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Three-point correlators of stress tensors in maximally-supersymmetric conformal theories in $d = 3$ and $d = 6$

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Abstract

We consider free superconformal theories of $\mathcal{N} = 8$ scalar multiplet in $d = 3$ and $(2,0)$ tensor multiplet in $d = 6$ and compute 2-point and 3-point correlators of their stress tensors. The results for the 2-point and the 3-point correlators for a single $d = 3$ and $d = 6$ multiplet differ from the “strong-coupling” AdS_4 and AdS_7 supergravity predictions by the factors $\frac{4\sqrt{2}}{3\pi} N^{3/2}$ and $4N^3$ respectively. These are the same factors as found earlier in hep-th/9703040 in the comparison of the brane free field theory and the $d = 11$ supergravity predictions for the absorption cross-sections of longitudinally polarized gravitons by N M2 and M5 branes. While the correspondence of the results for the cross-sections and 2-point functions was expected on the basis of unitarity, the fact that the same coefficients appear in the ratio of the free-theory and supergravity 3-point functions is non-trivial. Thus, like in the $d = 4$ SYM case, in both $d = 3$ and $d = 6$ theories the ratio of the 3-point and 2-point correlators $\langle TTT \rangle / \langle TT \rangle$ is exactly the same in the free field theory and in the interacting CFT as described (to leading order in large N) by the 11-dimensional supergravity on $AdS_{d+1} \times S^{10-d}$.

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1 Introduction and summary

In contrast to the low-energy theory on multiple D3-branes represented by $\mathcal{N} = 4$ SYM theory, the $d = 3$ and $d = 6$ superconformal theories describing a large number N of coincident M2 and M5 branes remain poorly understood (see, e.g., [1, 2] for reviews and references). The M2 brane CFT is expected to be described by an IR fixed point of $d = 3$ $U(N)$ SYM theory. The coincident M5 brane theory is expected to be a new kind of $d = 6$ CFT related to a theory of “tensionless strings” [3] (for its DLCQ description see [4]). At a generic point of moduli space, i.e. away from the M5-brane coincidence point, the M5 brane theory should be represented by N interacting tensor multiplets (see also [5] and refs. therein). Both $d = 3$ and $d = 6$ theories become free in the $N = 1$ limit, but for $N > 1$ they are still lacking a computationally useful description.

A few basic facts that follow from the properties of the corresponding classical supergravity solutions [6, 7] are:

- (i) The collective coordinates of a single M2 brane are represented by $\mathcal{N} = 8, d = 3$ scalar multiplet (8 scalars and 8 Majorana spinors) [8], while the collective coordinates of a single M5 brane by the (2,0) $d = 6$ multiplet (5 scalars, 2 Weyl spinors and an anti-selfdual 2-form) [9].
- (ii) While the entropy of N D3 branes scales in the usual N^2 way [10], the entropy of multiple M5 branes scales as N^3 and the entropy of M2 branes as $N^{3/2}$ [11]. This suggests that there is a corresponding enhancement of the number of light degrees of freedom when branes are put together. The guidance of the supergravity solution should probably be trusted more in the M5-brane case which is a non-singular background. This is the case we shall mostly concentrate on in what follows. The M-theory R^4 correction to the $d = 11$ supergravity action results [12] in a subleading $O(N)$ correction to the M5-brane entropy.
- (iii) The same $N^{3/2}$ and N^3 scalings dictated by the $d = 11$ supergravity description are found [13, 14] in the absorption rate of longitudinally polarized gravitons by M2-branes and M5-branes. The supergravity absorption cross-sections have the same form [13] as the cross-sections in the free theories of $d = 3$ and $d = 6$ multiplets, but, in addition to the $N^{3/2}$ and N^3 factors, the supergravity and the free field theory predictions differ also in the numerical coefficients [14]¹

$$\frac{\sigma_{2 \text{ sugra}}}{\sigma_{2 \text{ free f.t.}}} = \frac{4\sqrt{2}}{3\pi} N^{3/2}, \quad \frac{\sigma_{5 \text{ sugra}}}{\sigma_{5 \text{ free f.t.}}} = 4N^3. \quad (1.1)$$

¹The hep-th version of [14] used the M2 absorption cross-section which was off by the factor of 8 compared to the result of [15]. This was corrected in the published NPB version of [14].

This numerical discrepancy was absent in the D3 brane case where the precise agreement [13] between the free-theory and the supergravity absorption rates can be understood [16] as a consequence of the non-renormalization theorem for the correlator of the two stress tensors in $\mathcal{N} = 4$ SYM theory, and is intimately related [17] to the AdS/CFT correspondence [18, 17, 19].

Some $AdS_7 \times S^4$ supergravity predictions for the properties (spectrum, correlators, conformal anomaly) of the (2,0) non-abelian tensor multiplet theory were studied also in [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

Our prime aim below is to compute the 2- and 3-point correlators of the stress tensors in the free $d = 6$ tensor multiplet theory and to compare the result to the AdS supergravity correlators found in [33] ($\langle TT \rangle$) and [35] ($\langle TTT \rangle$). Since the classical supergravity absorption cross-section is expected to be related to the AdS supergravity correlator [17] and since the unitarity relates the free field theory graviton absorption amplitude to the imaginary part of the (Minkowski-space) correlator of the two stress tensors, one should find the same $4N^3$ result for ratio of the free-theory CFT and AdS 2-point correlators. We shall indeed confirm this by the explicit computation.

Moreover, we shall find that the 3-point AdS graviton correlators are again reproduced *exactly* by the $d = 6$ free theory $\langle TTT \rangle$ correlator, up to the *same* overall coefficient $4N^3!$ Let us note that the comparison of the 3-point correlators goes way beyond the absorption calculations in [14] – the AdS/CFT approach allows us to compute the multiple graviton (stress tensor) correlators in a systematic way, something that is hard to do in the context of the standard classical absorption calculations.

Similar conclusion will be reached in the case of the $d = 3$ theory: the ratios of the AdS and free $d = 3$ CFT predictions for $\langle TT \rangle$ and $\langle TTT \rangle$ will be again exactly the same as in (1.1) – $\frac{4\sqrt{2}}{3\pi} N^{3/2}$.

In the absence of a free coupling parameter, and thus of a “non-renormalization theorem” argument used in the $d = 4$ ($\mathcal{N} = 4$ SYM) case, one could expect that the 3-point correlators may have *different* structures in the free-field $d = 6$ theory and in the ‘true’ strongly coupled $d = 6$ CFT represented (to the leading order in large N) by the $d = 11$ supergravity on $AdS_7 \times S^4$. That this does not happen seems to be non-trivial. One plausible explanation is that the requirements of maximal superconformal symmetry are powerful enough to constrain the form of $\langle TTT \rangle$ so that it is reproduced by the free theory calculation, up to the overall coefficient determined by the coefficient in $\langle TT \rangle$ (cf. [34]). A possible point of view is that this coefficient may be N -dependent and should interpolate between the single free multiplet

field theory result for $N = 1$ and the non-trivial CFT (supergravity) result for $N \rightarrow \infty$.² As far as the 2-point and 3-point correlators of stress tensor multiplet states are concerned, the mysterious strongly coupled $d = 6$ CFT and the free $d = 6$ tensor multiplet theory thus happen to be in the same “universality class”.

The common overall numerical coefficient $4N^3$ of $\langle TT \rangle$ and $\langle TTT \rangle$ should have an important meaning. Since the only $d = 6$ CFT we explicitly know is the free tensor multiplet, we may try to “model” the CFT predicted, via AdS/CFT correspondence, by the supergravity M5 brane description by starting with a number of free tensor multiplets and assigning internal indices to them. In the $d = 4$ case the theory of free N^2 $\mathcal{N} = 4$ vector multiplets indeed reproduces the supergravity predictions for protected 2- and 3-point functions. The idea is then to try to fix the internal index structure of the tensor multiplet theory using the AdS supergravity results as a guide. The supergravity N^3 scaling may be formally reproduced by a “model (2,0) theory” where the 2-tensor, scalar and spinor fields carry *three* internal indices $i, j, k = 1, \dots, N$, i.e. the (selfdual) 2-form field strength is $H_{\mu\nu\lambda}^{ijk}$, etc. This theory may be describing the case when *three* (as opposed to two [3]) M5-branes are simultaneously put together.

The entropy and correlators of composite operators like the stress tensor would then scale as N^3 for large N (assuming there is no symmetry in internal indices which would reduce the coefficient N^3 by an integer factor). Since the field strength has non-zero dimension, introducing interactions would break classical conformal invariance. One may speculate that there may exist a new interacting $d = 6$ theory based on $H_{\mu\nu\lambda}^{ijk}$ (plus its superpartners) which is conformal at the *quantum* level. The hope is then that a non-trivial quantum dynamics should be responsible for the remaining factor of 4 mismatch between the free theory and the AdS supergravity predictions.³ Ignoring selfduality (and supersymmetry) constraints, it seems likely that if a consistent interacting theory of non-abelian antisymmetric tensors

²One could try to argue, as in [36], that the structure of the M-theory action on $AdS_7 \times S^4$ implies that both $\langle TT \rangle$ and $\langle TTT \rangle$ should not receive subleading $1/N$ corrections: higher-order corrections like R^4 (written in a specific “Weyl-tensor” scheme [12]) may not change the expressions for the 2-graviton and 3-graviton *AdS* correlators.

³ The assignment of the three internal indices to the antisymmetric tensor seems suggested by the heuristic explanation of the N^3 growth of the M5-brane entropy as being due to triple M5-brane connections by membranes of ‘pants’ shape (and is also related to the presence of the cubic $\int C_3 dC_3 dC_3$ term in the 11-d supergravity action, cf. [25]). Virtual triple connections are not dominant in the case of the open strings ending on D-branes (the 3-string interactions are subleading in the coupling) but are very natural for M5-branes connected by membranes: any membrane surface ending on several M5-branes may be cut into ‘pants’, with pair-wise (cylinder) connections being subleading at large N compared to the triple ones. The importance of similar triple M5 brane connections by membranes with 3 boundaries was suggested in [37] in order to explain the scaling of the entropy of the extremal 4-d black hole described by the 2555 intersecting M-brane configuration ($S \sim \sqrt{N_1 N_2 N_3}$ where N_i are charges of M5 branes).

in 6 dimensions $B_{\mu\nu}^{ijk}$ exists, its action should contain infinite number of terms with leading interaction being quartic in $B_{\mu\nu}^{ijk}$, and with $H_{\mu\nu\lambda}^{ijk}$ playing the role of an effective “coupling constant” or the structure tensor of the corresponding “soft” gauge algebra.⁴ Conformal invariance at the quantum level may be possible to achieve provided the dimension of $B_{\mu\nu}^{ijk}$ is shifted from its classical value 2.⁵

The structure of the paper is the following. To compute the free-theory correlators $\langle TT \rangle$ and $\langle TTT \rangle$ in (2,0) theory one needs to sum together the independent 2-form, scalar and spinor stress tensor contributions. The 2- and 3-point functions of stress tensors of the conformal scalar and spinor fields in an arbitrary dimension d were already computed in [39] (and refs. therein). The free k -form field theory is conformal in dimension $d = 2k + 2$. In section 2 we present the general results for the correlators of two and three stress tensors in such theory. Particular cases include the previously known $d = 4$ vector case and the new $d = 6$ 2-form case we are interested in. Our general $d = 2k + 2$ results may be useful in other contexts.

In section 3 we specialize to the case of the (2,0) tensor multiplet in $d = 6$. To find the contribution of the chiral 2-form field we employ the method similar to the one used in [40, 14] which is based on the use of the non-chiral 2-form propagator with the chirality projectors being inserted in the “vertices” (composite operators). Feynman rules obtained from actions for chiral bosons [41, 42, 45] should produce equivalent results, as it happens in the case of gravitational anomalies [43, 44]. We find that the 3-point correlator of the total free-theory stress tensor has exactly the same form as found in [35] from the AdS_7 supergravity description, apart from the overall coefficient. The latter coefficient $4N^3$ is the same as in the 2-point function. The overall scale of $\langle TTT \rangle$ is, in fact, related to that of $\langle TT \rangle$ by the conformal Ward identity.

In section 4 we repeat the same computation in the case of the free theory of 8 scalars and 8 Majorana spinors in $d = 3$ and show that again the free field theory 3-point function is the same as the AdS_4 one, apart from the overall coefficient which is the same as in the ratio of the 2-point functions or as in (1.1).

Thus, like in the $d = 4$ SYM case, in both $d = 3$ and $d = 6$ cases the ratio of the

⁴This may probably allow to avoid the “no-go” theorem of [38].

⁵Such interacting non-abelian (2,0) supersymmetric tensor multiplet theory may be a low-energy limit of a kind of “tensionless $d = 6$ string theory”. At a very speculative level, one may think of closed strings in $d = 6$ with *three* “Chan-Paton” indices which may originate from virtual membranes connecting *three* parallel M5-branes. When the distances between M5-branes reduce to zero, the membranes with 3 holes may produce a special kind of strings which somehow carry three internal indices (they may be visualized as ‘blown-up’ 3-string junctions, or “triangles”). The basic interaction at the boundary may then be described by $B_{\mu\nu}^{ijk}$.

3-point and 2-point stress tensor correlators is exactly the same in the free superconformal field theory and in the interacting CFT as described to leading order in large N by the 11-dimensional supergravity on $AdS_{d+1} \times S^{10-d}$,

$$\left(\frac{\langle TTT \rangle}{\langle TT \rangle} \right)_{free \ f.t.} = \left(\frac{\langle TTT \rangle}{\langle TT \rangle} \right)_{sugra}. \quad (1.2)$$

We expect that, as in the D3-brane case [46], similar results should hold also for all 2- and 3-point correlation functions of states belonging to the short multiplet of the stress tensor, i.e. all such correlators should be reproduced by the free field theory, apart from the same overall normalization factors.

2 Free conformal theory of k -form field in $d = 2k + 2$

We begin by considering a free nonchiral k -form field theory. The free field theory of a k -form $B_{\mu_1 \dots \mu_k}$ in the Minkowski space of $d = 2k + 2$ dimensions is described by the following action

$$S = -\frac{1}{2(k+1)!} \int d^d x H_{\mu_1 \dots \mu_{k+1}} H^{\mu_1 \dots \mu_{k+1}}, \quad (2.3)$$

where $H_{\mu_1 \dots \mu_{k+1}} = \partial_{\mu_1} B_{\mu_2 \dots \mu_{k+1}} \pm \text{cyclic permutations}$.

It is well known that due to the gauge invariance the number of physical degrees of freedom in the model is

$$n_{ph} = \frac{(2k)!}{(k!)^2}.$$

After coupling this theory to gravity in the minimal way one can check its Weyl invariance, which guarantees the conformal invariance in the flat space limit. Defining the stress-tensor as $T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}}$ one gets

$$T_{\lambda\nu} = \frac{1}{k!} H_{\lambda\mu_1 \dots \mu_k} H_\nu^{\mu_1 \dots \mu_k} - \frac{1}{2(k+1)!} \eta_{\lambda\nu} H^2, \quad (2.4)$$

which is obviously traceless.

Consider correlation functions of physical (gauge-invariant) observables which depend only on the field strength. To calculate them we need to know the propagator of the field strength. The simplest way to find it is to add to the action (2.3) the Lorentz gauge-fixing term $\frac{1}{2(k-1)!} (\partial^{\mu_1} B_{\mu_1 \mu_2 \dots \mu_k})^2$. Then the propagator of the k -form field is given by

$$\langle B_{\mu_1 \dots \mu_k}(x) B^{\nu_1 \dots \nu_k}(y) \rangle = \frac{\alpha_d k!}{(x-y)^{d-2}} \delta_{\mu_1 \dots \mu_k}^{\nu_1 \dots \nu_k},$$

where

$$\begin{aligned}\alpha_d &= \frac{1}{(d-2)\omega_{d-1}}, \\ \omega_{d-1} &= \frac{2\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} \quad (\text{volume of unit sphere } S^{d-1}), \\ \delta_{\mu_1 \dots \mu_k}^{\nu_1 \dots \nu_k} &= \delta_{[\mu_1}^{\nu_1} \dots \delta_{\mu_k]}^{\nu_k},\end{aligned}$$

with $[\dots]$ denoting antisymmetrization with unit strength. One then gets the following propagator for the gauge invariant field strength H

$$\langle H_{\mu_1 \dots \mu_{k+1}}(x) H^{\nu_1 \dots \nu_{k+1}}(y) \rangle = \frac{\alpha_d k!(d-2)(k+1)^2}{r^d} \left[\delta_{\mu_1 \dots \mu_{k+1}}^{\nu_1 \dots \nu_{k+1}} - d \hat{r}_{[\mu_1} \hat{r}^{\nu_1} \delta_{\mu_2 \dots \mu_{k+1}]^{\nu_2 \dots \nu_{k+1}}} \right] \quad (2.5)$$

with

$$r^\mu = x^\mu - y^\mu, \quad \hat{r}^\mu = \frac{r^\mu}{|r|}.$$

As is well known (see, e.g., [39] and refs. therein), the two-point function of the stress tensor in a d -dimensional conformal field theory is fixed by the conformal invariance up to a constant, and can be represented in the form

$$\langle T_{\alpha\beta}(x) T_{\gamma\delta}(y) \rangle = \frac{C_T}{r^{2d}} \mathcal{I}_{\alpha\beta,\gamma\delta}(r), \quad (2.6)$$

where

$$\mathcal{I}_{\alpha\beta,\gamma\delta} \equiv \frac{1}{2} J_{\alpha\gamma} J_{\beta\delta} + \frac{1}{2} J_{\alpha\delta} J_{\beta\gamma} - \frac{1}{d} \delta_{\alpha\beta} \delta_{\gamma\delta}, \quad J_{\alpha\beta} \equiv \delta_{\alpha\beta} - 2 \frac{r_\alpha r_\beta}{r^2}.$$

Thus all we need to know is the constant C_T . To find it we consider the correlators with the indices 1 and 2, and choose $y = 0$ and $x_\alpha = \delta_{1\alpha}$. Then a straightforward computation gives⁶

$$C_T = \frac{1}{\omega_{d-1}^2} \frac{d^2}{2} \frac{(2k)!}{(k!)^2} = \frac{1}{\omega_{d-1}^2} \frac{d^2}{2} n_{\text{ph}}. \quad (2.7)$$

As was shown in [39], the 3-point function of the stress tensor in a conformal field theory is parametrized by three independent constants, and can be written in the form [48]

$$\begin{aligned}T_{\alpha\beta,\gamma\delta,\rho\sigma} &= \frac{1}{|x-y|^d |y-z|^d |x-z|^d} \\ &\times \left[\begin{aligned} &E_{\alpha\beta,\alpha'\beta'} E_{\gamma\delta,\gamma'\delta'} E_{\rho\sigma,\rho'\sigma'} \left(\mathcal{A} J_{\alpha'\gamma'}(x-y) J_{\delta'\rho'}(y-z) J_{\beta'\sigma'}(z-x) \right. \\ &\left. + \mathcal{B} J_{\alpha'\gamma'}(x-y) J_{\delta'\rho'}(x-z) Y_{\beta'} Z_{\sigma'}(y-z)^2 + \text{cycl. perm.} \right) \end{aligned} \right]\end{aligned}$$

⁶After this paper was submitted for publication we were informed that the 2-point function for the antisymmetric tensors was computed previously in [47], with the equivalent result.

$$\begin{aligned}
& + \mathcal{C} \left(\mathcal{I}_{\alpha\beta,\gamma\delta}(x-y) \left(\frac{Z_\rho Z_\sigma}{Z^2} - \frac{1}{d} \delta_{\rho\sigma} \right) + \text{cycl. perm.} \right) \\
& + \mathcal{D} \left(E_{\alpha\beta,\alpha'\beta'} E_{\gamma\delta,\gamma'\delta'} X_\alpha Y_{\gamma'} (x-y)^2 J_{\beta'\delta'} (x-y) \left(\frac{Z_\rho Z_\sigma}{Z^2} - \frac{1}{d} \delta_{\rho\sigma} \right) + \text{cycl. perm.} \right) \\
& + \mathcal{E} \left(\frac{X_\alpha X_\beta}{X^2} - \frac{1}{d} \delta_{\alpha\beta} \right) \left(\frac{Y_\gamma Y_\delta}{Y^2} - \frac{1}{d} \delta_{\gamma\delta} \right) \left(\frac{Z_\rho Z_\sigma}{Z^2} - \frac{1}{d} \delta_{\rho\sigma} \right) \Big], \tag{2.8}
\end{aligned}$$

where

$$E_{\alpha\beta,\gamma\delta} = \frac{1}{2} \delta_{\alpha\gamma} \delta_{\beta\delta} + \frac{1}{2} \delta_{\alpha\delta} \delta_{\beta\gamma} - \frac{1}{d} \delta_{\alpha\beta} \delta_{\gamma\delta}$$

is the traceless symmetric projector and $X_\alpha, Y_\alpha, Z_\alpha$ are the conformal vectors

$$X_\alpha = \frac{(x-z)_\alpha}{(x-z)^2} - \frac{(x-y)_\alpha}{(x-y)^2}, \quad Y_\alpha = \frac{(y-x)_\alpha}{(y-x)^2} - \frac{(y-z)_\alpha}{(y-z)^2}, \quad Z_\alpha = \frac{(z-y)_\alpha}{(z-y)^2} - \frac{(z-x)_\alpha}{(z-x)^2}.$$

There are two linear relations [39, 48] between the 5 constants $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}$ entering (2.8) that allow to express two of them in terms of the remaining three

$$\begin{aligned}
(d^2 - 4)\mathcal{A} + (d+2)\mathcal{B} - 4d\mathcal{C} - 2\mathcal{D} &= 0, \\
(d-2)(d+4)\mathcal{B} - 2d(d+2)\mathcal{C} + 8\mathcal{D} - 4\mathcal{E} &= 0. \tag{2.9}
\end{aligned}$$

We choose \mathcal{A}, \mathcal{B} and \mathcal{C} as the three independent constants. To find them we take $z = 0$, $y_\alpha = \delta_{\alpha 1}$, $x_\alpha = 2\delta_{\alpha 1}$ and consider the following three correlators

$$\begin{aligned}
\langle T_{12}(x)T_{13}(y)T_{23}(z) \rangle &= 2^{-d} \tau \\
\langle T_{23}(x)T_{24}(y)T_{34}(z) \rangle &= 2^{-d} t \\
\langle T_{12}(x)T_{12}(y)T_{22}(z) \rangle &= 2^{-d} (\rho + 2\tau),
\end{aligned}$$

where τ, t, ρ are the coefficients in the collinear frame from eq.(4.21) of [39]. They are related to \mathcal{A}, \mathcal{B} and \mathcal{C} as follows (see eqs.(4.25) and (3.21) from [39], and the footnote on p.21 of [48])

$$\mathcal{A} = 8t \tag{2.10}$$

$$\mathcal{B} = 8(\tau + t) \tag{2.11}$$

$$\mathcal{C} = \frac{2}{d+1} [d(\rho + \tau) + (d^2 + d - 4)t]. \tag{2.12}$$

A straightforward calculation of the correlators gives

$$\tau = - \left(\frac{k+1}{\omega_{d-1}} \right)^3 \cdot \frac{(2k)!}{(k!)^2}, \quad t = -2 \left(\frac{k+1}{\omega_{d-1}} \right)^3 \cdot \frac{(2k-2)!}{k!(k-1)!}, \quad \rho = 0.$$

Using formulas (2.10)–(2.12) we thus obtain the values of the three independent constants

$$\mathcal{A} = -\frac{8}{2k-1} \left(\frac{k+1}{\omega_{2k+1}} \right)^3 \cdot n_{ph} \quad (2.13)$$

$$\mathcal{B} = -\frac{16k}{2k-1} \left(\frac{k+1}{\omega_{2k+1}} \right)^3 \cdot n_{ph} \quad (2.14)$$

$$\mathcal{C} = -\frac{8}{2k-1} \left(\frac{k+1}{\omega_{2k+1}} \right)^3 \cdot n_{ph} \quad (2.15)$$

A check of these formulas is provided by the conformal Ward identity relating the 2-point and 3-point correlators [48]

$$C_T = \frac{\omega_{d-1}}{d(d+2)} \left[\frac{1}{2}(d+2)(d-1)\mathcal{A} - \mathcal{B} - 2(d+1)\mathcal{C} \right]. \quad (2.16)$$

In terms of t , τ , ρ this identity can be rewritten in the form

$$C_T = -4\omega_{d-1} \left(\frac{1}{d+2}\rho + \frac{1}{d}\tau \right).$$

One can easily check the validity of this equation using (2.7). It is worth noting that the conformal Ward identity relation does not involve the constant t .

In the case of an even-rank $k = 2l$ field B the stress tensor (2.4) admits factorization into the sum of the “left” and “right” stress tensors depending on the anti selfdual and selfdual components of the field $H = dB$ respectively:

$$T_{\alpha\beta} = T_{\alpha\beta}^- + T_{\alpha\beta}^+, \quad T_{\alpha\beta}^\pm = \frac{1}{k!} H_{\alpha\mu\nu}^\pm H_\beta^{\pm\mu\nu}. \quad (2.17)$$

Moreover, one can easily show that the correlator of H^- and H^+ is proportional to the delta-function. We assume a regularization where contact terms proportional to delta functions are omitted, as this is consistent with conformal invariance, and so we can set both $\langle H^-(x)H^+(y) \rangle$ and $\langle T_{\alpha\beta}^-(x)T_{\gamma\delta}^+(y) \rangle$ to zero. This serves as a justification for the following prescription (essentially equivalent to the one originally used in [40] and also in [14]) to compute the correlation functions of operators depending on the field strength in a chiral model: one may use the non-chiral propagator (2.5) while replacing the field strength H by its (anti)selfdual part in the composite operators. In the case of the 2-point and 3-point correlation functions of the stress tensor $T_{\alpha\beta}^\pm$ the correlator of $T_{\alpha\beta}^-$ and $T_{\alpha\beta}^+$ vanishes, and thus the chiral correlators are equal to $\frac{1}{2}$ of the 2-point and 3-point “non-chiral” correlators of the full $T_{\alpha\beta}$.

3 The (2,0) tensor multiplet in $d = 6$

The (2,0) tensor multiplet in the 6-dimensional Minkowski space consists of 5 scalars X^i , 2 Weyl fermions ψ_L^I and an antisymmetric tensor $B_{\alpha\beta}$ with anti-selfdual strength. Covariant lagrangian descriptions of the 2-form part of the model exist [42], but are hard to work with at the quantum level, since one cannot easily implement a covariant gauge fixing for the gauge symmetry that gives the (anti)selfduality constraint. However, it is sufficient for our aims to use a Lagrangian containing a non-chiral 2-form with the prescription of projecting out its selfdual part in the relevant composite operators. Thus, the stress tensor of the system is given by the sum of the stress tensors of the fields

$$T_{\alpha\beta} = T_{\alpha\beta}^{H^-} + T_{\alpha\beta}^X + T_{\alpha\beta}^\psi , \quad (3.18)$$

with

$$T_{\alpha\beta}^{H^-} = \frac{1}{2} H_{\alpha\mu\nu}^- H_\beta^{-\mu\nu} \quad (3.19)$$

$$T_{\alpha\beta}^X = \partial_\alpha X^i \partial_\beta X^i - \frac{1}{5} \partial_\alpha \partial_\beta (X^i X^i) - \frac{1}{10} \eta_{\alpha\beta} \partial_\mu X^i \partial^\mu X^i \quad (3.20)$$

$$T_{\alpha\beta}^\psi = -\frac{i}{4} \bar{\psi}_L^I (\gamma_\alpha \partial_\beta + \gamma_\beta \partial_\alpha) \psi_L^I + \frac{i}{4} (\partial_\beta \bar{\psi}_L^I \gamma_\alpha \psi_L^I + \partial_\alpha \bar{\psi}_L^I \gamma_\beta \psi_L^I) , \quad (3.21)$$

where $i = 1, \dots, 5$ and ψ_L^I is the left component of a Dirac fermion. We took into account that the stress tensor for H can be represented in the form

$$T_{\alpha\beta}^H = \frac{1}{2} H_{\alpha\mu\nu} H_\beta^{\mu\nu} - \frac{1}{12} \eta_{\alpha\beta} H_{\mu\nu\rho}^2 = T_{\alpha\beta}^{H^-} + T_{\alpha\beta}^{H^+} .$$

The stress tensor (3.18) coincides, up to a factor, with the stress tensor for the (2,0) tensor multiplet found in [49] by using a different method. Note that the scalar stress tensor contains the improvement term as needed for conformal invariance (on-shell tracelessness).⁷

The 2- and 3-point correlation functions of the stress tensors of the free scalar and spinor theories in arbitrary number d of dimensions were previously computed in [39]. We extend those results by including the contributions of k -forms (with the understanding that this additional contribution is present only in the suitable “conformal” $d = 2k + 2$ dimensions). Thus, the corresponding constants C_T , \mathcal{A} , \mathcal{B} and \mathcal{C} in $\langle TT \rangle$ (2.6) and $\langle TTT \rangle$ (2.8) are

⁷The improvement term originates from the $\frac{(d-2)}{4(d-1)} RX^2$ term on a curved $d = 6$ background. This term was not included in the absorption calculation in [14] since it gives zero contribution to the tree-level 3-point amplitude with on-shell graviton. However, this term is crucial for the scalar stress tensor correlators to have the canonical CFT form described in the previous section.

given by

$$\begin{aligned}
C_T &= \frac{1}{\omega_{d-1}^2} \left(\frac{d}{d-1} n_S + \frac{d}{2} \tilde{n}_F + \frac{d^2}{2} \tilde{n}_B \right) \\
\mathcal{A} &= -\frac{1}{\omega_{d-1}^3} \left[-\frac{d^3}{(d-1)^3} n_S + \frac{d^3}{d-3} \tilde{n}_B \right] \\
\mathcal{B} &= -\frac{1}{\omega_{d-1}^3} \left[\frac{(d-2)d^3}{(d-1)^3} n_S + \frac{d^2}{2} \tilde{n}_F + \frac{(d-2)d^3}{d-3} \tilde{n}_B \right] \\
\mathcal{C} &= -\frac{1}{\omega_{d-1}^3} \left[\frac{(d-2)^2 d^2}{4(d-1)^3} n_S + \frac{d^2}{4} \tilde{n}_F + \frac{d^3}{d-3} \tilde{n}_B \right]. \tag{3.22}
\end{aligned}$$

Here $\tilde{n}_F = \text{tr } I \cdot n_F$ (tr is the Dirac spinor trace) and $\tilde{n}_B = \frac{(2k)!}{(k!)^2} n_B$, with n_S , n_F and n_B the numbers of scalars, Dirac spinors and k -forms, respectively. For Weyl fermions and chiral k -forms one should halve the corresponding numbers⁸.

Summing up the contributions of the anti-selfdual tensor, 5 scalars and 2 Weyl fermions, we finally obtain the following values of the four basic constants C_T , \mathcal{A} , \mathcal{B} and \mathcal{C} for the (2,0) tensor multiplet in $d = 6$

$$C_T = \frac{84}{\pi^6}, \tag{3.23}$$

$$\mathcal{A} = -\frac{2^6 \cdot 3^4}{5^2 \pi^9}, \quad \mathcal{B} = -\frac{181 \cdot 2^4 \cdot 3^2}{5^2 \pi^9}, \quad \mathcal{C} = -\frac{59 \cdot 2^3 \cdot 3^3}{5^2 \pi^9}. \tag{3.24}$$

We are now going to compare these *free field theory* results with the ones obtained from the 11-dimensional supergravity on the $AdS_7 \times S^4$ background describing the near-horizon limit of N coincident M5-branes. In units in which the radii of the two spaces are $R_{AdS} = 1$ and $R_{S^4} = \frac{1}{2}$ the 11-dimensional gravitational constant ($S = -\frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{g}R + \dots$) is [11, 37, 12]

$$\frac{1}{2\kappa_{11}^2} = \frac{2N^3}{\pi^5}. \tag{3.25}$$

Performing the dimensional reduction to seven dimensions, we get the 7-dimensional gravitational constant ($\text{Vol}(S^4) = \omega_4(\frac{1}{2})^4$)

$$\frac{1}{2\kappa_7^2} = \frac{N^3}{3\pi^3}. \tag{3.26}$$

In general, the constant C_T in (2.6) calculated by using the AdS_{d+1} supergravity description is given by [33]

$$C_T^{(ads)} = \frac{1}{2\kappa_{d+1}^2} \cdot \frac{2d(d+1)\Gamma(d)}{(d-1)\Gamma(d/2)\pi^{d/2}}. \tag{3.27}$$

⁸Except in $d = 2$ where the ϵ tensor gives a new structure in the stress tensor 2-point function and the chiral splitting happens in a different way.

We assumed that the coupling of $T_{\alpha\beta}(x)$ with $h_{\alpha\beta}(x)$ at the boundary of AdS_{d+1} has the standard form $\int d^d x \frac{1}{2} T_{\alpha\beta}(x) h^{\alpha\beta}(x)$. Taking into account the value of the gravitational constant κ_7 , we find that for $d = 6$

$$C_T^{(ads)} = 4N^3 \cdot \frac{84}{\pi^6}. \quad (3.28)$$

This differs by the factor $4N^3$ from the value (3.23) obtained from the free field theory. This factor coincides with the one obtained in [14] by comparing the absorption cross-sections calculated using the $d = 11$ supergravity and the free world-volume field theory descriptions. The constants \mathcal{A} , \mathcal{B} and \mathcal{C} were computed in the general case of AdS_{d+1} gravity in [35]. For $d = 6$ they are given by

$$\begin{aligned} \mathcal{A}^{(ads)} &= -4N^3 \cdot \frac{2^6 \cdot 3^4}{5^2 \pi^9}, \\ \mathcal{B}^{(ads)} &= -4N^3 \cdot \frac{181 \cdot 2^4 \cdot 3^2}{5^2 \pi^9}, \\ \mathcal{C}^{(ads)} &= -4N^3 \cdot \frac{59 \cdot 2^3 \cdot 3^3}{5^2 \pi^9}. \end{aligned} \quad (3.29)$$

Comparing these values with the ones obtained in the (2,0) tensor multiplet model (3.24) we see that they again differ by the same factor $4N^3$. The overall scale of these three constants is, of course, determined by C_T in view of the conformal Ward identity (2.16), but it is quite remarkable that the *relative* scales of $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are exactly the same in the free field theory and in the AdS_7 supergravity!

This suggests that all 2- and 3-point correlation functions of the states from the stress tensor short multiplet in the interacting (2,0) superconformal field theory coincide (in the large N limit) with the corresponding ones in the free theory of $4N^3$ (2,0) tensor multiplets. As a check of this expectation we can calculate the 2- and 3-point functions of the lowest chiral primary operators

$$\mathcal{O}^I = \mathcal{C}_{ij}^I X^i X^j, \quad (3.30)$$

where \mathcal{C}_{ij}^I is a symmetric traceless tensor, while the index I denotes a complete basis of such tensors, and compare them with the ones obtained from the 11-dimensional supergravity in [30]. Normalizing the operators in such a way that the ratio of the 2-point functions calculated in AdS_7 gravity and in the free theory is $4N^3$, we find again that the ratio of the 3-point functions is also $4N^3$.

4 M2-brane case: AdS_4 – free $d = 3$ CFT comparison

In this section we compare the 2- and 3-point functions of the stress tensor of the effective action for N M2-branes calculated from the 11-dimensional $AdS_4 \times S^7$ supergravity and in the 3-dimensional free field theory of 8 scalars and 8 Majorana fermions.

We choose units in which $R_{AdS} = 1$, so that $R_{S^7} = 2$, and thus

$$\frac{1}{2\kappa_{11}^2} = \frac{N^{\frac{3}{2}}}{2^9 \sqrt{2} \pi^5}, \quad \frac{1}{2\kappa_4^2} = \frac{N^{\frac{3}{2}}}{12\sqrt{2}\pi}. \quad (4.31)$$

Then from (3.27) and [35] we get

$$C_T^{(ads)} = \frac{4}{\pi^3} \cdot \frac{N^{\frac{3}{2}}}{\sqrt{2}}, \quad (4.32)$$

$$\mathcal{A}^{(ads)} = -\frac{27}{8\pi^4} \cdot \frac{N^{\frac{3}{2}}}{\sqrt{2}}, \quad \mathcal{B}^{(ads)} = -\frac{57}{8\pi^4} \cdot \frac{N^{\frac{3}{2}}}{\sqrt{2}}, \quad \mathcal{C}^{(ads)} = -\frac{99}{32\pi^4} \cdot \frac{N^{\frac{3}{2}}}{\sqrt{2}}. \quad (4.33)$$

By using (3.22) we obtain the following values of the 4 basic constants in the free $d = 3$ conformal field theory of 8 scalars and 8 Majorana fermions

$$C_T^{(free)} = \frac{3}{2\pi^2}, \quad (4.34)$$

$$\mathcal{A}^{(free)} = \frac{27}{64\pi^3}, \quad \mathcal{B}^{(free)} = -\frac{63}{64\pi^3}, \quad \mathcal{C}^{(free)} = -\frac{81}{256\pi^3}. \quad (4.35)$$

Although the constants (4.33) and (4.35) look different, this does not mean that the corresponding 3-point functions differ too. In fact, as was shown in [39], in *three* dimensions there are only *two* independent conformal tensor structures in $\langle TTT \rangle$, and, therefore, only two linear combinations of the constants in (2.8) have got an invariant meaning. These two independent constants may be expressed in terms of \mathcal{A} , \mathcal{B} and \mathcal{C} as follows

$$\mathcal{P} = 4\mathcal{A} + 3\mathcal{B} - 14\mathcal{C}, \quad \mathcal{Q} = \mathcal{A} - 2\mathcal{C}. \quad (4.36)$$

Then a straightforward calculation gives

$$\frac{C_T^{(ads)}}{C_T^{(free)}} = \frac{\mathcal{P}^{(ads)}}{\mathcal{P}^{(free)}} = \frac{\mathcal{Q}^{(ads)}}{\mathcal{Q}^{(free)}} = \frac{4\sqrt{2}}{3\pi} N^{\frac{3}{2}}. \quad (4.37)$$

Using the results of [30] and a simple free theory computation, one can also check that the ratio of the 2- and 3-point functions of the properly normalized chiral primary operators

(3.30) is again given by the same factor (4.37), which coincides also with the one obtained in the comparison of graviton absorption cross-sections in [14]. It seems natural to expect that all 2- and 3-point functions of operators from the short multiplet of the stress tensor in the effective theory of N M2-branes coincide, up to this overall factor, with the ones computed in the $\mathcal{N} = 8$ free field theory. The meaning of this irrational proportionality constant (which looks somewhat ugly compared to $4N^3$ in the $d = 6$ case) remains unclear.

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References

- [1] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” hep-th/9905111.
- [2] N. Seiberg, “Notes on theories with 16 supercharges,” Nucl. Phys. Proc. Suppl. **67**, 158 (1998) hep-th/9705117.
- [3] A. Strominger, “Open P-Branes,” Phys. Lett. **B383**, 44 (1996), hep-th/9512059; E. Witten, “Some comments on string dynamics,” hep-th/9507121; N. Seiberg and E. Witten, “Comments on String Dynamics in Six Dimensions,” Nucl. Phys. **B471**, 121 (1996), hep-th/9603003.
- [4] O. Aharony, M. Berkooz, S. Kachru, N. Seiberg and E. Silverstein, “Matrix description of interacting theories in six dimensions,” Adv. Theor. Math. Phys. **1**, 148 (1998), hep-th/9707079; O. Aharony, M. Berkooz and N. Seiberg, “Light-cone description of (2,0)

superconformal theories in six dimensions,” Adv. Theor. Math. Phys. **2**, 119 (1998), hep-th/9712117.

- [5] O. Ganor and L. Motl, “Equations of the (2,0) theory and knitted fivebranes,” JHEP **05**, 009 (1998) hep-th/9803108; O.J. Ganor, “Six-dimensional tensionless strings in the large N limit,” Nucl. Phys. **B489**, 95 (1997), hep-th/9605201.
- [6] M.J. Duff and K.S. Stelle, “Multimembrane solutions of $D = 11$ supergravity,” Phys. Lett. **B253**, 113 (1991).
- [7] R. Güven, “Black p-brane solutions of $D = 11$ supergravity”, Phys. Lett. **B276**, 49 (1992).
- [8] E. Bergshoeff, E. Sezgin and P.K. Townsend, “Supermembranes And Eleven- Dimensional Supergravity,” Phys. Lett. **B189**, 75 (1987).
- [9] G.W. Gibbons and P.K. Townsend, “Vacuum interpolation in supergravity via super p-branes,” Phys. Rev. Lett. **71**, 3754 (1993), hep-th/9307049; D.M. Kaplan and J. Michelson, “Zero Modes for the $D=11$ Membrane and Five-Brane,” Phys. Rev. **D53**, 3474 (1996), hep-th/9510053.
- [10] S.S. Gubser, I.R. Klebanov and A.W. Peet, “Entropy and Temperature of Black 3-Branes,” Phys. Rev. **D54**, 3915 (1996), hep-th/9602135.
- [11] I.R. Klebanov and A.A. Tseytlin, “Entropy of Near-Extremal Black p-branes,” Nucl. Phys. **B475**, 164 (1996), hep-th/9604089.
- [12] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, “Coupling constant dependence in the thermodynamics of $N = 4$ supersymmetric Yang-Mills theory,” Nucl. Phys. **B534**, 202 (1998), hep-th/9805156.
- [13] I.R. Klebanov, “World-volume approach to absorption by non-dilatonic branes,” Nucl. Phys. **B496**, 231 (1997), hep-th/9702076.
- [14] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, “String theory and classical absorption by three-branes,” Nucl. Phys. **B499**, 217 (1997), hep-th/9703040.
- [15] R. Emparan, “Absorption of scalars by extended objects,” Nucl. Phys. **B516**, 297 (1998), hep-th/9706204.
- [16] S.S. Gubser and I.R. Klebanov, “Absorption by branes and Schwinger terms in the world volume theory,” Phys. Lett. **B413**, 41 (1997), hep-th/9708005.
- [17] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. **B428**, 105 (1998), hep-th/9802109.
- [18] J. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. **2**, 231 (1998), hep-th/9711200.

- [19] E. Witten, “Anti-de Sitter space and holography,” *Adv. Theor. Math. Phys.* **2**, 253 (1998), hep-th/9802150.
- [20] O. Aharony, Y. Oz and Z. Yin, “M-theory on $AdS_p \times S^{11-p}$ and superconformal field theories,” *Phys. Lett.* **B430**, 87 (1998), hep-th/9803051.
- [21] S. Minwalla, “Particles on $AdS(4/7)$ and primary operators on $M(2/5)$ brane worldvolumes,” *JHEP* **10**, 002 (1998), hep-th/9803053.
- [22] R.G. Leigh and M. Rozali, “The large N limit of the (2,0) superconformal field theory,” *Phys. Lett.* **B431**, 311 (1998), hep-th/9803068.
- [23] E. Halyo, “Supergravity on $AdS(4/7) \times S(7/4)$ and M branes,” *JHEP* **04**, 011 (1998), hep-th/9803077.
- [24] M. Henningson and K. Skenderis, “The holographic Weyl anomaly,” *JHEP* **07**, 023 (1998), hep-th/9806087.
- [25] J.A. Harvey, R. Minasian and G. Moore, “Non-abelian tensor-multiplet anomalies,” *JHEP* **09**, 004 (1998), hep-th/9808060.
- [26] H. Awata and S. Hirano, “ $AdS(7)/CFT(6)$ correspondence and matrix models of M5-branes,” *Adv. Theor. Math. Phys.* **3**, 147 (1999), hep-th/9812218.
- [27] C.R. Graham and E. Witten, “Conformal anomaly of submanifold observables in AdS/CFT correspondence,” *Nucl. Phys.* **B546**, 52 (1999), hep-th/9901021; M. Henningson and K. Skenderis, “Weyl anomaly for Wilson surfaces,” *JHEP* **06**, 012 (1999), hep-th/9905163.
- [28] R. Corrado, B. Florea and R. McNees, “Correlation functions of operators and Wilson surfaces in the $d = 6$, (2,0) theory in the large N limit,” *Phys. Rev.* **D60**, 085011 (1999), hep-th/9902153.
- [29] H. Nastase, D. Vaman and P. van Nieuwenhuizen, “Consistent nonlinear KK reduction of 11d supergravity on $AdS(7) \times S(4)$ and selfduality in odd dimensions,” hep-th/9905075.
- [30] F. Bastianelli and R. Zucchini, Three-Point Functions of Chiral Primary Operators in $d = 3$, $\mathcal{N} = 8$ and $d = 6$, $\mathcal{N} = (2, 0)$ SCFT at Large N , hep-th/9907047.
- [31] F. Bastianelli and R. Zucchini, “Three point functions for a class of chiral operators in maximally supersymmetric CFT at large N,” hep-th/9909179.
- [32] M. Nishimura and Y. Tanii, “Local symmetries in the $AdS(7)/CFT(6)$ correspondence,” hep-th/9910192.
- [33] H. Liu and A.A. Tseytlin, “D = 4 super Yang-Mills, D = 5 gauged supergravity, and D = 4 conformal supergravity,” *Nucl. Phys.* **B533**, 88 (1998), hep-th/9804083.

- [34] J. Park, “Superconformal symmetry in six-dimensions and its reduction to four,” Nucl. Phys. **B539**, 599 (1999), hep-th/9807186; “Superconformal symmetry and correlation functions,” hep-th/9903230; “Superconformal symmetry in three-dimensions,” hep-th/9910199.
- [35] G. Arutyunov and S. Frolov, “Three-point Green function of the stress-energy tensor in the AdS/CFT correspondence,” Phys. Rev. **D60**, 026004 (1999), hep-th/9901121.
- [36] T. Banks and M.B. Green, “Non-perturbative effects in $AdS_5 \times S^5$ string theory and $d = 4$ SUSY Yang-Mills,” JHEP **05**, 002 (1998), hep-th/9804170.
- [37] I.R. Klebanov and A.A. Tseytlin, “Intersecting M-branes as Four-Dimensional Black Holes,” Nucl. Phys. **B475**, 179 (1996), hep-th/9604166.
- [38] X. Bekaert, M. Henneaux and A. Sevrin, “Deformations of chiral two-forms in six dimensions,” hep-th/9909094; X. Bekaert, “Interactions of chiral two-forms,” hep-th/9911109.
- [39] H. Osborn and A.C. Petkou, “Implications of conformal invariance in field theories for general dimensions,” Ann. Phys. **231**, 311 (1994), hep-th/9307010.
- [40] L. Alvarez-Gaume and E. Witten, “Gravitational Anomalies,” Nucl. Phys. **B234**, 269 (1984).
- [41] M. Henneaux and C. Teitelboim, “Dynamics Of Chiral (Selfdual) P Forms,” Phys. Lett. **B206**, 650 (1988).
- [42] P. Pasti, D. Sorokin and M. Tonin, “On Lorentz invariant actions for chiral p-forms,” Phys. Rev. **D55**, 6292 (1997), hep-th/9611100.
- [43] F. Bastianelli and P. van Nieuwenhuizen, “Gravitational Anomalies From The Action For Selfdual Antisymmetric Tensor Fields In $(4k+2)$ -Dimensions,” Phys. Rev. Lett. **63**, 728 (1989).
- [44] K. Lechner, “Self-dual tensors and gravitational anomalies in $4n+2$ dimensions,” Nucl. Phys. **B537**, 361 (1999), hep-th/9808025.
- [45] C. Van Den Broeck and K. Van Hoof, “Batalin-Vilkovisky gauge-fixing of a chiral two-form in six dimensions,” hep-th/9905117.
- [46] S. Lee, S. Minwalla, M. Rangamani and N. Seiberg, “Three-point functions of chiral operators in $D = 4$, $N = 4$ SYM at large N ,” Adv. Theor. Math. Phys. **2**, 697 (1998), hep-th/9806074.
- [47] D. Anselmi, “Higher-spin current multiplets in operator-product expansions,” hep-th/9906167.
- [48] J. Erdmenger and H. Osborn, “Conserved currents and the energy-momentum tensor in conformally invariant theories for general dimensions,” Nucl. Phys. **B483**, 431 (1997), hep-th/9605009.

- [49] E. Bergshoeff, E. Sezgin and A. Van Proeyen, (2,0) Tensor Multiplets and Conformal Supergravity in D=6, hep-th/9904085.