On the SO(N) symmetry of the chiral SU(N) Yang–Mills model

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Abstract

The posibility of quantizing the anomalous $SU(N)$ Yang–Mills model preserving the symmetry under the orthogonal subgroup is indicated. The corresponding Wess–Zumino action (1-cocycle) possesses the additional $SO(N)$ symmetry and can be expressed in terms of chiral fields taking values in the homogeneous space $SU(N)/SO(N)$. The modified anomaly and the constraints commutator (2-cocycle) are calculated.

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1 Introduction

It is known that chiral gauge models suffer from anomalies [1, 2, 3]. The calculation of the anomalies was performed both in the framework of perturbation theory and by algebraic and geometric methods [4, 5, 6, 7]. From algebraic point of view the anomaly corresponds to infinitesimal 1-cocycle on a group G. The global 1-cocycle as was indicated by Faddeev and Shatashvili [6, 7] is just the Wess–Zumino action [8], depending on the chiral fields with values in the group G. The anomaly leads also to the appearence of an additional term in the constraints commutator which is the infinitesimal 2-cocycle on the group G [6, 7, 9]. It was argued that it may change the physical content of the theory.

The particular form of anomaly, the corresponding Wess–Zumino term and 2-cocycle depend on the regularization used. Although the difference is a local term it may lead to important physical consequences. In the two dimensional case it was shown by Jackiw and Rajaraman [10] that different counterterms result in the different spectrum of the model. In the anomalous case there is no regularization preserving the full gauge symmetry of the theory and therefore a priori there is no unique choice of the particular form of anomaly. However it seems natural to choose a form keeping as much of classical gauge symmetries as possible. It is known that there exist some chiral gauge groups and the fermion representations for which the anomalies compensate, for example the orthogonal groups $SO(N)$, $N \neq 6$. Presumably for such groups and representations one can construct an invariant regularization explicitly preserving the gauge symmetry. In particular for the spinor representations of the $SO(N)$ groups such a regularization was proposed in ref.[11].

In the general case as we have already mentioned it is impossible to preserve the full gauge symmetry but one can construct a regularization invariant with respect to some nonanomalous subgroup. In this paper we shall consider the chiral $SU(N)$ gauge model with fermions in the fundamental representation. In this case one can maintain the invariance under the orthogonal subgroup $SO(N)$. The resulting anomaly and the Wess–Zumino action differ from the "standard" ones discussed in the papers mentioned above by local terms. The Wess–Zumino action possesses the additional $SO(N)$ invariance leading to vanishing of the anomaly on the $SO(N)$ subgroup. That means the Wess–Zumino action depends in fact on the chiral fields taking values not in the group $SU(N)$ as in the usual case, but in the homogeneous space $SU(N)/SO(N)$. Therefore the number of degrees of freedom is reduced. Naturally the 2-cocycle is also changed and the constraints commutator is anomaly free on the $SO(N)$ subgroup.

The Wess–Zumino action over homogeneous spaces was considered in several publications (see for example [12, 13]). However the authors of these papers were interested in the effective Nambu–Goldstone actions describing low energy QCD. In our case the Wess–Zumino action over homogeneous spaces arises in the process of quantization of chiral Yang–Mills model preserving a nonanomalous subgroup. We hope that it may be of interest for analyzing the spectrum of anomalous models.

In the second section we give two expressions for the Wess–Zumino action. The first one depends on the chiral fields with values in the group $SU(N)$ and has the additional $SO(N)$ invariance. Then we introduce chiral fields on the homogeneous space and rewrite the Wess–Zumino action in terms of these fields. Using this action we calculate anomaly. In the third section we apply the method proposed in ref.[14] to get the expression for the 2-cocycle, appearing in the anomalous constraints commutator.

2 The $SO(N)$ invariant Wess–Zumino action

We consider the model described by the Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu + A_\mu)\psi \tag{1}$$

where A_μ is a $SU(N)$ Yang–Mills field which in this section will be considered as an external one, $\psi \equiv \frac{1}{2}(1+\gamma_5)\psi$ is a chiral fermion in the fundamental representation. The fundamental representation of the Lie algebra $su(N)$ is generated by the antihermitian matrices λ^a :

$$\mathrm{tr}\, \lambda^a \lambda^b = -\frac{1}{2}\delta_{ab}; \quad [\lambda^a, \lambda^b] = f^{abc}\lambda^c \tag{2}$$

and $A_\mu = A_\mu^a \lambda^a$.

The gauge transformation looks as follows:

$$\begin{aligned} A_\mu &\rightarrow A_\mu^g = g^{-1}A_\mu g + g^{-1}\partial_\mu g \\ \psi &\rightarrow \psi^g = g^{-1}\psi, \qquad g \in SU(N) \end{aligned} \tag{3}$$

There is no $SU(N)$ invariant regularization for this Lagrangian however one can write the $SO(N)$ invariant regularized Lagrangian of the form:

$$\begin{aligned} \mathcal{L} = {} & i\bar{\psi}\gamma^\mu(\partial_\mu + A_\mu)\psi + \sum_{r=1}^{2K-1}[i\bar{\psi}_r\gamma^\mu(\partial_\mu + A_\mu)\psi_r - M_r\psi_r^T C\psi_r - M_r\bar{\psi}_r C\bar{\psi}_r^T] \\ & + i\sum_{r=1}^{2K}[(-1)^r\bar{\phi}_r\gamma^\mu(\partial_\mu + A_\mu)\phi_r - \sum_{s=1}^{2K}(M_{rs}\phi_r^T C\phi_s - M_{rs}\bar{\phi}_r C\bar{\phi}_s^T)] \end{aligned} \tag{4}$$

Here ψ_r are the anticommuting Pauli–Villars spinors and ϕ_r are the commuting ones. M_{rs} is an antisymmetric matrix. The standard Pauli–Villars conditions are assumed. The matrix C is the charge conjugation matrix. The only terms, which are not invariant under the gauge transformation (3) of all fields, are the mass term for the Pauli–Villars fields. The mass term transforms as follows

$$M_r\bar{\psi}_r C\bar{\psi}_r^T \rightarrow M_r\bar{\psi}_r C g g^T \bar{\psi}_r^T . \tag{5}$$

One sees that for $g \in SO(N)$, $gg^T = 1$ this mass term is invariant, and therefore the regularization preserves the $SO(N)$ gauge invariance.

The Wess–Zumino action $\alpha_1^{\mathrm{ort}}(A, g)$ is defined by usual formula

$$\mathrm{e}^{i\alpha_1^{\mathrm{ort}}(A,g)} = \frac{\det(\gamma^\mu(\partial_\mu + A_\mu^g))}{\det(\gamma^\mu(\partial_\mu + A_\mu))}. \tag{6}$$

Where it is understood that the determinant is calculated with the help of regularization (4) and necessary counterterms are introduced. It follows directly from eq.(6) that the Wess–Zumino action is a 1-cocycle and satisfies the condition:

$$\alpha_1^{\mathrm{ort}}(A, g_1) + \alpha_1^{\mathrm{ort}}(A^{g_1}, g_2) = \alpha_1^{\mathrm{ort}}(A, g_1 g_2) \qquad (mod\, 2\pi). \tag{7}$$

Due to $SO(N)$ gauge invariance of the regularized Lagrangian (4) the Wess–Zumino action has the additional invariance

$$\alpha_1^{\mathrm{ort}}(A, gh) = \alpha_1^{\mathrm{ort}}(A, g), \tag{8}$$

where $h \in SO(N)$.

Let us stress that in eq.(8) the field A is not transformed. Eq.(8) is a direct consequence of the invariance of the gauge transformed mass term (5) under the transformation

$$g \to gh, \qquad h \in SO(N). \tag{9}$$

Eq.(8) expresses the hidden symmetry of the Wess–Zumino action in our case. Hidden symmetries of this type in connection with models on homogeneous spaces were discussed in refs.[17, 18, 13]. It follows from eqs.(7,8) that the Wess–Zumino action vanishes if the chiral field g belongs to the orthogonal subgroup $SO(N)$

$$\alpha_1^{\rm ort}(A,h) = 0, \qquad h \in SO(N) \tag{10}$$

The geometric origin of the existence of such Wess–Zumino action is the triviality of the cohomology group $H^5(SO(N))$.

The Wess–Zumino action depends on the regularization scheme used. The difference is a trivial local 1-cocycle. We can use this fact to calculate the Wess–Zumino action corresponding to regularization (4) starting from the action given for example in the paper [7]. The action we are interested in may be presented in the form:

$$\alpha_1^{\rm ort}(A,g) = \alpha_1(A,g) + \alpha_0(A^g) - \alpha_0(A). \tag{11}$$

Here $\alpha_1(A,g)$ is the "standard" Wess–Zumino action

$$\begin{aligned} \alpha_1(A,g) \;\; &= \;\; \int d^4x \, [d^{-1}\kappa(g) - \frac{i}{48\pi^2}\epsilon^{\mu\nu\lambda\sigma} \, {\rm tr}\, [(A_\mu\partial_\nu A_\lambda \, + \, \partial_\mu A_\nu A_\lambda \, + \, A_\mu A_\nu A_\lambda)g_\sigma \, - \\ &\quad - \frac{1}{2}A_\mu g_\nu A_\lambda g_\sigma \, - \, A_\mu g_\nu g_\lambda g_\sigma]] \end{aligned} \tag{12}$$

and we use the notations

$$\int d^4x d^{-1}\kappa(g) \equiv -\frac{i}{240\pi^2}\int_{M_5} d^5x \, \epsilon^{pqrst} \, {\rm tr}\, (g_p g_q g_r g_s g_t) \tag{13}$$

$$g_\mu = \partial_\mu g g^{-1}. \tag{14}$$

In eq.(13) the integration goes over a five-dimensional manifold whose boundary is the usual four-dimensional space.

The functional $\alpha_0(A^g) - \alpha_0(A)$ is a trivial local 1-cocycle which can be determined from eq.(10). The explicit form of $\alpha_1(A,g)$ (eq.(12)) dictates the following ansatz for $\alpha_0(A)$:

$$\begin{aligned} \alpha_0(A) \;\; &= \;\; -\frac{i}{48\pi^2}\int d^4x \, \epsilon^{\mu\nu\lambda\sigma} \, {\rm tr}\, (a_1 A_\mu A_\nu A_\lambda A_\sigma^T + a_2 A_\mu A_\nu^T A_\lambda A_\sigma^T + \\ &\quad + a_3 A_\mu A_\nu A_\lambda^T A_\sigma^T + b_1\partial_\mu A_\nu A_\lambda A_\sigma^T + b_2\partial_\mu A_\nu A_\lambda^T A_\sigma + b_3\partial_\mu A_\nu A_\lambda^T A_\sigma^T) \end{aligned} \tag{15}$$

where A_μ^T is a transposed matrix A_μ.

Let us stress that to satisfy eq.(10) it is necessary to introduce the terms depending not only on A_μ but also on A_μ^T. Eq.(10) determines uniquely the coefficients a_i, b_i. As a result:

$$\begin{aligned} \alpha_0(A) \;\; &= \;\; -\frac{i}{48\pi^2}\int d^4x \, \epsilon^{\mu\nu\lambda\sigma} \, {\rm tr}\, (A_\mu A_\nu A_\lambda A_\sigma^T - \frac{1}{4}A_\mu A_\nu^T A_\lambda A_\sigma^T + \\ &\quad + \partial_\mu A_\nu A_\lambda A_\sigma^T + A_\mu\partial_\nu A_\lambda A_\sigma^T) \end{aligned} \tag{16}$$

Obviously one can add also any trivial local $SO(N)$ invariant 1-cocycle. The corresponding infinitesimal 1-cocycle (anomaly) is calculated in a standard way

$$\int d^4x\, \epsilon^a(x)\, \mathcal{A}^a_{ort}(A) = \alpha_1^{\rm ort}(A^h, h^{-1}g) - \alpha_1^{\rm ort}(A,g) \tag{17}$$

where $h = 1 + \epsilon^a \lambda^a$.

It looks as follows

$$\begin{aligned}\mathcal{A}^a_{ort}(A) = {} & \frac{i}{48\pi^2}\epsilon^{\mu\nu\lambda\sigma}\,{\rm tr}\,[(\lambda^a + \lambda^{a,T})(\partial_\mu(A_\nu\partial_\lambda A_\sigma + \\ & +\partial_\nu A_\lambda A_\sigma + A_\nu A_\lambda A_\sigma - A_\nu A^T_\lambda A_\sigma - \frac{1}{2}A^T_\nu \partial_\lambda A_\sigma - \frac{1}{2}\partial_\nu A_\lambda A^T_\sigma) - \\ & -\partial_\mu A_\nu A_\lambda A^T_\sigma - A_\mu \partial_\nu A_\lambda A^T_\sigma - A^T_\mu \partial_\nu A_\lambda A_\sigma - A^T_\mu A_\nu \partial_\lambda A_\sigma - \\ & -A_\mu A_\nu A_\lambda A^T_\sigma + \frac{1}{2}A_\mu A^T_\nu A_\lambda A^T_\sigma + \frac{1}{2}A^T_\mu A_\nu A^T_\lambda A_\sigma)]\end{aligned} \tag{18}$$

One sees that on the subgroup $SO(N)$ ($\lambda^a = -\lambda^{a,T}$) this anomaly vanishes. The Wess–Zumino consistensy condition is obviously satisfied because our anomaly differs from the standard one by the trivial 1-cocycle.

The additional $SO(N)$ invariance of the Wess–Zumino action $\alpha_1^{\rm ort}(A,g)$ means that it depends in fact not on all the elements of $SU(N)$ but only on the elements of the homogeneous space $SU(N)/SO(N)$. One can introduce coordinates on this homogeneous space and express the Wess–Zumino action in terms of these coordinates.

The natural coordinates are symmetric and unitary matrices

$$s = gg^T \tag{19}$$

This choice is suggested by the form of the mass term in the regularized Lagrangian (14). As follows from eq.(5) after the gauge transformation it depends only on the combination gg^T. The gauge group transforms the coordinates s in the following manner

$$s \to g^{-1} s g^{-1,T}. \tag{20}$$

In terms of these coordinates the Wess–Zumino action looks as follows:

$$\begin{aligned}\alpha_1^{\rm ort} = {} & \int d^4x\,[\frac{1}{2}d^{-1}\kappa(s) - \frac{i}{48\pi^2}\epsilon^{\mu\nu\lambda\sigma}\,{\rm tr}\,[(\partial_\mu A_\nu A_\lambda + A_\mu\partial_\nu A_\lambda + A_\mu A_\nu A_\lambda - \\ & -\frac{1}{2}\partial_\mu A_\nu s A^T_\lambda s^{-1} - \frac{1}{2}sA^T_\mu s^{-1}\partial_\nu A_\lambda - A_\mu s A^T_\nu s^{-1}A_\lambda)s_\sigma - \\ & -\frac{1}{2}A_\mu s_\nu A_\lambda s_\sigma + \frac{1}{2}(sA^T_\mu s^{-1}A_\nu - A_\mu s A^T_\nu s^{-1})s_\lambda s_\sigma - A_\mu s_\nu s_\lambda s_\sigma \\ & +\partial_\mu A_\nu A_\lambda s A^T_\sigma s^{-1} + A_\mu \partial_\nu A_\lambda s A^T_\sigma s^{-1} + A_\mu A_\nu A_\lambda s A^T_\sigma s^{-1} - \\ & -\frac{1}{4}A_\mu s A^T_\nu s^{-1} A_\lambda s A^T_\sigma s^{-1} - \alpha_0(A)]]\end{aligned} \tag{21}$$

where $s_\mu = \partial_\mu s s^{-1}$.

The derivation is straightforward but some comments are in order. Using the equality

$$g^T_\mu = s^{-1}(s_\mu - g_\mu)s \tag{22}$$

we express g^T_μ in terms of g_μ and s_μ and then comparing the terms of a given order in A_μ and applying again eq.(22) we find the expression (21). This action may be used for the construction of the symplectic form defining the integration messure in the path integral. It is worthwhile to emphasize that contrary to the standard case the action (21) depends not only on the chiral current $\partial_\mu s s^{-1}$, belonging to the Lie algebra of the group, but also on the coordinates of the homogeneous space $SU(N)/SO(N)$. It may be of importance for analyzing possible stationary points of the effective action.

3 Anomalous constraints commutator

In this section we shall calculate the 2-cocycle associated to the Wess–Zumino action (11). This 2-cocycle appears as the Schwinger term in the constraints commutator and can be calculated either by direct summation of the Feynman diagrams [15, 16] or by using the path integral representation for the commutator [14]. We use the second approach. According to the Bjorken–Johnson–Low (BJL) formula the matrix element of the equal time commutator may be expressed in terms of the expectation value of T-product as follows:

$$\lim_{q_0\to\infty} q_0 \int dt'\, \mathrm{e}^{iq_0(t'-t)} \langle\tilde{\varphi}|\, T A(x,t')B(y,t)\,|\varphi\rangle = i\langle\tilde{\varphi}|\,[A(x,t),B(y,t)]\,|\varphi\rangle \tag{23}$$

For the expectation value of T–product one can write the representation in terms of the path integral

$$\langle\tilde{\varphi}|\, T A(x,t')B(y,t)\,|\varphi\rangle = \int d\mu\, \mathrm{e}^{iS} A(x,t')B(y,t) \tag{24}$$

Here it is understood that the integration goes over the fields satisfying the boundary conditions corresponding to the initial and final states $|\varphi\rangle$ and $\langle\tilde{\varphi}|$. Following the approach of [14] we can consider the chiral $SU(N)$ Yang–Mills model in the Hamiltonian gauge $A_0 = 0$. In this gauge the S–matrix element can be written as the path integral

$$\langle\alpha|\beta\rangle = \int d\mu\, \delta(A_0)\mathrm{e}^{iS}, \tag{25}$$

where in the first order formalism

$$\begin{aligned} S \;&=\; \int d^4x\, [E^a_i \dot{A}^a_i - \frac{1}{2}(E^a_i)^2 - \frac{1}{4}(F^a_{ij})^2 + A^a_0 G^a + \\ &\quad + i\bar{\psi}\gamma_0\partial_0\psi - i\bar{\psi}\gamma_i(\partial_i - A_i)\psi] \end{aligned} \tag{26}$$

In the nonanomalous case the constraints G^a form a Lie algebra

$$[G^a(\mathbf{x}), G^b(\mathbf{y})] = i f^{abc} G^c(\mathbf{y})\delta(\mathbf{x}-\mathbf{y}) \tag{27}$$

However as was shown in refs. [14, 15, 16] in the anomalous theory this relation is violated and the Schwinger term arises.

To calculate this Schwinger term we make the gauge transformation of the variables in the integral (25). The transformed integral may be written in the form

$$\langle\tilde{\varphi}|\varphi\rangle = \int d\mu\, \delta(A_0)\mathrm{e}^{iS} \exp\{-i\int d^4x\, g^a_0 G^a(x) + i\alpha^{\mathrm{ort}}_1(A,g)\,|_{A_0=-g_0}\} \tag{28}$$

Here the 1-cocycle arises due to the noninvariance of the regularization in accordance with eq.(6).

Using the representation for the chiral field g: $g = \mathrm{e}^u$ and taking into account that the integral (28) does not depend on g we can put equal to zero variation of this integral over u. To the second order in u one has:

$$\begin{aligned}
&\frac{1}{2}\int d^4x d^4y\,\langle\tilde{\varphi}|T\widetilde{G}^a(x)\widetilde{G}^b(y)|\varphi\rangle\partial_0 u^a(x)\partial_0 u^b(y) + \\
&+\frac{i}{2}\int d^4x\, f^{abc}u^a(x)\partial_0 u^b(y)\langle\tilde{\varphi}|T\widetilde{G}^a(x)|\varphi\rangle + \\
&+\frac{1}{48\pi^2}\int d^4x\,\langle\tilde{\varphi}|\,\mathrm{tr}\,(\epsilon^{ijk}\partial_i A_j\{\partial_k u(x),\partial_0 u(x)\})|\varphi\rangle = 0. \qquad (29)
\end{aligned}$$

Here we introduced the notation

$$\begin{aligned}
\widetilde{G}^a(x) \;&=\; G^a(x) - \frac{i}{48\pi^2}\epsilon_{ijk}\,\mathrm{tr}\,[(\lambda^a+\lambda^{a,T})(A_i\partial_j A_k + \partial_i A_j A_k + A_i A_j A_k - A_i A_j^T A_k) \\
&\quad -\lambda^a\{\partial_i A_j, A_k^T\}] \qquad (30)
\end{aligned}$$

In the process of derivation of eqs.(29), (30) we used the explicit form of α_1^{ort} (11), (16) and made the shift of the variables E_i^a

$$E_i^a \to E_i^a + \frac{i}{48\pi^2}\epsilon_{ijk}\,\mathrm{tr}\,\lambda^a(\{A_j, g_k\} + g\{A_j^g, A_k^{T,g}\}g^{-1} - \{A_j, A_k^T\}) \qquad (31)$$

In eq.(29) we kept only the terms nonvanishing in the BJL limit.

To get the expression for the commutator of $\widetilde{G}$ we apply to eq.(29) the operator:

$$\lim_{(p_0-q_0)\to\infty}\frac{p_0-q_0}{p_0 q_0}\int dx_0 dy_0\,\mathrm{e}^{ip_0x_0+iq_0y_0}\frac{\delta}{\delta u^a(x)}\frac{\delta}{\delta u^b(y)} \qquad (32)$$

Taking the limit we get the result

$$[\widetilde{G}^a(\mathbf{x}),\widetilde{G}^b(\mathbf{y})] = if^{abc}\widetilde{G}^c(\mathbf{y})\delta(\mathbf{x}-\mathbf{y}) - \frac{1}{24\pi^2}\epsilon_{ijk}\,\mathrm{tr}\,(\partial_i A_j\{\lambda^a,\lambda^b\})\partial_k^x\delta(\mathbf{x}-\mathbf{y}). \qquad (33)$$

Let us note that the commutator of $\widetilde{G}$ (33) coincides with the analogous commutator obtained in ref.[14] with the different Wess–Zumino action. However the definition of $\widetilde{G}$ in our case is different. If one comes back to the constraints G one gets

$$[G^a(\mathbf{x}),G^b(\mathbf{y})] = if^{abc}G^c(\mathbf{y})\delta(\mathbf{x}-\mathbf{y}) + a_{2,ort}^{ab}(A;\mathbf{x},\mathbf{y}) \qquad (34)$$

Here $a_{2,ort}^{ab}$ is the ultralocal 2-cocycle

$$\begin{aligned}
a_{2,ort}^{ab}(A;\mathbf{x},\mathbf{y}) &= -\frac{1}{48\pi^2}\epsilon_{ijk}\,\mathrm{tr}\,([\lambda^a+\lambda^{a,T},\lambda^b+\lambda^{b,T}]\times \\
&\times(A_i\partial_j A_k + \partial_i A_j A_k + A_i A_j A_k - A_i A_j^T A_k + A_i^T\partial_j A_k + \partial_i A_j^T A_k) + \\
&+(\lambda^a+\lambda^{a,T})(\partial_i A_j - \partial_i A_j^T - A_i A_j^T - A_i^T A_j)(\lambda^b+\lambda^{b,T})A_k - \\
&-(\lambda^b+\lambda^{b,T})(\partial_i A_j - \partial_i A_j^T - A_i A_j^T - A_i^T A_j)(\lambda^a+\lambda^{a,T})A_k) \qquad (35)
\end{aligned}$$

This cocycle obviosly differs from the one obtaned in ref.[14]–[16]. In particular it vanishes if at least one of the constraints G^a corresponds to the subgroup $SO(N)$. We note that the adding to 1-cocycle any trivial 1-cocycle having topological nature does not change the commutator of modified constraints $\widetilde{G}$.

4 Conclusion

In this paper we showed that the anomalous $SU(N)$ gauge theory may be quantized in such a way that the resulting effective action possesses the residual $SO(N)$ symmetry. This action is described in a natural way in terms of the coordinates of the homogeneous space $SU(N)/SO(N)$. An analogous construction may be carried out for other gauge groups and representations having nonanomalous subgroups. A trivial local 1-cocycle in this case can be calculated using the same equation $\alpha(A, h) = 0$ where h is an element of the nonanomalous subgroup. The next problem is to try to investigate the physical content of this theory.

If one follows the approach of Faddeev and Shatashvili [9] to quantization of anomalous theories one should add to the original classical action the Wess–Zumino term. In general the physical content of the model depends on the particular form of modified Lagrangian.

The effective action we got and the constraints algebra differ from the ones obtained by Faddeev and Shatashvili. It would be very interesting to investigate if the number of physical degrees of freedom is the same or different in both cases. One possibility is that the variables corresponding to the $SO(N)$ subgroup in the Faddeev–Shatashvili action are not dynamical and integrating them out one would get our action. If it is not the case that means these two approaches lead to physically different models.

To analize the physical content of the theory one needs to develop the expansion of the path integral near some stationary point. At present it is an open problem, because in the four-dimensional case such a solution is not known, and in two dimensional models this effect is absent.

Let us note that in our case the effective action depends not only on the chiral currents and in principle allows constant solution for coordinates of the homogeneous space $SU(N)/SO(N)$.

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