount given on the horizontal axis. The cumulative distribution points are equivalent to head-count ratios in the sense that they represent the proportion of the population at and below a particular income level. Suppose we do not know the poverty line z, but we are sure it does not exceed z^{max}. As Ravallion (1994) states under these circumstances "poverty will fall between two dates if the poverty incidence curve for the latter date lies nowhere above that for the former date, up to z^{max}. This is called the First-order Dominance Condition (FOD)".[2] In other words if, for all poverty lines up to z^{max}

$$G(x) \geq F(x)$$

then P_0 will always be higher for the first distribution than the second, i.e. the poverty ranking of two distributions according to the headcount ratio is robust to all choices of the poverty line up to z_{max} if, and only if, one distribution stochastically dominates the other.

In terms of our diagrams in Figure 2 the distribution G(x) is everywhere above that of distribution F(x) and so poverty is higher for G(x) than F(x), no matter where the poverty line is drawn. This reflects the fact that the proportion of people consuming less than or equal to z_{max} is always greater with distribution G(x) than with distribution F(x). We can thus conclude that distribution F(x) first-order poverty dominates distribution G(x).

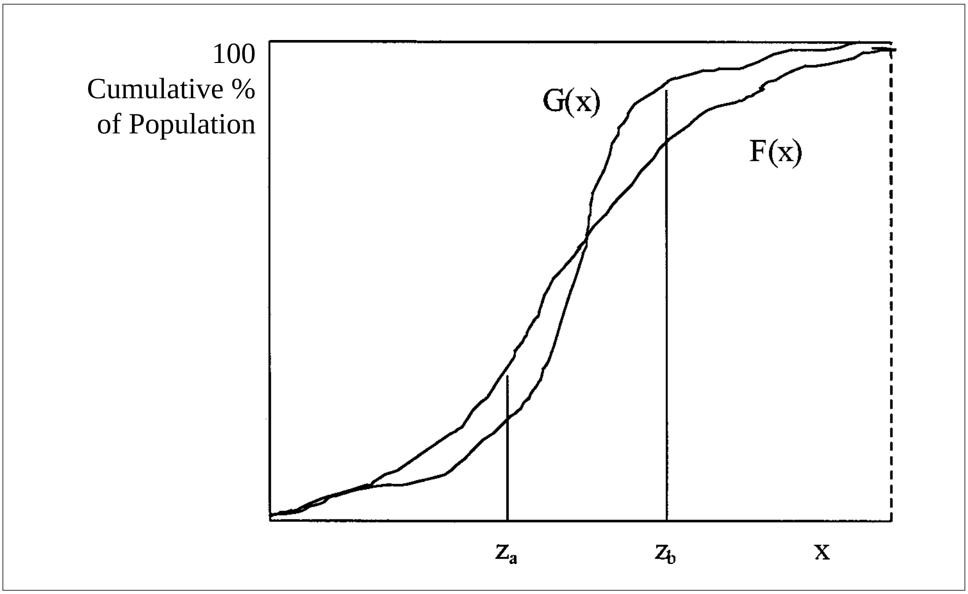
Figure 2: *First-order Poverty Dominance*



2. See Ravallion (1994, p. 69).

If the curves intersect, as in Figure 3 below, then the ranking is ambiguous. For example, if the poverty line was set at z_b, as in Figure 3, then distribution G(x) will lie above distribution F(x). If the poverty line however, is set at z_a then distribution F(x) will lie above distribution G(x). Poverty thus at z_b is higher with distribution G(x) but at z_a poverty is higher with distribution F(x). We cannot therefore unambiguously state that one distribution exhibits poverty dominance over the other as their ranking in terms of poverty changes depends on where the poverty line is drawn.

Figure 3: *Crossing of Poverty Incidence Curves*



In this case there are essentially two courses we can pursue if we wish to establish dominance. First we could restrict the range of the poverty line over which we search for dominance, i.e. look for dominance in an interval $z_{min} \leq z \leq z_{max}$. We will return to this approach later. Alternatively, we could impose more structure on our valuation function $\alpha(x)$ and hence equivalently on the range of admissible poverty measures.

This leads us on to the second type of stochastic dominance known as second-order stochastic dominance. We say that distribution F(x) second-order stochastically dominates distribution G(x) if and only if, for all monotone non-decreasing and *concave* functions $\alpha(x)$ the previous inequality holds, i.e.

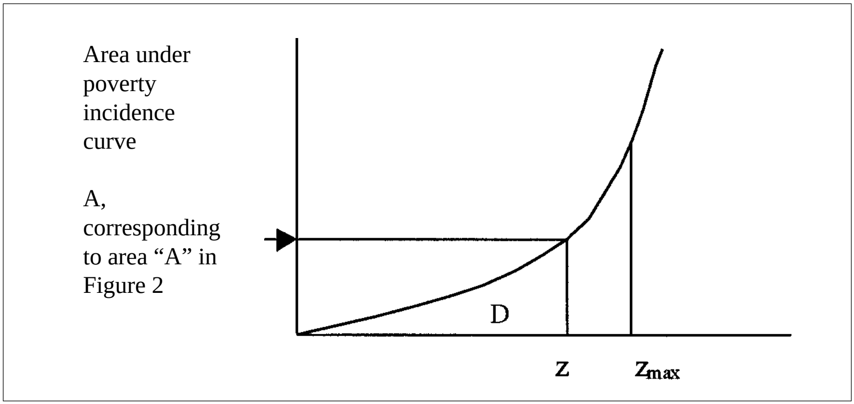
$$\int \alpha(x)dF(x) \geq \int \alpha(x)dG(x).$$

Once again second-order stochastic dominance can be expressed equivalently as

$$D_G(x) = \int^{x} G(t)dt \geq \int^{x} F(t)dt = D_f(x)$$

so that second-order stochastic dominance is checked, not by comparing the CDFs themselves, but by comparing the integrals below them.[3] When $\alpha(x)$ is concave, we can interpret the integrals beneath them as additive social welfare functions with $\alpha(x_i)$ the social valuation (utility) function for individual i. In the case of poverty analysis, this implies that second-order dominance holds for measures which are strictly decreasing and at least weakly convex in the incomes of the poor, i.e. measures which are sensitive to the depth of poverty such as the income gap ratio. We can then employ the Second-Order Dominance Condition above. To examine the robustness of the income gap ratio we must consider the "poverty deficit curve" which can be defined as the area under the CDF or the integrals of the CDF, as discussed above, up to some poverty line z (See Figure 4 below).

Figure 4: *Poverty Deficit Curve for F(x)*

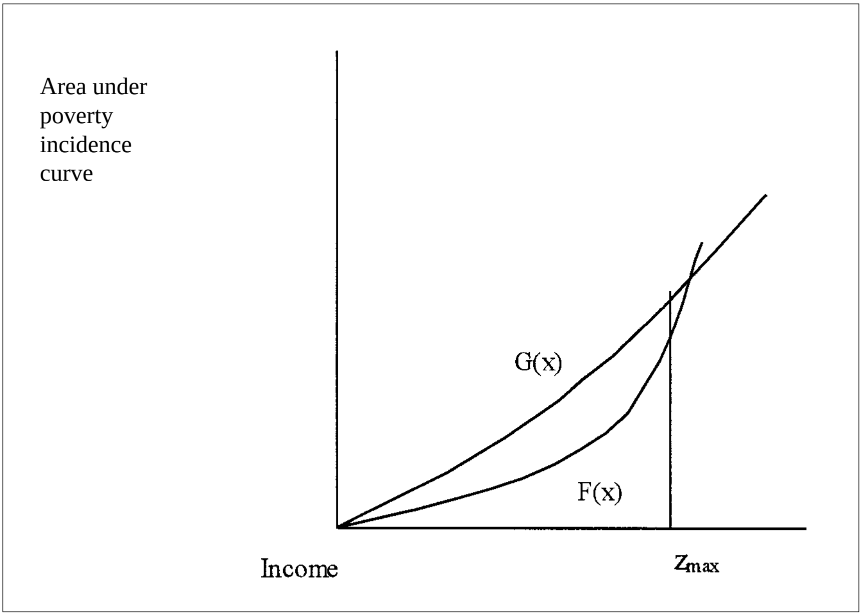


In this case a fall in poverty requires that the poverty deficit curve is nowhere lower for the earlier date at all points up to the maximum poverty line and at least somewhere higher (see Figure 5 below). Poverty is thus higher for G(x), in Figure 5, as its poverty deficit curve is above that for F(x) up to z_{max}. As previously mentioned, this result will hold for a measure that is sensitive to the depth of

3. Note that first order dominance necessarily implies second order dominance.

poverty such as the income gap ratio but not the head-count ratio. Second-order stochastic dominance is a concept that is weaker than first-order stochastic dominance as first-order stochastic dominance implies second-order dominance but not vice versa.

Figure 5: *Poverty Deficit Curves for F(x) and G(x)*



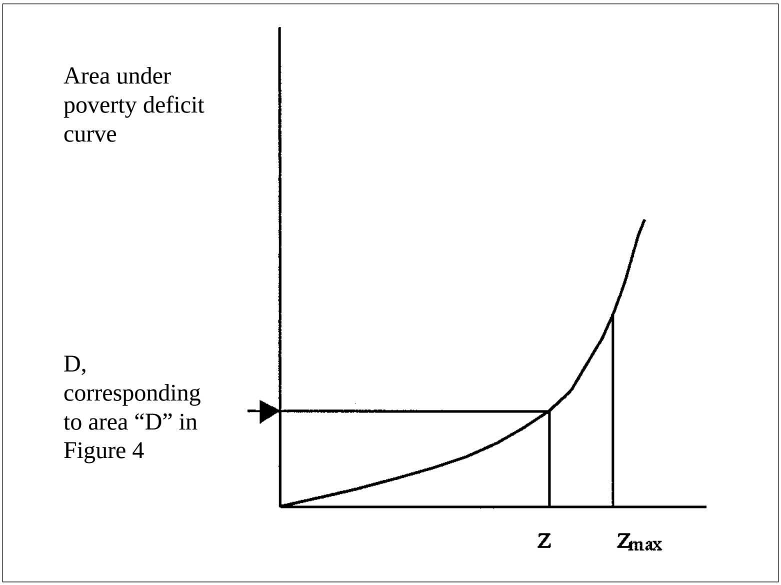
Once again, there is no guarantee that second-order dominance will hold and so once again, we might be interested in putting further restrictions on the valuation function α(x) or equivalently the underlying poverty measures. Suppose we add the further restriction to α(x) that its third derivative be non-negative, i.e. $\alpha''' \geq 0$.[4] This is the case of third-order stochastic dominance and it is equivalent to imposing the condition that our poverty measure places a higher weight on the poverty gaps of poorer households.[5] Thus once again we have the original dominance condition except that now we restrict α(x) to be monotone non-decreasing, concave and with a non-negative third derivative with the equivalent condition

4. For an interpretation of the third derivative of the utility function see Lambert (1992, p. 73).
5. These correspond to the FGT P_α measures referred to above where $\alpha > 1$.

$$S_G(x) = \int^{x} D_G(t)dt \geq \int^{x} D_F(t)dt = S_F(x)$$

i.e. we are now examining the areas under the poverty deficit curves which we label the *poverty severity curve*. Figure 6 below shows the poverty severity curve for distribution F(x).

Figure 6: *Poverty Severity Curve for F(x)*



In principle it is possible to examine higher orders of stochastic dominance but in practice it is rare to go beyond third order.

As discussed above, if any given order of dominance does not hold then it is always possible to investigate if a higher order holds. This is equivalent to imposing more restrictions of the valuation function (poverty measure). An alternative approach is to investigate if dominance holds over a restricted range of x. In the case of poverty dominance this involves restricting the range of the poverty line over which we search for dominance. Thus while in general we are interested in the entire distribution of the CDFs up to the maximum value of the poverty line z_{max}, we may also be concerned with the behaviour of the CDFs between a lower bound estimate, z_{min}, of the poverty line and an upper bound estimate z_{max}. It was Atkinson (1987) who first emphasised that in poverty analysis there is often a lower as well as an upper limit for the poverty line and it may turn out that the distributions do not cross between these limits, so that first-order dominance may hold for this range of income/expenditure, if not for all values of z up to z_{max}. This leads to another definition of stochastic dominance known as "poverty-mixed dominance". This definition is a combination of first and second-order dominance. "Poverty-mixed dominance" essentially requires that distribution F displays second-order dominance over distribution G from zero to the lower bound poverty line z_{min} and first-order dominance from z_{min} to z_{max}. An application of poverty mixed dominance to poverty in Chile can be found in Ferreira, Francisco and Litchfield (1998).

Before applying these techniques to Irish data it is worth emphasising again the link between poverty measures and social welfare. For example, by inserting a minus sign in front of H we obtain an unusual social welfare function which assigns a negative value of 1/n to every poor household (i.e. with income below z) and zero to every other household. This measure is non-decreasing in individual household incomes but the discontinuity at z implies that it is non-concave and does not obey the principle of transfers. Similarly, inserting a minus in front of a gap measure such as I will also give a form of social welfare function whereby the contribution of each individual to social welfare moves from –1 to zero and remains at zero for income values above z. Finally, a social welfare measure based upon a - P_α measure where $\alpha > 1$ is strictly concave below z and is thus sensitive to the degree of inequality among the poor. Alternatively, if we regard the valuation function $\alpha(x)$ as simply being a utility function u(x), then the stochastic dominance results outlined above can be interpreted as welfare dominance results (see Foster and Shorrocks, 1988).

It is also worth noting that dominance results in poverty may also be obtained using what Jenkins and Lambert term Three "I"s of Poverty (TIP) curves (Jenkins and Lambert, 1997). These curves are related to the poverty deficit curves outlined above and also to the Generalised Lorenz curves of Shorrocks (1983).

To summarise, if we find that the cumulative distribution functions do not cross, we need look no further because poverty rankings for all measures will be robust to the choice of poverty line. If they do cross but the poverty deficit curves do not, then rankings based on measures that are sensitive to the depth of poverty will be robust. If the poverty deficit curves cross but the poverty severity curves do not, then rankings based on poverty measures that are distribution sensitive will be robust. Finally, if first-order dominance holds between a lower and upper bound estimate of the poverty line and then second-order dominance holds for all values up to the lower bound estimate of the poverty line then "poverty-mixed dominance" holds. We will now apply these techniques to Irish data for 1987 and 1994.

III AN APPLICATION OF POVERTY DOMINANCE TO IRELAND

In this section we apply the ideas from Section II to data from the Irish Household Budget Surveys (HBS) of 1987 and 1994. These are nationally representative surveys carried out every seven years and collect a variety of information concerning the consumption patterns, income and demographic characteristics of in excess of 7,000 households. Fortunately, these years coincide with the years analysed by Callan *et al*. (1996) using the ESRI Living in Ireland Survey, which is the other major source of results on poverty in Ireland, and provides us with a useful basis for comparison. Since the results from Callan *et al*., are based upon a different survey from the one we are using, our results are not directly comparable. Nevertheless, we would expect to see a strong correspondence between the results, given that the Living in Ireland survey is also nationally representative (albeit with a smaller sample size) and that many of the measures used in Living in Ireland (e.g. disposable income) correspond with those used in the HBS.[6]

Before proceeding with the analysis we must first decide upon our definition of "income" or more particularly whether to use income or expenditure. [7] Broadly the issues are as follows:[8] certain components of income are difficult to measure e.g. income from self-employment. Furthermore, cross-section studies typically provide income measures which are snapshots in time and thus take no account of the difference between transitory and permanent income (which once again may be particularly pronounced for the self-employed). Since consumption/ expenditure decisions are usually made with reference to permanent income then expenditure measures may be preferable. However, such measures also have drawbacks. Expenditure on items such as alcohol and tobacco are typically under-reported. Also, as mentioned above, expenditure over a two-week period may not be a reliable measure of consumption, particularly for mature households who may have a large stock of durables from which they derive services. Our expenditure measure is total expenditure excluding repayments of loans other than house purchase mortgages, savings and taxes. It includes the value of home grown food consumed. The measure of disposable income which we use is gross income plus transfers less income tax and employee social insurance contributions. This corresponds to the measure employed by Callan *et al*. (1996).

A further problem specific to the HBS is that income observations are "top-coded", i.e. values of income in excess of £800 per week are simply entered as

6. For a comprehensive description of the Living in Ireland survey, see Callan *et al*. (1996).

7. For a recent discussion of poverty and inequality in Ireland which looks at measures of both income and consumption see O'Neill and Sweetman (1998).

8. For a detailed discussion see Blundell and Preston (1998).

£800 per week. Thus the distribution of income is censored on the right hand side at a value of £800. This causes problems when calculating a poverty line which is a certain percentage of mean income (it does not arise when using median income). However, since we do not wish to concentrate on a particular poverty line and are only interested in those parts of the distribution function where a poverty line could realistically lie (which is surely well below £800 per week), top-coding is not an issue of concern to us.

Finally, before presenting our results we wish to scale household income/expenditure to take account of differing household size and composition. There is an extensive literature on the appropriate choice of equivalence scale.[9] Here we use a scale which has been widely used in poverty studies in the EU. It is the same as scale "C" used by Callan *et al.* (1996) and is also used by O'Neill and Sweetman (1998). The weights are 1 for the first adult in the household, 0.7 for additional people aged over 14 and 0.5 for people aged less than 14. The overall results for calculation of specific poverty measures are not sensitive to the choice of equivalence scale used although the composition of poor families may change (see Smith, 1999). Also, we are interested in poverty purely at the household level. Even though percentages of households in poverty may remain unchanged it is possible that the numbers of individuals in poverty may not, if household size is changing. Overall household size fell between 1987 and 1994 but we choose not to take account of this in our analysis.

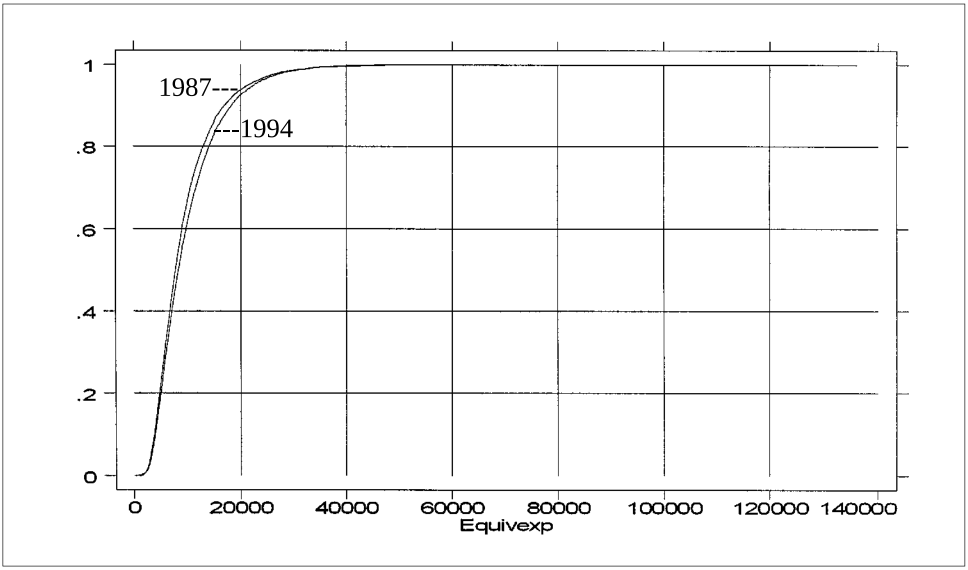
In this section we will examine whether first or second-order dominance exists over the entire range of expenditure values from zero to the upper bound estimate of the poverty line z_{max}. We also examine if first-order dominance exists between a lower bound estimate of the poverty line, z_{min}, and an upper bound estimate z_{max}. We also examine whether second-order dominance exists between zero and the lower bound estimate of the poverty line z_{min} in order to determine if "poverty-mixed dominance" occurs.

Figure 7 shows the cumulative distributions of household expenditure for the entire sample for 1987 and 1994. The x-axis represents equivalent expenditure where expenditure is displayed in pennies, i.e. 140,000 is £1,400 (all expenditure and income figures are in 1994 IR£). The y-axis represents the proportion of the population with at least x amount of expenditure which of course corresponds to the head-count ratio.

We now have to decide what constitutes a "reasonable" range for our poverty line. Many poverty studies employ poverty lines which are a certain percentage of mean or median income, with 40 per cent, 50 per cent and 60 per cent the

9. See Deaton and Muellbauer (1980) for a discussion.

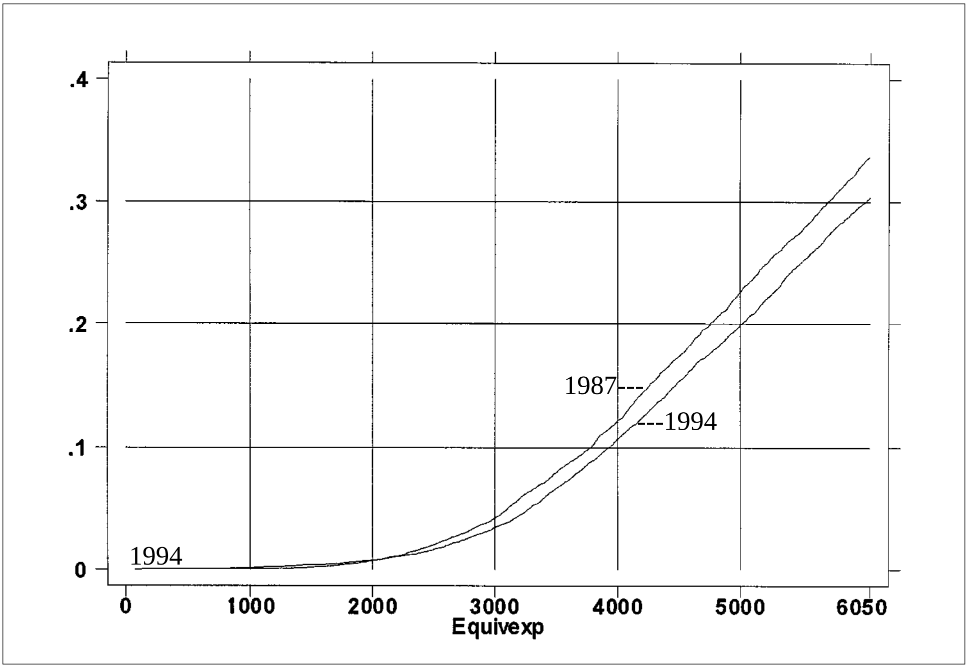
Figure 7: *Cumulative Distribution Functions for 1987 and 1994 Using HBS Expenditure Data*



most popular choices. Since the distribution of income/expenditure is typically not symmetrical then mean income usually exceeds median income. Thus we choose 40 per cent of median income in 1987 as our lower bound for the poverty line, z_{min} and 60 per cent of mean income in 1994 as our upper bound. This gives a lower bound of £30.97 and an upper bound of around £60.50. Figure 8 shows the two cumulative distribution functions up to £60.50.

So how do we determine whether dominance holds? One approach is to simply visually inspect the graphs. It is clear from Figure 8 that the CDF for 1987 and the CDF for 1994 appear to cross at very low levels of expenditure but that they do not cross from approx. £23 to £60 and above. This suggests that while first-order dominance does not hold for all values of expenditure up to z_{max} it does hold over the interval between z_{min} and z_{max}. However, mere visual inspection of CDFs overlooks the issue of sampling variation. Since the CDFs are based on samples there is the possibility that observed differences merely reflect sampling variation and are not significant in the statistical sense. Kakwani (1990) has derived formulae for the standard errors of a number of poverty measures including the headcount ratio (the formulae for the test statistic is given in the Appendix). Since the values for the CDF correspond to the values of the headcount ratio for different levels of expenditure Kakwani's test statistic can be applied in this case. However, here we are interested in testing for differences between

Figure 8: *Cumulative Distribution Functions to the Upper Bound Poverty Line of £60.29*



the distributions over a range rather than just for individual values of the distribution. To test for differences over the range of values between z_{min} and z_{max} we follow the approach of Bishop *et al.* (1991). They suggest that when testing for dominance we calculate test statistics for a number of ordinates within the relevant interval. Then if there is at least one positive significant difference and no negative significant differences between ordinates, dominance holds. Two distributions are ranked as equivalent if there are no significant differences, while the curves cross if the difference in at least one set of ordinates is positive and significant while at least one other set is negative and significant.

Table 1 shows a selection of CDF values for a variety of expenditure values, which lie between the lower and upper bound estimates of the poverty line. Confirming our visual inspection we see that all of the results are positive which suggests that poverty in 1994 was lower than that in 1987. To test for statistical significance we calculate standard errors and test statistics following Kakwani (1990). Out of the sixteen values chosen, as seen in Table 1, fourteen are statistically significant at the 1 per cent level and the remaining were significant at the 5 per cent level. Thus employing the Bishop *et al.* (1991) criterion first-order poverty dominance over the z_{min} to z_{max} is statistically significant.

Table 1: *Difference between Cumulative Distribution Functions of 1987 and 1994 for Various Poverty Lines Based on Expenditure*

Equivalent Expenditure £1987	*Cumulative Distribution 1987*	*Cumulative Distribution 1994*	*Difference Between 1987 and 1994*	*Test Statistics*
30.97	0.049448	0.038213	+.011235	3.42**
33.16	0.066061	0.053066	+.012995	3.43**
37.73	0.099546	0.087724	+.011822	2.53*
40.02	0.121350	0.106640	+.014710	2.89**
42.00	0.143673	0.123651	+.020022	3.67**
45.02	0.173524	0.153612	+.019912	3.36**
46.00	0.184556	0.163006	+.021550	3.55**
46.80	0.192992	0.171131	+.021861	3.56**
47.15	0.196496	0.173035	+.023461	3.77**
48.00	0.204543	0.180526	+.024017	3.80**
49.00	0.213628	0.189920	+.023708	3.69**
50.24	0.229851	0.201346	+.028505	4.33**
52.01	0.248151	0.218992	+.029159	4.30**
54.02	0.268657	0.241589	+.027068	3.88**
55.58	0.283452	0.266599	+.016853	2.36*
60.29	0.337560	0.301765	+.035795	4.79**

* significantly different at the 5 per cent level.
** significantly different at the 1 per cent level.

Figure 8 showed first-order dominance over a range from approximately £23-£60. But what about values of expenditure *less* than £23? Figure 9 below shows the CDFs for 1987 and 1994 for values of equivalised expenditure below £40. We can see that the curves cross at around £21 and for values of expenditure below £21 the CDF for 1987 lies below that for 1994. Application of the Bishop *et al.* (1991) criteria indicates that the crossing is significant (for the sake of brevity we do not present the precise values of the test statistic but they are available from authors on request).

This crossing below £21 suggests that the very poorest of the poor fared worse in 1994 than in 1987. How much consequence should we attach to this finding? Perhaps not too much. For both 1987 and 1994 the proportion of households with equivalised expenditure below £21 is less than 1 per cent. It is likely that measurement error has its most severe consequences at very low levels of expenditure, e.g. an absolute measurement error of, say £5, will have a much greater proportionate effect at low levels of expenditure. It may also be the case that we are picking up some infrequency of purchase here as well. Overall, our conclusion is that perhaps we should not put too much weight upon this crossing.

Finally, we repeat the analysis for income. Figure 10 shows the cumulative

Figure 9: *Cumulative Distribution for 1987 and 1994 Expenditure Values Less than £40*

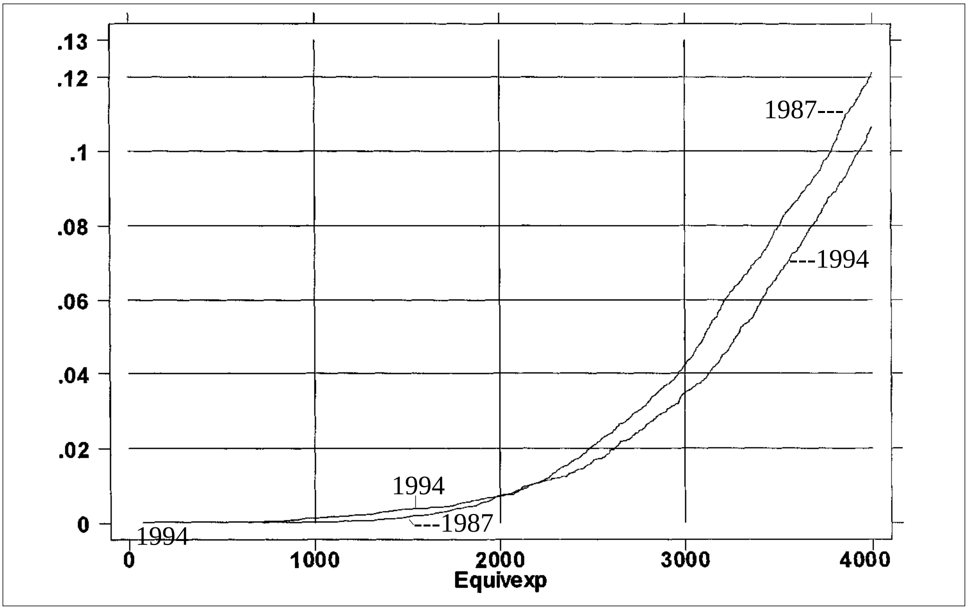


Figure 10: *Cumulative Distribution Functions for 1987 and 1994 Using HBS Income Data*

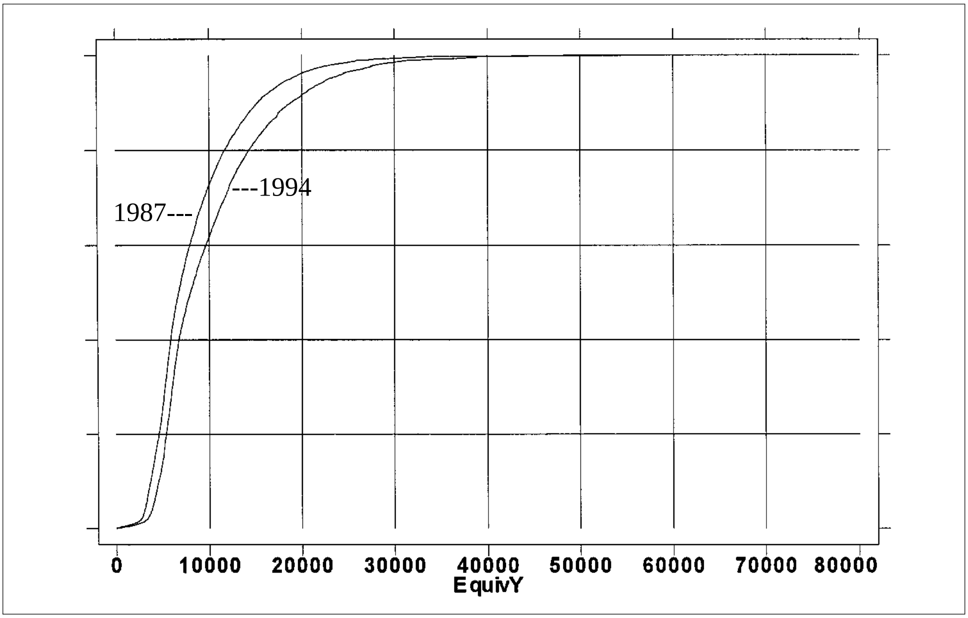


Table 2: *Difference Between Cumulative Distribution Functions of 1987 and 1994 for Various Poverty Lines Based on Income*

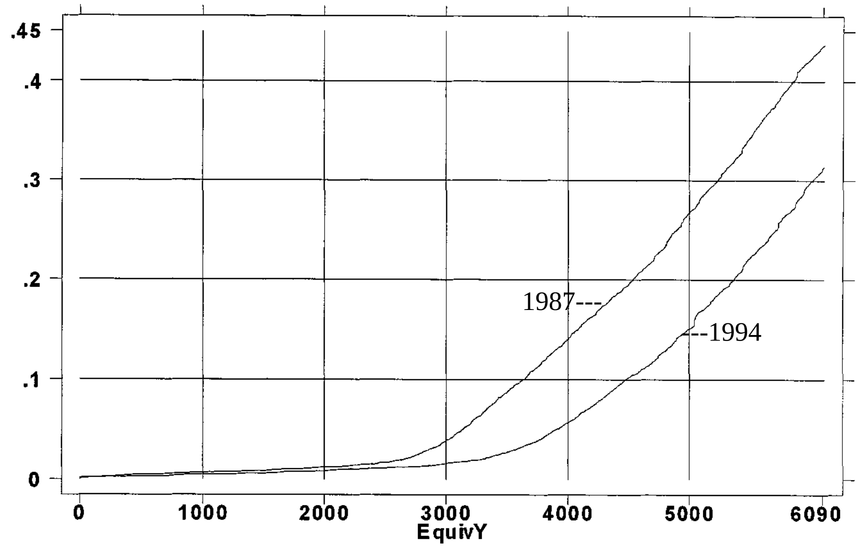
Equivalent Income £1987	*Cumulative Distribution 1987*	*Cumulative Distribution 1994*	*Difference Between Cumulatives*	*Test Statistics*
26.83	0.0207792	0.124603	+.0083189	4.06**
31.76	0.053636	0.018055	+.035581	11.96**
33.59	0.073117	0.022123	+.050994	15.0**
34.00	0.076623	0.023776	+.052847	15.2**
35.00	0.087533	0.027209	+.060324	16.3**
36.01	0.096883	0.031786	+.065097	16.7**
37.00	0.108312	0.036237	+.072075	17.5**
40.61	0.148831	0.061793	+.087038	17.83**
41.98	0.163117	0.073490	+.089627	17.45**
50.78	0.278182	0.167832	+.110350	16.67**
51.03	0.283636	0.169231	+.114405	17.20**
53.00	0.310779	0.192244	+.118535	17.19**
55.00	0.343507	0.222378	+.121129	16.92**
59.02	0.412857	0.282518	+.126509	17.18**
60.90	0.435844	0.313032	+.122812	15.95**

* significantly different at the 5 per cent level.
** significantly different at the 1 per cent level.

distribution functions or poverty incidence curves for the entire distributions of 1987 and 1994 household disposable income. Once again these graphs suggest that poverty in 1994 was lower than 1987. Figure 11 displays the CDFs up to the upper bound estimate of the poverty line, which is £60.90 (60 per cent of mean 1994 income). From this diagram, it seems clear that the CDFs do not cross over a very wide range of income values and indeed the distance between the distributions appears to be greater than for the case of expenditure (we return to this below). The particular range we are concerned with is between £26.83 (40 per cent of median income in 1987) and £60.90 (60 per cent of mean income in 1994). In order to confirm that the CDFs do not cross in this range, in Table 2 we carry out an identical exercise to that in Table 1. Once again we obtain significant differences between the CDFs indicating significant first-order poverty dominance for 1994 over 1987.

As can be seen from Figure 11 it is only in the left tail of the distributions where the CDFs cross. However, this is at such a low level of income, below £2, that once again we may put it down to measurement error. Indeed, given the problems associated with the income figures outlined above (most particularly the erratic pattern of income for the self-employed) it seems likely that this crossing can be dismissed.

Figure 11: *Cumulative Distribution Funcions to the Upper Bound Poverty Line of £60.90*



Thus so far we have established first-order poverty dominance for a fairly wide range of both income and expenditure values, but we have seen that for low values of expenditure and income dominance does not hold. Even allowing for the fact that the crossing of the distributions may be "spurious", is it possible that second-order dominance holds up to the lower bound of poverty so that in total mixed dominance holds?

In order to test for second-order dominance we need to compare the areas below the CDFs and determine if the area under one distribution is always greater than the area under another distribution, i.e. check that the poverty deficit curves do not cross. One way to determine this is to subtract the cumulative distribution points of the poverty incidence curves (CDFs) at particular expenditure values to calculate the differences or gaps between the areas of the two distributions. If we then find the cumulative gap by adding up all the individual gaps we have found an approximate estimate of the area between the 1987 distribution function and the 1994 distribution. In order for second-order stochastic dominance to hold we again use the Bishop *et al.*, criteria for dominance. For dominance to hold therefore, at least one of the cumulative gaps should be positive and significant with no negative and significant gaps. If all of the cumulative gaps are positive and at least one significant this indicates that the area of one distribution is everywhere above that of another.

Table 3: *Cumulative Gaps Between the Cumulative Expenditure Distributions of 1987 and 1994*

Cumulative Distribution for 1987	*Cumulative Distribution for 1994*	*Differences / Gap Between the Distributions*	*Cumulative Gaps*	*Test Statistic*
Cumulative Values from approx. £7 up to the Lower Bound Estimate of the Poverty Line:				
.00013	.000351	–.0002211		–0.09
.0005191	.0020312	–.0015121	–.0017332	–2.65**
.0011681	.0033007	–.0021326	–.0038658	–2.83**
.001557	.003682	–.0021242	–.00599	–2.60**
.002077	.003809	–.001732	–.007722	–2.00*
.003115	.004443	–.0013284	–.0090504	–1.35
.005451	.0064745	–.0010235	–.0100739	–0.83
.0085659	.0087597	–.0001938	–.0102677	–0.13
.0115509	.0111718	+.003791	–.0063685	+0.22
.0168722	.0135839	+.0032883	–.0000606	+1.68
.0205062	.0163768	+.0041294	+.0035219	+1.91
.0280337	.0225974	+.0054363	+.0089582	+2.16*
.0324465	.0267868	+.0056597	+.0146179	+2.08*
.0371188	.0300876	+.0070312	+.0216491	+2.43*
.0494484	.0382125	+.0112359	+.1640312	+3.42**
.066061	.0530659	+.0129951	+.1770263	+3.43**

* significantly different at the 5 per cent level.
** significantly different at the 1 per cent level.

Table 3 shows the cumulative gaps for a range of expenditure values from approximately £7 to the minimum poverty line. As we can see, the cumulative gaps change from negative to positive figures and some of the negative gaps are statistically significant, as are some of the positive gaps. This of course implies according to Bishop's criteria for dominance that second-order dominance does not hold as the area underneath the 1987 distribution is not always higher than the area under the 1994 distribution. Thus mixed dominance does not hold for expenditure between 1987 and 1994. Given the likelihood of measurement error associated with low values of income referred to above we do not carry out the analysis for income.

To summarise so far, we have discovered that first-order poverty dominance holds for the years 1987 and 1994 over what we regard as a reasonable range of both income and expenditure levels. Thus it is possible to say with a fair degree of confidence that poverty in 1994 was lower than in 1987 and that this finding is statistically significant. Note that given the finding of first-order dominance there is no need to search for second or third-order dominance since a finding of any degree of dominance implies that higher degrees of dominance must also hold.

IV RELATIVE POVERTY DOMINANCE

The dominance results in Section III are not strictly comparable with much of the analysis carried out in Callan *et al.* In Section III we compare CDFs for different values of income/expenditure. However, in all cases we are comparing the two CDFs for a *given* value of income/expenditure. Much of the analysis of Callan *et al*. uses purely relative poverty lines, i.e. a certain percentage of mean or median income. Of course this implies that they examine headcounts for 1987 and 1994 where the relevant poverty line *differs* between the two years (since mean/median income in 1994 was obviously higher than in 1987). In this section we repeat the analysis of Section III using purely relative poverty lines. We stress that there is no sense in which the results in this section should be seen as more or less accurate or relevant than those in Section III. That depends upon whether an absolute or relative poverty line is the appropriate route to take. The choice between a relative poverty line (the approach of Callan *et al.*) and a range of absolute ones (as in Section III) is an important issue in poverty analysis but one which we choose not to explore in this paper.[10]

To replicate the analysis in Section III but using relative poverty lines we normalise expenditure and income for 1987 and 1994 by dividing by mean income for the year in question. In Figures 12 and 13 below we show the CDFs for expenditure and income for a range of poverty lines up to 80 per cent of mean expenditure/income, on the basis that this is a reasonable upper limit to the poverty line.

While visual inspection can sometimes be untrustworthy, there is a clear crossing for income and the curves for expenditure appear to be too close to suggest dominance. This is confirmed by Tables 4 and 5 which reproduce Tables 1 and 2 for the case of relative poverty lines. Use of the Bishop *et al.*, criteria suggest that the expenditure distributions are equivalent while the incomes curves cross and the crossing is statistically significant.

Given that first-order dominance is not observed, what about second-order dominance? Figures 14 and 15 show the poverty deficit curves for expenditure and income respectively. Visual inspection suggests that there may be second-order dominance for expenditure but there is a crossing in the case of income. Given that the Bishop *et al.*, criteria call for at least one significant positive difference and no significant negative difference for dominance to hold, Table 6 confirms the existence of second-order poverty dominance for 1987 over 1994 for expenditure, using relative poverty lines. Table 7 confirms that there is no such dominance for income and so we check for third-order dominance. Figure 16 below shows the poverty severity curves for 1987 and 1994 for income.

10. For more on this see Callan *et al*. (1996), Foster (1998) and Madden (1999).

Figure 12: *Cumulative Distribution Function for Normalised Expenditure up to 80 Per Cent of Mean*

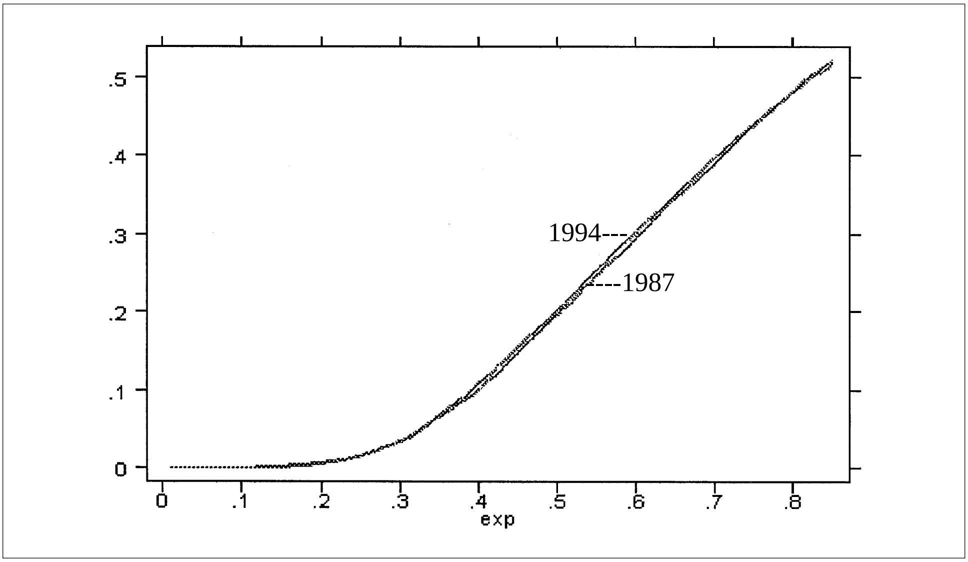


Figure 13: *Cumulative Distribution Function for Normalised Income up to 80 Per Cent of Mean*

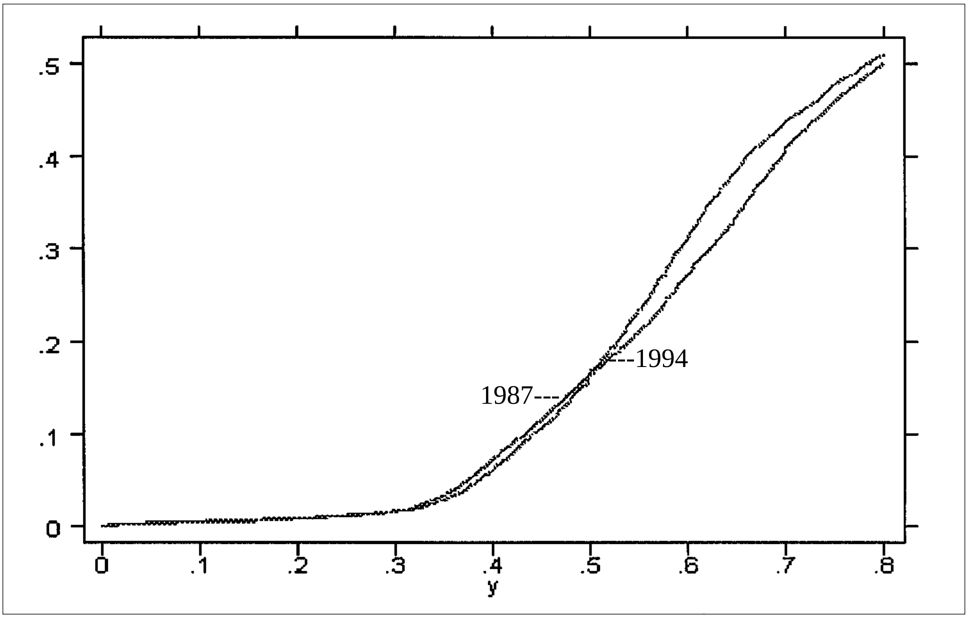


Table 4: *Difference Between Cumulative Distribution Functions of 1987 and 1994 for Various Relative Poverty Lines (Expenditure)*

Percentage of Mean Expenditure	*Cumulative Distribution 1987*	*Cumulative Distribution 1994*	*Difference Between 1987 and 1994*	*Test Statistics*
20	0.005191	0.00749	0.002299	1.809437
25	0.015315	0.016885	0.00157	0.778694
30	0.033874	0.03542	0.001546	0.527478
35	0.065023	0.067919	0.002897	0.725804
40	0.099676	0.10829	0.008615	1.761884
45	0.147956	0.155389	0.007433	1.293503
50	0.196626	0.201346	0.00472	0.737911
55	0.247242	0.253015	0.005773	0.831988
60	0.295522	0.301892	0.006369	0.868546
65	0.343803	0.349245	0.005442	0.71374
70	0.389358	0.396725	0.007367	0.941385
75	0.437249	0.439127	0.001878	0.236238
80	0.481506	0.480005	–0.0015	–0.18742

Table 5: *Difference Between Cumulative Distribution Functions of 1987 and 1994 for Various Relative Poverty Lines (Income)*

Percentage of Mean Income	*Cumulative Distribution 1987*	*Cumulative Distribution 1994*	*Difference Between 1987 and 1994*	*Test Statistics*
20	0.00987	0.0089	–0.00097	0.62728
25	0.012597	0.011443	–0.00115	0.66055
30	0.017013	0.016656	-0.00036	0.17303
35	0.033766	0.029371	–0.0044	1.56768
40	0.073247	0.06192	–0.01133	2.81409**
45	0.116234	0.106802	–0.00943	1.86882
50	0.163117	0.167832	0.004715	0.791551
55	0.21039	0.232931	0.022541	3.386964**
60	0.271299	0.31316	0.041861	5.748811**
65	0.337273	0.38487	0.047597	6.189805**
70	0.405974	0.437635	0.031661	4.001288**
75	0.46026	0.478195	0.017935	2.242054*
80	0.501299	0.510744	0.009445	1.178439

* significantly different at the 5 per cent level.

** significantly different at the 1 per cent level .

Figure 14: *Poverty Deficit Curve for Normalised Expenditure up to 80 Per Cent of Mean*

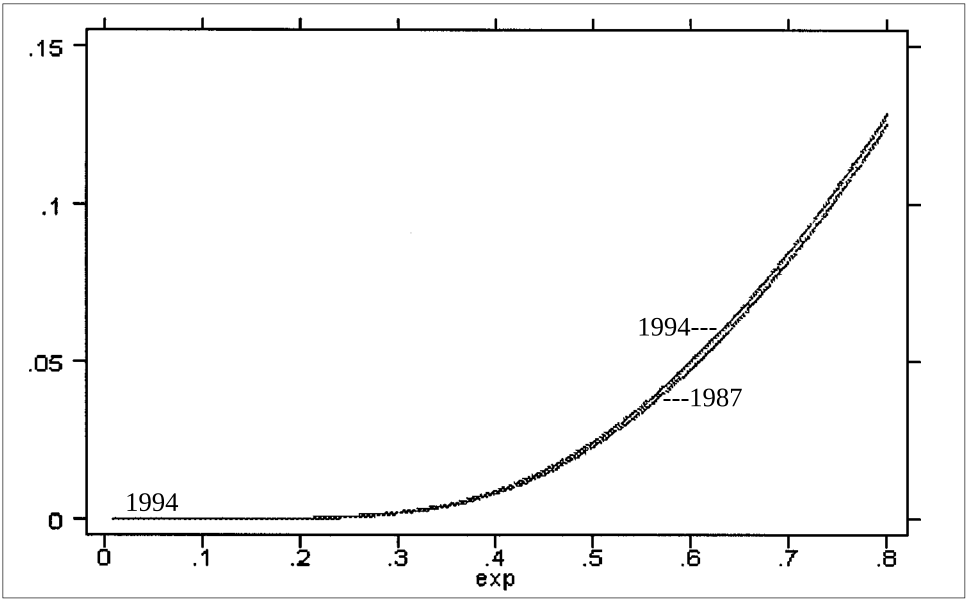


Figure 15: *Poverty Deficit Curve for Normalised Income up to 80 Per Cent of Mean*

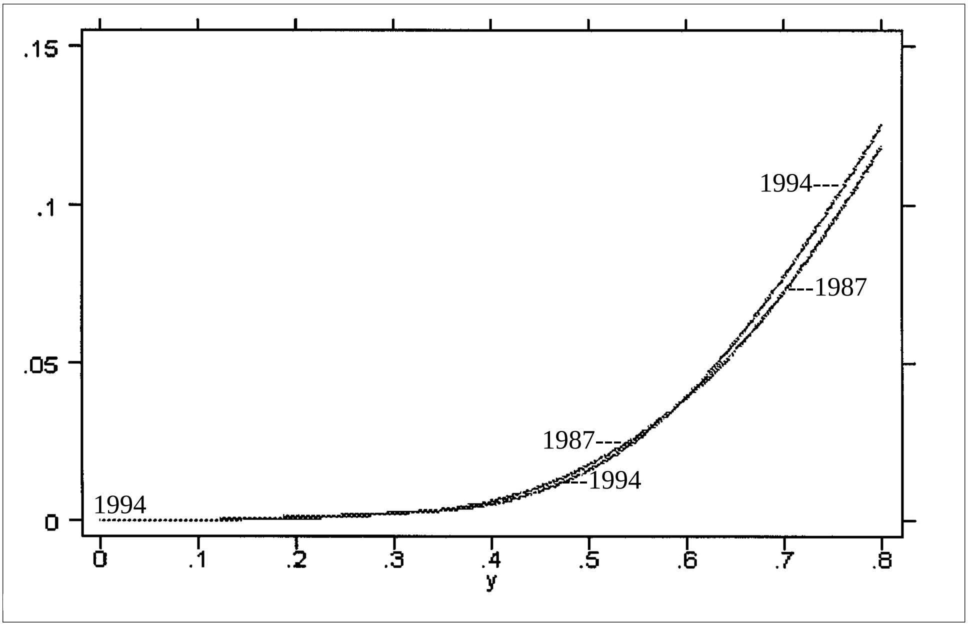


Table 6: *Difference Between Poverty Deficit Curves for 1987 and 1994 for Various Relative Poverty Lines (Expenditure)*

Percentage of Mean Expenditure	*Poverty Deficit Curve 1987*	*Poverty Deficit Curve 1994*	*Difference Between 1987 and 1994*	*Test Statistic*
20	0.0001555	0.0003816	-0.00023	3.25757**
25	0.0006144	0.0009513	-0.00034	2.52965*
30	0.0018167	0.0022151	-0.0004	1.73597
35	0.0042009	0.0047121	-0.00051	1.328
40	0.0083368	0.0090687	-0.00073	1.36941
45	0.0144934	0.015599	-0.00111	1.51748
50	0.0230934	0.0245387	-0.00145	1.45663
55	0.0340715	0.0358944	-0.00182	1.40038
60	0.0476286	0.0498389	-0.00221	1.40796
65	0.0636521	0.0661162	-0.00246	1.32574
70	0.0819716	0.0847209	-0.00275	1.29063
75	0.1027167	0.1056309	-0.00291	1.21576
80	0.125674	0.1286692	-0.003	1.09832

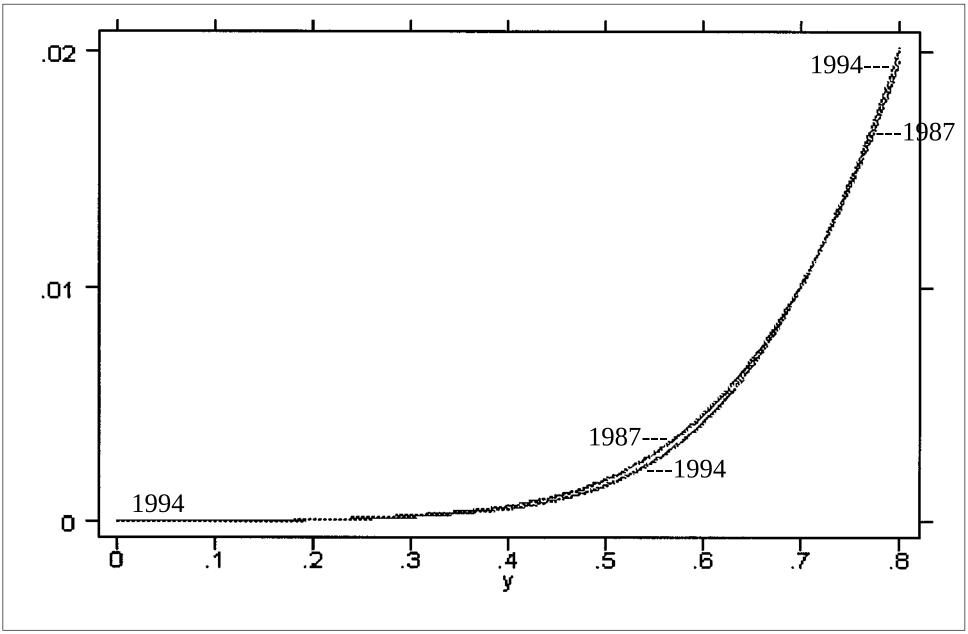
* significantly different at the 5 per cent level.
** significantly different at the 1 per cent level.

Table 7: *Difference Between Poverty Deficit Curves for 1987 and 1994 for Various Relative Poverty Lines (Income)*

Percentage of Mean Income	*Poverty Deficit Curve 1987*	*Poverty Deficit Curve 1994*	*Difference Between 1987 and 1994*	*Test Statistic*
20	0.0012402	0.0009396	0.000301	1.716878
25	0.0018131	0.001458	0.000355	1.497749
30	0.0025245	0.0021357	0.000389	1.378576
35	0.0037356	0.0032054	0.00053	1.326888
40	0.0062474	0.0053957	0.000852	1.885259
45	0.0109421	0.0096428	0.001299	2.197041*
50	0.0180079	0.0162994	0.001709	2.18012*
55	0.027326	0.0261648	0.001161	1.149705
60	0.0393379	0.0398792	-0.00054	0.26367
65	0.054503	0.0573333	-0.00283	1.70435
70	0.0730918	0.0780336	-0.00494	2.88522**
75	0.0948397	0.1008313	-0.00599	2.75581**
80	0.1188229	0.1254025	-0.00599	2.63791**

* significantly different at the 5 per cent level.
** significantly different at the 1 per cent level.

Figure 16: *Poverty Severity Curve for Normalised Income up to 80 Per Cent of Mean*



Once again visual inspection indicates a crossing of the curves and while this is confirmed by Table 8, the crossing is not statistically significant. Thus for a range of poverty lines up to about 70 per cent of average income third-order poverty dominance for 1994 over 1987 can be observed.

Thus we can summarise the results for poverty dominance for relative poverty lines as follows. There is second-order dominance for 1987 over 1994 for the case of expenditure. Any poverty measure which is sensitive to the depth of poverty such as the income gap ratio will show a rise in poverty over the 1987-1994 period. For the case of income, third-order dominance for 1994 is observed over a reasonable range of poverty lines so any measure which is sensitive to inequality amongst the poor will show a fall for 1987 compared to 1994.

These results are very much in line with those of Callan *et al.* Taking the case where the poverty line is held constant in real terms (an "absolute" poverty line) the dominance evident in Figures 10 and 11 are consistent with Callan *et al.*'s results showing substantial falls in poverty rates between 1987 and 1994. The results for relative poverty lines are also in accordance. Callan *et al.* find that the headcount ratio falls between 1987 and 1994 for a poverty line at 40 per cent of average income, but increases for the poverty line at 50 per cent and 60 per cent, which is pretty much what we find in Figure 13. They also find that when using gap measures or the FGT P_2 measure that poverty is higher in 1987 but that the gap between 1987 and 1994 narrows as the poverty line increases up to 60 per cent and beyond of average income, which is precisely what is evident in Figures 15 and 16.

Table 8: *Difference Between Poverty Severity Curves for 1987 and 1994 for Various Relative Poverty Lines (Income)*

Percentage of Mean Income	*Poverty Severity Curve 1987*	*Poverty Severity Curve 1994*	*Difference Between 1987 and 1994*	*Test Statistic*
20	0.0000893	0.000074	1.53E-05	1.872708
25	0.0001694	0.0001347	3.47E-05	1.786072
30	0.0002754	0.000224	5.14E-05	1.683435
35	0.0004325	0.0003551	7.74E-05	1.600447
40	0.0006748	0.0005637	0.000111	1.655943
45	0.0010943	0.0009308	0.000164	1.849048
50	0.0018106	0.0015641	0.000247	2.026564*
55	0.0029346	0.0026101	0.000325	2.006699*
60	0.0045901	0.0042571	0.000333	1.653588
65	0.0069212	0.0066653	0.000256	1.027146
70	0.0100984	0.0100486	4.98E-05	0.292101
75	0.0142932	0.0145022	-0.00021	0.34605
80	0.0195981	0.0201153	-0.00052	0.82804

* significantly different at the 5 per cent level.
** significantly different at the 1 per cent level.

The situation with regard to expenditure is somewhat different, however. When viewed on an absolute basis the fall in poverty is more pronounced for income than expenditure. When viewed on a relative basis, the rise in poverty is greater for expenditure than income (Callan *et al*., do not have expenditure data, but their results for income are very similar to our results for income). At this stage we can only speculate as to why this is so. It can be argued that we might expect changes in consumption to be less pronounced than changes in income since households may "smooth" their consumption following changes in income. Thus, if households believed that the boom in the Irish economy which began around 1993-94 (the advent of the so called "Celtic Tiger") was not permanent then it is to be expected that consumption would not rise as quickly as income. While it is plausible that some consumption smoothing is going on, it appears that more consumption smoothing is being carried out by low-income households. It is not clear why this effect should be more pronounced among poor households. It may be due to greater precautionary saving on behalf of poor households which is consistent with the notion of decreasing absolute risk aversion.[11] In future work we hope to investigate this in more detail.

V CONCLUSION

To conclude, this paper has advanced the analysis of poverty in Ireland in a number of directions. It employs Household Budget Survey data and permits comparison with previous work on poverty in Ireland which has used the ESRI Living in Ireland data. It also applies stochastic dominance techniques to overcome the difficulties associated with analysis which uses specific poverty lines. It also tests for the statistical significance of the results. We find that poverty in Ireland in 1994 was lower than in 1987 for a fairly wide range of absolute poverty lines. We also find that for the case of relative poverty lines no dominance can be found for headcount type measures but dominance of 1987 over 1994 can be found for gap type measures for the case of expenditure while for income, dominance is found for 1994 over 1987 for poverty measures which are sensitive to inequality among the poor.

The finding regarding the "absolute" approach to poverty lines is hardly controversial given Ireland's recent spectacular growth record (even allowing for the fact that much of the really high growth was recorded after 1994). However, we believe that the dominance results are important since they enable robust statements to be made regarding developments in poverty over the 1987-94 period. The more ambiguous results for relative poverty lines highlights the importance of the decision regarding relative or absolute poverty lines. We also believe that the different results regarding income and expenditure measures are notable and deserving of further investigation.

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11. Evidence from the US suggests that poor households are *less* able to carry out consumption smoothing. See Dynarksi and Gruber (1997). For a formal discussion of the link between risk-aversion and precautionary saving see Kimball (1990).

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