The scalar glueball from a tadpole-improved action

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The scalar glueball mass and the string tension are computed in lattice SU(3) gauge theory with the aim of establishing the effectiveness of the improved action approach in removing finite-spacing artifacts.

1. INTRODUCTION

The removal of lattice-spacing artifacts from numerical estimates of physical quantities is a central problem in lattice field theory. Taking the continuum limit by decreasing the coarseness of the mesh and extrapolating is a computationally-expensive task. The use of *improved actions* promises to be a much more efficient means of reducing cutoff contamination [1]. The purpose of this work is to examine the effectiveness of the improved action approach in lattice SU(3) gauge theory by studying the low-lying glueball spectrum.

In this work, we present results for the scalar glueball mass m_g and the string tension σ using the tadpole-improved Lüscher-Weisz action with lattice spacings $a \approx 0.25$ and 0.40 fm. Comparing to results from the standard Wilson action, we find a significant reduction in the finite-spacing errors in the ratio $m_g/\sqrt{\sigma}$ (setting the scale using the string tension). The calculation exposes two problems with using the Lüscher-Weisz action on a coarse lattice: first, the coarseness of the lattice in the temporal direction severely limits the number of statistically-useful correlator measurements and hampers the demonstration of plateaux in effective masses; secondly, the unusual properties of the transfer matrix can reduce the effectiveness of the variational method in diminishing excited-state contamination. These properties are due to the presence in the improved action of terms which couple fields separated by more than one time slice.

First, the improved action is described in Sec. 2. Simulation details are then presented in Sec. 3, followed by results and conclusions.

2. THE IMPROVED GAUGE ACTION

The perturbatively-improved QCD gauge action with tadpole improvement advocated in Ref. [1] was used in this study. The original perturbative improvement was carried out long ago in Refs. [2,3]; the application of tadpole improvement [4] to this action is more recent. The action is given by

$$\begin{aligned} S[U] &= \beta_{\rm pl} \sum_{\rm pl} \tfrac{1}{3}{\rm Re\ Tr}\,(1-U_{\rm pl}) \\ &+ \beta_{\rm rt} \sum_{\rm rt} \tfrac{1}{3}{\rm Re\ Tr}\,(1-U_{\rm rt}) \\ &+ \beta_{\rm pg} \sum_{\rm pg} \tfrac{1}{3}{\rm Re\ Tr}\,(1-U_{\rm pg}), \end{aligned} \qquad (1)$$

where $U_{\rm pl}$ denotes the usual plaquette, $U_{\rm rt}$ indicates the product of link variables about a planar 2×1 rectangular loop, and $U_{\rm pg}$ is the parallelogram loop. These terms are depicted in Fig. 1.

The coupling $\beta_{\rm pl}$ is the only input parameter. The other couplings are computed in tadpole-improved perturbation theory by removing $O(a^2)$ errors in spectral quantities order-by-order in the QCD coupling. To one-loop order, these couplings are given in SU(3) by

$$\beta_{\rm rt} = -\frac{\beta_{\rm pl}}{20u_0^2}\,(1+0.4805\,\alpha_\Box), \qquad (2)$$

$$\beta_{\rm pg} = -\frac{\beta_{\rm pl}}{u_0^2}\,0.03325\,\alpha_\Box, \qquad (3)$$

where the mean-field parameter u_0 and the QCD coupling $\alpha_\Box$ are given in terms of the measured expectation value of the plaquette as follows:

$$u_0 = \langle \tfrac{1}{3}{\rm ReTr}\,U_{\rm pl}\rangle^{1/4}, \qquad (4)$$

$$\alpha_\Box = -1.30362\,\ln u_0. \qquad (5)$$

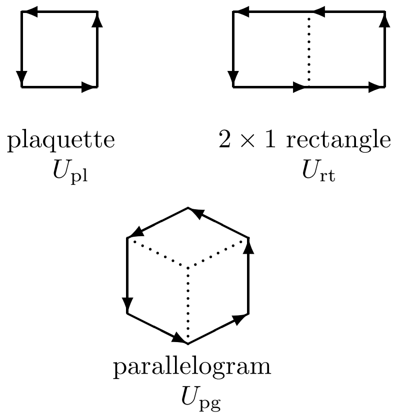

Figure 1. Wilson loops in the improved action.

Using identities from Ref. [2], the action $S[U]$ was found in Ref. [1] to be positive semi-definite for $\beta_{\rm pl} \geq 6.8$.

Since the input couplings depend on the measured quantity u_0, the appropriate value for u_0 must be determined by tuning for each value of $\beta_{\rm pl}$. There are many ways this could be done. We chose to fix the $\beta_{\rm pl}$ value and vary the input value of u_0 until agreement between the input and measured values is reached. An easy and efficient way to do this is to use the Ferrenberg-Swendsen technique [5] to simultaneously (in a single Markov chain) measure u_0 for a range of input values. These runs are very inexpensive. A sample of our tuning results follows: $u_0^4(6.8) = 0.467$, $u_0^4(6.9) = 0.480$, $u_0^4(7.0) = 0.494$, and $u_0^4(7.4) = 0.555$.

Due to the presence of terms which span two time slices in the action $S[U]$ given in Eq. 1, the concept of a transfer matrix is more complicated than for the simple Wilson action. Lüscher and Weisz [6] have shown that a transfer matrix can be defined, but it is in general not Hermitian; complex eigenvalues can occur, leading to damped oscillatory behaviour in correlation functions, and negative weight contributions can appear in the spectral decomposition of two-point functions. Nevertheless, the mass spectrum can still be obtained from the exponential decay of correlators for large temporal separations. Care must be exercised, however, in applying variational methods to extract masses from correlators at short time intervals: although invalid, strictly speaking, the variational method may still be practical whenever the coherence length is large enough such that the contributions from the problematic high-energy states are negligible.

3. SIMULATION DETAILS

A set of 18 scalar glueball creation operators $\{O_i(t)\}$ were constructed from combinations of Wilson loops on time-slice t with appropriate quantum numbers. Due to the coarseness of our lattice, fuzzing was not performed. We then obtained Monte Carlo estimates of the following matrix of correlators:

$$C_{ij}(\tau) = \sum_t \left\{ \langle O_i(t)\, O_j(t+\tau) \rangle - \langle O_i \rangle \, \langle O_j \rangle \right\}. \quad (6)$$

The simulations were run mainly on DEC-α and Hewlett-Packard workstations; 3000 CPU hours of Cray T3D time were also used. The T3D was employed essentially as a task farm consisting of 64 isolated workstations. The gauge fields were updated using both Cabbibo-Marinari [CM] pseudoheatbath and overrelaxation [OR] techniques. Updates with the improved action were found to be five times more costly than for the Wilson action. One CM and one OR sweep were performed between correlator measurements, and the results were combined into bins of 1000 to eliminate autocorrelations. Signals were enhanced by exploiting Euclidean lattice symmetries. Results were obtained using $\beta_{\rm pl} = 6.8$ on a 6^4 lattice (334 bins) and $\beta_{\rm pl} = 7.4$ on an 8^4 lattice (174 bins) with periodic boundary conditions. One simulation using the Wilson action with $\beta_W = 5.5$ was performed on an 8^4 lattice (104 bins).

To determine m_g, the variational method [7] was first used to select optimal combinations of glueball operators $O_{\rm opt} = \sum_j c_j O_j$; this involved choosing the c_j's such that $C_{\rm opt}(t_{\rm opt})/C_{\rm opt}(0)$ was a maximum, where $C_{\rm opt}$ is the correlator associated with $O_{\rm opt}$. The optimizations were performed using $t_{\rm opt} = 1, 2$. The effective masses associated with these optimal correlators were then examined. The coarseness of the lattice severely limited the number of time separations for which signals could be obtained; for this reason, a sophisticated correlated analysis was not possible.

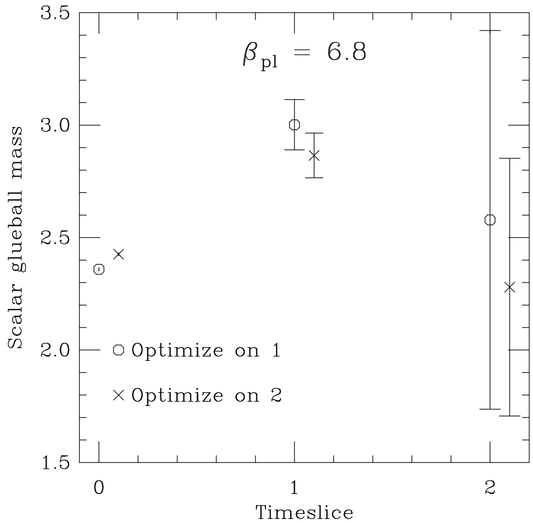

Figure 2. Effective mass for $\beta_{\rm pl} = 6.8$

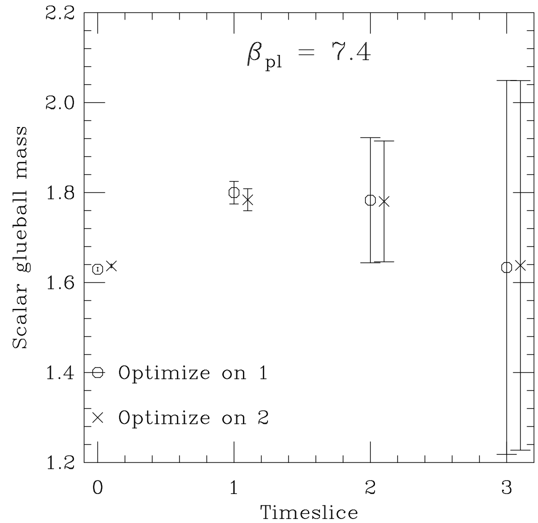

Figure 3. Effective mass for $\beta_{\rm pl} = 7.4$

Effective masses for glueballs in other channels were also studied, but no acceptable signals were obtained.

In order to set the scale, we also measured the string tension σ. This was done by extracting the static quark potential $V(\mathbf{r})$ for various $\mathbf{r}$ from ratios of appropriate Wilson loops. The results were then fit to the form

$$V(r) = \sigma r - \frac{\pi}{12r} + b, \qquad (7)$$

in order to obtain $a^2\sigma$. Only on-axis $\mathbf{r}$ vectors were used.

4. RESULTS

Our results for the on-axis string tension are $a^2\sigma(6.8) = 0.91(2)$ and $a^2\sigma(7.4) = 0.36(1)$. Systematic uncertainties associated with the form of $V(r)$ used to extract σ dominate over statistical uncertainties; the errors quoted are estimates of the systematic effects. Taking $\sqrt{\sigma} = 440$ MeV, the lattice spacing can be set, yielding $a(6.8) = 0.427(5)$ fm and $a(7.4) = 0.269(4)$ fm.

The effective glueball mass plots are shown in Figs. 2 and 3. Circles (crosses) indicate results extracted from the correlator of the glueball operator obtained by optimizing on time-slice 1 (2). Note that the effective masses do not decrease monotonically; this behaviour is a result of the presence of the two-time-step interaction in the improved action. Evidence of a plateau is clearly visible in the $\beta_{\rm pl} = 7.4$ plot; large errors shed some doubt on the existence of a plateau in the $\beta_{\rm pl} = 6.8$ plot. It is interesting to note that the plateau used in Ref. [8] to obtain m_g at $\beta_W = 6.4$ lies in the range 0.2–0.6 fm, whereas $t = 1$–3 corresponds to $0.3-0.8$ fm for our $\beta_{\rm pl} = 7.4$ results, and $t = 1-2$ corresponds to $0.4-0.9$ fm for the $\beta_{\rm pl} = 6.8$ data.

Finite-volume effects on our glueball masses are expected to be small. Using Lüscher's formula [9], we infer that our mass estimates lie within $\frac{1}{2}\%$ of their infinite volume limits. Note that our volumes $V(6.8) \approx (2.6\ {\rm fm})^4$ and $V(7.4) \approx (2.2\ {\rm fm})^4$ are of similar size to those used in Ref. [8].

We are confident that the $t = 2$ effective mass value shown in Fig. 3 gives a reliable determination of the glueball mass at $\beta_{\rm pl} = 7.4$. This value is shown in Fig. 4 where it is compared with glueball determinations from various earlier works using the Wilson action. Our glueball mass using the Wilson action with $\beta_W = 5.5$ is also shown in

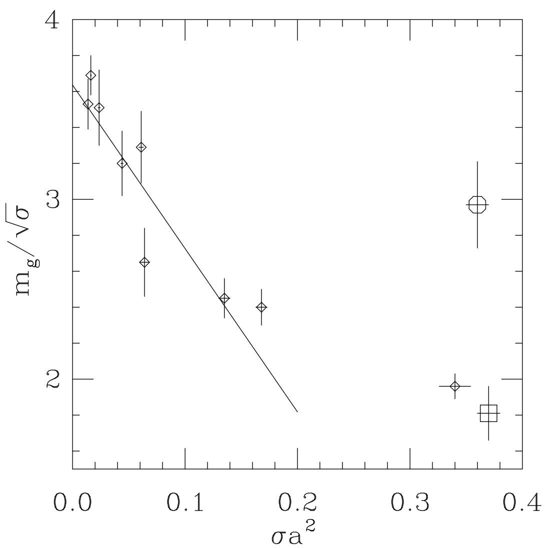

Figure 4. Glueball mass against string tension: Wilson and improved actions. The circle is our $\beta_{\rm pl} = 7.4$ result; the square indicates our $\beta_W = 5.5$ Wilson action result. The diamonds are Wilson action results from Refs. [10–12,8]. The straight line is a fit from Ref. [11] to the Wilson data.

this plot and agrees with a previous determination [12]. Comparing the Wilson $\beta_W = 5.5$ result with the $\beta_{\rm pl} = 7.4$ ratio from the improved action, one observes a remarkable three-fold reduction in the finite-a errors.

We are less confident that the $t = 1$ effective mass shown in Fig. 2 reliably represents the glueball mass at $\beta_{\rm pl} = 6.8$. However, if we use this value and convert to physical units using $\sqrt{\sigma} = 440$ MeV, we obtain $m_g(6.8) = 1.38(7)$ GeV, compared with $m_g(7.4) = 1.31(11)$ GeV. Results at more $\beta_{\rm pl}$ values are needed before one can safely comment about scaling properties. Note that the above values are low in comparison to the $a \to 0$ Wilson results.

5. CONCLUSION

We computed the scalar glueball mass and the string tension using the tadpole-improved Lüscher-Weisz SU(3) action on two coarse lattices and found a significant reduction in the finite lattice spacing errors in the ratio $m_g/\sqrt{\sigma}$. The large lattice spacings in the temporal direction limited the number of statistically-useful effective mass measurements. In the future, we intend to circumvent this problem by working on asymmetric coarse lattices in which the spacing in the time direction is reduced. We expect that the powerlaw increase in simulation cost will be more than offset by the exponential increase in signal-to-noise.

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