

Scalar Quartic Effective Action on AdS_5

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ABSTRACT: We review the recent results concerning the computation of cubic and quartic couplings of scalar fields in type IIB supergravity on $AdS_5 \times S^5$ background that are dual to (extended) chiral primary operators in $\mathcal{N} = 4$ SYM₄. We discuss the vanishing of certain cubic and quartic couplings and non-renormalization property of corresponding correlators in the conformal field theory.

1. Introduction

The AdS/CFT duality [1, 2, 3] provides the holographic relation between $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in four dimension (SYM₄) and type IIB supergravity on $AdS_5 \times S^5$ background. An important class of operators in SYM₄ playing the role of the BPS states is given by the chiral primary operators (CPO). Thus, the AdS/CFT correspondence gives a remarkable opportunity to study the dynamics of CPOs (BPS states) at the strong coupling regime.

The power of the superconformal symmetry in four dimensions is not enough to supply the Operator Product Expansion (OPE) of CPOs with the ring structure. In this respect the knowledge of correlation functions of CPOs provides us with new dynamical information about $\mathcal{N} = 4$ SYM₄ at the strong 't Hooft coupling (spectrum of operators in the OPE algebra of CPOs, their anomalous dimensions and so on). Comparison of the correlation functions at strong and weak couplings brings also insight previously unknown non-renormalization theorems.

An important problem that has not been solved yet is the computation of the 4-point correlation functions of CPOs in the supergravity approximation. To compute a 4-point correlation function in $\mathcal{N} = 4$ SYM₄ one has to derive

the relevant part of the gravity action on AdS_5 background up to the fourth order and then to compute the on-shell value of this action (with Dirichlet boundary conditions on gravity fields). By perturbation theory the second step can be represented as evaluation of exchange and contact Feynman diagrams. It is the absence of the well-defined 5d gravity action for fields that correspond to CPOs in SYM₄ that makes computation of the correlation functions of CPOs a non-trivial problem. So far the only known examples here are the 4-point functions of operators $\text{tr}(F^2 + \dots)$ and $\text{tr}(F\tilde{F} + \dots)$ that on the gravity side correspond to massless modes of dilaton and axion fields [4, 5], where the relevant part of the gravity action was known. These operators, however, are descendents of the CPOs that brings considerable complications both in perturbative analysis of the correlation functions, and in the study of their OPE from AdS gravity [6].

Recently we have constructed the quartic effective 5d action for scalar fields s^I dual at linear order to CPOs [7], which allows one to compute 4-point functions of *any* (extended) CPOs in the supergravity approximation. Here we present a brief review of the corresponding results. The quartic couplings of the 5d action possess a number of remarkable properties. In particular they

admit the consistent Kaluza-Klein reduction to the fields from the massless graviton multiplet and they vanish in the so-called extremal and sub-extremal cases. Each of these properties implies the existence of the non-renormalization theorem for the corresponding correlators of CPOs in SYM₄.

2. CPOs, field redefinition and operator mixing

In $\mathcal{N} = 4$ SYM₄ there are “short” or chiral multiplets generated by primary operators:

$$O^I(x) = \frac{(2\pi)^k}{\sqrt{k\lambda^k}} C_{i_1 \dots i_k}^I \text{tr} (: \phi^{i_1}(x) \dots \phi^{i_k}(x) :),$$

where $C_{i_1 \dots i_k}^I$ are totally symmetric traceless rank k orthonormal tensors of $SO(6)$:

$$\langle C^I C^J \rangle = C_{i_1 \dots i_k}^I C_{i_1 \dots i_k}^J = \delta^{IJ},$$

ϕ^i are scalars of SYM₄, the notation $: A_1 \dots A_n :$ is used to denote the normal-ordered product of the operators A_i and $\lambda = g_{YM}^2 N$ is the 't Hooft coupling.

Eight from sixteen supercharges annihilate $O^I(x)$ while the other eight ones generate chiral multiplets. A fundamental property of CPOs is that their conformal dimensions are protected. Thus, they may be regarded as BPS states preserving 1/2 of the supersymmetry.

The two- and three-point functions of CPOs computed in free theory (in the leading order in $1/N$) are [8]

$$\langle O^I(x) O^J(y) \rangle = \frac{\delta^{IJ}}{|x - y|^{2k}},$$

$$\langle O^{I_1}(x_1) O^{I_2}(x_2) O^{I_3}(x_3) \rangle = \frac{1}{N} \frac{\sqrt{k_1 k_2 k_3} C^{I_1 I_2 I_3}}{x_{12}^{2\alpha_3} x_{23}^{2\alpha_1} x_{13}^{2\alpha_2}},$$

where $x_{ij} = x_i - x_j$, $\alpha_i = \frac{1}{2}(k_j + k_l - k_i)$, $j \neq l \neq i$, and $C^{I_1 I_2 I_3}$ is the unique $SO(6)$ invariant obtained by contracting α_1 indices between C^{I_2} and C^{I_3} , α_2 indices between C^{I_3} and C^{I_1} , and α_3 indices between C^{I_2} and C^{I_1} .

Analysis of the superconformal transformations in SYM₄ allows one to conclude that CPOs $O^I(x)$ are dual to the scalars s^I of IIB supergravity compactified on $AdS_5 \times S^5$. Fields s^I appear

in spectrum of the compactified theory as a linear combination of the trace of the graviton on S^5 and of the 5-form field strength on S^5 . Index I runs the KK tower of scalar spherical harmonics.

In fact there are only two ways to tackle the problem of constructing an effective 5d action for the scalars s^I . The first way is to compactify on $AdS_5 \times S^5$ the covariant action for type IIB supergravity by [9]. Since the ten-dimensional theory contains a four-form with the self-dual field strength the manifest Lorenz covariance is achieved by introducing an auxiliary scalar field a . Fixing a to be some function of space-time variables and breaking thereby an additional gauge invariance associated with this field one gets a non-covariant action. The covariance w.r.t. the background isometry group is then restored by means of introducing some additional non-propagating fields. Namely this way was used in [10] to derive the quadratic action for physical fields of IIB supergravity on $AdS_5 \times S^5$ background. However, at the level of cubic and quartic actions the problem of solving the non-covariant constraints imposed by gauge symmetries becomes extremely complicated.

The second approach deals with the covariant equations of motion of IIB supergravity. The basic strategy here is to find quadratic and cubic corrections to equations of motion of gravity fields by decomposing the covariant equations up to the third order. The main problem here is that equations such obtained are non-Lagrangian and one has to perform very complicated and fine analysis to reduce them to the Lagrangian form. In what follows we undertake the second approach.

The way to bring the equation of motion of IIB supergravity to the Lagrangian form is to perform non-linear redefinitions of the original fields s^I in terms of the new fields s'^I . Here we restrict our attention to fields s^I but it should be noted that all other gravity fields require redefinitions of a similar type. Performing appropriate redefinitions one gets rid of the higher-derivative terms and simultaneously obtains the Lagrangian equations (see [8, 7, 11, 12]). Hence, in spite of the fact that the fields s^I correspond to the simplest single-trace operators $O^I(x)$ in

the Yang-Mills theory they are rather unnatural in the gravity description.

One may then wonder if the correspondence $O^I \leftrightarrow s'^I$ still holds. It turns out that in the extremal case, e.g., $k_3 = k_1 + k_2$, 3-point correlation functions of the corresponding CPOs computed in SYM₄ are non-singular when $x_1 \rightarrow x_2$, while on the gravity side the cubic couplings for s'^I vanish leading to the vanishing of the correlation functions. One natural conjecture [10] is that redefinitions of the gravity fields lead to the change of the basis of CPOs in conformal field theory. We will widely refer to the new basis as to the *extended* CPOs. Thus, redefined fields s'^I correspond to the extended CPOs. It is possible to construct the extended CPOs explicitly. Define the following operators in SYM₄:

$$\begin{aligned} \tilde{O}^{I_1}(x) &= O^{I_1}(x) \\ &- \frac{1}{2N} \sum_{I_2+I_3=I_1} \mathbf{C}^{I_1 I_2 I_3} : O^{I_2}(x) O^{I_3}(x) :, \end{aligned}$$

where $\mathbf{C}^{I_1 I_2 I_3} = \sqrt{k_1 k_2 k_3} C^{I_1 I_2 I_3}$. In the large N limit these operators have the normalized two-point functions, the same 3-point functions as CPOs if none of the relations

$$k_1 + k_2 = k_3, \quad k_2 + k_3 = k_1, \quad k_3 + k_1 = k_2,$$

is satisfied, and vanishing 3-point functions otherwise. Thus, the extended CPOs are constructed by mixing single- with double-trace operators. Still the expression for \tilde{O} is incomplete and it should be further extended by normal-ordered products of CPOs and their descendents.

In principle it seems possible to find an action for the scalars dual to CPOs by performing the field redefinitions reversed to the ones used to reduce the equations of motion to a Lagrangian form. The reversed transformations should be made at the level of the quartic action, but not at the level of equations of motion. However, the resulting action for the new scalars will be much more complicated and will contain higher-derivative terms with six derivatives. It's worth noting that the equations of motion derived from the new action certainly differ from the original ones despite the fact that one made reversed transformations. Hopefully the coincidence of the correlation functions of CPOs and of extended

CPOs for generic values of conformal dimensions and the existence of the analytic continuation procedure advocated in [13],[14] allows one to find the correlation functions of any CPOs by working with the gravity action for redefined fields.

3. Effective 5d action

The result of our study in [7] is the effective 5d action for scalars s^I needed to compute 4-point correlation functions of *any* (extended) CPOs in $\mathcal{N} = 4$ SYM₄ at large 't Hooft coupling:

$$S(s) = \frac{4N^2}{(2\pi)^5} \int d^5x \sqrt{-g_a} \times \left(\sum_i (\mathcal{L}_2(\Phi_i) + \mathcal{L}_3(\Phi_i)) + \mathcal{L}_4^{(0)} + \mathcal{L}_4^{(2)} + \mathcal{L}_4^{(4)} \right),$$

where g_a stands for the determinant of the AdS-metric. Here Φ_i denotes one of the fields from the following set

$$(s^I, t^I, \phi^I, \phi_{ab}^I, A_a^I, C_a^I).$$

In fact this are all the fields that appear in cubic interaction vertices $\mathcal{L}_3(\Phi_i)$ containing two fields s^I . Here the quadratic terms are given by [10]

$$\begin{aligned} \mathcal{L}_2(s) &= c_s \left(-\frac{1}{2} \nabla_a s_k \nabla^a s_k - \frac{1}{2} m^2 s_k^2 \right), \\ \mathcal{L}_2(t) &= c_t \left(-\frac{1}{2} \nabla_a t_k \nabla^a t_k - \frac{1}{2} m_t^2 t_k^2 \right), \\ \mathcal{L}_2(\phi) &= -\frac{1}{4} \nabla_a \phi_k \nabla^a \phi_k - \frac{1}{4} f(k) \phi_k^2, \\ \mathcal{L}_2(\varphi_{ab}) &= -\frac{1}{4} \nabla_c \varphi_{ab}^k \nabla^c \varphi_k^{ab} + \frac{1}{2} \nabla_a \varphi_k^{ab} \nabla^c \varphi_{cb}^k \\ &- \frac{1}{2} \nabla_a \varphi_c^{ck} \nabla_b \varphi_k^{ba} + \frac{1}{4} \nabla_c \varphi_a^{ak} \nabla^c \varphi_b^{bk} \\ &+ \frac{1}{4} (2 - f(k)) \varphi_{ab}^k \varphi_k^{ab} + \frac{1}{4} (2 + f(k)) (\varphi_a^{ak})^2, \\ \mathcal{L}_2(A_a) &= c_A \left(-\frac{1}{4} (F_{ab}(A^k))^2 - \frac{1}{2} m_A^2 (A_a^k)^2 \right), \\ \mathcal{L}_2(C_a) &= c_C \left(-\frac{1}{4} (F_{ab}(C^k))^2 - \frac{1}{2} m_C^2 (C_a^k)^2 \right), \end{aligned}$$

where the masses of the particles are

$$\begin{aligned} m^2 &= k(k-4), \quad m_t^2 = (k+4)(k+8), \\ m_A^2 &= k^2 - 1, \quad m_C^2 = (k+3)(k+5), \\ m_\phi^2 &= m_\varphi^2 = f(k) = k(k+4), \end{aligned}$$

$F_{ab}(A) = \partial_a A_b - \partial_b A_a$, and the normalization constants depending on k are found to be

$$c_s = \frac{32k(k-1)(k+2)}{k+1}, \quad c_A = \frac{k+1}{2(k+2)},$$

$$c_t = \frac{32(k+2)(k+4)(k+5)}{k+3}, \quad c_C = \frac{k+3}{2(k+2)}.$$

To simplify the notation we denote here s^{I_1} as s_k or simply as s^1 . The index $I \equiv I(k)$ runs the basis of a representation of $SO(6)$ specified by k . Terms \mathcal{L}_2 above represent the standard quadratic Lagrangians for fields of different spins.

The cubic terms were found in [8, 11, 12], and may be written as follows

$$\begin{aligned} \mathcal{L}_3(s) &= S_{123} s^1 s^2 s^3, \\ \mathcal{L}_3(t) &= T_{123} s^1 s^2 t^3, \\ \mathcal{L}_3(\phi) &= \Phi_{123} s^1 s^2 \phi^3, \\ \mathcal{L}_3(\varphi_{ab}) &= G_{123} \left(\varphi_{ab}^3 T^{ab}{}^{12} + \frac{f_3}{4} s^1 s^2 \varphi_c^3 \right) \\ \mathcal{L}_3(A_a) &= A_{123} s^1 \nabla^a s^2 A_a^3, \\ \mathcal{L}_3(C_a) &= C_{123} s^1 \nabla^a s^2 C_a^3, \end{aligned}$$

Here to describe the interaction with the massive graviton φ_{ab}^I we introduced the notation

$$T_{ab}^{12} = \nabla^a s^1 \nabla^b s^2 - \frac{\delta^{ab}}{2} \left(\nabla^c s^1 \nabla_c s^2 + \frac{1}{2} m_{12}^2 s^1 s^2 \right),$$

where the ‘‘mass’’ matrix is $m_{12}^2 = m_1^2 + m_2^2$. Here the summation over I_1, I_2, I_3 is assumed and $f_3 \equiv f(k_3)$. For the explicit values of the cubic couplings see [11].

Finally the quartic terms recently found in [7] are given by

$$\begin{aligned} \mathcal{L}_4^{(0)} &= S_{1234}^{(0)} s^1 s^2 s^3 s^4, \\ \mathcal{L}_4^{(2)} &= \left(S_{1234}^{(2)} + A_{1234}^{(2)} \right) s^1 \nabla_a s^2 s^3 \nabla^a s^4, \end{aligned}$$

with the symmetry

$$\begin{aligned} S_{1234}^{(2)} &= S_{2134}^{(2)} = S_{3412}^{(2)}, \\ A_{1234}^{(2)} &= -A_{2134}^{(2)} = A_{3412}^{(2)}; \end{aligned}$$

and by

$$\begin{aligned} \mathcal{L}_4^{(4)} &= \left(S_{1234}^{(4)} + A_{1234}^{(4)} \right) \\ &\quad \times s^1 \nabla_a s^2 \nabla_b^2 (s^3 \nabla^a s^4), \end{aligned}$$

where

$$\begin{aligned} S_{1234}^{(4)} &= S_{2134}^{(4)} = S_{3412}^{(4)}, \\ A_{1234}^{(4)} &= -A_{2134}^{(4)} = A_{3412}^{(4)}. \end{aligned}$$

Explicit values of the quartic couplings are given in [7]. Equations of motion that follow from this action are related with original non-Lagrangian equations of IIB supergravity on the $AdS_5 \times S^5$ background by a chain of *field redefinitions* and by the usage of *hidden relations* between different kinds of spherical harmonics. One of the novel features of the found couplings is the presence of the 2- and 4-derivative terms that can not be removed by any field redefinition without spoiling the minimal character of the interaction at the cubic level.

4. Reduction to the gauged $\mathcal{N} = 8$ 5-dimensional supergravity

The quartic couplings we found allow us to study the problem of the consistency of the Kaluza-Klein (KK) reduction down to five dimensions. It is customarily believed that the S^5 compactification of type IIB supergravity admits a consistent truncation to the massless multiplet, which can be identified with the field content of the gauged $\mathcal{N} = 8, d = 5$ supergravity [15, 16]. Consistency means that there is no term linear in massive KK modes in the untruncated supergravity action, so that all massive KK fields can be put to zero without any contradiction with equations of motion. From the AdS/CFT correspondence point of view the consistent truncation implies that *any* n -point correlation function of $n-1$ operators dual to the fields from the massless multiplet and one operator dual to a massive KK field vanishes because, as one can easily see there is no exchange Feynman diagram in this case.

Considering explicit expressions for the cubic couplings found in [11, 12] one sees immediately that they obey the consistency condition allowing therefore truncation to the fields from the massless multiplet at the level of the cubic action.

The truncation problem for the quartic couplings is a little bit more sophisticated. Recall that the gauged $\mathcal{N} = 8$ five-dimensional supergravity has in particular 42 scalars with 20 of

them forming the singlet of the global invariance group $SL(2, \mathbf{R})$. These 20 scalars comprise the **20** irrep. of $SO(6)$ and correspond to the IIB supergravity fields s^I with $k = 2$. The five-dimensional scalar Lagrangian consists of the kinetic energy and the potential. The maximal number of derivatives appearing in the Lagrangian is two and that is due to the non-linear sigma model type kinetic energy. We have however found the quartic 4-derivative vertices that can not be shifted away by any field redefinition. Thus, a highly non-trivial check of the relation between the compactification of the ten-dimensional theory and the gauged supergravity in five dimensions as well of the results obtained consists in showing that the 4-derivative vertices vanish for the modes from the massless multiplet. It turns out that this is indeed the case. Moreover, after an additional simple field redefinition the quartic vertices we found indeed vanish when one of the four fields is not from the massless multiplet, proving thereby the consistency of the reduction at the level of the quartic scalar couplings. This in particular provides an additional argument that the scalars s^I (and, in general, any supergravity field) correspond not to CPOs but rather to extended CPOs. Indeed, if we assume that the consistent truncation takes place at all orders in gravity fields, we get that correlators of the form $\langle O_2^{I_1} O_2^{I_2} \dots O_2^{I_{n-1}} O_k^{I_n} \rangle$ vanish for $k \geq 3$. This is certainly not the case for single-trace CPOs, and we are forced to conclude once more that supergravity fields are in general dual to extended operators which are admixtures of single-trace operators and multi-trace ones.¹ Since an extended operator is uniquely determined by a single-trace one, it is natural to assume that if a correlation function of extended operators vanishes then there exists a kind of a non-renormalization theorem for an analogous correlation function of single-trace operators. If we further assume that type IIB string theory on $AdS_5 \times S^5$ respects the consistent truncation, then the vanishing of n -point correlation functions of $n - 1$ extended operators dual to the

¹Note that the lowest modes s_2 may be dual only to single-trace CPOs. It is possible that any field from the massless supergravity multiplet is dual to a single-trace operator.

supergravity modes from the massless multiplet, and one extended operator dual to a massive KK mode seems to imply that [7]

at large N the n -point functions of the corresponding single-trace operators are independent of 't Hooft coupling $\lambda = g_{YM}^2 N$.

If the consistent truncation is valid at quantum level, that seems to be plausible because of a large amount of supersymmetry, then these n -point functions are independent of g_{YM} for any N .

In particular this conjecture is applied to n -point functions of $n - 1$ CPOs O_2 and a CPO O_4 . Very recently in [17] a non-renormalization property of the 4-point function of these operators was checked to first order in perturbation theory.

For the value of the quartic couplings for the scalars s^I belonging to the massless graviton multiplet we have found [7] the following 2-derivative vertex

$$\mathcal{L}_{AdS_5}^{(2)} = \frac{2^{14}}{9\pi^3} C_{I_1 I_2 I_3 I_4} \nabla_a (s^{I_1} s^{I_2}) \nabla^a (s^{I_3} s^{I_4})$$

and the vertex without derivatives:

$$\begin{aligned} \mathcal{L}_{AdS_5}^{(0)} = & -\frac{2^{16}}{3\pi^3} \left(C_{I_1 I_2 I_3 I_4} - \frac{1}{6} \delta^{I_1 I_2} \delta^{I_3 I_4} \right) \times \\ & \times s^{I_1} s^{I_2} s^{I_3} s^{I_4}. \end{aligned}$$

Here the shorthand notation $C_{I_1 I_2 I_3 I_4}$ for the trace product of four matrices C_{ij}^I was introduced. Note that C_{ij}^I are traceless symmetric matrices whose appearance here is due to the explicit description of the spherical harmonics on S^5 . One can also introduce the fields $s_{ij} \equiv C_{ij}^I s^I$ that provide the natural parametrization of the coset space $SL(6, \mathbf{R})/SO(6)$. Recently we have shown that the relevant part of the gauged $\mathcal{N} = 8$ 5-dimensional supergravity action coincides with the action [7] for scalars s^I from the massless graviton multiplet.

5. Quartic couplings for the extremal case

In [11] we argued that quartic couplings of the scalars s^I had to vanish in the extremal case when, say, $k_1 = k_2 + k_3 + k_4$. This conjecture

was based on the fact that all exchange Feynman diagrams vanished and contact Feynman diagrams had singularity in the extremal case, thus non-vanishing quartic couplings would contradict to the AdS/CFT correspondence. Although the vanishing of the found quartic couplings is not manifest, one can show that this important property does take place after an additional field redefinition. This means that 4-point extremal correlators of extended CPOs vanish, and also implies the non-renormalization theorem [14] for the corresponding extremal correlators of single-trace CPOs. It is clear that since the quartic couplings vanish then there should exist such a representation of the quartic couplings, that makes the vanishing explicit. An interesting problem is to find this representation.

The simplest example of the 4-point function of three CPOs O_2 and a CPO O_4 belongs, actually, to the class of so-called "sub-extremal" 4-point functions, for which $k_1 = k_2 + k_3 + k_4 - 2$. The non-renormalization of such correlation functions was shown in [18] to be the consequence of the superconformal Ward identities and of the constrained nature of the harmonic superfields. The non-renormalization theorem also implies the vanishing of the corresponding functions of extended CPOs and, since it is not difficult to show that there is no exchange diagram in this case, the corresponding sub-extremal quartic couplings of scalars s^I have to vanish too. Just recently we have checked that this remarkable property indeed takes place.

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