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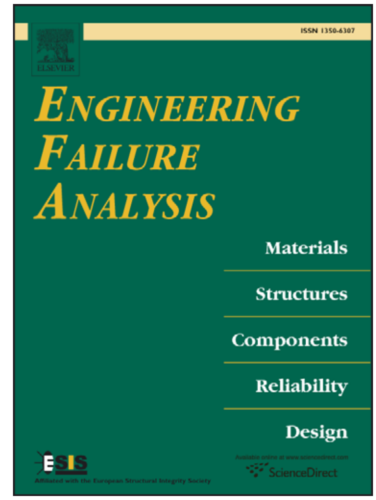

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Applications of the Theory of Critical Distances in Failure Analysis

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Abstract

The Theory of Critical Distances (TCD) is the name which I use to describe a group of methods employed for the prediction of failure in cases where stress concentrations are present and where the failure mode involves cracking, such as fatigue and brittle fracture. Some of these methods are more than fifty years old, some very recent. Precise predictions are possible in cases where accurate stress field information is available, for example using finite element analysis (FEA). In the present paper, however, I concentrate on the use of the TCD for approximate, order-of-magnitude predictions, because these can be very useful during failure analysis.

Two material constants are required: the critical distance L and (depending on which method is used) either a critical stress σ_o or a critical stress intensity K_C. Values of L in engineering materials can vary from microns to centimetres. The critical stress may be equal to the plain specimen strength (static or cyclic) but is often significantly higher.

In what follows I show through a series of examples and case studies how knowledge of the approximate values of L and σ_o can be very useful when conducting a failure analysis, in assessing the significance of defects and design features. I propose, for the first time in this article, a series of dimensionless numbers, composed of material constants and design variables, which I believe could usefully be adopted in fracture mechanics in the same spirit as they have been in other branches of engineering, such as fluid mechanics.

Keywords: Critical distance; fatigue; brittle fracture; defect; notch

Introduction

When conducting a failure analysis many factors have to be taken into account in order to arrive at the cause of the failure. Common causes of failure are defects such as pores or cracks introduced during manufacturing, and inappropriate design features which cause excessive stress concentration, such as sharp corners and poorly-designed joints. Failure can also occur due to local contact creating, for example, fretting fatigue. A common factor in all these causes is that they create high local stresses and also stress gradients, the stress decreasing with distance from the feature.

The Theory of Critical Distances (TCD) is a method for the prediction of failure arising from such situations. Failure commonly occurs due to the initiation and growth of a crack, through mechanisms such as brittle fracture, fatigue and stress corrosion cracking. In these cases it is well known that both the maximum stress and the stress gradient are important in determining whether failure will occur. It is also well known that materials possess inherent length scales which are related (in complex ways) to their microstructure and modes of deformation and damage. The interaction between the length scale and the stress gradient determines whether failure will occur from a given feature.

In recent years, many methods have been proposed to model this situation. The TCD is a name which I have given to a group of methods, some going back more than fifty years. These methods have one feature in common: the use of a material parameter with the units of length: the so-called critical distance L. This distance is taken to represent, in a simplified way, the inherent length scaling in the material as regards the fracture process under consideration. It can be used in various ways, which divide into two types of analysis. The first type are stress-based methods which consider the stress close to the feature of interest, e.g. the notch, corner or defect. Usually (though not always) a linear elastic stress analysis is conducted (e.g. using FEA) and the effective stress parameter is defined as either the stress at a given distance from the notch or the average stress over a line reaching out from the notch. These two methods we call the Point Method and Line Method. Failure is assumed to occur if the effective stress is greater than some material constant value σ_o or, in the case of fatigue failure, a stress range $\Delta\sigma_o$. The second type of analysis consists of some stress-intensity based methods in which standard fracture mechanics are used and the value of L appears as an imaginary crack or as a unit of crack extension.

I do not propose to deal in any more detail with the theory or its practical application, since this information can be found in many previous publications. A comprehensive treatment can be found in a recent book [1] and a brief overview in a recent journal article [2]. Suffice it to say that the theory has been tested against experimental data for many different materials, features and loading regimes; all of which has shown that it is capable of making precise predictions of failure, provided an accurate stress analysis is available. The method is becoming increasingly feasible for industrial components thanks to the power of modern computers and numerical methods.

In the present paper however I am concerned not with the precise use of the TCD, but with its approximate use. The failure analyst is not always in a position to carry out a detailed stress analysis, but nevertheless I will argue that knowledge of the TCD parameters, L and σ_o, can be very useful in arriving at an opinion as to the cause of failure. In the present article I will illustrate this point through various examples and, building on this experience, I will show that some useful dimensionless numbers can be obtained by normalizing the TCD parameters, allowing one to predict regimes of behaviour for different failure modes.

Comparing L to Defect Size

When examining a fracture surface one frequently observes defects such as casting pores, inclusions or contraction cracks (fig.1) and one may be able to deduce that a defect was located at the origin of the crack which caused the failure. However, this does not necessarily mean that the defect was the cause of failure. Cracks will of course tend to initiate from defects if they are present, but removing the defect (or reducing its size) will not necessarily improve the material's resistance to failure. An example of this phenomenon is the short crack effect, which has been very extensively studied. Fig.2 shows an example of data illustrating how the fatigue limit of a specimen changes with the length of a pre-existing crack. For large cracks the data lie on a line corresponding to the predictions of linear elastic fracture mechanics (LEFM), controlled by the stress intensity threshold for crack growth, ΔK_{th}. For very small cracks the crack has no effect, the fatigue limit being the same as that of a plain specimen, $\Delta\sigma_o$. For intermediate crack lengths the data curves between these two types of behaviour. Now it turns out that the value of L lies exactly in the middle of this curved region, as shown on fig.2. Thus we can say that if the crack length is very much less than L, the crack has no effect, i.e. a defect exists but the defect is harmless. If on the other hand the crack length is much greater than L, we can analyse the crack using standard LEFM procedures. For crack lengths similar to L neither approach works and a more detailed analysis using the TCD will be needed to give us an estimate of the safe working stress range.

It is very interesting to note that exactly the same type of behaviour, giving a graph of very similar appearance, occurs if we plot the strength of brittle ceramic specimens containing pre-existing cracks and defects, despite the fact that the physical mechanisms controlling failure in this case are very different from those controlling short fatigue crack growth. More details of this analysis can be found in a previous paper [3]. In this case again, L can be used to give an idea of the significance of the defect. The same approach also works if the defect is not a crack but a feature of some other shape, such as a circular hole, as shown by Whitney and Nuismer many years ago for the failure of fibre composite materials [4].

I have found the above to be very useful in my own failure analysis work, enabling me to make a quick assessment of the possible importance of a defect on a fracture surface and preventing me from jumping to conclusions at an early stage.

Comparing L to Notch Root Radius

Useful information can also be obtained by comparing the value of L to the root radius ρ of the notch or other stress concentration feature (e.g. the fillet radius at a corner) at which failure occurred. Some notches, even though they have a finite root radius, still behave as if they were cracks: this can be seen from the data on measured values of fracture toughness K_{IC} from specimens containing notches of different root radii. As shown for example in fig.3, if the root radius lies below a critical value then the result is the same as for a perfectly sharp crack. This critical value is given approximately by L, independently of the overall shape of the feature. For simple notches, the feature can be analysed by calculating the stress intensity factor K, for a crack of the same length. For other features, such as corners, an equivalent K factor can be estimated using a technique known as the Crack Modelling Method [5]. If, on the other hand, the root radius is much larger than L, then fatigue and brittle fracture loads can be predicted by simply using the elastic stress concentration factor, K_t. For intermediate values of ρ the load to failure would be greater than that for a crack of the same length but less than would be predicted by using K_t. A detailed TCD analysis will give the precise value in all cases, but simply knowing L and ρ without doing a detailed analysis can be very useful, as illustrated by the following examples:

Example 1: Failure of a Cast Iron Component

This analysis was reported in full in a previous publication in this journal [6]. It concerned a cast iron engine casing from a ship. Fatigue failure occurred from a corner which had a very small root radius of 0.3mm. The design was improved by increasing the radius to 3.2mm, but fatigue failures continued to occur. Normally such an increase in radius would be effective: the reason for the difficulty in the present case was that the value of L for high-cycle fatigue in this material turned out to be 3.8mm. This is a particularly large value, one of the largest which I have encountered for a metallic material. Typical mild steels have L values an order of magnitude smaller than this. So for this particular case a root radius significantly larger than 3.8mm would be needed in order to affect the fatigue strength.

Example 2: Notches in Bone

Orthopaedic surgeons often introduce stress concentrations into bone during operations. For example, an operation to replace a torn anterior cruciate ligament involves cutting a piece of bone from the patella (knee cap) of the other knee. Typically the surgeon will cut out the piece in such a way as to leave two sharp corners in the patella (fig.4); cracks will sometimes form at the these locations later on as a result of cyclic loading or impacts. We found that the critical distance for brittle fracture in bone is 0.35mm [7], so we advised that the bone should be cut in such a way as to leave rounded corners of radius 1mm or more. We confirmed, by impact tests carried out on pigs, that this significantly increased the strength of the remaining patella [8].

Example 3: The Modelling of Welded Joints

The assessment of fatigue failure in welded joints is a common problem in failure analysis, which in recent years has been greatly aided by the development of comprehensive standards such as the recent Eurocodes for welded steel and aluminium. However, cases often arise where the welded joint design is very different from anything found in the standards, and one must resort to a detailed analysis using FEA. But how to interpret the results? There are many different proposals regarding the creation of the model and its post-processing. One issue which arises is how accurately one must model the details of the weld. Cracking frequently starts at the point where the weld bead meets the base metal (fig.5): at this point the radius of curvature is small, but not zero, and if one models it to be zero then a singularity is created in the FE model, giving rise to stress values at that point which are meaningless. However, if using the TCD this is not important because we are not relying on the stress at that particular point, and provided the radius of curvature in the actual weld is less than L, we are allowed to use a zero radius in the model without affecting the results. For low and medium strength steels we found that the appropriate value of L is 0.43mm [9], so this simplification is allowable in most cases.

Comparing L to Body Size

Another length parameter which we might compare to the critical distance is the size of the entire body. A useful relevant parameter might be the width W of the specimen (or of the cross section of a component) in the direction of expected crack growth. One can easily appreciate that something strange will happen in cases where we attempt to use the TCD if W is the same as L, or smaller. In fact the TCD cannot be used in those cases, because we would be considering stresses at points which lie outside the body itself, which is obviously not appropriate. However we have developed a modified form of the TCD which can be used in these cases. The approach is more complex and involves a value of L which is no longer constant but varies with body size. This situation arises in cases where either the component itself is very small (e.g. small medical devices made from metallic materials) or the L value is particularly large (e.g. concrete) [1; 10].

Comparing σ_o to Material Strength

So far we have been thinking about the critical distance L. We now turn our attention to the other TCD parameter: the critical stress σ_o (or, for the case of fatigue, the stress range $\Delta\sigma_o$). In some cases this critical stress is simply equal to the strength of the material as measured in tests on plain (i.e. unnotched) samples. This is true for metal fatigue and for the static fracture of brittle ceramics (as implied in figs 1 and 2); it is also true for the static fracture of fibre composites [4]. However, for other cases the value of the critical stress is larger than the plain-specimen strength σ_u: these cases include fracture of polymers and metals and fatigue of polymers (though in the latter case we only have data for one polymer, PMMA). The fact that σ_o is larger than σ_u leads to some predictions which at first sight appear strange: for notches with K_t factors less than the ratio σ_o/σ_u the predicted strength of the notched specimen is greater than that of the plain specimen. This obviously cannot happen and in practice we find that the strength of these notches is identical to that of the plain specimen, as shown in fig.6. This gives rise to an interesting class of harmless notches, i.e. notches which do not affect the strength of the part. In the case of PMMA the ratio σ_o/σ_u is equal to 2, and in some other materials it can rise much higher. When we examine failed components in which failure occurred from some stress concentrating feature, it is common to blame the designer for introducing the feature in the first place, but sometimes we would be wrong, because some notches are harmless, as we see here, and this is a very useful thing to know.

Finding the Values of L and $\Delta\sigma_o$

In order to find the exact values of the two critical parameters, some experiments must be conducted, but these are relatively simple. Test results are needed from a minimum of two different specimen types, e.g. specimens with two different notches, one of which can be a crack. In cases where $\sigma_o/\sigma_u = 1$ the plain specimen can be used as one of the two specimen types. Some simple analysis shows that there is a relationship between L, σ_o and K_c as follows:

$$L = \frac{1}{\pi}\left(\frac{K_c}{\sigma_o}\right)^2 \tag{1}$$

This relationship also applies in fatigue, so if one knows the plain-specimen fatigue limit and crack propagation threshold for the material, L can be found without the need for further testing. However given the current controversy over the measurement of ΔK_{th}, where it seems that widely varying results can be obtained even when testing according to current standards, I would advise that endurance tests carried out on sharply-notched specimens give a more reliable and relevant estimate of the threshold and thus of L. Values of the TCD parameters have already been obtained for many materials and failure modes [1], so often a reasonable guess can be made for the material in question even without carrying out the tests.

Some Dimensionless Numbers

Dimensionless numbers are of great value in certain branches of engineering, such as the Reynolds number and other parameters in fluid mechanics. To date, such numbers have not been proposed in fracture mechanics, except the brittleness number suggested by Carpinteri [11] which is almost the same as L, normalized by the size of the body. Here I propose four dimensionless parameters obtained by using the TCD parameters as normalizing variables:

Normalised Defect Size: The length *a* of a crack (or other defect) normalized by L. If $a/L<<1$ then the defect is harmless and will not reduce the strength of the body. If $a/L>>1$ then the defect exerts its full effect: if it is a crack, we can use fracture mechanics, otherwise we can use its K_t factor (though see below regarding the limits on root radius). A suitable name for this parameter could be the ElHaddad number, since the critical length concept was first identified in a paper on short crack fatigue by ElHaddad et al [12], who called it a_o.

Normalised Notch Root Radius: The root radius ρ normalized by L. If ρ/L<1 then the notch will behave as a crack of the same length. If ρ/L>>1 then the notch can be analysed simply using its K_t factor. We might call this the Neuber number, since it was Neuber's pioneering work which first established the TCD for notches, in the form of the Line Method [13].

Normalised Body Size: A relevant body dimension such as the remaining ligament width W, normalized by L. If W/L>>1 then the TCD in its normal forms can be used, otherwise a modified form of the analysis is required. This number might be named after Carpinteri, given its similarity to his brittleness number.

Normalised Material Strength: The plain-specimen strength (in static loading or in cyclic loading) normalized by the critical stress σ_o (or $\Delta\sigma_o$). Notches having K_t less than this number are harmless. I would like to suggest a name for this parameter, but modesty forbids me!

Conclusions

1) Defects which are much smaller in size than the critical distance L can be assumed to be harmless, having no effect on the failure loads for the failure mechanism under consideration. Defects much larger than L can be treated using standard techniques such as LEFM for cracks or the stress concentration factor K_t for notches. A dimensionless number obtained by divided the linear size of the defect by L is thus useful in assessing defects.

2) Notches (and other stress concentration features in components) which have root radii smaller than L can be regarded as cracks. Features with root radii much larger than L exert the full effect of their K_t factor. A dimensionless number obtained by dividing the root radius by L is thus useful in assessing the effect of stress concentrations.

3) If the size of the body (defined by a relevant linear dimension) is similar to or less than L, the normal TCD methods cannot be used, and the failure behaviour can be expected to differ from that of larger bodies. Thus a dimensionless number consisting of the body size divided by L will be useful.

4) If the critical stress σ_o as used in the linear elastic TCD is greater than the plain-specimen strength of the material (in the cyclic or static loading modes as appropriate) then a notch having a K_t factor equal to or less than the ratio of critical stress to material strength will be harmless. Thus this ratio is a useful dimensionless number.

Figure Captions

Fig.1: The fracture surface of a component, made from a brittle polymer, showing a defect in the form of a spherical pore.

Fig.2: Experimental data showing the effect of initial crack size on fatigue strength for cyclically loaded specimens of a Cr-Mo steel: data taken from Lucas and Kunz [14]. The value of the critical distance L is indicated.

Fig.3: Experimental data showing the effect of notch root radius on the measured value of fracture toughness of a low alloy steel failing by brittle cleavage fracture at low temperature (original data from Malkin and Tetelman, reported by Hedner [15]). The value of L and the predictions of the point method (PM) and line method (LM) are shown.

Fig.4: The human patella, showing the sharp notches created when a piece is removed during surgery (black lines).

Fig.5: Welded joints frequently fail by fatigue cracking which initiates from the corner where the weld bead meets the base metal, as indicated here on this finite element model. In the TCD approach this corner can be modeled with zero root radius provided the actual root radius is less than L, which it usually is.

Fig.6: Data on the static strength of PMMA specimens containing notches, normalized by the static strength of the material (UTS, σ_u), as a function of the K_t factor for the notch. The prediction line for the TCD crosses the static strength at a K_t value below which notches are harmless. This occurs at $K_t = \sigma_o/\sigma_u$

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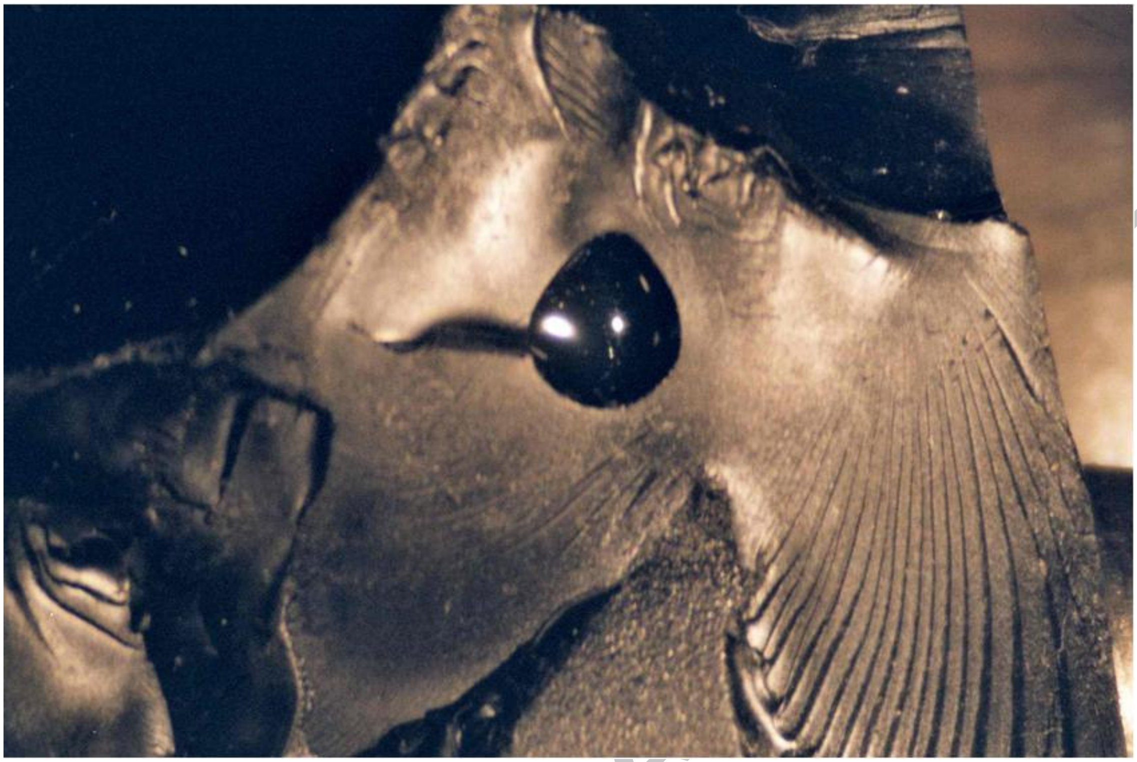

Fig1

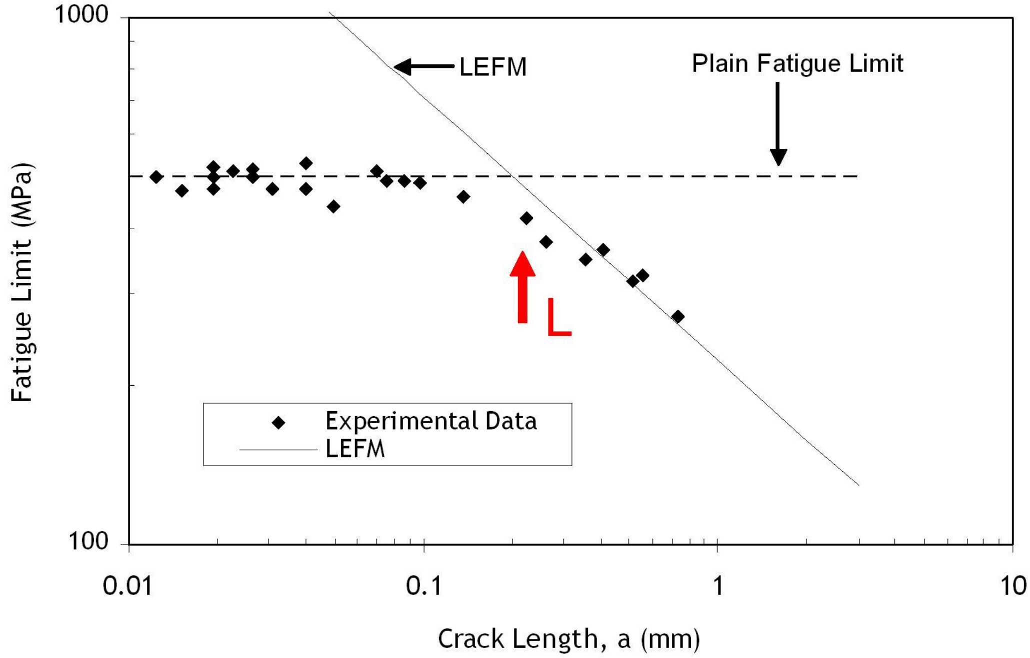
1000
100
Fatigue Limit (MPa)
LEFM
Plain Fatigue Limit
L
Experimental Data
LEFM
0.01
0.1
1
10
Crack Length, a (mm)

Fig2

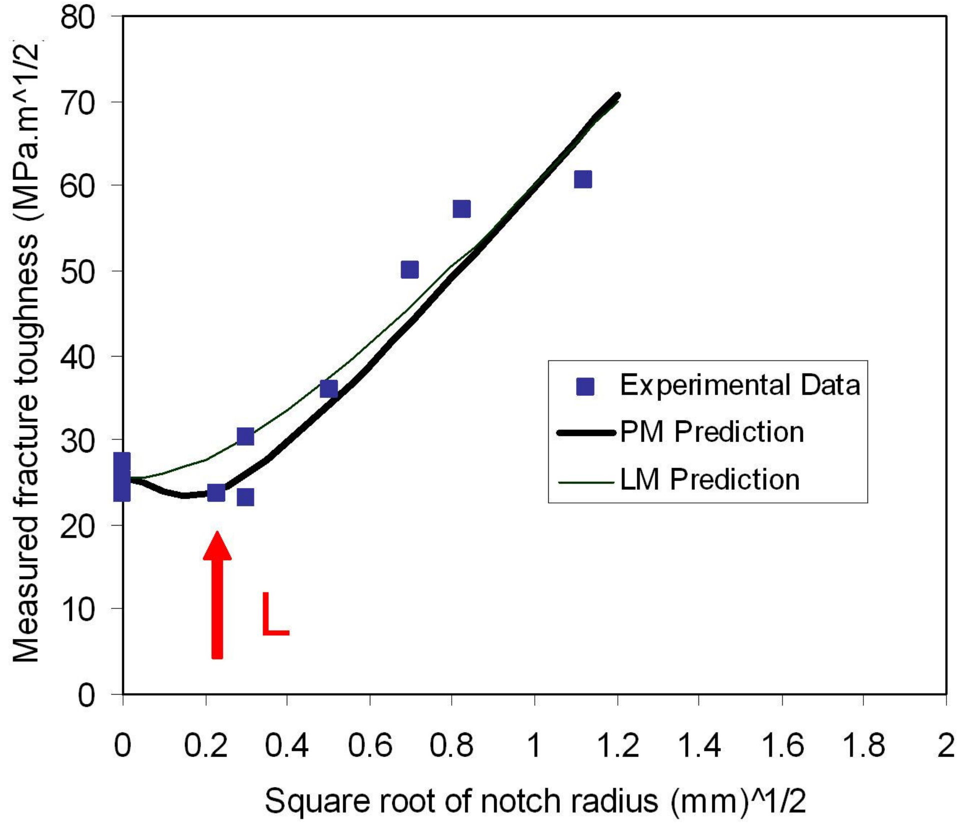
Measured fracture toughness (MPa.m^1/2
80
70
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Square root of notch radius (mm)^1/2
Experimental Data
PM Prediction
LM Prediction
L

Fig3

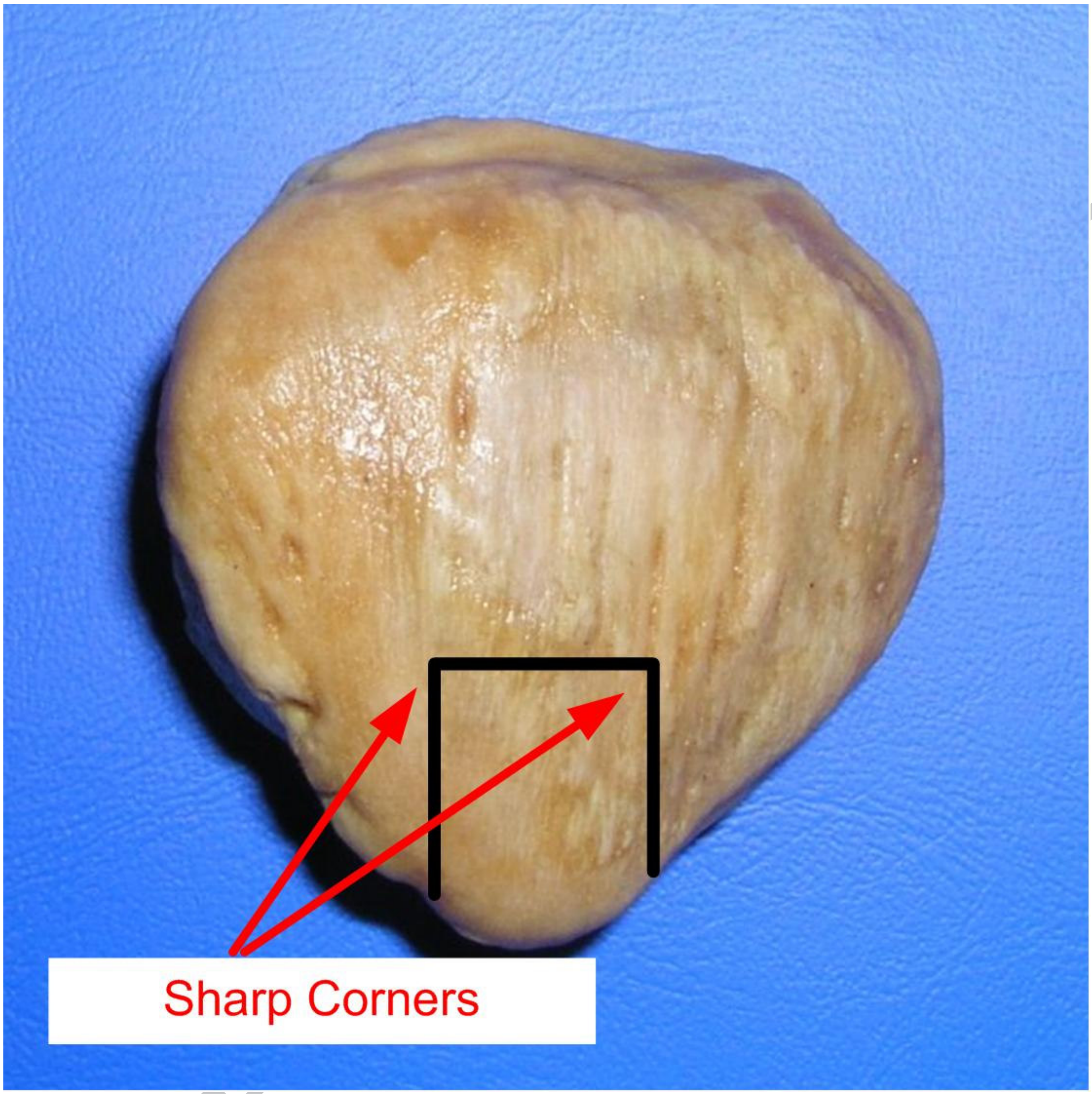

Fig4

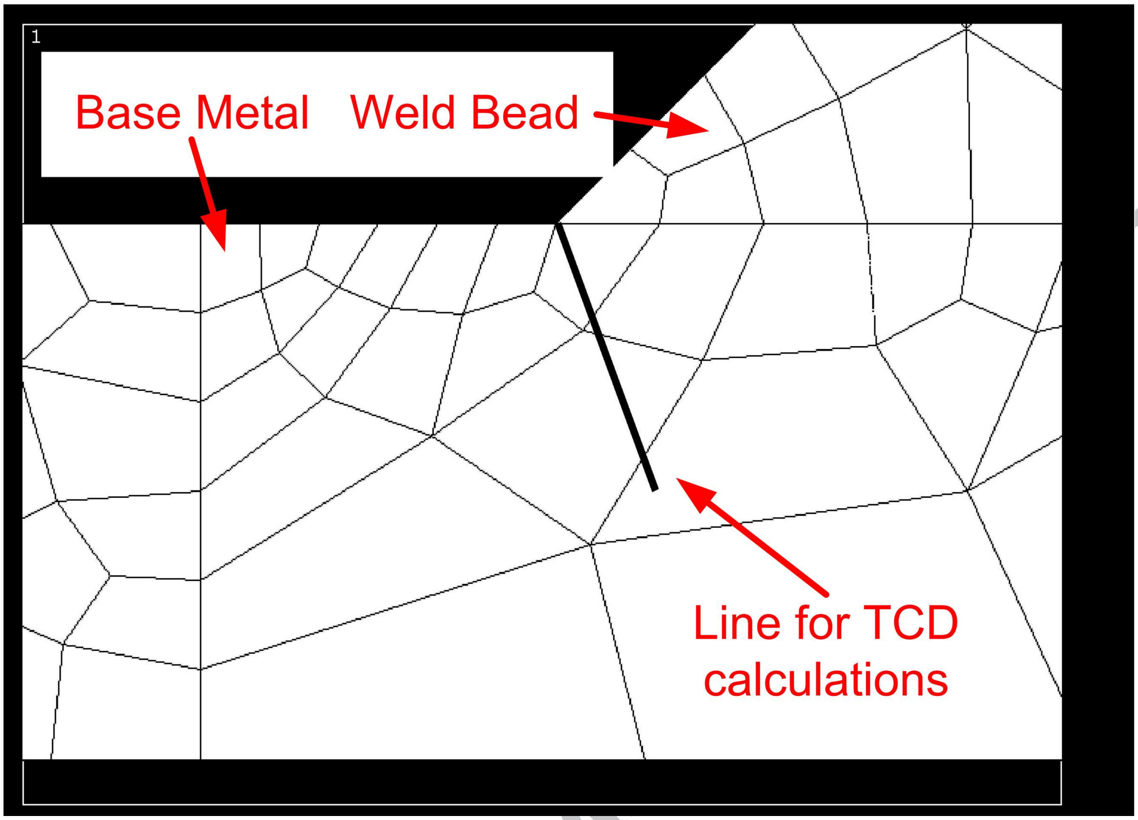

Fig5

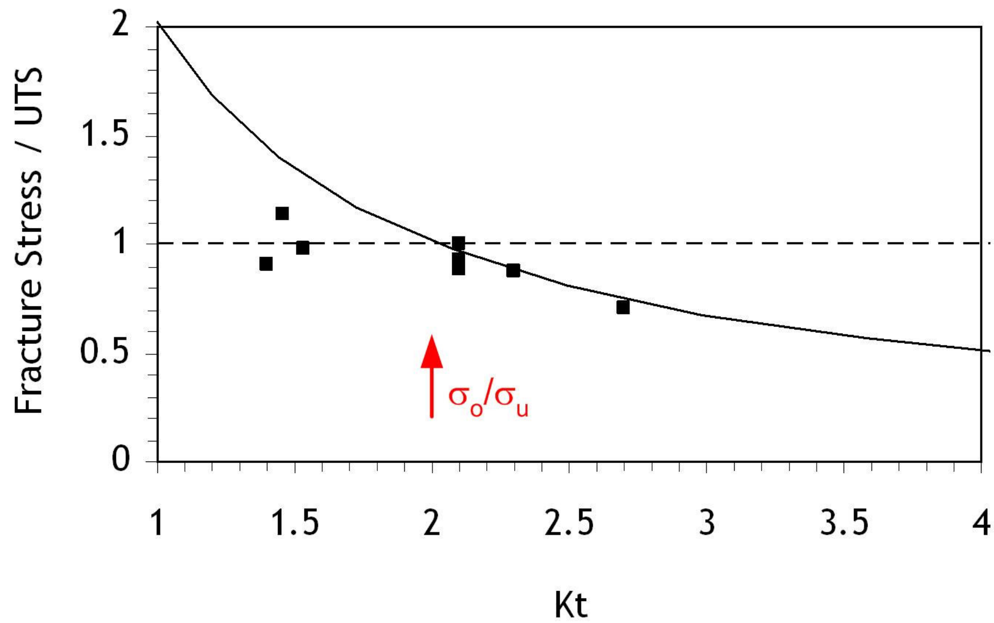

Fig6