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String Corrections to the Holographic RG Flow of Supersymmetric SU(N+M) x SU(N) Gauge Theory

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Abstract

We study leading string corrections to the type IIB supergravity solution dual to the $\mathcal{N}=1$ supersymmetric $SU(N+M)\times SU(N)$ gauge theory coupled to bifundamental chiral superfields A_i, B_j, $i,j=1,2$. This solution was found in hep-th/0007191, and its asymptotic form describing logarithmic RG flow was constructed in hep-th/0002159. The leading tree-level string correction to the type IIB string effective action is represented by the invariant of the form $\alpha'^3(R^4+...)$. Since the background contains 3-form field strengths, we need to know parts of this invariant that depend on them. By analyzing the 5-point superstring scattering amplitudes we show that only a few specific $R^3(H_3)^2$ and $R^3(F_3)^2$ terms are present in the effective action. Their contribution to the holographic RG flow turns out to be of the same order as of the R^4 terms. Using this fact we show that it is possible to have agreement between the α'-corrected radial dependence of the supergravity fields and the RG flow dictated by the NSVZ beta functions in field theory. The agreement with field theory requires that the anomalous dimension of the operators $\mathrm{Tr}(A_iB_j)$ is corrected by a term of order $(M/N)^4\lambda^{-1/2}$ from its value $-\frac{1}{2}$ found for $M=0$ (λ is the appropriate 't Hooft coupling which is assumed to be strong).

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1. Introduction

Investigations of D-branes on conifolds have produced interesting examples of gauge/string duality with $\mathcal{N} = 1$ supersymmetry. The first case to be considered involves a large number N of D3-branes placed at the singularity of the conifold, which is a Calabi-Yau cone described by the equation $\sum_{i=1}^{4} z_i^2 = 0$ in $\mathbf{C}^4$. The near-horizon geometry produced by the D3-branes is $AdS_5 \times T^{1,1}$ where $T^{1,1} = (SU(2) \times SU(2))/U(1)$ is the base of the cone. Type IIB string theory on this background is conjectured to be dual to the IR limit of the gauge theory on the stack of D3-branes, which is the $\mathcal{N} = 1$ supersymmetric $SU(N) \times SU(N)$ gauge theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2 [1,2].

This duality may be generalized by adding M fractional D3-branes (wrapped D5-branes) to the N regular D3-branes at the apex of the conifold [3,4]. The $SU(N+M) \times SU(N)$ gauge theory on such a stack is dual to a more complicated solution of type IIB supergravity [5,6]. As was realized in [6], in order to consistently extend the original singular solution of [5] to the small radius (IR) region, it is necessary to deform the conifold: $\sum_{i=1}^{4} z_i^2 = \epsilon^2$.

The presence of fractional branes destroys conformal invariance, and this $\mathcal{N} = 1$ gauge theory exhibits an intricate pattern of RG flows. The inverse-squared gauge couplings $1/g_1^2$ and $1/g_2^2$ flow logarithmically in opposite directions until the coupling of the bigger gauge group diverges. To continue past this point it is necessary to apply Seiberg duality to the bigger gauge group [6]. This transformation maps the original gauge theory to essentially the same theory with N replaced by $N - M$. After this, the pattern of the flow repeats itself. Thus, the RG flow involves a series of duality transformations. At the bottom of this duality cascade one finds a gauge theory which exhibits chiral symmetry breaking and confinement [6].

All these features of the RG flow are nicely encoded in the dual supergravity background. In the UV (for large radius) one finds a logarithmic flow of $1/g_1^2 - 1/g_2^2$ [4,5]. In fact, it was recently shown that the coefficient of the logarithm found in supergravity agrees exactly with the prediction of field theoretic NSVZ [7],[8] beta functions [9]. The reduction in the rank of the gauge group due to repeated cascade steps is reflected in the radial dependence of the 5-form flux [5]. In the IR the cascade is terminated by the deformation of the conifold which is responsible for the chiral symmetry breaking and confinement [6].

In this paper we examine modifications of the supergravity solutions found in [1,5,6] by the leading stringy effects encoded in the $O(\alpha'^3)$ corrections to the effective action. The paper has the following structure. In section 2 we examine the structure of the $O(\alpha'^3)$ terms in the type IIB string effective action. We pay special attention to terms that depend on the 3-form field strength. In particular, we show that certain $R^3(H_3)^2$ terms (R is the curvature and H_3 is the NS-NS 3-form), which were expected to be present in earlier literature, do not appear. The $R^3(H_3)^2$ terms that do appear in the action, (as well as other terms in NS-NS sector) turn out to contribute at the same order as the R^4 terms when evaluated on the KT [5] solution. [1] The necessary 5-point amplitude (Green-Schwarz light-cone gauge) calculation is delegated to Appendix A. We will not analyze other similar terms involving RR fields which are not explicitly known at present but will conjecture that since they are part of the same superinvariant, their contribution should be again the same as of R^4 terms.

In section 3 we show that the α'^3 correction does not modify the $AdS_5 \times T^{1,1}$ background. This is in line with expectations from the AdS/CFT correspondence [11,12,13]: the dual $SU(N) \times SU(N)$ gauge theory is conformal for all values of N and for all values of the gauge couplings. Thus, the radius of $AdS_5 \times T^{1,1}$, which is related to $g_s N$, must be a free parameter (a modulus of CFT) not only in the supergravity approximation but also in the full string theoretic treatment.

In section 4 we recall the structure of the supergravity solution [5] describing logarithmic RG flow in the dual $SU(N+M) \times SU(N)$ gauge theory, and review its comparison with the NSVZ β-functions. In section 5 we study the α'^3 corrections to this solution (their detailed analysis is presented in Appendix B). In contrast to the $AdS_5 \times T^{1,1}$ case, here the corrections modify the form of the solution. In particular, the dilaton, which was constant in the supergravity solution, acquires radial dependence due to the stringy effects. This translates into the RG flow of the sum of inverse-square couplings $1/g_1^2 + 1/g_2^2$. From the field theory point of view, this running is due to a correction to the anomalous dimension of the operator $\mathrm{Tr}(A_i B_j)$. For $M = 0$ this anomalous dimension is equal to $-1/2$ [1] but turning on M is expected to correct it by an even power of M/N [6]. We will see that, if $M \ll N$ and if the 't Hooft couplings are large, string theory predicts that this correction is of order $\left(\frac{M}{N}\right)^4 \left(\frac{1}{N g_1^2} + \frac{1}{N g_2^2}\right)^{1/2}$.

[1] We are indebted to K. Peeters, P. Vanhove and A. Westerberg for drawing our attention to a wrong statement concerning these terms in the first version of this paper and informing us about their covariant NSR computation of $R^3(H_3)^2$ terms in the type IIB string 1-loop effective action [10].

2. Structure of $O(\alpha'^3)$ terms in type IIB superstring effective action

In this section we recall the structure of α'^3 corrections to type IIB effective action. The classical type IIB supergravity action in the normalizations we use has the following form

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\Big[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}g_s^2(\partial C_0)^2 - \frac{1}{12}(e^{-\phi}H_3^2 + e^{\phi}g_s^2F_3^2) - \frac{1}{4\cdot 5!}g_s^2F_5^2 + ...\Big]. \tag{2.1}$$

Here $\kappa = 8\pi^{7/2}g_s\alpha'^2$ is the gravitational constant, ϕ is the dilaton, C_0 is the R-R scalar, $H_3 \equiv H = dB_2$ is the NS-NS 3-form, $F_3 \equiv F = dC_2$ is the R-R 3-form, and F_5 is the R-R self-dual 5-form. The leading α' corrections to the tree-level IIB string effective action implied by the structure of the Green-Schwarz 4-point massless string scattering amplitude in the NS-NS sector (i.e. depending on the metric, dilaton and the 2-form B_2) can be written as [14,15]

$$S_8 = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\ \mathcal{L}_8\ ,$$
$$\mathcal{L}_8 = c_1\alpha'^3 \mathrm{e}^{-\frac{3}{2}\phi}\left(t_8^{abcdefgh}t_8^{mnpqrstu} + \frac{1}{8}\epsilon_{10}^{abcdefghij}\epsilon_{10}^{mnpqrstu}{}_{ij}\right)\overline{C}_{abmn}\overline{C}_{cdpq}\overline{C}_{efrs}\overline{C}_{ghtu}\ , \tag{2.2}$$

where $c_1 = \frac{\zeta(3)}{3\cdot 2^{11}}$ and

$$\overline{C}_{ijkl} = C_{ijkl} + \frac{1}{2}\mathrm{e}^{-\frac{1}{2}\phi}\left(\nabla H\right)_{ijkl} - \frac{1}{4}\left(\nabla^2\phi\right)_{ijkl}\ , \tag{2.3}$$

$$\left(\nabla H\right)_{ijkl} = \nabla_i H_{jkl} - \nabla_j H_{ikl}\ , \tag{2.4}$$

$$\left(\nabla^2\phi\right)_{ijkl} = g_{ik}\nabla_j\nabla_l\phi - g_{jk}\nabla_i\nabla_l\phi - g_{il}\nabla_j\nabla_k\phi + g_{jl}\nabla_i\nabla_k\phi\ . \tag{2.5}$$

Here C^{hmnk} is the Weyl tensor, ϵ_{10} is the totally antisymmetric symbol (we use Minkowski notation for the metric, so that $\epsilon_{10}\epsilon_{10} = -10!$), and the tensor t_8 is defined in [16] (it involves only δ-symbols but not ϵ_8).

A few important clarifications are in order. The 4-point on-shell string scattering amplitude determines only terms of 4-th order in expansion of (2.2) near flat space (and modulo equation of motion terms not visible in on-shell amplitude). In particular, the $\epsilon_{10}\epsilon_{10}$ structure whose expansion starts with terms of 5-th order in the fields is not fixed by it. Fortunately, complementary information is provided [17] by the known 4-loop sigma model beta function [18] that allows us to restore the covariant non-linear form of the R^4 correction to the action (see also [19]). However, the sigma model calculation [18] was carried out only for $B_2 = 0$ [2] and shed no light on the H-dependent terms. In Appendix A we extract new information about such terms from the 5-point amplitude for antisymmetric tensors and gravitons.

As follows from [17], there exists a scheme in which the metric and dilaton dependent terms in the $O(\alpha'^3)$ action written in the string frame are given by[3]

$$S = -\frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G}\ e^{-2\phi}\ \left[R + 4(\partial\phi)^2 + \alpha'^3 c_1 J_0 \right] , \tag{2.6}$$

$$\begin{aligned} J_0 &= 3 \cdot 2^8 (R^{hmnk} R_{pmnq} R_h{}^{rsp} R^q{}_{rsk} \ + \ \frac{1}{2} R^{hkmn} R_{pqmn} R_h{}^{rsp} R^q{}_{rsk}) \\ &= (t_8 t_8 + \frac{1}{8} \epsilon_{10}\epsilon_{10}) RRRR + O(R_{mn}) \ . \end{aligned} \tag{2.7}$$

It is the action that follows from (2.6) upon transformation to the Einstein frame that is the correct metric-dilaton action beyond the 4-point order. The terms linear in the second derivative of the dilaton in (2.3) may then be understood as originating from conformal transformation to the Einstein frame $G_{mn} = e^{\phi/2} g_{mn}$.

Due to the field redefinition ambiguity [24,14], we can assume that all the terms in (2.2) depend only on the Weyl tensor, and there is no explicit dependence on the Ricci tensor. Only such a choice of the α'^3 corrections is directly compatible with the AdS/CFT correspondence (see also [25,26]).[4]

Ignoring the derivatives of the dilaton and the 3-form, eq. (2.2) may be written as

$$\mathcal{L}_8 = \frac{1}{8} \alpha'^3 \zeta(3) \mathrm{e}^{-\frac{3}{2}\phi} W \ , \qquad W = C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk} + \frac{1}{2} C^{hkmn} C_{pqmn} C_h{}^{rsp} C^q{}_{rsk} \ . \tag{2.8}$$

To obtain (2.8) one should use the well-known symmetries of the Weyl tensor. The tensor ∇H in (2.4) does not possess all of these symmetries, in particular, it is antisymmetric under the interchange of the first and second pairs of indices: $(\nabla H)_{ijkl} = -(\nabla H)_{klij}$. For this reason, a priori we are not allowed to use (2.8) with $C \to \overline{C}$ if the NS-NS 2-form does not vanish.

As follows from world-sheet parity considerations, the NS-NS part of tree-level type II string effective action must be even in B_2; in particular, it should not contain terms linear

[2] The generalization of the 4-loop beta function computation of [18] to the $B_2 \neq 0$ case is very complicated and was not done so far in full (see [20,21] for some partial results).

[3] For some useful relations between R^4 invariants see [22,23].

[4] Introducing Ricci-tensor dependent terms would imply that one would need a compensating field redefinition, i.e. a change of a scheme.

in $H_3 = dB_2$. [5] Thus, to find leading corrections to the backgrounds with vanishing H_3 we may ignore all H_3-dependent terms in the effective action.

The tensor $\nabla^2\phi$ in (2.5) possesses all the symmetries of the Weyl tensor but it is not traceless. Thus, if the dilaton does not vanish but the NS-NS 2-form does, the expressions (2.2) and (2.8) differ by the trace terms originating from the term $\epsilon_{10} \cdot \epsilon_{10} \overline{C}^4$. These trace terms can be readily taken into account in all the cases we shall consider.

The R-R scalar C_0 and the R-R 3-form F_3 can be easily included into the action (2.2) by using the $SL(2,\mathbf{Z})$-invariance of type IIB string theory. The $SL(2,\mathbf{Z})$-invariant form of the complete 4-point ($\alpha'^3 R^4 + ...$) effective action (including non-perturbative corrections) which depends on all of the type IIB massless bosonic fields except the 5-form was proposed in [27]. In particular, to account for the contribution of F_3 one can just add to the $\overline{C}$ in (2.3) the term

$$\frac{1}{2} \mathrm{e}^{\frac{1}{2}\phi} g_s \left(\nabla F\right)_{ijkl} \ , \tag{2.9}$$

obtained from the ∇H term in (2.3) by the change $\phi \to -\phi, \quad H \to g_s F$. We shall not explicitly include the terms depending on C_0 in the effective action: the action (2.1) is quadratic in C_0,[6] and C_0 is trivial for the backgrounds [1,5] we are studying; therefore, C_0-dependent terms cannot affect the equations of motion for other fields.

The quartic effective action may also contain terms dependent on F_5, e.g., $C^2(\nabla F_5)^2$, $(\nabla H_3)^2(\nabla F_5)^2$, $(\nabla F_3)^2(\nabla F)^2$, $(\nabla F_5)^4$, etc., which are not known at present. In what follows we will assume that such terms do not change the form of the leading corrections to the backgrounds we are studying.

The 8-derivative term in the effective action may contain also other non-linear structures that contribute to the S-matrix only starting at the 5-point or higher level. In particular, it was conjectured in [27] that there are the 5-point R^3H^2 terms of the form

$$\left(t_8 t_8 + \frac{1}{8} \epsilon_{10} \epsilon_{10} \right) RRR \, \overline{H}_3^2 \ , \tag{2.10}$$

where

$$\left(\overline{H}_3^2 \right)_{ijkl} = H_{ikm} H_{jlm} - H_{jkm} H_{ilm} \tag{2.11}$$

effectively replaces one of the four factors of R_{ijkl} in the R^4 term. As we will show in Appendix A, these particular terms are actually absent from the effective action. This

[5] The $hhhB_2$ 4-point amplitude corresponding to (2.2) indeed vanishes.

[6] The leading-order (supergravity) $C_0 H_3 \cdot F_3$ term vanishes on the backgrounds of [1,5].

is an important consequence of the supersymmetry of the theory. There is, nevertheless, the term of the form (2.10) with $\overline{H}_3^2$ (2.11) replaced by a different contraction of the two H-tensors

$$\left(H_3^2\right)_{ijkl} = H_{ijm}H_{klm} \ . \tag{2.12}$$

This term was found as a contribution to the 1-loop string effective action using the covariant NSR formalism in [10], and its presence is demonstrated through a light-cone gauge calculation in Appendix A.

We shall analyze the background values of the α'^3 correction terms in the NS-NS sector in Appendix B, and show that they behave in accord with the conjectured duality between the supergravity background of [5,6] and the $SU(N+M)\times SU(N)$ gauge theory.

3. Absence of corrections to $AdS_5 \times T^{1,1}$ background

In this section we study the $AdS_5 \times T^{1,1}$ background of type IIB supergravity [28,1], which has constant dilaton and the metric given by

$$ds_{10}^2 = \frac{L^2}{z^2}(dz^2 + dx_n dx_n) + L^2 ds^2_{T^{1,1}} \ . \tag{3.1}$$

With proper normalization included ($L^4 = \frac{27}{4}\pi g_s N\alpha'^2$) the 5-form field strength is given by [9]

$$F_5 = \mathcal{F}_5 + *\mathcal{F}_5 \ , \qquad \mathcal{F}_5 = 27\pi\alpha'^2 N \ \mathrm{vol}(T^{1,1}) \ ,$$

while the 3-forms are not turned on. To investigate possible leading higher derivative corrections to this background it is sufficient to analyze the terms of the form $C^4, C^3\nabla^2\phi$ in the effective action (2.2). As was explained in the previous section, all other terms, containing, e.g., the 3-form field strengths H_3 and F_3, are at least quadratic in the fields, and they cannot affect the equations of motion in leading-order perturbation theory.

By substituting the Weyl tensor for the metric (3.1) into the C^4 invariant W in (2.8) we have checked by a computer calculation that it vanishes. This means that the term $e^{-3\phi/2}W$ does not induce a dilaton tadpole. We further need to check that there is no dilaton tadpole induced by the terms of the form $e^{-3\phi/2}C^3\nabla^2\phi$. Luckily, such sources for the dilaton vanish as well. We will perform their calculation for the KT solution in Appendix B; the $AdS_5 \times T^{1,1}$ result may then be extracted in the limit $M \to 0$.

Let us note that the metric (3.1) can be brought by a Weyl transformation to the form

$$(ds_{10}^2)' = dx_n dx_n + dz^2 + z^2 ds^2_{T^{1,1}} \ , \tag{3.2}$$

which describes the direct product of the flat space $R^{3,1}$ and the conifold. The latter is a Ricci flat space of $SU(3)$ holonomy, i.e. its Weyl tensor has a vanishing eigenvalue. This fact is related to the vanishing of the invariant W (2.8).[7] Since W transforms under a Weyl transformation by an overall rescaling, its vanishing for the conifold implies its vanishing for $AdS_5 \times T^{1,1}$.

There are good reasons to believe that the conifold corresponds to an exact 2-d CFT, i.e. in contrast to generic CY spaces [30] it survives all α' corrections without deformation of its metric. Evidence for this is provided by the linear sigma model formulation which defines the conifold as an exact tree level string background [31]. Equipped with the explicit form of the leading higher-derivative correction to the effective action we can check a weaker statement: that the conifold survives a leading order perturbation in α'. We have already mentioned that W vanishes when evaluated on this background. It only remains to check that the same is true for $\delta W/\delta g_{mn}$, so that the Einstein equation continues to be satisfied in leading order perturbation theory. We have checked by a computer calculation that this is indeed true. A related statement is that the $E_6 = \epsilon_6\epsilon_6 RRR$ (6-d Euler density) correction [18,32] to the Kahler potential (and also to the dilaton) of the $\mathcal{N}=2$ sigma model for the conifold vanishes as well. Interestingly, this is no longer the case for the resolved and deformed conifolds.[8]

Similarly, we have checked that $\delta W/\delta g_{mn}$ vanishes for the $AdS_5 \times T^{1,1}$ space. This establishes that the leading perturbation in α' does not change the form of this background. A similar check for the $AdS_5 \times S^5$ background is trivial as there each factor of the Weyl tensor vanishes separately [25].[9] In the present $AdS_5 \times T^{1,1}$ case, the Weyl tensor is non-vanishing so that the direct confirmation of $\delta W/\delta g_{mn} = 0$ was necessary.

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[7] This is related to the fact that the on-shell $\mathcal{N}=1$, $D=10$ superinvariant [29,25] $\int d^{10}x d^{16}\theta\ \Phi^4 \to \int d^{10}x d^{16}\theta\ (\bar\theta\gamma^{mnk}\theta\bar\theta\gamma^{pq}{}_k\theta R_{mnpq})^4$ depends only on the Weyl tensor (because of the identity $\gamma^{mnk}\theta\bar\theta\gamma_{mnl}\theta \equiv 0$) and is proportional to W.

[8] While W still vanishes for the resolved and deformed conifolds (as, in fact, for any similar space of special holonomy), its variation and the cubic E_6 invariant do not, implying that the metric and dilaton receive α'^3-corrections (note that while the R^4 term in the dilaton equation vanishes, there is an extra correction term D^2R^3 that does not). One heuristic argument indicating why the singular conifold is not deformed by α'-corrections is based on the fact that the corresponding constrained sigma model $L = \frac{1}{4\pi\alpha'}[\partial^a z_i \partial_a z_i^* + (\Lambda z_i^2 + c.c)]$+fermions (here $i = 1, ..., 4$ and Λ is Lagrange multiplier field) has no intrinsic scale, i.e. is homogeneous and quadratic in z_i. Thus the dependence on α' can be absorbed into $z_i \to \sqrt{\alpha'} z_i$.

[9] A different argument for stability of $AdS_5 \times S^5$ based on its maximal supersymmetry was suggested in [33].

This conclusion is quite important from the point of view of the AdS/CFT correspondence. The $SU(N) \times SU(N)$ gauge theory dual to the type IIB $AdS_5 \times T^{1,1}$ background is conformal for all N, and for all values of the gauge couplings. The relation between the gauge couplings and the moduli of string theory is [1,2,4,9]

$$\frac{4\pi^2}{g_1^2} + \frac{4\pi^2}{g_2^2} = \frac{\pi}{g_s} e^{-\phi} , \tag{3.3}$$

$$\left(\frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2} \right) g_s e^{\phi} = \frac{1}{2\pi\alpha'} \int_{S^2} B_2 - \pi \quad (\mathrm{mod}\ 2\pi) . \tag{3.4}$$

If W were non-vanishing, then it would induce a radial variation of the dilaton which would have to be interpreted as being due to a non-vanishing beta function in the field theory. This would be in conflict with the vanishing of NSVZ beta functions for both gauge couplings. Thus, the fact that the $AdS_5 \times T^{1,1}$ background survives the leading perturbation in $\alpha'^3/L^6 \sim (g_s N)^{-3/2}$ is a new check of the correspondence with the field theory where $g_s N$ is a modulus of the CFT. We see that the AdS/CFT duality requires that all α' corrections and all string loop $(1/N)$ corrections vanish for this background. Proving this is quite a challenge. Perhaps the vanishing of all α' corrections for this metric is related to the ability to Weyl rescale it to the direct product (3.2) of $R^{3,1}$ and the conifold and use the fact that the conifold is an exact solution of string theory, as well as to the supersymmetry (with eight supercharges) of this background [28].

4. Fractional 3-branes on the conifold and RG flow of couplings

In this section we proceed to the more complicated case of the cascading theory [5,6] which, at the *bottom* of the cascade, has gauge group $SU(N+M) \times SU(N)$, $0 \le N < M$. For $N = 0$ the gravity dual of this theory is given by the solution of [6], while for $N > 0$ it is represented by an appropriate generalization which includes N additional D3-branes on the deformed conifold. In practice, we will only consider the asymptotic UV (large radius) form of these backgrounds derived in [5]; this asymptotic form encodes the logarithmic RG flow that may be compared with the NSVZ beta functions of the gauge theory.

First we review the solution of [5], complete with the normalization factors supplied in [9]. The 10-d metric is

$$ds_{10}^2 = h^{-1/2}(r) dx_n dx_n + h^{1/2}(r)(dr^2 + r^2 ds_{T^{1,1}}^2) , \tag{4.1}$$

where

$$h(r) = \frac{27\pi\alpha'^2}{4r^4}\ \left[g_s N + \frac{3}{2\pi}(g_s M)^2(\log(r/\tilde{r}) + \frac{1}{4})\right] \ . \tag{4.2}$$

The 5-form is given by

$$F_5 = dC_4 + B_2 \wedge F_3 = \mathcal{F}_5 + *\mathcal{F}_5 \ , \tag{4.3}$$

$$\mathcal{F}_5 = 27\pi\alpha'^2 \overline{N}_{\rm eff}(r) \text{vol}(T^{1,1}) \ , \tag{4.4}$$

where (cf. (4.2))

$$\overline{N}_{\rm eff}(r) \equiv N + \frac{3}{2\pi} g_s M^2 \log(r/\tilde{r}) \ . \tag{4.5}$$

The 3-form field strengths are determined by

$$F_3 = \frac{M\alpha'}{2}\omega_3 \ , \qquad\qquad B_2 = \frac{3g_s M\alpha'}{2}\omega_2 \log(r/\tilde{r}) \ , \tag{4.6}$$

$$H_3 = dB_2 = \frac{3g_s M\alpha'}{2r} dr \wedge \omega_2 \ ,$$

where ω_2 and ω_3 are the harmonic forms on $T^{1,1}$ given in [9]. This background [5] provides a nice illustration of the distinction between the gauge-invariant but not localized and not quantized charge defined by the generalized field strength,

$$\frac{1}{(2\pi)^4\alpha'^2}\int_{T^{1,1}} F_5 = \overline{N}_{\rm eff}(r) \ ,$$

and the quantized charge

$$\frac{1}{(2\pi)^4\alpha'^2}\int_{T^{1,1}} dC_4 = \frac{1}{(2\pi)^4\alpha'^2}\int_{T^{1,1}} (F_5 - B_2 \wedge F_3) = N \ . \tag{4.7}$$

The latter is the analog of the Page charge (see, e.g., [34]). It is quantized because a probe D3-brane couples directly to C_4; however, it is not invariant under the global gauge transformations of B_2 – it is defined modulo integer shifts, $N \to N + kM$. This is a reflection of the duality cascade jumps [6] and, in fact, is a general phenomenon in a system with different types of fluxes.

One can rewrite $h(r)$ in the form

$$h(r) = \frac{\text{L}^4}{r^4}\log(r/r_s), \qquad \text{L}^2 \equiv \frac{9g_s M\alpha'}{2\sqrt{2}} \ . \tag{4.8}$$

Performing the following change of coordinates

$$t = \log(r/r_s) \ , \qquad \tilde{x}_n = \frac{r_s}{\text{L}^2} x_n \ , \tag{4.9}$$

we can put the metric and B_2 into the form

$$ds_{10}^2 = \mathrm{L}^2 \big[\frac{\mathrm{e}^{2t}}{\sqrt{t}} d\tilde{x}_n d\tilde{x}_n + \sqrt{t}(dt^2 + ds_{T^{1,1}}^2)\big] \ , \tag{4.10}$$

$$B_2 = \frac{3}{2} g_s M \alpha' \left(t - \tilde{t}\right) \omega_2 \ , \tag{4.11}$$

where

$$\tilde{t} = \log(\tilde{r}/r_s) = \frac{2\pi N}{3 g_s M^2} + \frac{1}{4} \ . \tag{4.12}$$

Note that the scalar curvature is

$$R = \frac{2}{\mathrm{L}^2 t^{3/2}} \ ,$$

and no matter how small $\mathrm{L}^2/\alpha' \sim g_s M$ is, $\alpha' R$ can be made very small at large t. Thus, corrections to supergravity can be organized in inverse powers of $g_s M$ and t.

For $N = 0$ we recover the asymptotic large distance (τ) form of the KS solution [6]; in this limit $\tau \to 3t + \frac{1}{4}$. Note that $\overline{N}_{\text{eff}}(t)$ can be written in terms of L and g_s as

$$\overline{N}_{\text{eff}}(t) = \frac{4\mathrm{L}^4}{27\pi g_s \alpha'^2} \left(t - \frac{1}{4}\right) = \frac{3 g_s M^2}{2\pi} \left(t - \frac{1}{4}\right) \ . \tag{4.13}$$

Since F_3 and H_3 can be also expressed in terms of L (or M) and g_s, and $\frac{1}{2\pi\alpha'}\int_{S^2} B_2$ is an angular variable that takes values in the interval $[0, 2\pi]$, the solution has no explicit dependence on N.

The gravitational background describes the RG cascade of the $SU(N + (k+1)M) \times SU(N + kM) \equiv SU(N_{\text{eff}} + M) \times SU(N_{\text{eff}})$ gauge theories. The $SU(N_{\text{eff}} + M) \times SU(N_{\text{eff}})$ theory has the following symmetry

$$M \to -M \ , \quad N_{\text{eff}} \to N_{\text{eff}} + M \ , \qquad g_1 \to g_2 \ , \quad g_2 \to g_1 \ . \tag{4.14}$$

The combination

$$\overline{N} = N_{\text{eff}} + \frac{M}{2} \tag{4.15}$$

is invariant under this transformation, and, therefore, it is natural to organize the expansion in powers of $1/(g_s \overline{N})$ and $M/\overline{N}$. Let us define the point $\bar{t}$ by the equation

$$\overline{N}_{\text{eff}}(\bar{t}) = N + \frac{M}{2} \ , \qquad \bar{t} = \tilde{t} + \frac{\pi}{3 g_s M} = \frac{2\pi}{3 g_s M^2}\left(N + \frac{M}{2}\right) + \frac{1}{4} \ . \tag{4.16}$$

Then in the vicinity of the point $t_{\text{eff}} = \bar{t} + k\frac{2\pi}{3 g_s M}$ such that

$$\overline{N}_{\text{eff}}(t_{\text{eff}}) = N + kM + \frac{M}{2} = N_{\text{eff}} + \frac{M}{2} = \overline{N} \ , \tag{4.17}$$

the gravity background describes the $SU(N_{\rm eff}+M)\times SU(N_{\rm eff})$ gauge theory.[10] The background field strengths (curvatures) are small provided that $g_s\overline{N}\gg 1$.

Following [4,6,9] we will now use the relations (3.3),(3.4) to extract the supergravity prediction for the scale dependence of the gauge couplings. Thus, we assume that (3.3),(3.4) are valid not only for constant dilaton and B_2, but also when they are radially varying. Substituting the B_2 from (4.6) into (3.4) we find that [9]

$$\frac{4\pi^2}{g_1^2}-\frac{4\pi^2}{g_2^2}=3M(t-\bar{t})\quad(\text{mod }\frac{2\pi}{g_s})\ ,\qquad\qquad \frac{4\pi^2}{g_1^2}+\frac{4\pi^2}{g_2^2}=\frac{\pi}{g_s}\ . \tag{4.18}$$

Thus, any point $t_{\rm eff}$, in particular, $\bar{t}$, is also characterized by the requirement that the running gauge couplings $g_1(t)$ and $g_2(t)$ obey $g_1(t_{\rm eff})=g_2(t_{\rm eff})$. In fact, due to the symmetry (4.14) even if we include all possible corrections to the dilaton and B_2, there always exists such a point $t_{\rm eff}$ that $g_1(t_{\rm eff})=g_2(t_{\rm eff})$. It is not difficult to check by using (4.18) that at the point $t_1=t_{\rm eff}-\frac{\pi}{3g_sM}$ the gauge coupling g_1 of the bigger gauge group diverges, and that at the point $t_2=t_{\rm eff}+\frac{\pi}{3g_sM}$ the gauge coupling g_2 of the smaller gauge group does. These are the points where the RG cascade jumps occur.

Let us emphasize that the supergravity description is valid for large $g_sN_{\rm eff}$ even if g_sM is very small. The separation between the cascade steps is $\Delta t=\frac{2\pi}{3g_sM}$. Thus, there is a range of parameters where the cascade jumps are far from each other, and the supergravity calculation of the β-function (4.18) may be compared with the NSVZ β-functions for the $SU(N_{\rm eff}+M)\times SU(N_{\rm eff})$ theory:[11]

$$\frac{d}{d\log(\Lambda/\mu)}\left(\frac{4\pi^2}{g_1^2}+\frac{4\pi^2}{g_2^2}\right)=2\overline{N}\Delta\ , \tag{4.19}$$

[10] With our definitions, $N_{\rm eff}$ is automatically an integer. On the other hand, $\overline{N}_{\rm eff}(r)$ in (4.5) continuously varies with r. One may wonder how this continuous variation is consistent with the statement that the number of colors makes discrete jumps only at certain radii. We believe that $\overline{N}_{\rm eff}(r)$ can actually be interpreted as a measure of the number of degrees of freedom for all r. This is supported, for example, by the smooth temperature dependence of the Bekenstein-Hawking entropy for black holes embedded in the asymptotically KT geometry [35]. The logarithmic scale dependence of the effective number of colors in between the cascade jumps is presumably due to the interaction effects in the $SU(N_{\rm eff}+M)\times SU(N_{\rm eff})$ gauge theory. It would be interesting to study it directly in the gauge theory.

[11] Here by the NSVZ β-functions we actually mean the SV [8] β-functions for the holomorphic gauge couplings which differ from the NSVZ ones by the absense of the denominator factor (this point was implicit in [4] and was already mentioned [9]). Indeed, in comparing to the dual string/supergravity description one should be using Wilsonian effective action on the gauge theory side. With the "holomorphic" definition of gauge coupling as a factor in front of the $\mathcal{N}=1$

$$\frac{d}{d\log(\Lambda/\mu)}\left(\frac{4\pi^2}{g_1^2}-\frac{4\pi^2}{g_2^2}\right)=3M-M\Delta\ , \tag{4.20}$$

where Δ is the correction to the anomalous dimension γ of the operators $\mathrm{Tr}A_iB_j$:

$$\gamma=-\frac{1}{2}+\Delta\ .$$

One expects Δ to scale as some even positive power of $M/\overline{N}$ [6]. Therefore, far in the UV where $\overline{N}$ becomes large due to the cascading phenomenon, Δ naturally approaches zero. Hence, the dominant running of the couplings in the UV is given by

$$\frac{4\pi^2}{g_1^2}-\frac{4\pi^2}{g_2^2}\to 3M\log(\Lambda/\mu)\ ,\qquad \frac{4\pi^2}{g_1^2}+\frac{4\pi^2}{g_2^2}\to \mathrm{const}\ . \tag{4.21}$$

If we identify $t-t_{\rm eff}$ with $\log(\Lambda/\mu)$ then the supergravity result (4.18) is in perfect agreement with the field theory expectations.

The main purpose of this paper is to find out where the effects of Δ are encoded on the string theory side. We claim that these effects are provided precisely by the string higher derivative corrections to the supergravity action. In what follows we show that already the leading such correction makes the dilaton radius-dependent, in agreement with (4.19) which implies that $\frac{4\pi^2}{g_1^2}+\frac{4\pi^2}{g_2^2}$ runs due to the presence of Δ.

5. String correction to the KT solution and RG Flow

To determine the leading α'^3 correction to the dilaton we have to take into account possible mixing between linearized fluctuations of all of the relevant supergravity modes. In particular, we should expect that not only the dilaton but also the *2-form* B_2 gets correction at this order.

To find the mixing we use the following ansatz for the deformed metric, B_2 and dilaton:

$$ds_{10}^2=\mathrm{L}^2\left(\frac{\mathrm{e}^{2t}}{\sqrt{t}}\mathrm{e}^{2z}d\tilde{x}_n^2+\sqrt{t}\mathrm{e}^{-2z}\left[\mathrm{e}^{10y}dt^2+\mathrm{e}^{2y-8w}e_\psi^2+\mathrm{e}^{2y+2w}\left(e_{\theta_1}^2+e_{\theta_2}^2+e_{\phi_1}^2+e_{\phi_2}^2\right)\right]\right)\ , \tag{5.1}$$

supersymmetric kinetic term the corresponding β-function is effectively "one-loop" [8], i.e. does not have the denominator factor. This definition of the coupling is indeed consistent with the one based on D-brane probe action and interpreted as a quantum gauge theory effective action defined at some fixed scale. Let us mention also that the original expression [7],[8] for the NSVZ or SV β-functions does apply to the case of the non-semisimple gauge group like the one in the present example. We are grateful to M. Shifman for a clarifying discussion of these points.

$$B_2 = \frac{3}{2} g_s M \alpha' [t + b(t)] \omega_2 \ , \qquad \phi = \phi(t) \ . \tag{5.2}$$

Here the basis e_i is the same as in [35]. The functions z, y, w, b, ϕ depending only on the radial coordinate t represent the relevant fluctuations around the KT solution. This ansatz preserves the symmetry between the two spheres.

A few explanatory comments are in order. It may seem that the choice of the metric in (5.1) is too special. In general, one may always make a choice of radial coordinate u so that a metric with required symmetries will be

$$ds^2_{10} = \mathrm{e}^{2\hat{z}} d\tilde{x}^2_n + \mathrm{e}^{-2\hat{z}} [\mathrm{e}^{10\hat{y}} du^2 + \mathrm{e}^{2\hat{y}-8\hat{w}} e^2_\psi + \mathrm{e}^{2\hat{y}+2\hat{w}} \left(e^2_{\theta_1} + e^2_{\theta_2} + e^2_{\phi_1} + e^2_{\phi_2} \right)] \ . \tag{5.3}$$

Here the three functions will then have expansion in powers of α'. The metric (5.1) is obtained from (5.3) by extracting the leading-order solution for these functions and redefining $u \to t$. This implies that z, y, w in (5.1) should start with α'^3 correction terms.

We are assuming that F_3 is not modified and F_5 is expressed through B_2 and F_3 by the standard formula (4.3). Indeed, it is easy to see that since F_3 has purely magnetic form, the mixed "Weyl tensor – R-R 3-form" term $C^2 (\nabla F_3)^2$ in the action (2.2),(2.3),(2.9) does not modify the equation for C_2 (the metric is diagonal and has non-trivial dependence on t only). The same should be true for the terms depending on F_5.

To find the linearized equations of motion for the fluctuations we need to find the quadratic action for the fluctuations which follows from the type IIB supergravity action plus terms linear in fluctuations which follow from the α'^3 string correction, i.e. from S_8 (2.2). The computation of the leading quadratic term in the supergravity Lagrangian (2.1) is straightforward (prime denotes derivative with respect to t)

$$\begin{aligned} \mathcal{L}_2 = -\frac{\mathrm{L}^8 \mathrm{e}^{4t}}{2} \Big[& \phi'^2 + \frac{4}{t} b'^2 + 40\ w'^2 \ + 16\ z'^2 \ - 40\ y'^2 - \frac{8}{t} b' \ (\phi + 4w + 4y - 4z) \\ & + \frac{16}{t^2} b^2 + \frac{4}{t} \phi^2 + \frac{32}{t} \phi\ w + \frac{64}{t} w^2 + 480\ w^2 + \frac{32}{t} \phi\ y + \frac{128}{t} w\ y \\ & + \frac{64}{t} y^2 - 1280\ y^2 - \frac{64}{t^2} b\ z + \frac{256}{t} b\ z + \frac{32}{t^2} z^2 - \frac{192}{t} z^2 + 512\ z^2 \Big] \ . \end{aligned} \tag{5.4}$$

Adding the α'^3 correction term $S_8 \equiv \frac{1}{2\kappa^2} \bar{S}_8$ in (2.2) and taking the variational derivative of the action with respect to the fields, we derive the following equations of motion for small perturbations:

$$\begin{aligned} & \frac{8 \mathrm{L}^8 \mathrm{e}^{4t}}{t} \Big(\frac{1}{2} b'' + 2b' - \frac{2}{t} b' - \frac{2}{t} b - \frac{1}{2} \phi' + \frac{1}{2t} \phi - 2\phi - 2w' \\ & - 8w + \frac{2}{t} w - 2y' - 8y + \frac{2}{t} y + 2z' - 8z + \frac{2}{t} z \Big) + \frac{\delta \bar{S}_8}{\delta b} = 0 \ , \end{aligned} \tag{5.5}$$

$$\mathrm{L}^8\mathrm{e}^{4t}\Big(\phi''+4\phi'-\frac{4}{t}\phi+\frac{4}{t}b'-\frac{16}{t}y-\frac{16}{t}w\Big)+\frac{\delta\bar{S}_8}{\delta\phi}=0\ , \tag{5.6}$$

$$8\mathrm{L}^8\mathrm{e}^{4t}\Big(-5y''-20y'+160y-\frac{8}{t}y+\frac{2}{t}b'-\frac{2}{t}\phi-\frac{8}{t}w\Big)+\frac{\delta\bar{S}_8}{\delta y}=0\ , \tag{5.7}$$

$$16\mathrm{L}^8\mathrm{e}^{4t}\Big(z''+4z'-32z+\frac{12}{t}z-\frac{2}{t^2}z-\frac{1}{t}b'-\frac{8}{t}b+\frac{2}{t^2}b\Big)+\frac{\delta\bar{S}_8}{\delta z}=0\ , \tag{5.8}$$

$$8\mathrm{L}^8\mathrm{e}^{4t}\Big(5w''+20w'-60w-\frac{8}{t}w+\frac{2}{t}b'-\frac{2}{t}\phi-\frac{8}{t}y\Big)+\frac{\delta\bar{S}_8}{\delta w}=0\ . \tag{5.9}$$

Computing the α'^3 terms to linear order in the fluctuations, we find that all of the variational derivatives have the following leading behavior at large t:

$$\begin{gathered}\frac{\delta\bar{S}_8}{\delta\phi}=8\mathrm{L}^8\mathrm{e}^{4t}\frac{D_\phi}{t^{\frac{7}{2}}}\ ,\qquad \frac{\delta\bar{S}_8}{\delta b}=8\mathrm{L}^8\mathrm{e}^{4t}\frac{D_b}{t^{\frac{7}{2}}}\ ,\\ \frac{\delta\bar{S}_8}{\delta y}=8\mathrm{L}^8\mathrm{e}^{4t}\frac{D_y}{t^{\frac{5}{2}}}\ ,\qquad \frac{\delta\bar{S}_8}{\delta z}=8\mathrm{L}^8\mathrm{e}^{4t}\frac{D_z}{t^{\frac{5}{2}}}\ ,\qquad \frac{\delta\bar{S}_8}{\delta w}=8\mathrm{L}^8\mathrm{e}^{4t}\frac{D_w}{t^{\frac{5}{2}}}\ ,\end{gathered} \tag{5.10}$$

where D_{φ_i} are some coefficients proportional to α'^3/L^6 whose exact values are unknown due to the lack of information about the terms involving the 5-form F_5. We present a detailed analysis of these α'^3 corrections in Appendix B.

Comparing this behavior with the equations of motion (5.5)–(5.9), we find that the leading asymptotics are

$$b\sim\frac{b_0}{t^{\frac{3}{2}}},\qquad \phi\sim\frac{\phi_0}{t^{\frac{5}{2}}},\qquad z\sim\frac{z_0}{t^{\frac{5}{2}}},\qquad y\sim\frac{y_0}{t^{\frac{5}{2}}},\qquad w\sim\frac{w_0}{t^{\frac{5}{2}}}\ . \tag{5.11}$$

Let us pause and note that these asymptotics are the minimal ones compatible with the AdS/CFT correspondence between string theory on the $AdS_5\times T^{1,1}$ background and the $SU(N)\times SU(N)$ gauge theory. Indeed, the $AdS_5\times T^{1,1}$ background (3.1) can be obtained from the KT one (4.10)–(4.6) in the limit $M\to 0$ or, equivalently, $\mathrm{L}\to 0$. More precisely, one should first redefine the variable t as

$$t=\frac{2\pi N}{3g_sM^2}+\hat{t}=\frac{\alpha'^2}{\mathrm{L}^4}\frac{27\pi g_sN}{4}+\hat{t}\ , \tag{5.12}$$

and keep $\hat{t}$ fixed in the limit. Taking into account that the fluctuations are proportional to α'^3/L^6, we find that all of them except the function b vanish in the limit $\mathrm{L}\to 0$. While b itself goes to a constant, the 2-form B_2 vanishes in this limit because of the extra factor of M in (5.2). Thus the $AdS_5\times T^{1,1}$ background is not modified, in agreement with the discussion in Section 3. Note that if some of the fluctuations (other than b) would scale as $t^{-3/2}$, the $AdS_5 \times T^{1,1}$ background would acquire α'^3 corrections.

Substituting the asymptotics (5.11) into the equations of motion (5.5)–(5.9), we obtain

$$-5b_0 - 2\phi_0 - 8(z_0 + y_0 + w_0) = -D_b \ , \tag{5.13}$$

$$-\frac{3}{4}b_0 - \frac{7}{4}\phi_0 - 2w_0 - 2y_0 = -D_\phi \ , \tag{5.14}$$

$$-16b_0 - 64z_0 = -D_z \ , \tag{5.15}$$

$$y_0 = -\frac{D_y}{160} \ , \qquad w_0 = \frac{D_w}{60} \ . \tag{5.16}$$

Solving eqs. (5.13)–(5.15), we find

$$\phi_0 = \frac{4D_\phi}{5} - \frac{D_b}{5} + \frac{D_z}{40} \ , \tag{5.17}$$

$$b_0 = \frac{7D_b}{15} - \frac{8D_\phi}{15} - \frac{2D_w}{45} - \frac{7D_z}{120} + \frac{D_y}{60} \ , \tag{5.18}$$

$$z_0 = \frac{29D_z}{960} - \frac{7D_b}{60} + \frac{2D_\phi}{15} + \frac{D_w}{90} - \frac{D_y}{240} \ . \tag{5.19}$$

Let us now use the results for ϕ and b to compute the correction Δ to the anomalous dimension $\gamma = -\frac{1}{2} + \Delta$ which enters the beta functions (4.19),(4.20). Substituting (5.2) into (3.4), we get

$$\frac{4\pi^2}{g_1^2} + \frac{4\pi^2}{g_2^2} = \frac{\pi}{g_s} \mathrm{e}^{-\phi(t)} \ , \tag{5.20}$$

$$\frac{4\pi^2}{g_1^2} - \frac{4\pi^2}{g_2^2} = \mathrm{e}^{-\phi(t)} \left(3M \left[t - \bar{t} + b(t) \right] \quad (\mathrm{mod}\ \frac{2\pi}{g_s}) \right) . \tag{5.21}$$

To compute the beta functions we differentiate (5.20) and (5.21) over t. Comparing to (4.19) we get

$$\Delta = -\frac{\pi \phi'}{2g_s \overline{N}} \mathrm{e}^{-\phi}|_{t=t_{\mathrm{eff}}} = -\frac{2\pi^2 \phi'}{\lambda} \mathrm{e}^{-\phi}|_{t=t_{\mathrm{eff}}} \ , \qquad \lambda = 4\pi g_s \overline{N} \ . \tag{5.22}$$

This formula may also be written as

$$\Delta = -\frac{2\pi^2 \phi'}{\lambda_{\mathrm{eff}}} \ , \qquad \lambda_{\mathrm{eff}} = \lambda \mathrm{e}^{\phi}|_{t=t_{\mathrm{eff}}} \ . \tag{5.23}$$

Another expression for Δ is found from (4.20):

$$\Delta = 3\mathrm{e}^{-\phi} \left(\mathrm{e}^{\phi} - 1 - b' + b\phi' \right) |_{t=t_{\mathrm{eff}}} \ . \tag{5.24}$$

To analyze these expressions note that

$$\frac{\alpha'}{\mathrm{L}^2} = \frac{2\sqrt{2}}{9g_s M} = \frac{8\pi\sqrt{2}}{9\lambda}\left(\frac{\overline{N}}{M}\right) , \qquad \frac{1}{t_{\rm eff}} \sim \frac{3g_s M^2}{2\pi\overline{N}} = \frac{3\lambda}{8\pi^2}\left(\frac{M}{\overline{N}}\right)^2 . \tag{5.25}$$

Since according to (5.11), $\phi \sim \phi_0 t^{-5/2},\ b \sim b_0 t^{-3/2}$, where ϕ_0 and b_0 are proportional to α'^3/L^6, we obtain

$$\phi \sim b' \sim \frac{\alpha'^3}{\mathrm{L}^6 t_{\rm eff}^{\frac{5}{2}}} \sim \frac{1}{\lambda^{\frac{1}{2}}}\left(\frac{M}{\overline{N}}\right)^2 , \qquad \phi' \sim \frac{\alpha'^3}{\mathrm{L}^6 t_{\rm eff}^{\frac{7}{2}}} \sim \lambda^{\frac{1}{2}}\left(\frac{M}{\overline{N}}\right)^4 . \tag{5.26}$$

Thus, by using (5.22), we conclude that at large $t_{\rm eff}$ and at order α'^3

$$\Delta \sim \frac{1}{\lambda^{\frac{1}{2}}}\left(\frac{M}{\overline{N}}\right)^4 . \tag{5.27}$$

On the other hand, naively, eq.(5.24) seems to give a different expression $\Delta \sim \frac{1}{\lambda^{\frac{1}{2}}}\left(\frac{M}{\overline{N}}\right)^2$. A way to avoid contradiction is to assume that at order α'^3 the fluctuations b and ϕ satisfy the constraint

$$b'(t) = \phi(t) . \tag{5.28}$$

Then eq. (5.24) would imply the absence of corrections to this order. However, there are also higher order α'^5 corrections to the dilaton and B_2, and it is easy to see that assuming that the α'^5 corrections scale at large t in such a way that

$$b' - \phi \sim \frac{\alpha'^5}{t^{\frac{9}{2}}} , \tag{5.29}$$

one obtains from (5.24) a correction Δ with the same dependence on λ and $M/\overline{N}$ as in (5.27).

Working to leading order in $M/\overline{N}$ we may replace λ in (5.27) by $\lambda_{\rm eff}$. This latter expression is convenient because (5.20) directly relates $\lambda_{\rm eff}$ to the gauge couplings. Thus, we arrive at

$$\Delta = a_1 \left(\frac{M}{\overline{N}}\right)^4 \left(\frac{4\pi^2}{\overline{N}g_1^2} + \frac{4\pi^2}{\overline{N}g_2^2}\right)^{1/2} , \tag{5.30}$$

where a_1 is proportional to c_1 in (2.2) or $\zeta(3)$. With this expression for Δ, the β-function equation (4.19) assumes the form

$$\frac{d}{d\log(\Lambda/\mu)}\left(\frac{4\pi^2}{\overline{N}g_1^2} + \frac{4\pi^2}{\overline{N}g_2^2}\right)^{1/2} = a_1\left(\frac{M}{\overline{N}}\right)^4 . \tag{5.31}$$

Its solution is

$$\left(\frac{4\pi^2}{\overline{N}g_1^2}+\frac{4\pi^2}{\overline{N}g_2^2}\right)^{1/2}=2\pi\lambda_{\rm eff}^{-1/2}+a_1\left(\frac{M}{\overline{N}}\right)^4\log(\Lambda/\mu)\ . \tag{5.32}$$

Substituting this back into (5.30) we find

$$\Delta=2\pi a_1\left(\frac{M}{\overline{N}}\right)^4\lambda_{\rm eff}^{-1/2}+O[(M/\overline{N})^8\log(\Lambda/\mu)]\ . \tag{5.33}$$

Finally, substituting this into (4.20) and integrating we get

$$\frac{4\pi^2}{g_1^2}-\frac{4\pi^2}{g_2^2}=3M\log(\Lambda/\mu)\left(1-\frac{2}{3}\pi a_1\left(\frac{M}{\overline{N}}\right)^4\lambda_{\rm eff}^{-1/2}+O[(M/\overline{N})^8\log(\Lambda/\mu)]\right)\ . \tag{5.34}$$

It is remarkable that the solutions of the NSVZ equations including the effects of non-zero Δ have an expansion in powers of $\log(\Lambda/\mu)$. From the point of view of dual string theory this property of RG flow is guaranteed by the relations (5.20),(5.21), and by the fact that $t-t_{\rm eff}$ has to be identified with $\log(\Lambda/\mu)$. Moreover, the choice of the expansion point does not change the logarithmic character of the expansion. A closely related observation is that changing the variable according to (5.12), and expanding in small M, or, equivalently, in small $\hat{t}$ (i.e. expanding near $AdS_5\times T^{1,1}$ background dual to the conformal fixed point of [1]) we find that the α' corrections and the solutions of the resulting effective equations of motion can be represented in terms of power series in the logarithmic scaling variable $\hat{t}$. It would be very interesting to demonstrate the existence of higher powers of $\log(\Lambda/\mu)$ without invoking the gauge field/string duality.

6. Concluding Remarks

In this paper we studied the leading stringy corrections to two different solutions of type IIB supergravity, interpreting our results in terms of dual gauge theories. We were able to use the gauge/gravity duality in both directions, sometimes using constraints from field theory as a way of predicting properties of the string effective action.

For the $AdS_5\times T^{1,1}$ background we demonstrated that the $O(\alpha'^3)$ correction does not modify the form of the solution, in line with expectations from AdS/CFT duality. In fact, turning the duality around, we predicted a much stronger result: that the $AdS_5\times T^{1,1}$ background is exact to all orders in α' and g_s. We suspect that this property is tied to the exactness of the conifold background.

Our paper also contains a much more ambitious calculation of leading string-theoretic corrections to the solution of [5], which describes logarithmic RG flow and duality cascade in $\mathcal{N}=1$ supersymmetric $SU(N+M)\times SU(N)$ gauge theory. This calculation is made especially difficult by the fact that the complete structure of the $O(\alpha'^3)$ correction in the (tree-level) type IIB superstring effective action is not yet known. Nevertheless, we are able to demonstrate an interesting interplay between the NSVZ β-functions in field theory and the structure of stringy corrections. In fact, we may again turn the gauge/gravity duality around and use field theory to predict certain facts about the string theory effective action. The most basic fact is the absence from the effective action of terms which scale as $1/t^3$ at large t. If such a term were present, then agreement with the gauge/gravity duality would fail for this theory. We were able to check this prediction of gauge theory for string theory for all terms in the effective action from the NS-NS sector.

There were also more subtle predictions that we were unable to check completely, such as the relation (5.28) at order α'^3. The latter implies a relation between the coefficients D_{φ_i} in (5.10): $D_b=\frac{2}{15}D_w-\frac{1}{20}D_y+\frac{1}{8}D_z$. We concluded also that corrections to $\phi-b'$ at the order α'^5 should be completely determined by corrections to ϕ at order α'^3. We expect these relations to be consequences of the special structure of the string α'^3 correction to the effective action dictated by supersymmetry. [12]

Assuming that these properties hold, we are able to make a prediction for the correction Δ to the anomalous dimension of $\mathrm{Tr}(A_iB_j)$ in the gauge theory. The result is consistent with the expectation [6] that in field theory Δ must have an expansion in powers of $(M/\overline{N})^2$: our supergravity analysis suggests that the leading term has the form (5.30). The normalization factor a_1 in this formula is proportional to $\zeta(3)$. It would be very interesting, but probably hard, to understand this string-theory prediction directly on the gauge theory side.

Some methods developed in this paper may have applications in other related contexts. One may perform a similar study of leading α' corrections to the solutions considered in [36]. Our result (in Appendix A) about the absence of certain $R^3(H_3)^2$ terms in the $(R^4+...)$ superinvariant in type IIB 10-dimensional effective action may imply certain constraints on possible $R^3(F_4)^2$ terms in the 11-dimensional effective action, and this may

[12] The fact that there is only one anomalous dimension that enters the two NSVZ beta functions (r.h.s. of (4.19) and (4.20)) implies that there is a specific combination of the two gauge couplings (or of the dilaton and 2-form B_2) whose dependence on the scale (or r) is known exactly to all orders in the M/N expansion.

be important in the context of the discussions in [37,38]. Thus, a complete determination of the 8-derivative corrections to the type II supergravity action should have many useful applications.

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Appendix A. R^3H^2 terms in type II tree level effective action

The tree-level effective action of type IIB and type IIA theories is the same in the NS-NS (metric, B_2 and dilaton) sector. This action should be parity-even, and, apart from the central charge term, should have the universal dimension-independent structure.[13] Using sigma-model considerations, it is easy to see that the effective action should be even in H_{mnk}, i.e. it may contain terms of the form R^3H^2, R^2H^4, etc. Our aim is to study the possible presence of the R^3H^2 terms in the effective action. They may a priori accompany the R^4 terms on dimensional and (non-linear) $D = 10$ supersymmetry grounds.

In this Appendix we show that certain irreducible 5-point terms of the form R^nH^{5-n} are absent from the type II superstring effective action.[14] We shall directly compute the

[13] The corresponding NSR sigma model can be formulated in any dimension D and thus its beta-functions must have dimension-independent universal coefficients (in a standard dimensional regularization with minimal subtraction scheme). This is no longer so once we include the dependence on the R-R fields which are sensitive to $D = 10$.

[14] Here we call a term reducible if it can be represented as a sum of terms of the form $R^m(\nabla H)^n$ by using $[\nabla,\nabla]H \sim RH$. We shall consider R^nH^{5-n} for general $n = 1, ..., 5$ with understanding that, in view of the above remarks, the only potentially non-trivial cases are $n = 1, 3$ (the R^5-terms, i.e. $n = 0$, are absent as suggested by the result of [39] and also by the $\mathcal{N} = 2, D = 10$ supersymmetry).

corresponding 5-point terms in the effective action using the Green-Schwarz light-cone formulation of the superstring S-matrix [40].

Since we shall use the light-cone gauge approach in its standard most straightforward form, i.e. assuming that the components of the polarization tensors vanish in the two light-cone directions, we shall not be able to determine a special class of terms that involve antisymmetrization of *nine* (or more) space-time indices, e.g., $\epsilon_{km_1...m_9}\epsilon^{kn_1...n_9}R^{m_1m_2}_{n_1n_2}R^{m_3m_4}_{n_3n_4}R^{m_5m_6}_{n_5n_6}H^{m_7}{}_{n_7n_8}H_{n_9}{}^{m_8m_9}$.[15] However, it is possible to check that precisely because such terms involve antisymmetrization of nine indices, they do not change our conclusion about the $1/t^4$ scaling of the leading α'^3 corrections to our background.

In the notation of [41] the light-cone gauge vertex operators for the graviton h_{ij} and the NS-NS 2-form b_{ij} can be written as $(i, j, ... = 1, ..., 8)$

$$V(h) = \left(h_{ik}\dot{X}^i_L\dot{X}^k_R - \frac{i}{4}\Gamma_{ik,l}\dot{X}^i_L\tilde{S}\gamma^{kl}\tilde{S} - \frac{i}{4}\Gamma_{ki,j}\dot{X}^k_R S\gamma^{ij}S - \frac{1}{32}R_{ijkl}S\gamma^{ij}S\tilde{S}\gamma^{kl}\tilde{S}\right)\mathrm{e}^{ipX}, \tag{A.1}$$

$$V(b) = \left(b_{ij}\dot{X}^i_L\dot{X}^j_R + \frac{i}{8}H_{ikl}\dot{X}^i_L\tilde{S}\gamma^{kl}\tilde{S} + \frac{i}{8}H_{kij}\dot{X}^k_R S\gamma^{ij}S + \frac{i}{32}p_l H_{ijk}S\gamma^{ij}S\tilde{S}\gamma^{kl}\tilde{S}\right)\mathrm{e}^{ipX}. \tag{A.2}$$

Here $\Gamma_{ij,k}(p) = \frac{i}{2}(p_i h_{jk}(p) + p_j h_{ik}(p) - p_k h_{ij}(p))$ is a linearized Christoffel symbol, $R_{ijkl}(p) = -\frac{1}{2}(p_i p_l h_{jk}(p) - p_j p_l h_{ik}(p) + p_j p_k h_{il}(p) - p_i p_k h_{jl}(p))$ is a linearized Riemann tensor, and $H_{ijk}(p) = i(p_i b_{jk}(p) - p_j b_{ik}(p) + p_k b_{ij}(p))$ is the NS-NS 3-form. We also denote $\dot{X}^i_L \equiv \partial_- X^i_L$, $\dot{X}^k_R \equiv \partial_+ X^k_R$, and $\partial_\pm$ are derivatives over the world-sheet directions.

Our aim is to study the structure of some tree-level 5-point massless scattering amplitudes involving the graviton and 2-form field. Following the logic similar to the one in [14], we may use a short-cut argument. Due to the supersymmetry and the expected $SL(2,\mathbf{Z})$ invariance of the "massless" type IIB superstring effective action, for $\alpha'^3(R^4 + ...)$ terms in the tree-level effective action there should exist terms of the same structure in the one-loop effective action [42]. To get a nonzero one-loop amplitude one should saturate the integral over the fermionic zero modes – 8 modes S_0, and 8 modes $\tilde{S}_0$. Thus, the 5-point amplitude

[15] We are grateful to K. Peeters, P. Vanhove and A. Westerberg for pointing out to us the possible presence of such term in the (one-loop) type II effective action, as well as of the term $t_8t_8R^3H^2$ which we missed in the first version of this paper [10].

has to be given by a sum of the following terms

$$
\begin{aligned}
A_5 \sim\, & V_{i_1j_1k_1l_1}V_{i_2j_2k_2l_2}V_{i_3j_3k_3l_3}V_{i_4j_4k_4l_4}V_{i_5j_5k_5l_5}(t)_{10}^{i_1j_1\ldots i_5j_5}(t)_{10}^{k_1l_1\ldots k_5l_5}\int dp\mathrm{Tr}\left(\mathrm{e}^{ip_aX_a}w^{L_0}\bar{w}^{\tilde{L}_0}\right) \\
&+ V_{i_1j_1k_1l_1}V_{i_2j_2k_2l_2}V_{i_3j_3k_3l_3}V_{i_4j_4k_4l_4}V_{i_5k_5}(t)_8^{i_1j_1\ldots i_4j_4}(t)_8^{k_1l_1\ldots k_4l_4}\int dp\mathrm{Tr}\left(\dot{X}_L^{i_5}\dot{X}_R^{k_5}\mathrm{e}^{ip_aX_a}w^{L_0}\bar{w}^{\tilde{L}_0}\right) \\
&+ V_{i_1j_1k_1l_1}V_{i_2j_2k_2l_2}V_{i_3j_3k_3l_3}V_{i_5k_4l_4}V_{k_5i_4j_4}(t)_8^{i_1j_1\ldots i_4j_4}(t)_8^{k_1l_1\ldots k_4l_4}\int dp\mathrm{Tr}\left(\dot{X}_L^{i_5}\dot{X}_R^{k_5}\mathrm{e}^{ip_aX_a}w^{L_0}\bar{w}^{\tilde{L}_0}\right)
\end{aligned} \tag{A.3}
$$

Here V_{ijkl} is either R_{ijkl} or p_lH_{ijk}, V_{ikl} is either $\Gamma_{ik,l}$ or H_{ikl}, and V_{ik} is either h_{ik} or b_{ik}. The tensor $(t)_{2m}$ is defined as [41]

$$
(t)_{2m}^{i_1j_1\ldots i_mj_m} = 4^{-m}\mathrm{tr}\left(S_0\gamma^{i_1j_1}S_0\cdots S_0\gamma^{i_mj_m}S_0\right)\ , \tag{A.4}
$$

where the trace is over the spinor zero modes of S. One can show [41] that $(t)_8 = t_8 - \frac{1}{2}\epsilon_8$, where t_8 is given by the sum of products of the δ-symbols.

Since the light-cone set-up is essentially 8-dimensional, it does not actually allow one to determine the presence of, e.g., the $\epsilon_8\epsilon_8 RRRR$ term in the action directly, since this term is a total derivative in 8 dimensions to all orders in the graviton expansion. In general, fixing the 8-derivative 5-point terms depending on ϵ_8, i.e. $\epsilon_8\epsilon_8R^3H^2$ or $\epsilon_8t_8R^3H^2$, is subtle in the light-cone formulation (in particular, because of possible contact terms needed for space-time supersymmetry [43]). For this reason, we will only determine the terms with the t_8t_8 tensor structure.

It is understood that the trace of the operators $\mathrm{e}^{ip_aX_a}w^{L_0}\bar{w}^{\tilde{L}_0}$ and $\dot{X}_L^i\dot{X}_R^k\mathrm{e}^{ip_aX_a}w^{L_0}\bar{w}^{\tilde{L}_0}$ is taken over the nonzero modes of $X_L, X_R, S, \tilde{S}$. The integration over the positions of the 5 vertex operators and the modulus parameter w is implied. The integral over the zero mode momentum p can be regarded as the trace over the zero modes. It is because of this integral that the closed string amplitude can not be treated simply as a product of the two open string ones.

It is clear that the first term in (A.3) leads to terms of the form $R^n(\nabla H)^{5-n}$ in the effective action.

To analyze the contribution of the second and third terms to the 5-point amplitude we note that due to the $SO(8)$ invariance

$$
\int d^8p\mathrm{Tr}\left(\dot{X}_L^i\dot{X}_R^k\mathrm{e}^{ip_aX_a}w^{L_0}\bar{w}^{\tilde{L}_0}\right) = p_a^ip_b^kA^{ab} + \delta^{ik}D\ , \tag{A.5}
$$

where A^{ab}, D are some functions of momenta p_a, positions of the vertex operators and the modulus parameter w. Note that the δ^{ik} term appears only because of the integration over the zero mode momentum p, and because both $\dot{X}_L^i$ and $\dot{X}_R^k$ depend on p.

Now one can easily see that the second term in (A.3) cannot lead to a term $R^n H^{5-n}$ since the indices i_5, k_5 are contracted with $V_{i_5k_5}$. Moreover, if $V_{i_5k_5} = b_{i_5k_5}$ then the second term must be absent since it is not invariant under the gauge transformation $B_2 \to B_2 + d\zeta$ of the NS-NS 2-form while all other terms in (A.3) are. If $V_{i_5k_5} = h_{i_5k_5}$ then we get vanishing result since, as usual, one assumes that $h_m^m = 0$ in the on-shell vertex operator.

To analyze the third term in (A.3) we need to consider three cases. In all the cases the three vertices V_{ijkl} can be either R_{ijkl} or $p_l H_{ijk}$.

(i) $V_{i_5k_4l_4} = \Gamma_{i_5k_4,l_4},\ V_{k_5i_4j_4} = \Gamma_{k_5i_4,j_4}$. The tensor $p_a^{i_5} p_b^{k_5}$ in (A.5) leads to the terms of the form $R^n(\nabla H)^{3-n}\partial\Gamma\partial\Gamma$,[16] and because of the reparametrization invariance they can contribute either to the covariantization of the term $R^{n+1}(\nabla H)^{3-n}$ coming from the 4-point amplitude or to a term of the form $R^{n+2}(\nabla H)^{3-n}$. The $\delta^{i_5k_5}$ term in (A.5) gives $R^n(\nabla H)^{3-n}\Gamma\Gamma$ and contributes to the covariantization of the term $R^{n+1}(\nabla H)^{3-n}$.

(ii) $V_{i_5k_4l_4} = H_{i_5k_4l_4},\ V_{k_5i_4j_4} = \Gamma_{k_5i_4,j_4}$. This term is of the form $R^n(\nabla H)^{4-n}\partial\Gamma$ or $R^n(\nabla H)^{3-n}H\Gamma$, and the reparametrization invariance again requires that it contributes either to the term $R^n(\nabla H)^{4-n}$ or to a term of the form $R^{n+1}(\nabla H)^{4-n}$.

(iii) $V_{i_5k_4l_4} = H_{i_5k_4l_4},\ V_{k_5i_4j_4} = H_{k_5i_4j_4}$. Such term in the 5-point amplitude is gauge and reparametrization invariant. The tensor $p_a^{i_5} p_b^{k_5}$ in (A.5) leads to the terms of the form $R^n(\nabla H)^{5-n}$. However, we also find the contribution from the $\delta^{i_5k_5}$ term in (A.5), which is of the form $R^n(\nabla H)^{3-n}H^2$, where the contraction $\left(H^2\right)_{ijkl} = H_{ijm}H_{klm}$ is the same as in (2.12).

We conclude that, contrary to the earlier expectation [27], there is no term of the form (2.10),(2.11) in the effective action but there is still a $t_8 t_8 R^3 H^2$ term with H^2 given by (2.12). The presence of such a term is demonstrated using the NSR formalism in the forthcoming paper [10], to which we refer for the details of the covariant form of the R^3H^2 part of the 1-loop type IIB(A) string effective action.

Using similar considerations it may be possible to rule out most of the higher-dimensional terms of the form $R^n H^k$ where all H_3 factors are not covered by derivatives.[17]

[16] We use a shorthand notation $\partial\Gamma$ to denote $\partial^k\Gamma_{ki,j}$, where the derivative ∂_k may act on any of the fields in $R^n(\nabla H)^{3-n}\partial\Gamma\partial\Gamma$.

[17] Naively, one could expect [27] the presence of HH term (2.11) supplementing the curvature

Appendix B. Structure of α'^3 corrections to KT solution

To analyze α'^3 corrections it is convenient to use orthonormal zehnbeins E_m (or corresponding 1-forms) in terms of which our perturbed background (5.1),(5.2) may be written as ($n = 1,2,3,4$, $\alpha = 6,7,8,9$; we reserve index 0 for the radial direction t)

$$ds^2_{10} = \mathrm{L}^2 \left[\mathrm{e}^{2z} E_n^2 + \mathrm{e}^{-2z} \left(\mathrm{e}^{10y} E_0^2 + \mathrm{e}^{2y-8w} E_5^2 + \mathrm{e}^{2y+2w} E_\alpha^2 \right) \right] , \tag{B.1}$$

$$H_3 = \frac{9 g_s M \alpha'}{2\mathrm{L}^3} (1 + b') t^{-\frac{3}{4}} E_0 \wedge (E_6 \wedge E_8 - E_7 \wedge E_9) \ , \tag{B.2}$$

$$F_3 = \frac{9 M \alpha'}{2\mathrm{L}^3} t^{-\frac{3}{4}} E_5 \wedge (E_6 \wedge E_8 - E_7 \wedge E_9) \ . \tag{B.3}$$

Our aim is to compute the correction (2.2) in this case. We assume that all indices in (2.2) are tangent. A direct computation shows that the tensors appearing in (2.3) have the following t-dependence:

$$C_{ijkl} \sim C^{(1/2)}_{ijkl} t^{-1/2} + C^{(3/2)}_{ijkl} t^{-3/2} + C^{(5/2)}_{ijkl} t^{-5/2} \ , \tag{B.4}$$

where the coefficients $C^{(1/2)}_{ijkl}$ do not vanish only if $i,j,k,l = 6,7,8,9$;

$$(\nabla H)_{ijkl} \sim h^{(1)}_{ijkl} t^{-1} + h^{(2)}_{ijkl} t^{-2} \ , \tag{B.5}$$

where the coefficients $h^{(1)}_{ijkl}$ do not vanish only if two of the indices i,j,k,l take values from 6 to 9, and the other two indices take values from 1 to 6;

$$\left(\nabla^2 \phi\right)_{ijkl} \sim p^{(1/2)}_{ijkl} t^{-1/2} + p^{(3/2)}_{ijkl} t^{-3/2} \ , \tag{B.6}$$

where the coefficients $p^{(1/2)}_{ijkl}$ do not vanish only if at least two of the indices i,j,k,l take values from 1 to 6.

This t-dependence shows that if there were no cancellations, the C^4 and $C^3\nabla^2\phi$ terms would scale at large t as $1/t^2$, and the terms $C^2(\nabla H)^2$, $C(\nabla H)^2\nabla^2\phi$ and $C^3 H^2$ would scale as $1/t^3$.

---

in each factor in (2.3). This is motivated by the condition that the corrections to equations of motion should vanish for a group space (WZW model). However, it is easy to see that sigma model perturbation theory does not imply that all corrections to beta-functions and thus also the effective action should depend on H_3 only through the curvature of the generalized connection $\hat{\Gamma} = \Gamma \pm \frac{1}{2} H_3$ (see e.g. [44,20]).

We will see, however, that due to a special structure of these α'^3 terms (related to the supersymmetry of the underlying IIB theory) when evaluated on the KT solution they scale as $1/t^4$. The first variations of these terms with respect to the fluctuations y, z, w scale as $1/t^3$. Taking into account that

$$\sqrt{-g} = \mathrm{L}^{10}\mathrm{e}^{4t}\sqrt{t}\ \mathrm{e}^{10y-2z} \ , \tag{B.7}$$

we find that the variational derivatives of S_8 are indeed given by (5.10).

The computation of the invariant W (2.8) is straightforward and gives

$$W = \frac{1}{\mathrm{L}^8}\Big(\frac{40}{t^4} - \frac{36}{t^5} + \frac{452}{27\,t^6} - \frac{35}{6\,t^7} + \frac{85}{16\,t^8} - \frac{75}{32\,t^9} + \frac{225}{512\,t^{10}}\Big) \ . \tag{B.8}$$

Thus at large t we get $W \sim \frac{1}{t^4}$. This implies that in the limit $\mathrm{L} \to 0$, $t \sim 1/\mathrm{L}^4$ leading to the $AdS_5 \times T^{1,1}$ case the invariant W *vanishes*.

It is also not difficult to compute the terms in W which are linear in fluctuations y, z, w

$$\delta W = \frac{1}{\mathrm{L}^8}\left(\frac{640y}{t^3} + \frac{288y'}{t^3} + \frac{32y''}{t^3} - \frac{320z'}{t^3} - \frac{64z''}{t^3} - \frac{960w}{t^3} - \frac{192w'}{t^3} - \frac{48w''}{t^3}\right) \ , \tag{B.9}$$

where we kept only terms that are leading at large t. These terms give contributions to the equations of motion for y, z, w which are of order $t^{-5/2}$. Again, in the limit $\mathrm{L} \to 0$, $t \sim 1/\mathrm{L}^4$ these terms vanish.

Computing $C^3\nabla^2\phi$ by using the simplified formula (2.8), we find that all the terms with the $1/t^2$ scaling cancel. There are still two terms that scale as $1/t^3$

$$C^3\nabla^2\phi \sim \frac{1}{t^3}\,(\phi'' + 4\phi') \ . \tag{B.10}$$

It is easy to see, however, that being integrated over t with the measure $\sqrt{-g} = \mathrm{L}^{10}\mathrm{e}^{4t}\sqrt{t}$, these two terms cancel each other, and the leading contribution to the dilaton equation of motion is of order $t^{-7/2}$, as it was the case for the C^4 term. [18] Thus the $C^3\nabla^2\phi$ term only changes the coefficient in the r.h.s. of the dilaton equation.

[18] One can easily see that the trace terms originating from $\epsilon_{10}\cdot\epsilon_{10}C^3\nabla^2\phi$ are either proportional to the linearized dilaton equation of motion, and have the same form as (B.10), or are a total derivative, and do not contribute to the equation.

To show that the terms $C^2(\nabla H)^2$, $C(\nabla H)^2\nabla^2\phi$ and C^3H^2 do not contain terms of order $1/t^3$ we need to recall that the invariants formed from the t_8 and ϵ_{10} tensors can be written as follows

$$X \equiv t_8 t_8 \overline{C}^4 = 192 I_{41} + 384 I_{42} + 24 I_{43} + 12 I_{44} - 96 I_{45} - 96 \tilde{I}_{45} - 48 I_{46} - 48 \tilde{I}_{46} \ , \quad \text{(B.11)}$$

$$\frac{1}{8} Z \equiv -\frac{1}{8}\epsilon_{10}\epsilon_{10}\overline{C}^4 = 192 I'_{41} + 384 I'_{42} + 24 I'_{43} + 12 I'_{44} - 192 I'_{45} + 96 I'_{46} - 768 A_7 + \frac{1}{8} Z' \ . \quad \text{(B.12)}$$

Here Z' are trace terms, and the fundamental invariants are defined as

$$\begin{aligned}
I_{41} &= \mathrm{tr}\left(\overline{C}_{mn}\overline{C}_{nr}\overline{C}_{rs}\overline{C}_{sm}\right) \ , \\
I_{42} &= \mathrm{tr}\left(\overline{C}_{mn}\overline{C}_{nr}\overline{C}_{ms}\overline{C}_{sr}\right) \ , \\
I_{43} &= \mathrm{tr}\left(\overline{C}_{mn}\overline{C}_{rs}\right)\mathrm{tr}\left(\overline{C}_{mn}\overline{C}_{rs}\right) \ , \\
I_{44} &= \mathrm{tr}\left(\overline{C}_{mn}\overline{C}_{mn}\right)\mathrm{tr}\left(\overline{C}_{rs}\overline{C}_{rs}\right) \ , \\
I_{45} &= \mathrm{tr}\left(\overline{C}_{mn}\overline{C}_{nr}\right)\mathrm{tr}\left(\overline{C}_{rs}\overline{C}_{sm}\right) \ , \\
I_{46} &= \mathrm{tr}\left(\overline{C}_{mn}\overline{C}_{rs}\right)\mathrm{tr}\left(\overline{C}_{mr}\overline{C}_{ns}\right) \ , \\
Z &= \overline{C}_{mn}{}^{[mn}\overline{C}_{pq}{}^{pq}\overline{C}_{rs}{}^{rs}\overline{C}_{tu}{}^{tu]} \ , \\
A_7 &= \overline{C}_{pq}{}^{rs}\overline{C}_{ru}{}^{pt}\overline{C}_{tv}{}^{qw}\overline{C}_{sw}{}^{uv} \ .
\end{aligned} \quad \text{(B.13)}$$

The matrices $\overline{C}_{mn}$ are naturally defined by $(\overline{C}_{mn})^a{}_b \equiv \overline{C}_{mn}{}^{ab}$; the invariants $\tilde{I}_{ab}$ are defined by the same formulas with the replacement $\overline{C}_{mn} \to \overline{C}^T_{mn}$, where $(\overline{C}^T_{mn})^a{}_b = \overline{C}_{ab}{}^{mn}$; the invariants I'_{ab} are defined by replacing in the formulas (B.13) the second and the fourth tensors $\overline{C}$ with $\overline{C}^T$. When $\overline{C} = \overline{C}^T$ the above relations reduce to the ones given in [22,23].

First, we discuss the term $C^2(\nabla H)^2$. Since we want to show that it does not contribute to the order $1/t^3$ it is sufficient to consider the terms in C_{ijkl} that scale as $t^{-1/2}$, and the terms in $(\nabla H)_{ijkl}$ that scale as t^{-1}. As was mentioned above, in this case C_{ijkl} do not vanish only if $i,j,k,l = 6,7,8,9$, and $(\nabla H)_{ijkl}$ do not vanish only if two of the indices i,j,k,l take values from 6 to 9, and the other two indices take values from 1 to 6. Moreover, in any of the pairs i,j or k,l one of the indices takes values from 6 to 9, and the other takes values from 1 to 6. For this reason, we find that the $\epsilon_{10}\epsilon_{10}$ term vanishes. The t_8t_8 term has to be computed by using (B.11), and also gives zero. It is worth noting that in this consideration we have not assumed that the fluctuation b of B_2 vanishes. Thus, this term gives contributions of order $t^{-7/2}$ to both the equations of motion for the ϕ and b.

To show that $C(\nabla H)^2\nabla^2\phi$ does not contribute to the order $1/t^3$, we take C_{ijkl} and $(\nabla H)_{ijkl}$ with the same scaling as above, and $\nabla^2\phi$ scaling as $t^{-1/2}$. Then a straightforward computation shows that among the invariants I_{ab} only $I_{41} = I'_{41}$ does not vanish. This invariant, however, does not appear in the difference $X - \frac{1}{8}Z$ that gives the α'^3 correction. Therefore, we need to show the vanishing $-768A_7 +$ "trace terms" in (B.12). Computing A_7 and the "trace terms", we get

$$A_7 = -\frac{8}{t^3}(4\phi' - \phi'') \ , \qquad \frac{1}{8}Z' = -\frac{8 \cdot 768}{t^3}(12\phi' + \phi'') \ . \tag{B.14}$$

Thus, their total contribution is

$$768A_7 - \frac{1}{8}Z' = \frac{16 \cdot 768}{t^3}(4\phi' + \phi'') \ . \tag{B.15}$$

As was explained above, it leads to a contribution of order $t^{-7/2}$ to the dilaton equation of motion. This is of the same order as the contribution due to the C^4 term.

In [10] and in Appendix A it was shown that there are certain terms of the form $(t_8t_8 + \frac{1}{8}\epsilon_{10}\epsilon_{10})C^3H^2$ where H^2 is given by (2.12). It is again clear that the $\epsilon_{10}\epsilon_{10}$ term cannot contribute to the $1/t^3$ order. Computing the t_8t_8 term by using (B.11) and (B.13), we find that, although all the invariants I_{4n} contribute to the order $1/t^3$, the total contribution of the t_8t_8 term starts with $1/t^4$. [19]

Thus, we see that special properties of the 8-derivative corrections to the type IIB effective action lead to a number of cancellations when evaluated on the KT background. These cancellations are needed for consistency with the RG flow in the dual field theory.

[19] Though we have proved in Appendix A that the term (2.10) is not present in the effective action, we have checked that it also starts with $1/t^4$.

References

[1] I. R. Klebanov and E. Witten, "Superconformal field theory on threebranes at a Calabi-Yau singularity," Nucl. Phys. B **536**, 199 (1998) [hep-th/9807080].

[2] D. R. Morrison and M. R. Plesser, "Non-spherical horizons. I," Adv. Theor. Math. Phys. **3**, 1 (1999) [hep-th/9810201].

[3] S. S. Gubser and I. R. Klebanov, "Baryons and Domain Walls in an N=1 Superconformal Gauge Theory," Phys. Rev. **D58** 125025 (1998), [hep-th/9808075].

[4] I. R. Klebanov and N. A. Nekrasov, "Gravity duals of fractional branes and logarithmic RG flow," Nucl. Phys. B **574**, 263 (2000) [hep-th/9911096].

[5] I. R. Klebanov and A. A. Tseytlin, "Gravity Duals of Supersymmetric $SU(N) \times SU(N+M)$ Gauge Theories," Nucl. Phys. **B578**, 123 (2000), [hep-th/0002159].

[6] I. R. Klebanov and M. J. Strassler, "Supergravity and a Confining Gauge Theory: Duality Cascades and χSB-Resolution of Naked Singularities," JHEP **0008**, 052 (2000) [hep-th/0007191].

[7] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, "Supersymmetric Instanton Calculus: Gauge Theories With Matter," Nucl. Phys. B **260**, 157 (1985). "Beta Function In Supersymmetric Gauge Theories: Instantons Versus Traditional Approach," Phys. Lett. B **166**, 329 (1986).

[8] M. A. Shifman and A. I. Vainshtein, "Solution Of The Anomaly Puzzle In Susy Gauge Theories And The Wilson Operator Expansion," Nucl. Phys. B **277**, 456 (1986).

[9] C.P. Herzog, I.R. Klebanov and P. Ouyang, "Remarks on the Warped Deformed Conifold," `hep-th/0108101`.

[10] K. Peeters, P. Vanhove and A. Westerberg, to appear.

[11] J. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200].

[12] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428**, 105 (1998) [hep-th/9802109].

[13] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150].

[14] D. J. Gross and E. Witten, "Superstring Modifications Of Einstein's Equations," Nucl. Phys. B **277**, 1 (1986).

[15] D. J. Gross and J. H. Sloan, "The Quartic Effective Action For The Heterotic String," Nucl. Phys. B **291**, 41 (1987).

[16] J. H. Schwarz, "Superstring Theory," Phys. Rept. **89**, 223 (1982).

[17] M. T. Grisaru and D. Zanon, "Sigma Model Superstring Corrections To The Einstein-Hilbert Action," Phys. Lett. B **177**, 347 (1986). M. D. Freeman, C. N. Pope, M. F. Sohnius and K. S. Stelle, "Higher Order Sigma Model Counterterms And The Effective Action For Superstrings," Phys. Lett. B **178**, 199 (1986). Q.-H. Park and

D. Zanon, "More On Sigma Model Beta Functions And Low-Energy Effective Actions," Phys. Rev. D **35**, 4038 (1987).

[18] M. T. Grisaru, A. E. van de Ven and D. Zanon, "Four Loop Beta Function For The N=1 And N=2 Supersymmetric Nonlinear Sigma Model In Two-Dimensions," Phys. Lett. B **173**, 423 (1986). "Two-Dimensional Supersymmetric Sigma Models On Ricci Flat Kahler Manifolds Are Not Finite," Nucl. Phys. B **277**, 388 (1986).

[19] R. Myers, "Superstring Gravity And Black Holes," Nucl. Phys. B **289**, 701 (1987).

[20] A. A. Deriglazov and S. V. Ketov, "Four loop divergences of the two-dimensional (1,1) supersymmetric nonlinear sigma model with a Wess-Zumino-Witten term," Nucl. Phys. B **359**, 498 (1991).

[21] I. Jack, "The Twisted N=2 supersymmetric sigma model: A Four loop calculation of the beta function," Nucl. Phys. B **371**, 482 (1992).

[22] M. de Roo, H. Suelmann and A. Wiedemann, "The Supersymmetric effective action of the heterotic string in ten-dimensions," Nucl. Phys. B **405**, 326 (1993) [hep-th/9210099]. J. H. Suelmann, "Supersymmetry and string effective actions," Ph.D. Thesis, Groningen, 1994, RX-1510.

[23] K. Peeters, P. Vanhove and A. Westerberg, "Supersymmetric higher-derivative actions in ten and eleven dimensions, the associated superalgebras and their formulation in superspace," Class. Quant. Grav. **18**, 843 (2001) [hep-th/0010167].

[24] A. A. Tseytlin, "Ambiguity In The Effective Action In String Theories," Phys. Lett. B **176**, 92 (1986).

[25] T. Banks and M. B. Green, "Non-perturbative effects in $AdS_5 \times S^5$ string theory and d = 4 SUSY Yang-Mills," JHEP **9805**, 002 (1998) [hep-th/9804170].

[26] S. S. Gubser, I. R. Klebanov and A. A. Tseytlin, "Coupling constant dependence in the thermodynamics of N = 4 supersymmetric Yang-Mills theory," Nucl. Phys. B **534**, 202 (1998) [hep-th/9805156].

[27] A. Kehagias and H. Partouche, "The exact quartic effective action for the type IIB superstring," Phys. Lett. B **422**, 109 (1998) [hep-th/9710023].

[28] L. J. Romans, "New Compactifications Of Chiral N=2 D = 10 Supergravity," Phys. Lett. B **153**, 392 (1985).

[29] B.E.W. Nilsson and A.K. Tollsten, "Supersymmetrization of $\zeta(3)R^4$ in superstring theories," Phys. Lett. B **181**, 63 (1986). R. Kallosh, "Strings And Superspace," Phys. Scripta **T15**, 118 (1987).

[30] D. Nemeschansky and A. Sen, "Conformal Invariance Of Supersymmetric Sigma Models On Calabi-Yau Manifolds," Phys. Lett. B **178**, 365 (1986).

[31] E. Witten, "Phases of N = 2 theories in two dimensions," Nucl. Phys. B **403**, 159 (1993) [hep-th/9301042].

[32] M. D. Freeman and C. N. Pope, "Beta Functions And Superstring Compactifications," Phys. Lett. B **174**, 48 (1986). A. Sen, "Central Charge Of The Virasoro Algebra For

Supersymmetric Sigma Models On Calabi-Yau Manifolds," Phys. Lett. B **178**, 370 (1986).

[33] R. Kallosh and A. Rajaraman, "Vacua of M-theory and string theory," Phys. Rev. D **58**, 125003 (1998) [hep-th/9805041].

[34] D. Marolf, "Chern-Simons terms and the three notions of charge," hep-th/0006117.

[35] A. Buchel, C. P. Herzog, I. R. Klebanov, L. Pando Zayas and A. A. Tseytlin, "Non-extremal gravity duals for fractional D3-branes on the conifold," JHEP **0104**, 033 (2001) [hep-th/0102105]. S. S. Gubser, C. P. Herzog, I. R. Klebanov and A. A. Tseytlin, "Restoration of Chiral Symmetry: A Supergravity Perspective," JHEP **0105**, 028 (2001) [hep-th/0102172].

[36] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," hep-th/0105097.

[37] A. A. Tseytlin, "R^4 terms in 11 dimensions and conformal anomaly of (2,0) theory," Nucl. Phys. B **584**, 233 (2000) [hep-th/0005072].

[38] K. Becker and M. Becker, "Supersymmetry breaking, M-theory and fluxes," JHEP **0107**, 038 (2001) [hep-th/0107044].

[39] M. T. Grisaru, D. I. Kazakov and D. Zanon, "Five Loop Divergences For The N=2 Supersymmetric Nonlinear Sigma Model," Nucl. Phys. B **287**, 189 (1987).

[40] M. B. Green and J. H. Schwarz, "Supersymmetrical Dual String Theory," Nucl. Phys. B **181**, 502 (1981); "Supersymmetrical Dual String Theory. 2. Vertices And Trees," Nucl. Phys. B **198**, 252 (1982).

[41] M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vols. 1 and 2" *Cambridge Univ. Press (1987).*

[42] M. B. Green and S. Sethi, "Supersymmetry constraints on type IIB supergravity," Phys. Rev. D **59**, 046006 (1999) [hep-th/9808061]. M. B. Green, "Interconnections Between Type II Superstrings, M Theory And N = 4 Yang-Mills," hep-th/9903124.

[43] M. B. Green and N. Seiberg, "Contact Interactions In Superstring Theory," Nucl. Phys. B **299**, 559 (1988).

[44] R. R. Metsaev and A. A. Tseytlin, "Order alpha prime (two loop) equivalence ef the string equations of motion and the sigma model Weyl invariance conditions: dependence on the dilaton and the antisymmetric tensor," Nucl. Phys. B **293**, 385 (1987).