DISCUSSION

A comparison of the bearing capacity of flat and conical circular foundations on sand


T. Orr. Trinity College Dublin, Ireland

The authors have presented an interesting paper concerning a fundamental problem for geotechnical engineers: the bearing capacity of circular foundations on sand. In this paper the authors have included an equation for the factor $N_x$ in the bearing capacity equation for circular foundations on the ground surface, which has been determined from the results of centrifuge tests.

The authors’ paper is particularly relevant at this time as the new European Standard for Geotechnical Design, EN 1997-1: Eurocode 7 Geotechnical Design (CEN, 2004) is due to be implemented in March 2010 by the national standards organisations in the member countries of the Comité Européen de Normalisation (CEN). EN 1997-1 provides the principles for geotechnical design as code text and some calculation models in informative annexes, that is not as mandatory code text. One of these calculation models is an analytical method for calculating the bearing capacity of a spread foundation. As no exact solution exists for $N_x$, a number of equations have been proposed for $N_x$ since the bearing capacity equation was first introduced by Terzaghi (1943). The equation in EN 1997-1 for $N_x$ for a strip foundation with a rough base is

$$N_x = 2(N_q - 1)\tan \phi'$$

where

$$N_q = e^{\tan \phi'} \tan (45 + \phi'/2)$$

This equation for $N_x$ was obtained by Vesic (1973) and is based on the equation for $N_q$ in EN 1997-1, which was originally derived by Prandtl (1920). This later equation is an exact solution.

From the results of centrifuge model tests the authors provide the following equation for $N_x$ for a circular foundation with a flat rough base

$$N_x = 0.0286e^{0.2109\phi'}$$

It should be noted that this equation for $N_x$ is for a circular foundation and hence differs from the $N_q$ equations reported by most other authors, who are for strip foundations. An additional shape factor, $s_p$, is traditionally applied to $N_q$ to account for the foundation shape. A number of papers about the bearing capacity of spread foundations on sand have been published in Géotechnique over the years, for example Meyehof (1951), De Beer (1970), Cassidy and Houlsby (2002) and Lyamin et al. (2007). Some of the equations that have been proposed for $N_x$ and that differ from the EN 1997-1 equation include

$$N_x = 1.5(N_q - 1)\tan \phi' \text{ by Brinch Hansen (1970)}$$

$$N_x = (N_q - 1)\tan (1.4\phi') \text{ by Meyerhof (1963)}$$

Martin (2005) used the method of characteristics to obtain what he stated appear to be exact solutions for $N_x$ for various design situations. Salgado (2008) proposed the following equation that provides a good fit to Martin’s results

$$N_x = (N_q - 1)\tan (1.32\phi')$$

According to Smoltczyk (U. Smoltczyk (2008), personal communication), equation (7) was adopted for $N_x$ in EN 1997-1 because it had been used in DIN 4017 as an updating of earlier approximations, such as Brinch Hansen’s (1970) equation, on the basis of large-scale tests on sand carried out by DEGBO in Berlin (Muhs, 1971).

The authors’ equation for $N_x$ for a circular foundation can be converted to an equation for $N_x$ for a strip foundation by applying the following shape factor correction proposed by Lyamin et al. (2007)

$$s_p = 0.0336\phi'(1 + 0.002\phi')$$

so that

$$N_x = s_p N_x / (s_p + 1)$$

The $N_x$ values for a strip foundation for $\phi'$ ranging from $20^\circ$ to $40^\circ$ obtained using the authors’ converted equation (13) together with those obtained using the EN 1997-1 equation and the Meyerhof, Brinch Hansen and Salgado equations are tabulated in Table 3 and plotted in Fig. 11 for comparison. The $N_x$ values in Table 3 and Fig. 11 show that, for a strip foundation of $20^\circ \leq \phi' \leq 40^\circ$

(a) the EN 1997-1 $N_x$ values exceed all the other predicted $N_x$ values and hence, as previously shown by Orr (2008), are unconservative. For example, the EN 1997-1 $N_x$ value exceeds the authors’ value by 41% for $\phi' = 20^\circ$, reducing to 27% for $\phi' = 40^\circ$

(b) the authors’ values are very similar to Meyerhof’s values

(c) Salgado’s values based on Martin are similar to Brinch Hansen’s values and are more conservative than the authors’ values for $\phi' > 30^\circ$.

In the case of a circular foundation, the authors’ $N_x$ values are compared with the EN 1997-1 values obtained by multiplying equation (7) by the EN 1997-1 shape factor for a circular foundation, which is $s_p = 0.7$. These $N_x$ values are also given in Table 3 and plotted in Fig. 12. Fig. 12 shows that for $20 \leq \phi' \leq 40^\circ$, the EN 1997-1 $N_x$ values for a circular foundation exceed the authors’ values for $\phi' < 27.5^\circ$, for example by 42% for $\phi' = 20^\circ$, but are less than the authors’ values for $\phi' > 27.5^\circ$, for example by 44% for $\phi' = 40^\circ$. These differences for circular foundations are not as significant as those for strip foundations because for $\phi' > 27.5^\circ$ they are conservative, while for values of $\phi' < 27.5^\circ$, although they are unconservative, they are not normally relevant for foundations on sand.

The comparison between the authors’ and EN 1997-1 $N_x$ values for both circular and strip foundations shows that significant differences can occur and that the EN 1997-1...
Table 3. $N_q$ values for strip and circular foundations

<table>
<thead>
<tr>
<th>$\phi'$ (°)</th>
<th>$N_q$</th>
<th>$N_q$ – strip foundation</th>
<th>$N_q$ – circle foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_q$</td>
<td>EN 1997-1</td>
<td>Brinch Hansen</td>
</tr>
<tr>
<td>20</td>
<td>6.40</td>
<td>2.78</td>
<td>3.93</td>
</tr>
<tr>
<td>22.5</td>
<td>8.23</td>
<td>4.16</td>
<td>5.99</td>
</tr>
<tr>
<td>25</td>
<td>10.66</td>
<td>6.32</td>
<td>9.01</td>
</tr>
<tr>
<td>27.5</td>
<td>13.94</td>
<td>9.69</td>
<td>13.47</td>
</tr>
<tr>
<td>30</td>
<td>18.40</td>
<td>14.98</td>
<td>20.09</td>
</tr>
<tr>
<td>32.5</td>
<td>24.58</td>
<td>23.31</td>
<td>30.05</td>
</tr>
<tr>
<td>35</td>
<td>33.30</td>
<td>36.50</td>
<td>45.23</td>
</tr>
<tr>
<td>37.5</td>
<td>45.81</td>
<td>57.45</td>
<td>68.77</td>
</tr>
<tr>
<td>40</td>
<td>64.20</td>
<td>90.83</td>
<td>106.05</td>
</tr>
</tbody>
</table>

Fig. 11. $N_q$ values against $\phi'$ for a strip foundation

Fig. 12. $N_q$ values against $\phi'$ for a circular foundation

The discussion is concerned with the expression for the bearing capacity factor $N_q$ for rough circular footings that we provided (equation (2) in both the original paper and the discussion). It highlights the difference between this and other published solutions including in particular the equivalent expression adopted in Eurocode 7, which is based on a shape factor applied to an empirical expression for bearing resistance in their national annexes.

REFERENCES


Authors’ reply

We thank the discussers for their interest in our paper. The discussion is concerned with the expression for the bearing capacity factor $N_q$ for rough circular footings that we provided (equation (2) in both the original paper and the discussion). It highlights the difference between this and other published solutions including in particular the equivalent expression adopted in Eurocode 7, which is based on a shape factor applied to an empirical expression for $N_q$ for a strip footing. Our original paper was concerned only with circular foundations, but the discussion considers both plane strain (strip footing) and circular geometry. In this response we have therefore adopted the notation $N_q$-strip and $N_q$-circle to differentiate between these cases, which are linked by the shape factor, $N_q = N_q$-circle/$N_q$-strip (with similar notation applying for $N_q$ and $N_q$).
First a small clarification – the discusser perhaps implies that equation (2) was derived from centrifuge model tests but in fact it was fitted to theoretical lower bound plasticity solutions calculated using Martin’s analysis of bearing capacity (ABC) program (Martin 2003; 2004). The benefit of fitting an expression to these results is that it allows our procedure for capturing the stress-dependency of friction angle to be performed iteratively without the need for a look up table linking friction angle to \( N_{\text{circle}} \). The discrepancy between the theoretical values and our fitted expression is less than 5% over the range of friction angle 25° < \( \phi \) < 45°.

The status of the \( N_{\text{circle}} \) values on which our equation (2) is based is that they are lower bound plasticity solutions for which the consistent upper bound provides equal resistance (C. M. Martin (2009), personal communication). In order for the solutions to be established as formally exact, it must be demonstrated that the lower bound stress field can be extended without violating the yield criterion. This has not yet been done, but Lyamin et al. (2007) have produced extensible lower bound solutions by way of finite element limit analysis for a 24-sided polygon (nominally a circle) that are within a few percent of the ABC solutions, so it is highly likely that the ABC solutions will be extensible. For practical purposes the \( N_{\text{circle}} \) values on which equation (2) is based are definitive. The values of \( N_{\text{circle}} \) calculated by ABC are formally exact. The consistent upper bound mechanism provides the same resistance and the lower bound stress field is extensible (Martin, 2005).

There are idealisations associated with exact bearing capacity factors derived from plasticity theory. The soil is assumed to obey normality and progressive failure is neglected. Closed-form expressions for \( N_{\text{strip}} \) and \( N_{\text{circle}} \), based on these idealisations have long been established as exact (Shield, 1954), are generally accepted in practice, and are provided in the Eurocode. Therefore, for consistency we should also use expressions for \( N_{\text{strip}} \) and \( N_{\text{circle}} \) (and indeed \( N_{\text{circle}} \) and \( N_{\text{circle}} \) that have the same basis. Where a closed-form equation does not exist, an expression such as equation (2), which is fitted to theoretical solutions, must suffice. Similar expressions to equation (2) for other values of footing roughness and for plane strain (strip footing) conditions rather than circular geometry can easily be derived using ABC. The outdated empirical expressions such as those listed by the discusser should be expunged from design codes, textbooks and lecture notes now that more accurate relationships can be devised to replace them.

In practice, rather than using the individual bearing capacity factors, a more rigorous approach is to discard the (conservative) assumption of superposition and use a program such as ABC directly. In this way the effects of the soil strength, self-weight and surcharge are combined in the calculation of a single lower bound stress field and the resulting bearing capacity, rather than three terms – implying three different superimposed stress fields – being added together.

The remaining uncertainty, which was the principal focus of our paper, is the determination of appropriate input parameters – in particular the operative friction angle. The discusser highlights that our equation (2) indicates a value of \( N_{\text{circle}} \) that is 44% higher than the Eurocode value (\( s_r N_{\text{strip}} \)) for \( \phi = 40^\circ \). An alternative view of this discrepancy is to note that for \( N_{\text{circle}} \sim 75 \), equation (2) predicts a friction angle that is 7% lower than the Eurocode value (37.5° against 40°). In this range, a 1° adjustment of \( \phi \) (i.e. \( \sim 2.5\% \)) changes \( N_{\text{circle}} \) by more than 20%.

By viewing the discrepancy in terms of friction angle it is easier to reconcile the Eurocode expression with our more rigorous approach. The discusser states that the Eurocode expression for \( N_{\text{strip}} \) and the shape factor \( s_r \) were validated from large-scale model tests. In these tests, the link between \( \phi \) and \( N_{\text{circle}} \) (or \( s_r N_{\text{strip}} \)) was presumably assessed by comparing the measured \( N_{\text{circle}} \) with the friction angle measured in a soil element test – which is unaffected by progressive failure. As our paper highlights, progressive failure means that the operative friction angle beneath a foundation at failure is lower than the peak friction angle of any individual soil element. An operative friction angle of 37.5° in a large footing test compared with 40° for the same soil in an element test is perfectly plausible given the observations in our paper, and could explain the divergence in Fig. 12.

This observation does not provide an argument in support of the empirical Eurocode basis for \( N_{\text{circle}} \) instead of a theoretically rigorous one, because it applies equally to \( N_{\text{strip}} \) and \( N_{\text{strip}} \) (for which the theoretically exact solutions are firmly established). Instead, it highlights that designers must consider carefully the influence of progressive failure on the operative friction angle that is used to calculate bearing capacity factors.

It appears that the current set of bearing capacity factors in the Eurocode is a conflicting mixture of theoretical rigour based on idealised soil behaviour where closed-form expressions are available (e.g. \( N_{\text{circle}} \) and \( N_{\text{circle}} \)), and empiricism where they are not – perhaps inadvertently incorporating an adjustment of \( \phi \) for progressive failure. We suggest that the expressions currently given for \( N_{\text{strip}} \) and the shape factors should be revised to be consistent with theory. A simple improvement would be to adopt the empirical expression for \( N_{\text{strip}} \) given by Brinch Hansen (1970), which is identical with the current Eurocode expression but with an initial factor of 1.5 rather than 2 (see the discusser’s equation (9)). Brinch Hansen’s expression provides values that lie within 5% of the exact solutions. Our equation (2) could be used to define \( N_{\text{circle}} \) directly, rather than by way of a shape factor.

Equally importantly, it is necessary to develop guidance on the appropriate friction angle to input, accounting particularly for progressive failure.

We would encourage readers to download and make use of Martin’s ABC software, which he has generously made available through his website (http://www-civil.eng.ox.ac.uk/people/cmm/software/abc/).

REFERENCES

Brinch Hansen, J. (1970). A revised and extended formula for bearing capacity, Bulletin No. 28, Danish Geotechnical Institute, Lyngby


