

Causality Considerations for Missing Data Reconstruction in Image Sequences

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ABSTRACT

The 3D autoregressive (AR) model with a non-causal support region has been successfully employed in the reconstruction of texture and missing regions in image sequences [1]. This paper discusses the causality considerations when selecting the reconstruction model. When a distorted area to be reconstructed is large, a substantial computational load reduction can be obtained by implementing a predictor with a purely causal AR support. A novel reconstruction scheme which employs a selective causal/anti-causal (S-C/AC) AR model is presented. Experimental results suggest that the S-C/AC scheme produces a good trade-off between computational and reconstruction performance.

1. Introduction

Missing data is a common degradation found in archived motion pictures. This degradation known as 'Dirt and Sparkle' may consist of fairly extensive regions of corrupted signal which must first be detected and subsequently 'filled in' with useful information from data in the surrounding frames of the video sequence. This paper explores the use of the three dimensional autoregressive (3D AR) model for detail preserving reconstruction of missing data. Different causality modes of both the 3D AR model and data support regions are presented. Their relative merits are also discussed. A

novel selective causal/anti-causal (S-C/AC) predictor is proposed. This scheme provides a more robust reconstruction of missing data in regions undergoing occlusion and uncovering.

2. The 3D Autoregressive Reconstruction Model

The AR model attempts to make the best prediction of a pixel in the current frame based on a weighted linear combination of intensities at the pixels in a predefined support region [2]. This support region may occupy pixels in the current frame as well as past and future frames. The 3D AR model equation is given by

$$I(x, y, n) = \sum_{k=1}^P a_k I(x + q_{xk} + dx_{n, n+q_{nk}}(x, y), y + q_{yk} + dy_{n, n+q_{nk}}(x, y), n + q_{nk}) + \varepsilon(x, y, n) \quad (1)$$

where

$I(x, y, n)$ denote the pixel (x, y) at frame n

$a_k = \{ a_1, a_2, \dots, a_P \}$ are the AR coefficients

(q_{xk}, q_{yk}, q_{nk}) represents the offset vector which is the distance between the predicted pixel and the pixels in the supported 3D region.

$(dx_{n, n+q_{nk}}, dy_{n, n+q_{nk}})$ is the motion vector of frames n and $n+q_{nk}$

$\varepsilon(x, y, n)$ denote the error in prediction.

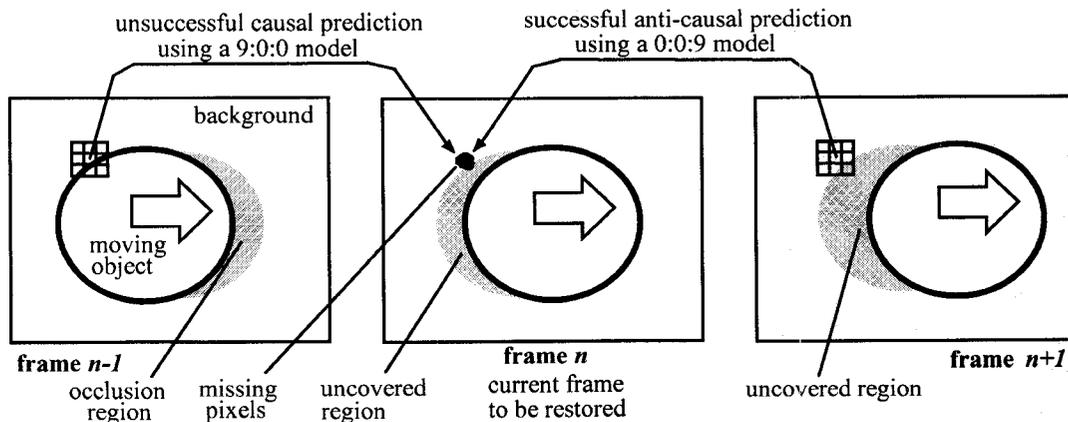


Figure 1 - Handling occlusion and uncovering using the S-C/AC predictor.

3. Missing Data Reconstruction

Consider the situation where a detection procedure [3] has selected a motion-compensated area of 'missing' pixels for replacement and the parameters for the 3D AR model have been estimated. What remains to be done in order to restore the degraded frame is to reconstruct the missing information within the area modelled. One approach is the use of an AR-based interpolator as proposed in [1]. The solution for the interpolated pixels are given by

$$\mathbf{i}_u = -[\mathbf{A}_u^T \mathbf{A}_u]^{-1} \mathbf{A}_u^T \mathbf{A}_k \mathbf{i}_k \quad (2)$$

where \mathbf{i}_u and \mathbf{i}_k represent the known and unknown pixel intensities, respectively. While \mathbf{A}_k and \mathbf{A}_u are the coefficient matrices corresponding to the known and unknown data vectors which satisfy the model equation at the considered points. In the case of a causal AR model, such as the 9:0:0 model [1], the resulting interpolator uses a *non-causal* data support region (i.e. region used to estimate the AR coefficients) which incorporates information from one frame both previous to, and following the current frame. It can be observed in equation 2, the interpolation of the missing pixel requires the solution of $[\mathbf{A}_u^T \mathbf{A}_u]^{-1}$ which is very computational demanding, especially if the size of the detected blotch is large (i.e. size of matrix \mathbf{A}_u is large).

A more economical approach for reconstructing the missing information is to use a purely causal AR-based predictor. Consider the situation when a causal 3D AR model (e.g. the 9:0:0 model) is used and data support for estimating the missing pixels are only extracted from the previous frame, then

$$\mathbf{A}_u = \mathbf{I} \quad (3)$$

where \mathbf{I} is the identity matrix. Therefore equation 2 reduces to

$$\mathbf{i}_u = \mathbf{A}_k \mathbf{i}_k \quad (4)$$

This implies that the intensities of the unknown pixels \mathbf{i}_u are those which are *predicted* from the intensities of the known pixels in the previous frame. This significantly reduces the computational requirements.

A temporally causal predictor has an inherent limitation in image regions which are being uncovered by an object moving in the foreground [4]. As illustrated in figure 1, there is no viable data support which can be used to predict the intensities of the degraded pixels in the current frame. On the other hand, an anti-causal predictor would be able to provide the necessary data support to reconstruct the missing information in the current frame. A selective causal/anti-causal (S-C/AC) AR based predictor is proposed to address the problem of occlusion and uncovering. In the S-C/AC predictor, a causal 3D AR

model (e.g. the 9:0:0 model) and an anti-causal 3D AR model (e.g. the 0:0:9 model) are used to generate a set of causal AR coefficients and anti-causal AR coefficients, respectively. The cumulative weighted squared residual error (ε_c) for the causal model within the $K \times K$ block is obtained from

$$\varepsilon_c = \sum_{x=0}^K \sum_{y=0}^K \left(w(x, y, -1) \sum_{k=0}^P a_k I(x, y, -1) \right)^2 \quad (5)$$

and the cumulative weighted squared residual error (ε_{ac}) for the anti causal model is then obtained from

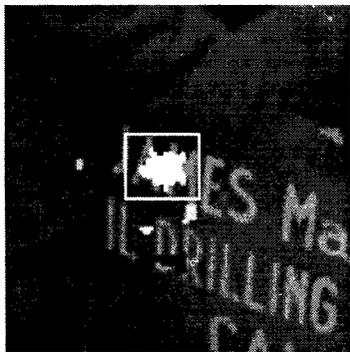
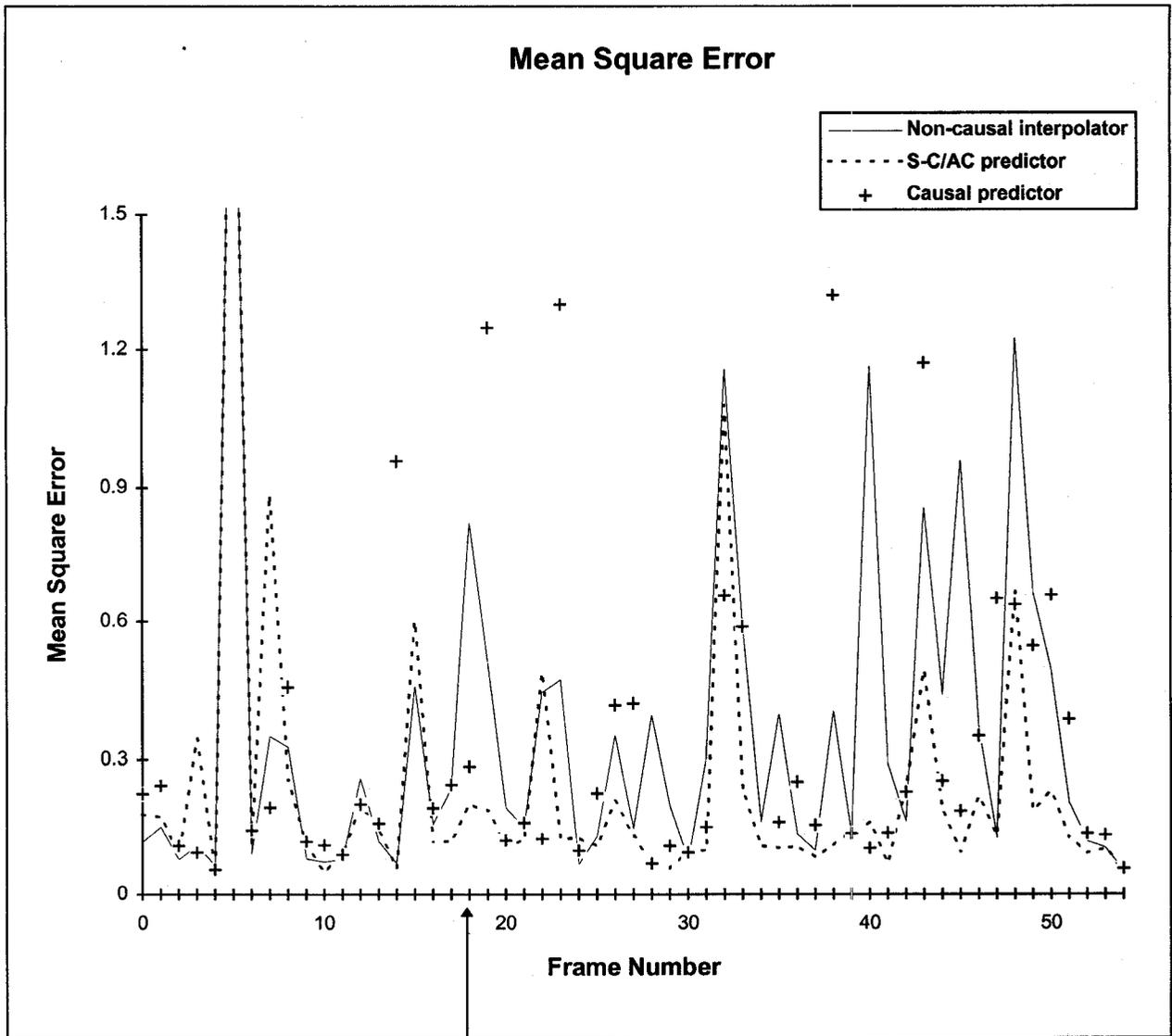
$$\varepsilon_{ac} = \sum_{x=0}^K \sum_{y=0}^K \left(w(x, y, 1) \sum_{k=0}^P a_k I(x, y, 1) \right)^2 \quad (6)$$

The causal/anti-causal selection is made based on whether ε_c or ε_{ac} is the smaller. The set of coefficients with the smaller cumulative weighted squared residual error in the $K \times K$ block is used to predict the intensities of the unknown pixels \mathbf{i}_u as given in equation 4.

4. Results and Discussion

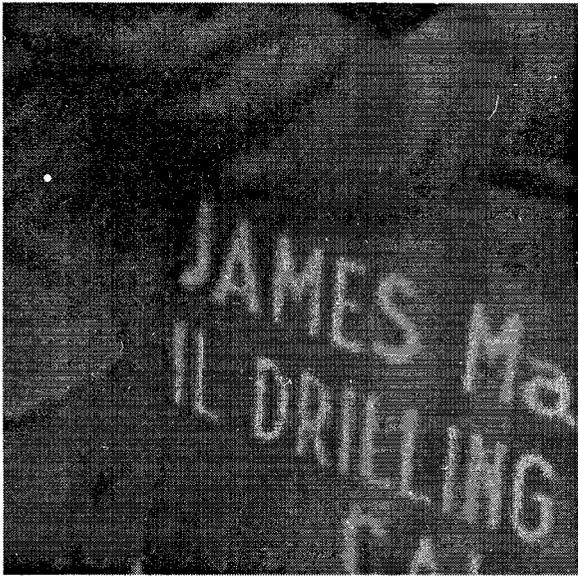
Figure 2 compares the reconstruction performances of the S-C/AC predictor against the non-causal interpolator using the Mean Squared Error (MSE) between the reconstructed and the original uncorrupted frames. In most cases, the S-C/AC predictor resulted in a smaller MSE. Figure 3 shows the reconstruction capabilities of the S-C/AC predictor in image regions which are affected by occlusions. Since the non-causal interpolator includes support regions from all three frames, bias caused by the occluded information in the previous or next frame, will result in artifacts such as those observed in figure 3(c). Figure 3(d) shows the S-C/AC predictor's ability to select the "best-fit" causal or anti-causal model for reconstruction of the missing data.

Figure 4 compares the computational performance of the non-causal interpolator, the causal predictor and the S-C/AC predictor for each separate frame of the WESTERN sequence. The graph confirms that the computational performance of the non-causal interpolator significantly degrades with the number of connected missing pixels. For example, in frame 5, the largest single blotch consist of 169 connected pixels and in that frame, the S-C/AC predictor was approximately 2.5 times faster than the non-causal interpolator. The causal predictor is generally 1.5 times faster than the S-C/AC predictor but given its poorer reconstruction ability, the S-C/AC predictor still represents a good choice in terms of computational and reconstruction performance.



Frame #18

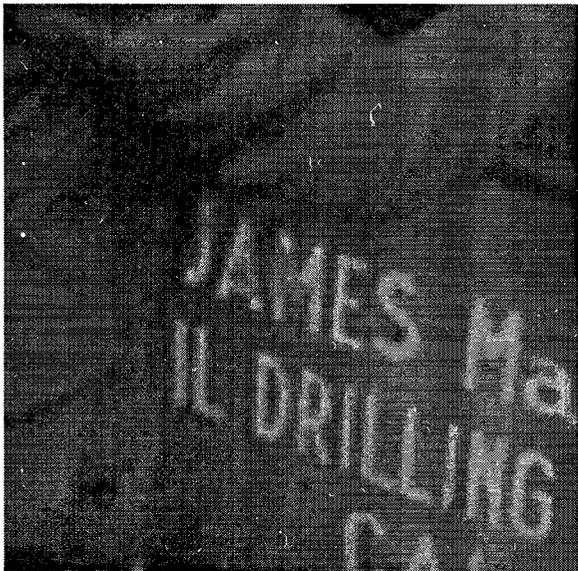
Figure 2 - Mean squared error (MSE) for the non-causal interpolator, S-C/AC predictor and causal predictor for 54 frames of the WESTERN sequence. The result for frame #18 is highlighted. This frame contains a large blotch (marked with a rectangle) which is affected by occlusion.



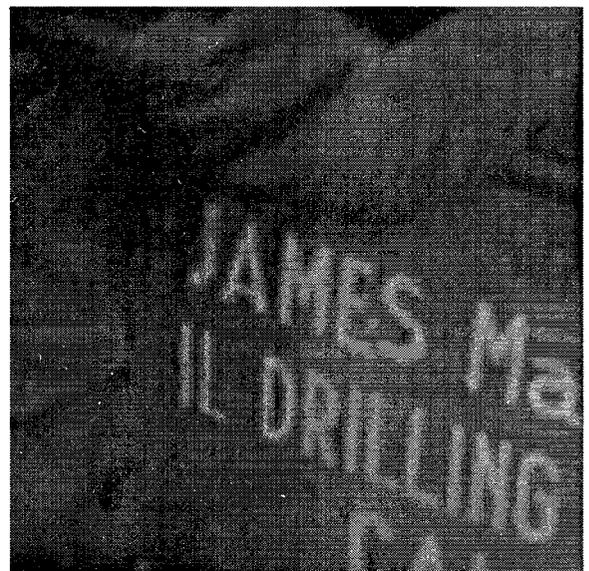
(a)



(b)



(c)



(d)

Figure 3 - Zoom in view of part of the frame #18 from the WESTERN sequence.
(a) The original clean frame,
(b) the corrupted frame with a large blotch covering the letters "AM",
(c) the restored frame using the non-causal interpolator and
(d) the restored frame using the S-C/AC predictor

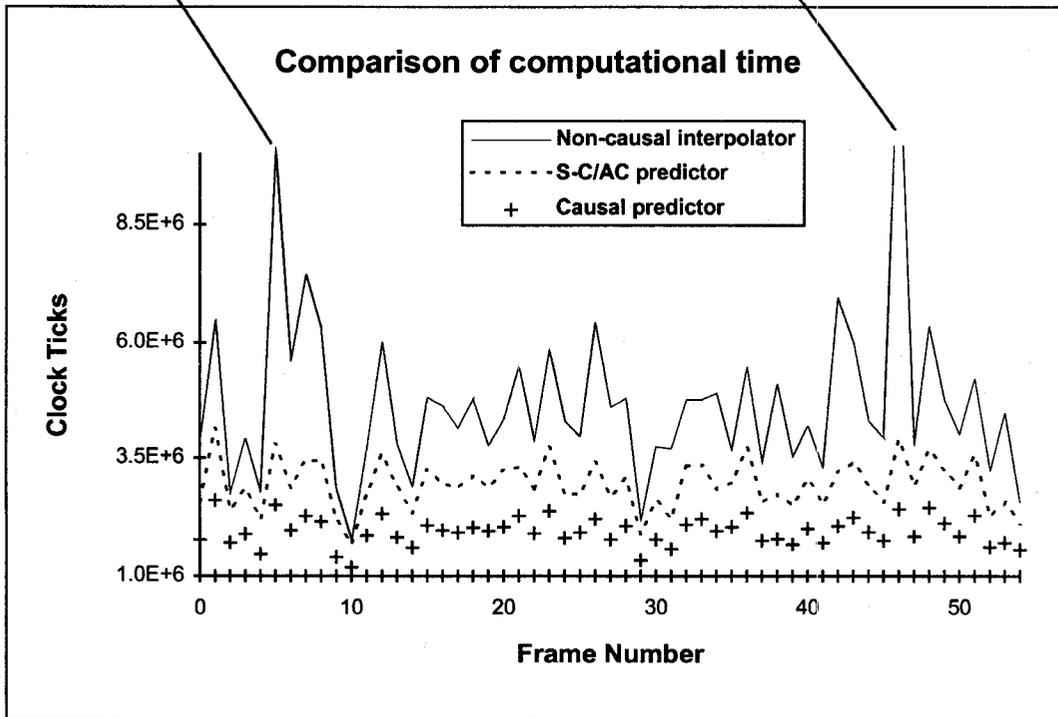
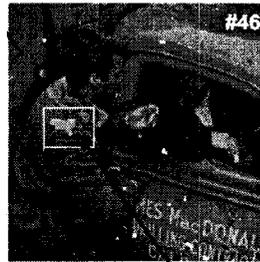
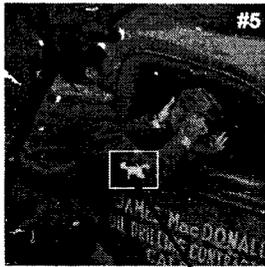


Figure 4 - Computational time for the various missing data reconstruction schemes applied to the WESTERN sequence. Two frames containing large blotches are shown.

5. Conclusions

Three different 3DAR based missing data reconstruction techniques employing different causality considerations have been presented. It has been shown that the selective causal/anti-causal (S-C/AC) predictor employing a temporally switchable causal or anti-causal data support produced the best results in sequences which contain regions undergoing occlusion and uncovering. Furthermore, the computational load of the S-C/AC predictor is also lower than the non-causal interpolator and has a more linear computational load relationship to the size of the corrupted region. This makes the S-C/AC predictor a suitable algorithm for use in video restoration systems requiring real-time performance.

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6. REFERENCES

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